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# Contributions to Relativistic Fundamental Astronomy and Tests of Gravitation in the Solar System

Christophe Le Poncin-Lafitte

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## Habilitation à Diriger les Recherches

PAR  
Christophe Le Poncin-Lafitte

SUJET

## Contributions to Relativistic Fundamental Astronomy and Tests of Gravitation in the Solar System

Soutenue le 28 Mars 2018 devant la commission :

Monsieur Luc Blanchet  
Monsieur Eric Gourghoulon  
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Monsieur Serge Reynaud  
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Je remercie également tous les collègues qui ont accepté de lire ces quelques pages et de faire partie de mon jury : Eric Gourghoulon, Luc Blanchet, Serge Reynaud, Gilles Métris, Benoît Famaey, Véronique Dehant et Anne Lemaître.

Enfin, comme il est souvent d'usage d'achever la rédaction d'un tel manuscrit par les remerciements, je m'avance ici vers les dernières frappes sur le clavier. je dédierai donc ces ultimes mots à ma mère, ce Docteur d'Etat qui me taquinait si souvent à propos de la HDR. *Per aspera ad astra*, j'aurais aimé que tu puisses lire ce texte.

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# Curriculum vitæ

## Expérience professionnelle

- Depuis 2009, Astronome-Adjoint au SYRTE, Observatoire de Paris, PSL Research University, CNRS, Sorbonne Universités, UPMC Univ. Paris 06, LNE.
- Entre 2008 et 2009, ATER au SYRTE, Observatoire de Paris, PSL Research University, CNRS, Sorbonne Universités, UPMC Univ. Paris 06, LNE.
- Entre 2006 et 2008, Chercheur post-doctoral au Lohrmann Observatory, Dresde, Allemagne

## Enseignements

### Cours dispensés

- Co-responsable avec Marie-Christine Angonin du cours *Physique Fondamentale dans l'Espace* du M2 de l'Observatoire de Paris
- TP de python en L3, Sorbonne Université.
- TD du cours Relativité et Temps du M1 de l'Observatoire de Paris.
- Depuis 2011, co-responsable, avec S. Lambert des méthodologies du parcours DSG du M2 de l'Observatoire de Paris
- Co-Encadrement, avec V. Lainey, de la méthodologie *Effets relativistes en navigation spatiale* au parcours DSG du M2 de l'Observatoire de Paris.
- Participation régulière aux jurys des stages de recherche au M2 de l'Observatoire de Paris.
- 2012-2014 : Co-Encadrement avec F. Pierret de la méthodologie *Mouvement relativiste de la Lune* au parcours DSG du M2 de l'Observatoire de Paris.

### Encadrement de thèses

- 2013-2016, co-encadrement (80%) avec Marie-Christine Angonin de la thèse d'Adrien Bourgoïn. Sujet : *Tests des théories alternatives de la gravitation avec les données de télémétrie laser sur la Lune.*
- 2010-2013, Co-encadrement (80%) avec Marie-Christine Angonin, de la thèse de Stefano Bertone. Sujet : *From Time Transfer Functions to a model for relativistic astrometry in the Gaia era.*

- 2009-2012, Participation à l'encadrement scientifique (10%) de la thèse d'Aurélien Hees dirigée par Peter Wolf et Véronique Dehant. Titre : *Signature of observables in alternative theories of gravity*.

## Encadrement de stages

- 2017. Encadrement du stage de recherche de M1 et M2 de Julien Frank et Valentin Decouesnes, respectivement.
- 2016. Encadrement du stage de recherche de M2 de Mansour Benbakoura. Encadrement des stages de L3 et de M1 de Nicolas Blonski et Valentin Decouesnes, respectivement.
- 2014. Encadrement du stage de recherche de M2 de Luc Senecal.
- 2013. Co-Encadrement, avec S. Mathis, du stage de recherche de M2 de Pierre Auclair-Desrotour.
- 2012. Encadrement du stage de recherche de M2 de Frédéric Pierret.
- 2011. Co-Encadrement, avec D. Hestroffer, du stage de recherche de M2 de Mélody Sylvestre.
- 2010. Encadrement des stages de recherche de M2 de Stefano Bertone et de Kazuhisa Oyama.
- 2009. Encadrement du stage de recherche de M2 de Meng-Hsun Chung.

## Implications dans la communauté nationale et internationale

- Directeur du Master PSL Sciences de l'Université et Technologies Spatiales, 2017.
- Membre du Groupe de travail de Physique Fondamentale, CERES du CNES (depuis Janvier 2015).
- Directeur-adjoint du département/laboratoire SYRTE, CNRS UMR 8630, LNE, Observatoire de Paris (depuis Juillet 2014).
- Membre de l'IERS (International Earth Rotation Service) Conventions Editorial Board (depuis Juin 2014).
- Membre de l'équipe internationale ISSI Encelade 2.0 : Constraining the Dynamical Timescale and Internal Processes of the Saturn System from Astrometry (depuis 2014).
- Membre de l'International Laser Ranging Service, ILRS (depuis 2013).
- Expert referee pour l'ANR (depuis 2013).
- Membre nommé à la section 17 du comité national du CNRS (2012-2016).
- Membre de l'Union Astronomique Internationale, divisions A et D. Membre du Bureau de l'Axe Thématique Gaia du Conseil Scientifique de l'Observatoire de Paris. Depuis 2014, Gaia est devenu un Axe Fédérateur du CS de l'Observatoire (depuis 2010).
- Participation au centre du SYRTE d'analyse des données de la mission ESA ACES-Pharao (depuis 2009).

- ✉ Membre du DPAC (Data Processing and Analysis Consortium) de la mission ESA Gaia. Responsable des modèles relativistes pour l'horloge embarquée entre 2009 et 2012. Membre du DPAC/CU3/REMAT (Relativistic modeling and tests) depuis 2006. Membre du DPAC/CU9 depuis 2010.
- ✉ Membre du groupe international Encelade (2009-2013).
- ✉ Membre du Relativity and reference frames working group de la mission Gaia (2003-2006).

## Publications

Au 20 Février 2018, le nombre total de publications dans des journaux à comité de lecture de rang A s'élevait à 35. Le nombre total de citations de mes articles atteint 1857 entrées (source : Nasa ADS), la figure 1 illustrant leur distribution temporelle. Dans les listes de publications qui suivent, les étudiants, sous ma direction directe et indirecte, sont soulignés par une couleur distincte (bleu, orange, rouge, magenta et violet).

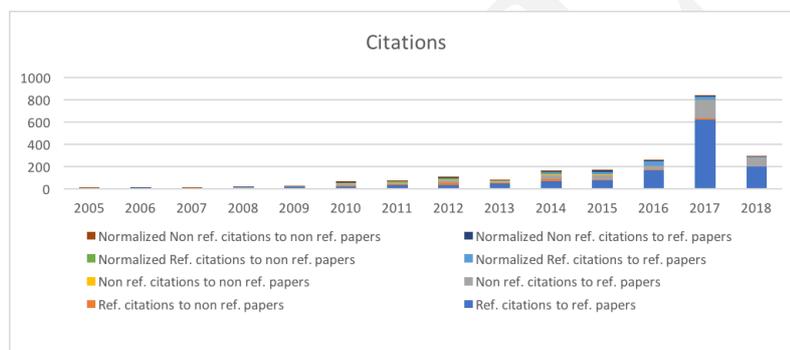


FIGURE 1 – Citations of my articles

## Articles dans des revues internationales avec comité de lecture

1. F. MEYNADIER, P. DELVA, C. LE PONCIN-LAFITTE, C. GUERLIN et P. WOLF. « Atomic clock ensemble in space (ACES) data analysis ». In : *Classical and Quantum Gravity* 35.3 (2018), p. 035018. URL : <http://stacks.iop.org/0264-9381/35/i=3/a=035018>
2. **S. BERTONE**, C. LE PONCIN-LAFITTE, P. ROSENBLATT, V. LAINEY, J.-C. MARTY et M.-C. ANGININ. « Impact analysis of the transponder time delay on radio-tracking observables ». In : *Advances in Space Research* 61 (jan. 2018), p. 89–96. DOI : [10.1016/j.asr.2017.09.003](https://doi.org/10.1016/j.asr.2017.09.003). arXiv : [1708.00546](https://arxiv.org/abs/1708.00546) [[astro-ph.EP](#)]
3. **S. BERTONE**, A. VECCHIATO, B. BUCCIARELLI, M. CROSTA, M. G. LATTANZI, L. BIANCHI, M.-C. ANGININ et C. LE PONCIN-LAFITTE. « Application of time transfer functions to Gaia's global astrometry. Validation on DPAC simulated Gaia-like observations ». In : *A&A* 608, A83 (déc. 2017), A83. DOI : [10.1051/0004-6361/201731654](https://doi.org/10.1051/0004-6361/201731654). arXiv : [1708.00541](https://arxiv.org/abs/1708.00541) [[astro-ph.IM](#)]
4. **A. BOURGOIN**, C. LE PONCIN-LAFITTE, **A. HEES**, S. BOUQUILLON, G. FRANCOU et M.-C. ANGININ. « Lorentz Symmetry Violations from Matter-Gravity Couplings with Lunar

- Laser Ranging ». In : *Physical Review Letters* 119.20, 201102 (nov. 2017), p. 201102. DOI : [10.1103/PhysRevLett.119.201102](https://doi.org/10.1103/PhysRevLett.119.201102). arXiv : [1706.06294](https://arxiv.org/abs/1706.06294) [gr-qc]
5. GAIA COLLABORATION, G. CLEMENTINI, L. EYER, V. RIPEPI, M. MARCONI, T. MURAVEVA, A. GAROFALO, L. M. SARRO, M. PALMER, X. LURI et et AL. « Gaia Data Release 1. Testing parallaxes with local Cepheids and RR Lyrae stars ». In : *A&A* 605, A79 (sept. 2017), A79. DOI : [10.1051/0004-6361/201629925](https://doi.org/10.1051/0004-6361/201629925). arXiv : [1705.00688](https://arxiv.org/abs/1705.00688) [astro-ph.SR]
  6. P. DELVA, J. LODEWYCK, S. BILICKI, E. BOOKJANS, G. VALLET, R. LE TARGAT, P.-E. POTTIE, C. GUERLIN, F. MEYNADIER, C. LE PONCIN-LAFITTE, O. LOPEZ, A. AMY-KLEIN, W.-K. LEE, N. QUINTIN, C. LISDAT, A. AL-MASOUDI, S. DORSCHER, C. GREBING, G. GROSCHE, A. KUHL, S. RAUPACH, U. STERR, I. R. HILL, R. HOBSON, W. BOWDEN, J. KRONJAGER, G. MARRA, A. ROLLAND, F. N. BAYNES, H. S. MARGOLIS et P. GILL. « Test of Special Relativity Using a Fiber Network of Optical Clocks ». In : *Phys. Rev. Lett.* 118 (22 juin 2017), p. 221102. DOI : [10.1103/PhysRevLett.118.221102](https://doi.org/10.1103/PhysRevLett.118.221102). URL : <https://link.aps.org/doi/10.1103/PhysRevLett.118.221102>
  7. GAIA COLLABORATION, F. VAN LEEUWEN, A. VALLENARI, C. JORDI, L. LINDEGREN, U. BASTIAN, T. PRUSTI, J. H. J. DE BRUIJNE, A. G. A. BROWN, C. BABUSIAUX et et AL. « Gaia Data Release 1. Open cluster astrometry : performance, limitations, and future prospects ». In : *A&A* 601, A19 (mai 2017), A19. DOI : [10.1051/0004-6361/201730552](https://doi.org/10.1051/0004-6361/201730552). arXiv : [1703.01131](https://arxiv.org/abs/1703.01131) [astro-ph.SR]
  8. GAIA COLLABORATION, A. G. A. BROWN, A. VALLENARI, T. PRUSTI, J. H. J. DE BRUIJNE, F. MIGNARD, R. DRIMMEL, C. BABUSIAUX, C. A. L. BAILER-JONES, U. BASTIAN et et AL. « Gaia Data Release 1. Summary of the astrometric, photometric, and survey properties ». In : *A&A* 595, A2 (nov. 2016), A2. DOI : [10.1051/0004-6361/201629512](https://doi.org/10.1051/0004-6361/201629512). arXiv : [1609.04172](https://arxiv.org/abs/1609.04172) [astro-ph.IM]
  9. GAIA COLLABORATION, T. PRUSTI, J. H. J. DE BRUIJNE, A. G. A. BROWN, A. VALLENARI, C. BABUSIAUX, C. A. L. BAILER-JONES, U. BASTIAN, M. BIERMANN, D. W. EVANS et et AL. « The Gaia mission ». In : *A&A* 595, A1 (nov. 2016), A1. DOI : [10.1051/0004-6361/201629272](https://doi.org/10.1051/0004-6361/201629272). arXiv : [1609.04153](https://arxiv.org/abs/1609.04153) [astro-ph.IM]
  10. A. BOURGOIN, A. HEES, S. BOUQUILLON, C. LE PONCIN-LAFITTE, G. FRANCOU et M. -C. ANGININ. « Testing Lorentz Symmetry with Lunar Laser Ranging ». In : *Phys. Rev. Lett.* 117 (24 déc. 2016), p. 241301. DOI : [10.1103/PhysRevLett.117.241301](https://doi.org/10.1103/PhysRevLett.117.241301). URL : <http://link.aps.org/doi/10.1103/PhysRevLett.117.241301>
  11. C. LE PONCIN-LAFITTE, A. HEES et S. LAMBERT. « Lorentz symmetry and very long baseline interferometry ». In : *Phys. Rev. D* 94.12, 125030 (déc. 2016), p. 125030. DOI : [10.1103/PhysRevD.94.125030](https://doi.org/10.1103/PhysRevD.94.125030). arXiv : [1604.01663](https://arxiv.org/abs/1604.01663) [gr-qc]
  12. V. LAINEY, R. A. JACOBSON, R. TAJEDDINE, N. J. COOPER, C. MURRAY, V. ROBERT, G. TOBIE, T. GUILLOT, S. MATHIS, F. REMUS, J. DESMARS, J.-E. ARLOT, J.-P. DE CUYPER, V. DEHANT, D. PASCU, W. THUILLOT, C. LE PONCIN-LAFITTE et J.-P. ZAHN. « New constraints on Saturn's interior from Cassini astrometric data ». In : *Icarus* 281 (jan. 2017), p. 286–296. DOI : [10.1016/j.icarus.2016.07.014](https://doi.org/10.1016/j.icarus.2016.07.014)
  13. S. MATHIS, P. AUCLAIR-DESROTOUT, M. GUENEL, F. GALLET et C. LE PONCIN-LAFITTE. « The impact of rotation on turbulent tidal friction in stellar and planetary convective regions ». In : *A&A* 592, A33 (juil. 2016), A33. DOI : [10.1051/0004-6361/201527545](https://doi.org/10.1051/0004-6361/201527545). arXiv : [1604.08570](https://arxiv.org/abs/1604.08570) [astro-ph.SR]

14. A. HEES, Q. G. BAILEY, C. LE PONCIN-LAFITTE, A. BOURGOIN, A. RIVOLDINI, B. LAMINE, F. MEYNADIER, C. GUERLIN et P. WOLF. « Testing Lorentz symmetry with planetary orbital dynamics ». In : *Phys. Rev. D* 92.6, 064049 (sept. 2015), p. 064049. DOI : [10.1103/PhysRevD.92.064049](https://doi.org/10.1103/PhysRevD.92.064049). arXiv : [1508.03478](https://arxiv.org/abs/1508.03478) [gr-qc]
15. P. AUCLAIR-DESROTOUR, S. MATHIS et C. LE PONCIN-LAFITTE. « Scaling laws to understand tidal dissipation in fluid planetary regions and stars I. Rotation, stratification and thermal diffusivity ». In : *A&A* 581, A118 (sept. 2015), A118. DOI : [10.1051/0004-6361/201526246](https://doi.org/10.1051/0004-6361/201526246). arXiv : [1506.07705](https://arxiv.org/abs/1506.07705) [astro-ph.EP]
16. A. HEES, S. BERTONE et C. LE PONCIN-LAFITTE. « Light propagation in the field of a moving axisymmetric body : Theory and applications to the Juno mission ». In : *Physical Review D* 90.8, 084020 (oct. 2014), p. 084020. DOI : [10.1103/PhysRevD.90.084020](https://doi.org/10.1103/PhysRevD.90.084020). arXiv : [1406.6600](https://arxiv.org/abs/1406.6600) [gr-qc]
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18. P. AUCLAIR-DESROTOUR, C. LE PONCIN-LAFITTE et S. MATHIS. « Impact of the frequency dependence of tidal Q on the evolution of planetary systems ». In : *Astronomy & Astrophysics* 561, L7 (jan. 2014), p. L7. DOI : [10.1051/0004-6361/201322782](https://doi.org/10.1051/0004-6361/201322782). arXiv : [1311.4810](https://arxiv.org/abs/1311.4810) [astro-ph.EP]
19. S. BERTONE, O. MINAZZOLI, M. CROSTA, C. LE PONCIN-LAFITTE, A. VECCHIATO et M.-C. ANGININ. « Time transfer functions as a way to validate light propagation solutions for space astrometry ». In : *Classical and Quantum Gravity* 31.1, 015021 (jan. 2014), p. 015021. DOI : [10.1088/0264-9381/31/1/015021](https://doi.org/10.1088/0264-9381/31/1/015021). arXiv : [1306.2367](https://arxiv.org/abs/1306.2367) [gr-qc]
20. A. HEES, B. LAMINE, S. REYNAUD, M.-T. JAEKEL, C. LE PONCIN-LAFITTE, V. LAINEY, A. FÜZFA, J.-M. COURTY, V. DEHANT et P. WOLF. « Radioscience simulations in general relativity and in alternative theories of gravity ». In : *Classical and Quantum Gravity* 29.23, 235027 (déc. 2012), p. 235027. DOI : [10.1088/0264-9381/29/23/235027](https://doi.org/10.1088/0264-9381/29/23/235027). arXiv : [1201.5041](https://arxiv.org/abs/1201.5041) [gr-qc]
21. J. OBERST, V. LAINEY, C. LE PONCIN-LAFITTE, V. DEHANT, P. ROSENBLATT, S. ULAMEC, J. BIELE, J. SPURMANN, R. KAHLE, V. KLEIN, U. SCHREIBER, A. SCHLICHT, N. RAMBAUX, P. LAURENT, B. NOYELLES, B. FOULON, A. ZAKHAROV, L. GURVITS, D. UCHAEV, S. MURCHIE, C. REED, S. G. TURYSHEV, J. GIL, M. GRAZIANO, K. WILLNER, K. WICKHUSEN, A. PASEWALDT, M. WÄHLISCH et H. HOFFMANN. « GETEMME : a mission to explore the Martian satellites and the fundamentals of solar system physics ». In : *Experimental Astronomy* 34 (oct. 2012), p. 243–271. DOI : [10.1007/s10686-012-9307-0](https://doi.org/10.1007/s10686-012-9307-0)
22. V. LAINEY, O. KARATEKIN, J. DESMARS, S. CHARNOZ, J.-E. ARLOT, N. EMELYANOV, C. LE PONCIN-LAFITTE, S. MATHIS, F. REMUS, G. TOBIE et J.-P. ZAHN. « Strong Tidal Dissipation in Saturn and Constraints on Enceladus' Thermal State from Astrometry ». In : *Astrophysical Journal* 752, 14 (juin 2012), p. 14. DOI : [10.1088/0004-637X/752/1/14](https://doi.org/10.1088/0004-637X/752/1/14). arXiv : [1204.0895](https://arxiv.org/abs/1204.0895) [astro-ph.EP]
23. S. CHARNOZ, A. CRIDA, J. C. CASTILLO-ROGEZ, V. LAINEY, L. DONES, O. KARATEKIN, G. TOBIE, S. MATHIS, C. LE PONCIN-LAFITTE et J. SALMON. « Accretion of Saturn's mid-sized moons during the viscous spreading of young massive rings : Solving

- the paradox of silicate-poor rings versus silicate-rich moons ». In : *Icarus* 216 (déc. 2011), p. 535–550. DOI : [10.1016/j.icarus.2011.09.017](https://doi.org/10.1016/j.icarus.2011.09.017). arXiv : [1109.3360](https://arxiv.org/abs/1109.3360) [astro-ph.EP]
24. S. B. LAMBERT et C. LE PONCIN-LAFITTE. « Improved determination of  $\gamma$  by VLBI ». In : *Astronomy & Astrophysics* 529, A70 (mai 2011), A70. DOI : [10.1051/0004-6361/201016370](https://doi.org/10.1051/0004-6361/201016370)
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DRAFT

# 1 | Document de synthèse

Le secret d'un bon discours, c'est d'avoir une bonne introduction et une bonne conclusion. Ensuite, il faut s'arranger pour que ces deux parties ne soient pas trop éloignées l'une de l'autre.

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George Burns

## 1.1 Introduction

The General Theory of Relativity (GR) was developed by Albert Einstein more than one century ago. Since its birth, GR has been tested in details, in particular in Solar System. The first three tests, proposed by Einstein himself in 1915, concerned the anomalous precession of Mercury perihelion, light deflection in gravitational fields and the gravitational redshift. Even if the precession of Mercury was already known before GR, the theory was able to explain it successfully. Experiments proving light bending was realized by Eddington in 1919, and evidence for gravitational redshift was claimed to be detected in 1925 by astrophysical measurement [2] and clearly confirmed by Pound & Rebka experiment in 1959 [144]. With the beginning of space exploration in the 1960s, additional tests began to be made starting with Shapiro's measurement of the relativistic time delay in Viking ranging [154, 155]. In 1974, with the discovery of binary pulsars, Hulse & Taylor [77] studied much stronger gravitational fields than those found in the Solar System, enabling the first indirect detection of gravitational waves. At the scale of the laboratory, the past few decades have seen dramatic progress in our ability to manipulate and coherently control matter-waves, leading also to new applications navigation and geophysics to tests of GR with an unprecedented sensitivity and accuracy [12]. Thus, GR is extremely well tested locally in both weak and strong field limit, and the story of testing GR opened a new windows in February 2016 with the announcement of Advanced LIGO & VIRGO teams of the direct detection of gravitational waves from a black hole merger [1]. However, we know that GR can not be renormalized as quantum theory and it is an ultimate goal to find a deviation in order to construct a new theory unifying all fundamental interactions, as string theories and/or quantum loop theory are aimed to do.

During the last 30 years, observational techniques in fundamental astronomy have progressed and we have seen a tremendous progress in technology. It has led to enormous improvements of accuracy in the disciplines of astrometry and time. Indeed, focusing on astrometry, it is

worth highlighting that between 1988 and 2013 we expected the same gain in accuracy (milli to micro-arcsecond, so 4 orders of magnitude) as that have been done during the whole history of astrometry, from Hipparchus until 1988. Other observation techniques like Interferometry, high-precision atomic clocks, Lunar and Satellite Laser Ranging, GNSS, Very Long Baseline Interferometry, Radio Science, etc., have also made progress in accuracy by several orders of magnitude.

Beyond some threshold of accuracy, any astronomical problem has to be formulated within the framework of Einstein's theory. An illuminating example is the light deflection at the limb of the Sun, which reaches 1.75as and decreases only as  $1/r$  with increasing impact parameter  $r$  of a light ray to the solar center. Thus, for light rays incident at about  $90^\circ$  from the Sun the angle of light deflection still amounts to 4 mas and has to be compared to the accuracy of the Gaia mission of several  $\mu$ as in positions, parallaxes and proper motions of billion stars [63]. It is the reason why the field of applied relativity has emerged about 40 years ago, when the growing accuracy of observations and the new observational techniques have made it necessary to take relativistic effects into account on a routine basis. Since that time applied relativity has evolved into one of the basic ingredients of fundamental astronomy, the discipline that includes celestial mechanics, astrometry, time scales and time dissemination etc. On the one hand, that development required significant theoretical efforts. Triggered also by the needs of applications at an engineering level, special theoretical techniques have been developed to construct the so-called local reference system and to derive the equations of translational and rotational motion of a system of  $N$  bodies having arbitrary composition and shape. On the other hand, astronomers and engineers had to rethink and to reformulate their problems in a language compatible with GR. The need to change the way of thinking from Newtonian *common sense* to relativistic is probably the source of many of the difficulties that non-experts have with relativity. In the same time the relativity itself is quite simple and elegant at least in the post-Newtonian approximation.

The International Astronomical Union (IAU) discussed in details the construction of a GR framework for modeling the high-accuracy astronomical observations, with a 20 years process from 1990 to 2000. By adopting new conventions concerning the definition and use of a set of relativistic reference systems [26, 157], IAU gave the essentials tools leading GR to become the standard theory of gravitation in the field of Fundamental Astronomy, astrophysics and experimental fundamental physics. It was the birth of the Fundamental Relativistic Astronomy which is perfectly summarized in Figure (1.1), from the construction of relativistic reference systems to their application to the determination of equations of motion, light propagation and the construction of astronomical observables. Nowadays Fundamental Relativistic Astronomy is essential for many practical goals in astrometry, deep space navigation, planetodesy, geodesy and time & frequency metrology, that is to say in all fields requiring high precision in the Solar System.

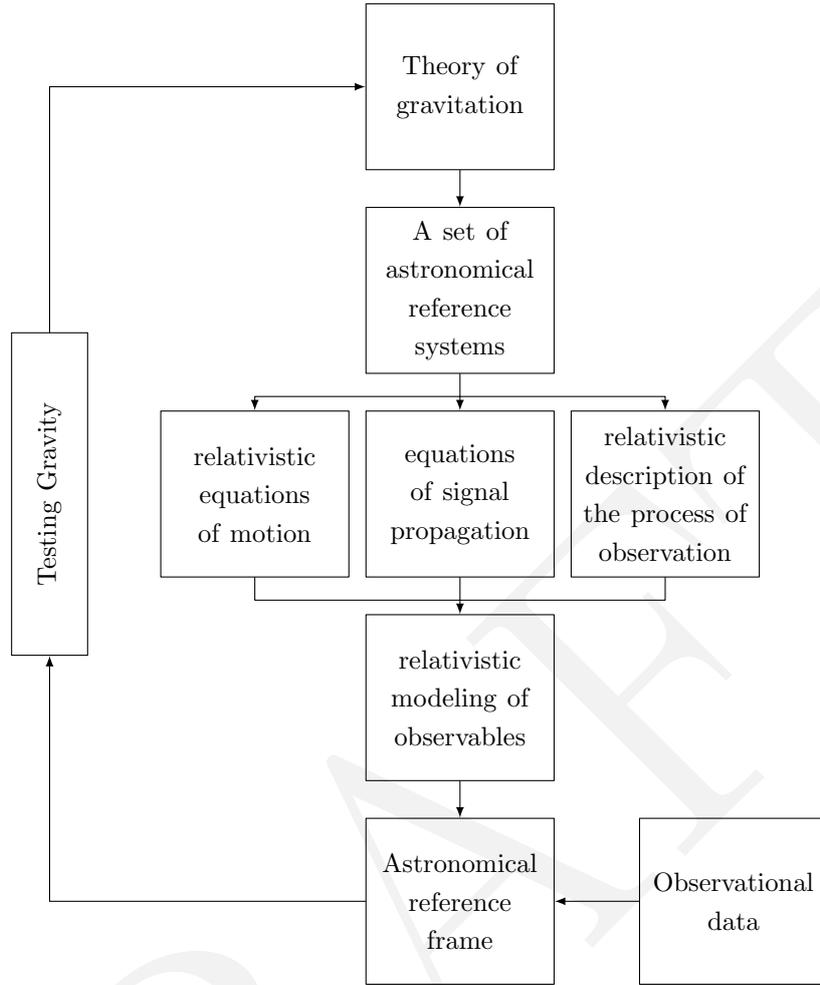


FIGURE 1.1 – Scheme of Relativistic Fundamental Astronomy. Inspired from [81].

Furthermore, I constructed all along these last years my research with respect to the different boxes of Figure (1.1), which allow me to decline the outline of the document along three kinds of contributions :

- I will consider first the description of light rays in weak fields; in almost all of the theoretical studies devoted to this problem, the properties of light rays are determined by integrating the differential equations of the null geodesics [82, 83, 87, 88, 89, 81, 41, 46, 40, 44, 182, 183, 184, 42]. This procedure is workable as long as one contents oneself with analyzing the effects of relativistic first order of corrections. However, analytical solution of the geodesic equations requires cumbersome calculations when terms of second order are taken into account, even in the case of a static, spherically symmetric space-time [58, 147, 148]. It is why I introduced several formalisms allowing to determine the deflection of light and the travel time of photons without integrating the geodesic equations. The first method was constructed on a general post-Minkowskian expansion of the Synge World Function [160] applied to light rays, leading to the determination of the so-called Time Transfer Functions. Then I found a way to determine the Time Transfer Functions

within a stand-alone method and obtained their general post-Minkowskian expansion. Some applications of the formalism are given in the context of global space astrometry and radioscience observables with example to the JUNO, BepiColombo and GRAIL missions.

- Then I will discuss my contributions dedicated to relativistic celestial mechanics. They are focused on the motion of deep space probe and the construction of ephemerides for planets and natural satellites. The concept of 4-dimensional ephemerides will be described. Then I will focus on planetary tides and dissipation. Indeed, the signature of tides and relativistic effects on the motion of the body are quite similar, a good example being the Earth-Moon System : I will summarize some results on the dissipation of rocky and fluid bodies, thanks to a new formalism I introduced in 2009.
- The last part of this report will be dedicated to the test of Fundamental Physics that one can expect nowadays with Solar System data. I will consider planetary ephemerides, Very Long Baseline Interferometry and Lunar-Laser Ranging. Two test phenomenologies will be used, *i.e.* the traditional Parametrized Post-Newtonian formalism and the Standard Model Extension, useful to study all kind of possible violations of the Lorentz symmetry.

As a *Habilitation à Diriger les Recherches* (HDR) is aiming at proving the capability to lead a research group and/or supervise students, I will emphasize on this point all along the following.

## 1.2 Light propagation in curved spacetime

### 1.2.1 Definition and fundamental properties of Synge's world function

During my PhD thesis and my post-doctoral position in Germany, so between fall 2002 and spring 2009, I developed a new formalism to describe light propagation through Solar System. Instead of solving, analytically and/or numerically, the equations of the null geodesics, we focused on the Synge World Function [160] and found a way to determine the Synge World Function as a general post-Minkowskian series, in the weak field approximation. From the World Function, we show how to determine explicitly the coordinate time of flight of a photon between two events located at finite distance from the origin of a coordinate frame. In contrast with null geodesic determinations, The two main advantages of this method is first to evaluate integrals along Minkowskian straight line, which is very simple, and second to be able to consider difficult gravitational field beyond the traditional first post-Minkowskian order.

First, we briefly recall how its fundamental properties can be straightforwardly derived from the variational principle defining the geodesic curves (for the characteristic function  $V = \sqrt{2} |\Omega|$ , see *e.g.* Buchdahl [29, 27, 28] ). We begin with recalling some useful results concerning the general variation of a functional defined as an action in classical mechanics.

Let  $x_A$  and  $x_B$  be two points of space-time connected by a differentiable curve  $C_{AB}$  defined by the parametric equations  $x^\alpha = x^\alpha(\zeta)$ , with  $\zeta_A \leq \zeta \leq \zeta_B$ . Given a Lagrangian function  $L(x^\alpha, \dot{x}^\beta)$  with  $\dot{x}^\beta(\zeta) = dx^\beta(\zeta)/d\zeta$ , one can define the functional  $\tilde{S}$  by

$$\tilde{S}[C_{AB}] = \int_{\zeta_A}^{\zeta_B} L(x^\alpha(\zeta), \dot{x}^\beta(\zeta)) d\zeta, \quad (1.1)$$

the integral being taken along  $C_{AB}$ . Let us consider now an arbitrarily neighbouring curve  $C'_{A'B'}$  connecting points  $x_{A'}$  and  $x_{B'}$ , represented by parametric equations  $x'^{\alpha} = x'^{\alpha}(\zeta')$ , with  $\zeta_{A'} \leq \zeta' \leq \zeta_{B'}$ . Let  $x_A^{\alpha} + \delta x_A^{\alpha} = x_{A'}^{\alpha}$  and  $x_B^{\alpha} + \delta x_B^{\alpha} = x_{B'}^{\alpha}$  be the coordinates of  $x_{A'}$  and  $x_{B'}$  respectively. Putting  $\bar{\delta}x^{\alpha}(\zeta) = x'^{\alpha}(\zeta) - x^{\alpha}(\zeta)$ , one can define a one-to-one correspondance between  $C_{AB}$  and  $C'_{A'B'}$  by the infinitesimal transformation

$$x^{\alpha}(\zeta) \rightarrow x'^{\alpha}(\zeta + \delta\zeta) = x^{\alpha}(\zeta) + \delta x^{\alpha}(\zeta), \quad (1.2)$$

where

$$\delta x^{\alpha}(\zeta) = \bar{\delta}x^{\alpha}(\zeta) + \dot{x}^{\alpha}(\zeta)\delta\zeta \quad (1.3)$$

with the boundary conditions

$$\delta x^{\alpha}(\zeta_A) = \delta x_A^{\alpha}, \quad \delta x^{\alpha}(\zeta_B) = \delta x_B^{\alpha}. \quad (1.4)$$

Performing an integration by parts leads directly to the following expression for the quantity  $\delta\tilde{S} = \tilde{S}[C'_{A'B'}] - \tilde{S}[C_{AB}]$  (see, e.g., [66])

$$\delta\tilde{S} = \int_{\zeta_A}^{\zeta_B} \left[ \frac{\partial L}{\partial x^{\alpha}} - \frac{d}{d\zeta} \left( \frac{\partial L}{\partial \dot{x}^{\alpha}} \right) \right] \bar{\delta}x^{\alpha}(\zeta) d\zeta + [p_{\alpha} \delta x^{\alpha} - H \delta\zeta]_A^B, \quad (1.5)$$

where  $p_{\alpha}$  is the 4-momentum belonging to  $x^{\alpha}$  and  $H$  is the Hamiltonian :

$$p_{\alpha} = \frac{\partial L}{\partial \dot{x}^{\alpha}}, \quad H = p_{\alpha} \dot{x}^{\alpha} - L. \quad (1.6)$$

In the following we assume that there exist domains  $\mathcal{D}$  of space-time such that whatever  $x_A \in \mathcal{D}$  and  $x_B \in \mathcal{D}$ ,  $x_A$  and  $x_B$  are linked by a unique curve  $\Gamma_{AB}$  realizing an extremum of the functional  $\tilde{S}$ . Then, describing  $\Gamma_{AB}$  by parametric equations  $x^{\alpha} = x^{\alpha}(\zeta)$ , we can associate to any domain  $\mathcal{D}$  a function  $S$  of  $x_A, x_B, \zeta_A, \zeta_B$  defined as

$$S(x_A, x_B, \zeta_A, \zeta_B) = \int_{\Gamma_{AB}} L(x^{\alpha}(\zeta), \dot{x}^{\beta}(\zeta)) d\zeta. \quad (1.7)$$

Extending a terminology currently used in mechanics, we shall call  $S$  the characteristic (or principal) function belonging to the Lagrangian  $L$  adapted to the domain  $\mathcal{D}$ . Since the functions  $x^{\alpha}(\zeta)$  satisfy the Euler-Lagrange equations

$$\frac{\partial L}{\partial x^{\alpha}} - \frac{d}{d\zeta} \left( \frac{\partial L}{\partial \dot{x}^{\alpha}} \right) = 0, \quad (1.8)$$

the total variation of  $S(x_A, x_B, \zeta_A, \zeta_B)$  reduces to the boundary terms in equation (1.5). Consequently, one has the relations

$$\frac{\partial S}{\partial x_A^{\alpha}} = -(p_{\alpha})_A, \quad \frac{\partial S}{\partial x_B^{\alpha}} = (p_{\alpha})_B, \quad (1.9)$$

$$\frac{\partial S}{\partial \zeta_A} = H_A, \quad \frac{\partial S}{\partial \zeta_B} = -H_B. \quad (1.10)$$

Considering that  $H$  is a function of  $x^\alpha$  and  $p_\alpha$ , then substituting for  $(p_\alpha)_A$  and  $(p_\alpha)_B$  from equations (1.9) into equations (1.10), it is easily seen that  $S(x_A, x_B, \zeta_A, \zeta_B)$  satisfies a Hamilton-Jacobi equation both at  $x_A^\alpha$  and at  $x_B^\alpha$

$$\frac{\partial S}{\partial \zeta_A} = H \left( x_A^\alpha, -\frac{\partial S}{\partial x_A^\alpha} \right), \quad \frac{\partial S}{\partial \zeta_B} = -H \left( x_B^\alpha, \frac{\partial S}{\partial x_B^\alpha} \right). \quad (1.11)$$

Let us apply these results to space-time  $V_4$  endowed with a Lorentzian metric  $g_{\mu\nu}$  and insert in equation (1.1) the Lagrangian defined by

$$L = \frac{1}{2} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta. \quad (1.12)$$

Each curve  $\Gamma_{AB}$  parametrized by  $\zeta$  for which  $\tilde{S}$  has an extremum is a geodesic path joining  $x_A$  and  $x_B$ . Parameter  $\zeta$  is said to be affine. According to (1.6), we have now

$$p_\alpha = g_{\alpha\beta} \dot{x}^\beta, \quad H = \frac{1}{2} g^{\alpha\beta} p_\alpha p_\beta, \quad (1.13)$$

from which it is easily deduced that  $H = L$  on any geodesic path. Owing to the fact that  $L$  does not contain  $\zeta$  as an explicit variable, this last equality implies that the Lagrangian  $L$  defined by (1.12) is a constant of the motion.

We are only concerned here with a weak-field metric represented by equation (1.41) throughout space-time. So we henceforth restrict our attention to the domains  $\mathcal{D} = \mathcal{N}_{x_A}, \mathcal{N}_{x_B}$  being defined in Introduction. Then whatever  $x_B \in \mathcal{N}_{x_A}$ , there exists a unique geodesic path  $\Gamma_{AB}$  connecting  $x_A$  and  $x_B$ . Let us denote by  $\hat{\Omega}$  the characteristic function belonging to the Lagrangian (1.12) adapted to  $\mathcal{N}_{x_A}$ . According to (1.7) we have

$$\hat{\Omega}(x_A, x_B, \zeta_A, \zeta_B) = \frac{1}{2} \int_{\zeta_A}^{\zeta_B} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta d\zeta, \quad (1.14)$$

the integral being taken along  $\Gamma_{AB}$ . Since  $L = H$  is a constant of the motion, we have  $L = H_A = H_B$  on  $\Gamma_{AB}$ . Inserting these relations into equation (1.14) yields

$$H_A = H_B = \frac{\hat{\Omega}(x_A, x_B, \zeta_A, \zeta_B)}{\zeta_B - \zeta_A}. \quad (1.15)$$

Substituting these expressions of  $H_A$  and  $H_B$  into equations (1.10), we get

$$\frac{\partial \hat{\Omega}}{\partial \zeta_A} = \frac{\hat{\Omega}}{\zeta_B - \zeta_A}, \quad \frac{\partial \hat{\Omega}}{\partial \zeta_B} = -\frac{\hat{\Omega}}{\zeta_B - \zeta_A}. \quad (1.16)$$

Integrating these equations, we find that  $\hat{\Omega}$  may be written as

$$\hat{\Omega}(x_A, x_B, \zeta_A, \zeta_B) = \frac{\Omega(x_A, x_B)}{\zeta_B - \zeta_A}, \quad (1.17)$$

where  $\Omega(x_A, x_B)$  is a function of  $x_A$  and of  $x_B$ . This two-point function, which is symmetric in  $x_A$  and  $x_B$ , is called the world function. Denoting by  $\lambda$  the unique affine parameter such that

$\lambda_A = 0$  and  $\lambda_B = 1$ , we infer from equations (1.14) and (1.17) that  $\Omega(x_A, x_B)$  may be written as

$$\Omega(x_A, x_B) = \frac{1}{2} \int_0^1 g_{\mu\nu}(x^\alpha(\lambda)) \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} d\lambda, \quad (1.18)$$

the integral being taken along  $\Gamma_{AB}$ . It can be easily seen that

$$\Omega(x_A, x_B) = \frac{1}{2} \epsilon_{AB} [s_{AB}]^2, \quad (1.19)$$

where  $\epsilon_{AB} = 1, 0, -1$  when  $\Gamma_{AB}$  is a timelike, a null or a spacelike geodesic, respectively, and  $s_{AB}$  is the geodesic distance between  $x_A$  and  $x_B$ , i.e.

$$s_{AB} = \int_{\Gamma_{AB}} \sqrt{|g_{\mu\nu} dx^\mu dx^\nu|}.$$

The relevance of the world function in the problems related to the light deflection, the time delay or the gravitational frequency shift is justified by the following properties, which are easily deduced from the above-mentioned theory.

**Property 1** *The covariant components of the vectors tangent to the geodesic path  $\Gamma_{AB}$  at  $x_A$  and  $x_B$  respectively, are given by*

$$\left( g_{\mu\nu} \frac{dx^\nu}{d\lambda} \right)_A = - \frac{\partial \Omega}{\partial x_A^\mu}(x_A, x_B), \quad (1.20)$$

$$\left( g_{\mu\nu} \frac{dx^\nu}{d\lambda} \right)_B = \frac{\partial \Omega}{\partial x_B^\mu}(x_A, x_B). \quad (1.21)$$

These fundamental formulae are immediately deduced from equations (1.9) and (1.13). They show that the vectors tangent to the geodesic path  $\Gamma_{AB}$  at  $x_A$  and  $x_B$  can be explicitly determined when the world function  $\Omega(x_A, x_B)$  is known.

Any other affine parameter  $\zeta$  along  $\Gamma_{AB}$  is such that

$$\zeta = (\zeta_B - \zeta_A)\lambda + \zeta_A, \quad (1.22)$$

where  $\zeta_A$  and  $\zeta_B$  are the values corresponding to  $x_A$  and  $x_B$ , respectively. As a consequence, we have for the tangent vector  $dx^\mu/d\zeta$  the general formulae

$$\left( g_{\mu\nu} \frac{dx^\nu}{d\zeta} \right)_A = - \frac{1}{\zeta_B - \zeta_A} \frac{\partial \Omega}{\partial x_A^\mu}(x_A, x_B), \quad (1.23)$$

$$\left( g_{\mu\nu} \frac{dx^\nu}{d\zeta} \right)_B = \frac{1}{\zeta_B - \zeta_A} \frac{\partial \Omega}{\partial x_B^\mu}(x_A, x_B). \quad (1.24)$$

It is clear that point  $x_B$  may be replaced in (1.24) by any point  $x(\zeta)$  on  $\Gamma_{AB}$  which differs from  $x_A$ . Returning to  $\lambda$  for the sake of simplicity, we find that the covariant components of the vector tangent to  $\Gamma_{AB}$  at point  $x(\lambda)$  are given by

$$\left( g_{\mu\nu} \frac{dx^\nu}{d\lambda} \right)_{x(\lambda)} = \frac{1}{\lambda} \frac{\partial \Omega}{\partial x^\mu}(x_A, x(\lambda)), \quad (1.25)$$

where  $\partial\Omega/\partial x^\mu$  denotes the partial derivative of  $\Omega(x_A, x)$  with respect to  $x^\mu$  at point  $x$ . It immediately follows from equations (1.25) that the system of equations

$$\left(\frac{dx^\mu}{d\lambda}\right)_{x(\lambda)} = \frac{1}{\lambda} g^{\mu\rho}(x(\lambda)) \frac{\partial\Omega}{\partial x^\rho}(x_A, x(\lambda)), \quad (1.26)$$

can be regarded as the first-order differential system governing the geodesic paths passing through a given point  $x_A$ . The regularity of this system at  $x_A$  is a direct consequence of the following property [160].

**Property 2** *The first-order partial derivatives of  $\Omega(x_A, x)$  may be expanded as*

$$\frac{\partial\Omega}{\partial x^\mu}(x_A, x) = g_{\mu\nu}(x_A)(x^\nu - x_A^\nu) + C_{\mu\alpha\beta}(x_A, x)(x^\alpha - x_A^\alpha)(x^\beta - x_A^\beta), \quad (1.27)$$

where the functions  $C_{\mu\alpha\beta}(x_A, x)$  remain bounded in the neighbourhood of  $x_A$ .

It results from this property that  $\partial\Omega/\partial x^\mu(x_A, x) \rightarrow 0$  as  $x \rightarrow x_A$  and that the r.h.s. of equations (1.25) and (1.26) remains bounded as  $\lambda \rightarrow 0$ .

Now, the following statement can be straightforwardly derived from equations (1.11), (1.13) and (1.15).

**Property 3** *The world function  $\Omega(x_A, x_B)$  satisfies the Hamilton-Jacobi equations*

$$\frac{1}{2} g^{\alpha\beta}(x_A) \frac{\partial\Omega}{\partial x_A^\alpha}(x_A, x_B) \frac{\partial\Omega}{\partial x_A^\beta}(x_A, x_B) = \Omega(x_A, x_B), \quad (1.28)$$

$$\frac{1}{2} g^{\alpha\beta}(x_B) \frac{\partial\Omega}{\partial x_B^\alpha}(x_A, x_B) \frac{\partial\Omega}{\partial x_B^\beta}(x_A, x_B) = \Omega(x_A, x_B). \quad (1.29)$$

As we shall see below, these equations and Property 2 enable to construct the world function in any post-Minkowskian approximation.

Properties 1, 2 and 3 are valid whatever the nature of the geodesic curve joining  $x_A$  and  $x_B$ . In the case of null geodesics, (1.18) and  $L = H_A = H_B = 0$  immediately lead to the following statement.

**Property 4** *Two points  $x_A$  and  $x_B$  are joined by a light ray if and only if the condition*

$$\Omega(x_A, x_B) = 0 \quad (1.30)$$

*is fulfilled.*

### 1.2.2 Time Transfer Functions Formalism

Thus,  $\Omega(x_A, x) = 0$  is the equation of the light cone  $\mathcal{C}_{x_A}$ . It follows from Property 4 that if  $\Omega(x_A, x_B)$  is known in  $\mathcal{N}_{x_A}$ , it is possible to determine the travel time  $t_B - t_A$  of a photon connecting two points  $x_A$  and  $x_B$  as a function of  $t_A$ ,  $\mathbf{x}_A$  and  $\mathbf{x}_B$  or as a function of  $t_B$ ,  $\mathbf{x}_A$  and  $\mathbf{x}_B$ . It must be pointed out, however, that solving the equation  $\Omega(ct_A, \mathbf{x}_A, ct_B, \mathbf{x}_B) = 0$  for  $t_B$  yields two distinct solutions  $t_B^+$  and  $t_B^-$  since the timelike curve  $x^i = x_B^i$  cuts the light cone  $\mathcal{C}_{x_A}$  at two points  $x_B^+$  and  $x_B^-$ ,  $x_B^+$  being in the future of  $x_B^-$ .

In the present document, we always consider  $x_A$  as the point of emission of the photon and  $x_B$  as the point of reception, and we focus our attention on the determination of  $t_B^+ - t_A$  (clearly, the determination of  $t_B^- - t_A$  comes within the same methodology). For the sake of brevity, we shall henceforth write  $t_B$  instead of  $t_B^+$ .

In general,  $t_B - t_A$  may be considered either as a function of the instant of emission  $t_A$ , and of  $\mathbf{x}_A$ ,  $\mathbf{x}_B$ , or as a function of the instant of reception  $t_B$  and of  $\mathbf{x}_A$ ,  $\mathbf{x}_B$ . So we are led to introduce two distinct (coordinate) time transfer functions  $\mathcal{T}_e$  and  $\mathcal{T}_r$  respectively defined by

$$t_B - t_A = \mathcal{T}_e(t_A, \mathbf{x}_A, \mathbf{x}_B), \quad (1.31)$$

and

$$t_B - t_A = \mathcal{T}_r(t_B, \mathbf{x}_A, \mathbf{x}_B). \quad (1.32)$$

We shall call  $\mathcal{T}_e(t_A, \mathbf{x}_A, \mathbf{x}_B)$  the emission time transfer function and  $\mathcal{T}_r(t_B, \mathbf{x}_A, \mathbf{x}_B)$  the reception time transfer function.

There exist direct relations between the time transfer functions and the components of the vector tangent to a null geodesic. Indeed, it results from equations (1.30) and (1.31) that whatever  $\mathbf{x}_B$ ,  $x_A^0$  and  $\mathbf{x}_A$ , one has the relation

$$\Omega(x_A^0, \mathbf{x}_A, x_A^0 + c\mathcal{T}_e(t_A, \mathbf{x}_A, \mathbf{x}_B), \mathbf{x}_B) \equiv 0. \quad (1.33)$$

Differentiating this identity with respect to  $x_A^0$ ,  $x_A^i$  and  $x_B^i$ , respectively, it is easily seen that the relations

$$\frac{\partial \Omega}{\partial x_A^0}(x_A, x_B) + \frac{\partial \Omega}{\partial x_B^0}(x_A, x_B) \left[ 1 + \frac{\partial \mathcal{T}_e}{\partial t_A}(t_A, \mathbf{x}_A, \mathbf{x}_B) \right] = 0, \quad (1.34)$$

$$\frac{\partial \Omega}{\partial x_A^i}(x_A, x_B) + c \frac{\partial \Omega}{\partial x_B^0}(x_A, x_B) \frac{\partial \mathcal{T}_e}{\partial x_A^i}(t_A, \mathbf{x}_A, \mathbf{x}_B) = 0, \quad (1.35)$$

$$c \frac{\partial \Omega}{\partial x_B^0}(x_A, x_B) \frac{\partial \mathcal{T}_e}{\partial x_B^i}(t_A, \mathbf{x}_A, \mathbf{x}_B) + \frac{\partial \Omega}{\partial x_B^i}(x_A, x_B) = 0, \quad (1.36)$$

hold for any couple of points  $(x_A, x_B)$  connected by a null geodesic. Of course, analogous relations may be derived from the identity

$$\Omega(x_B^0 - c\mathcal{T}_r(t_B, \mathbf{x}_A, \mathbf{x}_B), \mathbf{x}_A, x_B^0, \mathbf{x}_B) \equiv 0. \quad (1.37)$$

Comparing these relations with equations (1.20)-(1.21), we get the following theorem for the components of the vectors tangent to a light ray.

**Property 5** Consider a photon emitted at point  $x_A = (ct_A, \mathbf{x}_A)$  and received at point  $x_B = (ct_B, \mathbf{x}_B)$ . Denote by  $k^\mu$  the vector  $dx^\mu/d\zeta$  tangent to the photon path,  $\zeta$  being any affine parameter. Then, one has relations as follow for the covariant components of the vector tangent at  $x_A$  and  $x_B$

$$\left(\frac{k_i}{k_0}\right)_B = -c \frac{\partial \mathcal{T}_e}{\partial x_B^i} = -c \frac{\partial \mathcal{T}_r}{\partial x_B^i} \left[1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1}, \quad (1.38)$$

$$\left(\frac{k_i}{k_0}\right)_A = c \frac{\partial \mathcal{T}_e}{\partial x_A^i} \left[1 + \frac{\partial \mathcal{T}_e}{\partial t_A}\right]^{-1} = c \frac{\partial \mathcal{T}_r}{\partial x_A^i}, \quad (1.39)$$

$$\frac{(k_0)_B}{(k_0)_A} = \left[1 + \frac{\partial \mathcal{T}_e}{\partial t_A}\right]^{-1} = 1 - \frac{\partial \mathcal{T}_r}{\partial t_B}, \quad (1.40)$$

where  $\mathcal{T}_e$  and  $\mathcal{T}_r$  are taken at  $(t_A, \mathbf{x}_A, \mathbf{x}_B)$  and  $(t_B, \mathbf{x}_A, \mathbf{x}_B)$ , respectively.

These fundamental formulae show that all the theoretical problems related to the directions of light rays or to the frequency shifts may be solved as soon as at least one of the time transfer functions is explicitly determined. This property will be very useful in practice since extracting the time transfer formulae (1.31) or (1.32) from equation (1.30), next using equations (1.63)-(1.40) will be more straightforward than deriving the vectors tangent at  $x_A$  and  $x_B$  from equations (1.20)-(1.21), next imposing constraint (1.30).

### 1.2.3 General Post-Minkowskian Expansion of the World and Time Transfer Functions

With Pierre Teyssandier and Bernard Linet, I found a way in 2004 to express the World Function and the Time Transfer Functions as general post-minkowskian series in weak gravitational field, where perturbation are evaluated along a line integral. Then, assuming that space-time is globally regular with the topology  $\mathbb{R} \times \mathbb{R}^3$ , *i.e.* without event horizon, we can admit that the Lorentzian metric is represented at any point  $x$  by a series in ascending powers of  $G$

$$g_{\mu\nu}(x, G) = g_{\mu\nu}^{(0)} + \sum_{n=1}^{\infty} G^n g_{\mu\nu}^{(n)}(x), \quad (1.41)$$

where

$$g_{\mu\nu}^{(0)} = \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

in any quasi Cartesian coordinates. Neglecting all terms involving  $g_{\mu\nu}^{(n+1)}$ ,  $g_{\mu\nu}^{(n+2)}$ , ... defines the so-called  $n$ th post-Minkowskian approximation.

#### Post-Minkowskian expansion of the World Function

Taking equation (1.41) into account, it is possible to assume that  $\Omega(x_A, x_B)$  admits also an expansion as follows

$$\Omega(x_A, x_B) = \Omega^{(0)}(x_A, x_B) + \sum_{n=1}^{\infty} G^n \Omega^{(n)}(x_A, x_B). \quad (1.42)$$

It is trivial to find the expression  $\Omega^{(0)}(x_A, x_B)$ . In a Minkowskian background, the zeroth-order light ray trajectory is simply a Minkowskian straight line,  $\Gamma_{AB}^{(0)}$ , connecting  $x_A$  and  $x_B$  which can be parametrized as follow

$$x_{(0)}^\alpha(\lambda) = (x_B^\alpha - x_A^\alpha)\lambda + x_A^\alpha, \quad 0 \leq \lambda \leq 1. \quad (1.43)$$

$\Omega^{(0)}(x_A, x_B)$  can be obtained by replacing in Equation (1.18)  $g$  by  $\eta$  and  $x^\alpha$  by Eq. (1.43), respectively. We get immediately

$$\Omega^{(0)}(x_A, x_B) = \frac{1}{2}\eta_{\mu\nu}(x_B^\mu - x_A^\mu)(x_B^\nu - x_A^\nu). \quad (1.44)$$

In my 2004 paper [110], I found a recursive procedure for determining each term  $\Omega^{(n)}(x_A, x_B)$  within the  $n$ th post-Minkowskian approximation without calculating the geodesic  $\Gamma_{AB}$ , joining  $x_A$  and  $x_B$ . Indeed, all along the null geodesics joining these two point-events, the Hamilton-Jacobi are fulfilled; if we suppose  $x = (ct, \mathbf{x})$  as a *free parameter*, a point-event belonging to  $\Gamma_{AB}$ , we can introduce a new world function  $\Omega(x_A, x) = \bar{\Omega}(x)$  between  $x_A$  and  $x$ . Of course, we can also assume that  $\bar{\Omega}(x)$  admits a post-Minkowskian expansion

$$\bar{\Omega}(x) = \sum_{n=0}^{\infty} G^n \bar{\Omega}^{(n)}(x) \quad \text{with} \quad \bar{\Omega}(x_A) = 0. \quad (1.45)$$

$\bar{\Omega}(x)$  satisfies the Hamilton-Jacobi in  $x$ , so we get

$$g^{\alpha\beta}(x) \frac{\partial \bar{\Omega}(x)}{\partial x^\alpha} \frac{\partial \bar{\Omega}(x)}{\partial x^\beta} = g^{\alpha\beta}(x) \partial_\mu \bar{\Omega}(x) \partial_\nu \bar{\Omega}(x) = 2\bar{\Omega}(x). \quad (1.46)$$

I demonstrated that equation (1.46) can be replaced by an infinite set of ordinary differential equations for the perturbation terms  $\Omega^{(n)}(x_A, x)$  when  $x$  is constrained to move along the zeroth-order geodesic  $\Gamma_{AB}^{(0)}$  defined by Eq. (1.43). As a consequence, each term  $\Omega^{(n)}(x_A, x_B)$  is obtained in the form of a line integral along a straight line in the background Minkowski metric. Thus, I obtained a recursive procedure which completely avoids the calculation of the perturbation of the geodesic joining the given end points, leading to the following property :

**Property 6** *Assuming spacetime metric as a post-minkowskian expansion (1.41), the world function between two point-events  $x_A$  and  $x_B$  is given by*

$$\Omega(x_A, x_B) = \Omega^{(0)}(x_A, x_B) + \sum_{n=1}^{\infty} G^n \Omega^{(n)}(x_A, x_B),$$

where

$$\Omega^{(1)}(x_A, x_B) = -\frac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma}(x_B^\rho - x_A^\rho)(x_B^\sigma - x_A^\sigma) \int_0^1 g_{(1)}^{\mu\nu}(x_{(0)}(\lambda))d\lambda, \quad (1.47)$$

$$\begin{aligned} \Omega^{(n)}(x_A, x_B) &= -\frac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma}(x_B^\rho - x_A^\rho)(x_B^\sigma - x_A^\sigma) \int_0^1 g_{(n)}^{\mu\nu}(x_{(0)}(\lambda))d\lambda \\ &\quad -\frac{1}{2} \int_0^1 \sum_{p=1}^{n-1} g_{(p)}^{\mu\nu}(x_{(0)}(\lambda)) \left[ \eta_{\mu\rho}(x_B^\rho - x_A^\rho) \frac{1}{\lambda} \frac{\partial \Omega^{(n-p)}}{\partial x^\nu}(x_A, x_{(0)}(\lambda)) \right. \\ &\quad \left. + \sum_{q=1}^{n-p} \frac{1}{\lambda^2} \frac{\partial \Omega^{(q)}}{\partial x^\mu}(x_A, x_{(0)}(\lambda)) \frac{\partial \Omega^{(n-p-q)}}{\partial x^\nu}(x_A, x_{(0)}(\lambda)) \right] d\lambda \\ &\quad -\frac{1}{2} \int_0^1 \sum_{p=1}^{n-1} \frac{1}{\lambda^2} \eta^{\mu\nu} \frac{\partial \Omega^{(p)}}{\partial x^\mu}(x_A, x_{(0)}(\lambda)) \frac{\partial \Omega^{(n-p)}}{\partial x^\nu}(x_A, x_{(0)}(\lambda)) d\lambda, \end{aligned} \quad (1.48)$$

whatever  $n \geq 2$ , all integrals being evaluated along the Minkowskian straight line defined by Eq. (1.43).

