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# Cooperative and Discontinuous Consensus-based Control for Multi-agent Systems

Yunlong Zhang

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**CENTRALE LILLE**

**THESE**

Présentée en vue  
d'obtenir le grade de

**DOCTEUR**

En

**Spécialité : Automatique et Productique**

Par

**Yunlong ZHANG**

**DOCTORAT DELIVRE PAR CENTRALE LILLE**

Titre de la thèse :

**Contrôle Coopératif et Discontinu basé sur le Consensus des  
Systèmes Multi-agents**

**Cooperative and Discontinuous Consensus-based Control for  
Multi-agent Systems**

---

Soutenue le 13 Décembre 2021 devant le jury d'examen :

|                           |                    |  |
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Thèse préparée au Centre de Recherche en Informatique Signal et Automatique de Lille  
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*À mes parents,  
à toute ma famille,  
à mes professeurs,  
et à mes chère(s) ami(e)s.*

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*Yunlong ZHANG*

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# Chapter 1

## Introduction

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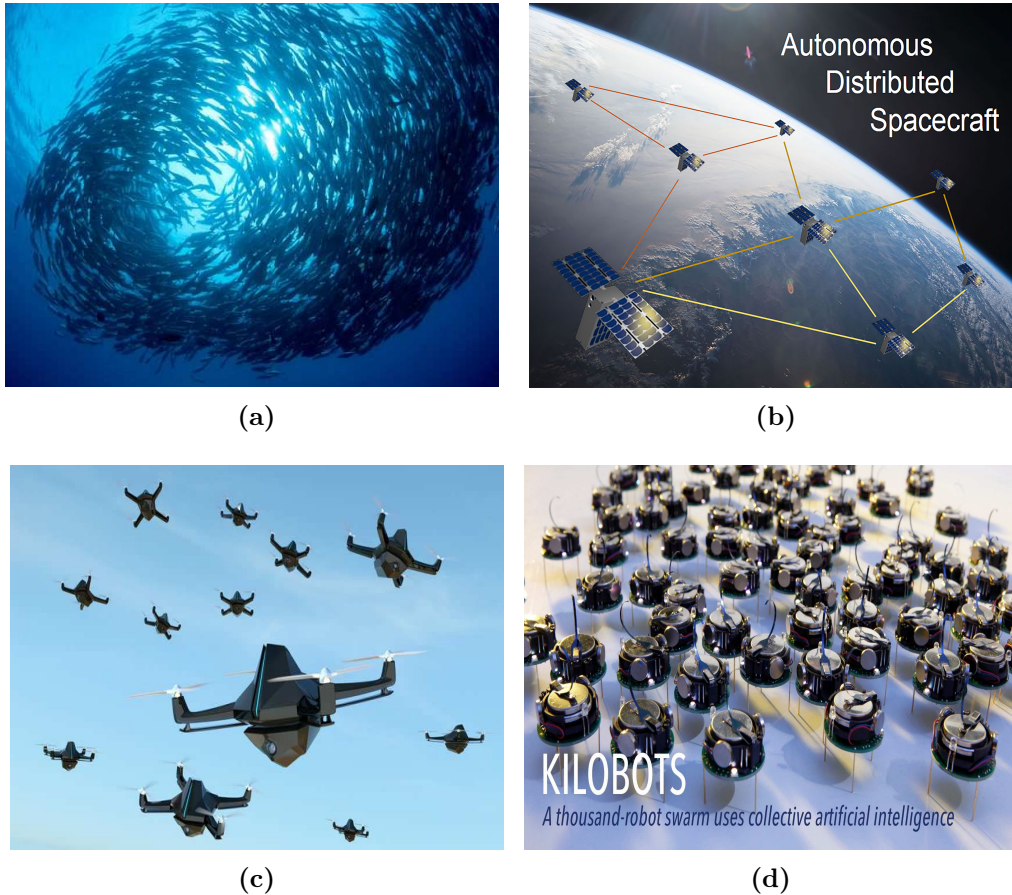
## 1.1 Background and motivation

In recent decades, much attention has been paid to the research of multi-agent systems (MASs) (Li & Liu, 2018) from various disciplines, such as applied mathematics, physics, computer sciences and control theory. There is no doubt that it is partly own to its broad applications, for example, search and rescue missions (Kantor *et al.*, 2003), spacecraft formation flying (Li *et al.*, 2020; Ren, 2007), cooperative surveillance (Olfati-Saber, 2006) (see Fig. 1.1). Different from individual agent which has limited sensing and computing ability, MASs are usually composed of numerous interacting agents with several sensors/actuators and it

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can perform or solve complex tasks in a coordinated fashion with many advantages, including improving scalability, increasing flexibility, reducing cost and avoiding a single-point of failure.



**Fig. 1.1.** Examples of multi-agent systems in practical applications

In the research field of cooperative control of MASs, such as formation (Yang *et al.*, 2018b), consensus, rendezvous (Su, 2015) and so on, the consensus problem is an important and critical issue, which aims at designing appropriate distributed control protocols by negotiating with their neighbors to reach an agreement. Up to now, numerous typical and profound results have been established in (Abdessameud & Tayebi, 2011; Li *et al.*, 2021a; Shi *et al.*, 2020).

It should be noted that most of literature about the consensus problem of MASs mainly relies on the assumption that the information is transmitted continuously among the neighbouring agents. However, in real applications, for

example, mobile robots and sensor networks, due to the limitations of communication obstacles, physical equipment failures, and sensing capabilities, the agents may interact with their neighbors with some communication constraints. On the other hand, in system analysis, an important subject is to minimise the communication and calculation. Therefore, how to design optimized control technologies is very crucial for reducing communication consumptions and saving resources.

Motivated by above discussions, this dissertation focus on the discontinuous control methods for the MASs with linear and nonlinear dynamics. The objective is to design appropriate control technologies for each agent according to different control tasks and different constraint conditions, such that all the agents can achieve consensus and meanwhile economize the control costs.

## 1.2 Overview of distributed coordination of MASs

Over the past decades, numerous works about the cooperative control problem of MASs have been obtained. In the following subsections, we will present an overview of related works based on the consensus problem and control technologies.

### 1.2.1 Consensus problem

As a fundamental topic in distributed coordination control, consensus problems have received a great deal of attention and obtained a lot of interesting results (Jadbabaie *et al.*, 2003; Olfati-Saber & Murray, 2004; Ren & Beard, 2005; Vicsek *et al.*, 1995). In the early work (Vicsek *et al.*, 1995), Vicsek et al. considered the phase transition of a group of self-driven particles based on a simple discrete-time model. (Jadbabaie *et al.*, 2003) gave a theoretical explanation for the obtained results in (Vicsek *et al.*, 1995) by using algebraic graph theory. In (Olfati-Saber & Murray, 2004), the authors discussed the consensus problem of MASs under fixed and switching topologies. In (Ren & Beard, 2005), the authors investigated consensus problem of MASs, where the information exchange is limited and unreliable because of dynamically changing interaction topologies. Note that the above publications on consensus problems mainly focus on MASs with first-order linear dynamics. Later, many researchers put their attention on second-order dynamics (Ren & Atkins, 2007; Wen *et al.*, 2012; Xie & Wang, 2007). In (Xie

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& Wang, 2007), the authors considered the average-consensus problem for the second-order MASs. In (Yu *et al.*, 2010), necessary and sufficient conditions were derived to guarantee the achievement of consensus for the second-order MASs. For the fixed directed topology, the second-order consensus with communication constraints was investigated in (Wen *et al.*, 2012). More recently, the consensus problem of MASs with high-order or general linear dynamics has been addressed. In (Wieland *et al.*, 2008), the consensus problem of high-order MASs was studied under a directed weighted graph. In addition, considering the effect of time-delays in both the communication network and control inputs, (Zhou & Lin, 2014) adopted a truncated predictor feedback approach to solve the consensus problem of high-order MASs.

It is worth noting that most of obtained results about the consensus problem of MASs focus on **complete consensus**, that is, the control inputs are designed to ensure that the agents are in a consistent state. However, in many real situations, due to some unanticipated situations such as the changes of cooperative tasks, external environment or event-time, all agents need to be split into several groups, namely, the agents that belong to the same group or cluster reach a common state, while the agents that belong to other group or cluster reach different convergence states, which is termed as **group consensus or cluster consensus**. As an extended concept about complete consensus, group/cluster consensus has various applications in military and civilian, such as battlefield assessment, reconnaissance, and surveillance. In the large scale complex networks, due to the specific requirements, it is easier to analysis and design by decomposing the large scale complex network into several smaller clusters; in nature, the birds, fish and bacteria colonies are often emerge several clusters; particularly in human society, some opinion formation models reveal that agents in the same cluster eventually form the same opinion (see Fig.1.2). A vivid example is given in (Ge *et al.*, 2018): multiple robots are required to keep different capabilities in order to search and rescue victims when a disaster occurs. Thus, it is necessary to divided all agents into different clusters according to the needs in practice. Up to now, many constructive results for the cluster consensus of MASs have been obtained from various aspects such as system dynamics, network topologies, time-delays (Aeyels & Smet, 2011; Chen *et al.*, 2019; Wen *et al.*, 2016a; Wenlian Lu, 2010). In (Aeyels & Smet, 2011), the authors investigated

## 1.2 Overview of distributed coordination of MASs

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cluster formation behavior for a time-varying MASs. In (Wenlian Lu, 2010), the authors studied cluster synchronization issues under the networks of coupled nonidentical dynamical systems. In (Xia *et al.*, 2016), the authors considered the group consensus of MASs with communication delays under fixed and switching topologies. Miao and Ma (Miao & Ma, 2015) proposed group consensus protocols for discrete-time and continuous-time MASs with nonlinear input constraints. In (Kang *et al.*, 2018), the authors investigated the cluster synchronization of a network system with nonidentical nodes, where the dynamics of agents are either general linear or nonlinear. In (Qin *et al.*, 2016), the authors addressed the cluster consensus problem for a second-order MAS by pinning leader-follower approach, where the communication topology is assumed to be weakly connected. In (Liu *et al.*, 2015), the authors investigated the adaptive group consensus of networked Euler-Lagrange systems. Meanwhile, some corresponding necessary and sufficient conditions for solving group consensus problem are established.

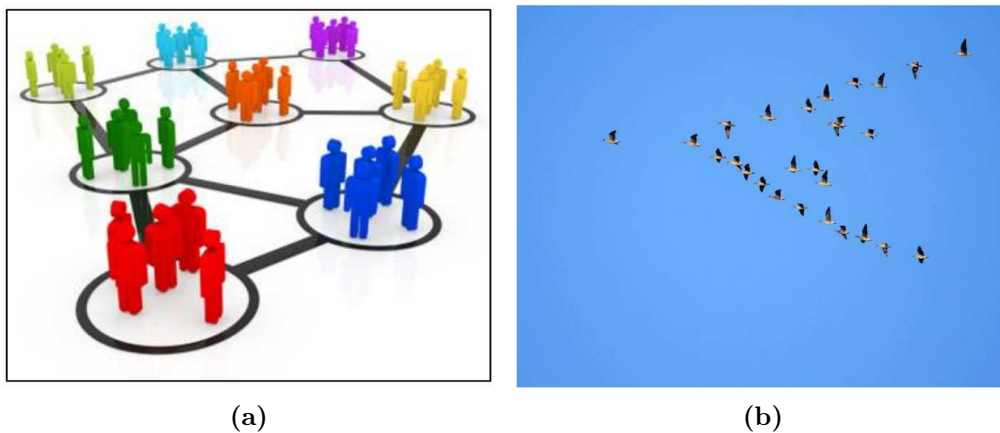


Fig. 1.2. Examples of group consensus of MASs

More recently, a more generalized counterpart of consensus is put forward by some scholars, that is **scaled consensus**. Compared with group consensus, scaled consensus means the states of all the agents reach assigned proportions instead of some common state values. Thus, group consensus can be considered as the special case of the scaled consensus. (Roy, 2015) referred to the concept of scaled consensus firstly. Scaled consensus has been paid more and more attention in recent years due to its broad applications, for example, water distribution systems (Ostfeld & Salomons, 2004), space cooperative vehicles, closed queuing



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networks (Reiser, 1981). In (Meng & Jia, 2015), the authors investigated the scaled consensus problem of first-order MASs under switching networks, and in order to guarantee the exponential convergence of agents, necessary and sufficient conditions were obtained. For high-order discrete-time MASs, the scaled consensus problem was considered in (Cheng *et al.*, 2020), in which there exists a time-varying delay in the progress of information transmission among agents.

**Bipartite consensus**, as a special case of scaled consensus, implies that the states of all agents converge to a consensus value with the same magnitude but opposite sign. In order to reach bipartite consensus, the sign graphs are often used to represent competitive-cooperative MASs. In (Altafini, 2012), negative weights were denoted to describe the antagonistic relationship among agents in MASs. In (Zhang & Chen, 2017), state feedback and output feedback control technologies were designed to achieve bipartite consensus. Assuming that the associated signed digraph is strongly connected, (Guo *et al.*, 2018) handled the bipartite consensus problems over signed digraphs with arbitrary finite communication delays. To specify multiple bipartite consensus behavior, by combining the characteristics of group consensus and bipartite consensus, (Liu *et al.*, 2020) introduced group-bipartite consensus of MASs over cooperative-competitive networks.

### 1.2.2 Control strategies

As far as we all know, the MASs can't reach consensus by itself. Thus, various control strategies have been constructed to guarantee the system performance and meanwhile save energy, such as pinning control, adaptive control, impulsive control, intermittent control, sampled-data control, event-triggered control and so on, which can be summarized as follows.

In practical application, especially for the large-scaled MASs, it is impossible for the leader to pin every follower. To overcome this drawback, **pinning control** can be used by controlling only a small fraction of agents rather than all the agents. Thus, pinning control is more economical and more effective for the large-scale leader-following MASs, which has been attracted much attention. (Wang & Wu, 2012) investigated two kinds of formation control problems for second-order nonlinear MASs by pinning control under fixed and switching topologies. In (Huang *et al.*, 2019), the authors applied pinning control scheme to heterogeneous MASs for solving the group consensus with reference. In (Wang & Li, 2017),

## 1.2 Overview of distributed coordination of MASs

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the authors presented an distributed pinning controller for robotic networks to realize cluster consensus, where the system dynamics are characterized by Euler Lagrangian equations. Moreover, based on matrix theory, algebraic graph theory and the stability theory of fractional system, (Yu *et al.*, 2015) employed adaptive pinning control to solve the consensus problem for fractional-order MASs.

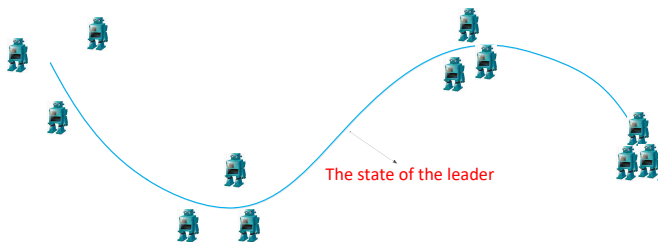
As a matter of fact, pinning control is an inseparable companion to leader-following consensus. **The leader-following consensus**, is also called as **tracking consensus**, which implies in a system all followers can track a prescribed state trajectory generated by a leader that is usually independent of all followers (see Fig.1.3). As it is referred in (Hummel, 1995), the leader-following configuration was an energy saving mechanism. Besides, the communication and orientation of the flock can be enhanced by leader-following method (Andersson & Wallander, 2004). Existing literature can be roughly divided into two sorts, i.e., leader-following consensus (Dai & Guo, 2018) and leaderless consensus. In (Kim *et al.*, 2014), the authors investigated the leaderless and leader-following consensus for discrete-time heterogeneous MASs with random link failures. In (Qiu *et al.*, 2015), by the aid of the perturbation analysis of matrices and quantization technique, the authors solved the quantized leaderless and leader-following consensus for a class of high-order MASs with limited data rate. Besides, for the MASs with multiple leaders, (Wang *et al.*, 2018) considered containment control problem, where controllers were designed to make all the followers can converge to a convex hull formed by the leaders.

It is worth noting that the control inputs of leaders are equal to zero in part of results on tracking consensus. However, in many circumstances, in order to avoid obstacles and achieve desirable objective states, the control inputs of leaders are expected to be nonzero and controllable. (Li *et al.*, 2012) handled the distributed tracking consensus problem, where the control input of leader is continuous and bounded, besides, it is not available to any follower. In (Ma *et al.*, 2017), the authors studied the tracking consensus problem by introducing a smart leader, where the leader can adjust the interaction strength between itself and the target point based on the tracking state errors.

Nevertheless, it is noted that the disadvantage for pinning control in existing literature is that the pinning strength is usually fixed and given in advance to

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**Fig. 1.3.** Leader-following consensus or tracking consensus problem.

avoid the appearance of the worst situation, which is unreasonable in real applications because the worst case rarely occurs. To overcome this drawback, many researchers adopt **adaptive control** approach to prevent the appearance of larger parameters than those required in practice and adjust the pinning weight during the process of dynamics evolution. For example, (Xu *et al.*, 2016) investigated cluster consensus problem of second-order nonlinear MASs by employing an adaptive pinning control method. Recently, a vast number of works on adaptive control strategy are derived. (Li *et al.*, 2015a) considered the output regulation problem of heterogeneous MASs via adaptive control under periodic switching topologies, where the feedback gains in control input are independent of the global information of underlying communication graph. (Jin & Haddad, 2019) proposed a novel distributed adaptive control architecture to address the tracking problem for MASs, where the adaptive control architecture can foil malicious sensor and actuator attacks when there exist exogenous stochastic disturbances. For a class of nonlinear MASs with unmodeled dynamics, (Shen *et al.*, 2019) designed a fully distributed neural-networks-based adaptive control technology to ensure that all the followers can track the leader within a prescribed level.

Notice that the former literatures about the consensus of MASs were mainly focused on the continuous control schemes with the assumption that all agents can transmit the information with their neighbours all the time. However, in the real engineering, due to the appearance of obstacles or the limitation of communication ability, the information communication may be discontinuous. Moreover, continuous information transmission will cause a heavy burden for the MASs with limited communication bandwidths. To mitigate these issues, many optimized and efficient control strategies are put forward.

**Impulsive control** can be regarded as a special discontinuous control method, where describes the state information of agents is just transmitted at some dis-

## 1.2 Overview of distributed coordination of MASs

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crete impulsive instants. Obviously, the frequency of state information transaction is reduced greatly. Thus, impulsive control strategy has many advantages including maintenance with low cost, more practicality and high efficiency. On the other hand, the impulsive dynamical systems describe continuous evolution with instantaneous state jumps and consist of three elements. The first element is continuous-time part, which is between system reset or impulsive, to control the motion of systems. The second element is discrete jumps, it controls the way that the system states are instantaneously changed when a resetting event occurs. The third is the criterion to determine the time that system can be reset. It is obvious that the impulsive control systems have a faster convergence speed than other continuous control systems. In addition, impulsive dynamical systems can capture properties of various complex systems like mechanical systems with impacts, orbital transfer of satellite, nanodevices with electron tunneling effects and so on. Up to date, impulsive control has gained considerable interests of many researchers. (Ning *et al.*, 2018) investigated the input-to-state stability (ISS) and integral ISS of impulsive systems by constructing a novel Lyapunov method which is based on an indefinite Lyapunov function instead of negative definite Lyapunov function. (He *et al.*, 2017) studied the tracking consensus problem of nonlinear MASs with network-induced delays via distributed impulsive control. (Tang *et al.*, 2015) considered the tracking consensus problem of MASs with multiple delays via impulsive control with several new characterizations, and the obtained results are verified in mechanical robotic systems. (Zhu *et al.*, 2017) analyzed the quantized consensus of second-order MASs via impulsive control. (Jiang *et al.*, 2011) introduced impulsive control strategy for linear dynamic MASs. Based on the theory of impulsive differential system and Lyapunov stability, (Ma *et al.*, 2020) proposed an impulsive control strategy with odd impulse sequences to address the consensus problem of MASs, which is more effective and flexible in real applications. Moreover, in the studies of synchronization analysis for complex dynamical networks, many results were obtained by using impulsive control (Ding *et al.*, 2020; Syed Ali *et al.*, 2020; Xu *et al.*, 2020).

**Intermittent control** strategy has been proposed and widely applied in transportation, manufacturing and communication. In actual application, the intermittent control strategy has the control time (or work time) and uncontrolled time (or rest time) alternately, that is, the controller will be activated within

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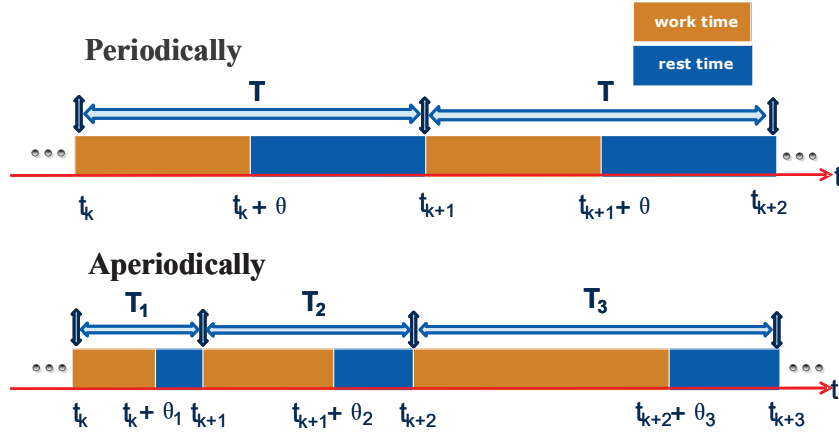
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certain nonzero time intervals and be off during other time intervals. Specially, when the control time tends to a time point, intermittent control becomes impulsive control. Compared with the impulsive control strategy that is activated instantaneously, intermittent control strategy is easier to be executed because its control width is nonzero. Therefore, the intermittent control can be viewed as an excellent junction between continuous control strategy and impulsive control method. Moreover, intermittent control can be divided into periodical intermittent control and aperiodically intermittent control (see Fig.1.4). In Fig.1.4,  $[t_k, t_k + \theta_1]$  is called as control interval,  $(t_k + \theta_1, t_{k+1})$  is called as rest interval,  $T_1$  is called as a period, where  $t_k$  and  $t_k + \theta_1$  represent the start time and end time of  $k$ th control interval,  $t_{k+1}$  is the end time of  $k$ th rest interval,  $\theta_1$  is the control width. Especially, when the control width and control period are fixed constants, the aperiodically intermittent control strategy become the periodically one. Compared with periodical intermittent control, aperiodically intermittent control is more reasonable and prevalent in practice because the conditions of periodical intermittent control is restricted to some extent, for instance, the wind power generation is obviously aperiodically intermittent. Numerous results have been obtained by employing intermittent control strategy in MASs (Gawthrop & Wang, 2009; Huang *et al.*, 2009; Li *et al.*, 2007; Zochowski, 2000). In (Zochowski, 2000), the author described the method of synchronizing slave to the master trajectory by using intermittent coupling. In (Huang *et al.*, 2009), the authors considered the exponential stabilization of delayed chaotic neural networks by using periodically intermittent control. In (Sader *et al.*, 2021), the authors investigated the  $H_\infty$  consensus problem for a class of nonlinear MASs with intermittent communications and actuator faults under the switching communication graph. Particularly, for the leader-following consensus of MASs by intermittent control, many results are derived in (Huang *et al.*, 2014; Wang & Wang, 2015; Zhang *et al.*, 2019b). In (Huang *et al.*, 2014), the authors considered the leader-following issue for second-order nonlinear MASs assuming that the communication among all agents is intermittent. By using aperiodically intermittent pinning control strategy, the authors in (Zhang *et al.*, 2019b) addressed partial component consensus for a nonlinear leader-following MASs. Furthermore, considering the MASs with general linear dynamics is more common and challenging, (Wang & Wang, 2015)

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investigated the semi-global consensus of general linear MASs via intermittent saturating actuator.



**Fig. 1.4.** Periodical and aperiodically intermittent control strategy.

The advantage of intermittent control is that it can shorten the working time of the controllers, while the deficiency is that the information updating rates of controllers cannot be reduced. For the sake of reducing the load of controllers updating, (Xiong *et al.*, 2016) employed intermittent sampled-data control to handle the synchronization problem for hierarchical time-varying neural networks. **Sampled-data control** scheme essentially adopts time-triggered mechanism, in which sampled data transmission happens at pre-designed constant sampling period or time-varying bounded sampling period (Gao *et al.*, 2009; Tang *et al.*, 2011). Commonly sampled-data control system describes a control system in which continuous-time plant is controlled with a digital device. In sampled-data control systems, control signals are constants during sampling intervals and are allowed to change only at sampling instants. An application of this technique is in the radio broadcasts of the live musical program (Chen *et al.*, 2016). Recently, various results have been reported for investigating the sampled-data synchronization/consensus problem of networked multi-agent systems. (Lee & Park, 2017) introduced a novel time-dependent discontinuous Lyapunov function to deal with stability problem of sampled-data systems. (Ozcan *et al.*, 2018) applied sampled-data control strategy to handle robust synchronization of uncertain Markovian jump complex dynamical networks with time-varying delays and reaction-diffusion terms. (Ali *et al.*, 2020) established a novel non-fragile

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sampled data control framework for nonlinear MASs with additive time varying delays and Markovian jump parameters. (Beikzadeh & Marquez, 2016) designed input-to-stable observer for nonlinear sampled-data systems. Under the switching topology, (Zhang & Zhang, 2017) addressed the consensus problem for general linear MASs via sampled-data control strategy. Considering random and deterministic packet losses, respectively, (Zhang *et al.*, 2016) solved the sampled-data consensus problem for linear MASs. (Du *et al.*, 2016) investigated the consensus problem for heterogeneous MASs composed of first-order MASs and second-order MASs with communication failure based on sampled-data information. (Zhang *et al.*, 2017a) studied the tracking consensus problem for a class of heterogeneous MASs by designing distributed output-feedback controller with the aperiodic sampled-data measurement subject to external disturbance. (Zhang *et al.*, 2017b) developed a time-varying sampled-data strategy to solve the consensus problem for Euler-Lagrange systems, where the controller can either work or rest during each sampling interval, avoiding over provisioning of system hardware compared with the traditional sampled-data strategies. (Yu *et al.*, 2017b) gave the necessary and sufficient criteria for leaderless consensus and leader-following consensus of fractional-order MASs based on sampled-data control strategy.

In sampled-data control, in order to avoid packet dropouts and network congestion as well as guarantee the performance, the data sampling or control action update is more frequency than that it is actually required, which leads to a waste of communication and computational resources. Different from the sampled-data control scheme, under the **event-triggered control** (ETC) scheme (Nowzari *et al.*, 2019), agents depend on a predefined triggering function rather than a fixed sampling period to determine when to sample data among the neighboring agents and update their controllers. Thus, energy consumption can be cut down greatly. Up to date, a vast amount of literature on MASs via ETC strategy has been obtained (Li *et al.*, 2021b; Luo & Ye, 2019; Wang *et al.*, 2020) and a variety of triggering functions have been designed. In (Garcia *et al.*, 2014), the centralized and decentralized ETC techniques were adopted respectively to handle the consensus problem for general linear MASs subject to undirected graph. So as to further decrease the information communication and minimize the number of controller updates, (Zhou *et al.*, 2015) used the combinational measurements method to design the event-triggered control mechanism for MASs. By using only local

information instead of global information, (Cheng & Li, 2018) investigated the fully distributed ETC strategy for the general linear MASs by designing the time-varying weights in both triggering conditions and the control laws. The designed adaptive ETC strategy was robust to some bounded external disturbances. By introducing sampled-data method, event-triggered sampled-data control strategy was presented in (Guo *et al.*, 2014), where the triggering function only needs to be detected at the sampling instants and the minimum inter-event time could be guaranteed to be no less than one sampling period. For multi-vehicle systems with nonholonomic constraints, (Chu *et al.*, 2019) considered the distributed formation tracking control problem via event-triggered sampled-data control strategy.

## 1.3 Preliminaries

**Notations:** Throughout this thesis,  $N = \{1, 2, \dots\}$  represents the set of positive integer. For the given positive integers  $p$  and  $q$ .  $R^q$  and  $R^{p \times q}$  denote  $q$ -dimensional real column vectors and  $p \times q$  real matrices, respectively.  $\|\cdot\|$  not only stands for Euclidean norm for vector, but also the induced matrix 2-norm. For a real matrix  $P$ ,  $\lambda_{max}(P)$ ,  $\lambda_{min}(P)$ ,  $P^{-1}$  and  $P^T$ , mean respectively, maximum eigenvalue, minimum eigenvalue, the inverse, and transpose of  $P$ .  $P > 0$  indicates  $P$  is positive definite.  $I_N$  is the identity matrix.  $\otimes$  denotes Kronecker product. The  $diag\{\dots\}$  stands for a block-diagonal matrix.

### 1.3.1 Graph theory

In this thesis, it is assumed that the communication topology of MASs is characterized by a triplet  $\mathcal{G}=(\mathcal{V}, \mathcal{E}, \mathcal{A})$ , in which  $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$  stands for the set of nodes, and  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  denotes the set of edges. For a directed topology,  $e_{ij}$  is an edge and  $e_{ij} = (v_i, v_j) \in \mathcal{E}$  indicates that the information flows can be transformed from agent  $v_j$  to agent  $v_i$  (See Fig.1.5), while for the undirected topology,  $e_{ij} = (v_i, v_j) \in \mathcal{E}$  represents the information flows that can be transformed mutually between agent  $v_i$  and agent  $v_j$ .

$\mathcal{A} = [a_{ij}]_{N \times N}$  is used to represent the weighted **adjacency matrix** associated with the communication topology and it is supposed to satisfy the following properties:  $a_{ij} > 0$  implies  $e_{ji} \in \mathcal{E}$  ;  $a_{ij} = 0$  otherwise. It should be pointed out that the topology has no self loops or parallel edges, that is,  $a_{ii} = 0$ . Besides, in this



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**Fig. 1.5.** Information flow from  $v_j$  to  $v_i$

thesis, we assume that the adjacency matrices corresponding to the topologies have 0-1 weight. The weighted adjacency matrix is represented as

$$\mathcal{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \in \mathcal{R}^{N \times N}.$$

Furthermore, the communication topology can be categorized in two classes according to the information flows, that is, undirected and directed topology. A **directed path** corresponding to the directed topology is a finite ordered nodes sequence  $v_{i_1}, \dots, v_{i_k}$  such that  $(v_{i_l}, v_{i_{l+1}}) \in \mathcal{E}$ , where  $l = 1, \dots, k - 1$ . The definition about a **weak path** is that there exists a finite ordered nodes sequence  $v_{i_1}, \dots, v_{i_k}$  such that  $(v_{i_l}, v_{i_{l+1}})$  or  $(v_{i_{l+1}}, v_{i_l}) \in \mathcal{E}$ . For a directed topology  $\mathcal{G}$ , if it contains a **directed spanning tree**, which means there exists at least one node that is called as root node having a directed path to any remaining nodes. Furthermore, a directed topology  $\mathcal{G}$  is called **weakly connected** if there exists at least one weak path for any two distinct agents, which means its corresponding undirected graph is connected and each agent can either obtain information from other agents or send information to other agents. Apparently, the weakly connected topology is more universal than the topology with a directed spanning tree. The Laplacian matrix  $L = [l_{ij}]_{N \times N}$  of topology  $\mathcal{G}$  satisfies

$$l_{ij} = \begin{cases} -a_{ij} & \text{if } i \neq j, \\ \sum_{j=1, j \neq i}^N a_{ij} & \text{if } i = j, \end{cases}$$

which makes sure that the diffusion property holds good, that is  $\sum_{j=1}^N l_{ij} = 0$ .

Divide the  $N$  followers into  $m$  clusters and suppose the  $i$ th cluster has  $m_i$  nodes, that is  $\mathcal{V}_i = \{v_{m_1+m_2+\dots+m_{i-1}+1}, \dots, v_{m_1+m_2+\dots+m_{i-1}+m_i}\}$ . As a result, we

have  $\mathcal{V} = \{v_1, v_2, \dots, v_N\} = \mathcal{V}_1 \cup \mathcal{V}_2 \cup \dots \cup \mathcal{V}_m$  and  $\mathcal{V}_p \cap \mathcal{V}_q = \emptyset$ , where  $p \neq q$ . Assume that each cluster has a virtual leader. If the  $i$ th agent is in the cluster that can be denoted as  $\mathcal{V}_{\hat{i}}$ , then  $i \in \mathcal{V}_{\hat{i}}$ , where  $\hat{i}$  is the subscript. In addition, the subscript set of the virtual leaders and followers are denoted as  $\Omega = \{1, 2, \dots, m\}$  and  $V = \{1, 2, \dots, N\}$  respectively. For convenience of expression, there is no difference for the following descriptions  $i \in V$ ,  $i \in \mathcal{V}$ , and  $v_i \in \mathcal{V}$ . Obviously, the followers of the  $j$ th virtual leader are in  $\mathcal{V}_j$ ,  $j \in \Omega$ . Refer to (Da *et al.*, 2018) for the definition of **inter-act agents** and **intra-act agents**.  $\tilde{\mathcal{V}}_{\hat{i}}$  represents the subset of  $\mathcal{V}_{\hat{i}}$  and the agents in  $\tilde{\mathcal{V}}_{\hat{i}}$  means that they can receive information from the agents in other cluster, that is, for any  $v_i \in \tilde{\mathcal{V}}_{\hat{i}}$ , there exists at least one  $v_j \in \mathcal{V}_{\hat{i}} \setminus \tilde{\mathcal{V}}_{\hat{i}}$  such that  $a_{ij} \neq 0$ . Furtherly, if  $v_i \in \tilde{\mathcal{V}}_{\hat{i}}$ , the node  $v_i$  is termed as the inter-act agent, otherwise,  $v_i$  is termed as the intra-act agent if  $v_i \in \mathcal{V}_{\hat{i}} \setminus \tilde{\mathcal{V}}_{\hat{i}}$ .

Moreover, the interaction topology composed of  $N$  followers and  $m$  leaders can be characterized by a digraph  $\bar{\mathcal{G}}$ , where there is no information flow from followers to leaders. Define **pinning matrix**  $D = \text{diag}\{d_1, d_2, \dots, d_N\}$ , where  $d_i > 0$ , if leader can transmit information to follower  $i$ , and  $d_i = 0$  otherwise. Then, define  $\tilde{L} = L + D$  for subsequent use.

In what follows, three examples are presented to specify the fixed undirected graph, directed graph containing a spanning tree and weakly connected graph, respectively. In addition, the adjacency matrixes  $\mathcal{A}$ , pinning matrix  $D$  and Laplace matrix  $\mathcal{L}$  are obtained according to the definitions.

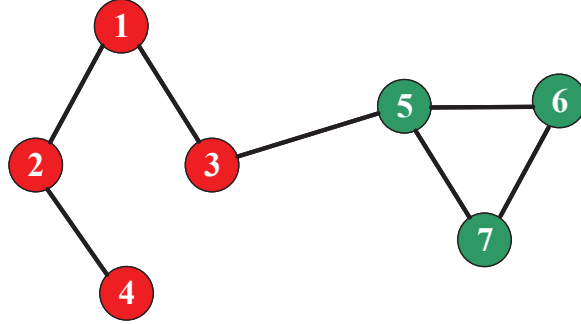
**Example 1.1** Consider the fixed undirected communication topology as Fig. 1.6, its adjacency matrixes  $\mathcal{A}$  and Laplace matrix  $L$  can be given as follows:

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}, L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}.$$

**Example 1.2** Consider the fixed directed communication topology which contains a directed spanning tree with agent 1 as the root node (See Fig. 1.7), its adjacency

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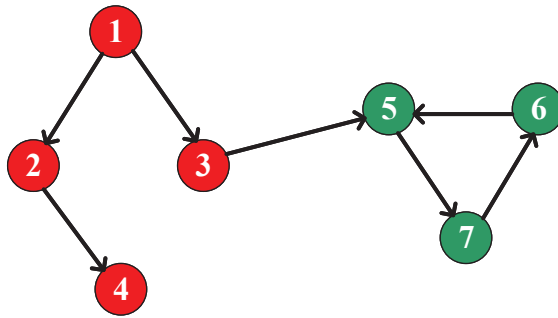
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**Fig. 1.6.** A fixed undirected communication topology with seven agents.

matrixes  $\mathcal{A}$  and Laplace matrix  $L$  can be given as follows:

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$



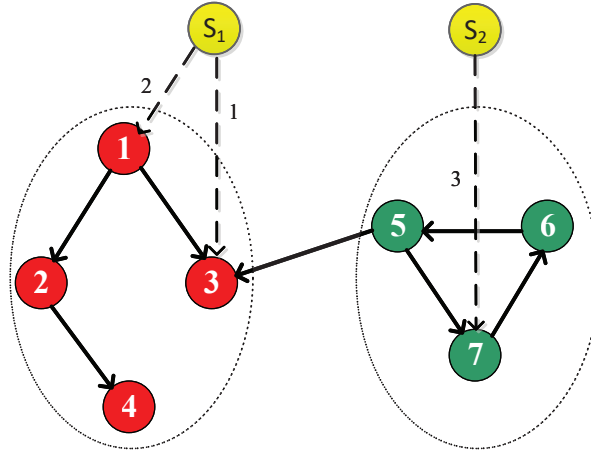
**Fig. 1.7.** A fixed directed communication topology contains a spanning tree with agent 1 as the root node.

