# Frictional labor markets and policy interventions: dynamics and welfare implications 

Alessandra Pizzo

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# Frictional Labor markets and policy interventions: DYNAMICS AND WELFARE IMPLICATIONS 

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A Francesca, Giovanni e Stefano.
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## Avertissement

Mis à part l'introduction et la conclusion de cette thèse, les différents chapitres sont issus d'articles de recherche rédigés en anglais et dont la structure est autonome. Par conséquent, des termes "papier" ou "article" y font référence, et certaines informations, notamment la littérature, sont répétées d'un chapitre à l'autre.

## Notice

Except the general introduction and the conclusion, all chapters of this thesis are selfcontaining research articles. Consequently, terms "paper" or "article" are frequently used. Moreover, some explanations, like corresponding literature, are repeated in different places of the thesis.

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## Introduction générale

Le dysfonctionnement du marché du travail et son corollaire, le chômage, est l'une des sources de préoccupation les plus importantes pour nos sociétés.

Dans les économies développées, les performances du marché du travail sont assez hétérogènes. Certains économistes vantent le fonctionnement du marché du travail des États-Unis d'Amérique appréciant son dynamisme et sa capacité à créer des emplois en période de reprise, alors que certaines économies européennes souffrent d'un taux de chômage durablement plus élevé, depuis maintenant une trentaine d'années. D'autres, nuancent ce jugement sur le marché du travail des États-Unis en rappelant les coûts, en termes de bien-être, associés aux fortes fluctuations qui le caractérisent, les arguments avancés étant que ce marché serait caractérisé par un défaut d'assurance, face à ces fluctuations importantes : la diminution récente du taux de chômage, par exemple, est observée après une très forte hausse consécutive à la crise de 2008.

La question qui me préoccupe est la compréhension du fonctionnement de ce marché si particulier - où le bien échangé est le travail -, afin d'établir un diagnostic quant au rôle de régulation potentiel d'une autorité publique dans ce marché. L'objectif sous-jacent aux trois chapitres qui composent cette thèse concerne donc mon intérêt profond pour l'étude des interventions publiques sur le marché du travail.

Dès lors que l'on considère le marché du travail comme étant régi par les règles de la concurrence pure et parfaite, il n'y a pas beaucoup de place pour l'intervention d'une autorité. Un des outils de la théorie économique récente, qui est largement mobilisé pour expliquer le fonctionnement du marché du travail, est le modèle d'appariement à la Mortensen et Pissarides ${ }^{1}$ : dans cette vision, le marché du travail serait caractérisé par des imperfections ou "frictions".

Recruter des travailleurs nécessite du temps et des ressources ; le comportement de recrutement et de recherche d'emploi génère des externalités négatives; le niveau du chômage frictionnel résultant d'une économie décentralisée ne correspond donc pas forcement à celui qu'un planificateur bienveillant choisirait. Cette imperfection légitimise théoriquement l'intervention publique dans le fonctionnement du marché.

[^0]Cette théorie offre donc un cadre théorique pertinent pour analyser le fonctionnement du marché du travail, en relation avec les interventions de l'autorité publique au sens large, en incluant a priori tant l'autorité fiscale que monétaire.

Les questions que je pose dans cette thèse sont liées à la compréhension de certains mécanismes affectant le fonctionnement du marché du travail, avec une attention particulière portée au rôle que les politiques publiques jouent, ou devraient jouer. Les questions énoncées sont passionnantes et d'intérêt général; je n'ai pas toutefois la prétention d'apporter des réponses générales, ni des recettes miracles. L'objectif modeste de cette thèse est de mettre en lumière des mécanismes qui pourraient être utiles tant à la compréhension des faits qu'à la proposition de solutions alternatives.

J'ai adopté une approche macroéconomique dans le but d'analyser le comportement de certaines variables agrégées en tenant compte des effets d'équilibre général. J'ai aussi choisi d'analyser un éventail réduit de variables de politique économique. En termes de politiques fiscales, j'ai pris en compte la taxation, que ce soit sur le revenu du travail, du capital ou de la consommation.

En termes de variables institutionnelles, j'ai considéré les allocations chômage et le pouvoir de négociation des travailleurs et des entreprises. Enfin, en termes de politique monétaire, j'ai pris en compte le taux d'intérêt nominal, fixé à travers une règle de Taylor. L'attention portée aux politiques fiscales et institutionnelles est largement prépondérante dans cette thèse, bien que dans le premier chapitre je me concentre sur des dynamiques de plus court terme, où la règle monétaire de fixation du taux d'intérêt joue un rôle.

Si le point commun aux trois chapitres qui composent la thèse est donc la question de savoir comment les acteurs institutionnels influencent le fonctionnement du marché du travail, et comment, éventuellement, on peut imaginer qu'ils devraient plutôt intervenir, les trois chapitres se concentrent néanmoins sur des questions différentes.

Dans le premier chapitre, j'analyse, d'un point de vue purement "positif", la capacité du modèle avec frictions d'appariement à répliquer les fluctuations de court terme de variables du marché du travail, avec une attention particulière sur les États-Unis. Le cadre d'analyse est celui d'un modèle de fluctuations de court terme, avec rigidité de prix. Dans le premier chapitre, la règle de conduite de la politique monétaire détermine le mécanisme de transmission d'un choc technologique, dans le court terme.

Dans le deuxième chapitre (co-écrit avec F. Langot), nous étudions les déterminants des évolutions du taux d'emploi sur les cinquante dernières années, ainsi que les heures travaillées par employé. Nous soutenons que l'évolution du coin fiscal ${ }^{2}$, et l'évolution de deux variables reflétant l'évolution du cadre institutionnel ${ }^{3}$, permettent d'expliquer les différentes

[^1]trajectoires du taux d'emploi et des heures travaillées observées aux États-Unis et dans les économies européennes (on se concentre en particulier sur la France, l'Allemagne et le Royaume-Uni). Nous nous servons de cet outil pour évaluer les conséquences en termes de croissance, ainsi que de "bien-être d'un agent représentatif", des choix de politiques économiques de la France.

Dans le troisième chapitre, j'analyse une économie simplifiée où le risque de chômage génère de l'épargne de précaution. Le rôle de l'autorité publique peut donc consister à fournir une assurance publique, qui s'ajoute aux mécanismes d'assurance privée qui découle de l'accumulation du capital. Ce modèle stylisé permet de conclure qu'un système fiscal redistributif, caractérisé par une taxe progressive, qui finance des transferts vers les agents à bas revenus, est préférable en termes de bien-être moyen, à un système qui propose une assurance publique au travers d'allocations chômage. Ces résultats proviennent du fait que, dans le cadre d'un marché du travail frictionnel, la progressivité de la taxe aide à corriger certaines frictions qui dégradent sa performance, notamment en termes de taux d'emploi.

Même si les questions posées dans les trois chapitres diffèrent, ainsi que l'horizon temporel de référence et le cadre théorique adopté (fluctuations de courte période, transition vers le sentier de croissance équilibrée et agents hétérogènes), le modèle de base est commun aux trois papiers : un modèle avec frictions d'appariement et offre de travail individuelle.

Dans la suite, je décris de manière plus détaillée la problématique de chaque chapitre, la relation avec la littérature et ma contribution.

## Fluctuations du marché du travail : l'impact des coûts d'ouverture des postes vacants avec une application à un modèle Nouveau Keynésien

Dans ce chapitre, j'analyse la capacité du modèle d'appariement à reproduire les fluctuations de court terme des variables du marché du travail. Le point de départ de mon analyse repose sur la critique que Shimer (2005) adresse au modèle d'appariement : il soutient que ce modèle ne permet pas de reproduire les volatilités relatives des variables du marché du travail, ni leurs corrélations avec la productivité. Shimer (2005) se réfère aux deux problèmes respectivement en termes de "manque d'amplification" et "manque de propagation".

L'article de Shimer (2005) a donné lieu à une littérature foisonnante cherchant à résoudre le "puzzle" de la volatilité induite des variables du marché du travail.

Une partie de la littérature s'est focalisée sur les alternatives en termes de modélisation cherchant à pallier les failles du modèle à la Mortensen et Pissarides. C'est le cas par exemple, sans être exhaustif, des travaux de Hall (2005), Krause and Lubik (2006) or Hall and Milgrom (2008).
et le pouvoir de négociation des travailleurs .

D'un autre côté, de nombreux articles se sont concentrés sur la méthode de calibration du modèle de Shimer (2005). Ainsi Hagedorn and Manovskii (2008) montrent qu'en calibrant le coût d'opportunité de l'emploi à une valeur très élevée (en combinaison avec un pouvoir de négociation des travailleurs calibré à une valeur très basse), ils peuvent obtenir une amplification beaucoup plus forte du choc technologique.

Cet article est donc articulé comme suivant. Dans un premier temps, je montre que l'amplitude du problème souligné par Shimer (2005) diminue déjà fortement en adoptant la stratégie de calibration des modèles de cycle réel. En effet, lorsque l'on se place dans un contexte d'équilibre qénéral, comme c'est le cas par exemple de Andolfatto (1996), la stratégie de calibration permet déjà de redimensionner le problème. Je montre qu'une stratégie de calibration cohérente avec les ratios macroéconomiques, en équilibre général, implique des coûts d'ouverture des postes vacants moins élevés que dans Shimer (2005) : la conséquence est une élasticité de la tension du marché du travail par rapport à la productivité beaucoup plus élevée.

En deuxième lieu, je mets en évidence l'importance de considérer le mécanisme de transmission d'un choc technologique, dans le contexte du problème que Shimer (2005) nomme "manque de propagation". Je compare les mécanismes de transmission d'un choc technologique quand les prix ne sont pas parfaitement flexibles, avec le cas d'absence de frictions nominales. Pour ce faire, le modèle que je développe dans ce chapitre présente certaines caractéristiques importantes en termes de choix de modélisation. J'ai notamment introduit une dynamique du marché du travail selon laquelle les nouveaux appariements deviennent instantanément productifs (comme dans Blanchard and Galí (2010) par exemple) et des préférences, non-séparables entre consommation et loisir, telles qu'il n'y ait pas d'effets richesse sur l'offre de travail ${ }^{4}$.

Une première partie du chapitre est donc consacrée à expliquer les différences entre la stratégie de calibration que j'adopte avec celle de Shimer (2005). Plutôt que de fixer la valeur du non-emploi, je calibre le ratio des coûts d'ouverture des postes vacants en termes de production totale, comme c'est le cas dans la plupart des modèles de cycle réel. Ce choix, qui repose sur des faits empiriques ${ }^{5}$, implique, à travers de restrictions sur les variables endogènes d'équilibre, une valeur du non-emploi inférieure à celle proposée par Shimer. L'auteur interprète cette valeur comme donnée par les allocations chômage et il la calibre à $40 \%$ du revenu du travail.

Ma stratégie de calibration va donc dans le sens opposé à celui suggéré par Hagedorn and Manovskii (2008). Dans cet article, les auteurs réussissent à générer des fluctuations sur

[^2]le marché du travail en considérant un différentiel très étroit entre la valeur de l'emploi et du non-emploi ${ }^{6}$. Je montre aussi qu'en suivant la stratégie de calibration de Shimer (2005) dans le contexte de mon modèle, on obtient un ratio des coûts totaux d'ouverture de postes vacants presque six fois plus élevés que dans l'économie de référence. L'élasticité de la tension du marché du travail par rapport à la productivité, qui est l'une des statistiques rapportées par Shimer (2005), devient alors dix fois plus élevée en suivant ma stratégie de calibration, qu'en suivant la procédure de Shimer, appliquée à mon modèle.

Bien que les résultats en termes d'élasticité d'état stationnaire soient indicatifs, il est aussi nécessaire de considérer la dimension transitoire des chocs technologiques.

Le deuxième apport du chapitre est d'illustrer le mécanisme de transmission d'un choc technologique (temporaire) en comparant le cas analysé dans Shimer (2005), c'est-à-dire dans le contexte d'un modèle de cycle réel, avec le cas où les prix ne sont pas parfaitement flexibles (le cadre Nouveau Keynésien).

Pourquoi considérer la présence de frictions nominales sur les prix? Dans un contexte où les prix sont rigides, à court terme, le mécanisme de transmission d'un choc technologique est différent de celui décrit par Shimer (2005), qui est lui, commun aux modèles de cycle réel. Considérer la présence de rigidités des prix à court terme, permet d'expliquer un fait empirique confirmé à plusieurs reprises depuis Galí (1999) ${ }^{7}$, c'est-à-dire le fait que, suite à un choc technologique, les heures de travail dans l'économie globale baissent, à court terme.

Le modèle Nouveau Keynésien présente un ajustement non-monotone des postes vacants et de l'emploi : à l'impact du choc technologique, le nombre des postes vacants baisse, pour remonter ensuite, au fur et à mesure que les prix s'ajustent. Ce modèle introduit ainsi une source additionnelle de volatilité, par rapport au modèle de cycle réel. En outre, la présence de ces mouvements additionnels, c'est-à-dire que suite à un choc technologique, la tension du marché du travail initialement baisse et pour augmenter ensuite, permet de mieux expliquer les corrélations de variables du marché du travail avec la productivité. Toutefois, dans le chapitre, je rappelle aussi les prédictions potentiellement problématiques du modèle avec frictions nominales, notamment ses implications en termes d'autocorrélations des postes vacants et de tension du marché du travail.

À partir du papier de Galí (1999), une vaste littérature empirique a mis en évidence que dans le court terme, un choc technologique peut avoir un effet récessif sur les heures travaillées. Même si ces résultats ont donné lieu à un vaste débat, le papier de Fève and Guay (2009) semble confirmer la robustesse de ces résultats indépendamment de la manière dont les heures sont considérées (en niveau ou en différences). Plus récemment, les papiers de Balleer (2012) et Barnichon (2010) ont montré que le taux de chômage augmentait suite à un choc technologique, pour diminuer ensuite.

[^3]Les modèles Nouveaux Keynésiens peuvent expliquer un effet récessif d'un choc technologique dans le court terme : si les entreprises ne peuvent pas adapter leur prix à tout moment, la demande agrégée qui leur est adressée ne varie pas. Elles ont donc intérêt à produire la même quantité de biens en utilisant moins de travail comme facteur de production, et leur taux de marge augmente.

Dans le chapitre, je montre que dans un modèle avec frictions nominales, dans lequel l'autorité monétaire suit une règle de Taylor pour fixer son taux directeur, la transmission d'un choc technologique est non-monotone : la tension sur le marché du travail diminue au début, pour augmenter ensuite.

Dans la deuxième partie de ce chapitre, j'analyse en détail le mécanisme de transmission d'un choc technologique, à l'aide des fonctions de réponse et des diagrammes de phase : je montre que, dans le contexte d'un modèle où l'autorité monétaire réagit aux mouvements de la production et de l'inflation, sur la base d'une règle de Taylor, la présence de frictions nominales implique un effet non-monotone d'ajustement des variables.

De nombreux papiers ont fait référence à ce mécanisme, on peut citer par exemple Sveen and Weinke (2008) ou Barnichon (2014). Sveen and Weinke (2008) considèrent un modèle avec une dynamique standard sur les nouveaux appariements qui deviennent productifs à la période qui suit leur constitution. En outre, ils considèrent différents types de chocs (de demande, technologique, monétaire), alors que je me concentre exclusivement sur le choc de productivité.

Barnichon (2014) analyse un modèle avec rigidités des prix et considère l'importance de la calibration de la règle monétaire, afin d'obtenir l'effet négatif sur le chômage, d'un choc technologique à court terme, dans le contexte d'une dynamique non-traditionnelle pour les flux sur le marché du travail. Néanmoins, le mécanisme principal dans son modèle passe à travers l'effort endogène dans l'offre de travail. Je reprends le papier de Van Zandweghe (2010) pour expliciter les différentes implications du modèle avec rigidités nominales, selon l'hypothèse retenue pour la dynamique du fonctionnement du marché du travail.

Le chapitre contient une section consacrée aux tests de robustesse, où je montre que dans un modèle Nouveau Keynésien caractérisé par une dynamique standard des flux sur le marché du travail, l'introduction de rigidités nominales exacerbe le problème de Shimer (2005), comme dans Sveen and Weinke (2008). Le contraire est vrai quand la dynamique adoptée est celle qui considère les nouveaux appariements comme étant instantanément productifs. À l'origine de ces différences, on trouve le comportement des postes vacants : dans le cas du modèle Nouveau Keynésien consideré comme référence, un choc technologique engendre une baisse du nombre des postes vacants, puis ensuite une hausse. Ceci explique le fait qu'ils présentent une autocorrélation négative. Je montre aussi que le choix de la dynamique du marché du travail a un effet beaucoup moins important quand les prix sont parfaitement flexibles.

Enfin, les tests de robustesse montrent que les résultats ne sont pas affectés lorsque les heures par travailleur peuvent aussi varier. Ils confirment aussi l'importance du choix de la forme des préférences, l'idée étante de ne pas introduire des mécanismes additionnels par rapport à Shimer (2005), liés à l'effet richesse.

## Expliquer les disparités de l'offre de travail globale

Dans ce chapitre, coécrit avec F. Langot, nous considérons l'évolution des heures travaillées au cours des cinquante dernières années aux États-Unis et dans trois économies européennes, qui sont la France, l'Allemagne et le Royaume-Uni.

En effet, les heures travaillées présentent une évolution très différente au sein des économies développées: elles sont stables ou en légère augmentation aux États-Unis, alors que dans les deux économies de l'Europe continentale elles montrent une baisse marquée. Le Royaume-Uni se place entre ces deux expériences extrêmes, avec une évolution qui est similaire à celle de la France et de l'Allemagne au début de l'échantillon, puis en se rapprochant plutôt de l'expérience des États-Unis dans les dernières années.

L'évolution des heures totales travaillées peut, en outre, être décomposée en différents facteurs. Nous choisissons de considérer les heures travaillées comme données par le nombre d'employés (marge extensive) multiplié par les heures travaillées par employé (marge intensive). L'évolution de ces deux marges montre une tendance très forte à la baisse des heures travaillées par employé en Allemagne et France, alors que les performances en termes de taux d'emploi de l'Allemagne et du Royaume-Uni se rapprochent beaucoup sur la dernière partie de l'échantillon. La France est caractérisée à la fois par la baisse des heures par employé, et par une performance relativement mauvaise du taux d'emploi, surtout à partir des années quatre-vingt.

L'hétérogénéité dans les évolutions des heures travaillées parmi les économies considérées nous amène à nous poser une question : quelles sont les politiques économiques derrière ces évolutions majeures? Comment peut-on évaluer leur impact sur la croissance globale ou le bien être agrégé?

Les différentes variables des politiques économiques susceptibles de jouer un rôle sur les heures travaillées, peuvent se décomposer de la manière suivante : $i$ différences en termes de politiques fiscales (taxes sur le revenu du travail, sur le capital, mais aussi sur la consommation) ; ii différences institutionnelles du marché du travail, en particulier sur la compensation du non-emploi (sur les allocations chômages), ainsi que le pouvoir de négociation des salariés, pour ce qui concerne la fixation de la compensation salariale (salaire horaire et heures travaillées).

Pour répondre aux questions énoncées, nous développons un modèle structurel que nous utilisons pour révéler certains paramètres, et ensuite réaliser des expériences contrefactuelles :
nous imaginons des scénarios alternatifs en regardant des évolutions différentes des variables de politique économique.

Les résultats de ce chapitre montrent que, pour la France, les coûts en termes de PIB "perdu" ainsi qu'en termes de bien-être agrégé, induits par le coin fiscal, sont plus importants que ceux dus aux institutions du marché du travail. En outre, nous montrons aussi qu'il existe de "complémentarités" entre, d'un côté les reformes structurelles ayant un impact sur la fiscalité, de l'autre les institutions du marché du travail, car elles affectent à la fois les deux marges (intensive et extensive) qui composent le facteur de production agrégé travail.

Il existe une littérature importante qui a analysé l'impact des taxes sur les évolutions des heures travaillées. Souvent, toutefois, c'est la totalité des heures travaillées à être considérée (sans donc différencier la marge intensive et extensive).

Prescott (2004) soutient que l'évolution de la politique fiscale explique la totalité des différences des heures travaillées entre deux points dans le temps (la moitié des années 1970 et la moitié des années 1990) dans les économies développées de son échantillon. Il évalue les gains de bien-être pour la France d'une baisse de son coin fiscal de vingt points (de sorte à atteindre le niveau de taxation des États-Unis) à dix-neuf points de pourcentage (pp) en termes de consommation.

Ohanian et al. (2008) réalisent le même exercice que Prescott (2004), mais en incluant cinquante pays de l'OCDE sur toute la période allant des années 1950 au début des années 2000.

Ljungqvist and Sargent (2007) critiquent les résultats de Prescott (2004) : si l'on introduit aussi une mesure de l'ampleur de la couverture sociale (c'est-à-dire une mesure de l'indemnisation du non-emploi fournie par le Gouvernement), qui est plus élevée dans les pays Européens que ce n'est le cas aux États-Unis, le modèle de Prescott prédirait une offre de travail pour les pays du Vieux Continent encore moins élevée que dans les données. Ljungqvist and Sargent (2007) considèrent en fait que c'est l'ampleur de l'indemnisation du non-emploi qui influence les heures travaillées, et non pas les taxes.

Un autre courant de la littérature a considéré en plus des taxes, d'autres variables "institutionnelles" comme étant à l'origine des évolutions des heures travaillées. Nous pouvons citer par exemple Layard and Nickell (1999) ou Blanchard and Wolfers (2000). Dans le premier papier, les auteurs rapportent les conclusions de la littérature autour de l'impact de cinq types de variables institutionnelles (taxes sur le travail, protection de l'emploi, rôle des syndicats et salaire minimum ; générosité du système d'indemnisation du chômage et politiques actives pour l'emploi ; éducation et formation). Le deuxième papier cité considère l'interaction de ces variables avec les chocs macroéconomiques. Par rapport à cette littérature, nous choisissons de nous concentrer sur deux groupes de variables institutionnelles au-delà de la taxation : la générosité de compensation du non-emploi et le pouvoir de négociation des travailleurs. Les conclusions de cette littérature empirique ne sont pas toujours
sans ambigüité. Toutefois, l'on peut retenir que la générosité du système de compensation du chômage semble affecter négativement le taux de chômage. Un effet négatif sur le taux d'emploi serait aussi attribué au pouvoir des organisations syndicales, dans un contexte de négociation partiellement décentralisée.

Dans ce chapitre, nous étudions donc l'impact des variables institutionnelles, y compris la taxation, sur l'évolution des heures totales travaillées, exprimées comme le produit entre le taux d'emploi et les heures par employé. A la différence de Layard and Nickell (1999) et Blanchard and Wolfers (2000), nous développons un modèle structurel avec croissance équilibrée, pour expliquer l'évolution des heures tout au longs des cinquante dernières années.

Nous utilisons en fait la méthodologie proposée par McDaniel (2011), qui développe un modèle de croissance équilibrée pour étudier l'évolution des heures travaillées, en utilisant comme variables explicatives la taxation ainsi que la productivité (exogène). Par rapport à McDaniel (2011), nous ne considérons que les heures marchandes, mais nous introduisons des frictions sur le marché du travail, ce qui nous permet d'intégrer les variables institutionnelles telles que les allocations chômage et le pouvoir de négociation des travailleurs et des entreprises.

Notre analyse d'état stationnaire est construite en suivant Fang and Rogerson (2009) : les auteurs développent en fait un modèle avec chômage frictionnel et heures travaillées pour analyser les interactions entre les deux marges, intensive et extensive. Ils montrent que les deux marges fonctionnent comme des "substituts" pour les ménages et les entreprises. Leur modèle, toutefois, ne considère pas le capital ; leur exercice consistant à fournir des résultats analytiques pour des comparaisons qualitatives. Nous partons de leur intuition pour construire un modèle qui donne des prédictions quantitatives sur la dynamique des heures, en plus des variations de l'état stationnaire.

Nous partons du travail de Langot and Quintero Rojas (2008), qui développent un modèle comprenant les deux marges (extensive et intensive) de l'offre de travail, dans le contexte d'un marché du travail frictionnel en équilibre général, avec aussi la dynamique du capital physique. Les auteurs toutefois utilisent la méthodologie de Ohanian et al. (2008) : ils considèrent les conditions d'équilibre (statiques) du modèle pour calculer les différences entre les conditions d'optimalité de premier ordre du modèle et celles vérifiées dans les données. En outre, comme dans Prescott (2004), ils ont recours à une élasticité de l'offre de travail plus élevée que normalement acceptée d'après les études micro-économétriques.

Pour résumer, le modèle développé dans ce chapitre inclut donc les frictions sur le marché du travail et les heures travaillées par employé. Les heures et le salaire horaire sont fixés à travers une négociation entre travailleur et entreprise. Les variables considérées comme exogènes sont les institutions du marché du travail, les taxes et la productivité (mesurée par résidu de Solow). Le modèle est calibré en utilisant les données de quatre pays : en particulier, nous révélons, à travers la méthode des moments simulés, les paramètres de préférences par
rapport à la consommation et le loisir, en imposant qu'ils soient les mêmes pour les quatre pays. Les différences de comportement peuvent donc être interprétées comme induites par les variables de politique économique, et non pas par les différences de préférences ${ }^{8}$.

En suivant Ohanian et al. (2008) et McDaniel (2011), nous intégrons aussi la présence d'un terme de consommation de subsistance dans les préférences, de manière à tenir compte de la différence, en termes de conditions initiales de richesse, entre les États-Unis et les pays européens (au début des années Soixante, les pays européens étaient encore en phase de reconstruction, après la Seconde Guerre Mondiale).

Une fois le modèle calibré, nous testons sa capacité à reproduire le sentier des variables d'intérêt au cours des cinquante dernières années; le modèle est capable de reproduire les évolutions des heures travaillées et du taux d'emploi. La performance du modèle, mesurée en termes d'erreur quadratique moyenne, est meilleure pour les États-Unis et la France que pour le Royaume-Uni et l'Allemagne. Dans la suite de ce chapitre, nous utilisons le modèle pour réaliser des expériences contrefactuelles en simulant la trajectoire de la France avec les variables de politiques économiques caractéristiques des États-Unis. Cet exercice nous permet d'évaluer les conséquences des élasticités estimées.

Finalement, nous évaluons les conséquences des politiques menées en France en termes de bien-être agrégé et de croissance du PIB : nous comparons les évolutions historiques des heures travaillées et du taux d'emploi en France avec les séries contrefactuelles, obtenues en supposant un optimum de premier rang comme mesure de référence (c'est-à-dire une situation où la condition d'optimalité de Hosios est respectée).

Les conclusions que nous dérivons de notre exercice sont alors les suivantes : les coûts en termes de PIB ainsi que de bien-être de l'agent représentatif impliqués par le coin fiscal sont plus élevés que ceux induits par les institutions du marché du travail proprement dites; ils existent des complémentarités, car les effets induits de réformes simultanées sont plus importants que les bénéfices liés à la somme de réformes opérées de façon indépendante. Nous évaluons les gains que procurerait l'élimination du coin fiscal à neuf pp (en moyenne sur toute la période considérée) en termes de consommation pérenne ${ }^{9}$.

Notre évaluation est donc bien plus réduite que celle proposée par Prescott (2004) ${ }^{10}$.

[^4]
## Les effets en termes de bien-être d'un système fiscal progressif et de l'assurance publique dans le contexte d'un marché du travail frictionnel

Dans le troisième chapitre, j'analyse la performance et la désirabilité de deux systèmes alternatifs de sécurité sociale. Je considère un cadre dans lequel l'assurance "privée" n'est pas parfaite : les agents sont soumis à un risque de chômage, contre lequel ils ne peuvent pas s'assurer complètement. Un des rôles du gouvernement peut donc consister à intervenir en tant que fournisseur d'assurance.

J'étudie dans ce chapitre deux systèmes: d'un côté un système fiscal redistributif, basé sur une taxe progressive, qui finance des transferts pour les agents à bas revenu; de l'autre, des transferts pour les individus qui deviennent chômeurs, financés par une taxe proportionnelle. En effet, je considère un système où les deux mécanismes (taxe progressive et assurance chômage) peuvent aussi être présents au même moment.

L'objectif de ce chapitre est de montrer les effets des deux types de politiques considérées en termes de performance de l'économie, c'est-à-dire en termes de conséquences sur le taux de chômage, les heures travaillées, l'accumulation du capital, ainsi que d'évaluer les deux politiques en termes de bien-être.

Les résultats de ce chapitre montrent que le système fiscal progressif est supérieur, en termes de bien-être agrégé, à l'assurance fournie à travers des allocations chômage. En termes de performance du marché du travail, une taxe progressive contribue en fait à réduire le taux de chômage, alors que l'assurance chômage a l'effet contraire.

Le modèle utilisé dans ce chapitre est une extension de celui développé par Krusell et al. (2010) : le cadre comprend une économie caractérisée à la fois par un marché financier incomplet et un marché du travail frictionnel. Les agents sont soumis au risque de devenir chômeurs, et ils ne peuvent pas s'assurer contre ce risque. Une partie de l'épargne privée est donc une épargne de précaution : à travers la richesse accumulée, les agents s'assurent contre la réduction de revenu anticipée en cas de chômage.

Mon intérêt se concentre sur un cadre sans incertitude agrégée, alors que Krusell et al. (2010) étudient aussi les fluctuations en cas d'incertitude macroéconomique : dans mon modèle, les agents sont donc soumis seulement au risque idiosyncratique de chômage. J'introduis dans ce cadre la possibilité pour les agents de moduler leur offre de travail, s'ils sont employés. Comme dans le modèle du chapitre précèdent, les heures de travail sont fixées à travers une négociation entre le salarié et l'entreprise ${ }^{11}$.

Dans le cadre d'un marché du travail frictionnel comme celui analysé, une taxe progres-

[^5]sive peut avoir des effets positifs sur le fonctionnement du marché, en particulier un impact bénéfique sur le taux de chômage. Le mécanisme à l'œuvre a été étudié par la littérature sur les marchés frictionnels, dans le cadre d'un agent représentatif, par exemple par Pissarides (1998), Sørensen (1999) ou Holmlund and Kolm (1995). Dans le processus de négociation salariale classique, l'entreprise et le travailleur doivent s'accorder sur la division du surplus généré par la formation de l'emploi. La mise en place d'une taxe progressive réduit le coût pour le travailleur de renoncer à une partie de ce surplus, et elle augmente celui pour l'entreprise de fournir au salarié une augmentation du salaire net, de façon qu'il y ait une convergence d'intérêts vers des revendications salariales mineures. Les pressions à la baisse sur les salaires ont comme conséquence une baisse du chômage.

Plus récemment, le papier de Parmentier (2006) a étudié les conséquences d'une taxation progressive dans le cadre d'un marché à la Mortensen et Pissarides, où l'auteur introduit aussi les heures travaillées. Les résultats théoriques de Parmentier (2006) montrent que, $a$ priori, l'effet de la progressivité de la taxe sur le taux de chômage est ambigu, dépendant de la manière dont est modélisée la compensation du chômage. Ses exercices numériques montrent que, dans presque tous les cas analysés, le taux marginal de taxation sur le revenu du travail (à taux moyen inchangé) a un effet de réduction du taux de chômage, ainsi qu'un effet non-monotone sur l'efficacité économique (il existe donc un taux marginal optimal).

Dans ce chapitre, le point de départ de mon analyse repose sur le mécanisme illustré par la littérature du marché du travail, pour ce qui concerne l'effet d'une taxe progressive, et j'introduis deux dimensions nouvelles. En premier lieu, je considère la dimension de l'hétérogénéité du niveau de richesse accumulée (liée au defaut d'assurance) ; en deuxième lieu, je considère la totalité des effets d'équilibre général, qui passent aussi à travers l'épargne (de précaution).

Je considère dans ce chapitre la même forme fonctionnelle pour le revenu disponible que Sørensen (1999) and Holmlund and Kolm (1995) : dans le cas où les agents ont des niveaux hétérogènes de revenus (et richesse), cette forme fonctionnelle implique la présence de "taxes négatives", c'est-à-dire de transferts, pour les agents à bas revenu.

Les effets d'équilibre général, dans le modèle développé dans ce chapitre, résultent du fait que les agents peuvent épargner en accumulant un actif (capital physique), qui rentre dans la fonction de production, comme dans le modèle classique de Aiyagari (1994). En outre, le taux marginal de la taxation ne peut pas changer sans que le taux moyen soit aussi changé : le budget de l'État est toujours à l'équilibre.

L'autre courant de la littérature inspirant ce chapitre inclut des travaux qui analysent l'impact des politiques économiques considérées (la progressivité de la taxe d'un côté, l'assurance chômage de l'autre), dans le cadre d'économies caractérisées par des marchés financiers imparfaits. Pour ce qui concerne les travaux sur la progressivité de la taxe dans le contexte d'assurance incomplète, Heathcote et al. (2014) développent un modèle théorique avec offre
de travail et investissement en capital humain, en utilisant la même famille de taxes et de transferts que Sørensen (1999). Ils montrent que, une fois calibré pour les États-Unis, le modèle implique un niveau optimal de progressivité inférieur à celui observé.

Le niveau de progressivité d'un système fiscal, dans mon modèle, comme dans celui de Heathcote et al. (2014) et de Sørensen (1999), est mesuré en termes du "coefficient de progression du revenu résiduel".

Bakış et al. (2015) développent un modèle avec générations imbriquées et héritage; ils considèrent aussi la même famille de taxes et de transferts utilisée par Heathcote et al. (2014), et analysent non seulement les équilibres d'état stationnaire, mais aussi le sentier de transition. Ils montrent qu'en regardant seulement l'état stationnaire, l'optimum serait atteint avec un système de taxation régressif, alors qu'en considérant les coûts de la transition en termes d'épargne additionnelle, le niveau optimal de progressivité est pratiquement égal à celui observé aux États-Unis.

Par rapport à ces travaux, le modèle développé dans ce chapitre est plus simple : il ne considère pas de chocs sur la productivité, ni d'épargne pour motif de transferts intergénérationnels ou de l'accumulation de capital humain. Il ne peut donc pas reproduire les caractéristiques de la distribution de la richesse. J'introduis en revanche un marché du travail frictionnel, pour étudier les effets de la progressivité de la taxe et de l'assurance chômage dans ce contexte.

Krusell et al. (2010) introduisent un marché du travail frictionnel à la Mortensen et Pissarides dans une économie à la Aiyagari (1994) : je pars donc de leur modèle, et je considère en plus une offre de travail avec choix du nombre d'heures travaillées. Krusell et al. (2010) utilisent leur modèle, en état stationnaire, pour étudier l'optimalité du niveau de l'allocation chômage ; leur conclusion est que l'assurance privée étant assez efficace, l'assurance publique optimale est plutôt basse.

Le modèle de ce chapitre confirme les résultats de Krusell et al. (2010) quant au fait que l'impact de l'assurance chômage sur le bien-être agrégé est plutôt négatif, en considérant de simples comparaisons d'état stationnaire. Dans ce chapitre, en premier lieu, je décris les effets d'un système de taxes et transferts progressifs sur les variables macroéconomique et le bien-être. En deuxième lieu, j'analyse les effets de l'assurance chômage, quand les dépenses publiques sont financées par une taxe proportionnelle. Enfin, je considère un système qui permet d'intégrer l'assurance chômage au système fiscal progressif. Je compare donc les états stationnaires de l'économie qui résultent des différentes combinaisons de politiques. Les résultats montrent que le plannificateur préfère toujours l'outil progressif seul.

Une première limite de ce chapitre repose sur la caractéristique statique du modèle : la transition entre différents états stationnaires n'est pas prise en compte. Toutefois, l'introduction de cet aspect devrait plutôt renforcer les conclusions qui découlent de l'analyse de statique comparative, vu que le niveau optimal de la progressivité implique un niveau
de capital agrégé inférieur au cas avec, par exemple, une taxe proportionnelle : pendant la transition, les agents vont donc pouvoir consommer plus.

La deuxième limite, plus importante, est que les agents sont homogènes en termes de productivité : une taxe progressive ainsi n'a pas l'effet négatif de décourager l'offre de travail des agents les plus productifs. Toutefois, un volet de la littérature, dans le cadre d'un marché du travail Walrasien, a aussi mis en évidence que, si les agents sont soumis à de chocs idiosyncratiques de productivité, l'offre de travail individuelle s'avère être "trop" élevée, par rapport au cas avec marchés complets. Pijoan-Mas (2006) parle donc d'une "offre de travail de précaution", en relation avec le phénomène de l'épargne de précaution. Alonso-Ortiz and Rogerson (2010) considèrent ce phénomène pour conclure qu'un système de taxes (proportionnelles) et de transferts, dans le cadre d'un modèle avec marchés financiers incomplets, peut donc être optimal. Prendre en considération cette dimension d'hétérogénéité est donc important pour évaluer dans son ensemble les effets de la progressivité.

Les résultats de ce chapitre sont donc à considérer comme une ouverture sur un projet de recherche plus vaste. Deux extensions possibles seraient donc de reprendre les résultats en introduisant une dimension d'hétérogénéité en termes de productivité ainsi qu'une dimension dynamique.

Le chapitre contient aussi une partie dédiée à des tests de robustesse, pour mettre en évidence l'impact de certaines hypothèses faites lors de la calibration. Je montre en particulier que plus l'élasticité de l'offre de travail est élevée, plus le niveau optimal de progressivité est faible. L'intuition de ce résultat est que plus les heures de travail réagissent à des variations de la taxe, plus l'offre de travail sera faible lorsque la progressivité augmente. Un système fiscal plus progressif réduit plus fortement l'offre de travail, quand les heures sont davantage élastiques.

Résultat sans doute moins intuitif, je montre que si les heures travaillées sont fixes, la taxe optimale n'est pas progressive (mais régressive). Dans ce cas, les heures travaillées ne baissent pas, suite à une augmentation de la progressivité, mais l'effet positif sur le taux de chômage est aussi moins fort, et la taxe n'a pas pour effet positif d'augmenter le loisir. La progressivité de la taxe "compense" les frictions préexistantes sur le marché du travail : les tests de robustesse montrent que si le pouvoir de négociation des travailleurs est plus (moins) élevé que le coefficient avec lequel le chômage rentre dans la fonction d'appariement, le niveau optimal de progressivité est plus (moins) élevé, par rapport au cas où les deux paramètres sont calibrés à la même valeur. Enfin, plus le niveau de frictions (sous la forme du coût d'ouverture des postes vacants) est élevé, plus le niveau optimal de progressivité est élevé.

## Bibliographie

John M. Abowd and Francis Kramarz. The costs of hiring and separations. Working Paper 6110, National Bureau of Economic Research, July 1997.

S Rao Aiyagari. Uninsured Idiosyncratic Risk and Aggregate Saving. The Quarterly Journal of Economics, 109(3) :659-84, August 1994.

Alberto F. Alesina, Edward L. Glaeser, and Bruce Sacerdote. Work and Leisure in the U.S. and Europe : Why So Different? In NBER Macroeconomics Annual 2005, Volume 20, NBER Chapters, pages 1-100. National Bureau of Economic Research, Inc, June 2006.

Jorge Alonso-Ortiz and Richard Rogerson. Taxes, transfers and employment in an incomplete markets model. Journal of Monetary Economics, 57(8) :949-958, November 2010.

David Andolfatto. Business Cycles and Labor-Markand Search. American Economic Review, 86(1) :112-32, March 1996.

Ozan Bakış, Barış Kaymak, and Markus Poschke. Transitional dynamics and the optimal progressivity of income redistribution. Review of Economic Dynamics, 18(3) :679-693, 2015.

Almut Balleer. New evidence, old puzzles : Technology shocks and labor market dynamics. Quantitative Economics, 3(3):363-392, 2012.

Regis Barnichon. Productivity and unemployment over the business cycle. Journal of Monetary Economics, 57(8) :1013-1025, November 2010.

Regis Barnichon. The shimer puzzle and the endogeneity of productivity. mimeo, 2014.
Olivier Blanchard and Jordi Galí. Labor markets and monandary policy : A new keynesian model with unemployment. American Economic Journal : Macroeconomics, 2(2) :1-30, 2010.

Olivier Blanchard and Justin Wolfers. The Role of shocks and institutions in the rise of European unemployment : The aggregate evidence. The Economic Journal, 110(462) : C1-C33, March 2000.

Lei Fang and Richard Rogerson. Policy Analysis In A Matching Model With Intensive And Extensive Margins. International Economic Review, 50(4) :1153-1168, November 2009.

Patrick Fève and Alain Guay. The response of hours to a technology shock : A two-step structural var approach. Journal of Money, Credit and Banking, 41(5), 2009.

Jordi Galí. Technology, employment, and the business cycle : Do technology shocks explain aggregate fluctuations? American Economic Review, 89(1) :249-271, 1999.

Marcus Hagedorn and Iourii Manovskii. The cyclical behavior of equilibrium unemployment and vacancies revisited. American Economic Review, 98(4) :1692-1706, 2008.

Robert E. Hall. Employment fluctuations with equilibrium wage stickiness. American Economic Review, 95(1) :50-65, 2005.

Robert E. Hall and Paul R. Milgrom. The limited influence of unemployment on the wage bargain. American Economic Review, 98(4) :1653-74, 2008.

Jonathan Heathcote, Kjetil Storesletten, and Giovanni L. Violante. Optimal tax progressivity : An analytical framework. Working Paper 19899, National Bureau of Economic Research, February 2014.
B. Holmlund and A.S. Kolm. Progressive Taxation, Wage Setting, and Unemployment , Theory and Swedish Evidence. Technical report, 1995.

Michael U. Krause and Thomas A. Lubik. The cyclical upgrading of labor and on-the-job search. Labour Economics, 13(4) :459-477, August 2006.

Per Krusell, Toshihiko Mukoyama, and Ayşegül Şahin. Labour-market matching with precautionary savings and aggregate fluctuations. 77(4) :1477-1507, 2010.

François Langot and Coralia Quintero Rojas. Explaining the Evolution of Hours Worked and Employment across OECD Countries : An Equilibrium Search Approach. IZA Discussion Papers 3364, Institute for the Study of Labor (IZA), February 2008.

Richard Layard and Stephen Nickell. Labor market institutions and economic performance. In O. Ashenfelter and D. Card, editors, Handbook of Labor Economics, volume 3 of Handbook of Labor Economics, chapter 46, pages 3029-3084. Elsevier, 1999.

Lars Ljungqvist and Thomas J. Sargent. Do Taxes Explain European Employment? Indivisible Labor, Human Capital, Lotteries, and Savings. In NBER Macroeconomics Annual 2006, Volume 21, NBER Chapters, pages 181-246. National Bureau of Economic Research, Inc, 2007.

Cara McDaniel. Forces shaping hours worked in the OECD, 1960-2004. American Economic Journal : Macroeconomics, 3(4) :27-52, October 2011.

Dale T. Mortensen and Christopher A. Pissarides. New developments in models of search in the labor market. In O. Ashenfelter and D. Card, editors, Handbook of Labor Economics, volume 3 of Handbook of Labor Economics, chapter 39, pages 2567-2627. Elsevier, 1999.

Lee Ohanian, Andrea Raffo, and Richard Rogerson. Long-term changes in labor supply and taxes : Evidence from OECD countries, 1956-2004. Journal of Monetary Economics, 55 (8) :1353-1362, November 2008.

Alexis Parmentier. The effects of the marginal tax rate in a matching model with endogenous labor supply. Discussion papers (econ - département des sciences economiques), Université catholique de Louvain, Département des Sciences Economiques, April 2006.

Josep Pijoan-Mas. Precautionary Savings or Working Longer Hours? Review of Economic Dynamics, 9(2) :326-352, April 2006.

Christopher A. Pissarides. The impact of employment tax cuts on unemployment and wages ; the role of unemployment benefits and tax structure. European Economic Review, 42(1) : $155-183,1998$.

Christopher A. Pissarides. Equilibrium Unemployment Theory, 2nd Edition, volume 1 of MIT Press Books. The MIT Press, June 2000.

Edward C. Prescott. Why do Americans work so much more than Europeans? Quarterly Review, (Jul) :2-13, 2004.

Robert Shimer. The Cyclical Behavior of Equilibrium Unemployment and Vacancies. American Economic Review, 95(1) :25-49, March 2005.

José Ignacio Silva and Manuel Toledo. Labor turnover costs and the cyclical behavior of vacancies and unemployment. Macroeconomic Dynamics, 13, 52009.
P.B. Sørensen. Optimal tax progressivity in imperfect labour markets. Labour Economics, $6(3): 435-452,1999$.

Tommy Sveen and Lutz Weinke. New keynesian perspectives on labor market dynamics. Journal of Monandary Economics, 55(5) :921-930, 2008.

Willem Van Zandweghe. On-the-job search, sticky prices, and persistence. Journal of Economic Dynamics and Control, 34(3) :437-455, March 2010.

## Chapter 1

## Labor market fluctuations: the impact of vacancy posting costs with an application to a New Keynesian model

### 1.1 Introduction

Models with labor market frictions à la Mortensen and Pissarides ${ }^{1}$ became the workhorse to explain the behavior of labor market variables. However, Shimer (2005) in a seminal paper questions the ability of such models to reproduce credible fluctuations of labor market variables: in particular, Shimer (2005) considers productivity and separation shocks and states (p.40): "Not only is there little amplification, but there is also no propagation of the labor productivity shock in the model."

A vast literature followed ${ }^{2}$ : some researchers focused on the calibration strategy of the model adopted by Shimer (2005) (for example, Hagedorn and Manovskii (2008)); another research strand focused on the modelling strategy, by adding new model features: for example, as a non-exhaustive list, Hall (2005) considers wage rigidities, Krause and Lubik (2006) introduce endogenous job destruction, Hall and Milgrom (2008) propose a different bargaining rule.

In addition to models in the real business cycle tradition, search and matching frictions in the labor market have also been extensively introduced in New Keynesian models, allowing to study the interactions between labor market variables, inflation and the conduct of monetary

[^6]policy ${ }^{3}$. The New Keynesian framework has also been studied to check if the interaction of labor frictions with nominal frictions could create more volatility in labor market variables, as for example in Andres et al. (2012).