### Post-Minkowskian expansion of the TTF from the World Function

In 2004, the first method to determine the Time Transfer Functions was to deduce them from the Synge World Function. Let be  $x_A = (ct_A, \mathbf{x}_A)$  et  $x_B = (ct_B, \mathbf{x}_B)$  two point-events on a manifold endowed by a metric admitting the general post-Minkowskian expansion (1.41) and supposing that  $\Omega$  is also admitting an expansion as

$$\Omega(x_A, x_B) = \Omega^{(0)}(x_A, x_B) + \Omega^{(PM)}(x_A, x_B), \text{ where } \Omega^{(PM)}(x_A, x_B) = \sum_{n=1}^{\infty} G^n \Omega^{(n)}(x_A, x_B), \quad (1.49)$$

and  $\Omega^{(0)}(x_A, x_B)$  is the World Function in Minkowski spacetime as defined by (1.44), we can easily deduce from (1.44)

$$\Omega^{(0)}(x_A, x_B) = \frac{1}{2}\eta_{\mu\nu}(x_B^\mu - x_A^\mu)(x_B^\nu - x_A^\nu) = c^2(t_B - t_A)^2 - R_{AB}^2, \quad (1.50)$$

$R_{AB}$  being defined by  $R_{AB}^2 = \delta_{ij}(x_B^i - x_A^i)(x_B^j - x_A^j)$ . Imposing  $\Omega = 0$  in (1.49) and substituting (1.50) to  $\Omega^{(0)}(x_A, x_B)$ ,  $\mathcal{T}_e$  and  $\mathcal{T}_r$  to  $t_B - t_A$ , respectively, we get

$$c^2\mathcal{T}_e^2 = R_{AB}^2 - 2 \sum_{n=1}^{\infty} G^n \Omega^{(n)}(ct_A, \mathbf{x}_A, ct_A + c\mathcal{T}_e, \mathbf{x}_B), \quad (1.51)$$

and

$$c^2\mathcal{T}_r^2 = R_{AB}^2 - 2 \sum_{n=1}^{\infty} G^n \Omega^{(n)}(ct_B - c\mathcal{T}_r, \mathbf{x}_A, ct_B, \mathbf{x}_B). \quad (1.52)$$

The question is now to inverse  $\Omega^{(n)}$ , depending themselves on  $\mathcal{T}_e$  et  $\mathcal{T}_r$ . This kind of problems have been treated by Lagrange in celestial mechanics, in particular concerning the Kepler equation; let us consider a function  $u(x, \varepsilon)$  defined by

$$u^2 = x^2 + \sum_{n=1}^{\infty} \varepsilon^n h_n(u) \quad (1.53)$$

satisfying

$$u(x, \varepsilon)|_{\varepsilon=0} = x, \quad (1.54)$$

$\varepsilon$  being a small parameter (which is the case for the Gravitational constant  $G$ ) in order to satisfy the convergence of the series. Using a MacLaurin expansion of  $u$  in the neighborhood of  $\varepsilon = 0$ , the solution of (1.53) can be written

$$u(x, \varepsilon) = x + \sum_{n=1}^{\infty} \varepsilon^n u_n(x),$$

where

$$u_n(x) = \frac{1}{n!} \left. \frac{\partial^n u(x, \varepsilon)}{\partial \varepsilon^n} \right|_{\varepsilon=0}.$$

One has then to calculate successive derivatives of (1.53) with respect to  $\varepsilon$  whilst taking into account (1.54). A straightforward calculation gives  $u_n(x)$  whatever  $n$ . The first elements can be obtained as follows

$$\begin{aligned} u_1(x) &= \frac{h_1(x)}{2x}, \\ u_2(x) &= \frac{1}{2x} [h_2(x) + h_1'(x)u_1(x) - u_1^2(x)], \\ u_3(x) &= \frac{1}{2x} \left[ h_3(x) + h_2'(x)u_1(x) + \frac{1}{2}h_1''(x)u_1^2(x) + h_1'(x)u_2(x) - 2u_1(x)u_2(x) \right]. \end{aligned} \quad (1.55)$$

Applying this procedure to (1.51), it is possible to determine explicitly  $\mathcal{T}_e$  at all pos-Minkowskian order of approximation. First, let us introduce two useful notations as follow

$$\tilde{\Omega}_e^{(n)}(t_A, \mathbf{x}_A, \mathbf{x}_B) = \Omega^{(n)}(ct_A, \mathbf{x}_A, ct_A + R_{AB}, \mathbf{x}_B) \quad (1.56)$$

and

$$\tilde{\Omega}_{e|k}^{(n)}(t_A, \mathbf{x}_A, \mathbf{x}_B) = \frac{\partial^k \Omega^{(n)}}{(\partial ct_B)^k}(ct_A, \mathbf{x}_A, ct_A + R_{AB}, \mathbf{x}_B) \quad (1.57)$$

with  $k = 1, 2, 3, \dots$ . Taking into account (1.55)-(1.57) and substituting  $\mathcal{T}_e$  to  $u$ ,  $R_{AB}$  to  $x$  and  $\Omega^{(n)}$  to  $h_n$  in (1.53), we obtain the following property

**Property 7** *At the third post-Minkowskian order of approximation, the Time Transfer Function in emission, characterizing the coordinate time of flight of a photon between a point-event of emission  $x_A$  and a point-event of reception  $x_B$  is given by*

$$\mathcal{T}_e(t_A, \mathbf{x}_A, \mathbf{x}_B) = \frac{1}{c}R_{AB} + \sum_{n=1}^3 G^n \mathcal{T}_e^{(n)}(t_A, \mathbf{x}_A, \mathbf{x}_B) + O(G^4), \quad (1.58)$$

where

$$\mathcal{T}_e^{(1)} = -\frac{\tilde{\Omega}_e^{(1)}}{cR_{AB}}, \quad (1.59)$$

$$\mathcal{T}_e^{(2)} = -\frac{1}{cR_{AB}} \left[ \tilde{\Omega}_e^{(2)} + c\mathcal{T}_e^{(1)}\tilde{\Omega}_{e|1}^{(1)} + \frac{1}{2}c^2(\mathcal{T}_e^{(1)})^2 \right], \quad (1.60)$$

$$\mathcal{T}_e^{(3)} = -\frac{1}{cR_{AB}} \left[ \tilde{\Omega}_e^{(3)} + c\mathcal{T}_e^{(1)}\tilde{\Omega}_{e|1}^{(2)} + \frac{1}{2}c^2(\mathcal{T}_e^{(1)})^2\tilde{\Omega}_{e|2}^{(1)} + c\mathcal{T}_e^{(2)}\tilde{\Omega}_{e|1}^{(1)} + c^2\mathcal{T}_e^{(1)}\mathcal{T}_e^{(2)} \right], \quad (1.61)$$

quantities  $\widetilde{\Omega}_e^{(n)}$  and  $\widetilde{\Omega}_{e|k}^{(n)}$  being defined by (1.56) and (1.57), respectively.

A similar reasoning can be performed for  $\mathcal{T}_r$ .

### Stand-alone post-Minkowskian expansion of the Time Transfer Functions

It is worth to say that the calculation of the Time Transfer Functions from the determination of the Synge's World Function  $\Omega(x_A, x_B)$  and then deducing light travel time from the equation  $\Omega(x_A, x_B) = 0$  is a procedure which works quite well, but presents an unpleasant drawback : once the general post-Minkowskian expansion of the world function is known, obtaining the corresponding expansion of light travel time still requires a lot of additional calculations, as we demonstrated in [165].

It is why I reconsider the problem with Pierre Teyssandier in order to find a stand-alone method which totally avoids the calculation of the World Function. Going back to the property 5, let us focus on two particular relations, *i.e.*,

$$\left(\frac{k_i}{k_0}\right)_{x_A} = c \frac{\partial \mathcal{T}_r(\mathbf{x}_A, t_B, \mathbf{x}_B)}{\partial x_A^i} \quad (1.62)$$

and

$$\left(\frac{k_i}{k_0}\right)_{x_B} = -c \frac{\partial \mathcal{T}_e(t_A, \mathbf{x}_A, \mathbf{x}_B)}{\partial x_B^i}. \quad (1.63)$$

The covariant components of the vector tangent to  $\Gamma_{AB}$  at  $x_A$  satisfy the equation

$$(g^{\mu\nu} k_\mu k_\nu)_{x_A} = 0. \quad (1.64)$$

Dividing equation (1.64) side by side by  $[(k_0)_A]^2$ , and then taking equation (1.62) into account yield<sup>1</sup>.

$$g^{00}(x_B^0 - c\mathcal{T}_r, \mathbf{x}_A) + 2c g^{0i}(x_B^0 - c\mathcal{T}_r, \mathbf{x}_A) \frac{\partial \mathcal{T}_r}{\partial x_A^i} + c^2 g^{ij}(x_B^0 - c\mathcal{T}_r, \mathbf{x}_A) \frac{\partial \mathcal{T}_r}{\partial x_A^i} \frac{\partial \mathcal{T}_r}{\partial x_A^j} = 0 \quad (1.65)$$

The same reasoning using  $(g^{\mu\nu} k_\mu k_\nu)_{x_B} = 0$  and equation (1.63) leads to

$$g^{00}(x_A^0 + c\mathcal{T}_e, \mathbf{x}_B) - 2c g^{0i}(x_A^0 + c\mathcal{T}_e, \mathbf{x}_B) \frac{\partial \mathcal{T}_e}{\partial x_B^i} + c^2 g^{ij}(x_A^0 + c\mathcal{T}_e, \mathbf{x}_B) \frac{\partial \mathcal{T}_e}{\partial x_B^i} \frac{\partial \mathcal{T}_e}{\partial x_B^j} = 0. \quad (1.66)$$

Equations (1.65) and (1.66) are nothing else that Hamilton-Jacobi-like partial differential equation. In [163], I showed that these two relations can be rewritten as two integro-differential equations on which a post-Minkowskian can be performed without difficulties. Indeed I obtained a recursive procedure at the  $n$ th post-Minkowskian approximation which spares the trouble of solving the geodesic differential equations and avoids determining Synge's world function, any  $n$ th-order perturbation term being an integral taken along a zeroth-order null straight line.

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1. The covariant component  $k_0$  of a null vector  $k$  cannot vanish in the chosen coordinate system since the vector  $\partial/\partial x^0$  is assumed to be timelike everywhere.

### 1.2.4 Some applications with the TTF formalism

Several projects emerged from this work on TTF formalism. The first main idea was to develop a complete relativistic ray tracing modeling based on TTF, dedicated to space astrometry. A second idea was to consider the TTF formalism as the basic algebra to express accurately the radioscience observables, used for deep space navigation and planeto-dynamics.

With a Ph.D. student, Stefano Bertone, I constructed a TTF-based astrometric model for observable Gaia modeling when Solar System deflecting bodies are considered to be punctual in uniform motion [16]. We compared our approach to the two ray tracing models commonly used by the Data Processing and Analysis Consortium (DPAC) of the Gaia mission : GREM [81] and RAMOD [43], validating our approach at the high level of  $0.1\mu\text{as}$ . We continued this work in a very close collaboration with the Torino Observatory, which hosts a Gaia data center. By merging our 2014 model with the Italian astrometric data analysis pipeline called GSR (Global Sphere Reconstruction) [170], we obtained a new pipeline, GSR-TTF, now fully operational in Torino [17].

These years of work have taught me a very important lesson : all groups (mine, Torino and Dresden) have noticed the emergence of divergent terms in their first-order relativistic formulation. So we need to push our calculations to the second order of corrections in order to construct a reliable model [84, 115], even if a priori the first order seems to be sufficient. By analogy, it is a situation quite similar to perturbative work in celestial mechanics.

Furthermore, space astrometry is not the only very high precision technique currently available. It is also the case for deep space navigation. Indeed Range accuracy improved by one order of magnitude during the last 10 years (from 1 meter for the NASA Cassini probe to 10 cm for the ESA BepiColombo mission [127, 67]). Thanks to the development of X- and Ka-band transponders, Doppler accuracy increased drastically from  $\approx 10$  mHz for Pioneer Venus Orbiter [85] (at 60 s integration time) to the  $\mu\text{Hz}$  level for BepiColombo and for Juno [64] at 1000 s integration time. Improvements in the technical accuracy of these observables result in better constraints on their scientific interpretation and have consequences in several domains. For this reason, a continuous effort is necessary to keep up the modeling with the increasing accuracy of instruments and mission goals. It is of course the case for relativistic modeling of light propagation in radioscience. I decided then to extend explicitly the TTF formalism to the second and even the third order of post-Minkowskian order of corrections.

First of all, I completed with my students a complete post-Newtonian first-order theory in the gravitational field of uniformly moving extended bodies (axisymmetric case). We obtained a very general result that we applied to the JUNO mission, showing that the Jupiter quadrupole moment (*i.e.* its flattening) has a significant influence on signal propagation [74]. Indeed, the level of correction is of the same order of magnitude than the accuracy of the measurements and must be considered in the data analysis. As illustrated on Figure (1.2), the Doppler signal (left plot) can reach the level of several  $10^{-5}$  m/s and the Range signal is roughly about the centimeter level (right plot). I communicated this result to NASA experts involved in the radioscience of JUNO (W. Folker and B. Jacobson at Jet Propulsion Laboratory) who successfully implemented my new formula in their navigation data analysis.

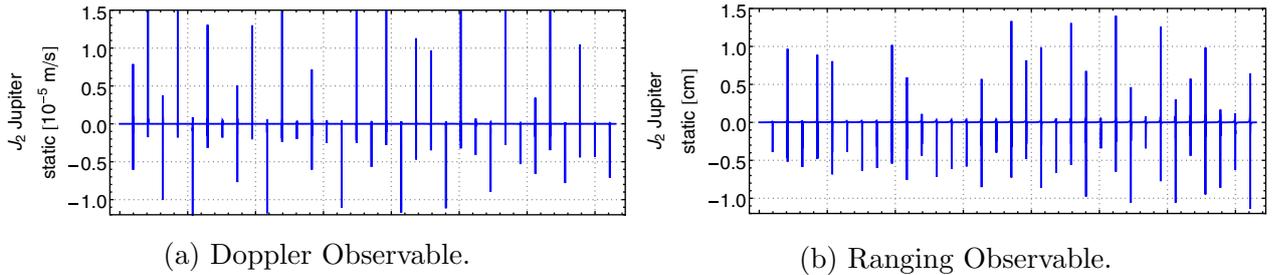


FIGURE 1.2 – Simulation of impact of Jupiter flattening on the radioscience observables of the Juno mission.

Moreover, I studied with S. Bertone the impact of transponder delay in the formulation of Doppler and Range [15]. We modified the standard approach developed by Théo Moyer [128] by including a new term, modeling explicitly the transponder internal delay, in the computation of the light-time equation. We considered several interesting cases such as an Earth swing-by, NASA GRAIL [185] and ESA BepiColombo missions. We used the planetary extension of the Bernese GNSS software [45] to simulate two-way X-band Doppler for BepiColombo Mercury Planetary Orbiter (MPO) nominal orbit retrieved from ESA Spice SPK for 08/04/2025. We first computed Doppler data as observed by the Deep Space Network antennas following the standard formulation by [128]. Then, we included the transponder delay in the light-time modeling used for the simulation. We computed the resulting Doppler signal for several values of transponder delay in the range  $10^{-6} - 10^{-3}$  s and show the differences with respect to [128] in Fig. 1.3.

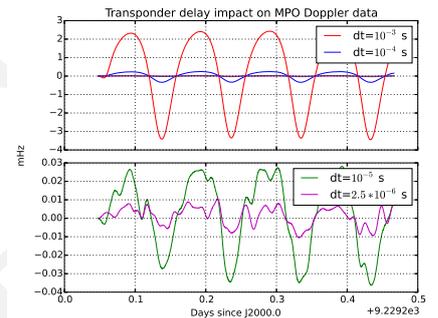


Fig. 1.3 – Doppler difference for the nominal MPO orbit around Mercury on 08/04/2025 for different values of the transponder delay  $dt$  ( $1 \text{ mHz} \approx 0.035 \text{ mm/s} @ 8.4 \text{ GHz}$ ).

Second-order of relativistic post-Minkowskian corrections were considered with caution. My goal was to obtain either new useful analytical formulas and/or the simplest possible numerical formulation. It is exactly what we did with my students in [75]. After expressing TTF at second PM order in the most simplest way possible, we proposed a numerical scheme to calculate these functions. We applied our result to the GAME project [62] and the BepiColombo mission. We were able to highlight an interesting enhanced term to be considered in the future data analysis of the radioscience BepiColombo MORE experiment [78]. The accuracy of the MORE instrument being around 10cm, Figure (1.4) illustrates that, *modulo* particular orbital configurations of the probe, relativistic peaks of 2nd order of 80cm can appear and therefore must be cleaned.

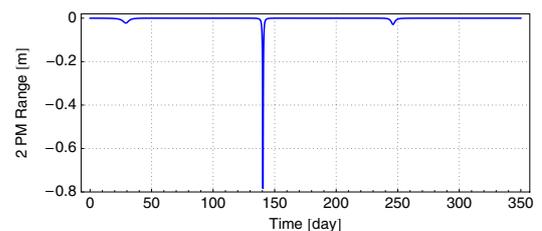


Fig. 1.4 – 2PM Simulation of Mercury Range during Bepi-Colombo mission.

All of this works led me to be involved in several space projects. Indeed, in 2012, I was co-Pi of the M3 GETEMME proposal [137]. This project was aimed to study in details the

martian satellites, Phobos and Deimos, by using space geodesy techniques applied to the martian system. This proposal was complementary with PHOBOS Grunt mission which was not launched successfully and the spacecraft were lost quickly. I also became responsible for the *astrometric core processing* workpackages of the Theia consortium [166] when answering to ESA M4 and M5 calls. I am currently member of the Science Team of the E-GRASP proposal to the ESA Earth Explorer 10 call.

### Related Publications :

1. S. BERTONE, C. LE PONCIN-LAFITTE, P. ROSENBLATT, V. LAINEY, J.-C. MARTY et M.-C. ANGININ. « Impact analysis of the transponder time delay on radio-tracking observables ». In : *Advances in Space Research* 61 (jan. 2018), p. 89–96. DOI : [10.1016/j.asr.2017.09.003](https://doi.org/10.1016/j.asr.2017.09.003). arXiv : [1708.00546](https://arxiv.org/abs/1708.00546) [astro-ph.EP]
2. S. BERTONE, A. VECCHIATO, B. BUCCIARELLI, M. CROSTA, M. G. LATTANZI, L. BIANCHI, M.-C. ANGININ et C. LE PONCIN-LAFITTE. « Application of time transfer functions to Gaia’s global astrometry. Validation on DPAC simulated Gaia-like observations ». In : *A&A* 608, A83 (déc. 2017), A83. DOI : [10.1051/0004-6361/201731654](https://doi.org/10.1051/0004-6361/201731654). arXiv : [1708.00541](https://arxiv.org/abs/1708.00541) [astro-ph.IM]
3. A. HEES, S. BERTONE et C. LE PONCIN-LAFITTE. « Light propagation in the field of a moving axisymmetric body : Theory and applications to the Juno mission ». In : *Physical Review D* 90.8, 084020 (oct. 2014), p. 084020. DOI : [10.1103/PhysRevD.90.084020](https://doi.org/10.1103/PhysRevD.90.084020). arXiv : [1406.6600](https://arxiv.org/abs/1406.6600) [gr-qc]
4. A. HEES, S. BERTONE et C. LE PONCIN-LAFITTE. « Relativistic formulation of coordinate light time, Doppler, and astrometric observables up to the second post-Minkowskian order ». In : *Physical Review D* 89.6, 064045 (mar. 2014), p. 064045. DOI : [10.1103/PhysRevD.89.064045](https://doi.org/10.1103/PhysRevD.89.064045). arXiv : [1401.7622](https://arxiv.org/abs/1401.7622) [gr-qc]
5. S. BERTONE, O. MINAZZOLI, M. CROSTA, C. LE PONCIN-LAFITTE, A. VECCHIATO et M.-C. ANGININ. « Time transfer functions as a way to validate light propagation solutions for space astrometry ». In : *Classical and Quantum Gravity* 31.1, 015021 (jan. 2014), p. 015021. DOI : [10.1088/0264-9381/31/1/015021](https://doi.org/10.1088/0264-9381/31/1/015021). arXiv : [1306.2367](https://arxiv.org/abs/1306.2367) [gr-qc]
6. P. TEYSSANDIER et C. LE PONCIN-LAFITTE. « General post-Minkowskian expansion of time transfer functions ». In : *Classical and Quantum Gravity* 25.14, 145020 (juil. 2008), p. 145020. DOI : [10.1088/0264-9381/25/14/145020](https://doi.org/10.1088/0264-9381/25/14/145020). arXiv : [0803.0277](https://arxiv.org/abs/0803.0277) [gr-qc]
7. C. LE PONCIN-LAFITTE, B. LINET et P. TEYSSANDIER. « World function and time transfer : general post-Minkowskian expansions ». In : *Classical and Quantum Gravity* 21 (sept. 2004), p. 4463–4483. DOI : [10.1088/0264-9381/21/18/012](https://doi.org/10.1088/0264-9381/21/18/012). eprint : [gr-qc/0403094](https://arxiv.org/abs/gr-qc/0403094)

### 1.3 Celestial mechanics, ephemerides and tidal effects

The preparation of the Gaia mission led to a modification of planetary ephemerides. Indeed, all available ray tracing modeling used as timescale TCB (Coordinate Barycentric Time) of IAU conventional BCRS (Barycentric Celestial Reference System). Usually, planetary ephemerides realized their own timescale TDB (Dynamical Barycentric Time). An adequate timescale transformation, from TDB to TCB, was then necessary. It was possible by using algorithms developed in [48, 80, 71]. However these procedures were not fully satisfactory because they are performed *a posteriori* from a given ephemeride, generally by using different gravitational fields to calculate the time dilation. It led to an incoherence. In 2005 at the Institut de Mécanique Céleste et de Calcul des Ephémérides (IMCCE), Jacques Laskar and collaborators prepared a new numerical ephemeride called INPOP (Intégrateur Numérique Planétaire de l'Observatoire de Paris). Dr. Laskar invited me to participate to this adventure during several months between fall 2005 and Spring 2006, just before I got my post-doc position in Germany. I participated to the project with several dedicated tasks concerning the relativistic modeling used by INPOP : equations of motion, conservation laws and timescales.

In consequence, I co-authored the publication of INPOP08 where my contribution was first to check the relativistic equations of motion used in INPOP and to determine exactly the motion of the barycentre. I proposed also a way to calculate, simultaneously with the equation of motions, the difference between TCB and TDB. It led to the production of the first 4D planetary ephemerides [53]. Fig. (1.5) shows the quality of the stability of INPOP08 barycentre, which is constrained at a level below 0.1 millimeter over two centuries.

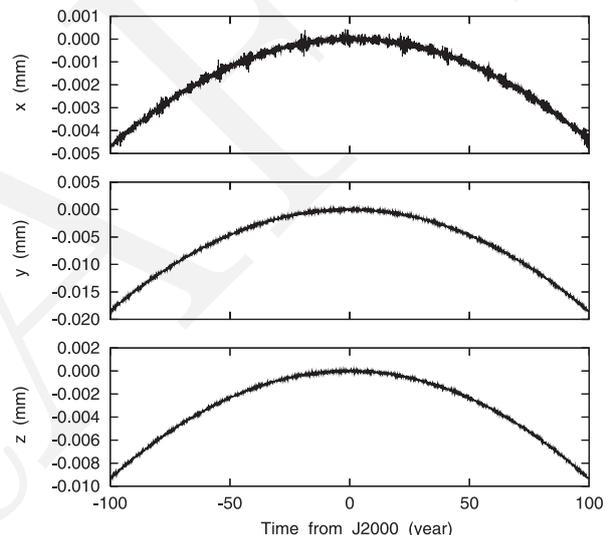


Fig. 1.5 — The INPOP Center of mass drift in mm.

When I have been hired as an assistant astronomer in 2009, Valéry Lainey contacted me. At that time, he published a paper in Nature [99] demonstrating that the tidal dissipation, traditionally called the  $Q$  factor [69], inside the Io-Jupiter system was substantial. The same year, using Cartesian symmetric trace free tensor, I published with Stéphane Mathis a very general paper on tidal effect going beyond the punctual approximation for the tidal perturber. I noticed indeed a strong correlation between relativistic and tidal effects when fitting ephemerides. Having already in mind GR tests with Lunar Laser Ranging, I decided to study more deeply tidal effects between two extended bodies and, more precisely, the interaction between mass multipole moments of their gravitational fields and the associated tidal phenomena, obtaining the most general formalism to study tides [120].

Dr. Lainey was interested by saturnian moons with an objective to combine ground optical astrometry over long periods (*i.e.* centuries) and the Cassini data. He detected indeed for the first time tidal dissipation in this system and his goal was to build an extremely accurate

ephemeris of the Saturnian moons of Saturn to confirm this new result. Valéry therefore formed a group of experts from different fields to study this discovery, the international group *Encelade 2.0*. In this collaboration, my role was to be his expert in GR and tide. I validated all modelings concerning observation process and dynamical equations of motion, in order to avoid any a conceptual mistake (or a missing term), in particular concerning GR effects. I co-authored the articles of the collaboration where I had a substantial contribution [33, 102, 100].

We continued with Stéphane Mathis to study tidal dynamics, in particular by focusing on the rheology of the bodies. We co-supervised Pierre Auclair-Desrotour during his M2 internship. We asked P. Auclair-Desrotour to study the contribution of inertial and gravito-inertial waves on the tide, so the answer of a superficial fluid layer. This led to the publication of several papers [6, 5, 119]. We computed the tidal kinetic energy dissipated by viscous friction and thermal diffusion in a rotating local fluid Cartesian section of a star, planet, or moon submitted to a periodic tidal forcing. The properties of tidal gravito-inertial waves excited by the perturbation were derived analytically as explicit functions of the tidal frequency and local fluid parameters (*i.e.* the rotation, the buoyancy frequency characterizing the entropy stratification, viscous and thermal diffusivities) for periodic normal modes. We demonstrated the strong impact of the internal structure and of the rheology of the central body on the orbital evolution of the tidal perturber : a smooth frequency-dependence of the tidal dissipation causing a smooth orbital evolution while a peaked dissipation can produce erratic orbital behavior.

As shown on Figure (1.6), the influence of the rheology of the dissipative body can be very varied according to its internal structure. Indeed, this figure summarizes a *Gedankenexperiment* where we imagined the fall of Phobos on a rocky Mars (blue dotted reference curve with constant  $Q$  factor) and on a completely fluid planet with a similar mass than Mars (green curve). In this situation, the upper envelope of our fictitious planet is excited by Phobos leading to the birth of inertial and gravito-inertial waves with resonances at wave excitation frequencies. It leads to *shake* the evolution of Phobos semi-major axis with erratic jumps. It is also worth highlighting that in this case, when the Phobos orbital period leaves the excitation interval of waves, the fall of Phobos stops. This is the difference between equilibrium [180, 181] and dynamical [68] tides.

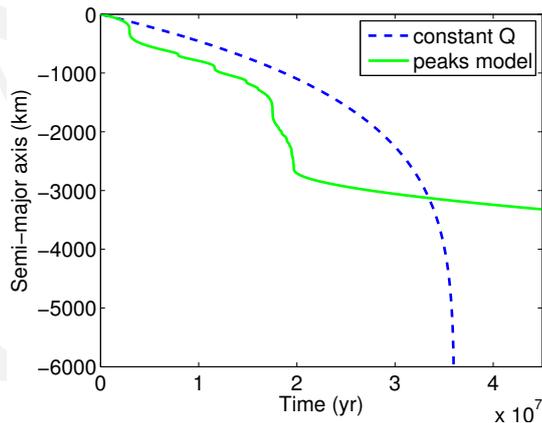


Fig. 1.6 — Evolution of the Phobos semi-major axis  $a$  over time with a  $Q$  factor proportional to inertial wave dissipation in fluids (green curve), and with a constant  $Q$  factor (blue dashed curve). The abscissa represents time in years, the vertical axis measures the evolution of the semi-major axis from its actual value.

To obtain these results, I developed a numerical code called ESPER (Evolution Séculaire des orbites Planétaires En Relativité) which has never been published. Post-Newtonian GR effects are correctly taken into account up to second post-Newtonian order of correction. A complete module for tide have been implemented. Recently, with S. Brun and S. Mathis, we supervised Mansour Benbakoura during its M2 internship. Our objective was to implement magnetism inside ESPER, following the interesting results presented in [159]. It led us to produce a patch to ESPER called ESPERM (Evolution Séculaire des orbites Planétaires avec Magnétisme) which

will be presented in a forthcoming publication [14].

All of this work gave me a solid approach to tidal effects necessary for GR tests with Lunar Laser Ranging, which I started later with a PhD student, A. Bourgoïn (see section 1.5.3).

### Related Publications :

1. M. BENBAKOURA, V. RÉVILLE, A. S. BRUN, C. LE PONCIN-LAFITTE et S. MATHIS. « Evolution of star-planet systems under magnetic braking and tidal interaction ». In : *to be submitted to A&A* (2018)
2. V. LAINEY, R. A. JACOBSON, R. TAJEDDINE, N. J. COOPER, C. MURRAY, V. ROBERT, G. TOBIE, T. GUILLOT, S. MATHIS, F. REMUS, J. DESMARS, J.-E. ARLOT, J.-P. DE CUYPER, V. DEHANT, D. PASCU, W. THUILLOT, C. LE PONCIN-LAFITTE et J.-P. ZAHN. « New constraints on Saturn’s interior from Cassini astrometric data ». In : *Icarus* 281 (jan. 2017), p. 286–296. DOI : [10.1016/j.icarus.2016.07.014](https://doi.org/10.1016/j.icarus.2016.07.014)
3. S. MATHIS, P. AUCLAIR-DESROTOUR, M. GUENEL, F. GALLET et C. LE PONCIN-LAFITTE. « The impact of rotation on turbulent tidal friction in stellar and planetary convective regions ». In : *A&A* 592, A33 (juil. 2016), A33. DOI : [10.1051/0004-6361/201527545](https://doi.org/10.1051/0004-6361/201527545). arXiv : [1604.08570](https://arxiv.org/abs/1604.08570) [astro-ph.SR]
4. P. AUCLAIR-DESROTOUR, S. MATHIS et C. LE PONCIN-LAFITTE. « Scaling laws to understand tidal dissipation in fluid planetary regions and stars I. Rotation, stratification and thermal diffusivity ». In : *A&A* 581, A118 (sept. 2015), A118. DOI : [10.1051/0004-6361/201526246](https://doi.org/10.1051/0004-6361/201526246). arXiv : [1506.07705](https://arxiv.org/abs/1506.07705) [astro-ph.EP]
5. P. AUCLAIR-DESROTOUR, C. LE PONCIN-LAFITTE et S. MATHIS. « Impact of the frequency dependence of tidal Q on the evolution of planetary systems ». In : *Astronomy & Astrophysics* 561, L7 (jan. 2014), p. L7. DOI : [10.1051/0004-6361/201322782](https://doi.org/10.1051/0004-6361/201322782). arXiv : [1311.4810](https://arxiv.org/abs/1311.4810) [astro-ph.EP]
6. V. LAINEY, O. KARATEKIN, J. DESMARS, S. CHARNOZ, J.-E. ARLOT, N. EMELYANOV, C. LE PONCIN-LAFITTE, S. MATHIS, F. REMUS, G. TOBIE et J.-P. ZAHN. « Strong Tidal Dissipation in Saturn and Constraints on Enceladus’ Thermal State from Astrometry ». In : *Astrophysical Journal* 752, 14 (juin 2012), p. 14. DOI : [10.1088/0004-637X/752/1/14](https://doi.org/10.1088/0004-637X/752/1/14). arXiv : [1204.0895](https://arxiv.org/abs/1204.0895) [astro-ph.EP]
7. S. CHARNOZ, A. CRIDA, J. C. CASTILLO-ROGEZ, V. LAINEY, L. DONES, O. KARATEKIN, G. TOBIE, S. MATHIS, C. LE PONCIN-LAFITTE et J. SALMON. « Accretion of Saturn’s mid-sized moons during the viscous spreading of young massive rings : Solving the paradox of silicate-poor rings versus silicate-rich moons ». In : *Icarus* 216 (déc. 2011), p. 535–550. DOI : [10.1016/j.icarus.2011.09.017](https://doi.org/10.1016/j.icarus.2011.09.017). arXiv : [1109.3360](https://arxiv.org/abs/1109.3360) [astro-ph.EP]
8. A. FIENGA, J. LASKAR, T. MORLEY, H. MANCHE, P. KUCHYNKA, C. LE PONCIN-LAFITTE, F. BUDNIK, M. GASTINEAU et L. SOMENZI. « INPOP08, a 4-D planetary ephemeris : from asteroid and time-scale computations to ESA Mars Express and Venus Express contributions ». In : *Astronomy & Astrophysics* 507 (déc. 2009), p. 1675–1686. DOI : [10.1051/0004-6361/200911755](https://doi.org/10.1051/0004-6361/200911755). arXiv : [0906.2860](https://arxiv.org/abs/0906.2860) [astro-ph.EP]
9. S. MATHIS et C. LE PONCIN-LAFITTE. « Tidal dynamics of extended bodies in planetary systems and multiple stars ». In : *Astronomy & Astrophysics* 497 (avr. 2009), p. 889–910. DOI : [10.1051/0004-6361/20079054](https://doi.org/10.1051/0004-6361/20079054)

## 1.4 Tests with the Parametrized Post-Newtonian formalism

The post-Newtonian approximation of GR is a method to solve Einstein's field equations for physical systems in which motions are slow compared to the speed of light and where gravitational fields, inside and around bodies, are weak. It is exactly the case for Solar System bodies. The underlying idea is to treat spacetime as being that of flat Minkowski spacetime as the zeroth approximation, and to modify it by successive corrections. This means that one can characterize a gravitational system in question by a small parameter  $\epsilon$  as

$$\epsilon \sim (v/c)^2 \sim GM/rc^2 \sim p/\rho c^2, \quad (1.67)$$

where  $v$ ,  $M$  and  $r$  denote the characteristic velocity, mass and size or separation within the system;  $p$  and  $\rho$  are the characteristic pressure and density within the bodies,  $G$  and  $c$  being Newton's gravitational constant and the speed of light in a vacuum, respectively.

Consequently, the components of the metric tensor can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (1.68)$$

where  $\eta$  is the Minkowski flat spacetime metric and,  $|h_{\mu\nu}| \ll 1$ , a perturbation. A consistent construction of  $h$  requires determination of  $h_{00}$  correct through  $\mathcal{O}(\epsilon^2)$ ,  $h_{0i}$  through  $\mathcal{O}(\epsilon^{3/2})$  and  $h_{ij}$  correct through  $\mathcal{O}(\epsilon)$ .

The only way that one metric theory differs from another is in the numerical values of the coefficients that appear in front of the metric potentials. It is the purpose of the Parametrized post-Newtonian Formalism (PPN) where one inserts parameters in place of these coefficients, parameters whose values depend on the theory under study. PPN formalism was pioneered by Kenneth Nordtvedt [131, 133], who studied the post-Newtonian metric of a system of gravitating point masses, extending earlier work by Eddington, Robertson and Schiff. Will [168, 174, 175] generalized the framework to perfect fluids. A general and unified version of the PPN formalism was developed by Will and Nordtvedt [177, 136]. The metric tensor can be expressed as follows

$$\begin{aligned} g_{00} &= -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 \\ &\quad + 2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2U \\ &\quad - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^i V_i, \\ g_{0i} &= -\frac{1}{3}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{3}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i \\ &\quad - \frac{1}{3}(\alpha_1 - 2\alpha_2)w^i U - \alpha_2 w^j U_{ij}, \\ g_{ij} &= + (1 + 2\gamma U)\delta_{ij}, \end{aligned}$$

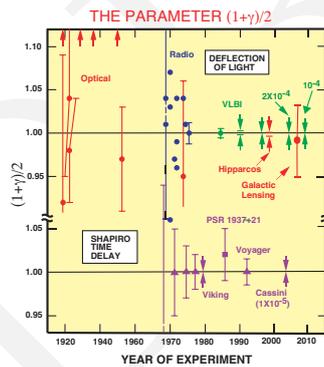
where  $\beta$ ,  $\gamma$ ,  $\xi$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\zeta_1$ ,  $\zeta_2$ ,  $\zeta_3$  et  $\zeta_4$  are the PPN parameters,  $w^i$  being the  $i$ th component of velocity relative to a preferred frame,  $U$  the Newtonian gravitational potential,  $\Phi_W$ ,  $\Phi_1$ ,  $\Phi_2$ ,  $\Phi_3$ ,  $\Phi_4$ ,  $\mathcal{A}$ ,  $U_{ij}$ ,  $V_i$  and  $W_i$  being post-Newtonian potentials.

The parameters  $\gamma$  and  $\beta$  are the Eddington-Robertson-Shiff parameters used to describe *classical tests* as light deflection and perihelion precession, and are in some sense the most important.

TABLE 1.1 – PPN parameters and significance.  $\alpha_3$  is indicated twice since it is a measure of 2 effects.

Parameter	What is measured relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories
$\gamma$	How much space-curvature produced by unit rest mass?	1	$\gamma$	$\gamma$
$\beta$	How much <i>nonlinearity</i> in the superposition law for gravity?	1	$\beta$	$\beta$
$\xi$	Preferred-location effects?	0	$\xi$	$\xi$
$\alpha_1$	Preferred-frame effects?	0	$\alpha_1$	0
$\alpha_2$		0	$\alpha_2$	0
$\alpha_3$		0	0	0
$\alpha_3$	Violation of conservation of total momentum?	0	0	0
$\zeta_1$		0	0	0
$\zeta_2$		0	0	0
$\zeta_3$		0	0	0
$\zeta_4$		0	0	0

They are the only non-zero parameters in GR and scalar-tensor theories. The parameter  $\xi$  is non-zero in any theory of gravity predicting preferred-location effects such as an anisotropy in the local gravitational constant (the Whitehead effects);  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  measure whether or not theory predicting post-Newtonian preferred-frame effects;  $\alpha_3$ ,  $\zeta_1$ ,  $\zeta_2$ ,  $\zeta_3$  and  $\zeta_4$  measure whether or not violations of global conservation laws for momentum. I summarize in Table (1.1) the values of these parameters in GR, in any theory that possesses 6 global conservation laws for angular momentum, the *fully conservative* scenario and in any theory that possesses conservation laws for angular momentum, the *fully conservative* scenario.



(a) Constraints on PPN  $\gamma$ .

Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	$2.3 \times 10^{-5}$	Cassini tracking
	light deflection	$2 \times 10^{-4}$	VLBI
$\beta - 1$	perihelion shift	$8 \times 10^{-5}$	$J_{2\odot} = (2.2 \pm 0.1) \times 10^{-7}$
	Nordtvedt effect	$2.3 \times 10^{-4}$	$\eta_N = 4\beta - \gamma - 3$ assumed
$\xi$	spin precession	$4 \times 10^{-9}$	millisecond pulsars
$\alpha_1$	orbital polarization	$10^{-4}$	Lunar laser ranging
		$7 \times 10^{-5}$	PSR J1738+0333
$\alpha_2$	spin precession	$2 \times 10^{-9}$	millisecond pulsars
$\alpha_3$	pulsar acceleration	$4 \times 10^{-20}$	pulsar $\dot{P}$ statistics
$\zeta_1$	—	$2 \times 10^{-2}$	combined PPN bounds
$\zeta_2$	binary acceleration	$4 \times 10^{-5}$	$\ddot{P}_b$ for PSR 1913+16
$\zeta_3$	Newton's 3rd law	$10^{-8}$	lunar acceleration
$\zeta_4$	—	—	not independent

(b) Techniques used to constrain PPN parameters.

FIGURE 1.7 – Actual constraints on PPN parameters.

Figure (1.7) gives an overview of the actual constraints on the PPN parameters. Fig. (1.7b) is a table describing which what techniques the constraint has been obtained. Fig. (1.7a) is dedicated

on the determination of PPN  $\gamma$ ; it illustrates the constant improvement of its determination with time and the variety of techniques used, from optical light deflection, VLBI and radio-science experiments.

### 1.4.1 Contributions with planetary ephemerides

The analysis of the motion of the planet Mercury around the Sun was historically the first evidence in favor of GR with the explanation of the famous advance of the perihelion in 1915. From there, planetary ephemerides have always been a very powerful tool to constrain GR and alternative theories of gravitation.

When I collaborated with the INPOP team between 2006 and 2009, we used the INPOP08 ephemerides [55] to perform some tests of gravitation. At that time, we considered the advance of perihelion of a planet given by

$$\Delta\omega = \frac{2\pi(2\gamma - \beta + 2)GM_{sun}}{a(1 - e^2)c^2} + \frac{3\pi J_2 R_{sun}^2}{a^2(1 - e^2)^2}, \quad (1.69)$$

where  $G$  and  $c$  are the Newtonian gravitational constant and the speed of light in a vacuum, respectively;  $J_2$  and  $R_{sun}$  are the Sun oblateness and equatorial radius;  $a$  and  $e$  are the semi-major axis and the eccentricity of the precessing planet. Nevertheless if  $\gamma$  plays a role in the equations of motion, it is worth to note that light propagation is only sensible to that parameter. PPN  $\gamma$  can be then estimated with high accuracy by light deflection measurements by VLBI, by time delay during an interplanetary roundtrip and by Doppler tracking data of a space mission. This is also why, in the following, we put  $\gamma = 1$  in order to only test the sensitivity of PPN  $\beta$  on the perihelion's advance of planets. However, Eq. (1.69) demonstrates that we can immediately suspect that it is not possible to do a relevant estimation of PPN  $\beta$  without considering Sun  $J_2$ . Furthermore, it may be not possible to decorrelate safely these two quantities with only one planet, as Mercury.

With INPOP08 [55], MEX and VEX tracking data have lead to an important improvement of Mars and Venus orbits, respectively. But their impact was not only limited to the improvement of the planet dynamics, playing also a role in the determination of parameters such as asteroid masses, the oblateness of the Sun and the PPN parameter  $\beta$ . It was then suitable to take advantage of this new situation by attempting to decorrelate these parameters.

The ratio between the uncertainties of the observations and the sensitivity of the observed orbit to the GR modifications was evaluated by dividing the cumulative advance of the perihelion over a period of time corresponding to the time span of observations by the angle uncertainty of INPOP and presented in table 1.2. If the amplitude of the advance of the perihelion on Venus and Mars orbits is considered for a set of observations of equivalent accuracy, Venus data will be seven times more efficient to test GR and to estimate the sun  $J_2$  than Mars. If VEX mission is prolonged from 2 years to 4 years and if VLBI observations are done from the tracking of the spacecraft with an accuracy of about 1 mas, VEX data will be then as important for the PPN testing and Sun  $J_2$  estimations as the direct 800-meter accuracy radar ranging on Mercury. Besides, the Mars data are still very important because of the long time span of observations of very good quality obtained since the Viking mission in 1978.

TABLE 1.2 – The first 2 columns give the *a priori* INPOP uncertainties in geocentric angles and distances limited by the observation accuracies. In the third column, one may find the estimation of the general relativity and sun oblateness effect of the advance in perihelion,  $\dot{\omega}$ , on the Mercury, Venus and Mars perihelia per year. The fourth column gives the S/N ratio estimated over the period of time given in years in column 5.

Planets	INPOP angle	accuracy distance	$\dot{\omega}$ "/yr	S/N	period years
Venus	0.001"	4m	0.086	172	2
				344	4
Mars	0.001"	2m	0.013	130	10
				390	30
Mercury	0.050"	1km	0.43	300	35

However, The advance of the perihelion induced by GR and sun  $J_2$  has an impact very similar to the advance induced by the main-belt asteroids on inner planet orbit. In INPOP08, a ring was fixed to average the perturbations induced by the main-belt asteroids which cannot have their signal fitted individually on tracking observations. This ring has its physical characteristics (mass and distance to the sun) estimated independently from the fit by considering the albedos and physical properties of 24635 asteroids [98].

Two different but complementary analysis and determination of PPN  $\beta$  and Sun  $J_2$  have been done with a fixed model of asteroid perturbations (same values of asteroid and ring masses and of densities as INPOP08). The first approach was based on a least square estimate of parameters during the fit of planet equations of motion to observations. We estimated what was the impact of each datasets in the determination of  $J_2$  and  $\beta$  : several fits of the initial condition of planets and the parameters  $J_2$  and  $\beta$  have been made using different sets of observations. This led to 32 adjustments based on INPOP08. For each fit, changes were made in the selection of Mars and Venus data in order to estimate the impact of each important set of observations in the fit of the Sun  $J_2$  and PPN  $\beta$ . We looked at the variations in the estimation errors of the 2 parameters and we use the 1- $\sigma$  given by the least squares as indicator of this uncertainty. With this method, we were then able to quantify the influence of each data sets on the determination of the pair  $(\beta, J_2)$  as well as the stability of the determinations of the parameters. Indeed these variations in the error's estimation of the pair  $(\beta, J_2)$  are a relevant indicator of the uncertainty of the fit of  $\beta$  and  $J_2$ . To take into account the correlation between  $J_2$  and  $\beta$ , we used two modes of adjustments : in the *mode 1*,  $\beta$  or  $J_2$  are fitted alone with the initial conditions of planets; in the *mode 2*, both parameters are fitted simultaneously with the initial conditions of planets.

The results are summarized in table 1.3. One can first notice that the determinations of sun  $J_2$  and  $\beta$  made separately (*i. e.* mode 1) give better  $\sigma$  than fits including simultaneous  $(\beta, J_2)$  determination (mode 2). This is obviously consistent with the expected result relative to the determination of correlated parameters. The best results for a correlated determination of  $J_2$  and  $\beta$  (mode 2) are then obtained when only the most accurate observations of Mars (MGS/MO, MEX and Viking) and Venus (VEX) are used simultaneously.

TABLE 1.3 – 1- $\sigma$  least squares obtained for  $J_2$  and  $\beta$  using several sets of observations.

	mode	$J_2$ $\times 10^7$	$(1 - \beta)$ $\times 10^3$		mode	$J_2$ $\times 10^7$	$(1 - \beta)$ $\times 10^3$
Modern Mars	1	0.181		Impact of VEX	1	0.144	
MEX + MGS/MO	1		0.042	Mars + VEX	1		0.025
	2	0.367	0.085		2	0.208	0.037
Impact of Vkg	1	0.161		Impact of old Venus	1	0.188	
MEX + MGS/MO + Vkg	1		0.040	Mars + old Venus	1		0.040
= Mars	2	0.302	0.076		2	0.283	0.060

A second and original strategy to study the sensitivity of the planetary ephemerides to  $J_2$  and PPN  $\beta$  was to estimate how does an ephemeris built using different values for  $J_2$  and PPN  $\beta$  and fitted on the same set of observations as INPOP08 differ from INPOP08. Such differences give an indication on how observations are sensitive to these parameters and with which accuracy we can estimate a parameter such as  $\beta$ .

To test such sensitivity, we focused our attention on the postfit residuals of the most accurate dataset used in INPOP08 adjustment : the Mercury direct range, because of its sensitivity to GR and to the Sun  $J_2$ ; VEX, MEX and MGS/MO data because of their high accuracy and simulated  $S/N$  presented on table 1.2; Jupiter Galileo data and Saturn Cassini normal point. These 2 latest data sets were selected because they induce a global improvement of the planetary ephemerides and especially of the Earth orbit.

To estimate the sensitivity of these 7 most accurate sets of data used in INPOP08 adjustment to the variations of values of  $J_2$  and PPN  $\beta$ , we estimated the ratio  $S/N$  defined as :

$$S/N = \frac{\sigma_{i,j} - \sigma_{0,0}}{\sigma_{0,0}}$$

where  $\sigma_{i,j}$  is the 1-sigma dispersion of the postfit residuals of an ephemerides based on INPOP08 but with values of  $J_2$  and PPN  $\beta$  different from the ones used in INPOP08 and fitted on all the INPOP08 data sets and  $\sigma_{0,0}$  is the 1-sigma dispersion of the postfit INPOP08 residuals. We have used 9 values of  $J_2$  varying from  $1.45 \times 10^{-7}$  to  $3.05 \times 10^{-7}$  with a 0.2 step and 24 values of PPN  $\beta$ , building then 192 different ephemerides.

TABLE 1.4 –  $\beta$  intervals in which the residuals stay below the 5% limit. The value of  $\beta$  given here is estimated for  $\gamma = 1$ .

Data	$\beta$ min	$\beta$ max	Data	$\beta$ min	$\beta$ max	Data	$\beta$ min	$\beta$ max
MGS/MO+MEX	0.99995	1.0002	Jupiter VLBI	0.9996	1.0002	Viking	0.9995	1.0002
VEX	0.99990	1.0002	Saturn Ranging	0.9998	1.0005	Mercury	0.9985	1.005

On Table (1.4), we have gathered minimum and maximum values of PPN  $\beta$  defining the sensitivity interval of the different datasets. The sensitivity interval is the interval of PPN  $\beta$  for which the  $S/N$  remains below 5%. Values of PPN  $\beta$  greater than the maximum value given in table (1.4) or smaller than the minimum value cannot be seen as realistic in comparison to modern observations. It appeared that the MGS/MO and MEX data provide the most narrow interval of sensitivity with  $0.99995 < \beta < 1.0002$ .

Nowadays, INPOP team is still pursuing this kind of experiment, by using new available data.

For example, recently, by using the Messenger tracking data and introducing Monte-Carlo method, they succeed to improve drastically the sensitivity to PPN  $\beta$  [171, 50].

#### Related Publications :

- ✦ A. FIENGA, J. LASKAR, P. KUCHYNKA, C. LE PONCIN-LAFITTE, H. MANCHE et M. GASTINEAU. « Gravity tests with INPOP planetary ephemerides ». In : *IAU Symposium*. Sous la dir. de S. A. KLIONER, P. K. SEIDELMANN, & M. H. SOFFEL. T. 261. IAU Symposium. Jan. 2010, p. 159–169. DOI : [10.1017/S1743921309990330](https://doi.org/10.1017/S1743921309990330). arXiv : [0906.3962](https://arxiv.org/abs/0906.3962) [gr-qc]

### 1.4.2 Contributions with Very Long Baseline Interferometry

Very Long Baseline Interferometry (VLBI) is a geometric technique which measures the time difference in the arrival of a radio wavefront emitted by a distant radio source (typically a quasar) between at least two Earth-based radio telescopes, with a precision of a few picoseconds. Knowing the group delay and the angular separation between the baseline between the antennas of the telescopes and the line of sight of the observation, the distance between the telescopes can be determined and consequently VLBI tracks the orientation of the Earth in an inertial reference frame provided by the very distant quasars, determining accurate terrestrial and celestial reference frames.

Geodetic VLBI observations are run daily since 1979 and the database contains nowadays almost 6000 24hours sessions, corresponding actually to 10 millions group-delay observations, with a present precision of a few picoseconds. One of the principal goals of VLBI observations is the kinematical monitoring of Earth rotation with respect to a global inertial frame realized by a set of quasars, the International Celestial Reference Frame [49], as defined by the International Astronomical Union [157]. The International VLBI Service for Geodesy and Astrometry (IVS) organizes sessions of observation, data storage and products distribution, in particular the Earth Orientation Parameters. Because of this precision, VLBI is also a very interesting tool to test gravitation in the Solar System [158]. Indeed, the gravitational fields of the Sun and the planets are responsible of relativistic effects on the quasar light beam through the propagation of the signal to the observing station. VLBI is able to detect these effects very accurately through the gravitational group delay, given by [56]

$$\tau_g = (\gamma + 1) \frac{GM}{c^3} \log \left( \frac{|\vec{r}_1| + \vec{r}_1 \cdot \vec{k}}{|\vec{r}_2| + \vec{r}_2 \cdot \vec{k}} \right), \quad (1.70)$$

where  $\vec{r}_i$  stands for the position vector of the  $i$ th station and  $\vec{k}$  the unit vector pointing towards the radio source, both referring to the center of mass of the deflecting body. For a typical VLBI baseline between Westford (Massachusetts) and Wettzell (Germany) of  $\sim 6,000$  km,  $\tau_g$  is  $\sim 170$  nanoseconds (ns) for a source at the Sun's limb, rapidly decreases to  $\sim 10$  ns at  $4^\circ$  away from the Sun, and remains close to the accuracy of VLBI measurements (nowadays around 10 ps), even for elongations close to  $180^\circ$  (see Fig. 1.8).

In the past years, VLBI data were used in various attempts to determine  $\gamma$ . Using less than 4 years of observations, [149] found  $\gamma$  consistent with GR within 0.005. Using 10 years of observations, [150] estimated a standard error of 0.002. [113] got  $0.9996 \pm 0.0017$  after observations of the relative deflection of 3C 273B and 3C 279. [156] obtained  $0.99983 \pm 0.00026$  (statistical

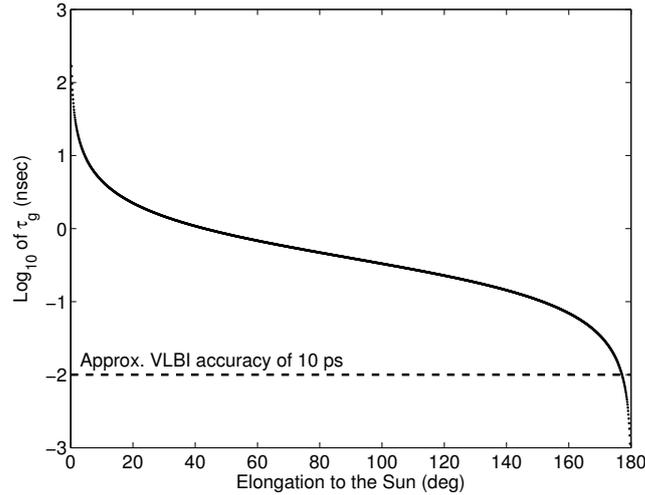


FIGURE 1.8 – Gravitational delay  $\tau_g$  as a function of the elongation angle  $\phi$  to the Sun for the baseline Westford–Wetzell.

standard error) with VLBI observations before 1999. The current best estimate of  $\gamma$ , however, was not obtained with VLBI : it is consistent with GR with an error of  $2 \times 10^{-5}$ , and was obtained by [18] who derived it from spacecraft tracking experiments.

Errors reported in the various papers are often formal errors obtained from the propagation through the adjustment procedure of an initial SNR-derived standard error on the delays. They might therefore not directly compare to one another. Though all these works, except [18], deal with deflection of the radio waves by the Sun, it must be mentioned that special VLBI sessions were carried out to measure the deflection close to Jupiter or other planets [151].

Since 1998, SYRTE maintains a VLBI data analysis center, OPAR lead by Dr. Sébastien Lambert. This center has multiple objectives : determination of Earth Orientation Parameters and the realization of terrestrial and celestial frames. Its operational activity is realized in the framework of the International VLBI Service (IVS). OPAR is analyzing every day Intensive VLBI observations, enabling us to determine the rotation angle of the Earth, namely UT1. The astrometric accuracy of these observations is roughly some dozen of microarcseconds, representing a precision of a few millimeters concerning the absolute positioning of the Earth pole. Between 2009 and 2011, I collaborated with S. Lambert to produce a new generation of GR tests with VLBI data. In our works, we estimate  $\gamma$  from routine geodetic VLBI observations, using the additio-

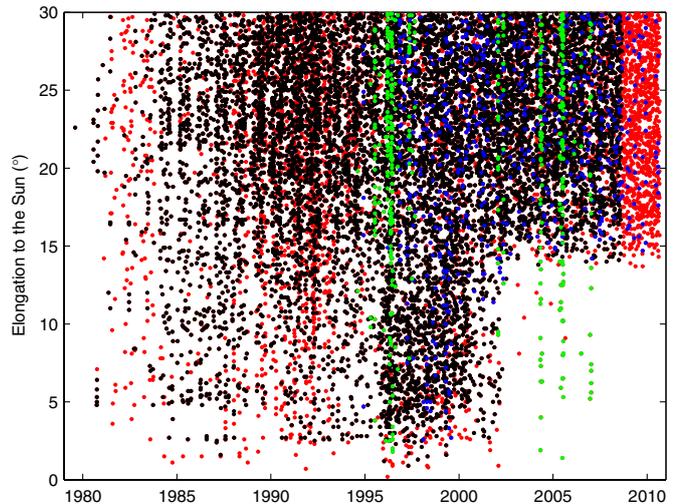


Fig. 1.9 – The main plot displays the observational history of the sources at less than  $30^\circ$  from the Sun (black : observations treated in [104]; red : additional observations of routine experiments not processed in [104] excluding VLBA+ and VLBA; blue : VLBA+; green : VLBA)

nal 1999-2008 time period with respect to [156]. We compared estimates and errors obtained over several time spans and using various analysis schemes in order to address the accuracy and to point out some systematics and limitations. In particular, we stressed the importance of having new observations close to the sun, which had not been done roughly since 2000 as shown in Fig. 1.9. Concerning our determination of PPN  $\gamma$ , it was possible to obtain a new constraint, with the complete observations database, at the level of  $1.2 \times 10^{-4}$  [105, 106], which is still today the best estimate possible with this technique. With S. Lambert, we have since convinced IVS to make new observations close to the Sun ; since 2011, IVS decided to run new observations close to the Sun and the first data are now available, their processing being under study.

#### Related Publications :

- ✦ S. B. LAMBERT et C. LE PONCIN-LAFITTE. « Improved determination of  $\gamma$  by VLBI ». In : *Astronomy & Astrophysics* 529, A70 (mai 2011), A70. DOI : [10.1051/0004-6361/201016370](https://doi.org/10.1051/0004-6361/201016370)
- ✦ S. B. LAMBERT et C. LE PONCIN-LAFITTE. « Determining the relativistic parameter  $\gamma$  using very long baseline interferometry ». In : *Astronomy & Astrophysics* 499 (mai 2009), p. 331–335. DOI : [10.1051/0004-6361/200911714](https://doi.org/10.1051/0004-6361/200911714). arXiv : [0903.1615](https://arxiv.org/abs/0903.1615) [gr-qc]

## 1.5 Testing Lorentz symmetry

Lorentz invariance is one of the fundamental symmetry of relativity, one of the corner stones of both GR and the Standard Model of particle physics. It states that the outcome of any local experiment is independent of the velocity and of the orientation of the laboratory in which the experiment is performed. If one considers non-gravitational experiments, Lorentz symmetry is part of the Einstein Equivalence Principle (EEP). A breaking of Lorentz symmetry implies that the equations of motion, the particle thresholds, etc... may be different when the experiment is boosted or rotated with respect to a background field [38]. More precisely, it is related to a violation of the invariance under "particle Lorentz transformations" [38] which are the boosts and rotations that relate the properties of two systems within a specific oriented inertial frame (or in other words they are boosts and rotations on localized fields but not on background fields). On the other hand, the invariance under coordinates transformations known as "observer Lorentz transformations" [38] which relate observations made in two inertial frames with different orientations and velocities is always preserved. Considering the broad field of applicability of this symmetry, searches for Lorentz symmetry breaking provide a powerful test of fundamental physics. Moreover, it has been suggested that Lorentz symmetry may not be a fundamental symmetry of Nature and may be broken at some level. While some early motivations came from string theories [94, 95, 92], breaking of Lorentz symmetry also appears in loop quantum gravity [65, 3, 124, 130], non commutative geometry [72, 31], multiverses [20], brane-world scenarios [30, 61, 37] and others (see for example [162, 123]).

Tests of Lorentz symmetry have been performed since the time of Einstein but the last decades have seen the number of tests increased significantly [93] in all fields of physics. In particular, a dedicated effective field theory has been developed in order to systematically consider all hypothetical violations of the Lorentz invariance. This framework is known as the Standard-Model Extension (SME) [38, 39] and covers all fields of physics. It contains the Standard Model

of particle physics, GR and all possible Lorentz-violating terms that can be constructed at the level of the Lagrangian, introducing a large numbers of new coefficients that can be constrained experimentally.

In this document, we focus on the gravitational sector of the SME which parametrizes deviations from GR. GR is built upon two principles [169, 176, 173] : (i) the EEP and (ii) the Einstein field equations that derive from the Einstein-Hilbert action. The EEP gives a geometric nature to gravitation allowing this interaction to be described by spacetime curvature. From a theoretical point of view, the EEP implies the existence of a spacetime metric to which all matter minimally couples [167]. A modification of the matter part of the action will lead to a breaking of the EEP. In SME, such a breaking of the EEP is parametrized (amongst others) by the matter-gravity coupling coefficients  $\bar{a}_\mu$  and  $\bar{c}_{\mu\nu}$  [97, 161]. From a phenomenological point of view, the EEP states that [176, 173] : (i) the universality of free fall (also known as the weak equivalence principle) is valid, (ii) the outcome of any local non-gravitational experiment is independent of the velocity of the free-falling reference frame in which it is performed and (iii) the outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed. The second part of Einstein theory concerns the purely gravitational part of the action (the Einstein-Hilbert action) which is modified in SME to introduce hypothetical Lorentz violations in the gravitational sector. This section focuses exclusively on this kind of Lorentz violations and not on breaking of the EEP.