**Example 1.3** Consider the communication topology  $\bar{\mathcal{G}}$  composed of followers 1 to 7 and two leaders  $s_1, s_2$  as Fig. 1.8. Its corresponding subgraph  $\mathcal{G}$  composed of followers 1 to 7 is weakly connected. The adjacency matrixes  $\mathcal{A}$ , pinning matrix

$D$  and Laplace matrix  $L$  of  $\mathcal{G}$  can be given as follows:

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix},$$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$



**Fig. 1.8.** A weakly connected communication topology composed of seven followers from 1 to 7 and two leaders  $s_1$  and  $s_2$ .

**Lemma 1.4** (*Ren & Beard, 2008*) Suppose the undirected communication topology  $\bar{\mathcal{G}}$  which contains  $N$  followers and one leader is connected, then  $M = L + D$  is symmetric and positive definite.

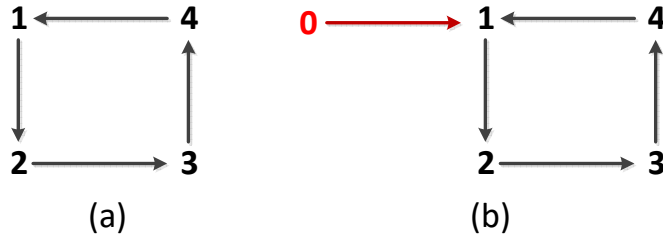
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**Lemma 1.5** (*Li et al., 2015b*) Suppose the directed communication topology  $\bar{\mathcal{G}}$  composed of the followers and the leader has a directed spanning tree with the leader as the root node (See Fig. 1.9), then all the eigenvalues of matrix  $\tilde{L} = L + D$  own positive real parts. In addition, there exists a positive definite diagonal matrix  $E$  satisfying  $E\tilde{L} + \tilde{L}^T E > 0$ , where  $E = \text{diag}\{\xi_1, \dots, \xi_N\} > 0$ . Furthermore, the equality  $E\tilde{L} + \tilde{L}^T E > \gamma E$  exists with a positive constant  $\gamma$ . The matrix  $E$  can be constructed as follows:

$$E = \text{diag}\{\xi_1, \xi_2, \dots, \xi_N\},$$

$$\xi = [\xi_1, \xi_2, \dots, \xi_N]^T = \tilde{L}^{-T} \mathbf{1}_N.$$



**Fig. 1.9.** The directed topologies of the MASs: (a) without leader; (b) with a leader

**Definition 1.6** (*Signed graph*) The graph describing the communication topology of MASs with antagonistic links is called as the signed graph (see Fig. 1.10). To describe the competitive-cooperative interaction, the positive edge denotes the cooperative interaction between nodes  $i$  and  $j$ , and in the same way, the negative edge denotes the competitive interaction.

### 1.3.2 Mathematical knowledge

**Lemma 1.7** (*(Roger & Charles, 1994)*). The definition of Kronecker product is: if  $P_1 = [p_{ij}]$  is an  $m \times n$  matrix and  $P_2$  is a  $p \times q$  matrix, then the Kronecker product  $P_1 \otimes P_2$  is an  $mp \times nq$  block matrix as follows:

$$P_1 \otimes P_2 = \begin{bmatrix} p_{11}B & \cdots & p_{1n}B \\ \vdots & \ddots & \vdots \\ p_{m1}B & \cdots & p_{mn}B \end{bmatrix}.$$

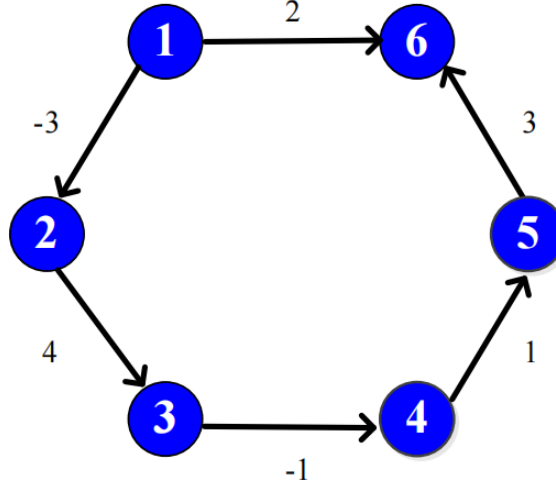


Fig. 1.10. The signed graph.

The properties of Kronecker product are: for the matrices  $P_1, P_2, P_3$ , and  $P_4$  with appropriate dimensions, the following equalities are established:

$$\begin{aligned} (P_1 \otimes P_2)^T &= P_1^T \otimes P_2^T \\ (P_1 + P_2) \otimes P_3 &= (P_1 \otimes P_3) + (P_2 \otimes P_3) \\ (P_1 \otimes P_2)(P_3 \otimes P_4) &= (P_1 P_3) \otimes (P_2 P_4) \\ (P_1 \otimes P_2)^{-1} &= P_1^{-1} \otimes P_2^{-1} \\ \lambda_{\max}(P_1 \otimes P_2) &= \lambda_{\max}(P_1) \lambda_{\max}(P_2). \end{aligned}$$

**Lemma 1.8** ((Boyd et al., 1994)). The linear matrix inequality  $\begin{pmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{pmatrix} < 0$ , where  $Q(x) = Q(x)^T$  and  $R(x) = R(x)^T$ , is equivalent to any one of the following conditions:

- (1)  $Q(x) < 0, R(x) - S(x)^T Q(x)^{-1} S(x) < 0$ ,
- (2)  $R(x) < 0, Q(x) - S(x) R(x)^{-1} S(x)^T < 0$ .

**Lemma 1.9** ((Qian et al., 2014)) For a given symmetric matrix  $G \in R^{N \times N}$  and a diagonal matrix  $D = \text{diag}\{d_1, \dots, d_l, 0, \dots, 0\}_{N \times N}$ , where  $d_i > 0, i = 1, \dots, l (1 \leq l \leq N)$ . Then  $G - D = \begin{pmatrix} R - \tilde{D} & S \\ S^T & G_l \end{pmatrix}$ , where  $\tilde{D} = \text{diag}\{d_1, \dots, d_l\}$ ,

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$G_l$  satisfies the definition in notation that is obtained by removing the first  $l$  row-column pairs. Matrices  $G$ ,  $R$  and  $S$  have the appropriate dimensions. If  $d_i > \lambda_{\max}(R - SG_l^{-1}S^T)$ , then  $G - D < 0$  is equivalent to  $G_l < 0$ .

**Lemma 1.10** (*Cao et al., 2005*) For any real matrixes  $X, Y$ ,  $\Xi = \Xi^T > 0$  and scalar  $\xi > 0$ , it holds

$$X^T Y + Y^T X \leq \xi X^T \Xi X + \xi^{-1} Y^T \Xi^{-1} Y. \quad (1.1)$$

**Lemma 1.11** (*(Boyd et al., 1994)*) Suppose that  $P_1 \in R^{n \times n}$  is a positive definite matrix and  $P_2 \in R^{n \times n}$  is symmetric. Then, for any vector  $x \in R^n$ , the following inequality holds:

$$\lambda_{\min}(P_1^{-1}P_2)x^T P_1 x \leq x^T P_2 x \leq \lambda_{\max}(P_1^{-1}P_2)x^T P_1 x \quad (1.2)$$

**Definition 1.12** (*Signum Function*): The notation  $\text{sgn}(x)$  represents the signum function, and it is defined as follows:

$$\text{sgn}(x) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases}$$

## 1.4 Contributions and outline of dissertation

This thesis focus on discontinuous control methods for multi-agent systems with linear and nonlinear dynamics. The communication topology is assumed to be fixed undirected or directed. The main contributions are summarized as follows.

**Chapter 2:** Notice that the relevant works on consensus problem for MASs mainly focus on complete consensus. However, in many physical applications, due to some unanticipated situations such as the changes of cooperative tasks, external environment or event-time, the agents are usually required to be divided into several clusters. Therefore, an extended concept about complete consensus is proposed to deal with such complex phenomena, that is **cluster consensus**, which means the agents in same cluster share a common state while there is no consensus behavior among different clusters. Secondly, in the most of prior works, the controller for all the agents is commonly assumed to be continuous. However, in real engineering, due to the presence of obstacles, the limitation of

## 1.4 Contributions and outline of dissertation

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computing and communication ability, or some other factors, the controller may be discontinuous. To mitigate these issues, the **intermittent control strategy** has been introduced and widely applied in transportation, manufacturing and communication.

Motivated by all the above-mentioned discussions, this Chapter addresses the cluster consensus problem via intermittent adaptive pinning control for MASs with general linear or nonlinear dynamics, respectively, where each cluster has a virtual leader whose state is available to only a small part of followers on some disconnected time intervals because of communication constraints. To our best knowledge, there are few works to investigate the problem. The main contributions of this chapter can be summarized as follows. In order to solve the cluster consensus problem, the agents in each cluster are categorized into three types. Specifically, the agents can only receive information from their own cluster, or they can receive information from other clusters or cannot receive any information from other agents. Hence, a class of intermittent adaptive pinning control protocols is proposed for the different type of agents. Correspondingly, some sufficient consensus criteria are derived to guarantee that the agents in the same cluster asymptotically can reach consensus while the agents in different clusters can reach different consensus. Rigorous proofs are given by the aid of Lyapunov stability theory and matrix theory. Finally, a numerical simulation example is presented to validate the main results.

Compared with the existing literatures, the main advantages are given as follows: Firstly, in contrast to the dynamics of integrator MASs, the dynamics of general linear MASs are much more complicated, and some integrator MASs such as, single integral ones and double integral ones can be seen as the special case of general linear MASs. Moreover, the dynamics of integrator MASs only depends on the coupling of the agents, the dynamics of general linear MASs depends not only on the coupling of the agents, but also the self-dynamics governing the evolution of each isolated agent. This makes the cluster consensus of general linear MASs technically more challenging than the case for integrator MASs. Secondly, different from continuous control protocols, the pinning gains in proposed control protocols are designed to be intermittent adaptive and with an exponential convergence rate, which can effectively reduce communication costs, avoid the



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pinning gains being larger than those needed in practice. Meanwhile, it guarantees the pinning gains quickly converge to steady value. Thirdly, in contrast to the directed network topology required to be in-degree balanced, strongly connected or contain a directed spanning tree, a weakly connected topology is taken into account, which is more universal in practice.

**Chapter 3:** The results obtained in above Chapter 2 assume that the local relative state information among all agents is available. However, in many actual systems, due to the states of agents are not always available or measured directly in expensive cost, it is quite necessary to design an **observer** to detect the agent's real-time state. In addition, the **time delays** universally exist in many physical systems and time delay can cause oscillations or instabilities. For example, in the consensus of migrating geese or locust population migration, all agents in the same group almost reach the place at the same time, but the different groups arrive at the place in different times. That is, not all agents arrive at the same place simultaneously, but the arrival time of the agents may be different. Besides, proper time delays between different vehicles in the way can keep the road safe and orderly. Otherwise, congestion often occurs.

Therefore, based on the above discussion and Chapter 2, in this Chapter, we investigate the observer-based intra-cluster lag consensus problems of multi-agent systems (MASs) with general linear dynamics and nonlinear dynamics via intermittent adaptive pinning control. The so-called **intra-cluster lag consensus** means that the followers in the same cluster can achieve lag consensus asymptotically while the followers in different clusters can achieve different agreements. The interaction network is still considered to be weakly connected, i.e., it is not necessary to be strongly connected, in-degree balanced or contain a directed spanning tree. To realise the intra-cluster lag consensus, a class of observers is designed to estimate the states of followers. Then a class of observer-based intermittent adaptive pinning control protocols is proposed according to the difference that the agents receive information source. Correspondingly, some sufficient consensus criteria are derived and rigorous proofs are given based on matrix theory and Lyapunov stability theory. Finally, the effectiveness for the proposed intermittent adaptive pinning control strategy is validated by a numerical simulation.

The main contributions and comparisons of this Chapter are listed as follows. Firstly, since the relevant full state information of each follower is not

always available, the Luenberger observers are designed for general linear and nonlinear MASs respectively. Hence, the control protocols in this chapter only use the observed states information of followers instead of their real-time states information. Secondly, to realize the intra-cluster lag consensus, the followers in each cluster are classified into three types. Specifically: the followers which can only receive information from the followers in their own cluster, the ones receiving information from the followers in other clusters, and finally, those who cannot receive information from any followers. Then, the distributed intermittent adaptive pinning control protocols are further designed according to the different categories of followers, which make sure that the followers in the same cluster achieve intra-cluster lag consensus whereas the followers in the different clusters achieve different consensus. Thirdly, compared with the most of existing results about cluster consensus where each block unit in the weighted adjacency matrix of network topology is assumed to be a zero-row-sum matrix or an equal-row-sum matrix, which is relative conservative in actual applications, in this chapter a weakly connected topology is considered and all the coupling weights of network topology are positive, which is more universal in practice.

**Chapter 4:** Note that the intermittent controller in Chapter 3 still requires to be updated continuously when it is executed in the work time interval, which is a waste of energy and resource. To mitigate this issue, the **event-triggered control** provides a novel strategy because it can avoid continuous update of the controller, which not only enhances the utilization of the network resources but also extends the lifetime of network components. Under an event-triggered mechanism, each agent exchanges information or controller makes corresponding update only at data transmission instants, which are determined by a predefined triggering function. In this sense, unnecessary consumption of resources is minimised as much as possible. Moreover, intermittent control can be divided into periodical intermittent control and **aperiodically intermittent control**. Compared with periodical intermittent control, aperiodically intermittent control is more reasonable and prevalent in practice due to the conditions of periodical intermittent control is restricted to some extent, for example, the generation of wind power is obviously aperiodically intermittent.

Inspired by the above discussion, this Chapter investigates the leader-following consensus of general linear MASs via aperiodically intermittent adaptive event-

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triggered control. The aperiodically intermittent adaptive event-triggered control inherits the respective advantages of aperiodically intermittent control strategy, event-triggered control strategy and adaptive control strategy, which improves communication efficiency, reduces control update frequency and is closer to the practical situations. The major advances of this Chapter lie in following aspects: Firstly, to reach leader-following consensus and save more control resources, a distributed aperiodically intermittent ETC protocol is devised, in which the transmission channels among agents only open if the local event-trigger condition is satisfied in predefined time intervals. Secondly, in order to get rid of continuous inter-agent communication for monitoring the triggering condition, a more general triggering mechanism is presented, in which discrete-time combinational measurement is adopted instead of using continuous-time tracking error directly. Thirdly, to overcome the unexpected large feedback gains in real applications and appropriately tune the feedback gains, the aperiodically intermittent adaptive event-triggered controller is further devised. With aid of the matrix theory, stability of switching systems and Lyapunov function, some sufficient criteria are deduced. Moreover, the analyses of excluding the Zeno behavior are included by showing explicit positive lower bounds between any two consecutive triggered events.

Compared with the existing literatures, there are the following main differences. Firstly, several authors have integrated intermittent strategy and event-triggered strategy together to address the consensus problem, however, the aperiodically intermittent adaptive event-triggered control method has not been considered. Secondly, some existing works on leader-following consensus of MASs by intermittent event-triggered control, continuous communication among agents is still required to check the triggering condition, this requirement is removed in this Chapter. Thirdly, the dynamics of agents in this Chapter is general linear rather than integral-order, therefore, some works about leader-following consensus can be regarded as special cases of this Chapter.

**Chapter 5:** In Chapter 4, we introduce the adaptive event-triggered control method into aperiodically intermittent control. It should be pointed out that the thresholds in Chapter 4 were state-dependent. When the measurement error equals or exceeds the threshold, the events are triggered, which can be regarded

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as the static triggering conditions. At the beginning, the static triggering conditions will effectively reduce the communication cost, as they are not easy to be satisfied. However, as time goes, it will be triggered frequently since the threshold becomes smaller and smaller, which causes unnecessary triggered instants. Developing more flexible event-triggering conditions to further reduce the communication cost and the number of the control updates is in great demand. By introducing an internal dynamic variable, a new class of event-triggering mechanisms is presented, that is, **dynamic event-triggered control**.

Therefore, motivated by the above-mentioned consideration, in this Chapter, to further reduce the communication cost and the number of the control updates, a hybrid aperiodically intermittent adaptive dynamic event-triggered control is put forward. The main contributions can be summarized as follows: Firstly, the dynamic adaptive event-triggered control is designed to further reduce the communication cost and the number of the control updates. Compared with the traditional static event-triggered control in Chapter 4, the time-varying threshold ensures less triggering instants. Secondly, based on matrix theory and Lyapunov function, the corresponding parameters are obtained. We modify the event triggering mechanism that depends on combined measurement approach in Chapter 4, continuous monitoring of neighbors' states are avoided for the triggering mechanism in this Chapter. Finally, numerical simulations are provided to illustrate the effectiveness of the theoretical results.

**Conclusions and perspectives:** In this chapter, the results are summarized and several possible directions for our future research are shared.

## 1. INTRODUCTION

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### Publications:

- **Yunlong Zhang**, Guoguang Wen, Ahmed Rahmani, Zhaoxia Peng and Wei Hu. Cluster consensus of multi-agent systems with general linear and nonlinear dynamics via intermittent adaptive pinning control. *Transactions of the Institute of Measurement and Control, Vol 43(6): pp. 1337-1346, 2021.*
- **Yunlong Zhang**, Guoguang Wen, Ahmed Rahmani, Zhaoxia Peng and Shiping Wen. Observer-based intra-cluster lag consensus of multi-agent systems via intermittent adaptive pinning control. *Journal of Systems Science and Complexity*, (Under 2<sup>nd</sup> review after minor revisions).
- **Yunlong Zhang**, Guoguang Wen, Ahmed Rahmani, and Xiaoqin Zhai. Aperiodically intermittent adaptive event-triggered control for linear multi-agent systems. *International Journal of Control, Automation and Systems*, (Accepted).
- **Yunlong Zhang**, Guoguang Wen, Ahmed Rahmani, and Xiaoqin Zhai. Observer-based cluster consensus of general linear multi-agent systems via aperiodically intermittent adaptive dynamic event-triggered control (Under preparation to be submitted to Journal).

# Chapter 2

## Cluster consensus of multi-agent systems with linear and nonlinear dynamics via intermittent adaptive pinning control

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### 2.1 Introduction

There are two critical factors that influence the consensus of MASs. One is the individual dynamics behavior. In (Liang-Hao & Xiao-Feng, 2013), MASs are

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mainly modeled by integral dynamics of each agent including first-order MASs model (Fan *et al.*, 2013), second-order MASs model (Liu *et al.*, 2016) and high-order MASs model (Tian *et al.*, 2016), etc. More recently, many researchers in (Dong & Hu, 2016) focus their attention on MASs with general linear dynamics due to the fact that the MASs with integral dynamical model can be viewed as a special situation of general linear MASs (Wen *et al.*, 2015). The other factor is the communication topology structures of systems, such as undirected topology and directed topology, fixed topology and switching topology, etc. In order to model different practical systems better, the researchers considered the MASs under different kinds of topology structures (Liu & Huang, 2019; Zhang *et al.*, 2020). Note that the previous results (Movric & Lewis, 2013; Yu & Wang, 2009) are based on the assumption that the communication topologies are undirected connected, directed strong connected, or containing a directed spanning tree. In fact, the assumptions are very conservative in practice, in other words, the communication topology may be weakly connected. In addition, as mentioned in Chapter 1, in many cases, agents may be divided into multiple subgroups due to different environments or tasks, and in each subgroup can reach different consistent state, this is so-called group consensus or cluster consensus. Although numerous available results are aimed to consider group/cluster consensus, to the best of authors knowledge, none of them investigate the cluster consensus of MASs with linear and Lipschitz nonlinear dynamics under a weakly connected graph.

It is well known that the control techniques are especially important for MASs to reach objective state. In the most of prior works, the communication among all the agents is commonly assumed to be continuous. However, in real engineering, due to the presence of obstacles, the limitation of computing and communication ability, or some other factors, the information communication may be discontinuous. To mitigate these issues, the intermittent control strategy has been introduced and widely applied in transportation, manufacturing and communication. Reference (Yang *et al.*, 2018a) analysed the cluster lag synchronization issue for heterogeneous complex networks by employing a intermittent control to reduce control cost and information communication. In (Yu *et al.*, 2017a), an intermittent sampled data control mechanism was employed, which not only shortens the working time, but also decreases the load of controllers' updating rate.

Motivated by all the above-mentioned discussions, this chapter addresses the cluster consensus problem via intermittent adaptive pinning control for MASs with general linear or nonlinear dynamics, respectively. To our best knowledge, there are few works to investigate the problem. The main contributions of this chapter can be summarized as follows. In order to solve the cluster consensus problem, the agents in each cluster are categorized into three types. Specifically, the agents can only receive information from their own cluster, or they can receive information from other clusters or cannot receive any information from other agents. Hence, a class of intermittent adaptive pinning control protocols are proposed for the different type of agents. Correspondingly, some sufficient consensus criteria are derived to guarantee that the agents in the same cluster asymptotically can reach consensus while the agents in different clusters can reach different consensus. Rigorous proofs are given by the aid of Lyapunov stability theory and matrix theory. Finally, a numerical simulation example is presented to validate the main results. Compared with the existing literatures, this chapter has the following two main advantages. Firstly, in contrast to the dynamics of integrator MASs, the dynamics of general linear MASs are much more complicated, and some integrator MASs such as, single integral ones and double integral ones can be seen as the special case of general linear MASs. Moreover, the dynamics of integrator MASs only depends on the coupling of the agents, the dynamics of general linear MASs depends not only on the coupling of the agents, but also the self-dynamics governing the evolution of each isolated agent. This makes the cluster consensus of general linear MASs technically more challenging than the case for integrator MASs. Secondly, different from continuous control protocols, the pinning gains in proposed control protocols are designed to be intermittent adaptive and with an exponential convergence rate, which can effectively reduce communication costs, avoid the pinning gains being larger than those needed in practice. Meanwhile, it guarantees the pinning gains quickly converge to steady value. Thirdly, in contrast to the directed network topology required to be in-degree balanced, strongly connected or contain a directed spanning tree, a weakly connected topology is taken into account, which is more universal in practice.

The remainder of this chapter is organized as follows. The problem is formulated in Section 2.2. Main results on cluster consensus of MASs via intermittent



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adaptive pinning control are introduced in Section 2.3. To demonstrate the theoretical results, a simulation example is presented in Section 2.4. Finally, a short conclusion is drawn in Section 2.5.

### 2.2 Problem formulation

To study the cluster consensus via intermittent adaptive pinning control for linear MASs and a class of nonlinear MASs, in this section, the model and problem objective are briefly introduced.

*Case I (Linear Systems):*

Consider a general linear MASs with  $N$  followers and  $m$  leaders ( $N > m \geq 2$ ). For each follower, the dynamic can be modeled as

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), i \in \{1, 2, \dots, N\} \triangleq \mathcal{V}, \quad (2.1)$$

where  $x_i \in R^n$  and  $u_i \in R^p$  represent the state and control input, respectively.

The leaders' dynamics are assumed to be:

$$\dot{s}_j(t) = As_j(t), j \in \{1, 2, \dots, m\} \triangleq \Omega, \quad (2.2)$$

where  $s_j(t) \in R^n$  represents the  $j$ th leader's state.

*Case II (Nonlinear Oscillators):*

Next, consider a MASs with nonlinear dynamics, in which the dynamics of each follower can be modeled as:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + f(x_i(t)), i \in \mathcal{V}. \quad (2.3)$$

where  $x_i \in R^n$  and  $u_i \in R^p$  represent the state and control input respectively, and  $f(x_i(t))$  is nonlinear function.

The leaders' dynamics are described by:

$$\dot{s}_j(t) = As_j(t) + f(s_j(t)), j \in \Omega. \quad (2.4)$$

where  $s_j(t) \in R^n$  represents the  $j$ th leader's position state.

A necessary assumption of the nonlinear function  $f(\cdot)$  is introduced below, which will be useful for the subsequent analysis.

**Assumption 2.1** *There exists a constant  $\gamma > 0$ , such that for any vectors  $x, y \in R^m$ , the nonlinear function  $f(\cdot)$  satisfies  $\|f(x) - f(y)\| \leq \gamma \|x - y\|$ .*

**Remark 2.2** *Compared with the consistent continuous, Assumption 2.1 is a smoother condition, which is called the Lipschitz condition.*

**Definition 2.3** *Given the MASs (2.1) and (2.2), (2.3) and (2.4) the cluster consensus is reached by designing an appropriate control input such that  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$  for any initial values, where  $e_i(t) = x_i(t) - s_{\hat{i}}(t)$ ,  $i \in \mathcal{V}$ ,  $\hat{i}$  is the subscript of the the index set of the cluster which the  $i$ th follower belongs, that is  $v_i \in \mathcal{V}_{\hat{i}}$ . See graph theory 1.3.1.*

**Remark 2.4** *Note that the trajectories of all leaders are determined by system matrix  $A$  owing to  $s_j(t) = s_j(0)e^{At}$ . If  $A$  is a Hurwitz matrix, then the states of all leaders will asymptotically reach zeros, which goes against our objective. Therefore, in this chapter, we assume the matrix  $A$  is not Hurwitz. By giving different initial value for system (2.2) and (2.4), the states of leaders will be different, that is, for any  $l \neq k$ ,  $\lim_{t \rightarrow \infty} \|s_l(t) - s_k(t)\| \neq 0$ , where  $l, k \in \Omega$ .*

## 2.3 Main results

### 2.3.1 Cluster consensus of MASs with general linear dynamics via intermittent adaptive pinning control

In practical application, especially for the large-scaled MASs, it is impossible for the leader to pin every follower. To overcome this drawback, pinning control can be used by controlling only a small fraction of agents rather than all the agents. Besides, to prevent the appearance of larger pinning gain than those required in practice, adaptive control method can be introduced to pinning control. Furthermore, considering the appearance of obstacles or the limitations of sensing ranges in some situations, the leaders may only pinning the followers over some disconnected time intervals, in other words, the pinning time may be intermittent, not continuous. Motivated by the above discussion, in this chapter the following

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intermittent adaptive pinning control is proposed to solve the cluster consensus problem,

$$u_i(t) = \begin{cases} K \sum_{j=1}^N a_{ij}(x_j(t) - x_i(t)) - d_i(t)(x_i(t) - s_i(t)) + K \sum_{j=1}^N l_{ij}s_j(t), & \text{if } i \in \tilde{\mathcal{V}}_i, \\ K \sum_{j=1}^N a_{ij}(x_j(t) - x_i(t)) - d_i(t)(x_i(t) - s_i(t)), & \text{if } i \in \mathcal{V}_i \setminus \tilde{\mathcal{V}}_i \text{ and } \deg(i)_{in} = 0, \\ K \sum_{j=1}^N a_{ij}(x_j(t) - x_i(t)), & \text{otherwise,} \end{cases} \quad (2.5)$$

$$d_i(t) = \begin{cases} \sigma e^{\sigma_1 t} (x_i(t) - s_i(t))^T \Gamma (x_i(t) - s_i(t)), & \text{if } t \in [mT, mT + \delta), \\ 0, & \text{if } t \in [mT + \delta, (m+1)T), \end{cases} \quad (2.6)$$

where  $\sigma$  and  $\sigma_1$  are positive constants,  $m = 0, 1, 2, \dots$ ,  $K$  and  $\Gamma$  represent the feedback control gain matrices which will be designed later in Theorem 2.8. The parameter  $d_i(t)$  satisfies the following conditions:  $d_i(t) > 0$  if the node  $v_i$  is pinned and  $d_i(t) = 0$  otherwise. In addition, the control period and the control width are denoted as  $T > 0$  and  $\delta \in (0, T)$ , respectively. Denote the control rate  $\theta = \delta/T$ , which will be designed later in Theorem 2.8.

**Remark 2.5** *In order to solve the cluster consensus problem, the agents in each cluster are categorized into three types. Specifically, the agents can only receive information from their own cluster, or they can receive information from other clusters or cannot receive any information from other agents. Hence, a class of intermittent adaptive pinning control protocols are proposed for the different type of agents. Moreover, the cluster consensus is aimed at reaching consensus in the same cluster and reaching different consensus among different clusters. The term  $K \sum_{j=1}^N a_{ij}(x_j(t) - x_i(t))$  in the control protocol (2.5) reflects the interaction between agent  $i$  and its neighbours. The term  $d_i(t)(x_i(t) - s_i(t))$  reflects that whether the agent  $i$  will be pinned. In fact, if the agent  $i$  can receive information from other clusters, or it can not receive information from any agents (that is to say, it is with zero in-degree), then it should be pinned. The term  $K \sum_{j=1}^N l_{ij}s_j(t)$  is used to counteract the influence among clusters.*

**Remark 2.6** *The design of (2.6) is enlightened as follows: in practice, the pinning gains between leaders and followers are not always fixed due to the existence*

## 2.3 Main results

of external disturbance and expensive control costs. Therefore, the pinning gains in proposed control protocols are designed to be intermittent adaptive and with an exponential convergence rate, which can effectively reduce communication costs, avoid the pinning gains being larger than those needed in practice. Meanwhile, it guarantees the pinning gains quickly converge to steady value.

**Lemma 2.7** *If  $v_i$  is the node that belongs to  $\mathcal{V}_i \setminus \tilde{\mathcal{V}}_i$ , then  $\sum_{j=1}^N l_{ij}s_j(t) = 0$ .*

**Proof:** From the definition of  $\tilde{\mathcal{V}}_i$ , if  $v_i \in \mathcal{V}_i \setminus \tilde{\mathcal{V}}_i$ , one has  $l_{ij} = 0$  for  $\forall v_j \notin \mathcal{V}_i$ . Note from the fact  $\sum_{j=1}^N l_{ij} = 0$ , it follows that

$$\begin{aligned} \sum_{j=1}^N l_{ij}s_j(t) &= \sum_{v_j \in \mathcal{V}_i} l_{ij}s_j(t) + \sum_{v_j \notin \mathcal{V}_i} l_{ij}s_j(t) = \sum_{v_j \in \mathcal{V}_i} l_{ij}s_i(t) \\ &= \left( \sum_{v_j \in \mathcal{V}_i} l_{ij} + \sum_{v_j \notin \mathcal{V}_i} l_{ij} \right) s_i(t) = \sum_{j=1}^N l_{ij}s_i(t) = 0. \end{aligned} \quad (2.7)$$

For convenience, the agents in each cluster are categorized into three types and defined as follows.  $\mathcal{V}_1^i$  is the set of the nodes that belong to  $\tilde{\mathcal{V}}_i$ ;  $\mathcal{V}_2^i$  is the set of the nodes that belong to  $\mathcal{V}_i \setminus \tilde{\mathcal{V}}_i$  and  $\deg(i)_{in} = 0$ ;  $\mathcal{V}_3^i$  is the set of the rest of nodes. Recalling the definition of  $e_i(t)$ , one has,

$$\dot{e}_i(t) = \begin{cases} Ae_i(t) - BK \sum_{j=1}^N l_{ij}e_j(t) - d_i(t)BK e_i(t), & \text{if } i \in \mathcal{V}_1^i \cup \mathcal{V}_2^i, \\ Ae_i(t) - BK \sum_{j=1}^N l_{ij}e_j(t), & \text{if } i \in \mathcal{V}_3^i. \end{cases} \quad (2.8)$$

Furthermore, without loss of generality, the followers can be rearranged the order. Let the first  $l$  ( $1 \leq l \leq N$ ) agents be pinned in MASs. Then  $e_i(t)$  can be rewritten as:

$$\dot{e}_i(t) = Ae_i(t) - BK \sum_{j=1}^N l_{ij}e_j(t) - d_i(t)BK e_i(t), \text{ if } i \in \mathcal{V}_1^i \cup \mathcal{V}_2^i \cup \mathcal{V}_3^i, \quad (2.9)$$

$$\dot{d}_i(t) = \begin{cases} \sigma e^{\sigma_1 t} e_i^T(t) \Gamma e_i(t), & \text{if } t \in [mT, mT + \delta), \\ 0, & \text{if } t \in [mT + \delta, (m+1)T). \end{cases} \quad (2.10)$$

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**Theorem 2.8** *The MASs (2.1) and (2.2) with the control input (2.5) can reach cluster consensus, that is to say,  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$  for  $i \in \mathcal{V}$ , if the following conditions are satisfied:*

(i) *the pair  $(A, B)$  is stabilizable,*

(ii)  *$\bar{L} + D > 0$ ,*

(iii)  *$K = \mu_1 B^T P$  and  $\Gamma = 2\mu_1 P B B^T P$ ,*

(iv)  *$\theta_1 = \delta/T > \frac{\beta_1}{\alpha_1 + \beta_1}$ ,*

*where  $D = \text{diag}\{d_1, \dots, d_l, 0, \dots, 0\}_{N \times N}$ ,  $\alpha_1 = \min\{\frac{-\lambda_1}{\lambda_{\max}(P)}, \sigma_1\}$ ,  $\lambda_1 = \lambda_{\max}(A^T P + PA - \mu_1 P B B^T P \lambda_{\min}(2\bar{L} + 2D))$ ,  $\beta_1 = \max\{\frac{\lambda_2}{\lambda_{\min}(P)}, \sigma_1\}$ ,  $\lambda_2 = \max\{\lambda_{\max}(I_N \otimes (A^T P + PA) - 2\mu_1 \bar{L} \otimes P B B^T P), \epsilon\}$ ,  $\sigma$ ,  $\sigma_1$  and  $\epsilon$  are any positive constants,  $P = P^T > 0$  and  $\mu_1$  satisfies the following equations:*

$$A^T P + PA - \mu_1 P B B^T P \lambda_{\min}(2\bar{L} + 2D) < 0, \quad (2.11)$$

$$\mu_1 \cdot \lambda_{\min}(2\bar{L} + 2D) > 1 \quad (2.12)$$

**Proof:** Choose the following Lyapunov function candidate:

$$V(t) = \sum_{i=1}^N e_i^T(t) P e_i(t) + \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}, \quad (2.13)$$

where  $\sigma$  is positive constant. Matrix  $P$  and the parameters  $d_i$ ,  $\sigma_1$  are defined in the conditions of Theorem 2.8.

(1) When  $t \in [mT, mT + \delta)$ , take the time derivative of (5.15) as

$$\begin{aligned} \dot{V}(t) = & 2 \sum_{i=1}^N e_i^T(t) P (A e_i(t) - B K \sum_{j=1}^N l_{ij} e_j(t) - d_i(t) B K e_i(t)) \\ & + \frac{1}{2} (-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + (d_i(t) - d_i) \sum_{i=1}^N e_i^T(t) \Gamma e_i(t). \end{aligned} \quad (2.14)$$

Denote  $e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$ ,  $D(t) = \text{diag}\{d_1(t), \dots, d_l(t), 0, \dots, 0\}_{N \times N}$ ,  $K = \mu_1 B^T P$ ,  $\Gamma = 2\mu_1 P B B^T P$ ,  $D = \text{diag}\{d_1, \dots, d_l, 0, \dots, 0\}_{N \times N}$ , where  $d_i >$

$0, i = 1, \dots, l (1 \leq l \leq N)$ . The derivative of  $V(t)$  can be written as,

$$\begin{aligned}
 \dot{V}(t) &= e^T(t)(I_N \otimes A^T P - L^T \otimes K^T B^T P - D(t) \otimes K^T B^T P)e(t) \\
 &\quad + e^T(t)(I_N \otimes PA - L \otimes PBK - D(t) \otimes PBK)e(t) \\
 &\quad + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + e^T(t)(D(t) - D) \otimes (2\mu_1 PBB^T P)e(t) \\
 &= e^T(t)(I_N \otimes (A^T P + PA) - 2\mu_1 \bar{L} \otimes PBB^T P - 2\mu_1 D(t) \otimes PBB^T P)e(t) \\
 &\quad + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + e^T(t)(D(t) - D) \otimes (2\mu_1 PBB^T P)e(t) \\
 &= e^T(t)(I_N \otimes (A^T P + PA) - \mu_1(2\bar{L} + 2D) \otimes PBB^T P)e(t) \\
 &\quad + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}.
 \end{aligned} \tag{2.15}$$

Next, we will show that the matrix  $I_N \otimes (A^T P + PA) - \mu_1(2\bar{L} + 2D) \otimes PBB^T P$  is

negative. Since the condition  $2\bar{L} + 2D > 0$ , it follows that there exists a constant

$\mu_1$  such that  $\mu_1 \cdot \lambda_{\min}(2\bar{L} + 2D) > 1$ . Furtherly, on the basis of linear systems the-

ory, there exists a positive matrix  $P$  such that  $A^T P + PA - \mu_1 PBB^T P \lambda_{\min}(2\bar{L} +$

$2D) < 0$ , namely, matrix  $I_N \otimes (A^T P + PA) - \mu_1(2\bar{L} + 2D) \otimes PBB^T P$  is negative.