The objectives of this paper are twofold: first of all, I stress the importance of the calibration strategy and its implications for the vacancy posting costs. To do so, I use a model which is as close as possible to Shimer (2005), while still allowing for general equilibrium effects (and potentially allowing for real effects of nominal frictions). I state that following the real business cycle models tradition, as for example Andolfatto (1996), in calibrating vacancy posting costs already reduces the problem of the amplification mechanism of a productivity shock ${ }^{4}$. In terms of steady state elasticity of labor market tightness, the model implies a value ten times higher than if the model is calibrated following the approach of Shimer (2005) ${ }^{5}$.

Secondly, I apply this calibration procedure to a stripped-down version of a model with frictions on the labor market and monopolistic competition in the final goods market, to check the performance of the model in terms of some informative moments (the relative volatilities and correlations with respect to productivity, as well as autocorrelations). This basic model allows to study the transmission mechanism of a productivity shock in a framework which embeds flexible as well sticky prices: I use this model in order to highlight the different implications of the two settings in answering the "propagation" as well as the "amplification" issues raised by Shimer (2005). This exercise can be interpreted as a critical review of some results of the literature, and an extension to it in the effort of disentangling the mechanisms and isolating the effects of different hypothesis.

In considering the model with nominal rigidities, I take into consideration all the literature which, starting from Galí (1999), has stressed the idea that productivity shock, in the short run, can have a negative effect on labor input ${ }^{6}$ : in particular, as Balleer (2012) writes, it seems appropriate to talk about a "job finding puzzle", referring to the fact that empirical evidence seems to indicate a decrease in the job finding rate (and an increase in unemployment) after a positive technological innovation. In this regard, I close the model by supposing a simple Taylor rule for the conduct of monetary policy, and in calibrating its parameters, I consider

[^7]that the monetary authority reacts to both inflation and output. I follow Clarida et al. (2000) in choosing the values of the parameters of the interest rate rule as those identified for the Volcker-Greenspan period: this calibration allows to obtain a recessionary effect of a positive productivity shock on employment ${ }^{7}$.

I analyse the performance of the two versions of the model (with flexible and sticky prices) in terms of their implications for not only the relative volatilities of labor market variables, but also their unconditional correlations with productivity and their (first order) autocorrelations. The volatility of labor market tightness implied by the model with flexible prices is half of its correspondent in the data, while it is four fifths in the model with sticky prices. The two models have also different implications for what it regards the correlations and the autocorrelations ${ }^{8}$.

It is important to stress that I consider exclusively a productivity shock, while I am also aware that a part of the literature stressed the importance of studying the volatilities and correlations of labor market variables, conditionally on the type of shock ${ }^{9}$ : in general, works in the New Keynesian literature highlight the need of introducing also demand shocks (as monetary policy shocks, preference shocks and so on).

I choose to focus only on productivity shocks because my objective is to clarify the implications of the different features of the model, once I consider my preferred approach for the calibration of vacancy posting costs: the purpose is neither to match all the business cycles moments of labor market series, nor to solve the puzzles raised by Shimer (2005) by adding other types shocks ${ }^{10}$.

Lastly, the present work contains a number of checks done with alternative modelling hypothesis, to show in a transparent way the importance of each assumption made in the baseline case ${ }^{11}$.

For what it regards the previous literature, I build on Balleer (2012), Barnichon (2010a) and Barnichon (2014) who stress the importance of considering the transmission mechanism of a productivity shock and of including demand side shocks. Differently from them, I do

[^8]not include other shocks than the productivity, and from the point of view of the model I do not consider variable effort as in Barnichon (2010a) and Barnichon (2014).

My modelling choices are closer to the simpler setting in Kurozumi and Zandweghe (2010), henceforth KZ (2010): they study the stability of the Taylor rule parameters in a setting with only inflation expectation shocks, while I focus on the implications of the transmission mechanism of a productivity shock.

### 1.2 The Shimer (2005) puzzle(s) in the data

### 1.2.1 The values of relative volatilities

The first Shimer puzzle consists in the fact that the labor market variables, obtained by his theoretical model, are ten times less volatile than in the data. I report in Table 1.1 the empirical moments identified by Shimer (2005) as the targets to reproduce. The lack of amplification of a productivity shock is evident by comparing the first row of Table 1.2 to the first row of Table 1.1.

Table 1.1 - Volatilities of labor market variables, Shimer (2005)

| Summary statistics, quarterly US data, $1951-2003$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| St dev | $U$ | $V$ | $\theta$ | $p$ | productivity |
| Correlation matrix | 0.190 | 0.202 | 0.382 | 0.118 | 0.020 |
| $U$ | 1 | -0.894 | -0.971 | -0.949 | -0.408 |
| $V$ |  | 1 | 0.975 | 0.897 | 0.364 |
| $\theta$ |  |  | 1 | 0.948 | 0.396 |
| $p$ |  |  |  | 1 | 0.396 |
| productivity |  |  |  |  | 1 |

( $U$ stands for unemployment, $V$ for vacancies, $\theta$ for labor market tightness, $p$ is job finding rate and productivity is the average labor productivity -the seasonally adjusted real average output per person in the non-farm business sectorconstructed by BLS; variables are expressed in logs as deviations from HP trend with smoothing parameter $10^{5}$ )

Summary statistics: relative volatilities

|  | $U$ | $V$ | $\theta$ | $p$ | productivity |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{\sigma(X)}{\sigma(\text { productivity })}$ | 9.5 | 10.1 | 19.1 | 5.9 | 1 |

# Table 1.2 - Model volatilities conditional to labor productivity shocks, Shimer 

 (2005)| Simulated moments, smoothing parameter $=10^{5}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $U$ | $V$ | $\theta$ | $p$ | productivity |
| St dev | 0.009 | 0.027 | 0.035 | 0.010 | 0.020 |
| Correlation matrix | $U$ | $V$ | $\theta$ | $p$ | productivity |
| $U$ | 1 | -0.927 | -0.958 | -0.958 | -0.958 |
| $V$ |  | 1 | 0.996 | 0.996 | 0.995 |
| $\theta$ |  |  | 1 | 1 | 0.999 |
| $p$ |  |  |  | 1 | 0.999 |
| productivity |  |  |  |  | 1 |

Summary statistics: relative volatilities

|  | $U$ | $V$ | $\theta$ | $p$ | productivity |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{\sigma(X)}{\sigma(\text { productivity })}$ | 0.5 | 1.4 | 1.8 | 0.5 | 1 |

### 1.2.2 The values of correlations

There is another failure in the search and matching model with only productivity shocks, in addition to the lack of amplification of the shock: the second puzzle, as expressed by Shimer (2005), is the lack of propagation of the shock.

The correlations of labor market variables with productivity implied by the model are, in fact, too high: considering the whole data sample from 1951 to 2003, Shimer (2005) reports empirical (unconditional) correlations within the 0.35-0.4 range ${ }^{12}$, while the values implied by the model are above 0.95 (in absolute value), as it can be seen by looking at the last column of Table 1.2. Shimer (2005) moreover highlights that the empirical values can hide a switch in sign around the mid-1980s ${ }^{13}$.

Considering the period 1951-2011, I find values in line with those reported by Shimer (2005) and Michaillat (2012) for the correlations between vacancies, unemployment and tightness with labor productivity. Considering the two sub-periods 1951-1984 and 1985-2011 to compute the summary statistics implies a somehow different picture, as it can be seen in Table 1.3: the correlation of labor market variables with productivity are characterised by lower (absolute) values ${ }^{14}$. For example Barnichon (2010a) in his article argues that the correlation between unemployment and productivity hides a switch in sign in the mid-1980s.

[^9]Figure 1.1 - Productivity and labor market tightness 1951-2011


Labor productivity is, as in Shimer (2005), the seasonally adjusted real average output per person in the non-farm business sector as constructed by the LBS;
labor market tightness is the ratio of vacancies and unemployment expressed in levels; for vacancies I use the composite Help Wanted Index proposed by Barnichon (2010b),
the series of unemployment is taken from BLS.

In line with previous considerations, there exists a large literature that has documented the presence of a break in many macroeconomic series after the arrival of Volcker at the Federal Reserve Bank: I therefore decide to consider as my empirical counterpart the period post-1984, for which the conduct of monetary policy is generally considered as stable.

### 1.3 The Galí (1999) evidence

The question about the reaction of hours to a technological shock raised by Galí (1999) has been the object of a vast discussion in macroeconomics.

Without attempting to give here an exhaustive discussion, I just want to summarize the conclusions I retain to be of interest for this paper; in doing this, I rely heavily on the clear exposition one can find in Fève and Guay (2009).

Basu et al. (2006) and Francis and Ramey (2005) confirm the findings of Galí (1999):

Table 1.3 - Summary statistics U.S quarterly data

|  | Michaillat (2012) <br> $\mathbf{1 9 6 4 - 2 0 0 9}$ | $\mathbf{3 9 5 1 - 2 0 1 1}$ | my calculations <br> $\mathbf{1 9 5 1 - 1 9 8 4}$ | $\mathbf{1 9 8 5 - 2 0 1 1}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\sigma_{Y}$ | 0.030 | 0.033 | 0.034 | 0.031 |
| $\sigma_{A}$ | 0.020 | 0.020 | 0.023 | 0.016 |
| $\frac{\sigma(U)}{\sigma(A)}$ | 8.5 | 9.533 | 9.330 | 10.345 |
| $\frac{\sigma(V)}{\sigma(A)}$ | 9.2 | 9.354 | 8.824 | 10.514 |
| $\frac{\sigma(\theta)}{\sigma(A)}$ | 17.1 | 18.317 | 17.295 | 20.142 |
| $\frac{\sigma(w)}{\sigma(A)}$ | 1.050 | 1.104 | 1.256 | 1.009 |
| $\operatorname{corr}(U, A)$ | -0.561 | -0.414 | -0.486 | -0.118 |
| $\operatorname{corr}(V, A)$ | 0.524 | 0.400 | 0.525 | 0.074 |
| $\operatorname{corr}(\theta, A)$ | 0.559 | 0.419 | 0.517 | 0.098 |
| $\operatorname{corr}(w, Y)$ | 0.502 | 0.352 | 0.421 | 0.333 |
| $\operatorname{corr}(Y, A)$ | 0.891 | 0.707 | 0.765 | 0.551 |
| $\operatorname{cor}(U, V)$ | -0.889 | -0.883 | -0.905 | -0.859 |
| $\operatorname{corr}(U, \pi)$ | - | -0.266 | -0.397 | -0.195 |

( $Y$ stands for production, $A$ is labor productivity, $U$ stands for unemployment, $V$ for vacancies, $\theta$ for labor market tightness, $w$ is real wage, $\pi$ is inflation)

All series are provided by the BLS: labor productivity $(A)$ is real average output per person in the non-farm business sector, $U$ is the unemployment level from CPS, for vacancies $(V)$ I use the composite Help Wanted Index proposed by Barnichon (2010b), real wage $(w)$ is the real hourly average earnings of production and non supervisory employees (available from 1964), production is non-farm business sector output, the rate of inflation is calculated from the CPI for all urban consumers. All variables are detrended with an HP filter with smoothing parameter of $10^{5}$ and reported in $\log$ as deviation from the trend, as in Shimer (2005) and Michaillat (2012).
hours decrease after a positive technological shock in the U.S. ${ }^{15}$. Numbers of other papers have also confirmed these findings, for example Balleer (2012) or Barnichon (2010a). Nevertheless, the paper by Christiano et al. (2003) questions these results: the authors find that, using a SVAR where hours enter in levels, the response of this variable after a technological shock is positive and hump-shaped.

The debate then focused on the stationarity of the series of hours, and thus on the choice of introducing them in levels or in first difference, but the presence of a unit root in the series cannot be stated or denied easily.

The paper by Fève and Guay (2009) tries to give an answer with an alternative methodology: they propose a VAR with a two-step estimation; first the technological shocks are identified without using the hours worked as a variable, and then hours are regressed on the technological shocks, for different lags. The results of the authors are interesting since they embed the previous findings of both Galí (1999) (and followers) and Christiano et al. (2003):

[^10]according to the authors in fact the two-step methodology indicates that hours decrease after a positive technological shock in the short run and increase (with an hump-shape) afterwards. According to them, the results are not sensitive to the choice of introducing hours (in the second step) in levels or in first differences, even if the IRFs are more precisely estimated when hours enter in first differences. I report in Figure 1.2 the main findings of the authors. I use this evidence in the following of the paper to claim that the reaction of hours after a positive technological shock is negative, at least in the short run.

Figure 1.2 - IRFs of hours to a technological improvement, Fève and Guay (2009)


From Fève and Guay (2009) p. 1003

### 1.4 The model

I set up a very simple model with monopolistic competition in the product market and labor market frictions, in the tradition of models such as Trigari (2006), Blanchard and Galí (2010) and KZ (2010).

This type of model allows to compare the transmission mechanism of a productivity shock in the presence of nominal frictions (in the form of sticky prices) with the mechanism
studied instead by Shimer (2005): when the monopolistically competitive firms do not face any nominal frictions in setting their prices, the model behaves as in the real business cycle framework adopted by Shimer (2005).

I make two modelling choices in order to be closer to the environment studied by Shimer (2005), so that the comparisons of the mechanisms are more transparent: firstly, I abstract from the intensive labor margin, i.e. I consider an indivisible labor framework, such that if the agent is employed, she can supply $h$ (fixed) hours. Secondly, I choose a particular set of preferences such that there are no wealth effects on labor supply: in this respect, my model remains comparable to Shimer (2005), who considers risk-neutral agents with linear utility in consumption. In particular, the preferences I adopt are the Greenwood-HercowitzHuffman (GHH) preferences, developed in Greenwood et al. (1988), which are non-separable in consumption and leisure ${ }^{16}$. For the sake of completeness, in Section (1.B) in the Appendix, I provide the comparison of the baseline modelling choices with the alternative hypothesis: in particular, in Section (1.B) I check that the introduction of variable hours does not change qualitatively the results. In Section (1.B) instead I check the consequences of allowing for a traditional set of separable preferences ${ }^{17}$.

In choosing the timing assumption for functioning of the labor market, my approach in describing the economy is in line with the "Keynesian" interpretation: in the short run prices are rigid, so that quantities adjust after a shock; since hours worked per worker are fixed, it is the extensive margin which has to be able to react. I therefore adopt a different description for the dynamics of labor market flows, with respect to the canonical search and matching model: in line with papers such as, among others, Blanchard and Galí (2010) or KZ (2010), I consider in fact that newly hired workers become immediately productive. This convention changes the definition of employment: as in KZ (2010), I can say that $N_{t}$ is not anymore a state variable, while $N_{t-1}$ remains a predetermined state variable.

Van Zandweghe (2010) in his paper points out that the adoption of the instantaneous hirings hypothesis has different implications if nominal frictions are present or not. For the sake of clarity, in Section (1.B) in the Appendix, I compare the functioning of the model under the two timing assumptions, when prices are flexible or not. The results are in line with

[^11]the mechanisms explained by Van Zandweghe (2010): with sticky prices, the instantaneous hirings hypothesis implies a different behavior of labor market variables, after a productivity shock, with respect to a model with traditional timing ${ }^{18}$, while in a RBC model the adoption of the traditional timing assumption does not change qualitatively the results (even if the implied volatilities of labor market variables is lower).

The decisions about prices are kept separated from those about posting vacancies, i.e. I assume that there are three sectors: wholesale firms produce output using only labor ${ }^{19}$; these firms need to post vacancies in order to hire workers, since the labor market is frictional, but they can sell their output in a perfectly competitive one. The second sector is composed by retail firms, who are monopolistically competitive: these firms buy the output of wholesale firms, differentiate it and sell to a perfectly competitive final producer, who has a CES aggregation function and who sells the final production to households ${ }^{20}$.

For what it regards the nominal frictions, I suppose that the retail firms cannot change freely the price of their output, since they face nominal frictions à la Calvo: they are able to reset their price optimally each period only with a certain probability $(1-\alpha)$. Alternatively, they will leave it unchanged. Finally, I will close the model by supposing a monetary authority which adopts a Taylor rule, to set the nominal interest rate.

I look at the decentralized equilibrium (the only public authority here is the monetary authority) in a perfect risk sharing framework. The hypothesis about the behavior of monetary authority, with respect to deviations of output from its steady state, is of fundamental importance, since it affects the behavior of the agents in the model, and so the response of variables to the shocks ${ }^{21}$.

Many authors have emphasized the different characteristics of the "Great Moderation" period with respect to the past, claiming that the conduct of monetary policy changed around the mid 1980s: I therefore choose to consider the post-1984 period as reference (so that the hypothesis of a stable monetary policy rule is quite acceptable).

### 1.4.1 Labor market flows

I suppose that new hirings become immediately productive, so that the law of motion of employment is given by the equation:

[^12]\[

$$
\begin{equation*}
N_{t}=(1-s) N_{t-1}+\Upsilon S_{t}^{1-\psi} V_{t}^{\psi} \tag{1.1}
\end{equation*}
$$

\]

with $S_{t}=1-(1-s) N_{t-1}, \Upsilon>0,0<s<1$ and $0<\psi<1$.
$S_{t}=1-(1-s) N_{t-1}$ indicates the agents who are actually looking for a job. The definition of job finding rate and labor market tightness take into account the actual searching agents, and not the unemployed ones, who are defined as all those who are not employed $\left(U_{t}=1-N_{t}\right)$; if I call $M_{t}=\Upsilon S_{t}^{1-\psi} V_{t}^{\psi}$, then $p_{t}=\frac{M_{t}}{S_{t}}$ and $\theta_{t}=\frac{V_{t}}{S_{t}}$ are respectively the job finding rate and the labor market tightness; finally the job filling rate is given by $\Phi_{t}=\frac{M_{t}}{V_{t}}$.

When new hired workers are immediately productive, a reformulation of the search and matching model in discrete time is necessary. In this setting, in each period there are two subperiods: the market for input (which is only labor) and then production. At the beginning of the first sub-period, some matches are destroyed, but the agents who have been separated from their position can immediately look again for a job; the firm then decides how many workers to hire, and in the second sub-period it produces using the "new hired workers". The pool of job seekers in the first sub-period is therefore given by the unemployed workers coming from the previous period, plus those $(s N)$ whose matches have been destroyed: those workers can therefore look for a job, find one and become productive before the end of the period.

This sequence of events implies that those who look for a job constitute a larger set than normally considered to estimate the job finding rate, since some of them are "employed" at the end of the previous period, and at the beginning of the following period, but in the meantime they have been "job seekers".

For what it regards the separation rate, the same gap with the usual measure also exists: during the period some matches are destroyed but new ones are formed before the end of the period, leading to an underestimation of the separation rate, when only transitions from employment to unemployment are considered. Intuitively, if this second underestimation of the separation rate is less important, the corresponding job finding rate will be smaller than that one measured by Shimer (2005) for the same level of unemployment rate ${ }^{22}$.

### 1.4.2 Households and preferences

The representative household provides full insurance to its members, as in Merz (1995) and Andolfatto (1996): a proportion $N_{t}$ of its members can work $h$ (fixed) hours ${ }^{23}$, while

[^13]the remaining $1-N_{t}$ are unemployed. The household can save by buying bonds so that her problem in terms of value function can be written as:
\[

$$
\begin{align*}
& W\left(N_{t-1}\right)=\max \left\{N_{t} U\left(C_{t}^{e}, \Gamma^{e}\right)+\left(1-N_{t}\right) U\left(C_{t}^{u}, \Gamma^{u}\right)+\beta E_{t} W\left(N_{t}\right)\right\}  \tag{1.2}\\
& \text { s.t. } \begin{cases}N_{t} C_{t}^{e}+\left(1-N_{t}\right) C_{t}^{u}+\frac{B_{t+1}}{P_{t}} \frac{1}{\left(1+r_{t}^{n}\right)} & =w_{t} N_{t} h+\frac{B_{t}}{P_{t}}+\frac{D_{t}}{P_{t}} \\
N_{t+1} & =(1-s) N_{t}+p_{t+1} S_{t+1} \\
p_{t} & =\frac{M_{t}}{S_{t}} \\
S_{t} & =1-(1-s) N_{t-1}\end{cases}
\end{align*}
$$
\]

where $C_{t}^{e}$ and $C_{t}^{u}$ represent respectively the consumption of the employed and the unemployed, $B_{t+1}$ is the amount of bonds expressed in currency units, $r_{t}^{n}$ is the nominal interest rate, $w_{t}$ is the real wage and $D_{t}$ the nominal profits eventually rebated by firms; $P_{t}$ is the price of the consumption good. The state variable now is $N_{t-1}$, so it appears in the value function with the right timing ${ }^{24}$.

With respect to the utility function, I consider the GHH preferences.
In Greenwood et al. (1988) the utility function is given by $U(c, 1-h)=\varphi[c+\nu(1-h)]$, where $\varphi, \nu$ are strictly increasing and concave functions, $c$ is consumption, $h$ is hours of work, with $h \in\{0,1\}$.

The functional form is then the standard one:

$$
\begin{equation*}
U\left(C_{t}^{z}, L_{t}^{z}\right)=\log \left(C_{t}^{z}+\Gamma_{t}^{z}\right) \tag{1.3}
\end{equation*}
$$

for $z=e, u$, with $\gamma, \varepsilon>0$, where $\Gamma_{t}^{e}=\gamma \frac{\left(1-h_{t}\right)^{1-\varepsilon}}{1-\varepsilon}$ and $\Gamma_{t}^{u}=\Gamma^{u}$.
Since I set non-variable hours, $\Gamma_{t}^{e}=\Gamma^{e}=\gamma \frac{(1-\varepsilon)^{1-\varepsilon}}{1-\varepsilon}$ and $\Gamma_{t}^{u}=\Gamma^{u}$, with $h$ fixed at the steady state value ${ }^{25}$.

I suppose, as in Cheron and Langot (2004), that at the steady state the unemployed worker values leisure more than the employed one, i.e. $\Gamma^{u}-\Gamma^{e}>0$.

The FOCs for the households problem imply that the marginal utility of consumption is the same for both agents, so

$$
\begin{equation*}
\lambda_{t}=\frac{1}{C_{t}^{e}+\Gamma_{t}^{e}}=\frac{1}{C_{t}^{u}+\Gamma^{u}} \tag{1.4}
\end{equation*}
$$

which implies that $C_{t}^{e}=C_{t}^{u}+\Gamma^{u}-\Gamma^{e}$ so that $C_{t}^{e}>C_{t}^{u}$, under the assumption $\Gamma^{u}-\Gamma^{e}>0$.

[^14]Considering the utility gap, I obtain

$$
\begin{equation*}
U\left(C_{t}^{e}, \Gamma^{e}\right)-U\left(C_{t}^{u}, \Gamma^{u}\right)=\log \left(C_{t}^{e}+\Gamma^{e}\right)-\log \left(C_{t}^{u}+\Gamma^{u}\right)=0 \tag{1.5}
\end{equation*}
$$

i.e. the unemployed are as well off as the employed.

When preferences are separable and take the following standard form:

$$
\begin{equation*}
U\left(C_{t}^{z}, L_{t}^{z}\right)=\log \left(C_{t}^{z}\right)+\Gamma_{t}^{z} \tag{1.6}
\end{equation*}
$$

for $z=e, u$, with $\gamma, \varepsilon>0$, where $\Gamma_{t}^{e}=\gamma \frac{\left(1-h_{t}\right)^{1-\varepsilon}}{1-\varepsilon}$ and $\Gamma_{t}^{u}=\Gamma^{u}$, the implication is that $C_{t}^{e}=C_{t}^{u}$ and therefore $U\left(C_{t}^{u}, \Gamma^{u}\right)>U\left(C_{t}^{e}, \Gamma^{e}\right)$, i.e. employed agents are worse off than the unemployed.

To sum up, the non-separable preferences à la Greenwood et al. (1988) with $\gamma>0$ imply that consumption and leisure are substitutes, so that employed workers consume more than the unemployed, and their utility level is the same as that of the unemployed. On the contrary, the traditional separable preferences imply that unemployed workers are better off than the employed, which is at odd with common sense and with all the well-being literature ${ }^{26}$.

### 1.4.3 Intermediate firms

The intermediate representative firm produces its output using only labor with a linear production function $Y_{t}=A_{t} N_{t} h$, where $A_{t}$ is the stochastic technological process. This firm faces a frictional labor market: in order to hire workers, it must post vacancies; it sells its output to retail firms in a perfectly competitive environment.

The problem of the firm in terms of value function is therefore:

$$
\begin{align*}
V\left(N_{t-1}\right)= & \max _{V_{t}}\left\{x_{t} A_{t} N_{t} h-w_{t} N_{t} h-\omega V_{t}+\beta E_{t}\left[\frac{\lambda_{t+1}}{\lambda_{t}} V\left(N_{t}\right)\right]\right\}  \tag{1.7}\\
\text { s.t. } & \begin{cases}N_{t}=(1-s) N_{t-1}+\Phi_{t} V_{t} \\
\Phi_{t} & =\frac{M_{t}}{V_{t}}\end{cases}
\end{align*}
$$

where the problem is expressed in real terms: $x_{t}$ is the relative price of the intermediate output i.e. $x_{t}=\frac{P_{t}^{\text {intermed }}}{P_{t}} ; \omega$ is the real cost (in terms of consumption good) of posting a vacancy ( i.e. in nominal terms I would have $\omega P_{t}$ ).

[^15]The first order condition, combined with the expression of the marginal value of one additional worker for the firm, gives the vacancy opening condition:

$$
\begin{equation*}
\frac{\omega}{\Phi_{t}}=\left(x_{t} A_{t} h-w_{t} h\right)+\beta E_{t}\left[\frac{\lambda_{t+1}}{\lambda_{t}} \frac{\omega}{\Phi_{t+1}}(1-s)\right] \tag{1.8}
\end{equation*}
$$

As it is standard, the firm posts vacancies till the cost it incurs (given by $\omega$ during the time the vacancy is open) is equal to the benefit, given by the net production obtained and the "saved" cost of recruiting if the job remains filled.

### 1.4.4 Wage equation

The total surplus of a match is defined as Surplus ${ }_{t}=\frac{\partial V\left(N_{t-1}\right)}{\partial N_{t-1}}+\frac{1}{\lambda_{t}} \frac{\partial W\left(N_{t-1}\right)}{\partial N_{t-1}}$, where the marginal value of a worker for the household is expressed in terms of consumption goods.

I assume a Nash-bargaining framework, so that the sharing rule is obtained by solving the Nash product maximization problem:

$$
\begin{equation*}
\max _{w_{t}}\left(\frac{1}{\lambda_{t}} \frac{\partial W\left(N_{t-1}\right)}{\partial N_{t-1}}\right)^{1-\xi}\left(\frac{\partial V\left(N_{t-1}\right)}{\partial N_{t-1}}\right)^{\xi} \tag{1.9}
\end{equation*}
$$

where the parameter $\xi$ represents the bargaining power of the firm.
I derive the sharing rule and I finally obtain the wage equation by using the expressions for the marginal value of employment for the firm and the worker ${ }^{27}$ :

$$
\begin{equation*}
w_{t} h=(1-\xi)\left[x_{t} A_{t} h+\beta(1-s) \omega E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} \theta_{t+1}\right]+\xi\left[\left(\Gamma^{u}-\Gamma^{e}\right)\right] \tag{1.10}
\end{equation*}
$$

Let me briefly consider the first square bracket in the RHS of the wage equation, which represents the contribution of the worker to production from the firm's point of view: the first element is the real marginal product of $N_{t}$, while the second is the saving coming from the fact that the firm does not need to post a vacancy in the future, if the match is not destroyed.

The real marginal product of a worker is given by her productivity, $A_{t}$, multiplied by the relative price of the homogeneous intermediate good $x_{t}$; this term is also the marginal cost of the retailer: the behavior of marginal cost in determining the behavior of a model with nominal frictions has been analysed in, for example, Walsh (2005), Trigari (2009) or Thomas (2011). I briefly remind to the reader that in a New Keynesian model, under certain hypothesis for the monetary policy rule, the real marginal cost decreases after a positive technological shock (i.e. when $A_{t}$ increases), so that the overall effect on the term $x_{t} A_{t} h$ is not unambiguous, and it depends on the relative strength of the two effects; this is

[^16]the crucial element which brings non-monotonicity in the dynamics of adjustment of labor market variables after a productivity shock.

The important point is that, once there exist nominal rigidities, the dynamics of vacancies and unemployment are not any more unambiguous, as in the real model of Shimer (2005): due to the presence of nominal frictions, according to the behavior of the monetary authority, it is possible to have a recessionary or an expansionary effect of a positive productivity shock on employment (it depends on the fact that monetary policy is enough accommodating with respect to output) ${ }^{28}$ : in the short run, it is the "Keynesian" reaction of vacancies and employment which prevails (they both decrease), while as prices adjust (so that the effect of the decrease of the marginal cost vanishes), the effect of productivity prevails, and vacancies increase.

Let me consider now the second element of the RHS of the equation: it represents the outside options of the worker, i.e. the threatening point should she leave the bargaining.

With non-separable preferences the term inside the bracket is not changing along the cycle: I find here the equivalent of what Shimer (2005) and Hagedorn and Manovskii (2008), henceforth HM (2008), call " $z$ ", i.e. the value of leisure.

In my model this value comes from steady state conditions, since I follow the calibration strategy as in Andolfatto (1996): I target a value for the costs of posting vacancies in terms of output, and therefore I derive the value of non-market activity from steady state conditions, as we will see in details in Subsection 1.4.8; in my calibration exercise the value of leisure corresponds to a replacement rate of $30 \%$, therefore even lower than that one in Shimer (2005).

### 1.4.5 Retailers

There is a continuum of monopolistically competitive retailer firms $i \in[0,1]$ which buy the intermediate firm good and differentiate it at no cost; they sell their output to the final producer, which buys the differentiated goods and aggregate them according to a CES production function, and finally sells to the households its output.

The problem of the retailer $i$ can be therefore written in the following way ${ }^{29}$ :

$$
\begin{equation*}
\max _{P_{t}(i)} E_{t} \sum_{j=0}^{\infty} \alpha^{j} v_{t+j}\left[P_{t}(i) Y_{t+j}(i)-P_{t+j} x_{t+j} Y_{t+j}(i)\right] \tag{1.11}
\end{equation*}
$$

[^17]$$
\text { s.t. } \quad Y_{t+j}(i)=\left(\frac{P_{t}(i)}{P_{t+j}}\right)^{-\eta} C_{t+j}
$$
where $\alpha$ is the probability that the retailer cannot reset optimally its price, $P_{t} x_{t}$ is therefore the nominal marginal cost and $v_{t+j}=\beta^{j} \frac{\lambda_{t+j} \frac{1}{P_{t+j}}}{\lambda_{t} \frac{1}{P_{t}}}$.

The first order condition for this problem gives the pricing setting rule:

$$
P_{t}(i)=\frac{E_{t} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+j} C_{t+j} P_{t+j}^{\eta} \frac{\eta}{\eta-1} x_{t+j}}{E_{t} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+j} C_{t+j} P_{t+j}^{\eta-1}}
$$

where $\frac{\eta}{\eta-1}$ is the price mark-up.
As it is standard in sticky price literature, the expression for the evolution of the price index is given by:

$$
P_{t}^{1-\eta}=\left[(1-\theta) P_{t}(i)^{(1-\eta)}+\theta P_{t-1}^{(1-\eta)}\right]
$$

The market clearing condition implies the total production of retailers to be equal to total demand for their goods expressed by the final producer:

$$
y_{t} \equiv \int_{0}^{1} Y_{t}(i) d i=\int_{0}^{1}\left(\frac{P_{t}(i)}{P_{t}}\right)^{-\eta} C_{t} d i=\Delta_{t} C_{t}
$$

where $\Delta_{t} \equiv \int_{0}^{1}\left(\frac{P_{t}(i)}{P_{t}}\right)^{-\eta} d i$ is the price distortion.
As it is standard in the sticky price literature, the price distortion can be expressed as:

$$
\Delta_{t}=P_{t}^{\eta}\left[\left(\int_{0}^{1} P_{t}(i)^{-\eta} d i\right)^{-\frac{1}{\eta}}\right]^{-\eta}=P_{t}^{\eta}\left(P_{t}^{*}\right)^{-\eta}=\left(\frac{P_{t}^{*}}{P_{t}}\right)^{-\eta}
$$

which implies that $P_{t}^{*} \equiv\left(\int_{0}^{1} P_{t}(i)^{-\eta} d i\right)^{-\frac{1}{\eta}}$, so that the expression for the law of motion of the price distortion is given by:

$$
P_{t}^{*}=\left[(1-\alpha) P_{t}(i)^{-\eta}+\alpha\left(P_{t-1}^{*}\right)^{-\eta}\right]^{\frac{-1}{\eta}}
$$

In steady state $\Delta=1$ and, up to a first order linear approximation around a zero steady state inflation, the log-linearized price dispersion is always null, so that I will ignore it in my solution.

### 1.4.6 Aggregate relations

The aggregate relations which hold in the economy are the aggregate production function of the intermediate firms and economy wide resource constraint:

$$
\begin{gather*}
Y_{t}=A_{t} N_{t} h  \tag{1.12}\\
Y_{t}=y_{t}+\omega V_{t}=\Delta_{t} C_{t}+\omega V_{t} \tag{1.13}
\end{gather*}
$$

### 1.4.7 Technology and monetary policy

The stochastic technological process is specified as a standard AR (1) process:

$$
A_{t}=A_{t-1}^{\varrho_{A}} A^{\left(1-\varrho_{A}\right)} e^{\varepsilon_{t}^{A}}
$$

where $0<\varrho_{A}<1$ and $\varepsilon_{t}^{A} \sim N\left(0, \sigma_{\varepsilon_{A}}\right)$.
I suppose the existence of a monetary policy authority which adopts a Taylor rule. I take into consideration the results by KZ (2010), who study indeterminacy issues in a model similar to mine, where nonetheless the only source of aggregate uncertainty is given by sunspot shocks to inflation expectations: the main conclusion of the authors is that a Taylor rule which is targeting only future inflation gives almost always rise to indeterminacy. Their study justifies the widespread approach in the literature ${ }^{30}$ to adopt a Taylor rule of the type proposed by Clarida et al. (2000), which includes a smoothing element and the reaction of monetary policy to both (contemporaneous) inflation and output ${ }^{31}$.

The chosen specification of the Taylor rule is then the following:

$$
\begin{equation*}
\frac{R_{t}^{n}}{R^{n}}=\left(\frac{R_{t-1}^{n}}{R^{n}}\right)^{\varrho_{m}}\left(\frac{\pi_{t}}{\pi}\right)^{\left(1-\varrho_{m}\right) \gamma_{\pi}}\left(\frac{Y_{t}}{Y}\right)^{\left(1-\rho_{m}\right) \gamma_{y}} e^{\varepsilon_{t}^{m}} \tag{1.14}
\end{equation*}
$$

where $\varepsilon_{t}^{m} \sim N\left(0, \sigma_{\varepsilon_{m}}\right)$.

### 1.4.8 Calibration

The period of reference is considered to be a quarter, as it is standard in the DSGE literature ${ }^{32}$.

In order to be comparable with the labor market literature, I choose to target the high value for employment adopted by Shimer (2005) of $N=0.9455$ which, together with the

[^18]choice of the widely accepted value for the separation rate of $s=0.1$ at a quarterly level, allows me to pin down the job finding rate ${ }^{33}$.

The choice of these values implies that the labor market tightness and the job finding rate in steady state are respectively $\theta=0.7049, p=0.6344$.

For the worker bargaining power, Shimer (2005) sets it to $1-\xi=0.72$, but in the literature the more commonly adopted values stay in the range $0.4-0.6,{ }^{34}$ so I set $1-\xi=0.6$.

For what it regards vacancy costs in terms of output, the RBC literature has adopted values in the range $0.5 \%-1 \%$. To set this value, I follow Cheron and Langot (2004) who consider the evidence in Abowd and Kramarz (1997): according to the statistics of this paper, total hiring costs account for $1.03 \%$ of total labor costs ${ }^{35}$; this value, in terms of my model, can be expressed as $\frac{\frac{\omega V}{q}}{w h N}=0.0103$, where total wage costs can be recovered from the intermediate firm program as $w h N=x A h N-\omega V^{36}$.

Considering the value for the job filling rate of $\Phi=0.9$, I end up with a value for $\frac{\omega V}{Y}$ between $0.7 \%$ and $0.8 \%$. I therefore choose a value of $\frac{\omega V}{Y}=0.8 \%$, which belongs to the range of possible values typically found in the RBC literature.

Another widely cited paper in the literature with respect to the calibration of the vacancy posting cost is that by Silva and Toledo (2009): the authors provide evidence of the fact that total hiring costs account for around $14 \%$ of the quarterly compensation of an employee. In terms of my model, this calibration would imply a ratio of total vacancy costs in terms of output of $\frac{\omega V}{Y}=1.2 \%^{37}$, so that I consider my calibration choice not in contrast to this alternative view.

The steady state value of the consumption ratio will be determined by steady state restrictions. ${ }^{38}$

The calibrated value for vacancy costs implies therefore a consumption ratio of 0.2405 , for the benchmark case of non-separable preferences; the "value of leisure" (in Shimer's terms)

[^19]is 0.25 , which means a replacement rate in terms of hourly wage of $\frac{\left(C^{e}-C^{u}\right)}{w}=0.3{ }^{39}$.
As I stressed in the Introduction, my calibration procedure is in line with the RBC tradition, starting from Andolfatto (1996), while Shimer (2005) sets the value for the outside options and derives from the steady state restrictions the value for the parameter $\omega$.

For what it regards the calibration of the value of leisure, Shimer (2005) calibrates $z=0.4$ : considering that he normalizes productivity to the unity, and that his mean labor income has a value of 0.993 , the choice of the value of $z$ implies a replacement rate of $40 \%$, when interpreted as unemployment benefit. HM (2008) stress that with a higher value for $z$ (together with a low value for the bargaining power of the worker), the standard search and matching model can replicate the relative volatilities of labor market variables ${ }^{40}$.

In the light of previous considerations, if one wants to address the Shimer puzzle about volatilities without using the HM (2008) calibration approach, it is necessary to be careful about the calibration of the value of leisure.

If I had to follow Shimer's procedure, setting a value for non-market activity as $40 \%$ of the wage ${ }^{41}$, I would obtain a value for the parameter of the vacancy posting cost of $\omega=0.13$ and therefore a ratio of $\frac{\omega V}{Y}=4.46 \%$, which is above any value usually considered in the literature.

As expected, a model calibrated with such a high cost of posting vacancies delivers much less volatility of labor market variables, with respect to a model where the recruiting costs are lower, as it will be shown in Section 1.5.1.

The value for the elasticity of substitution implies a steady state mark-up of $20 \%$. For what it concerns nominal frictions, the Calvo parameter, about which there is also a considerable uncertainty, and which has a fundamental impact on the reaction of employment to a technology shock, is set to a value of $\alpha=0.5$, which implies an average price spell of almost 6 months ${ }^{42}$.

The parameters on the Taylor rule are standard: in line with Clarida et al. (2000) for the Volcker-Greenspan period, in my baseline calibration I suppose that monetary policy reacts to output and inflation; I also include a component of interest rate smoothing.

[^20]The calibrated values I use for quarterly data in the benchmark case are reported in the Table 1.4.

Table 1.4 - Baseline calibration for nominal model

| $h$ | $\varepsilon$ | $\varrho_{A}$ | $\sigma_{\varepsilon_{A}}$ | $\omega \frac{V}{Y}$ | $\frac{C^{u}}{C^{e}}$ | $\beta$ | $\eta$ | $\gamma_{\pi}$ | $\gamma_{Y}$ | $\alpha$ | $\varrho_{m}$ | $N$ | $\Phi$ | $\xi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 / 3$ | 4 | 0.9 | 0.009 | $0.8 \%$ | 0.24 | 0.985 | 6 | 1.5 | 0.5 | 0.5 | 0.8 | 0.946 | 0.9 | 0.4 |

In order to solve the model, I calibrate the model and log-linearize it around a zero steady state inflation to obtain IRFs and theoretical moments, using the software Dynare ${ }^{43}$.

For what it regards data sources used to compute first and second order moments, all series except hours per worker are provided by the BLS. Employment (N) and hours per worker (h) are taken from the dataset of Ohanian and Raffo (2012); employment is total employment from the BLS and the hours per worker are the results of their estimation.

### 1.5 Results

The reasoning behind my quantitative results is the following: once I depart from the calibration procedure of Shimer (2005), I can obtain a model which already considerably reduces the issue about the steady state elasticity of labor market tightness.

Considering the model with the different calibration strategy, I then analyze the importance of the transmission mechanism when prices are flexible or there is some degree of stickiness, i.e. I analyze the quantitative importance of the different reaction of employment, in the short and in the long run.

For the sake of clarity, I report the simulated moments characterising both variables $U_{t}$ (the measure of unemployed agents) and $S_{t}$ (the measure of job seekers in the model), the instantaneous hirings hypothesis implying this distinction between the two variables.

In addition to the main simulations, I perform some additional exercises, to check the robustness of the results to three hypothesis I adopted: in particular, I check the impact of allowing for variable hours, the implications of using separable preferences in consumption and leisure and finally the consequences of using the standard timing assumption for the new hirings. I report these exercises in Section (1.B) of the Appendix ${ }^{44}$.

[^21]
### 1.5.1 The value of the vacancy posting cost

First of all I present the quantitative properties of my model considering the two different calibration procedures (à la Andolfatto (1996) and à la Shimer (2005)), and therefore the two different values for the vacancy posting costs.

For exposition concerns, I consider a framework in which nominal rigidities are almost nonexistent, in order to be exactly in the framework of Shimer (2005) for what it regards the transmission mechanism of a positive labor productivity shock: this means that I set my Calvo parameter to a value of $\alpha=0.01$.

## Steady state elasticities

In order to be directly comparable with Shimer (2005), I compute the steady state elasticities ${ }^{45}$.

Let me remember firstly Shimer (2005) set up: the equilibrium in the labor market can be summarized by two equations, i.e. the job creation condition (the labor demand, eq. 1.15) and the wage equation (the labor supply, eq. 1.16):

$$
\begin{gather*}
\frac{\omega}{\Phi(\theta)}=\frac{A-w}{r+s}  \tag{1.15}\\
w=\xi z+(1-\xi)(A+\omega \theta) \tag{1.16}
\end{gather*}
$$

where $\omega$ is the cost of opening a vacancy, $A$ is the marginal product of a filled job, $\Phi(\theta)=\Upsilon \theta^{\psi-1}$ is the job filling rate, $r$ is the real interest rate, $s$ is the fixed separation rate, $\xi$ is the bargaining power of the firm and $z$ represents the outside options (unemployment benefit or home production) or the value of leisure ${ }^{46}$. Combining these two equations, I obtain:

$$
\begin{equation*}
\frac{(r+s)}{\Phi(\theta)}+(1-\xi) \theta=\xi \frac{A-z}{\omega} \tag{1.17}
\end{equation*}
$$

and by total differentiation I obtain an expression for the elasticity of tightness with

[^22]respect to productivity:
\[

$$
\begin{equation*}
\epsilon_{\theta, A}=\frac{d \theta}{d A} \frac{A}{\theta}=\frac{\xi}{\omega\left[(1-\xi)-(r+s) \frac{\partial \Phi(\theta)}{\partial \theta} \frac{1}{(\Phi(\theta))^{2}}\right]} \frac{A}{\theta} \tag{1.18}
\end{equation*}
$$

\]

This expression enables me to say what happens to labor market tightness when productivity increases (permanently), all the other variables remaining constant: with the benchmark calibration of Shimer $(2005)^{47}$, this elasticity has a value of 1.012: if the productivity of a filled job increases, the incentive of a firm to increase vacancies increases, but also the wage, so that the overall effect is that quantities barely move.

Since I want to perform the same exercise than Shimer (2005), I derive the expressions for elasticity and consider the quantitative differences which come from the calibration strategy.

If I combine the labor demand and labor supply (equations 1.8 and 1.10) at the steady state, I obtain equation 1.19, which is the equivalent to equation 1.17 in Shimer (2005). If I differentiate it, I get the expression for elasticity which is given by equation 1.20 :

$$
\begin{align*}
& \frac{1-\beta(1-s)}{\Phi(\theta)}+(1-\xi)(1-s) \beta \theta=\frac{\xi\left[x A h-\left(\Gamma^{u}-\Gamma^{e}\right)\right]}{\omega}  \tag{1.19}\\
\epsilon_{\theta, A}= & \frac{d \theta}{d A} \frac{A}{\theta}=\frac{\xi x h}{\omega\left[(1-\xi) \beta(1-s)-[1-\beta(1-s)] \frac{\partial \Phi(\theta)}{\partial \theta} \frac{1}{(\Phi(\theta))^{2}}\right]} \frac{A}{\theta} \tag{1.20}
\end{align*}
$$

With my benchmark calibration ${ }^{48}$, I get $\epsilon_{\theta, A}=10.274$.
The value I get is therefore ten times bigger than the corresponding one in Shimer (2005) ; the reason behind this result lies in two aspects: first of all, I set the bargaining power of the firm to a higher value with respect to Shimer (2005) ${ }^{49}$; secondly, and mostly important, the cost of posting a vacancy is obtained in my case by a different calibration procedure, which gives a much lower value than in Shimer (2005) .

## Labor market variables volatilities

Table 1.5 presents the moments obtained by simulating the models characterised by the two different values for the vacancy posting cost.

Let me consider the fifth row of Table 1.5, which reports the moments characterising labor market tightness: in the last column it can be seen that, when the general equilibrium model is calibrated following the procedure of Shimer (2005), for what it regards the value of

[^23]the outside options (and therefore the vacancy posting cost), the volatility of labor market tightness is ten times lower than its empirical counterpart. Adopting a calibration in line with the RBC tradition, for the vacancy posting cost, implies instead a value of volatility that is only two times lower than its empirical counterpart.