### 1.5.1 Postfit analysis versus full modeling

Since the last decade, several studies aimed to find upper limits on SME coefficients in the gravitational sector. A lot of these studies are based on the search of possible signals in post-fit residuals of experiments. This was done with LLR [13], GPB [9], binary pulsars [152, 153] or Solar System planetary motions [79, 73]. However, I proposed two new works focused on a direct fit to data with LLR [25] and VLBI [109], which are more satisfactory.

Indeed, in the case of a post-fit analysis, a simple modeling of extra terms containing SME coefficients are least square fitted in the residuals, attempting to constrain the SME coefficients of a testing function in residual noise obtained from a pure GR analysis, where of course Lorentz symmetry is assumed. It comes out correlations between SME coefficients and other global parameters previously fitted (masses, position and velocity...) cannot be assessed in a proper way. In others words, searching hypothetical SME signals in residuals, i.e. in noise, can lead to an overestimated formal error on SME coefficients, as illustrated in the case of VLBI [109], and without any chance to learn something about correlations with other parameters, as for example demonstrated in the case of LLR [25]. Let us consider the VLBI example to illustrate this fact. The VLBI analysis is described in Section 1.5.2. Including the SME contribution within the full VLBI modeling and estimating the SME coefficient  $\bar{s}^{TT}$  altogether with the other parameters fitted in standard VLBI data reduction leads to the estimate  $\bar{s}^{TT} = (-5 \pm 8) \times 10^{-5}$ . A postfit analysis performed by fitting the SME contribution within the VLBI residuals obtained after a pure GR analysis leads to  $\bar{s}^{TT} = (-0.6 \pm 2.1) \times 10^{-8}$  [109]. This example shows that a postfit analysis can lead to results with overoptimistic uncertainties and one needs to be extremely careful when using such results.

In the following, I am using a standard canonical frame used in the SME framework is a Sun-

centered celestial equatorial frame [90], which is approximately inertial over the time scales of most observations. This frame is asymptotically flat and comoving with the rest frame of the Solar System. This special frame will help to deal with the different techniques presented below (VLBI, LLR and planetary ephemerides), in order to compare all the corresponding results, we need to report them in the same canonical inertial frame. The Cartesian coordinates related to this frame are denoted by capital letters

$$X^\Xi = (cT, X^J) = (cT, X, Y, Z). \quad (1.71)$$

The  $Z$  axis is aligned with the rotation axis of the Earth, while the  $X$  axis points along the direction from the Earth to the Sun at vernal equinox. The origin of the coordinate time  $T$  is given by the time when the Earth crosses the Sun-centered  $X$  axis at the vernal equinox. These conventions are depicted in Figure 2 from [11].

### 1.5.2 VLBI

Following what I have done previously with VLBI (see section 1.4.2), we tried with S. Lambert and A. Hees to use VLBI group delay to constrain SME parameters. Our first concern was about a formulation of light propagation in SME to construct the VLBI group delay in this phenomenology. Indeed, the propagation time of a photon emitted at the event  $(cT_e, \mathbf{X}_e)$  and received at the position  $\mathbf{X}_r$  can be computed in the SME formalism using the time transfer function formalism [111, 164, 112, 75, 74] and is given by [11, 8]

$$\begin{aligned} \mathcal{T}(\mathbf{X}_e, T_e, \mathbf{X}_r) &= \frac{R_{er}}{c} + 2\frac{G_N M}{c^3} \left[ 1 - \frac{2}{3}\bar{s}^{TT} - \bar{s}^{TJ} N_{er}^J \right] \ln \frac{R_e - \mathbf{N}_{er} \cdot \mathbf{X}_e}{R_r - \mathbf{N}_{er} \cdot \mathbf{X}_r} \\ &+ \frac{G_N M}{c^3} \left( \bar{s}^{TJ} P_{er}^J - \bar{s}^{JK} N_{er}^J P_{er}^K \right) \frac{R_e - R_r}{R_e R_r} \\ &+ \frac{G_N M}{c^3} \left[ \bar{s}^{TJ} N_{er}^J + \bar{s}^{JK} \hat{P}_{er}^J \hat{P}_{er}^K - \bar{s}^{TT} \right] (\mathbf{N}_r \cdot \mathbf{N}_{er} - \mathbf{N}_e \cdot \mathbf{N}_{er}), \end{aligned} \quad (1.72)$$

where the terms  $a_1$  and  $a_2$  from [8] are taken as unity (which corresponds to using the harmonic gauge, which is the one used for VLBI data reduction),  $R_e = |\mathbf{X}_e|$ ,  $R_r = |\mathbf{X}_r|$ ,  $R_{er} = |\mathbf{X}_r - \mathbf{X}_e|$  with the central body located at the origin and where we introduce the following vectors

$$\mathbf{K} = \frac{\mathbf{X}_e}{R_e}, \quad \mathbf{N}_{ij} \equiv \frac{\mathbf{X}_{ij}}{R_{ij}} = \frac{\mathbf{X}_j - \mathbf{X}_i}{|\mathbf{X}_{ij}|}, \quad \mathbf{N}_i = \frac{\mathbf{X}_i}{|\mathbf{X}_i|}, \quad \mathbf{P}_{er} = \mathbf{N}_{er} \times (\mathbf{X}_r \times \mathbf{N}_{er}), \quad (1.73)$$

and

$$\hat{\mathbf{P}}_{er} = \frac{\mathbf{P}_{er}}{|\mathbf{P}_{er}|}, \quad (1.74)$$

$G_N$  being the observed Newton constant, measured by considering the orbital motion of bodies, and defined by [11, 9, 73]

$$G_N = G \left( 1 + \frac{5}{3}\bar{s}^{TT} \right). \quad (1.75)$$

Equation (1.72) is the generalization of the well-known Shapiro time delay including Lorentz violation. VLBI is actually measuring the difference of the time of arrival of a signal received

by two different stations. This observable is therefore sensitive to a differential time delay. Assuming a radio-signal emitted by a quasar at event  $(T_e, \mathbf{X}_e)$  and received by two different VLBI stations at events  $(T_1, \mathbf{X}_1)$  and  $(T_2, \mathbf{X}_2)$  (all quantities being expressed in a barycentric reference frame), respectively, I obtained the VLBI group-delay  $\Delta\tau_{(\text{SME})}$  in SME formalism as follows [109]

$$\Delta\tau_{(\text{SME})} = 2\frac{G_N M}{c^3}\left(1 - \frac{2}{3}\bar{s}^{TT}\right)\ln\frac{R_1 + \mathbf{K}\cdot\mathbf{X}_1}{R_2 + \mathbf{K}\cdot\mathbf{X}_2} + \frac{2}{3}\frac{G_N M}{c^3}\bar{s}^{TT}(\mathbf{N}_2\cdot\mathbf{K} - \mathbf{N}_1\cdot\mathbf{K}), \quad (1.76)$$

where I only kept the  $\bar{s}^{TT}$  contribution (see Eq. (7) from [109] for the full expression) and we use the same notations as in [57] by introducing three unit vectors

$$\mathbf{K} = \frac{\mathbf{X}_e}{|\mathbf{X}_e|}, \quad \mathbf{N}_1 = \frac{\mathbf{X}_1}{|\mathbf{X}_1|}, \quad \text{and} \quad \mathbf{N}_2 = \frac{\mathbf{X}_2}{|\mathbf{X}_2|}. \quad (1.77)$$

Ten million VLBI delay observations between August 1979 and mid-2015 have been used to estimate the  $\bar{s}^{TT}$  coefficient. First, VLBI observations are corrected from delay due to the radio wave crossing of dispersive media by using 2 GHz and 8 GHz recordings. Then, we used only the 8 GHz delays and the Calc/Solve geodetic VLBI analysis software, developed at NASA Goddard Space Flight Center and coherent with the latest standards of the International Earth Rotation and Reference Systems Service [141]. We added the partial derivative of the VLBI delay with respect to  $\bar{s}^{TT}$  from Eq. (1.76) to the software package using the USERPART module of Calc/Solve. We turned to a global solution in which we estimated  $\bar{s}^{TT}$  as a global parameter together with radio source coordinates. We obtained

$$\bar{s}^{TT} = (-5 \pm 8) \times 10^{-5}, \quad (1.78)$$

with a postfit root mean square of 28 picoseconds and a  $\chi^2$  per degree of freedom of 1.15. Correlations between radio source coordinates and  $\bar{s}^{TT}$  are lower than 0.02, the global estimate being consistent with the mean value obtained with the session-wise solution with a slightly lower error.

In conclusion, VLBI is an incredible tool to test Lorentz symmetry, especially the  $\bar{s}^{TT}$  coefficient. This coefficient has an isotropic impact on the propagation speed of gravitational waves as can be noticed from Eq. (9) from [96] or Eq. (11) from [91]. The analysis performed in [109] includes the SME contribution in the modeling of VLBI observations and includes the  $\bar{s}^{TT}$  parameter in the global fit with other parameters. It is therefore a robust analysis that produced the first reliable estimate on the  $\bar{s}^{TT}$  parameter. This constrain has been drastically improved very recently with the observation by Advanced LIGO and Virgo detectors of the gravitational-wave event GW170817 on 2017 August 17. With its electromagnetic counterpart, in particular the gamma-ray burst GRB 170817A observed by Fermi,  $\bar{s}^{TT}$  has been constrained at the level of  $10^{-15}$  [64], so ten orders of magnitude better than with VLBI.

### 1.5.3 Lunar Laser Ranging

On August, 20th 1969, after ranging to the lunar retro-reflector placed during the Apollo 11 mission, the first LLR echo was detected at the McDonald Observatory in Texas. Currently,

there are five stations spread over the world which have realized laser shots on five lunar retro-reflectors. Among these stations four are still operating : Mc Donald Observatory in Texas, Observatoire de la Côte d'Azur in France, Apache point Observatory in New Mexico and Matera in Italy while one on Maui, Hawaii has stopped lunar ranging since 1990. Concerning the lunar retro-reflectors three are located at sites of the Apollo missions 11, 14 and 15 and two are French-built array operating on the Soviet roving vehicle Lunakhod 1 and 2.

LLR is used to conduct high precision measurements of the light travel time of short laser pulses emitted at time  $t_1$  by a LLR station, reflected at time  $t_2$  by a lunar retro-reflector and finally received at time  $t_3$  at a station receiver. The data are presented as normal points which combine time series of measured light travel time of photons, averaged over several minutes to achieve a higher signal-to-noise ratio measurement of the lunar range at some characteristic epoch. Each normal-point is characterized by one emission time ( $t_1$  in universal time coordinate – UTC), one time delay ( $\Delta t_c$  in international atomic time – TAI) and some additional observational parameters as laser wavelength, atmospheric temperature and pressure *etc.* According to [32], the theoretical pendent of the observed time delay ( $\Delta t_c = t_3 - t_1$  in TAI) is defined as

$$\Delta t_c = \left[ T_3 - \Delta \tau_t(T_3) \right] - \left[ T_1 - \Delta \tau_t(T_1) \right], \quad (1.79)$$

where  $T_1$  is the emission time expressed in barycentric dynamical time (TDB) and  $\Delta \tau_t$  is a relativistic correction between the TDB and the terrestrial time (TT) at the level of the station. The reception time  $T_3$  expressed in TDB is defined by the following two relations

$$T_3 = T_2 + \frac{1}{c} \left\| \mathbf{X}_{o'}(T_3) - \mathbf{X}_r(T_2) \right\| + \Delta \mathcal{T}_{(\text{grav})} + \Delta \tau_a, \quad (1.80a)$$

$$T_2 = T_1 + \frac{1}{c} \left\| \mathbf{X}_r(T_2) - \mathbf{X}_o(T_1) \right\| + \Delta \mathcal{T}_{(\text{grav})} + \Delta \tau_a, \quad (1.80b)$$

with  $T_2$  the time in TDB at the reflection point  $\mathbf{X}_o$  and  $\mathbf{X}_{o'}$  are respectively the barycentric position vector at the emitter and the reception point,  $\mathbf{X}_r$  is the barycentric position vector at the reflection point,  $\Delta \mathcal{T}_{(\text{grav})}$  is the one way gravitational time delay correction and  $\Delta \tau_a$  is the one way tropospheric correction.

LLR measurements are used to produce the Lunar ephemeris but also provide a unique opportunity to study the Moon's rotation, the Moon's tidal acceleration, the lunar rotational dissipation, etc [47]. In addition, LLR measurements have turn the Earth-Moon system into a laboratory to study fundamental physics and to conduct tests of the gravitation theory. Nordtvedt was the first to suggest that LLR can be used to test GR by testing one of its pillar : the Strong Equivalence Principle [132, 134, 135]. He showed that precise laser ranging to the Moon would be capable of measuring precisely the ratio of gravitational mass to inertial mass of the Earth to an accuracy sufficient to constrain a hypothetical dependence of this ratio on the gravitational self-energy. He concluded that such a measurement could be used to test Einstein's theory of gravity and others alternative theories as scalar tensor theories. The best current test of the Strong Equivalence Principle is provided by a combination of torsion balance measurements with LLR analysis and is given by [179, 178, 125]

$$\eta = (4.4 \pm 4.5) \times 10^{-4}, \quad (1.81)$$

where  $\eta$  is the Nordtvedt parameter that is defined as  $m_G/m_I = 1 + \eta U/mc^2$  with  $m_G$  the gravitational mass,  $m_I$  the inertial mass and  $U$  the gravitational self-energy of the body. Using

the Cassini constraint on the  $\gamma$  PPN parameter [19] and the relation  $\eta = 4\beta - \gamma - 3$  leads to a constraint on  $\beta$  PPN parameter at the level  $\beta - 1 = (1.2 \pm 1.1) \times 10^{-4}$  [178].

In addition to tests of the Strong Equivalence Principle, many other tests of fundamental physics were performed with LLR analysis. For instance, LLR data can be used to search for a temporal evolution of the gravitational constant  $\dot{G}/G$  [179] and to constrain the fifth force parameters [129]. In addition, LLR has been used to constrain violation of the Lorentz symmetry in the PPN framework. [129] deduced from LLR data analysis constraints on the preferred frame parameters  $\alpha_1$  and  $\alpha_2$  at the level  $\alpha_1 = (-7 \pm 9) \times 10^{-5}$  and  $\alpha_2 = (1.8 \pm 2.5) \times 10^{-5}$ .

Considering all the successful GR tests performed with LLR observations, it is quite natural to use them to search for Lorentz violations in the gravitation sector. In the SME framework, [13] used the lunar orbit to provide estimates on the SME coefficients. Using a perturbative approach, the main signatures produced by SME on the lunar orbit have analytically been computed in [11]. These computations give a first idea of the amplitude of the signatures produced by a breaking of Lorentz symmetry. Nevertheless, these analytical signatures have been computed assuming the lunar orbit to be circular and fixed (i.e. neglecting the precession of the nodes for example). These analytical signatures have been fitted to LLR residuals obtained from a data reduction performed in pure GR [13]. They determined a “*realistic*” error on their estimates from a similar postfit analysis performed in the PPN framework. The results obtained by this analysis are presented in Table 1.5. It is important to note that this analysis uses projections of the SME coefficients into the lunar orbital plane  $\bar{s}^{11}, \bar{s}^{22}, \bar{s}^{0i}$  (see Section V.B.2 of [11]) while the standard SME analyses uses coefficients defined in a Sun-centered equatorial frame (and denoted by capital letter  $\bar{s}^{IJ}$ ).

TABLE 1.5 – Estimation of SME coefficients from LLR postfit data analysis from [13]. No correlations coefficients have been derived in this analysis. The coefficients  $\bar{s}^{ij}$  are projections of the  $\bar{s}^{IJ}$  into the lunar orbital plane (see Eq. (107) from [11]) while the linear combinations  $\bar{s}_{\Omega_{\oplus c}}$  and  $\bar{s}_{\Omega_{\oplus s}}$  are given by Eq. (108) from [11].

Coefficient	
$\bar{s}^{11} - \bar{s}^{22}$	$(1.3 \pm 0.9) \times 10^{-10}$
$\bar{s}^{12}$	$(6.9 \pm 4.5) \times 10^{-11}$
$\bar{s}^{01}$	$(-0.8 \pm 1.1) \times 10^{-6}$
$\bar{s}^{02}$	$(-5.2 \pm 4.8) \times 10^{-7}$
$\bar{s}_{\Omega_{\oplus c}}$	$(0.2 \pm 3.9) \times 10^{-7}$
$\bar{s}_{\Omega_{\oplus s}}$	$(-1.3 \pm 4.1) \times 10^{-7}$

However, as discussed in Section 1.5.1 and in [109, 25], a postfit search for SME signatures into residuals of a data reduction previously performed in pure GR is not fully satisfactory. First of all, the uncertainties obtained by a postfit analysis based on a GR data reduction can be underestimated by up to two orders of magnitude. This is mainly due to correlations between SME coefficients and others global parameters (masses, positions and velocities, ...) that are neglected in this kind of approach.

In addition, in the case of LLR data analysis, the oscillating signatures derived in [11] and used in [13] to determine pseudo-constraints are computed only accounting for short periodic oscillations, typically at the order of magnitude of the mean motion of the Moon around the Earth. Therefore, this analytic solution remains only valid for few years while LLR data spans over 45 years (see also the discussions in footnote 2 from [73] and page 22 from [11]).

Regarding LLR data analysis, a more robust strategy consists in including the SME modeling in the complete data analysis and to estimate the SME coefficients in a global fit along with others parameters by taking into account short and long period terms and also correlations (see [25]). In order to perform such an analysis, a new numerical lunar ephemeris named "Ephéméride Lunaire Parisienne Numérique" (ELPN) has been developed within the SME framework, during the Ph.D. thesis of A. Bourgoïn supervised by myself and M.-C. Angonin. The dynamical model of ELPN is similar to the DE430 one [60] but includes the Lorentz symmetry breaking effects arising on the orbital motion of the Moon. The SME contribution to the lunar equation of motion has been derived in [11] and is given by

$$a_{\text{SME}}^J = \frac{G_N M}{r^3} \left[ \bar{s}_t^{JK} r^K - \frac{3}{2} \bar{s}_t^{KL} \hat{r}^K \hat{r}^L r^J + 2 \frac{\delta m}{M} \left( \bar{s}^{TK} \hat{v}^K r^J - \bar{s}^{TJ} \hat{v}^K r^K \right) + 3 \bar{s}^{TK} \hat{V}^K r^J - \bar{s}^{TJ} \hat{V}^K r^K - \bar{s}^{TK} \hat{V}^J r^K + 3 \bar{s}^{TL} \hat{V}^K \hat{r}^K \hat{r}^L r^J \right], \quad (1.82)$$

where  $G_N$  is the observed Newtonian constant defined by Eq. (1.75),  $M$  is the mass of the Earth-Moon barycenter,  $\delta m$  is the difference between the Earth and the lunar masses;  $\hat{r}^J$  being the unit position vector of the Moon with respect to the Earth;  $\hat{v}^J = v^J/c$  with  $v^J$  being the relative velocity vector of the Moon with respect to the Earth;  $\hat{V}^J = V^J/c$  with  $V^J$  being the Heliocentric velocity vector of the Earth-Moon barycenter and the 3-dimensional traceless tensor defined by

$$\bar{s}_t^{JK} = \bar{s}^{JK} - \frac{1}{3} \bar{s}^{TT} \delta^{JK}. \quad (1.83)$$

These equations of motion as well as their partial derivatives are integrated numerically in ELPN. In addition to the orbital motion, effects of a violation of Lorentz symmetry on the light travel time of photons is also considered. More precisely, the gravitational time delay  $\Delta \mathcal{T}_{(\text{grav})}$  appearing in Eq. (1.79) is given by the gravitational part of Eq. (1.72) [8].

Estimates on the SME coefficients are obtained by a standard chi-squared minimization : the LLR residuals are minimized by an iterative weighted least squares fit using partial derivatives previously computed from variational equations in ELPN. After an adjustment of 82 parameters including the SME coefficients a careful analysis of the covariance matrix shows that LLR data does not allow to estimate independently all the SME coefficients but that they are sensitive to the following three linear combinations :

$$\bar{s}^{XX} - \bar{s}^{YY}, \quad \bar{s}^{TY} + 0.43 \bar{s}^{TZ}, \quad \bar{s}^{XX} + \bar{s}^{YY} - 2 \bar{s}^{ZZ} - 4.5 \bar{s}^{YZ}. \quad (1.84)$$

The estimations on the 6 SME coefficients derived in [25] is summarized in Table 1.6. In particular, it is worth emphasizing that the quoted uncertainties are the sum of the statistical uncertainties obtained from the least-square fit with estimations of systematics uncertainties obtained with a Jackknife resampling method [116, 70].

TABLE 1.6 – Estimation of SME coefficients from a full LLR data analysis from [25] and associated correlation coefficients.

Coefficient	Estimates	Correlation coefficients					
$\bar{s}^{TX}$	$(-0.9 \pm 1.0) \times 10^{-8}$	1					
$\bar{s}^{XY}$	$(-5.7 \pm 7.7) \times 10^{-12}$	-0.06	1				
$\bar{s}^{XZ}$	$(-2.2 \pm 5.9) \times 10^{-12}$	-0.04	0.29	1			
$\bar{s}^{XX} - \bar{s}^{YY}$	$(0.6 \pm 4.2) \times 10^{-11}$	0.58	-0.12	-0.16	1		
$\bar{s}^{TY} + 0.43 \bar{s}^{TZ}$	$(6.2 \pm 7.9) \times 10^{-9}$	0.16	-0.01	-0.09	0.25	1	
$\bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ} - 4.5 \bar{s}^{YZ}$	$(2.3 \pm 4.5) \times 10^{-11}$	0.07	-0.10	-0.13	-0.10	0.03	1

In summary, LLR is a powerful experiment to constrain gravitation theory and in particular hypothetical violation of the Lorentz symmetry. A first analysis based on a postfit estimations of the SME coefficients have been performed [13] which is not satisfactory regarding the neglected correlations with other global parameters as explained in Section 1.5.1. A full analysis including the integration of the SME equations of motion and the SME contribution to the gravitational time delay has been done in [25]. The resulting estimates on some SME coefficients are presented in Table 1.6. In addition, some SME coefficients are still correlated with parameters appearing in the rotational motion of the Moon as the principal moment of inertia, the quadrupole moment, the potential Stockes coefficient  $C_{22}$  and the polar component of the velocity vector of the fluid core [25]. A very interesting improvement regarding this analysis would be to produce a joint GRAIL (Gravity Recovery And Interior Laboratory) [86, 114, 4] and LLR data analysis that would help in decorrelating the SME parameters from the lunar potential Stockes coefficients of degree 2 and therefore improve marginalized estimations of the SME coefficients. Finally, in [13] and [25], the effects of SME on the translational lunar equations of motion are considered and used to derive constraints on the SME coefficients. It would be also interesting to extend these analyses by considering the modifications due to SME on the rotation of the Moon. A first attempt has been proposed in Section V. A. 2. of [11] but needs to be extended.

#### 1.5.4 Planetary ephemerides

A violation of Lorentz symmetry within the gravity sector of SME induces different types of effects that can have implications on planetary ephemerides analysis : effects on the orbital dynamics and effects on the light propagation. Simulations using TTF [164, 75, 74] based on the software presented in [76] have shown that only the  $\bar{s}^{TT}$  coefficients produce a non-negligible effect on the light propagation (while it has impact only at the next post-Newtonian level on the orbital dynamics [11, 97]). On the other hand, the other coefficients produce non-negligible effects on the orbital dynamics [11] and can therefore be constrained using planetary ephemerides data. In the linearized gravity limit, the contribution from SME to the 2-body equations of motion within the gravitational sector of SME are given by the first line of Eq. (1.82) (*i.e.* for a vanishing  $V^k$ ). The coefficient  $\bar{s}^{TT}$  is completely unobservable in this context since absorbed in a rescaling of the gravitational constant (see the discussion in [11, 9]).

Ideally, in order to perform a solid estimation of the SME coefficients using planetary ephemerides, one should include the full SME equations in the integration of the planets motion and fit

them simultaneously with the other estimated parameters (positions and velocities of planets,  $J_2$  of the Sun, ...). This solid analysis within the SME formalism has not been performed so far.

As a first step, a postfit analysis has been performed with A. Bourgoïn and A. Hees [79, 73]. The idea of this analysis is to derive the analytical expression for the secular evolution of the orbital elements produced by the SME contribution to the equations of motion. Using the Gauss equations, secular perturbations induced by SME on the orbital elements have been computed in [11] (see also [79] for a similar calculations done for the  $\bar{s}^{TJ}$  coefficients only). In particular, the secular evolution of the longitude of the ascending node  $\Omega$  and the argument of the perihelion  $\omega$  is given by

$$\left\langle \frac{d\Omega}{dt} \right\rangle = \frac{n}{\sin i (1 - e^2)^{1/2}} \left[ \frac{\varepsilon}{e^2} \bar{s}_{kP} \sin \omega + \frac{(e^2 - \varepsilon)}{e^2} \bar{s}_{kQ} \cos \omega - \frac{\delta m}{M} \frac{2na\varepsilon}{ec} \bar{s}^k \cos \omega \right] \quad (1.85a)$$

$$\left\langle \frac{d\omega}{dt} \right\rangle = -\cos i \left\langle \frac{d\Omega}{dt} \right\rangle - n \left[ -\frac{\varepsilon^2}{2e^4} (\bar{s}_{PP} - \bar{s}_{QQ}) + \frac{\delta m}{M} \frac{2na(e^2 - \varepsilon)}{ce^3(1 - e^2)^{1/2}} \bar{s}^Q \right], \quad (1.85b)$$

where  $a$  is the semimajor axis,  $e$  the eccentricity,  $i$  the orbit inclination (with respect to the ecliptic),  $n = (G_N m_\odot / a^3)^{1/2}$  is the mean motion,  $\varepsilon = 1 - (1 - e^2)^{1/2}$ ,  $\delta m$  the difference between the two masses and  $M$  their sum (in the cases of planets orbiting the Sun, one has  $M \approx \delta m$ ). In all these expressions, the coefficients for Lorentz violation with subscripts  $P$ ,  $Q$ , and  $k$  are understood to be appropriate projections of  $\bar{s}^{\mu\nu}$  along the unit vectors  $P$ ,  $Q$ , and  $k$ , respectively. For example,  $\bar{s}^k = k^i \bar{s}^{Ti}$ ,  $\bar{s}_{PP} = P^i P^j \bar{s}^{ij}$ . The unit vectors  $P$ ,  $Q$  and  $k$  define the orbital plane (see [11] or Eq. (8) from [73]).

Instead of including the SME equations of motion in planetary ephemerides, the postfit analysis uses estimations of supplementary advances of perihelia and nodes derived from ephemerides analysis [143, 142, 52] to fit the SME coefficients through Eq. (1.85). In [73], estimations of supplementary advances of perihelia and longitude of nodes from INPOP (see Table 5 from [52]) are used to fit a posteriori the SME coefficients. This analysis suffers from large correlations due to the fact that the planetary orbits are very similar to each other : nearly eccentric orbit and very low inclination orbital planes. In order to deal properly with these correlations a Bayesian Monte Carlo inference has been used [73]. The posterior probability distribution function can be found on Figure 1 from [73]. The intervals corresponding to the 68% Bayesian confidence levels are given in Table 1.7 as well as the correlation matrix. It is interesting to mention that a decomposition of the normal matrix in eigenvectors allows one to find linear combinations of SME coefficients that are uncorrelated with the planetary ephemerides analysis (see Eq. (15) and Table IV from [73]).

In summary, planetary ephemerides offer a great opportunity to constrain hypothetical violations of Lorentz symmetry. So far, only postfit estimations of the SME coefficients have been performed [79, 73]. In this analysis, estimations of secular advances of perihelia and longitude of nodes obtained with the INPOP planetary ephemerides [52] are used to fit a posteriori the SME coefficients using the Eqs. (1.85). The 68% marginalized confidence intervals are given in Table 1.7. This analysis suffers highly from correlations due to the fact that the planetary orbits are very similar. A very interesting improvement regarding this analysis would be to perform a full analysis by integrating the planetary equations of motion directly within the

TABLE 1.7 – Estimations of the SME coefficients from a postfit data analysis based on planetary ephemerides from [73]. The uncertainties correspond to the 68% Bayesian confidence levels of the marginal posterior probability distribution function. The associated correlation coefficients can be found in Table III from [73].

Coefficient	
$\bar{s}^{XX} - \bar{s}^{YY}$	$(-0.8 \pm 2.0) \times 10^{-10}$
$\bar{s}^{XX} + \bar{s}^{YY} - 2 \bar{s}^{ZZ}$	$(-0.8 \pm 2.7) \times 10^{-10}$
$\bar{s}^{XY}$	$(-0.3 \pm 1.1) \times 10^{-10}$
$\bar{s}^{XZ}$	$(-1.0 \pm 3.5) \times 10^{-11}$
$\bar{s}^{YZ}$	$(5.5 \pm 5.2) \times 10^{-12}$
$\bar{s}^{TX}$	$(-2.9 \pm 8.3) \times 10^{-9}$
$\bar{s}^{TY}$	$(0.3 \pm 1.4) \times 10^{-8}$
$\bar{s}^{TZ}$	$(-0.2 \pm 5.0) \times 10^{-8}$

Correlation coefficients							
1							
0.99	1						
0.99	0.99	1					
0.98	0.98	0.99	1				
-0.32	-0.24	-0.26	-0.26	1			
0.99	0.98	0.98	0.98	-0.32	1		
0.62	0.67	0.62	0.59	0.36	0.60	1	
-0.83	-0.86	-0.83	-0.81	-0.14	-0.82	-0.95	1

SME framework and by fitting the SME coefficients simultaneously with the other parameters fitted during the ephemerides data reduction.

### Related Publications :

1. **A. BOURGOIN**, C. LE PONCIN-LAFITTE, **A. HEES**, S. BOUQUILLON, G. FRANCOU et M.-C. ANGININ. « Lorentz Symmetry Violations from Matter-Gravity Couplings with Lunar Laser Ranging ». In : *Physical Review Letters* 119.20, 201102 (nov. 2017), p. 201102. DOI : [10.1103/PhysRevLett.119.201102](https://doi.org/10.1103/PhysRevLett.119.201102). arXiv : [1706.06294](https://arxiv.org/abs/1706.06294) [gr-qc]
2. **A. HEES**, Q. BAILEY, **A. BOURGOIN**, H. PIHAN-LE BARS, C. GUERLIN et C. LE PONCIN-LAFITTE. « Tests of Lorentz Symmetry in the Gravitational Sector ». In : *Universe* 2 (déc. 2016), p. 30. DOI : [10.3390/universe2040030](https://doi.org/10.3390/universe2040030). arXiv : [1610.04682](https://arxiv.org/abs/1610.04682) [gr-qc]
3. C. LE PONCIN-LAFITTE, **A. HEES** et S. LAMBERT. « Lorentz symmetry and very long baseline interferometry ». In : *Phys. Rev. D* 94.12, 125030 (déc. 2016), p. 125030. DOI : [10.1103/PhysRevD.94.125030](https://doi.org/10.1103/PhysRevD.94.125030). arXiv : [1604.01663](https://arxiv.org/abs/1604.01663) [gr-qc]
4. **A. BOURGOIN**, **A. HEES**, S. BOUQUILLON, C. LE PONCIN-LAFITTE, G. FRANCOU et M. -C. ANGININ. « Testing Lorentz Symmetry with Lunar Laser Ranging ». In : *Phys. Rev. Lett.* 117 (24 déc. 2016), p. 241301. DOI : [10.1103/PhysRevLett.117.241301](https://doi.org/10.1103/PhysRevLett.117.241301). URL : <http://link.aps.org/doi/10.1103/PhysRevLett.117.241301>
5. **A. HEES**, Q. G. BAILEY, C. LE PONCIN-LAFITTE, **A. BOURGOIN**, A. RIVOLDINI, B. LAMINE, F. MEYNADIER, C. GUERLIN et P. WOLF. « Testing Lorentz symmetry with

planetary orbital dynamics ». In : Phys. Rev. D 92.6, 064049 (sept. 2015), p. 064049.  
DOI : [10.1103/PhysRevD.92.064049](https://doi.org/10.1103/PhysRevD.92.064049). arXiv : [1508.03478](https://arxiv.org/abs/1508.03478) [gr-qc]

DRAFT

## 1.6 Perspectives : Research Projects

With the direct detection of gravitational waves and especially the observation of their electromagnetic counterparts, the determination of the SME  $\bar{s}^{TT}$  parameter has been improved by ten orders of magnitude with respect to my VLBI constrain. It illustrates brilliantly the impact of this emerging *gravitational astronomy* for fundamental physics, VLBI seems then not any more competitive in the future. However, Gaia will bring a very large amount of accurate data, covering at least a timespan of 5 years of observation. GSR-TTF astrometric sphere reconstruction can be an interesting tool to test Lorentz invariance. I will therefore develop the analytic model of the astrometric observable in minimal SME. In the forthcoming months, It will be implemented inside GSR-TTF and used with the Gaia data, in particular those concerning stars.

I will pursue studies of the minimal SME in its gravity-matter sector. I plan to strengthen my collaboration with Valéry Lainey, actually senior fellow at JPL. Indeed, we agreed recently to test Lorentz invariance with ephemerides of natural satellites. The rheological variety of natural satellites is such that my LLR results, in the context of gravity-matter sector of minimal SME, can be greatly improved. This work will be put into relation with the recent invitation of W. Folkner, from JPL, in order to collaborate on DE planetary ephemerides.

### 1.6.1 Gaia data and Solar System Objects dynamics

The second release of the Gaia catalog (DR2) will be published end of April 2018. Observations of more than 14 000 Solar System Object (SSO) will be published, with a very good astrometric accuracy. During the last two years, with A. Hees, we developed a simulation pipeline of SSO orbit propagation, whatever the metric theory of gravitation. My primary objective is to perform fundamental physics tests with SSOs. The great interest of these objects is their wide variety in semi-major axes, eccentricity and especially in inclination. With the large amount of Gaia data, this interests is even stronger because it will improve *de facto* the statistics after a global fit. Moreover, the variety in inclination ensures interesting results with respect to planetary ephemerides in the context of the minimal SME in its gravity sector. Indeed, this large variety of orbits can lead to a complete decorrelation of all the nine SME parameters of the gravity sector, which will be done for the first time.

Some preliminary simulations have been already performed and the results are encouraging. That is why I am actively collaborating with Daniel Hestroffer and Pedro David (IMCCE), both responsible for part of the official SSOs Gaia pipeline, to implement our simulation tool in data analysis.

Finally, I am interested with the Radar data from a long observation program at Arecibo telescope, led by Prof. J.-L. Margot at UCLA. A collaboration, with A. Hees, is ongoing in order to combine the Gaia and Radar data. I am expecting an improvement PPN parameter  $\beta$  as well as a significative extension of the exclusion zone concerning the 5th force formalism [59].

### 1.6.2 ACES-PHARAO

As part of my astronomer's duties, I am participating in the implementation of the data analysis software of the microwave link of the ACES-PHARAO mission. This link allows to compare the PHARAO clock on board the International Space Station (ISS) with ground clocks located in metrology laboratories involved in the project. We recently finalized an article describing the data analysis algorithm [126] and the impact of the orbit error of the pallet on mission specifications.

The near future is now to develop Einstein's gravitational redshift test. It will be necessary to take into account any possible noise on board the ISS, on the link microwave and on ground. We already know that each metrology laboratory will be able to communicate in direct view with the ISS during 5 minutes every 90 minutes. We must therefore quantify how many laboratories and how many ISS passes be needed to perform the redshift test with the best possible sensitivity, while ensuring the best control of systematics and biases. For example, we have to study which adjustment algorithm, direct least squares or Monte-Carlo, is the most convenient.

### 1.6.3 Satellite Laser Ranging

I used almost all astrometric and astro-geodesic techniques to constrain alternative phenomenologies to GR, except one of them : Satellite Laser Ranging (SLR) data. But, with the large number of SLR satellites orbiting the Earth at different semi-major axes and inclination, the large amount of data over decades, SLR techniques seems to be an interesting tool to test gravity. It is worth to highlighting that SLR data, from LAGEOS satellite, has been used to detect the Lense-Thirring effect [36]. The situation is even favorable with the recent launch of LARES satellite in 2013 [35], aiming to improve this fundamental physics test.

As member of ILRS (International Laser Ranging Service) since 2013, I have access to SLR data. In France, we have a powerful tool : the GINS software (Géodésie par Intégrations Numériques Simultanées) which is developed and maintained by the CNES Space Geodesy team in Toulouse. It is a precise orbit determination software. It includes on the one hand a process of numerical integration of the differential equations of motion of a satellite or a constellation of satellites in an inertial frame, taking into account all the gravitational and surface forces acting on the satellite as well as its specified attitude thanks to a macro modeling. On the second hand GINS allows a least-squares fitting procedure to produce a precise restituted orbit with a simultaneous determination of global and local parameters (gravity field, radiation pressure, etc.). GINS can deal with GNSS (GPS, Galileo, GLONAS, etc.), SLR, LLR, DORIS, altimetry and even VLBI data. GINS is also a tool for planetary geodesy by calculating orbits of deep space probes with Ranging and Doppler DSN (Deep Space Navigation) data.

In collaboration with Florent Deleflie (IMCCE) who is in charge of a SLR analysis center at Paris Observatory, Pascal Rosenblatt and Jean-Charles Marty (GINS developers), I intend to modify deeply the software, either on the dynamics and the processing of the data, to be able to consider any alternative phenomenologies to GR, in particular the SME formalism. The first steps will be to consider the equations of motion, light propagation and clocks in SME and 5th force formalisms.

### 1.6.4 Non-minimal SME

I presented previously my main results concerning a search for potential Lorentz symmetry violations, by considering different sectors of SME. For the gravity sector, I got the best current constraints. Is it sufficient? Without going into theoretical details, in fact the community has tried to accurately test the first-order corrections in SME, which corresponds to a linear Lagrangian with the Riemann tensor. This is called the minimal SME and. It motivates me to consider the second order of corrections, which is called the non-minimal SME. It is a question here of taking into account terms in the Lagrangian that can be quadratic and/or proportional to the covariant derivatives of the Riemann tensor. The first theoretical works, describing this regime, just appeared in the literature, notably by Prof. Q. Bailey at Embry-Riddle University in Arizona [7, 10].

In these pioneering works, Bailey does not yet conclude on the equations of motion of a  $N$  bodies system. On the other hand, equations of light propagation have been obtained. As a preliminary work, I used the spacetime interval proposed by Bailey to calculate TTF. I was able to retrieve his result concerning the propagation delay of photons and I have extended its result to the deflection of light rays. I am therefore considering, as a first step, to use these preliminary results to theoretically construct the gravitational delay of VLBI observable and to confront this new formulation with the IVS-OPAR data to constrain, for the first time, the parameters of this new non-minimal gravity sector.

Then, a long-standing collaboration with Quentin Bailey being established, I plan to work with his team to theoretically establish the equations of motion of  $N$  bodies system, ultimately to confront them with the LLR data within the ELPN model.

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## 2 | Note d'accompagnement

Accompagner quelqu'un, c'est se placer  
ni devant, ni derrière, ni à la place.  
C'est être à côté.

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Joseph Templier

### 2.1 Scientific and administrative responsibilities

Since my recruitment, I have been involved in the administration of research. Thus, between 2012 and 2016, I was member of Section 17 of the CNRS National Committee. This experience first allowed me to discover the evaluation of CNRS research units and researchers. I was able to appreciate the role of the section in recruiting young researchers. In 2014, I was proposed by the director of SYRTE, Arnaud Landragin, as deputy director of the laboratory, which was validated by the Council of the laboratory and then by INSU. I am primarily concerned with the space division of the laboratory, that is to say, the supervisory of two teams : *Earth Rotation and Space Geodesy* and *Celestial Reference System*. As SYRTE Deputy Director, I am responsible for representing the laboratory at the DIM/ACAV+ Steering Committee. In addition, I have been an evaluator for several funding proposals, for example french ANR and american NSF. Finally, since 2016, I became member of the Fundamental Physics Group of the CERES at CNES.

I am also involved in scientific society. Since 2012, I am a member of the International Astronomical Union and since 2013 of the International Laser Ranging Service.

I have participated in the proposal of space projects and answered to calls for ESA-M missions. So, in 2012, I was Co-PI of the GETEMME proposal to the M3 call. The objective of this project was high-precision planetodesy in the Martian Moon system. In 2014 and 2016, I participated in the Theia consortium in to the call M4 and M5. Theia is a high precision differential astrometry project (sub- $\mu$ as) and an open observatory. For the M5 call, I took charge of workpackages concerning relativistic modeling of the astrometric observable. Unfortunately, none of his answers were accepted by the European Space Agency. Finally, more recently, I became a member of the Science Team of the E-GRASP mission proposal, which is submitted, and still in competition, to the European Space Agency at the call [Earth Explorer Mission EE-10](#). In this context, I am in charge of workpackages concerning relativistic geodesy and possible fundamental physics tests with E-GRASP.

Some of my scientific activities are related to consortia. Thus, I belong to the DPAC consortium of the Gaia mission since 2006 and I am member of the data analysis center of the ESA ACES/PHARAO mission. I also participated in the international group Encelade 2.0, initially funded by Sorbonne Université, which is now an ISSI international team.

## 2.2 Astronomer's duties

Since 2006, I am member of the Data Processing and Analysis Consortium (DPAC) of the Gaia mission of the European Space Agency (ESA).

Between 2008 and 2012, I was responsible for timescales on board the Gaia satellite, that is to say observation time tagging thanks to an onboard rubidium clock. Given the astrometric accuracy of the satellite of a few  $\mu\text{as}$ , the onboard timescale has to be maintained with a relative stability of  $1\mu\text{s}$  over the timespan of the mission. It led me to develop a relativistic modeling of time transfer in order to calculate precisely the relation between proper and coordinated time along the satellite orbit, as well as the management of the realization onboard timescale thanks to the rubidium clock.

Since 2010, I am contributing to the data analysis pipeline of Solar System Objects. In collaboration with Daniel Hestroffer, who is responsible of a part of that pipeline, we are preparing softwares needed to perform fundamental physics tests with these observations.

Finally, at the local scale of the Paris Observatory, I am member since 2011 of the Steering Committee of the Gaia Specific Action, created by the Scientific Council of Paris Observatory.

Since 2009, I also joined the ACES group within SYRTE *Theory and Metrology* team. My role is to participate in the PHARAO/ACES microwave link modeling and analysis code. To do this, I studied and quantified the multi-path reflections between a ground station and the ACES platform integrated on the Columbus module of the International Space Station. I also contributed to the software for reference frames transformations, in particular the transformations between terrestrial (ITRF) and celestial (ICRF) frames. I also modeled the attitude of the International Space Station using quaternion, as well as the precise positioning of the two antennas (in S and Ku band) of the ACES payload.

Finally, since the end of 2016, I have taken the responsibility of the ANO1/PHARAO national observation service.

## 2.3 Thesis supervisory

### 2009-2012 : Aurélien Hees.

Aurélien Hees realized his Ph.D. thesis under the direction of Peter Wolf, SYRTE, and Véronique Dehant, Royal Observatory of Belgium. I took part at the level of 10% with a scientific supervisory. My involvement was important concerning Aurélien's learning of my TTF Formalism, which was finally very useful to Aurélien's work. After three post-docs, at JPL, at Rhodes University in South Africa and at the University of Los Angeles (UCLA), Aurélien joined the CNRS as permanent researcher in 2017. He is now one of my closest collaborator.

**Selected related Publications :**

1. A. HEES, Q. BAILEY, A. BOURGOIN, H. PIHAN-LE BARS, C. GUERLIN et C. LE PONCIN-LAFITTE. « Tests of Lorentz Symmetry in the Gravitational Sector ». In : *Universe* 2 (déc. 2016), p. 30. DOI : [10.3390/universe2040030](https://doi.org/10.3390/universe2040030). arXiv : [1610.04682](https://arxiv.org/abs/1610.04682) [gr-qc]
2. C. LE PONCIN-LAFITTE, A. HEES et S. LAMBERT. « Lorentz symmetry and very long baseline interferometry ». In : *Phys. Rev. D* 94.12, 125030 (déc. 2016), p. 125030. DOI : [10.1103/PhysRevD.94.125030](https://doi.org/10.1103/PhysRevD.94.125030). arXiv : [1604.01663](https://arxiv.org/abs/1604.01663) [gr-qc]
3. A. HEES, Q. G. BAILEY, C. LE PONCIN-LAFITTE, A. BOURGOIN, A. RIVOLDINI, B. LAMINE, F. MEYNADIER, C. GUERLIN et P. WOLF. « Testing Lorentz symmetry with planetary orbital dynamics ». In : *Phys. Rev. D* 92.6, 064049 (sept. 2015), p. 064049. DOI : [10.1103/PhysRevD.92.064049](https://doi.org/10.1103/PhysRevD.92.064049). arXiv : [1508.03478](https://arxiv.org/abs/1508.03478) [gr-qc]
4. A. HEES, S. BERTONE et C. LE PONCIN-LAFITTE. « Light propagation in the field of a moving axisymmetric body : Theory and applications to the Juno mission ». In : *Physical Review D* 90.8, 084020 (oct. 2014), p. 084020. DOI : [10.1103/PhysRevD.90.084020](https://doi.org/10.1103/PhysRevD.90.084020). arXiv : [1406.6600](https://arxiv.org/abs/1406.6600) [gr-qc]
5. A. HEES, S. BERTONE et C. LE PONCIN-LAFITTE. « Relativistic formulation of coordinate light time, Doppler, and astrometric observables up to the second post-Minkowskian order ». In : *Physical Review D* 89.6, 064045 (mar. 2014), p. 064045. DOI : [10.1103/PhysRevD.89.064045](https://doi.org/10.1103/PhysRevD.89.064045). arXiv : [1401.7622](https://arxiv.org/abs/1401.7622) [gr-qc]
6. A. HEES, B. LAMINE, S. REYNAUD, M.-T. JAEKEL, C. LE PONCIN-LAFITTE, V. LAINEY, A. FÜZFA, J.-M. COURTY, V. DEHANT et P. WOLF. « Radioscience simulations in general relativity and in alternative theories of gravity ». In : *Classical and Quantum Gravity* 29.23, 235027 (déc. 2012), p. 235027. DOI : [10.1088/0264-9381/29/23/235027](https://doi.org/10.1088/0264-9381/29/23/235027). arXiv : [1201.5041](https://arxiv.org/abs/1201.5041) [gr-qc]

**2009-2012 : Stefano Bertone.**

Stefano Bertone realized his Ph.D. thesis under the direction of Marie-Christine Angonin (50%) and me (50%). The goal was to construct a new astrometric modeling, based on the TTF formalism. Stefano was able to show the equivalence between this new modeling and the Gaia's DPAC modeling GREM. A close collaboration with Torino Observatory, which provided us with a probe attitude model, led to build a complete celestial sphere reconstruction, named GSR-TTF, which is today fully operational. Since 2013, Stefano is postdoctoral fellow at Bern University in Switzerland, where he is developing new space geodesy methods for the study of the Moon gravitational field with the GRAIL data.

**Related Publications :**

1. S. BERTONE, C. LE PONCIN-LAFITTE, P. ROSENBLATT, V. LAINEY, J.-C. MARTY et M.-C. ANGININ. « Impact analysis of the transponder time delay on radio-tracking observables ». In : *Advances in Space Research* 61 (jan. 2018), p. 89–96. DOI : [10.1016/j.asr.2017.09.003](https://doi.org/10.1016/j.asr.2017.09.003). arXiv : [1708.00546](https://arxiv.org/abs/1708.00546) [astro-ph.EP]
2. S. BERTONE, A. VECCHIATO, B. BUCCIARELLI, M. CROSTA, M. G. LATTANZI, L. BIANCHI, M.-C. ANGININ et C. LE PONCIN-LAFITTE. « Application of time transfer

- functions to Gaia's global astrometry. Validation on DPAC simulated Gaia-like observations ». In : *A&A* 608, A83 (déc. 2017), A83. DOI : [10.1051/0004-6361/201731654](https://doi.org/10.1051/0004-6361/201731654). arXiv : [1708.00541](https://arxiv.org/abs/1708.00541) [[astro-ph.IM](#)]
3. [A. HEES](#), [S. BERTONE](#) et C. LE PONCIN-LAFITTE. « Light propagation in the field of a moving axisymmetric body : Theory and applications to the Juno mission ». In : *Physical Review D* 90.8, 084020 (oct. 2014), p. 084020. DOI : [10.1103/PhysRevD.90.084020](https://doi.org/10.1103/PhysRevD.90.084020). arXiv : [1406.6600](https://arxiv.org/abs/1406.6600) [[gr-qc](#)]
  4. [A. HEES](#), [S. BERTONE](#) et C. LE PONCIN-LAFITTE. « Relativistic formulation of coordinate light time, Doppler, and astrometric observables up to the second post-Minkowskian order ». In : *Physical Review D* 89.6, 064045 (mar. 2014), p. 064045. DOI : [10.1103/PhysRevD.89.064045](https://doi.org/10.1103/PhysRevD.89.064045). arXiv : [1401.7622](https://arxiv.org/abs/1401.7622) [[gr-qc](#)]
  5. [S. BERTONE](#), O. MINAZZOLI, M. CROSTA, C. LE PONCIN-LAFITTE, A. VECCHIATO et M.-C. ANGININ. « Time transfer functions as a way to validate light propagation solutions for space astrometry ». In : *Classical and Quantum Gravity* 31.1, 015021 (jan. 2014), p. 015021. DOI : [10.1088/0264-9381/31/1/015021](https://doi.org/10.1088/0264-9381/31/1/015021). arXiv : [1306.2367](https://arxiv.org/abs/1306.2367) [[gr-qc](#)]

### 2012-2016 : Adrien Bourgoïn.

Adrien Bourgoïn realized his Ph.D. thesis under the direction of Marie-Christine Angonin (20%) and myself (80%), with the scientific participation of Sébastien Bouquillon and Gérard Francou from Paris Observatory Lunar Analysis Center (POLAC at SYRTE). The purpose of Adrien's thesis was to construct, from scratch, a new dynamical modeling of the Earth-Moon System in alternative theories to GR. The objective was to be able to simulate the Earth-Moon distance in any metric theory of gravity. In a second step, Adrien deeply modified the existing data analysis software of LLR measurements (CAROL, written actually in GR, using IERS conventions). The goal was to be able to consider any light-time equation whatever the gravitational theory used. When completed, Adrien attempted to consider a complete study of Lorentz invariance with LLR data within the SME formalism. Two important articles have been published in *Physical Review Letters* : the first concerns the gravitational sector and the second a gravity-matter coupling. Adrien is since January 2017 in post-doc at the University of Bologna.

#### Related Publications :

1. [A. BOURGOÏN](#), C. LE PONCIN-LAFITTE, [A. HEES](#), S. BOUQUILLON, G. FRANCOU et M.-C. ANGININ. « Lorentz Symmetry Violations from Matter-Gravity Couplings with Lunar Laser Ranging ». In : *Physical Review Letters* 119.20, 201102 (nov. 2017), p. 201102. DOI : [10.1103/PhysRevLett.119.201102](https://doi.org/10.1103/PhysRevLett.119.201102). arXiv : [1706.06294](https://arxiv.org/abs/1706.06294) [[gr-qc](#)]
2. [A. HEES](#), Q. BAILEY, [A. BOURGOÏN](#), H. PIHAN-LE BARS, C. GUERLIN et C. LE PONCIN-LAFITTE. « Tests of Lorentz Symmetry in the Gravitational Sector ». In : *Universe* 2 (déc. 2016), p. 30. DOI : [10.3390/universe2040030](https://doi.org/10.3390/universe2040030). arXiv : [1610.04682](https://arxiv.org/abs/1610.04682) [[gr-qc](#)]
3. [A. BOURGOÏN](#), [A. HEES](#), S. BOUQUILLON, C. LE PONCIN-LAFITTE, G. FRANCOU et M.-C. ANGININ. « Testing Lorentz Symmetry with Lunar Laser Ranging ». In : *Phys. Rev. Lett.* 117 (24 déc. 2016), p. 241301. DOI : [10.1103/PhysRevLett.117.241301](https://doi.org/10.1103/PhysRevLett.117.241301). URL : <http://link.aps.org/doi/10.1103/PhysRevLett.117.241301>

4. A. HEES, Q. G. BAILEY, C. LE PONCIN-LAFITTE, A. BOURGOIN, A. RIVOLDINI, B. LAMINE, F. MEYNADIER, C. GUERLIN et P. WOLF. « Testing Lorentz symmetry with planetary orbital dynamics ». In : *Phys. Rev. D* 92.6, 064049 (sept. 2015), p. 064049. DOI : [10.1103/PhysRevD.92.064049](https://doi.org/10.1103/PhysRevD.92.064049). arXiv : [1508.03478](https://arxiv.org/abs/1508.03478) [gr-qc]

## 2.4 Licence/Master internships

Regularly, I have supervised students of licence and of first/second year master. For some of them, it was even possible to enhance their internship with one (or even two) publication(s) in rank A refereed journals.

- 2017. M2 internship of Valentin Decouesnes. M1 internship of Julien Frank.
- 2016. M2 internship of Mansour Benbakoura. L3 and M2 internships of Nicolas Blonski and Valentin Decouesnes, respectively.

### Related Publications :

- (a) M. BENBAKOURA, V. RÉVILLE, A. S. BRUN, C. LE PONCIN-LAFITTE et S. MATHIS. « Evolution of star-planet systems under magnetic braking and tidal interaction ». In : *to be submitted to A&A* (2018)

- 2014. M2 internship of Luc Senecal.

- 2013. M2 internship of Pierre Auclair- Desrotour (co-director : S. Mathis).

### Related Publications :

- (a) P. AUCLAIR-DESROTOUT, C. LE PONCIN-LAFITTE et S. MATHIS. « Impact of the frequency dependence of tidal Q on the evolution of planetary systems ». In : *Astronomy & Astrophysics* 561, L7 (jan. 2014), p. L7. DOI : [10.1051/0004-6361/201322782](https://doi.org/10.1051/0004-6361/201322782). arXiv : [1311.4810](https://arxiv.org/abs/1311.4810) [astro-ph.EP]

- (b) P. AUCLAIR-DESROTOUT, S. MATHIS et C. LE PONCIN-LAFITTE. « Scaling laws to understand tidal dissipation in fluid planetary regions and stars I. Rotation, stratification and thermal diffusivity ». In : *A&A* 581, A118 (sept. 2015), A118. DOI : [10.1051/0004-6361/201526246](https://doi.org/10.1051/0004-6361/201526246). arXiv : [1506.07705](https://arxiv.org/abs/1506.07705) [astro-ph.EP]

- 2012. M2 internship of Frédéric Pierret.

- 2011. M2 internship of Mélody Sylvestre (co-director : D. Hestroffer).

- 2010. M2 internships of Stefano Bertone and Kazuhisa Oyama.

### Related Publications :

- (a) S. BERTONE, C. LE PONCIN-LAFITTE, P. ROSENBLATT, V. LAINEY, J.-C. MARTY et M.-C. ANGININ. « Impact analysis of the transponder time delay on radio-tracking observables ». In : *Advances in Space Research* 61 (jan. 2018), p. 89–96. DOI : [10.1016/j.asr.2017.09.003](https://doi.org/10.1016/j.asr.2017.09.003). arXiv : [1708.00546](https://arxiv.org/abs/1708.00546) [astro-ph.EP]

- 2009. M2 internship of Meng-Hsun Chung.

## 2.5 Teaching

Since 2017, I co-supervised with Daniel Hestroffer a methodology in the *Dynamics of Gravitational Systems* at M2 of the Observatory.

Since 2016, I am in charge of TP of L3 *numerical modeling for physics in python*, whose head is Pacôme Delva at Sorbonne Université.

Since 2015, I participate in the lectures *Metrology and Fundamental Physics*, whose head is Marie-Christine Angonin, at M2 of Observatory.

Since 2009, I am teaching at M1 of the Observatory, responsible of the TD *Relativity and Time*.

From 2009 to 2016, I co-supervised with Valery Lainey a methodology in the *Dynamics of Gravitational Systems* at M2 of the Observatory.

In addition, I am regularly invited to teach in national and/or international schools. For example, I gave two lectures during the last GRGS (Groupe de Recherche en Géodésie Spatiale) summer school in 2016 at Les Houches. I gave also a lecture in 2016 in Bad Honeff in Germany on Time Transfer Functions and more recently, in September 2017, I gave two lectures at Gif School of IN2P3. I also co-organized, with Jean Souchay, a CNRS school in 2008 in Barèges, south part of France near the Pyrénées.

I also have teaching administration activities. Thus, since 2011, I co-organize all *méthodologies* of the *Dynamics of Gravitational Systems* at M2 of the Observatory.

More recently, in September 2017, I am the new director of the Observatory Master's Mention, now endorsed by PSL, whose name being SUTS (Sciences de l'Univers et Technologies Spatiales) and is one of the most important Master in Astronomy & Astrophysics in France.

## A | Selected Publications

DRAFT

## Improved determination of $\gamma$ by VLBI (Research Note)

S. B. Lambert and C. Le Poncin-Lafitte

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### ABSTRACT

**Aims.** This study revisits the estimate of the post-Newtonian relativistic parameter  $\gamma$  reported previously. We use (i) improved physical and astronomical modeling in the analysis software package, and (ii) a higher number of observations, a large part of which come from a relatively small number of VLBA experiments at 8 GHz.

**Methods.** We analyzed more than seven million group delays measured by very long baseline interferometry between August 1979 and August 2010. The parameter  $\gamma$  was least squares fitted to delays as a global parameter over the entire observational time period.

**Results.** The most complete solution of this study yielded  $\gamma = 0.99992 \pm 0.0001$ , whereas it was  $0.99984 \pm 0.00015$  in our 2009 paper. The item (i), which is recognized as important for geodesy and reference frame realization, provides estimates of  $|\gamma - 1|$  that are smaller than  $10^{-4}$ . As expected, the formal error in  $\gamma$  decreases when additional sessions are processed. In particular, we demonstrate that the inclusion of more than 1.7 million observations from the VLBA (mainly from the RDV and VLBA calibrator survey experiments) in the analysis decreases the formal error in the estimate of  $\gamma$  by about 15% with respect to our previous determination.

**Key words.** astrometry – techniques: interferometric – geodesy

### 1. Introduction

In an earlier paper (Lambert & Le Poncin-Lafitte 2009, hereafter LL09), we estimated the post-Newtonian parameter  $\gamma$  by analyzing group delays recorded by astrometric and geodetic very long baseline interferometry (VLBI) at 8 GHz with an accuracy of  $1.5 \times 10^{-4}$ .

In a parallel study, Fomalont et al. (2009) found agreement with General Relativity of  $10^{-4}$  using phase-referenced VLBA observations of 3C 279 and 3C 273 at 15, 22, and 43 GHz. Though the accuracy of this measurement does not compete with other studies, the authors pointed out that the precision of the determination could be improved by scheduling specially designed VLBA experiments.

Since then, a number of efforts in the geodesy and astronomical communities led to the inclusion of substantially improved geophysical and astrometric models in the VLBI data reduction. Moreover, we completed the Paris Observatory VLBI analysis center observational database by adding a large number of new observations that had not been processed in LL09, and by upgrading the analysis software to new models. For these reasons, we thought that it was worthwhile to re-launch the LL09 analyses to appreciate the impact of all the improvements listed above.