Denote  $\lambda_1 = \lambda_{\max}(A^T P + PA - \mu_1 PBB^T P \lambda_{\min}(2\bar{L} + 2D))$ , one has

$$\begin{aligned}
 \dot{V}(t) &\leq \lambda_1 e^T(t)e(t) + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
 &\leq \frac{\lambda_1}{\lambda_{\max}(P)} e^T(t)(I_N \otimes P)e(t) + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
 &\leq -\alpha_1 V(t),
 \end{aligned} \tag{2.16}$$

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where  $\alpha_1 = \min\{\frac{-\lambda_1}{\lambda_{\max}(P)}, \sigma_1\}$ .

(2) When  $t \in [mT + \delta, (m+1)T)$ , the time derivative of (5.15) is given as

$$\begin{aligned}
\dot{V}(t) &= 2 \sum_{i=1}^N e_i^T(t) P (A e_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t)) + \frac{1}{2} (-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&= e^T(t) (I_N \otimes (A^T P + PA) - \mu_1 (2\bar{L}) \otimes P B B^T P) e(t) - \frac{1}{2} \sigma_1 \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq \lambda_{\max}(I_N \otimes (A^T P + PA) - 2\mu_1 \bar{L} \otimes P B B^T P) e^T(t) e(t) + \sigma_1 \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}
\end{aligned} \tag{2.17}$$

Denote  $\lambda_2 = \max\{\lambda_{\max}(I_N \otimes (A^T P + PA) - 2\mu_1 \bar{L} \otimes P B B^T P), \epsilon\}$  with  $\epsilon > 0$ , one has,

$$\begin{aligned}
\dot{V}(t) &\leq \lambda_2 e^T(t) e(t) + \sigma_1 \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq \frac{\lambda_2}{\lambda_{\min}(P)} e^T(t) (I_N \otimes P) e(t) + \sigma_1 \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq \beta_1 V(t),
\end{aligned} \tag{2.18}$$

where  $\beta_1 = \max\{\frac{\lambda_2}{\lambda_{\min}(P)}, \sigma_1\}$ . By induction, one obtains,

$$\begin{aligned}
V((m+1)T) &\leq V(mT + \delta) e^{\beta_1(T-\delta)} \leq V(mT) e^{-\alpha_1 \delta} \cdot e^{\beta_1(T-\delta)} \\
&= V(mT) e^{-\alpha_1 \delta + \beta_1(T-\delta)} \leq \dots \leq V(0) e^{(-\alpha_1 \delta + \beta_1(T-\delta))(m+1)}.
\end{aligned} \tag{2.19}$$

To ensure that  $V((m+1)T) \rightarrow 0$  as  $m \rightarrow \infty$ , let  $-\alpha_1 \delta + \beta_1(T-\delta) < 0$ . Then one has  $\theta_1 = \delta/T > \frac{\beta_1}{\alpha_1 + \beta_1}$ . Thus,  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ . It means that cluster consensus of general linear MASs via intermittent adaptive pinning control is reached. This completes the proof.

**Remark 2.9** *The proof of Theorem 2.8 is divided into two parts  $t \in [mT, mT + \delta)$  and  $t \in [mT + \delta, (m+1)T)$ . When  $t \in [mT, mT + \delta)$ , the derivative of  $V(t)$  is negative due to the influence of leaders, which indicates the  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ . However, when  $t \in [mT + \delta, (m+1)T)$ , the derivative of  $V(t)$  cannot be determined to be negative, while it is only obtained that  $\dot{V}(t) \leq \beta_1 V(t)$  by choosing the appropriate positive constants  $\lambda_2, \beta_1$ . Hence, it is critical to design the intermittent control rate  $\theta_1$  in (2.19) for ensuring  $V((m+1)T) \rightarrow 0$  as  $m \rightarrow \infty$ , that is, cluster consensus is achieved.*

The general criterions are presented in Theorem 2.8 to reach leader-following cluster consensus of MASs. However, how to construct an appropriate gain matrix  $D$  to satisfy the condition  $2\bar{L} + 2D > 0$  is not easy. Hence, the following theorem will be given to solve this problem.

**Theorem 2.10** *The matrix  $\bar{L} + D$  is a positive if the following conditions hold:*

$$\lambda_{\max}(-\bar{L}_l) < 0, \quad (2.20)$$

$$d_i > \lambda_{\max}(R - S(-\bar{L}_l)^{-1}S^T). \quad (2.21)$$

**Proof:** In Lemma 1.9, let  $G = -\bar{L}$ , and  $-\bar{L} - D = \begin{pmatrix} R - \tilde{D} & S \\ S^T & -\bar{L}_l \end{pmatrix}$ , where  $-\bar{L}_l$  is the minor matrix of  $-\bar{L}$  by removing its first  $l$  row-column pairs,  $\tilde{D} = \text{diag}\{d_1, \dots, d_l\}$ ,  $R$  and  $S$  represent the matrices with appropriate dimensions. Thanks to Lemma 1.9, we have  $-\bar{L} - D < 0$ , that is,  $\bar{L} + D$  is a positive matrix. This completes the proof.

**Remark 4.** It is worth noting that most literatures on cluster consensus assume that adjacency matrix  $A$  satisfies  $\sum_{j \in \mathcal{V}_k} a_{ij} = 0$ , or  $\sum_{j \in \mathcal{V}_k} a_{ij} = \beta$ ,  $\forall k \neq \hat{i}$ , where  $\beta$  is a constant. In fact, the assumption is conservative. In this chapter, we take a weakly connected graph into consideration, in where the Laplace matrix just needs to satisfy the general diffusion property, that is,  $\sum_{j=1}^N l_{ij} = 0$ . Moreover, by choosing an appropriate low bound pinning gain  $d_i$ , one can get  $\bar{L} + D > 0$ . For an undirected connected graph,  $\bar{L} + D > 0$  can hold naturally due to the Laplace matrix is a real symmetric. Therefore, the MASs with undirected connected graph can be considered as a special case only by choosing the inter-act followers to be pinned.

### 2.3.2 Cluster consensus of MASs with a class of nonlinear dynamics via intermittent adaptive pinning control

It well known that most of physical systems are inherent nonlinear in reality. Particularly, for the MASs, the nonlinear dynamics play an important role in determining the final consensus states (Cheng *et al.*, 2019). To satisfy the requirement for the application of cluster problem in the real physical systems, this chapter further presents an intermittent adaptive pinning approach for a group



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of nonlinear MASs given by (2.3) and (2.4) subject to both plant uncertainties and external disturbances.

Let us consider the system (2.3)-(2.4) and control input (2.5). Taking the control input (2.5) for system (2.3) and combining with the definition of  $e_i(t)$ , one has,

$$\dot{e}_i(t) = \begin{cases} Ae_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t) - d_i(t) BK e_i(t) + f(x_i(t)) - f(s_i(t)), \\ \text{if } i \in \mathcal{V}_1^i \cup \mathcal{V}_2^i, \\ Ae_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t) + f(x_i(t)) - f(s_i(t)), \text{if } i \in \mathcal{V}_3^i, \end{cases} \quad (2.22)$$

where  $K$  is the feedback control matrix will be recalculated in Theorem 2.11,  $d_i(t)$  is defined in (2.6) and  $f(\cdot)$  is the nonlinear function. Similarity, by rearranging the order of pinned followers, that is, let the first  $l$  ( $1 \leq l \leq N$ ) agents are pinned in MASs. Combine the definition of  $d_i(t)$ ,  $e_i(t)$  can be changed as:

$$\begin{aligned} \dot{e}_i(t) &= Ae_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t) - d_i(t) BK e_i(t) + f(x_i(t)) - f(s_i(t)), \\ i &\in \mathcal{V}_1^i \cup \mathcal{V}_2^i \cup \mathcal{V}_3^i, \end{aligned} \quad (2.23)$$

$$\dot{d}_i(t) = \begin{cases} \sigma e^{\sigma_1 t} e_i^T(t) \Gamma e_i(t), \text{if } t \in [mT, mT + \delta), \\ 0, \text{if } t \in [mT + \delta, (m+1)T). \end{cases} \quad (2.24)$$

**Theorem 2.11** *The MASs (2.3) and (2.4) with the control protocol (2.5) reach cluster consensus, that is to say,  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$  for  $i \in \mathcal{V}$ , if the following conditions are satisfied:*

- (i)  $(A, B)$  is stabilizable,
- (ii)  $\bar{L} + D > 0$ ,
- (iii)  $K = \mu_2 B^T P$  and  $\Gamma = 2\mu_2 P B B^T P$ ,
- (iv)  $\theta_2 = \delta/T > \frac{\beta_2}{\alpha_2 + \beta_2}$ ,

where  $D = \text{diag}\{d_1, \dots, d_l, 0, \dots, 0\}_{N \times N}$ ,  $\alpha_2 = \min\{-\lambda_3 \cdot \frac{\lambda_{\min}(P^{-1})^2}{\lambda_{\max}(P)}, \sigma_1\}$ ,  $\beta_2 = \max\{\frac{\lambda_4}{\lambda_{\min}(P)}, \sigma_1\}$ ,  $\lambda_3 = \lambda_{\max}(AQ + QA^T + \gamma^2 I + Q^2 - \tau B B^T)$ ,  $\Lambda = A^T P + P A + I + \gamma^2 P^2$ ,  $\lambda_4 = \max\{\lambda_{\max}(I_N \otimes \Lambda - 2\mu_2 \bar{L} \otimes P B B^T P), \epsilon\}$ ,  $\sigma$ ,  $\sigma_1$  and  $\epsilon$  is any

## 2.3 Main results

positive constants,  $\tau$ ,  $P = P^T = Q^{-1} > 0$  and  $\mu_2$  satisfy the following equations:

$$\begin{pmatrix} AQ + QA^T - \tau BB^T + \gamma^2 I & Q \\ Q^T & -I \end{pmatrix} < 0 \quad (2.25)$$

$$\mu_2 \cdot \lambda_{\min}(2\bar{L} + 2D) > \tau \quad (2.26)$$

**Proof:** Choose the following Lyapunov function candidate:

$$V(t) = \sum_{i=1}^N e_i^T(t) P e_i(t) + \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}. \quad (2.27)$$

(1) When  $t \in [mT, mT + \delta)$ , take the time derivative of (2.27) as,

$$\begin{aligned} \dot{V}(t) &= 2 \sum_{i=1}^N e_i^T(t) P (A e_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t) - d_i(t) BK e_i(t) + f(x_i(t)) - f(\hat{s}_i(t))) \\ &\quad + \frac{1}{2} (-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + (d_i(t) - d_i) \sum_{i=1}^N e_i^T(t) \Gamma e_i(t). \end{aligned} \quad (2.28)$$

Denote the compact vector

$$F(t) = (f^T(x_1(t)) - f^T(s_1(t)), f^T(x_2(t)) - f^T(s_2(t)), \dots, f^T(x_N(t)) - f^T(s_N(t)))^T.$$

Based on the Assumption 2.1, one has,

$$2e_i^T(t) P (f(x_i(t)) - f(\hat{s}_i(t))) \leq 2\gamma \|P e_i(t)\| \cdot \|e_i(t)\| \leq e_i^T(t) (I + \gamma^2 P^2) e_i(t). \quad (2.29)$$

Combining (2.29), the derivative of  $V(t)$  can be written as,

$$\begin{aligned} \dot{V}(t) &= e^T(t) (I_N \otimes A^T P - L^T \otimes K^T B^T P - D(t) \otimes K^T B^T P) e(t) \\ &\quad + e^T(t) (I_N \otimes PA - L \otimes PBK - D(t) \otimes PBK) e(t) + 2e^T(t) (I_N \otimes P) F(t) \\ &\quad + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + e^T(t) (D(t) - D) \otimes (2\mu_2 P B B^T P) e(t). \end{aligned} \quad (2.30)$$

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Upon (2.29), we can get

$$\begin{aligned}
\dot{V}(t) &\leq e^T(t)(I_N \otimes (A^T P + PA + I + \gamma^2 P^2) \\
&\quad - \mu_2(L^T + L) \otimes PBB^T P - 2\mu_2 D(t) \otimes PBB^T P)e(t) \\
&\quad + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + e^T(t)(D(t) - D) \otimes (2\mu_2 PBB^T P)e(t) \\
&\leq e^T(t)(I_N \otimes (A^T P + PA + I + \gamma^2 P^2) - \mu_2(2\bar{L} + 2D) \otimes PBB^T P)e(t) \\
&\quad + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}.
\end{aligned} \tag{2.31}$$

According to the condition (ii) in Theorem 2.11, there exists a constant  $\mu_2$  satisfies

$\mu_2 \cdot \lambda_{\min}(2\bar{L} + 2D) > \tau$ . Let  $Q = P^{-1}$ ,  $\eta(t) = (I_N \otimes P)e(t)$ , one has,

$$\begin{aligned}
\dot{V}(t) &\leq \eta^T(t)(I_N \otimes (AQ + QA^T + \gamma^2 I + Q^2) - I_N \otimes \tau BB^T)\eta(t) \\
&\quad + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}.
\end{aligned} \tag{2.32}$$

Further, according to the Lemma 1.8 and the conditions in Theorem 2.11, the

matrix  $AQ + QA^T + \gamma^2 I + Q^2 - \tau BB^T < 0$ . Denote  $\lambda_3 = \lambda_{\max}(AQ + QA^T +$

$\gamma^2 I + Q^2 - \tau BB^T)$ , one has,

$$\begin{aligned}
\dot{V}(t) &\leq \lambda_3 \eta^T(t)\eta(t) + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&= \lambda_3 e^T(t)(I_N \otimes P^{-T})(I_N \otimes P^{-1})e(t) + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq \lambda_3 \cdot \frac{\lambda_{\min}(P^{-1})^2}{\lambda_{\max}(P)} e^T(t)(I_N \otimes P)e(t) + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq -\alpha_2 V(t),
\end{aligned} \tag{2.33}$$

where  $\alpha_2 = \min\{-\lambda_3 \cdot \frac{\lambda_{\min}(P^{-1})^2}{\lambda_{\max}(P)}, \sigma_1\}$ .

(2) When  $t \in [mT + \delta, (m + 1)T)$ , take the time derivative of (2.27) as

$$\begin{aligned} \dot{V}(t) = & 2 \sum_{i=1}^N e_i^T(t) P (A e_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t)) + 2 \sum_{i=1}^N e_i^T(t) P (f(x_i(t)) - f(\hat{s}_i(t))) \\ & + \frac{1}{2} (-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}. \end{aligned} \quad (2.34)$$

Based on (2.29), we have

$$\begin{aligned} \dot{V}(t) \leq & e^T(t) (I_N \otimes (A^T P + PA + I + \gamma^2 P^2) - \mu_2 (L^T + L) \otimes PBB^T P) e(t) \\ & - \frac{1}{2} \sigma_1 \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\ \leq & \lambda_4 e^T(t) e(t) + \sigma_1 \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\ \leq & \frac{\lambda_4}{\lambda_{\min}(P)} e^T(t) (I_N \otimes P) e(t) + \sigma_1 \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\ \leq & \beta_2 V(t), \end{aligned} \quad (2.35)$$

where  $\beta_2 = \max\{\frac{\lambda_4}{\lambda_{\min}(P)}, \sigma_1\}$ ,

$\lambda_4 = \max\{\lambda_{\max} I_N \otimes \Lambda - \mu_2 (L^T + L) \otimes PBB^T P, \epsilon\}$ ,  $\Lambda = A^T P + PA + I + \gamma^2 P^2$ .

Similarly, by induction, one obtains,

$$\begin{aligned} V((m + 1)T) \leq & V(mT + \delta) e^{\beta_2(T - \delta)} \leq V(mT) e^{-\alpha_2 \delta} \cdot e^{\beta_2(T - \delta)} \\ = & V(mT) e^{-\alpha_2 \delta + \beta_2(T - \delta)} \leq \dots \leq V(0) e^{(-\alpha_2 \delta + \beta_2(T - \delta))(m + 1)}. \end{aligned} \quad (2.36)$$

To ensure that  $V((m + 1)T) \rightarrow 0$  as  $m \rightarrow \infty$ , let  $-\alpha_2 \delta + \beta_2(T - \delta) < 0$ . Then one has  $\theta_2 = \delta/T > \frac{\beta_2}{\alpha_2 + \beta_2}$ . Thus,  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ , it means that cluster consensus of MASs with nonlinear dynamics via intermittent adaptive pinning control can be reached. This completes the proof.

## 2.4 Simulation results

Two examples are given in this section to illustrate the effectiveness of obtained results. Consider a MASs with three clusters, Fig. 2.1 is the communication topology. The leaders are labelled as  $S_1$ ,  $S_2$  and  $S_3$ , and the followers are labelled

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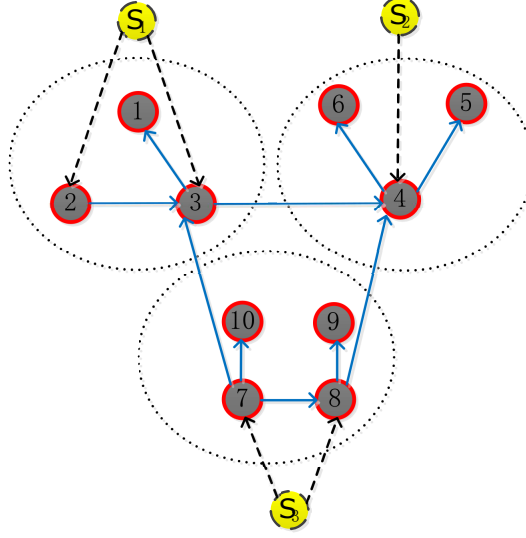
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as 1 to 10, where followers 1, 2, 3 are in the first cluster, followers 4, 5, 6 are in the second cluster, followers 7, 8, 9, 10 are in the third cluster. The dotted line denotes the effects that the leaders impose on followers. From Fig.2.1, we can see that followers 2, 3, 4, 7 and 8 should be pinned according to the control strategy (2.5). Specifically, due to followers 3 and 4 can receive information from other clusters, the first term of controller (2.5) should be applied to these followers; followers 2, 7 and 8 cannot receive any information from other clusters, the second term of controller (2.5) should be applied to these followers; the rest of followers only can receive information from their own cluster, the third term of controller (2.5) should be applied to these followers.

From the graph theory, the matrices  $\mathcal{A}$  and  $L$  can be derived as follows:

$$\mathcal{A} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{and } L = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}.$$



**Fig. 2.1.** The communication topology of the MASs, where  $s_j$  denotes the  $j$ th leader, and  $i$  denotes the  $i$ th follower,  $j = 1, 2, 3; i = 1, 2, \dots, 10$ .

**Example 1: (linear case)** For the linear case, two kinds of system matrices  $A$  and  $B$  are presented to show the influence of system matrix for system performance.

**Linear case (a):** For the linear MASs (2.1) and (2.2), we choose  $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$ ,

$B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ , respectively. Obviously,  $(A, B)$  is stabilizable. And the positive constants can be chosen as:  $\sigma = 1$ ,  $\sigma_1 = 1$ ,  $\epsilon = 0.1$ . Control period  $T$  is given as  $T = 1s$ . By calculating the equations (2.11) and (2.12), we can obtain

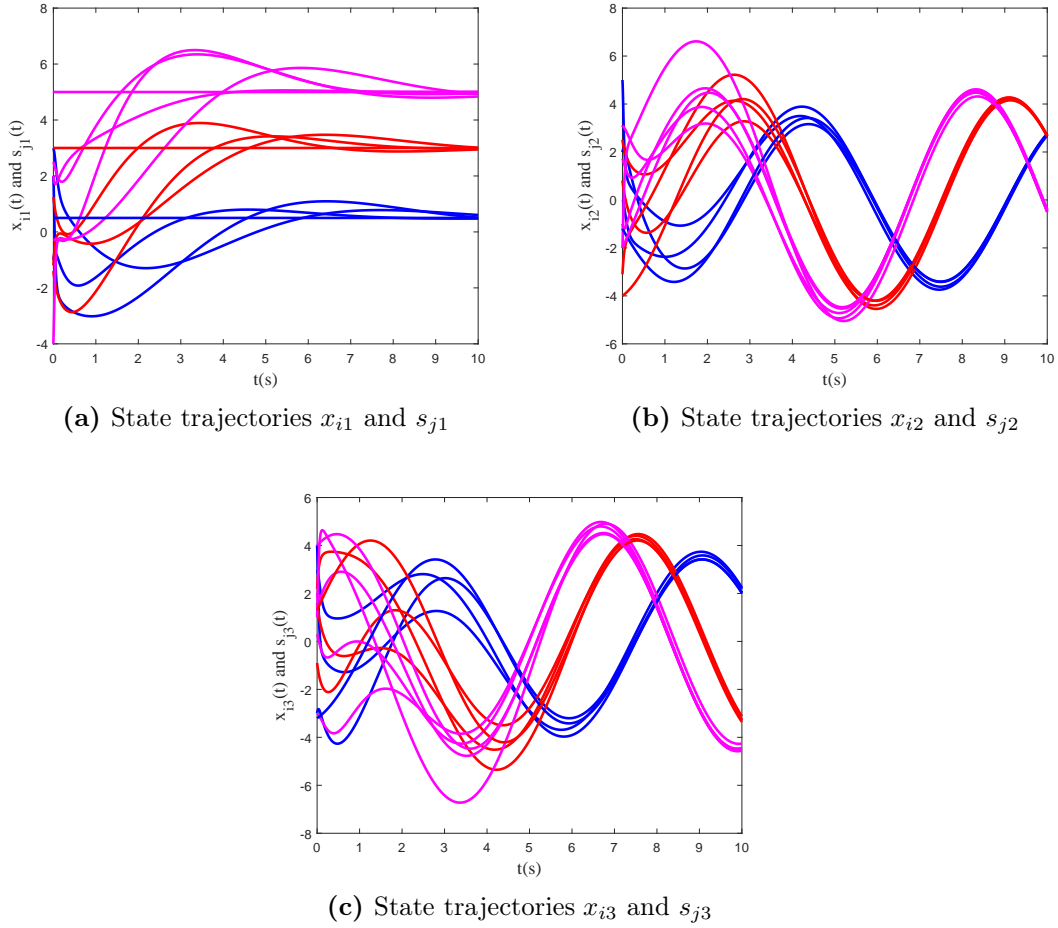
$P = \begin{pmatrix} 2.3483 & -0.0488 & -1.4134 \\ -0.0488 & 1.8000 & -0.4988 \\ -1.4134 & -0.4988 & 1.8689 \end{pmatrix}$ ,  $K = ( 1.7722 \quad 2.5048 \quad -0.0865 )$ ,  $\mu_1 = 2$ ,  $d_i = 1$ ,  $i = 2, 3, 4, 7, 8$ ,  $\delta = 0.98$ . Fig. 2.2 is the state trajectories of leaders and followers with three dimensions. Fig. 2.3 is the time evolution of

$d_i(t)$  with  $i = 2, 3, 4, 7, 8$  and state error  $e_{ij}(t)$  for  $j = 1, 2, 3, i = 1, 2, \dots, 10$ .

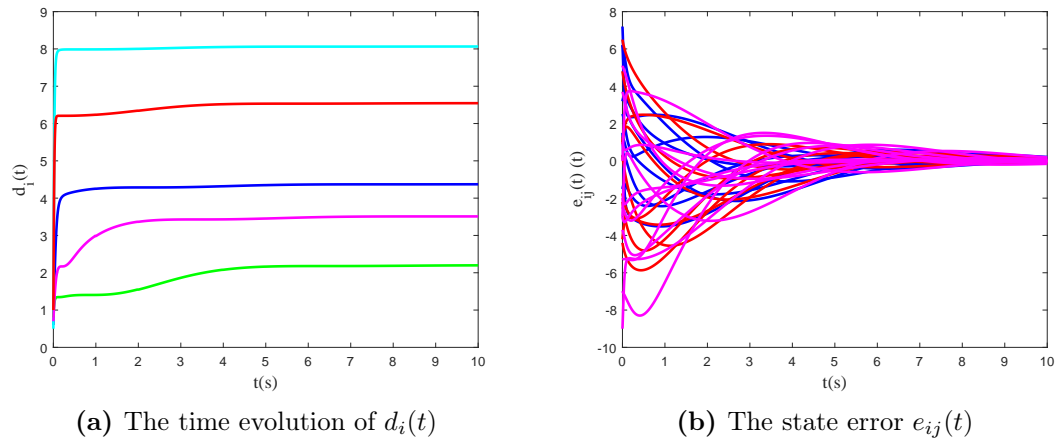
**Linear case (b):** Choose  $A = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$  as the system matrices for linear MASs (2.1) and (2.2), respectively. Fig. 2.4 is the state trajectories, Fig. 2.5 is the time evolution of  $d_i(t)$  and state error  $e_{ij}(t)$ .

## 2. CLUSTER CONSENSUS OF MULTI-AGENT SYSTEMS WITH LINEAR AND NONLINEAR DYNAMICS VIA INTERMITTENT ADAPTIVE PINNING CONTROL

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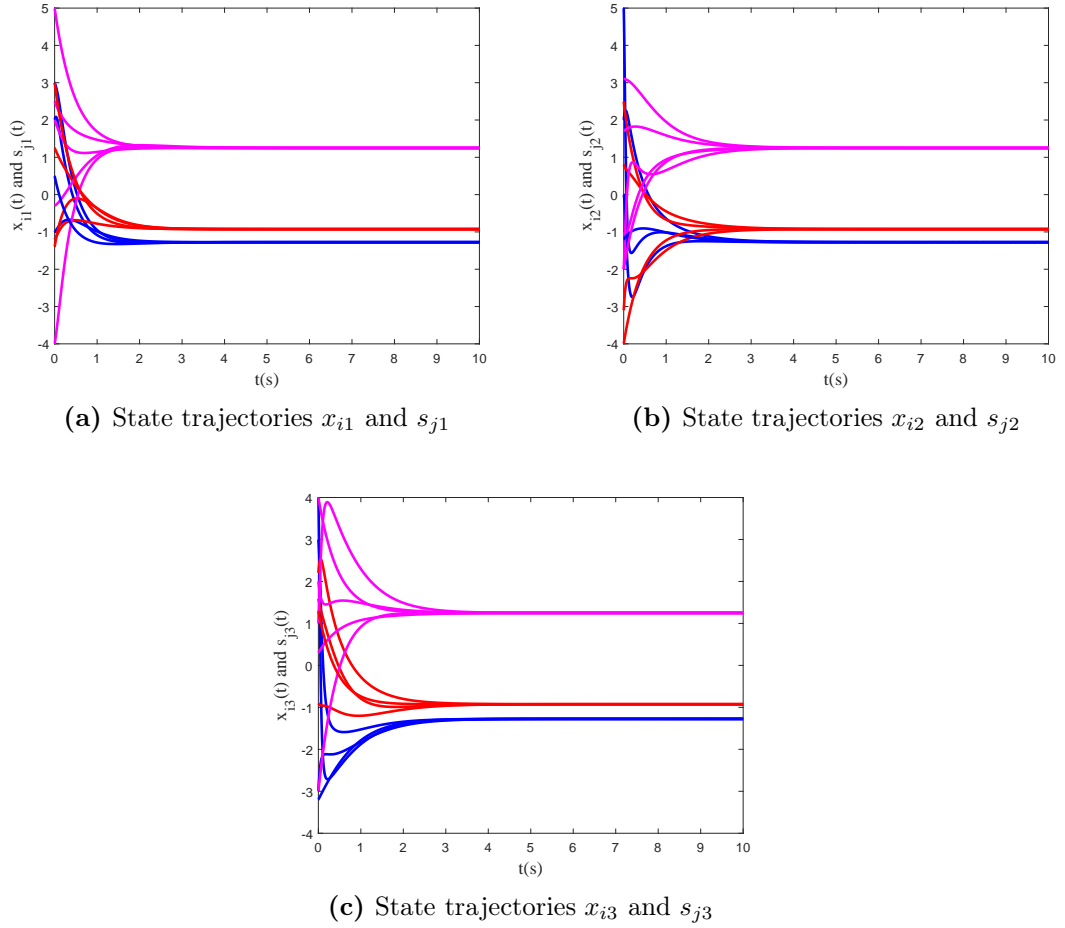


**Fig. 2.2.** State trajectories of all agents for linear case (a).

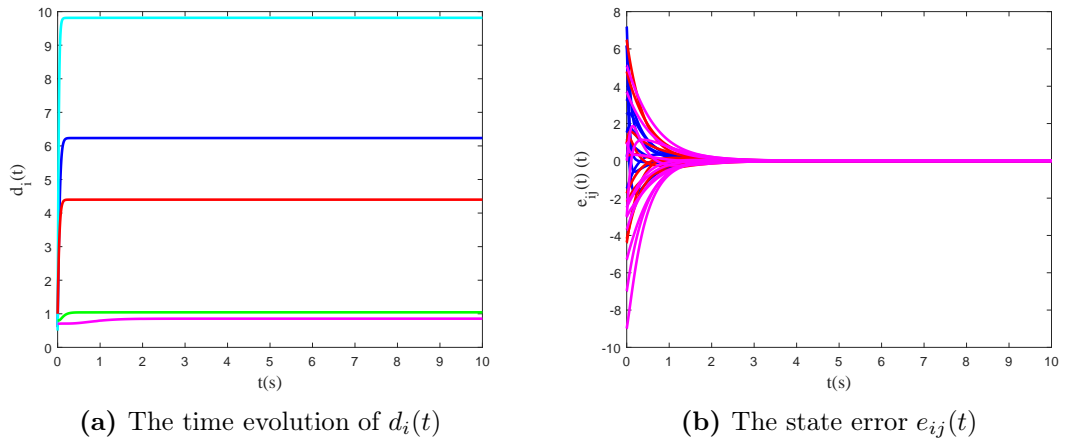


**Fig. 2.3.** The time evolution of  $d_i(t)$  and state error  $e_{ij}(t)$  for linear case (a).

## 2.4 Simulation results



**Fig. 2.4.** State trajectories of all agents for linear case (b).



**Fig. 2.5.** The time evolution of  $d_i(t)$  and state error  $e_{ij}(t)$  for linear case (b).

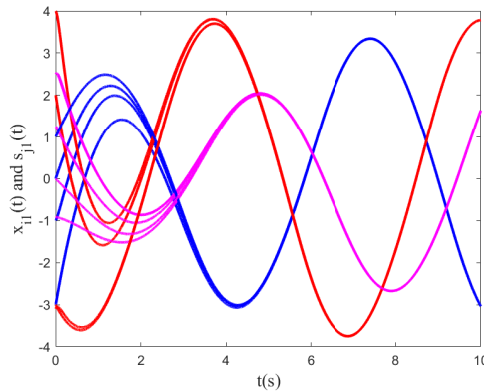


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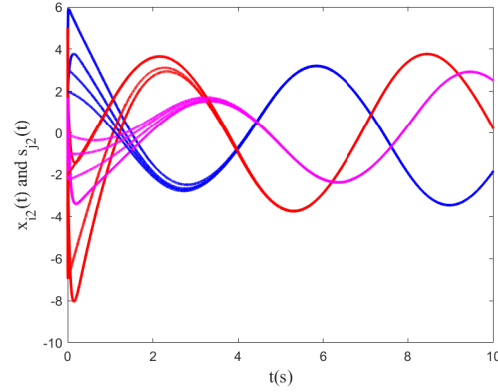
**Example 2:** For the dynamics of nonlinear MASs (2.3) and (2.4), we choose  $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ ,  $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ , and  $f(x)=0.2\sin(x)$  as system matrices and nonlinear function, respectively. Obviously,  $(A, B)$  is stabilizable and nonlinear function  $f(\cdot)$  satisfies the Assumption 2.1. And the positive constants can be chosen as:  $\gamma = 0.2$ ,  $\tau = 1$ ,  $\sigma = 1$ ,  $\sigma_1 = 0.1$ ,  $\epsilon = 0.01$ . Control period  $T$  is given as  $T = 1s$ . By calculating the equations (2.25), (2.26) and (2.21), we can obtain  $P = Q^{-1} = \begin{pmatrix} 20 & 10 \\ 10 & 10 \end{pmatrix}$ ,  $K = (20 \ 20)$ ,  $\mu_2 = 2$ ,  $d_i = 1$ ,  $i = 2, 3, 4, 7, 8$ ,  $\delta = 0.96$ . Fig. 2.6 and Fig. 2.7 are the first component and second component of the state trajectories of the all agents respectively. Fig. 2.8 is the time evolution of  $d_i(t)$ . Fig. 2.9 is the state error  $e_i(t)$ .

From the above figures, we can conclude that the followers in the same cluster achieve leader-following consensus and there is no consensus among the different clusters for both linear system and a class of nonlinear system, that is, the control inputs are effective, the cluster consensus of MASs is realised via intermittent adaptive pinning control.

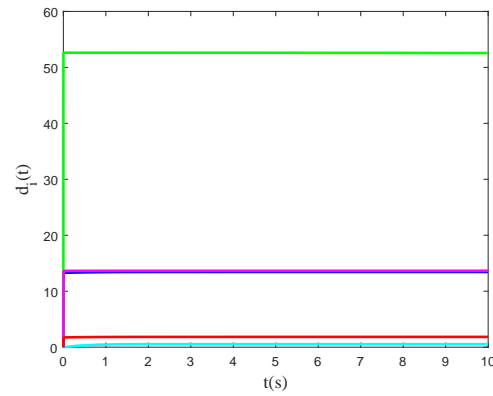


**Fig. 2.6.** The first component  $x_{i1}$  and  $s_{j1}$  of the state trajectories of the all agents, where  $j = 1, 2, 3$ ;  $i = 1, 2, \dots, 10$ .

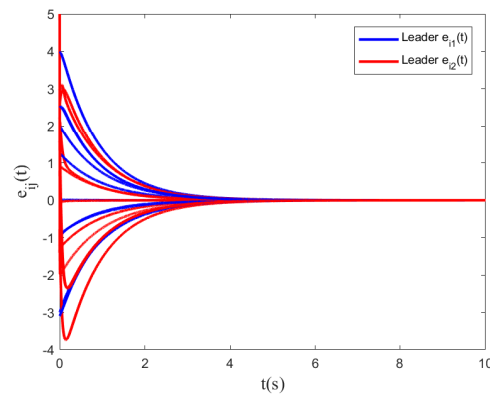
## 2.4 Simulation results



**Fig. 2.7.** The second component  $x_{i2}$  and  $s_{j2}$  of the state trajectories of the all agents, where  $j = 1, 2, 3; i = 1, 2, \dots, 10$ .



**Fig. 2.8.** The time evolution of  $d_i(t)$ , where  $i = 2, 3, 4, 7, 8$ .



**Fig. 2.9.** The state error  $e_{ij}(t)$ , where  $j = 1, 2; i = 1, 2, \dots, 10$ .

## 2.5 Conclusion

This chapter has investigated the cluster consensus of MASs via intermittent adaptive pinning control. Firstly, for the linear MASs and a class of nonlinear MASs, we have employed intermittent adaptive pinning control schemes respectively to accomplish the cluster consensus. Secondly, for the network topology of MASs, we take a weakly connected topology into consideration, which is more universal in practice in contrast to the directed network topology required to be balanced or contain a directed spanning tree. Thirdly, a rigorous proof have been given for the intermittent adaptive pinning control input based on the Lyapunov stability theory and the corresponding sufficient conditions have been derived. Finally, simulation examples are presented to verify the effectiveness of the main results. It is well known that there still exists numerous challenging problems to be studied such as time-delay influence, state observer, which will be studied in our next chapter.