Even if I am in the same line of HM (2008), who stress the importance of the calibration of some key parameters, my calibration strategy is very far from their choices.

It is important to stress the importance of the timing assumption: the variable measuring job seekers in the model $\left(S_{t}=1-(1-s) N_{t-1}\right)$ has a much lower volatility than the measure of unemployed agents $\left(U_{t}=1-N_{t}\right)$.

The middle part of Table 1.5 reports the correlations of labor market variables with productivity: in both versions of the model, they are in absolute values higher than in the data, as remarked by Shimer (2005) (the lack of propagation puzzle). The model, as expected, cannot reproduce the negative correlation between unemployment and inflation (the Phillips curve). It is instead able to reproduce the high autocorrelations of labor market variables, as it can been noticed looking at the bottom part of Table 1.5.

### 1.5.2 The transmission mechanism: flexible versus sticky prices

Once clarified the importance of the calibration strategy, I consider the consequences of the transmission mechanism of a positive productivity shock.

The discussion about the Shimer puzzles should be conditional on the transmission mechanism of a productivity shock in the economy. In general, the effect of a "supply" shock (positive technology shock) on total hours (here employment) in a New Keynesian model with sticky prices can be ambiguous. As it is well known, the Galí (1999) effect, i.e. the fact that employment decreases after a positive productivity shock, is linked to the formulation of the Taylor rule: for a given level of nominal rigidities, with a non-accommodating monetary policy with respect to output, employment can decrease after a positive productivity shock, therefore the calibration of the parameter $\gamma_{Y}$ in the Taylor rule is of fundamental importance.

In the specification with price stickiness (when $\alpha=0.5$ ), a positive technology shock has a recessionary effect on employment: as it can be seen in the IRFs in Figure 1.3, in the very short run job seekers increase, vacancies and tightness decrease. Then, as soon as the effect of the technological shock prevails on that one of the marginal cost, vacancies and tightness increase, unemployment decreases. This different transmission mechanism implies a lower correlation of labor market variables with productivity, as it can be seen in the middle part of Table 1.6. Two points characterising the model with sticky prices, which arise from the instantaneous hirings hypothesis, are worth noting: firstly, the additional volatility of tightness, with respect to the model with flexible prices, is originated by a sharp

## Table 1.5 - The importance of the vacancy posting costs (model with flexible prices)


decrease in vacancies: the model implies then a negative autocorrelation for vacancies and tightness; moreover, the timing definition (if i.e. one considers the variable $S_{t}$ or the variable $1-N_{t}$ ) has important consequences for the Beveridge curve (the value of correlation with vacancies is positive once the variable $S_{t}$ is considered). The fact that a technology shock is not able to generate a Beveridge or a Phillips curve, in the model with sticky prices is not very meaningful in itself: Cheron and Langot (2000) for example, in a model with search and matching frictions and price adjustment costs à la Rotemberg, show that a monetary shock is needed in order to originate the Phillips curve. The point I want to make is thus to stress the different implications on the functioning of the model of the hypothesis about the presence (or not) of nominal frictions.

When I consider the model with flexible prices (when $\alpha=0.01$ ), I obtain an important quantitative reaction of labor market variables to a productivity shock, as it has already been noticed in the previous Section. However, I also obtain very high (absolute) values for the correlations between productivity and labor market variables (as it can be seen in middle
part of Table 1.6).
Finally, it can be noticed that this Keynesian propagation mechanism makes the wages less pro-cyclical than usually obtained in RBC models.

Table 1.6 - Model with non-separable preferences

|  | U.S. data <br> $\mathbf{1 9 8 5 - 2 0 1 1}$ | $\alpha=0.5$ <br> Tech shocks | $\alpha=0.01$ <br> Tech shock |
| :---: | :---: | :---: | :---: |
| $\sigma_{Y}$ | 0.031 | 0.020 | 0.025 |
| $\sigma_{A}$ | 0.016 | 0.016 | 0.016 |
| $\frac{\sigma(U)}{\sigma(A)}\left(\frac{\sigma(S)}{\sigma(A)}\right)$ | 10.345 | $11.491(3.783)$ | $9.801(3.224)$ |
| $\frac{\sigma(V)}{\sigma(A)}$ | 10.514 | 16.640 | 7.603 |
| $\frac{\sigma(\theta)}{\sigma(A)}$ | 20.142 | 15.975 | 10.044 |
| $\frac{\sigma(w)}{\sigma(A)}$ | 1.009 | 1.391 | 0.907 |
| $\rho(U, A)(\rho(S, A))$ | -0.118 | $-0.060(-0.029)$ | $-0.979(-0.816)$ |
| $\rho(V, A)$ | 0.074 | 0.044 | 0.975 |
| $\rho(\theta, A)$ | 0.098 | 0.052 | 0.999 |
| $\rho(w, Y)$ | 0.333 | 0.564 | 0.998 |
| $\rho(Y, A)$ | 0.551 | 0.845 | 0.997 |
| $\rho(U, V)(\rho(S, V))$ | -0.859 | $-0.845(0.286)$ | $-0.909(-0.665)$ |
| $\rho(U, \pi)(\rho(S, \pi))$ | -0.195 | $-0.533(0.042)$ | $0.622(0.267)$ |
| Autocorr $(U)$ | 0.94 | 0.27 | 0.916 |
| Autocorr $(V)$ | 0.94 | -0.306 | 0.709 |
| Autocorr $(\theta)$ | 0.94 | -0.099 | 0.846 |
| Autocorr $(Y)$ | 0.93 | 0.897 | 0.871 |

( $Y$ stands for production, $A$ for labor productivity, $U$ for unemployment -i.e. $U_{t}=1-N_{t}, S$ indicates the job seekers according to the definition of the model, i.e. $S_{t}=1-(1-s) N_{t-1}, V$ stands for vacancies, $\theta$ for labor market tightness, $w$ for real wage, $\pi$ for inflation)

## The main mechanism: a simple graphical illustration

In this section I provide a graphical illustration of the mechanisms behind my quantitative results: I look at the IRFs of unemployment and labor market tightness and I show the differences between the two cases with and without nominal rigidities, using the tool of a phase diagram.

Let me consider what happens in the models with and without nominal rigidities, when preferences are non-separable, i.e. without wealth effects.

The job creation condition and wage equation are given by:

$$
\begin{equation*}
\frac{\omega}{\Phi(\theta)}[1-\beta(1-s)]=x A h-w h \tag{1.21}
\end{equation*}
$$

Figure 1.3 - IRFs after a positive technological shock when $\alpha=0.5$


Figure 1.4 - IRFs after a positive technological shock when $\alpha=0.01$

and

$$
\begin{equation*}
w h=(1-\xi)[x A h+\beta(1-s) \omega \theta]+\xi\left(\Gamma^{u}-\Gamma^{e}\right) \tag{1.22}
\end{equation*}
$$

Looking at the equilibrium conditions, two additional elements intervene after a (transitory) productivity shock, with respect to a partial equilibrium framework with perfect competition: the real interest rate (since $\beta=\frac{1}{1+r}$ ) and the mark-up.

In a demand-constrained setting, i.e. when prices are sticky, when firms are more productive, they see their marginal cost to decrease (i.e. the mark-up increases), since they cannot cut prices and the demand is fixed. When prices are flexible, the marginal cost does
not change.
The overall effect on the term $x A h$ (the marginal product of one additional employee in value), which enters in both equations 1.21 and 1.22 , depends on the relative strength of the two opposed forces moving $x$ and $A$ : in my framework, the effect of marginal cost prevails on impact; then, after few periods, that one of technology wins, as it can be seen in the blue line IRFs in the top left panel of Figure $1.5^{50}$. This overall (even if short lasting) negative shock on the marginal product of employment pushes the intermediate firm to decrease vacancies, and therefore employment.

Figure 1.5 - Selected IRFs with and without nominal rigidities


> In green the IRFs when there are almost no nominal rigidities and in blue the IRFs of the benchmark model with $\alpha=0.5$

I show the mechanisms underlying my results in terms of the phase diagram in the space $(S, \theta)$, where I remember that $S_{t}$ indicates the pool of job seekers ${ }^{51}$ and the variable on the y -axis is the labor market tightness.

First of all let me combine the job creation condition and the wage equation in order to obtain the following equation:

[^24]\[

$$
\begin{equation*}
\frac{\omega}{\Phi\left(\theta_{t}\right)}=\xi\left(x_{t} A_{t} h-\left(\Gamma^{u}-\Gamma^{e}\right)\right)+\beta(1-s) \omega\left[E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} \frac{1}{\Phi\left(\theta_{t+1}\right)}-(1-\xi) E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} \theta_{t+1}\right] \tag{1.23}
\end{equation*}
$$

\]

The dynamic system I represent, following Pissarides (2000) and Miao (2014), is composed by two equations: the one which gives the dynamics of employment (equation 1.1, that I re-write in terms of job seekers) and equation 1.23.

$$
\left\{\begin{array}{l}
S_{t+1}=s+(1-s)\left[1-\Phi\left(\theta_{t}\right) \theta_{t}\right] S_{t}  \tag{1.24}\\
\frac{\omega}{\Phi\left(\theta_{t}\right)}=\xi\left(x_{t} A_{t} h-\left(\Gamma^{u}-\Gamma^{e}\right)\right)+\beta(1-s) \omega\left[E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} \frac{1}{\Phi\left(\theta_{t+1}\right)}-(1-\xi) E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} \theta_{t+1}\right]
\end{array}\right.
$$

The graphical representation in Figure 1.6 is qualitatively equivalent to the standard one of a transitory technological shock, which can be found in the textbook of Pissarides (2000).

Figure 1.6 - The Shimer puzzle in a NK framework: the phase diagram


Figure 1.7 reports the "empirical" phase diagram I obtain if I plot the simulated values for the pool of job seekers and labor market tightness obtained after a positive technological shock.

At the beginning, when the negative effect of the marginal cost $x_{t}$ prevails, so that the marginal product of employment $A_{t} x_{t} h$ decreases, labor market tightness jumps down (looking at Figure 1.6, it corresponds to the movement of the locus $\dot{\theta}=0$ to the locus $\dot{\theta}=0(1))$. Then the adjustment begins (the movement along the trajectory indicated by the arrows from the locus $\dot{\theta}=0(1)$ to $\dot{\theta}=0$ ), but, at this point, the effect of the positive technology shock prevails on that on the marginal cost, so that $A_{t} x_{t} h$ increases, and the locus $\dot{\theta}=0$ is shifted till $\dot{\theta}=0(2)$. The adjustment of the system therefore follows the path

## Figure 1.7 - The Shimer puzzle in a NK framework: the "empirical" phase diagram


which characterizes a RBC model (the trajectory of the arrows from the locus $\dot{\theta}=0(2)$ till the original locus $\dot{\theta}=0$ ).

The adjustment path characterising a RBC model can be observed in Figures 1.8 and 1.9: the first one reports the "theoretical" phase diagram, the second one the "empirical one I obtain by simulating the model in which the nominal rigidities are absent, i.e. when the Calvo parameter is set to $\alpha=0.01$. In a framework with no nominal rigidities, a positive technology shock implies an increase in the marginal product of employment $A_{t} x_{t} h$, as it can be seen in the top left panel of Figure 1.5, green dashed line. This effect corresponds, in Figure 1.8, to a movement of the locus $\dot{\theta}=0$ till $\dot{\theta}=0(1)$, followed by the adjustment along the trajectory, as the effect of the shock disappears. It can be noticed that this movement is the same that happens in the second stage of the model with nominal frictions.

Figure 1.8 - The Shimer puzzle in a NK framework: the phase diagram with no nominal rigidities


Figure 1.9 - The Shimer puzzle in a NK framework: the "empirical" phase diagram with no nominal rigidities


### 1.6 Concluding remarks

In this work, I analyse a simple model with labor market frictions and monopolistic competition, which allows to study the transmission mechanism of a productivity shock in a framework in which prices are flexible or sticky.

The modelling choices respond to the willingness to remain as close as possible to the original model by Shimer (2005), while allowing for general equilibrium effects: hours per worker are considered fixed, preferences are such that there are no wealth effects affecting the determination of wage.

I use the model which is the closest to Shimer (2005) (the version with flexible prices) to highlight the consequences of the calibration strategy of the vacancy posting costs in determining the (lack of) elasticity of labor market tightness with respect to productivity: if vacancy posting costs are disciplined as in the RBC tradition, i.e. as in Andolfatto (1996) for example, the steady state elasticity of labor market tightness is ten times higher than in the version of the model which follows Shimer (2005) calibration strategy.

I then apply this new calibration procedure to the model, distinguishing the case with flexible and sticky prices: the presence of a positive (but not very high) level of nominal frictions, combined with a Taylor rule according to which the monetary authority responds to the output gap, implies an additional source of volatility of labor market variables.

Since demand is sticky, in fact, in the short run firms decrease labor input, after a positive productivity shock, so that tightness first decreases and then increases. This characteristic of the model allows to better replicate not only the relative volatilities of labor market variables, but also the unconditional correlations of labor market variables with productivity; however, the additional volatility comes mainly from the reaction of vacancies, which implies a negative autocorrelation for the simulated vacancies and tightness.

The model with flexible prices, instead, reproduces the highly persistent labor market variables.

The mechanisms of the model are tested in presence of alternative modelling hypothesis to assess the robustness: in the Appendix, I report the results of additional experiments, which confirm some results of the literature while stressing others too: (i) the introduction of the intensive margin (hours per worker) does not affect qualitatively the results (but the model is not able to replicate the relative volatilities of hours and employment); (ii) preferences which are separable in consumption and leisure worsen the critique of Shimer (2005) about the lack of amplification of a productivity shock, as highlighted by Cheron and Langot (2004) in a RBC framework, but they do not have much consequence if prices are sticky; (iii) the instantaneous hirings hypothesis is of fundamental importance, if prices are sticky, in determining the reaction of vacancies and therefore the value of autocorrelation.

## Bibliography

John M. Abowd and Francis Kramarz. The costs of hiring and separations. Working Paper 6110, National Bureau of Economic Research, July 1997.

David Andolfatto. Business Cycles and Labor-Market Search. American Economic Review, 86(1): 112-32, March 1996.

Javier Andres, Rafael Domenech, and Javier Ferri. Price Rigidity and the Volatility of Vacancies and Unemployment. Working papers, International Economics Institute, University of Valencia, June 2012.

Almut Balleer. New evidence, old puzzles: Technology shocks and labor market dynamics. Quantitative Economics, 3(3):363-392, 2012.

Regis Barnichon. Productivity and unemployment over the business cycle. Journal of Monetary Economics, 57(8):1013-1025, November 2010a.

Regis Barnichon. Building a composite help-wanted index. Economics Letters, 109(3):175 - 178, 2010b.

Regis Barnichon. The shimer puzzle and the endogeneity of productivity. mimeo, 2014.
Susanto Basu, John G. Fernald, and Miles S. Kimball. Are technology improvements contractionary? American Economic Review, 96(5):1418-1448, 2006.

Mark Bils and Peter J. Klenow. Some Evidence on the Importance of Sticky Prices. Journal of Political Economy, 112(5):947-985, October 2004.

Olivier Blanchard and Jordi Galí. Labor markets and monetary policy: A new keynesian model with unemployment. American Economic Journal: Macroeconomics, 2(2):1-30, 2010.

Arnaud Cheron and Francois Langot. The Phillips and Beveridge curves revisited. Economics Letters, 69(3):371-376, December 2000.

Arnaud Cheron and François Langot. Labor Market Search and Real Business Cycles: Reconciling Nash Bargaining with the Real Wage Dynamics. Review of Economic Dynamics, 7(2):476-493, April 2004.

Lawrence J. Christiano, Martin Eichenbaum, and Robert Vigfusson. What Happens After a Technology Shock? NBER Working Papers 9819, National Bureau of Economic Research, Inc, July 2003.

Kai Christoffel and Keith Kuester. Resuscitating the wage channel in models with unemployment fluctuations. Journal of Monetary Economics, 55(5):865-887, July 2008.

Richard Clarida, Jordi Galí, and Mark Gertler. Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. The Quarterly Journal of Economics, 115(1):147-180, 2000.

Neville Francis and Valerie A. Ramey. Is the technology-driven real business cycle hypothesis dead? shocks and aggregate fluctuations revisited. Journal of Monetary Economics, 52(8):1379 - 1399, 2005.

Patrick Fève and Alain Guay. The response of hours to a technology shock: A two-step structural var approach. Journal of Money, Credit and Banking, 41(5), 2009.

Jordi Galí. Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? American Economic Review, 89(1):249-271, 1999.

Jeremy Greenwood, Zvi Hercowitz, and Gregory W Huffman. Investment, Capacity Utilization, and the Real Business Cycle. American Economic Review, 78(3):402-17, June 1988.

Marcus Hagedorn and Iourii Manovskii. The cyclical behavior of equilibrium unemployment and vacancies revisited. American Economic Review, 98(4):1692-1706, 2008.

Robert E. Hall. Employment fluctuations with equilibrium wage stickiness. American Economic Review, 95(1):50-65, 2005.

Robert E. Hall and Paul R. Milgrom. The limited influence of unemployment on the wage bargain. American Economic Review, 98(4):1653-74, 2008.

Robert G. King and Sergio T. Rebelo. Resuscitating real business cycles. In J. B. Taylor and M. Woodford, editors, Handbook of Macroeconomics, volume 1 of Handbook of Macroeconomics, chapter 14, pages 927-1007. Elsevier, 1999.

Michael U. Krause and Thomas A. Lubik. The cyclical upgrading of labor and on-the-job search. Labour Economics, 13(4):459-477, August 2006.

Takushi Kurozumi and Willem Van Zandweghe. Labor market search, the taylor principle, and indeterminacy. Journal of Monetary Economics, 57(7):851 - 858, 2010.

Monika Merz. Search in the labor market and the real business cycle. Journal of Monetary Economics, 36(2):269-300, November 1995.

Jianjun Miao. Economic dynamics in discrete time. MIT Press, 2014.
Pascal Michaillat. Do matching frictions explain unemployment? not in bad times. American Economic Review, 102(4):1721-50, 2012.

Claudio Michelacci and David Lopez-Salido. Technology shocks and job flows. The Review of Economic Studies, 74(4):1195-1227, 2007.

Tommaso Monacelli and Roberto Perotti. Fiscal policy, wealth effects, and markups. Working Paper 14584, National Bureau of Economic Research, December 2008.

Dale Mortensen and Eva Nagypal. More on Unemployment and Vacancy Fluctuations. Review of Economic Dynamics, 10(3):327-347, July 2007.

Dale T. Mortensen and Christopher A. Pissarides. Job creation and job destruction in the theory of unemployment. The Review of Economic Studies, 61(3):397-415, 1994.

Makoto Nakajima. Business Cycles In The Equilibrium Model Of Labor Market Search And SelfInsurance. International Economic Review, 53(2):399-432, 052012.

Emi Nakamura and Jon Steinsson. Price Rigidity: Microeconomic Evidence and Macroeconomic Implications. Annual Review of Economics, 5(1):133-163, 052013.

Lee E. Ohanian and Andrea Raffo. Aggregate hours worked in OECD countries: New measurement and implications for business cycles. Journal of Monetary Economics, 59(1):40-56, 2012.

Barbara Petrongolo and Christopher A. Pissarides. Looking into the black box: A survey of the matching function. Journal of Economic Literature, 39(2):390-431, 2001.

Christopher A. Pissarides. Equilibrium Unemployment Theory, 2nd Edition, volume 1 of MIT Press Books. The MIT Press, June 2000.

Richard Rogerson and Randall Wright. Involuntary unemployment in economies with efficient risk sharing. Journal of Monetary Economics, 22(3):501-515, 1988.

Stephanie Schmitt-Grohé and Martín Uribe. What's news in business cycles. Econometrica, 80(6): 2733-2764, 2012.

Robert Shimer. The cyclical behavior of equilibrium unemployment and vacancies. American Economic Review, 95(1):25-49, 2005.

Robert Shimer. Comment on: \"New Keynesian perspectives on labor market dynamics\" by Tommy Sveen and Lutz Weinke. Journal of Monetary Economics, 55(5):931-935, July 2008.

José Ignacio Silva and Manuel Toledo. Labor turnover costs and the cyclical behavior of vacancies and unemployment. Macroeconomic Dynamics, 13, 52009.

Tommy Sveen and Lutz Weinke. New keynesian perspectives on labor market dynamics. Journal of Monetary Economics, 55(5):921 - 930, 2008.

Carlos Thomas. Search frictions, real rigidities, and inflation dynamics. Journal of Money, Credit and Banking, 43(6):1131-1164, 2011.

Antonella Trigari. The Role of Search Frictions and Bargaining for Inflation Dynamics. Technical report, 2006.

Antonella Trigari. Equilibrium unemployment, job flows, and inflation dynamics. Journal of Money, Credit and Banking, 41(1):1-33, 2009.

Willem Van Zandweghe. On-the-job search, sticky prices, and persistence. Journal of Economic Dynamics and Control, 34(3):437-455, March 2010.

Carl E. Walsh. Labor Market Search, Sticky Prices, and Interest Rate Policies. Review of Economic Dynamics, 8(4):829-849, October 2005.

## 1.A The model in detail

I provide a detailed derivation of the nominal model.

## Households

The original problem of the households can be written as:

$$
\begin{equation*}
W\left(\Omega_{t}^{H}\right)=\max _{C_{t}^{e}, C_{t}^{u}, B_{t+1}^{e}, B_{t+1}^{u}, I n s_{t}}\left\{N_{t}\left[U\left(C_{t}^{e}, \Gamma^{e}\right)+\beta E_{t} W\left(\Omega_{t+1}^{H, e}\right)\right]+\left(1-N_{t}\right)\left[U\left(C_{t}^{u}, \Gamma^{u}\right)+\beta E_{t} W\left(\Omega_{t+1}^{H, u}\right)\right]\right\} \tag{A.1}
\end{equation*}
$$

$$
\text { s.t } \begin{cases}\frac{B_{t}}{P_{t}}+\frac{D_{t}}{P_{t}}+w_{t} h-C_{t}^{e}-\tau_{t} \text { Ins }_{t}-\frac{B_{t+1}^{e}}{P_{t}}\left(1+r_{t}^{n}\right)^{-1} & =0  \tag{A.2}\\ \frac{B_{t}}{P_{t}}+\frac{D_{t}}{P_{t}}+\text { Ins }_{t}-C_{t}^{u}-\tau_{t} \text { Ins }_{t}-\frac{B_{t+1}}{P_{t}}\left(1+r_{t}^{n}\right)^{-1} & =0\end{cases}
$$

where I suppose that there exists an insurance company which sells a contract denoted by $I n s_{t}$; the zero profit condition for this insurance firm is given by $\prod_{t}^{i n s}=\tau_{t} I n s_{t}-\left(1-N_{t}\right) I n s_{t}=$ 0 and I assume also that employed and unemployed agents start with the same amount of bonds $B_{t}$.

I write the FOCs:

$$
\begin{cases}\frac{\partial L}{\partial C_{t}^{e}}= & 0 \rightarrow \lambda_{t}^{e}=U_{1}\left(C_{t}^{e}, \Gamma^{e}\right)  \tag{A.3}\\ \frac{\partial L}{\partial C_{t}^{u}}= & 0 \rightarrow \lambda_{t}^{u}=U_{1}\left(C_{t}^{u}, \Gamma^{u}\right) \\ \frac{\partial L}{\partial I n s_{t}}= & 0 \rightarrow\left(1-N_{t}\right) \lambda_{t}^{u}\left(1-\tau_{t}\right)=N_{t} \lambda_{t}^{e} \tau_{t} \rightarrow \lambda_{t}^{u}=\lambda_{t}^{e}=\lambda_{t} \\ \frac{\partial L}{\partial B_{t+1}^{e}}=0 \rightarrow \lambda_{t}^{e}\left[P_{t}\left(1+r_{t}^{n}\right)\right]^{-1}=\beta E_{t} \frac{\partial W\left(\Omega_{t+1}^{H, e}\right)}{\partial B_{t+1}^{e}} \\ \frac{\partial L}{\partial B_{t+1}^{u}}=0 \rightarrow \lambda_{t}^{u}\left[P_{t}\left(1+r_{t}^{n}\right)\right]^{-1}=\beta E_{t} \frac{\partial W\left(\Omega_{t+1}^{H, u}\right)}{\partial B_{t+1}^{u}}\end{cases}
$$

therefore $E_{t} \frac{\partial W\left(\Omega_{t+1}^{H, e}\right)}{\partial B_{t+1}^{e}}=E_{t} \frac{\partial W\left(\Omega_{t+1}^{H, u}\right)}{\partial B_{t+1}^{u}} \rightarrow B_{t+1}^{e}=B_{t+1}^{u}=B_{t+1}$ because of the continuity and concavity of the value function.

Therefore I can write the problem of the households with only one budget constraint for both types of agents.

I then report the value function for the households and derive the FOCs:

$$
\begin{align*}
& W\left(\Omega_{t}^{H}\right)=\max _{C_{t}^{e}, C_{t}^{u}, B_{t+1}}\left\{N_{t}\left[U\left(C_{t}^{e}, \Gamma^{e}\right)\right]+\left(1-N_{t}\right)\left[U\left(C_{t}^{u}, \Gamma^{u}\right)\right]+\beta E_{t} W\left(\Omega_{t+1}^{H}\right)\right\}  \tag{A.4}\\
& \text { s.t. } \begin{cases}N_{t} C_{t}^{e}+\left(1-N_{t}\right) C_{t}^{u}+\frac{B_{t+1}}{P_{t}} \frac{1}{\left(1+r_{t}^{n}\right)} & =w_{t} N_{t} h+\frac{B_{t}}{P_{t}}+\frac{D_{t}}{P_{t}} \\
N_{t+1} & =(1-s) N_{t}+p_{t+1} S_{t+1} \\
p_{t} & =\frac{M_{t}}{S_{t}} \\
S_{t} & =1-(1-s) N_{t-1}\end{cases} \tag{A.5}
\end{align*}
$$

where $\Omega_{t}^{H}=\left\{N_{t-1}, B_{t}\right\}$.

The FOCs are:

$$
\begin{equation*}
\frac{\partial L}{\partial C_{t}^{e}}=0 \rightarrow \lambda_{t}=U_{1}\left(C_{t}^{e}, \Gamma^{e}\right) \frac{\partial L}{\partial C_{t}^{u}}=0 \rightarrow \lambda_{t}=U_{1}\left(C_{t}^{u}, \Gamma^{u}\right) \tag{A.6}
\end{equation*}
$$

where $\lambda_{t}$ is the Lagrangian multiplier of the budget constraint, so it expresses the marginal value of real wealth (so the marginal value not of one unit of currency, as it would be if the budget constraint had been written in nominal terms, but the marginal value of the wealth necessary to have one unit more of the consumption good, i.e. the marginal value of $P_{t}$ units of nominal income).

$$
\begin{equation*}
\frac{\partial L}{\partial B_{t+1}}=0 \rightarrow \frac{\lambda_{t}}{P_{t}\left(1+r_{t}^{n}\right)}=\beta E_{t} \frac{\partial W\left(\Omega_{t+1}^{H}\right)}{\partial B_{t+1}} \tag{A.7}
\end{equation*}
$$

I apply the envelope theorem to find:

$$
\begin{equation*}
\lambda_{t}=\beta E_{t} \lambda_{t+1} \frac{\left(1+r_{t}^{n}\right)}{\frac{P_{t+1}}{P_{t}}} \rightarrow \lambda_{t}=\beta E_{t} \lambda_{t+1}\left(1+r_{t}\right) \tag{A.8}
\end{equation*}
$$

where therefore $r_{t}$ indicates the real net interest rate and the gross rate of inflation is given by $\pi_{t+1}=\frac{P_{t+1}}{P_{t}}$ : the Fisher parity condition $\left(1+r_{t}\right)=\frac{\left(1+r_{t}^{n}\right)}{\pi_{t+1}}$ holds.

Then, the first order conditions for the households problem as reported in the text are:

$$
\left\{\begin{array}{l}
\lambda_{t}=\frac{1}{C_{t}^{e}+\Gamma^{e}}  \tag{A.9}\\
\lambda_{t}=\frac{1}{C_{t}^{u}+\Gamma^{u}} \\
\lambda_{t}=\beta E_{t} \lambda_{t+1}\left(1+r_{t}\right)
\end{array}\right.
$$

which imply that $C_{t}^{e}=C_{t}^{u}+\Gamma^{u}-\Gamma^{e}$; since I assume $\Gamma^{u}-\Gamma^{e}>0$, I have that the consumption of the employed is higher than that one of the unemployed.

## Firms

The hypothesis that markets are complete brings me to derive the expression of the discount factor that is used by firms to evaluate their future flow of profits. I assume that intermediate firms are owned by households, so that their budget constraint be written, more precisely, as

$$
\begin{equation*}
N_{t} C_{t}^{e}+\left(1-N_{t}\right) C_{t}^{u}+\frac{Q_{t}}{P_{t}} B_{t+1}+\frac{Q_{t}^{s}}{P_{t}} \tilde{s}_{t+1}=w_{t} N_{t} h+\frac{B_{t}}{P_{t}}+\frac{\left(Q_{t}^{s}+D_{t}\right)}{P_{t}} \tilde{s}_{t} \tag{A.10}
\end{equation*}
$$

where $Q_{t}=\frac{1}{\left(1+r_{t}^{n}\right)}$ and I explicitly consider that at the beginning of the period the household holds a number $\tilde{s}_{t}$ of the firm's share, whose price is given by $Q_{t}^{s}$, and that she decides how many shares to hold for the following period.

The problem of the household can be written, for the sake of completeness, as:

$$
\begin{align*}
& \qquad \max _{C_{t}^{e}, C_{t}^{u}, B_{t+1}} E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[N_{t} U\left(C_{t}^{e}\right)+\left(1-N_{t}\right) U\left(C_{t}^{u}, \Gamma^{u}\right)\right]  \tag{A.11}\\
& \text { s.t. } \begin{cases}N_{t} C_{t}^{e}+\left(1-N_{t}\right) C_{t}^{u}+\frac{Q_{t}}{P_{t}} B_{t+1}+\frac{Q_{t}^{s}}{P_{t}} \tilde{s}_{t+1} & =w_{t} N_{t}+\frac{B_{t}}{P_{t}}+\frac{\left(Q_{t}^{s}+D_{t}\right)}{P_{t}} \tilde{s}_{t} \\
N_{t+1} & =(1-s) N_{t}+p_{t+1} S_{t+1} \\
p_{t} & =\frac{M_{t}}{S_{t}} \\
S_{t} & =1-(1-s) N_{t-1}\end{cases} \tag{A.12}
\end{align*}
$$

In addition to the previous FOCs, I have also that one for $\widetilde{s}_{t+1}$, which gives:

$$
\begin{equation*}
\beta E_{t} \lambda_{t+1} \frac{\left(Q_{t+1}^{s}+D_{t+1}\right)}{P_{t+1}}=\lambda_{t} \frac{Q_{t}^{s}}{P_{t}} \tag{A.13}
\end{equation*}
$$

By iterating forward the previous equation for one period:

$$
\begin{equation*}
\lambda_{t+1} \frac{Q_{t+1}^{s}}{P_{t+1}}=\beta E_{t+1} \lambda_{t+2} \frac{\left(Q_{t+2}^{s}+D_{t+2}\right)}{P_{t+2}} \rightarrow \lambda_{t} \frac{Q_{t}^{s}}{P_{t}}=\beta E_{t}\left[\beta E_{t+1} \lambda_{t+2} \frac{\left(Q_{t+2}^{s}+D_{t+2}\right)}{P_{t+2}}+\lambda_{t+1} \frac{D_{t+1}}{P_{t+1}}\right] \tag{A.14}
\end{equation*}
$$

so by continuing, with the addition of the transversality condition, according to which I eliminate the possibility of bubbles, i.e. $\lim _{j \rightarrow \infty} E_{t} \beta^{j} \lambda_{t+j} \frac{Q_{t+j}^{s}}{P_{t+j}}=0$, it obtains:

$$
\begin{equation*}
\lambda_{t} \frac{Q_{t}^{s}}{P_{t}}=E_{t} \sum_{j=1}^{\infty} \beta^{j} \lambda_{t+j} \frac{D_{t+j}}{P_{t+j}} \tag{A.15}
\end{equation*}
$$

which is the pricing condition of the share of the firm: the price (in real terms) of a share is given by the present value of the (real) dividends it promises, where the discount factor, in terms of marginal value of real consumption, is given by $\beta^{j} \frac{\lambda_{t+j}}{\lambda_{t}}$.

A firm which wants to maximize its real value, then, has to solve the problem:

$$
\begin{equation*}
\max \frac{\left(Q_{t}^{s}+D_{t}\right)}{P_{t}} \tilde{s}_{t}=\max \left(E_{t} \sum_{j=1}^{\infty} \beta^{j} \frac{\lambda_{t+j}}{\lambda_{t}} \frac{D_{t+j}}{P_{t+j}}+\frac{D_{t}}{P_{t}}\right) \tilde{s}_{t} \tag{A.16}
\end{equation*}
$$

Normalizing the number of shares as $\tilde{s}_{t}=\tilde{s}_{t+1}=1$, the problem can be written as:

$$
\begin{equation*}
\max E_{t} \sum_{j=0}^{\infty} \beta^{j} \frac{\lambda_{t+j}}{\lambda_{t}} \frac{D_{t+j}}{P_{t+j}} \tag{A.17}
\end{equation*}
$$

i.e. the firm maximizes the value of the real dividends using as discount factor the term $Z_{t+j} \equiv \beta^{j} \frac{\lambda_{t+j}}{\lambda_{t}}$, which means that at the end the real dividends are evaluated according to the marginal utility of consumption they provide to the households.

## Wage equation

The sharing rule which derives form the maximization of the Nash product is:

$$
\begin{equation*}
(1-\xi)\left(1-p_{t}\right) \frac{\partial V\left(N_{t-1}\right)}{\partial N_{t-1}}=\xi \frac{1}{\lambda_{t}} \frac{\partial W\left(N_{t-1}\right)}{\partial N_{t-1}} \tag{A.18}
\end{equation*}
$$

The expressions of the marginal values of employment for the firm and the household are given by:

$$
\begin{equation*}
\frac{\partial V\left(N_{t-1}\right)}{\partial N_{t-1}}=(1-s) \frac{\omega}{\Phi_{t}} \tag{A.19}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial W\left(N_{t-1}\right)}{\partial N_{t-1}}=\left[U\left(C_{t}^{e}, \Gamma^{e}\right)-U\left(C_{t}^{u}, \Gamma^{u}\right)+\lambda_{t}\left[w_{t} h-C_{t}^{e}+C_{t}^{u}\right]+\beta E_{t} \frac{\partial W\left(N_{t}\right)}{\partial N_{t}}\right](1-s)\left(1-p_{t}\right) \tag{A.20}
\end{equation*}
$$

From the maximization of the Nash product I get the FOC:

$$
\begin{equation*}
(1-\xi)\left(1-p_{t}\right) \frac{\partial V\left(N_{t-1}\right)}{\partial N_{t-1}}=\xi \frac{1}{\lambda_{t}} \frac{\partial W\left(N_{t-1}\right)}{\partial N_{t-1}} \tag{A.21}
\end{equation*}
$$

Since $\frac{\partial V\left(N_{t-1}\right)}{\partial N_{t-1}}=(1-s) \frac{\omega}{\Phi_{t}}$, this implies that by considering one period forward it obtains:

$$
\begin{equation*}
\frac{\partial W\left(N_{t}\right)}{\partial N_{t}}=\frac{(1-\xi)}{\xi} \lambda_{t+1}(1-s)\left(1-p_{t+1}\right) \frac{\omega}{\Phi_{t+1}} \tag{A.22}
\end{equation*}
$$

## Final producer and the retailers

The problem of the final producer is

$$
\begin{gather*}
\qquad \max P_{t} Y_{t}^{F}-\int_{0}^{1} Y_{t}(i) P_{t}(i) d i  \tag{A.23}\\
\text { s.t. }\left\{Y_{t}^{F}=\left\{\int_{0}^{1} Y_{t}(i)^{\frac{\eta-1}{\eta}} d i\right\}^{\frac{\eta}{\eta-1}}\right. \tag{A.24}
\end{gather*}
$$

where $Y_{t}^{F}$ is the production of the final producer, and since it sells to households I also know that $Y_{t}^{F}=C_{t} ; Y_{t}(i)$ is the production of retailer $i$ whose price is $P_{t}(i)$ desired by the final producer and $\eta$ is the elasticity of substitution.

The solution of this problem gives the demand for good $i$ which is faced by the retailer:

$$
\begin{equation*}
Y_{t}(i)=\left(\frac{P_{t}(i)}{P_{t}}\right)^{-\eta} C_{t} \tag{A.25}
\end{equation*}
$$

By substituting back the demand function I can derive the expression of the price index:

$$
\begin{equation*}
P_{t}=\left\{\int_{0}^{1} P_{t}(i)^{1-\eta} d i\right\}^{\frac{1}{1-\eta}} \tag{A.26}
\end{equation*}
$$

whose evolution is given by the expression:

$$
\begin{equation*}
P_{t}^{1-\eta}=\left[(1-\alpha) P_{t}(i)^{(1-\eta)}+\alpha P_{t-1}^{(1-\eta)}\right] \tag{A.27}
\end{equation*}
$$

I write here explicitly the equations needed to derive the Phillips curve, as it is standard in sticky price literature; the FOC for price setting decision is given by:

$$
\begin{equation*}
P_{t}(i)=\frac{E_{t} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+j} \frac{1}{P_{t+j}} C_{t+j} P_{t+j}^{(\eta+1)} \frac{\eta}{\eta-1} x_{t+j}}{E_{t} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+j} \frac{1}{P_{t+j}} C_{t+j} P_{t+j}^{(\eta-1)}} \tag{A.28}
\end{equation*}
$$

I can also re-write the FOC in the following way:

$$
\begin{equation*}
E_{t} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+j} \frac{1}{P_{t+j}} C_{t+j} P_{t+j}^{(\eta+1)}\left[\left(\frac{P_{t}(i)}{P_{t}} \frac{1}{P_{t+j} / P_{t}}\right)-\frac{\eta}{\eta-1} x_{t+j}\right]=0 \tag{A.29}
\end{equation*}
$$

so that if I define:

$$
\begin{gather*}
\frac{P_{t}(i)}{P_{t}}=p_{t}^{*}(i)  \tag{A.30}\\
X_{t, j}= \begin{cases}\frac{1}{\pi_{t+j} \ldots \pi_{t+1}} & j \geq 1 \\
1 & j=0\end{cases} \tag{A.31}
\end{gather*}
$$

it implies that $X_{t, j}=X_{t+1, j-1} \frac{1}{\pi_{t+1}}$. I can rewrite the previous expression as:

$$
\begin{equation*}
E_{t} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+j} C_{t+j} X_{t, j}^{(-\eta)}\left[p_{t}^{*}(i) X_{t, j}-\frac{\eta}{\eta-1} x_{t+j}\right]=0 \tag{A.32}
\end{equation*}
$$

which gives

$$
\begin{equation*}
p_{t}^{*}(i)=\frac{E_{t} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+j} C_{t+j} X_{t, j}^{(-\eta)} \frac{\eta}{\eta-1} x_{t+j}}{E_{t} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+j} C_{t+j} X_{t, j}^{(1-\eta)}}=\frac{R_{t}}{F_{t}} \tag{A.33}
\end{equation*}
$$

At this point I need to find the expressions for $R_{t}$ and $F_{t}$ : simply by taking out from the sum the first element, and using the definition of $X_{t, j}$, I can re-write the expression of $R_{t}$ as follows:

$$
\begin{equation*}
R_{t}=\lambda_{t} C_{t} x_{t} \frac{\eta}{\eta-1}+\beta \theta E_{t} \pi_{t+1}^{\eta} E_{t+1} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+1+j} C_{t+1+j} X_{t+1, j}^{(-\eta)} \frac{\eta}{\eta-1} x_{t+1+j} \tag{A.34}
\end{equation*}
$$

which gives the following relation:

$$
\begin{equation*}
R_{t}=\lambda_{t} C_{t} x_{t} \frac{\eta}{\eta-1}+\beta \theta E_{t} \pi_{t+1}^{\eta} R_{t+1} \tag{A.35}
\end{equation*}
$$

Similarly I obtain for $F_{t}$ the following expression:

$$
\begin{equation*}
F_{t}=\lambda_{t} C_{t}+\beta \theta E_{t} \pi_{t+1}^{\eta-1} F_{t+1} \tag{A.36}
\end{equation*}
$$

I notice that here, since I do not suppose any efficient subsidy to the labor input, I find the term which represents the distortion coming from monopolistic competition: $\frac{\eta}{\eta-1}$.

If I consider the dynamics for the aggregate price index, $P_{t}^{1-\eta}=\left[(1-\theta) P_{t}(i)^{(1-\eta)}+\theta P_{t-1}^{(1-\eta)}\right]$, I can re-write this expression as following:

$$
\begin{equation*}
1=(1-\theta) p_{t}^{*}(i)^{(1-\eta)}+\theta \pi_{t}^{(\eta-1)} \tag{A.37}
\end{equation*}
$$

so that finally

$$
\begin{equation*}
p_{t}^{*}(i)=\left\{\frac{1}{(1-\theta)}\left[1-\theta \pi_{t}^{(\eta-1)}\right]\right\}^{\frac{1}{1-\eta}} \tag{A.38}
\end{equation*}
$$

## The equations of the model

1. $N_{t+1}=(1-s) N_{t}+\Upsilon S_{t+1}^{1-\psi} V_{t+1}^{\psi}$
2. $S_{t}=1-(1-s) N_{t-1}$
3. $\lambda_{t}=\beta E_{t} \lambda_{t+1}\left(1+r_{t}\right)$
4. $r_{t}=\frac{r_{t}^{n}}{E_{t} \pi_{t+1}}$
5. $\lambda_{t}=\frac{1}{C_{t}^{u}+\Gamma^{u}}$
6. $C_{t}=C_{t}^{e} N_{t}+C_{t}^{u}\left(1-N_{t}\right)$
7. $C_{t}^{e}=C_{t}^{u}+\Gamma^{u}-\Gamma^{e}$
8. $\frac{\omega}{\Phi_{t}}=\left(x_{t} A_{t} h-w_{t} h\right)+\beta E_{t}\left[\frac{\lambda_{t+1}}{\lambda_{t}} \frac{\omega}{\Phi_{t+1}}(1-s)\right] \rightarrow x_{t}=\frac{w_{t}}{A_{t}}+\frac{\frac{\omega}{\Phi_{t}}-\beta E_{t}\left[\frac{\lambda_{t+1}}{\lambda_{t}} \frac{\omega}{\Phi_{t+1}}(1-s)\right]}{A_{t} h}$
9. $Y_{t}=A_{t} N_{t} h$
10. $Y_{t}=\Delta_{t} C_{t}+\omega V_{t}$
11. $w_{t} h=(1-\xi)\left[x_{t} A_{t} h+\beta(1-s) \omega E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} \theta_{t+1}\right]+\xi\left[\Gamma^{u}-\Gamma^{e}\right]$
12. $p_{t}^{*}(i)=\frac{E_{t} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+j} C_{t+j} X_{t, j}^{(-\eta)} \frac{\eta}{\eta-1} x_{t+j}}{E_{t} \sum_{j=0}^{\infty} \theta^{j} \beta^{j} \lambda_{t+j} C_{t+j} X_{t, j}^{(1-\eta)}}=\frac{R_{t}}{F_{t}}$
13. $R_{t}=\lambda_{t} C_{t} x_{t} \frac{\eta}{\eta-1}+\beta \theta E_{t} \pi_{t+1}^{\eta} R_{t+1}$
14. $F_{t}=\lambda_{t} C_{t}+\beta \theta E_{t} \pi_{t+1}^{\eta-1} F_{t+1}$
15. $p_{t}^{*}(i)=\left\{\frac{1}{(1-\theta)}\left[1-\theta \pi_{t}^{(\eta-1)}\right]\right\}^{\frac{1}{1-\eta}}$
16. $\frac{R_{t}^{n}}{R^{n}}=\left(\frac{R_{t-1}^{n}}{R^{n}}\right)^{\varrho_{m}}\left(\frac{\pi_{t}}{\pi}\right)^{\left(1-\varrho_{m}\right) \gamma_{\pi}}\left(\frac{Y_{t}}{Y}\right)^{\left(1-\rho_{m}\right) \gamma_{y}} e^{\varepsilon_{t}^{m}}$
17. $A_{t}=A_{t-1}^{\varrho_{A}} A^{\left(1-\varrho_{A}\right)} e^{\varepsilon_{t}^{A}}$

Log-linearization

1. $\hat{N}_{t}=(1-s) \hat{N}_{t-1}+s\left[\psi \hat{V}_{t}+(1-\psi) \hat{S}_{t}\right]$
2. $S \hat{S}_{t}+(1-s) \hat{N}_{t-1}=0$
3. $\hat{\theta}_{t}=\hat{V}_{t}-\hat{S}_{t}$
4. $\hat{p}_{t}=\psi \hat{\theta}_{t}$
5. $\hat{\Phi}_{t}=(\psi-1) \hat{\theta}_{t}$
6. $\hat{\lambda}_{t}=E_{t} \hat{\lambda}_{t+1}+\hat{r}_{t}^{n}-E_{t} \pi_{t+1}$
7. $\left[\frac{1}{C^{u}+\Gamma^{u}}\right] \hat{\lambda}_{t}+\frac{c^{u}}{\left(C^{u}+\Gamma^{u}\right)^{2}} \hat{C}_{t}^{u}=0$
8. $\frac{C}{N} \hat{C}_{t}^{e}+\left(C^{e}-C^{u}\right) \hat{N}_{t}-C^{e} \hat{C}_{t}^{e}-\frac{1-N}{N} C^{u} \hat{C}_{t}^{u}=0$
9. $\hat{C}_{t}^{e}=\frac{C^{u}}{C^{e}} \hat{C}_{t}^{u}$
10. $\hat{Y}_{t}=\hat{A}_{t}+\hat{N}_{t}$
11. $\hat{Y}_{t}=\frac{C \Delta}{Y}\left(\hat{C}_{t}+\hat{\Delta}_{t}\right)+\frac{\omega V}{Y} \hat{V}_{t}$ with $\Delta=1$ and $\hat{\Delta}_{t}=0 \forall t$
12. $\frac{\omega}{\Phi} \beta(1-s)\left(E_{t} \hat{\Phi}_{t+1}-\frac{1}{\beta(1-s)} \hat{\Phi}_{t}+\hat{\lambda}_{t}-E_{t} \hat{\lambda}_{t+1}\right)=x A h\left(\hat{x}_{t}+\hat{A}_{t}\right)-w h \hat{w}_{t}$
13. $w h \hat{w}_{t}=(1-\xi)\left[x A h\left(\hat{x}_{t}+\hat{A}_{t}\right)+\beta(1-s) \omega\left(E_{t} \hat{\theta}_{t+1}+E_{t} \hat{\lambda}_{t+1}-\hat{\lambda}_{t}\right)\right]+\xi\left[\left(C^{e} \hat{C}_{t}^{e}-C^{u} \hat{C}_{t}^{u}\right)\right]$
14. $\hat{\pi}_{t}=\frac{(1-\beta \theta)(1-\theta)}{\theta} \hat{x}_{t}+\beta E_{t} \hat{\pi}_{t+1}$
15. $\hat{A}_{t}=\varrho^{A} \hat{A}_{t-1}+\hat{\varepsilon}_{t}^{A}$
16. $\hat{r}_{t}^{n}=\varrho^{m} \hat{r}_{t-1}^{n}+\left(1-\varrho^{m}\right)\left[\gamma_{y} \hat{Y}_{t}+\gamma_{\pi} \hat{\pi}_{t}\right]+\hat{\varepsilon}_{t}^{m}$

## 1.B Alternative modelling hypothesis

## Variable hours

In the main text, I considered that the hours per worker are fixed and equal to $h$, which is calibrated to the steady state value of $1 / 3$.