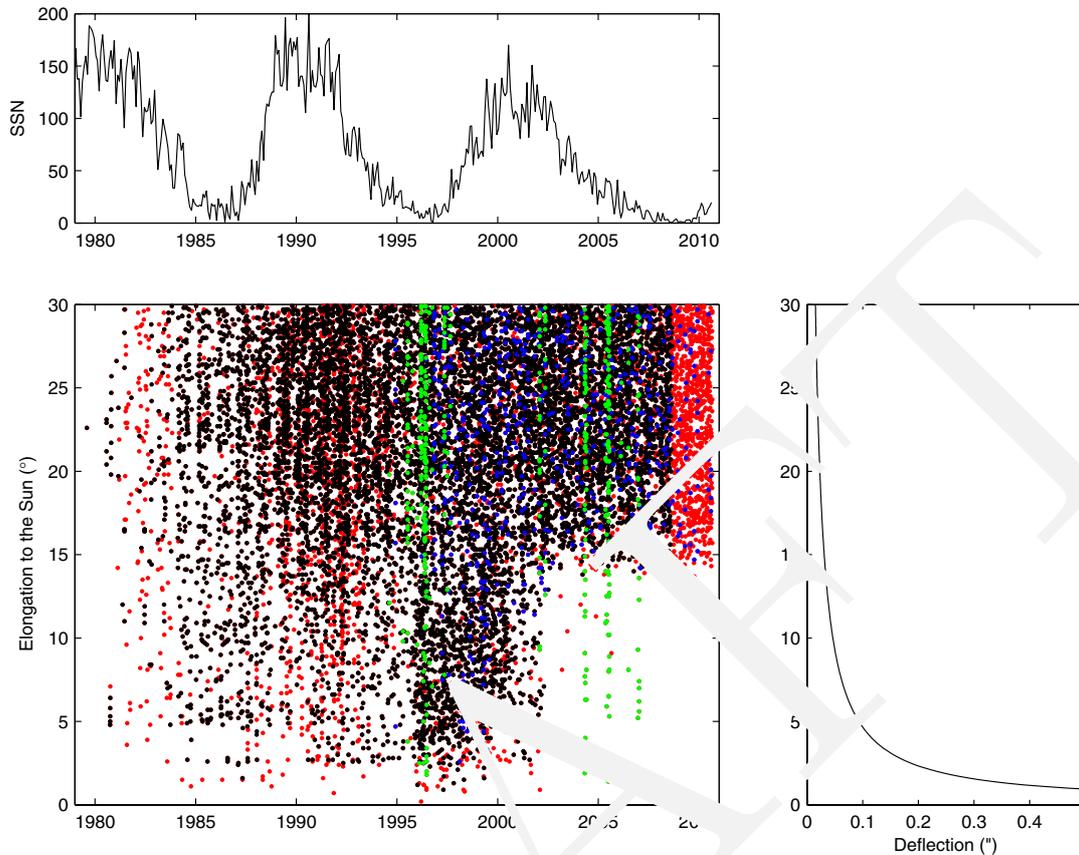
### 2. Analysis configuration

The session list now includes a number of sessions that were not processed in LL09 (Fig. 1). Most of them are generally designed for geodetic application such as Earth orientation parameter monitoring or station coordinate determination. After 2009, most of the additional sessions are the IVS rapid turn around experiments R1 and R4, scheduled every Monday and Thursday, respectively, that are designed to provide twice weekly Earth

orientation parameters and continuity with the NEOS and CORE sessions that were operated before 2002. A relatively small number of additional sessions after 1994 used of the 10-station North American Very Long Baseline Array (VLBA). The observing configuration of the VLBA network allows one to image sources and determine highly accurate station and source positions (see, e.g., Petrov et al. 2009). The VLBA can be used either alone (these sessions will be referred to as VLBA sessions in the following) or together with additional overseas antennas (denoted by VLBA+ sessions in the following) that push baseline lengths to more than 10 000 km. The former category includes the VLBA Calibrator Survey (VCS) programs 1 to 6 (Beasley et al. 2002; Fomalont et al. 2003; Petrov et al. 2005, 2006; Kovalev et al. 2007; Petrov et al. 2008) that were scheduled between 1994 and 2007. They contain observations as close as  $1.4^\circ$  to the Sun, which are indicated as green, vertical bands in Fig. 1. In the latter category, one finds the sessions known as RDV experiments using the VLBA plus up to ten additional geodetic stations located worldwide (in blue in Fig. 1). It appears that VLBA+ sessions stopped observing at less than  $15^\circ$  from the Sun after 2002, like all other routine VLBI experiments. VLBA and VLBA+ sessions usually have a number of observations larger than 10 000 and a postfit rms delay in the range 5–30 ps.

The most complete solution in this study processed 5055 sessions between 3 August 1979 and 30 August 2010, totalling more than 7.3 millions ionosphere-free group delay measurements at 8 GHz. The calculations used the Calc 10.0/Solve 2010.05.21 geodetic VLBI analysis software package<sup>1</sup>, developed and maintained at NASA Goddard Space Flight Center, and were carried out at the VLBI analysis center of the Paris Observatory (Gontier et al. 2006), which is part of

<sup>1</sup> <http://gemini.gsfc.nasa.gov/solve>



**Fig. 1.** The *main plot* displays the observation history of the sources at less than  $30^\circ$  from the Sun (black: observations treated in LL09; red: additional observations of routine experiments not processed in LL09 excluding VLBA+ and VLBA; blue: VLBA+; green: VLBA). The *upper plot* gives the Sun spot number (SSN, Clette et al. 2007). The *right plot* displays the deflection angle predicted by General Relativity.

the International VLBI Service for Geodesy and Astrometry (IVS, Schlüter & Behrend 2007), north orientation parameters, their rates, and station coordinates were estimated once per session. Loose no-net rotation (NNR) and translation constraints per session were uniformly applied to the positions of all stations including Fort Davis (Texas), Pie Island (New Mexico), Fairbanks (Alaska), and the VGO antenna in Concepción, Chile because of strong non-linear displacements (the latter two stations experienced post-seismic relaxation effects after large earthquakes on the Denali fault in 2003, and between Talca and Concepción in early 2010, respectively). Antenna thermal deformations were mapped using the values of Nothnagel (2009) whereas antenna axis offsets were estimated as global parameters over the full observational time span for a set of 66 stations. The cut-off elevation angle was set to  $5^\circ$ . Zenith wet delays were estimated as a continuous piecewise linear function at 30-min intervals. Troposphere gradients were estimated as 6-hr east and north piecewise functions at all stations except a set of 110 stations with poor observational history.

Station heights were corrected for atmospheric pressure and oceanic tidal loading. The relevant loading quantities were deduced from surface pressure grids from the U. S. NCEP/NCAR reanalysis project atmospheric global circulation model (Kalnay et al. 1996; Petrov & Boy 2004) and from the FES 2004 ocean tide model (Lyard et al. 2004). A critical change with respect to LL09 is the use of atmospheric pressure loading coefficients

at diurnal and semi-diurnal frequencies computed with a non-inverted barometer (NIB) hypothesis. In this hypothesis, oceans do not react to any atmospheric pressure variations and any increment in the atmospheric sea level pressure is fully and instantaneously transmitted to the ocean bottom. The NIB ocean is assumed to be static on time scales around and below a few days (e.g., Willebrand et al. 1980; Wunsch & Stammer 1997). In LL09, we assumed that the ocean reacted to balance atmospheric pressure variations at the sea level.

We used a priori source coordinates of the second realization of the International Celestial Reference Frame (ICRF2, Fey et al. 2010), adopted by the International Astronomical Union (IAU) in August 2009 as the fundamental realization of the International Celestial Reference System (Feissel & Mignard 1997) and a replacement of the ICRF (Ma et al. 1998). The ICRF2 contains the coordinates of 3414 extragalactic sources determined after VLBI observations at 8 GHz during 1979–2009. With respect to the first ICRF, the noise floor ( $40 \mu\text{as}$ ) has improved by a factor of five, and the axis stability ( $10 \mu\text{as}$ ) by a factor of two. Nevertheless, in LL09, we mentioned that the a priori radio source catalogue does not influence the results at a significant level since source coordinates are estimated during the analysis. Therefore, we do not expect the use of the ICRF2 to be the source of the improvement yielded in this paper. As in LL09, we loosely constrained the sources to the ICRF2 catalogue. With respect to LL09, three sources identified

**Table 1.** Characteristics of the solutions and estimates of  $\gamma$ . Details of the various solutions are given in the text.

	No. sessions	No. delays	No. sources	Postfit rms delay (ps)	$\gamma$	$\chi^2/f$
1979–2008 LL09	3937	4 386 112	916	25.0	$0.99984 \pm 0.00015$	0.86
1979–2008 S1	3937	4 400 315	913	24.8	$1.00004 \pm 0.00015$	0.84
1979–2008 S2	4643	4 952 152	1022	24.8	$0.99998 \pm 0.00015$	0.84
1979–2010 S3	4930	5 584 232	1077	24.8	$1.00004 \pm 0.00014$	0.84
1979–2010 S4	5012	6 997 297	1343	23.3	$0.99993 \pm 0.00013$	0.83
1979–2010 S5	5055	7 351 000	3706	23.2	$0.99992 \pm 0.00012$	0.84

as gravitational lenses were removed: 0218+357 and a component 0218+35A (observed in one session each), and PKS 1830–211 (observed in two sessions).

In the new software release, the a priori precession and nutation comply with the IAU 2000/2006 resolutions, which incorporate the nutation model of Mathews et al. (2002) in a way that is consistent with the precession of Capitaine et al. (2003b) and the non-rotating origin-based coordinate transformation between the terrestrial and celestial coordinate systems (Capitaine et al. 2003a).

As a major change with respect to LL09, the Niell (1996) mapping functions (NMF) were replaced by the Vienna mapping functions 1 (VMF1, Böhm et al. 2006), where two of the three coefficients of the continued fraction expressing the static mapping functions are computed from the 40-yr reanalysis (ERA-40) of the European Center for Medium-Range Weather Forecast (ECMWF) data. Unlike the previously available mapping functions (Niell 1996; Niell 2000; Böhm & Schuh 2001), they depend on the day of the year, and are no longer symmetric with respect to the equator. Tesmer et al. (2007) showed that VMF1 lead to station height repeatability that is 5.7% better than other mapping functions. They also mention that, compared with NMF, the effect on the radio source declinations is  $\sim 10 \mu\text{s}$  for declinations below  $-20^\circ$ .

In the next section, we present various solutions designed to test the influence of the changes with respect to LL09 listed above.

### 3. Analysis and results

Table 1 summarizes the solution information and displays estimates of  $\gamma$  and the reduced  $\chi^2$  of the solution. The first line of Table 1 reproduces the most complete solution of LL09. In solution S1, we considered the same session list as in LL09 to check the effects of the improved geophysical and astronomical modeling and the slight changes in the analysis configuration, as listed in Sect. 2. The number of delays in S1 appears to be higher by less than 0.005% than in LL09 because of the use of different versions for a few sessions. It turns out that the formal errors in  $\gamma$  are expected to be comparable, at about  $1.5 \times 10^{-4}$ . However, it appears that  $\gamma$  is closer to unity:  $|\gamma - 1|$  was  $2 \times 10^{-4}$  in LL09, and less than  $10^{-4}$  in S1.

Taken individually, none of the analysis configuration changes listed in the previous section are able to fully explain the small shift in the central value of  $\gamma$  toward unity. It results from the combined effects of all the geophysical and astronomical model improvements.

Solution S2 included additional VLBI sessions between 1979 and 2008 that had not been processed in LL09 (excluding the VLBA and VLBA+ sessions). It therefore checked the

effect of completing the observational database, but not extending the observing time span. The next three solutions were based on processed sessions until the end of August 2010. Solution S3 included additional VLBI sessions between 2000 and 2010, and checked the ability of the current geodetic VLBI observations (excluding the VLBA and VLBA+ sessions) to constrain  $\gamma$  if the current observing strategy were continued in the future. As for S1, the estimated value of  $|\gamma - 1|$  in S2 and S3 are well below  $10^{-4}$ . Solution S3 processed about 12% more observations than S2. The formal error decreased by 6%. Routine VLBI sessions scheduled every three days on average are only able to decrease the error in  $\gamma$  at a rate of about  $10^{-6}$  per year.

Solution S4 is similar to S3, except that it included an additional 72 VLBA+ sessions scheduled bi-monthly in average). The error was reduced to  $1.5 \times 10^{-4}$ . Finally, solution S5, which is the most complete solution considered in this study, included both VLBA and VLBA+ sessions. We obtained  $\gamma = 0.99992 \pm 0.00012$ . Although the lengths of the S3 and S5 session lists differ by less than 1%, the addition of the rather sparse VLBA sessions increases the number of delays by 30% and the number of sources by a factor of four. Compared with LL09 and other solutions that used routine VLBI experiments only (S1 to S3), solution S5 has the formal error that is smaller by  $\sim 15\%$ , where  $|\gamma - 1|$  is below  $10^{-4}$ , and shows that the VLBA experiments, although scheduled sparsely, have a good potential for General Relativity tests.

### 4. Concluding remarks

Although we do not challenge the results of Bertotti et al. (2003) from Cassini spacecraft measurements ( $\gamma = 1.00002 \pm 0.00002$ ), the small improvement presented in this study is notable because it illustrates the capability of certain geodetic/astrometric VLBI networks and observing configurations to increase the sensitivity to  $\gamma$ . The VLBI observational data base provides a very flexible way to test General Relativity in the Solar System at the level of  $10^{-4}$  thanks to the public availability of the data and the low CPU time taken by the solutions.

This data base is still increasing with new observations of very good quality thanks to a great, joint effort of worldwide radio astronomical observatories and space agencies. The upcoming VLBI 2010 will be designed in particular to reduce systematic errors, including possible source structure corrections thanks to faster antennas, larger networks, and higher data rates resulting in a  $uv$  coverage that is much better than in the current geodetic experiments (Petračenko et al. 2008). This new VLBI network will likely lead to improved ground-based tests of General Relativity (Heinkelmann & Schuh 2010). We therefore encourage VLBI observing program committees to schedule

observations of sources close to the Sun as in the VLBA calibrator survey sessions.

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**Relativistic formulation of coordinate light time, Doppler, and astrometric observables up to the second post-Minkowskian order**A. Hees,<sup>1,\*</sup> S. Bertone,<sup>2,3,†</sup> and C. Le Poncin-Lafitte<sup>2,‡</sup><sup>1</sup>*Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena California 91109, USA*<sup>2</sup>*Observatoire de Paris, SYRTE, CNRS/UMR 8630, UPMC, 61 avenue de l'Observatoire F-75014 Paris, France*<sup>3</sup>*INAF, Astrophysical Observatory of Torino, University of Torino, Via Osservatorio 20, 10025 Pino Torinese (Torino), Italy*

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Given the extreme accuracy of modern space science, a precise relativistic modeling of observations is required. In particular, it is important to properly describe light propagation through the Solar System. For two decades, several modeling efforts based on the solution of the null geodesic equations have been proposed, but they are mainly valid only for the first-order post-Newtonian approximation. However, with the increasing precision of ongoing space missions such as Gaia, Gaia, BepiColombo, JUNO, and JUICE, we know that some corrections up to the second order have to be taken into account for future experiments. We present a procedure to compute the relativistic coordinate time delay, Doppler, and astrometric observables avoiding the integration of the null geodesic equation. This is possible using the time transfer function formalism, a powerful tool providing key quantities such as the time of flight of a light signal between two point events and the tangent vector to its null geodesic. Indeed, we show how to compute the time transfer functions and their derivatives (and thus range, Doppler, and astrometric observables) up to the second post-Minkowskian order. We express these quantities as quadratures of some functions that depend only on the metric and its derivatives evaluated along a Minkowskian straight line. This method is particularly well adapted for numerical estimations. As an illustration, we provide explicit expressions in static and spherically symmetric space-time up to second post-Minkowskian order. Then we give the order of magnitude of these corrections for the range and Doppler on the BepiColombo mission and for astrometry in a GAME-like observation.

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**I. INTRODUCTION**

During the last 20 years, space science has made stunning progress. Indeed, the accuracy of tracking of probes increased drastically. For example, the Cassini spacecraft reached the level of centimeters for the range and  $3 \times 10^{-6}$  m/s for the Doppler [1]. In the near future, the BepiColombo mission should reach an accuracy of 10 cm and  $10^{-6}$  m/s for range and Doppler, respectively [4,5]. On the other hand, within the next few years, Gaia's astrometric catalogue is expected to get positions, parallaxes and proper motions of a billion celestial objects with a precision of several microarcseconds [6], improving by a factor of 100 what was accomplished with HIPPARCOS [7]. However, we know that these high-precision observations need to be reduced and interpreted in a complete relativistic framework [8]. Several key points need to be considered, in particular a precise modeling for the propagation of the observed signal. In the limit of geometrical

optics, it is well known that light rays follow null geodesics. Thus, radioscience (range & Doppler) and astrometric observables are traditionally analyzed by determining the full light trajectory, by solving the null geodesic equations. This method works quite well within the first post-Newtonian (1PN) and post-Minkowskian (1PM) approximations, as it is shown by the results obtained in [9–16]. In the context of the Schwarzschild-like geometry, a solution of the null geodesic equations has been derived at the post-post-Minkowskian (2PM) order in [17,18]. The case of a static, spherically symmetric space-time has also been considered in [19] where a solution of the eikonal equation is found.

However, finding an analytical solution of the geodesic equations is a challenging task that requires complex calculations, in particular when one has to take into account the presence of mass multipoles and/or the effects due to the planetary motions. Moreover, calculations become quite complicated in the 2PM approximation [20] especially when space-time is not stationary [21]. Nevertheless, it has been recently demonstrated that this task is not at all mandatory and can be replaced by another approach, initially based on the Sygne World Function [22] and then on the time transfer functions (TTF) [23].

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Indeed, within the TTF formalism, the solution of the null geodesic equation is advantageously replaced by the determination of the TTF and its first derivatives. In general, the determination of the TTF is as challenging as the integration of the null geodesic equations. Nevertheless, this task is really easier in a weak gravitational field. In particular, an algorithmic method to compute a PM expansion of the TTF at any order has been presented in [23], the determination of the TTF being done by performing integrals of some functions of the space-time metric evaluated along a Minkowskian segment between the emitter and the receiver of the signal. Moreover, from a computational point of view, the quadrature of a function taken along a straight line is easier than the full determination of the photon trajectory, which is a boundary value problem [24].

In this paper, we take advantage of the properties of the TTF formalism in order to construct a straightforward modeling of radioscience and astrometric observables. We present here a method to compute the TTF and its derivatives (and therefore coordinate time delay, frequency shift and astrometric observables) at 2PM order. Our method is particularly well adapted for numerical computation of radioscience and astrometric observables from the space-time metric. It can be used in General Relativity as well as in any alternative theories of gravity, where the light propagation is described by the null geodesic equations at the geometric optic approximation, improving what at two of us presented up to 1PM order in [25].

The paper is organized as follows. In Sec. II, we present the notations and the conventions used through this paper. In Sec. III, we introduce briefly the TTF formalism and recall how to determine the TTF when the emitter and the receiver of the light ray are in motion. In Sec. IV, we present a straightforward modeling of the radioscience and astrometric observables from the TTF. At this point, no expansion nor approximation is made and the observables are expressed in terms of the TTF and its first derivatives. In Sec. V, we show how to compute the TTF and its derivatives up to 2PM order. In Sec. VI, we specify our formulas for a static, spherically symmetric space-time and apply them to a Schwarzschild-like geometry. We compute the order of magnitude of the 2PM terms in two cases. First, we compute the values of the range and Doppler for BepiColombo and compare our results with those obtained in [26]. Second, considering a GAME-like observation [27], we simulate absolute and relative astrometric observations near the limb of the Sun to make evident the 2PM contribution to light deflection and aberration. In Sec. VII, we give our conclusions.

## II. NOTATION AND CONVENTIONS

In this paper  $c$  is the speed of light in a vacuum and  $G$  is the Newtonian gravitational constant. The Lorentzian metric of space-time  $V_4$  is denoted by  $g$ .

The signature adopted for  $g$  is  $(+---)$ . We suppose that space-time is covered by some global quasi-Galilean coordinate system  $(x^\mu) = (x^0, \mathbf{x})$ , where  $x^0 = ct$ ,  $t$  being a time coordinate, and  $\mathbf{x} = (x^i)$ . We assume that the curves of equation  $x^i = \text{constants}$  are timelike, which means that  $g_{00} > 0$  anywhere. We employ the vector notation  $\mathbf{a}$  in order to denote  $(a^1, a^2, a^3) = (a^i)$ . Considering two such quantities  $\mathbf{a}$  and  $\mathbf{b}$  we use  $\mathbf{a} \cdot \mathbf{b}$  to denote  $a^i b^i$  (Einstein convention on repeated indices is used). The quantity  $|\mathbf{a}|$  stands for the ordinary Euclidean norm of  $\mathbf{a}$ . For any quantity  $f(x^\mu)$ ,  $f_{,\alpha}$  denotes the partial derivative of  $f$  with respect to  $x^\alpha$ . In this paper, we are dealing with post-Minkowskian (PM) expansions. We suppose each quantity can be represented as a series in ascending power of  $G$ . The indices in parentheses characterize the order of perturbation. They can set up or down, depending on the convenience. For example, the space-time metric can be expanded as

$$g_{\mu\nu} = \eta_{\mu\nu} + \sum_{n=1}^{\infty} g_{\mu\nu}^{(n)}, \quad (1)$$

where  $g_{\mu\nu}^{(n)}$  is of order  $\mathcal{O}(G^n)$ .

## III. TIME TRANSFER FUNCTION FORMALISM

We thus consider two observers  $\mathcal{O}_A$  and  $\mathcal{O}_B$  located at points  $\mathbf{x}_A$  and  $\mathbf{x}_B$ , respectively. We suppose that the past null cone at a given point  $x_B = (ct_B, \mathbf{x}_B)$  intersects the world line  $\mathcal{O}_A$  at any one point  $x_A = (ct_A, \mathbf{x}_A)$  (see Fig. 1). The difference  $t_B - t_A$  is the coordinate travel time of a light ray connecting the emission point  $x_A$  with the reception point  $x_B$ . This quantity may be written as a time transfer function [22,28–30],

$$t_B - t_A = \mathcal{T}_r(\mathbf{x}_A, t_B, \mathbf{x}_B) = \mathcal{T}_e(t_A, \mathbf{x}_A, \mathbf{x}_B), \quad (2)$$

where  $\mathcal{T}_r$  and  $\mathcal{T}_e$  are the time transfer functions (TTF) at reception and at emission, respectively. In the following,

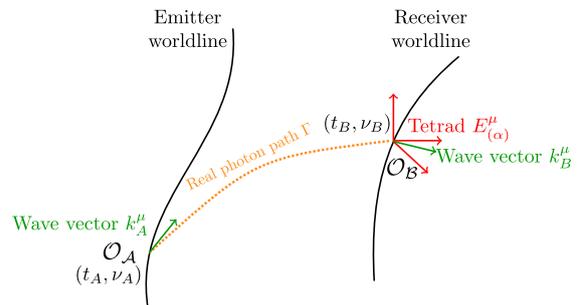


FIG. 1 (color online). Representation of the general geometry studied in this paper: a light signal of frequency  $\nu_A$  is emitted by  $\mathcal{O}_A$  with a wave four-vector of components  $k_A^\mu$  and received by  $\mathcal{O}_B$  at a frequency  $\nu_B$  and with a wave four-vector of components  $k_B^\mu$ .

we consider only the case of the TTF at reception, but the discussion can be done in the same way by using the TTF at emission. TTF directly gives the coordinate propagation time of an electromagnetic signal and is therefore closely related to the Range observable [25]. The determination of the TTF is as challenging as the integration of the null geodesic equation [111], but, in the weak field approximation, a general PM expansion of the TTF has been presented in [23] and will be used in Sec. V to derive explicit equations up to the 2PM order. Generally speaking, neither the emitter  $\mathcal{O}_A$  nor the receiver  $\mathcal{O}_B$  of an electromagnetic signal are static. Instead, they are following a trajectory  $\mathbf{x}_A(t)$  and  $\mathbf{x}_B(t)$  usually parametrized by a coordinate time  $t$ . In this case, Eq. (2) becomes an implicit relation since  $\mathbf{x}_A$  depends on  $t_A$ . In the weak field approximation, Eq. (2) must then be read as

$$\begin{aligned} t_B - t_A &= \mathcal{T}_r(\mathbf{x}_A(t_A), t_B, \mathbf{x}_B(t_B)) \\ &= \frac{|\mathbf{x}_B(t_B) - \mathbf{x}_A(t_A)|}{c} + \frac{1}{c} \Delta_r(\mathbf{x}_A(t_A), t_B, \mathbf{x}_B(t_B)), \end{aligned} \quad (3)$$

with  $\Delta_r/c$  the so-called “delay function” [23]. From an experimental point of view, the position of the emitter  $\mathcal{O}_A$  may be recorded at the time of emission  $t_B$  rather than at the time of reception  $t_A$ , i.e. we may have more direct access to  $\mathbf{x}_A(t_B)$  rather than  $\mathbf{x}_A(t_A)$ . Two approaches are possible. First an analytical solution can be derived by following the procedure presented in [31]. For any quantity  $Q_A(t)$  defined along the world line of the observer  $\mathcal{O}_A$ , let us put  $\tilde{Q}_A = Q_A(t_B)$ . Thus we may write  $\tilde{\mathbf{x}}_A$  for  $\mathbf{x}_A(t_B)$ ,  $\tilde{t}_A$  for  $t_A(t_B)$ , etc. The idea is now to expand the position of  $\mathcal{O}_A$  recorded at time  $t_A$  with respect to coordinate time  $t_B$  by a Taylor expansion as follows,

$$\begin{aligned} \mathbf{x}_A(t_A) &= \tilde{\mathbf{x}}_A + (t_A - t_B)\tilde{\mathbf{v}}_A + \frac{1}{2}(t_A - t_B)^2\tilde{\mathbf{a}}_A \\ &\quad + \frac{1}{6}(t_A - t_B)^3\tilde{\mathbf{b}}_A + \dots, \end{aligned} \quad (4)$$

where  $\tilde{\mathbf{v}}_A = \mathbf{v}_A(t_B) = \frac{d\mathbf{x}_A}{dt}|_{t_B}$ ,  $\tilde{\mathbf{a}}_A = \mathbf{a}_A(t_B) = \frac{d^2\mathbf{x}_A}{dt^2}|_{t_B}$  and  $\tilde{\mathbf{b}}_A = \mathbf{b}_A(t_B) = \frac{d^3\mathbf{x}_A}{dt^3}|_{t_B}$ . The introduction of this expansion in (3) leads

$$\begin{aligned} t_B - t_A &= \frac{\tilde{D}_{AB}}{c} + (t_B - t_A) \frac{\tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{D}}_{AB}}{c\tilde{D}_{AB}} + \frac{(t_B - t_A)^2}{2c\tilde{D}_{AB}} \left[ \tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{D}}_{AB} - \left( \frac{\tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{D}}_{AB}}{\tilde{D}_{AB}} \right)^2 \right] \\ &\quad + \frac{(t_B - t_A)^3}{2c\tilde{D}_{AB}} \left[ \frac{1}{3}\tilde{\mathbf{b}}_A \cdot \tilde{\mathbf{D}}_{AB} - \tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{a}}_A - \frac{\tilde{v}_A^2 \tilde{\mathbf{v}}_A \cdot \mathbf{L}_{AB}}{\tilde{D}_{AB}^2} + \frac{(\mathbf{v}_A \cdot \tilde{\mathbf{v}}_A)(\tilde{\mathbf{a}}_A \cdot \tilde{\mathbf{D}}_{AB})}{\tilde{D}_{AB}^2} + \frac{(\tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{D}}_{AB})^3}{\tilde{D}_{AB}^4} \right] + \frac{1}{c} \Delta_r(\tilde{\mathbf{x}}_A, t_B, \mathbf{x}_B) \\ &\quad - \frac{(t_B - t_A)}{c} \frac{\partial \Delta_r(\tilde{\mathbf{x}}_A, t_B, \mathbf{x}_B)}{\partial x_A^i} \tilde{v}_A^i + \frac{(t_B - t_A)^2}{c} \left[ \frac{1}{2} \frac{\partial \Delta_r(\tilde{\mathbf{x}}_A, t_B, \mathbf{x}_B)}{\partial x_A^i} \tilde{a}_A^i + \frac{1}{c} \frac{\partial \Delta_r(\tilde{\mathbf{x}}_A, t_B, \mathbf{x}_B)}{\partial x_A^i \partial x_A^j} \tilde{v}_A^i \tilde{v}_A^j \right] + \dots, \end{aligned} \quad (5)$$

where  $\tilde{\mathbf{D}}_{AB} = \mathbf{x}_B(t_B) - \mathbf{x}_A(t_B)$ ,  $\tilde{D}_{AB} = |\tilde{\mathbf{D}}_{AB}|$  and  $v_A = |\mathbf{v}_A|$ . An iterative solution of Eq. (5) gives

$$\begin{aligned} \mathcal{T}_r(\mathbf{x}_A(t_A), t_B, \mathbf{x}_B(t_B)) &= \frac{\tilde{v}_A^2}{c^2} + \frac{\tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{D}}_{AB}}{c^2} + \frac{1}{2c} \left[ \tilde{v}_A^2 + \left( \frac{\tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{D}}_{AB}}{\tilde{D}_{AB}} \right)^2 - \tilde{\mathbf{a}}_A \cdot \tilde{\mathbf{D}}_{AB} \right] \\ &\quad + \frac{1}{c^4} \left[ \tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{D}}_{AB} (\tilde{v}_A^2 - \tilde{\mathbf{a}}_A \cdot \tilde{\mathbf{D}}_{AB}) + \frac{1}{6} \tilde{D}_{AB}^2 \tilde{\mathbf{b}}_A \cdot \tilde{\mathbf{D}}_{AB} - \frac{1}{2} \tilde{D}_{AB}^2 \tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{a}}_A \right] + \frac{1}{c} \Delta_r(\tilde{\mathbf{x}}_A, t_B, \mathbf{x}_B) \\ &\quad - \frac{\tilde{D}_{AB}}{c^2} \frac{\partial \Delta_r(\tilde{\mathbf{x}}_A, t_B, \mathbf{x}_B)}{\partial x_A^i} \tilde{v}_A^i + \frac{\tilde{\mathbf{v}}_A \cdot \tilde{\mathbf{D}}_{AB}}{c^2 \tilde{D}_{AB}} \Delta_r(\tilde{\mathbf{x}}_A, t_B, \mathbf{x}_B) + O(1/c^5). \end{aligned} \quad (6)$$

Equation (6) is a post-Newtonian (PN) formula since the TTF is expanded in terms of quantities such as  $\tilde{v}_A/c$ ,  $(\tilde{\mathbf{D}}_{AB} \cdot \tilde{\mathbf{a}}_A)/c^2$  that should be small in order to assure the convergence of the series. It should be noted that for this PN expansion  $\Delta_r$  is considered of order  $G/c^2$ . This computation can be continued to higher orders if necessary. This analytical expansion includes what is usually referred to as Sagnac-like terms [12,32]. However, this expansion has the disadvantage of being valid for small velocities/accelerations only, which is not problematic in the Solar System but can be limiting in other applications like binary

pulsars. Moreover, in this expansion, derivatives of  $\Delta_r$  appear (and higher derivatives appear at higher orders), and these terms can become difficult to compute.

That is why a second approach, based on a numerical iterative process, is more practical. This procedure is standard and can be written as

$$\text{Start : } t_A^{(0)} = t_B - \mathcal{T}_r(\mathbf{x}_A(t_B), t_B, \mathbf{x}_B(t_B)) \quad (7a)$$

$$\text{Loop : } t_A^{(i+1)} = t_B - \mathcal{T}_r(\mathbf{x}_A(t_A^{(i)}), t_B, \mathbf{x}_B(t_B)) \quad (7b)$$

$$\text{End : when } |t_A^{(i+1)} - t_A^{(i)}| < \varepsilon, \quad (7c)$$

with  $\varepsilon$  the desired accuracy. Each step of this iterative procedure requires one to evaluate the TTF. In practice, at least for Solar System applications, this procedure converges very quickly after two or three iterations. The main advantages of this procedure are that no PN approximation is done and it is easy to implement.

These two procedures allow us to compute  $t_A$ , the coordinate emission time of the signal emitted along the world line  $\mathbf{x}_A(t)$ , from the reception coordinate time  $t_B$  and the coordinate of the receiver  $\mathbf{x}_B$ . The analytical expansion (6) is a PN expansion of  $t_A$  up to  $\mathcal{O}(1/c^4)$  while the iterative procedure (7) is valid up to any order.

#### IV. DOPPLER AND ASTROMETRIC OBSERVABLES FROM THE TIME TRANSFER FUNCTION

Here, we derive exact relativistic formulas to model the Doppler and astrometric observables as functions of the TTF and its partial derivatives. We consider three different observers  $\mathcal{O}_A$ ,  $\mathcal{O}_{A'}$  and  $\mathcal{O}_B$ . We assume that  $\mathcal{O}_A$  and  $\mathcal{O}_{A'}$  are emitting light rays at coordinates  $(t_A, \mathbf{x}_A)$  and  $(t_{A'}, \mathbf{x}_{A'})$ , respectively. We assume also that these signals are received by  $\mathcal{O}_B$  at coordinates  $(t_B, \mathbf{x}_B)$ .  $\mathcal{O}_B$  is equipped with a comoving tetrad of components  $E$ . Figure 1 illustrates the specific case of a light ray of frequency  $\nu_A$  emitted by  $\mathcal{O}_A$  with a wave four-vector of components  $k_A^\mu$  and received by  $\mathcal{O}_B$  at a frequency  $\nu_B$  and with a wave four-vector of components  $k_B^\mu$ .

##### A. Frequency shift observables

First, we focus on the one-way frequency shift between  $\mathcal{O}_A$  and  $\mathcal{O}_B$ . Let us define it as follows:

$$\frac{\Delta\nu}{\nu} \Big|_{A \rightarrow B}^{\text{one-way}} = \frac{\nu_B}{\nu_A} - 1.$$

It is well known that the ratio  $\nu_B/\nu_A$  can be expressed as [33,34]

$$\frac{\nu_B}{\nu_A} = \frac{u_B^\mu k_\mu^B}{u_A^\nu k_\nu^A} = \frac{u_{B,i}^i}{u_{A,0}^0} \frac{1 + \beta_B^i \hat{k}_i^B}{1 + \beta_A^i \hat{k}_i^A}, \quad (9)$$

where  $u_A^\mu = (dx^\mu/ds)_A$  and  $u^{\mu} = (dx^\mu/ds)_B$  are the four-velocity of  $\mathcal{O}_A$  and  $\mathcal{O}_B$ ,  $\beta_A^i = dx_A^i/cdt$  and  $\beta_B^i = dx_B^i/cdt$  are their coordinate velocities,  $\hat{k}_i^A = (k_i^A/k_0^A)$  and  $\hat{k}_i^B = (k_i^B/k_0^B)$ , while  $u_{A,0}^0$  and  $k_\mu^B$  are the wave vectors tangents to the light ray at the point of emission  $x_A$  and at the point of reception  $x_B$ , respectively.

The TTF formalism provides a direct way of defining the ratio of the spatial and temporal covariant components of the tangent vector to a photon trajectory  $k^\mu = dx^\mu/d\sigma$ ,  $\sigma$  being an affine parameter, at  $\mathcal{O}_A$  and  $\mathcal{O}_B$  as [22]

$$(\hat{k}_i)_A = \left( \frac{k_i}{k_0} \right)_A = c \frac{\partial \mathcal{T}_r}{\partial x_A^i} = -N_{AB}^i + \frac{\partial \Delta_r}{\partial x_A^i}, \quad (10a)$$

$$\begin{aligned} (\hat{k}_i)_B &= \left( \frac{k_i}{k_0} \right)_B = -c \frac{\partial \mathcal{T}_r}{\partial x_B^i} \left[ 1 - \frac{\partial \mathcal{T}_r}{\partial t_B} \right]^{-1} \\ &= - \left( N_{AB}^i + \frac{\partial \Delta_r}{\partial x_B^i} \right) \times \left[ 1 - \frac{1}{c} \frac{\partial \Delta_r}{\partial t_B} \right]^{-1}, \end{aligned} \quad (10b)$$

$$\frac{(k_0)_B}{(k_0)_A} = 1 - \frac{\partial \mathcal{T}_r}{\partial t_B} - \frac{1}{c} \frac{\partial \Delta_r}{\partial t_B}, \quad (10c)$$

where  $N_{AB}^i = \frac{R_{AB}^i}{R_{AB}}$  with  $R_{AB}^i = x_B^i - x_A^i$  and  $R_{AB} = |\mathbf{x}_B - \mathbf{x}_A|$ . Noting that

$$\frac{1}{c} \frac{\partial \Delta_r}{\partial t_B} = [g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j]_{A/B}^{-1/2}, \quad (11)$$

it is then straightforward to define the one-way frequency shift  $\delta\nu$  as a function of  $\Delta_r$  and its partial derivatives. Substituting for  $\hat{k}_i$  from Eq. (10) and inserting it in relation (9) using (11), one gets the exact expression [25,34,35]

$$\begin{aligned} \frac{\nu_B}{\nu_A} &= \frac{[g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j]_{A/B}^{1/2}}{[g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j]_B^{1/2}} \\ &\times \frac{1 - N_{AB}^i \beta_B^i - \beta_B^i \frac{\partial \Delta_r}{\partial x_B^i} - \frac{1}{c} \frac{\partial \Delta_r}{\partial t_B}}{1 - N_{AB}^i \beta_A^i + \beta_A^i \frac{\partial \Delta_r}{\partial x_A^i}}. \end{aligned} \quad (12)$$

This modeling can be extended easily to a multiway frequency shift. For example, let us consider a signal emitted with a frequency  $\nu_A$  by an observer  $\mathcal{O}_A$ , transmitted by an observer  $\mathcal{O}_B$  and then received by an observer  $\mathcal{O}_C$  at a frequency  $\nu_C$ , which can eventually be  $\mathcal{O}_A$  for a two-way frequency shift. The frequency shift between  $\mathcal{O}_A$  and  $\mathcal{O}_C$  is defined in the same way as for the one-way,

$$\frac{\Delta\nu}{\nu} \Big|_{A \rightarrow C} = \frac{\nu_C}{\nu_A} - 1. \quad (13)$$

The ratio  $\nu_C/\nu_A$  can be decomposed as follows,

$$\frac{\nu_C}{\nu_A} = \frac{\nu_C}{\nu_{B,e}} \delta\nu_B \frac{\nu_{B,r}}{\nu_A}, \quad (14)$$

where  $\nu_{B,r}$  is the proper frequency received by the observer  $\mathcal{O}_B$ , while  $\nu_{B,e}$  is the proper frequency emitted by the same observer. The factor  $\delta\nu_B \equiv \nu_{B,e}/\nu_{B,r}$  stands for any frequency shift, i.e. due to a transponder, introduced between the reception and reemission of the signal. The computation of the multiway frequency shift is straightforward: the two terms  $\nu_C/\nu_{B,e}$  and  $\nu_{B,r}/\nu_A$  from (14) are one-way frequency shifts and can be computed using (12). This procedure can be generalized easily if more links are needed.

### B. Astrometric observables

The goal of astrometry is to determine the position of celestial bodies from angular observations. We focus on two main approaches. First, we consider the modeling of the direction of incidence of a light ray in a given reference frame, which gives an absolute positioning of the studied object on a celestial sphere. Second, we consider the case of the angular separation of two light sources.

One way to get a covariant definition of the absolute positioning of a light source is to use the tetrad formalism [36–39] thus giving the direction of observation of an incoming light ray in a tetrad  $E$  comoving with the observer  $O_B$  (see Fig. 1). Let us note  $E_{(\alpha)}^\mu$ , the components of this tetrad, where  $(\alpha)$  corresponds to the tetrad index and  $\mu$  is a normal tensor index that can be lowered and raised by use of the metric. The tetrad is assumed to be orthonormal so that

$$g_{\mu\nu} E_{(\alpha)}^\mu E_{(\beta)}^\nu = \eta_{(\alpha)(\beta)}. \quad (15)$$

Vector  $E_{(0)}^\nu$  is chosen unit and timelike, and consequently  $E_{(i)}^\nu$  are unit and spacelike. The components of the tetrad allow us to transform the coordinates of the wave vector from the global coordinate frame to the tetrad frame,

$$k_{(\alpha)} = E_{(\alpha)}^\mu k_\mu,$$

where  $k_\mu$  are the coordinates of the wave vector in the global frame (represented on Fig. 1) while  $k_{(\alpha)}$  are the coordinates of the same vector in the tetrad frame. The incident direction of the light ray in the tetrad frame (which is a relativistic observable) is given by the normalization

$$n^{(i)} = \frac{k^{(i)}}{\sqrt{g_{jk} k^j k^k}} = \frac{k^{(i)}}{k^{(0)}} = -\frac{k_{(i)}}{k_{(0)}}, \quad (17)$$

where we used the properties of a null-vector  $k^{(i)}$  and the fact that the metric tensor has a Minkowskian form in the tetrad frame. Using the transformation law (16) into Eq. (17), one gets

$$n^{(i)} = -\frac{E_{(i)}^0 k_0 + E_{(i)}^j k_j}{E_{(0)}^0 k_0 + E_{(0)}^j k_j} = -\frac{E_{(i)}^0 + F_{(i)}^j \hat{k}_j}{E_{(0)}^0 + E_{(0)}^j \hat{k}_j}, \quad (18)$$

where  $\hat{k}_j$  are the deflection functions at  $O_B$  defined in (10b). This expression is consistent with the one derived in [40]. Using the relation (17) one can then express the incoming direction of the light ray in terms of the reception delay function and its derivatives [41,42] as

$$n^{(i)} = -\frac{E_{(i)}^0 \left(1 - \frac{1}{c} \frac{\partial \Delta_r}{\partial t_B}\right) - E_{(i)}^j N^j - E_{(i)}^j \frac{\partial \Delta_r}{\partial x_B^j}}{E_{(0)}^0 \left(1 - \frac{1}{c} \frac{\partial \Delta_r}{\partial t_B}\right) - E_{(0)}^j N^j - E_{(0)}^j \frac{\partial \Delta_r}{\partial x_B^j}}, \quad (19)$$

which is an exact formula.

Let us now examine the second kind of astrometric observations, namely the modeling of angular distance between two celestial bodies. This observable can also be computed within the TTF formalism. We assume that two different light sources  $O_A$  and  $O_{A'}$  are emitting a light ray  $\Gamma$  and  $\Gamma'$ , respectively. These light rays are received simultaneously by  $O_B$  at coordinates  $(t_B, \mathbf{x}_B)$ . We denote by  $k$  and  $k'$  the wave vectors of  $\Gamma$  and  $\Gamma'$  at  $O_B$ , respectively. Using expression (10'), we construct the ratio  $(\hat{k}_j)_B$  corresponding to  $\Gamma$  and  $(\hat{k}'_j)_B$  describing  $\Gamma'$ , which require an expression for the derivatives of the TTF whose expression up to the 2PM order will be given in Sec. V. It is straightforward to show that the angular distance  $\phi$  between  $O_A$  and  $O_{A'}$ , as observed by a receiving observer  $O_B$ , can be written as [45]

$$\sin^2 \frac{\phi}{2} = \frac{1}{4} \left[ \frac{(g_{00} + 2g_{0k} \beta^k + g_{kl} \beta^k \beta^l) g^{ij} (\hat{k}'_i - \hat{k}_i) (\hat{k}_j - \hat{k}'_j)}{(1 + \beta^m \hat{k}_m)(1 + \beta^l \hat{k}'_l)} \right]_B, \quad (20)$$

where  $\beta_B^i = dx^i/c dt_B$  is the coordinate velocity of  $O_B$  at coordinates  $(t_B, \mathbf{x}_B)$ .

### V. POST-MINKOWSKIAN EXPANSION OF THE TIME TRANSFER FUNCTION AND ITS DERIVATIVES

In Sec. IV we have presented a method to compute Doppler and astrometric observables in an exact form depending explicitly on the expression of the TTF and its derivatives. In this section, we present a way to derive these quantities up to 2PM order as integrals of some functions of the space-time metric taken along a straight line. In the weak field approximation, the expression of  $\mathcal{T}_r$  as a formal PM series has been derived by [23] and can be written in ascending powers of  $G$  as

$$\mathcal{T}_r(\mathbf{x}_A, t_B, \mathbf{x}_B) = \frac{R_{AB}}{c} + \frac{1}{c} \sum_{n=1}^{\infty} \Delta_r^{(n)}(\mathbf{x}_A, t_B, \mathbf{x}_B), \quad (21)$$

where  $\Delta_r^{(n)}$  is of the order  $\mathcal{O}(G^n)$ . The goal of this section is then to derive analytical formulas for the delay functions  $\Delta_r^{(1)}$ ,  $\Delta_r^{(2)}$  and their derivatives [44] up to 2PM order.

#### A. Notations and variables used

In the following, we provide some useful notations used throughout this paper. First of all, the Minkowskian path between the emitter and the receiver (which is a straight line) is parametrized by  $\lambda$  (whose values are between 0 and 1) and is given by

$$z^0(\lambda) = ct_B - \lambda R_{AB} \quad (22a)$$

$$z(\lambda) = \mathbf{x}_B - \lambda \mathbf{R}_{AB} = \mathbf{x}_B(1 - \lambda) + \lambda \mathbf{x}_A. \quad (22b)$$

We introduce the derivatives of these expressions with respect to the variables  $x_{A/B}$ , i.e. the quantities

$$z_{,(Ai)}^0(\lambda) = \frac{\partial z^0(\lambda)}{\partial x_A^i} = \lambda N_{AB}^i, \quad (23a)$$

$$z_{,(Bi)}^0(\lambda) = \frac{\partial z^0(\lambda)}{\partial x_B^i} = -\lambda N_{AB}^i, \quad (23b)$$

$$z_{,(Ai)}^j(\lambda) = \frac{\partial z^j(\lambda)}{\partial x_A^i} = \lambda \delta_i^j, \quad (23c)$$

$$z_{,(Bi)}^j(\lambda) = \frac{\partial z^j(\lambda)}{\partial x_B^i} = (1 - \lambda) \delta_i^j, \quad (23d)$$

$$z_{,(Ak)(Al)}^0(\lambda) = \frac{\partial^2 z^0}{\partial x_A^k \partial x_A^l} = \frac{\lambda}{R_{AB}} (N_{AB}^k N_{AB}^l - \delta_{kl}), \quad (23e)$$

$$z_{,(Ak)(Bl)}^0(\lambda) = \frac{\partial^2 z^0}{\partial x_B^k \partial x_B^l} = -\frac{\lambda}{R_{AB}} (N_{AB}^k N_{AB}^l - \delta_{kl}). \quad (23f)$$

We will use the functions  $p$  and  $p_{(n)}$  defined from the PM expansion of the space-time metric as follows,

$$\begin{aligned} p_{(n)}(\lambda) &= p[g_{(n)}^{\mu\nu}(z^\beta(\lambda)), N_{AB}^i, R_{AB}] \\ &= \frac{R_{AB}}{2} [g_{(n)}^{00} - 2N_{AB}^k g_{(n)}^{0k} + N_{AB}^k N_{AB}^l g_{(n)}^{kl}]_{z^\beta(\lambda)}. \end{aligned} \quad (24)$$

We also define a similar expression with the metric replaced by its derivatives

$$\begin{aligned} p_{(n)\alpha}(\lambda) &= p[g_{(n)\alpha}^{\mu\nu}(z^\beta(\lambda)), N_{AB}^i, R_{AB}] \\ &= \frac{R_{AB}}{2} [g_{(n)\alpha}^{00} - 2N_{AB}^k g_{(n)\alpha}^{0k} + N_{AB}^k N_{AB}^l g_{(n)\alpha}^{kl}]_{z^\beta(\lambda)}. \end{aligned}$$

It is worth noting that the last definition corresponds to the derivative of  $p_{(n)}$  with respect to  $z^\alpha$  by keeping  $N_{AB}^i$  and  $R_{AB}$  constants,

$$p_{(n)\alpha}(\lambda) = \left. \frac{\partial p_{(n)}}{\partial z^\alpha} \right|_{N_{AB}^i, R_{AB} = \text{cst}}. \quad (25)$$

We will also use the functions  $q_{(n)}^j$  that are defined by the derivative of  $p_{(n)}$  with respect to  $x_A^i$  by keeping  $z^\beta$  constant,

$$\begin{aligned} q_{(n)}^j(\lambda) &= \frac{\partial p_{(n)}}{\partial x_A^i} \Big|_{z^\beta = \text{cst}} \\ &= \frac{1}{2} [-N_{AB}^j g_{(n)}^{00} + 2g_{(n)}^{0j} - 2g_{(n)}^{jk} N_{AB}^k \\ &\quad + N_{AB}^k N_{AB}^l N_{AB}^j g_{(n)}^{kl}]_{z^\beta(\lambda)}. \end{aligned} \quad (26a)$$

It is then straightforward to show that

$$q_{(n)}^j(\lambda) = \left. \frac{\partial p_{(n)}}{\partial x_B^i} \right|_{z^\beta = \text{cst}}. \quad (26b)$$

We define a similar expression by replacing the metric by its derivatives,

$$\begin{aligned} q_{(n)\alpha}^j(\lambda) &= \frac{\partial p_{(n)\alpha}}{\partial x_A^i} \Big|_{z^\beta = \text{cst}} = \left. \frac{\partial p_{(n)\alpha}}{\partial x_B^i} \right|_{z^\beta = \text{cst}} \\ &= \frac{1}{2} [-N_{AB}^j g_{(n)\alpha}^{00} + 2g_{(n)\alpha}^{0j} - 2g_{(n)\alpha}^{jk} N_{AB}^k \\ &\quad + N_{AB}^k N_{AB}^l N_{AB}^j g_{(n)\alpha}^{kl}]_{z^\beta(\lambda)}. \end{aligned} \quad (26c)$$

Finally, the derivatives of  $q_{(n)}^j$  with respect to  $x_A^k$  keeping  $z^\beta$  constant are denoted by

$$\begin{aligned} s_{(n)}^{ij}(\lambda) &= \left. \frac{\partial q_{(n)}^j}{\partial x_A^k} \right|_{z^\alpha = \text{cst}} = \left. \frac{\partial q_{(n)}^j}{\partial x_B^k} \right|_{z^\alpha = \text{cst}} \\ &= \frac{1}{2R_{AB}} [g_{(n)}^{ij} - N_{AB}^i N_{AB}^j + 2g_{(n)}^{ij} - 2N_{AB}^k (g_{(n)}^{ik} N_{AB}^j \\ &\quad + g_{(n)}^{jk} N_{AB}^i) + N_{AB}^k N_{AB}^l (3N_{AB}^i N_{AB}^j - \delta^{ij})]_{z^\alpha(\lambda)}. \end{aligned} \quad (26d)$$

### PM expansion at first PM order

The expression of  $\Delta_r^{(1)}$  is given in [23] as an integral taken along  $z^\alpha(\lambda)$  of the components of the metric tensor,

$$\begin{aligned} \Delta_r^{(1)}(\mathbf{x}_A, t_B, \mathbf{x}_B) &= \frac{R_{AB}}{2} \int_0^1 [g_{(1)}^{00} - 2N_{AB}^i g_{(1)}^{0i} \\ &\quad + N_{AB}^i N_{AB}^j g_{(1)}^{ij}]_{z^\alpha(\lambda)} d\lambda. \end{aligned} \quad (27)$$

Using the notations (24), we rewrite Eq. (27) as

$$\begin{aligned} \Delta_r^{(1)}(\mathbf{x}_A, t_B, \mathbf{x}_B) &= \int_0^1 p[g_{(1)}^{\mu\nu}(z^\beta(\lambda)), N_{AB}^i, R_{AB}] d\lambda \\ &= \int_0^1 p_{(1)}(\lambda) d\lambda. \end{aligned} \quad (28)$$

The derivatives of  $\Delta_r^{(1)}$  can then be computed from (28) by inverting the integral and the partial derivative and by using the chain rules,

$$\begin{aligned} \frac{\partial \Delta_r^{(1)}}{\partial x_A^i}(\mathbf{x}_A, t_B, \mathbf{x}_B) &= \int_0^1 \left[ \left. \frac{\partial p_{(1)}(\lambda)}{\partial z^\alpha} \right|_{N_{AB}^i, R_{AB} = \text{cst}} \frac{\partial z^\alpha}{\partial x_A^i} \right. \\ &\quad \left. + \left. \frac{\partial p_{(1)}(\lambda)}{\partial x_A^i} \right|_{z^\beta = \text{cst}} \right] d\lambda. \end{aligned} \quad (29)$$

This can be rewritten using relations (23), (25) and (26a) as

$$\frac{\partial \Delta_r^{(1)}}{\partial x_A^i}(\mathbf{x}_A, t_B, \mathbf{x}_B) = \int_0^1 [p_{(1)\alpha}(\lambda) z_{,(Ai)}^\alpha(\lambda) + q_{(1)}^i(\lambda)] d\lambda, \quad (30a)$$

while a similar reasoning leads to

$$\frac{\partial \Delta_r^{(1)}}{\partial x_B^i}(\mathbf{x}_A, t_B, \mathbf{x}_B) = \int_0^1 [p_{(1)\alpha}(\lambda) z_{,(Bi)}^\alpha(\lambda) - q_{(1)}^i(\lambda)] d\lambda, \quad (30b)$$

$$\frac{\partial \Delta_r^{(1)}}{\partial t_B}(\mathbf{x}_A, t_B, \mathbf{x}_B) = c \int_0^1 p_{(1)0}(\lambda) d\lambda. \quad (30c)$$

Equations (29)–(30) are equivalent to those derived in [25]. When replaced into Eq. (12), Eq. (19) and Eq. (20), they give a full description of Doppler and astrometric observables at 1PM.

Some other quantities will be useful for the computations at the 2PM order. In particular,  $\Delta_r^{(1)}(\mathbf{z}(\lambda), t_B, \mathbf{x}_B)$  is defined similarly to (27) as

$$\begin{aligned} \Delta_r^{(1)}(\mathbf{z}(\lambda), t_B, \mathbf{x}_B) &= \frac{R_{zB}}{2} \int_0^1 [g_{(1)}^{00} - 2N_{zB}^i g_{(1)}^{0i} \\ &\quad + N_{zB}^i N_{zB}^j g_{(1)}^{ij}]_{y^\alpha(\mu)} d\mu \\ &= \int_0^1 p[g_{(1)}^{\mu\nu}(y^\alpha(\mu)), N_{zB}^i, R_{zB}] d\mu, \end{aligned} \quad (31)$$

where the integral is performed over a straight line joining  $z^\alpha(\lambda)$  to  $x_B^\alpha$ . This path is parametrized by  $y^\alpha(\mu)$ , whose components are given by

$$y^0(\mu) = ct_B - \mu |\mathbf{x}_B - \mathbf{z}(\lambda)| \quad (32a)$$

$$\mathbf{y}(\mu) = \mathbf{x}_B - \mu (\mathbf{x}_B - \mathbf{z}(\lambda)). \quad (32b)$$

Using the parametrization of  $z^\alpha$  given in Eq. (22), the last expression becomes

$$y^\alpha(\mu) = z^\alpha(\mu\lambda). \quad (33)$$

Incorporating this relation in (31) and noticing that

$$R_{zB} = \lambda R_{AB} \quad (34a)$$

and

$$N_{zB} = N_{AB}, \quad (34b)$$

one gets

$$\Delta_r^{(1)}(\mathbf{z}(\lambda), t_B, \mathbf{x}_B) = \int_0^1 p[g_{(1)}^{\mu\nu}(z^\beta(\lambda\mu)), N_{AB}^i, \lambda R_{AB}] d\mu.$$

Since  $p$  is linear with respect to  $R_{AB}$ , the last expression becomes

$$\begin{aligned} \Delta_r^{(1)}(\mathbf{z}(\lambda), t_B, \mathbf{x}_B) &= \lambda \int_0^1 p[g_{(1)}^{\mu\nu}(z^\beta(\lambda\mu)), N_{AB}^i, R_{AB}] d\mu \\ &= \lambda \int_0^1 p_{(1)}(\lambda\mu) d\mu. \end{aligned} \quad (35)$$

We shall also need the quantity  $\frac{\partial \Delta_r^{(1)}(\mathbf{z}(\lambda), t_B, \mathbf{x}_B)}{\partial x_A^i}$  (where the derivative is taken with respect to the first argument). To compute it, we apply the chain rule in Eq. (31) so that we get

$$\begin{aligned} \frac{\partial \Delta_r^{(1)}(\mathbf{z}(\lambda), t_B, \mathbf{x}_B)}{\partial x_A^i} &= \int_0^1 \left[ \frac{\partial p[g_{(1)}^{\mu\nu}(y^\beta(\mu)), N_{zB}^i, R_{zB}]}{\partial y^\alpha} \frac{\partial y^\alpha}{\partial z^i} \right]_{N_{zB}^i, R_{zB} = \text{cst}} \\ &\quad + \frac{\partial p[g_{(1)}^{\mu\nu}(y^\beta(\mu)), N_{zB}^i, R_{zB}]}{\partial z^i} \Big|_{y^\beta = \text{cst}} \Big] d\mu. \end{aligned} \quad (36)$$

The first part of the integrand gives

$$\begin{aligned} \frac{\partial p[g_{(1)}^{\mu\nu}(y^\beta(\mu)), N_{zB}^i, R_{zB}]}{\partial y^\alpha} \Big|_{N_{zB}^i, R_{zB} = \text{cst}} &= p_{(1),\alpha}(\mu, y^\beta(\mu), N_{zB}^i, R_{zB}) \\ &= \lambda p_{(1),\alpha}(z^\beta(\lambda\mu), N_{AB}^i, R_{AB}) = \lambda p_{(1)\alpha}(\lambda\mu), \end{aligned} \quad (37)$$

while the derivative of  $y^\alpha$  is given by

$$\frac{\partial y^0(\mu)}{\partial z^i} = \mu N_{zB}^i = \mu N_{AB}^i, \quad \frac{\partial y^j(\mu)}{\partial z^i} = \mu \delta_i^j.$$

The comparison of these expressions with (23a) and (23c) gives then

$$\lambda \frac{\partial y^\alpha(\mu)}{\partial z^i} = \frac{\partial z^\alpha(\lambda\mu)}{\partial x_A^i} = z_{,(Ai)}^\alpha(\lambda\mu). \quad (38)$$

The computation of the last quantity in the integrand is also straightforward. We get

$$\begin{aligned} \frac{\partial p[g_{(1)}^{\mu\nu}(y^\beta(\mu)), N_{zB}^i, R_{zB}]}{\partial z^i} \Big|_{y^\beta = \text{cst}} &= \frac{1}{2} [-N_{zB}^i g_{(1)}^{00} + 2g_{(1)}^{0i} - 2g_{(1)}^{ik} N_{zB}^k + N_{zB}^k N_{zB}^l N_{zB}^i g_{(1)}^{kl}]_{y^\beta(\mu)}. \end{aligned} \quad (39)$$

Using (34) and (33) into Eq. (39) and comparing to Eq. (26), one can finally set

$$\left. \frac{\partial p[g_{(1)}^{\mu\nu}(y^\beta(\mu)), N_{zB}^i, R_{zb}]}{\partial z^i} \right|_{y^\beta=\text{cst}} = q_{(1)}^i(\lambda\mu). \quad (40)$$

We can now use (37), (38) and (40) into (36) to get

$$\frac{\partial \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)}{\partial z^i} = \int_0^1 [p_{(1)\alpha}(\lambda\mu) z_{(Ai)}^\alpha(\lambda\mu) + q_{(1)}^i(\lambda\mu)] d\mu. \quad (41a)$$

The next quantities of interest are the derivatives of  $\Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)$  with respect to  $x_{A/B}^i$  and  $t_B$ . Once again, we apply the chain rules to (35) so that

$$\frac{\partial \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)}{\partial x_A^i} = \lambda \int_0^1 [p_{(1)\alpha}(\lambda\mu) z_{(Ai)}^\alpha(\lambda\mu) + q_{(1)}^i(\lambda\mu)] d\mu, \quad (41b)$$

$$\frac{\partial \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)}{\partial x_B^i} = \lambda \int_0^1 [p_{(1)\alpha}(\lambda\mu) z_{(Bi)}^\alpha(\lambda\mu) - q_{(1)}^i(\lambda\mu)] d\mu, \quad (41c)$$

$$\frac{\partial \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)}{\partial t_B} = c \lambda \int_0^1 p_{(1)0}(\lambda\mu) d\mu. \quad (41d)$$

Finally, we make explicit the second derivatives of  $\Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)$  using the chain rules from Eq. (41) as

$$\frac{\partial^2 \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)}{\partial x_A^i \partial x^j} = \int_0^1 [p_{(1)\alpha\beta} z_{(Aj)}^\alpha z_{(Ai)}^\beta + q_{(1)\alpha} z_{(Aj)}^\alpha + s_{(1)j} z_{(Ai)}^\alpha + s_{(1)i} z_{(Aj)}^\alpha] d\mu, \quad (42a)$$

$$\frac{\partial^2 \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)}{\partial x_B^i \partial x^j} = \int_0^1 [p_{(1)\alpha\beta} z_{(Ai)}^\alpha z_{(Bj)}^\beta - q_{(1)\alpha} z_{(Aj)}^\alpha + s_{(1)j} z_{(Ai)}^\alpha + s_{(1)i} z_{(Bj)}^\alpha] d\mu, \quad (42b)$$

$$\frac{\partial^2 \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)}{\partial t_B \partial x^j} = c \int_0^1 [p_{(1)0\alpha}(\lambda\mu) z_{(Aj)}^\alpha(\lambda\mu) + q_{(1)0}^j(\lambda\mu)] d\mu. \quad (42c)$$

To summarize, the expansion of the TTF and its derivatives at 1PM order are given by (28) and (30). The quantities

$$\frac{\partial \mathcal{I}_1(\lambda)}{\partial x_A^i} = p_{(2)\alpha}(\lambda) z_{(Ai)}^\alpha(\lambda) + q_{(2)}^i(\lambda) - \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B) [p_{(1)0\alpha}(\lambda) z_{(Ai)}^\alpha(\lambda) + q_{(1)0}^i(\lambda)] - p_{(1)0}(\lambda) \frac{\partial \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)}{\partial x_A^i}, \quad (46a)$$

(35), (41) and (42) will be useful in the calculation of the 2PM expansion of the TTF presented in the following.