# Chapter 3

## Observer-based intra-cluster lag consensus of multi-agent systems via intermittent adaptive pinning control

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### 3.1 Introduction

In real applications, it is often difficult to avoid the occurrence of time delays due to the finite chemical reaction times, finite switching speed of amplifiers, memory effects, finite signal propagation time in biological networks and so on. Thus, it is extremely important and necessary to investigate cluster consensus of MASs with

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time delays. Recently, the topic of cluster consensus with time delays has been extensively studied in many aspects. In (Ma *et al.*, 2016), the authors proposed the concept of cluster-delay consensus in first-order MASs with nonlinear dynamics via pinning control. In (Da *et al.*, 2018), the authors considered the cluster lag consensus of nonlinear MASs with two types of time-delays. In addition, by using frequency-domain analysis method and matrix theory, the dynamics group consensus problem of heterogenous MASs with time delay was investigated in (Wen *et al.*, 2016b). To the best of our knowledge, the intra-cluster lag consensus of multi-agent systems via intermittent adaptive pinning control has not been addressed before.

Furthermore, due to the states of many actual systems are not always available or measured directly in expensive cost, it is quite necessary to design an observer to estimate the agent's real-time state. Up to now, various issues are addressed by observer-based method. (Rosaldo-Serrano *et al.*, 2019) investigated the formation tracking problem, where suitable Luenberger observers are used for the agents to estimate linear and angular velocities. (Han *et al.*, 2019) studied the containment control issue of general linear MASs with exogenous disturbances, where the disturbances are estimated by disturbance observer-based control scheme. In order to estimate the state of leader and deal with fixed-time stability problem, (Zuo *et al.*, 2019) considered the consensus problem for MASs with high-order integrator dynamics by designing a distributed consensus observer. To our best knowledge, so far the observer-based intra-cluster lag consensus of MASs has not been investigated. Moreover, compared with a single system or integrator MASs, the observer-based intra-cluster lag consensus of MASs with general linear dynamics and nonlinear dynamics is more challenging.

Inspired by the aforementioned these considerations, this chapter investigates the observer-based intra-cluster lag consensus (OBICLC) of general linear MASs and nonlinear MASs via intermittent adaptive pinning control (IAPC), where each cluster has a virtual leader whose state is available to only a small part of followers, and the pinning gains are changeable because of communication constraints. The so-called intra-cluster lag consensus (ICLC) means that the followers in the same cluster can achieve lag consensus asymptotically while the followers in different clusters can achieve different agreements. The main contributions of this work are listed as follows.

- Firstly, considering the relevant full state information of each follower is not always available, the Luenberger observers are designed for general linear and nonlinear MASs respectively to estimate the states of followers. Hence, the control protocols in this chapter only use the observed states information of followers instead of their real-time states information.
- Secondly, similar with above chapter, to realize the intra-cluster lag consensus, the followers in each cluster are classified into three types. Specifically, the followers can only receive information from the followers in their own cluster, the ones receive information from the followers in other clusters, and the others cannot receive information from any followers. Then, the distributed intermittent adaptive pinning control protocols are further designed according to the different categories of followers, which make sure that the followers in the same cluster achieve intra-cluster lag consensus whereas the followers in the different clusters achieve different consensus.
- Thirdly, compared with the most of existing results about cluster consensus where each block unit in the weighted adjacency matrix of network topology is assumed to be a zero-row-sum matrix or an equal-row-sum matrix, which is relative conservative in actual applications, in this chapter a weakly connected topology is still considered and all the coupling weights of network topology are positive, which is more universal in practice.
- Fourthly, different from continuous control protocols in (Qian *et al.*, 2014) and (Huang *et al.*, 2019), the pinning gains in proposed control protocols are designed to be intermittent adaptive and with an exponential convergence rate, which can effectively reduce communication costs, avoid the pinning gains being larger than those needed in practice. Meanwhile, it guarantees the pinning gains quickly converge to steady value.

## 3.2 Problem formulation

To study the OBICLC problem via IAPC strategy for linear MASs and a class of nonlinear MASs, in this section, the model and problem objective are briefly introduced.

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*Case I (Generic Linear Systems):*

Consider a general linear MASs containing  $N$  followers and  $m$  virtual leaders ( $N > m \geq 2$ ). The  $i$ th follower's dynamics is

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) \\ y_i(t) = Cx_i(t) \end{cases}, \quad i \in \{1, 2, \dots, N\} \triangleq \mathcal{V}, \quad (3.1)$$

where  $x_i \in R^n$ ,  $u_i \in R^p$ ,  $y_i \in R^q$  denote the state, control protocol and output measurement of  $i$ th follower, respectively. The constant matrices  $A \in R^{n \times n}$ ,  $B \in R^{n \times p}$  and  $C \in R^{q \times n}$  have appropriate dimensions. Suppose that the matrix triple  $(A, B, C)$  is stabilizable and detectable.

The  $j$ th virtual leader's dynamics is modeled by

$$\begin{cases} \dot{s}_j(t) = As_j(t) \\ y_j(t) = Cs_j(t) \end{cases}, \quad j \in \{1, 2, \dots, m\} \triangleq \Omega, \quad (3.2)$$

where  $s_j(t) \in R^n$ ,  $y_j \in R^q$  denote the state and output measurement, respectively.

*Case II (Nonlinear Oscillators):*

The dynamics of the  $i$ th follower is characterized by

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) + f(x_i(t)) \\ y_i(t) = Cx_i(t) \end{cases}, \quad i \in \mathcal{V}, \quad (3.3)$$

and the virtual leader  $j$ 's dynamics is described by

$$\begin{cases} \dot{s}_j(t) = As_j(t) + f(s_j(t)) \\ y_j(t) = Cs_j(t) \end{cases}, \quad j \in \Omega, \quad (3.4)$$

where the nonlinear function  $f(\cdot)$  holds the following Assumption 3.1.

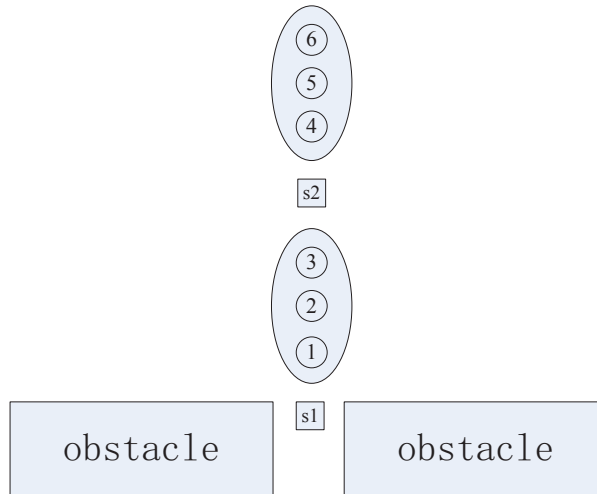
**Assumption 3.1** *It is assumed that the nonlinear function  $f(\cdot)$  satisfies the Lipschitz condition, i.e., for any vectors  $x, y \in R^m$ , there exists a constant  $\gamma > 0$  such that  $\|f(x) - f(y)\| \leq \gamma\|x - y\|$ .*

**Remark 3.2** *The Lipschitz condition assumption is widely used in the analysis of nonlinear systems. With the help of Lipschitz condition assumption, the existence and uniqueness of nonlinear systems' solutions can be guaranteed. Refer to literatures (Wen et al., 2013) and (Ekramian, 2017) for more details.*

## 3.2 Problem formulation

**Definition 3.3** For arbitrary initial states, the MASs (3.1)-(3.2) and (3.3)-(3.4) achieve the ICLC if  $\lim_{t \rightarrow \infty} \|\eta_i(t)\| = 0$  and  $\lim_{t \rightarrow \infty} \|s_l(t) - s_k(t)\| \neq 0$ , where  $l \neq k$ ,  $l, k \in \Omega$ ,  $\eta_i(t) = x_i(t) - s_{\hat{i}}(t - \tau_{\hat{i}})$ ,  $i \in \mathcal{V}$ ,  $\hat{i} \in \Omega$  is the subscript of the leader that the  $i$ th follower will track with,  $\tau_{\hat{i}}$  denotes the time delay in  $\hat{i}$  cluster between the  $i$ th follower and its own leader.

**Remark 3.4** Note that the trajectories of all virtual leaders are determined by system matrix  $A$  owing to  $s_j(t) = s_j(0)e^{At}$ . If  $A$  is a Hurwitz matrix, then the states of all leaders will asymptotically reach zeros, which goes against our objective. Therefore, in this chapter, we assume the matrix  $A$  is not Hurwitz. By giving different initial values for systems (3.2) and (3.4), the states of virtual leaders will be different, that is, for any  $l \neq k$ ,  $\lim_{t \rightarrow \infty} \|s_l(t) - s_k(t)\| \neq 0$ , where  $l, k \in \Omega$ .



**Fig. 3.1.** An example of MASs with time delays.

**Remark 3.5** Note that the existence of time-delays in many practical systems is ubiquitous and inevitable due to communication, calculation, actuation. Therefore, more and more researchers have focused their attention on the consensus problem of MASs with time delays. Lag consensus, including consensus as a special case, which means the followers' states are consistent with the delayed states of leaders. For example, proper time delays between different vehicles in the way can keep the road safe and orderly. Otherwise, congestion often occurs. Taking



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into account lag consensus can mitigate network or traffic congestion problem, in this paper, we consider the intra-cluster lag consensus of MASs, which means that the followers in the same cluster can achieve lag consensus asymptotically while the followers in different clusters can achieve different agreements. In order to clarify the problem more clearly, For example, two isolated groups of vehicles follow the leader and pass across the obstacle, obviously, they cannot pass across the obstacle at the same time (see Fig. 3.1), but we can design a suitable protocol to make the vehicles pass across the obstacle orderly.

## 3.3 Main results

### 3.3.1 Observer-based intra-cluster lag consensus for linear MASs

Before proceeding, some explanations are presented for the reason why we propose the following control protocol. In the former works about MASs (Zhang *et al.*, 2019a), the cluster lag consensus was achieved based on two common assumptions. Firstly, the local relative state information among all agents is available. Secondly, information between the leaders and followers can be transmitted continuously. However, in many circumstances, full state measurements and communication information are not always available due to physical constrain. Therefore, in this chapter, assuming that each follower can receive the relative output information instead of its neighbors' state information, the following observer-based control protocol for the follower  $i$  in (3.1) is proposed,

$$u_i(t) = \begin{cases} K \sum_{j=1}^N a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) - d_i(t)(\hat{x}_i(t) - s_i(t - \tau_i)) + K \sum_{j=1}^N l_{ij}s_j(t - \tau_j), \\ \text{if } i \in \tilde{\mathcal{V}}_i \\ K \sum_{j=1}^N a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) - d_i(t)(\hat{x}_i(t) - s_i(t - \tau_i)), \\ \text{if } i \in \mathcal{V}_i \setminus \tilde{\mathcal{V}}_i \text{ and } \deg(i)_{in} = 0 \\ K \sum_{j=1}^N a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)), \text{ otherwise,} \end{cases} \quad (3.5)$$

where  $\hat{x}_i(t) \in R^n$  is the observer state,  $\hat{i}$  and  $\hat{j}$  are the subscripts of the leaders in Theorem 3.9,  $\tau_{\hat{i}}$  and  $\tau_{\hat{j}}$  are the time delays,  $K$  represents the feedback control gain matrix. The observer state  $\hat{x}_i(t)$  and adaptive law  $d_i(t)$  are defined as follows:

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + F(\hat{y}_i(t) - y_i(t)) \\ \hat{y}_i(t) = C\hat{x}_i(t), \end{cases} \quad (3.6)$$

$$\dot{d}_i(t) = \begin{cases} \sigma e^{\sigma_1 t} (\hat{x}_i(t) - s_i(t - \tau_i))^T \Gamma (\hat{x}_i(t) - s_i(t - \tau_i)), \text{ if } t \in [mT, mT + \delta), \\ 0, \text{ if } t \in [mT + \delta, (m + 1)T), \end{cases} \quad (3.7)$$

where matrices  $F$  and  $\Gamma$  will be determined later in Theorem 3.9,  $\sigma$  and  $\sigma_1$  are positive constants,  $T > 0$  and  $\delta \in (0, T)$  represent the control period and control width respectively,  $[mT, mT + \delta)$  and  $[mT + \delta, (m + 1)T)$  are called as work time interval and rest time interval.

**Remark 3.6** *Note that the followers in each cluster can be divided into three types. Correspondingly, the controller (3.5) is designed according to the special property of each follower. For all the followers, The term  $K \sum_{j=1}^N a_{ij} (\hat{x}_j(t) - \hat{x}_i(t))$  in the control protocol (3.5) is necessary, which reflects the interaction between agent  $i$  and its neighbours. For the followers which can receive information from other clusters or can not receive information from any followers, the term  $d_i(t)(\hat{x}_i(t) - s_i(t - \tau_i))$  is introduced to obtain the information of leaders and the time delay  $\tau_i$  can be different among different clusters. Besides, for the followers that can receive information from other clusters, the term  $K \sum_{j=1}^N l_{ij} s_j(t - \tau_j)$  is used to counteract the influence among clusters. The purpose of controller (3.5) is to make sure that the followers in the same cluster achieve ICLC whereas the followers in the different clusters achieve different consensus.*

**Remark 3.7** *The design of controller (3.5) is partly inspired by (Da et al., 2018; Qian et al., 2014), where the pinning control gain  $d_i(t)$  is a fixed constant all the time or in every time interval. However, due to the change of external environment or in the consideration of reducing control cost, it is more reasonable and effective to design the pinning control gain to be intermittent adaptive. Furthermore, in order to make the pinning control gain quickly converge to steady value,*

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the pinning control gain  $d_i(t)$  possesses an exponential convergence rate. Based on above analysis, the derivative of  $d_i(t)$  is designed to be the form of equation (3.7) if  $i$ th follower is pinned;  $d_i(t) = 0$  otherwise.

**Lemma 3.8** *If  $v_i$  is the follower that belongs to  $\mathcal{V}_i \setminus \tilde{\mathcal{V}}_i$ , then  $\sum_{j=1}^N l_{ij}s_j(t - \tau_j) = 0$ .*

**Proof:** From the definition of  $\tilde{\mathcal{V}}_i$ , if  $v_i \in \mathcal{V}_i \setminus \tilde{\mathcal{V}}_i$ , one has  $l_{ij} = 0$  for  $\forall v_j \notin \mathcal{V}_i$ . Besides, if  $v_j \in \mathcal{V}_i$ , one has  $s_j(t - \tau_j) = s_i(t - \tau_i)$ . Combining the fact  $\sum_{j=1}^N l_{ij} = 0$ , it follows that,

$$\begin{aligned} \sum_{j=1}^N l_{ij}s_j(t - \tau_j) &= \sum_{v_j \in \mathcal{V}_i} l_{ij}s_j(t - \tau_j) + \sum_{v_j \notin \mathcal{V}_i} l_{ij}s_j(t - \tau_j) = \sum_{v_j \in \mathcal{V}_i} l_{ij}s_i(t - \tau_i) \\ &= \left( \sum_{v_j \in \mathcal{V}_i} l_{ij} + \sum_{v_j \notin \mathcal{V}_i} l_{ij} \right) s_i(t - \tau_i) = \sum_{j=1}^N l_{ij}s_i(t - \tau_i) = 0. \end{aligned} \quad (3.8)$$

Combining the definition  $e_i(t) = \hat{x}_i(t) - s_i(t - \tau_i)$  and the formulas (3.2) ~ (3.7), one has,

$$\dot{e}_i(t) = \begin{cases} Ae_i(t) - BK \sum_{j=1}^N l_{ij}e_j(t) - d_i(t)BK e_i(t) + F(\hat{y}_i(t) - y_i(t)), \\ \quad \text{if } i \in \mathcal{V}_i^1 \cup \mathcal{V}_i^2, \\ Ae_i(t) - BK \sum_{j=1}^N l_{ij}e_j(t) + F(\hat{y}_i(t) - y_i(t)), \text{ if } i \in \mathcal{V}_i^3, \end{cases} \quad (3.9)$$

where  $\mathcal{V}_i^1, \mathcal{V}_i^2, \mathcal{V}_i^3$  is defined in Chapter 2, see equation (2.8).

$$d_i(t) = \begin{cases} \sigma e^{\sigma t} e_i(t)^T \Gamma e_i(t), \text{ if } t \in [mT, mT + \delta), \\ 0, \text{ if } t \in [mT + \delta, (m+1)T). \end{cases} \quad (3.10)$$

Let the first  $l$  ( $l = 1, 2, \dots, N$ ) followers be pinned by rearranging the followers' order. Due to the fact that the derivative of  $d_i(t)$  satisfies equation (3.10), that is, for any time  $t$ ,  $d_i(t) > 0$  if the  $i$ -th follower is pinned;  $d_i(t) = 0$  otherwise. When

$v_i \in \mathcal{V}_i^3$ , that means  $d_i(t) = 0$ , then the error system (3.9) can be reformulated as

$$\begin{aligned} \dot{e}_i(t) &= Ae_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t) - d_i(t) B K e_i(t) + F(\hat{y}_i(t) - y_i(t)), \\ i &\in \mathcal{V}_i^1 \cup \mathcal{V}_i^2 \cup \mathcal{V}_i^3. \end{aligned} \quad (3.11)$$

**Theorem 3.9** *Suppose that the matrix triple  $(A, B, C)$  is stabilizable and detectable, there exists a diagonal matrix  $D$ , a positive constant  $\mu_1$  and a positive definite matrix  $P = P^T > 0$  such that the following equations:*

$$\bar{L} + D > 0, \quad (3.12)$$

$$\mu_1 \cdot \lambda_{\min}(2\bar{L} + 2D) > 1, \quad (3.13)$$

$$A^T P + PA - \mu_1 P B B^T P \lambda_{\min}(2\bar{L} + 2D) < 0, \quad (3.14)$$

where  $\bar{L} = \frac{L+L^T}{2}$ ,  $D = \text{diag}\{d_1, \dots, d_l, 0, \dots, 0\}_{N \times N}$  is a diagonal matrix with  $d_1, \dots, d_l > 0, 1 \leq l \leq N$ . Then letting  $K = \mu_1 B^T P$ ,  $\Gamma = 2\mu_1 P B B^T P$ , and designing a matrix  $F$  such that  $A + FC$  is Hurwitz matrix, the OBICLC of MASs (3.1)-(3.2) can be reached by control input (3.5) with the following condition:

$$-\alpha_1 \delta + \beta_1 (T - \delta) < 0, \quad (3.15)$$

where  $\alpha_1 = \min\{\frac{-\lambda_1}{\lambda_{\max}(P)}, \sigma_1\}$ ,  $\lambda_1 = \lambda_{\max}(A^T P + PA - \mu_1 P B B^T P \lambda_{\min}(2\bar{L} + 2D))$ ,  $\beta_1 = \max\{\frac{\lambda_2}{\lambda_{\min}(P)}, \sigma_1\}$ ,  $\lambda_2 = \max\{\lambda_{\max}(I_N \otimes (A^T P + PA) - 2\mu_1 \bar{L} \otimes P B B^T P), \epsilon\}$ ,  $\sigma_1$  and  $\epsilon$  are positive constants.

**Proof:** Theorem 3.9 can be proved by two steps: the first step is to show the effectiveness of state observer, i.e.,  $\lim_{t \rightarrow \infty} \|x_i(t) - \hat{x}_i(t)\| = 0$ , and the second step is to prove the asymptotic stability of error system (3.11), that is,  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ .

**Step 1:** Let  $\xi_i(t) = x_i(t) - \hat{x}_i(t)$ , one has,

$$\begin{aligned} \dot{\xi}_i(t) &= Ax_i(t) + Bu_i(t) - A\hat{x}_i(t) - Bu_i(t) - F(\hat{y}_i(t) - y_i(t)) \\ &= A(x_i(t) - \hat{x}_i(t)) + FC(x_i(t) - \hat{x}_i(t)) \\ &= (A + FC)\xi_i(t). \end{aligned} \quad (3.16)$$

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Based on the condition in Theorem 3.9:  $A + FC$  is Hurwitz matrix, we have

$$\lim_{t \rightarrow \infty} \|\xi_i(t)\| = \lim_{t \rightarrow \infty} \|x_i(t) - \hat{x}_i(t)\| = 0, \text{ which indicates the state observer is feasi-}$$

ble. Furthermore, one has  $F(\hat{y}_i(t) - y_i(t)) = FC\xi_i(t) = 0$  as  $t \rightarrow \infty$ . In equation

(3.11), owing to  $\xi_i(t)$  is decoupled from the error  $e_i(t)$ , the stability of (3.11) is

identical with that of the following system:

$$\dot{e}_i(t) = Ae_i(t) - BK \sum_{j=1}^N l_{ij}e_j(t) - d_i(t)BK e_i(t), v_i \in \mathcal{V}_i^1 \cup \mathcal{V}_i^2 \cup \mathcal{V}_i^3. \quad (3.17)$$

**Step 2:** Construct the Lyapunov function for equation (3.17):

$$V(t) = \sum_{i=1}^N e_i^T(t) P e_i(t) + \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}, \quad (3.18)$$

where matrix  $P$  and the parameters  $\sigma$ ,  $d_i$ ,  $\sigma_1$  are defined in Theorem 3.9. Evi-

dently,  $V(t) \geq 0$ .

(1) When  $t \in [mT, mT + \delta)$ , differentiating  $V(t)$  along (3.17) yields

$$\begin{aligned} \dot{V}(t) = & 2 \sum_{i=1}^N e_i^T(t) P (Ae_i(t) - BK \sum_{j=1}^N l_{ij}e_j(t) - d_i(t)BK e_i(t)) \\ & + \frac{1}{2}(-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + (d_i(t) - d_i) \sum_{i=1}^N e_i^T(t) \Gamma e_i(t). \end{aligned} \quad (3.19)$$

Let  $D(t) = \text{diag}\{d_1(t), \dots, d_l(t), 0, \dots, 0\}_{N \times N}$ ,  $e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$ .

With  $K = \mu_1 B^T P$  and  $\Gamma = 2\mu_1 P B B^T P$ , the derivative of  $V(t)$  can be expressed

as,

$$\begin{aligned}
 \dot{V}(t) &= e^T(t)(I_N \otimes A^T P - L^T \otimes K^T B^T P - D(t) \otimes K^T B^T P)e(t) \\
 &\quad + e^T(t)(I_N \otimes PA - L \otimes PBK - D(t) \otimes PBK)e(t) \\
 &\quad + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + e^T(t)(D(t) - D) \otimes (2\mu_1 PBB^T P)e(t) \\
 &= e^T(t)(I_N \otimes (A^T P + PA) - 2\mu_1 \bar{L} \otimes PBB^T P - 2\mu_1 \cdot D(t) \otimes PBB^T P)e(t) \\
 &\quad + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + e^T(t)(D(t) - D) \otimes (2\mu_1 PBB^T P)e(t) \\
 &= e^T(t)(I_N \otimes (A^T P + PA) - \mu_1(2\bar{L} + 2D) \otimes PBB^T P)e(t) \\
 &\quad + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
 &\leq \lambda_1 e^T(t)e(t) + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
 &\leq \frac{\lambda_1}{\lambda_{\max}(P)} e^T(t)(I_N \otimes P)e(t) + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
 &\leq -\alpha_1 V(t),
 \end{aligned} \tag{3.20}$$

where  $\lambda_1 = \lambda_{\max}(A^T P + PA - \mu_1 PBB^T P \lambda_{\min}(2\bar{L} + 2D))$ ,  $\alpha_1 = \min\{\frac{-\lambda_1}{\lambda_{\max}(P)}, \sigma_1\} > 0$ .

(2) When  $t \in [mT + \delta, (m+1)T)$ , the derivative of (3.18) can be obtained:

$$\begin{aligned}
 \dot{V}(t) &= 2 \sum_{i=1}^N e_i^T(t) P (A e_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t)) + \frac{1}{2} (-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
 &= e^T(t)(I_N \otimes (A^T P + PA) - \mu_1(2\bar{L}) \otimes PBB^T P)e(t) \\
 &\quad - \frac{1}{2} \sigma_1 \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}, \\
 &\leq \lambda_{\max}(I_N \otimes (A^T P + PA) - 2\mu_1 \bar{L} \otimes PBB^T P) e^T(t)e(t) \\
 &\quad + \sigma_1 \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
 &\leq \lambda_2 e^T(t)e(t) + \sigma_1 \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}
 \end{aligned} \tag{3.21}$$

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furtherly,

$$\begin{aligned}\dot{V}(t) &\leq \frac{\lambda_2}{\lambda_{\min}(P)} e^T(t)(I_N \otimes P)e(t) + \sigma_1 \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\ &\leq \beta_1 V(t),\end{aligned}\quad (3.22)$$

where  $\lambda_2 = \max\{\lambda_{\max}(I_N \otimes (A^T P + PA) - 2\mu_1 \bar{L} \otimes PBB^T P), \epsilon\}$ ,  
 $\beta_1 = \max\{\frac{\lambda_2}{\lambda_{\min}(P)}, \sigma_1\} > 0$ .

Accordingly, when  $t \in [0, \delta)$ , one has

$$V(t) \leq V(0)e^{-\alpha_1 t}, \quad (3.23)$$

when  $t \in [\delta, T)$ , one has

$$V(t) \leq V(\delta)e^{\beta_1(t-\delta)} \leq V(0)e^{-\alpha_1 \delta + \beta_1(t-\delta)}. \quad (3.24)$$

By induction, one obtains, when  $t \in [mT, mT + \delta)$ ,

$$V(t) \leq V(0)e^{-\alpha_1(m\delta+t-mT)+\beta_1 m(T-\delta)} \leq V(0)e^{-\alpha_1 m\delta+\beta_1 m(T-\delta)} = V(0)e^{m(-\alpha_1 \delta + \beta_1(T-\delta))}. \quad (3.25)$$

Similarly, when  $t \in [mT + \delta, (m+1)T)$ ,

$$\begin{aligned}V(t) &\leq V(0)e^{-\alpha_1(m+1)\delta+\beta_1(m(T-\delta)+t-T_{2k+1})} \leq V(0)e^{-\alpha_1(m+1)\delta+\beta_1(m(T-\delta)+(T-\delta))} \\ &= V(0)e^{(m+1)(-\alpha_1 \delta + \beta_1(T-\delta))}.\end{aligned}\quad (3.26)$$

According to the equation (3.15) in Theorem 3.9:  $-\alpha_1 \delta + \beta_1(T - \delta) < 0$ , one has  $V(t) \rightarrow 0$  and  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$  as  $m \rightarrow \infty$ . Combining the first step and the second step, one has  $\lim_{t \rightarrow \infty} \|\eta_i(t)\| = \lim_{t \rightarrow \infty} \|x_i(t) - \hat{x}_i(t) + \hat{x}_i(t) - s_i(t - \tau_i)\| = 0$ . It means that ICLC of general linear MASs via IAPC can be reached. This proof is completed.

**Remark 3.10** *The proof of Theorem 3.9 is divided into two parts. The first step is to show that state observer is valid by only using the relevant output information of system (3.1). In the second step, when  $t \in [mT, mT + \delta)$ , the derivative of  $V(t)$  is negative due to the effect of adaptive pinning gain  $d_i(t)$ , which indicates the  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ . However, when  $t \in [mT + \delta, (m+1)T)$ , the derivative of  $V(t)$  cannot be determined to be negative, while it is only obtained that  $\dot{V}(t) \leq \beta_1 V(t)$*

by choosing the appropriate positive constants  $\epsilon, \lambda_2, \beta_1$ . Hence, the equation (3.15) is critical to ensure  $V((m+1)T) \rightarrow 0$  as  $m \rightarrow \infty$ . To make it more clear, define the control rate  $\theta = \frac{\delta}{T-\delta}$ , where  $\delta$  is the control width,  $T-\delta$  is the rest width, then equation (3.15) can be rewritten as:  $\theta > \frac{\beta_1}{\alpha_1}$ . It can be seen that the control rate  $\theta$  plays an essential role in realizing intra-cluster lag consensus. In consideration of this fact, in real applications, we can select the control period  $T$  according to our request.

**Remark 3.11** The condition (3.14) is equivalent to the following Riccati equation condition:  $A^T P + PA - \mu_1 P B B^T P \lambda_{\min}(2\bar{L} + 2D) + Q = 0$  where  $Q$  is any positive definite matrix. The purpose for the form (3.14) is to make a comparison with the following condition (3.34) in Theorem 3.14.

The general criteria are presented in Theorem 3.9 to reach intra-cluster lag consensus of MASs. However, how to construct an appropriate gain matrix  $D$  to satisfy the condition (3.12):  $\bar{L} + D > 0$  is not obvious. Hence, we give Theorem 3.12 to solve this problem.

**Theorem 3.12** The matrix  $\bar{L} + D$  is positive definite with the following prerequisites

$$\lambda_{\max}(-\bar{L}_l) < 0, \quad (3.27)$$

$$d_i > \lambda_{\max}(R - S(-\bar{L}_l)^{-1}S^T). \quad (3.28)$$

**Proof:** In Lemma 1.8, let  $G = -\bar{L}$ , one has  $-\bar{L} - D = \begin{pmatrix} R - \tilde{D} & S \\ S^T & -\bar{L}_l \end{pmatrix}$ , where  $\tilde{D} = \text{diag}\{d_1, \dots, d_l\}$ ,  $-\bar{L}_l$  is obtained by removing the first  $l$  row and column,  $R$  and  $S$  represent the matrices with appropriate dimensions. Thanks to Lemma 1.8 and combine with (3.27), (3.28), we have  $-\bar{L} - D < 0$ , that is,  $\bar{L} + D$  is a positive definite matrix. This completes the proof.

**Remark 3.13** In (Cai et al., 2015; Kang et al., 2018; Tan et al., 2011), in order to achieve cluster synchronization or group consensus, the topology contains negative weight couplings  $a_{ij} < 0$  and is required to satisfy the following assumption:  $\sum_{j \in \mathcal{V}_k} a_{ij} = 0$ , or  $\sum_{j \in \mathcal{V}_k} a_{ij} = \beta, \forall k \neq \hat{i}$ , which is too conservative in practical applications. In order to remove the restrictions, (Qian et al., 2014) and (Da et al., 2018) employ pinning leader-following control scheme to reach cluster consensus of MASs under weakly connected graph. Inspired by the works,



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in this paper, the intermittent adaptive pinning control is designed for general linear MASs and Lipschitz nonlinear MASs under weakly connected graph, in which the Laplace matrix is just supposed to satisfy the general diffusion property, that is,  $\sum_{j=1}^N l_{ij} = 0$ . Moreover, by choosing an appropriate low bound pinning gain  $d_i$ , one can get  $\bar{L} + D > 0$ . For an undirected connected graph,  $\bar{L} + D > 0$  can hold naturally due to the Laplace matrix is a real symmetry. Therefore, the MASs with undirected connected graph can be considered as a special case only by choosing the inter-act followers to be pinned.

#### 3.3.2 Observer-based intra-cluster lag consensus for nonlinear MASs

In this section, the OBICLC of MASs with nonlinear dynamics via intermittent adaptive pinning control is further considered, which is more challenging due to the complexity of the system structure. Before going on, the state observer for the  $i$ th nonlinear follower agent is designed as follows,

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + f(\hat{x}_i(t)) + F(\hat{y}_i(t) - y_i(t)) \\ \hat{y}_i(t) = C\hat{x}_i(t) \end{cases}, \quad i \in \mathcal{V}. \quad (3.29)$$

In this section, we still use the control protocol (3.5) with (3.7), taking the control protocol (3.5) for system (3.3)-(3.4) and combining to  $e_i(t) = \hat{x}_i(t) - s_i(t - \tau_i)$ , one has,

$$\dot{e}_i(t) = \begin{cases} Ae_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t) - d_i(t) BK e_i(t) + F_i(t) + F(\hat{y}_i(t) - y_i(t)), \\ \quad \text{if } i \in \mathcal{V}_i^1 \cup \mathcal{V}_i^2, \\ Ae_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t) + F_i(t) + F(\hat{y}_i(t) - y_i(t)), \text{ if } i \in \mathcal{V}_i^3, \end{cases} \quad (3.30)$$

$$\dot{d}_i(t) = \begin{cases} \sigma e^{\sigma t} e_i(t)^T \Gamma e_i(t), \text{ if } t \in [mT, mT + \delta), \\ 0, \text{ if } t \in [mT + \delta, (m+1)T), \end{cases} \quad (3.31)$$

where  $F_i(t) = f(x_i(t)) - f(s_i(t))$ .

**Theorem 3.14** *Suppose that the matrix triple  $(A, B, C)$  is stabilizable and detectable, there exists a diagonal matrix  $D$ , a positive constant  $\mu_2$  and a positive definite matrix  $P = P^T = Q^{-1} > 0$  such that the following equations:*

$$\bar{L} + D > 0, \quad (3.32)$$

$$\mu_2 \cdot \lambda_{\min}(2\bar{L} + 2D) > \tau, \quad (3.33)$$

$$\begin{pmatrix} AQ + QA^T - \tau BB^T + \gamma^2 I & Q \\ Q^T & -I \end{pmatrix} < 0, \quad (3.34)$$

where  $\bar{L} = \frac{L+L^T}{2}$ ,  $D = \text{diag}\{d_1, \dots, d_l, 0, \dots, 0\}_{N \times N}$  is a diagonal matrix with  $d_1, \dots, d_l > 0, 1 \leq l \leq N$ . Then letting  $K = \mu_2 B^T P$ ,  $\Gamma = 2\mu_2 P B B^T P$ , and designing a matrix  $F$  such that  $A + \gamma I + FC$  is Hurwitz matrix, the OBICLC of MASs (3.3)-(3.4) can be reached by control input (3.5) with the following condition:

$$-\alpha_2 \delta + \beta_2 (T - \delta) < 0, \quad (3.35)$$

where  $\alpha_2 = \min\{-\lambda_3 \cdot \frac{\lambda_{\min}(P^{-1})^2}{\lambda_{\max}(P)}, \sigma_1\}$ ,  $\lambda_3 = \lambda_{\max}(AQ + QA^T + \gamma^2 I + Q^2 - \tau BB^T)$ ,  $\lambda_4 = \max\{\lambda_{\max}(I_N \otimes (A^T P + PA + I + \gamma^2 P^2) - 2\mu_2 \bar{L} \otimes P B B^T P), \epsilon\}$ ,  $\beta_2 = \max\{\frac{\lambda_4}{\lambda_{\min}(P)}, \sigma_1\}$ ,  $\sigma_1$  and  $\epsilon$  are positive constants.

**Proof:** The proof is still divided into two parts, the first part is to show the effectiveness of state observer, i.e.,  $\lim_{t \rightarrow \infty} \|x_i(t) - \hat{x}_i(t)\| = 0$ . And the second part is to prove  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ .

**The first step:** Let  $\xi_i(t) = x_i(t) - \hat{x}_i(t)$ , one has,

$$\begin{aligned} \dot{\xi}_i(t) &= Ax_i(t) + Bu_i(t) - A\hat{x}_i(t) - Bu_i(t) + f(x_i(t)) - f(\hat{x}_i(t)) - F(\hat{y}_i(t) - y_i(t)) \\ &= A(x_i(t) - \hat{x}_i(t)) + FC(x_i(t) - \hat{x}_i(t)) + f(x_i(t)) - f(\hat{x}_i(t)) \\ &= (A + FC)\xi_i(t) + f(x_i(t)) - f(\hat{x}_i(t)). \end{aligned} \quad (3.36)$$

Construct the following Lyapunov function candidate:

$$V(t) = \frac{1}{2} \sum_{i=1}^N \xi_i^T(t) \xi_i(t). \quad (3.37)$$

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Taking the time derivative of (3.37) yields

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^N \xi_i^T(t) ((A + FC)\xi_i(t) + f(x_i(t)) - f(\hat{x}_i(t))) \\ &\leq \sum_{i=1}^N \xi_i^T(t) (A + FC + \gamma I)\xi_i(t).\end{aligned}\quad (3.38)$$

Based on the condition  $A + FC + \gamma I$  is Hurwitz matrix, one has  $\dot{V}(t) < 0$ . Thus,  $\lim_{t \rightarrow \infty} \|\xi_i(t)\| = \lim_{t \rightarrow \infty} \|x_i(t) - \hat{x}_i(t)\| = 0$ , as well  $F(\hat{y}_i(t) - y_i(t)) = FC\xi_i(t) = 0$ . In equation (3.30), owing to  $\xi_i(t)$  is decoupled from the error  $e_i(t)$ , the stability of (3.30) is identical with the stability of the following system:

$$\dot{e}_i(t) = \begin{cases} Ae_i(t) - BK \sum_{j=1}^N l_{ij}e_j(t) - d_i(t)BK e_i(t) + F_i(t), \text{ if } i \in \mathcal{V}_i^1 \cup \mathcal{V}_i^2, \\ Ae_i(t) - BK \sum_{j=1}^N l_{ij}e_j(t) + F_i(t), \text{ if } i \in \mathcal{V}_i^3. \end{cases}\quad (3.39)$$

**The second step:** Construct the Lyapunov function candidate:

$$V(t) = \sum_{i=1}^N e_i^T(t) P e_i(t) + \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}.\quad (3.40)$$

(1) When  $t \in [mT, mT + \delta)$ , the derivative of (3.40) is

$$\begin{aligned}\dot{V}(t) &= 2 \sum_{i=1}^N e_i^T(t) P (Ae_i(t) - BK \sum_{j=1}^N l_{ij}e_j(t) - d_i(t)BK e_i(t) + f(\hat{x}_i(t)) - f(\hat{s}_i(t))) \\ &\quad + \frac{1}{2} (-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + (d_i(t) - d_i) \sum_{i=1}^N e_i^T(t) \Gamma e_i(t).\end{aligned}\quad (3.41)$$

Denote  $F(t) = (f^T(\hat{x}_1(t)) - f^T(s_1(t)), f^T(\hat{x}_2(t)) - f^T(s_2(t)), \dots, f^T(\hat{x}_N(t)) - f^T(s_N(t)))^T$ . Based on the Assumption 3.1, one has

$$2e_i^T(t) P (f(\hat{x}_i(t)) - f(\hat{s}_i(t))) \leq 2\gamma \|P e_i(t)\| \cdot \|e_i(t)\| \leq e_i^T(t) (I + \gamma^2 P^2) e_i(t).\quad (3.42)$$

### 3.3 Main results

Combining (3.42) and the conditions in Theorem 3.14,  $\dot{V}(t)$  can be written as

$$\begin{aligned}
\dot{V}(t) = & e^T(t)(I_N \otimes A^T P - L^T \otimes K^T B^T P - D(t) \otimes K^T B^T P)e(t) \\
& + e^T(t)(I_N \otimes PA - L \otimes PBK - D(t) \otimes PBK)e(t) + 2e^T(t)(I_N \otimes P)F(t) \\
& + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + e^T(t)(D(t) - D) \otimes (2\mu_2 PBB^T P)e(t),
\end{aligned} \tag{3.43}$$

furtherly,

$$\begin{aligned}
\dot{V}(t) \leq & e^T(t)(I_N \otimes (A^T P + PA + I + \gamma^2 P^2) - 2\mu_2(\bar{L}) \otimes PBB^T P \\
& - 2\mu_2 D(t) \otimes PBB^T P)e(t) \\
& + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} + e^T(t)(D(t) - D) \otimes (2\mu_2 PBB^T P)e(t) \\
\leq & e^T(t)(I_N \otimes (A^T P + PA + I + \gamma^2 P^2) - \mu_2(2\bar{L} + 2D) \otimes PBB^T P)e(t) \\
& + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}.
\end{aligned} \tag{3.44}$$

Based on the condition in Theorem 3.14, the matrix  $2\bar{L} + 2D > 0$ , there exist a

constant  $\mu_2$  satisfies  $\mu_2 \lambda_{\min}(2\bar{L} + 2D) > \tau$ . Let  $Q = P^{-1}$ ,  $\eta(t) = (I_N \otimes P)e(t)$ ,

one has,

$$\begin{aligned}
\dot{V}(t) \leq & \eta^T(t)(I_N \otimes (AQ + QA^T + \gamma^2 I + Q^2) - I_N \otimes \tau BB^T)\eta(t) \\
& + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma}.
\end{aligned} \tag{3.45}$$

Furthermore, since the conditions :  $\begin{pmatrix} AQ + QA^T - \tau BB^T + \gamma^2 I & Q \\ Q^T & -I \end{pmatrix} < 0$  in Theorem 3.14, and with the help of Schur complement lemma (Boyd *et al.*, 1994),

one has  $AQ + QA^T + \gamma^2 I + Q^2 - \tau BB^T < 0$ . Denote  $\lambda_3 = \lambda_{\max}(AQ + QA^T +$

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$\gamma^2 I + Q^2 - \tau BB^T$ ), it follows

$$\begin{aligned}
\dot{V}(t) &\leq \lambda_3 \eta^T(t) \eta(t) + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq \lambda_3 \lambda_{\min}(P) e^T(t) (I_N \otimes P) e(t) + (-\sigma_1) \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq -\alpha_2 V(t),
\end{aligned} \tag{3.46}$$

where  $\alpha_2 = \min\{-\lambda_3 \lambda_{\min}(P), \sigma_1\} > 0$ .

(2) When  $t \in [mT + \delta, (m+1)T)$ , the derivative of (3.40) follows

$$\begin{aligned}
\dot{V}(t) &= 2 \sum_{i=1}^N e_i^T(t) P (A e_i(t) - BK \sum_{j=1}^N l_{ij} e_j(t)) + 2 \sum_{i=1}^N e_i^T(t) P (f(x_i(t)) - f(\hat{s}_i(t))) \\
&\quad + \frac{1}{2} (-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq e^T(t) (I_N \otimes (A^T P + PA + I + \gamma^2 P^2) - \mu_2 (L^T + L) \otimes PBB^T P) e(t) \\
&\quad - \frac{1}{2} \sigma_1 \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq \lambda_4 e^T(t) e(t) + \sigma_1 \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq \frac{\lambda_4}{\lambda_{\min}(P)} e^T(t) (I_N \otimes P) e(t) + \sigma_1 \cdot \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(d_i(t) - d_i)^2}{\sigma} \\
&\leq \beta_2 V(t),
\end{aligned} \tag{3.47}$$

where  $\lambda_4 = \max\{\lambda_{\max} I_N \otimes (A^T P + PA + I + \gamma^2 P^2) - \mu_2 (L^T + L) \otimes PBB^T P, \epsilon\} > 0$ ,  $\beta_2 = \max\{\frac{\lambda_4}{\lambda_{\min}(P)}, \sigma_1\} > 0$ .

Similarly, by induction, one obtains,

$$\begin{aligned}
V((m+1)T) &\leq V((mT + \delta) e^{\beta_2(T-\delta)}) \\
&\leq V(mT) e^{-\alpha_2 \delta} \cdot e^{\beta_2(T-\delta)} \\
&= V(mT) e^{-\alpha_2 \delta + \beta_2(T-\delta)} \leq \dots \\
&\leq V(0) e^{(-\alpha_2 \delta + \beta_2(T-\delta))(m+1)}.
\end{aligned} \tag{3.48}$$

Next, by the same operations with (3.23)-(3.26) and combining to the condition

in Theorem 3.14:  $-\alpha_2\delta + \beta_2(T - \delta) < 0$ , one has  $V(t) \rightarrow 0$  and  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$  as  $m \rightarrow \infty$ . Combine the first step and the second step, one has  $\lim_{t \rightarrow \infty} \|\eta_i(t)\| = \lim_{t \rightarrow \infty} \|x_i(t) - \hat{x}_i(t) + \hat{x}_i(t) - s_i(t - \tau_i)\| = 0$ . It means that ICLC of MASs with nonlinear dynamics via IAPC can be reached. This completes the proof.

### 3.4 Simulation results

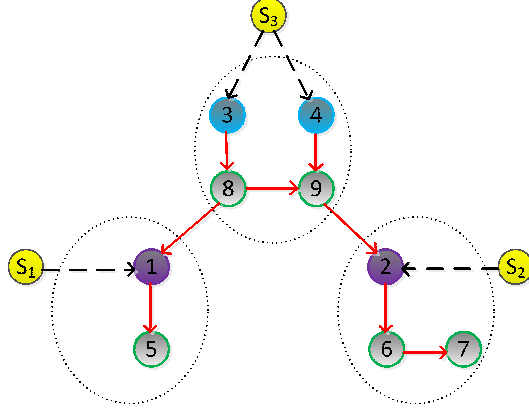
For conveniently, we just present the simulation result of Theorem 3.14. Consider a nonlinear MAS composed of three clusters, where the virtual leaders are denoted as  $S_1$ ,  $S_2$  and  $S_3$ , and the followers are denoted as 1 to 9. Fig.3.2 is the communication topology. In Fig.3.2, followers 1 and 2 can receive information from other clusters, followers 3 and 4 can not receive information from any followers. Therefore, according to the control strategy (3.5), followers 1, 2, 3, 4 should be pinned. In addition, from the graph theory, the matrices  $\mathcal{A}$  and  $L$  can be derived as follows:

$$\mathcal{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{and } L = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}.$$

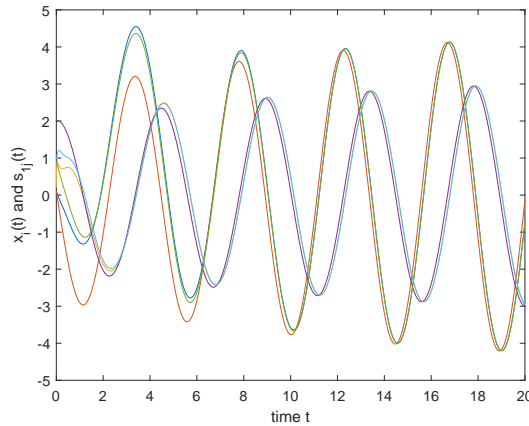
### 3. OBSERVER-BASED INTRA-CLUSTER LAG CONSENSUS OF MULTI-AGENT SYSTEMS VIA INTERMITTENT ADAPTIVE PINNING CONTROL

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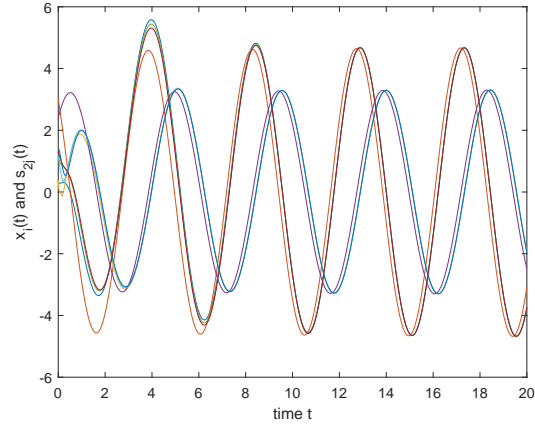
**Fig. 3.2.** The interaction topology of MASs with  $s_1, s_2, s_3$  being leaders and 1 to 9 being followers.

Furthermore, for the dynamics of nonlinear MASs (3.3) and (3.4), we choose  $A = \begin{pmatrix} 0 & -2 \\ 1 & 0 \end{pmatrix}$ ,  $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ,  $C = (1 \ 1)$ , and  $f(x)=0.1\cos(x)$ . Meanwhile, by calculating, the conditions in Theorem 3.14 are shown as follows:  $P = Q^{-1} = \begin{pmatrix} 3.9453 & -3.2213 \\ -3.2213 & 6.5765 \end{pmatrix}$ ,  $F = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$ ,  $K = 2 \cdot \begin{pmatrix} -3.2213 & 6.5765 \end{pmatrix}$ ,  $d_i = 4$ ,  $i = 1, 2, 3, 4$ ,  $\tau = 1.5521$ ,  $\mu_2 = 2$ ,  $\sigma = 1$ ,  $\sigma_1 = 0.1$ ,  $\epsilon = 0.01$ .  $T=1s$ ,  $\delta = 0.96$ . Specially, the time delays are given as:  $\tau_1 = 0.1, \tau_2 = 0.15, \tau_3 = 0.12$ . Fig. 3.3, Fig. 3.4 and Fig. 3.5 are state trajectories of leader and followers in each cluster respectively. Fig. 3.6 is the time evolution of  $d_i(t)$ . Fig. 3.7 is the observer error  $\xi_i(t)$ . From the above figures, we can conclude that the ICLC of MASs is realised via IAPC.

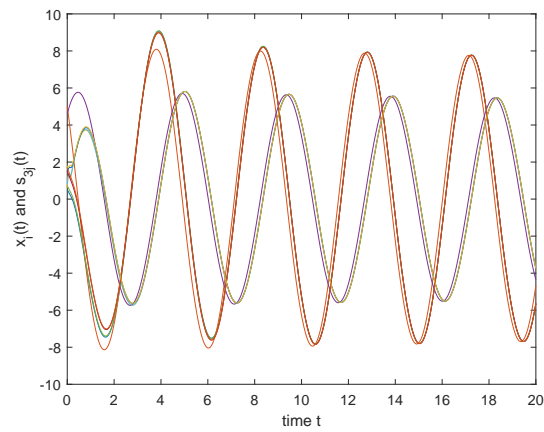


**Fig. 3.3.** The state trajectories of leader  $s_1$  and followers in first cluster, where  $j = 1, 2; i = 1, 5$ .

### 3.4 Simulation results



**Fig. 3.4.** The state trajectories of leader  $s_2$  and followers in second cluster, where  $j = 1, 2; i = 2, 6, 7$ .

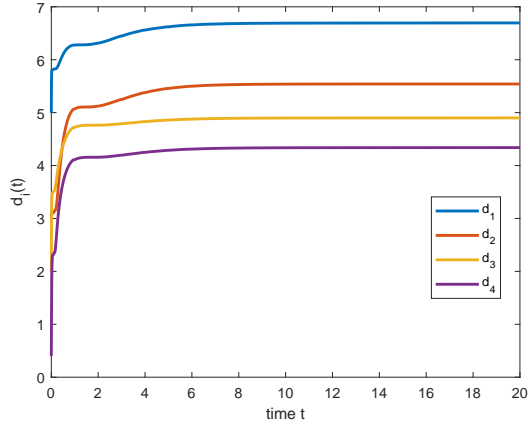


**Fig. 3.5.** The state trajectories of leader  $s_3$  and followers in third cluster, where  $j = 1, 2; i = 3, 4, 8, 9$ .

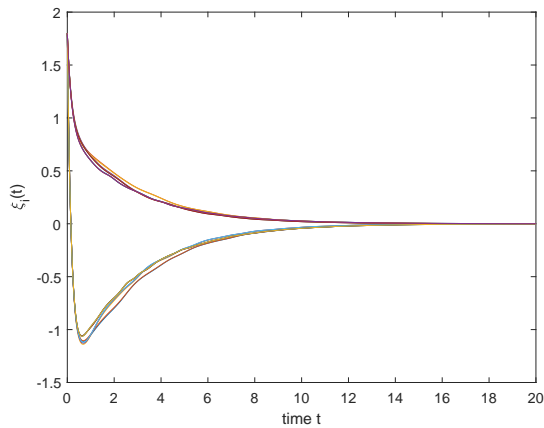


### 3. OBSERVER-BASED INTRA-CLUSTER LAG CONSENSUS OF MULTI-AGENT SYSTEMS VIA INTERMITTENT ADAPTIVE PINNING CONTROL

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**Fig. 3.6.** The time evolution of  $d_i(t)$ , where  $i = 1, 2, 3, 4$ .



**Fig. 3.7.** The observer error  $\xi_i(t)$ , where  $i = 1, 2, \dots, 9$ .

## 3.5 Conclusion

In this chapter, the OBICLC problem for general linear MASs and nonlinear MASs under a weakly connected graph via aperiodically IAPC has been investigated. Considering the relevant full state information of each follower is not always available, a class of observers has been designed to estimate the states of followers. Then a class of observer-based IAPC protocols is proposed according to the difference that the agents receive information source. Moreover, the pinning gains have been designed to be intermittent adaptive and with an exponential convergence rate, which will effectively reduce communication costs,

### 3.5 Conclusion

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avoid the pinning gains being larger than those needed in practice and guarantees the pinning gains quickly converge to steady value. Finally, rigorous proofs and numerical simulations have been provided to guarantee the correctness of the theoretical results. Future works will focus on furtherly reducing communication among agents by employing intermittent event-triggered control mechanism.

**3. OBSERVER-BASED INTRA-CLUSTER LAG CONSENSUS OF  
MULTI-AGENT SYSTEMS VIA INTERMITTENT ADAPTIVE  
PINNING CONTROL**

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# Chapter 4

## Aperiodically intermittent adaptive event-triggered control for linear multi-agent systems

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### 4.1 Introduction

Note that the intermittent controller still requires to be updated continuously when it is executed in the work time interval, which is a waste of energy and resource. To mitigate this issue, the ETC scheme presents a flexible and efficient strategy for the communication data transmission and control update. The combination of intermittent ETC strategy has been designed in (Hu & Cao, 2017; Lei *et al.*, 2017; Liu *et al.*, 2021; Yang *et al.*, 2020). In (Liu *et al.*, 2021), the

#### 4. APERIODICALLY INTERMITTENT ADAPTIVE EVENT-TRIGGERED CONTROL FOR LINEAR MULTI-AGENT SYSTEMS

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authors designed intermittent event-triggered method to solve leader-following mean square consensus for stochastic MASs, where the topology was assumed to be directed. In (Hu & Cao, 2017), the authors considered tracking consensus problem for a class of nonlinear MASs, where both centralised and decentralised event-triggered strategies were introduced into intermittent control. Based on the error state and three indices of threshold-value, check-period, and control-free index, (Yang *et al.*, 2020) constructed event-triggered aperiodic intermittent control to deal with synchronization problem for chaotic systems. In (Lei *et al.*, 2017), the authors studied leader-following output consensus problem for heterogeneous MASs, and the result is further extended to the formation control problem. However, as far as the authors know, the tracking consensus of MASs by aperiodically intermittent adaptive ETC has not been fully investigated yet.

Inspired by the above discussion, this chapter investigates the tracking consensus of general linear MASs via aperiodically intermittent adaptive ETC approach. The aperiodically intermittent adaptive event-triggered control inherits the respective advantages of aperiodically intermittent control strategy, ETC strategy and adaptive control strategy, which improves communication efficiency, reduces control update frequency and is closer to the practical situations. The major advances of this chapter lie in following aspects: Firstly, to reach tracking leader-following consensus and save more control resources, a distributed aperiodically intermittent ETC protocol is devised, in which the transmission channels among agents only open if the local event-trigger condition is satisfied in predefined time intervals. Secondly, in order to get rid of continuous inter-agent communication for monitoring the triggering condition, a more general triggering mechanism is presented, in which discrete-time combinational measurement is adopted instead of using continuous-time tracking error directly. Thirdly, to overcome the unexpected large feedback gains in real applications and appropriately tune the feedback gains, the aperiodically intermittent adaptive event-triggered controller is further devised. With the aid of the matrix theory, stability of switching systems and Lyapunov function, some sufficient criteria are deduced. Moreover, the analyses of excluding the Zeno behavior are included by showing explicit positive lower bounds between any two consecutive triggered events.

## 4.2 Problem formulation

For a general linear MASs consisting of  $N$  followers and one leader, the dynamics of each follower is modeled as

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), i \in \mathcal{V}, \quad (4.1)$$

in which  $x_i \in R^n$  and  $u_i \in R^m$  represent the state and controller, respectively.  $A \in R^{n \times n}$ ,  $B \in R^{n \times m}$  are constant matrices. The virtual leader's dynamics is:

$$\dot{s}_0(t) = As_0(t), \quad (4.2)$$

in which  $s_0(t) \in R^n$  is the state of virtual leader.

**Assumption 4.1** *The pair  $(A, B)$  is stabilizable.*

**Assumption 4.2** *The topology  $\bar{G}$  for the leader  $s_0$  and all followers is fixed, undirected and there exists at least one follower can receive the information from the leader  $s_0$ .*

**Lemma 4.3** (*Menard et al., 2020*) *If Assumption 4.2 holds, there exists a positive definite diagonal matrix  $E$  satisfying  $E\tilde{L} + \tilde{L}^T E > 0$  where  $E = \text{diag}\{\xi_1, \dots, \xi_N\}$ . Furthermore, the equality  $E\tilde{L} + \tilde{L}^T E > \gamma E$  exists with a positive constant  $\gamma$ .*

**Definition 4.4** *For given MASs (4.1)-(4.2), the tracking consensus is achieved if for any initial values, there exists  $\lim_{t \rightarrow \infty} \|\delta_i(t)\| = 0$ , where  $\delta_i(t) = x_i(t) - s_0(t)$ ,  $i \in \mathcal{V}$ .*

## 4.3 Main results

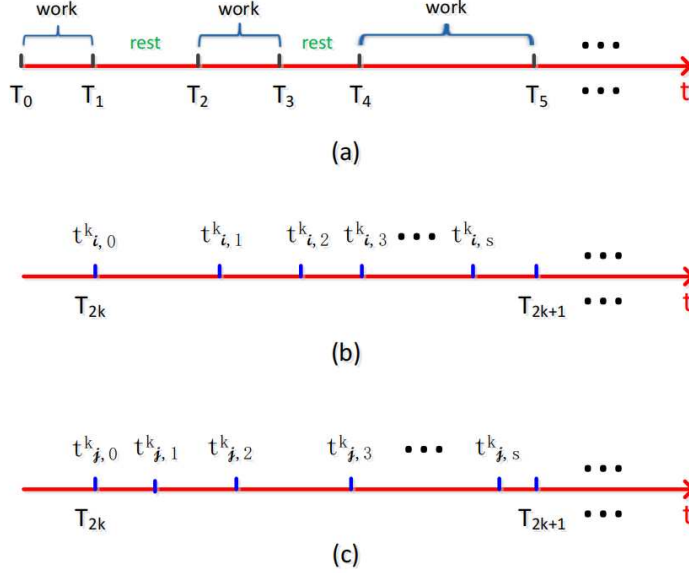
### 4.3.1 Aperiodically intermittent ETC for linear MASs

To address tracking consensus, the controller for  $i$ th follower is proposed

$$u_i(t) = \begin{cases} cKq_i(t_{i,s}^k), & t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1}), \\ 0, & t \in [T_{2k+1}, T_{2k+2}), \end{cases} \quad (4.3)$$

#### 4. APERIODICALLY INTERMITTENT ADAPTIVE EVENT-TRIGGERED CONTROL FOR LINEAR MULTI-AGENT SYSTEMS

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**Fig. 4.1.** (a) sketch map of time division; (b) event-triggered instants of  $i$ th follower; (c) event-triggered instants of  $j$ th follower.

$$q_i(t_{i,s}^k) = \sum_{j=1}^N a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + d_i(\hat{s}_0(t) - \hat{x}_i(t)), \quad (4.4)$$

in which  $K$  represents the feedback gain matrix, which will be determined thereafter in Theorem 4.5. Constant  $c$  is positive.  $t_{i,s}^k$  is the  $s$ th triggering instant of follower  $i$  in control time  $[T_{2k}, T_{2k+1})$ .  $\hat{x}_i(t) = e^{A(t-t_{i,s}^k)}x_i(t_{i,s}^k)$ ,  $\hat{x}_j(t) = e^{A(t-t_{j,s'}^k)}x_j(t_{j,s'}^k)$ ,  $\hat{s}_0(t) = s_0(t)$ . In addition, the width of rest time and work time, in the form, are denoted as  $\Delta_k = T_{k+1} - T_k, k = 0, 1, \dots$ , commonly (see Fig.4.1).

For the  $i$ th follower, the measurement error can be depicted by

$$e_i(t) = \hat{x}_i(t) - x_i(t). \quad (4.5)$$

Substituting the control input (4.3) into (4.1), and combining the definition

of measurement error  $e_i(t)$ , the tracking error system is addressed below:

$$\dot{\delta}_i(t) = \begin{cases} A\delta_i(t) + cBK \sum_{j=1}^N a_{ij}(e_j(t) - e_i(t) + \delta_j(t) - \delta_i(t)) - d_i(e_i(t) + \delta_i(t)), \\ t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1}), \\ A\delta_i(t), \quad t \in [T_{2k+1}, T_{2k+2}). \end{cases} \quad (4.6)$$

Further, let  $x(t) = (x_1^T(t), x_2^T(t), \dots, x_N^T(t))^T$ ,  $\delta(t) = (\delta_1^T(t), \delta_2^T(t), \dots, \delta_N^T(t))^T$ ,  $e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$ , then the tracking error can be reformulated in a compact version, i.e.,

$$\dot{\delta}(t) = \begin{cases} (I_N \otimes A)\delta(t) - c(\tilde{L} \otimes BK)(e(t) + \delta(t)), t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1}), \\ (I_N \otimes A)\delta(t), t \in [T_{2k+1}, T_{2k+2}). \end{cases} \quad (4.7)$$

**Theorem 4.5** *The proposed intermittent ETC strategy (4.3) can guarantee the achievement of tracking consensus for linear MASs (4.1) and (4.2) with the conditions below:*

(i) Under Assumptions 4.1 and 4.2, the feedback matrix  $K$  is obtained by the following two steps:

1) Find a solution  $P > 0$  satisfying the algebraic Riccati equation below:

$$PA + A^T P - c\gamma PBB^T P + k_1 I_n = 0, \quad (4.8)$$

where  $c > 0$  is a parameter in (4.3),  $\gamma > 0$  is denoted in Lemma 4.3,  $k_1$  is a positive constant.

2) Compute the feedback matrix  $K$  as follows:  $K = B^T P$ .

(ii) The triggering function for  $i$ th follower is devised as:

$$h_i(t) = e_i^T(t)e_i(t) - \eta_i \delta_i^T(t)\delta_i(t) \quad (4.9)$$

where  $\eta_i < \frac{2p}{c}(\lambda_1 - \frac{pc}{2}\lambda_2^2\lambda_3^2)$ ,  $\lambda_1 = k_1\lambda_{\min}(E) > 0$ ,  $\lambda_2 = \lambda_{\max}(E\tilde{L} + \tilde{L}^T E) > 0$ ,  $\lambda_3 = \lambda_{\max}(PBB^T P) > 0$ , and  $p$  satisfies the following condition:  $\lambda_2^2\lambda_3^2pc < 2\lambda_1$ ;

(iii) There exist positive scalars  $\tau_1$  and  $\tau_2$  such that  $\begin{cases} \Delta_{2k} \geq \tau_1 \\ \Delta_{2k+1} \leq \tau_2 \end{cases}$  and  $\frac{\tau_1}{\tau_2} > \frac{\beta_1}{\alpha_1}$ ,

where  $\Delta_{2k}$  is the the width of work time,  $\Delta_{2k+1}$  is the width of rest time,  $\alpha_1 = \frac{\lambda_1 - \frac{pc}{2}\lambda_2^2\lambda_3^2 - \frac{c}{2p}\bar{\eta}}{\lambda_{\max}(E \otimes P)} > 0$ ,  $\bar{\eta} = \max\{\eta_1, \eta_2, \dots, \eta_N\}$ ,  $\beta_1 = \frac{s_1}{\lambda_{\min}(E \otimes P)}$ ,  $s_1 = \max\{\lambda_{\max}(PA +$



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$A^T P), \varepsilon_0\}$ ,  $\varepsilon_0$  is a small positive constant.

**Proof:**

Choose the Lyapunov function candidate below:

$$V(t) = \delta^T(t)(E \otimes P)\delta(t), \quad (4.10)$$

in which matrices  $E$  and  $P$  are shown as mentioned above.

(1) When  $t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1})$ , for any given  $s$ , the derivative of (4.10) obeys

$$\begin{aligned} \dot{V}(t) &= \delta^T(t)(E \otimes P)[(I_N \otimes A)\delta(t) - c(\tilde{L} \otimes BK)(\delta(t) + e(t))] \\ &\quad + [\delta^T(t)(I_N \otimes A^T) - c(\delta^T(t) + e^T(t))(\tilde{L}^T \otimes K^T B^T)](E \otimes P)\delta(t) \\ &= \delta^T(t)(E \otimes PA + E \otimes A^T P)\delta(t) - c\delta^T(t)(E\tilde{L} \otimes PBK)\delta(t) \\ &\quad - c\delta^T(t)(\tilde{L}^T E \otimes K^T B^T P)\delta(t) - c\delta^T(t)(E\tilde{L} \otimes PBK)e(t) \\ &\quad - ce^T(t)(\tilde{L}^T E \otimes K^T B^T P)\delta(t) \\ &= \delta^T(t)(E \otimes (PA + A^T P))\delta(t) - c\delta^T(t)[(E\tilde{L} + \tilde{L}^T E) \otimes PBK]\delta(t) \\ &\quad - c\delta^T(t)[(E\tilde{L} + \tilde{L}^T E) \otimes PBK]e(t), \end{aligned} \quad (4.11)$$

in which  $K = B^T P$ . Upon Lemma 4.3, a positive constant  $\gamma$  is existent to satisfy:

$$E\tilde{L} + \tilde{L}^T E > \gamma E, \quad (4.12)$$

then, one gets

$$\begin{aligned} \dot{V}(t) &\leq \delta^T(t)(E \otimes (PA + A^T P - c\gamma PBB^T P))\delta(t) \\ &\quad - c\delta^T(t)((E\tilde{L} + \tilde{L}^T E) \otimes PBB^T P)e(t) \\ &\leq -\lambda_1 \delta^T(t)\delta(t) - c\delta^T(t)((E\tilde{L} + \tilde{L}^T E) \otimes PBB^T P)e(t), \end{aligned} \quad (4.13)$$

where  $\lambda_1 = k_1 \lambda_{\min}(E)$ ,  $k_1 > 0$ . Furthermore, noting Young's inequality  $x^T y \leq \frac{p}{2}x^2 + \frac{1}{2p}y^2$ , the last term in (4.13) leads to

$$\begin{aligned} & -\delta^T(t)[(E\tilde{L} + \tilde{L}^T E) \otimes PBB^T P]e(t) \\ & \leq \frac{p}{2}\delta^T(t)[(E\tilde{L} + \tilde{L}^T E)^2 \otimes (PBB^T P)^2]\delta(t) + \frac{1}{2p}e^T(t)e(t) \\ & \leq \frac{p}{2}\lambda_2^2 \lambda_3^2 \delta^T(t)\delta(t) + \frac{1}{2p}e^T(t)e(t), \end{aligned} \quad (4.14)$$

where  $\lambda_2 = \lambda_{\max}(E\tilde{L} + \tilde{L}^T E) > 0$ ,  $\lambda_3 = \lambda_{\max}(PBB^T P) > 0$ . Substituting (4.14) into (4.13), one arrives at

$$\dot{V}(t) \leq \sum_{j=1}^N (-\lambda_1 + \frac{pc}{2} \lambda_2^2 \lambda_3^2) \delta_i^T(t) \delta_i(t) + \sum_{j=1}^N \frac{c}{2p} e_i^T(t) e_i(t), \quad (4.15)$$

where  $p$  satisfies the following condition:  $\lambda_2^2 \lambda_3^2 pc < 2\lambda_1$ . In review of the triggering condition:  $h_i(t) = e_i^T(t) e_i(t) - \eta_i \delta_i^T(t) \delta_i(t) \leq 0$ . Consider the fact:  $\lambda_{\min}(E \otimes P) \sum_{j=1}^N \delta_i^T(t) \delta_i(t) \leq V(t) \leq \lambda_{\max}(E \otimes P) \sum_{j=1}^N \delta_i^T(t) \delta_i(t)$ . Let  $\bar{\eta} = \max\{\eta_1, \eta_2, \dots, \eta_N\}$ , one has

$$\begin{aligned} \dot{V}(t) &\leq \sum_{j=1}^N (-\lambda_1 + \frac{pc}{2} \lambda_2^2 \lambda_3^2 + \frac{c}{2p} \eta_i) \delta_i^T(t) \delta_i(t) \\ &\leq (-\lambda_1 + \frac{pc}{2} \lambda_2^2 \lambda_3^2 + \frac{c}{2p} \bar{\eta}) \sum_{j=1}^N \delta_i^T(t) \delta_i(t) \\ &\leq (-\lambda_1 + \frac{pc}{2} \lambda_2^2 \lambda_3^2 + \frac{c}{2p} \bar{\eta}) \frac{V(t)}{\lambda_{\max}(E \otimes P)} \\ &\leq -\alpha_1 V(t), \end{aligned} \quad (4.16)$$

where  $\alpha_1 = \frac{\lambda_1 - \frac{pc}{2} \lambda_2^2 \lambda_3^2 - \frac{c}{2p} \bar{\eta}}{\lambda_{\max}(E \otimes P)} > 0$  and  $\bar{\eta} < \frac{2p}{c} (\lambda_1 - \frac{pc}{2} \lambda_2^2 \lambda_3^2)$ .

(2) When  $t \in [T_{2k+1}, T_{2k+2})$ , the derivative of (4.10) follows

$$\begin{aligned} \dot{V}(t) &= \delta^T(t) (E \otimes (PA + A^T P)) \delta(t) \leq \lambda_{\max}(E \otimes (PA + A^T P)) \delta^T(t) \delta(t) \\ &\leq \frac{\varsigma_1}{\lambda_{\min}(E \otimes P)} \delta^T(t) (E \otimes P) \delta(t) \leq \beta_1 V(t), \end{aligned} \quad (4.17)$$

where  $\beta_1 = \frac{\varsigma_1}{\lambda_{\min}(E \otimes P)}$ ,  $\varsigma_1 = \max\{\lambda_{\max}(E \otimes (PA + A^T P)), \varepsilon_0\}$ ,  $\varepsilon_0$  is a small positive constant.

Especially, when  $t \in [T_0, T_1)$ , one has

$$V(t) \leq V(T_0) e^{-\alpha_1(t-T_0)}, \quad (4.18)$$

when  $t \in [T_1, T_2)$ , one has

$$V(t) \leq V(T_0) e^{-\alpha_1 \Delta_0 + \beta_1(t-T_1)}, \quad (4.19)$$

where  $\Delta_0 = T_1 - T_0$ .

By induction and combining the conditions in Theorem 4.5, one obtains, when

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$t \in [T_{2k}, T_{2k+1})$ ,

$$\begin{aligned}
V(t) &\leq V(0)e^{-\alpha_1(\Delta_0+\Delta_2+\dots+\Delta_{2k-2}+t-T_{2k})+\beta_1(\Delta_1+\Delta_3+\dots+\Delta_{2k-1})} \\
&\leq V(0)e^{-\alpha_1(k\tau_1+t-T_{2k})+\beta_1k\tau_2} \\
&\leq V(0)e^{-\alpha_1k\tau_1+\beta_1k\tau_2} \\
&= V(0)e^{k(-\alpha_1\tau_1+\beta_1\tau_2)}.
\end{aligned} \tag{4.20}$$

Similarly, when  $t \in [T_{2k+1}, T_{2k+2})$ ,

$$\begin{aligned}
V(t) &\leq V(0)e^{-\alpha_1(\Delta_0+\Delta_2+\dots+\Delta_{2k})+\beta_1(\Delta_1+\Delta_3+\dots+\Delta_{2k-1}+t-T_{2k+1})} \\
&\leq V(0)e^{-\alpha_1(k+1)\tau_1+\beta_1(k\tau_2+t-T_{2k+1})} \\
&\leq V(0)e^{-\alpha_1(k+1)\tau_1+\beta_1(k\tau_2+\tau_2)} \\
&= V(0)e^{(k+1)(-\alpha_1\tau_1+\beta_1\tau_2)},
\end{aligned} \tag{4.21}$$

where  $\Delta_k = T_{k+1} - T_k$ . Then when  $t \in [T_k, T_{k+1}]$ , for the sake of guaranteeing  $V(t) \rightarrow 0$  when  $k \rightarrow \infty$ , let  $-\alpha_1\tau_1 + \beta_1\tau_2 < 0$ , one has  $\frac{\tau_1}{\tau_2} > \frac{\beta_1}{\alpha_1}$ . Thus,  $\lim_{t \rightarrow \infty} \|\delta_i(t)\| = 0$ . This indicates that the tracking consensus for linear MASs (4.1) and (4.2) is reached under intermittent ETC strategy.