One can wonder if my results continue to be valid once the hours per worker are also a variable of choice. In this case, in fact, there are two margins of adjustment of the labor input, the number of employees $\left(N_{t}\right)$ and the hours worked by each worker $\left(h_{t}\right)$ : if the adjustment on the second margin is more important than that one on the first margin, I could again incur in the critique of Shimer (2005). I therefore extend the benchmark model in order to have variable hours.

There are at least two possible ways of modelling: one consists in considering the "efficient bargaining model", in which hours are bargained at the same time in which the wage is chosen by the worker and the firm. The second one consists in considering the "right-to-manage" model, in which the firm chooses unilaterally the hours worked, after the wage has been set through the bargaining procedure.

I choose the efficient bargaining setup, in order to have results which are directly comparable to what has been mostly used in the literature ${ }^{52}$. In this case, the only difference with respect to my benchmark model consists in the fact that in the Nash product maximization problem there are now two variables of choice, $w_{t}$ and $h_{t}$.

The Nash product problem is therefore:

$$
\begin{equation*}
\max _{w_{t}, h_{t}}\left(\frac{1}{\lambda_{t}} \frac{\partial W\left(N_{t-1}\right)}{\partial N_{t-1}}\right)^{1-\xi}\left(\frac{\partial V\left(N_{t-1}\right)}{\partial N_{t-1}}\right)^{\xi} \tag{B.1}
\end{equation*}
$$

from which I obtain the wage and the hours equations:

$$
\begin{gather*}
w_{t} h_{t}=(1-\xi)\left[x_{t} A_{t} h_{t}+\beta(1-s) \omega E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} \theta_{t+1}\right]+\xi\left[\left(\Gamma^{u}-\Gamma_{t}^{e}\right)\right]  \tag{B.2}\\
\gamma\left(1-h_{t}\right)^{-\varepsilon}=A_{t} x_{t} \tag{B.3}
\end{gather*}
$$

where I remind that $\Gamma_{t}^{e}=\gamma \frac{\left(1-h_{t}\right)^{1-\varepsilon}}{1-\varepsilon}$.
The calibration strategy and therefore the steady state of the model are exactly equal to those of the benchmark model.

I check the IRFs and the moments implied by the model with productivity shocks.
As it can be seen in Figure 1.a, when hours per worker are also a variable of choice (left panel), employment decreases less than in the benchmark case, so that the volatilities of the job market variables are expected to be lower than in the benchmark case.

The relative importance of the intensive (hours worked per worked) and extensive (number of employees) margins in accounting for the total change of labor input across the business

[^25]Figure 1.a - Selected IRFs with variable hours (left) and fixed hours (right)

cycle has been widely studied: the recent paper by Ohanian and Raffo (2012) confirms a result known since King and Rebelo (1999), i.e. the fact that the extensive margin accounts for more than two thirds of the total labor input variations over the business cycle. This consideration has led to the use of models which do not explicitly model the intensive margin.

Before analyzing the quantitative properties of the model with variable hours in Table 7, I precise that I relied on the database created by Ohanian and Raffo (2012) for the series on hours worked per worker and employment, which anyway are taken from the Bureau of Labor Statistics website.

Since the reaction of the labor input is given by both the extensive and the extensive margin, the volatilities of the employment rate (and so of the unemployment) and of the vacancy rate are lower than in the benchmark case. However, as it can be seen by looking at unemployment, the quantitative result is far from the order of magnitude of the Shimer's critique.

The model anyway presents an important drawback: if one compares the relative volatilizes of the extensive and the intensive margins, it appears that the model implies an intensive margin which is more volatile than the extensive one, something which is at odd with what it is found in the data ${ }^{53}$.

## Separable preferences

In order to stress the importance of the choice of the non-separable preferences, in line with the point raised by Cheron and Langot (2004), I proceed to check quantitatively the relevance of the wealth effects on the movements of labor supply, when the traditional set of separable preferences are considered.

I report for convenience the functional form of the utility function in this case:

[^26]Table 7 - Model with variable hours: comparison with fixed hours (flex and sticky prices)

|  |  | Flexible prices | Flexible prices | Sticky prices | Sticky prices |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | U.S. data | Variable hours | $h=1 / 3$ | Variable hours | $h=1 / 3$ |
|  | $\mathbf{1 9 8 5 - 2 0 1 1}$ | Tech shock | Tech shocks | Tech shocks | Tech shocks |
| $\sigma_{Y}$ | 0.031 | 0.033 | 0.025 | 0.026 | 0.020 |
| $\sigma_{A}$ | 0.016 | 0.016 | 0.016 | 0.016 | 0.016 |
| $\frac{\sigma(U)}{\sigma(A)}\left(\frac{\sigma(S)}{\sigma(A)}\right)$ | 10.345 | $9.814(3.230)$ | $9.801(3.224)$ | $8.658(2.851)$ | $11.491(3.783)$ |
| $\frac{\sigma(V)}{\sigma(A)}$ | 10.514 | 7.603 | 7.603 | 10.689 | 16.640 |
| $\frac{\sigma(\theta)}{\sigma(A)}$ | 20.142 | 10.050 | 10.044 | 10.932 | 15.975 |
| $\frac{\sigma(N)}{\sigma(A)}$ | 0.615 | 0.565 | 0.565 | 0.497 | 0.665 |
| $\frac{\sigma(h)}{\sigma(A)}$ | 0.185 | 0.497 | - | 0.745 | - |
| $\frac{\sigma(w)}{\sigma(A)}$ | 1.009 | 0.913 | 0.907 | 0.963 | 1.391 |
| $\rho(U, A)(\rho(S, A))$ | -0.118 | $-0.979(-0.815)$ | $-0.979(-0.816)$ | $-0.157(-0.108)$ | $-0.060(-0.029)$ |
| $\rho(V, A)$ | 0.074 | 0.975 | 0.975 | 0.113 | 0.044 |
| $\rho(\theta, A)$ | 0.098 | 0.999 | 0.999 | 0.139 | 0.052 |
| $\rho(w, Y)$ | 0.338 | 0.999 | 0.998 | 0.851 | 0.564 |
| $\rho(Y, A)$ | 0.551 | 0.998 | 0.997 | 0.648 | 0.845 |
| $\rho(U, V)(\rho(S, V))$ | -0.859 | $-0.910(-0.667)$ | $-0.909(-0.665)$ | $-0.831(0.047)$ | $-0.845(0.286)$ |
| $\rho(U, \pi)(\rho(S, \pi))$ | -0.195 | $0.624(0.271)$ | $0.622(0.267)$ | $-0.301(-0.010)$ | $-0.533(0.042)$ |
| Autocorr $(U)$ | 0.94 | 0.916 | 0.916 | 0.517 | 0.27 |
| Autocorr $(V)$ | 0.94 | 0.712 | 0.709 | -0.066 | -0.306 |
| Autocorr $(\theta)$ | 0.94 | 0.847 | 0.846 | 0.188 | -0.099 |
| Autocorr $(Y)$ | 0.93 | 0.865 | 0.871 | 0.736 | 0.897 |

( $Y$ stands for production, $A$ for labor productivity, $U$ for unemployment -i.e. $U_{t}=1-N_{t}, S$ indicates the job seekers according to the definition of the model, i.e. $S_{t}=1-(1-s) N_{t-1}, V$ stands for vacancies, $\theta$ for labor market tightness, $w$ for real wage, $\pi$ for inflation)

$$
\begin{equation*}
U\left(C_{t}^{z}, L_{t}^{z}\right)=\log \left(C_{t}^{z}\right)+\Gamma^{z} \tag{B.4}
\end{equation*}
$$

for $z=e, u$, where $\Gamma^{e}=\gamma \frac{(1-h)^{1-\varepsilon}}{1-\varepsilon}$ and $\gamma, \varepsilon>0$, where the superscripts $e$ and $u$ refer respectively to the employed and the unemployed agent.

If I substitute in the model these preferences and I derive the wage equation, the differences with respect to the behavior of the outside options of the workers are evident.

Let me consider the expression for the wage equation:

$$
\begin{equation*}
w_{t} h=(1-\xi)\left[x_{t} A_{t} h+\beta(1-s) \omega E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} \theta_{t+1}\right]+\xi\left[\frac{\Gamma^{u}-\Gamma^{e}}{\lambda_{t}}\right] \tag{B.5}
\end{equation*}
$$

The element in the second square bracket in the RHS of the equation implies that wealth
effects have an impact on wage: when $\lambda_{t}$ decreases (i.e. when consumption increases), there is a upward pressure on the real wage.

In the version of the model with low nominal rigidities, therefore, the mechanism described by Shimer (2005) is worsened: since wages increase, firms profits are squeezed so that the effect of the increase in productivity is dampened even more. The quantitative impact in this case is very relevant, as it can be seen in the last column of Table $8^{54}$.

Table 8 - Comparison between separable and non-separable preferences

| Theoretical moments, smoothing parameter $=10^{5}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Flexible prices | Flexible prices | Sticky prices | Sticky prices |
|  | $\begin{aligned} & \text { U.S. data } \\ & \text { 1985-2011 } \end{aligned}$ | Non separable pref. <br> Tech shock | Separable pref. Tech shocks | Non separable pref. <br> Tech shocks | Separable pref. <br> Tech shocks |
| $\sigma_{Y}$ | 0.031 | 0.025 | 0.017 | 0.020 | 0.014 |
| $\sigma_{A}$ | 0.016 | 0.016 | 0.016 | 0.016 | 0.016 |
| $\frac{\sigma(U)}{\sigma(A)}\left(\frac{\sigma(S)}{\sigma(A)}\right)$ | 10.345 | 9.801 (3.224) | 0.733 (0.242) | 11.491 (3.783) | 5.025 (1.652) |
| $\frac{\sigma(V)}{\sigma(A)}$ | 10.514 | 7.603 | 0.571 | 16.640 | 7.242 |
| $\frac{\sigma(\theta)}{\sigma(A)}$ | 20.142 | 10.044 | 0.752 | 15.975 | 6.963 |
| $\frac{\sigma(w)}{\sigma(A)}$ | 1.009 | 0.907 | 1.006 | 1.391 | 0.851 |
| $\rho(U, A)(\rho(S, A))$ | -0.118 | -0.979 (-0.816) | -0.981 (-0.817) | -0.060 (-0.029) | 0.607 (0.526) |
| $\rho(V, A)$ | 0.074 | 0.975 | 0.971 | 0.044 | -0.314 |
| $\rho(\theta, A)$ | 0.098 | 0.999 | 0.999 | 0.052 | -0.451 |
| $\rho(w, Y)$ | 0.333 | 0.998 | 1 | 0.564 | 0.773 |
| $\rho(Y, A)$ | 0.551 | 0.997 | 1 | 0.845 | 0.964 |
| $\rho(U, V)(\rho(S, V))$ | -0.859 | -0.909 (-0.665) | -0.908 (-0.658) | -0.845 (0.286) | -0.845 (0.281) |
| $\rho(U, \pi)(\rho(S, \pi))$ | -0.195 | 0.622 (0.267) | 0.616 (0.260) | -0.533 (0.042) | -0.929 (-0.313) |
| Autocorr ( $U$ ) | 0.94 | 0.916 | 0.913 | 0.27 | 0.277 |
| Autocorr (V) | 0.94 | 0.709 | 0.710 | -0.306 | -0.237 |
| Autocorr ( $\theta$ ) | 0.94 | 0.846 | 0.844 | -0.099 | -0.052 |
| Autocorr (Y) | 0.93 | 0.871 | 0.841 | 0.897 | 0.930 |

to the definition of the model, i.e. $S_{t}=1-(1-s) N_{t-1}, V$ stands for vacancies, $\theta$ for labor market tightness, $w$ for real wage,
$\pi$ for inflation)

## Traditional timing

The quantitative impact of the adoption of the instantaneous hirings hypothesis is not of fundamental importance in the model with no nominal rigidities, while it affects dramatically the results when prices are sticky.

[^27]I report in Table 9 the quantitative results for two models which differ for the dynamics of labor market flows: in the case with traditional timings, obviously there is no difference between unemployed and job seekers in the model.

The different dynamics implies that the same values for the separation rate and the job finding rate are consistent with different values of the employment rate: in my benchmark specification $N=0.9455$, while with the traditional dynamics the implied rate of employment is $N=0.8638$.

If one considers the model with flexible prices, the instantaneous hirings hypothesis brings some additional volatility: labor market tightness is 2.5 higher in the data than in the simulated model with the traditional timing assumption, while the alternative hypothesis implies a volatility which is half the empirical one. The most important impact, however, is found when prices are sticky: as already noticed by Van Zandweghe (2010), with the traditional timing all the reaction to productivity shocks is absorbed by movements in prices ${ }^{55}$, so that the instantaneous hirings hypothesis is of fundamental importance in a framework with price stickiness.

[^28]Table 9 - The impact of the "instantaneous hirings" hypothesis

|  |  | Flexible prices | Flexible prices | Sticky prices | Sticky prices |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { U.S. data } \\ & \text { 1985-2011 } \end{aligned}$ | Instantan. hirings Tech shock | Traditional timing Tech shocks | Instantan. hirings Tech shocks | Traditional timing Tech shocks |
| $\sigma_{Y}$ | 0.031 | 0.025 | 0.022 | 0.020 | 0.016 |
| $\sigma_{A}$ | 0.016 | 0.016 | 0.016 | 0.016 | 0.016 |
| $\frac{\sigma(U)}{\sigma(A)}\left(\frac{\sigma(S)}{\sigma(A)}\right)$ | 10.345 | 9.801 (3.224) | 2.733 (-) | 11.491 (3.783) | 0.025 (-) |
| $\frac{\sigma(V)}{\sigma(A)}$ | 10.514 | 7.603 | 6.081 | 16.640 | 0.056 |
| $\frac{\sigma(\theta)}{\sigma(A)}$ | 20.142 | 10.044 | 8.230 | 15.975 | 0.075 |
| $\frac{\sigma(w)}{\sigma(A)}$ | 1.009 | 0.907 | 0.907 | 1.391 | 11.764 |
| $\rho(U, A)(\rho(S, A))$ | -0.118 | -0.979 (-0.816) | -0.816 (-) | -0.060 (-0.029) | 0.769 (-) |
| $\rho(V, A)$ | 0.074 | 0.975 | 0.984 | 0.044 | -0.998 |
| $\rho(\theta, A)$ | 0.098 | 0.999 | 0.998 | 0.052 | -0.966 |
| $\rho(w, Y)$ | 0.333 | 0.998 | 0.998 | 0.564 | -0.453 |
| $\rho(Y, A)$ | 0.551 | 0.997 | 0.984 | 0.845 | 1 |
| $\rho(U, V)(\rho(S, V))$ | -0.859 | -0.909 (-0.665) | -0.701 (-) | -0.845 (0.286) | -0.806 (-) |
| $\rho(U, \pi)(\rho(S, \pi))$ | -0.195 | 0.622 (0.267) | 0.377 (-) | -0.533 (0.042) | -0.193 (-) |
| Autocorr ( $U$ ) | 0.94 | 0.916 | 0.915 | 0.27 | 0.932 |
| Autocorr (V) | 0.94 | 0.709 | 0.738 | -0.306 | 0.801 |
| Autocorr ( $\theta$ ) | 0.94 | 0.846 | 0.866 | -0.099 | 0.897 |
| Autocorr (Y) | 0.93 | 0.871 | 0.910 | 0.897 | 0.900 |

( $Y$ stands for production, $A$ for labor productivity, $U$ for unemployment -i.e. $U_{t}=1-N_{t}, S$ indicates the job seekers according to the definition of the model, i.e. $S_{t}=1-(1-s) N_{t-1}$, $V$ stands for vacancies, $\theta$ for labor market tightness, $w$ for real wage, $\pi$ for inflation)

## Chapter 2

## Accounting for Labor Gaps (joint with F. Langot)

### 2.1 Introduction

The evolution of total hours worked during the post WWII period was characterized by sharp differences across developed economies: if we look at the evolution of this aggregate, by analysing in particular the US and three selected European countries ${ }^{1}$ (France, Germany ${ }^{2}$ and the UK), we can observe a sharp decline in hours in the two continental European countries, at least until the mid-eighties, while in the UK the decline was less significant and in the US total hours marginally increased (see Figure 2.1, top panel). Moreover, the gaps between the US and the selected European countries have continuously increased since the mid-seventies: whereas in the 1960s, a European employee worked $15 \%$ more than an American worker, his descendant works $30 \%, 20 \%$, or $10 \%$ less than his American counterpart, if he lives, respectively, in France, Germany or the UK (see Figure 2.1, bottom panel).

The heterogeneity in the evolution of aggregate worked hours has already been highlighted in the literature ${ }^{3}$. The main finding of these contributions is the following: "[...] I determine the importance of tax rates in accounting for these differences in labor supply for the major advanced industrial countries and find that tax rates alone account for most of these differences in labor supply", Prescott (2004).

According to this view, the welfare impact of a tax reform in France could therefore be considerable: if France were to adopt American tax rates on labor income (i.e. reduce the

[^29]Figure 2.1 - Total hours 1960-2010

effective tax rate on labor by 20 percentage points) "the welfare of the French people would increase by 19 percent in terms of lifetime consumption equivalents".

Such large welfare gains would undoubtedly call for cutting French (and more broadly, European) taxes down to US levels. At general equilibrium, the corollary of such a policy would naturally be to reduce government expenditures and transfers, as is suggested by Prescott (2004) ${ }^{4}$.

A simple neoclassical growth model with endogenous labor supply, as that adopted by Prescott (2004), can be considered as a parsimonious approach to quantitatively evaluate the impact of a tax reform on the level of aggregate hours worked at general equilibrium: in this case, the labor wedge is reduced to the tax wedge.

However, this approach does not allow a distinction to be made between hours worked per employee and the number of employees, whereas in fact these two margins experienced different evolutions (see Figure 2.2): while the American "jobs miracle", characterized by an increase in the chance of being employed and a marginal decline in the hours worked per employee, seems to be a peculiar feature of only the US, until the end of the 2000s, Germany

[^30]and the UK show comparable evolutions to each other.
In particular, it seems possible to observe the effects on the employment rate of the Hartz reforms implemented in Germany from the mid nineties, as well as the Thatcher reforms, applied in UK from the end of the seventies. By contrast, France performs particularly bad. France shares Germany's substantial decline in hours worked per employee (the gap between a French and an American worker is equal to $-10 \%$ at the end of the sample). However, in France the chance of being employed falls from $65 \%$ to $60 \%$ (leading to a gap in the chance of being employed equal to 20 percentage points compared to an American worker), following the Mitterrand reforms.

Finally, it can be observed that these long-run shifts are much larger than those observed during periods (from the 1960s to 1990s) when governments followed expansionary Keynesian policy recommendations. These facts suggest that the long-run trends of the European economies have been driven by a negative supply shift.

Figure 2.2 - The decomposition of total hours 1960-2010


The objective of this paper is to develop a dynamic general equilibrium model with search and matching frictions, in order to decompose the labor wedges into two components: the "hours worked per employee wedge" and the "employment wedge".

From a theoretical point of view, the originality of our paper is that it proposes a theory for the allocation of time that makes it possible to identify the relative contribution of taxes
and changes to labor market institutions, by decomposing the aggregate hours of work into the intensive and extensive margins.

We then propose to analyze the decentralised allocation of a general equilibrium model with matching frictions, wage bargaining and efficient bargaining on the number of hours worked per employee ${ }^{5}$.

With this type of theory of hours and employment allocation, we can distinguish between the contribution of taxes and the contribution of changes in labor market institutions (such as unemployment benefit systems and the type of the wage bargaining arrangements) on the labor wedge ${ }^{6}$.

An important strand of literature focuses on the low employment rate in Europe.
These studies encompass those that focus on unemployment ${ }^{7}$ and those that analyse non-participation ${ }^{8}$.

The main result of these studies is that incomes received during inactivity create a large distortion to the employment rate: unemployment benefits, pensions or subsidised education induce an implicit tax on the labor supply. Hence, beyond the unemployment gap, the European countries are characterised by an employment gap.

This leads us to focus on the employment rate, rather than on the restrictive measure of the unemployment rate ${ }^{9}$, and on the impact of the distortions implied by the shifts in labor market institutions.

One possible shortcoming of our approach is linked to the "representative agent hypothesis": behind our modelling choice, there is the hypothesis that there exists a representative agent, so that the average tax rates we consider, as well as the labor market institutions, are relevant to explain his choices. The labor market institutions we consider, in particular, apply to a typical "insider" of labor market:a representative adult agent. We are aware of the fact that the employment rate of the young or of the old are characterised by different evolutions than those of the middle aged worker: our summary measure of replacement rate of

[^31]income in non-employment, in particular, does not properly take into account the variables which affect labor choices at the end of the career ${ }^{10}$.

Our approach therefore consists of distinguishing the elasticities of the hours worked per worker and of the employment rates with respect to long-run changes in taxation and labor market institutions: as is suggested by Ljungqvist and Sargent (2007), the standard neoclassical growth model used by Prescott (2004) cannot account for the observed impact of both taxes and labor market institutions.

Our approach also enables us to overcome the doubts expressed by Nickell (1997) about the puzzling fact that, while in Europe labor market institutions remained almost unchanged between the sixties and the nineties, the performance of countries in terms of unemployment was often reversed at the end of this period versus the beginning of it ${ }^{11}$.

First of all, we show that labor market institutions did indeed change; secondly, we emphasise the relative elasticities of both hours and employment to these changes, highlighting that it is indeed possible to reconcile the long-run evolution of the employment rate with the evolution of the labor market institutions.

A point to be emphasized is that we abstract from the "cultural" interpretation in explaining differences in hours worked across countries: our hypothesis is that agents, across countries, share the same "preferences" for leisure and work. This is somehow a simplifying assumption; however, more than relying of intrinsical differences between peoples on the two sides of the Ocean, we find more convincing the approach proposed by Alesina et al. (2006), when they speak about externalities in the utility from leisure at a society level (or the existence of a "social multiplier).

From a methodological point of view, we depart from Prescott (2004) and Ohanian et al. (2008): they only compute wedges in the static first-order condition governing labor supply, in a calibrated version of the growth model ${ }^{12}$.

In our paper, we instead follow McDaniel (2011): rather than simply focusing on the static first-order conditions, we solve for the time series of choice variables given countryspecific tax rates, labor market institutions and productivity series, with perfect foresight ${ }^{13}$.

What is interesting about this approach is that it tests the theory, given the restrictions it implies on agent expectations. This point is particularly important from a long-run perspective, where the objective is to explain the structural shifts on labor market allocation, linked to permanent drifts in taxation and institutions. From an "econometric" point of view, this

[^32]type of test is more demanding with respect to the theory, because the complete resolution of the model leads to an accumulation of the forecast errors of the static first-order conditions. With this method, we then include measures based on static wedges and others, such as the one proposed by Pissarides (2007), where the steady state equilibrium variables depend on the growth rate of the exogenous variables.

With this general equilibrium approach, we take into account the dynamics of the Solow residual and of taxes on capital: these two components cannot be disregarded in an analysis of the long-run evolutions of the input factors.

Finally, we also depart from Prescott (2004) when it comes to the choice of modelling of government expenditure. Indeed, Prescott's evaluation is performed under the simplifying extreme assumption that all government expenditure can be substituted by private consumption: hence, in Prescott's view, the government size is "excessive" by definition, because its optimal size is zero.

This view is contestable: one can distinguish between individual goods public expenditure (education, health, etc.) and those that are intrinsically collective (army, justice, collective equipment).

Unlike the first category, the optimal size of collective public spending cannot be zero because it is not a perfect substitute to private consumption, since it cannot be made by the household herself ${ }^{14}$. Hence, in our evaluation of a tax-cut reform, we will only reduce the individual government spending, which induces a misallocation of consumption. In the model, we then distinguish between these two types of government spending, by postulating that the optimal size of the collective good is strictly positive.

What do we learn from our methodology? Firstly, from a theoretical point of view, we show that:
(i) there is a non-trivial interaction between the two labor margins, leading to a substitution between hours per worker and employment, and
(ii) taxes and labor market institutions do not have the same impact on these substitutions.

A change in the labor market institution has a direct impact solely on the extensive margin: lowering wage pressures (by reducing the bargaining power of workers or their nonemployment benefits) increases the level of hirings. This is perceived as a positive wealth effect, leading households to reduce their hours worked by employee. Hence, the magnitude of such a change in institutions can be significant for the two labor margins ${ }^{15}$.

[^33]This is not the case for a reduction in tax. Indeed, this policy provides incentives to increase effort at work (intensive margin), with the side effect being to raise disutility at work, and thus the reservation wage. This last effect counteracts the positive impact of the tax decrease on the labor costs. Thus, the employment rate is less sensitive to tax reforms, whereas the hours per worker rate is highly sensitive to them.

Secondly, from a quantitative point of view, several points must be stressed:
(i) the choice of these countries allows us to control for certain factors: the US workers constitute the "non-treated" group because the tax wedge and the labor market institutions are stable over the whole period, whereas the French workers constitute the group experiencing different treatment. In France, the taxes increase over the period, and the labor market institutions shift in favor of the worker, at the beginning of the first socialist government (the beginning of the 1980s). We also use German and UK data in order to check that contrasting reforms in Europe produce different model predictions. In particular, in Germany the taxes increase over the period, and the labor market institutions shift in favor of firms, during the nineties, with the Kohl government and with the Hartz reforms (after 2003). In UK, taxes are low, and the labor market reforms implemented by the Thatcher government, at the beginning of the eighties, shift power in favor of firms.
(ii) Given our calibration strategy, which only restricts the averages of the simulated series of hours per worker and employment to match their empirical counterparts, we show that the model enables to predict the slope of the continuous decline in hours per worker in all European countries, and the considerable changes in employment rates. France, Germany and the UK exhibit contrasting evolutions, since these countries did not implement the same reforms at the same time.
(iii) Finally, we can compute the welfare gains associated with a change in policy. The size of the distortions induced by the high tax wedge in France overcomes that one induced by the distortions on the labor market. Moreover, a complementarity exists between the two sets of reforms, originating from the fact that each reform affects, at the same time, both the intensive and the extensive margin: the sum of the gains coming from each reform is lower than the overall gains coming from implementing both reforms at the same time.

This paper is organised as follows. Section 2.2 documents the data used. In Section 2.3, the search and matching model is outlined. Section 2.4 applies the model to the data, after calibrating the key parameters of the model. In Section 2.5 we perform counterfactual experiments. Section 2.6 sets out our conclusion.

### 2.2 The basic facts

In this section, we describe the data we use for the US, France, Germany and the UK. We consider the UK to be an "intermediate" case, lying somewhere in between the US and the continental cases. With respect to Germany, it is important to remember that most of the data have been reconstructed, so as to correspond to the reunified country after 1989. However, the variables which measure labor market arrangements are typically not comparable between the two economies of West and East Germany: we therefore decided to perform a "check" of the explanatory power of our model, by looking at its predictions for Germany only for the last 20 years (i.e. starting from 1990). We can therefore demonstrate why the choice of these countries is interesting, from the perspective of a test of the theory: since the time series of the exogenous variables (taxes, labor market institutions and technological progress) do not have the same dynamics, we expect that the same will apply for the endogenous variables, which are the hours per worker and the employment rate.

### 2.2.1 Hours per worker and employment rates

In this section, we present the basic facts we want to take into account in our model ${ }^{16}$. Figures 2.1 and 2.2 give a complete description of the dynamics of hours worked and employment in the selected countries. Table 2.1 summarises these data ${ }^{17}$.

Table 2.1 - Ratios

| Ratios in 2008 relative to 1960 | US | FR | UK | GE |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{N h}{P o p * 365 * 14}$ | 1.00 | 0.66 | 0.85 | 0.70 |
| $\frac{N}{P o p}$ | 1.10 | 0.95 | 1.03 | 1.06 |
| $\frac{h}{365 * 14}$ | 0.91 | 0.70 | 0.82 | 0.66 |

For the UK we report the ratios in 2008 relative to 1971

The evolution of total hours worked differs widely over the last fifty years in the selected countries: while hours remained flat in the US, we observe a considerable decline in the European continental countries (France and Germany), with the evolution in the UK being in between that of the US and its continental neighbours. This evolution is in fact caused, as we have seen, by the path of the two margins which compose total labor input: the intensive margin and the extensive margin. In the US, the employment rate showed an increasing trend

[^34]between the mid-eighties and the end of the nineties, while in France the employment rate decreased between the eighties and the mid-nineties. The German performance is markedly better than that of France, with a continuously increasing employment rate starting from the mid-nineties. As we can see in the bottom right panel of Figure 2.2, the UK recently closed the "employment gap" with respect to the US that started to be observed in the mid-seventies.

If we look at the intensive margin, in the top left panel of Figure 2.2, we observe a sharp decline in France and Germany, while in the US hours per worker declined only marginally. The UK seems to share the evolution of its continental neighbour, but only until the mideighties, when the decline slowed down.

From the mid-nineties, with the Kohl reforms, the employment rate continuously increases in Germany. This feature is magnified from the beginning of the implementation of the Hartz reforms ${ }^{18}$. In the UK, the reforms implemented by the Thatcher governments seem to have a significant impact on the probability of being employed: the employment rate has been rising since the mid-eighties, with a current level equal to the one observed in 1960. From the beginning of the eighties, the number of hours worked per employee is also stable and equal to that of an American worker. We can measure the total-hours-worked gap with the US, and the contribution of employment rates and hours per worker to this gap. We then replicate the counterfactual exercise proposed by Rogerson (2006) to assess the explanatory role of both the hours worked per employee and the employment rate in the dynamic of total employment. The procedure used is as follows:

1. Consider a reference year $t_{0}$, say 1981.
2. Consider a different year $t$. For each country $i=\{$ France, Germany, UK, US $\}$, compute the change in the employment rate between $t_{0}$ and $t: \Delta_{i, t}^{N}=N_{i, t}-N_{i, t_{0}}$.
3. Compute the change for country $i \neq$ US minus the change for the US: $r \Delta_{i, t}^{N}=\Delta_{i, t}^{N}-$ $\Delta_{U S, t}^{N} . r \Delta_{i, t}^{N}$ denotes the differential in country $i \neq$ US's employment rate relative to the US (and to the reference year $t_{0}$ ).
4. Consider the hypothetical case in which the change in country $i \neq$ US's relative employment did not happen: the chance of being employed evolves as in the US. Instead, assume that the $r \Delta_{i \neq U S, t}^{N}(\%)$ individuals were employed in $t$ and worked the same number of hours as an individual in country $i \neq \mathrm{US}$, that is $h_{i, t}$. This would raise total hours in country $i \neq$ US by an amount equal to $\Delta_{i, t}^{N h, N}=-r \Delta_{i, t}^{N} \times h_{i, t}$. The series

[^35]$\Delta_{i, t}^{N h, N}$ are the number of additional hours worked that economy $i$ would have at date $t$ if its employment rate were the same as in the US.
5. The comparison of $\Delta_{i, t}^{N h, N}$ with the observed differential in relative total hours gives us a measure of the employment rate contribution. The observed differential is computed as $\Delta_{i, t}^{o b s}=N_{i, t} h_{i, t}-N_{U S, t} h_{U S, t}$. If the contribution of employment to the total hours gap is significant, we expect a series of hypothetical hours $\left(\Delta_{i, t}^{N h, N}\right)$ close to the actual hours $\left(\Delta_{i, t}^{o b s}\right)$.

Figure 2.3 - A decomposition of total hours gaps


In each panel of Figure 2.3, for each country, the "gap" is $\Delta_{i, t}^{o b s}$. The time series "N contribution" and "h contribution" are computed respectively using $\Delta_{i, t}^{N h, N}$ and $\Delta_{i, t}^{N h, h}$. The reference year $\left(t_{0}\right)$ is 1981 . Total hours $N h$ are multiplied by $365^{*} 14$ : the gaps are then the differences between the US and European countries for one year.

We also assess how much of the gap between each country and the US, in total hours worked, is due to the intensive margin. To this end, we compare the contribution of the additional hours that European countries would have if all employed workers were working as much as American workers $\left(\Delta_{i, t}^{N h, h}\right)$. Given that $\Delta_{i, t}^{N h, N}$ measures the additional hours that European countries would have if the employment rates were the same as in the US, we have
$\Delta_{i, t}^{N h, N}+\Delta_{i, t}^{N h, h}=\Delta_{i, t}^{\text {obs }}$, where $\Delta_{i, t}^{N h, N}$ and $\Delta_{i, t}^{N h, h}$ are the relative contribution of extensive and intensive margins in the observed gap $\left(\Delta_{i, t}^{\text {obs }}\right)$.

In Figure 2.3 we report the results for France, Germany and the UK, where the reference year is $t_{0}=1981$. As expected, the size of the gap is smaller in the UK than in France and Germany. From the beginning of the 1980s, the three European countries exhibit differing experiences. In Germany, the contribution of the employment rate is minimal, showing that this country provides the same chance of being employed as the US. This dynamics, in terms of the relative influence of the employment rate on the hours gap, is shared by the UK. Thus, for these two countries, the gap with the US comes from a smaller number of hours worked per employee: the losses, in Germany and in the UK, are 200 hours and 100 hours respectively. In France, the experience is different: from 1985 to 2000, the most significant contribution to the hours gap is the low chance of employment provided by the French economy. During this period, the losses due to this "under-employment" are equal to 150 hours per year and per participant. At the same time, we also observe in France, that the hours per worker make a strong contribution to the hours gap (approximately 150 hours per worker on average since 1980). Thus, both gaps in France are significant and of similar magnitude.

### 2.2.2 Taxes, labor market institutions and technological progress

In our model we have three sets of exogenous variables: the set of tax rates (on labor income, on consumption, on capital revenues and on investment), a set of variables summing up the labor market institutions (the replacement rate and an indicator of the level of unionisation, representing the bargaining power of the worker), and finally the Solow residual of the production function, representing the technological progress in labor productivity.

## Taxes on labor

In terms of data sources, the tax rates are taken from McDaniel (2007) ${ }^{19}$. In Figure 2.4, we report the evolution of this first exogenous variable. We observe that the tax wedge is stable over the total period in the US. In 1960, it was necessary to produce $\$ 1.4$ in order to consume $\$ 1$. In 2010, the situation is unchanged. By contrast, in France, these tax wedges increase rapidly between 1960 and 1985, and continue to grow afterwards, but at a lower rate. Whereas in 1960 it was necessary to earn $\$ 1.6$ in order to consume $\$ 1$, in 2010, it would be necessary to earn for a amount of $\$ 2.2$ in order to obtain $\$ 1$ of consumption. In the UK, the evolution of the tax wedge is comparable to what happened in France until the mid-seventies, while after that date it ceases to grow, and its level today remains in between

[^36]Figure 2.4 - Tax wedge $\frac{1+\tau_{c}}{1-\tau_{w}}$

that of the US and France. In Germany the tax wedge increased through the seventies, while a lower growth in the tax rate is observed in the sub-sample 1980-2010.

## Taxes on capital

Given that our model is a general equilibrium one, it is important not to omit taxes on capital. These taxes modify the relative demand between capital and employment. There are two type of taxes: those on the revenues of existing capital, and those on investment goods. The tax rates on capital revenues and on investment are taken from McDaniel (2007) ${ }^{20}$. In Figures 2.5 we report the evolution of this set of exogenous variables. The policy choices are not the same on the two sides of the Atlantic. In the US and the UK, the taxes are based largely on capital incomes, whereas in France and Germany taxes are heavier on investment. Note that the gaps in the capital income taxes decline over the entire sample, whereas the gaps in the investment taxes are persistent.

## Labor market institutions

The data on bargaining power are taken from the Database on Institutional Characteristics of Trade Unions, Wage Settings, State Intervention and Social Pact (ICTWSS) ${ }^{21}$. The bargaining power of the worker is considered as an average of two indicators, the union density and the union coverage. The replacement rate is taken from the OECD. Since it is available only for uneven years, we linearly interpolated the missing values.

[^37]Figure 2.5 - Taxes on capital and investment 1960-2010


Tax rate on capital revenues


Tax rate on investment

Two statistical indicators are available, to give an indirect measure of the bargaining power of the employee during the wage bargaining process: the union coverage and the union density. These two indicators are closely linked to the bargaining power: a wide union coverage or a high union density enable the worker to make counter-offers during the bargaining process. We choose to evaluate the worker bargaining power by the average of the union coverage and the union density. Indeed, even if we observe a decline in union density, the institutional agreements can conserve an "historical" coverage (the memory effect). It should be remembered that for Germany, the data prior to the reunification correspond only to West Germany.

In Figure 2.6 we report the evolution of these exogenous variables. A brief look at these data suggests that the influence of the labor market institutions is very different in our selected countries: the unemployment benefits are more than two times lower, in 2008, in the two English-speaking countries than in France; the path of the unemployment benefit level in the UK is striking: while at the beginning of the sample the replacement rate was more similar to the French one, from the eighties onwards we observe a strong policy intervention towards decreasing it. In France we observe a substantial rise in generosity at the beginning of the first socialist government (the beginning of the 1980s). It is also important to notice that this increase in generosity of unemployment benefits is also accompanied by an increase in eligibility: older workers ( 55 years old and above) can obtain benefits until their retirement age, without the constraint of searching for a job, and from 1988, non-employed workers can receive a "social minimum income", which is larger than the unemployment benefits in the UK. Given that this rise in the replacement rate is accompanied by less stringent eligibility

Figure 2.6 - Labor market institutions 1960-2010


Replacement rate


Bargaining power of the workers
rules, it can be viewed as an increase of the non-wage incomes for all 16-65 year old. This French policy experience can thus be analysed in a model which does not distinguish between unemployment and non-participation (retirement, pre-retirement etc).

In terms of the bargaining power of workers, we observe a continuous decline in the US, while the UK shows a sharp change of direction in the eighties, in comparison to the evolution observed in the seventies. Germany and France do not share the same dynamics of labor market institutions over the whole sample, nor the same labor market outcomes. Indeed, at the beginning of the nineties, with the Kohl government, Germany simultaneously experiences a decline in the replacement rate and in the bargaining power of workers. Between 1993 and 1997, three laws lead to a reduction in the eligibility of unemployment insurance, and in the replacement rate. The first Schröder government chose to backtrack on these labor market reforms, but the labor market outcomes leads the second Schröder government to re-introduce more flexibility. After 2002 and the implementation of the Hartz reforms, the decline of the replacement rate is greater. In France, we observe a continuous increase in the bargaining power of workers from 1960 to 1990 , followed by a stabilisation at a high level. Finally, in the UK, reforms to the labor market began sooner, with the first Thatcher government. The direction of these reforms is the opposite to that of France: the replacement rate largely declines, as well as the bargaining power of workers.

## Technological progress

We recover the Solow residuals, measuring the labor augmenting technological process, from the production function, as $A=\left(\frac{Y}{K^{1-\alpha}}\right)^{1 / \alpha} \frac{1}{N h}$. Figure 2.7 shows the logarithm of total
factor productivity (TFP) time series, the raw series (left panel) and the deflated series (right panel). The left panel suggests that there are some "breaks" in these time series.

Figure 2.7 - Solow residuals in log


As far as the "European" economies are concerned, there is a great deal of literature asserting that there was a period of technological "catching-up", after the material destructions of the WWII period. In order to capture this feature, we proceed in a very simplistic manner: we identify a linear trend for the TFP using only data starting from the half (for France) or the end (for the UK) of the eighties, and then use it to deflate the whole sample data. In this way, our deflated TFP can track the trajectory of the technological "catch-up". The approach to identifying technological progress in Europe then consists of computing the expectations of agents, by setting the growth rate of technological progress at its long-run value, with the catch-up period being a transitory period, during which the level of technological progress is under its long-run value. The convergence toward this long-run trend is effective in the mid-eighties. For the US, we observe a break in 1990: the TFP seems to be higher after this year. Hence, as in the case for the catch-up story in France and Germany, we assume that the TFP rate of growth is not affected by this episode. The growth rate is estimated over the period 1960-1990. We then identify 1990-2010 as a transitory period where the level of the TFP is above its long-run value.

### 2.3 Employment rates and hours per worker: a theoretical model

The model we use is a neoclassical growth model with search and matching frictions in the labor market. It is composed of a representative household, a representative firm and a government that runs a balanced budget every period. We present the equilibrium under the assumption of dynamic perfect foresight.

### 2.3.1 Labor market

In the labor market the evolution of the stock of employment is given by the new matches $M_{t}$ which add to the "non-destroyed" jobs $(1-s) N_{t}: N_{t+1}=(1-s) N_{t}+M_{t}$, where the matching function is $M_{t}=\Upsilon V_{t}^{\psi}\left(1-N_{t}\right)^{1-\psi}$. We highlight here that the separation rate $s$ is fixed and differs from country to country. The labor market tightness is given by $\theta_{t}=\frac{V_{t}}{1-N_{t}}$, while $f_{t}=\Upsilon \theta_{t}^{\psi}$ and $q_{t}=\Upsilon \theta_{t}^{\psi-1}$ indicate respectively the job finding and the job filling probability.

### 2.3.2 Households

The economy is populated by a large number of households, and each of them consists of a continuum of identical infinitely-lived agents. Each agent can be either employed or non-employed (thus free to occupy a job). Agents pool their incomes inside the household, so that they are fully insured against non-employment idiosyncratic risk. Agents consume and save by accumulating physical capital, that they rent to firms. Agents pay taxes on their wage income, capital income, investment decisions and consumption. When they are non-employed, they receive benefits from the government. Investment is subject to capital adjustment costs ${ }^{22} \mathrm{~A}$ distinction can be made between roughly two different types of government expenditure $\left(G_{t}\right)$ : collective services expenditure and individual services expenditure. We consider then that the part of consumption that government uses for individual services $\left(G_{t}^{i n d}\right)$ is a perfect substitute for private consumption, while the part that is offered to collective services $\left(G_{t}^{c o l}\right)$ enters in the utility function of a household, but in a separate way. The term $\bar{c}$ indicates the presence of a "subsistence" term in consumption: this term is important to match the "catching-up" of the European countries, compared to the development levels of the US in the aftermath of WWII ${ }^{23}$. The benefits received during periods of non-employment are expressed as a fraction (given by the replacement rate $\rho$ ) of the wage bill. The program

[^38]of a household is given by:
\[

$$
\begin{aligned}
& W^{h}\left(N_{t}, K_{t}\right)=\max _{c_{t}, K_{t+1}}\left\{\begin{array}{l}
\log \left(c_{t}+\gamma G_{t}^{\text {ind }}-\bar{c}\right)+\chi \log \left(G_{t}^{c o l}\right)+N_{t}\left(-\sigma_{l} \frac{h^{1+\eta}}{1+\eta}\right)+\left(1-N_{t}\right) \Gamma^{u} \\
+\beta W^{h}\left(N_{t+1}, K_{t+1}\right)
\end{array}\right\} \\
& \text { s.t. } \quad I_{t}\left(1+\tau_{i, t}\right)+c_{t}\left(1+\tau_{c, t}\right)+\frac{\Phi}{2}\left(K_{t+1}-(1+g) K_{t}\right)^{2}=\left(1-\tau_{w, t}\right)\left[w_{t} h_{t} N_{t}+\left(1-N_{t}\right) \widetilde{b}_{t}\right] \\
&+\pi_{t}+\left(1-\tau_{k, t}\right) r_{t} K_{t} \\
& K_{t+1}= K_{t}(1-\delta)+I_{t} \\
& N_{t+1}=(1-s) N_{t}+f_{t}\left(1-N_{t}\right)
\end{aligned}
$$
\]

where, at the symmetric equilibrium, we have $\widetilde{b}_{t}=\rho_{t} w_{t} h_{t}$. The "subsistence" term changes the individual choices: it reduces the wealth effect when the economy is above its long run steady state. Indeed, with exogenous growth, this component disappears, because $C_{t} \rightarrow \infty$ when $t \rightarrow \infty$, whereas $\bar{c}$ is constant.