### C. Expansion at second PM order

The expression of  $\Delta_r^{(2)}$  can also be derived from [23] and rewritten with our notations as

$$\Delta_r^{(2)}(\mathbf{x}_A, t_B, \mathbf{x}_B) = \int_0^1 [\mathcal{I}_1(\lambda) + \mathcal{I}_2(\lambda) + \mathcal{I}_3(\lambda)] d\lambda \quad (43)$$

with

$$\begin{aligned} \mathcal{I}_1(\lambda) &= p_{(2)\alpha}(\lambda) z_{(Ai)}^\alpha(z(\lambda), t_B, \mathbf{x}_B) p_{(1)0}(\lambda) \\ &= p_{(2)\alpha}(\lambda) - \lambda p_{(1)0}(\lambda) \int_0^1 p_{(1)}(\lambda\mu), \end{aligned} \quad (44a)$$

where we have used (35),

$$\begin{aligned} \mathcal{I}_2(\lambda) &= [p_{(1)\alpha}(\lambda) q_{(1)}^{0i} - R_{AB}^k z_{(Ai)}^k] \times \frac{\partial \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)}{\partial x^i} \\ &= [R_{AB}^k z_{(Ai)}^k - R_{AB}^k g_{(1)}^{ik}] z_{(Aj)}^\alpha \\ &\quad \times \int_0^1 [p_{(1)\alpha}(\lambda\mu) z_{(Aj)}^\alpha(\lambda\mu) + q_{(1)}^j(\lambda\mu)] d\mu, \end{aligned} \quad (44b)$$

where we have used (41) and

$$\begin{aligned} \mathcal{I}_3(\lambda) &= -\frac{R_{AB}^3}{2} z_{(Ai)}^3 \left[ \frac{\partial \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)}{\partial x^j} \right]^2 \\ &= -\frac{R_{AB}^3}{2} \sum_{j=1}^3 \left\{ \int_0^1 [p_{(1)\alpha}(\lambda\mu) z_{(Aj)}^\alpha(\lambda\mu) + q_{(1)}^j(\lambda\mu)] d\mu \right\}^2, \end{aligned} \quad (44c)$$

where we have used the relation (41).

Applying extensively the chain rules, we can now derive the expression of the partial derivatives of Eq. (43) as

$$\begin{aligned} \frac{\partial \Delta_r^{(2)}(\mathbf{x}_A, t_B, \mathbf{x}_B)}{\partial x_{A/B}^i} &= \int_0^1 \left[ \frac{\partial \mathcal{I}_1}{\partial x_{A/B}^i}(\lambda) + \frac{\partial \mathcal{I}_2}{\partial x_{A/B}^i}(\lambda) \right. \\ &\quad \left. + \frac{\partial \mathcal{I}_3}{\partial x_{A/B}^i}(\lambda) \right] d\lambda \end{aligned} \quad (45a)$$

$$\frac{\partial \Delta_r^{(2)}(\mathbf{x}_A, t_B, \mathbf{x}_B)}{\partial t_B} = \int_0^1 \left[ \frac{\partial \mathcal{I}_1}{\partial t_B}(\lambda) + \frac{\partial \mathcal{I}_2}{\partial t_B}(\lambda) + \frac{\partial \mathcal{I}_3}{\partial t_B}(\lambda) \right] d\lambda, \quad (45b)$$

where the derivatives can be written as follows,

$$\frac{\partial \mathcal{I}_1(\lambda)}{\partial x_B^i} = p_{(2)\alpha}(\lambda) z_{(Bi)}^\alpha(\lambda) - q_{(2)}^i(\lambda) - \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B) [p_{(1)0\alpha}(\lambda) z_{(Bi)}^\alpha(\lambda) - q_{(1)0}^i(\lambda)] - p_{(1)0}(\lambda) \frac{\partial \Delta_r^{(1)}}{\partial x_B^i}(z(\lambda), t_B, \mathbf{x}_B), \quad (46b)$$

and with  $\Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B)$  and its derivatives given by (35), (41b) and (41c). Similarly, we also compute

$$\begin{aligned} \frac{\partial \mathcal{I}_2(\lambda)}{\partial x_A^i} &= [-N_{AB}^i g_{(1)}^{0j} + g_{(1)}^{ij} + (R_{AB} g_{(1),\alpha}^{0j} - g_{(1),\alpha}^{jk} R_{AB}^k) z_{(Ai)}^\alpha]_{z^\beta(\lambda)} \times \frac{\partial \Delta_r^{(1)}}{\partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \\ &+ [R_{AB} g_{(1)}^{0j} - R_{AB}^k g_{(1),z^\beta}^{jk}]_{z^\beta(\lambda)} \times \frac{\partial^2 \Delta_r^{(1)}}{\partial x_A^i \partial x^j}(z(\lambda), t_B, \mathbf{x}_B), \end{aligned} \quad (46c)$$

$$\begin{aligned} \frac{\partial \mathcal{I}_2(\lambda)}{\partial x_B^i} &= [N_{AB}^i g_{(1)}^{0j} - g_{(1)}^{ij} + (R_{AB} g_{(1),\alpha}^{0j} - g_{(1),\alpha}^{jk} R_{AB}^k) z_{(Bi)}^\alpha]_{z^\beta(\lambda)} \times \frac{\partial \Delta_r^{(1)}}{\partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \\ &+ [R_{AB} g_{(1)}^{0j} - R_{AB}^k g_{(1),z^\beta}^{jk}]_{z^\beta(\lambda)} \times \frac{\partial^2 \Delta_r^{(1)}}{\partial x_B^i \partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \end{aligned} \quad (46d)$$

and

$$\frac{\partial \mathcal{I}_3(\lambda)}{\partial x_A^i} = \frac{N_{AB}^i}{2} \sum_{j=1}^3 \left( \frac{\partial \Delta_r^{(1)}}{\partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \right)^2 - R_{AB} \sum_{j=1}^3 \left[ \frac{\partial \Delta_r^{(1)}}{\partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \cdot \frac{\partial^2 \Delta_r^{(1)}}{\partial x_A^i \partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \right], \quad (46e)$$

$$\frac{\partial \mathcal{I}_3(\lambda)}{\partial x_B^i} = -\frac{N_{AB}^i}{2} \sum_{j=1}^3 \left( \frac{\partial \Delta_r^{(1)}}{\partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \right)^2 - R_{AB} \sum_{j=1}^3 \left[ \frac{\partial \Delta_r^{(1)}}{\partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \cdot \frac{\partial^2 \Delta_r^{(1)}}{\partial x_B^i \partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \right], \quad (46f)$$

where the derivatives of  $\Delta_r^{(1)}$  are given by (41) and the second derivatives are given explicitly in Eq. (42). Finally, the derivatives of  $\mathcal{I}_j(\lambda)$  with respect to  $t_B$  are given by

$$\frac{\partial \mathcal{I}_1}{\partial t_B} = c p_{(2)0}(\lambda) - c p_{(1)00}(\lambda) \Delta_r^{(1)}(z(\lambda), t_B, \mathbf{x}_B) - p_{(1)0}(\lambda) \frac{\partial \Delta_r^{(1)}}{\partial t_B}(z(\lambda), t_B, \mathbf{x}_B), \quad (46g)$$

$$\frac{\partial \mathcal{I}_2}{\partial t_B} = c [R_{AB} g_{(1),0}^{ji} - R_{AB}^k g_{(1),0}^{jk}]_{z^\beta(\lambda)} \times \frac{\partial \Delta_r^{(1)}}{\partial x^i}(z(\lambda), t_B, \mathbf{x}_B) + [R_{AB} g_{(1)}^{0i} - R_{AB}^k g_{(1),z^\beta}^{ik}]_{z^\beta(\lambda)} \times \frac{\partial^2 \Delta_r^{(1)}}{\partial t_B \partial x^i}(z(\lambda)), \quad (46h)$$

$$\frac{\partial \mathcal{I}_3}{\partial t_B} = -\frac{c}{2} \sum_{j=1}^3 \left( \frac{\partial \Delta_r^{(1)}}{\partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \right)^2 - R_{AB} \sum_{j=1}^3 \frac{\partial \Delta_r^{(1)}}{\partial x^j}(z(\lambda), t_B, \mathbf{x}_B) \cdot \frac{\partial^2 \Delta_r^{(1)}}{\partial t_B \partial x^j}(z(\lambda), t_B, \mathbf{x}_B) d\lambda, \quad (46i)$$

where  $\Delta_r^{(1)}$  is given by (35), its first derivative by (41d) and (41e), and where the expression of the second derivatives are given by Eq. (42).

The relations given above provide the TTF and its derivatives up to the 2PM order in  $\epsilon$  in integral form particularly adapted for numerical evaluation from any metric. When replaced into Eq. (12), Eq. (1c) and Eq. (20), they give a full description of Doppler and astrometric observables at 2PM.

## VI. APPLICATIONS TO A STATIC, SPHERICALLY SYMMETRIC SPACE-TIME

The results presented above will be illustrated through the case of a static, spherically symmetric space-time. In isotropic coordinates, the line element can be written

$$ds^2 = A(r) c^2 dt^2 - B(r) \delta_{ij} dx^i dx^j. \quad (47)$$

As mentioned in [45], the light rays of metric (47) are the same as the light rays of any  $d\tilde{s}^2$  conformal to (47). We can thus simplify the calculations by choosing  $d\tilde{s}^2 = A^{-1}(r) ds^2$  and deal with the following line element:

$$d\tilde{s}^2 = c^2 dt^2 - \frac{B(r)}{A(r)} \delta_{ij} dx^i dx^j = c^2 dt^2 - U(r) \delta_{ij} dx^i dx^j. \quad (48)$$

We can now consider a PM expansion of the function  $U(r) = 1 + U^{(1)}(r) + U^{(2)}(r) + \dots$ . This procedure

will simplify the results shown in Sec. V. Let us assume that a light ray is emitted by  $\mathcal{O}_A$  at coordinates  $(ct_A, \mathbf{x}_A)$  and received by an observer  $\mathcal{O}_B$  at coordinates

$(ct_B, \mathbf{x}_B)$ . Using Eqs. (28)–(30), the reception delay function and its first derivatives at 1PM order can be written as

$$\Delta_r^{(1)}(\mathbf{x}_A, \mathbf{x}_B) = \frac{R_{AB}}{2} \int_0^1 U^{(1)}(z(\lambda)) d\lambda, \quad (49a)$$

$$\frac{\partial \Delta_r^{(1)}}{\partial x_A^i}(\mathbf{x}_A, \mathbf{x}_B) = -\frac{U^{(1)}(r_A)}{2} N_{AB}^i + \left[ \frac{R_{AB}}{2} x_B^i + \frac{N_{AB}^i}{4} (r_A^2 - R_{AB}^2 - r_B^2) \right] \times \int_0^1 \frac{\lambda}{z(\lambda)} \frac{\partial U^{(1)}}{\partial r}(z(\lambda)) d\lambda, \quad (49b)$$

$$\begin{aligned} \frac{\partial \Delta_r^{(1)}}{\partial x_B^i}(\mathbf{x}_A, \mathbf{x}_B) &= \frac{U^{(1)}(r_A)}{2} N_{AB}^i + \frac{R_{AB} x_B^i}{2} \int_0^1 \frac{1}{z(\lambda)} \frac{\partial U^{(1)}}{\partial r}(z(\lambda)) d\lambda \\ &\quad - \left[ \frac{R_{AB}}{2} x_B^i + \frac{N_{AB}^i}{4} (r_A^2 + R_{AB}^2 - r_B^2) \right] \times \int_0^1 \frac{\lambda}{z(\lambda)} \frac{\partial U^{(1)}}{\partial r}(z(\lambda)) d\lambda, \end{aligned} \quad (49c)$$

where  $z(\lambda) = |z(\lambda)|$ ,  $z(\lambda)$  being given by Eq. (22) and where we use the notations  $r_A = |\mathbf{x}_A|$ ,  $r_B = |\mathbf{x}_B|$ ,  $R_{AB} = |\mathbf{x}_B - \mathbf{x}_A|$  and  $N_{AB}^i = (x_B^i - x_A^i)/R_{AB}$ . Similarly, using (41a), one can show that

$$\begin{aligned} \frac{\partial \Delta_r^{(1)}}{\partial x^i}(z(\lambda), \mathbf{x}_B) &= -U^{(1)}(z(\lambda)) \frac{N_{AB}^i}{2} \\ &\quad + \lambda \left[ \frac{R_{AB}}{2} x_B^i + \frac{N_{AB}^i}{4} (r_A^2 - R_{AB}^2 - r_B^2) \right] V(\lambda), \end{aligned} \quad (50)$$

with

$$V(\lambda) \equiv \int_0^1 \frac{\lambda}{z(\lambda)} \frac{\partial U^{(1)}}{\partial r}(z(\lambda\mu)) d\mu. \quad (51)$$

Substituting now for the metric tensor from Eq. (48), and using Eq. (43), the 2PM order of the reception delay function is given by

$$\Delta_r^{(2)}(\mathbf{x}_A, \mathbf{x}_B) = \frac{R_{AB}}{2} \int_0^1 [U^{(2)}(z(\lambda)) + \bar{\mathcal{I}}_3(\lambda)] d\lambda, \quad (52)$$

where we define  $\bar{\mathcal{I}}_3(\lambda) \equiv 2\mathcal{I}_3(\lambda)/R_{AB}$ . Using Eq. (50), one gets

$$\begin{aligned} \mathcal{I}_3(\lambda) &= -\sum^3 \left[ \frac{\partial \Delta_r^{(1)}(z(\lambda))}{\partial x^i} \right]^2 \\ &= -\frac{1}{4} \left\{ \left[ \frac{\partial \Delta_r^{(1)}(z(\lambda))}{\partial x^i} \right]^2 \right. \\ &\quad \left. + \frac{V^2(\lambda)}{4} [4r_B^2 R_{zB}^2 - (z^2(\lambda) - R_{zB}^2 - r_B^2)^2] \right\} \end{aligned} \quad (53)$$

with  $R_{zB} \equiv |\mathbf{x}_B - z(\lambda)|$ . In the last relation, it can sometimes be useful to replace  $z(\lambda)^2 - R_{zB}^2 - r_B^2 = \lambda(r_A^2 - R_{AB}^2 - r_B^2)$  or  $4R_{zB}^2 r_B^2 - (z^2(\lambda) - R_{zB}^2 - r_B^2)^2 = -\lambda^2[(r_A + r_B)^2 - R_{AB}^2] \times [(r_A - r_B)^2 - R_{AB}^2]$  and use  $V(\lambda)$  as defined by (51). From Eqs. (45)–(46), the derivatives of  $\Delta_r^{(2)}$  are then given by

$$\frac{\partial \Delta_r^{(2)}}{\partial x_A^i} = -\frac{N_{AB}^i}{R_{AB}} \Delta_r^{(2)} + \frac{R_{AB}}{2} \int_0^1 \left[ \frac{\lambda z^i(\lambda)}{z(\lambda)} \frac{\partial U^{(2)}}{\partial r}(z(\lambda)) + \frac{\partial \bar{\mathcal{I}}_3}{\partial x_A^i}(\lambda) \right] d\lambda, \quad (54a)$$

$$\frac{\partial \Delta_r^{(2)}}{\partial x_B^i} = \frac{N_{AB}^i}{R_{AB}} \Delta_r^{(2)} + \frac{R_{AB}}{2} \int_0^1 \left[ \frac{(1-\lambda)z^i(\lambda)}{z(\lambda)} \frac{\partial U^{(2)}}{\partial r}(z(\lambda)) + \frac{\partial \bar{\mathcal{I}}_3}{\partial x_B^i}(\lambda) \right] d\lambda, \quad (54b)$$

with

$$\begin{aligned} \frac{\partial \bar{\mathcal{I}}_3}{\partial x_A^i}(\lambda) &= -\frac{1}{4} \left\{ 2\lambda \frac{z^i(\lambda)}{z(\lambda)} U^{(1)}(z(\lambda)) \frac{\partial U^{(1)}}{\partial r}(z(\lambda)) - \lambda^2 \frac{V(\lambda)}{2} \frac{\partial V}{\partial x_A^i}(\lambda) [(r_A + r_B)^2 - R_{AB}^2] [(r_A - r_B)^2 - R_{AB}^2] \right. \\ &\quad \left. - \lambda^2 V^2(\lambda) [2r_B^2 R_{AB}^i + (r_A^2 - R_{AB}^2 - r_B^2) x_B^i] \right\}, \end{aligned} \quad (55a)$$

$$\frac{\partial \tilde{I}_3}{\partial x_B^i}(\lambda) = -\frac{1}{4} \left\{ 2(1-\lambda) \frac{z^i(\lambda)}{z(\lambda)} U^{(1)}(z(\lambda)) \frac{\partial U^{(1)}}{\partial r}(z(\lambda)) - \lambda^2 \frac{V(\lambda)}{2} \frac{\partial V}{\partial x_B^i}(\lambda) [(r_A + r_B)^2 - R_{AB}^2] [(r_A - r_B)^2 - R_{AB}^2] \right. \\ \left. + \lambda^2 V^2(\lambda) [(r_A^2 + r_B^2 - R_{AB}^2) R_{AB}^i + (r_A^2 + R_{AB}^2 - r_B^2) x_B^i] \right\}, \quad (55b)$$

and where the derivatives of  $V(\lambda)$  can be computed as

$$\frac{\partial V}{\partial x_A^i}(\lambda) = \int_0^1 \left[ \frac{\partial^2 U^{(1)}}{\partial r^2}(z(\lambda\mu)) \frac{\lambda\mu^2 z^i(\lambda\mu)}{z^2(\lambda\mu)} - \lambda\mu^2 \frac{\partial U^{(1)}}{\partial r}(z(\lambda\mu)) \frac{z^i(\lambda\mu)}{z^3(\lambda\mu)} \right] d\mu, \quad (56a)$$

$$\frac{\partial V}{\partial x_B^i}(\lambda) = \int_0^1 \left[ \frac{\partial^2 U^{(1)}}{\partial r^2}(z(\lambda\mu)) \frac{(1-\lambda\mu)\mu z^i(\lambda\mu)}{z^2(\lambda\mu)} - (1-\lambda\mu)\mu \frac{\partial U^{(1)}}{\partial r}(z(\lambda\mu)) \frac{z^i(\lambda\mu)}{z^3(\lambda\mu)} \right] d\mu. \quad (56b)$$

Let us now study a Schwarzschild-like metric, whose expansion in isotropic coordinates is

$$ds^2 = \left( 1 - 2\frac{m}{r} + 2\beta\frac{m^2}{r^2} + \dots \right) c^2 dt^2 - \left( 1 + 2\gamma\frac{m}{r} + \frac{3}{2}\epsilon\frac{m^2}{r^2} + \dots \right) \delta_{ij} dx^i dx^j, \quad (57)$$

and  $U(r)$  is given by

$$U(r) = 1 + 2(1+\gamma)\frac{m}{r} + 2\kappa\frac{m^2}{r^2} + \dots, \quad (58)$$

where  $\kappa = 2(1+\gamma) - \beta + \frac{3}{4}\epsilon$ .

Introducing  $U(r)$  from Eq. (58) into (49) leads to

$$\Delta_r^{(1)} = R_{AB}(1+\gamma)m \int_0^1 \frac{d\lambda}{z(\lambda)} \\ = (\gamma+1)m \ln \left( \frac{r_A + r_B + R_{AB}}{r_A + r_B - R_{AB}} \right), \quad (59a)$$

$$\frac{\partial \Delta_r^{(1)}}{\partial x_A^i} = -(1+\gamma) \frac{m}{r_A} N_{AB}^i \\ + \left[ \frac{R_{AB}}{2} x_B^i + \frac{N_{AB}^i}{4} (r_A^2 - R_{AB}^2 - r_B^2) \right] \\ \times \frac{-4(1+\gamma)m}{(r_A + r_B)^2 - R_{AB}^2} \\ = -\frac{2(1+\gamma)m}{(r_A + r_B)^2 - R_{AB}^2} \left[ \frac{R_{AB}}{r_A} x_A^i + N_{AB}^i (r_A + r_B) \right], \quad (59b)$$

$$\frac{\partial \Delta_r^{(1)}}{\partial x_B^i} = (1+\gamma) \frac{m}{r_A} N_{AB}^i - \frac{R_{AB}^2}{2} \left[ \frac{4(1+\gamma)m \left( \frac{1}{r_A} + \frac{1}{r_B} \right)}{(r_A + r_B)^2 - R_{AB}^2} \right] \\ - \left[ \frac{N_{AB}^i}{2} (r_A^2 - R_{AB}^2 - r_B^2) + \frac{N_{AB}^i}{4} (r_A^2 + R_{AB}^2 - r_B^2) \right] \\ \times \frac{-4(1+\gamma)m}{r_A [(r_A + r_B)^2 - R_{AB}^2]} \\ = -\frac{2(1+\gamma)m}{(r_A + r_B)^2 - R_{AB}^2} \left[ \frac{R_{AB}}{r_B} x_B^i - N_{AB}^i (r_A + r_B) \right]. \quad (59c)$$

One should note that Eq. (59a) is equivalent to the expression of time delay found by Shapiro [46], while the two derivatives (59b) and (59c) are in agreement with results found in [12].

The computation at 2PM order is more cumbersome. Substituting for  $U(r)$  from Eq. (58) into Eq. (52), one gets

$$\Delta_r^{(2)}(\mathbf{x}_A, \mathbf{x}_B) = \kappa m^2 \int_0^1 \frac{d\lambda}{z^2(\lambda)} + \frac{R_{AB}}{2} \int_0^1 \tilde{I}_3(\lambda) d\lambda, \quad (60)$$

where, using  $V(\lambda)$  as determined from Eq. (51),

$$V(\lambda) = -2(1+\gamma)m \int_0^1 \frac{\mu}{z^3(\lambda\mu)} d\mu \\ = -\frac{4(1+\gamma)m}{z(\lambda) [(z(\lambda) + r_B)^2 - \lambda^2 R_{AB}^2]}, \quad (61)$$

and substituting into Eq. (53), we obtain  $\tilde{I}_3(\lambda)$  as

$$\tilde{I}_3(\lambda) = -\frac{4(1+\gamma)^2 m^2 r_B}{z(\lambda) [(z(\lambda) + r_B)^2 - \lambda^2 R_{AB}^2]} \\ = -4(1+\gamma)^2 m^2 \frac{d}{d\lambda} \left[ \frac{\lambda}{(z(\lambda) + r_B)^2 - \lambda^2 R_{AB}^2} \right]. \quad (62)$$

Replacing this expression in Eq. (60) and integrating, one gets

$$\Delta_r^{(2)}(\mathbf{x}_A, \mathbf{x}_B) = m^2 \frac{R_{AB}}{r_A r_B} \left[ \kappa \arccos \mu - \frac{(1+\gamma)^2}{1+\mu} \right], \quad (63)$$

with  $\mu = (\mathbf{n}_A \cdot \mathbf{n}_B)$  and where  $\mathbf{n}_{A/B} = \mathbf{x}_{A/B} / r_{A/B}$ . Substituting for  $\Delta_r$  from Eqs. (49)–(63) into Eq. (21), we finally get an expression for the TTF in a Schwarzschild-like metric and up to 2PM as

$$\begin{aligned} \mathcal{T}_r(\mathbf{x}_A, t_B, \mathbf{x}_B) &= t_B - t_A \\ &= \frac{R_{AB}}{c} + \frac{(\gamma + 1)m}{c} \ln \left( \frac{r_A + r_B + R_{AB}}{r_A + r_B - R_{AB}} \right) \\ &\quad + \frac{m^2 R_{AB}}{c r_A r_B} \left[ \frac{\kappa \arccos \mu}{\sqrt{1 - \mu^2}} - \frac{(1 + \gamma)^2}{1 + \mu} \right]. \end{aligned} \quad (64)$$

We recover a result previously derived by different approaches [19,20,22,23,45] (see also [47] in the case where  $\beta = \gamma = \varepsilon = 1$ ).

We can now compute the derivatives of  $\Delta_r^{(2)}$ . As an example, we will only focus on the derivative with respect to  $x_A^i$ ; the other derivative (with respect to  $x_B^i$ ) can be computed similarly. Using Eq. (58) into Eq. (56), one gets

$$\begin{aligned} \frac{\partial V}{\partial x_A^i}(\lambda) &= \frac{8(1 + \gamma)m\lambda}{z^3(\lambda)[(z(\lambda) + r_B)^2 - \lambda^2 R_{AB}^2]^2} \\ &\quad \times \{ [z^2(\lambda) + 2z(\lambda)r_B + z(\lambda) \cdot \mathbf{x}_B] x_B^i \\ &\quad - \lambda R_{AB}^i [2z(\lambda)r_B + z(\lambda) \cdot \mathbf{x}_B] \}. \end{aligned} \quad (65)$$

Replacing this result in Eq. (55a) then leads to

$$\begin{aligned} \frac{\partial \bar{\mathcal{L}}_3}{\partial x_A^i} &= \frac{4(1 + \gamma)^2 m^2 r_B \lambda}{z^3(\lambda)[(z(\lambda) + r_B)^2 - \lambda^2 R_{AB}^2]^2} \\ &\quad \times \{ x_B^i [r_B^2 + 4r_B z(\lambda) + 3z^2(\lambda) - \lambda^2 R_{AB}^2] \\ &\quad - \lambda R_{AB}^i [r_B^2 + 4r_B z(\lambda) + z^2(\lambda) - \lambda^2 R_{AB}^2] \}, \end{aligned} \quad (66)$$

which, after some lengthy but straightforward calculations, can be written as

$$\frac{\partial \bar{\mathcal{L}}_3}{\partial x_A^i} = 8(1 + \gamma)^2 \frac{d}{d\lambda} \left[ \lambda \frac{(z(\lambda) + r_B)x_B^i - \lambda R_{AB}^i}{z(\lambda)[(z(\lambda) + r_B)^2 - \lambda^2 R_{AB}^2]^2} \right]. \quad (67)$$

Finally, one needs to compute the integral corresponding to the second term of Eq. (54a), namely

$$\begin{aligned} \frac{R_{AB}}{2} \int_0^1 \left[ \frac{\lambda z^i(\lambda)}{z(\lambda)} \frac{\partial U^{(2)}}{\partial r}(z(\lambda)) \right] d\lambda &= -\gamma R_{AB} m^2 \int_0^1 \frac{\lambda z^i(\lambda)}{z^4(\lambda)} d\lambda \\ &= \frac{\kappa m^2 \gamma R_{AB}}{r_A^2 r_B (\sqrt{1 - \mu^2})^2} (n_A^i + \mu n_B^i) - \frac{\kappa m^2 \gamma R_{AB}}{r_A^2 r_B (\sqrt{1 - \mu^2})} (n_B^i - \mu n_A^i). \end{aligned} \quad (68)$$

Now, substituting from Eq. (63), Eq. (67) and Eq. (68) into Eq. (54a), one gets

$$\begin{aligned} \frac{\partial \Delta_r^{(2)}}{\partial x_A^i} &= \frac{\kappa m^2}{r_A r_B} \left\{ \frac{\gamma}{\sqrt{1 - \mu^2}} \left[ -N_{AB}^i - \frac{R_{AB}}{r_A(1 - \mu^2)} (n_A^i - \mu n_B^i) \right] - \frac{R_{AB}}{r_A(1 - \mu^2)} (n_B^i - \mu n_A^i) \right\} \\ &\quad + \frac{(1 + \gamma)^2 m^2}{r_B(1 + \mu)} \left\{ \lambda \frac{d}{d\lambda} \left[ \frac{x_B^i + \frac{R_{AB}}{r_A(1 + \mu)} (n_A^i - \mu n_B^i)}{z(\lambda)[(z(\lambda) + r_B)^2 - \lambda^2 R_{AB}^2]^2} \right] \right\}, \end{aligned} \quad (69a)$$

while a similar reasoning for  $\frac{\partial \Delta_r^{(2)}}{\partial x_B^i}$  leads to

$$\begin{aligned} \frac{\partial \Delta_r^{(2)}}{\partial x_B^i} &= \frac{\kappa m^2}{r_B} \left\{ \frac{\gamma}{\sqrt{1 - \mu^2}} \left[ N_{AB}^i - \frac{R_{AB}}{r_B(1 - \mu^2)} (n_B^i - \mu n_A^i) \right] - \frac{R_{AB}}{r_B(1 - \mu^2)} (n_A^i - \mu n_B^i) \right\} \\ &\quad + \frac{(1 + \gamma)^2 m^2}{r_A(1 + \mu)} \left\{ -N_{AB}^i + \frac{R_{AB}}{r_B(1 + \mu)} (n_A^i + n_B^i) \right\}. \end{aligned} \quad (69b)$$

Some algebra allows us to put the last two results in the same form as the one found in [48], which serves as verification of our approach. Of course, in the case of the Schwarzschild metric the analytical derivation of Eq. (63) is much simpler than the above calculations to get Eq. (69) and can be used to check our calculation. Nevertheless the method presented here is very efficient for numerical evaluations of the derivatives of the TTF, necessary when using more complex metrics and for the test of alternative theories of gravity, when the integrals are no

longer analytic. As an example, we will present in this section several applications of our formulae to future space missions.

### A. Application to BepiColombo

The future BepiColombo mission will reach an impressive level of accuracy on its measurements: 10 cm on the range and  $10^{-6}$  m/s on the Doppler [4,5]. Such an accuracy needs a light propagation model that includes the influence of some of the 2PM terms coming from the Sun [26]. As an

example of how the equations presented in this paper can be applied to a real measurement, we simulate a one year Mercury-Earth Doppler link taking into account only the gravitational contribution from the Sun. The Earth and Mercury orbits used here come from the JPL ephemerides [49,50] obtained using the SPICE toolkit [51].

Substituting for the metric,  $\Delta_r$ , and its derivatives from Eq. (57), Eq. (59) and Eq. (69), respectively, into Eq. (12), one can write the expression of the Doppler around a spherical mass as

$$\frac{\nu_B}{\nu_A} = \frac{\sqrt{1 - 2\frac{m}{r_A} + 2\beta\frac{m^2}{r_A^2} - \frac{3}{2}\beta_3\frac{m^3}{r_A^3} - \frac{v_A^2}{c^2} - 2\gamma\frac{v_A^2 m}{c^2 r_A} - \frac{3}{2}\epsilon\frac{m^2 v_A^2}{r_A^2 c^2}}}{\sqrt{1 - 2\frac{m}{r_B} + 2\beta\frac{m^2}{r_B^2} - \frac{3}{2}\beta_3\frac{m^3}{r_B^3} - \frac{v_B^2}{c^2} - 2\gamma\frac{v_B^2 m}{c^2 r_B} - \frac{3}{2}\epsilon\frac{m^2 v_B^2}{r_B^2 c^2}}} \times \frac{q_B}{q_A}, \quad (70)$$

where we defined

$$q_A = 1 - \frac{N_{AB} \cdot v_A}{c} - \frac{(1+\gamma)m}{cr_A r_B (1+\mu)} [(r_A + r_B)N_{AB} \cdot v_A + R_{AB} n_A \cdot v_A] + \frac{\kappa m^2}{cr_A r_B} \left[ \frac{\arccos \mu}{\sqrt{1-\mu^2}} \left( -N_{AB} \cdot v_A - \frac{R_{AB}}{r_A(1-\mu^2)} (n_A \cdot v_A - \mu n_B \cdot v_A) \right) - \frac{R_{AB}}{r_A(1-\mu^2)} (n_B \cdot v_A - \mu n_A \cdot v_A) \right] + \frac{(1+\gamma)^2 m^2}{cr_A r_B (1+\mu)} \left[ N_{AB} \cdot v_A + \frac{R_{AB}}{r_A(1+\mu)} (n_A \cdot v_A + n_B \cdot v_A) \right] \quad (71a)$$

and

$$q_B = 1 - \frac{N_{AB} \cdot v_B}{c} - \frac{(1+\gamma)m}{cr_A r_B (1+\mu)} [(r_A + r_B)N_{AB} \cdot v_B - R_{AB} n_B \cdot v_B] + \frac{\kappa m^2}{cr_A r_B} \left[ \frac{\arccos \mu}{\sqrt{1-\mu^2}} \left( -N_{AB} \cdot v_B + \frac{R_{AB}}{r_B(1-\mu^2)} (n_B \cdot v_B - \mu n_A \cdot v_B) \right) + \frac{R_{AB}}{r_B(1-\mu^2)} (n_A \cdot v_B - \mu n_B \cdot v_B) \right] + \frac{(1+\gamma)^2 m^2}{cr_A r_B (1+\mu)} \left[ N_{AB} \cdot v_B - \frac{R_{AB}}{r_B(1+\mu)} (n_A \cdot v_B - n_B \cdot v_B) \right] \quad (71b)$$

We use relation (64) and Eqs. (71a) and (71b) to estimate the order of magnitude of the first and second PM contributions to the Mercury-Earth range and Doppler as illustrated in Fig. 2. The different peaks correspond to Solar conjunctions in the geometry of the observation.

Moreover, we would like to stress the fact that the expression of the time transfer used in the standard modeling of radioscience measurements (see for example [52]) is only an approximation of the relation (64) given by

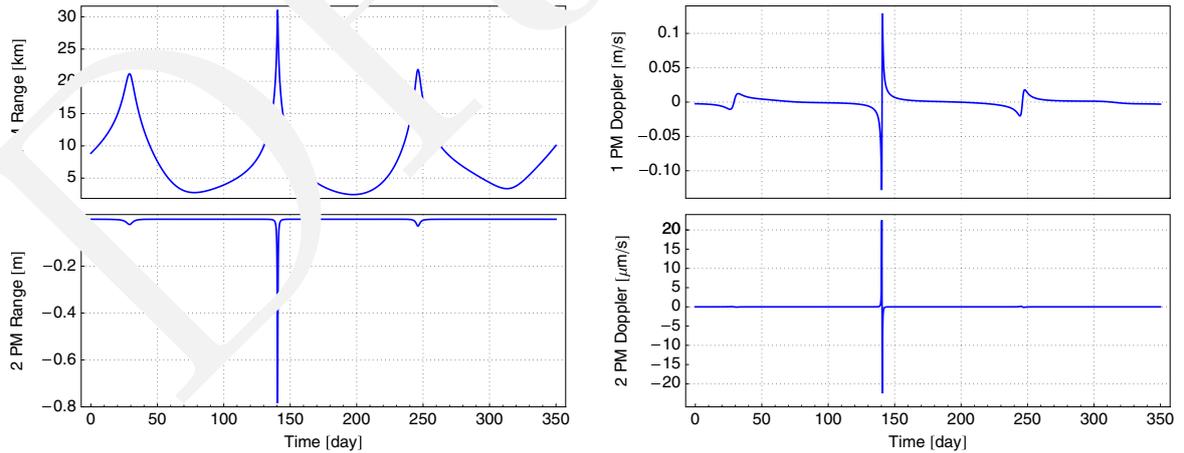


FIG. 2 (color online). First and second post-Minkowskian contributions to the range and the Doppler for a one-year Mercury-Earth radioscience link.

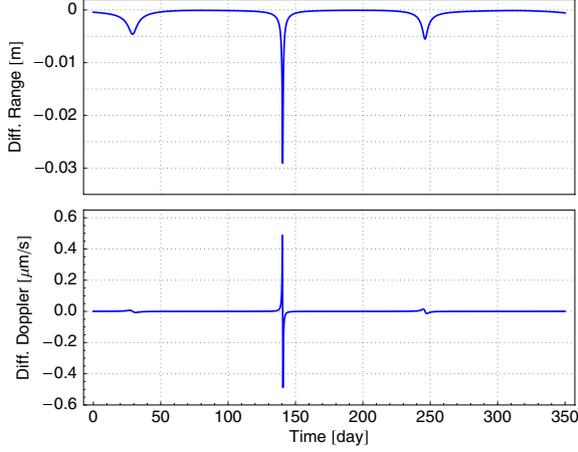


FIG. 3 (color online). Difference between the standard formulation of the Range/Doppler used in radioscience modeling (72) and the exact 2PM expression (64).

$$\begin{aligned}
 T_r(\mathbf{x}_A, t_B, \mathbf{x}_B) &= t_B - t_A \\
 &= \frac{R_{AB}}{c} + \frac{(\gamma + 1)m}{c} \\
 &\quad \times \ln \left( \frac{r_A + r_B + R_{AB} + (1 + \gamma)m}{r_A + r_B - R_{AB} + (1 + \gamma)m} \right). \quad (72)
 \end{aligned}$$

A comparison of range and Doppler simulations obtained using expressions based on the approximation (72) and on expression (64), which is complete up to 2PM order, is shown in Fig. 3. To quantify the accuracy of the standard radioscience modeling, we get results just below BepiColombo accuracy. Nevertheless, future space missions will aim at increasing the level of accuracy on radioscience measurements so that the current modeling shall be improved to include the full 2PM correction on light propagation.

### B. Direction of a light ray emitted by a star and observed on Earth

In order to simulate an astrometric observable, one can specify the reference frame used to give the incident direction of a light ray. As shown in Sec. IV B, this reference frame is mathematically modeled by a tetrad  $E_{(\alpha)}^\mu$ , which explicitly appears in the computation of the astrometric observables (19). We develop here the expression of a kinematically nonrotating tetrad comoving with an observer in the case of a static spherically symmetric space-time described by the metric (47). This tetrad is called “kinematically nonrotating” in the sense that the spatial coordinates’ transformation between the global and the local coordinate frames does not depend on a time-dependent orthogonal matrix [53]. This kind of local coordinate system is currently used in the definition of

the Celestial Geocentric Reference System [54] and is extensively used in the context of the Gaia mission [40]. Defining  $\partial_\alpha$  the vectors of the natural coordinate basis and  $e_{(\alpha)}$  the basis vectors of the tetrad, the transformation between these two bases is noted  $E_{(\alpha)}^\mu$  and is given by

$$e_{(\alpha)} = E_{(\alpha)}^\mu \partial_\mu. \quad (73)$$

The great advantage of such a basis is that the tetrad is locally orthonormal. This transformation physically corresponds to a change of basis in the tangent space of the differential manifold. From the point of view of the metric, we can easily show the link between the vectors of the natural coordinate basis and  $\eta_{(\alpha)(\beta)}$  using Eq. (73):

$$\begin{aligned}
 \eta_{(\alpha)(\beta)} &= \mathbf{g}(e_{(\alpha)}, e_{(\beta)}) = \mathbf{g}(E_{(\alpha)}^\mu \partial_\mu, E_{(\beta)}^\nu \partial_\nu) \\
 &= E_{(\alpha)}^\mu E_{(\beta)}^\nu \mathbf{g}(\partial_\mu, \partial_\nu) = E_{(\alpha)}^\mu E_{(\beta)}^\nu g_{\mu\nu}. \quad (74)
 \end{aligned}$$

All indices related to the tetrad (between angle brackets) are raised and lowered using Minkowski metric tensor, while natural coordinate basis indexes are set up and down using the  $g_{\mu\nu}$  metric.

We can split the transformation between the natural coordinate basis and the local comoving basis of the tetrad into two parts  $\lambda_{(\alpha)}^\mu = \Lambda_{(\alpha)}^\mu{}_{\hat{\kappa}} \hat{\lambda}^{\hat{\kappa}}$  [37]. The first step (parametrized by  $\hat{\Lambda}_{\hat{\kappa}}^\mu$ ) consists in orthogonalizing the natural coordinate basis to obtain a local orthonormal coordinate basis static with respect to the coordinate system used. The second part of the transformation (parametrized by  $\Lambda_{(\alpha)}^{\hat{\kappa}}$ ) consists in applying a Lorentz boost to this orthonormal basis to make it comoving with the observer. Quantities related to the final tetrad will be denoted with indices between angle brackets while quantities expressed in the intermediate tetrad will be denoted with a hat. Since the space-time metric (47) is diagonal, it is straightforward to orthonormalize the basis,

$$\tilde{\Lambda}_0^0 = \frac{1}{\sqrt{A(r)}}, \quad \tilde{\Lambda}_0^i = \tilde{\Lambda}_i^0 = 0, \quad \tilde{\Lambda}_i^j = \frac{\delta_i^j}{\sqrt{B(r)}}. \quad (75)$$

The second step consists in a Lorentz boost of the previous tetrad in order to make it comoving with the observer. We will note the quadrivelocity of the observer (expressed in the global coordinate system) by  $u^\alpha = dx^\alpha/ds$ . This velocity can also be expressed in terms of coordinates related to the intermediate tetrad  $\hat{u}^{\hat{\alpha}} = d\hat{x}^{\hat{\alpha}}/ds = \tilde{\Lambda}_{\hat{\mu}}^{\hat{\alpha}} u^\mu = (\sqrt{A(r)}u^0, \sqrt{B(r)}u^i)$ . Finally, the coordinate velocity of the observer will be denoted by  $\beta^i = \frac{1}{c} \frac{dx^i}{dt}$ . The same quantity expressed in the intermediate tetrad is  $\hat{\beta}^i = \frac{1}{c} \frac{d\hat{x}^i}{dt} = \sqrt{\frac{B(r)}{A(r)}} \beta^i$ . The second matrix transformation is thus simply given by a standard Lorentz transformation matrix whose inverse is given by

$$\Lambda_{(0)}^0 = \hat{\gamma}, \quad \Lambda_{(i)}^0 = \Lambda_{(0)}^i = -\hat{\gamma}\hat{\beta}^i, \quad \Lambda_{(j)}^i = \delta_{ij} + \frac{\hat{\gamma}^2}{\hat{\gamma}+1}\hat{\beta}^i\hat{\beta}^j \quad (76)$$

with

$$\hat{\gamma} = (1 - \hat{\beta}^2)^{-1/2} = \left(1 - \frac{B(r)}{A(r)}\beta^2\right)^{-1/2}. \quad (77)$$

The combination of Eq. (75) and Eq. (76) gives

$$E_{(0)}^0 = \frac{\hat{\gamma}}{\sqrt{A(r)}} = \frac{1}{\sqrt{A(r) - B(r)\beta^2}}, \quad (78a)$$

$$E_{(0)}^i = -\frac{\hat{\gamma}\hat{\beta}^i}{\sqrt{B(r)}} = -\frac{\beta^i}{\sqrt{A(r) - B(r)\beta^2}}, \quad (78b)$$

$$E_{(j)}^0 = -\frac{\hat{\gamma}\hat{\beta}^j}{\sqrt{A(r)}} = -\sqrt{\frac{B(r)}{A(r)}} \frac{\beta^j}{\sqrt{A(r) - B(r)\beta^2}}, \quad (78c)$$

$$E_{(j)}^i = \frac{\delta_{ij} + \frac{\hat{\gamma}^2}{\hat{\gamma}+1}\hat{\beta}^i\hat{\beta}^j}{\sqrt{B(r)}} = \frac{\delta_{ij}}{\sqrt{B(r)}} + \frac{\sqrt{B(r)}\beta^i\beta^j}{\sqrt{A^2(r) - A(r)B(r)\beta^2 + A(r) - B(r)\beta^2}}. \quad (78d)$$

Equation (78) is the exact expression of a kinematically nonrotating tetrad comoving with a given observer in a static, spherically symmetric space-time. It can be expanded to 2PM order if necessary using Eqs. (57)–(58).

We then consider a hypothetical star located far away from the Solar System and nearly in the Earth's orbital plane. We compute the incident direction of the light ray emitted by this star and observed on Earth. The reference frame used to give the incident direction is given by a comoving kinematically nonrotating tetrad. The only gravitational interaction considered is the one of the Sun described by the metric (6). The incident direction of the light ray can be computed using Eq. (78) and Eq. (69b) into Eq. (19), and the expression of the tetrad (78). The incident direction of the light ray with respect to the tetrad is denoted by  $n^{(i)}$  and can be parametrized by two angles  $\alpha$  and  $\delta$  usually called right ascension and declination:

$$n^{(i)} = (\cos \alpha \cos \delta, \sin \alpha \cos \delta, \sin \delta). \quad (79)$$

Figure 4 represents the 1PM and 2PM contributions to  $\alpha$  and  $\delta$  as well as the total deflection angle. As one can see from relation (65) the 2PM correction to the angular measurement depends on two terms: a first term proportional to  $\kappa$  and a second one proportional to  $(1 + \gamma)^2$ , both of them being formally of order 2PM. Nevertheless, it is known that the term proportional to  $(1 + \gamma)^2$  can be absorbed into the 1PM term by a change of variable and it

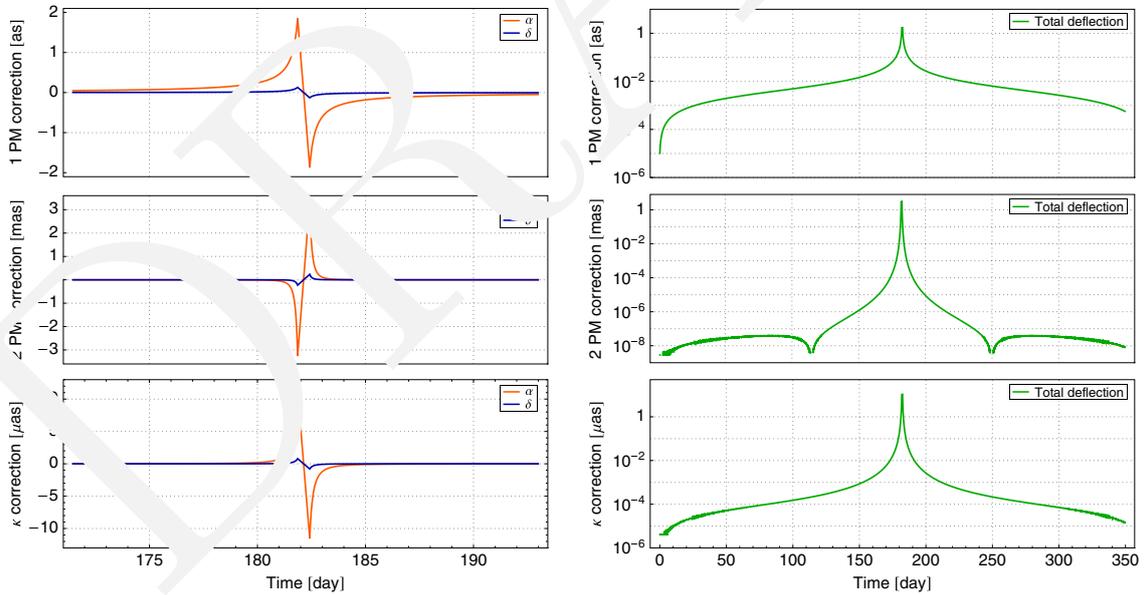


FIG. 4 (color online). Contributions to the observed direction of an incident light ray coming from a star. Left: contributions expressed for the right ascension and declination in the tetrad [see relation (79)]. Right: contribution to the total angular deflection. The 2PM contribution is the total formal 2PM contribution (including the so-called enhanced 2PN terms). The  $\kappa$  contribution represents the  $\kappa$  term in (69b).

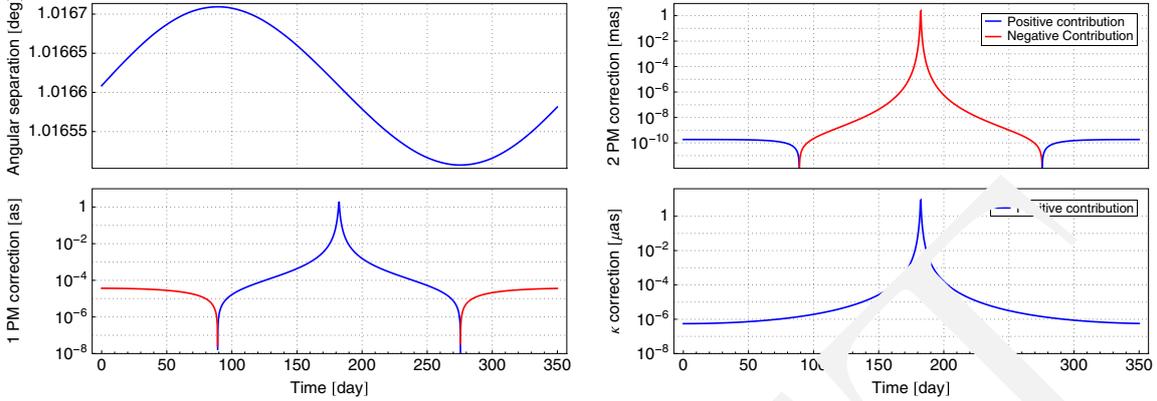


FIG. 5 (color online). Contributions to the angular separation between two incident light rays coming from two stars as observed from Earth. The 2PM contribution is the total formal 2PM contribution (including the so-called enhanced 2PN terms). The  $\kappa$  contribution is the contribution proportional to the  $\kappa$  term in (69b).

is therefore usually called “enhanced 2PN term” (for further details, see [18,48]). The enhanced 2PN term has a contribution of the order of few milliarcseconds (mas) while the second-order contribution proportional to  $\kappa$  has a contribution of 10 microarcseconds ( $\mu\text{as}$ ) only.

### C. Angular distance between two stars as measured from Earth

For this application, we consider two hypothetical stars located far away from the Solar System nearly in the Earth’s orbital plane and we compute the angular separation between these two stars as measured from Earth. This representation can be used as a very simplified model of the GAME space mission [45,55–57]. The only gravitational interaction considered here is the one due to the Sun. Relation (20), giving the angular separation between two incident light rays, can be simplified in the case of static and spherically geometry described by the space-time metric (47). The observed angle between two stars can then be written as

$$\sin^2 \frac{\phi}{2} = \frac{1}{4} \left[ \frac{(A(r_B) - \beta^2) |\hat{k}' - \hat{k}|}{B(r_B)(1 + \beta^2/c^2)(1 + \beta^2 k'_t)} \right]_B, \quad (80)$$

where  $(\hat{k})_B$  and  $(\hat{k}'_t)_B$  are the components of the deflection functions of the two incident light rays expressed in global coordinates and can be computed using the expression (10b),  $A(r)$  and  $B(r)$  are the functions parametrizing the metric (47) and  $\beta = v/c$  is the coordinate velocity of the observer. We apply the last expression in a Schwarzschild geometry. The functions  $A(r)$  and  $B(r)$  are then given by (57) and the  $\hat{k}$  vectors are determined by (10b), once Eq. (59c) and Eq. (69b) have been introduced. Figure 5 represents the evolution of the angular separation (80) with respect to time and the contribution of the 1PM and 2PM

correction. As for the direction of the incident light ray (see previous section), the 2PM correction to the angular measurement depends on two terms: a first term proportional to  $\kappa$  and a second one proportional to  $(1 + \gamma)^2$ . In this case too, the so-called “enhanced 2PN” term has a contribution of the order of few mas, while the 2PM contribution, proportional to  $\kappa$ , has a contribution of 10  $\mu\text{as}$  only.

We should recall that the accuracy aimed by modern astrometric missions is about the  $\mu\text{as}$  level, so that most 2PM order effects are observable near the Sun, while the “enhanced 2PN term” also needs to be taken into account when observing near Jupiter or Saturn.

## VII. CONCLUSIONS

In this paper, we use the time transfer function in order to compute range, Doppler and two kinds of astrometric observables: the absolute incident direction of light rays in a given frame and the angular separation between two incident light rays. The formulation presented in Sec. IV is very general and can be used at any order. All the observables depend on the TTF and its derivatives. We also show how to numerically compute the TTF and its derivatives up to 2PM order. This is done in the form of integrals of functions of the metric and its derivatives taken along a straight line. This method is particularly efficient from a numerical point of view. On one hand, it does not require one to numerically derive the TTF (which can lead to numerical error). On the other hand, it does not require the computation of the full trajectory of the photon in curved space-time, which is a boundary value problem (see [24]). This approach can be applied to any metric and therefore can also be used to determine observables in alternative theories of gravity (as long as the light propagation is governed by a null geodesic). We also present a version of the formalism valid in the case

of a static, spherically symmetric space-time. As a validation of our method, we explicitly compute analytically the TTF and its derivatives in the case of a Schwarzschild-like geometry and compare our expressions with well-established results from [48]. Finally, we apply our formulae to compute the Range and Doppler for a BepiColombo-like space mission and to simulate different configurations of a Gaia-like and GAME-like astrometric observations. We show that the standard model used for radioscience measurements is accurate at a level just below BepiColombo accuracy. We also highlight that modern  $\mu$ as-astrometry needs to take into account second-order relativistic corrections for observations near the limb of the Sun and of giant Solar System planets.

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## Light propagation in the field of a moving axisymmetric body: Theory and applications to the Juno mission

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Given the extreme accuracy of modern space science, a precise relativistic modeling of observations is required. We use the time transfer function formalism to study light propagation in the field of uniformly moving axisymmetric bodies, which extends the field of application of previous works. We first present a space-time metric adapted to describe the geometry of an ensemble of uniformly moving bodies. Then, we show that the expression of the time transfer functions in the field of a uniformly moving body can be easily derived from its well-known expression in a stationary field by using a change of variables. We also give a general expression of the time transfer function in the case of an ensemble of arbitrarily moving point masses. This result is given in the form of an integral that is easily computable numerically. We also provide the derivatives of the time transfer function in this case, which are mandatory to compute Doppler and astrometric observables. We particularize our results in the case of moving axisymmetric bodies. Finally, we apply our results to study the different relativistic contributions to range and Doppler tracking for the Juno mission in the Jovian system.

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### I. INTRODUCTION

In modern times, the accuracy of spacecraft tracking requires a very detailed modeling of the light propagation in order to compute range and Doppler observables. For example, the Cassini spacecraft reaches the level of few meters accuracy for the range and  $3 \times 10^{-6}$  m/s for the Doppler [1–3], while the future BepiColombo mission should reach an accuracy of 10cm on the range and  $10^{-6}$  m/s on the Doppler [4,5]. Similar accuracies are expected for the Juno mission [6], which will reach the Jovian system by mid-2016.

The computation of radioscience observables, as well as the determination of astrometric observables (very large baseline interferometry [7]) requires determining the propagation of light in a curved spacetime. In this context, several approaches exist. Assuming that the metric is known, solving the null geodesic equations [8] or the eikonal equation [9] is the standard method allowing one to get all the information about light propagation between two point events. Many solutions have been proposed in the post-Newtonian (PN) and in the post-Minkowskian (PM) approximations when dealing with the bending effects due to the mass multipole moments of the bodies in the Solar

System [10–17]. On the other hand, the effects of the motion of multipoles on the light propagation have also been studied [10,16]. A different approach is also available, initially based on the Synge world function [22–24] and then on the time transfer functions (TTFs) [25,26]. In this formalism, the computation of the coordinate light time, the frequency shift and the light deflection can be computed as integrals of functions of the components of the metric tensor over a straight line joining the emitter and the receiver of the signal [25,26]. This method has already been successfully used to compute the propagation of light in different configurations. For example, the TTF in the field of a stationary axisymmetric body has been determined at the first post-Newtonian (1PN) approximation [22,27]. The light propagation in the field of moving monopoles at 1.5 post-Newtonian order has also been treated [28]. Finally, the TTFs in the field of a static monopole up to the second and third post-Minkowskian (3PM) approximation have also been determined [26,29–31].

In this paper, we use the time transfer function formalism to compute the coordinate propagation time, the frequency shift and the deflection of light in the field of uniformly moving axisymmetric bodies and in the field of arbitrarily moving point masses. In Sec. III, we briefly review how the radioscience and astrometric observables can be determined from the TTF and its derivatives. Then, in Sec. IV, we determine the space-time metric describing the geometry in the field of a uniformly moving axisymmetric body, and we note the metric describing the field of arbitrarily

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moving point masses. In Sec. V, we use these metrics to develop a general expression of the TTF. A general result is given in the form of an integral that is computable numerically. Moreover, an analytical result is developed in the case of a uniform motion. The derivatives of the TTF are also determined. In Sec. VI, we particularize our results in the case of a uniformly moving axisymmetric body by analytically determining the contribution of each multipole to the TTF. Finally, in Sec. VII, we apply our results to determine the different relativistic contributions to the radioscience tracking of the Juno spacecraft in the Jovian system. The contributions of the Sun and Jupiter moving monopoles and of the Jupiter moving  $J_2$  are identified. Finally, we give our conclusions and general remarks in Sec. VIII.

## II. NOTATION AND CONVENTIONS

In this paper,  $c$  is the speed of light in a vacuum and  $G$  is the Newtonian gravitational constant. The Lorentzian metric of space-time  $V_4$  is denoted by  $g$ . The signature adopted for  $g$  is  $(+---)$ . We suppose that space-time is covered by some global quasi-Galilean coordinate system  $(x^\mu) = (x^0, \mathbf{x})$ , where  $x^0 = ct$ , with  $t$  being a time coordinate, and  $\mathbf{x} = (x^i)$ . We assume that the curves of equations  $x^i = \text{const}$  are timelike, which means that  $g_{00} > 0$  anywhere. We employ the vector notation  $\mathbf{a}$  in order to denote  $(a^1, a^2, a^3) = (a^i)$ . Considering two such quantities  $\mathbf{a}$  and  $\mathbf{b}$ , we use  $\mathbf{a} \cdot \mathbf{b}$  to denote  $a^i b^i$  (Einstein convention on repeated indices is used). The quantity  $|\mathbf{a}|$  stands for the ordinary Euclidean norm of  $\mathbf{a}$ . For any quantity  $f(x^i)$ ,  $f_{,\alpha}$  denotes the partial derivative of  $f$  with respect to  $x^\alpha$ .