Next, we will show Zeno behavior is completely excluded, which means the intermittent ETC is feasible. Let  $z(t) = \frac{\|e(t)\|}{\|\delta(t)\|}$ , since the event will be triggered only when  $t \in [T_{2k}, T_{2k+1})$ , the time derivative of  $z(t)$  in each  $t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1})$  is

$$\dot{z}(t) = \frac{d}{dt} \left( \frac{\|e(t)\|}{\|\delta(t)\|} \right) = \frac{\|\dot{e}(t)\| \|\delta(t)\| - \|e(t)\| \|\dot{\delta}(t)\|}{\|\delta(t)\|^2} \leq \frac{\|\dot{e}(t)\|}{\|\delta(t)\|} + z(t) \frac{\|\dot{\delta}(t)\|}{\|\delta(t)\|}. \tag{4.22}$$

According to the first equation of (4.7),

$$\begin{aligned}
\|\dot{\delta}(t)\| &= \|(I_N \otimes A)\delta(t) - c(\tilde{L} \otimes BK)(e(t) + \delta(t))\| \\
&\leq (\|I_N \otimes A\| + \|c(\tilde{L} \otimes BK)\|) \|\delta(t)\| + \|c(\tilde{L} \otimes BK)\| \|e(t)\| \\
&\leq r_1 \|\delta(t)\| + r_2 \|e(t)\|,
\end{aligned} \tag{4.23}$$

where  $r_1 = \|I_N \otimes A\| + \|c(\tilde{L} \otimes BK)\|$ ,  $r_2 = \|c(\tilde{L} \otimes BK)\|$ , obviously,

$r_1 > r_2 > 0$ . In addition, since  $e_i(t) = \hat{x}_i(t) - x_i(t)$ ,  $e_i(t)$  can be rewritten as:

$$\begin{aligned} \|\dot{e}(t)\| &= \|(I_N \otimes A)e(t) + c(\tilde{L} \otimes BK)(e(t) + \delta(t))\| \\ &\leq (\|(I_N \otimes A)\| + \|c(\tilde{L} \otimes BK)\|) \|e(t)\| + \|c(\tilde{L} \otimes BK)\| \|\delta(t)\| \\ &\leq r_1 \|e(t)\| + r_2 \|\delta(t)\|. \end{aligned} \tag{4.24}$$

Combining (4.23) and (4.24), it yields that:

$$\begin{aligned} \dot{z}(t) &\leq r_1 z(t) + r_2 + z(t)(r_1 + r_2 z(t)) = r_2 z^2(t) + 2r_1 z(t) + r_2 \\ &\leq r_1 z^2(t) + 2r_1 z(t) + r_1 = r_1 (z(t) + 1)^2. \end{aligned} \tag{4.25}$$

Let  $s(t, s_0)$  be the solution of

$$\dot{s}(t) = r_1 (1 + s(t))^2, s(0, s_0) = s_0, \tag{4.26}$$

then,  $z(t)$  satisfies the bound

$$z(t) \leq s(t, s_0), \tag{4.27}$$

the solution of  $\dot{s}(t) = r_1 (1 + s(t))^2$  is given as

$$s(\varepsilon, 0) = \frac{r_1 \varepsilon}{1 - r_1 \varepsilon}. \tag{4.28}$$

Based on the event-triggered condition (4.9), one has  $s(\varepsilon, 0) = \sqrt{\eta_i}$ . Consequently,  $\sqrt{\eta_i} = \frac{r_1 \varepsilon}{1 - r_1 \varepsilon} \Rightarrow \varepsilon = \frac{\sqrt{\eta_i}}{(1 + \eta_i)r_1} > 0$ . Thus, we obtain the lower bounded  $\varepsilon$ , which means the Zeno behavior can be avoided. This finalizes the derivation.

Note that the triggering function (4.9) still needs to be detected continuously for each agent and the leader, which cannot save energy and resource efficiently. Besides, in reality, not whole followers can directly obtain the state information of leader. In order to mitigate these disadvantages, in the following, we present a more general triggering mechanism with combinational measurement (4.30), which not only avoids the requirement for continuous monitoring but also decreases the amount of triggering instants (Zhou *et al.*, 2015).

**Theorem 4.6** *The proposed intermittent ETC strategy (4.3) can guarantee the achievement of tracking consensus for linear MASs (4.1) and (4.2) if the following conditions hold:*

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(i) Under Assumptions 4.1 and 4.2, the feedback matrix  $K$  is obtained by the following two steps:

1) Find a solution  $P > 0$  such that algebraic Riccati equation below:

$$PA + A^T P - \frac{1}{2}c\gamma P B B^T P + k_2 I_n = 0, \quad (4.29)$$

where  $c > 0$  is a parameter in (4.3),  $\gamma > 0$  is defined in Lemma 4.3, constant  $k_2$  is positive.

2) Compute the feedback matrix  $K$  as follows:  $K = B^T P$ .

(ii) The distributed triggering function for  $i$ th follower is devised as:

$$\tilde{h}_i(t) = e_i^T(t)e_i(t) - \tilde{\eta}_i q_i^T(t_{i,s}^k)q_i(t_{i,s}^k), \quad (4.30)$$

where  $\tilde{\eta}_i < \frac{\tilde{\lambda}_1 - \frac{\omega_{12}}{2\tilde{p}}}{\lambda_{2\tilde{p}} + \tilde{\lambda}_3 - \omega_{12}}$ ,  $\omega_1 = k_2 \lambda_{\min}(E)$ . Let  $\omega_1 = \omega_{11} + \omega_{12}$ , and  $\omega_{11}, \omega_{12} > 0$ ,  $\tilde{\lambda}_1 = \omega_{12} \lambda_{\min}((\tilde{L}^{-T} \tilde{L}^{-1}) \otimes I_n) > 0$ ,  $\tilde{\lambda}_2 = \omega_{12} \lambda_{\max}((\tilde{L}^{-T} + \tilde{L}^{-1})^2 \otimes I_n) > 0$ ,  $\tilde{\lambda}_3 = \frac{1}{2}c \lambda_{\max}((E \tilde{L}^T + \tilde{L} E) \otimes P B B^T P) > 0$ , and  $\tilde{p}$  satisfies the following condition:  $\tilde{\lambda}_1 - \frac{\omega_{12}}{2\tilde{p}} > 0$ ;

(iii) There exist positive constants  $\tilde{\tau}_1$  and  $\tilde{\tau}_2$  such that  $\begin{cases} \Delta_{2k} \geq \tilde{\tau}_1 \\ \Delta_{2k+1} \leq \tilde{\tau}_2 \end{cases}$  and  $\frac{\tilde{\tau}_1}{\tilde{\tau}_2} > \frac{\beta_1}{\alpha_2}$ ,

where  $\Delta_{2k}$  is still the the width of work time,  $\Delta_{2k+1}$  is still the the width of rest time,  $\alpha_2 = \frac{\omega_{11}}{\lambda_{\max}(E \otimes P)} > 0$ ,  $\beta_1 = \frac{\varsigma_1}{\lambda_{\min}(E \otimes P)}$ ,  $\varsigma_1 = \max\{\lambda_{\max}(PA + A^T P), \varepsilon_0\}$ ,  $\varepsilon_0$  is a small positive constant.

**Proof:**

Design the Lyapunov function for (4.7):

$$V(t) = \delta^T(t)(E \otimes P)\delta(t), \quad (4.31)$$

where matrix  $P$  is defined as mentioned above in Theorem 4.6.

(1) When  $t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1})$ , the derivative of (4.31) obeys

$$\begin{aligned} \dot{V}(t) &= \delta^T(t)(E \otimes P)[(I_N \otimes A)\delta(t) - c(\tilde{L} \otimes BK)(\delta(t) + e(t))] \\ &\quad + [\delta^T(t)(I_N \otimes A^T) - c(\delta^T(t) + e^T(t))(\tilde{L}^T \otimes K^T B^T)](E \otimes P)\delta(t) \\ &= \delta^T(t)(E \otimes (PA + A^T P))\delta(t) - c\delta^T(t)[(E \tilde{L} + \tilde{L}^T E) \otimes PBK]\delta(t) \\ &\quad - c\delta^T(t)[(E \tilde{L} + \tilde{L}^T E) \otimes PBK]e(t), \end{aligned} \quad (4.32)$$

where  $K = B^T P$ . Based on the Young's inequality, it admits

$$\begin{aligned} & -c\delta^T(t)[(E\tilde{L} + \tilde{L}^T E) \otimes PBB^T P]e(t) \\ & \leq \frac{1}{2}c\delta^T(t)[(E\tilde{L} + \tilde{L}^T E) \otimes PBB^T P]\delta(t) + \frac{1}{2}ce^T(t)[(E\tilde{L} + \tilde{L}^T E) \otimes PBB^T P]e(t). \end{aligned} \quad (4.33)$$

By the aid of Lemma 4.3 and (4.29), one obtains,

$$\begin{aligned} \dot{V}(t) & \leq \delta^T(t)[E \otimes (PA + A^T P - \frac{1}{2}c\gamma PBB^T P)]\delta(t) \\ & \quad + \frac{1}{2}ce^T(t)[(E\tilde{L} + \tilde{L}^T E) \otimes PBB^T P]e(t) \\ & = -\omega_1\delta^T(t)\delta(t) + \frac{1}{2}ce^T(t)[(E\tilde{L} + \tilde{L}^T E) \otimes PBB^T P]e(t), \end{aligned} \quad (4.34)$$

where  $\omega_1 = k_2\lambda_{\min}(E)$ . Let  $\omega_1 = \omega_{11} + \omega_{12}$ , where  $\omega_{11}, \omega_{12} > 0$ ,  $H = \tilde{L}^{-1} \otimes I_n$ ,  $q(t_s^k) = (q_1^T(t_{1,s}^k), q_2^T(t_{2,s}^k), \dots, q_N^T(t_{N,s}^k))^T$ . Combining with the definition of  $e_i(t)$ ,  $\delta_i(t)$  and  $q_i(t_{i,s}^k)$ , one obtains

$$q(t_s^k) = -(\tilde{L} \otimes I_n)(\delta(t) + e(t)), \quad (4.35)$$

$$\delta(t) = -Hq(t_s^k) - e(t). \quad (4.36)$$

Then, we have

$$\begin{aligned} \delta^T(t)\delta(t) & = (Hq(t_s^k) + e(t))^T(Hq(t_s^k) + e(t)) \\ & = q^T(t_s^k)(\tilde{L}^{-T}\tilde{L}^{-1} \otimes I_n)q(t_s^k) + e^T(t)[(\tilde{L}^{-T} + \tilde{L}^{-1}) \otimes I_n]q(t_s^k) + e^T(t)e(t). \end{aligned} \quad (4.37)$$

Furthermore, the derivative of  $V(t)$  obeys

$$\begin{aligned} \dot{V}(t) & \leq -\omega_{11}\delta^T(t)\delta(t) - \omega_{12}q^T(t_s^k)(\tilde{L}^{-T}\tilde{L}^{-1} \otimes I_n)q(t_s^k) \\ & \quad - \omega_{12}e^T(t)[(\tilde{L}^{-T} + \tilde{L}^{-1}) \otimes I_n]q(t_s^k) - \omega_{12}e^T(t)e(t) \\ & \quad + \frac{1}{2}ce^T(t)[(E\tilde{L} + \tilde{L}^T E) \otimes PBB^T P]e(t). \end{aligned} \quad (4.38)$$

Based on the Young inequality, one has

$$-e^T(t)[(\tilde{L}^{-T} + \tilde{L}^{-1}) \otimes I_n]q(t_s^k) \leq \frac{\tilde{p}}{2}e^T(t)[(\tilde{L}^{-T} + \tilde{L}^{-1})^2 \otimes I_n]e(t) + \frac{1}{2\tilde{p}}q^T(t_s^k)q(t_s^k). \quad (4.39)$$

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Let

$$\begin{aligned}\tilde{\lambda}_1 &= \omega_{12} \lambda_{\min}((\tilde{L}^{-T} \tilde{L}^{-1}) \otimes I_n), \\ \tilde{\lambda}_2 &= \omega_{12} \lambda_{\max}((\tilde{L}^{-T} + \tilde{L}^{-1})^2 \otimes I_n), \\ \tilde{\lambda}_3 &= \frac{1}{2} c \lambda_{\max}((E \tilde{L}^T + \tilde{L} E) \otimes P B B^T P),\end{aligned}$$

one has

$$\begin{aligned}\dot{V}(t) &\leq -\omega_{11} \delta^T(t) \delta(t) - \tilde{\lambda}_1 q^T(t_s^k) q(t_s^k) + \tilde{\lambda}_2 \frac{\tilde{p}}{2} e^T(t) e(t) \\ &\quad + \frac{\omega_{12}}{2\tilde{p}} q^T(t_s^k) q(t_s^k) - \omega_{12} e^T(t) e(t) + \tilde{\lambda}_3 e^T(t) e(t) \\ &\leq -\omega_{11} \delta^T(t) \delta(t) + \left(\frac{\omega_{12}}{2\tilde{p}} - \tilde{\lambda}_1\right) q^T(t_s^k) q(t_s^k) + \left(\tilde{\lambda}_2 \frac{\tilde{p}}{2} + \tilde{\lambda}_3 - \omega_{12}\right) e^T(t) e(t).\end{aligned}\tag{4.40}$$

Enforcing the triggering mechanism  $\tilde{h}_i(t) < 0$ , i.e.,  $e_i^T(t) e_i(t) \leq \tilde{\eta}_i q_i^T(t_{i,s}^k) q_i(t_{i,s}^k)$ . Furthermore, since the condition  $\tilde{\lambda}_1 - \frac{\omega_{12}}{2\tilde{p}} - \tilde{\eta}_i (\tilde{\lambda}_2 \frac{\tilde{p}}{2} + \tilde{\lambda}_3 - \omega_{12}) > 0$ , it follows

$$\begin{aligned}\dot{V}(t) &\leq -\omega_{11} \delta^T(t) \delta(t) - \sum_{j=1}^N \left(\tilde{\lambda}_1 - \frac{\omega_{12}}{2\tilde{p}} - \tilde{\eta}_j (\tilde{\lambda}_2 \frac{\tilde{p}}{2} + \tilde{\lambda}_3 - \omega_{12})\right) q_j^T(t_{j,s}^k) q_j(t_{j,s}^k) \\ &\leq -\omega_{11} \frac{V(t)}{\lambda_{\max}(E \otimes P)} \leq -\alpha_2 V(t),\end{aligned}\tag{4.41}$$

where  $\alpha_2 = \frac{\omega_{11}}{\lambda_{\max}(E \otimes P)} > 0$ .

(2) When  $t \in [T_{2k+1}, T_{2k+2})$ , the proof is same as (4.17), we omit it here.

In what follows, by the analogous operations as (4.18)-(4.21), under the triggering function (4.30), the tracking consensus for MASs (4.1) and (4.2) is realized via intermittent ETC protocol (4.3).

Next, we will show there is no agent will exhibit Zeno behavior, which means the intermittent event-triggered control is feasible. Let  $\tilde{z}(t) = \frac{\|e(t)\|}{\|q(t_s^k)\|}$ ,  $\tilde{\eta} = \max\{\tilde{\eta}_1, \tilde{\eta}_2, \dots, \tilde{\eta}_N\}$ . Since the event will be triggered only when  $t \in [T_{2k}, T_{2k+1})$ , the time derivative of  $\tilde{z}(t)$  in each  $t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1})$  satisfies

$$\dot{\tilde{z}}(t) = \frac{d}{dt} \left( \frac{\|e(t)\|}{\|q(t_s^k)\|} \right) = \frac{\|\dot{e}(t)\| \| \|q(t_s^k)\| - \|e(t)\| \|\dot{q}(t_s^k)\|}{\|q(t_s^k)\|^2} \leq \frac{\|\dot{e}(t)\|}{\|q(t_s^k)\|} + \tilde{z}(t) \frac{\|\dot{q}(t_s^k)\|}{\|q(t_s^k)\|}.\tag{4.42}$$

Since  $\dot{e}(t) = (I_N \otimes A)e(t) - c(I_N \otimes BK)q(t_s^k)$  and  $\dot{q}(t_s^k) = (I_N \otimes A)q(t_s^k)$ , it follows

that:

$$\dot{\tilde{z}}(t) \leq \| I_N \otimes A \| \tilde{z}(t) - \| c(I_N \otimes BK \| + \tilde{z}(t) \| I_N \otimes A \| \leq \tilde{r}_1 \tilde{z}(t) + \tilde{r}_2, \quad (4.43)$$

where  $\tilde{r}_1 = 2 \| I_N \otimes A \|$ ,  $\tilde{r}_2 = \| c(I_N \otimes BK \|$ . Thus, the evolution of  $\tilde{z}(t)$  satisfies  $\tilde{z}(t) \leq \tilde{s}(t, \tilde{s}_0)$ , in which  $\tilde{s}(t, \tilde{s}_0)$  is the solution to

$$\dot{\tilde{s}}(t) = \tilde{r}_1 \tilde{s}(t) + \tilde{r}_2, \tilde{s}(0, \tilde{s}_0) = \tilde{s}_0. \quad (4.44)$$

The solution of  $\dot{\tilde{s}}(t) = \tilde{r}_1 \tilde{s}(t) + \tilde{r}_2$  is given as

$$\tilde{s}(\tilde{\varepsilon}, 0) = \frac{1}{\tilde{r}_1} \ln\left(\frac{\tilde{r}_1 \tilde{\eta}}{\tilde{r}_2} + 1\right). \quad (4.45)$$

Based on the event-triggered condition (4.30), the evolution time of  $\tilde{z}(t)$  from 0 to  $\tilde{\eta}$  is no less than  $\tilde{\varepsilon} = \frac{1}{\tilde{r}_1} \ln\left(\frac{\tilde{r}_1 \tilde{\eta}}{\tilde{r}_2} + 1\right)$ . Since  $\tilde{r}_1, \tilde{r}_2, \tilde{\eta} > 0$ , one has  $\tilde{\varepsilon} > 0$ , which shows any two time intervals have explicit positive lower bounds. The proof is completed.

### 4.3.2 Aperiodically intermittent adaptive ETC for linear MASs

Note that the coupling weight  $c$  in control input (4.3) is fixed and needs to be given in advance. In this section, we introduce the adaptive control into intermittent event-triggered control to adjust the coupling strength. Compared with the constant coupling strength, the time-varying one is more flexible and practical. Based on the above discussions, the following aperiodically adaptive intermittent ETC strategy is presented to handle the tracking consensus problem, which is characterised by

$$u_i(t) = \begin{cases} c_i(t) K \tilde{q}_i(t_{i,s}^k), & t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1}), \\ 0, & t \in [T_{2k+1}, T_{2k+2}), \end{cases} \quad (4.46)$$

$$\dot{\tilde{q}}_i(t) = \sum_{j=1}^N a_{ij} (x_j(t) - x_i(t)) + d_i (s_0(t) - x_i(t)), \quad (4.47)$$



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$$\dot{c}_i(t) = \text{proj}(c_i(t)) = \begin{cases} 0, & \text{if } c_i(t) \geq \bar{c}_i, \\ \sigma e^{\sigma_1 t} q_i^T(t) \Gamma q_i(t), & \text{if } c_i(t) < \bar{c}_i, \end{cases} \quad (4.48)$$

where  $K, \Gamma$  still represent the gain matrices.  $\bar{c}_i$  is a positive constant. In addition, the width of rest time and work time, in the form, are denoted as  $\Delta_k = T_{k+1} - T_k, k = 0, 1, \dots$ , commonly.

The combination measurement error for each follower  $i$  is represented as:

$$\tilde{e}_i(t) = \tilde{q}_i(t_{i,s}^k) - \tilde{q}_i(t). \quad (4.49)$$

Substituting the control input (4.46) into (4.1), and using the definition of combination measurement error  $e_i(t)$ , the tracking error system for  $i$ th follower is obtained as

$$\dot{\delta}_i(t) = \begin{cases} A\delta_i(t) + c_i(t)BK\tilde{q}_i(t_{i,s}^k), & t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1}), \\ A\delta_i(t), & t \in [T_{2k+1}, T_{2k+2}). \end{cases} \quad (4.50)$$

Let  $C(t) = \text{diag}\{c_1(t), c_2(t), \dots, c_N(t)\}$ ,  $C = \text{diag}\{\bar{c}_1, \bar{c}_2, \dots, \bar{c}_N\}$ ,  $x(t) = (x_1^T(t), x_2^T(t), \dots, x_N^T(t))^T$ ,  $\delta(t) = (\delta_1^T(t), \delta_2^T(t), \dots, \delta_N^T(t))^T$ ,  $\tilde{e}(t) = (\tilde{e}_1^T(t), \tilde{e}_2^T(t), \dots, \tilde{e}_N^T(t))^T$ ,  $\tilde{q}(t) = (\tilde{q}_1^T(t), \tilde{q}_2^T(t), \dots, \tilde{q}_N^T(t))^T$ , then the tracking error can be reformulated in a compact version, i.e.,

$$\dot{\delta}(t) = \begin{cases} (I_N \otimes A)\delta(t) + (C(t) \otimes BK)(\tilde{e}(t) + \tilde{q}(t)), & t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1}), \\ (I_N \otimes A)\delta(t), & t \in [T_{2k+1}, T_{2k+2}). \end{cases} \quad (4.51)$$

**Theorem 4.7** *The proposed intermittent adaptive ETC strategy (4.46)~(4.48) can guarantee the achievement of tracking consensus for linear MASs (4.1) and (4.2) with the conditions below:*

(i) *Suppose Assumption 4.1 holds and suppose there exists a diagonal matrix  $E > 0$  satisfying:  $E\tilde{L}^{-1} + (\tilde{L}^{-1})^T E > I_N$ , the feedback matrix  $K$  is obtained by the following two steps:*

1) *Find a solution  $P > 0$  satisfying the algebraic Riccati equation below:*

$$PA + A^T P - \frac{\tilde{c}}{\lambda_{\max}(\Phi_1)} P B B^T P + \frac{\tilde{c}}{\lambda_{\max}(\Phi_1)} I_n = 0, \quad (4.52)$$

where  $\tilde{c} = \min\{\bar{c}_1, \bar{c}_2, \dots, \bar{c}_N\}$ ,  $\Phi_1 = (\tilde{L}^{-1})^T E \tilde{L}^{-1}$ ,  $\bar{c}_i$  is defined in (4.48).

### 4.3 Main results

2) Compute the feedback matrix  $K$  as follows:  $K = B^T P$ .

(i) The distributed triggering mechanism for  $i$ th follower is devised as:

$$\hat{h}_i(t) = \|\tilde{e}_i(t)\| - \hat{\eta}_i \|\tilde{q}_i(t)\|, \quad (4.53)$$

where  $\hat{\eta}_i < \sqrt{2a(\tilde{c} - \frac{a}{2}\hat{c}\hat{\lambda})}$ ,  $\tilde{c} = \min\{\bar{c}_1, \bar{c}_2, \dots, \bar{c}_N\}$ ,  $\hat{c} = \max\{\bar{c}_1, \bar{c}_2, \dots, \bar{c}_N\}$ ,  $\hat{\lambda} = \lambda_{\max}\{\Phi_2^2 \otimes (PBB^T P)^2\}$ ,  $\Phi_2 = E\tilde{L}^{-1} + (\tilde{L}^{-1})^T E$ ,  $a$  is a positive constant satisfying:  $\tilde{c} - \frac{a}{2}\hat{c}\hat{\lambda} > 0$ ;

(ii) There exist positive constants  $\hat{\tau}_1$  and  $\hat{\tau}_2$  such that  $\begin{cases} \Delta_{2k} \geq \hat{\tau}_1 \\ \Delta_{2k+1} \leq \hat{\tau}_2 \end{cases}$  and  $\frac{\hat{\tau}_1}{\hat{\tau}_2} > \frac{\beta}{\alpha_3}$ , where  $\Delta_{2k}$  is the the width of work time,  $\Delta_{2k+1}$  is the the width of rest time,  $\alpha_3 = -\frac{h_1}{h_2} > 0$ ,  $h_1 = -\tilde{c} + \frac{a}{2}\hat{c}^2\hat{\lambda} + \frac{\hat{\eta}^2}{2a} < 0$ ,  $\hat{\eta} = \max\{\hat{\eta}_1, \hat{\eta}_2, \dots, \hat{\eta}_N\}$ ,  $h_2 = \lambda_{\max}((\tilde{L}^{-1})^T E\tilde{L}^{-1} \otimes P) > 0$ ,  $\beta = \frac{\varsigma_1}{\lambda_{\min}(E \otimes P)}$ ,  $\varsigma_1 = \max\{\lambda_{\max}(E \otimes (PA + A^T P)), \varepsilon_0\}$ ,  $\varepsilon_0$  is a small positive constant.

**Proof:** Construct the Lyapunov function for (4.51):

$$V(t) = \delta^T(t)(E \otimes P)\delta(t) + \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma}, \quad (4.54)$$

where matrix  $E$  and  $P$  are defined as mentioned above.

(1) When  $t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1})$ , take the derivative of (5.15) as

$$\begin{aligned} \dot{V}(t) &= \delta^T(t)(E \otimes (PA + A^T P))\delta(t) + 2\delta^T(t)[EC(t) \otimes PBK]\tilde{e}(t) \\ &\quad + 2\delta^T(t)[EC(t) \otimes PBK]\tilde{q}(t) + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\ &\quad + \sum_{i=1}^N (c_i(t) - \bar{c}_i) \tilde{q}_i^T(t) \Gamma \tilde{q}_i(t). \end{aligned} \quad (4.55)$$

Based on (4.47), we arrive at

$$\tilde{q}(t) = -(\tilde{L} \otimes I_n)\delta(t). \quad (4.56)$$

Furtherly, one obtains  $\delta(t) = -(\tilde{L}^{-1} \otimes I_n)\tilde{q}(t)$ . Combining  $K = B^T P$  and  $\Gamma =$

#### 4. APERIODICALLY INTERMITTENT ADAPTIVE EVENT-TRIGGERED CONTROL FOR LINEAR MULTI-AGENT SYSTEMS

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$PBB^T P$ , then

$$\begin{aligned}
\dot{V}(t) &= \tilde{q}^T(t)((\tilde{L}^{-1})^T E \tilde{L}^{-1} \otimes (PA + A^T P))\tilde{q}(t) \\
&\quad - \tilde{q}^T(t)[(E \tilde{L}^{-1} + (\tilde{L}^{-1})^T E)C(t) \otimes PBB^T P]\tilde{q}(t) \\
&\quad - \tilde{q}^T(t)[(E \tilde{L}^{-1} + (\tilde{L}^{-1})^T E)C(t) \otimes PBB^T P]\tilde{e}(t) \\
&\quad + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} + \sum_{i=1}^N (c_i(t) - \bar{c}_i) \tilde{q}_i^T(t) \Gamma \tilde{q}_i(t).
\end{aligned} \tag{4.57}$$

Let  $\Phi_1 = (\tilde{L}^{-1})^T E \tilde{L}^{-1}$ ,  $\Phi_2 = E \tilde{L}^{-1} + (\tilde{L}^{-1})^T E$ . Since the fact  $E \tilde{L}^{-1} + (\tilde{L}^{-1})^T E > I_N$ , the following equation holds

$$\begin{aligned}
\dot{V}(t) &= \tilde{q}^T(t)(\Phi_1 \otimes (PA + A^T P))\tilde{q}(t) - \tilde{q}^T(t)[\Phi_2 C(t) \otimes PBB^T P]\tilde{q}(t) \\
&\quad - \tilde{q}^T(t)[\Phi_2 C(t) \otimes PBB^T P]\tilde{e}(t) + \tilde{q}^T(t)[(C(t) - C) \otimes PBB^T P]\tilde{q}(t) \\
&\quad + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma}.
\end{aligned} \tag{4.58}$$

Based on the conditions:  $\lambda_{\min}(\Phi_2) \geq 1$  and in light of (4.52), one has

$$\begin{aligned}
\dot{V}(t) &\leq \lambda_{\max}(\Phi_1) \tilde{q}^T(t)(I_N \otimes (PA + A^T P - \frac{\tilde{c}}{\lambda_{\max}(\Phi_1)} PBB^T P))\tilde{q}(t) \\
&\quad - \tilde{q}^T(t)[\Phi_2 C(t) \otimes PBB^T P]\tilde{e}(t) + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\
&\leq -\tilde{c} \tilde{q}^T(t)\tilde{q}(t) - \tilde{q}^T(t)[\Phi_2 C(t) \otimes PBB^T P]\tilde{e}(t) + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma}.
\end{aligned} \tag{4.59}$$

Further, by the inequality  $x^T y \leq \frac{a}{2} x^T x + \frac{1}{2a} y^T y$ , one has

$$\begin{aligned}
&\quad - \tilde{q}^T(t)[\Phi_2 C(t) \otimes PBB^T P]\tilde{e}(t) \\
&\leq \frac{a}{2} \tilde{q}^T(t)[(C(t))^2 \Phi_2^2 \otimes (PBB^T P)^2]\tilde{q}(t) + \frac{1}{2a} \tilde{e}^T(t)\tilde{e}(t).
\end{aligned} \tag{4.60}$$

Since  $(C(t))^2 \Phi_2^2 \otimes (PBB^T P)^2 \leq \hat{c}^2 \Phi_2^2 \otimes (PBB^T P)^2$ , one has

$$\dot{V}(t) \leq -\tilde{c} \tilde{q}^T(t)\tilde{q}(t) + \frac{a}{2} \hat{c}^2 \tilde{q}^T(t)\tilde{q}(t) + \frac{1}{2a} \tilde{e}^T(t)\tilde{e}(t) + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma}, \tag{4.61}$$

where  $\hat{c} = \max\{\bar{c}_1, \bar{c}_2, \dots, \bar{c}_N\}$ ,  $\hat{\lambda} = \lambda_{\max}\{\Phi_2^2 \otimes (PBB^T P)^2\}$ . Considering the

event-triggered function:  $\|\tilde{e}_i(t)\| \leq \hat{\eta}_i \|\tilde{q}_i(t)\|$ , it admits

$$\begin{aligned} \dot{V}(t) &\leq (-\tilde{c} + \frac{a}{2}\hat{c}^2\hat{\lambda} + \frac{\bar{\eta}^2}{2a})\tilde{q}^T(t)\tilde{q}(t) + (-\sigma_1)\frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\ &\leq h_1 \tilde{q}^T(t)\tilde{q}(t) + (-\sigma_1)\frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma}, \end{aligned} \quad (4.62)$$

where  $h_1 = -\tilde{c} + \frac{a}{2}\hat{c}^2\hat{\lambda} + \frac{\bar{\eta}^2}{2a} < 0$ ,  $\bar{\eta} = \max\{\hat{\eta}_1, \hat{\eta}_2, \dots, \hat{\eta}_N\}$ . On the other hand,

note that

$$\begin{aligned} V(t) &= \tilde{q}^T(t)((\tilde{L}^{-1})^T \otimes I_n)(E \otimes P)(\tilde{L}^{-1} \otimes I_n)\tilde{q}(t) + \frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\ &= \tilde{q}^T(t)((\tilde{L}^{-1})^T E \tilde{L}^{-1} \otimes P)\tilde{q}(t) + \frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\ &\leq \lambda_{\max}((\tilde{L}^{-1})^T E \tilde{L}^{-1} \otimes P)\tilde{q}^T(t)\tilde{q}(t) + \frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\ &\leq h_2 \tilde{q}^T(t)\tilde{q}(t) + \frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma}, \end{aligned} \quad (4.63)$$

where  $h_2 = \lambda_{\max}((\tilde{L}^{-1})^T E \tilde{L}^{-1} \otimes P) > 0$ . Obviously,  $\tilde{q}^T(t)\tilde{q}(t) \geq \frac{1}{h_2}(V(t) - \frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma})$ . Thus,

$$\begin{aligned} \dot{V}(t) &\leq \frac{h_1}{h_2}(V(t) - \frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma}) + (-\sigma_1)\frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\ &\leq \frac{h_1}{h_2}V(t) - (\frac{h_1}{2h_2} + \frac{\sigma_1}{2})\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma}. \end{aligned} \quad (4.64)$$

By the aid of the condition:  $\sigma_1 > -\frac{h_1}{h_2}$ , and let  $\alpha_3 = -\frac{h_1}{h_2}$ , it leads to  $\dot{V}(t) \leq$

$-\alpha_3 V(t)$ .

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(2) When  $t \in [T_{2k+1}, T_{2k+2})$ , the derivative of (5.15) obeys

$$\begin{aligned}
\dot{V}(t) &= \delta^T(t)(E \otimes (PA + A^T P))\delta(t) + (-\sigma_1)\frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\
&\quad + \sum_{i=1}^N (c_i(t) - \bar{c}_i) \tilde{q}_i^T(t) \Gamma \tilde{q}_i(t) \\
&\leq \lambda_{\max}(E \otimes (PA + A^T P))\delta^T(t)\delta(t) + (-\sigma_1)\frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\
&\leq \frac{\varsigma_1}{\lambda_{\min}(E \otimes P)} \delta^T(t)(E \otimes P)\delta(t) + (-\sigma_1)\frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \leq \beta V(t),
\end{aligned} \tag{4.65}$$

where  $\beta = \frac{\varsigma_1}{\lambda_{\min}(E \otimes P)}$ ,  $\varsigma_1 = \max\{\lambda_{\max}(E \otimes (PA + A^T P)), \varepsilon_0\}$ ,  $\varepsilon_0$  is a small positive constant.

In what follows, by the analogous operations as (4.18)-(4.21), under the triggering function (4.53), the linear MASs (4.1) and (4.2) can reach tracking consensus via intermittent adaptive ETC strategy (4.46).

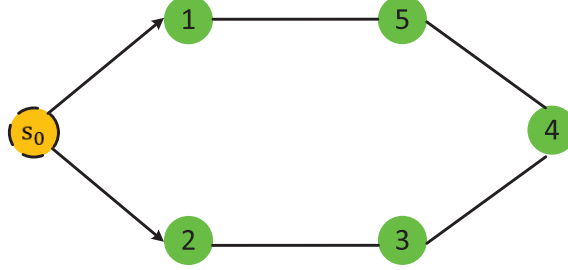
Next, we will show there is no agent will exhibit Zeno behavior, which means the intermittent event-triggered control is feasible. Let  $\hat{z}(t) = \frac{\|\tilde{e}(t)\|}{\|\tilde{q}(t)\|}$ , since the event will be triggered only when  $t \in [T_{2k}, T_{2k+1})$ , the time derivative of  $z(t)$  in each  $t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1})$  satisfies  $\dot{\hat{z}}(t) \leq \frac{\|\dot{\tilde{e}}(t)\|}{\|\tilde{q}(t)\|} + \hat{z}(t) \frac{\|\dot{\tilde{q}}(t)\|}{\|\tilde{q}(t)\|}$ .

According to the equations of (4.49), (4.51) and (4.56), one obtains

$$\begin{aligned}
\|\dot{\hat{z}}(t)\| &= \|\dot{\tilde{e}}(t)\| = \|(\tilde{L} \otimes I_n)\dot{\delta}(t)\| \\
&= \|(I_N \otimes A - \tilde{L}C(t) \otimes BK)\tilde{q}(t) + (\tilde{L}C(t) \otimes BK)\tilde{e}(t)\| \\
&\leq (\|I_N \otimes A\| + \|\tilde{L}C(t) \otimes BK\|) \|\tilde{q}(t)\| + \|\tilde{L}C(t) \otimes BK\| \|\tilde{e}(t)\| \\
&\leq (\|I_N \otimes A\| + \|\hat{c}\tilde{L} \otimes BK\|) \|\tilde{q}(t)\| + \|\hat{c}\tilde{L} \otimes BK\| \|\tilde{e}(t)\|.
\end{aligned} \tag{4.66}$$

Let  $r_3 = \|I_N \otimes A + \hat{c}\tilde{L} \otimes BK\|$ ,  $r_4 = \|\hat{c}\tilde{L} \otimes BK\|$ . Obviously,  $r_3 > r_4 > 0$ . It follows from (4.66) that

$$\begin{aligned}
\dot{\hat{z}}(t) &\leq \|I_N \otimes A + \hat{c}\tilde{L} \otimes BK\| + \|\hat{c}\tilde{L} \otimes BK\| \hat{z}(t) \\
&\quad + \hat{z}(t)(\|I_N \otimes A + \hat{c}\tilde{L} \otimes BK\| + \|\hat{c}\tilde{L} \otimes BK\| \hat{z}(t)) \\
&= r_3 + r_4 \hat{z}(t) + \hat{z}(t)(r_3 + r_4 \hat{z}(t)) \leq r_3(\hat{z}(t) + 1)^2.
\end{aligned} \tag{4.67}$$



**Fig. 4.2.** The interaction topology of MASs with  $s_0$  being the leader and 1 to 5 being followers.

By the similar analysis with Theorem 4.5, the lower bounded  $\hat{\varepsilon} = \frac{\sqrt{\hat{\eta}_i}}{(1+\hat{\eta}_i)r_3} > 0$  can be obtained, which indicates the Zeno behavior is avoided. This finalizes the derivation.