### 2.3.3 Firms

The representative firm produces using a Cobb-Douglas technology combining capital $K_{t}$ and labor input $N_{t} h_{t}: Y_{t}=K_{t}^{1-\alpha}\left(A_{t} N_{t} h_{t}\right)^{\alpha}$. The technological progress $A_{t}$ is labor augmenting, according to a balanced growth path. In order to hire workers the firm posts vacancies $V_{t}$, and the unit cost of keeping a vacancy open is given by $\omega$, so that the total costs paid by the firm are given by the wage bill, the rental cost of capital and the vacancy posting costs. The firm's program is given by

$$
V^{f}\left(N_{t}\right)=\max _{V_{t}, K_{t}}\left\{K_{t}^{1-\alpha}\left(A_{t} N_{t} h_{t}\right)^{\alpha}-w_{t} h_{t} N_{t}-r_{t} K_{t}-\omega_{t} V_{t}+\beta \frac{\lambda_{t+1}}{\lambda_{t}} V^{f}\left(N_{t+1}\right)\right\}
$$

$$
\text { s.t. } \quad N_{t+1}=N_{t}(1-s)+q_{t} V_{t}
$$

### 2.3.4 Wage bargaining

Wages and hours are set by the firm and the worker simultaneously, according to a Nash bargaining scheme. In contrast to most models, we allow for a time-varying bargaining power of the firm $\left(\epsilon_{t}\right)$

$$
\max _{w_{t}, h_{t}}\left(\frac{\partial W^{h}}{\partial N_{t}}\right)^{1-\epsilon_{t}}\left(\frac{\partial V^{f}}{\partial N_{t}} \lambda_{t}\right)^{\epsilon_{t}}
$$

The result of the bargaining process is given by the wage and hour equations:

$$
\begin{align*}
w_{t} h_{t}= & \left(1-\epsilon_{t}\right) \underbrace{\left[\alpha \frac{Y_{t}}{N_{t}}+\omega_{t}\left\{\frac{(1-s)}{q_{t}}\left(1-\frac{\phi_{t+1}}{\phi_{t}} \frac{\left(1-\tau_{w, t+1}\right)}{\left(1-\tau_{w, t}\right)}\right)+\frac{\phi_{t+1}}{\phi_{t}} \frac{\left(1-\tau_{w, t+1}\right)}{\left(1-\tau_{w, t}\right)} \theta_{t}\right\}\right]}_{R S} \\
& +\epsilon_{t} \underbrace{\left[\frac{\left(1+\tau_{c, t}\right)}{\left(1-\tau_{w, t}\right)}\left(C_{t}-\bar{c}\right)\left(\Gamma^{u}+\sigma_{l} \frac{h_{t}^{1+\eta}}{1+\eta}\right)+\rho_{t} w_{t} h_{t}\right]}_{R W}  \tag{2.1}\\
\sigma_{l} h_{t}^{\eta=} & \alpha \frac{Y_{t}}{N_{t} h_{t}} \frac{\left(1-\tau_{w, t}\right)}{\left(1+\tau_{c, t}\right)} \frac{1}{C_{t}-\bar{c}} \tag{2.2}
\end{align*}
$$

where $\phi_{t}=\frac{1-\epsilon_{t}}{\epsilon_{t}}$. The marginal labor cost per employee $\left(w_{t} h_{t}\right)$ expresses the opportunity cost of working as the sum of the bargained surplus $(B S)$ and the reservation wage ( $R W$ ). The $B S$ is made up of two components: the marginal productivity of the employee and the cost of the search activity ${ }^{24}$. During the bargaining process, the firm-worker pair shares the returns on the search process. For the worker, this is equal to the discounted time necessary to find a job offer, while for the firm, returns are instead equivalent to the discounted time necessary to find a worker. These relative time spans cannot be proxied by the ratio of the average duration for these two search processes $\left(\theta_{t}=\frac{f_{t}}{q_{t}}\right)$ - as would be the case if bargaining power and taxes were constant ${ }^{25}$. Indeed, if workers expect that tomorrow their bargaining powers are close to zero ( $\phi_{t+1} \approx 0$ ), the evaluation of the current match surplus is only driven by the search costs saved by the firm if the job is not destroyed: $(1-s) \frac{\omega}{q_{t}}$. On the contrary, when the bargaining power of the worker increases $\left(\phi_{t+1}>\phi_{t}\right)$, the match value must be depreciated by the firm (it expects a decrease of its bargaining power), whereas the relative time spans must be over-evaluated by the worker because her bargaining power increases. Thus, the value of the search cost is a function of the bargaining power, which itself changes over time, and is affected by the time-varying distortions induced by taxes: this explains why $B S$ is a function of dynamics of $\epsilon$ and $\tau$. The $R W$ is given by the sum of the marginal rate of substitution of consumption for employment $\left(C_{t}-\bar{c}\right)\left(\Gamma^{u}+\sigma_{l} \frac{h_{t}^{1+\eta}}{1+\eta}\right)$ and the non-employment benefits $\rho_{t} w_{t} h_{t}$. In the basic case, where the bargaining power of the workers is nil $\left(\epsilon_{t}=1, \forall t\right)$, a gap remains, equal to $\frac{1}{1-\rho_{t}}$, between the real wage and the marginal rate of substitution of consumption for employment, because the non-employment benefits are proportional to the average wage. By raising the labor costs, this gap reduces the equilibrium employment rate.

Since we are assuming an efficient bargaining process, the equilibrium number of hours

[^39](the intensive labor supply) is determined jointly with wages. Equation (2.2) shows that, at the symmetric equilibrium, the solution is such that the marginal rate of substitution of consumption for an hour worked is equal to the marginal product of an hour worked, net of the tax wedge. This expression does not introduce any labor market institutions, because we assume an efficient bargaining process over the hours worked, so that the hours contracts are only directly affected by the taxes.

### 2.3.5 Equilibrium

To complete the model, the market clearing conditions on the goods market must be satisfied:

$$
\left.Y_{t}=C_{t}+G_{t}^{c o l}+I_{t}+\omega_{t} V_{t}+\frac{\Phi}{2}\left(K_{t+1}-(1+g) K_{t}\right)\right)^{2} \quad \text { with } C_{t}=c_{t}+G_{t}^{i n d}
$$

whereas the government budget constraint is balanced at each date through lump-sum transfers given to the agents:
$T R_{t}=\tau_{c, t} c_{t}+\tau_{w, t}\left(w_{t} h_{t} N_{t}+\rho_{t} w_{t} h_{t}\left(1-N_{t}\right)\right)+\tau_{i, t} I_{t}+\tau_{k, t} r_{t} K_{t}-\rho_{t} w_{t} h_{t}\left(1-N_{t}\right)-G_{t}^{c o l}-G_{t}^{\text {ind }}$
The model we described is a neoclassical growth model which allows for a balanced growth path. We have two sources of growth in the economy: population, which is growing at rate $g_{n}$, and technological progress, which is growing at the constant rate $g_{A}$. Each of the three countries is characterised by a different growth rate, but here we simply stress that in order to have a stationary model, we deflate all growing variables by the total growth rate $g=g_{A}+g_{n}{ }^{26}$.

### 2.4 Quantitative results

The model is solved with perfect foresight: the path of all exogenous variables is known to agents from the beginning ${ }^{27}$. First of all, we present our calibration strategy. Secondly, once the model is simulated with the identified parameters, and the exogenous variables are specific to each country, we plot the simulated series and the actual series for the four countries ${ }^{28}$. We then choose to focus mainly on the two countries at the ends of the spectrum: the US and France. We evaluate the functioning of the model by considering a

[^40]steady state version to study the impact of a permanent change in policy variable on both the intensive margin (hours per worker) and the extensive margin (in this case represented by labor market tightness).

### 2.4.1 Identification of parameters

In order to solve the model, we need to identify some parameters. The set of all parameters is given by $\Theta=\left\{\beta, \delta, \sigma_{l}, \eta, \alpha, g_{A}, \bar{c}, \Upsilon, \psi, \omega, \Gamma^{u}, s, g_{n}, \Phi, \frac{G^{c o l}}{Y}\right\}$. We choose to calibrate the following subset of parameters $\Theta_{1}=\left\{\beta, \delta, \alpha, \eta, \psi, s, g_{A}, g_{n}, \frac{G^{c o l}}{Y}, \Phi\right\}$.

We set $\beta=0.98, \delta=0.05$ and $\alpha=0.3$ according to standard values in the literature, when the period of reference is one year. In particular, for evidence about the depreciation rate we follow Gomme and Rupert (2007). In the long run, we have the following restrictions: i) $1=\frac{\beta}{1+g}\left[1-\delta+\frac{1-\tau_{k}}{1+\tau_{i}} r\right]$, ii) $r=(1-\alpha) \frac{Y}{K}$ and iii) $I=(\delta+g) K$. For an observed value of $E[I / Y] \approx 0.17$ in the US, we obtain a gross interest rate $r \approx 12.35 \%$, and thus $r-\delta \approx 7.35 \%$, using $i i$ ) and $i i i$ ) (the demand of capital). Note that our value of $\alpha$ is such that the first equation (the supply of capital) is also satisfied, for the average values of the tax rates on capital and investment. Nevertheless, this standard calibration leads, as usual, to an overestimation of the interest rate ${ }^{29}$. We therefore choose to reduce the interest rate by an amount which corresponds to the risk premium ( $\kappa=20 \%$ ), which is paid by the firm when the uncertainty on its investment projects is taken into account by the financing contract. This leads us to $r(1-\kappa)-\delta \approx 4.88 \%$, which is closer to the long-run value of the asset returns.

Our parameter $\eta$ is set to a value of 2 , which implies a Frisch labor supply elasticity of 0.5 , a widely adopted value in the literature, in range with the estimates derived from microdata reported by, for example, Chetty et al. (2011). The parameter $\psi$, which represents the elasticity of the matching function with respect to vacancies, is set to a value of 0.5 , which is an average of the range of possible values identified by Pissarides and Petrongolo (2001), and is widely adopted in literature. In terms of the country-specific separation rate $s$, we use the estimation results in Elsby et al. (2008).

We then use information from our data to give a value to $g_{A}$ and $g_{n}$. Using our computation of the Solow residual for each country (see Section 2.2.2), we fit a linear trend which is our $g_{A}$. Similarly, we fit a linear trend to the population $15-64$ and call it $g_{n}$. The total rate of growth of all non-stationary variables will therefore be $g=g_{A}+g_{n}$.

Data for the ratio of government expenditure in collective goods are taken from the OECD ${ }^{30}$.

As documented in Figure 2.8, France, Germany and the UK feature collective public

[^41]
## Figure 2.8 - A decomposition of government expenditure (OECD data)



In each panel of Figure 2.8, for each year, the central mark is the median value over a sample of 32 OECD countries. The edges of the box are the $25^{t h}$ and $75^{t h}$ percentiles, the whiskers extend to the most extreme data points not considered outliers, and
outliers are plotted individually.
spending (in proportion to GDP) comparable to other OECD countries ${ }^{31}$, while the US is characterized by a higher share of collective public spending ${ }^{32}$. By contrast, individual government spending in France, Germany and the UK is much larger than the average of the OECD countries, while in the US the share of individual government expenditure is minimal.

Table 2.2 - Calibrated parameters I

|  | $\beta$ | $\delta$ | $\eta$ | $\alpha$ | $\psi$ | $\Phi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Common | 0.98 | 0.05 | 2 | 0.3 | 0.5 | 10 |
|  | $s$ | $g_{A}$ | $g_{n}$ | $\frac{G^{c o l}}{Y}$ |  |  |
| US | 0.12 | 0.017 | 0.0136 | 0.098 |  |  |
| FR | 0.15 | 0.019 | 0.0069 | 0.085 |  |  |
| UK | 0.17 | 0.024 | 0.0033 | 0.090 |  |  |
| GE | 0.12 | 0.018 | 0.004 | 0.069 |  |  |

We are then left with the following set of parameters to identify: $\Theta_{2}=\left\{\sigma_{l}, \Gamma^{u}, \bar{c}_{i}, \Upsilon_{i}, \omega_{i}\right\}$ for $i=U S, F R, U K, G E R$. Our restrictions are that the asymptotic preferences (represented by $\sigma_{l}$ and $\lambda$ ) are common to all countries, whereas two sets of parameters are countryspecific: the scale parameters of the matching functions $(\Upsilon)$ and the vacancy posting cost

[^42]$(\omega)$. The value of $\bar{c}$ should be different from zero only for countries which are not the US: as in Rogerson (2006), Ohanian et al. (2008) and McDaniel (2011), we introduce a consumption subsistence term, in order to capture the fact that the level of hours worked was higher, at the beginning of the sample, in the countries that experienced a lower level of productivity compared to the US ${ }^{33}$. The calibration procedure finds parameter values $\Theta_{2}$ that minimise the distance between theoretical and observed moments $\Psi^{\text {theo }}\left(\Theta_{2}\right)-\Psi$. The function $\Psi^{\text {theo }}\left(\Theta_{2}\right)$ comes from the numerical solution of the model. The moments provided by the data are $\Psi=\left\{\bar{N}_{i}, \bar{h}_{i}, \theta_{i}, \overline{\Delta h}_{j}\right\}$, for $i=U S, F R, U K, G E R$ and $j=F R, U K, G E R$ and where $\bar{X}=\frac{1}{T-t_{0}} \sum_{t=t_{0}}^{T} X_{t}$, with $T=2007 \forall X=N, h$ and $\overline{\Delta h}=h_{1975}-h_{1960}$. To build the times series of $\theta$ for the the US we use the Composite Help-Wanted Index developed by Barnichon (2010) ${ }^{34}$; for France the data used to compute the average value of tightness over the period come from DARES ${ }^{35}$; because of data issues for the UK, we prefer to consider for the average labor market tightness the same target as for the US ${ }^{36}$. The targets of $\theta=\frac{V}{1-N}$ are $0.7,0.5$ and 0.7 for the US, France and the UK respectively ${ }^{3738}$. In Table 2.3 we report the values of the identified parameters.

Table 2.3 - Identified parameters

| parameter | US | FR | UK | GE |
| :---: | :---: | :---: | :---: | :---: |
| $\sigma_{l}$ | 31.1 | 31.1 | 31.1 | 31.1 |
| $\Gamma^{u}$ | 0.03 | 0.03 | 0.03 | 0.03 |
| $\Upsilon$ | 0.36 | 0.39 | 0.48 | 0.42 |
| $\bar{c}$ | 0 | 0.36 | 0.15 | 0.24 |

With these restrictions, any difference in the behavior of the economic variables predicted by the model will therefore be guided by differences in policy variables or "technological" conditions (namely the Solow' residuals, the matching technology efficiency and the search costs).

[^43]
### 2.4.2 The fit of the model

Given that the parameters are set in order to match certain average values $\{h ; N\}$ and the slope over the first years of $h$ in France, Germany and the UK, the ability of the model to fit the observed data must be tested using additional information: we check that the model correctly predicts the long run evolution of the variables of interest (the intensive and extensive margins). We report additional information in Appendix 2.H: in Section 2.H, the simulated values of the investment/output ratio and of total hours, and, in Section 2.H, a decomposition of the contribution of each exogenous set of variables in the model outcomes.

Initially, for the US, taxes remain stable, whereas the labor market institutions (LMI) shift slowly in favor of firms. When the simulation is performed using these country-specific policy variables, the model then predicts an increase in the employment rate and a small decline in the hours per worker. The composition of these adjustments of the two labor market margins leads to predicted total hours that marginally increase during the period. These results for the US economy are reported in Figure 2.9 and are compared to the observed data. Given that there is no break point in the time series of the taxes and the LMI in the US, the model reproduces the small and continuous changes observed simultaneously in the hours per worker and employment rate. The Mean Square Errors (MSEs) for the hours worked and for employment are respectively equal to $6.3686 \cdot 10^{-5}$ and $3.1536 \cdot 10^{-4}$.

Figure 2.9 - The US economy


In France, the tax wedge experienced at least three regimes. At the beginning of the sample, until 1985, it increases rapidly. Between 1985 and 2000, its increase is less marked, whereas after 2000, we observe a significant decline (see Figure 2.4). In response to these tax rates, the model predicts that each French employee works fewer hours, with a small
recovery after 2000. This prediction is not contradicted by the data (see Figure 2.10).
Figure 2.10 - The French economy


In the French labor market, while the bargaining power of the workers remains stable over the total sample, this is not the case for the replacement rate: it largely increases in 1981 and 1985, then it remains stable, before beginning to decline from 2002 (see Figure 2.6). In response to these substantial changes in the LMI, the employment rate predicted by the model largely declines at the beginning of the eighties, and increases at the end of the sample. These predictions are consistent with the data (see Figure 2.10), even if the elasticity of the model slightly over-estimates the changes in the employment rate in France. The MSEs for hours worked and for employment are equal to 0.0012 and $4.9340 \cdot 10^{-4}$, respectively.

The overall fit for the UK is worse than for the previous two economies: the MSEs for the hours worked and for employment are equal to $6.0647 \cdot 10^{-4}$ and 0.0032 , respectively. We can see that the model does a relatively good job of reproducing the dynamics for hours but only starting from the mid eighties, once the tax wedge on consumption and labor income stabilizes. In terms of the employment rate, the model captures its tendency to decrease until the end of the seventies, and a recovery afterwards, mirroring the change in the evolution of labor market institutions; however, it overestimates the elasticity of the employment rate to these changes, and it does not capture the evolutions of the nineties.

With respect to Germany, the simulation (starting from the reunification period) does capture the decrease in the intensive margin which continues during the nineties and the positive trend in the employment rate. The overall fit of the model in terms of MSEs, for hours and employment is respectively $3.3486 \cdot 10^{-5}$ and 0.0039 .

Figure 2.11 - The UK economy


Figure 2.12 - The German economy


### 2.4.3 Analysing the mechanisms of the model

In this subsection, we propose to analyse the mechanisms of the model by focusing on a steady state analysis ${ }^{39}$. Due to concerns about the effectiveness of our message, for the steady state experiments (as well as for the counterfactual experiments in the Section 2.5 below), we have decided to concentrate on the two countries which lie at the extreme ends of the spectrum in terms of the evolution of policy variables: the US and France ${ }^{40}$.

## Steady state analysis

Before considering the counterfactual experiments in the fully dynamic model, we look at the final steady state and study the comparative static effects of a change in a policy variable (tax rates, replacement rate or bargaining power). In doing this, we follow Fang and Rogerson (2009), who also give a diagrammatic representation of the equilibrium of a search and matching model with both the extensive and intensive margins of labor input. In contrast to their exercise, our model includes capital, as well as the replacement rate, and is calibrated to match empirical data. At the steady state, we can reduce the system of three steady state equations for three unknowns $\{C, h, \theta\}^{41}$. The equation (2.5) implicitly defines the consumption $C(\theta, h)$ as a function of $\theta$ and $h$, which can be integrated into the two other relationships.

$$
\begin{align*}
C h^{1+\eta} & =\frac{\alpha}{\sigma_{l}} \frac{\left(1-\tau_{w}\right)}{\left(1+\tau_{c}\right)}\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}} A h  \tag{2.3}\\
\frac{\omega \theta^{1-\psi}}{\Upsilon}\left[\frac{1}{\beta}-(1-s)\right]+\left(\frac{1-\epsilon}{1-\rho \epsilon}\right) \omega \theta & =\frac{\epsilon}{1-\rho \epsilon}\left[\begin{array}{c}
(1-\rho) \alpha A h\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}} \\
-C\left(\frac{1+\tau_{c}}{1-\tau_{w}}\right)\left(\Gamma^{u}+\sigma_{l} \frac{h^{1+\eta}}{1+\eta}\right)
\end{array}\right]  \tag{2.4}\\
C+\omega \theta\left(1-\frac{1}{1+\frac{n+s}{\Upsilon \theta^{\psi}}}\right) & =A h\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}}\left(\frac{\left(\frac{r}{1-\alpha}\right)-(g+\delta)}{1+\frac{n+s}{\Upsilon \theta^{\psi}}}\right) \tag{2.5}
\end{align*}
$$

[^44]We therefore obtain the following system:

$$
\begin{aligned}
\sigma_{l} h^{1+\eta} C(\theta, h) & =\alpha A h \frac{\left(1-\tau_{w}\right)}{\left(1+\tau_{c}\right)}\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}} \quad(L s) \\
\frac{\omega \theta^{1-\psi}}{\Upsilon}\left[\frac{1}{\beta}-(1-s)\right]+\left(\frac{1-\epsilon}{1-\rho \epsilon}\right) \omega \theta & =\left(\frac{\epsilon}{1-\rho \epsilon}\right)\left\{\begin{array}{c}
(1-\rho) \alpha A h\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}} \\
-\left(\Gamma^{u}+\sigma_{l} \frac{h^{1+\eta}}{1+\eta}\right) \frac{\left(1+\tau_{c}\right)}{\left(1-\tau_{w}\right)} C(\theta, h)
\end{array}\right\} \quad(J C)+(W E)
\end{aligned}
$$

The first equation can be interpreted as the locus where the "intensive margin" is at equilibrium, whereas the second is the locus where the "extensive margin" is at equilibrium. These two relationships can be interpreted as showing a trade-off between the two margins of labor input for households as well as firms, at general equilibrium.

Proposition 1 For an equilibrium employment rate greater than $1 / 3$, the resource constraint (2.5) always implies that $\epsilon_{C \mid h}>0$ and $\epsilon_{C \mid \theta}>0$

Proof. See Appendix 2.C. ■ Given that an employment rate equal to $1 / 3$ is largely below what has been observed for all countries along the total time span, we can confidently say that in our model an increase in tightness or in hours per worker implies an increase in consumption.

Proposition 2 If the resource constraint (2.5) leads to $\epsilon_{C \mid h}>0$ and $\epsilon_{C \mid \theta}>0$, then the equilibrium intensive margin (equation 2.3) defines a negative relationship between hours worked $h$ and the labor market tightness $\theta$.

Proof. Differentiating the equations (2.3) and (2.5) leads to $\left(\eta+\epsilon_{C \mid h}\right) \frac{d h}{h}=-\epsilon_{C \mid \theta} \frac{d \theta}{\theta}$, where $\epsilon_{C \mid h}=C_{h}^{\prime} \frac{h}{C(\theta, h)}$ and $\epsilon_{C \mid \theta}=C_{\theta}^{\prime} \frac{\theta}{C(\theta, h)}$ where $C(\theta, h)$ is the consumption compatible with the resource constraint. ■ The optimal choice of the intensive margin shows that the labor market tightness acts as a wealth effect or agent decisions: a high $\theta$ implies a high employment rate, therefore a lower incentives for each individual in the household to work.

Proposition 3 For $\frac{\eta}{1+\eta}>\rho$, there exists a value for $\Gamma^{u}<0$ such that the equilibrium extensive margin (equation 2.4) defines a negative relationship between hours worked $h$ and the labor market tightness $\theta$.

Proof. See Appendix 2.C. ■ The optimal choice of the extensive margin shows that a high $h$ implies a higher gap between disutility at work and at home, leading to lower incentives for an additional worker in the household to work. This can be viewed as an increase of the wage reservation due to the scarcity of leisure when $h$ increases.

Comparative statics: counterfactual experiments. Let us now perform a comparative statics analysis: what would the impact be of a reduction in tax rates on the final steady state (SS)? We can, for example, apply the US tax rates on consumption and labor income to France, and check the functioning of the model. We expect both the labor supply and the labor market equilibrium curves to shift upward. The overall effect on hours per worker is unequivocally positive, while the effect on the extensive margin depends on the relative movements of the curves. We can see this mechanism graphically in the left panel of Figure 2.13. Indeed, for our calibration, the tax reduction gives incentives to work longer and at the same time it also reduce the labor costs, inducing a rise in tightness.

Figure 2.13 - SS comparative statics - France


What if we simulate the French economy with the unemployment benefits system of the US? We see in the graphical representation in the right panel of Figure 2.13 that a change in the replacement rate does not affect the labor supply curve, but that it does affect the labor market equilibrium curve: the latter shifts towards the right so that the effect on tightness is strongly positive. Note that when the workers who work are numerous to work, the generated wealth effect leads them to reduce their individual effort at work. Thus, the reservation wage is reduced, and then the impact of the reduction of the replacement rate is amplified. Considering that the "intensive margin" curve is very flat, the effect on hours per worker (which decrease overall) is quantitatively less significant.

### 2.5 Counterfactual experiments: alternative policies

Once we have understood the forces at work at steady state, we can perform a counterfactual experiment with the fully fledged dynamic version. We concentrate on the two countries which lie at the extremes of the spectrum in terms of the evolution of policy variables, the US and France:

- what would the evolution have been of the two margins of labor input in France with the path of policy variables which characterized the post WWII history of the US?


### 2.5.1 The impact of policies on the two margins

Let us start by comparing the evolution of the simulated variables for France, when the economy is fed with the complete set of policy variables which characterize the US. This means that in our first experiment the two economies differ only with respect to the path of technological progress and a few parameters: the separation rate, the population growth rate and the matching efficiency. In Figure 2.14 we see the results of the simulations of the French economy with both US taxes and labor market institutions.

On average, the fictive French employee works longer and has a higher chance of being employed. If we look at the evolution of the employment rate, it seems that with the labor market institutional arrangements which characterize the US, France would have observed a spectacularly high employment rate: at the end of the simulation, the employment is $85 \%$ in this "fictive" France, whereas it is equal to $62.5 \%$ in the "actual" economy.

Figure 2.14 - Counterfactual: US taxes and LMI in France


Let us consider a situation in which we have the US tax rate evolution in France, while keeping the labor market arrangements as they are; we see in Figure 2.15 that the amount of hours worked would be higher. The important point here is that a simple reduction of the tax rates does not have a significant impact on employment, because the decline of the labor cost they induce is compensated by the rise in reservation wage of the workers, who now work longer. These results are the counterparts, in a dynamic framework, of the comparative static results summarised in Propositions 2 and 3.

Figure 2.15 - Counterfactual: France with US taxes


If we look at Figure 2.16, which shows the hypothetical case of France applying US labor market institutions, but its own taxation system, we see that in this instance we would have observed a very high employment level, with a contemporaneous decrease in hours worked. Agents in France, when strongly taxed, choose to work less than an American worker. The general equilibrium effects magnify these two direct effects. Firstly, the large "chance" of employment is perceived by the agent as a wealth effect that reduces his incentive to work longer. Secondly, when a worker reduces his hours worked, his reservation wage decreases, leading to a magnification of the rise in the employment rate. This also echoes the results obtained with the comparative statics analysis in Propositions 2 and 3.

### 2.5.2 Do the French work less to be happier?

To evaluate the impact of policies on welfare, we compute the welfare gap in Frances induced by the "distortive" taxes and labor market institutions, with respect to a "reference" value of welfare that would be chosen by a benevolent social planner. In this case, the government consumption expenditure in collective goods would be financed in a non-distortive way

Figure 2.16 - Counterfactual: France with US LMI

through lump-sum taxes ${ }^{42}$. The planner observes the same dynamics of the technological shocks as the private agents do. The two counterfactual experiments which are performed are the following: in one case, we set the proportional taxes rates as null, government expenditure in collective goods being financed by lump-sum transfers; in the other, we eliminate instead the distortions on the labor market institutions.

We then compute the rate at which we should "tax" the social planner in order to have an equivalent welfare level to that of the market economy at each date. We define a factor $\lambda_{t}$ entering the social welfare function as the following:

$$
\left.\begin{array}{c}
W_{t}^{a c t u a l} \\
W_{t}^{c f}
\end{array}\right\}=\log \left(\left(1-\lambda_{t}\right) C_{t}\right)+\zeta \log \left(G_{t}^{c o l}\right)+N_{t}\left(-\sigma_{l} \frac{h_{t}^{1+\eta}}{1+\eta}\right)+\left(1-N_{t}\right) \Gamma^{u}+\beta W_{t+1}
$$

The factor $\lambda_{t}$ gives the losses in consumption units implied by the market allocations (actual or $c f$ as counterfactual). The value of $\zeta$ is derived by considering the choice that a social planner would make, i.e. considering the first order conditions (FOCs) with respect to privately consumed goods and collective goods: $\zeta=\frac{G_{t}^{\text {col }}}{C_{t}}$. In order to find the value of $\zeta$ we consider the time series for $G_{t}^{c o l}$ and we compute the mean value over the period, i.e. $\zeta=\frac{\bar{G}_{\bar{C}}^{\text {col }}}{\bar{C}}{ }^{43}$, obtaining 0.139 for France.

If we look at the left panel of Figure 2.17, we can see that the gains originated by a

[^45]Figure 2.17 - Factor $\lambda_{t}$ and output loss for France

tax reform are higher than those coming from a labor market reform (around 9 percentage points of life-time consumption would be gained at the end of the period in the first case, in contrast to around 4 percentage points in the second case). The intuition can come from the analysis of the steady state in Figure 2.13: starting from a situation in which hours and the employment rate are lower than the optimal level, a fiscal reform which reduces the tax burden, contributes in helping the employment rate too (left panel of Figure 2.13), while a reform of labor market institution "worsens" the performance in hours (right panel of Figure 2.13), even if in this case the overall elasticity of hours is quite low. This first result suggests that a French worker is not better off than an American, even if he works less.

In comparison to Prescott (2004), our evaluation of the welfare losses is lower ${ }^{44}$. This mainly results from (i) the assumption that government expenditures are not a pure waste and (ii) a much lower elasticity of the labor supply ${ }^{45}$.

Another interesting point regards the complementarities that can arise in a general equilibrium framework in which the intensive and the extensive margin of labor supply interact: if we simply sum up the gains of implementing one reform at a time we obtain the dashed black line in the left panel of Figure 2.17: we can see that the sum of the costs of the two distortions is lower than the actual cost experienced in the economy, or in other words the

[^46]gains of implementing both reforms at the same time are higher.
Finally, we can also consider the percentage points of output (in efficiency units) that have been "lost" in France, compared to what could have happened with a set of policy variables such as those chosen by the planner: in the right panel of Figure 2.17, we plot the history of output in the actual French economy and in the two counterfactual experiments normalized at their respective level in 1980. Given the size of these "structural" losses each year, measuring the losses in the "potential" output, it seems that the Keynesian "Okun gap" is negligible for France: all the efforts of policy-makers should be devoted to the reduction of the "Harberger triangles", which are increasing, representing today between 3 and 4 percentage points of the output.

### 2.6 Conclusions

In this paper we developed a dynamic perfect foresight model of neoclassical growth with labor market frictions, that can account for the long run evolutions of both the extensive and the intensive margin of labor supply. We calibrated it to reproduce the evolutions of these two margins for four representative countries, namely the United States, France, the United Kingdom and Germany. We then focused on the differences between the two countries which represent the "extreme" cases in terms of policy choices: the US and France. These two countries in fact showed extremely different evolutions with respect to the aggregate labor supply. We highlighted that there are non-trivial interactions between the two margins, and we confirmed that quantitatively the evolution of the tax wedge can explain the path of hours worked per worker while labor market institutions ${ }^{46}$ must be taken into account if we want to explain the evolution of employment.

The appeal of the model lies in the fact that it allows us to perform counterfactual experiments and to evaluate the welfare losses or gains of implementing certain reforms: in this instance the country of interest is France, and we ask ourselves what the welfare gains would be of switching towards an "optimal" system for taxes and institutions. One of the potential policy implications of our exercise is that it seems pointless to advocate a "liberalization" of labor market institutions if we do not consider diminishing the tax wedge at the same time.

[^47]
## Bibliography

Alberto F. Alesina, Edward L. Glaeser, and Bruce Sacerdote. Work and Leisure in the U.S. and Europe: Why So Different? In NBER Macroeconomics Annual 2005, Volume 20, NBER Chapters, pages 1-100. National Bureau of Economic Research, Inc, June 2006.

David Andolfatto. Business cycles and labor market search. American Economic Review, 86(1): 112-32, March 1996.

Regis Barnichon. Building a composite help-wanted index. Economics Letters, 109(3):175 - 178, 2010.

Olivier Blanchard and Justin Wolfers. The Role of shocks and institutions in the rise of European unemployment: The aggregate evidence. The Economic Journal, 110(462):C1-C33, March 2000.

Sveinbjörn Blöndal and Stefano Scarpetta. The retirement decision in oecd countries. OECD Economics Department Working Papers 202, OECD Publishing, 1999.

Michael Burda and Mark Weder. Payroll Taxes, Social Insurance and Business Cycles. 2010.
Raj Chetty, Adam Guren, Day Manoli, and Andrea Weber. Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins. American Economic Review, 101(3):471-75, May 2011.

Gabriel Chodorow-Reich and Loukas Karabarbounis. The cyclicality of the opportunity cost of employment. Working Paper 19678, National Bureau of Economic Research, November 2013.

Michael Elsby, Bart Hobijn, and Aysegul Sahin. Unemployment dynamics in the oecd. Working Paper 14617, National Bureau of Economic Research, December 2008.

Lei Fang and Richard Rogerson. Policy Analysis In A Matching Model With Intensive And Extensive Margins. International Economic Review, 50(4):1153-1168, November 2009.

Paul Gomme and Peter Rupert. Theory, measurement and calibration of macroeconomic models. Journal of Monetary Economics, 54(2):460-497, March 2007.

Jonathan Gruber and David Wise. Social security programs and retirement around the world: Fiscal implications, introduction and summary. Working Paper 11290, National Bureau of Economic Research, May 2005.

Marcus Hagedorn and Iourii Manovskii. The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited. American Economic Review, 98(4):1692-1706, September 2008.

Jean-Olivier Hairault, François Langot, and Thepthida Sopraseuth. Why is Old Workers' Labor Market More Volatile? Unemployment Fluctuations over the Life-Cycle. IZA Discussion Papers 8076, Institute for the Study of Labor (IZA), March 2014.

Robert E. Hall. Employment Fluctuations with Equilibrium Wage Stickiness. American Economic Review, 95(1):50-65, March 2005.

François Langot. Unemployment and Real Business Cycle : A Matching Model. In P.Y. Henin, editor, Advances in Economic Business Cycles Theory. Springer Verlag, 1995.

François Langot and Coralia Quintero Rojas. Explaining the Evolution of Hours Worked and Employment across OECD Countries: An Equilibrium Search Approach. IZA Discussion Papers 3364, Institute for the Study of Labor (IZA), February 2008.

François Langot, Lise Patureau, and Thepthida Sopraseuth. Optimal Fiscal Devaluation. Banque de france working papers, Banque de France, 2014.

Lars Ljungqvist and Thomas J. Sargent. Do taxes explain European employment? Indivisible labor, human capital, lotteries, and savings. In NBER Macroeconomics Annual 2006, Volume 21, NBER Chapters, pages 181-246. National Bureau of Economic Research, Inc, 2007.

Cara McDaniel. Average tax rates on consumption, investment, labor and capital in the OECD 1950-2003. Working papers, Arizona State University, December 2007.

Cara McDaniel. Forces shaping hours worked in the OECD, 1960-2004. American Economic Journal: Macroeconomics, 3(4):27-52, October 2011.

Monika Merz. Search in the labor market and the real business cycle. Journal of Monetary Economics, 36(2):269-300, November 1995.

Dale T. Mortensen and Christopher A. Pissarides. New developments in models of search in the labor market. In O. Ashenfelter and D. Card, editors, Handbook of Labor Economics, volume 3 of Handbook of Labor Economics, chapter 39, pages 2567-2627. Elsevier, 1999.

Stephen Nickell. Unemployment and labor market rigidities: Europe versus North America. Journal of Economic Perspectives, 11(3):55-74, Summer 1997.

Lee Ohanian, Andrea Raffo, and Richard Rogerson. Long-term changes in labor supply and taxes: Evidence from OECD countries, 1956-2004. Journal of Monetary Economics, 55(8):1353-1362, November 2008.

Lee E. Ohanian and Andrea Raffo. Aggregate hours worked in OECD countries: New measurement and implications for business cycles. Journal of Monetary Economics, 59(1):40-56, 2012.

Christopher A. Pissarides. Unemployment And Hours Of Work: The North Atlantic Divide Revisited. International Economic Review, 48(1):1-36, 022007.

Christopher A. Pissarides and Barbara Petrongolo. Looking into the Black Box: A Survey of the Matching Function. Journal of Economic Literature, 39(2):390-431, June 2001.

Edward C. Prescott. Why do Americans work so much more than Europeans? Quarterly Review, (Jul):2-13, 2004.

Kelly S. Ragan. Taxes and Time Use:Fiscal Policy in a Household Production Model. American Economic Journal: Macroeconomics, 1(5):168-92, 2013.

Richard Rogerson. Understanding differences in hours worked. Review of Economic Dynamics, 9 (3):365-409, July 2006.

Richard Rogerson. Taxation and Market Work: Is Scandinavia an Outlier? Nber working papers, National Bureau of Economic Research, Inc, February 2007.

Robert Shimer. The Cyclical Behavior of Equilibrium Unemployment and Vacancies. American Economic Review, 95(1):25-49, March 2005.

## 2.A Stationarized FOCs

We report the equations that compose the model, where the convention adopted is to indicate with $\widehat{X}$ a variable $X$, for $X \in\{A, C, Y, K, I, w, \omega\}$ which is deflated by the rate of growth, ie. $\widehat{X}_{t}=X_{t} /(1+g)^{t}$. The set of equations used to solve the dynamic paths of the model is the following:

$$
\begin{aligned}
(1+n) N_{t+1}= & (1-s) N_{t}+\min \left[\min \left(V_{t}, 1-N_{t}\right), \Upsilon V_{t}^{\psi}\left(1-N_{t}\right)^{1-\psi}\right] \\
(1+g) \widehat{K}_{t+1}= & (1-\delta) \widehat{K}_{t}+\widehat{I}_{t} \\
\widehat{Y}_{t}= & \widehat{K}_{t}^{1-\alpha}\left(\widehat{A}_{t} N_{t} h_{t}\right)^{\alpha} \\
\widehat{Y}_{t}= & \widehat{C}_{t}+\widehat{I}_{t}+\widehat{G_{t}^{c o l}}+\widehat{\omega} V_{t} \\
\frac{\left(\widehat{C}_{t+1}-\bar{c}\right)}{\left(\widehat{C}_{t}-\bar{c}\right)}= & \frac{\beta}{(1+g)} \frac{\left(1+\tau_{c, t}\right)}{\left(1+\tau_{c, t+1}\right)\left(1+\tau_{i, t+1}\right)}\left[r_{t+1}\left(1-\tau_{k, t+1}\right)+(1-\delta)\left(1+\tau_{i, t+1}\right)\right] \\
r_{t}= & (1-\alpha) \widehat{K}_{t}^{-\alpha}\left(\widehat{A}_{t} N_{t} h_{t}\right)^{\alpha} \\
\frac{\widehat{\omega} \theta_{t}}{f_{t}}= & \frac{\beta}{(1+g)} \frac{\left(\widehat{C}_{t}-\bar{c}\right)\left(1+\tau_{c, t}\right)}{\left(\widehat{C}_{t+1}-\bar{c}\right)\left(1+\tau_{c, t+1}\right)}\left[\alpha \frac{\widehat{Y}_{t+1}}{N_{t+1}}-\widehat{w}_{t+1} h_{t+1}+(1-s) \frac{\widehat{\omega} \theta_{t+1}}{f_{t+1}}\right] \\
\widehat{w}_{t} h_{t}= & \frac{\left(1-\epsilon_{t}\right)}{\left(1-\rho_{t} \epsilon_{t}\right)}\left[\alpha \frac{\widehat{Y}_{t}}{N_{t}}+\widehat{\omega}\left(\frac{1-s}{q_{t}}\left(1-\left(\frac{\phi_{t+1}}{\phi_{t}} \frac{\left(1-\tau_{w, t+1}\right)}{\left(1-\tau_{w, t}\right)}\right)+\frac{\phi_{t+1}}{\phi_{t}} \frac{\left(1-\tau_{w, t+1}\right)}{\left(1-\tau_{w, t}\right)} \theta_{t}\right)\right)\right] \\
& +\frac{\epsilon_{t}}{1-\rho_{t} \epsilon_{t}} \frac{\left(1+\tau_{c, t}\right)}{\left(1-\tau_{w, t}\right)}\left(\widehat{C}_{t}-\bar{c}\right)\left(\Gamma^{u}+\sigma_{l} \frac{h_{t}^{(1+\eta)}}{(1+\eta)}\right) \\
f_{t}= & \min \left(\frac{\Upsilon V_{t}^{\psi}\left(1-N_{t}\right)^{1-\psi}}{1-N_{t}}, 1\right) \\
q_{t}= & \min \left(\frac{\Upsilon V_{t}^{\psi}\left(1-N_{t}\right)^{1-\psi}}{V_{t}}, 1\right) \\
\theta_{t}= & \frac{V_{t}}{U_{t}}
\end{aligned}
$$

In order to ensure that the job finding rate and the job filling rate are in $[0,1]$, we take the minimum between the unconstrained definition of these rates and 1. In accordance with these constraints, the matching function is also redefined.

## 2.B Steady state analysis

We firstly report all the equations which compose the model:

$$
\begin{aligned}
(n+s) N & =q(\theta) V \\
(g+\delta) K & =I \\
Y & =K^{1-\alpha}(A N h)^{\alpha} \\
Y & =C+I+G^{c o l}+\omega V \\
1 & =\frac{\beta}{1+g}\left[r \frac{\left(1-\tau_{k}\right)}{\left(1+\tau_{i}\right)}+1-\delta\right] \\
r & =(1-\alpha)\left(\frac{K}{A N h}\right)^{-\alpha} \\
\frac{\omega \theta}{f(\theta)} & =\beta\left[\alpha \frac{Y}{N}-w h+(1-s) \frac{\omega \theta}{f(\theta)}\right] \\
w h & =\frac{1-\epsilon}{1-\rho \epsilon}\left(\alpha \frac{Y}{N}+\omega \theta\right)+\frac{\epsilon}{1-\rho \epsilon} \frac{1+\tau_{c}}{1-\tau_{w}}(C-\bar{c})\left(\Gamma^{u}+\sigma_{l} \frac{h^{1+\eta}}{1+\eta}\right) \\
\theta & =\frac{V}{U} \\
f(\theta) & =\Upsilon \theta^{\psi} \\
q(\theta) & =\frac{\Upsilon}{\theta^{1-\psi}} \\
\sigma_{l} h^{1+\eta} & =\alpha \frac{Y}{N} \frac{1-\tau_{w}}{1+\tau_{c}} \frac{1}{C-\bar{c}}
\end{aligned}
$$

We then report for clarity an intermediate step in the substitution. Let us define the two following values:

$$
\begin{aligned}
r & =\left[\frac{1+g}{\beta}-(1-\delta)\right] \frac{1+\tau_{i n v}}{1-\tau_{k}} \\
K & =\left(\frac{r}{1-\alpha}\right)^{-\frac{1}{\alpha}} A N h
\end{aligned}
$$

We can therefore reduce the system of steady state equations to the following one ${ }^{47}$ :

$$
\begin{aligned}
N(\theta) & =\left(\frac{n+s}{\Upsilon \theta^{\psi}}+1\right)^{-1} \\
Y(\theta, h) & =\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}} A N(\theta) h \\
K(\theta, h) & =\left(\frac{r}{1-\alpha}\right)^{-\frac{1}{\alpha}} A N(\theta) h \\
I(\theta, h) & =(g+\delta) K(\theta, h) \\
C(\theta, h) & =\frac{Y(\theta, h)}{N(\theta)} \frac{\alpha}{\sigma_{l}} \frac{\left(1-\tau_{w}\right)}{\left(1+\tau_{c}\right)} \frac{1}{h^{1+\eta}} \\
\frac{\omega \theta}{\Upsilon \theta^{\psi}}\left[\frac{1}{\beta}-(1-s)\right]+\frac{1-\epsilon}{1-\rho \epsilon} \omega \theta & =\frac{\epsilon}{1-\rho \epsilon}[1-\rho) \alpha \frac{Y(\theta, h)}{N(\theta, h)} \\
Y(\theta, h) & =C(\theta, h)+I(\theta, h)+\omega \theta(1-N(\theta))
\end{aligned}
$$

By continuing in substituting, we arrive to the following three equations which represent the labor supply equation, the combination of the wage equation and the job opening condition and the aggregate market clearing, respectively.

$$
\begin{aligned}
C h^{1+\eta} & =\frac{\alpha}{\sigma_{l}} \frac{\left(1-\tau_{w}\right)}{\left(1+\tau_{c}\right)}\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}} A h \\
\frac{\omega \theta^{1-\psi}}{\Upsilon}\left[\frac{1}{\beta}-(1-s)\right]+\left(\frac{1-\epsilon}{1-\rho \epsilon}\right) \omega \theta & =\frac{\epsilon}{1-\rho \epsilon}\left[\begin{array}{c}
(1-\rho) \alpha A h\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}} \\
-C\left(\frac{1+\tau_{c}}{1-\tau_{w}}\right)\left(\Gamma^{u}+\sigma_{l} \frac{h^{1+\eta}}{1+\eta}\right)
\end{array}\right] \\
C+\omega \theta\left(1-\left(\frac{n+s}{\Upsilon \theta^{\psi}}+1\right)^{-1}\right) & =A h\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}}\left(\frac{n+s}{\Upsilon \theta^{\psi}}+1\right)^{-1}\left(\left(\frac{r}{1-\alpha}\right)-(g+\delta)\right)
\end{aligned}
$$

## 2.C Proofs of proposition

## Proof of proposition 1

Let $\epsilon_{C \mid h}=C_{h}^{\prime} \frac{h}{C(\theta, h)}$ and $\epsilon_{C \mid \theta}=C_{\theta}^{\prime} \frac{\theta}{C(\theta, h)}$ where $C(\theta, h)$ is the consumption compatible with the resource constraint. Differentiating equation (2.5) with respect to $h$, we obtain $d C=\frac{-A\left(\delta+g-\frac{r}{1-\alpha}\right)}{\left(\frac{n+s}{r \theta^{W}}+1\right)\left(\frac{r}{1-\alpha}\right)^{\frac{1}{\alpha}}} d h$. Let us substitute the expression of the equilibrium real interest

[^48]rate, which is given by $r=\left(\frac{1+\tau_{i n v}}{1-\tau_{k}}\right)\left(\frac{1+g}{\beta}-(1-\delta)\right)$, to obtain that
$$
C_{h}^{\prime} \frac{h}{C(\theta, h)}=\frac{-A\left[\delta+g-\left(\frac{1+\tau_{i n v}}{1-\tau_{k}}\right)\left(\frac{1+g}{\beta}-(1-\delta)\right)\left(\frac{1}{1-\alpha}\right)\right]}{\left(\frac{n+s}{\gamma \theta^{\psi}}+1\right)\left(\frac{r}{1-\alpha}\right)^{\frac{1}{\alpha}}} \frac{h}{C(\theta, h)}>0
$$
because the term inside parenthesis in the numerator is always negative. ${ }^{48}$ If we now check the derivative with respect to tightness, we find the following expression:
$$
\frac{\partial C}{\partial \theta}=\omega\left[\frac{\Upsilon \theta^{\psi}}{n+s}(1+\psi)-1\right]+\left[\frac{-A h \psi(n+s)\left(\delta+g-\frac{r}{1-\alpha}\right)}{\Upsilon \theta^{1+\psi}\left(\frac{n+s}{\Upsilon \theta^{\psi}}+1\right)^{2}\left(\frac{r}{1-\alpha}\right)^{\frac{1}{\alpha}}}\right]
$$

The second term in square brackets is always positive, so that the overall sign depends on the conditions on the first term in square bracket; we find that a sufficient condition to have an overall positive sign at the numerator is that the first term in square brackets is positive too, which is satisfied if $\frac{\Upsilon \theta^{\psi}}{n+s}(1+\psi)>1$. Since we know that in steady state $\frac{f}{n+s}=\frac{N}{1-N}$, the previous condition reduces to $\frac{N}{1-N}(1+\psi)>1$, i.e. $N>\frac{1}{2+\psi}$. In the most "restrictive" case $(\psi=1)$, the condition would be satisfied for an employment rate at least equal to $1 / 3$.