## III. TIME TRANSFER FUNCTION AND OBSERVABLES

Let  $x_A = (ct_A, \mathbf{x}_A)$  and  $x_B = (ct_B, \mathbf{x}_B)$  be two events of space-time that are supposed to be connected by a unique light ray. They denote the emission and reception point of the electromagnetic signal. The coordinate light time of a photon connecting  $x_A$  and  $x_B$  is given by the TTF [23,25,32,33] as

$$t_B - t_A = \mathcal{T}(\mathbf{x}_A, t_B, \mathbf{x}_B) = \frac{R_{AB}}{c} + \frac{1}{c} \Delta(\mathbf{x}_A, t_B, \mathbf{x}_B), \quad (1)$$

where  $\mathcal{T}(\mathbf{x}_A, t_B, \mathbf{x}_B)$  is the TTF and  $R_{AB} = |\mathbf{x}_B - \mathbf{x}_A|$  and  $\Delta(\mathbf{x}_A, t_B, \mathbf{x}_B)$  is the so-called ‘delay function’.<sup>2</sup>

As developed in detail in [26], the range, Doppler and astrometric observables can all be computed from the TTF. The range is directly related to the coordinate time of flight

<sup>1</sup>In this paper, we used the reception TTF. Similar results can be obtained using the emission TTF, which depends on  $t_A$  instead of  $t_B$  [25].

<sup>2</sup>In this paper, we call for simplicity  $\Delta(\mathbf{x}_A, t_B, \mathbf{x}_B)$  a ‘‘delay function’’ even though it has the dimension of a distance.

of the photon through a coordinate transformation (see also [34]).

The frequency shift is given by [24,34,35]

$$\frac{\nu_B}{\nu_A} = \frac{[g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j]_A^{1/2}}{[g_{00} + 2g_{0i}\beta^i + g_{ij}\beta^i\beta^j]_B^{1/2}} \times \frac{1 - N_{AB}^i \beta_B^i - \beta_B^i \frac{\partial}{\partial x_B^i} - \frac{1}{c} \frac{\partial \Delta}{\partial t_B}}{1 - N_{AB}^i \beta_A^i + \beta_A^i \frac{\partial \Delta}{\partial x_A^i}}, \quad (2)$$

where  $\beta_{A/B}^i = dx_{A/B}^i / c dt$  is the coordinate velocity.

The astrometric observables are analytically related to the TTF through the use of [23]

$$\begin{aligned} (\hat{k}_i)_B &= \left( \frac{k_i}{k_0} \right)_B = -c \frac{\partial \mathcal{T}}{\partial x_B^i} \left[ 1 - \frac{\partial \mathcal{T}}{\partial t_B} \right]^{-1} \\ &= - \left( N_{AB}^i + \frac{\partial \Delta}{\partial x_B^i} \right) \times \left[ 1 - \frac{1}{c} \frac{\partial \Delta}{\partial t_B} \right]^{-1}, \end{aligned} \quad (3)$$

where  $k_\mu$  are the covariant components of the tangent vector to the photon trajectory  $(k^\mu)_B = dx^\mu / d\lambda|_B$  ( $\lambda$  being an affine parameter) and  $N_{AB}^i = \frac{R_{AB}^i}{R_{AB}} = \frac{x_B^i - x_A^i}{R_{AB}}$ .

Finally, the angular distance between two light rays coming from two different sources can also be related to  $(\hat{k}_i)_B$  [26,36].

Therefore, the computation of the TTF (or equivalently of the delay function) and its derivatives is crucial in order to analyze direct effects on observations that are done using light propagation.

## IV. METRIC AT FIRST POST-MINKOWSKIAN APPROXIMATION

### A. Uniformly moving axisymmetric body

Let us suppose that the gravitational field is generated by an ensemble of axisymmetric bodies. We are interested in calculating the contributions of the mass multipoles and of the motion of the bodies on light propagation. The first step is to consider the metric describing such a space-time. The metric for each of the bodies at 1PM order in its own local reference system is given by  $G_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu}$ , where  $H_{\mu\nu}$  is given by [37]

$$H_{00} = -2 \frac{W(X^\alpha)}{c^2} + \mathcal{O}(G^2), \quad (4a)$$

$$H_{0i} = 0, \quad (4b)$$

$$H_{ab} = -2\delta_{ab} \frac{W(X^\alpha)}{c^2} + \mathcal{O}(G^2), \quad (4c)$$

with the spin multipoles being neglected. Let us stress that the potential  $W$  depends on the local coordinate  $X^\alpha = (cT, \mathbf{X})$ .

We can now perform a Poincaré transformation in order to obtain the metric in the case of a uniformly moving body. The procedure is similar to what is developed in [38]. The coordinate transformation is given by

$$x^\mu = b^\mu + \Lambda_\alpha^\mu X^\alpha, \quad (5)$$

where  $x^\mu = (ct, \mathbf{x})$  are the coordinates of the global reference system and  $\Lambda_\alpha^\mu$  is given by

$$\begin{aligned} \Lambda_0^0 &= \gamma_p, & \Lambda_i^0 &= \Lambda_0^i = \gamma_p \beta_p^i, \\ \Lambda_i^j &= \delta_{ij} + \frac{\gamma_p^2}{1 + \gamma_p} \beta_p^i \beta_p^j, \end{aligned} \quad (6)$$

where  $\beta_p^i = v_p^i/c$ ,  $v_p^i$  is the coordinate velocity of the body and  $\gamma_p = 1/\sqrt{1 - \beta_p^2}$  with  $\beta_p = |\boldsymbol{\beta}_p|$ . Note that  $b^\mu$  is a constant four-vector that specifies the origin of the coordinate system: it points from the origin of the global reference system to the origin of the comoving frame at  $T = 0$  [38]. We have

$$b^i = x_p^i(t_0) \quad \text{and} \quad b^0 = ct_0, \quad (7)$$

and the trajectory of the moving body in the global frame is given by

$$\mathbf{x}_p(t) = \mathbf{x}_p(t_0) + c\boldsymbol{\beta}_p(t - t_0). \quad (8)$$

The inverse coordinate transformation is given by

$$X^\alpha = \tilde{\Lambda}_\mu^\alpha (x^\mu - b^\mu), \quad (9)$$

where  $\tilde{\Lambda}_\mu^\alpha$  is the inverse of  $\Lambda_\alpha^\mu$  and is given by

$$\begin{aligned} \tilde{\Lambda}_0^0 &= \gamma_p, & \tilde{\Lambda}_i^0 &= -\gamma_p \beta_p^i, \\ \tilde{\Lambda}_i^j &= \delta_{ij} - \frac{\gamma_p^2}{1 + \gamma_p} \beta_p^i \beta_p^j. \end{aligned} \quad (10)$$

The metric transformation is given by

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu} = \Lambda_\alpha^\mu \Lambda_\beta^\nu G^{\alpha\beta} = \Lambda_\alpha^\mu \Lambda_\beta^\nu (\eta^{\alpha\beta} + H^{\alpha\beta}), \quad (11)$$

which leads to

$$h^{\mu\nu} = \Lambda_\alpha^\mu \Lambda_\beta^\nu H^{\alpha\beta}. \quad (12)$$

From Eq. (4), we have  $H^{\alpha\beta} = \frac{2W}{c^2} \delta^{\alpha\beta}$ . The introduction of this expression and the expression of  $\Lambda_\alpha^\mu$  given by Eq. (6) into Eq. (12) leads to

$$h^{00} = \frac{2W(X^\alpha)}{c^2} \gamma_p^2 (1 + \beta_p^2) + \mathcal{O}(G^2), \quad (13a)$$

$$h^{0i} = \frac{4W(X^\alpha)}{c^2} \beta_p^i \gamma_p^2 + \mathcal{O}(G^2), \quad (13b)$$

$$h^{ij} = \frac{2W(X^\alpha)}{c^2} (\delta_{ij} + 2\beta_p^i \beta_p^j \gamma_p^2) + \mathcal{O}(G^2). \quad (13c)$$

It is worth mentioning that this metric is a generalization of the international astronomical union (IAU) metric [37] which can be recovered in the limit of small  $\beta_p$ . This limit is explicitly developed in Appendix E. Let us also stress that  $W$  still depends on the local coordinates  $X^\alpha$ . Therefore, we still need to use the coordinate transformation (9) to express the potential  $W$  as a function of the global coordinates  $x^\mu$ . More precisely, we get

$$W = W(X^\alpha) = W(\tilde{\Lambda}_\mu^\alpha (x^\mu - b^\mu)). \quad (14)$$

The metric (13) describes the geometry generated by a uniformly moving body at 1PM. The metric describing the geometry due to an ensemble of  $N$  uniformly moving bodies is then given by

$$h^{00} = \sum_{p=1}^N \frac{2W_p}{c^2} \gamma_p^2 (1 + \beta_p^2) + \mathcal{O}(G^2), \quad (15a)$$

$$h^i = \sum_{p=1}^N \frac{4W_p}{c^2} \beta_p^i \gamma_p^2 + \mathcal{O}(G^2), \quad (15b)$$

$$h^{ij} = \sum_{p=1}^N \frac{2W_p}{c^2} (\delta_{ij} + 2\beta_p^i \beta_p^j \gamma_p^2) + \mathcal{O}(G^2). \quad (15c)$$

In the case of an axisymmetric body, the Newtonian potential can be decomposed in a multipolar expansion

$$W_p(X^i) = \frac{GM_p}{R} \left[ 1 - \sum_{n=2}^{\infty} J_{np} \left( \frac{r_{pe}}{R} \right)^n P_n \left( \frac{\mathbf{k}_p \cdot \mathbf{X}}{R} \right) \right], \quad (16)$$

where  $\mathbf{k}_p$  denotes the unit vector along the symmetry axis of the body  $p$ ,  $M_p$  is the mass of the body  $p$ ,  $J_{np}$  are its mass multipole moments,  $P_n$  are the Legendre polynomials,  $r_{pe}$  is the equatorial radius of body  $p$  and  $R = |\mathbf{X}|$ . In this paper, we assume that the symmetry axis of the body  $\mathbf{k}_p$  is time independent, which means we neglect the precession and nutation of the body.

## B. Arbitrarily moving point masses

The determination of the metric describing the geometry around an arbitrarily moving extended body at the

<sup>3</sup>Notice that we define  $H^{\alpha\beta} = G^{\alpha\beta} - \eta^{\alpha\beta}$ , which at the linear order is given by  $H^{\alpha\beta} = -\eta^{\alpha\mu} \eta^{\beta\nu} H_{\mu\nu}$ .

post-Minkowskian approximation is very complex. In particular, one cannot simply use an instantaneous Lorentz transformation, but a local accelerated reference system has to be defined (see the discussion in the conclusion of [38]). This is beyond the scope of this paper. Nevertheless, the metric for arbitrarily moving point masses at the post-Minkowskian approximation has already been determined using the Liénard-Wiechert potentials [18,19,38]. In Appendix A, we briefly note how to compute this metric.

The space-time metric describing the geometry around an arbitrarily moving point mass can be written as [18,19,38] (see also Appendix A)

$$h^{00} = \frac{2W(X^i)}{c^2} \gamma_{pr}^2 (1 + \beta_{pr}^2) + \mathcal{O}(G^2), \quad (17a)$$

$$h^{0i} = \frac{4W(X^i)}{c^2} \beta_{pr}^i \gamma_{pr}^2 + \mathcal{O}(G^2), \quad (17b)$$

$$h^{ij} = \frac{2W(X^i)}{c^2} (\delta_{ij} + 2\beta_{pr}^i \beta_{pr}^j \gamma_{pr}^2) + \mathcal{O}(G^2), \quad (17c)$$

with  $W(X^i) = GM/R$  (since this metric is only valid for point masses),

$$X^i = -\beta_{pr}^i \gamma_{pr} r_{pr} + r_{pr}^i + \frac{\gamma_{pr}^2}{1 + \gamma_{pr}} \beta_{pr}^i (\boldsymbol{\beta}_{pr} \cdot \mathbf{r}_{pr}), \quad (18)$$

and where the index  $r$  denotes quantities that have to be evaluated at the retarded time  $t_r$ , defined by

$$t_r = t - \frac{|\mathbf{x} - \mathbf{x}_p(t_r)|}{c} = t - \frac{r_{pr}}{c}, \quad (19)$$

where  $r_{pr} = |\mathbf{x} - \mathbf{x}_p(t_r)|$  and  $\mathbf{x}_p(t_r)$  is the position of the body  $p$  at the retarded time. The expression of the potential  $W(X^i)$  can then be explicitly written as

$$W(X^i) = \frac{G}{|X^i|} \frac{GM_p}{(r_{pr} - (\mathbf{r}_{pr} \cdot \boldsymbol{\beta}_{pr}))}, \quad (20)$$

In the limit of small velocities, the expression of the IAU metric is recovered (see Appendix B 2). Finally, the metric for an ensemble of masses is the sum of the metrics generated by each body.

## V. TIME TRANSFER FUNCTION AT GENERALIZED 1PM APPROXIMATION

In [24], a PM expansion of the TTF is presented. It develops the TTF in terms of integrals of functions of the metric components over a straight line between the emitter and the receiver of a light signal. At 1PM order, the delay function is given by Ref. [35] as

$$\begin{aligned} \Delta(\mathbf{x}_A, t_B, \mathbf{x}_B) &= \frac{R_{AB}}{2} \int_0^1 [h^{00} - 2N_{AB}^i h^{0i} + N_{AB}^i N_{AB}^j h^{ij}]_{z^\alpha(\lambda)} d\lambda + \mathcal{O}(G^2), \end{aligned} \quad (21)$$

where the integral is taken along a straight line parametrized by

$$z^0(\lambda) = ct = \lambda R_{AB}, \quad (22a)$$

$$\mathbf{z}(\lambda) = \mathbf{x}_B - \lambda \mathbf{r}_{AB} = \mathbf{x}_B(1 - \lambda) + \lambda \mathbf{x}_A, \quad (22b)$$

### A. General expression in the case of uniform motion

As can be seen from the expression of the metric (17),  $\Delta$  can be written as a sum of delay functions generated by each individual body  $\Delta = \sum_{p=1}^N \Delta_p$ . Replacing the expression of the metric (15) in (21) gives

$$\begin{aligned} \Delta_p(\mathbf{x}_A, t_B, \mathbf{x}_B) &= \frac{2R_{AB}}{c^2} \int_0^1 [\gamma_p - N_{AB} \cdot \boldsymbol{\beta}_p]^2 W_p(\tilde{\Lambda}_\mu^i(z^\mu(\lambda) - b^\mu)) d\lambda. \end{aligned} \quad (23)$$

It is useful to express the argument appearing in the expression of the potential  $W_p$  in the right-hand side of (23) as

$$\begin{aligned} \tilde{\Lambda}_\mu^i(z^\mu(\lambda) - b^\mu) &= -\beta_p^i \gamma_p c(t - t_0) + z^i(\lambda) - x_{p0}^i \\ &\quad + \frac{\gamma_p^2}{1 + \gamma_p} \beta_p^i \boldsymbol{\beta}_p \cdot (z(\lambda) - \mathbf{x}_{p0}) \\ &= x_B^i + \frac{\gamma_p^2}{1 + \gamma_p} \beta_p^i [\boldsymbol{\beta}_p \cdot (\mathbf{x}_B - \mathbf{x}_{p0})] \\ &\quad - x_{p0}^i - \gamma_p v_p^i (t_B - t_0) - \lambda R_{AB} \\ &\quad \times [N_{AB}^i - \gamma_p \beta_p^i + \frac{\gamma_p^2}{1 + \gamma_p} \beta_p^i (\boldsymbol{\beta}_p \cdot \mathbf{N}_{AB})], \end{aligned} \quad (24)$$

by using Eqs. (8), (10) and (22) and by denoting  $\mathbf{x}_{p0} \equiv \mathbf{x}_p(t_0)$ . It is also possible to rewrite Eq. (24) in a more compact form as

$$\tilde{\Lambda}_\mu^i(z^\mu(\lambda) - b^\mu) = R_{pB}^i - \lambda G_{AB}^i \quad (25)$$

by setting

$$\begin{aligned} \mathbf{R}_{pX} &= \mathbf{x}_X + \frac{\gamma_p^2}{1 + \gamma_p} \boldsymbol{\beta}_p [\boldsymbol{\beta}_p \cdot (\mathbf{x}_X - \mathbf{x}_{p0})] \\ &\quad - \mathbf{x}_{p0} - \gamma_p \mathbf{v}_p (t_B - t_0), \end{aligned} \quad (26a)$$

$$\begin{aligned} \mathbf{G}_{AB} &= R_{AB} \left[ N_{AB} - \gamma_p \boldsymbol{\beta}_p + \frac{\gamma_p^2}{1 + \gamma_p} \boldsymbol{\beta}_p (\boldsymbol{\beta}_p \cdot N_{AB}) \right] \\ &= R_{AB} \mathbf{g}_{pAB} \end{aligned} \quad (26b)$$

and

$$\mathbf{g}_{pAB} = N_{AB} - \gamma_p \boldsymbol{\beta}_p + \frac{\gamma_p^2}{1 + \gamma_p} \boldsymbol{\beta}_p (\boldsymbol{\beta}_p \cdot N_{AB}). \quad (26c)$$

Let us denote by  $I$  the integral appearing in the TTF expression (23) in the case where the body  $p$  is static. Therefore,  $I$  is given by

$$I(\mathbf{x}_{pA}, \mathbf{x}_{pB}) = \tilde{I}(\mathbf{R}_{AB}, \mathbf{x}_{pB}) = \int_0^1 W_p(\mathbf{x}_{pB} - \lambda \mathbf{R}_{AB}) d\lambda, \quad (27)$$

where  $\mathbf{x}_{pA} = \mathbf{x}_A - \mathbf{x}_p$  and  $\mathbf{x}_{pB} = \mathbf{x}_B - \mathbf{x}_p$ . Usually, the solution of this integral is given in terms of  $\mathbf{x}_{pA}$  and  $\mathbf{x}_{pB}$ , but formally, the integral depends on  $\mathbf{R}_{AB}$  and  $\mathbf{x}_{pB}$ . The transition between the two expressions of  $I$  and  $\tilde{I}$  in the static case is trivial because  $\mathbf{x}_{pA} = \mathbf{x}_{pB} - \mathbf{R}_{AB}$ . However, this transition no longer applies in the moving case, and it has to be replaced in Eq. (23) by

$$\tilde{I}(\mathbf{G}_{AB}, \mathbf{R}_{pB}) = \int_0^1 W_p(\mathbf{R}_{pB} - \lambda \mathbf{G}_{AB}) d\lambda \quad (28)$$

with the two variables defined by Eqs. (26) and similarly to what was proposed in [28].

Therefore, all the results in the moving case can be derived from the expression used in the static case by replacing  $\mathbf{x}_{pB}$  by  $\mathbf{R}_{pB}$  (26c) and  $\mathbf{R}_{AB}$  by  $\mathbf{G}_{AB}$  (26b). We can use the conversions given below, where for each “static case” quantity on the left we give the “moving case” equivalent on the right. We have

$$\begin{aligned} \mathbf{x}_{pB} &\rightarrow \mathbf{R}_{pB} = \mathbf{x}_B - \mathbf{x}_p + \frac{\gamma_p^2}{1 + \gamma_p} \boldsymbol{\beta}_p (\boldsymbol{\beta}_p \cdot (\mathbf{x}_B - \mathbf{x}_p)) \\ &\quad - \mathbf{x}_{p0} - \gamma_p (\mathbf{x}_B - \mathbf{x}_p - t_0), \end{aligned} \quad (29a)$$

$$r_{pB} = |\mathbf{x}_{pB}| \rightarrow r_{pB} = |\mathbf{R}_{pB}|, \quad (29b)$$

$$\mathbf{n}_{pB} \rightarrow N_{pB} = \frac{\mathbf{R}_{pB}}{R_{pB}}, \quad (29c)$$

$$\begin{aligned} \mathbf{R}_{AB} &\rightarrow \mathbf{G}_{AB} = \mathbf{g}_{pAB} \\ &= R_{AB} \left[ N_{AB} - \gamma_p \boldsymbol{\beta}_p + \frac{\gamma_p^2}{1 + \gamma_p} \boldsymbol{\beta}_p (\boldsymbol{\beta}_p \cdot N_{AB}) \right], \end{aligned} \quad (29d)$$

$$R_{AB} \rightarrow R_{AB} \gamma_p (1 - \boldsymbol{\beta}_p \cdot N_{AB}), \quad (29e)$$

$$N_{AB} \rightarrow \frac{\mathbf{g}_{pAB}}{g_{pAB}} = \frac{\mathbf{g}_{pAB}}{\gamma_p (1 - \boldsymbol{\beta}_p \cdot N_{AB})}, \quad (29f)$$

$$\mathbf{x}_{pA} = \mathbf{x}_{pB} - \mathbf{R}_{AB} \rightarrow \mathbf{R}_{pB} - \mathbf{G}_{AB} = \mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \quad (29g)$$

$$r_{pA} = |\mathbf{x}_{pA}| \rightarrow R_{pA} = |\mathbf{R}_{pA}|, \quad (29h)$$

$$\mathbf{n}_{pA} \rightarrow \frac{\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}}{|\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}|}, \quad (29i)$$

with

$$g_{pA} = |\mathbf{R}_{pA}| = \gamma_p (1 - \boldsymbol{\beta}_p \cdot N_{AB}), \quad (30)$$

and  $\mathbf{R}_{pX}$  given by Eq. (26a). Therefore, we can write Eq. (27) as

$$\begin{aligned} \Delta_p(\mathbf{x}_A, t_B, \mathbf{x}_B) &= \frac{R_{AB}}{c^2} \gamma_p^2 (1 - N_{AB} \cdot \boldsymbol{\beta}_p)^2 \\ &\quad \times \int_0^1 W_p(R_{pB} - \lambda \mathbf{G}_{AB}) d\lambda. \end{aligned} \quad (31)$$

Then, using the definition of  $I$  from Eqs. (27) and (28) and the correspondences (29), we are able to express the moving form of the TTF in the field of moving bodies as

$$\begin{aligned} \Delta_p(\mathbf{x}_A, t_B, \mathbf{x}_B) &= \frac{2R_{AB}}{c^2} \gamma_p^2 (1 - N_{AB} \cdot \boldsymbol{\beta}_p)^2 \\ &\quad \times I(\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \mathbf{R}_{pB}), \end{aligned} \quad (32)$$

where  $\mathbf{R}_{pX}$  given by (26a). The last expression can also be written as

$$\begin{aligned} \Delta_p(\mathbf{x}_A, t_B, \mathbf{x}_B) &= \frac{\gamma_p^2 (1 - N_{AB} \cdot \boldsymbol{\beta}_p)^2}{g_{pAB}} \\ &\quad \times \tilde{\Delta}_p(\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \mathbf{R}_{pB}) \end{aligned} \quad (33)$$

$$\begin{aligned} &= \gamma_p (1 - N_{AB} \cdot \boldsymbol{\beta}_p) \\ &\quad \times \tilde{\Delta}_p(\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \mathbf{R}_{pB}), \end{aligned} \quad (34)$$

where  $\tilde{\Delta}_p(\mathbf{x}_{pA}, \mathbf{x}_{pB})$  is the expression of the static TTF. This particularly simple equation is very useful since it allows one to determine the TTF of a uniformly moving body from the corresponding static TTF.

The derivatives of the TTF, needed to compute the frequency shift (2) and the astrometric direction (3), can be computed from (34), keeping in mind Eq. (26). In the case of a uniformly moving body, their expressions are given by

$$\begin{aligned} \frac{\partial \Delta_p(\mathbf{x}_A, t_B, \mathbf{x}_B)}{\partial x_A^i} &= \gamma_p (1 - N_{AB} \cdot \boldsymbol{\beta}_p) \tilde{\Delta}_{p,jA}(\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \mathbf{R}_{pB}) \left[ \delta_{ij} - \gamma_p \beta_p^j \left( N_{AB}^i - \frac{\gamma_p}{1 + \gamma_p} \beta_p^i \right) \right] \\ &\quad + \gamma_p \frac{\beta_p^i - N_{AB}^i \boldsymbol{\beta}_p \cdot N_{AB}}{R_{AB}} \tilde{\Delta}_p(\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \mathbf{R}_{pB}), \end{aligned} \quad (35a)$$

$$\begin{aligned} \frac{\partial \Delta_p(\mathbf{x}_A, t_B, \mathbf{x}_B)}{\partial x_B^i} &= \gamma_p (1 - N_{AB} \cdot \boldsymbol{\beta}_p) \left\{ \tilde{\Delta}_{p,jB}(\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \mathbf{R}_{pB}) \left[ \delta_{ij} + \frac{\gamma_p^2}{1 + \gamma_p} \beta_p^i \beta_p^j \right] + \gamma_p \beta_p^j N_{AB}^i \tilde{\Delta}_{p,jA}(\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \mathbf{R}_{pB}) \right\} \\ &\quad - \gamma_p \frac{\beta_p^i - N_{AB}^i \boldsymbol{\beta}_p \cdot N_{AB}}{R_{AB}} \tilde{\Delta}_p(\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \mathbf{R}_{pB}), \end{aligned} \quad (35b)$$

$$\frac{\partial \Delta_p(\mathbf{x}_A, t_B, \mathbf{x}_B)}{\partial t_B} = -c \gamma_p^2 (1 - N_{AB} \cdot \boldsymbol{\beta}_p) \beta_p^i [\tilde{\Delta}_{p,iA}(\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \mathbf{R}_{pB}) + \Delta_{p,iL}(\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}, \mathbf{R}_{pB})], \quad (35c)$$

where  $\tilde{\Delta}_{p,iX}(\mathbf{x}_A, \mathbf{x}_B)$  is the expression of the derivative of the static TTF with respect to  $\mathbf{x}_X$ ,

$$\tilde{\Delta}_{p,iX}(\mathbf{x}_A, \mathbf{x}_B) = \frac{\partial \tilde{\Delta}_p(\mathbf{x}_A, \mathbf{x}_B)}{\partial x_X^i}. \quad (36)$$

It is worth mentioning that in the static case, we have the relation

$$\tilde{\Delta}_p(\mathbf{x}_A, \mathbf{x}_B) = \tilde{\Delta}_p(\mathbf{x}_B, \mathbf{x}_A) \quad (37)$$

and consequently

$$\tilde{\Delta}_{p,iB}(\mathbf{x}_A, \mathbf{x}_B) = \tilde{\Delta}_{p,iA}(\mathbf{x}_B, \mathbf{x}_A). \quad (38)$$

Therefore, the expressions of the derivatives of the TTF in the moving case is also obtained by inserting in Eqs. (35) the static TTF and its derivatives. We present an application in the field of moving axisymmetric bodies in Sec. V.

### B. Case of nonuniform motion

The previous section gave the exact solution of the TTF in the field of uniformly moving bodies. If the bodies undergo acceleration, it is still possible to use the previous formalism which corresponds to neglecting higher order terms related to the acceleration of the body. In this case, the choice of the parameter  $t_0$  introduced in Eq. (24) becomes critical. It has been shown [10,12,21] that a good choice of  $t_0$  (i.e. which minimizes the approximation error) is given by the time of the closest approach of the photon with respect to the body which is given by

$$t_0 = \max \left( t_A, t_B - \max \left( 0, \frac{\mathbf{g} \cdot (\mathbf{x}_B - \mathbf{x}_p(t_B))}{c|\mathbf{g}|^2} \right) \right) \quad (39)$$

with  $\mathbf{g} = N_{AB} - \boldsymbol{\beta}_p(t_B)$ .

In the case of arbitrarily moving point masses, it is possible to numerically integrate the TTF (21) using the metric (17). This approach has the convenience to be strictly valid at 1PM order, whatever the motion of the bodies. Inserting (17) in the expression (21) gives

$$\begin{aligned} \Delta_p(\mathbf{x}_A, t_B, \mathbf{x}_B) &= \frac{2R_{AB}}{c^2} \int_{t_r}^{t_p} \dot{\rho}_r (1 - N_{AB} \cdot \boldsymbol{\beta}_{pr})^2 \\ &\quad \times \gamma_{pr} \left( z^i(\lambda) - x_p^i(t_r) + \frac{\gamma_{pr}^2}{1 + \gamma_{pr}} \beta_{pr}^i \beta_{pr} \right. \\ &\quad \left. \cdot (z(\lambda) - \mathbf{x}_p(t_r)) - c \beta_{pr}^i \gamma_{pr} (t - t_r) \right) d\lambda, \end{aligned} \quad (40)$$

where  $\gamma_{pr}$  and  $\beta_{pr}$  depend on the retarded time coordinate  $t_r$  that is related to  $t$  through (19). The integral in Eq. (40) can then be evaluated numerically, whatever the motion of the body  $\mathbf{x}_p(t)$ .

### C. Moving emitter

In the previous sections, we handle the case where the source of the gravitational field is moving. In general, the emitter and the receiver of the electromagnetic signal are also moving. In this case, the determination of the time transfer requires solving Eq. (1), which is now implicit,

$$\begin{aligned} t_B - t_A &= \mathcal{T}(\mathbf{x}_A(t_A), t_B, \mathbf{x}_B) \\ &= \frac{|\mathbf{x}_B - \mathbf{x}_A(t_A)|}{c} + \frac{1}{c} \Delta(\mathbf{x}_A(t_A), t_B, \mathbf{x}_B). \end{aligned}$$

In practice, the solution of this implicit equation can be determined by an iterative procedure to find  $t_A$  [for example, see Eq. (7) of [26]]. Another solution consists in a post-Newtonian expansion of  $t_A$  from the TTF [for

example, see Eq. (6) of [26]]. Let us denote by  $\bar{t}_A$  the Minkowskian coordinate time of the emission solution of

$$t_B - \bar{t}_A = \frac{|\mathbf{x}_B - \mathbf{x}_A(\bar{t}_A)|}{c}, \quad (41)$$

and then we can write, at first order in  $\bar{\beta}_A = \mathbf{v}_A(\bar{t}_A)/c$ ,

$$t_B - t_A = \frac{\bar{R}_{AB}}{c} + \Delta(\mathbf{x}_A(\bar{t}_A), t_B, \mathbf{x}_B) - \frac{\bar{R}_{AB}}{c} \bar{\beta}_A^i \frac{\partial \Delta(\mathbf{x}_A(\bar{t}_A), t_B, \mathbf{x}_B)}{\partial x_A^i}, \quad (42)$$

where the “bar” denotes quantities evaluated at  $\bar{t}_A$  like  $\bar{R}_{AB} = |\mathbf{x}_B - \mathbf{x}_A(\bar{t}_A)|$ . The contribution proportional to  $\bar{\beta}_A$  is also known as a Sagnac term. It has the same form as the contribution from the velocity of the source of the gravitational field at first post-Newtonian order as can be seen from Eq. (50). The order of magnitude of this contribution can reach a few meters for a Juno–Earth signal, as can be seen from Fig. 4. Therefore, when iteratively solving the light-time equation, one needs to include the relativistic perturbations or take into account the Sagnac terms to avoid the risk of significant errors.

## VI. CASE OF UNIFORMLY MOVING AXISYMMETRIC BODIES

We can now use the general procedure presented in the previous section in the case of uniformly moving axisymmetric bodies whose potential is given by the multipole expansion (16). The TTF in the case of a static axisymmetric body has been computed in [27] and is given by

$$\Delta_p(\mathbf{x}_{pA}, \mathbf{x}_{pB}) = \Delta_{Mp}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) + \Delta_{Jpn}(\mathbf{x}_{pA}, \mathbf{x}_{pB}), \quad (43)$$

where  $\Delta_{Mp}$  represents the mass multipole contribution and  $\Delta_{Jpn}$  represents the mass multipole contribution.

The TTF corresponding to a static multipole is well known [22] and is given by

$$\tilde{\Delta}_p(\mathbf{x}_{pA}, \mathbf{x}_{pB}) = \gamma \frac{GM_p}{c^2} \ln \frac{r_{pA} + r_{pB} + R_{AB}}{r_{pA} + r_{pB} - R_{AB}}, \quad (44)$$

By inserting (44) into (34) and using the substitutions (29), we obtain the TTF in the field of monopoles in uniform motion as

$$\Delta_M(\mathbf{x}_A, t_B, \mathbf{x}_B) = 2 \frac{GM_p}{c^2} \gamma_p (1 - N_{AB} \cdot \beta_p) \ln \frac{|\mathbf{R}_{pA} + \gamma_p \beta_p R_{pA}| + R_{pB} + \gamma_p R_{AB} (1 - \beta_p \cdot N_{AB})}{|\mathbf{R}_{pA} + \gamma_p \beta_p R_{pA}| - R_{pB} - \gamma_p R_{AB} (1 - \beta_p \cdot N_{AB})}, \quad (45)$$

with  $R_{pX}$  given by (26a). On the other hand, the mass multipole contribution  $\Delta_{Jpn}$  has been computed in [27] as

$$\tilde{\Delta}_{Jpn}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) = K_{pn} \sum_{m=1}^n \left[ \frac{1}{(r_{pA} + r_{pB} - R_{AB})^{n-m+1}} - \frac{1}{(r_{pA} + r_{pB} + R_{AB})^{n-m+1}} \right] \Theta_{nm}(\mathbf{x}_{pA}, \mathbf{x}_{pB}), \quad (46a)$$

with  $K_{pn} \equiv 2GM_p J_{np} r_{pe}^n / c^2$  and

$$\Theta_{nm}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) = (-1)^{n-m} \sum_{i_1, \dots, i_m} \frac{(n-m)!}{i_1! i_2! \dots i_m!} \prod_{l=1}^m [S_l(\mathbf{x}_{pA}, \mathbf{x}_{pB})]^{i_l}, \quad (46b)$$

where the sum  $\sum_{i_1, \dots, i_m}$  denotes the summation over the sets of non-negative integers  $i_1, i_2, \dots, i_m$  satisfying the pair of equations

$$\begin{cases} i_1 + i_2 + 3i_3 + \dots + mi_m = n \\ i_1 + i_2 + \dots + i_m = n - m + 1 \end{cases} \quad (46c)$$

and where  $S_l(\mathbf{x}_{pA}, \mathbf{x}_{pB})$  is defined by

$$S_l(\mathbf{x}_{pA}, \mathbf{x}_{pB}) = \frac{1}{r_{pA}^{l-1}} C_l^{(-1/2)} \left( \frac{\mathbf{k}_p \cdot \mathbf{x}_{pA}}{r_{pA}} \right) + \frac{1}{r_{pB}^{l-1}} C_l^{(-1/2)} \left( \frac{\mathbf{k}_p \cdot \mathbf{x}_{pB}}{r_{pB}} \right), \quad (46d)$$

with  $C_l^{(-1/2)}(x)$  the Gegenbauer polynomial of degree  $l$  and of parameter  $-1/2$ .

Therefore, the multipole term of the TTF for the case of moving axisymmetric bodies is given by inserting (46) into the relation (34) and using the substitutions (29),

$$\begin{aligned} \Delta_{J_{np}}(\mathbf{x}_A, t_B, \mathbf{x}_B) &= \frac{2GM_p J_{np} r_{pe}^n}{c^2} \gamma_p (1 - \mathbf{N}_{AB} \cdot \boldsymbol{\beta}_p) \\ &\times \sum_{m=1}^n \left[ \frac{1}{(|\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}| + R_{pB} - R_{AB} \gamma_p (1 - \boldsymbol{\beta}_p \cdot \mathbf{N}_{AB}))^{n-m+1}} \right. \\ &\left. - \frac{1}{(|\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}| + R_{pB} + R_{AB} \gamma_p (1 - \boldsymbol{\beta}_p \cdot \mathbf{N}_{AB}))^{n-m+1}} \right] \Theta_{nm}(\mathbf{R}_{pA} + \boldsymbol{\beta}_p R_A, \mathbf{R}_{pB}), \end{aligned} \quad (47)$$

with  $\mathbf{R}_{pX}$  given by (26a).

In order to compute the derivatives of the TTF in the case of moving bodies from Eq. (35), one also needs the derivatives of the TTF in the static case. The derivative of the TTF in the case of a static monopole is known, for example, [39], and it is given by

$$\tilde{\Delta}_{Mp,iA}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) = -\frac{4GM_p N_{AB}^i (r_{pA} + r_{pB}) + R_{AB} n_{pA}^i}{c^2 (r_{pA} + r_{pB})^2} \kappa_{AB}, \quad (48a)$$

$$\tilde{\Delta}_{Mp,iB}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) = +\frac{4GM_p N_{AB}^i (r_{pA} + r_{pB}) - R_{AB} n_{pB}^i}{c^2 (r_{pA} + r_{pB})^2} = \tilde{\Delta}_{p,iA}(\mathbf{x}_{pB}, \mathbf{x}_{pA}). \quad (48b)$$

Also, the derivatives of Eq. (46) can be computed as

$$\begin{aligned} \tilde{\Delta}_{J_{pn,iA}}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) &= K_{pn} \sum_{m=1}^n \left\{ -(n-m+1) \left[ \frac{\mathbf{n}_{pA} + \mathbf{N}_{AB}}{(r_{pA} + r_{pB} - R_{AB})^{n-m+2}} - \frac{-\mathbf{N}_{AB}}{(r_{pA} + r_{pB} + R_{AB})^{n-m+2}} \right] \Theta(\mathbf{x}_{pA}, \mathbf{x}_{pB}) \right. \\ &\left. + \left[ \frac{1}{(r_{pA} + r_{pB} - R_{AB})^{n-m+1}} - \frac{1}{(r_{pA} + r_{pB} + R_{AB})^{n-m+1}} \right] \Upsilon_{A|nm}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) \right\}, \end{aligned} \quad (49a)$$

$$\begin{aligned} \tilde{\Delta}_{J_{pn,iB}}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) &= K_{pn} \sum_{m=1}^n \left\{ -(n-m+1) \left[ \frac{\mathbf{n}_{pB} + \mathbf{N}_{AB}}{(r_{pA} + r_{pB} - R_{AB})^{n-m+2}} - \frac{\mathbf{n}_{pB} + \mathbf{N}_{AB}}{(r_{pA} + r_{pB} + R_{AB})^{n-m+2}} \right] \Theta(\mathbf{x}_{pA}, \mathbf{x}_{pB}) \right. \\ &\left. + \left[ \frac{1}{(r_{pA} + r_{pB} - R_{AB})^{n-m+1}} - \frac{1}{(r_{pA} + r_{pB} + R_{AB})^{n-m+1}} \right] \Upsilon_{B|nm}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) \right\}, \end{aligned} \quad (49b)$$

where

$$\begin{aligned} \Upsilon_{X|nm}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) &= (-1)^{n-m} \sum_{i_1, \dots, i_m} \frac{m!}{i_1! i_2! \dots i_m!} \sum_{l=1}^m i_l [S_l(\mathbf{x}_{pA}, \mathbf{x}_{pB})]^{i_l-1} \\ &\times \prod_{q=1}^m \mathbf{S}_q(\mathbf{x}_{pA}, \mathbf{x}_{pB})^{i_q} \frac{[P_{l-1}(\mathbf{k}_p \cdot \mathbf{n}_{pX}) \mathbf{k}_p - P_l(\mathbf{k}_p \cdot \mathbf{n}_{pX}) \mathbf{n}_{pX}]}{r_{pX}^l}. \end{aligned} \quad (49c)$$

The derivatives of the TTF function in the case of a moving axisymmetric body are then given by combining Eqs. (44) and (49) with Eq. (43) and by using it together with the combination of Eqs. (48) and (49) with Eqs. (35) [using the correspondences (29)].

### A. Particular case: Post-Newtonian expansion

Section VA gives a way to compute the TTF in the field of uniformly moving bodies. The obtained expressions are exact at any order in  $\boldsymbol{\beta}_p$ . Nonetheless, a post-Newtonian expression can sometimes be more practical to use in the case of slowly moving bodies. Therefore, we present here an expansion of the previous results in terms of the small parameter  $\boldsymbol{\beta}_p$ . An expansion of (32) gives

$$\Delta_p(\mathbf{x}_A, t_B, \mathbf{x}_B) = (1 - \boldsymbol{\beta}_p \cdot \mathbf{N}_{AB}) \tilde{\Delta}_p(\mathbf{x}_{pA}, \mathbf{x}_{pB}) + (R_{AB} - c(t_B - t_0)) \beta_p^i \tilde{\Delta}_{p,iA}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) - c(t_B - t_0) \beta_p^i \tilde{\Delta}_{p,iB}(\mathbf{x}_{pA}, \mathbf{x}_{pB}), \quad (50)$$

with  $\mathbf{x}_{pX} = \mathbf{x}_X - \mathbf{x}_p(t_0)$ . For example, the use of this formula in the case of the moving monopoles leads to

$$\begin{aligned}
& \Delta_p(\mathbf{x}_A, t_B, \mathbf{x}_B) \\
&= 2 \frac{GM_p}{c^2} (1 - \boldsymbol{\beta}_p \cdot \mathbf{N}_{AB}) \ln \frac{r_{pA} + r_{pB} + R_{AB}}{r_{pA} + r_{pB} - R_{AB}} \\
&\quad - 4 \frac{GM_p R_{AB}}{c^2} \frac{(r_{pA} + r_{pB}) \mathbf{N}_{AB} \cdot \boldsymbol{\beta}_p + R_{AB} \mathbf{n}_{pA} \cdot \boldsymbol{\beta}_p}{(r_{pA} + r_{pB})^2 - R_{AB}^2} \\
&\quad + 4 \frac{GM_p R_{AB}}{c} (t_b - t_0) \frac{\boldsymbol{\beta}_p \cdot (\mathbf{n}_{pA} + \mathbf{n}_{pB})}{(r_{pA} + r_{pB})^2 - R_{AB}^2} + \mathcal{O}(c^{-4}), \tag{51}
\end{aligned}$$

with  $\mathbf{n}_{pX} = \frac{\mathbf{x}_{pX}}{r_{pX}} = \frac{\mathbf{x}_{pX}}{|\mathbf{x}_{pX}|}$ . This expression is equivalent to the one given by Eq. (20) of [28]. Obtaining this result in such a straightforward way illustrates the effectiveness of the TTF approach.

### B. Particular case: The quadrupolar term

An explicit calculation for each of the multipoles is straightforward given the above formulas. As an example,

let us explicitly develop the expression for the quadrupolar term  $J_2$ . The only sets of integer solutions to Eqs. (46c) are  $i_1 = 2$  for  $m = 1$  and  $\{i_1 = 0, i_2 = 1\}$  for  $m = 2$ . As shown in [27], we obtain

$$\begin{aligned}
\tilde{\Delta}_{J_{p2}}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) &= \frac{GM_p J_{p2} r_{pe}^2}{c^2} \frac{R_{AB}}{r_{pA} r_{pB} (1 + \mathbf{n}_{pA} \cdot \mathbf{n}_{pB})} \\
&\quad \times \left[ \frac{1 - (\mathbf{k}_p \cdot \mathbf{n}_{pA})^2}{r_{pA}} + \frac{1 - (\mathbf{k}_p \cdot \mathbf{n}_{pB})^2}{r_{pB}} \right. \\
&\quad \left. - \left( \frac{1}{r_{pA}} + \frac{1}{r_{pB}} \right) \frac{[\mathbf{k}_p \cdot (\mathbf{n}_{pA} + \mathbf{n}_{pB})]^2}{1 + \mathbf{n}_{pA} \cdot \mathbf{n}_{pB}} \right]. \tag{52}
\end{aligned}$$

Therefore, inserting (52) into (34) and using the substitutions (49) we obtain

$$\begin{aligned}
\Delta_{J_{p2}}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) &= \frac{GM_p}{c^2} \gamma_p^2 (1 - \mathbf{N}_{AB} \cdot \boldsymbol{\beta}_p)^2 \frac{J_{p2} r_{pe}^2}{|\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}| R_{pB} (1 + \tilde{\mathbf{N}}_A \cdot \mathbf{N}_{pB})} \frac{R_{AB}}{R_{pB}} \\
&\quad \times \left[ \frac{1 - (\mathbf{k}_p \cdot \tilde{\mathbf{N}}_A)^2}{|\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}|} + \frac{1 - (\mathbf{k}_p \cdot \mathbf{n}_{pB})^2}{R_{pB}} \left( \frac{1}{|\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}|} + \frac{1}{R_{pB}} \right) \frac{[\mathbf{k}_p \cdot (\tilde{\mathbf{N}}_A + \mathbf{N}_{pB})]^2}{1 + \tilde{\mathbf{N}}_A \cdot \mathbf{N}_{pB}} \right] \tag{53}
\end{aligned}$$

and

$$\tilde{\mathbf{N}}_A = \frac{\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}}{|\mathbf{R}_{pA} + \gamma_p \boldsymbol{\beta}_p R_{AB}|}. \tag{54}$$

The derivative of (52) with respect to  $\mathbf{x}_{pA}$  can be computed using Eq. (49a) and is given by

$$\begin{aligned}
\tilde{\Delta}_{J_{p2,iA}}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) &= 2 \frac{GM_p J_{p2} r_{pe}^2}{c^2} \frac{R_{AB}}{r_{pA} r_{pB}} \left\{ [\mathbf{k}_p \cdot (\mathbf{n}_{pA} + \mathbf{n}_{pB})]^2 \left[ \frac{\mathbf{n}_{pA} + \mathbf{N}_{AB}}{(r_{pA} + r_{pB} - R_{AB})^3} - \frac{\mathbf{n}_{pA} - \mathbf{N}_{AB}}{(r_{pA} + r_{pB} + R_{AB})^3} \right] \right. \\
&\quad \left. - \frac{1}{2} \left[ \frac{1 - (\mathbf{k}_p \cdot \mathbf{n}_{pA})^2}{r_{pA}} + \frac{1 - (\mathbf{k}_p \cdot \mathbf{n}_{pB})^2}{r_{pB}} \right] \left[ \frac{\mathbf{n}_{pA} + \mathbf{N}_{AB}}{(r_{pA} + r_{pB} - R_{AB})^2} - \frac{\mathbf{n}_{pA} - \mathbf{N}_{AB}}{(r_{pA} + r_{pB} + R_{AB})^2} \right] \right. \\
&\quad \left. - \frac{R_{AB} (r_{pA} + r_{pB})}{r_p^2} \frac{\mathbf{k}_p \cdot (\mathbf{n}_{pA} + \mathbf{n}_{pB})}{(1 + \mathbf{n}_{pA} \cdot \mathbf{n}_{pB})^2} [\mathbf{k}_p - (\mathbf{k}_p \cdot \mathbf{n}_{pA}) \mathbf{n}_{pA}] \right. \\
&\quad \left. - \frac{1}{2r_p^3} \frac{R_{AB}}{r_{pB}} \frac{2(\mathbf{k}_p \cdot \mathbf{n}_{pA}) \mathbf{k}_p + [1 - 3(\mathbf{k}_p \cdot \mathbf{n}_{pA})^2] \mathbf{n}_{pA}}{1 + \mathbf{n}_{pA} \cdot \mathbf{n}_{pB}} \right\}, \tag{55a}
\end{aligned}$$

while the derivatives with respect to  $\mathbf{x}_{pB}$  can be obtained by symmetry as

$$\tilde{\Delta}_{J_{p2,iB}}(\mathbf{x}_{pA}, \mathbf{x}_{pB}) = \tilde{\Delta}_{J_{p2,iA}}(\mathbf{x}_{pB}, \mathbf{x}_{pA}). \tag{55b}$$

In order to evaluate the contribution of the moving quadrupole to the derivatives of the time transfer, it is then sufficient to combine Eqs. (55) and (52) as shown in Eqs. (35).

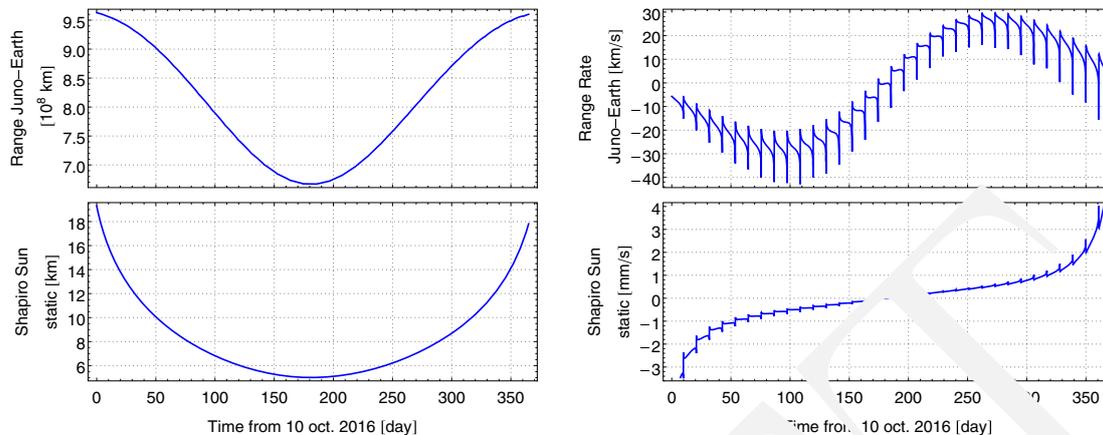


FIG. 1 (color online). Representation of different contributions on the range (left panels) and range rate (right panels) between Juno and Earth over one year. The contributions represented are the lower order contribution (the actual value of the observables) and the corrections produced by the Shapiro due to the monopole of the Sun.

**VII. APPLICATION TO JUNO**

As an example, we use the equations presented in previous sections to give estimates of the relativistic corrections on the observables for the Juno mission. Juno is currently on its way to Jupiter, which it will reach in 2016. The spacecraft will orbit Jupiter during one year. Some of the relativistic perturbations on the Juno orbit have been studied in [40,41]. The main goal of this section is to assess the order of magnitude produced by different effects due to the Sun and Jupiter on the time transfer. We shall use the nominal orbit of the mission around Jupiter obtained using the Naif SPICE toolkit [42] and kernels, as well as the DE430 planetary ephemeris [43]. The expected accuracy for Juno is of the order of 10 cm on the range and  $10^{-6}$  m/s

on the Doppler [46]. In the following, we present different relativistic contributions to the two-way coordinate light time between Earth and Juno and the corresponding range rate. The range rate has been computed with an integration time of 10 seconds.

In the following figures, all the time scales are given in terms of the coordinate time, which is similar to the barycentric coordinate time introduced in the IAU conventions (see [37]). The observations, done in terms of local time, can be derived by a relativistic coordinate transformation, which is conventional [37,44]. Nevertheless, this transformation will not significantly change the figures presented below.

Figure 1 represents the lower order time transfer and range rate between Juno and Earth, as well as the relativistic

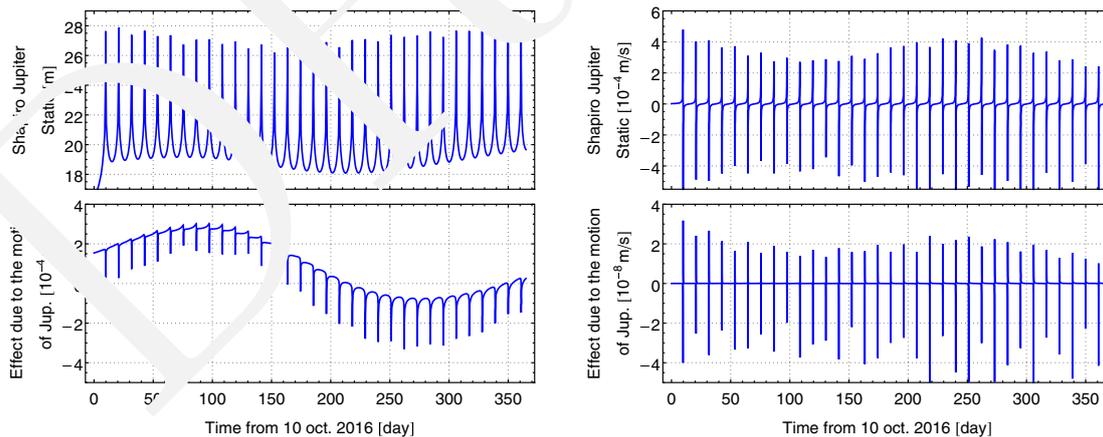


FIG. 2 (color online). Representation of different contributions on the range (left panels) and range rate (right panels) between Juno and Earth over one year. The contributions represented are the corrections produced by the Shapiro due to the monopole of Jupiter (top panels) and the contributions due to the velocity of Jupiter (bottom panels).

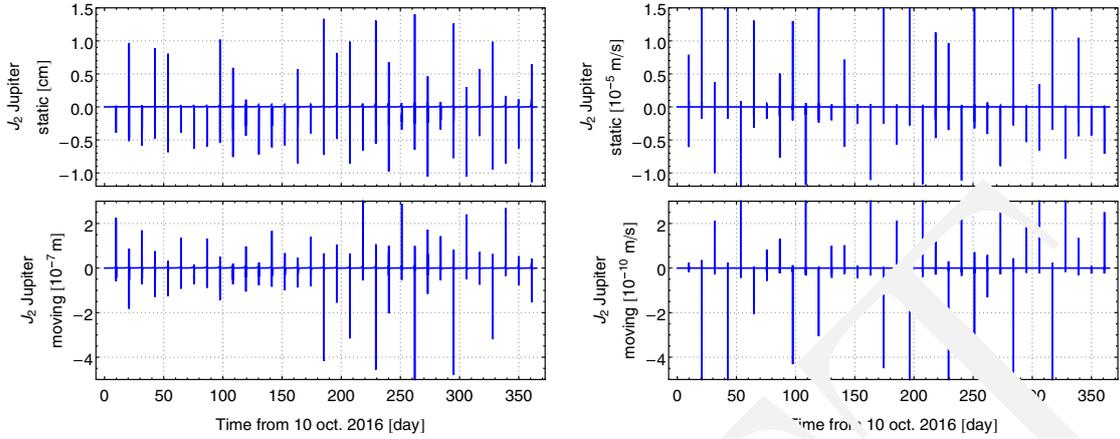


FIG. 3 (color online). Representation of different contributions on the range (left panels) and range rate (right panels) between Juno and Earth over one year. The contributions represented are the corrections produced by the  $J_2$  of Jupiter (top panels) and the contributions produced by the fact that the  $J_2$  is moving (bottom panels)

Shapiro correction from the Sun. These corrections are standard.

Figure 2 represents the contributions of the mass monopole of Jupiter on the range and on the range rate. These contributions have been split into two parts: one related to the case where Jupiter is static and a contribution proportional to Jupiter's velocity  $\beta_{Jup}$ . The static part is computed using (44) with the position of Jupiter taken at the critical time  $t_0$  given by Eq. (39). The contribution relative to the velocity is computed by taking the difference between the relations (45) and (44). One can see, the contributions relative to the motion of Jupiter are 2 orders of magnitude below the expected accuracy of Juno and can

safely be neglected in the modeling of the time transfer. A similar conclusion holds for the motion of the Sun around the Solar System barycenter, which is even smaller. Note that the analytical results presented in these graphs have been checked by numerically integrating the TTF (40).

Figure 3 represents the contributions of the quadrupole of Jupiter ( $J_2$ ) on the range and on the range rate of Juno. As above, we have split these contributions into two parts: one related to the case where Jupiter is static and one proportional to Jupiter's velocity  $\beta_{Jup}$ . The static part is computed using (52) with the position of Jupiter taken at the critical time  $t_0$  given by Eq. (39). The contribution relative to the velocity is computed by taking the difference

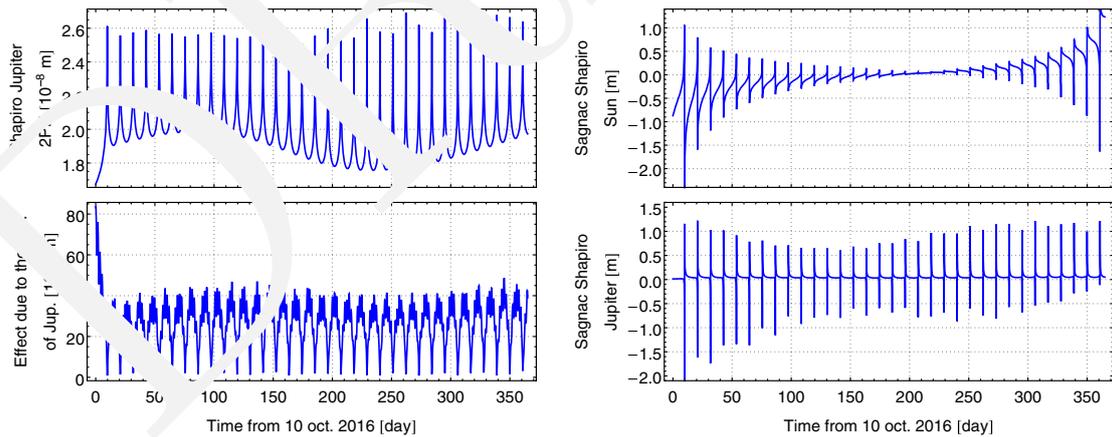


FIG. 4 (color online). Representation of different contributions on the range between Juno and Earth over one year. Top left panel: The 2PN contribution from the monopole of Jupiter (contribution proportional to  $\beta_{Jup}^2$ ). Bottom left panel: The contribution proportional to the acceleration of Jupiter. Right panels: The Sagnac contributions (proportional to the Sun mass and to Jupiter mass) due to the motion of Juno.

between the relations (53) and (52). As one can see, the contributions relative to the  $J_2$  of Jupiter are of the same order as the Juno's expected accuracy. Therefore, the effect of the  $J_2$  should be taken into account in the reduction of the tracking data. The contribution related to the velocity of the  $J_2$  is far beyond the current tracking accuracy. Once again, the analytical results presented in these graphs have been checked by numerically integrating the TTF (40). It is important to notice that the curves highly depend on the geometry of the probe orbit. Since Juno has a polar orbit and is never in conjunction with Jupiter; the velocity effects are not detectable. Therefore, the situation can be different for another space mission like JUICE [45].

Figure 4 is given for illustrative purposes, and it shows more effects on the range of Juno. First of all, the effects of the second order in  $\beta_{\text{Jup}}$  are represented. They are computed by making the difference between the formula valid at all orders in  $\beta_{\text{Jup}}$  (45) and the 1PN expansion (50). This shows that one can safely use the PN expansion presented in Sec. VIA within the Solar System. The effect of the acceleration of Jupiter on the range is also presented. This is computed by making the difference between the numerical integration of the TTF in which we are using the real Jupiter trajectory (40) and the result valid at all orders in the velocity (45). The small rapid oscillations come from oscillations in Jupiter's acceleration, which results from perturbations due to the Galilean satellites.

Finally, on the right of Fig. 4 the Sagnac effects due to the motion of Juno are represented. The contributions represented are due to the Shapiro of the Sun and Jupiter, which has been computed using (42). These contributions should be included in the analysis of tracking data either as a perturbation or when solving the light-time iterations.

## VIII CONCLUSIONS

In this paper, we computed the TTF and its derivatives in the field of uniformly moving axisymmetric bodies and in the field of arbitrarily moving point masses, which is useful in order to evaluate range, Doppler and astrometric observables. First, in Sec. IV, we computed a metric adapted to describe the space-time geometry due to  $n$  bodies in a global reference system by using a Poincaré transformation. Then, we presented a general method to compute the TTF and its derivatives in the case where the bodies generating the gravitational field are in uniform motion. We showed that the TTF in the case of uniform motion can be directly deduced from the static TTF, as can be seen from Eqs. (34) and (35). This result is very powerful and valid for any velocities. Moreover, in Sec. VB, we developed a general expression of the TTF in the case where the gravitational field is generated by arbitrarily moving point masses. The result is given as an integral over a straight line between the emitter and the receiver (40), which can be computed numerically. This general formulation has been used to numerically check our analytical derivations but is

also useful to assess the effects due to the acceleration of the body on the light propagation.

Then, in Sec. VI, we showed how our method can be easily applied to the metric presented in Sec. IV to analytically compute the TTF and its derivatives (and thus the range, frequency shift and astrometric direction) for a light signal propagating in the field of one or more axisymmetric bodies in uniform motion. The results of this paper complete the work of [27,28] and, in general, extend the field of applicability of the TTF formalism [25].

Finally, as an example of our method, we computed the range and Doppler for the Juno mission during its orbit around Jupiter and studied in detail the different perturbations due to the Sun and Jupiter on light propagation. In particular, we showed that in addition to the standard Shapiro contributions due to the mass monopoles of Jupiter and of the Sun, the contribution of Jupiter  $J_2$  is also important at the level of accuracy expected for Juno. The motion of the Sun and of Jupiter produces effects that are too small compared to Juno accuracy. Nevertheless, this conclusion highly depends on the geometry of Juno orbit and it should be assessed carefully for other space missions (JUICE, for example [45]).