## 4.4 Simulation results

An example is introduced in this section to demonstrate the validity of given theoretical results. Consider the following directed interaction topology of MASs composed of 5 followers and one leader in Fig. 4.2. From the graph theory, the matrix  $\tilde{L}$  is:

$$\mathcal{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix} \text{ and } L = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}. \text{ Furthermore,}$$

we can obtain  $\tilde{L}$  is:

$$\tilde{L} = \begin{pmatrix} 2 & 0 & 0 & 0 & -1 \\ 0 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}. \text{ Furthermore, the dynamics of followers and}$$

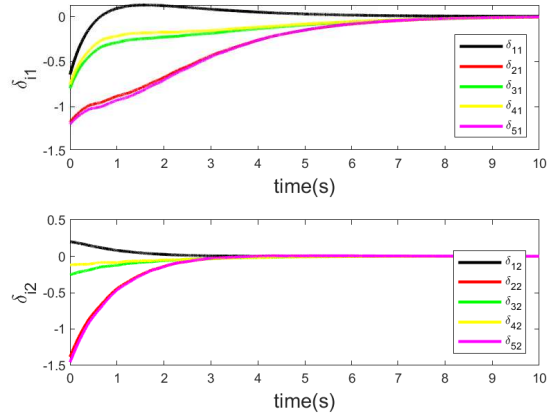
leader satisfy (4.1) and (4.2), where  $A = \begin{pmatrix} -0.5 & 1 \\ 0 & -1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ . Obviously, the pair  $(A, B)$  is stabilisable.

**Case 1:** The verification for Theorem 4.5.

Fig. 4.3 is the tracking errors of MASs. It is obvious that tracking consensus is reached. Fig. 4.4 is the measurement errors of followers of MASs. It can be concluded the measurement error  $e_i(t)$  will converge to zero eventually when

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**Fig. 4.3.** The tracking errors of MASs for Case 1.

consensus is reached. Fig. 4.5 is the graphical representations of controllers. Fig. 4.6 is the event-triggered instants of followers. Fig. 4.7 is the intermittent control progress of MASs. It can be clearly seen the intermittent control is aperiodic.

**Case 2:** The verification for Theorem 4.6.

Fig. 4.8 is the tracking errors of MASs, which indicates the leader-following consensus is realized. Fig. 4.9 is the measurement errors of followers of MASs. Fig. 4.10 is controllers. Fig. 4.11 is the event-triggered instants of followers. Fig. 4.12 is the intermittent control progress of MASs.

**Case 3:** The verification for Theorem 4.7.

Fig. 4.13 is the tracking errors of MASs. Fig. 4.14 is the measurement errors of followers of MASs. Fig. 4.15 is intermittent adaptive event-triggered controllers. Fig. 4.16 is the event-triggered instants of each follower under the triggering function (4.53). Fig. 4.17 is the adaptive coupling weights. Fig. 4.18 is the intermittent control progress of MASs.

## 4.5 Conclusion

This Chapter has investigated the aperiodically intermittent adaptive ETC strategy for general linear MASs. Firstly, a simple intermittent centralized ETC strategy is presented to motivate the problem, where the transmission channels among agents only open if the local event-trigger condition is satisfied in predefined time intervals. Then, the focus is placed on designing a more general triggering mech-

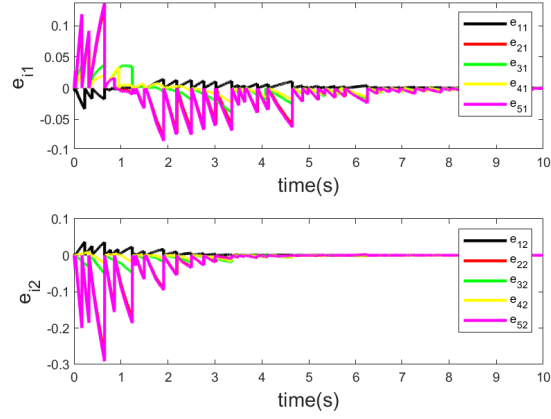


Fig. 4.4. The measurement errors of followers for Case 1.

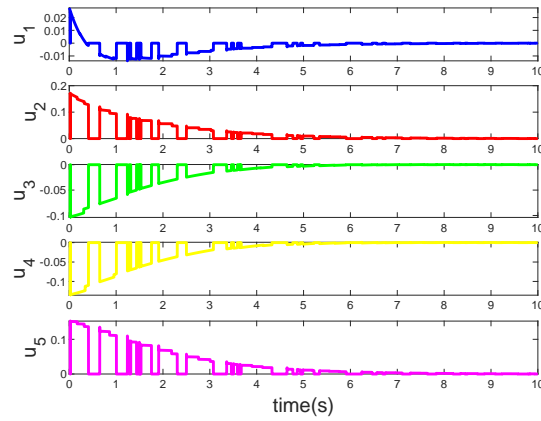


Fig. 4.5. The intermittent event-triggered controllers for Case 1.

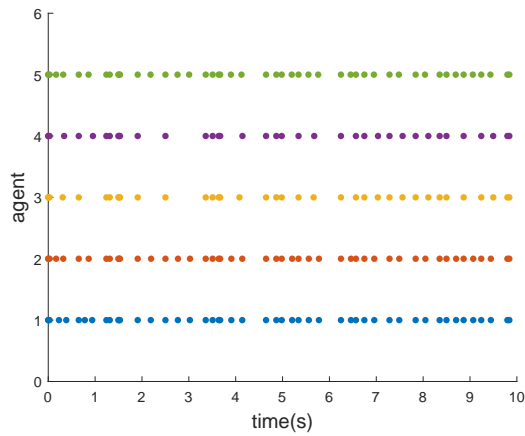


Fig. 4.6. The triggering instants of all followers under the triggering function (4.9) for Case 1.



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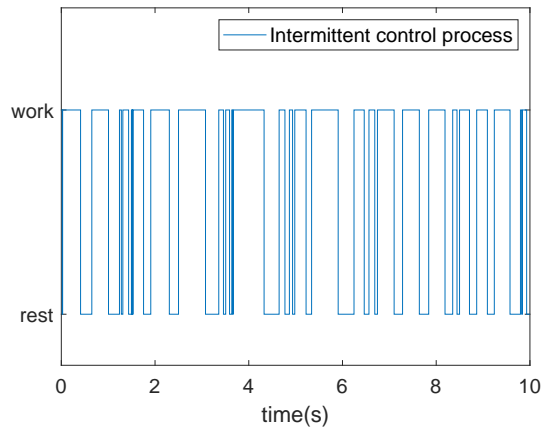


Fig. 4.7. The intermittent control progress for Case 1.

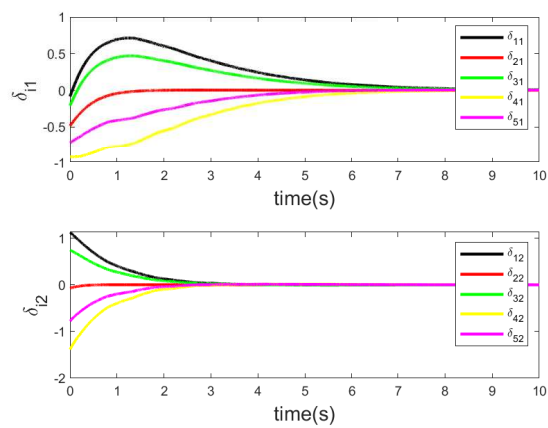


Fig. 4.8. The tracking errors of MASs for Case 2.

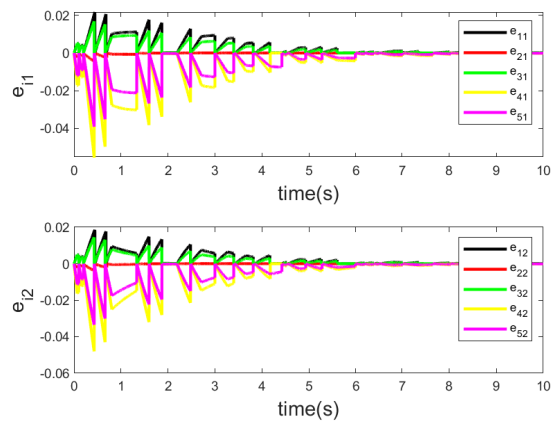


Fig. 4.9. The measurement errors of followers for Case 2.

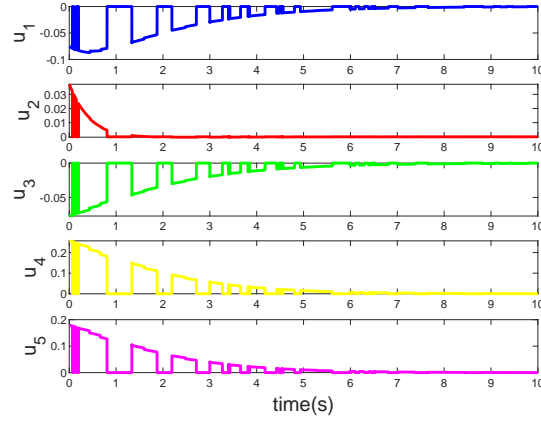


Fig. 4.10. The intermittent event-triggered controllers for Case 2.

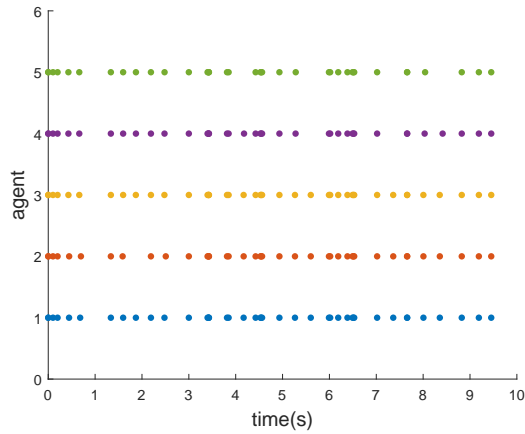


Fig. 4.11. The triggering instants of all followers under the triggering function (4.30) for Case 2.

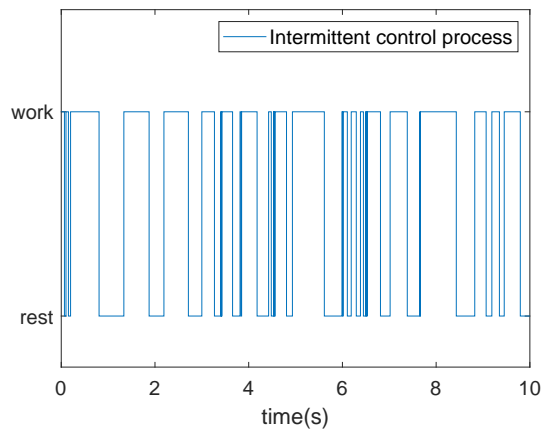


Fig. 4.12. The intermittent control progress for Case 2.

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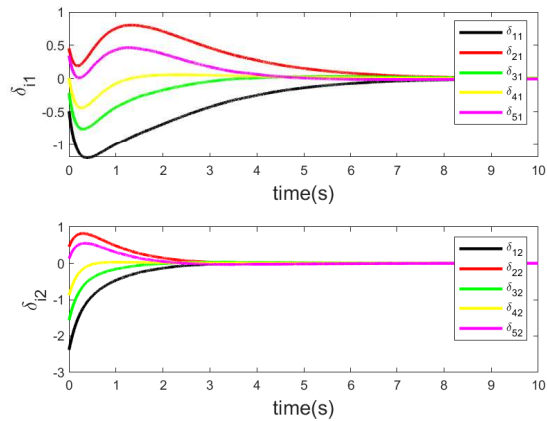


Fig. 4.13. The tracking errors of MASs for Case 3.

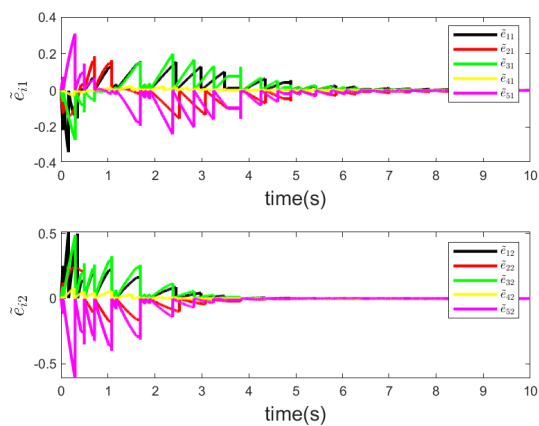


Fig. 4.14. The combinational measurement errors of followers for Case 3.

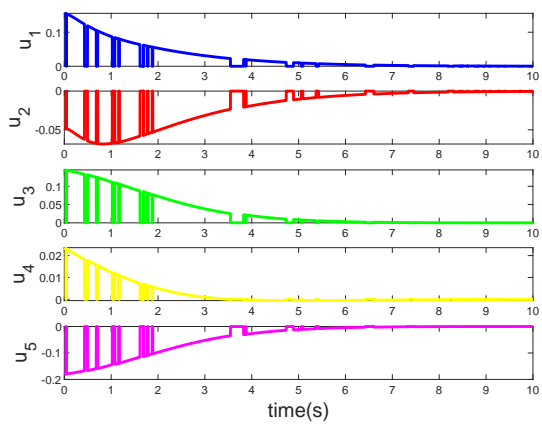
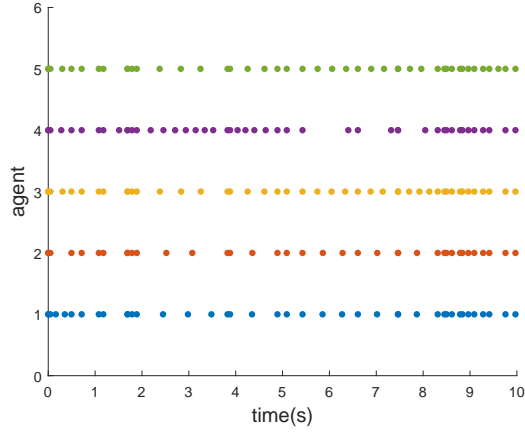
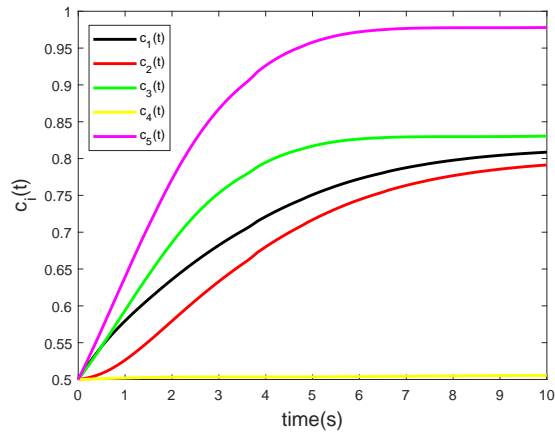


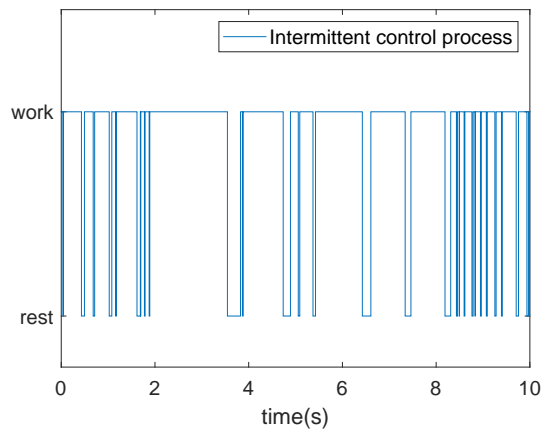
Fig. 4.15. The intermittent adaptive event-triggered controllers for Case 3.



**Fig. 4.16.** The triggering instants of all follower under the triggering function (4.53) for Case 3.



**Fig. 4.17.** The adaptive coupling weights in Case 3.



**Fig. 4.18.** The intermittent control progress for Case 3.

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anism to get rid of continuous inter-agent communication for monitoring the triggering condition. Next, to overcome the unexpected large feedback gains in real applications and appropriately tune the feedback gains, the aperiodically intermittent adaptive event-triggered controller is further devised. For each case, detailed analysis is provided to avoid Zeno behavior. Finally, the validity for the devised control schemes is substantiated by simulations. Note that under the intermittent adaptive ETC strategy, continuous monitoring of triggering function is still required and the frequency of triggering numbers will increase when consensus is approaching, how to improve the control strategy is the mainly focus in the future.

# Chapter 5

## Aperiodically intermittent adaptive dynamic event-triggered control for linear multi-agent systems

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### 5.1 Introduction

In Chapter 4, we introduced the adaptive event-triggered control method into aperiodically intermittent control. According to the triggering mechanism, three different types of control inputs have been designed. For example, in Theorem 4.5, the triggering function related to controller (4.3) is designed as follows:  $\hat{h}_i(t) = \|\tilde{e}_i(t)\| - \hat{\eta}_i \|\tilde{q}_i(t)\|$ , where  $e_i(t)$  is the measurement error,  $q_i(t)$  is combinational measurement error. It should be pointed out that this triggering condition is related to system states and a prescribed threshold parameter  $\hat{\eta}_i$ . When

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$\hat{\eta}_i$  change, the time and frequency of data transmissions will change. Therefore, threshold parameters play the key role in the event-triggered control (ETC) strategy. Nevertheless, in some practical situations, it may be unreasonable to permanently fix the threshold parameter. For instance, in order to maintain an anticipated formation, inter-agent communication still needs to be kept even when the formation is successfully achieved. This requires further transmissions of agents' states. Whereas, if  $\hat{\eta}_i$  is fixed all the time, some data packets which contain useful information to preserve the formation may not be triggered (Ge & Han, 2017). Besides, when the threshold parameter  $\hat{\eta}_i$  is fixed, the event will be triggered as follows: At the beginning, the static triggering conditions will effectively reduce the communication cost, as they are not easy to be satisfied. However, as time goes, it will be triggered frequently since the triggering conditions are easier to be satisfied, which causes unnecessary triggered instants. Inspired by above considerations, the threshold parameter should dynamically change over time to reflect such an engineering practice of time-varying data transmission rates. Therefore, in this Chapter, we will develop dynamic event-triggered conditions to further reduce the communication cost and the number of the control updates.

Compared with traditional static ETC, the dynamic ETC strategy depends on an auxiliary function to adjust the triggering mechanism. Owing to some promising feature (e.g., non-negativeness) of the introduced auxiliary or internal dynamic variable, an event-trigger under the dynamic ETC can felicitously extend the average inter event time. Thus, the consumption of resources can be reduced (Ge *et al.*, 2020). In recent years, the dynamic ETC strategy has received intensive interests. (Girard, 2014) constructed the dynamic triggering mechanisms for the first time by introducing an internal dynamic variable into ETC. In (Ge & Han, 2017), the authors employed the dynamic ETC strategy to handle a distributed resource-efficient formation control problem. In (Yi *et al.*, 2018), the authors investigated the average consensus problem for first-order MASs via two novel distributed controllers, that is, dynamic ETC and self-triggered control. In (Li *et al.*, 2018), the authors considered synchronization problem for a class of discrete time-delay complex dynamical networks by applying dynamic ETC strategy. Further, (Yu *et al.*, 2021) investigated intermittent dynamic event-based state estimation problem for a class of delayed complex dynamic networks, where the

estimate is implemented based on the measurements from a fraction of network nodes. In (Hu *et al.*, 2021), the authors addressed the tracking consensus problem for a class of nonlinear MASs by intermittent ETC strategy. To the best of our knowledge, the tracking consensus for linear MASs via aperiodically intermittent adaptive dynamic ETC has not investigated yet.

Motivated by the above-mentioned consideration, in this Chapter, a hybrid aperiodically intermittent adaptive dynamic event-triggered control is put forward to solve the tracking problem. The proposed control input inherits the respective advantages of aperiodically intermittent control strategy, dynamic ETC strategy and adaptive control strategy. Therefore, it is more general, practical and economical. The main contributions can be summarized as follows: Firstly, the aperiodically intermittent adaptive dynamic event-triggered control is designed and the corresponding parameters are obtained. Compared with the controllers in Chapter 4, the proposed control input in this Chapter ensures less triggering instants. Thus, the communication cost and the number of the control updates can be reduced furtherly. Secondly, in Chapter 4, the aperiodically intermittent adaptive ETC was designed. However, continuous inter-agent communication for monitoring the triggering condition is still required. To overcome this drawback, discrete-time combinational measurement is constructed for the triggering mechanism in this Chapter. Therefore, continuous monitoring of neighbors' states are avoided. Finally, numerical simulations are provided to illustrate the effectiveness of the theoretical results.

## 5.2 Problem formulation

Let us consider MASs composed of a group of followers and a virtual leader. The dynamic of followers can be modelled as

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), i \in \mathcal{V}, \quad (5.1)$$

where  $x_i \in R^n$  and  $u_i \in R^m$  represent the position state and control input, respectively.

The virtual leader's dynamics is assumed to be:

$$\dot{s}_0(t) = As_0(t), \quad (5.2)$$



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where  $s_j(t) \in R^n$  represents the  $j$ th virtual leader's position state.

**Assumption 5.1** *The matrix pair  $(A, B)$  is stabilizable.*

**Assumption 5.2** *The communication topology  $\mathcal{G}$  composed of the all followers is supposed fixed and its corresponding graph is undirected and connected.*

**Assumption 5.3** *Assume there exists a positive definite diagonal matrix  $E$  satisfying  $\tilde{L}^{-T}E + E\tilde{L}^{-1} - a_2E > I_N$  where  $E = \text{diag}\{\rho_1, \dots, \rho_N\} > 0$ , where  $\tilde{L} = L + D$ ,  $D = \text{diag}\{d_1, d_2, \dots, d_N\}$ ,  $d_i > 0$ , if a leader can transmit information to follower  $i$ , and  $d_i = 0$  otherwise.  $\rho_i > 0$ .*

**Remark 5.4** *Based on the graph theory, matrix theory and above assumptions, we can obtain  $L = L^T$ ,  $\tilde{L}^{-1} = \tilde{L}^{-T}$ . However, for giving the comparison with directed graph, we still use the notation  $\tilde{L}^{-1}$  and  $\tilde{L}^{-T}$ .*

**Definition 5.5** *Given the leader-following MASs (5.1) and (5.2), the leader-following consensus is reached by designing an appropriate control input such that  $\lim_{t \rightarrow \infty} \|\delta_i(t)\| = 0$  for any initial values, where  $\delta_i(t) = x_i(t) - s_0(t)$ ,  $i \in \mathcal{V}$ .*

### 5.3 Main results

#### 5.3.1 Aperiodically intermittent adaptive dynamic ETC for linear MASs

The following control input is proposed to solve the leader-following consensus problem, which is characterised by

$$u_i(t) = \begin{cases} c_i(t)Kq_i(t_{i,s}^k), & t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1}), \\ 0, & t \in [T_{2k+1}, T_{2k+2}), \end{cases} \quad (5.3)$$

$$q_i(t_{i,s}^k) = \sum_{j=1}^N a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + d_i(\hat{s}_0(t) - \hat{x}_i(t)), \quad (5.4)$$

$$\dot{c}_i(t) = \text{proj}(c_i(t)) = \begin{cases} 0, & \text{if } c_i(t) \geq \bar{c}_i, \\ \sigma e^{\sigma_1 t} q_i^T(t_{i,s}^k) \Gamma q_i(t_{i,s}^k), & \text{if } c_i(t) < \bar{c}_i, \end{cases} \quad (5.5)$$

where  $\hat{x}_i(t) = e^{A(t-t_{i,s}^k)} x_i(t_{i,s}^k)$ ,  $K, \Gamma$  represent the feedback control gain matrices, which will be designed in Theorem 5.6.  $\bar{c}_i, \sigma, \sigma_1$  are positive constants. In

addition, the width of work time and rest time, in the form, are denoted as  $\Delta_k = T_{k+1} - T_k, k = 0, 1 \dots$ , commonly.

For each follower  $i$ , the combination measurement error can be represented as:

$$e_i(t) = \hat{x}_i(t) - x_i(t), \quad (5.6)$$

Substituting the control input (5.3) into (5.1), and combining the definition of combination measurement error  $e_i(t)$ , one has the tracking error system

$$\dot{\delta}_i(t) = \begin{cases} A\delta_i(t) + c_i(t)BKq_i(t_{i,s}^k), t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1}), \\ A\delta_i(t), t \in [T_{2k+1}, T_{2k+2}). \end{cases} \quad (5.7)$$

Further, let  $x(t) = (x_1^T(t), x_2^T(t), \dots, x_N^T(t))^T$ ,  $\delta(t) = (\delta_1^T(t), \delta_2^T(t), \dots, \delta_N^T(t))^T$ ,  $e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$ ,  $q(t) = (q_1^T(t), q_2^T(t), \dots, q_N^T(t))^T$ ,  $q(t_s^k) = (q_1^T(t_{1,s}^k), q_2^T(t_{2,s}^k), \dots, q_N^T(t_{N,s}^k))^T$ ,  $C = \text{diag}\{c_1, c_2, \dots, c_N\}$ ,  $C(t) = \text{diag}\{\bar{c}_1(t), \bar{c}_2(t), \dots, \bar{c}_N(t)\}$ , then we can obtain the following equations:

$$q(t_s^k) = -(\tilde{L} \otimes I_n)(\delta(t) + e(t)), \quad (5.8)$$

$$\dot{\delta}(t) = \begin{cases} (I_N \otimes A)\delta(t) + (C(t) \otimes BK)q(t_s^k), t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1}), \\ (I_N \otimes A)\delta(t), t \in [T_{2k+1}, T_{2k+2}). \end{cases} \quad (5.9)$$

**Theorem 5.6** *Combining with Assumption 5.1, 5.2 and 5.3, the proposed intermittent dynamic ETC strategy (5.3)-(5.5) can guarantee the achievement of leader-following consensus for linear MASs (5.1) and (5.2) if the following conditions hold:*

(i) Find a solution  $P > 0$  satisfying the algebraic Riccati equation below:

$$PA + A^T P - \gamma P B B^T P + I_n = 0, \quad (5.10)$$

where  $\gamma > 0$  is any positive constant. Then compute the feedback matrix  $K$  as follows:  $K = B^T P$ .

(ii) The triggering function for  $i$ th follower is devised as:

$$\theta_i(\lambda_1 e_i^T(t) \Gamma e_i(t) - (\bar{c}_i - \lambda_2) q_i^T(t_{i,s}^k) \Gamma q_i(t_{i,s}^k)) \geq \eta_i(t), \quad (5.11)$$

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$$\dot{\eta}_i(t) = \begin{cases} -\gamma_i \eta_i(t) + \xi_i((\bar{c}_i - \lambda_2) q_i^T(t_{i,s}^k) \Gamma q_i(t_{i,s}^k) - \lambda_1 e_i(t) \Gamma e_i(t)), \\ \quad t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1}), \\ 0, \\ \quad t \in [T_{2k+1}, T_{2k+2}). \end{cases} \quad (5.12)$$

where  $\lambda_1 = \lambda_{\max}(\Phi_1)$ , and  $\Phi_1 = \gamma(E + \frac{1}{2a_1}(\tilde{L}^{-T}E + E\tilde{L}^{-1})) + \frac{\bar{c}}{a_2}E$ ,  $\lambda_2 = \lambda_{\max}(\Phi_2)$ ,  $\Phi_2 = \gamma(\tilde{L}^{-T}E\tilde{L}^{-1} + \frac{a_1}{2}(\tilde{L}^{-T}E + E\tilde{L}^{-1}))$ ,  $\bar{c} = \max\{\bar{c}_1, \bar{c}_2, \dots, \bar{c}_N\}$  and  $a_1, a_2$  are positive constants,  $\theta_i, \gamma_i, \xi_i$  and  $\bar{c}_i$  satisfy the following equations:

$$\bar{c}_i > \lambda_2, \quad (5.13)$$

$$\gamma_i - \frac{1 - \xi_i}{\theta_i} > 0. \quad (5.14)$$

(iii) There exist positive constants  $\tau_1$  and  $\tau_2$  such that  $\begin{cases} \Delta_{2k} \geq \tau_1 \\ \Delta_{2k+1} \leq \tau_2 \end{cases}$  and  $\frac{\tau_1}{\tau_2} > \frac{\beta_1}{\alpha_1}$ , where  $\Delta_{2k}$  is the the width of work time,  $\Delta_{2k+1}$  is the width of rest time,  $\alpha_1 = \min\{\frac{\gamma}{\lambda_{\max}(P)}, \sigma_1, \gamma_i - \frac{1 - \xi_i}{\theta_i}\}$ ,  $\beta_1 = \frac{\zeta_1}{\lambda_{\min}(E \otimes P)}$ ,  $\zeta_1 = \max\{\lambda_{\max}(E) \cdot \lambda_{\max}(PA + A^T P), \varepsilon_0\}$ .  $\gamma, \sigma_1, \varepsilon_0$  are positive constants.

**Proof:**

Choose the following Lyapunov function candidate:

$$V(t) = \delta^T(t)(E \otimes P)\delta(t) + \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} + \sum_{i=1}^N \eta_i(t), \quad (5.15)$$

where matrix  $E$  and  $P$  are defined as mentioned above.

(1) When  $t \in [t_{i,s}^k, t_{i,s+1}^k) \cap [T_{2k}, T_{2k+1})$ , let  $V_1(t) = \delta^T(t)(E \otimes P)\delta(t)$ ,  $V_2(t) = \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma}$ ,  $V_3(t) = \sum_{i=1}^N \eta_i(t)$ , take the time derivative of  $V_1(t)$  along the first equation of (5.9) yields

$$\begin{aligned} \dot{V}_1(t) &= \delta^T(t)(E \otimes (PA + A^T P))\delta(t) + 2\delta^T(t)[EC(t) \otimes PBK]q(t_s^k) \\ &= \delta^T(t)(E \otimes (PA + A^T P) - E \otimes \gamma\Gamma)\delta(t) + \gamma\delta^T(t)(E \otimes \Gamma)\delta(t) \\ &\quad + 2\delta^T(t)(EC(t) \otimes PBK)q(t_s^k). \end{aligned} \quad (5.16)$$

By the aid of (5.8), one obtains

$$\begin{aligned}
& \gamma \delta^T(t)(E \otimes \Gamma) \delta(t) \\
&= \gamma(-1)[q^T(t_s^k)(\tilde{L}^{-T} \otimes I_n) + e^T(t)](E \otimes \Gamma)(-1)[(\tilde{L}^{-1} \otimes I_n)q(t_s^k) + e(t)] \\
&= \gamma q^T(t_s^k)(\tilde{L}^{-T} E \tilde{L}^{-1} \otimes \Gamma)q(t_s^k) + \gamma e^T(t)(E \otimes \Gamma)e(t) + 2\gamma q^T(t_s^k)(\tilde{L}^{-T} E \otimes \Gamma)e(t).
\end{aligned} \tag{5.17}$$

By the Young's inequality  $x^T y \leq \frac{a_1}{2} x^2 + \frac{1}{2a_1} y^2$ , the last term of (5.17) can be written as

$$\begin{aligned}
& 2\gamma q^T(t_s^k)(\tilde{L}^{-T} E \otimes \Gamma)e(t) \\
&= \gamma q^T(t_s^k)[(\tilde{L}^{-T} E + E \tilde{L}^{-1}) \otimes \Gamma]e(t) \\
&\leq \frac{\gamma}{2} a_1 q^T(t_s^k)[(\tilde{L}^{-T} E + E \tilde{L}^{-1}) \otimes \Gamma]q(t_s^k) \\
&\quad + \frac{\gamma}{2a_1} e^T(t)[(\tilde{L}^{-T} E + E \tilde{L}^{-1}) \otimes \Gamma]e(t).
\end{aligned} \tag{5.18}$$

Substituting (5.18) into (5.17), results in

$$\begin{aligned}
\gamma \delta^T(t)(E \otimes \Gamma) \delta(t) &\leq \gamma q^T(t_s^k)[(\tilde{L}^{-T} E \tilde{L}^{-1} + \frac{a_1}{2}(\tilde{L}^{-T} E + E \tilde{L}^{-1})) \otimes \Gamma]q(t_s^k) \\
&\quad + \gamma e^T(t)[(E + \frac{1}{2a_1}(\tilde{L}^{-T} E + E \tilde{L}^{-1})) \otimes \Gamma]e(t).
\end{aligned} \tag{5.19}$$

Similar to the equation (5.18), via the Young's inequality  $x^T y \leq \frac{a_2}{2} x^2 + \frac{1}{2a_2} y^2$ , one has

$$\begin{aligned}
& 2\delta^T(t)(EC(t) \otimes PBK)q(t_s^k) \\
&= -2[q^T(t_s^k)(\tilde{L}^{-T} \otimes I_n) + e^T(t)](EC(t) \otimes \Gamma)q(t_s^k) \\
&= -2q^T(t_s^k)(\tilde{L}^{-T} EC(t) \otimes \Gamma)q(t_s^k) - 2e^T(t)(EC(t) \otimes \Gamma)q(t_s^k) \\
&\leq -2q^T(t_s^k)(\tilde{L}^{-T} EC(t) \otimes \Gamma)q(t_s^k) + a_2 q^T(t_s^k)(EC(t) \otimes \Gamma)q(t_s^k) \\
&\quad + \frac{1}{a_2} e^T(t)(EC(t) \otimes \Gamma)e(t) \\
&\leq \frac{1}{a_2} e^T(t)(EC(t) \otimes \Gamma)e(t) - q^T(t_s^k)[(\tilde{L}^{-T} E + E \tilde{L}^{-1} - a_2 E)C(t) \otimes \Gamma]q(t_s^k).
\end{aligned} \tag{5.20}$$

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Substituting (5.19) and (5.20) into (5.16), we have

$$\begin{aligned}
\dot{V}_1(t) &\leq -\gamma\delta^T(t)(E \otimes I_n)\delta(t) + \gamma q^T(t_s^k)[(\tilde{L}^{-T}E\tilde{L}^{-1} + \frac{a_1}{2}(\tilde{L}^{-T}E + E\tilde{L}^{-1})) \otimes \Gamma]q(t_s^k) \\
&\quad + e^T(t)[(\gamma(E + \frac{1}{2a_1}(\tilde{L}^{-T}E + E\tilde{L}^{-1})) + \frac{1}{a_2}EC(t)) \otimes \Gamma]e(t) \\
&\quad - q^T(t_s^k)[(\tilde{L}^{-T}E + E\tilde{L}^{-1} - a_2E)C(t) \otimes \Gamma]q(t_s^k).
\end{aligned} \tag{5.21}$$

$E$  and  $a_2$  satisfy the following condition:

$$\tilde{L}^{-T}E + E\tilde{L}^{-1} - a_2E > I_N. \tag{5.22}$$

Let  $\Phi_1 = \gamma(E + \frac{1}{2a_1}(\tilde{L}^{-T}E + E\tilde{L}^{-1})) + \frac{\bar{c}}{a_2}E$ , and  $\lambda_1 = \lambda_{\max}(\Phi_1)$ ,

$\Phi_2 = \gamma(\tilde{L}^{-T}E\tilde{L}^{-1} + \frac{a_1}{2}(\tilde{L}^{-T}E + E\tilde{L}^{-1}))$ , and  $\lambda_2 = \lambda_{\max}(\Phi_2)$ .