## Proof of proposition 3

Differentiating equation (2.4) leads to

$$
\left\{\begin{array}{l}
(1-\psi) \frac{\omega \theta^{1-\psi}}{\Upsilon}\left[\frac{1}{\beta}-(1-s)\right] \\
+\left(\frac{1-\epsilon}{1-\rho \epsilon}\right) \omega \theta+\left(\frac{\epsilon}{1-\rho \epsilon}\right) \Gamma^{u} \frac{\left(1+\tau_{c}\right)}{\left(1-\tau_{w}\right)} C \epsilon_{C \mid \theta}
\end{array}\right\} \frac{d \theta}{\theta}=\left(\frac{\epsilon}{1-\rho \epsilon}\right)\left\{\begin{array}{c}
\frac{\eta-\rho(1+\eta)}{1+\eta} \alpha A h\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}} \\
-\Gamma^{u} \frac{\left(1+\tau_{c}\right)}{\left(1-\tau_{w}\right)} C \epsilon_{C \mid h}
\end{array}\right\} \frac{d h}{h}
$$

With $\Gamma_{u}<0$ and $\eta-\rho(1+\eta)>0$, the RHS is positive whereas the sign of the LHS is undermined. Its sign is negative iff

$$
(1-\psi) \frac{\omega \theta^{1-\psi}}{\Upsilon}\left[\frac{1}{\beta}-(1-s)\right]+\left(\frac{1-\epsilon}{1-\rho \epsilon}\right) \omega \theta+\left(\frac{\epsilon}{1-\rho \epsilon}\right) \Gamma^{u} \frac{\left(1+\tau_{c}\right)}{\left(1-\tau_{w}\right)} C \epsilon_{C \mid \theta}<0
$$

If we assume that $\Gamma_{u}=-\sigma_{l} \frac{e^{1+\eta}}{1+\eta}$ with $e<h$, we then have $\Gamma_{u}=-\mu \sigma_{l} \frac{h^{1+\eta}}{1+\eta}$ with $\mu<1$. Using (2.3), we deduce that $\Gamma^{u} \frac{\left(1+\tau_{c}\right)}{\left(1-\tau_{w}\right)} C=-\mu \frac{\alpha}{1+\eta} \frac{Y}{N}$. Hence the previous restriction can be rewritten

[^49]as follows:
$$
(1-\psi) \frac{\omega \theta^{1-\psi}}{\Upsilon}\left[\frac{1}{\beta}-(1-s)\right]+\left(\frac{1-\epsilon}{1-\rho \epsilon}\right) \omega \theta<\left(\frac{\epsilon}{1-\rho \epsilon}\right) \mu \frac{\alpha}{1+\eta} \frac{Y}{N} \epsilon_{C \mid \theta}
$$

Which is, when we assume for simplicity that $n \rightarrow 0$ and $\beta \rightarrow 1$

$$
\frac{\omega \theta(1-N)}{Y}\left[1-\psi+\frac{1-\epsilon}{1-\rho \epsilon} \frac{N}{1-N}\right]<\left(\frac{\epsilon}{1-\rho \epsilon}\right) \mu \frac{\alpha}{1+\eta} \frac{\frac{\alpha}{1-\alpha}(g+\delta)}{\frac{\alpha}{1-\alpha}(g+\delta)-\frac{\omega \theta(1-N)}{Y}}
$$

given that

$$
\begin{aligned}
\frac{d C}{d h} \frac{h}{C} & =A h\left(\frac{r}{1-\alpha}\right)^{-\frac{(1-\alpha)}{\alpha}} \frac{1}{C}\left(\frac{\left(\frac{r}{1-\alpha}\right)-(g+\delta)}{1+\frac{n+s}{\Upsilon \theta^{\psi}}}\right) \\
\Leftrightarrow \epsilon_{C \mid h} & =\frac{Y}{C}\left[\frac{r}{1-\alpha}-(g+\delta)\right]=\frac{Y\left[\frac{r}{1-\alpha}-(g+\delta)\right]}{Y\left[\frac{r}{1-\alpha}-(g+\delta)\right]-\omega \theta(1-N)}
\end{aligned}
$$

Assume that $x=\frac{\omega \theta(1-N)}{Y}<1$ is given, we have

$$
x\left[1-\psi+\frac{1-\epsilon}{1-\rho \epsilon} \frac{N}{1-N}\right]<\left(\frac{\epsilon}{1-\rho \epsilon}\right) \mu \frac{\alpha}{1+\eta} \frac{\frac{\alpha}{1-\alpha}(g+\delta)}{\frac{\alpha}{1-\alpha}(g+\delta)-x}
$$

where the largest value of the LHS is obtained for $N=1 / 2$. Hence a sufficient condition is

## 2.D Unexpected policy change

In order to check the impact of the perfect foresight hypothesis, we perform the following experiment: we suppose that at the beginning of the sample (in 1960), agents in France correctly anticipated the path of all exogenous variables but the replacement rate (we choose this policy change because the break in the evolution of the replacement rate is the largest among all changes of exogenous variables). The agents consider that the replacement rate evolves till 1980 and then remains constant at its 1981 level. When the government changes in 1981 and so does the policy about the replacement rate, agents are surprised. At this point, the full new path of the replacement rate is revealed. The results of this experiment are shown in Figure 2.a.

Until 1978, both scenarii, under perfect forecast (black line with triangles) or with unexpected change (green line with stars), produce the same results. After 1979, the firms begin to reduce their vacancies if the policy change is correctly anticipated. Thus, the reduction of the employment rate is slightly smoothed until 1984, year after which the employment

Figure 2.a - Surprise in 1981 for replacement rate

rate becomes again the same, as we can see looking at the lines which represent the perfect foresight case (black line with triangles) and the reaction of agents when they are surprised (red line with circles). Nevertheless, the gap between the two scenarii does never exceed 1.5 percentage point, which can be considered small, given the size of the policy change.

## 2.E The role of the subsistence term of consumption $\bar{c}$

In this section we provide a discussion about the role of the term $\bar{c}$ : as already highlighted in the main text, the term $\bar{c}$ captures the "exceptionality" of the period between 1960s and 1970s for the economies which undertook a "reconstruction" period after WWII. In order to check the importance of this hypothesis, we consider an alternative version of the model in which $\bar{c}$ is set equal to zero: the model would require therefore another set of parameters for both countries to be found by matching the same four moments as in Section 2.4.1.

Table 4 - Identified parameters

| parameter | US | FR |
| :---: | ---: | ---: |
| $\sigma_{l}$ | 29.2 | 29.2 |
| $\Gamma^{u}$ | 0.01 | 0.01 |
| $\Upsilon$ | 0.35 | 0.38 |
| $\bar{c}$ | 0 | 0 |

With this alternative calibration, we show the performance of the model in replicating the variables of interest for France. As we can see in Figure 2.b, without the subsistence
term $\bar{c}$ the model underestimates the level of hours till the mid 1980s; the important point is that it predicts a decrease in hours worked, even if with a lower "speed".

Figure 2.b - The French economy


We can also compute a simple measure of the "lost" in the fit of the model by considering the Mean Squared Errors (MSEs) for hours for the two models: the MSEs for hours with the alternative model is 2.33 times higher than that one of the benchmark model (the values are respectively 0.0028 and 0.0012 ).

## 2.F The flow value of non-employment

As it has been pointed out by Shimer (2005) and Hall (2005) the performance of the search and matching model in reproducing the variability of employment is tightly linked to the mechanisms underlying the wage process, in particular the evolution of the so called "flow value of non-employment" ${ }^{49}$. We thus check the evolution over time of the implied flow value of non-employment produced by our model. We report for the reader's convenience the term entering the wage equation which represents the flow value of non-employment (or reservation wage):

$$
R W_{t}=\underbrace{\left(\Gamma^{u}+\sigma_{l} \frac{h_{t}^{1+\eta}}{1+\eta}\right)\left(C_{t}-\bar{c}\right)\left(\frac{1+\tau_{c}}{1-\tau_{w}}\right)}_{\begin{array}{c}
\text { Value of leisure } \\
\text { in consumption goods }
\end{array}}+\underbrace{\rho_{t} w_{t} h_{t}}_{\begin{array}{c}
\text { Non-employment } \\
\text { benefits }
\end{array}}
$$

[^50]We plot the results implied by our model ${ }^{50}$ in Figure 2.c.
Figure 2.c - Flow value of non-employment


Figure 2.c puts in evidence two things. First of all, the implied flow value of nonemployment remains inside some range which is widely accepted in the literature which is [0.4; 0.943], the highest value being provided by Hagedorn and Manovskii (2008) whereas the lowest being the one proposed by Shimer (2005). Nevertheless, it seems that the general equilibrium value of this flow value of non-employment is closer to the one proposed by Hagedorn and Manovskii (2008): this is not surprising because our model does not restrict this value to the replacement rate, as it is suggested in Shimer (2005).

[^51]Firstly, remark that the levels of flow value of non-employment are quite similar among countries. But it is important to have in mind that in the US and in the UK, the bargaining power of the workers is very low, leading the real wage to be close to this flow value of nonemployment ${ }^{51}$, whereas in France, the bargaining power of the workers is large, implying that the real wage is larger than this flow value of non-employment.

Secondly we remark that the evolution of the flow value of non-employment for our countries is driven by different forces: in France the weight of the non-employment benefit is much more significant while the the value of "home staying" decreased (market hours decreased). In the US, the flow value of non-employment increases because the number of hours worked per employee rises over the sample: this endogenously increases the reservation wage of the US workers. In the UK the overall flow value of non-employment started to decrease after the Thatcher reforms (the beginning of the 1980s), mainly driven by the decrease of the non-employment benefit. In Germany, the dynamics of the overall reservation wage are mainly driven by the replacement rate, which increased temporarily during the first Schröder government (between the end 1990s and the mid-2000s), whereas it had decreased during the Kohl government and has decreased during the second Schröder government (Hartz reforms). At the opposite, in France, the flow value of non-employment decreases over the whole sample: before the large increase of the replacement ratio at the beginning and at the end of the 1980s (the Mitterrand reforms), this decline of the reservation wage explains the small increase of the employment rate. After the 1980s, this component of the flow value of the non-employment is dominated by the dynamics of the replacement rate: the employment rate declines after the beginning of the 1980s. It is only during the 2000s that the large decline of the reservation wage, due to the dynamics of the hours worked, explains the small increase of the employment rate. It is interesting to remark that the decline of the hours worked per employee en France and in Germany has not the same impact on the reservation wage in these two countries: in Germany, the high level of the employment rate allows families to have a higher wealth, that maintains the relative value between the consumption and the leisure, whereas in France, the decline of the employment rate reduces the consumption and thus, reduces the relative leisure value. Thus, the endogeneity of the reservation wage, and more precisely its strong link with the number of hours worked, underline the interest of our general equilibrium approach.

## 2.G Non-employment income and the employment rate by age

The model we developed includes only two possible states: either a member of the representative family is employed, either he is unemployed, and thus he receives a "nonemployment" income. The measure of unemployed is actually a measure including adult (between 16 and 64 years old) unemployed, inactive and retired members of the family. What is the meaning of the "replacement rate" in our framework ${ }^{52}$ ?

[^52]If we restrict ourself to the unemployment benefits, this measure corresponds to the average gains during an unemployment spell, given the rule of the insurance scheme. Hence, we consider that in each family, there exists a representative group of unemployed workers that gains the "average" unemployment benefits. The degressivity and the loss of eligibility explain the low level of the replacement rate of a "representative" unemployed worker.

Beyond the heterogeneity between the unemployed workers, there also exists an heterogeneity between the insurance possibilities, which are contingent to the worker ages. In particular, this is the case for the pensions schemes or the early retirement benefits: for what it concerns the older workers (55-64 years old), our modelling assumption is equivalent to say that the revenue of a retiree (till 65 years old) is equivalent to that of an unemployed stricto sensu.

Is this hypothesis acceptable?
If we look at the evolution of the replacement rate in France (left panel of Figure 2.6), we can see that the biggest change occurred between the end of 1970s till 1982/1983: the synthetic measure of the replacement rate passed from 0.25 to around 0.35 . Behind this change, we find from the one hand, the increase in the generosity of the support to the unemployed, from the other the generosity of compensation of the old.

Let us consider the data provided by Blöndal and Scarpetta (1999) in their study: in 1975 the official age of retirement was 65 years old, but the average age to leave the labor market was 63 years old ${ }^{53}$; for the remaining years until 65 , the authors report an "expected old-age pension gross replacement rate" of $62.5 \%^{54}$.

In 1995, the normal age of retirement was 60 years old, the estimated age of transition of inactivity was 59.2 years and the gross old-age pension replacement rate was $64.8 \%$.

The overall pension replacement rates in 1975 and in 1995 can therefore be computed as following:

$$
\begin{gathered}
p_{1975}=0.625 *(2 / 10)=0.13 \\
p_{1995}=0.65 *(5 / 10)=0.33
\end{gathered}
$$

where $2 / 10$ and $5 / 10$ represent the average number of years between 55 and 65 during which the worker perceives a pension (respectively $(65-63) /(65-55)$ and (65-60)/(65-55)). The measures are thus not far from the overall replacement rate.

For the sake of completeness, we also report the evolution of the employment rate for different age groups.

Figure 2.d shows the decomposition of the employment rate with respect to three different age groups: the young (between 15 and 24 years old), the middle age (between 25 and 54 years old) and the old (between 54 and 64 years old) for the US and France.

The usual definition of the employment rate for age group considers, at the denominator, the total population of the age group of interest; here we consider a "weighted" employment
and-wages-statistics.htm.
${ }^{53}$ Blöndal and Scarpetta (1999), Table II.1.
${ }^{54}$ Blöndal and Scarpetta (1999), Table III.3.
rate, i.e. the employment rate per age group weighted for the share of the age group with respect to the total working age population ${ }^{55}$.

Figure 2.d - Employment rate decomposition by age: US and FR



First of all, it can be noticed that the long run evolution of the weighted employment rates for the young and the old, across the two considered, show some similar pattern: starting from the mid 1970s, the share of young people in employment fell (there persists however a difference in level: today the employment rate of the young is around 5 percentage points higher in the US than in France).

The weighted employment rate of the old instead started to increase in both countries from the beginning of the last decade; still a difference persists in terms of level.

Figure 2.e - Employment rate of the 55-64: US and FR


Our model does not include the tools to account for the evolution of the employment rate of the youngest: considerations about the organization and financing of the education system would play a fundamental role and we admittedly do not include them in our study.

[^53]
## 2.H The fit of the model

## Additional model predictions

We report the complete set of information about the fit of the model: Figures 2.f-2.i show for each country not only the hours per worker and the employment rate as in the main text, but also total hours and the investment/output ratio.

## Figure 2.f - The US economy



## The driving forces of the model

In this section we analyze the impact of the different driving forces of the model. In order to disentangle the effect of the evolution of the exogenous variables, we proceed as in

Figure 2.g - The French economy


Figure 2.h - The UK economy


Figure 2.i - The German economy


Hours per worker


Total hours


Employment rate


I/Y

McDaniel (2011) by "switching off" the effects caused by the different variables: we compare the outcome of the "full" benchmark model with that of
(i) a model in which only taxes vary, but labor market institutions and TFP do not;
(ii) a model in which only labor market institutions evolve, but taxes and TFP do not.

In particular we fix the level of the constant exogenous variables to that one they attain in 2010 , so that all the versions of the model share the same final steady state.

Figure 2.j - Disentangling the contribution of exogenous variables (France)


Hours if only taxes vary


Hours if only LMI vary


Employment if only taxes vary


Employment if only LMI vary

If we look at the evolution of hours for France in the top left panel of Figure 2.j, we see clearly the explicative power of the tax wedge: hours remain flat till 1975 and then they
start to decrease, reflecting the evolution of the tax wedge that we can see in Figure 2.4. On the other hand, the evolution of employment is not at all explained in this case.

If we look at labor market institutions as the driving force of the model, we confirm the insight coming from the steady state analysis that they explain the evolution of the employment rate while the overall effect of hours worked is very limited, due to the low elasticity of the labor supply locus (see the right panel of Figure 2.13).

## Chapter 3

## The welfare effects of tax progressivity and public insurance in a frictional labor market

### 3.1 Introduction

Most developed economies have redistributive fiscal system. In order to raise tax revenues, the state asks to the rich to contribute relatively more, while the poor generally receive transfers. These transfers can take various forms: they are mostly conditional, either on the level of income (as in the case of a negative income tax), or on a specific status (as in the case of the unemployment insurance scheme).

Redistributive taxation is rooted into considerations about the role of social insurance that the state should play, and the type of "social contract" desired by a society; nevertheless, redistributive taxation can also increase efficiency, depending on which distortions exist in the decentralised economy, prior to government intervention.

Among these possible types of distortions, are the presence of incomplete financial markets (in which agents cannot perfectly insure themselves), or frictional labor markets.

The effect of progressive taxation in the context of frictional labor markets has been extensively analysed in the literature since the late 1980s, and the 1990s, within a representative agent framework ${ }^{1}$; more recently, this effect has been analysed by authors including Parmentier (2006) and Hüngerbuhler et al. (2006).

The conclusion of this strand of literature is that, in the context of frictional labor markets, a more progressive tax schedule can have an impact on the rate of unemployment, through its effect on wage bargaining: by decreasing the part of the surplus going

[^54]to the worker, a more progressive tax schedule contributes to downward wage pressures, which in turn have a beneficial effect on the rate of unemployment. Parmentier (2006) adds considerations about the ambiguous results of progressive taxes on wage, and therefore on unemployment, once the hours of work are variable (and set through Nash bargaining).

Another strand of literature studies the optimal level of tax progressivity. In a context of incomplete financial markets (but perfectly competitive labor markets), progressive taxation can be desirable, because it provides insurance and allows better consumption smoothing; however, it has some negative effects on economic efficiency, most notably through its decrease of hours of work (which brings its own negative effects; e.g., decreasing human capital accumulation), and crowding out private savings.

Recently some authors, as Heathcote et al. (2014) or Bakış et al. (2015), have analysed the optimal level of tax progressivity in a heterogeneous agents framework. Bakış et al. (2015) develop a general equilibrium model with risk adverse agents and savings (as well as bequests); the conclusion of the authors is that, without taking into account the transition path toward a new tax policy setting, the optimal tax schedule for the US is slightly regressive; in contrast, taking into consideration the welfare costs through the whole transition path implies an optimal progressive tax schedule (slightly less progressive than the actual level that characterizes the US, according to the estimations of the authors).

In this paper, I analyse the efficiency losses and gains induced by a progressive tax system, in a framework in which the labor market is frictional and financial markets are incomplete: risk-averse agents are subject only to unemployment risk ${ }^{2}$, they save to insure themselves and end up with different levels of assets, due to the sequence of employment/unemployment spells.

I thus aim to encompass both the effects of tax progressivity highlighted by the labor market literature, which focuses on the effects on the unemployment rate, and those made evident by the heterogeneous agents framework, looking in particular at the effect on total capital.

My results show that an utilitarian welfare criterion calls for a progressive tax system (even considering just steady state comparisons). Moreover, tax progressivity is always preferred to an alternative system to provide public insurance, based on unemployment insurance.

Different trade-offs are at stake: on the one hand, tax progressivity reduces individual labor supply and savings, thus reducing the total size of the economy; however, the decrease in labor supply is valuable in terms of gained leisure time ${ }^{3}$, and the decrease in savings implies a higher level of consumption.

[^55]On the other, tax progressivity, by exerting a downward pressure on wage (through Nash bargaining), has a decreasing effect on unemployment, thus contributing to increase labor input.

In contrast, unemployment benefits, even if they allow constrained agents to better smooth consumption, increase unemployment rate, thus making the total cost of unemployment insurance too high.

The progressive tax system which is studied is, more precisely, a tax and transfer system ${ }^{4}$, which allows for negative taxes (i.e. transfers) to be paid to agents under a certain income threshold. Although the chosen function has to be interpreted as only an approximation to reality, it allows to study social welfare systems characterised by a "negative income tax", financed through tax revenues raised with increasing marginal tax rates.

In terms of modelling choices, I start from the workhorse model of Krusell et al. (2010), which combines the Bewley-Huggett-Aiyagari (BHA) model of incomplete markets with the search and matching model à la Mortensen and Pissarides; I extend it by allowing for variable hours of labor and progressive taxation.

Krusell et al. (2010) consider the optimality of unemployment insurance, in their framework characterised by search frictions and incomplete financial markets: their steady state comparisons indicate that, according to an utilitarian criterion, the optimal level of unemployment benefit is very low ${ }^{5}$. Their results differ from what suggested by the basic Aiyagari (1994) model: if unemployment risk is exogenous, therefore not affected by the unemployment insurance level, it would be optimal to provide perfect insurance (i.e. a level equal to the wage). The reason is that, as explained by Krusell et al. (2010), self-insurance through savings is quite effective, while the distortions induced by the unemployment insurance (an increase in the unemployment rate) are important ${ }^{6}$.

As anticipated, by considering simply steady state comparison, I find that a positive level of progressivity is optimal ${ }^{7}$. I also explore an alternative welfare system, where unemployment insurance is financed through a proportional tax, and I conclude that tax progressivity is always preferred to unemployment benefits ${ }^{8}$.

Finally, I provide some additional results about the sensitivity to alternative hypothesis:

[^56]in particular, I show that: (i) the higher the elasticity of hours, the lower the optimal level of progressivity; (ii) if hours are not allowed to vary, the result about optimal tax progressivity is no longer valid; (iii) the optimal level of progressivity is lower (higher) if the bargaining power of workers is higher (lower) than the elasticity of the matching function; (iv) the higher the level of search frictions, the higher the optimal level of progressivity.

The paper is organised as follows: Section 3.2 presents the model, when unemployment benefits are included or not; Section 3.3.1 discusses the quantitative results for the baseline case, and Section 3.3.2 presents the results of the variant with unemployment benefits of the baseline model. Section 3.4 contains some robustness checks of the model, to assess the stability of the results depending on different calibrations. Section 3.5 concludes.

### 3.2 A two-states model with labor supply and progressive taxation

I develop a model with search and matching frictions in the labor market and individual labor supply. Agents' utility depends on consumption and leisure, the worker and the firm set wage and hours through Nash bargaining; agents can be either employed or unemployed (i.e. I abstract from the participation choice), the separation rate is exogenous, and the job-finding probability, as well as labor market tightness, are endogenous.

The tax schedule I adopt follows, among others, Sørensen (1999), Heathcote et al. (2014) and Bakış et al. (2015). It is actually a tax and transfer scheme, that allows for different levels of progressivity; the particular case of a proportional tax schedule is also embedded. The tax revenues are used to finance the transfers to the poor agents. The same tax schedule is adopted in the variant of the model with unemployment benefits.

### 3.2.1 Labor market flows

The flows on the labor market are modelled following the workhorse model of Diamond, Mortensen and Pissarides. There are many firms, one for each job; firms posts vacancies to fill their job. They are randomly matched with unemployed workers who look for a job; the matching function has the standard Cobb-Douglas form $M=M(v, u)=\chi v^{1-\eta} u^{\eta}$, and labor market tightness is defined as the ratio between the vacancy and the unemployment rates: $\theta=\frac{v}{u}$. After a match is created, the firm and worker bargain over the wage bill (they set the wage and the hours worked). It is important to note that agents are heterogeneous in assets and therefore in their reservation wages. The steady state unemployment rate is given by the equalization between the flows in and out of the stock of employment; i.e., $f(\theta) u=s(1-u)$, which implies that steady state unemployment is given by:

$$
\begin{equation*}
u=\frac{f(\theta)}{s+f(\theta)} \tag{3.1}
\end{equation*}
$$

where $f(\theta)=M / u$ and $q(\theta)=M / v=f(\theta) / \theta$ indicate respectively the job-finding and job-filling probabilities.

### 3.2.2 The financial structure

The financial structure of the model follows Krusell et al. (2010). Consumers can hold two types of assets: physical capital $k$ or equity, denoted as $x$; the total amount of equity is normalised to one, while the equity price is given by the actual value of the future gain, which is given by the dividend and the price of the equity itself (if it were to be sold in the future): $p=\frac{p+d}{1+r}$. Because of the arbitrage condition, the return on equity or on physical capital must be the same; a variable "asset" can therefore be considered, given by the combination of the two types of assets, so that the portfolio combination choice can be discarded.

$$
\begin{equation*}
a=k+p x \tag{3.2}
\end{equation*}
$$

so that

$$
\begin{equation*}
a^{\prime}=k(1+r)+p x \frac{p+d}{p}=(1+r)(k+p x) \tag{3.3}
\end{equation*}
$$

The profit of the firm coming from one matched job pair is given by the production less the costs of capital and labor:

$$
\begin{equation*}
\pi(a)=F(k, h)-(r+\delta) k-w(a) h(a) \tag{3.4}
\end{equation*}
$$

The dividend is paid on the profit net of the vacancy posting cost, which is given by the cost of opening a vacancy $(\omega)$ multiplied by the number of vacancies $(v)$ :

$$
\begin{equation*}
d=\int \pi(a) P_{e}(a) d a-\omega v \tag{3.5}
\end{equation*}
$$

### 3.2.3 The households

I consider a simple way to introduce non-linear labor income taxation, that has been adopted from Sørensen (1999), Bénabou (2002), Heathcote et al. (2014) and recently by Bakış et al. (2015), among others; it is a particular two-parameters functional form for the tax schedule, that allows for negative taxes (i.e. transfers) to low income agents. It presents some convenient properties, already identified by Jakobsson (1976), notably the fact that the Coefficient of Residual Income Progression is constant.

The CRIP, defined originally by Musgrave and Thin (1948), is one of the most-used measure of progressivity ${ }^{9}$ : it represents the elasticity of post-tax income to pre-tax income. A tax schedule is considered progressive if the CRIP is strictly less than one, regressive if it is bigger than one; in the case of a flat tax, the CRIP is equal to one. One advantage of this measure of tax progressivity is that it is also defined when the average tax rate is null. The chosen functional form for the tax schedule defines disposable income as:

$$
\begin{equation*}
y^{d}=(1-\tau) y^{1-\lambda} \tag{3.6}
\end{equation*}
$$

so that total (net) taxes are given by:

$$
\begin{equation*}
T(y)=y-(1-\tau) y^{1-\lambda} \tag{3.7}
\end{equation*}
$$

The coefficient $\lambda$ governs the level of progressivity, while $\tau$ is a shift parameter that serves to balance the government budget. The CRIP relates to the marginal and average tax rates as follows:

$$
\begin{equation*}
\operatorname{CRIP}(\mathrm{y})=\frac{\partial y^{d}}{\partial y} \frac{y}{y^{d}}=\frac{1-T^{\prime}(y)}{1-T(y) / y} \tag{3.8}
\end{equation*}
$$

Considering the chosen tax schedule, the expression for the CRIP is given by:

$$
\begin{equation*}
\operatorname{CRIP}(\mathrm{y})=(1-\lambda) \tag{3.9}
\end{equation*}
$$

so that if $\lambda$ is zero the tax schedule is flat, while if $0<\lambda<1$ the tax schedule is progressive ${ }^{10}$. The chosen tax schedule is actually a tax and transfer scheme: it allows for "negative" taxes if the revenue is lower than a certain threshold ${ }^{11}$. Moreover, there exists a minimum for the net tax schedule, which implies that transfers are non-monotonic in income:

$$
\begin{equation*}
T^{\prime}(y)=0 \leftrightarrow y=[(1-\tau)(1-\lambda)]^{1 / \lambda}=y_{00} \tag{3.10}
\end{equation*}
$$

The definition of revenue $y$ is different for the employed and the unemployed agent: in the baseline case, the term $y$ refers to private income, i.e., to the labor and interest rate income for the employed and to the sole interest income for the unemployed.

In the policy variant, I allow for an additional income source: the unemployed agents receive an unemployment benefit (UB), $\mu^{12}$, which is encompassed within the definition of

[^57]taxable income.
To sum up, in the baseline case:
\[

$$
\begin{align*}
y_{e}^{d}= & (1-\tau)(w h+r a)^{1-\lambda}  \tag{3.11}\\
y_{u}^{d} & =(1-\tau)(r a)^{1-\lambda} \tag{3.12}
\end{align*}
$$
\]

When the welfare system allows for unemployment benefits, the disposable income becomes:

$$
\begin{align*}
& y_{e}^{d}=(1-\tau)(w h+r a)^{1-\lambda}  \tag{3.13}\\
& y_{u}^{d}=(1-\tau)(r a+\mu)^{1-\lambda} \tag{3.14}
\end{align*}
$$

I turn to the definition of the value functions of the employed and unemployed agent in the baseline economy:

$$
\begin{align*}
W(a) & =u\left(c_{e}(a), 1-h(a)\right)+\beta\left[(1-s) W\left(g_{e}(a)\right)+s U\left(g_{u}(a)\right)\right]  \tag{3.15}\\
U(a) & =u\left(c_{u}(a), \Gamma^{u}\right)+\beta\left[f(\theta) W\left(g_{e}(a)\right)+(1-f(\theta)) U\left(g_{u}(a)\right)\right] \tag{3.16}
\end{align*}
$$

where the decision rules for savings (and therefore for consumption) of the employed and unemployed are respectively:

$$
\begin{gather*}
a_{e}^{\prime}=g_{e}(a)  \tag{3.17}\\
a_{u}^{\prime}=g_{u}(a)  \tag{3.18}\\
c_{e}=a+(1-\tau)(w(a) h(a)+r a)^{(1-\lambda)}-a_{e}^{\prime}  \tag{3.19}\\
c_{u}=a+(1-\tau)(r a)^{(1-\lambda)}-a_{u}^{\prime} \tag{3.20}
\end{gather*}
$$

The specific form of the instantaneous utility function that I adopt is the following:

$$
\begin{equation*}
u(c, h)=\ln (c)+\Gamma^{z} \tag{3.21}
\end{equation*}
$$

where $z=\{e, u\}, \Gamma^{e}=\sigma_{l} \frac{(1-h)^{1-\nu}}{1-\nu}$ and $\Gamma^{u}$ is a constant. The Euler equations are given by:

$$
\frac{\partial u(c, 1-h)}{\partial c}=\beta[(1-s) \frac{\partial u\left(c^{\prime}, 1-h^{\prime}\right)}{\partial c^{\prime}}(1+(1-\tau)(1-\lambda) y_{e}^{-\lambda}(\frac{\overbrace{\frac{\partial w\left(a^{\prime}\right)}{\partial a^{\prime}} h\left(a^{\prime}\right)+w\left(a^{\prime}\right) \frac{\partial h\left(a^{\prime}\right)}{\partial a^{\prime}}}^{\text {strategic bargaining }}+r)}{})+
$$

and

$$
\begin{align*}
& \frac{\partial u(c, 0)}{\partial c}=\beta\left[f(\theta) \frac{\partial u\left(c^{\prime}, 1-h^{\prime}\right)}{\partial c^{\prime}}\left(\begin{array}{l}
1+(1-\tau)(1-\lambda) y_{e}^{-\lambda}(\overbrace{\frac{\partial w\left(a^{\prime}\right)}{\partial a^{\prime}} h\left(a^{\prime}\right)+w\left(a^{\prime}\right) \frac{\partial h\left(a^{\prime}\right)}{\partial a^{\prime}}}^{\text {strategic bargaining }}+r)
\end{array}\right)+\right. \\
&\left.(1-f(\theta)) \frac{\partial u\left(c^{\prime}, 0\right)}{\partial c^{\prime}}\left(1+r^{2}(1-\tau)(1-\lambda) a^{-\lambda}\right)\right] \quad(3.23) \tag{3.23}
\end{align*}
$$

By looking at the Euler equations, two important elements can be discerned: if $\lambda=0$ the expressions are equivalent to those obtained by Reichling $(2006)^{13}$, who defined the "strategic bargaining" component of savings: if the agent takes into consideration that hourly wage and hours are function of wealth, he will also save in order to able to obtain a higher salary when employed ${ }^{14}$. Secondly, the new part of the Euler equation connected to the progressive taxation (when $0<\lambda<1$ ) shows that, ceteris paribus, the incentive to save is lower as $\lambda$ increases (I remind that the interest rate income is taxed too).

### 3.2.4 The firm

Firms produce using capital and labor: they post vacancies to hire workers, and they rent capital from the households. After a match is created, the firm and the worker set the wage and the hours. The firm chooses to open vacancies by taking into consideration the value of filling a job, which is given by:

[^58]\[

$$
\begin{equation*}
J(a)=\max _{k}\left(\frac{k}{h}\right)^{\alpha} h(a)-(r+\delta) k-w(a) h(a)+\frac{1}{1+r}\left[s V+(1-s) J\left(a_{e}^{\prime}\right)\right] \tag{3.24}
\end{equation*}
$$

\]

The filled job produces an amount of output given by $\left(\frac{k}{h}\right)^{\alpha} h(a)$. The firm has to pay the rental cost of capital and the wage bill; in addition, the firm takes into account that the job has a probability of being destroyed, described as the separation rate. In this case, the firm is left with the value of opening a vacancy, indicate by $\mathrm{V}^{15}$.

The term $\left(\frac{k}{h}\right) \equiv \tilde{k}$ represents the capital per labor ratio. In equilibrium, it must be the same across all matched firms, because the capital market is perfectly competitive; this means that each firm must set the capital labor ratio as $\tilde{k}=\frac{K}{H}$, where $K$ is aggregate capital and $H$ represents total effective labor supply:

$$
\begin{equation*}
H=\int h(a) P_{e}(a) d a=\left(\int h(a) \frac{P_{e}}{(1-u)} d a\right)(1-u) \tag{3.25}
\end{equation*}
$$

The interest rate is given by:

$$
\begin{equation*}
r=\alpha\left(\frac{k}{h}\right)^{(\alpha-1)}-\delta \tag{3.26}
\end{equation*}
$$

The value of a vacancy is given by:

$$
\begin{equation*}
V=-\omega+\frac{1}{1+r}\left[q(\theta) \int J\left(a_{u}^{\prime}\right) \frac{P_{u}(a)}{u} d a+(1-q(\theta)) V\right] \tag{3.27}
\end{equation*}
$$

The value of opening a vacancy depends on the vacancy posting cost $\omega$ and on the probability $q(\theta)$ of filling the vacancy. It must be noted that since it is not a directed search model, the firm cannot distinguish among workers before filling the vacancy; as agents are heterogeneous in terms of their assets, and therefore their reservation wage, but not in terms of their productivity, the firm faces an uncertainty with respect to how much it will have to pay a worker (and how many hours he will work for a certain hourly wage). This is the reason that the "average" value of a filled job $\int J\left(a_{u}^{\prime}\right) \frac{P_{u}(a)}{u} d a$ appears in the value of a vacancy.

In equilibrium, the value of opening a vacancy has to be null; setting $V=0$ implies the following value for the job filling rate:

$$
\begin{equation*}
q(\theta)=\omega(1+r)\left(\int J\left(a_{u}^{\prime}\right) \frac{P_{u}(a)}{u} d a\right)^{-1} \tag{3.28}
\end{equation*}
$$

[^59]
### 3.2.5 Wage bargaining

The wage and hours are fixed through Nash-bargaining between the firm and the worker: the fact that the tax on labor income is progressive has an impact on the bargaining, and therefore on the equilibrium choices, of wage and hours. The firm and the worker maximise the Nash product of their respective evaluation of the value of the job:

$$
\max _{w, h}(W(a)-U(a))^{\gamma}(J(a)-V)^{1-\gamma}
$$

where I remind the reader that the expressions that define the surplus of the worker are given by:

$$
\begin{align*}
W(a) & =u\left(c_{e}(a), 1-h(a)\right)+\beta\left[(1-s) W\left(g_{e}(a)\right)+s U\left(g_{u}(a)\right)\right]  \tag{3.29}\\
U(a) & =u\left(c_{u}(a), \Gamma^{u}\right)+\beta\left[f(\theta) W\left(g_{e}(a)\right)+(1-f(\theta)) U\left(g_{u}(a)\right)\right] \tag{3.30}
\end{align*}
$$

and

$$
\begin{align*}
& c_{e}=(1-\tau)[w(a) h(a)+r a]^{(1-\lambda)}-g_{e}(a)  \tag{3.31}\\
& c_{u}=(1-\tau)(r a)^{(1-\lambda)}-g_{u}(a) \tag{3.32}
\end{align*}
$$

The value of a filled job for the firm is:

$$
\begin{equation*}
J(a)=\tilde{k}^{\alpha} h(a)-w(a) h(a)-(r+\delta) k+\frac{1}{1+r}\left[s V+(1-s) J\left(a^{\prime}\right)\right] \tag{3.33}
\end{equation*}
$$

The FOC with respect to the wage gives:

$$
\begin{equation*}
\gamma(W(a)-U(a))^{\gamma-1} \frac{\partial W(a)}{\partial w}(J(a))^{1-\gamma}+(1-\gamma)(J(a))^{-\gamma} \frac{\partial J(a)}{\partial w}(W(a)-U(a))^{\gamma}=0 \tag{3.34}
\end{equation*}
$$

where the derivative of the value for a worker of a higher (hourly) wage is given by:

$$
\begin{equation*}
\frac{\partial W(a)}{\partial w}=\frac{\partial u\left(c_{e}, h\right)}{\partial c_{e}}(1-\tau)(1-\lambda)(w(a) h(a)+r a)^{(-\lambda)} h \tag{3.35}
\end{equation*}
$$

If $\lambda=0$ (a proportional income tax), equation (3.35) takes the form of a traditional expression: the value of an increase in wage is given by the additional (net) consumption it allows, evaluated through its marginal utility. When $\lambda$ increases, ceteris paribus, the value of an increase in wage for the worker decreases.

The derivative of the value of a filled job for the firm with respect to the wage is given
by:

$$
\begin{equation*}
\frac{\partial J(a)}{\partial w}=-h(a) \tag{3.36}
\end{equation*}
$$

The expression becomes:

$$
\begin{equation*}
\frac{(W(a)-U(a))}{\left(\frac{\partial W(a)}{\partial w}\right)} \frac{1-\gamma}{\gamma}=-\frac{J(a)}{\frac{\partial J(a)}{\partial w}} \tag{3.37}
\end{equation*}
$$

and therefore ${ }^{16}$ :

$$
\begin{equation*}
(W(a)-U(a)) \frac{c_{e}(a)[w(a) h(a)+r a]^{\lambda}}{(1-\tau)(1-\lambda)}=\frac{\gamma}{1-\gamma} J(a) \tag{3.38}
\end{equation*}
$$

The FOC with respect to hours gives:

$$
\begin{equation*}
\gamma \frac{1}{(W(a)-U(a))} \frac{\partial W(a)}{\partial h} J(a)+(1-\gamma) \frac{\partial J(a)}{\partial h}=0 \tag{3.39}
\end{equation*}
$$

The derivative of the value function with respect to hours is given by:

$$
\begin{equation*}
\frac{\partial W(a)}{\partial h}=\frac{\partial u\left(c_{e}, 1-h\right)}{\partial c_{e}} w(1-\tau)(1-\lambda)(w h+r a)^{-\lambda}+\frac{\partial u\left(c_{e}, 1-h\right)}{\partial h} \tag{3.40}
\end{equation*}
$$

Similarly to what has been observed for equation (3.35), the effect of more progressive taxation (an increasing value for $\lambda$ ) is to decrease the convenience to work additional hours: when $\lambda \rightarrow 1$ (i.e., a situation involving perfect pooling of income and redistribution among agents), the only consequence of working an additional hour would be the disutility of enjoying less leisure time.

The derivative of the value for the firm of an additional hour is given by:

$$
\begin{equation*}
\frac{\partial J(a)}{\partial h}=\tilde{k}^{\alpha}-w \tag{3.41}
\end{equation*}
$$

Substituting the previous expressions in equation (3.39), it becomes:

$$
\begin{equation*}
\frac{(W(a)-U(a))}{\left(\frac{\partial u\left(c_{e}, 1-h\right)}{\partial c_{e}} w(1-\tau)(1-\lambda)(w h+r a)^{-\lambda}+\frac{\partial u\left(c_{e}, 1-h\right)}{\partial h}\right)}=\frac{\gamma}{1-\gamma} \frac{J(a)}{\left(w-\tilde{k}^{\alpha}\right)} \tag{3.42}
\end{equation*}
$$

[^60]Combining the FOC on hours and wage, I finally obtain the following condition:

$$
\begin{equation*}
(1-h(a))^{-\nu}=\frac{\tilde{k}^{\alpha}}{\sigma_{l}} \frac{(1-\tau)(1-\lambda)}{c_{e}(a)[w(a) h(a)+r a]^{\lambda}} \tag{3.43}
\end{equation*}
$$

Once again, by setting $\lambda=0$ the usual hours equation appears; as it has been remarked, the parameter $\lambda$ has a negative effect on hours, ceteris paribus.

### 3.2.6 Government budget constraint

The government runs a balanced budget: it collects taxes which are used to finance its transfers. In the alternative policy settings, the Government finances also a public insurance, in the form of unemployment benefits.

In the numerical simulations, the value of the degree of progressivity $(\lambda)$ is exogenous, while the parameter governing the level of tax revenues $(\tau)$ varies endogenously, in order to balance the government budget.

I recall the definition of tax revenues, which consist of the difference between total (pretax) income and the disposable (after-tax) income:

$$
\begin{equation*}
T(y)=y-y^{d}=y-(1-\tau) y^{1-\lambda} \tag{3.44}
\end{equation*}
$$

In the baseline economy, the net taxes have to be null (all the tax collected finance the negative taxes paid by income-poor agents). In the alternative setting, there are additional expenditures given by the amount of the unemployment benefit $\mu$, which is paid to a fraction $u$ of the population. I write the general formulation including $\lambda$, in order to be able to analyse not only a flat income tax, used to finance the public insurance schemes, but also a combination of tax progressivity and additional transfers in the form of UB.

I briefly recall the definition of disposable income in the two alternative policy frameworks, and the corresponding government budget constraint.

## Baseline economy: Tax on wage and capital income

Disposable income for worker:

$$
\begin{equation*}
y_{d}^{e}=(1-\tau)(w h+r a)^{(1-\lambda)} \tag{3.45}
\end{equation*}
$$

Disposable income for unemployed:

$$
\begin{equation*}
y_{d}^{u}=(1-\tau)(r a)^{(1-\lambda)} \tag{3.46}
\end{equation*}
$$

Government budget constraint:

$$
\begin{align*}
(1-u)\left[\int(w h+r a) \frac{d P_{e}}{(1-u)}-(1-\tau)\right. & \left.f(w h+r a)^{(1-\lambda)} \frac{d P_{e}}{(1-u)}\right]+ \\
& u\left[\int(r a) \frac{d P_{u}}{u}-(1-\tau) \int(r a)^{(1-\lambda)} \frac{d P_{u}}{u}\right]=0 \tag{3.47}
\end{align*}
$$

## Alternative case: Tax and unemployment benefit (UB)

Disposable income for worker:

$$
\begin{equation*}
y_{d}^{e}=(1-\tau)(w h+r a)^{(1-\lambda)} \tag{3.48}
\end{equation*}
$$

Disposable income for unemployed:

$$
\begin{equation*}
y_{d}^{u}=(1-\tau)(\mu+r a)^{(1-\lambda)} \tag{3.49}
\end{equation*}
$$

Government budget constraint:

$$
\begin{align*}
&(1-u)\left[\int(w h+r a) \frac{d P_{e}}{(1-u)}\right.\left.-(1-\tau) \int(w h+r a)^{(1-\lambda)} \frac{d P_{e}}{(1-u)}\right]+ \\
& u\left[\int(\mu+r a) \frac{d P_{u}}{u}-(1-\tau) \int(\mu+r a)^{(1-\lambda)} \frac{d P_{u}}{u}\right]=\mu u \tag{3.50}
\end{align*}
$$

### 3.3 Calibration and quantitative results

The model is calibrated for a period of six weeks. The parameter values chosen for the discount factor and the separation rate imply an annual interest rate of $4.1 \%$ and a monthly separation rate of 0.031 . I choose to calibrate the elasticity of the matching function with respect to vacancies and the bargaining power of the firm to the same value (0.5), and I fix the vacancy posting cost, as well as the matching efficiency in order to target the values of the job-finding and job-filling rates. In regards to the parameters of the utility function, I choose to use a logarithmic function for consumption. In the presence of progressive taxation on labor income, the Frisch elasticity of labor supply is not independent of the parameter that affects the CRIP ${ }^{17}$. The parameters that govern the taste for leisure and the curvature of the labor supply imply that in the benchmark economy the "traditional" Frisch elasticity ${ }^{18}$ belongs to the interval [0.002;0.08]: the asset-poor agents show a very inelastic labor supply,

[^61]while the rich have the highest level ${ }^{19}$.
I choose as a "benchmark economy" a case that can be considered close to the actual state of the US economy: the policy parameter $\lambda$ is set to $0.16^{20}$.