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## APPENDIX A: METRIC OF ARBITRARILY MOVING POINT MASSES

The standard way to compute the metric for an arbitrarily moving point mass is to use the Liénard-Wiechert potentials as in [18,19,38]. Based on the analogy between the Maxwell equations and the linearized Einstein field equations [46], the guidelines of classical electromagnetism (see Chapter 8 of [47]) follow. This procedure is described in detail in [38]. According to the formulas for the retarded potentials, the field at the point of observation at time  $t$  is determined by the state of motion of the body at the earlier time  $t_r$  which is determined by (19) (in the following, the index  $r$  denotes quantities evaluated at the retarded time  $t_r$ ).

We can introduce a reference system comoving with the body at the retarded time, whose temporal origin coincides with the retarded time. The coordinates with respect to this frame will be denoted by  $X^\alpha$ , and they can be derived by the instantaneous Lorentz transformation

$$X^\alpha = \tilde{\Lambda}^\alpha_{r\mu}(x^\mu - x^\mu_{pr}), \quad (\text{A1})$$

where  $x^0_{pr} = ct_r$  and  $\mathbf{x}_{pr} = \mathbf{x}_p(t_r)$  are the coordinates of the body at the retarded time. It is important to notice that the Lorentz transformation is done at the retarded time [i.e.

$\tilde{\Lambda}_{r\mu}^\alpha = \tilde{\Lambda}_\mu^\alpha(t_r)$ . The four-vector  $X^\alpha$  is a null four-vector [ $\eta_{\alpha\beta}X^\alpha X^\beta = \eta_{\mu\nu}(x^\mu - x_r^\mu)(x^\nu - x_r^\nu) = c^2(t - t_r)^2 - |\mathbf{x} - \mathbf{x}_p(t_r)|^2 = 0$ ]. In this frame, the space-time metric is known,  $H^{\alpha\beta} = \frac{2W(X^i)}{c^2}\delta^{\alpha\beta}$ , which can be written in a manifestly covariant way as  $H^{\alpha\beta} = \frac{4W(X^i)}{c^2}(U_{pr}^\alpha U_{pr}^\beta - \frac{1}{2}\eta^{\alpha\beta})$  with  $U_{pr}^\alpha$  the four-velocity of the body at the retarded time (in the comoving frame  $U_{pr}^\alpha = \delta^{\alpha 0}$ ). Since the expression of the metric is manifestly covariant, we can express it in the global frame by using the local transformation (A1),

$$h^{\mu\nu} = \frac{4W(X^i)}{c^2} \left( u_{pr}^\alpha u_{pr}^\beta - \frac{1}{2}\eta^{\alpha\beta} \right), \quad (\text{A2})$$

where the  $X^i$  are given by the transformation (A1) and where  $u_{pr}^\mu$  is now given by  $u_{pr}^\mu = \gamma_{pr}(1, \boldsymbol{\beta}_{pr})$  with  $\gamma_{pr} = \gamma_p(t_r)$  and  $\mathbf{v}_{pr} = d\mathbf{x}_p/dt|_{t_r}$ .

We can write the space-time metric as (17) with  $X^i$  given by (A1), which can be explicitly written as (18). For example, in the case of a point mass ( $W = GM/R$ ), one gets

$$R = |X^i| = \gamma_{pr}(r_{pr} - (\mathbf{r}_{pr} \cdot \boldsymbol{\beta}_{pr}))$$

and the metric can be written as

$$h^{00} = \frac{2GM}{c^2(r_{pr} - (\mathbf{r}_{pr} \cdot \boldsymbol{\beta}_{pr}))} \gamma_{pr}(1 + \beta_{pr}^2) + \mathcal{O}(G^2), \quad (\text{A3a})$$

$$h^{0i} = \frac{4GM}{c^2(r_{pr} - (\mathbf{r}_{pr} \cdot \boldsymbol{\beta}_{pr}))} \beta_{pr}^i \gamma_{pr} + \mathcal{O}(G^2), \quad (\text{A3b})$$

$$h^{ij} = \frac{2GM}{c^2(r_{pr} - (\mathbf{r}_{pr} \cdot \boldsymbol{\beta}_{pr}))} (\delta_{ij} + \gamma_{pr}^2 \beta_{pr}^i \beta_{pr}^j) + \mathcal{O}(G^2), \quad (\text{A3c})$$

This expression is exactly the same as what we found in [19,38]. In the limit of small velocities, the expressions of the IAU metric are recovered (see Appendix B 2).

## APPENDIX B: CORRESPONDENCE WITH THE IAU METRIC

### 1. Case of uniformly moving bodies

In Sec. IV A, we derive the post-Minkowskian metric related to uniformly moving bodies (13). It is interesting to show that the post-Minkowskian limit of the IAU metric [37] is recovered in the limit of the small velocities. In order to show this, we first need to develop the argument appearing in the potential  $W$  from Eq. (14) as

$$\begin{aligned} \tilde{\Lambda}_\mu^i(x^\mu - b^\mu) &= -\gamma_p \beta_p^i c(t - t_0) + x^i - x_p^i(t_0) \\ &\quad + \frac{\gamma_p^2}{1 + \gamma_p} \beta_p^i \boldsymbol{\beta} \cdot (\mathbf{x} - \mathbf{x}_p(t_0)). \end{aligned} \quad (\text{B1})$$

Using the fact that the motion of the body is uniform, this expression can also be written as

$$\tilde{\Lambda}_\mu^i(x^\mu - b^\mu) = x^i - x_p^i(t) + \frac{\gamma_p^2}{1 + \gamma_p} \beta_p^i \boldsymbol{\beta} \cdot (\mathbf{x} - \mathbf{x}_p(t)). \quad (\text{B2})$$

In the limit of the small velocities ( $\beta_p \ll 1$ ), we have

$$\tilde{\Lambda}_\mu^i(x^\mu - b^\mu) = x^i - x_p^i(t) + \frac{1}{2} \beta_p^i \boldsymbol{\beta}_p \cdot (\mathbf{x} - \mathbf{x}_p(t)) + \mathcal{O}(\beta_p^4) \quad (\text{B3})$$

Using an expansion in the limit of small velocities, the expression of the potential  $W$  appearing in the metric (13) becomes

$$\begin{aligned} W[\tilde{\Lambda}_\mu^i(x^\mu - b^\mu)] &= \frac{1}{2} W_{,j} [x^i - x_p^i(t)] + \frac{1}{2} W_{,j} [x^i - x_p^i(t)] \beta_p^j \boldsymbol{\beta}_p \\ &\quad \cdot (\mathbf{x} - \mathbf{x}_p(t)) + \mathcal{O}(\beta_p^4), \end{aligned} \quad (\text{B4})$$

where  $W_{,j} = \partial W / \partial X^j$ . Introducing this expression in the metric (13), it leads to

$$\begin{aligned} h^{00} &= \frac{2W(x^i - x_p^i)}{c^2} + 4 \frac{W(x^i - x_p^i)}{c^2} \beta_p^2 \\ &\quad + \frac{\boldsymbol{\beta}_p \cdot (\mathbf{x} - \mathbf{x}_p(t))}{c^2} \beta_p^j W_{,j} (x^i - x_p^i(t)) + \mathcal{O}(G^2) \\ &\quad + \mathcal{O}(\beta_p^4/c^2), \end{aligned} \quad (\text{B5a})$$

$$h^{0i} = \frac{4W(x^i - x_p^i)}{c^2} \beta_p^i + \mathcal{O}(G^2) + \mathcal{O}(\beta_p^3/c^2), \quad (\text{B5b})$$

$$h^{ij} = \frac{2W(x^i - x_p^i)}{c^2} \delta_{ij} + \mathcal{O}(G^2) + \mathcal{O}(\beta_p^2/c^2). \quad (\text{B5c})$$

For example, in the case of a point mass ( $W = \frac{GM}{R}$ ), the last expression becomes

$$\begin{aligned} h^{00} &= 2 \frac{GM}{r_p c^2} + 4 \frac{GM}{r_p c^2} \beta_p^2 - \frac{GM}{c^2 r_p^3} (\mathbf{r} \cdot \boldsymbol{\beta}_p)^2 + \mathcal{O}(G^2) \\ &\quad + \mathcal{O}(\beta_p^4/c^2), \end{aligned} \quad (\text{B6a})$$

$$h^{0i} = 4 \frac{GM}{r_p c^2} \beta_p^i + \mathcal{O}(G^2) + \mathcal{O}(\beta_p^3/c^2), \quad (\text{B6b})$$

$$h^{ij} = 2 \frac{GM}{r_p c^2} \delta_{ij} + \mathcal{O}(G^2) + \mathcal{O}(\beta_p^2/c^2), \quad (\text{B6c})$$

with  $r_p = |\mathbf{r}_p|$  and  $\mathbf{r}_p = \mathbf{x} - \mathbf{x}_p(t)$ . This expression is exactly the one recommended in the IAU conventions [see Eqs. (8) and (51–55) from [37] or Resolutions B1.5. in the Appendix of the same paper].

## 2. Case of arbitrarily moving point masses

The metric (17) or (A3) describes the space-time geometry around an arbitrarily moving point mass at the first post-Minkowskian approximation. It is interesting to show that the post-Minkowskian limit of the IAU metric [37] is recovered in the limit of the small velocities.

We need to express the quantities at the retarded time  $t_r$  as a function of the quantities at the time  $t$ . Since

$$t_r - t = -\frac{r_{pr}}{c}, \quad (\text{B7})$$

we have

$$\begin{aligned} \mathbf{r}_p(t) &= \mathbf{r}_p = \mathbf{x} - \mathbf{x}_p(t) \\ &= \mathbf{x} - \mathbf{x}_p(t_r) - (t - t_r)\mathbf{v}_p(t_r) - \frac{(t - t_r)^2}{2}\mathbf{a}_p(t_r) \\ &\quad + \mathcal{O}((t - t_r)^3) \\ &= \mathbf{r}_{pr} - \frac{r_{pr}}{c}\mathbf{v}_{pr} - \frac{r_{pr}^2}{2c^2}\mathbf{a}_{pr} + \mathcal{O}(c^{-3}). \end{aligned} \quad (\text{B8})$$

It is useful to notice that in the last term of this expression, we can replace  $t_r$  by  $t$  (this will introduce a higher order correction). A simple calculation leads to

$$\begin{aligned} r_p^2 + (\boldsymbol{\beta}_p \cdot \mathbf{r}_p)^2 &= (r_{pr} - (\boldsymbol{\beta}_p \cdot \mathbf{r}_{pr}))^2 + \frac{r_{pr}^2}{c^2} \\ &\quad - \frac{r_p^2}{c^2} \mathbf{r}_p + \mathcal{O}(c^{-3}). \end{aligned} \quad (\text{B10})$$

This leads to

$$\begin{aligned} r_{pr} - (\boldsymbol{\beta}_{pr} \cdot \mathbf{r}_{pr}) &= r_p \left[ 1 - \frac{\beta_{pr}^2}{2} + \frac{(\boldsymbol{\beta}_p \cdot \mathbf{r}_p)^2}{2r_p^2} + \frac{\mathbf{a}_p \cdot \mathbf{r}_p}{2c^2} \right] \\ &\quad + \mathcal{O}(c^{-3}). \end{aligned} \quad (\text{B11})$$

Since

$$\gamma_{pr} = \gamma_p + \mathcal{O}(c^{-3}) = 1 + \frac{\beta_p^2}{2} + \mathcal{O}(c^{-3}), \quad (\text{B12})$$

we have

$$\frac{1}{\gamma_{pr}(r_{pr} - (\boldsymbol{\beta}_{pr} \cdot \mathbf{r}_{pr}))} = \frac{1}{r_p} \left[ 1 - \frac{(\boldsymbol{\beta}_p \cdot \mathbf{r}_p)^2}{2r_p^2} - \frac{\mathbf{a}_p \cdot \mathbf{r}_p}{2c^2} \right]. \quad (\text{B13})$$

Introducing Eq. (B13) in the space-time metric (A3), leads to

$$\begin{aligned} h^{00} &= \frac{2M}{c^2 r_p} + \frac{2M}{c^2 r_p} \beta_p^2 - \frac{GM}{c^2 r_p^3} (\mathbf{r}_p \cdot \boldsymbol{\beta}_p)^2 \\ &\quad - \frac{GM}{c^4 r_p} (\mathbf{a}_p \cdot \mathbf{r}_p) + \mathcal{O}(G^2) + \mathcal{O}(1/c^5), \end{aligned} \quad (\text{B14a})$$

$$h^{0i} = 4 \frac{GM}{r_p^2 c^2} \beta_p^i + \mathcal{O}(G^2) + \mathcal{O}(1/c^4), \quad (\text{B14b})$$

$$h^{ij} = 2 \frac{GM}{r_p c^2} \delta_{ij} + \mathcal{O}(G^2) + \mathcal{O}(1/c^3). \quad (\text{B14c})$$

The only additional term with respect to the metric (B6) is the term proportional to the acceleration in  $g_{00}$ . This term is exactly the one appearing in the IAU metric [see Eq. (54) of [37]] as already noticed in [48].

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Planetary ephemerides are a very powerful tool to constrain deviations from the theory of general relativity (GR) using orbital dynamics. The effective field theory framework called the Standard-Model Extension (SME) has been developed in order to systematically parametrize hypothetical violations of Lorentz symmetry (in the Standard Model and in the gravitational sector). In this communication, we use the latest determinations of the supplementary advances of the perihelia and of the nodes obtained by planetary ephemerides analysis to constrain SME coefficients in the pure gravity sector and also from gravity-matter couplings. Our results do not show any deviation from GR and they improve current constraints. Moreover, combinations with existing constraints from Lunar Laser Ranging and from atom interferometry gravimetry allow us to disentangle contributions from the pure gravity sector from the gravity-matter couplings.

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**I. INTRODUCTION**

The Solar System has proven to be an efficient laboratory to discover new phenomena from gravitational observations. Historically, one can mention the discovery of “dark” components [such as the planet Neptune predicted by Le Verrier] or evidence towards non-Newtonian gravity theories [for example the perihelion advance of Mercury which pointed towards general relativity (GR)]. The Solar System remains the most precise laboratory to test the theory of gravity, that is to say GR.

Constraints on deviations from GR can only be obtained in an extended theoretical framework that parametrizes such deviations. The constraints that are obtained from observations are framework dependent. In the past decades, two frameworks were widely used in the literature at the scale of the Solar System, namely the parametrized post-Newtonian (PPN) formalism [1] and the fifth force framework [2]. Stringent constraints have been obtained for these formalisms [1,3–8]. More recently, other phenomenological frameworks have been developed like the Standard-Model Extension (SME). The SME is an extensive formalism that allows a systematic description of Lorentz symmetry violations in all sectors of physics, including gravity [9–11]. Violations of Lorentz symmetry are possible in a number of

scenarios considered in the literature. While some early motivation came from string theory [12], Lorentz violations can also appear in loop quantum gravity, noncommutative field theory and others [13,14]. The SME is an effective field theory aiming at making phenomenological connections between fundamental theories and experiments.

In particular, a hypothetical Lorentz violation in the gravitational sector naturally leads to an expansion at the level of the action [11,15] which in the minimal SME writes

$$S_{\text{grav}} = \int d^4x \frac{\sqrt{-g}}{16\pi G} (R - uR + s^{\mu\nu} R_{\mu\nu}^T + t^{\alpha\beta\mu\nu} C_{\alpha\beta\mu\nu}) + S'[s^{\mu\nu}, t^{\alpha\beta\mu\nu}, g_{\mu\nu}], \quad (1)$$

with  $G$  the gravitational constant,  $g$  the determinant of the metric,  $R$  the Ricci scalar,  $R_{\mu\nu}^T$  the trace-free Ricci tensor,  $C_{\alpha\beta\mu\nu}$  the Weyl tensor and  $u$ ,  $s^{\mu\nu}$  and  $t^{\alpha\beta\mu\nu}$  the Lorentz violating fields. To avoid conflicts with the underlying Riemann geometry, we assume spontaneous symmetry breaking so that the Lorentz violating coefficients need to be considered as dynamical fields. The last part of the action  $S'$  contains the dynamical terms governing the evolution of the SME coefficients. In the linearized gravity limit, the metric depends only on  $\bar{u}$  and  $\bar{s}^{\mu\nu}$  which are the vacuum expectation value of  $u$  and  $s^{\mu\nu}$  [15]. The coefficient  $\bar{u}$  is unobservable since it can be absorbed in a rescaling of the gravitational constant. The so obtained post-Newtonian

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metric differs from the one introduced in the PPN formalism [15]. In addition to the minimal SME action given by Eq. (1), there exist some higher order Lorentz-violating curvature couplings in the gravity sector (nonminimal SME) [16] that have been constrained by short range experiments [17]. These terms are not considered in this communication.

In addition to Lorentz symmetry violations in the pure-gravity sector, violations of Lorentz symmetry can also arise from gravity-matter couplings. In [18], it has been shown that gravity-matter couplings violation of Lorentz symmetry can be parametrized by the following classical point mass action:

$$S_{\text{mat}} = \int d\lambda (-m\sqrt{g_{\mu\nu} + 2c^{\mu\nu}u_\mu u_\nu} - (a_{\text{eff}})^\mu u_\mu), \quad (2)$$

where  $u^\mu$  is the four-velocity of the particle,  $m$  is its mass and  $c^{\mu\nu}$  and  $(a_{\text{eff}})^\mu$  are Lorentz violating fields. In this action, spin-coupled Lorentz violation is effectively set to zero. The new fields  $c^{\mu\nu}$  and  $(a_{\text{eff}})^\mu$  depend on the composition of the point particle [18]. This modification of the action produces two different types of effects: (i) a modification of the way gravity is sourced and (ii) a violation of the three facets of the Einstein equivalence principle. The first effect will result in a modification of the space-time metric solution of the field equations. Modifications of the metric in the linearized approximation depend on  $(\bar{a}_{\text{eff}}^S)^\mu$  coefficients, the background values of the coefficients  $(a_{\text{eff}})^\mu$  from the source body [18]. On the other hand, the violation of the equivalence principle generated by the action (2) leads to a deviation from the geodesic motion depending at first order on the coefficients  $\bar{c}_T^{\mu\nu}$  and  $(\bar{a}_{\text{eff}}^T)^\mu$ , the background values of the Lorentz violating fields of the test mass.

Up to now, several studies have constrained pure-gravity SME coefficients  $\bar{s}^{\mu\nu}$  for example Lunar Laser Ranging [19], atom interferometry gravimetry [20,21], short range experiments [22], planetary orbital dynamics [23], Gravity Probe B [24] and recently binary pulsars [25]. The  $(\bar{a}_{\text{eff}}^w)^\mu$  coefficients are currently poorly constrained by [26–29]. On the opposite, some of the  $\bar{c}^{\mu\nu}$  coefficients are severely constrained (see for example [26,27,30,31]). A list of current constraints on all SME coefficients can be found in [32]. In this study, we will concentrate on the impact of  $\bar{s}^{\mu\nu}$  and  $(\bar{a}_{\text{eff}}^w)^\mu$  coefficients on planetary orbital dynamics and neglect the  $\bar{c}$  coefficients and leave them for future work.

In this communication we show that planetary orbital dynamics can be used to derive stringent constraints on the SME coefficients. Indeed, SME modifications of gravity induce a secular variation of some orbital elements [15,18] such as the longitude of the ascending node and the argument of perihelia. These variations are introduced in Sec. II. In Sec. III, we compare these variations with the present level of residuals coming from INPOP10a

(Intégrateur Numérique Planétaire de l’Observatoire de Paris) ephemerides [33]. We use a Bayesian inversion to infer the posterior probability density function (pdf) on the SME coefficients. From the pdf, we estimate correlations between the coefficients. We estimate realistic confidence intervals and also determine linear combinations of the SME coefficients that can be determined independently from planetary orbital dynamics. In Sec. IV, we combine our results with previous results obtained by Lunar Laser Ranging analysis and atom interferometry gravimetry. Finally, in Sec. V, we discuss the obtained results and present several ideas that may improve the current analysis.

## II. EFFECTS OF SME ON ORBITAL DYNAMICS

In the linearized gravity limit, the gravity sector of the minimal SME is parametrized by a symmetric rank-2 tensor  $\bar{s}^{\mu\nu}$  and by a scalar  $\bar{u}$  that is unobservable since it corresponds to a rescaling of the gravitational constant [15]. Furthermore, the matter-gravity coupling is parametrized amongst others by the  $(a_{\text{eff}})^\mu$  coefficients which depend on the composition of the different bodies. The components of these coefficients depend on the observer coordinate system. The standard frame used in the SME formalism labeled by  $(T, X, Y, Z)$  corresponding with the Solar System, the spatial axes are defined by equatorial coordinates (see Fig. 1 of [15]) and the origin of time is given by the time when Earth crosses the Sun-centered  $X$  axis at the vernal equinox. The planetary orbital elements are defined with respect to the ecliptic coordinate system. The two coordinate systems differ by a rotation  $\mathcal{R}$  of angle  $\epsilon = 23.44^\circ$  (the Earth obliquity) around the  $X$  axis. Therefore, the transformation of the tensor  $\bar{s}^{\mu\nu}$  is given by  $\bar{s}^{ij} = \mathcal{R}^i{}_{j'} \bar{s}^{j'k'} \mathcal{R}^{k'j}$  and  $\bar{s}^{0i} = \mathcal{R}^i{}_{j'} \bar{s}^{0j'}$  where capital letters refer to the equatorial reference system and lower case letters refer to the ecliptic one. Similarly, the transformation of the  $(\bar{a}_{\text{eff}})^\mu$  vector is given by  $(\bar{a}_{\text{eff}})^i = \mathcal{R}^i{}_{j'} (\bar{a}_{\text{eff}})^{j'}$ .

SME modifications of gravity induce different types of effects (for an extensive review, see [15,18]). Two important effects can have implications on planetary ephemerides analysis: effects on the orbital dynamics and effects on the light propagation. Simulations using the time transfer formalism [34] based on the software presented in [35] have shown that only the  $\bar{s}^{TT}$  and  $(\bar{a}_{\text{eff}})^T$  coefficients produce a non-negligible effect on the light propagation (while it has impact only at the next post-Newtonian level on the orbital dynamics [15,18]). Since in this analysis we concentrate on orbital dynamics, these coefficients are not considered and will be neglected. This can safely be done since the signatures from the  $\bar{s}^{TT}$  and  $(\bar{a}_{\text{eff}})^T$  coefficients on the light propagation are similar to the logarithmic standard Shapiro delay, which is not correlated to orbital dynamics effects.

The equations of motion in the SME formalism are given in [15,18]. Neglecting the  $\bar{c}_{\mu\nu}$  contributions, the two-body equation of motion reads

$$\begin{aligned} \frac{d^2 r^j}{dt^2} = & -\frac{G_N M}{r^3} r^j + \frac{G_N M}{r^3} \left[ \bar{s}^{jk} r^k - \frac{3}{2} \bar{s}^{kl} \frac{r^k r^l}{r^2} r^j \right. \\ & + 2 \frac{\delta m}{M} \left( \bar{s}^{0k} + \sum_{w=e,p,n} \frac{n_2^w}{\delta m} \alpha(\bar{a}_{\text{eff}}^w)^k \right) v^k r^j \\ & \left. - 2 \frac{\delta m}{M} \left( \bar{s}^{0j} + \sum_{w=e,p,n} \frac{n_2^w}{\delta m} \alpha(\bar{a}_{\text{eff}}^w)^j \right) v^k r^k \right], \quad (3) \end{aligned}$$

where  $G_N$  is the observed Newton constant,  $M = m_1 + m_2$  is the total mass of the two bodies,  $\delta m = m_2 - m_1$  is the difference of the two masses,  $r^j = r_1^j - r_2^j$  is the relative position of the two masses and

$$n_2^w = N_1^w - N_2^w, \quad (4)$$

with  $N_{1,2}^w$  the number of particles of species  $w$  in the body 1, 2. The coefficient  $\bar{s}^{TT}$  is completely unobservable in this context since absorbed in a rescaling of the gravitational constant (see the discussion in [15,24]). The coefficient  $(\bar{a}_{\text{eff}}^w)^T$  can also be absorbed in a rescaling of the gravitational constant that depends on the composition of each planet [18]. In this context, one would observe a different  $G_N$  with the different planets. Nevertheless, this effect is expected to be very small [18] and would not produce any supplementary advances of the perihelia and of the nodes and therefore is neglected in this analysis.

In Eq. (3), the sums on  $w$  need to be done on the electrons, protons and neutrons. In the case of a Sun-planet system, we have  $M = m_p + m_\odot$  and  $\delta m = m_\odot - m_p \approx m_\odot$  and  $n_2^w = N_p^w - N_\odot^w \approx -N_\odot^w$ . The point that we are neglecting  $N_p^w$  means that we are neglecting effects produced by the violation of the universality of free fall. Under these assumptions, the equations of motion depend on

$$\bar{s}^{0j} \approx -\sum_w \frac{N_\odot^w}{m_\odot} \alpha(\bar{a}_{\text{eff}}^w)^j \quad (5a)$$

$$\approx \bar{s}^{0j} - 0.9\alpha(\bar{a}_e)^j - 0.1\alpha(\bar{a}_{\text{eff}}^n)^j, \quad (5b)$$

where we used a simple model for the composition of the Sun characterized by  $N_\odot^e/m_\odot = N_\odot^p/m_\odot \approx 0.9(\text{GeV}/c^2)^{-1}$  and  $N_\odot^n/m_\odot \approx 0.1(\text{GeV}/c^2)^{-1}$  as described in [21] (with  $c$  the speed of light in vacuum). In this paper,  $\alpha(\bar{a}_{\text{eff}}^w)^j$  is always expressed in  $\text{GeV}/c^2$  and

$$(\bar{a}_{\text{eff}}^w)^j = (\bar{a}_{\text{eff}}^e)^j + (\bar{a}_{\text{eff}}^p)^j. \quad (6)$$

Using the Gauss equations, secular perturbations induced by SME on the orbital elements can be computed similarly to what is done in [15,23]. The two orbital elements needed for our analysis are the longitude of the

ascending node  $\Omega$  and the argument of the perihelion  $\omega$ . The secular change in these two elements is given by

$$\begin{aligned} \left\langle \frac{d\Omega}{dt} \right\rangle = & \frac{n}{\sin i(1-e^2)^{1/2}} \left[ \frac{\varepsilon}{e^2} \bar{s}_{kP} \sin \omega \right. \\ & \left. + \frac{(e^2 - \varepsilon)}{e^2} \bar{s}_{kQ} \cos \omega - \frac{2na\varepsilon}{ec} \bar{s}^k \cos \omega \right], \quad (7a) \end{aligned}$$

$$\begin{aligned} \left\langle \frac{d\omega}{dt} \right\rangle = & -\cos i \left\langle \frac{d\Omega}{dt} \right\rangle - n \left[ \frac{(e^2 - 2\varepsilon)}{e^2} (\bar{s}_{PP} - \bar{s}_{QQ}) \right. \\ & \left. + \frac{2na(e^2 - \varepsilon)}{ce^3(1-e^2)} \bar{s}_{\odot}^Q \right], \quad (7b) \end{aligned}$$

where  $a$  is the semi-major axis,  $e$  the eccentricity,  $i$  the orbit inclination (with respect to the ecliptic),  $n = (G_N m_\odot / a^3)^{1/2}$  is the mean motion and  $\varepsilon = 1 - (1 - e^2)^{1/2}$ . In all these expressions, the coefficients for Lorentz violation with subscripts  $P$ ,  $Q$ , and  $k$  are understood to be appropriate projections of  $\bar{s}^{uv}$  along the unit vectors  $P$ ,  $Q$ , and  $k$ , respectively. For example,  $\bar{S}_{\odot}^k = k^i \bar{S}_{\odot}^{Ti}$ ,  $\bar{s}_{PP} = P^i P^j \bar{s}^{ij}$ . The unit vectors  $P$ ,  $Q$  and  $k$  define the orbital plane

$$\begin{aligned} \vec{P} = & (\cos \Omega \cos \omega - \cos i \sin \Omega \sin \omega) \vec{e}_x \\ & + (\sin \Omega \cos \omega + \cos i \cos \Omega \sin \omega) \vec{e}_y + \sin i \sin \omega \vec{e}_z, \quad (8a) \end{aligned}$$

$$\begin{aligned} \vec{Q} = & -(\cos \Omega \sin \omega + \cos i \sin \Omega \cos \omega) \vec{e}_x \\ & + (\cos i \cos \Omega \cos \omega - \sin \Omega \sin \omega) \vec{e}_y + \sin i \cos \omega \vec{e}_z, \quad (8b) \end{aligned}$$

$$k = \sin i \sin \Omega \vec{e}_x - \sin i \cos \Omega \vec{e}_y + \cos i \vec{e}_z, \quad (8c)$$

where  $\vec{e}_{x,y,z}$  define the basis of the ecliptic reference system. The relations (7) are generalizations of Eqs. (168)–(171) from [15] that do not include the  $(\bar{a}_{\text{eff}}^w)^j$  terms.

### III. ANALYSIS AND RESULTS

Planetary ephemerides analysis uses an impressive number of different observations to produce high accurate planetary and asteroid trajectories. The observations used to produce ephemerides comprise radioscience observations of spacecraft that orbited around Mercury, Venus, Mars and Saturn, flyby tracking of spacecraft close to Mercury, Jupiter, Uranus and Neptune and optical observations of all planets [4,6,7,33,36–43]. Estimations of supplementary advances of perihelia with the Russian Ephemerides of Planets and the Moon (EPM) are presented in [6,42]. The INPOP ephemerides have produced estimations of supplementary advances of perihelia and nodes. Table I gives

TABLE I. Values of supplementary longitude of nodes and argument of perihelia estimated by INPOP10a (see Table 5 from [33]). These values are estimated in [33] as the interval in which the differences of postfit residuals are below 5%.

Planet	$\dot{\Omega}$ (mas $\times$ cy $^{-1}$ )	$\dot{\omega}$ (mas $\times$ cy $^{-1}$ )
Mercury	$1.4 \pm 1.8$	$0.4 \pm 0.6$
Venus	$0.2 \pm 1.5$	$0.2 \pm 1.5$
EMB	$0.0 \pm 0.9$	$-0.2 \pm 0.9$
Mars	$-0.05 \pm 0.13$	$-0.04 \pm 0.15$
Jupiter	$-40 \pm 42$	$-41 \pm 42$
Saturn	$-0.1 \pm 0.4$	$0.15 \pm 0.65$

estimations obtained by INPOP10a [33] on supplementary longitude of nodes  $\dot{\Omega}$  and on supplementary argument of perihelia<sup>1</sup>  $\dot{\omega}$ .

Since  $\bar{s}^{TT}$  and  $(\bar{a}_{\text{eff}}^w)^T$  do not play any role in the orbital dynamics and  $\bar{s}^{\mu\nu}$  is trace free, the observations depend on eight independent fundamental coefficients:  $\bar{s}^{XX} - \bar{s}^{YY}$ ,  $\bar{s}^Q = \bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}$ ,  $\bar{s}^{XY}$ ,  $\bar{s}^{XZ}$ ,  $\bar{s}^{YZ}$  and  $\bar{S}_{\odot}^{JJ}$  (these coefficients will be denoted as  $p_i$  in the following). In this communication, we perform a Bayesian inversion to infer knowledge on these eight independent coefficients using a Monte Carlo Markov chain (MCMC) algorithm. The approach is very similar to the one used for pulsar data [25]. The observations are assumed to be independent and the errors to be normally distributed. The pdf describing the likelihood (i.e. the probability to obtain observations  $O_i$  given certain values of the SME coefficients  $p_k$ ) is given by

$$L(O_i|p_1, p_2, \dots, p_n) = \text{cste} e^{-\chi^2/2} \quad (9)$$

where the  $\chi^2$  is computed

$$\chi^2 = \sum_{pl} \frac{(\dot{\omega}_{pl, \text{SME}}(p_k) - \dot{\omega}_{pl, \text{INPOP}})^2}{\sigma_{\dot{\omega}_{pl}}^2} + \frac{(\dot{\Omega}_{pl, \text{SME}}(p_k) - \dot{\Omega}_{pl, \text{INPOP}})^2}{\sigma_{\dot{\Omega}_{pl}}^2} \quad (10)$$

where the index  $pl$  of the sum is running over the six different planets from Table I,  $\dot{\omega}_{pl, \text{INPOP}}$ ,  $\dot{\omega}_{pl, \text{SME}}(p_k)$  and the corresponding  $\sigma$  are from Table I and where  $\dot{\omega}_{pl, \text{SME}}(p_k)$  and  $\dot{\Omega}_{pl, \text{SME}}(p_k)$  are simulated values depending on the SME coefficients  $p_k$ . The posterior pdf of the SME coefficients is given by

<sup>1</sup>In [33],  $\dot{\omega}$  is noted  $\dot{\varpi}$  which is commonly used for the longitude of the perihelion but the estimated values correspond to supplementary argument of perihelia and not to longitude of perihelia (usually noted by  $\varpi$ ) [44].

$$P(p_1, p_2, \dots, p_n | O_i) = \mathcal{C} L(O_i | p_1, \dots, p_n) \pi(p_1, \dots, p_n), \quad (11)$$

where  $\pi(p_1, \dots, p_n) = \pi(p_1) \dots \pi(p_n)$  is the prior pdf on the SME coefficients  $p_k$  and  $\mathcal{C}$  a constant. We use a uniform prior pdf on the SME coefficients and the MCMC algorithm used is a standard Metropolis-Hasting algorithm [45]. We run the Metropolis-Hastings sampler until  $10^6$  samples have been generated. The convergence of the MC is ascertained by monitoring the estimated Bayesian confidence intervals of the parameter. Finally, to diminish the effect of the starting configuration, we discard the first 1000 samples.

The marginal pdf of a single SME coefficient  $p_j$  is given by

$$P(p_j | O_i) = \int dp_1 \dots \int dp_n P(p_1, \dots, p_n | O_i) \quad (12)$$

where the integrals are performed over all the SME coefficients  $p_k$  except  $p_j$ .

The figure shows that the coefficients of our model are highly correlated, see Fig. 1. We have used the correlation matrix estimator to assess the strength of the parameters correlations, see Table III. These correlations are mainly due to the fact that all planets have very similar, low inclination, orbital plane. Nevertheless, we can produce marginal 1D posterior distribution for each of the eight SME coefficients. The histograms corresponding to these

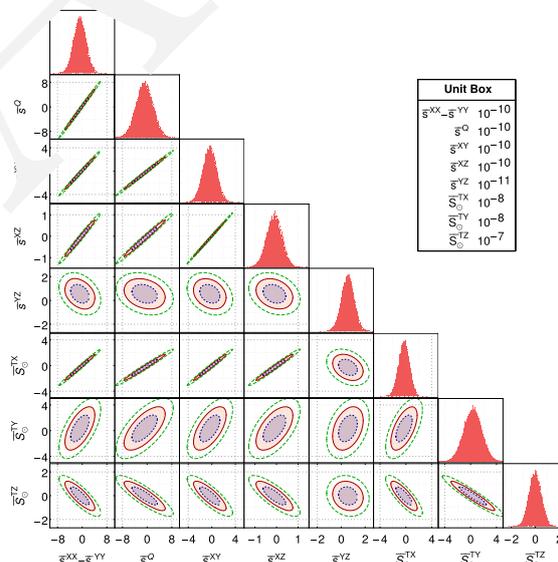


FIG. 1 (color online). 2D marginal posterior pdf (useful to assess the correlations). On the 2D plots, the blue dotted contours represent the 67% Bayesian confidence area, the red continuous contour represent the 95% Bayesian confidence area and the dashed green contours represent the 99.7% Bayesian confidence area. The histograms represent the marginal pdf of the SME coefficients.

TABLE II. Estimations of the SME coefficients. These estimations are still correlated and the correlation matrix is given in Table III. The uncertainties correspond to the 68% Bayesian confidence levels of the marginal pdf.

SME coefficients	Estimation
$\bar{s}^{XX} - \bar{s}^{YY}$	$(-0.8 \pm 2.0) \times 10^{-10}$
$\bar{s}^Q = \bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}$	$(-0.8 \pm 2.7) \times 10^{-10}$
$\bar{s}^{XY}$	$(-0.3 \pm 1.1) \times 10^{-10}$
$\bar{s}^{XZ}$	$(-1.0 \pm 3.5) \times 10^{-11}$
$\bar{s}^{YZ}$	$(5.5 \pm 5.2) \times 10^{-12}$
$\bar{S}_{\odot}^{TX}$	$(-2.9 \pm 8.3) \times 10^{-9}$
$\bar{S}_{\odot}^{TY}$	$(0.3 \pm 1.4) \times 10^{-8}$
$\bar{S}_{\odot}^{TZ}$	$(-0.2 \pm 5.0) \times 10^{-8}$

distributions are presented in Fig. 1. The corresponding Bayesian confidence intervals are presented in Table II.

Another approach (based on the first run) to avoid highly correlated coefficients is to find the independent linear combinations of the SME coefficients that can be determined by planetary ephemerides analysis. This can be done numerically by performing a normalized Cholesky decomposition of the covariance matrix,

$$C = K^T D^2 K,$$

where  $C$  is the covariance matrix of the SME coefficients estimated from our first run,  $K$  is an upper triangular matrix whose diagonal elements are unity and  $D$  is a diagonal matrix. Then the linear combinations  $\mathbf{b}$  of the *fundamental* SME coefficients (noted  $\mathbf{p}$ ) given by

$$\mathbf{b} = K^{-T} \mathbf{p}, \quad (14)$$

with  $K^{-T}$  the inverse of the transpose of  $K$ , can be determined completely independently by the analysis of planetary orbital dynamics. In our case, this Cholesky decomposition ( $K^{-T}$ ) is given by

$$b_1 = (\bar{s}^{XX} - \bar{s}^{YY}), \quad (15a)$$

$$b_2 = -1.37b_1 + \bar{s}^Q, \quad (15b)$$

$$b_3 = 0.15b_1 - 0.31\bar{s}^Q + \bar{s}^{XY}, \quad (15c)$$

$$b_4 = 0.01b_1 + 0.064\bar{s}^Q - 0.4\bar{s}^{XY} + \bar{s}^{XZ}, \quad (15d)$$

$$b_5 = 0.26b_1 - 0.005\bar{s}^Q + 0.51\bar{s}^{XY} - 1.67\bar{s}^{XZ} + \bar{s}^{YZ} \quad (15e)$$

$$b_6 = -35.5b_1 + 9.2\bar{s}^Q - 22.67\bar{s}^{XY} - 33.95\bar{s}^{XZ} + 7.83\bar{s}^{YZ} + \bar{S}_{\odot}^X, \quad (15f)$$

$$b_7 = 1641.4b_1 - 2101.1\bar{s}^Q + 4939.9\bar{s}^{XY} - 8846.8\bar{s}^{XZ} + 4810.6\bar{s}^{YZ} - 0.89\bar{S}_{\odot}^X + \bar{S}_{\odot}^Y, \quad (15g)$$

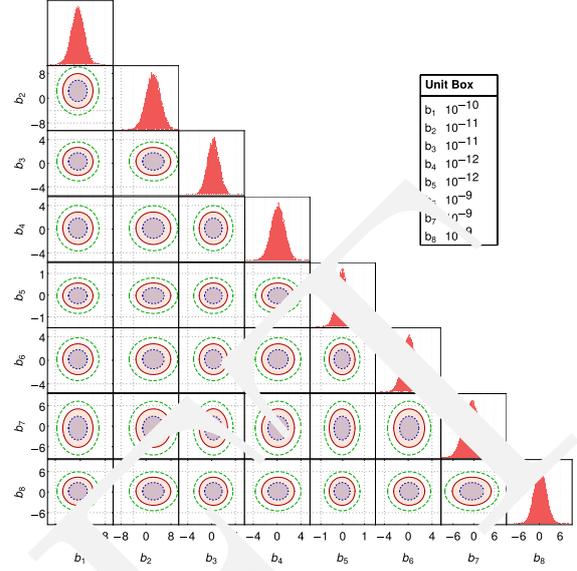


FIG. 2 (color online). 2D marginal posterior pdf (useful to assess the correlation) of the linear combinations  $b_i$  of the SME coefficients given by Eq. (15). On the 2D plots, the blue dotted contours represent the 68% Bayesian confidence area, the red continuous contour represents the 95% Bayesian confidence area and the dashed green contours represent the 99.7% Bayesian confidence area. The 1D histograms represent the marginal pdf of the SME linear combinations  $b_i$ .

$$b_8 = 44.5b_1 + 47.1\bar{s}^Q - 580.1\bar{s}^{XY} + 1041.3\bar{s}^{XZ} + 231.5\bar{s}^{YZ} + 3.43\bar{S}_{\odot}^X + 2.56\bar{S}_{\odot}^Y + \bar{S}_{\odot}^Z, \quad (15h)$$

with the expression of  $\bar{S}_{\odot}^j$  given by Eq. (5). We can now use the linear combinations  $b_i$  as fundamental parameters for our analysis. Performing a new MC run (using the same prior and likelihood as previously), we show that these combinations can be estimated without any correlation. This can be seen in Fig. 2 where the 2D marginal posterior pdf on the  $b_i$  combinations are presented. More quantitatively, the computation of the correlation matrix shows that the  $b_i$  combinations are completely decorrelated by planetary ephemerides analysis since the absolute values of the correlation parameters never exceed 0.03. The 1D posterior pdf of the  $b_i$  combinations are also represented in Fig. 2. The estimated mean and standard deviation are given in Table IV. The obtained uncertainties are much smaller than those given in Table II.

We want to emphasize the fact that the results from both approaches presented above are completely equivalent. They are two ways to represent the same results. One is free to choose which approach is more appropriate: to work with the fundamental SME coefficients determined by Table II at the price of including the covariance matrix

TABLE III. Estimations of the correlations coefficients between the different SME coefficients:  $\bar{s}^{XX} - \bar{s}^{YY}$ ,  $\bar{s}^Q$ ,  $\bar{s}^{XY}$ ,  $\bar{s}^{XZ}$ ,  $\bar{s}^{YZ}$ ,  $\bar{s}_{\odot}^{TX}$ ,  $\bar{s}_{\odot}^{TY}$  and  $\bar{s}_{\odot}^{TZ}$ .

1							
0.99	1						
0.99	0.99	1					
0.98	0.98	0.99	1				
-0.32	-0.24	-0.26	-0.26	1			
0.99	0.98	0.98	0.98	-0.32	1		
0.62	0.67	0.62	0.59	0.36	0.60	1	
-0.83	-0.86	-0.83	-0.81	-0.14	-0.82	-0.95	1

TABLE IV. Estimations of the independent linear combinations  $b_i$  of the SME coefficients. The expressions of the combinations  $b_i$  are given by Eq. (15). The uncertainties correspond to the 68% Bayesian confidence levels of the marginal pdf.

SME linear combinations	Estimation
$b_1$	$(-0.8 \pm 2.0) \times 10^{-10}$
$b_2$	$(2.3 \pm 2.3) \times 10^{-11}$
$b_3$	$(3.0 \pm 9.7) \times 10^{-12}$
$b_4$	$(0.2 \pm 1.1) \times 10^{-12}$
$b_5$	$(-0.3 \pm 2.4) \times 10^{-13}$
$b_6$	$(0.2 \pm 1.1) \times 10^{-9}$
$b_7$	$(-0.6 \pm 2.3) \times 10^{-9}$
$b_8$	$(0.3 \pm 1.7) \times 10^{-9}$

(or equivalently the correlation matrix from Table III) in the analysis or to work with uncorrelated linear combinations of the SME coefficients that are determined by Table IV. The results provided by both approaches describe the same physical information. Therefore, they are completely equivalent.

#### IV. COMBINATION WITH LUNAR LASER RANGING AND ATOM INTERFEROMETRY GRAVIMETRY

It is interesting to compare the results obtained in the last section with constraints available in the literature. In particular, Lunar Laser Ranging (LLR) data have been used to constrain the pure gravity sector of SME [19]. Similarly, atomic gravimetry data have also been used to constrain the  $\bar{s}^{\mu\nu}$  coefficients [2, 21]. We will first combine our results from Sec. III with LLR results to produce constraints on the SME pure gravity sector alone. This will highlight the improvement brought by the planetary ephemerides data. In a second step, we will consider both the pure gravity sector and the gravity-matter couplings coefficients. We will demonstrate that the combination of planetary ephemerides data, LLR data and atom interferometry gravimetry data allows one to completely disentangle all the SME coefficients  $\bar{s}^{\mu\nu}$  and  $(\bar{a}_{\text{eff}}^w)^J$ .

The procedure to combine different types of analysis is standard and consists of performing a global least squares fit of all the estimations available. Obviously, the planetary estimations given by Table II are not independent. To take into account the correlation between the coefficients estimated in Sec. III, we use the parameter covariance matrix from Table III as a weight in the least squares fit. Similarly, the coefficients estimated in the LLR analysis are weighted by their standard deviation in the least squares fit. Since no covariance matrix can be found in the literature, we assume these estimations to be independent (this corresponds to a worst case scenario). Instead of working with results given in Table II that are correlated, we can equivalently use the linear combinations given by Eq. (15) and we then use the estimated standard deviations from Table IV to weight the least squares fit. In that approach, the weight matrix in the fit is diagonal. We insist on the fact that both approaches lead to the same results. In the following we provide the mean and the standard deviation of the SME coefficients as given from the least square fit.

#### Pure gravity sector

First, let us focus on the pure gravity sector alone and neglect the  $(\bar{a}_{\text{eff}}^w)^J$  coefficients. It has been shown in [15] that the main oscillations of the radial distance between the Earth and the Moon due to the  $\bar{s}^{\mu\nu}$  coefficients depend on six independent combinations:  $\bar{s}^{11} - \bar{s}^{22}$ ,  $\bar{s}^{12}$ ,  $\bar{s}^{01}$ ,  $\bar{s}^{02}$ ,  $\bar{s}_{\odot\oplus c}$  and  $\bar{s}_{\odot\oplus s}$ . They can be expressed in terms of the standard SME coefficients expressed in an Earth equatorial frame and in terms of the longitude of the ascending node  $\alpha$  and of the inclination  $\beta$  of the Moon's orbit with respect to the equator. These combinations are given by Eqs. (107) and (108) from [15]. The longitude of the ascending node  $\alpha$  with respect to the equator oscillates around 0. This oscillation is due to the secular advance of the longitude of the ascending node with respect to the ecliptic. Similarly, the inclination of the Moon's orbit with respect to the equator oscillates around  $\beta = 23.44^\circ$ . As a consequence, the transformation of the LLR linear combinations to the standard SME coefficients is given by

$$\bar{s}_{\text{LLR}}^A = \bar{s}^{11} - \bar{s}^{22} = 0.92(\bar{s}^{XX} - \bar{s}^{YY}) + 0.08(\bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}) - 0.73\bar{s}^{YZ}, \quad (16a)$$

$$\bar{s}_{\text{LLR}}^B = \bar{s}^{12} = 0.92\bar{s}^{XY} + 0.40\bar{s}^{XZ}, \quad (16b)$$

$$\bar{s}_{\text{LLR}}^C = \bar{s}^{02} = 0.92\bar{s}^{TY} + 0.40\bar{s}^{TZ}, \quad (16c)$$

$$\bar{s}_{\text{LLR}}^D = \bar{s}^{01} = \bar{s}^{TX}, \quad (16d)$$

$$\bar{s}_{\text{LLR}}^E = \bar{s}_{\odot\oplus c} = -3.21\bar{s}^{TY} - 1.39\bar{s}^{TZ}, \quad (16e)$$

$$\bar{s}_{\text{LLR}}^F = \bar{s}_{\odot\oplus s} = -3.50\bar{s}^{TX}. \quad (16f)$$

TABLE V. Estimations of the SME coefficients derived from LLR analysis from [19].

SME linear combination	Estimation
$\bar{s}_{\text{LLR}}^A$	$(1.3 \pm 0.9) \times 10^{-10}$
$\bar{s}_{\text{LLR}}^B$	$(6.9 \pm 4.5) \times 10^{-11}$
$\bar{s}_{\text{LLR}}^C$	$(-5.2 \pm 4.8) \times 10^{-7}$
$\bar{s}_{\text{LLR}}^D$	$(-0.8 \pm 1.1) \times 10^{-6}$
$\bar{s}_{\text{LLR}}^E$	$(0.2 \pm 3.9) \times 10^{-7}$
$\bar{s}_{\text{LLR}}^F$	$(-1.3 \pm 4.1) \times 10^{-7}$

Note that the above transformations are different from those used in [21]. In that paper, the authors have used  $\alpha = 125^\circ$ , which corresponds to the transformation between the lunar plane and the ecliptic plane at the date J2000 while the reference frame used in the SME framework is the equatorial plane (and not the ecliptic one). Therefore, the value of  $\alpha$  and  $\beta$  needs to be taken with respect to the equatorial plane at the moment where the experiment was performed, or as their average value if they vary during the experiment.<sup>2</sup>

In [19], Battat *et al.* have fitted the amplitudes related to the signature of the six SME combinations (16) on residuals of LLR analysis. As a result, they obtained constraints given in Table V.

Combining these constraints with those obtained in the previous section from planetary ephemerides lead to estimations of the pure gravity SME coefficients given in Table VI. One can see that the  $\bar{s}^{XX} - \bar{s}^{YY}$  and the three coefficients  $\bar{s}^{JK}$  (with  $J \neq K$ ) are improved by the

TABLE VI. Estimated mean and  $1\sigma$  uncertainty of the SME coefficients  $\bar{s}^{\mu\nu}$  by combining planetary ephemerides analysis from Sec. III and LLR analysis [19]. It has been assumed that the  $(\bar{a}_{\text{eff}}^w)^J$  coefficients vanish.

SME coefficients	Estimation
$\bar{s}^{XX} - \bar{s}^{YY}$	$(9.6 \pm 5.6) \times 10^{-11}$
$\bar{s}^Q = \bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}$	$(1.1 \pm 0.78) \times 10^{-10}$
$\bar{s}^{XY}$	$(6.5 \pm 3.2) \times 10^{-11}$
$\bar{s}^{XZ}$	$(2.0 \pm 1.0) \times 10^{-11}$
$\bar{s}^{YZ}$	$(4.1 \pm 5.0) \times 10^{-12}$
$\bar{s}^{TX}$	$(4.3 \pm 2.5) \times 10^{-9}$
$\bar{s}^{TY}$	$(1.1 \pm 1.1) \times 10^{-8}$
$\bar{s}^{TZ}$	$(1.8 \pm 3.0) \times 10^{-8}$

combination of the data. This is mainly due to the fact that the correlations are reduced. It is also worth mentioning that this combined analysis improves the combined LLR and atom interferometry gravimetry analysis from [21] by 2 or more orders of magnitude.

## B. Gravity sector and matter-gravity couplings

In order to use LLR analysis to constrain simultaneously the  $\bar{s}^{\mu\nu}$  and  $(\bar{a}_{\text{eff}}^w)^J$  coefficients, we need to identify the contributions of the  $(\bar{a}_{\text{eff}}^w)^J$  coefficients to the amplitudes of the Earth-Moon distance oscillations. The SME contribution to the Earth-Moon system of motion of the Moon-Earth system can be found in [15,18] and is given by

$$\left. \frac{d^2 r^J}{dt^2} \right|_{\text{SME}} = \frac{G_N M}{r^3} \left[ \bar{s}^{JK} r^K - \frac{3\bar{s}^{KL} r^K r^L r^J}{2r^2} + \left( \bar{s}^{TK} - \sum_{w=e,p,n} \frac{2n_2^w}{3M} \alpha(\bar{a}_{\text{eff}}^w)^K \right) v^K r^J - v^K r^K \bar{s}^{TJ} - v^J \bar{s}^{TK} r^K \right. \\ \left. + \frac{3v^K \bar{s}^{KL} r^K r^J}{r^2} - 2 \frac{\delta m}{M} \left( \bar{s}^{TK} + \sum_{w=e,p,n} \frac{n_2^w}{M} \alpha(\bar{a}_{\text{eff}}^w)^K \right) v^K r^J - 2 \frac{\delta m}{M} \left( \bar{s}^{TJ} + \sum_{w=e,p,n} \frac{n_2^w}{M} \alpha(\bar{a}_{\text{eff}}^w)^J \right) v^K r^K \right], \quad (17)$$

where  $M = m_\zeta + m_\oplus$ ,  $N_\zeta^w = m_\oplus - m_\zeta$ ,  $N_\oplus^w = m_\oplus$ ,  $v^J$  is the position of the Moon with respect to the Earth,  $v^J$  is the relative velocity of the Moon with respect to the Earth,  $V^K$  is the barycentric velocity of the Earth-Moon barycenter and

$$n_2^w = N_\zeta^w - N_\oplus^w, \approx -N_\oplus^w, \quad (18a)$$

$$= \frac{N_\zeta^w}{m_\zeta} + \frac{N_\oplus^w}{m_\oplus}, \quad (18b)$$

<sup>2</sup>Note that [15] advised caution on this point: "For definiteness and to acquire insight, we adopt the values  $\alpha = 125^\circ$  and  $\beta = 23.5^\circ$ . However, these angles vary for the Moon due to comparatively large Newtonian perturbations, so some caution is needed in using the equations that follow".

where  $N_i^w$  is the number of particles of species  $w$  in the body  $i$ . Following the approach described in Appendix A of [15] (see also [46,47]), we expand the equations of motion around a reference circular orbit and perform a Fourier analysis to obtain the contributions of the  $(\bar{a}_{\text{eff}}^w)^J$  terms to the oscillations of the Earth-Moon distance. The term proportional to  $V^K r^J$  in the first line of Eq. (17) leads to an oscillation at the Earth orbital frequency  $\Omega_\oplus$ . The  $(\bar{a}_{\text{eff}}^w)^J$  coefficient modifies the expression of  $\bar{s}_{\Omega_\oplus,1}$  in Eq. (A20) from [15]. Similarly, the modifications of the terms proportional to  $\delta m$  in Eq. (17) change the expression for  $\bar{s}^{01}$  and  $\bar{s}^{02}$ . To summarize, we find that the  $(\bar{a}_{\text{eff}}^w)^J$  coefficients will modify the combinations appearing in LLR oscillations as  $(\bar{s}_{\text{LLR}}^A)$  and  $(\bar{s}_{\text{LLR}}^B)$  being unchanged)

$$\begin{aligned}\bar{s}_{\text{LLR}}^C &= \bar{s}^{02} + \sum_w \frac{n_2^w}{\delta m} \alpha (\bar{a}_{\text{eff}}^w)^2 \\ &\approx \bar{s}^{02} - \sum_w \frac{N_{\oplus}^w}{m_{\oplus}} \alpha (\bar{a}_{\text{eff}}^w)^2,\end{aligned}\quad (19a)$$

$$\begin{aligned}\bar{s}_{\text{LLR}}^D &= \bar{s}^{01} + \sum_w \frac{n_2^w}{\delta m} \alpha (\bar{a}_{\text{eff}}^w)^1 \\ &\approx \bar{s}^{01} - \sum_w \frac{N_{\oplus}^w}{m_{\oplus}} \alpha (\bar{a}_{\text{eff}}^w)^1,\end{aligned}\quad (19b)$$

$$\begin{aligned}\bar{s}_{\text{LLR}}^E &= \bar{s}_{\Omega_{\oplus}c} + 2 \sum_w \frac{n_3^w}{M} (\cos \eta \alpha (\bar{a}_{\text{eff}}^w)^Y + \sin \eta \alpha (\bar{a}_{\text{eff}}^w)^Z) \\ &= \bar{s}_{\Omega_{\oplus}c} + 2 \sum_w \left( \frac{N_{\oplus}^w}{m_{\oplus}} + \frac{N_{\oplus}^w}{m_{\oplus}} \right) (\cos \eta \alpha (\bar{a}_{\text{eff}}^w)^Y \\ &\quad + \sin \eta \alpha (\bar{a}_{\text{eff}}^w)^Z),\end{aligned}\quad (19c)$$

$$\begin{aligned}\bar{s}_{\text{LLR}}^F &= \bar{s}_{\Omega_{\oplus}s} + 2 \sum_w \frac{n_3^w}{M} \alpha (\bar{a}_{\text{eff}}^w)^X \\ &= \bar{s}_{\Omega_{\oplus}s} + 2 \sum_w \left( \frac{N_{\oplus}^w}{m_{\oplus}} + \frac{N_{\oplus}^w}{m_{\oplus}} \right) \alpha (\bar{a}_{\text{eff}}^w)^X,\end{aligned}\quad (19d)$$

where  $\bar{s}_{\Omega_{\oplus}c}$  and  $\bar{s}_{\Omega_{\oplus}s}$  are given by Eq. (108) of [15] or by Eqs. (16e) and (16f). A simple model for the composition of the Earth leads to  $N_{\oplus}^c/m_{\oplus} = N_{\oplus}^p/m_{\oplus} \approx N_{\oplus}^n/m_{\oplus} \approx 0.5(\text{GeV}/c^2)^{-1}$  [18]. Similarly, the model for the composition of the Moon from [18] leads to  $N_{\oplus}^c/m_{\oplus} = N_{\oplus}^p/m_{\oplus} \approx N_{\oplus}^n/m_{\oplus} \approx 0.5(\text{GeV}/c^2)^{-1}$ . Using these values, the combinations (16c)–(16f) appearing in LLR data analysis are modified by the  $(\bar{a}_{\text{eff}}^w)^J$  coefficients as follows:

$$\begin{aligned}\bar{s}_{\text{LLR}}^C &= 0.92(\bar{s}^{TY} - 0.5\alpha(\bar{a}_{\text{eff}}^{e+p})^Y - 0.5\alpha(\bar{a}_{\text{eff}}^n)^Y) \\ &\quad + 0.08(\bar{s}^{TZ} - 0.5\alpha(\bar{a}_{\text{eff}}^{e+p})^Z - 0.5\alpha(\bar{a}_{\text{eff}}^n)^Z),\end{aligned}\quad (20a)$$

$$\bar{s}_{\text{LLR}}^F = \bar{s}^{TX} - 0.5\alpha(\bar{a}_{\text{eff}}^{e+p})^X - 0.5\alpha(\bar{a}_{\text{eff}}^n)^X,\quad (20b)$$

$$\begin{aligned}\bar{s}_{\text{LLR}}^E &= 21\bar{s}^{TY} - 1.39\bar{s}^{TZ} + 1.4\alpha(\bar{a}_{\text{eff}}^{e+p})^Y \\ &\quad + 1.4\alpha(\bar{a}_{\text{eff}}^n)^Y + 0.8\alpha(\bar{a}_{\text{eff}}^{e+p})^Z + 0.8\alpha(\bar{a}_{\text{eff}}^n)^Z,\end{aligned}\quad (20c)$$

$$\bar{s}_{\text{LLR}}^F = -3.50\bar{s}^{TX} + 3.5\alpha(\bar{a}_{\text{eff}}^{e+p})^X + 2\alpha(\bar{a}_{\text{eff}}^n)^X.\quad (20d)$$

Atom interferometry gravimetry has also been used to constrain SME coefficients [20,21]. A violation of Lorentz symmetry induces periodic variations of the local acceleration that can be measured by atom gravimetry. Amplitudes of

these oscillations have been partially computed in [15] for the  $\bar{s}^{\mu\nu}$  coefficients (see Table IV) and in [18] for the  $(\bar{a}_{\text{eff}}^w)^J$  coefficients (see Table IV). An improved calculation shows that the  $(\bar{a}_{\text{eff}}^w)^J$  coefficients modify only two of the amplitudes constrained in [20,21]:

$$\begin{aligned}C_{\omega} &= \frac{i_4}{2} \bar{s}^{XZ} \sin 2\chi - 2 \frac{V_L}{c} i_5 \bar{s}^{TY} \\ &\quad + \frac{4V_L}{3c} \sum_{w=e,p,n} \left[ i_{\oplus} \frac{N_T^w}{m_T} + \frac{3N_{\oplus}^w}{2m_{\oplus}} \right] \alpha (\bar{a}_{\text{eff}}^w)^Y \\ &= \frac{i_4}{2} \bar{s}_{\text{AI}}^A \sin 2\chi,\end{aligned}\quad (21a)$$

$$\begin{aligned}D_{\omega} &= \frac{i_4}{2} \bar{s}^{YZ} \sin 2\chi + 2 \frac{V_L}{c} i_5 \bar{s}^{TX} \\ &\quad - \frac{4V_L}{3c} \sum_{w=e,p,n} \left[ i_{\oplus} \frac{N_T^w}{m_T} + \frac{3N_{\oplus}^w}{2m_{\oplus}} \right] \alpha (\bar{a}_{\text{eff}}^w)^X \\ &= \frac{i_4}{2} \bar{s}_{\text{AI}}^B \sin 2\chi,\end{aligned}\quad (21b)$$

where  $i_{\oplus} = I_{\oplus}/m_{\oplus}R_{\oplus}^2 \approx 1/2$  (with  $I_{\oplus}$  the Earth spherical inertial moment and  $R_{\oplus}$  the Earth radius),  $i_4 = 1 - 3i_{\oplus} \approx -1/2$ ,  $i_5 = 1 - 2i_{\oplus}/3 \approx 4/3$ , the subscripts refer to the test body,  $V = \omega_{\oplus}R_{\oplus} \sin \chi$  is the velocity of the laboratory due to Earth rotation ( $\omega_{\oplus}$  being the angular velocity of Earth rotation) and  $\chi$  is the geographical latitude of the location where the experiment is performed. In the last expressions, we introduced two linear combinations given by

$$\begin{aligned}\bar{s}_{\text{AI}}^A &= \bar{s}^{XZ} - \frac{4}{i_4 \sin 2\chi} \frac{V_L}{c} i_5 \bar{s}^{TY} \\ &\quad + \frac{8V_L}{3c} \frac{1}{i_4 \sin 2\chi} \sum_{w=e,p,n} \left[ i_{\oplus} \frac{N_T^w}{m_T} + \frac{3N_{\oplus}^w}{2m_{\oplus}} \right] \alpha (\bar{a}_{\text{eff}}^w)^Y,\end{aligned}\quad (22a)$$

$$\begin{aligned}\bar{s}_{\text{AI}}^B &= \bar{s}^{YZ} + \frac{4}{i_4 \sin 2\chi} \frac{V_L}{c} i_5 \bar{s}^{TX} \\ &\quad - \frac{8V_L}{3c} \frac{1}{i_4 \sin 2\chi} \sum_{w=e,p,n} \left[ i_{\oplus} \frac{N_T^w}{m_T} + \frac{3N_{\oplus}^w}{2m_{\oplus}} \right] \alpha (\bar{a}_{\text{eff}}^w)^X.\end{aligned}\quad (22b)$$

For the experiment performed by [20,21], we have  $\chi = 42.3^\circ$  and  $V_L/c \approx 1.04 \times 10^{-6}$ . Moreover, numerical estimations for a cesium atom interferometer lead to  $N_{\text{Cs}}^c/m_{\text{Cs}} = N_{\text{Cs}}^p/m_{\text{Cs}} = 0.44(\text{GeV}/c^2)^{-1}$ ,  $N_{\text{Cs}}^n/m_{\text{Cs}} = 0.63(\text{GeV}/c^2)^{-1}$ . Finally, the values for the Earth are given in [18] and are mentioned above after Eq. (19). Using these values gives

$$\begin{aligned}\bar{s}_{\text{AI}}^A &= \bar{s}^{XZ} + 1.12 \times 10^{-5} \bar{s}^{TY} \\ &\quad - 5.43 \times 10^{-6} \alpha (\bar{a}_{\text{eff}}^{e+p})^Y - 5.96 \times 10^{-6} \alpha (\bar{a}_{\text{eff}}^n)^Y,\end{aligned}\quad (23a)$$

TABLE VII. Estimations of the SME coefficients derived from atom interferometry gravimetry by [20,21].