Hence, one has

$$\begin{aligned}
\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \\
&\leq -\gamma\delta^T(t)(E \otimes I_n)\delta(t) + \lambda_1 e^T(t)(I_N \otimes \Gamma)e(t) + \lambda_2 q^T(t_s^k)(I_N \otimes \Gamma)q(t_s^k) \\
&\quad - q^T(t_s^k)[(\tilde{L}^{-T}E + E\tilde{L}^{-1} - a_2E)C(t) \otimes \Gamma]q(t_s^k) \\
&\quad + (-\sigma_1)\frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} + q^T(t_s^k)[(C(t) - C) \otimes \Gamma]q(t_s^k) + \sum_{i=1}^N \dot{\eta}_i(t) \\
&\leq -\gamma\delta^T(t)(E \otimes I_n)\delta(t) + \lambda_1 e^T(t)(I_N \otimes \Gamma)e(t) - q^T(t_s^k)[(C - \lambda_2 I_N) \otimes \Gamma]q(t_s^k) \\
&\quad + (-\sigma_1)\frac{1}{2}\sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} + \sum_{i=1}^N \dot{\eta}_i(t).
\end{aligned} \tag{5.23}$$

According to the derivative of  $\eta_i(t)$  and combining with the following equation:

$$\begin{aligned}
&\lambda_1 e^T(t)(I_N \otimes \Gamma)e(t) - q^T(t_s^k)[(C - \lambda_2 I_N) \otimes \Gamma]q(t_s^k) \\
&= \sum_{i=1}^N \lambda_1 e_i^T(t)\Gamma e_i(t) - (c_i - \lambda_2)q_i^T(t_{i,s}^k)\Gamma q_i(t_{i,s}^k),
\end{aligned} \tag{5.24}$$

one has

$$\begin{aligned}
 \dot{V}(t) &= -\gamma\delta^T(t)(E \otimes I_n)\delta(t) + \sum_{i=1}^N \lambda_1 e_i^T(t)\Gamma e_i(t) - (c_i - \lambda_2)q_i^T(t_{i,s}^k)\Gamma q_i(t_{i,s}^k) \\
 &\quad + \frac{1}{2}(-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\
 &\quad + \sum_{i=1}^N -\gamma_i \eta_i(t) + \xi_i((c_i - \lambda_2)q_i^T(t_{i,s}^k)\Gamma q_i(t_{i,s}^k) - \lambda_1 e_i(t)\Gamma e_i(t)) \\
 &= -\gamma\delta^T(t)(E \otimes I_n)\delta(t) + \frac{1}{2}(-\sigma_1) \sum_{j=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} + \sum_{i=1}^N -\gamma_i \eta_i(t) \\
 &\quad + (1 - \xi_i) \left( \sum_{i=1}^N \lambda_1 e_i^T(t)\Gamma e_i(t) - (c_i - \lambda_2)q_i^T(t_{i,s}^k)\Gamma q_i(t_{i,s}^k) \right) \\
 &\leq -\gamma\delta^T(t)(E \otimes I_n)\delta(t) + \frac{1}{2}(-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\
 &\quad + \left(-\gamma_i + \frac{1 - \xi_i}{\theta_i}\right) \sum_{i=1}^N \eta_i(t),
 \end{aligned} \tag{5.25}$$

With the help of Lemma 1.11, we can obtain

$$\begin{aligned}
 \dot{V}(t) &\leq \frac{-\gamma}{\lambda_{\max}(P)} \delta^T(t)(E \otimes P)\delta(t) + \frac{1}{2}(-\sigma_1) \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\
 &\quad + \left(-\gamma_i + \frac{1 - \xi_i}{\theta_i}\right) \sum_{i=1}^N \eta_i(t) \\
 &\leq -\alpha_1 V(t),
 \end{aligned} \tag{5.26}$$

where  $\alpha_1 = \min\left\{\frac{\gamma}{\lambda_{\max}(P)}, \sigma_1, \gamma_i - \frac{1 - \xi_i}{\theta_i}\right\}$ .

(2) When  $t \in [T_{2k+1}, T_{2k+2})$ , the time derivative of (5.15) is given as

$$\begin{aligned}
 \dot{V}(t) &= \delta^T(t)(E \otimes (PA + A^T P))\delta(t) + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\
 &\quad + \sum_{i=1}^N (c_i(t) - \bar{c}_i) q_i^T(t_s^k) \Gamma q(t_s^k).
 \end{aligned} \tag{5.27}$$

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Since  $c_i(t) < \bar{c}_i$ , one has

$$\begin{aligned}
\dot{V}(t) &\leq \lambda_{\max}(E \otimes (PA + A^T P)) \delta^T(t) \delta(t) + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\
&\leq \frac{\zeta_1}{\lambda_{\min}(E \otimes P)} \delta^T(t) (E \otimes P) \delta(t) + (-\sigma_1) \frac{1}{2} \sum_{i=1}^N e^{-\sigma_1 t} \frac{(c_i(t) - \bar{c}_i)^2}{\sigma} \\
&\quad + \frac{\zeta_1}{\lambda_{\min}(E \otimes P)} \sum_{i=1}^N \eta_i(t) \\
&\leq \beta_1 V(t),
\end{aligned} \tag{5.28}$$

where  $\zeta_1 = \max\{\lambda_{\max}(E) \cdot \lambda_{\max}(PA + A^T P), \varepsilon_0\}$ ,  $\beta_1 = \frac{\zeta_1}{\lambda_{\min}(E \otimes P)}$ .

Accordingly, when  $t \in [T_0, T_1)$ , one has

$$V(t) \leq V(T_0) e^{-\alpha_1(t-T_0)}, \tag{5.29}$$

when  $t \in [T_1, T_2)$ , one has

$$V(t) \leq V(T_0) e^{-\alpha_1 \Delta_0 + \beta_1(t-T_1)}, \tag{5.30}$$

where  $\Delta_0 = T_1 - T_0$ .

By induction and combining the conditions in Theorem 5.6, one obtains, when  $t \in [T_{2k}, T_{2k+1})$ ,

$$\begin{aligned}
V(t) &\leq V(0) e^{-\alpha_1(\Delta_0 + \Delta_2 + \dots + \Delta_{2k-2} + t - T_{2k}) + \beta_1(\Delta_1 + \Delta_3 + \dots + \Delta_{2k-1})} \\
&\leq V(0) e^{-\alpha_1(k\tau_1 + t - T_{2k}) + \beta_1 k\tau_2} \\
&\leq V(0) e^{-\alpha_1 k\tau_1 + \beta_1 k\tau_2} \\
&= V(0) e^{k(-\alpha_1\tau_1 + \beta_1\tau_2)}.
\end{aligned} \tag{5.31}$$

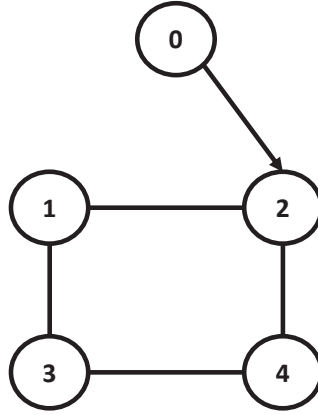
Similarly, when  $t \in [T_{2k+1}, T_{2k+2})$ ,

$$\begin{aligned}
V(t) &\leq V(0) e^{-\alpha_1(\Delta_0 + \Delta_2 + \dots + \Delta_{2k}) + \beta_1(\Delta_1 + \Delta_3 + \dots + \Delta_{2k-1} + t - T_{2k+1})} \\
&\leq V(0) e^{-\alpha_1(k+1)\tau_1 + \beta_1(k\tau_2 + t - T_{2k+1})} \\
&\leq V(0) e^{-\alpha_1(k+1)\tau_1 + \beta_1(k\tau_2 + \tau_2)} \\
&= V(0) e^{(k+1)(-\alpha_1\tau_1 + \beta_1\tau_2)},
\end{aligned} \tag{5.32}$$

where  $\Delta_k = T_{k+1} - T_k$ . Then when  $t \in [T_k, T_{k+1}]$ , in order to ensure that  $V(t) \rightarrow 0$  as  $k \rightarrow \infty$ , let  $-\alpha_1\tau_1 + \beta_1\tau_2 < 0$ , one has  $\frac{\tau_1}{\tau_2} > \frac{\beta_1}{-\alpha_1}$ . Thus,  $\lim_{t \rightarrow \infty} \|\delta_i(t)\| = 0$ . It means that leader-following consensus of MASs via intermittent adaptive dynamic event-triggered control is reached. The proof to exclude Zeno behavior is similar with the method in Chapter 4, we omit it here.

## 5.4 Simulation results

In this section, a numerical example is presented to illustrate the effectiveness of obtained result in Theorem 5.6. Consider the linear MASs composed by a leader (0) and four followers (1, 2, 3, 4). The fixed undirected communication topology is given by the following graph Fig. 5.1.



**Fig. 5.1.** The communication topology of MASs.

From the graph theory, the matrices  $\mathcal{A}$  and  $L$  can be derived as follows:  
 $\mathcal{A} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$  and  $L = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}$ . The system matrices of the considered MAS is given by

$$A = \begin{bmatrix} -3 & 4 & 2 \\ -5 & 1 & 2.5 \\ 1.56 & 3.78 & -8 \end{bmatrix}, B = \begin{bmatrix} 5 & 6 \\ 1 & 5 \\ 2 & 4 \end{bmatrix}.$$



## 5. APERIODICALLY INTERMITTENT ADAPTIVE DYNAMIC EVENT-TRIGGERED CONTROL FOR LINEAR MULTI-AGENT SYSTEMS

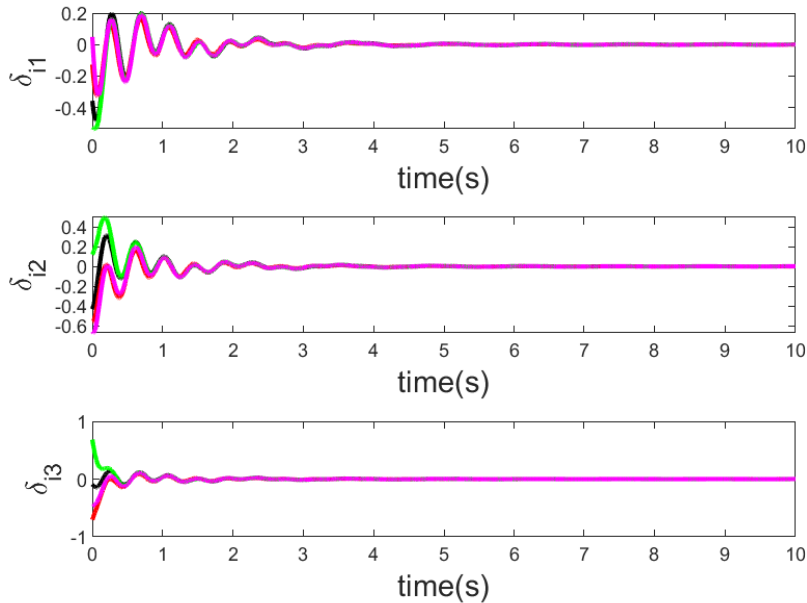
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By solving the Ricatti equation (5.10), we can obtain

$$P = \begin{bmatrix} 0.2383 & -0.1687 & -0.0260 \\ -0.1687 & 0.2851 & 0.0473 \\ -0.0260 & 0.0473 & 0.0636 \end{bmatrix}.$$

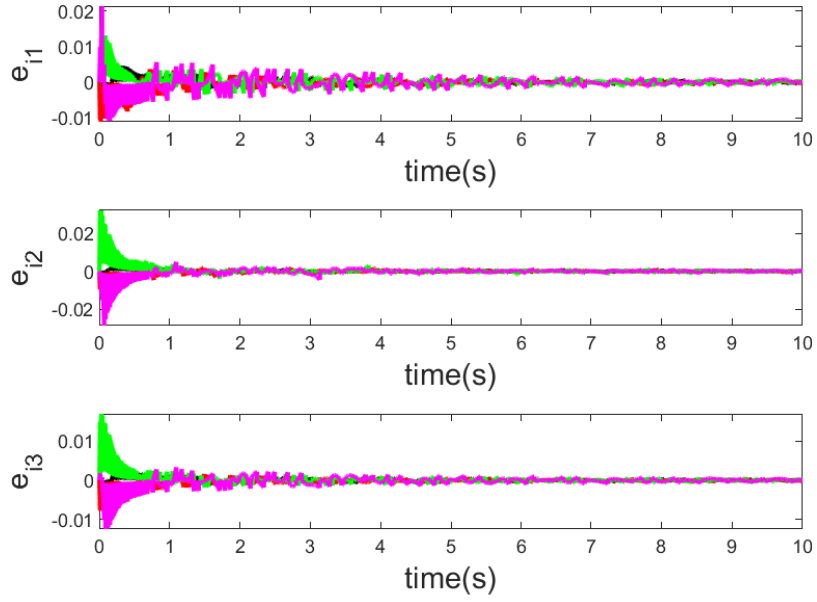
According to the conditions in Theorem 5.6, by calculating equations (5.13) and (5.14), we can choose the adaptive parameters  $\bar{c}_i = [200, 201, 202, 200.5, 200.8]$ ,  $\sigma = 1$ , and  $\sigma_1 = 1$ .  $\theta = [300, 600, 900, 1200]$ ,  $\xi = [400, 600, 800, 1000]$ ,  $\gamma_i = 0.05, 0.08, 0.1, 0.2$ .

Fig. 5.2 is state errors between all followers and the leader along different dimensions. Fig. 5.3 is the measurement errors of followers of MASs. Fig. 5.4 is the state trajectories of all agents. Fig. 5.5 denotes the total number of triggered instants for each follower. Fig. 5.6 shows the intermittent interval of MASs. It can be concluded the measurement error  $e_i(t)$  will converge to zero eventually when consensus is reached. Fig. 5.7 is the adaptive coupling weights. From Fig. 5.2, the MASs achieving consensus and the Theorem 5.6 is verified.

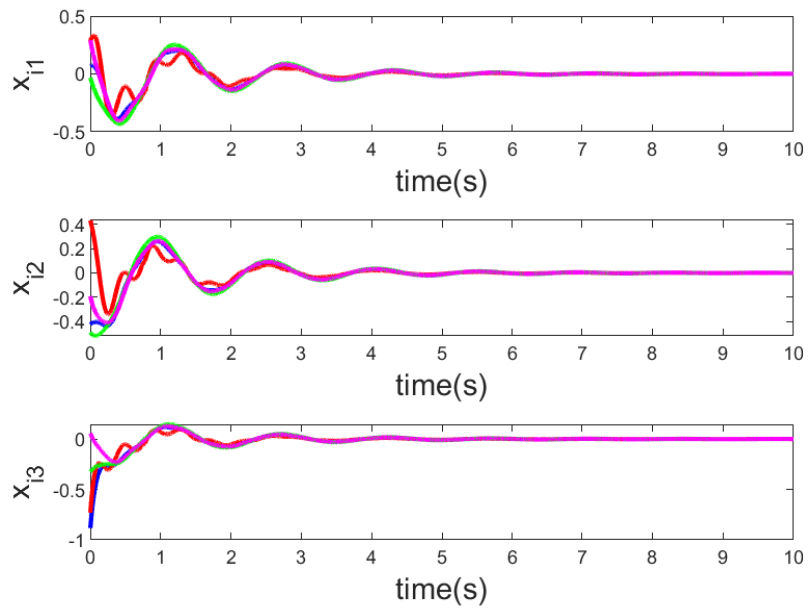


**Fig. 5.2.** The tracking errors  $\delta_i(t)$  of MASs.

## 5.4 Simulation results



**Fig. 5.3.** The measurement errors  $e_i(t)$  of followers.



**Fig. 5.4.** The state trajectories of followers.

## 5. APERIODICALLY INTERMITTENT ADAPTIVE DYNAMIC EVENT-TRIGGERED CONTROL FOR LINEAR MULTI-AGENT SYSTEMS

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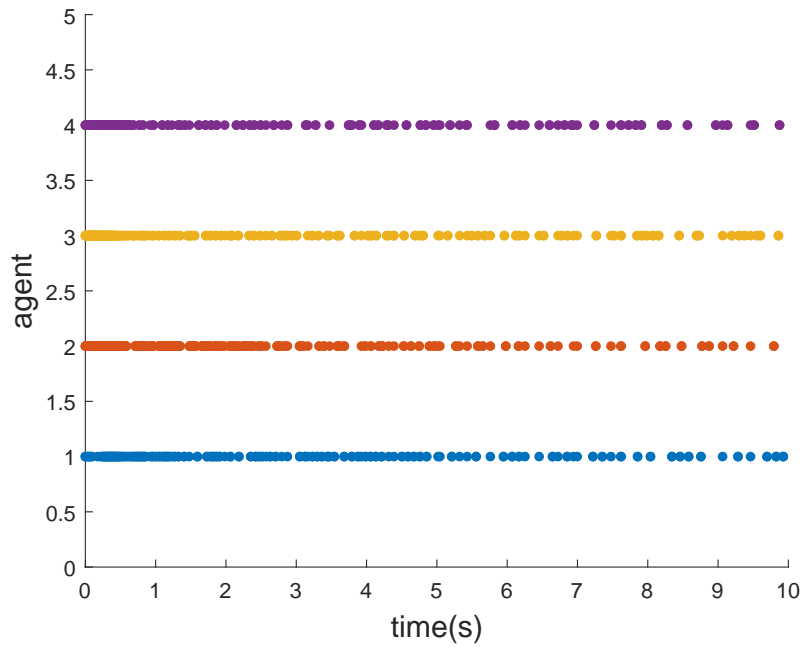


Fig. 5.5. Triggered instants for followers.

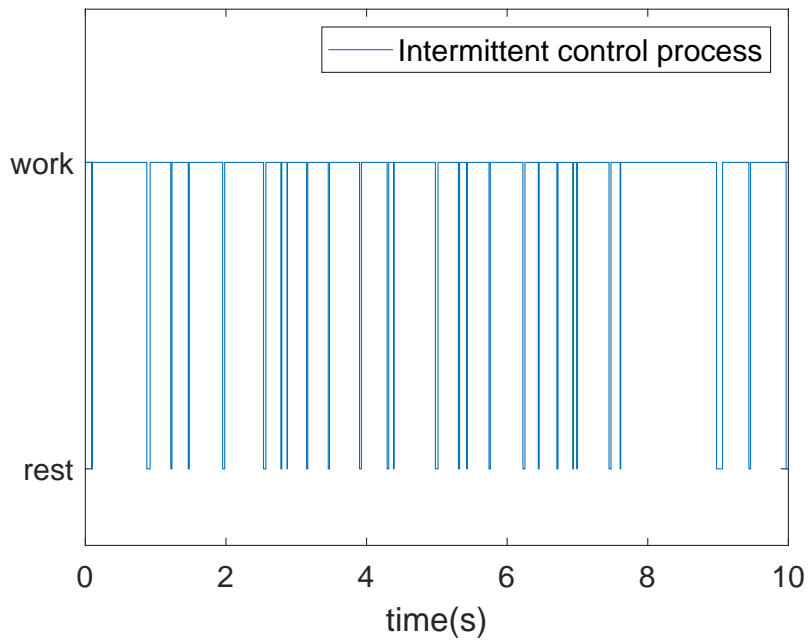


Fig. 5.6. The intermittent control progress.

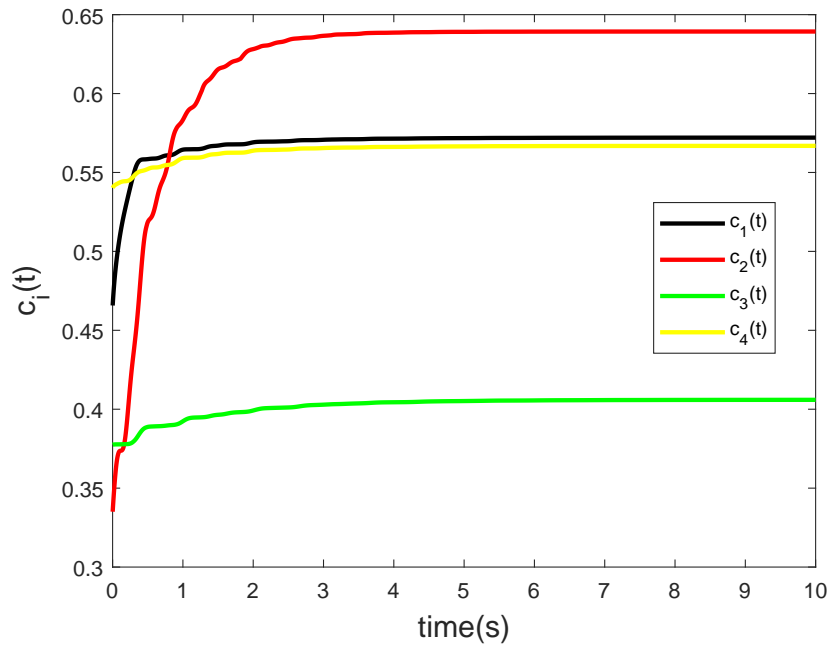


Fig. 5.7. The adaptive coupling weights  $c_i(t)$ .

## 5.5 Conclusion

In this chapter, the tracking consensus problem was investigated via aperiodically intermittent adaptive dynamic ETC strategy. Firstly, the dynamic adaptive ETC strategy was designed to further reduce the communication cost and the number of the control updates. Secondly, the corresponding parameters were obtained based on matrix theory and Lyapunov function. Compared with the traditional static event-triggered control in Chapter 4, the time-varying threshold ensures less triggering instants. Thirdly, we modified the event triggering mechanism that depends on combined measurement approach in Chapter 4, continuous monitoring of neighbors' states were avoided. Finally, numerical simulations were provided to illustrate the effectiveness of the theoretical results.

**5. APERIODICALLY INTERMITTENT ADAPTIVE DYNAMIC  
EVENT-TRIGGERED CONTROL FOR LINEAR MULTI-AGENT  
SYSTEMS**

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# Conclusions and Perspectives

## Summary of main results

This dissertation handles the consensus problems of MASs via discontinuous control methods. According to different control tasks and different constraint conditions, the appropriate control technologies are designed for each agent, such that all the agents can track their own leader and meanwhile economize the control costs.

In chapter 2, the cluster consensus of MASs via intermittent adaptive pinning control has investigated. Firstly, for the linear MASs and a class of nonlinear MASs, we have employed intermittent adaptive pinning control schemes respectively to accomplish the cluster consensus. Secondly, for the network topology of MASs, we take a weakly connected topology into consideration, which is more universal in practice in contrast to the directed network topology required to be balanced or contain a directed spanning tree. Thirdly, a rigorous proof have been given for the intermittent adaptive pinning control input based on the Lyapunov stability theory and the corresponding sufficient conditions have been derived. Finally, simulation examples are presented to verify the effectiveness of the main results.

In chapter 3, the observer-based intra cluster lag consensus problem for general linear MASs and nonlinear MASs under a weakly connected graph via aperiodically intermittent adaptive pinning control has been investigated. Considering the relevant full state information of each follower is not always available, a class of observers has been designed to estimate the states of followers. Then a class of observer-based intermittent adaptive pinning control protocols is proposed according to the topological property of nodes. Moreover, the pinning gains have been designed to be intermittent adaptive and with an exponential conver-

## CONCLUSIONS AND PERSPECTIVES

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gence rate, which will effectively reduce communication costs, avoid the pinning gains being larger than those needed in practice and guarantees the pinning gains quickly converge to steady value. Finally, rigorous proofs and numerical simulations have been provided to guarantee the correctness of the theoretical results.

In chapter 4, the tracking consensus problem for general linear MASs has investigated the aperiodically intermittent adaptive event-triggered control strategy. Firstly, a simple intermittent centralized event-triggered control strategy is presented to motivate the problem, where the transmission channels among agents only open if the local event-triggered condition is satisfied in predefined time intervals. Then, the focus is placed on designing a more general triggering mechanism to get rid of continuous inter-agent communication for monitoring the triggering condition. Next, to overcome the unexpected large feedback gains in real applications and appropriately tune the feedback gains, the aperiodically intermittent adaptive event-triggered controller is further devised. For each case, detailed analysis is provided to avoid Zeno behavior. Finally, the validity for the devised control schemes is substantiated by simulations.

In chapter 5, the tracking consensus problem was investigated via aperiodically intermittent adaptive dynamic event-triggered control strategy. Firstly, the dynamic adaptive event-triggered control strategy was designed to further reduce the communication cost and the number of the control updates. Secondly, the corresponding parameters were obtained based on matrix theory and Lyapunov function. Compared with the traditional static event-triggered control in Chapter 4, the time-varying threshold ensures less triggering instants. Thirdly, we modified the event triggering mechanism that depends on combined measurement approach in Chapter 4, continuous monitoring of neighbors' states were avoided. Finally, numerical simulations were provided to illustrate the effectiveness of the theoretical results.

## Future works

The following directions will be explored in the future

- Note that the obtained results on consensus problem in this thesis focus on the homogeneous MASs, namely, all agents share identical dynamical behaviours. However, in reality, especially in engineering applications, due

## CONCLUSIONS AND PERSPECTIVES

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to the restriction of external impact, the dynamics of coupled agents are expected to be different. Therefore, it is very meaningful to deal with the consensus problem of heterogeneous MASs via intermittent event-triggered control strategy.

- On the other hand, there is very few of the previous research results concerns fractional-order dynamics via discontinuous control methods. In recent years, many researchers try their best to extend the theory and application of conventional integer-order differential systems to the situation of fractional order in various fields, such as chemical processing systems (Flores-Tlacuahuac & Biegler, 2014), physics and engineering (Sabatier *et al.*, 2007), fractional neural networks (Yu *et al.*, 2014), to name a few. What is noteworthy is that fractional-order models can describe the systems more precise than the integer-order models do due to its advantage in the description of memory and hereditary properties of various substance and processes. Many practical behaviour, such as ground vehicles moving on top of carpet, submarine underwater robots in the bottom of the sea with a large number of microorganisms and unmanned aerial vehicles operating in an environment where the influence of particles in air cannot be ignored (e.g. high-speed light in dust storm, rain, or snow) will be better described by fractional-order dynamics. Therefore, in the future, we will consider the consensus problem and stability problem for fractional-order switched MASs.
- In Chapter 2 and Chapter 3, the all weight couplings corresponding the topology of MASs were assumed to be positive, that is,  $a_{ij} > 0$  implies the cooperative relationship among agents, which is unrealistic in many practical situation. For example, in biological, social and technological networks, there actually exist both cooperative and competitive relationship. Inspired by this fact, the scaled consensus problem for fractional-order MASs via intermittent dynamic event-triggered control will be a topic for future research.



## CONCLUSIONS AND PERSPECTIVES

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# Résumé Etendu

Cette thèse porte sur le contrôle discontinu, basé sur le consensus, des systèmes multi-agents, à dynamique linéaire et non linéaire. Dans un premier temps, nous proposons des stratégies de contrôle d'épinglage adaptatif intermittent pour des systèmes multi-agents, linéaires et pour une classe non linéaire. L'objectif de ce control est d'atteindre un consensus de cluster et un consensus de décalage intra-cluster basé sur des observateurs, sous contrainte de graphe faiblement connecté, respectivement. Ensuite, un contrôle adaptatif déclenché par événement adaptatif de manière intermittente est construit pour les systèmes multi agents linéaires afin d'obtenir un consensus de suivi. L'événement ne sera déclenché que pendant l'intervalle de contrôle. En outre, afin de réduire la fréquence de mise à jour de la commande, un protocole de commande plus efficace, c'est-à-dire une commande dynamique adaptative déclenchée par événement adaptatif intermittente aperiodiquement, est conçu pour gérer le problème de consensus de suivi. En utilisant les théories, des matrices, des graphes algébriques et la stabilité des systèmes de commutation, les conditions suffisantes correspondantes sont proposées. Ces conditions garantissent les performances du système en termes de robustesse élevée, de convergence rapide et de maîtrise de la complexité de calcul. Tous nos résultats ont été démontrés avec et validés par de simulations numériques.

Dans le premier chapitre, le contexte, la motivation et la vue d'ensemble de la coordination distribuée des MASs, y compris le problème de consensus et les stratégies de contrôle, sont présentés. Par la suite, quelques préliminaires, tels que la théorie des graphes et des connaissances mathématiques sont fournis.

Dans le chapitre 2, le problème de consensus de cluster via un contrôle d'épinglage adaptatif intermittent pour les MASs avec une dynamique générale linéaire ou non linéaire est abordé, respectivement. Chaque cluster a un leader virtuel dont

l'état n'est disponible que pour une petite partie des suiveurs sur certains intervalles de temps déconnectés en raison de contraintes de communication. Afin de résoudre le problème du consensus de cluster, les agents de chaque cluster sont classés en trois types. Par conséquent, une classe de protocoles de contrôle d'épinglage adaptatif intermittent est proposée pour les différents types d'agents. En conséquence, des critères de consensus suffisants sont dérivés pour garantir que les agents d'un même cluster peuvent atteindre un consensus asymptotique tandis que les agents de différents clusters peuvent atteindre un consensus différent.

Par rapport à la littérature existante, les principaux avantages sont donnés comme suit: premièrement, contrairement à la dynamique des MASs intégrateurs, la dynamique des MASs linéaires généraux est beaucoup plus compliquée, et certains MASs intégrateurs tels que les MASs intégraux simples et les MASs doubles intégraux peut être considéré comme le cas particulier des MASs linéaires généraux. De plus, la dynamique des MASs intégrateurs ne dépend que du couplage des agents, la dynamique des MASs linéaires généraux dépend non seulement du couplage des agents, mais aussi de l'auto-dynamique régissant l'évolution de chaque agent isolé. Cela rend le consensus de cluster des MASs linéaires généraux techniquement plus difficile que le cas des MASs intégrateurs. Deuxièmement, à la différence des protocoles de contrôle continu, les gains d'épinglage dans les protocoles de contrôle proposés sont conçus pour être adaptatifs intermittents et avec un taux de convergence exponentiel, ce qui peut réduire efficacement les coûts de communication, éviter que les gains d'épinglage ne soient plus importants que ceux nécessaires dans la pratique. Pendant ce temps, il garantit que les gains d'épinglage convergent rapidement vers une valeur stable. Troisièmement, contrairement à la topologie de réseau dirigée qui doit être équilibrée en degré, fortement connectée ou contenir un arbre couvrant dirigé, une topologie faiblement connectée est prise en compte, ce qui est plus universel en pratique.

De plus, dans de nombreux systèmes réels, le retard est généralement inévitable en raison de la communication, du calcul et de l'actionnement. De plus, en raison des contraintes physiques ou des coûts de mise en IJuvre, il est assez difficile de détecter les états en temps réel des agents. Inspirés par ces discussions, dans le chapitre 3, les problèmes de consensus de décalage intra-cluster basés sur l'observateur sont étudiés plus en détail. Le soi-disant consensus de décalage intra-cluster signifie que les suiveurs d'un même cluster peuvent parvenir à un

consensus de décalage de manière asymptotique tandis que les suiveurs de différents clusters peuvent parvenir à des accords différents. Le réseau d'interaction est considéré comme faiblement connecté, c'est-à-dire qu'il n'est pas nécessaire d'être fortement connecté, équilibré en degré ou contenir un arbre couvrant dirigé. Pour réaliser le consensus de décalage intra-cluster, une classe d'observateurs est conçue pour les MASs linéaires et non linéaires généraux respectivement pour estimer les états des suiveurs. Ensuite, une classe de protocoles de contrôle d'épinglage adaptatif intermittent basés sur l'observateur est proposée. En conséquence, certains critères de consensus suffisants sont dérivés et des preuves rigoureuses sont données sur la base de la théorie des matrices et de la théorie de la stabilité de Lyapunov. Enfin, l'efficacité de la stratégie de contrôle d'épinglage adaptatif intermittent proposée est validée par une simulation numérique.

Notez que le contrôleur intermittent du chapitre 3 nécessite toujours d'être mis à jour en continu lorsqu'il est exécuté dans l'intervalle de temps de travail, ce qui est un gaspillage d'énergie et de ressources. De plus, le contrôle intermittent aperiodique est plus raisonnable et répandu dans la pratique. Inspiré par la discussion ci-dessus, le chapitre 4 étudie le consensus suivant le leader des MASs linéaires généraux via un contrôle déclenché par événement adaptatif intermittent aperiodique. La commande déclenchée par événement adaptative intermittente aperiodique hérite des avantages respectifs de la commande intermittente aperiodique, de la commande déclenchée par événement et de la commande adaptative, ce qui améliore l'efficacité de la communication, réduit la fréquence de mise à jour de la commande et est plus proche des situations pratiques.

Par rapport aux littératures existantes, il existe les principales différences suivantes. Premièrement, plusieurs auteurs ont intégré la stratégie intermittente et la stratégie déclenchée par un événement pour résoudre le problème du consensus, cependant, la méthode de contrôle déclenchée par un événement adaptatif aperiodiquement intermittent n'a pas été envisagée. Deuxièmement, certains travaux existants sur le consensus de suivi du leader des MASs par un contrôle intermittent déclenché par un événement, une communication continue entre les agents est toujours nécessaire pour vérifier la condition de déclenchement, cette exigence est supprimée dans ce chapitre. Troisièmement, la dynamique des agents dans ce chapitre est linéaire plutôt que d'ordre intégral, par conséquent, certains travaux

## Résumé Etendu

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sur le consensus suivant le leader peuvent être considérés comme des cas spatiaux de ce chapitre.

Enfin, il convient de souligner que les seuils de la fonction de déclenchement du chapitre 4 étaient dépendants de l'état. Lorsque l'erreur de mesure est égale ou supérieure au seuil, les événements sont déclenchés, ce qui peut être considéré comme les conditions de déclenchement statiques. Au début, les conditions de déclenchement statiques réduiront efficacement le coût de la communication, car elles ne sont pas faciles à satisfaire. Cependant, au fil du temps, il se déclenchera fréquemment puisque le seuil devient de plus en plus petit, ce qui provoque des instants de déclenchement inutiles. Par conséquent, dans le chapitre 5, pour réduire davantage le coût de communication et le nombre de mises à jour de contrôle, un contrôle dynamique adaptatif déclenché par événement hybride aperiodiquement intermittent est proposé. Par rapport à la commande statique traditionnelle déclenchée par un événement au chapitre 4, le seuil variable dans le temps dans la fonction de déclenchement garantit moins d'instantanés de déclenchement. De plus, nous modifions le mécanisme de déclenchement d'événement qui dépend de l'approche de mesure combinée au chapitre 4, la surveillance continue des états des voisins est évitée pour le mécanisme de déclenchement de ce chapitre.

**Titre:** Contrôle Coopératif et Discontinu basé sur le Consensus pour des Systèmes Multi-agents

**Résumé:** Cette thèse porte sur le contrôle discontinu, basé sur le consensus, des systèmes multi-agents, à dynamique linéaire et non linéaire. Dans un premier temps, nous proposons des stratégies de contrôle d'épinglage adaptatif intermittent pour des systèmes multi-agents, linéaires et pour une classe non linéaire. L'objectif de ce contrôle est d'atteindre un consensus de cluster et un consensus de décalage intra-cluster basé sur des observateurs, sous contrainte de graphe faiblement connecté, respectivement. Ensuite, un contrôle adaptatif déclenché par événement adaptatif de manière intermittente est construit pour les systèmes multi agents linéaires afin d'obtenir un consensus de suivi. L'événement ne sera déclenché que pendant l'intervalle de contrôle. En outre, afin de réduire la fréquence de mise à jour de la commande, un protocole de commande plus efficace, c'est-à-dire une commande dynamique adaptative déclenchée par événement adaptatif intermittente aperiodiquement, est conçu pour gérer le problème de consensus de suivi. En utilisant les théories, des matrices, des graphes algébriques et la stabilité des systèmes de commutation, les conditions suffisantes correspondantes sont proposées. Ces conditions garantissent les performances du système en termes de robustesse élevée, de convergence rapide et de maîtrise de la complexité de calcul. Tous nos résultats ont été démontrés avec et validés par de simulations numériques.

**Mots-clés:** Systèmes multi-agent, contrôle discontinu, contrôle intermittent, suivi du consensus distribué/suivi du leader, consensus de décalage, observateurs, dynamique nonlinéaire, paramètres inconnus, déclenché par événement adaptatif.

**Title:** Discontinuous cooperative and consensus-based control for multi-agent systems

**Abstract:** This thesis focus on discontinuous control methods for multi-agent systems with linear and nonlinear dynamics. Firstly, intermittent adaptive pinning control strategies are designed for both the linear MASs and a class of nonlinear MASs to reach cluster consensus and observer-based intra-cluster lag consensus under weakly connected graph, respectively. Then, aperiodically intermittent adaptive event-triggered control is constructed for linear MASs to achieve tracking consensus. The event only will be triggered during the control interval. Furthermore, in order to reduce the control update frequency, a more efficient control protocol, that is, aperiodically intermittent adaptive dynamic event-triggered control is designed to handle the tracking consensus problem. Finally, several numerical simulations are given to demonstrate the effectiveness of the obtained results.

**Keywords:** Multi-agent systems, intermittent control, adaptive control, pinning control, dynamic event-triggered control, tracking consensus, lag consensus, cluster consensus, time delays, nonlinear dynamics, state observer.