Table 3.1 contains the calibrated values of the parameters, and Table 3.2 the steady state results ${ }^{21}$.

Table 3.1 - Calibrated parameters

| Symbol | Interpretation | Value |
| :---: | :---: | :---: |
| $\beta$ | discount factor | 0.995 |
| $\delta$ | depreciation rate | 0.01 |
| $\alpha$ | capital share of income | 0.33 |
| $\sigma$ | risk aversion parameter | 1 |
| $\sigma_{l}$ | distaste for work | 0.15 |
| $\nu$ | curvature of labor supply | 0.45 |
| $\Gamma^{u}$ | utility cost of unemployment | -0.022 |
| $s$ | separation rate | 0.046 |
| $\gamma$ | bargaining power | 0.5 |
| $\eta$ | matching elasticity | 0.5 |
| $\chi$ | matching efficiency | 0.55 |
| $\omega$ | vacancy posting costs | 0.02 |

Table 3.2 - Steady state results

| u | $\theta$ | j finding r | j filling r | mean w | $\tau$ | mean a |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $6.17 \%$ | 1.62 | 0.70 | 0.43 | 2.97 | -0.21 | 85.30 |

### 3.3.1 Welfare effect of tax progressivity

In this section, I illustrate the first result of the paper: in an economy characterised by the tax and transfer schedule in equation (3.44), there exists a positive optimal level of tax progressivity, measured through the Coefficient of Residual Income Progression ${ }^{22}$, once social welfare is evaluated using a utilitarian criterion. The utilitarian welfare criterion is obtained

[^62]by simply summing up the welfare level of each agent, expressed by his value function. I report the general formula for social welfare which allows, at the limit, for an evaluation according to the Rawls criterion ${ }^{23}$ :
\[

$$
\begin{equation*}
\text { Welfare }=\left(\int W(a)^{1-\xi} P_{e}(a)+\int U(a)^{1-\xi} P_{u}(a)\right)^{\frac{1}{1-\xi}} \tag{3.51}
\end{equation*}
$$

\]

According to the simulations obtained by varying only the parameter $\lambda$, which are shown in the left panel of Figure 3.1, the optimal level of CRIP is $1-\lambda^{*}=0.8$.

Figure 3.1 - Utilitarian welfare and unemployment rate: the effects of $\lambda$


To evaluate the gains (or losses) of moving along the curve representing welfare in Figure 3.1, I calculate the equivalence between the welfare levels in terms of average consumption variations $\Delta$. Considering that preferences are separable in consumption and leisure, I define the consumption variation as follows:

$$
\begin{equation*}
\ln (1+\Delta) \frac{1}{1-\beta}=\int \tilde{W} \tilde{P}_{e}+\int \tilde{U} \tilde{P}_{u}-\left(\int W P_{e}+\int U P_{u}\right) \tag{3.52}
\end{equation*}
$$

where $W$ and $\tilde{W}$ ( $U$ and $\tilde{U}$ ) stand for the value function of the employed (unemployed) in the original and experiment economies.

In Table 3.3, I report the values of $\Delta$, where the original economy is the one with the optimal level of the parameter $\lambda\left(\lambda=\lambda^{*}=0.2\right)$ and the two experiments are $\lambda=0.0$ and $\lambda=0.25$ respectively. Since I set the reference point as the optimal value, moving either to the right or left of the point implies a cost in terms of consumption.

[^63]
## Table 3.3 - Welfare gains/losses in terms of consumption

| $\lambda$ | $\Delta$ total |
| :---: | :---: |
| 0.0 | $-0.89 \%$ |
| 0.2 | $0 \%$ |
| 0.25 | $-0.08 \%$ |

## The mechanisms behind tax progressivity

The effects of $\lambda$ on the functioning of the economy pass through the wage and hours negotiation.

In the context of a representative agent model, Sørensen (1999) summarises the impact of marginal tax rate on the unemployment rate and economic efficiency: he looks at different frameworks characterised by labor market frictions, among which the search and matching model and the union bargaining model.

Parmentier (2006) extends the analysis to a case with search and matching frictions in which hours are elastic, and fixed through Nash bargaining between the worker and the firm.

Sørensen (1999) states that a higher progressive tax implies a lower unemployment rate: since the increased progressivity decreases the part of the surplus going to the worker, and makes more costly for the employer to assign part of the surplus to the worker, there is a convergence of interest in lowering wage pressure. This contributes to the decrease of unemployment. In a union model, he shows that the higher the labor supply elasticity, the lower is the optimal level of progressivity: this is because a higher progressivity distorts effort (or labor supply), since the gain of each hour worked is lower.

Parmentier (2006) stresses that introducing variable hours implies an ambiguity in the reaction of unemployment, depending on how the utility in unemployment is considered ${ }^{24}$, and on the value of some key parameters. Moreover, the effects on economic efficiency are not monotonic. In his numerical simulations, Parmentier (2006) shows that the key parameters affecting the results are the level of utility in unemployment, and labor supply elasticity: in his benchmark economy, unemployment decreases with the increase of marginal tax rate, and economic efficiency has an inverted u-shape.

My results are in line with those of Sørensen (1999) and Parmentier (2006) ${ }^{25}$ : the numerical simulations show that as the measure of tax progressivity increases, both the negotiated wage and hours worked decrease, for each level of asset.

Both these effects can be seen playing a role in the model by looking at Figure 3.2. The

[^64]Figure shows the resulting hours and wage functions for two experiments, one in which $\lambda=0$, and the other in which $\lambda=0.16$ : for all levels of assets, hours worked decrease (left panel), while the wage is lower (right panel), when $\lambda$ increases.

The unemployment rate is decreasing in $\lambda$, as it can be seen in the right panel of Figure 3.1. The overall effect on total labor, defined as $H=(1-u) \int h(a) \frac{P e(a)}{(1-u)}$ remains positive, driven by the movement in the rate of unemployment.

Figure 3.2 - Hours and wage function: the effects of $\lambda$


An additional effect that lacks in the model of Sørensen (1999) and Parmentier (2006) regards capital accumulation and its effects on labor productivity. Once agents are allowed to save (and the revenues of savings are taxed), aggregate capital reacts to changes in tax progressivity, as in the model of Bakış et al. (2015) ${ }^{26}$ : in a general equilibrium framework with risk-averse agents and precautionary savings, aggregate capital can be crowded out, if agents have access to other means of insurance, and their capital income is taxed to finance the insurance system.

The effects on saving decisions and on asset inequality can be seen in Figure 3.3: with respect to an economy with no tax and transfers, the employed agents save less while the unemployed are able to dissave less.

Overall, the amount of capital is lower and the distribution of assets is more concentrated, as it is shown in the right panel of Figure $3.3^{27}$.

[^65]Since capital enters in the production function, the decrease in capital implies, ceteris paribus, a lower labor productivity (in terms of the model, the capital per effective labor ratio, $\tilde{k}$, decreases), therefore a lower size of the overall economy. The effect of $\lambda$ on aggregate capital is shown in the left panel of Figure 3.4.

In the evaluation of welfare, the decrease in savings allows to increase consumption, up to a certain level; in addition to consumption, there is also the effect of (increased) leisure time and better job-finding probabilities.

Figure 3.3 - Saving decisions and densities: the effects of $\lambda$


Figure 3.4 - Aggregate capital and labor: the effects of $\lambda$


Which are the general equilibrium effects of higher progressivity in terms of total fiscal
effort? Figure 3.5 shows the marginal and average tax rates ${ }^{28}$ for the employed (left panel) and unemployed agents (right panel), while Figure 3.6 reports the total taxes (if they are negative, they correspond to transfers received by agents). The marginal tax rate, in a progressive system (when $\lambda>0$ ) is always higher than the average tax rate, and it is increasing in the income level. The employed agents save less if $\lambda$ increases, because they pay more taxes (excepting the poorest, who receive transfers), while the disposable income of all unemployed agents increase (they all receive transfers). This allows them to dissave less (those with very low levels of assets they actually dissave more, since they know that they have access to transfers and the perception of the job market is better).

Figure 3.6 clearly illustrates one unappealing feature of the considered tax and transfer schedule, as it has been pointed out in Section 3.2.3: the negative taxes are non monotonic in the pre-tax income, i.e. for agents with an income lower than the threshold $y_{00}$, the poorer they are, the lower transfer they receive ${ }^{29}$.

Figure 3.5 - Marginal and average tax rates for employed and unemployed: the effects of $\lambda$



### 3.3.2 Welfare effect of unemployment benefits (UB)

As it has been shown at the end of the previous section, the tax and transfer schedule that has been adopted presents an unappealing feature, for what it concerns the transfers to the poorest (unemployed) agents. Moreover, since in this framework the only risk is linked

[^66]Figure 3.6 - Total taxes for employed and unemployed: the effects of $\lambda$

to the probability of becoming unemployed, it is natural to study the welfare implication of an unemployment insurance scheme, as an instrument to improve consumption smoothing. The question I ask in this section is therefore: which are the welfare effects of unemployment benefits ${ }^{30}$, when they are financed through a proportional tax? This is the same question that Krusell et al. (2010) ask in their paper: my framework differs from theirs, because I add leisure in the utility function, and I allow for variable individual labor supply. The introduction of leisure time, however, does not change the results, which are in line with the conclusions expressed by Krusell et al. (2010): the welfare costs of unemployment insurance outweigh the benefits that can come from improved consumption smoothing. The unemployment insurance scheme, in fact, has detrimental effects on the unemployment rate, as it is known from the labor market literature: this effect is due to the fact that unemployment benefits increase the reservation wage, and therefore push up wages: this reduces the profitability of vacancies. However, in a framework with capital accumulation, additional insurance crowds out private savings: capital decreases, so that capital labor ratio is negatively affected ${ }^{31}$. The contribution of the worker for the firm (the term $\tilde{k}=K / H$ in equation 3.33) decreases, and this puts a downward pressure on wage.

These two counteracting effects on wage are at work in a framework in which the reservation wage is linked to the (heterogeneous) level of assets of each agent. The right panel of Figure 3.8 shows that the upward pressure on wage prevail only for the lowest levels of wages. In the experiment with a positive value for the unemployment benefit, its value is

[^67]kept quite low (the maximum value for the replacement rate, i.e. for the ratio $\mu / w$, is $7 \%$ ): the overall negative effect on the unemployment rate is an increase from $8 \%$ to $8.3 \%$. The conclusion in terms of average welfare are not affected, however: unemployment benefits do not seem to be a good choice.

Table 3.4 - Steady state values: the effects of UB $(\lambda=0)$

|  | $\mu=0$ | $\mu=0.2$ |
| :---: | :---: | :---: |
| $\max \mu / w$ | 0 | $7 \%$ |
| unemploym. rate | $8 \%$ | $8.3 \%$ |

Figure 3.7 - Utilitarian welfare and unemployment rate: the effects of UB $(\lambda=0)$


The welfare detrimental effect of unemployment benefit come from the fact that the higher the level of $\mu$, the higher the unemployment rate, and therefore the higher the need to raise taxes, in order to finance the cost of the unemployment insurance scheme: the average tax rate increases from $0.03 \%$ to $0.51 \%$.

### 3.3.3 General welfare comparisons: progressive tax schedule and UB

To sum up the findings of the previous sections, I showed firstly that allowing for a progressive tax and transfer schedule (even if constrained to a specific functional form) can improve welfare, with respect to a system with no tax and transfers (Section 3.3.1).

Secondly, I showed that the evaluation in terms of average welfare of an alternative tax and transfer system (where the fiscal revenues, collected through proportional income tax,

Figure 3.8 - Hours and wage function: the effects of UB $(\lambda=0)$

are used to finance a scheme of unemployment insurance) calls for transfers which are null, and therefore for a null level of taxation: in this case, the costs in terms of efficiency of the publicly provided insurance scheme are overwhelming.

In this Section, I allow for a combination of the previous schemes, i.e. I consider a tax schedule as in equation (3.7), where the CRIP can take values between 0 and 1 , and at the same time I allow for unemployment benefits.

This comprehensive experiment is motivated by the fact that it allows to evaluate the relative weight of the effects of progressivity and of the UB on the evolution of welfare: I do expect to find a positive level of progressivity at the optimum, and no need for additional transfers, in the form of UB, but it is important to see the "relative" strength of welfare cost (i.e. the derivative of welfare with respect to the different policy parameters ${ }^{32}$ ).

In addition, this comprehensive experiment also allows to compare the results with those in Sørensen (1999): in his paper in fact, he looks for the "optimal" level of progressivity ${ }^{33}$, conditionally on different levels of unemployment benefits. His conclusions is that the higher the level of unemployment benefit, the higher the distortion on the labor market that can be corrected by the progressive tax schedule: thus, the optimal level of the CRIP is lower.

The simulations of the comprehensive experiment in terms of average welfare are showed

[^68]in the left panel of Figure 3.9: the optimum is reached for a level of $\lambda=0.2$, and no additional transfers (the black circle). If the level of the unemployment benefit $\mu$ is fixed to 0.2 (which corresponds to $7 \%$ of the average wage), the optimal level of $\lambda$ is slightly higher (the triangle on the yellow surface), as in Sørensen (1999) ${ }^{34}$.

The variation of welfare due to changes in the transfer $\mu$ is less important than that implied by varying the parameter $\lambda$; for low levels of progressivity, the negative welfare effects of the unemployment benefits are more pronounced.

The right panel of Figure 3.9 sums up the effects of the policy parameters on the unemployment rate: the marginal effect of the degree of progressivity seems to be more important than that one of the level of unemployment benefit, similarly to what has been observed for average welfare.

Figure 3.9 - Utilitarian welfare and unemployment rate


Figure 3.10 sumps up the evolution of the production factors: aggregate labor increases with the level of progressivity, but it decreases with the generosity of unemployment benefits. Aggregate capital is mostly crowded out when the tax and transfer schedule is progressive. The higher average tax rates necessary to finance a UB system explain the relative stronger decrease of aggregate consumption for high levels of progressivity (high $\lambda$ ) and additional transfers, as it can be seen in the left panel of Figure 3.11.

Finally, it can be observed, in the right panel of Figure 3.11, that a more progressive tax and transfer system implies a substantial redistribution and a strong decline of the Gini coefficient for wealth.

[^69]Figure 3.10 - Factors of production: L and K


Figure 3.11 - Aggregate consumption and Gini coefficient for wealth



### 3.4 Sensitivity analysis

In this section, I check the robustness of the model's predictions to alternative hypothesis. In particular, I consider the results in relation to: (i) the value of the labor supply elasticity; (ii) the fact to switch off the intensive margin of labor supply; (iii) the consequences of calibrating the bargaining power of workers to a different value than the elasticity of the matching function; (iv) the level of search frictions.

### 3.4.1 Labor supply elasticity

In this section, I check the effects of allowing hours to vary more or less.
My conclusions are in line with Sørensen (1999): the higher the variability of hours, the lower the optimal value of progressivity (i.e. the closer the optimal CRIP is to the unity). This result comes from the fact that the higher the elasticity, the stronger is the reaction of agents in decreasing labor supply, when tax progressivity is increased. The crowding out effect of labor supply implies a lower total labor input, and therefore production, which cannot be compensated by the increased utility coming from leisure time.

I report the values for the optimal level of the parameter $\lambda$ for two alternative calibrations of the model, which imply a lower or higher elasticity of hours ${ }^{35}$. I remind that, being hours a (decreasing) function of assets, the elasticity takes a different value at each value of wealth; in Table 3.5, I report the average value for the elasticity ${ }^{36}$.

Table 3.5 - Optimal tax policy values for different levels of labor supply elasticity

|  | Low elasticity | Baseline | High elasticity |
| :---: | :---: | :---: | :---: |
| Elasticity $($ for $\lambda=0)$ | 0.0059 | 0.0158 | 0.0690 |
| Optimal $\lambda$ | 0.24 | 0.2 | 0.18 |

### 3.4.2 Fixed individual hours

In this Section, I check the consequences of the hypothesis of variables hours.
I thus consider the same baseline economy as in section 3.3.1(i.e. I start with policy values of $\lambda=0.16$ and $\mu=0$ ), and I shut down the intensive margin: hours are fixed at the average of the baseline model, conditionally on being employed, i.e. $h=\bar{h}=\int h(a) \frac{P_{e}(a)}{1-u}$. I perform the same experiments than in the main text, i.e. I allow the policy parameters $\lambda$

[^70]and $\mu$ to vary, and I compute the average welfare. The results on average welfare and the effects of unemployment rate are shown in the right panels of Figures 3.12 and 3.13.

The consequences in terms of unemployment of tax progressivity are qualitatively the same as when hours are variable: I confirm the results of the (simpler) model with no intensive margin of Sørensen (1999). However, the elasticity of unemployment to progressivity is lower: starting from the situation where $\lambda=0.16$ (when the economies with fixed and variable hours are characterised by an unemployment rate of, respectively, $6.25 \%$ and $6.17 \%$ ) and comparing it to the situation when $\lambda=0$, unemployment increases to $6.76 \%$ if hours are fixed, while it increase to $8.01 \%$ if hours can vary. When hours are variable, in fact, an increase in progressivity decreases hours too, which contributes to the downward pressure on wage, and further decreases the unemployment rate.

The consequences in terms of welfare are profoundly different: welfare is maximised in the absence of (progressive) taxation ${ }^{37}$. When hours are fixed, in fact, there is no additional utility coming from an increase in leisure, when progressivity increases. Moreover, labor market conditions still ameliorate when progressivity increases, but less than in the case with variable hours.

Figure 3.12 - Hours functions and unemployment rate


Overall, the results point to the importance of introducing variable hours. These results have also to be considered in relation to the literature which has stressed the importance of "precautionary" hours ${ }^{38}$.

[^71]Figure 3.13 - Aggregate consumption and utilitarian welfare



The results of this Section can additionally be considered in relation to what stated by Reichling (2006), when analysing unemployment benefit generosity. The author states that when hours are variable, the optimal level of replacement ratio is higher than when the intensive margin is fixed. As he says, an increase in the unemployment benefit allows the utility from leisure to increase, when hours are variable; when hours are fixed, the costs coming from unemployment benefits are the same, while the benefits diminish.

### 3.4.3 When $\gamma \neq \eta$

In calibrating the baseline economy, I made the assumption that the bargaining power of the workers $(\gamma)$ is equal to the value of the elasticity of the matching function with respect to unemployment $(\eta)$.

The importance of the value assigned to the bargaining power of the workers has been widely analysed in the literature about labor market fluctuations. The choice of calibration of the two parameters has important consequences also in terms of efficiency: in a standard representative agent model, it is known that the decentralised equilibrium is efficient if the Hosios condition is respected, i.e. if $\gamma=\eta$ (and there are no unemployment benefits).

As Krusell et al. (2010) state, the respect of the Hosios condition does not guarantee, however, that the equilibrium is constrained efficient, because of the externality coming from
ary savings, to smooth consumption. Building on his results, Alonso-Ortiz and Rogerson (2010) show that a tax and transfer system with positive tax rates can be welfare improving. Both papers analyse a framework with Walrasian labor markets and idiosyncratic productivity shocks. Marcet et al. (2007) consider a model where agents, if given the opportunity to work, decide how many hours to work. In their case, agents work fewer hours than in a complete market economy, because they are overall richer (and labor supply depends negatively on wealth).
capital accumulation, as it has been stated by Dávila et al. (2012).
If the bargaining power of the worker is lower (higher) than the elasticity of the matching function, the equilibrium unemployment rate is lower (higher) than in the benchmark case of equality. The value of optimal tax progressivity goes in the direction of "correcting" the distortion: therefore, as it can be seen in Table 3.6, when the parameter $\gamma$ is lower (higher) than $\eta$, the optimal value of $\lambda$ is lower (higher).

Table 3.6 - Optimal tax policy values when $\gamma \neq \eta$

|  | $\gamma=0.4<\eta$ | Baseline $\gamma=0.5=\eta$ | $\gamma=0.7>\eta$ |
| :---: | :---: | :---: | :---: |
| Unempl. rate $(\lambda=0)$ | $7.29 \%$ | $8.01 \%$ | $9.68 \%$ |
| Optimal $\lambda^{*}$ | 0.19 | 0.2 | 0.24 |
| Unempl. rate $\left(\lambda=\lambda^{*}\right)$ | $5.05 \%$ | $5.70 \%$ | $6.95 \%$ |

### 3.4.4 The level of unemployment risk

In this section, I perform an experiment to check if the conclusion about the desirability of progressive tax and transfer schedule, without additional transfers in the form of UB, is robust to alternative values for the unemployment risk. I therefore perform the same type of simulations of Section 3.3.3: I consider two alternative economies, showing a lower or a higher level of search frictions. For the case with low unemployment risk, the vacancy posting costs are lower and the matching efficiency higher than in the baseline case, and viceversa for the case with high unemployment risk. The resulting unemployment rate in the starting setting (i.e. with $\lambda=0.16$ and no unemployment benefits) takes a value of $4.49 \%$ in the "low unemployment risk" case, and it takes a value of $18.8 \%$ in the "high unemployment risk" case. Then I simulate the economies, by letting the policy parameters (the level of progressivity and the generosity of the unemployment compensation) to vary.

Table 3.7 sums up the differences in the calibration and their impacts in terms of steady state results.

Table 3.7 - Calibrated parameters and s.s. unemployment rate

| Symbol | Interpretation | Low unempl. risk | Baseline | High unempl. risk |
| :---: | :---: | :---: | :---: | :---: |
| $\chi$ | matching efficiency | 0.6 | 0.55 | 0.4 |
| $\omega$ | vacancy cost $(\omega v / Y)$ | $0.01(0.03 \%)$ | $0.02(0.05 \%)$ | $0.45(0.6 \%)$ |
| $u$ | unemployment rate | $4.49 \%$ | $6.34 \%$ | $18.80 \%$ |
| $f$ | job finding rate | 0.98 | 0.7 | 0.2 |

The optimal level of progressivity is higher (the CRIP is lower) in the high unemployment risk case, because the labor market "distortions" prior to Government intervention are
greater, so that there is more room for improvement through progressive taxation. The same reasoning applies to the case with low unemployment risk: the optimal level of $\lambda$ in this case is lower, since initial distortions are lower.

Table 3.8 - Optimal tax policy values for different levels of unemployment risk

| Optimal $\lambda$ | Low unempl. risk | Baseline | High unempl. risk |
| :---: | :---: | :---: | :---: |
| $\lambda^{*}$ | 0.17 | 0.2 | 0.29 |

### 3.5 Conclusion

In this paper, I study alternative social insurance systems, in terms of their implications for macroeconomic aggregates and welfare.

I consider a framework in which agents are heterogeneous in terms of assets, as a result of their specific sequence of employment/unemployment spells: financial markets are incomplete, so that agents cannot perfectly insure against unemployment risk, and the functioning of the labor market is characterised by searching frictions.

This framework allows me to make evident different effects of a progressive tax and transfer schedule: on the one hand, progressivity has a beneficial effect on the unemployment rate and job-finding probability (as has been stressed in the labor market literature, once wage and hours are bargained over by the worker and the firm). On the other, progressivity reduces individual labor supply and crowds out savings (as has been stressed in the literature on consumption/savings decisions and Walrasian labor markets). The combination of the frictional labor market literature and the incomplete market settings allows me to analyse the different effects of progressivity on capital and labor.

The first set of result concerns the desirability of a positive level of progressivity, measured by the Coefficient of Residual Income Progression, once the tax and transfer schedule takes a specific functional form, as in Sørensen (1999), Heathcote et al. (2014) and Bakış et al. (2015). Here, a utilitarian welfare criterion calls for a positive level of progressivity. I then analyse an alternative welfare system, in which a flat income tax is used to finance unemployment benefits to the unemployed agents.

The second set of results in the paper shows that the losses in terms of efficiency, caused by the unemployment insurance, are much more important than those implied by the previously considered progressive tax schedule.

I also illustrate some robustness results: I show that allowing for variable hours has a fundamental impact in terms of welfare evaluation; I illustrate that the higher is the labor supply elasticity, the lower is the optimal level of progressivity. Finally, I analyse the effects of the level of search frictions on the optimal level of progressivity, as well as the consequences
of calibrating the bargaining power of workers to a different value than the elasticity of matching function: in both cases, the higher the level of distortions which are present prior to Government intervention, the higher the optimal level of tax progressivity.

The paper highlights interesting mechanisms, but lacks the ability to reproduce the moments which characterise the wealth distribution of an actual economy; in my settings, agents are too homogeneous: the only sources of heterogeneity are their employment status and assets level.

One interesting direction future research could take involves enriching the model to allow for different levels of labor productivity. As an example, Lifschitz et al. (2016) extend the model developed by Krusell et al. (2010) to allow for four types of skills/productivity levels. This feature implies that the optimal level of unemployment benefit is higher than if agents are homogeneous, because additional heterogeneity (and therefore possibility of redistribution) is introduced.

A model set-up which allows for additional heterogeneity could help to match some characteristics of the income distribution, in order to be able to evaluate a quantitatively credible optimal level of progressivity. Heterogeneity in educational/skills level, moreover, allows to study the labor supply elasticities of different groups of agents.

## Bibliography

S. Rao Aiyagari. Uninsured Idiosyncratic Risk and Aggregate Saving. The Quarterly Journal of Economics, 109(3):659-84, August 1994.

Jorge Alonso-Ortiz and Richard Rogerson. Taxes, transfers and employment in an incomplete markets model. Journal of Monetary Economics, 57(8):949-958, November 2010.

Ozan Bakış, Barış Kaymak, and Markus Poschke. Transitional dynamics and the optimal progressivity of income redistribution. Review of Economic Dynamics, 18(3):679 - 693, 2015.

Roland Bénabou. Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency? Econometrica, 70(2):481-517, March 2002.

Raj Chetty. Bounds on Elasticities With Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply. Econometrica, 80(3):969-1018, 052012.

Arnaud Chéron. Allocation universelle vs. indemnité chômage: Évaluation quantitative dans un modèle d'appariement. Revue économique, 53(5):951-964, 2002.

Julio Dávila, Jay H. Hong, Per Krusell, and José-Víctor Ríos-Rull. Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks. Econometrica, 80 (6):2431-2467, 2012.

Jonathan Heathcote, Kjetil Storesletten, and Giovanni L. Violante. Optimal tax progressivity: An analytical framework. Working Paper 19899, National Bureau of Economic Research, February 2014.

Mathias Hüngerbuhler, Etienne Lehmann, Alexis Parmentier, and Bruno Van Der Linden. Optimal Redistributive Taxation in a Search Equilibrium Model. Review of Economic Studies, 73(3):743-767, 2006.
B. Holmlund and A.S. Kolm. Progressive Taxation, Wage Setting, and Unemployment, Theory and Swedish Evidence. Technical report, 1995.

Ulf Jakobsson. On the measurement of the degree of progression. Journal of Public Economics, 5(1-2):161-168, 1976.

Per Krusell, Toshihiko Mukoyama, and Ayşegül Şahin. Labour-market matching with precautionary savings and aggregate fluctuations. 77(4):1477-1507, 2010.

Avihai Lifschitz, Ofer Setty, and Yaniv Yedid-Levi. Who cares about unemployment insurance? Technical report, mimeo, 2016.

Ben Lockwood and Alan Manning. Wage setting and the tax system theory and evidence for the United Kingdom. Journal of Public Economics, 52(1):1-29, August 1993.

Albert Marcet, Francesc Obiols-Homs, and Philippe Weil. Incomplete markets, labor supply and capital accumulation. Journal of Monetary Economics, 54(8):2621-2635, November 2007.

Toshihiko Mukoyama. Understanding the welfare effects of unemployment insurance policy in general equilibrium. Journal of Macroeconomics, 38, Part B(0):347-368, 2013. ISSN 0164-0704.
R. A. Musgrave and Tun Thin. Income tax progression, 1929-48. Journal of Political Economy, 56(6):498-514, 1948.

Alexis Parmentier. The effects of the marginal tax rate in a matching model with endogenous labor supply. Discussion papers (econ - département des sciences economiques), Université catholique de Louvain, Département des Sciences Economiques, April 2006.

Josep Pijoan-Mas. Precautionary Savings or Working Longer Hours? Review of Economic Dynamics, 9(2):326-352, April 2006.

Christopher A. Pissarides. Equilibrium Unemployment Theory, 2nd Edition, volume 1 of MIT Press Books. The MIT Press, June 2000.

Andreas Pollak. Optimal unemployment insurance with heterogeneous agents. European Economic Review, 51(8):2029-2053, November 2007.

Knut Røed and Steinar Strøm. Progressive taxes and the labour market: Is the trade-off between equality and efficiency inevitable? Journal of Economic Surveys, 16(1):77-110, 2002.

Felix Reichling. Optimal Unemployment Insurance in Labor Market Equilibrium when Workers can Self-Insure. Mpra paper, University Library of Munich, Germany, November 2006.
P.B. Sørensen. Optimal tax progressivity in imperfect labour markets. Labour Economics, 6(3):435-452, 1999.

## 3.A Some analytical insights: risk-neutral agents

In order to provide some analytical insight about the different effects implied by the introduction of progressive taxation, and the unemployment benefit, I simplify the model and assume that agents are risk-neutral ${ }^{39}$. I derive the expression of the "wage equation", for the two cases, in order to provide some intuition about the effects, in a partial equilibrium setting, of the various parameters.

## The effects of progressivity

In this section I focus on the effects of progressivity on the wage bargaining, while keeping linear utility in consumption. For simplicity, I assume that only wage income (and not interest rate income) is taxed, to remain comparable to the previous literature, in particular to Parmentier (2006).

The problem of the agent is:

$$
\begin{gather*}
\max \sum \beta^{t}\left(c_{t}+\Gamma^{z}\right)  \tag{A.1}\\
\text { s.t. } c+a^{\prime}=a(1+r)+z
\end{gather*}
$$

where

$$
\Gamma^{z}= \begin{cases}\Gamma^{e}=\sigma_{l} \frac{(1-h)^{\nu}}{(1-\nu)}, & \text { if employed } \\ \Gamma^{u}, & \text { if unemployed }\end{cases}
$$

The disposable income definition is given by:

$$
z= \begin{cases}(1-\tau)(w h)^{1-\lambda}, & \text { if employed } \\ (1-\tau) \mu^{1-\lambda}, & \text { if unemployed }\end{cases}
$$

Because of linear utility, I can write that:

1. $\beta=\frac{1}{1+r}$;
2. $a^{\prime}=a$
and therefore without loss of generality: $c=(1-\tau)(w h)^{1-\lambda}$ for the employed and $c=(1-\tau) \mu^{1-\lambda}$ for the unemployed.

The expressions for the value functions are therefore:

[^72]\[

$$
\begin{align*}
W & =(1-\tau)(w h)^{1-\lambda}+\frac{1}{1+r}[(1-s) W+s U]  \tag{A.2}\\
U & =(1-\tau) \mu^{1-\lambda}+\frac{1}{1+r}[f(\theta) W+(1-f(\theta)) U]  \tag{A.3}\\
J & =(y-w) h+\frac{1}{1+r}[(1-s) J+s V]  \tag{A.4}\\
V & =-\omega+\frac{1}{1+r}[q(\theta) J+(1-q(\theta)) V] \tag{A.5}
\end{align*}
$$
\]

where the term $y$ refers to the (fixed) productivity of one hour of labor. In equilibrium $V=0$, so that I can write:

$$
\begin{align*}
& \frac{J}{1+r}=\left(\frac{(y-w) h}{(r+s)}\right)  \tag{A.6}\\
& \frac{J}{1+r}=\frac{\omega}{q(\theta)} \tag{A.7}
\end{align*}
$$

Solving for the value functions I obtain:

$$
W-U=\left[(1-\tau)\left[(w h)^{1-\lambda}-\mu^{1-\lambda}\right]+\Gamma^{u}-+\Gamma^{e}\right] \frac{(1+r)}{(r+s+f)}
$$

The sharing rule in the actual framework is given by ${ }^{40}$ :

$$
\begin{equation*}
\frac{W-U}{\gamma} \frac{(w h)^{\lambda}}{(1-\tau)(1-\lambda)}=\frac{J}{1-\gamma} \tag{A.8}
\end{equation*}
$$

By substituting the expressions for the value functions, the sharing rule becomes:

$$
\frac{(1-\gamma)}{(r+f+s)}\left[(1-\tau)\left[(w h)^{1-\lambda}-\mu^{1-\lambda}\right]+\Gamma^{u}-+\Gamma^{e}\right] \frac{(w h)^{\lambda}}{(1-\tau)(1-\lambda)}=\frac{\gamma}{(r+s)}(y-w) h
$$

The expression for the wage bill is then given by:

$$
w h=\frac{(1-\lambda)}{(1-\gamma \lambda)} \gamma(y h+\omega \theta)+\frac{(1-\gamma)}{(1-\tau)} \frac{(w h)^{\lambda}}{(1-\lambda)}\left[(1-\tau) \mu^{1-\lambda}+\Gamma^{u}-\Gamma^{e}\right]
$$

The wage equation takes the usual form: when $\lambda=0$, the expression comes back to the standard form, as it can be found in Pissarides (2000).

[^73]The hours equation is given by ${ }^{41}$ :

$$
\begin{equation*}
\sigma_{l}(1-h)^{-\nu}=y(1-\tau)(1-\lambda)(w h)^{-\lambda} \tag{A.9}
\end{equation*}
$$

The factor $\frac{(w h)^{\lambda}}{(1-\lambda)}$ in the wage equation can be substituted, by using the expression of the hours equation, to obtain:

$$
\begin{equation*}
w h=\frac{(1-\lambda)}{(1-\gamma \lambda)} \gamma(y h+\omega \theta)+\frac{(1-\gamma)}{(1-\tau)} \frac{y(1-\tau)(1-h)^{\nu}}{\sigma_{l}}\left[(1-\tau) \mu^{1-\lambda}+\Gamma^{u}-\Gamma^{e}\right] \tag{A.10}
\end{equation*}
$$

Regarding the first multiplicative factor on the right side of equation (A.10):

$$
\begin{equation*}
\frac{\partial \frac{(1-\lambda)}{(1-\gamma \lambda)}}{\partial \lambda}=-\frac{(1-\gamma)}{(1-\gamma \lambda)^{2}}<0 \text { since } 0<\gamma \leq 1 \tag{A.11}
\end{equation*}
$$

The effect of the level of progressivity $\lambda$ is then to decrease the part of the surplus coming from the firm and appropriated by the worker.

By looking at the second term on the rhs of eq. (A.10), an opposite force intervenes; by looking at the multiplicative factor, it can be seen that:

$$
\begin{equation*}
\frac{y(1-\tau)(1-h)^{\nu}}{\sigma_{l}}>1 \leftrightarrow h<1-\left(\frac{\sigma_{l}}{y(1-\tau)}\right)^{\frac{1}{\nu}} \tag{A.12}
\end{equation*}
$$

moreover,

$$
\begin{equation*}
\frac{\partial \mu^{1-\lambda}}{\partial \lambda}=-\ln (\mu) \mu^{1-\lambda}>0 \leftrightarrow \mu<1 \tag{A.13}
\end{equation*}
$$

i.e. under certain parameters restrictions $\left(h<1-\left(\frac{\sigma_{l}}{y(1-\tau)}\right)^{\frac{1}{\nu}}\right.$ and $\left.\mu<1\right)$, a higher progressivity (higher $\lambda$ ) puts a upward pressure on wage. Regarding the hours worked, considering the wage rate $w$ as fixed, it can be seen that the factor $\lambda$ has a negative effect on labor supply:

$$
\frac{d h}{d \lambda}=-\frac{y(1-\tau)\left[w^{-\lambda}+(1-\lambda) \lambda w^{-\lambda-1}\right]}{\lambda h^{\lambda-1}(1-h)^{-\nu}+h^{\lambda} \nu(1-h)^{-\nu-1}}<0
$$

In conclusion, there is at least one force pushing down the wage as $\lambda$ increases, for every level of hours ${ }^{42}$.

[^74]
## The effects of UB

In this section I focus on the effects of UB in the wage bargaining. For simplicity I assume that taxes are raised through a flat income tax.

The problem of the agent is therefore as before:

$$
\begin{gathered}
\max \sum \beta^{t}\left(c_{t}+\Gamma^{z}\right) \\
\text { s.t. } \quad c+a^{\prime}=a(1+r)+z
\end{gathered}
$$

where

$$
\Gamma^{z}= \begin{cases}\Gamma^{e}=\sigma_{l} \frac{(1-h)^{\nu}}{(1-\nu)}, & \text { if employed } \\ \Gamma^{u}, & \text { if unemployed }\end{cases}
$$

and

$$
z= \begin{cases}w h(1-\tau), & \text { if employed } \\ \mu(1-\tau), & \text { if unemployed }\end{cases}
$$

Because of linear utility, I can write that:

1. $\beta=\frac{1}{1+r}$;
2. $a^{\prime}=a$
and therefore, without loss of generality: $c=w h(1-\tau)$ for the employed and $c=\mu(1-\tau)$ for the unemployed.

The expressions for the value functions are exactly as in previous section.
The sharing rule takes the simple form:

$$
\begin{equation*}
\frac{W-U}{\gamma(1-\tau)}=\frac{J}{1-\gamma} \tag{A.15}
\end{equation*}
$$

which by substituting for the expression of the value functions becomes:

$$
\frac{(1-\gamma)}{(r+f+s)(1-\tau)}\left[(w h-\mu)(1-\tau)+\Gamma^{u}-\Gamma^{e}\right]=\frac{\gamma}{(r+s)}(y-w) h
$$

By substituting the expression for the value function of the firm, I obtain again the well known expression for the wage bill:

$$
\begin{equation*}
w h=\gamma(y h+\omega \theta)+\frac{(1-\gamma)}{(1-\tau)}\left[\mu(1-\tau)+\Gamma^{u}-\Gamma^{e}\right] \tag{A.16}
\end{equation*}
$$

I report the hours equation, obtained by computing the FOC with respect to hours and by using the wage equation (see eq. 3.43 in the main text):

$$
\begin{equation*}
\sigma_{l}(1-h)^{-\nu}=y(1-\tau) \tag{A.17}
\end{equation*}
$$

In this simplified version of the model, the effect of an increase of the unemployment benefit is an upward pressure on wages. The differences with respect to the complete model come from: (i) the fact that labor productivity is fixed and equal to $y$, while in the model with capital accumulation is endogenous and it depends on the level of aggregate capital; (ii) in the complete model, marginal utility affects the evaluation of the value of unemployment.

## 3.B Frisch elasticity with progressive taxation

I compute the Frisch elasticity of labor supply starting from the FOCs of the Lagrangian for the consumer as in a Walrasian market:

$$
\begin{equation*}
L=\sum \beta^{t} u\left(c_{t}, 1-h_{t}\right)+\zeta_{t}\left[\left(1+r_{t}\right) a_{t}+(1-\tau)\left(w_{t} h_{t}+r_{t} a_{t}\right)^{(1-\lambda)}\right] \tag{B.18}
\end{equation*}
$$

FOC wrt consumption:

$$
\begin{equation*}
\zeta_{t}=\frac{\partial u(c, 1-h)}{\partial c_{t}} \tag{B.19}
\end{equation*}
$$

FOC wrt hours:

$$
\begin{equation*}
-\frac{\partial u(c, 1-h)}{\partial h_{t}}=\zeta_{t}(1-\lambda)(1-\tau) w_{t}\left(w_{t} h_{t}+r_{t} a_{t}\right)^{(-\lambda)} \tag{B.20}
\end{equation*}
$$

I derive the two conditions wrt to the wage $w_{t}$ to obtain the following (I discard the time subscript):

$$
\left\{\begin{array}{l}
u_{c c} \frac{\partial c}{\partial w}+u_{c h} \frac{\partial h}{\partial w}=0  \tag{B.21}\\
-u_{c h} \frac{\partial c}{\partial w}-u_{h h} \frac{\partial h}{\partial w}=\zeta(1-\lambda)(1-\tau)\left[(w h+r a)^{-\lambda}\left(1-\lambda w h(w h+r a)^{-1}\right)\right]
\end{array}\right.
$$

I substitute the expression for $\frac{\partial c}{\partial w}$ in the second equation to obtain:

$$
\begin{equation*}
\frac{\partial h}{\partial w}=\frac{u_{h}}{\left(u_{h h}-\frac{u_{c h}^{2}}{u_{c c}}\right) \frac{1}{w}\left(1-\lambda \frac{w h}{(w h+r a)}\right)} \tag{B.22}
\end{equation*}
$$

The elasticity of hours with respect to (pre-tax) wage is thus given by:

$$
\begin{equation*}
\epsilon_{h, w}=\frac{\partial h}{\partial w} \frac{w}{h}=\frac{u_{h}}{h *\left(u_{h h}-\frac{u_{c h}^{2}}{u_{c c}}\right)}\left(1-\lambda \frac{w h}{(w h+r a)}\right) \tag{B.23}
\end{equation*}
$$

If $\lambda=0$ (flat tax schedule), the expression comes back to the standard expression for Frisch elasticity; with the chosen function for instantaneous utility $u(c, 1-h)=\ln (c)+$ $\sigma_{l} \frac{(1-h)^{(1-\nu)}}{(1-\nu)}$, the expression becomes:

$$
\begin{equation*}
\epsilon_{h, w}=\left(\frac{1-h}{h}\right) \frac{1}{\nu} \tag{B.24}
\end{equation*}
$$

## 3.C Welfare with the Rawls criterion

The welfare function I considered in the main text assigns equal weight to each agent value function. The resulting conclusion is that a progressive tax schedule has to be privileged, without additional transfers in the form of UB.

Focusing on the welfare of the least well-off agent ${ }^{43}$ usually brings different conclusions than those obtained by looking at average welfare. Considering the fact that the negative taxes are non-monotonic in income, it can be expected that the additional transfer to the assets-poor agents, in terms of UB, can now be desirable, from the point of view of a Rawlsian social welfare function.

The left panel of Figure 3.a shows that high levels of both tax progressivity and transfers are desirable from the point of view of the unluckiest agent. The right panel of the same Figure 3.a shows the shape of the net tax function of equation (3.7) for the unemployed agents in two economies: the first one is characterised by the policy parameters which are defined as optimal, according to the utilitarian criterion, i.e. $\lambda=0.2, \mu \approx 0$; the second one instead is characterised by higher levels of progressivity and a higher level of (unconditional) transfer, i.e. $\lambda=0.25, \mu=0.2$. The overall effect is a "shift" of the tax schedule: the poorest agent in terms of assets pays a negative tax (i.e. he receives a transfer), while in the economy with $\mu \approx 0$ the poorest agents has almost zero transfer. The interval of levels of income for which the negative taxes decrease (as the agent gets poorer) is also more limited.

## Figure 3.a - Welfare with Rawls criterion and the effect of UB



[^75]
## 3.D Algorithm

In this section, I briefly describe the algorithm used to solve the model with labor supply. Note that solving the model amounts to solving a fixed functional problem, because we are not looking for single values of wage and hours, but for empirical functional forms $w=w(a), h=h(a)$.

1. Use a grid of $\mathrm{N}=1000$ points, for asset levels $a \in[0,400]$; this is a log linearly spaced in order to have more points at the bottom of the distribution.
2. Guess a level of the interest rate, labor income tax rate, labor market tightness, and wage and hours functions $w=w(a), h=h(a)$.
3. Inner loops (savings and wage function) Given the previous values, solve for the consumer problem to find the savings decision rules $a_{e}^{\prime}=g_{e}(a, w)$ and $a_{u}^{\prime}=g_{u}(a)$ and the value functions when employed and unemployed; compute the value of the firm implied by the Nash bargaining on wage in eq. (3.38). Compute the the wage $w=w(a)$ implied by the value function for the firm in eq. (3.24). Compare the implied wage function to the initial guess; if the distance is bigger than the tolerance value, update the initial guess to a new value $w_{i+1}=\xi_{w} w_{i}+\left(1-\xi_{w}\right) w_{\text {implied }}$, where the subscript $(i+1)$ refers to the $(i+1)^{t h}$ iteration.
4. First intermediate level loop (hours function) Once the saving and wage functions have been found, look for the hours function implied by the FOC on hours in eq.(3.43): this is the very same equation that in a Walrasian labor market. Iterate until convergence, updating the guess for the labor market tightness in a similar way as that of the wage function.
5. Second intermediate level loop (tightness) Once the wage and hours equation are obtained (given the guessed tightness, tax rate and interest rate), check the implication for the value of filling a job (and therefore for the labor market tightness, since $\theta=$ $q^{-1}(\theta)$ ), using the value function of a vacancy in eq. (3.28); if the distance between the guessed and the implied tightness is bigger than the tolerance level, update the guess for the labor market tightness in a similar way as for the wage function: $\theta_{i+1}=$ $\xi_{\theta} \theta_{i}+\left(1-\xi_{\theta}\right) \theta_{\text {implied }}$.
6. Third intermediate level loop (tax rate) Using the condition that government budget constraint must be in equilibrium, i.e. eq. (3.47), look for the equilibrium tax rate.
7. Outer loop (interest rate) Once the model is solved, check the interest rate implied by the aggregation of all savings: for agent $i$, the total asset is given by $\int a_{i} d i=$ $\int k_{i} d i+p \int x_{i} d i$, where $\int k_{i} d i=\bar{K}$ and $\int x_{i} d i=1$. Since aggregate savings can be computed as $\int a_{i} d i$, compare the implied interest rate (which is a function of the implied $\bar{K}$, as it is shown in eq. (3.26) to the initial guess, and update it.