SME linear combination	Estimation
$\bar{s}^{XX} - \bar{s}^{YY}$	$(4.4 \pm 11) \times 10^{-9}$
$\bar{s}^{XY}$	$(0.2 \pm 3.9) \times 10^{-9}$
$\bar{s}_{\text{AI}}^A$	$(-2.6 \pm 4.4) \times 10^{-9}$
$\bar{s}_{\text{AI}}^B$	$(-0.3 \pm 4.5) \times 10^{-9}$
$\bar{s}^{TX}$	$(-3.1 \pm 5.1) \times 10^{-5}$
$\bar{s}^{TY}$	$(0.1 \pm 5.4) \times 10^{-5}$
$\bar{s}^{TZ}$	$(1.4 \pm 6.6) \times 10^{-5}$

$$\bar{s}_{\text{AI}}^B = \bar{s}^{YZ} - 1.12 \times 10^{-5} \bar{s}^{TX} + 5.43 \times 10^{-6} \alpha(\bar{a}_{\text{eff}}^{e+p})^X + 5.96 \times 10^{-6} \alpha(\bar{a}_{\text{eff}}^n)^X, \quad (23b)$$

with  $(\bar{a}_{\text{eff}}^{e+p})^J$  given by Eq. (6).

Therefore, the experiment from [20,21] is sensitive to the last two combinations and not to  $\bar{s}^{XZ}$  and  $\bar{s}^{YZ}$  alone. The results from [21] are presented in Table VII.

In our final analysis, we combine the three analysis with both the  $\bar{s}^{\mu\nu}$  and  $(\bar{a}_{\text{eff}}^w)^J$  coefficients: (i) planetary ephemerides analysis given by Table II with the correlation matrix from Table III [or equivalently the results from Table II on the linear combinations given by Eqs. (16a), (16b), and (20) and (ii) LLR data analysis from [19] summarized in Table IV with linear combinations given by Eqs. (16a), (16b), and (20) and (iii) atom interferometry gravimetry analysis from [20,21] presented in Table VII with the linear combination given by Eq. (23). The (marginalized) results of this fit are presented in Table VIII.

The resulting estimations do not show any significant deviations from GR. The combinations of the three data analyses allow one to estimate each of the coefficients

TABLE VIII. Estimated mean and  $1\sigma$  uncertainty on the SME coefficients obtained with a fit combining results from Solar System LLR data analysis from [19] and atom interferometry gravimetry experiment [20,21].

SME coefficients	Estimation
$\bar{s}^{XX} - \bar{s}^{YY}$	$(9.6 \pm 5.6) \times 10^{-11}$
$\bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}$	$(1.6 \pm 0.7) \times 10^{-10}$
$\bar{s}^{XY}$	$(6.5 \pm 3.2) \times 10^{-11}$
$\bar{s}^{XZ}$	$(2.0 \pm 1.0) \times 10^{-11}$
$\bar{s}^{YZ}$	$(4.1 \pm 5.0) \times 10^{-12}$
$\bar{s}^{TX}$	$(-7.4 \pm 8.7) \times 10^{-6}$
$\bar{s}^{TY}$	$(-0.8 \pm 2.5) \times 10^{-5}$
$\bar{s}^{TZ}$	$(0.8 \pm 5.8) \times 10^{-5}$
$\alpha(\bar{a}_{\text{eff}}^e)^X + \alpha(\bar{a}_{\text{eff}}^p)^X$	$(-7.6 \pm 9.0) \times 10^{-6} \text{ GeV}/c^2$
$\alpha(\bar{a}_{\text{eff}}^e)^Y + \alpha(\bar{a}_{\text{eff}}^p)^Y$	$(-6.2 \pm 9.5) \times 10^{-5} \text{ GeV}/c^2$
$\alpha(\bar{a}_{\text{eff}}^e)^Z + \alpha(\bar{a}_{\text{eff}}^p)^Z$	$(1.3 \pm 2.2) \times 10^{-4} \text{ GeV}/c^2$
$\alpha(\bar{a}_{\text{eff}}^n)^X$	$(-5.4 \pm 6.3) \times 10^{-6} \text{ GeV}/c^2$
$\alpha(\bar{a}_{\text{eff}}^n)^Y$	$(4.8 \pm 8.2) \times 10^{-4} \text{ GeV}/c^2$
$\alpha(\bar{a}_{\text{eff}}^n)^Z$	$(-1.1 \pm 1.9) \times 10^{-3} \text{ GeV}/c^2$

individually. The spatial part of  $\bar{s}^{JK}$  is completely determined by the combination of planetary ephemerides and LLR data. The atom interferometry gravimetry is not accurate enough to provide any significant improvement on the uncertainty of these coefficients. With an improvement of 2 orders of magnitude, the atom gravimetry data would become significant to estimate the  $\bar{s}^{JK}$  coefficients. On the other hand, the three data sets are required in order to decorrelate the  $\bar{s}^{TJ}$  and the  $(\bar{a}_{\text{eff}}^w)^J$  coefficients. The uncertainties on  $\bar{s}^{TJ}$  are much larger than those shown in Table VI where the coefficients  $\alpha(\bar{a}_{\text{eff}}^w)^J$  have been neglected. This reflects the fact that the individual coefficients are still highly correlated.

## V. DISCUSSION

First of all, the accuracy of the constraints on the SME coefficients obtained in Table II (planetary orbital dynamics alone) are of the same order of magnitude as the binary pulsars [19] constraints on the SME coefficients with an improvement of 1 order of magnitude on the coefficients  $\bar{s}^{YZ}$ . Nevertheless, it is known that nonperturbative effects (similar to those computed in [49]) may arise in binary pulsar systems. The nonperturbative effects depend highly on the fundamental theory (for example, see [50] for nonperturbative calculation in Einstein-Aether theory or in Lorentz gravity). In general, the results from [25] are effective constraints on the strong field version of the  $\bar{s}^{\mu\nu}$  that may include nonperturbative strong field effects and one should be careful when comparing strong field tests and weak field tests as the one performed in Sec. III. The results shown in Table IV improve the current Solar System constraints [32] by 1 to 3 orders of magnitude. Furthermore, the analysis combining planetary orbital dynamics and LLR from Table VI improves by 2 to 3 orders of magnitude the previous results that combined LLR and atom interferometry. This shows the high impact provided by planetary ephemerides analysis.

As mentioned in Sec. III, our results show that the estimated SME coefficients are highly correlated. The correlations are due to the similarity of the orbital planes of all the planets. Therefore, one way to improve the results by reducing the correlations is to use bodies with different orbital planes like e.g. asteroids. This can be achieved for example with Gaia observations similar to what is proposed in [51].

The constraints obtained in Sec. III are mainly due to the internal planets. For instance, Jupiter has absolutely no influence on the results shown in Table II. This is a consequence of its not so well-known orbit. An improvement by a factor 10 on the knowledge of Jupiter's orbit is required for that planet to play a significant role in this analysis. Therefore, the improvement of Jupiter's trajectory expected from the analysis of Juno's radioscience and very long baseline interferometry data [52] may improve the

result of our analysis. In particular, it will reduce some of the correlations which will lead to an improvement of the estimations of the SME coefficients. In the same spirit, the influence of Saturn is weak but nevertheless highly important to decorrelate the coefficients. Furthermore, an improvement of Mercury's orbit by a factor 10 (which can be regarded as the improvement by Messenger's data that are not yet included in INPOP10a analysis [33]) will lead to an improvement on the estimations of  $\bar{s}^{YZ}$  by a factor 2 and to a 10% improvement on the coefficients  $\bar{s}_{\odot}^{TY}$  and  $\bar{s}_{\odot}^{TZ}$  (but to no improvement at all on the other coefficients). In summary, the best way to improve the current analysis is to improve the trajectory of the "badly" determined planetary orbits in order to improve the decorrelation instead of improving more the planets that are already very well determined.

As mentioned in Sec. II, the influence of the  $\bar{s}^{TT}$  and the  $(\bar{a}_{\text{eff}}^w)^T$  coefficients on the orbital dynamics only appears at the next post-Newtonian order and these coefficients are therefore not constrained by our analysis. Nevertheless, these coefficients will play an important role in the light propagation [53,54]. Therefore, planetary ephemerides may potentially constrain this coefficient by considering the effect of  $\bar{s}^{TT}$  on the light time of the radioscience Range observables used in the analysis. Other opportunities to constrain this coefficient are to consider a conjunction experiment like the one performed with the Cassini spacecraft [3] (or to analyze Cassini data within the SME formalism as proposed in [55]) or to consider Very Long Baseline Interferometry observations similar to what has been done for the  $\gamma$  post-Newtonian parameter [5].

The multiplication of the numbers of SME coefficients that need to be considered leads to an increase in the uncertainties on each individual coefficient. This is due to the correlations between the different coefficients that appear when their number is increased. Therefore, it is highly important to increase the number of analyses to constrain SME. In this communication, we have shown how a combination of these analyses can disentangle the different coefficients. Nevertheless, the correlations shown in Table VIII are still highly correlated, especially in the  $(\bar{c}^{\mu\nu})^J$  sector. One way to reduce these correlations is to use more observations that are sensitive to other combinations of the  $(\bar{c}^{\mu\nu})^J$  coefficients. This can be done in two ways: (i) to consider different source bodies that generate the gravitational field and (ii) to use more orbital geometry like e.g. asteroids dynamics as already mentioned. The first point is related to the fact that the  $(\bar{a}_{\text{eff}}^w)^J$  coefficients enter the equations of motion essentially through the properties of the source body. In this communication, only two source bodies have been used: the Sun (in the planetary orbital dynamics analysis) and the Earth (in LLR and in atom interferometry gravimetry). Considering more source bodies with different compositions can help to reduce

correlations. In this sense, a test using the satellites around the different planets would be highly relevant.

Finally, we would like to soften the results presented here. First of all, we insist on the fact that the constraints obtained in Sec. III correspond to the intervals in which the differences of INPOP10a postfit residuals are below 5%, as they are obtained directly from the limits of Table I coming from [33]. As such, they do not directly represent the usual  $1\sigma$  confidence interval. A cleaner approach would be to include the SME equations of motion directly in the planetary ephemerides software used to estimate the SME coefficients directly from the raw data which corresponds to the approach usually used for estimating the PPN coefficients [4,6,7] or more recently to constrain the Modified Newtonian Dynamics theory [56]. Our analysis demonstrates the impact of such an analysis and therefore, provides a strong incentive.

In addition, the LLR data analysis has been performed by fitting some oscillating signatures in the LLR data residuals. This approach is not optimal since it suffers from two drawbacks. First, the oscillating signatures derived in [15] have been computed analytically using several approximations. They can be used to estimate an order of magnitude on the different effects produced by SME but they are not optimal for a real data analysis (furthermore, the signatures used in [15] include only the dominant oscillations, several other frequencies are produced by SME and ignored in the data analysis). Second, fitting in the residuals is not optimal since it does not allow one to analyze the correlations between the SME coefficients and the other parameters that are usually fitted in a standard LLR data analysis. For these reasons, a cleaner analysis would include the SME equations of motion directly in the software used to reduce LLR data. Results obtained in [19] and in this communication give strong motivations to perform such an analysis.

Finally, the atom interferometry gravimetry analysis should be interpreted with caution. The atom interferometry gravimeter results from [20,21] assume a model of the local solid Earth tides. While such models can be partly analytically based, it is known that the many frequencies of the Earth tides include all of the frequencies in the SME signal [57]. If any aspect of the tidal model includes fitting sinusoidal functions to local gravimetry measurements or global measurements of, for example, the ocean heights [58], the signal for the SME may be partly subtracted due to the strong correlation with the tidal signal.

## VI. CONCLUSION

In this communication, we have shown that the planetary orbital dynamics allow one to constrain a violation of Lorentz symmetry with an impressive accuracy. In Sec. III, we use the current limits on supplementary advances of perihelia and nodes provided by INPOP10a [33] to estimate the SME coefficients  $\bar{s}^{\mu\nu}$  and  $(\bar{a}_{\text{eff}}^w)^J$ . In this analysis, the coefficients  $\bar{c}^{\mu\nu}$  have been neglected since they are already

constrained with a high level of accuracy [32] but they can be considered in a future work. Our analysis has been performed using a standard Bayesian inversion. Results on the SME coefficients are given in Table II. No significative deviation from GR is observed. As mentioned in Sec. III, these estimations are highly correlated (see Table III or Fig. 1). We have identified numerically the linear combinations of the SME coefficients that can be estimated independently from planetary ephemerides. The estimations on these combinations are given in Table IV. These two results are completely equivalent (as long as one uses the correlation matrix with the first estimation). Our results produce uncertainties similar to those obtained from binary pulsars data [25] on most of the coefficients and improve the constraints on  $\bar{s}^{YZ}$  by 1 order of magnitude. Moreover, we improve the current best weak field tests by 2 to 3 orders of magnitude.

We also perform a combined estimation of the SME coefficients using results from three different analyses: (i) the planetary ephemerides analysis performed in Sec. III, (ii) the LLR data analysis performed in [19] and (iii) the atom interferometry gravimetry analysis realized in [20,21]. The combination of LLR and planetary ephemerides leads to the best current estimations on the pure gravity SME coefficients as shown in Table VI [when neglecting

the  $(\bar{a}_{\text{eff}}^{w})^J$  coefficients]. In these three analyses, we also take into account potential effects produced by a Lorentz violation in the matter-gravity coupling which is parametrized by the  $(\bar{a}_{\text{eff}}^{w})^J$  coefficients. Finally, the combinations of the results from the three data analyses lead to the first independent estimations of the  $\bar{s}^{\mu\nu}$  and  $(\bar{a}_{\text{eff}}^{w})^J$  coefficients. The results are presented in Table VIII. The obtained uncertainties are relatively large, which is due to the numbers of coefficients considered and to the remaining correlations. Some ideas to reduce these correlations are proposed in Sec. IV.

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# Lorentz symmetry and Very Long Baseline Interferometry

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Lorentz symmetry violations can be described by an effective field theory framework that contains both General Relativity and the Standard Model of particle physics called the Standard-Model extension (SME). Recently, post-fit analysis of Gravity Probe B and binary pulsars lead to an upper limit at the  $10^{-4}$  level on the time-time coefficient  $\bar{s}^{TT}$  of the pure-gravity sector of the minimal SME. In this work, we derive the observable of Very Long Baseline Interferometry (VLBI) in SME and then we implement it into a real data analysis code of geodetic VLBI observations. Analyzing all available observations recorded since 1979, we compare estimates of  $\bar{s}^{TT}$  and errors obtained with various analysis schemes, including global estimations over several time spans and with various Sun elongation cut-off angles, and with analysis of radio source coordinate time series. We obtain a constraint on  $\bar{s}^{TT} = (-5 \pm 8) \times 10^{-5}$ , directly fitted to the observations and improving by a factor 5 previous post-fit analysis estimates.

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Historically, the measurement of the bending of light due to the gravitational mass of the Sun is one of the most important and precise test of General Relativity (GR). Within the Parameterized Post-Newtonian (PPN) formalism [1], this effect has been constrained by Very Long Baseline Interferometry (VLBI) observations [2, 3], space astrometry with Hipparcos [4] and the Cassini radio-science experiment [5], the latter being the most stringent constraint on the PPN  $\gamma$  parameter.

The SME framework has been developed to be an extensive formalism that allows a systematic description of Lorentz symmetry violations in all sectors of physics, including gravity [6–8]. The motivations came first from string theory [9, 10] which can possibly produce Lorentz violations, this statement appears also in loop quantum gravity, non commutative field theory and others [11, 12].

A hypothetical Lorentz violation in the gravitational sector naturally leads to an expansion at the level of the action [8, 13] which in the minimal SME writes

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} (R - u_{\mu\nu} + s^{\mu\nu} R_{\mu\nu}^T - t^{\alpha\beta\mu\nu} C_{\alpha\beta\mu\nu}) + S'[s^{\mu\nu}, t^{\alpha\beta\mu\nu}, g_{\mu\nu}], \quad (1)$$

with  $G$  the gravitational constant,  $g$  the determinant of the space-time metric  $g_{\mu\nu}$ ,  $R$  the Ricci scalar,  $R_{\mu\nu}^T$  the trace-free Ricci tensor,  $C_{\alpha\beta\mu\nu}$  the Weyl tensor and  $u_{\mu\nu}$ ,  $s^{\mu\nu}$  and  $t^{\alpha\beta\mu\nu}$  the Lorentz violating fields. To avoid conflicts with the underlying Riemann geometry, we assume spontaneous symmetry breaking so that the Lorentz violating coefficients need to be considered as dynamical fields [13]. The last part of the action  $S'$  contains the dynamical terms governing the evolution of the SME coefficients. In the linearized gravity limit, the metric depends only on  $\bar{u}$  and  $\bar{s}^{\mu\nu}$  which are the vacuum expect-

ation value of  $u_{\mu\nu}$  and  $s^{\mu\nu}$  [13]. The coefficient  $\bar{u}$  is unobservable since it can be absorbed in a rescaling of the gravitational constant. The  $\bar{s}^{\mu\nu}$  obtained post-Newtonian metric differs from the  $\bar{s}^{\mu\nu}$  introduced in the PPN formalism [13]. In addition to Lorentz symmetry violations in the pure-gravity sector, violations of Lorentz symmetry can also come from gravity-matter couplings [14], but we do not consider them in this work. Hence SME is an effective field theory making possible confrontations of fundamental theories and experiments. Indeed, since the last decade, several studies aimed to find upper limit on SME coefficients by searching possible signals in post-fit residuals of experiments. This was done for pure-gravity SME coefficients with Lunar Laser Ranging [15], atom interferometry [16], Gravity probe B [17], binary pulsars [18], Solar System planetary motions [19, 20], cosmic ray observations [21] or even very recently with gravitational waves detection [22]. However, all these works are post-fit analysis based originally on pure GR and consequently their approach is not fully satisfactory in the sense that correlations in the determination of SME coefficients and other global parameters (masses, position and velocity...) can not be assessed. Then in the best case, a simple modeling of extra terms containing SME coefficients are least square fitted in the residuals of the experiment. In a more correct approach, SME modeling must be included in the complete data analysis and its coefficients must be determined as global parameters. It is exactly what we present here in the case of VLBI observations.

VLBI is a geometric technique which measures the time difference in the arrival of a radio wavefront emitted by a distant radio source (typically a quasar) between at least two Earth-based radio telescopes, with a preci-

sion of a few picoseconds. Knowing the group delay and the angular separation between the baseline between the antennas of the telescopes and the line of sight of the observation, the distance between the telescopes can be determined and consequently VLBI tracks the orientation of the Earth in an inertial reference frame provided by the very distant quasars, determining accurate terrestrial and celestial reference frames.

Let us write the VLBI group delay in the International Celestial Reference Frame (ICRF) as defined by the International Astronomical Union (IAU) [23] with coordinates  $(x^\mu) = (x^T, \mathbf{x})$ , where  $x^T = ct$ ,  $t$  being a time coordinate, and  $\mathbf{x} = (x^I)$  is the spatial position. We consider a quasar as source with as coordinates of the emission event  $(t_e, \mathbf{x}_e)$ . This signal is received by two different VLBI stations at events  $(t_1, \mathbf{x}_1)$  and  $(t_2, \mathbf{x}_2)$ , respectively. Using the same notations as in [24], we introduce three unit vectors

$$\mathbf{k} = \frac{\mathbf{x}_e}{|\mathbf{x}_e|}, \quad \mathbf{n}_{ij} \equiv \frac{\mathbf{x}_{ij}}{r_{ij}} = \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_{ij}|}, \quad \text{and} \quad \mathbf{n}_i = \frac{\mathbf{x}_i}{|\mathbf{x}_i|}. \quad (2)$$

We denote by  $t_r - t_e = \mathcal{T}(\mathbf{x}_e, t_e, \mathbf{x}_r)$  the coordinate propagation time of a photon between an emission event whose coordinates are given by  $(t_e, \mathbf{x}_e)$  and a reception event whose coordinates are given by  $(t_r, \mathbf{x}_r)$ . We consider simply the VLBI group delay  $\Delta\tau$  from

$$\Delta\tau(\mathbf{x}_e, t_e, \mathbf{x}_1, \mathbf{x}_2) = \mathcal{T}(\mathbf{x}_e, t_e, \mathbf{x}_2) - \mathcal{T}(\mathbf{x}_e, t_e, \mathbf{x}_1). \quad (3)$$

For the observation of a quasar, we then use the limit

$$\begin{aligned} \Delta\tau_{(\text{grav})}(\mathbf{k}, \mathbf{x}_1, \mathbf{x}_2) &= 2 \frac{GM}{c^3} [1 + \bar{s}^T + \bar{s}^{TJ} k^J] \ln \frac{r_1 + \mathbf{k} \cdot \mathbf{x}_1}{r_2 + \mathbf{k} \cdot \mathbf{x}_2} \\ &\quad + \frac{GM}{3} [\bar{s}^{TT} + \bar{s}^{JJ} n_{er}^J] (\mathbf{n}_2 \cdot \mathbf{k} - \mathbf{n}_1 \cdot \mathbf{k}) \\ &\quad + \frac{GM}{c^3} [\bar{s}^{TJ} + \bar{s}^{JK} k^K] (\mathbf{n}_2 - \mathbf{n}_1)^J + \frac{GM}{c^3} [\bar{s}^{JK} \hat{p}_1^J \hat{p}_1^K (\mathbf{n}_1 \cdot \mathbf{k} - 1) - \bar{s}^{JK} \hat{p}_2^J \hat{p}_2^K (\mathbf{n}_2 \cdot \mathbf{k} - 1)], \end{aligned} \quad (7)$$

where the subscript (grav) refers to the gravitational part of the group delay and where

$$\mathbf{p}_i = \mathbf{k} \times (\mathbf{x}_i \times \mathbf{k}) = \mathbf{x}_i - (\mathbf{k} \cdot \mathbf{x}_i) \mathbf{k}, \quad (8)$$

and  $\hat{p}_i = \frac{\mathbf{p}_i}{|\mathbf{p}_i|}$ . Moreover, a simplified formula can be used for practical utilisation considering a typical accuracy of a VLBI observation of the order of 10 ps and that  $GM/c^3 \sim 5 \times 10^{-6}$  s. Since the coefficients  $\bar{s}^{TJ}$  are already constrained and are smaller than  $\sim 10^{-7}$  [20, 28], all terms  $GM/c^3 \bar{s}^{TJ}$  are too small to be detected and can be neglected. The coefficients  $\bar{s}^{IJ}$  with  $I \neq J$  are also constrained by previous studies and are smaller than  $10^{-10}$  [18, 20, 28]. Therefore, we can also neglect terms that are proportional to  $GM/c^3 \bar{s}^{IJ}$  with  $I \neq J$ .

$r_e \equiv |\mathbf{x}_e| \rightarrow \infty$  and the VLBI time delay is given by

$$\Delta\tau(\mathbf{k}, \mathbf{x}_1, \mathbf{x}_2) = \lim_{r_e \rightarrow \infty} [\mathcal{T}(\mathbf{x}_e, t_e, \mathbf{x}_2) - \mathcal{T}(\mathbf{x}_e, t_e, \mathbf{x}_1)]. \quad (4)$$

The coordinate propagation time can be computed from the linearized SME metric from [13, 25] using the time transfer functions formalism [26]. In SME, it has been computed in [27] (see Eq. (2.1)) for the pure gravity sector and is given by

$$\begin{aligned} \mathcal{T}(\mathbf{x}_e, t_e, \mathbf{x}_r) &= \frac{r_{er}}{c} + \frac{GM}{c^3} \left( \bar{s}^{TJ} p_{er}^J - \bar{s}^{JK} n_{er}^J p_{er}^K \right) \frac{r_e - r_r}{r_e r_r} \\ &\quad + 2 \frac{GM}{c^3} [1 + \bar{s}^{TT} + \bar{s}^{TJ} n_{er}^J] \ln \frac{r_e - \mathbf{x}_e \cdot \mathbf{n}_{er}}{r_r - \mathbf{x}_e \cdot \mathbf{n}_{er}} \\ &\quad + \frac{GM}{c^3} [\bar{s}^{TJ} n_{er}^J + \bar{s}^{JK} \hat{p}_{er}^J \hat{p}_{er}^K - \bar{s}^{TT}] (\mathbf{n}_r \cdot \mathbf{n}_{er} - \mathbf{n}_e \cdot \mathbf{n}_{er}) \end{aligned} \quad (5)$$

where the terms  $a_1$  and  $a_2$  from [27] are taken as unity (which corresponds to using the harmonic gauge, which is the one used for VLBI data reduction) and where

$$\begin{aligned} \mathbf{p}_{er} &= \mathbf{n}_e \times (\mathbf{x}_r \times \mathbf{n}_{er}) = \mathbf{x}_r - (\mathbf{n}_{er} \cdot \mathbf{x}_r) \mathbf{n}_{er} \\ &= \mathbf{n}_{er} \times (\mathbf{x}_e \times \mathbf{n}_{er}) = \mathbf{x}_e - (\mathbf{n}_{er} \cdot \mathbf{x}_e) \mathbf{n}_{er}, \end{aligned} \quad (6)$$

and where  $\hat{p}_{er} = \frac{\mathbf{p}_{er}}{|\mathbf{p}_{er}|}$ .

We can now give the expression of the group delay between two VLBI stations. We are using the assumptions that the sources are located at infinity ( $r_e \rightarrow \infty$ ). We need to introduce (5) into (4), which leads to

Finally, since we know that  $|\bar{s}^{XX} - \bar{s}^{YY}| < 10^{-10}$  and  $|\bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}| < 10^{-10}$  [28], we can safely say that at the level of accuracy required  $\bar{s}^{XX} \approx \bar{s}^{YY} \approx \bar{s}^{ZZ}$  in Eq. (7). Under these assumptions and using the fact that  $\bar{s}^{\mu\nu}$  is traceless, the VLBI group delay can be written

$$\begin{aligned} \Delta\tau_{(\text{grav})} &= 2 \frac{GM}{c^3} (1 + \bar{s}^{TT}) \ln \frac{r_1 + \mathbf{k} \cdot \mathbf{x}_1}{r_2 + \mathbf{k} \cdot \mathbf{x}_2} \\ &\quad + \frac{2}{3} \frac{GM}{c^3} \bar{s}^{TT} (\mathbf{n}_2 \cdot \mathbf{k} - \mathbf{n}_1 \cdot \mathbf{k}). \end{aligned} \quad (9)$$

It is important to notice that the bare  $GM$  parameter appearing in the post-Newtonian metric do not correspond to the observed  $\widehat{GM}$  parameter measured with orbital dynamics (using planetary motion for the Sun). There is a rescaling between the two parameters given by

$\widetilde{GM} = GM(1 + 5/3\bar{s}^{TT})$  (see Sec. IV of [17] or [13, 20]). Using the observed mass parameter leads to

$$\Delta\tau_{(\text{grav})} = 2\frac{\widetilde{GM}}{c^3}\left(1 - \frac{2}{3}\bar{s}^{TT}\right)\ln\frac{r_1 + \mathbf{k}\cdot\mathbf{x}_1}{r_2 + \mathbf{k}\cdot\mathbf{x}_2} + \frac{2\widetilde{GM}}{3c^3}\bar{s}^{TT}(\mathbf{n}_2\cdot\mathbf{k} - \mathbf{n}_1\cdot\mathbf{k}). \quad (10)$$

This last formula is the one used to fit the  $\bar{s}^{TT}$  coefficient using VLBI observations.

From August 1979 to mid-2015, almost 6000 VLBI 24-hr sessions (correspondingly 10 million delays) have been scheduled for primary goal of monitoring the Earth's rotation and determining reference frames. The International VLBI Service for Geodesy and Astrometry (IVS) [29]<sup>1</sup> imposed a minimal distance to the Sun of  $15^\circ$  after 2002 in order to avoid potential degradation of geodetic products due to radio wave crossing of the Solar corona. This limit was recently removed (Fig. 1).

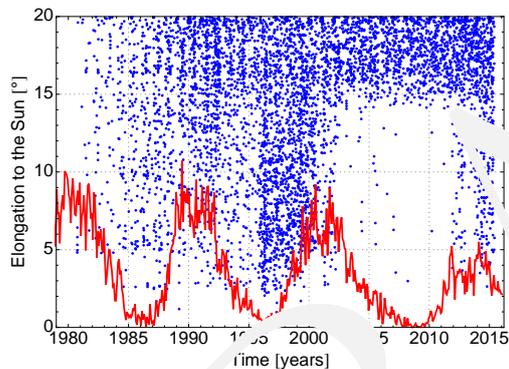


FIG. 1. Observational history of the sources closer than  $20^\circ$  to the Sun (blue dots) and minimum solar separation number (red curve, rescaled to fit in the plot; [30]).

In our analysis, the VLBI delays were corrected from delay due to the radio wave crossing of the conservative regions in the signal propagation path in a preliminary step that include use of 2 GHz and 8 GHz recordings. Then, we only used the 8 GHz delays to fit the parameters listed hereafter. We used the Calc/Solve geodetic VLBI analysis software developed at NASA Goddard Space Flight Center, in which the astrometric modelling of VLBI time delay is compliant with the latest standards of the International Earth Rotation and Reference Systems Service (IERS) [31]. We added the partial derivative of the VLBI delay with respect to  $\bar{s}^{TT}$  from Eq. (10) to the software package using the USERPART module of Calc/Solve.

We ran a first solution in which we estimated  $\bar{s}^{TT}$ , all source and station coordinates and all five Earth orientation parameters once per session. A priori zenith delays were determined from local pressure values [32], which were then mapped to the elevation of the observation using the Vienna mapping function [33]. Wet zenith delays and clock drifts were estimated at intervals of ten and thirty minutes, respectively. Troposphere gradients were estimated at intervals of 6 hours. Suitable loose constraints were applied to source and station coordinates to avoid global rotation of the celestial frame and global rotation and translation of the terrestrial frame. Sites undergoing strong non-linear motions (due to, e.g., post-seismic relaxation), were excluded from the constraint. This preliminary solution allowed us to identify a half-dozen of sessions with abnormally high post-residuals (generally higher than 1 ns). The distribution  $\bar{s}^{TT}$  estimated by its error also reveals a few points clearly lying outside the distribution (see Fig. 2). These data corresponds to the 26 sessions of the CONT08 campaign (August 2008), representing 0.41% of the dataset. Without the CONT08 sessions, we obtained  $\bar{s}^{TT} = (-5 \pm 11) \times 10^{-5}$ . Keeping the CONT08 sessions moves the mean value to  $7 \times 10^{-5}$ .

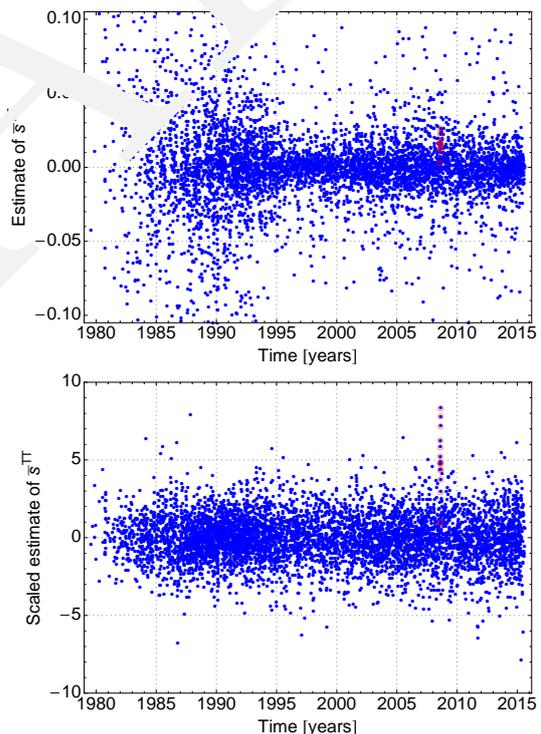


FIG. 2. Session-wise estimates of  $\bar{s}^{TT}$  (Top) and  $\bar{s}^{TT}$  scaled by its error (Bottom) for 5895 sessions (blue dots). The red circles highlight the 26 CONT08 sessions.

<sup>1</sup> The IVS operates regular geodetic VLBI since 1998.

A spectral analysis of the time series revealed no significant peak. We computed  $\bar{s}^{TT}$  over 1000 random subsets containing three quarters of the 5895 sessions to check the stability of the mean value.  $\bar{s}^{TT}$  stays around 0 within  $8 \times 10^{-5}$ . We also addressed the sensitivity to Solar activity. To do so, we used the Sun spot number (SSN) monthly data to separate VLBI sessions into two groups: each group contains sessions occurring when the SSN is higher or lower than its median value computed over our observational time span, that is 2947 sessions in each group. We obtained  $\bar{s}^{TT} = (3 \pm 16) \times 10^{-5}$  for the high activity periods, and  $\bar{s}^{TT} = (-12 \pm 15) \times 10^{-5}$  for low activity period, giving no clue on the influence of Solar activity.

We turned to a global solution in which we estimated  $\bar{s}^{TT}$  as a global parameter together with radio source coordinates. Station coordinates were left as session parameters. Constraints remained unchanged. We obtained  $\bar{s}^{TT} = (-5 \pm 8) \times 10^{-5}$ , with a global postfit rms of 28 ps and a  $\chi^2$  per degree of freedom of 1.15. Correlations between radio source coordinates and  $\bar{s}^{TT}$  remain lower than 0.02. The global estimate is consistent with the mean value obtained with the session-wise solution with a slightly lower error.

In this letter, we have presented a test of Lorentz symmetry performed using 36 years of VLBI data. Contrary to previous studies of Lorentz symmetry in the gravity sector, our work is not based on a post-fit analysis on residuals obtained after a GR analysis but rather on a full SME modelling in the VLBI data reduction process. Our analysis leads to a constraint on the  $\bar{s}^{TT}$  coefficient at the level of  $10^{-5}$ . This coefficient is particularly important since it controls the speed of gravity in the SME framework [22]. Our result improves the best current constraint on this coefficient [17, 18] by a factor of five. In the future, the accumulation of VLBI data in a framework of the permanent geodesy monitoring program will let us expect improvements of this constraint as well as extended tests.

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# Testing Lorentz symmetry with Lunar Laser Ranging

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Lorentz symmetry violations can be parametrized by an effective field theory framework that contains both General Relativity (GR) and the Standard-Model of particle physics called the Standard-Model-Extension (SME). We present new constraints on SME parameters of the pure gravity sector, obtained analysing Lunar Laser Ranging (LLR) normal points. We have built a new numerical lunar ephemeris computed in the SME framework and we have performed a LLR data analysis using a set of 20603 normal points extended from August 1969 to December 2013. We found no evidence for Lorentz violation at the level of  $10^{-8}$ ,  $10^{-12}$ ,  $10^{-12}$ ,  $10^{-11}$ ,  $10^{-9}$ ,  $10^{-10}$ ,  $10^{-10}$  and  $10^{-10}$ , respectively  $\bar{s}^{TX}$ ,  $\bar{s}^{XY}$ ,  $\bar{s}^{XZ}$ ,  $\bar{s}^A$ ,  $\bar{s}^C$ ,  $\bar{s}^D$ ,  $\bar{s}^E$  and  $\bar{s}^F$ . Four parameters estimates are improved until three order of magnitude compared to previous post-fit residuals analysis conducted on theoretical grounds. This work is the first LLR experimental measurement computed in SME framework.

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Since his establishment in 1915 by Einstein, GR has survived one century of experimental and theoretical scrutiny. During this century, foundations of the theory have been tested spanning all scales, from the Solar System to the edge of the early Universe. Current Solar System remains the most precise opened laboratory to put constraints on GR. Those constraints can only be computed in an extended framework parametrising deviations to GR. In the past decades, two frameworks were widely used in the literature at the scale of the Solar System, namely the Parametrized Post-Newtonian (PPN) [1] and the fifth force formalism [2]. However, some motivations are given to look for deviations to GR in other frameworks than the two extensively considered. One of those is the SME framework [3–5] aiming to systematically parametrized any violation of the Lorentz symmetry in all sectors of physics from particles physics to gravity.

Lorentz symmetry or Local Lorentz Invariance (LLI) is a fundamental principle of space-time following from the special theory of relativity applied from particles physics to GR. It states that the outcome of any local test experiment (gravitational or not) is independent of the velocity of the local freely falling frame in which the experiment is realized [1]. Considering the broad field of applicability of the LLR testing Lorentz symmetry violations provide a powerful tool of fundamental physics. In addition, many scenarios in the literature expect some Lorentz violation at different energy level. String theory, loop quantum gravity and non-commutative fields theory are some examples of such scenarios [6, 7]. In this context, Colladay and Kostelecký have built an effective fields theory making possible confrontations between fundamental theories and experiments called the SME.

Following from [5, 8], an hypothetical violation of the Lorentz symmetry in the gravitational sector naturally

leads to an extension at the level of the action which is given in the minimal SME as

$$S_g = -\frac{c^3}{16\pi G} \int d^4x \sqrt{-g} (R - u R + s^{\mu\nu} R_{\mu\nu}^T + t^{\alpha\beta\mu\nu} C_{\alpha\beta\mu\nu}) + S'[s^{\mu\nu}, t^{\alpha\beta\mu\nu}, g_{\mu\nu}] \quad (1)$$

Here  $\kappa$  is the gravitational constant,  $g$  is the determinant of the metric tensor  $g_{\mu\nu}$ ,  $R$  is the Ricci scalar,  $R_{\mu\nu}^T$  is the trace free Ricci tensor,  $C_{\alpha\beta\mu\nu}$  is the Weyl conformal tensor,  $u$ ,  $s^{\mu\nu}$  and  $t^{\alpha\beta\mu\nu}$  being the Lorentz violating fields.  $S'$  contains the dynamical terms governing the evolution of SME coefficients. From experimental evidences the violating fields have to be small quantities, therefore we will work in the linearised gravity limit where the metric depends only on  $\bar{u}$  and  $\bar{s}^{\mu\nu}$  which are the vacuum expectation values of  $u$  and  $s^{\mu\nu}$  [8]. The coefficient  $\bar{u}$  will be not take into account since it can be absorbed in a rescaling of the gravitational constant. As mentioned by [8], the so obtain post-Newtonian metric differs from the one introduced in the PPN formalism.

Using this wide formalism, many studies aimed to constrain SME parameters of the pure gravity sector by searching possible theoretical signals in post-fit residuals of experiments obtained in pure GR. This procedure has been applied in many experiments such atom interferometry [9], Gravity Probe B [10], binary pulsars [11], planetary ephemerides [12, 13], cosmic ray observations [14], gravitational waves detection [15] and LLR [16].

However, this kind of approach is not fully satisfactory since correlations between SME coefficients and others global parameters (masses, positions and velocities, ...) are neglected. Moreover, the first order analytical terms characterising Lorentz violation, which are looked for in this kind of post-fit residuals analysis, are always at the same frequency than natural frequencies appearing in the

fundamental problem governing the evolution of the experiment. Consequently, after a fit in pure GR, signals at the natural frequencies are absorbed in the redefinition of initial conditions and physical constants. Therefore, it could be problematic to look for first order analytical Lorentz violating signal in post-fit residuals since it could have been absorbed in a redefinition of one or more physical parameters. Finally, in the case of LLR data analysis, the oscillating signatures derived in [8], are computed only accounting for short periodic oscillations; typically at the order of magnitude of the mean motion of the Moon around the Earth. For instance, the precession motion in 18.6 years, of the lunar orbit on the ecliptic plane is neglected. Therefore, this analytic solution remains only available for few years compared to the 45 years of LLR data span. Consequently, approach in [16] is not the most fair to estimate SME coefficients. In a more correct form, they have to be estimated in a global fit with others parameters in data analysis. This approach, have been realized recently in a study using Very Long Baseline Interferometry data [17] to improve  $\bar{s}^{TT}$ . In this letter, we present the first global experimental measurement in SME framework with LLR observations.

LLR is used to conduct high-precision measurements of the round-trip travel time, or the separation between a LLR station on Earth and a corner cube retroreflectors on the lunar surface. Modifications of the round-trip travel time contain a lot of informations about the Earth-Moon system leading to many different fields of investigations as astronomy, lunar science, geodesy, geodynamics and gravitational physics. In addition, the determination of physical or gravitational parameters fits from the 45 years of LLR data span and from the technology improvement which leads to the observational accuracy at the sub-centimetric level.

The theoretical observation is defined in [16], the light travel time from a LLR station transmitter ( $\mathbf{r}_o$ ) at time  $T_1$  to a lunar retroreflector ( $\mathbf{r}_r$ ) at  $T_2$  and then the way back to a station receiver ( $\mathbf{r}_{o'}$ ) at time  $T_3$ . So, the computed value is given as follow

$$\Delta t_c = [T_3 - \Delta\tau_{t_3}] - [T_1 - \Delta\tau_{t_1}] \quad (2)$$

with

$$T_3 = T_1 + \frac{\|\mathbf{r}_r(T_2) - \mathbf{r}_{o'}(T_3)\|}{c} + \Delta\tau_s + \Delta\tau_a + \dots \quad (3)$$

$$T_2 = T_1 + \frac{\|\mathbf{r}_r(T_2) - \mathbf{r}_o(T_1)\|}{c} + \Delta\tau_s + \Delta\tau_a + \dots \quad (4)$$

where  $\Delta t_c$  is the computed round-trip travel time in TAI (International Atomic Time),  $\mathbf{r}_o$  is the barycentric position vector of one of the 5 LLR stations (Mc Donald Observatory, Texas, Observatoire de la Côte d'Azur, France, Haleakala Observatory, Hawaiï, Apache point Observatory, New Mexico and Matera, Italy),  $\mathbf{r}_r$  is the barycentric position vector of one of the 5 lunar retroreflectors

(located at the Apollo 11, 14, 15 and Lunokhod 1, 2),  $T_i$  with  $i = 1, 2$  and 3 is the Barycentric Dynamical Time (TDB) at epoch  $i$ ,  $c$  is the speed of light in vacuum,  $\Delta\tau_s$  is the one-way light time delay due to gravity (equal to Shapiro effect at first order),  $\Delta\tau_a$  is the one-way time delay due to the troposphere and  $\Delta\tau_t$  is a relativistic time scale correction from TDB to TAI. Ellipses in the second member of Eq. (3) and (4) represent some additional possible corrections to  $T_2$  and  $T_3$ . In practice,  $\Delta t_c$  is changed in a one-way length quantity defined as the distance  $\rho_c = \frac{c}{2}\Delta t_c$ .

In order to simulate  $\rho_c$  in the SME framework, we have built a new numerical lunar ephemeris, ELPN (Éphéméride Lunaire Française Numérique) which computes the orbit and the rotational motion of the Moon and planets (published soon). In addition, ELPN computes the relativistic time scale difference between the TT (Terrestrial Time) and TDB and also the angular velocity of the Moon's liquid core considering a damping term between the core and the lunar mantle. Moreover, we integrate partials at the same time than the equations of motion directly from variational equations unlike a purely numerical method. These are approximately 6000 equations which are integrated at the same time, during the computation. The dynamical model is very closed to the one in E430 [19] since the post-fit uncertainty, on the Moon's orbit remains below 5cm on the radial Moon distance and below 50cm on the longitude, during the time span of LLR data. The main dynamical differences are the non-spherical potential of the Earth and the Moon modelled only until the 5<sup>th</sup> degree in ELPN and the number of accounted asteroids, since only the 70 most massive are integrated in ELPN.

The most important difference, is the Lorentz violating contribution arising from the Earth-Moon system alone, implemented with the associated partials. The additional SME acceleration to the Earth-Moon vector is given in [8] (see Eq. (104)) and is expressed as

$$a_{LV}^J = \frac{\bar{G}M}{r^3} \left[ \bar{s}_t^{JK} \hat{r}^K - \frac{3}{2} \bar{s}_t^{KL} \hat{r}^K \hat{r}^L r^J + 3 \bar{s}^{TK} \hat{V}^K r^J - \bar{s}^{TJ} \hat{V}^K r^K - \bar{s}^{TK} \hat{V}^J r^K + 3 \bar{s}^{TL} \hat{V}^K \hat{r}^K \hat{r}^L r^J + 2 \frac{\delta m}{M} \left( \bar{s}^{TK} \hat{v}^K r^J - \bar{s}^{TJ} \hat{v}^K r^K \right) \right], \quad (5)$$

where  $\bar{G}$  is the observed Newtonian constant,  $M$  is the mass of the Earth-Moon barycentre,  $\delta m$  is the difference between the Earth mass and the lunar mass;  $\hat{r}^J$  being the unit position vector of the Moon with respect to the Earth;  $\hat{v}^J = v^J/c$  with  $v^J$  being the relative velocity vector of the Moon with respect to the Earth;  $\hat{V}^J = V^J/c$  with  $V^J$  being the Heliocentric velocity vector of the Earth-Moon barycentre. Latin indices represents space-coordinate ( $X, Y, Z$ ) and  $T$  represents the coordinate time (TDB) as in Eq. (2). We have introduced the 3-dimensional traceless tensor  $\bar{s}_t^{JK} = \bar{s}^{JK} - \frac{1}{3} \bar{s}^{TT} \delta^{JK}$

which leads to a rescaling of the Newtonian constant as  $\bar{G} = G(1 + \frac{5}{3}\bar{s}^{TT})$  [10]. From now until the end, we will omit the subscript "t", when we will speak about  $\bar{s}_t^{JK}$ .

The numerical ephemeris provides orientation, positions and velocities of bodies' centres of mass, time scales transformation and all the associated partials. The remaining quantities needed for the evaluation of Eq. (2) are computed using an existing software at POLAC (Paris Observatory Lunar Analysis Centre) based on IERS (International Earth Rotation System) conventions 2003 [20]. This software has been uploaded in order to integrate the SME time delay formulae (see [21], Eq. (24)) of the pure gravity sector, expressed in standard harmonic gauge ( $a_1 = a_2 = 1$ ). This expression have to take into account the rescaled Newtonian constant  $\bar{G}$ , defined previously in the orbital part. However, considering that the current accuracy over the measured observable is  $\rho_o \gtrsim 1mm$ , and computing the largest value of  $\rho_c$  arising from time delay at 0 order, we conclude that only  $|\bar{s}^{TT}| \gtrsim 3 \times 10^{-4}$ ,  $|\bar{s}^{TJ}| \gtrsim 5 \times 10^{-4}$  and  $|\bar{s}^{JK}| \gtrsim 5 \times 10^{-4}$  are attainable with the LLR time delay derived in SME framework. Considering the current available constraints on  $\bar{s}^{\mu\nu}$  (see [22]), the SME time delay can be safely reduced to the well known Shapiro time delay in pure GR (putting all the  $\bar{s}^{\mu\nu} = 0$ ). However, the full expression could help, decorrelating the  $\bar{s}^{\mu\nu}$  parameters since they not only appear in the orbital part any more. So we kept the full expression deduced from [21].

Therefore, integrating Eq. (5) and computing the time delay, let to evaluate Eq. (2) in order to determine residuals which are then minimized by a square fit using analytical partials numerically integrate in ELPN.

First of all, we have built a reference solution computed in pure GR by fitting a set of 76 parameters including initial conditions as the geocentric positions of LLR stations, the selenocentric positions of lunar retroreflectors, the barycentric Earth-Moon position and velocity vectors at J2000, the lunar libration angles (Euler angles) with their time derivatives at J2000, and the rotation vector of the Moon fluid core at J2000. We estimated also, the masses of the Earth-Moon barycentre and the one of the Moon, the Earth rotational time-lag for diurnal and semi-diurnal deformation, the potential Love number of degree 2, 3 and 4 of the Moon, the Moon time-lag for solid-body tide of degree 2, the total moment of inertia of the Moon, the ratio of polar moment of inertia of core to the mean total moment of inertia of the Moon, the flattening of the Moon core and the damping term between the solid mantle and the fluid core of the Moon.

This new lunar solution constitutes the starting point of the current analysis. From it, we have built a second one by fitting the exact same parameters including the following SME coefficients  $\bar{s}^{TX}$ ,  $\bar{s}^{XY}$ ,  $\bar{s}^{XZ}$ ,  $\bar{s}^A$ ,  $\bar{s}^C$ ,  $\bar{s}^D$ ,  $\bar{s}^E$  and  $\bar{s}^F$ . Coefficients from  $\bar{s}^A$  to  $\bar{s}^F$  are linear combination

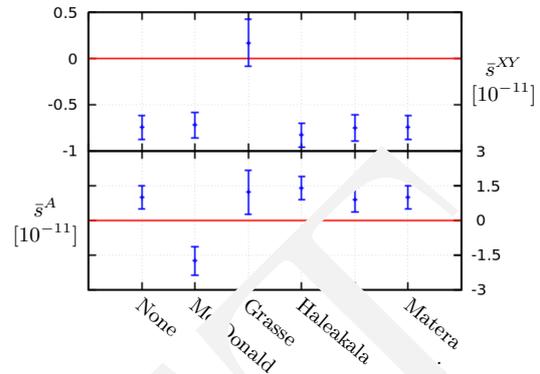


FIG. 1. Estimations of  $\bar{s}^{XY}$  and  $\bar{s}^A$  as a function of 5 subsamples from data. Error bars are those provided by the chi-square fit at 1 standard deviation ( $\sigma$ ). The  $x$ -axis shows the name of the corresponding LLR station which are deleted from the sample, also "None" means the all set of observations. The red line corresponds to the theoretical values of the SME coefficients in GR framework.

of SME parameters, which

$$\bar{s}^A = \bar{s}^{XX} - \bar{s}^{YY} \quad (6a)$$

$$\bar{s}^B = \bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ} \quad (6b)$$

$$-C = \bar{s}^{TY} + 0.43\bar{s}^{TZ} \quad (6c)$$

$$\bar{s}^D = \bar{s}^{TZ} - 0.43\bar{s}^{TY} \quad (6d)$$

$$\bar{s}^E = \bar{s}^{YZ} - 22.2\bar{s}^B \quad (6e)$$

$$\bar{s}^F = \bar{s}^B + 22.2\bar{s}^{YZ}. \quad (6f)$$

Evaluating  $\bar{s}^A$  and  $\bar{s}^B$  instead of  $\bar{s}^{JK}$  for  $J=K$ , let to maintain the 3-dimensional traceless condition ( $\bar{s}^{JJ}=0$ ), after successive fits. Concerning the choice of fitting  $\bar{s}^C$ ,  $\bar{s}^D$ ,  $\bar{s}^E$  and  $\bar{s}^F$  instead of  $\bar{s}^{TY}$ ,  $\bar{s}^{TZ}$ ,  $\bar{s}^{YZ}$ , and  $\bar{s}^B$ , it is related to high correlations arising between pure SME coefficients and it will be discussed in the continuation.

As mentioned by [23] analysis of LLR data may suffers from neglected systematic uncertainties in model parameter-estimates. Such omissions may arise from observations or from modelling, for instance from individual data weights or from neglected correlation between observations of each LLR stations. As a consequence, the standard deviation reported by the chi-square fit solution (labelled  $\sigma$ ), could underestimates the true model parameter-estimates uncertainties. Therefore, it is essential to quantify the order of magnitude of such ignored systematics in the data analysis. Resampling methods, as Jackknife [24], could let to determine bias using parameter estimators on subsamples from data.

In this study, we have assumed that neglected systematics may arise from observations and more specifically from each LLR station. This assumption is highlighted

SME	Other works	This work	$\sigma_r/\sigma$
$\bar{s}^{TX}$	$(+0.5 \pm 6.2) \times 10^{-7}$	$(-0.4 \pm 1.2) \times 10^{-8}$	4.9
$\bar{s}^{XY}$	$(-0.6 \pm 1.5) \times 10^{-9}$	$(-7.5 \pm 7.6) \times 10^{-12}$	6.0
$\bar{s}^{XZ}$	$(-2.7 \pm 1.4) \times 10^{-9}$	$(-1.7 \pm 4.6) \times 10^{-12}$	5.4
$\bar{s}^A$	$(-1.2 \pm 1.6) \times 10^{-9}$	$(+1.0 \pm 2.4) \times 10^{-11}$	4.7
$\bar{s}^C$	$(-0.1 \pm 2.9) \times 10^{-6}$	$(+3.7 \pm 7.6) \times 10^{-9}$	3.9
$\bar{s}^D$	$(-0.4 \pm 4.6) \times 10^{-6}$	$(-3.5 \pm 3.3) \times 10^{-7}$	4.1
$\bar{s}^E$	$(-0.4 \pm 8.5) \times 10^{-7}$	$(-6.4 \pm 8.8) \times 10^{-10}$	6.5
$\bar{s}^F$	$(+1.5 \pm 6.9) \times 10^{-8}$	$(+0.3 \pm 1.0) \times 10^{-7}$	4.1

TABLE I. Table of estimated values of SME parameters of the minimal SME with LLR. *Second column* : results deduced from [22] where authors mentioned the work of [16] and [25] who provide estimations deduced on theoretical grounds. *Third column* : results obtained from experimental measurement by the current LLR data analysis. Uncertainties are distributed at 1 realistic standard deviation ( $\sigma_r$ ).

with Fig 1, since values of  $\bar{s}^{XY}$  and  $\bar{s}^A$  are strongly influenced by respectively Grasse and Mc Donald observations. However, SME coefficients are universal parameters and should not depend on subsets. Therefore, for these two coefficients we conclude that their estimation are biased by respectively Grasse and Mc Donald LLR stations.

Also, in order to estimate systematics arising from stations, we have built 5 subsamples labelled as  $X_{[i]}$  for  $i = 1$  to 5, each one deleting all observation of one LLR station from the sample  $X$ , and we have computed the associated pseudovalues labelled as  $ps_i^j$ , for all the  $j$  SME parameters. Pseudovalues are associated with parameter estimators thanks data analysis as

$$ps_i^j = \phi^j(X) - 4 \left[ \phi^j(X) - \phi^j(X_{[i]}) \right], \quad (7)$$

where  $\phi^j(X)$  is the estimation of parameter  $j$ , deduced for sample  $X$ . Then, Jackknife technique is used to treat each pseudovalues as if they were independent random variables with mean  $\phi^j(X)$ . Also, with the central limit theorem, we compute for each SME parameter the mean and the sample variance of  $ps_i^j$  (labelled as  $\sigma_r^j$ ). In order to get a confidence interval accounting for bias. Finally, the realistic error is given for each SME parameters by the following expression :  $\sigma_r = \sqrt{\sigma_r^j + \sigma^j}$ . The resulting parameter values and their realistic errors, are reported in Tab. I, with the corresponding ratio of  $\sigma_r/\sigma$ .

From the chi-square solution we are able to deduced correlations between SMF parameters and others global parameters. First of all, let's point out that fitting  $\bar{s}^C$  and  $\bar{s}^D$  instead of  $\bar{s}^{YZ}$  and  $\bar{s}^{TZ}$  is absolutely necessary, since the anticorrelation between couple of coefficients goes from -0.99 to -0.08 which is more acceptable. At the same time, the correlation between  $\bar{s}^{YZ}$  and  $\bar{s}^B$  goes from 0.99 to 0.07 once changed in  $\bar{s}^E$  and  $\bar{s}^F$ . Second of all, while contribution of SME parameters have been included only in the orbital part and in the time delay,  $\bar{s}^A$

and  $\bar{s}^F$  are strongly correlated with parameters appearing in the rotational motion of the Moon as the principal moment of inertia, the quadrupole moment, the potential Stockes coefficient  $C_{22}$  and the polar component of the velocity vector of the fluid core. Those parameters have an impact on the rotational motion of the Moon which affects the orbital motion through the effect of the lunar potential. Therefore, a possible explanation is that signature of SME coefficients  $\bar{s}^A$  and  $\bar{s}^F$  arise at the same frequency than effect of the lunar potential.

In conclusion, we have analysed a set of 20603 data spanning 44 years of LLR by evaluating the range thanks ELPN a new numerical lunar ephemeris computed in SME framework. We found no evidence of Lorentz violation and we have improved all the previous estimations deduced on theoretical grounds, until 3 orders of magnitude for  $\bar{s}^{XY}$ ,  $\bar{s}^{XZ}$ ,  $\bar{s}^C$  and  $\bar{s}^E$ .

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