The algorithm to solve the model without labor supply is the same as the previous one with one difference: I keep hours fixed (in particular I set $h=\bar{h}=1$ and therefore I switch off the loop which uses the FOC for hours. I start with guesses on interest rate, tax rate, tightness, and the wage function; after having solved for the policy and wage function, I proceed with the second and third intermediate loops before continuing to the outer loop.

# Marché du travail frictionnel et interventions publiques: dynamique et évaluation de bien-être 

Mots clés

Frictions d'appariement, heures travaillées, chômage, taxation, politiques publiques, fluctuations de court et moyen terme, marchés financiers incomplets.

## Résumé

L'objectif sous-jacent aux trois chapitres qui composent cette thèse est la compréhension du fonctionnement du marché du travail, afin d'établir un diagnostic quant au rôle de régulation potentiel d'une autorité publique dans ce marché.

Si le point commun aux trois chapitres est donc la question de savoir comment les acteurs institutionnels influencent le fonctionnement du marché du travail, et comment, éventuellement, on peut imaginer qu'ils devraient plutôt intervenir, les trois chapitres se concentrent néanmoins sur des questions différentes.

Dans le premier chapitre, j'analyse, d'un point de vue purement "positif", la capacité du modèle avec frictions d'appariement à répliquer les fluctuations de court terme de variables du marché du travail, avec une attention particulière sur les États-Unis. Le cadre d'analyse est celui d'un modèle de fluctuations de court terme, avec rigidité de prix. Dans le premier chapitre, la règle de conduite de la politique monétaire détermine le mécanisme de transmission d'un choc technologique, dans le court terme.

Le point de départ de mon analyse repose sur la critique que Shimer (2005) adresse au modèle d'appariement: il soutient que ce modèle ne permet pas re reproduire les volatilités relatives des variables du marché du travail, ni leurs corrélations avec la productivité. Dans un premier temps je montre que l'amplitude du problème souligné par Shimer (2005) diminue déjà fortement en adoptant la stratégie de calibration des modèles de cycle réel. Une première partie du chapitre est donc consacrée à expliquer les différences entre la stratégie de calibration que j'adopte avec celle de Shimer (2005). Plutôt que de fixer la valeur du nonemploi, je calibre le ratio des coûts d'ouverture des postes vacants en termes de production totale, comme c'est le cas dans la plupart des modèles de cycle réel. Dans la deuxième partie de ce chapitre, j'analyse en détail le mécanisme de transmission d'un choc technologique, à l'aide des fonctions de réponse et des diagrammes de phase: je montre que, dans le contexte d'un modèle où l'autorité monétaire réagit aux mouvements de la production et de l'inflation, sur la base d'une règle de Taylor, la présence de frictions nominales implique un effet non-monotone sur d'ajustement des variables.

Le modèle Nouveau Keynésien présente un ajustement non-monotone des postes vacants et de l'emploi : à l'impact du choc technologique, le nombre des postes vacants baisse, pour remonter ensuite, au fur et à mesure que les prix s'ajustent. Ce modèle introduit ainsi une source additionnelle de volatilité, par rapport au modèle de cycle réel. En outre, la présence de ces mouvements additionnels, c'est-à-dire que suite à un choc technologique, la tension du marché du travail initialement baisse et pour augmenter ensuite, permet de mieux expliquer les corrélations de variables du marché du travail avec la productivité. Toutefois, dans le chapitre, je rappelle aussi les prédictions potentiellement problématiques du modèle avec frictions nominales, notamment ses implications en termes d'autocorrélations des postes vacants et de tension du marché du travail.

Dans le deuxième chapitre (co-écrit avec F. Langot), nous étudions les déterminants des évolutions du taux d'emploi sur les cinquante dernières années, ainsi que les heures travaillées par employé. Nous soutenons que l'évolution du coin fiscal, et l'évolution de deux variables reflétant l'évolution du cadre institutionnel (la générosité du taux de remplacement du revenu en cas de "non-emploi" et le pouvoir de négociation des travailleurs), permettent d'expliquer les différentes trajectoires du taux d'emploi et des heures travaillées observées aux États-Unis et dans trois économies européennes qui sont la France, l'Allemagne et le Royaume-Uni.

Nous partons du travail de Langot et Quintero-Rojas (2008), qui développent un modèle comprenant les deux marges (extensive et intensive) de l'offre de travail, dans le contexte d'un marché du travail frictionnel en équilibre général, avec aussi la dynamique du capital physique. Contrairement aux auteurs, nous utilisons en fait la méthodologie proposée par McDaniel (2011), qui développe un modèle de croissance équilibrée pour étudier l'évolution des heures travaillées, en utilisant comme variables explicatives la taxation ainsi que la productivité (exogène).

Nous nous servons de cet outil pour évaluer les conséquences en termes de croissance, ainsi que de "bien-être" d'un agent représentatif, des choix de politique économique de la France en termes de politique fiscale et institutions du marché du travail. Les conclusions que nous dérivons de notre exercice sont alors les suivantes: les coûts en termes de PIB ainsi que de bien-être de l'agent représentatif impliqués par le coin fiscal sont plus élevés que ceux induits par les institutions du marché du travail proprement dites; ils existent des complémentarités, car les effets induits de réformes simultanées sont plus importants que les bénéfices liés à la somme de réformes opérées de façon indépendante.

Dans le troisième chapitre, j'analyse la performance et la désirabilité de deux systèmes alternatifs de sécurité sociale. Je considère un cadre dans lequel l'assurance "privée" n'est pas parfaite: les agents sont soumis à un risque de chômage, contre lequel ils ne peuvent pas s'assurer complètement. Un des rôles du gouvernement peut donc consister à intervenir en tant que fournisseur d'assurance.

J'étudie dans ce chapitre deux systèmes: d'un côté un système fiscal redistributif, basé sur
une taxe progressive, qui finance des transferts pour les agents à bas revenu; de l'autre, des transferts pour les individus qui tombent au chômage, financés par une taxe proportionnelle. En effet, je considère un système où les deux mécanismes (taxe progressive et assurance chômage) peuvent aussi être présents au même moment. Le modèle utilisé dans ce chapitre est une extension de celui développé par Krusell et al. (2010): le cadre comprend une économie caractérisée à la fois par un marché financier incomplet et un marché du travail frictionnel. Les agents sont soumis au risque de tomber chômeurs, et ils ne peuvent pas s'assurer contre ce risque. J'introduis dans ce cadre la possibilité pour les agents de moduler leur offre de travail, s'ils sont employés, et la présence d'un système de taxes et transferts progressif.

Dans le cadre d'un marché du travail frictionnel comme celui analysé, une taxe progressive peut avoir des effets positifs sur le fonctionnement du marché, en particulier elle peut avoir un impact bénéfique sur le taux de chômage. Le mécanisme à l'œuvre a été étudié par la littérature sur les marchés frictionnels, dans le cadre d'un agent représentatif. Dans ce chapitre, le point de depart de mon analyse repose sur le mécanisme illustré par la littérature du marché du travail, pour ce qui concerne l'effet d'une taxe progressive, et j'introduis deux dimensions nouvelles. En premier lieu, je considère la dimension de l'hétérogénéité par rapport au niveau de richesse accumulée (liée au defaut d'assurance) ; en deuxième lieu, je considère la totalité des effets d'équilibre général, qui résultent du mécanisme de l'épargne de précaution.

Ce modèle stylisé permet de conclure que le système fiscal progressif est supérieur, en termes de bien-être agrégé, à l'assurance fournie à travers des allocations chômage. En termes de performance du marché du travail, une taxe progressive contribue en fait à réduire le taux de chômage, alors que l'assurance chômage a l'effet contraire.

# Frictional labor markets and policy interventions: dynamics and welfare implications 

## Keywords

Search and matching frictions, hours worked, unemployment, taxation, public policies, short term fluctuations and medium run evolutions, incomplete financial markets.


#### Abstract

The objective underlying the three chapters of this thesis is the understanding of the functioning of the labor market, with the aim to establish a diagnosis of the potential regulatory role of a public authority in this market. The common feature to the three chapters of the thesis is the question of how institutional actors influence the functioning of the labor market, and how possibly we can imagine they should rather intervene. Still, the three chapters still focus on different issues.

In the first chapter, I analyze, from a purely " positive " perspective, the ability of the model with matching frictions to track short-term fluctuations of labor market variables, with a focus on the United States. The framework is that of a model of short-term fluctuations with sticky prices. In the first chapter, the monetary policy rule determines the transmission mechanism of a technology shock in the short term.

The starting point is the criticism that Shimer (2005) addresses to the matching model: the author considers that this model can neither explain the relative volatilities of labor market variables, nor their correlation with productivity. In this chapter, I show that the problem highlighted by Shimer (2005) already loses its importance if the adopted calibration strategy follows the business cycle models tradition. The first part of the chapter aims at explaining the differences between the calibration strategy that I adopted and the one of Shimer (2005): rather than fixing the value of non-employment, I follow the tradition of real business cycle models, which calibrate the ratio of the opening costs of vacancies in terms of total production. In the second part of the first chapter, I analyze the transmission mechanism of a technology shock, using impulse response functions and phase diagrams. I show that when the monetary authority fixes the nominal interest rate using a Taylor rule, that reacts to movements in both output and inflation, the presence of nominal frictions involves an over-shooting of variables, during the dynamics of adjustment.

In this version of the New Keynesian model, that presents a non-monotonic adjustment of vacancies and employment (on impact vacancies fall, and then, as soon as prices are adjusted, they increase), introduces an additional source of volatility relative to the actual cycle model. However, in the chapter, I also recall the problematic predictions of the model with nominal


frictions, in particular its implications in terms of the autocorrelations of vacancies and labor market tightness.

In the second chapter (co-written with F. Langot), we study the determinants of variations in the employment rate and individual worked hours over the past fifty years, in the US, France, Germany and the UK. We argue that the evolution of the tax wedge, as well as the evolution of two variables reflecting the evolution of the institutional framework (the generosity of income replacement rate in case of " non-employment " and the bargaining power of workers), can explain the different trajectories of the employment rate and hours worked observed in the United States and the three European economies we consider.

We start from the paper by Langot and Quintero-Rojas (2008), who developed a model that includes both the extensive and intensive margins of labor supply, in a general equilibrium model with labor market frictions and physical capital. Contrary to the authors, we use the methodology proposed by McDaniel (2011): we consider a balanced growth model to study the dynamics of the variables of interest. We suppose that the policy and technology variables are the exogenous drivers of the movements of the endogenous variables, in a perfect foresight framework.

We use this tool to assess the implications of the economic policy choices of France. We evaluate them in terms of production growth, as well as of the " well-being " of a representative agent. The results of this chapter show that, in the case of France, the costs in terms of GDP loss and aggregate welfare, induced by the tax wedge, are more important than those due to labor market institutions. In addition, we also show that there exist " complementarities" between the structural reforms that have an impact on taxation and the labor market institutions, as they both affect the two margins (intensive and extensive) that compose the aggregate labor input.

In the third chapter, I analyze the performance and desirability of two alternative systems of social security. I consider a framework in which the individual " insurance " is not perfect: agents are subject to unemployment risk, against which they cannot insure completely. One of the roles of the Government may therefore be that of act as insurance provider. I study in this chapter two social security systems: on the one hand, a redistributive tax system, based on a progressive tax, which funds transfers for low-income workers; on the other, a system based on transfers to individuals who become unemployed, financed by a proportional tax. Indeed, I consider a system where both mechanisms (progressive taxes and unemployment insurance) may also be present at the same time.

The model used in this chapter is an extension of the one developed by Krusell et al. (2010): the framework includes an economy characterized by both an incomplete financial market and frictional labor market. Agents are subject to the risk of becoming unemployed, and they cannot insure against this risk. I introduce in this context the possibility for agents to adjust their labor supply, if they are employed, and a progressive system of taxes
and transfers. In a frictional labor market, a progressive tax can have positive effects on the functioning of the market, in particular it can have a beneficial impact on the unemployment rate. The mechanism at work was studied by the literature on the frictional markets, and it is based on the downward wage pressure implied by the presence of progressive taxation.

In this third chapter, I start from the mechanism illustrated by the labor market literature, for what it concerns the effect of a progressive tax on unemployment, and I introduce two new dimensions: firstly, I consider the dimension of heterogeneity in terms of accumulated wealth (due to the lack of insurance). Secondly, I consider general equilibrium effects, which pass through the (precautionary) saving behavior. This stylized model leads to conclude that a redistributive tax system, characterized by a progressive tax, which funds transfers to low-income workers, is preferable in terms of average welfare, to a system that offers a public insurance through unemployment benefits. These results stem from the fact that the progressivity of the tax helps correct some frictions which distort the labor market performance, particularly in terms of unemployment rates.


[^0]:    ${ }^{1}$ Mortensen and Pissarides (1999), Pissarides (2000).

[^1]:    ${ }^{2}$ Le coin fiscal introduit une distorsion dans la condition d'égalité entre le taux marginal de substitution entre consommation et loisir et la productivité marginal du travail.
    ${ }^{3}$ En particulier, nous considérons la générosité du taux de remplacement du revenu en cas de 'non-emploi'

[^2]:    ${ }^{4}$ Dans l'économie de référence, les heures par travailleur sont considérées comme fixes, toutefois le chapitre contient une extension où la marge intensive est active; dans les deux cas, la forme de préférences est identique.
    ${ }^{5}$ Les papiers de Abowd and Kramarz (1997) ou Silva and Toledo (2009) par exemple, quantifient les coûts de recrutement à partir des données.

[^3]:    ${ }^{6}$ Ils calibrent la valeur du non-emploi comme étant le $95 \%$ de celle de l'emploi.
    ${ }^{7}$ Voir par exemple récemment Balleer (2012) et Barnichon (2010).

[^4]:    ${ }^{8}$ On est conscient que les économies considérées peuvent être caractérisées par des valeurs différentes en tant que sociétés. Le travail d'Alesina et al. (2006) par exemple propose le concept d'externalités dans le loisir qui sont à l'origine d'un "multiplicateur social". Nous ne suivons pas cette direction et ainsi notre travail peut être considéré comme complémentaire.
    ${ }^{9}$ Dans cette expérience contrefactuelle, tous les revenus du gouvernement, utilisés pour financer les allocations chômage ainsi que les dépenses en biens collectifs, sont obtenus à travers des taxes forfaitaires.
    ${ }^{10}$ Dix-neuf pp de consommation additionnelle pour la France si le coin fiscal était abaissé au niveau de celui des États-Unis, c'est-à-dire d'environs vingt pp.

[^5]:    ${ }^{11}$ Dans le cadre que j'analyse, où les agents peuvent être employés ou non, la présence du choix du nombre d'heures travaillées a aussi une importance fondamentale dans l'évaluation en termes de bien-être d'un système fiscal progressif.

[^6]:    ${ }^{1}$ See Mortensen and Pissarides (1994) and Pissarides (2000).
    ${ }^{2}$ A first review of the literature, close in time to the article by Shimer (2005), is provided by Mortensen and Nagypal (2007).

[^7]:    ${ }^{3}$ As a non-exhaustive list, consider the works by Trigari (2006), Sveen and Weinke (2008) or Blanchard and Galí (2010).
    ${ }^{4}$ In my calibration exercise I set the ratio of recruiting costs in terms of output to $\frac{\omega V}{Y}=0.8 \%$ while following the procedure of Shimer (2005) would imply a ratio of around $4.5 \%$; the range for this ratio accepted in the RBC literature goes from $0.5 \%$ to $1 \%$.
    ${ }^{5}$ Shimer (2005) approximates the quantitative implications of a transitory technological shock via the elasticities of labor market variables at steady state. See also Mortensen and Nagypal (2007) for a discussion about the robustness of this approximation of the short run dynamics of the real matching model after a transitory shock.
    ${ }^{6}$ Among others, I recall the works of Michelacci and Lopez-Salido (2007), Barnichon (2010a), Balleer (2012).

[^8]:    ${ }^{7}$ As clearly reminded by Barnichon (2010a), sticky prices by themselves are not enough to have a contractionary effect on labor input of a positive productivity shock: it is important how accommodative monetary policy is.
    ${ }^{8}$ In this respect, my results are in line with the considerations expressed in Van Zandweghe (2010).
    ${ }^{9}$ For example, Balleer (2012) or Barnichon (2010a).
    ${ }^{10}$ The functioning of the model with nominal frictions remains anyway prone to the criticism expressed by Shimer (2008): supposing a monetary authority which reacts to inflation and output gaps already introduces, by itself, a distortion and a source of variability in the model.
    ${ }^{11}$ In particular, the baseline economy presents fixed individual hours, non-separable preferences in consumption and leisure and instantaneous hirings; in the Appendix, I report the results for a version of the model with variable individual hours, separable preferences in consumption and leisure, and the traditional timing for the flows on the labor market. My results are in line with what expressed in Cheron and Langot (2004) for the consequences of separable preferences in a RBC model, as well as with the results found in Van Zandweghe (2010) about the different consequences of the instantaneous hirings hypothesis.

[^9]:    ${ }^{12}$ Michaillat (2012) reports slightly higher values, in the range 0.5-0.65, for the period 1964-2009.
    ${ }^{13}$ Shimer (2005), p. 33 : "From 1951 to 1985, the contemporaneous correlation between detrended labor productivity and and the v-u ratio was 0.57 and the peak correlation was 0.74 . From 1986 to 2003, however, the contemporaneous and peak correlations are negative, -0.37 and -0.43 , respectively.[...]"
    ${ }^{14}$ If I consider the correlation between labor market tightness and productivity for the same sub-period analyzed by Shimer (2005), i.e. 1986-2003, I find that it is indeed negative, as stated by the author.

[^10]:    ${ }^{15}$ The former paper uses a direct measure of "purified technology" while the latter, as Galí (1999), uses a Structural VAR where hours enter in first difference.

[^11]:    ${ }^{16}$ Although these preferences are not consistent with balanced growth path, they have been recently used by Nakajima (2012) in the context of a model with heterogeneous agents and labor market frictions; as the author reminds, these preferences have been adopted in the literature of open economies and news, as for example in Monacelli and Perotti (2008) and Schmitt-Grohé and Uribe (2012), because of their ability to better reproduce data characteristics.
    ${ }^{17}$ This second robustness check can be considered as an extension of the discussion provided in Cheron and Langot (2004): the authors in fact study the implications of movements of the marginal value of wealth in shaping labor market fluctuations after a productivity shock, using a RBC model. They compare the traditional separable preferences to a particular set on non-separable preferences developed by Rogerson and Wright (1988). I compare instead the implications of traditional separable preferences with the GHH preferences adopted in the benchmark case, using a model which allows for both flexible and sticky prices.

[^12]:    ${ }^{18}$ In particular, the performance of the model is improved in terms of volatilities, but the autocorrelation of vacancies becomes negative.
    ${ }^{19}$ I abstract from the presence of physical capital, again to analyze a framework close to Shimer (2005).
    ${ }^{20}$ As we know, it would have been the same to suppose that households have CES preferences for the differentiated goods produced by retail firms.
    ${ }^{21}$ As pointed out by Barnichon (2014), the behavior of the monetary authority in reaction to a productivity shock is of fundamental importance in shaping the reaction of labor market variables: if the Central Bank is not accommodating, a positive productivity shock can imply a reaction in labor input.

[^13]:    ${ }^{22}$ In my calibration I set the quarterly separation rate to $s=0.1$ and the employment rate to $N=0.9455$, in line with Shimer (2005); these values, together with the "immediate hiring" hypothesis, imply a job finding probability of $p=0.6344$, which corresponds to a monthly probability of $x_{m}=0.285$, thus in line with Hall (2005), computed in order to verify $x_{m}+\left(1-x_{m}\right) x_{m}+\left(1-x_{m}\right)^{2} x_{m}=0.6344$.
    ${ }^{23}$ See Subsection 1.B for the case with variable hours.

[^14]:    ${ }^{24}$ See the Appendix for more details about the derivation of a unique budget constraint.
    ${ }^{25}$ I use this convention in order to easily allow for variable hours in the extension of the model that I develop in Subsection 1.B.

[^15]:    ${ }^{26}$ When preferences are separable, labor supply depends also on consumption, so that wealth effects have an impact on labor supply choices: in Section 1.B, I show that with separable preferences the marginal utility of wealth $\lambda_{t}$ enters in the wage equation, and that it has important quantitative consequences.

[^16]:    ${ }^{27}$ See the Appendix for details.

[^17]:    ${ }^{28}$ This feature of the monetary policy implies that aggregate demand will be less or more sticky. Once the retailers fix their price, demand is given and they have to satisfy it; if there is a positive technological shock which increases the productivity of labor, the retailer which cannot change its price will see its mark-up to increase, which means that the relative price of intermediate firms output decreases, so that those firms can be induced to decrease employment, since each worker is now more productive.
    ${ }^{29}$ See the Appendix for a detailed derivation.

[^18]:    ${ }^{30}$ For example in Trigari (2006), Andres et al. (2012).
    ${ }^{31}$ Notice however that KZ (2010) use a Taylor rule which includes directly employment and not output.
    ${ }^{32}$ In the Appendix I check that a model calibrated on a monthly basis does not give different results.

[^19]:    ${ }^{33}$ I remember that, due to the different dynamics of employment, the steady state value of employment is given by $N=\frac{p}{s+(1-s) p}$.
    ${ }^{34}$ Shimer (2005) imposes the Hosios condition, setting the same value of 0.72 for the elasticity of the matching function with respect to unemployment while, according to Petrongolo and Pissarides (2001), the range of admissible values for this parameter is $0.3-0.7$. The value proposed by Hagedorn and Manovskii (2008) for the worker bargaining weight is 0.052 .
    ${ }^{35}$ From Abowd and Kramarz (1997), Table 1 p. 22, $\frac{\text { Hiring Costs per Hire } * \text { Total Hirings }}{\text { Aver Labor cost } * \text { Total Employment }}=0.0103$.
    ${ }^{36}$ One can alternatively think of the total wage costs as approximately $80 \%$ of total output, since there is no physical capital, and find the same result as in the main text.
    ${ }^{37}$ The average cost of recruiting is $14 \%$ of the compensation of an employee, which in terms of my model can be written as $\frac{\omega}{\Phi}=0.14 w h$; using the job creation condition, this implies that $\omega=\frac{x A h}{[(1 / 0.14)-\beta(1-s)+1]} \Phi$, which gives a value for $\omega$ of 0.0345 and therefore a ratio of $\frac{\omega V}{Y}=0.0115$.
    ${ }^{38}$ From the firm's foc for vacancies I have $w=x \cdot A+\left(\frac{\omega}{\Phi}\right)(1 / h)[(\beta(1-s))-1]$ and from the wage equation I have that $\left.C^{e}-C^{u}=\left(\frac{1}{\xi}\right) *\left\{w \cdot h-(1-\xi)\left[x \cdot A \cdot h+\beta(1-s)\left(\frac{\omega}{\Phi}\right)\right) p\right]\right\}$ : once I calibrate the parameters of the labor market and the vacancies costs, I derive the value for the consumption gap.

[^20]:    ${ }^{39}$ In Shimer (2005) this ratio has a value of 0.4.
    ${ }^{40} \mathrm{HM}$ (2008) normalize the value of productivity to the unit as Shimer (2005) and set $z=0.955$ and $1-\xi=0.052$.
    ${ }^{41}$ Shimer (2005) normalizes labor productivity to the unity and he sets the value of leisure to $z=0.4$; since the mean wage is $w=0.993$, these values imply the following ratios: $\frac{z}{w}=0.4028$ and $\frac{w}{y}=0.993$. If I want to follow Shimer's calibration procedure in my model, I set the value of leisure, i.e. ( $C^{e}-C^{u}$ ) so that I keep the same ratios of Shimer (2005), i.e. $\left(C^{e}-C^{u}\right) / w=0.4$ and $\frac{w}{x A h}=0.993$, which imply a value of leisure of $\left(C^{e}-C^{u}\right)=0.11$ and a wage of $w=0.28$.
    ${ }^{42}$ In the empirical literature estimates for the duration of the price spell range from 1.5 quarters according to Bils and Klenow (2004) to 3 or 4 quarters, according to Nakamura and Steinsson (2013); the calibration strategies adopted in the DSGE literature reflect this heterogeneity, so that one can find calibrated values for the price spells going from 4.5 months (Thomas (2011)) to almost 20 months (Trigari (2009)), passing through a value between 9 and 12 months chosen by Blanchard and Galí (2010).

[^21]:    ${ }^{43}$ Stéphane Adjemian, Houtan Bastani, Michel Juillard, Frédéric Karamé, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto and Sébastien Villemot (2011), "Dynare: Reference Manual, Version 4," Dynare Working Papers, 1, CEPREMAP.
    ${ }^{44}$ The main messages can be resumed as following: (i) the overall conclusions of the baseline model are not invalidated by allowing for variable hours, even if, obviously, if both the intensive margin (hours per worker) and the extensive margin (the employment rate) can react, the relative volatility of employment is lower than if the sole extensive margin has to account for all the variations; (ii) the traditionally adopted set of preferences, which are separable in consumption and leisure, worsen the performance of the (real) model in

[^22]:    terms of volatilities, as already pointed out by Cheron and Langot (2004); in a sticky prices framework, the effect on labor market variables volatilities is less important, but they exacerbate the contraction, in the short run, of labor input after a positive productivity shock (which is reflected in the unconditional correlations of labor market variables with productivity); (iii) Assuming that the new hirings become productive a period after their recruitment does not change much the behavior of the model, if prices are flexible, while it is a fundamental assumption in the sticky prices framework: with the traditional timing, prices, and not labor input, react more, absorbing all the productivity shock, as highlighted by Van Zandweghe (2010).
    ${ }^{45}$ In the case in which nominal rigidities are absent, the transmission mechanism is as in the real business cycle framework, so that it is meaningful to compute steady state elasticities.
    ${ }^{46}$ In terms of Shimer (2005) notation, I actually have $\frac{c}{q(\theta)}=\frac{p-z}{r+s}$ and $w=(1-\beta) z+\beta(p+c \theta)$.

[^23]:    ${ }^{47}$ For quarterly values he sets $A=1, z=0.4, \xi=0.28, \omega=0.213, r=0.012, s=0.1, \theta=1, \Upsilon=1.355, \psi=$ 0.28 .
    ${ }^{48}$ I already discussed the calibration and steady state restrictions, but I report here the values for convenience: $A=1, \xi=0.4, \psi=0.4, \omega=0.024, \beta=0.985, s=0.1, \theta=0.7, \Upsilon=0.7296, x=0.83, h=0.33$.
    ${ }^{49} \xi=0.4$ instead of $\xi=0.28$.

[^24]:    ${ }^{50}$ See also Thomas (2011) for a clear discussion about the reaction of the term $x_{t} A_{t} h$ entering the wage equation and determining the movements of labor market variables.
    ${ }^{51} S_{t}=\left(1-N_{t-1}\right)+s N_{t-1}$.

[^25]:    ${ }^{52}$ The right-to-manage approach has been introduced in Trigari (2006) and adopted by, for example, Christoffel and Kuester (2008).

[^26]:    ${ }^{53}$ This is a problem shared with other models, as for example in Thomas (2008).

[^27]:    ${ }^{54}$ I solve and simulate a model which is calibrated as the benchmark one: the steady state values are the same, in particular for the "value of leisure" I have that $\frac{\Gamma^{u}-\Gamma^{e}}{\lambda}$ in the model with separable preferences is exactly equivalent to $\left(\Gamma^{u}-\Gamma^{e}\right)$ in the benchmark model.

[^28]:    ${ }^{55}$ The author also shows that the higher the level of price stickiness, the lower the volatility of labor market tightness.

[^29]:    ${ }^{1}$ These countries are also among those retained by Prescott (2004) in his seminal paper on the labor wedge and taxes.
    ${ }^{2}$ Data for Germany refer to the actual country as it exists from 1990; the series for the years which precede the reunification are reconstructed using data from West and East Germany.
    ${ }^{3}$ See for example Prescott (2004), Ohanian et al. (2008) or McDaniel (2011).

[^30]:    ${ }^{4}$ From a methodological point of view, Prescott's evaluation of the welfare gains arising from tax cuts takes into account the contraction in government expenditure and the transition necessary to reach a new steady state with a smaller welfare State.

[^31]:    ${ }^{5}$ The model is close to the first contributions of Langot (1995), Merz (1995) and Andolfatto (1996).
    ${ }^{6}$ Ohanian et al. (2008) show that the labor wedge computed with a model that merges hours worked per employee and the employment rate is not independent from the labor market institutions, such as the unemployment benefit system or the type of wage bargaining arrangements. Nevertheless, these authors shows that such shifts in labor market institutions do not have strong explanatory power in relation to the dynamics of the labor wedge. However, there is a break in the data: since the end of the seventies, the differences across countries in aggregate hours are mainly due to quantitatively significant differences along the extensive margin. Moreover, it is interesting to distinguish the elasticities of the two margins (extensive and intensive) in order to account for the relative importance of taxes and labor market institutions to the dynamics of the aggregate hours.
    ${ }^{7}$ See e.g. Mortensen and Pissarides (1999), Blanchard and Wolfers (2000) or Ljungqvist and Sargent (2007).
    ${ }^{8}$ See e.g. Gruber and Wise (2005) and Hairault et al. (2014).
    ${ }^{9}$ In this respect, we agree with the view presented in Rogerson (2006): the unemployment rate alone cannot be the only factor to explain the gap between Europe and the US. Given the institutional arrangements, only the employment rate can account for this gap.

[^32]:    ${ }^{10}$ In section 2.G of the Appendix, we report a decomposition of the employment rate by age, to show that although not perfect, the "representative agent hypothesis" is however acceptable.
    ${ }^{11}$ We are grateful to Alexandre Janiak for bringing this to our attention.
    ${ }^{12}$ Prescott (2004) computes this static wedge at two points (the early seventies and the mid-nineties), whereas Ohanian et al. (2008) compute this wedge with an annual frequency, on a much larger set of OECD countries.
    ${ }^{13}$ In this last point, we depart from Langot and Quintero Rojas (2008) who propose a search and matching model, but only account for the static wedges of FOCs.

[^33]:    ${ }^{14}$ This view finds some empirical support in Ragan (2013) and Rogerson (2007). They show that it is necessary to introduce this "collective" public spending in the utility function of the agent to account for labor market outcomes heterogeneity among OECD countries.
    ${ }^{15}$ Our analysis then extends the one by Fang and Rogerson (2009) who study, from a qualitative point of view, the implications of a model of labor supply and search and matching frictions (without capital accumulation) on the interactions between the two margins of labor input at steady state.

[^34]:    ${ }^{16}$ We use the dataset constructed by Ohanian and Raffo (2012). We are extremely grateful to A. Raffo for sharing the latest version of this dataset with us. A version of it can be found at: http://www.sciencedirect.com/science/article/pii/S0304393211001139.
    ${ }^{17}$ We choose 2008 as the terminal point of the sample in order to remove the effect of the last recession from our basic statistics.

[^35]:    ${ }^{18}$ These new laws on the labor market make part-time work easier, and also reduce the labor costs. Thus, the hours per worker do not increase, whereas at the end of the sample, the employment rate is greater than the one observed in the US, for the first time since the end of WWII.

[^36]:    ${ }^{19}$ latest update: 2012; http://www.caramcdaniel.com/researchpapers.

[^37]:    ${ }^{20}$ latest update: 2012; http://www.caramcdaniel.com/researchpapers.
    ${ }^{21}$ The database is compiled by the Amsterdam Institute for Advanced Labour Studies (AIAS).

[^38]:    ${ }^{22}$ While the presence of these costs helps to smooth the reaction of the variable physical capital, the results of the model remain even if the costs are not present.
    ${ }^{23}$ See for example Rogerson (2006) Rogerson (2006).

[^39]:    ${ }^{24}$ Note that in the simple case where bargaining power and taxes are constant over time, we simply have $B S=\alpha \frac{Y_{t}}{N_{t}}+\omega \theta_{t}$.
    ${ }^{25}$ See for example Burda and Weder (2010) for a discussion about the implications for business cycle fluctuations of the presence of a time-varying tax wedge in the wage equation.

[^40]:    ${ }^{26}$ See Appendix 2.A for a complete description of the equation of the stationarised model.
    ${ }^{27}$ In appendix 2.D we show that the paths are not greatly modified when the policy changes are unexpected. Given this robustness check, we prefer to present the results based on perfect forecasts because they avoid introducing the arbitrary timing of the market participants' "knowledge" of the reforms.
    ${ }^{28}$ As noted earlier, for Germany we will use our model to simulate only the period starting from 1990.

[^41]:    ${ }^{29}$ See Gomme and Rupert (2007) for a more complete discussion on this point.
    ${ }^{30}$ Table COFOG, government expenditure by function.

[^42]:    ${ }^{31}$ Data come from Langot et al. (2014).
    ${ }^{32}$ In which defense is included.

[^43]:    ${ }^{33}$ The presence of the term $\bar{c}$ is important from a quantitative point of view to match the fact that in countries which were relatively poorer compared to the US, hours worked were much higher at the beginning of the sample period and decreased strongly in the catching-up period; see Appendix 2.E for a more detailed discussion.
    ${ }^{34}$ Available through the author's website https://sites.google.com/site/regisbarnichon/research.
    ${ }^{35}$ Direction de l'Animation de la Recherche, des Etudes et des Statistiques, which is attached to the French Ministry of Labor, www.emploi.gouv.fr.
    ${ }^{36}$ We consider that the official number of vacancies is underestimated, because according to the data provided by the OECD, under the denomination "Registered unemployed and job vacancies", the resulting average of tightness would be of 0.3 , even lower than the value for France.
    ${ }^{37}$ Our actual implied values of tightness are, respectively, $\theta=0.8,0.5$ and 0.8.
    ${ }^{38}$ Remark that $\operatorname{dim}\left(\Theta_{2}\right)=12$ (with $\omega_{G E R}=\omega_{F R}$ ) whereas $\operatorname{dim}(\Psi)=14$ (because $\theta_{G E R}$ is unknown): there are over-identifying restrictions. As well as the graphical analysis, we report in the text the value of the MSE of each time series in order to have a statistical measure of the fit.

[^44]:    ${ }^{39}$ In Appendix 2.F, we analyze the dynamics of the outside value of employment, which is usually presented as a key variable of the Shimer (2005) puzzle. It is shown that the general equilibrium approach puts this "puzzle" into perspective.
    ${ }^{40}$ Consider moreover that the simulation for Germany relates to fewer years and the performance of the model for the UK is worse.
    ${ }^{41}$ When we consider the steady state of the model, we refer to an "asymptotic" steady state, which is obtained when the subsistence consumption term is null, so that in the following $\bar{c}=0$.

[^45]:    ${ }^{42}$ In the labor market, the bargaining power of workers would be constant and equal to the elasticity of the matching function with respect to unemployment, while the unemployment benefit would be null.
    ${ }^{43}$ Given that this ratio is not constant over the period, our calibration procedure ensures that the first best allocation matches the observed time series of the collective government expenditures.

[^46]:    ${ }^{44}$ In his paper Prescott evaluates the welfare gains for France of shifting to a "US style" tax system to $19 \%$ of life-time consumption.
    ${ }^{45}$ Prescott (2004) estimates the labor supply elasticity and find it " [...] large, nearly 3 when the fraction of time allocated to the market is in the neighborhood of the current U.S. level." (Prescott (2004), p. 11), but it is important to remember that in his model there is no unemployment, so his elasticity refers at the same time to both the intensive and the extensive margin.

[^47]:    ${ }^{46} \mathrm{We}$ limit ourselves to consider only the bargaining power of workers and the replacement rate.

[^48]:    ${ }^{47}$ When we consider the steady state of the model, we refer to an "asymptotic" steady state, which is obtained when the subsistence consumption term is null, so that in the following $\bar{c}=0$.

[^49]:    ${ }^{48}$ This can be seen more easily if we re-arrange it as following

    $$
    \delta\left(1-\frac{1}{1-\alpha}\left(\frac{1+\tau_{i n v}}{1-\tau_{k}}\right)\right)+\frac{1}{1-\alpha}\left(\frac{1+\tau_{i n v}}{1-\tau_{k}}\right)\left(1-\frac{1}{\beta}\right)+g\left(1-\frac{1}{\beta(1-\alpha)\left(\frac{1+\tau_{i n v}}{1-\tau_{k}}\right)}\right)<0
    $$

[^50]:    ${ }^{49}$ Chodorow-Reich and Karabarbounis (2013) focus on the empirical counterparts of the elements we found in the flow value of non-employment when wages are set through a Nash bargaining mechanism.

[^51]:    ${ }^{50}$ We plot the measure of the reservation wage normalized with respect to the wage bill $w_{t} h_{t}$.

[^52]:    ${ }^{51}$ The observation of low bargaining power also supports the views of Hagedorn and Manovskii (2008) who calibrate this parameter at a value equals to 0.061 .
    ${ }^{52}$ The "replacement rate" for non-employment income is a comprehensive measure calculated by the OECD, referring to an average 40 years old person, which can be found at http://www.oecd.org/els/benefits-

[^53]:    ${ }^{55}$ According to the adopted decomposition: employm. rate $=\frac{N}{P_{0 \text { p }}^{15-64}}=\frac{N_{15-24}}{P_{15} p_{15-64}}+\frac{N_{25-54}}{\text { Pop }_{15-64}}+\frac{N_{55-64}}{\text { Pop }_{15-64}}$.

[^54]:    ${ }^{1}$ Consider for example, as a non-exhaustive list, the papers by Lockwood and Manning (1993), Holmlund and Kolm (1995), Sørensen (1999), Røed and Strøm (2002).

[^55]:    ${ }^{2}$ There is no aggregate uncertainty, only idiosyncratic unemployment risk.
    ${ }^{3}$ The fact that labor supply can be inefficiently high, in an incomplete market framework, has been shown by Pijoan-Mas (2006).

[^56]:    ${ }^{4}$ I adopt a widely used functional form for expressing the disposable income, used by for example Sørensen (1999), Bénabou (2002), Heathcote et al. (2014) and Bakış et al. (2015); the convenience of such a functional form lies also in the fact that a unique measure of progressivity, the so called Coefficient or Residual Income progression, can be defined.
    ${ }^{5}$ Earlier contributions about the optimal level of unemployment benefits in similar models are those by Pollak (2007) and Reichling (2006).
    ${ }^{6}$ Mukoyama (2013) considers the effects of unemployment benefits in the framework developed by Krusell et al. (2010), including the transitional path to different steady states.
    ${ }^{7}$ As a steady state comparison cannot be considered as fully satisfying, in a follow-up of the present work I plan to analyse the transitional dynamics too.
    ${ }^{8}$ In the Appendix 3.C, I show that a social welfare function based on the Rawls criterion, by privileging the poorest unemployed agent, calls for both tax progressivity and unemployment benefits.

[^57]:    ${ }^{9}$ See Røed and Strøm (2002) for a comprehensive survey.
    ${ }^{10}$ It has to be understood that the "optimal" level of progressivity (which could be in theory be null or negative) is conditional on the chosen functional form of the tax and transfer schedule; this one, however, is not the optimal one, as already noted in Sørensen (1999) p. 439.
    ${ }^{11}$ I define the value of revenue which is associated with a zero net tax as $y_{0}=(1-\tau)^{1 / \lambda}$; if the agent's revenue is lower than this threshold, i.e. if $y<y_{0}$, the agent pays a negative tax.
    ${ }^{12}$ The unemployment insurance scheme is very simplistic, since the unemployment benefit consists of a fixed amount, for all agents.

[^58]:    ${ }^{13}$ The small differences with respect to the equations reported in the technical appendix of Reichling (2006) come from the fact that the author considers as taxable income only the labor income, and not the interest rate income.
    ${ }^{14}$ According to the author, the strategic saving component significantly increases total savings in the economy.

[^59]:    ${ }^{15}$ Notice that $v$ indicates the number of vacancies while $V$ the value for the firm of opening one vacancy.

[^60]:    ${ }^{16}$ By setting $\lambda=0$ and by considering linear utility, equation (3.38) reverts to to the standard DMP form. In Appendix 3.A, I provide some analytical insight, under the simplifying hypothesis that agents are risk neutral, about the effects that the transfer (the UB) has on wage bargaining; I also analyse the effect of $\lambda$ in the wage equation.

[^61]:    ${ }^{17}$ See Appendix 3.B for the derivation of the Frisch elasticity with progressive taxation.
    ${ }^{18}$ The Frisch elasticity for the adopted separable preferences in consumption and leisure is given by $\epsilon=\frac{1-h(a)}{h(a)} \frac{1}{\nu}$.

[^62]:    ${ }^{19}$ The empirical estimates about labor supply elasticity have found very different values, but it is not uncommon to find in the literature values as low as 0.1 for the labor supply elasticity of adult men: for an example, see Chetty (2012).
    ${ }^{20}$ Intermediate value between the estimates obtained by Heathcote et al. (2014), who consider a value of 0.15 , and Bakış et al. (2015) who provide 0.17 .
    ${ }^{21}$ The interest rate in the benchmark experiment is $\mathrm{r}=0.006$, while in the case without any taxes $(\lambda=0)$ the interest rate is $0.005 \%$, corresponding to the discount rate.
    ${ }^{22}$ I remind that the CRIP is given by $(1-\lambda)$.

[^63]:    ${ }^{23}$ If in fact $\xi=0$, social welfare is calculated according to the utilitarian criterion, while for $\xi \rightarrow \infty$, social welfare is evaluated according to the Rawls criterion (which assigns all the weight to the least well-off agent).

[^64]:    ${ }^{24}$ If instantaneous utility in unemployment is perfectly linked to net wages, through a fixed replacement ratio, or if it is instead fixed and disconnected from net wages.
    ${ }^{25}$ To have an intuition about the mechanisms, since I do not have analytical results for the complete model, I solve, in Section 3.A of the Appendix, for the wage equation of a model with linear utility.

[^65]:    ${ }^{26}$ In their paper, risk-averse agents save and leave bequests, in a situation where the labor market is perfectly competitive.
    ${ }^{27}$ The existence of an ergodic distribution is evident in the case of $\lambda=0.2$, since there exists no agent with a wealth higher than (approximately) 150; in the case of $\lambda=0$, aggregate capital does not tend to infinity because an agent always consumes his savings, if he becomes unemployed.

[^66]:    ${ }^{28}$ I remind that total taxes are defined as $T(y)=y-y^{d}=y-(1-\tau) y^{(1-\lambda)}$; the marginal tax rate is therefore given by $T^{\prime}(y)=1-(1-\tau)(1-\lambda) y^{-\lambda}$, while the average tax rate is given by $T(y) / y=1-(1-\tau) y^{-\lambda}$.
    ${ }^{29}$ This consideration already suggests that a welfare criterion privileging the poorest agent would call for additional instrument, in addition to tax progressivity, if available: for example, the unemployment benefit.

[^67]:    ${ }^{30}$ I restrict my analysis to a very simplistic unemployment insurance scheme, i.e. one in which all unemployed agents receive the same (fixed) amount (denoted $\mu$ ) during their unemployment spell.
    ${ }^{31}$ The overall effect on capital labor ratio is due to the fact that both the numerator (total capital) and the denominator (total labor) decrease.

[^68]:    ${ }^{32}$ The result that the best policy, in terms average welfare, is a progressive tax and transfer schedule, without additional transfers, is robust to alternative calibrations of the unemployment risk. In Section 3.4.4, I perform two experiments in which I vary the level of search frictions (in particular, I change the value of vacancy posing costs and the matching efficiency). The baseline unemployment rate (i.e., when $\lambda=0.16$ and $\mu=0$ ) is $18.8 \%$ in one case, and $4.49 \%$ in the other (as a reminder, in the baseline case the unemployment rate is $6.3 \%$ ).
    ${ }^{33}$ Note again that the specific functional form is the same as in equation (3.7)

[^69]:    ${ }^{34}$ In Appendix 3.C, I consider a social welfare function based on the Rawls criterion, which privileges the poorest (unemployed) agent: in this case, the optimal policy combination calls for a positive level of both tax progressivity and unemployment benefits.

[^70]:    ${ }^{35}$ To avoid confusion, I measure hours elasticity for the case in which $\lambda=0$, so that elasticity $=\left(\frac{1-h}{h}\right) \frac{1}{\nu}$. In Appendix 3.B, I report the definition of labor supply elasticity in the presence of progressive taxation.
    ${ }^{36}$ The low values are, in part, implied by the fact that the distribution of agents does not present a fat right tail: the agents who in theory present a more elastic labor supply do not exist in the economy.

[^71]:    ${ }^{37}$ I remind that when $\lambda=0$ and $\mu=0$ there is no government transfer to finance, so that the average tax rate is also null.
    ${ }^{38}$ Pijoan-Mas (2006) showed that, in a framework with imperfect financial markets and precautionary savings, poor agents work more than they would in a complete market framework, in addition to precaution-

[^72]:    ${ }^{39}$ Chéron (2002), Parmentier (2006) and Krusell et al. (2010) assume risk neutral agents to obtain analytical results.

[^73]:    ${ }^{40}$ See equation (3.38) in the main text.

[^74]:    ${ }^{41}$ See equation (3.43) in the main text.
    ${ }^{42}$ Equation (A.10) recalls equation (9) in Parmentier (2006); as the author puts to evidence, to know the overall effect of the marginal tax rate on the equilibrium wage it is necessary to take into consideration also the reaction of hours as well, but it is not my interest to obtain such a general equilibrium result.

[^75]:    ${ }^{43}$ I remind that welfare with Rawls criterion is computed using equation (3.51) with $\xi \rightarrow \infty$.

