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Utilisation efficace des échantillons dans le rendu photo-réaliste

Efficient Sample Usage in Photo-Realistic Rendering

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Abstract

Rendering photo-realistic images of a virtual scene requires the simulation of how light interacts with it. This typically requires a large number of samples detailing the received and outgoing radiance between surfaces. As this is an expensive process, it is important to be efficient with this information. This thesis investigates two different areas for this. The first is the Low-Rank Matrix Completion problem, which looks to fill in the missing entries of some partially observed low-rank matrix. We apply this to the Many-Lights problem, a direct lighting problem that involves rendering scenes with large numbers of lights. This problem is particularly suited to Low-Rank Matrix Completion as the matrix formulation of it is shown to be approximately low-rank. We propose a modified version of the Adaptive Matrix Completion algorithm, which we term Boolean Adaptive Matrix Completion, to reduce the number of visibility computations between lights and surfaces, allowing significantly faster renderings of the final images. The second area we investigate is the efficient reuse of Monte-Carlo samples when the probability measure has changed. This is important as naively combining samples taken from different probability distributions can lead to higher variance, or even bias. We propose a hybrid method, which we term Reweighting-Augmentation, which aims to address this. We investigate this method in two different application areas. The first is progressive path tracing with animated cameras, where the sampling distribution changes on glossy surfaces due to a change in the incident direction. The second is the Practical Path Guiding algorithm, where sampling distributions change as the irradiance function of a scene is iteratively learned. We found that our proposed method showed promising results for the former, allowing for reduced variance in the final generated images. Although we did not obtain positive results for the latter, we believe that the problem should still be investigated further and there is reason to remain positive.

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Nomenclature

IR	Instant Radiosity
VPL	Virtual Point Light
VSL	Virtual Spherical Light
VSL	Virtual Disc Light
SVD	Singular Value Decomposition
LRMC	Low-Rank Matrix Completion
AMC	Adaptive Matrix Completion
BAMC	Boolean Adaptive Matrix Completion
RMSE	Root Mean Squared Error
<i>ESS</i>	Effective Sample Size
\widehat{ESS}	Effective Sample Size surrogate
SMC	Sequential Monte Carlo
RIS	Resampled Importance Sampling
BSDF	Bidirectional Scattering Distribution Function
CDF	Cumulative Distribution Function
PPG	Practical Path Guiding

Part I

Matrix Completion with the Many-Lights Problem

Part II

Using Samples Across Probability Measures in Monte Carlo Rendering

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Appendix C

Additional Results for Practical Path Guiding

C.1 Comparisons by Samples Per Pixel

C.2 Comparisons by Static Spatial Tree Depth

We provide the full results for our Practical Path Guiding Experiments when varying the depth of the spatial tree. These are as follows:

- Figure C.1 shows renders for the Fractal Reflectance scene,
- Figure C.2 shows renders for the Kitchen scene,
- Table C.1 shows statistics for our experiments.

		12	14	16	18
Fractal Reflectance	Rew-Aug	0.02955(0.0002)	0.02802(0.00027)	0.03019(0.00064)	0.03346(0.00118)
	Rew	0.0308(0.00036)	0.02896(0.00029)	0.03135(0.00057)	0.03521(0.00081)
	None	0.03035(0.0003)	0.02856(0.00032)	0.02965(0.00066)	0.03249(0.00072)
	Double	0.0302(0.00045)	0.02845(0.00035)	0.02804(0.00026)	0.02833(0.00047)
Kitchen	Rew-Aug	0.08089(0.00169)	0.08132(0.00111)	0.08065(0.00099)	0.081(0.00256)
	Rew	0.08056(0.00185)	0.08187(0.00199)	0.0797(0.00134)	0.08065(0.00243)
	None	0.08117(0.00127)	0.08184(0.00169)	0.08057(0.00163)	0.08183(0.00085)
	Double	0.08104(0.00238)	0.08153(0.00278)	0.08033(0.00165)	0.08127(0.00262)

TABLE C.1: Statistics for Progressive Path Tracing by spatial tree depth across all scenes. Shown in parentheses are the standard deviations.

C.3 Intermediate Render Combination Results

We provide the full results for our Practical Path Guiding Experiments when combining intermediate renders to generate the final image. These are as follows:

- Figure C.3 shows renders for the Fractal Reflectance scene,
- Figure C.4 shows renders for the Kitchen scene,

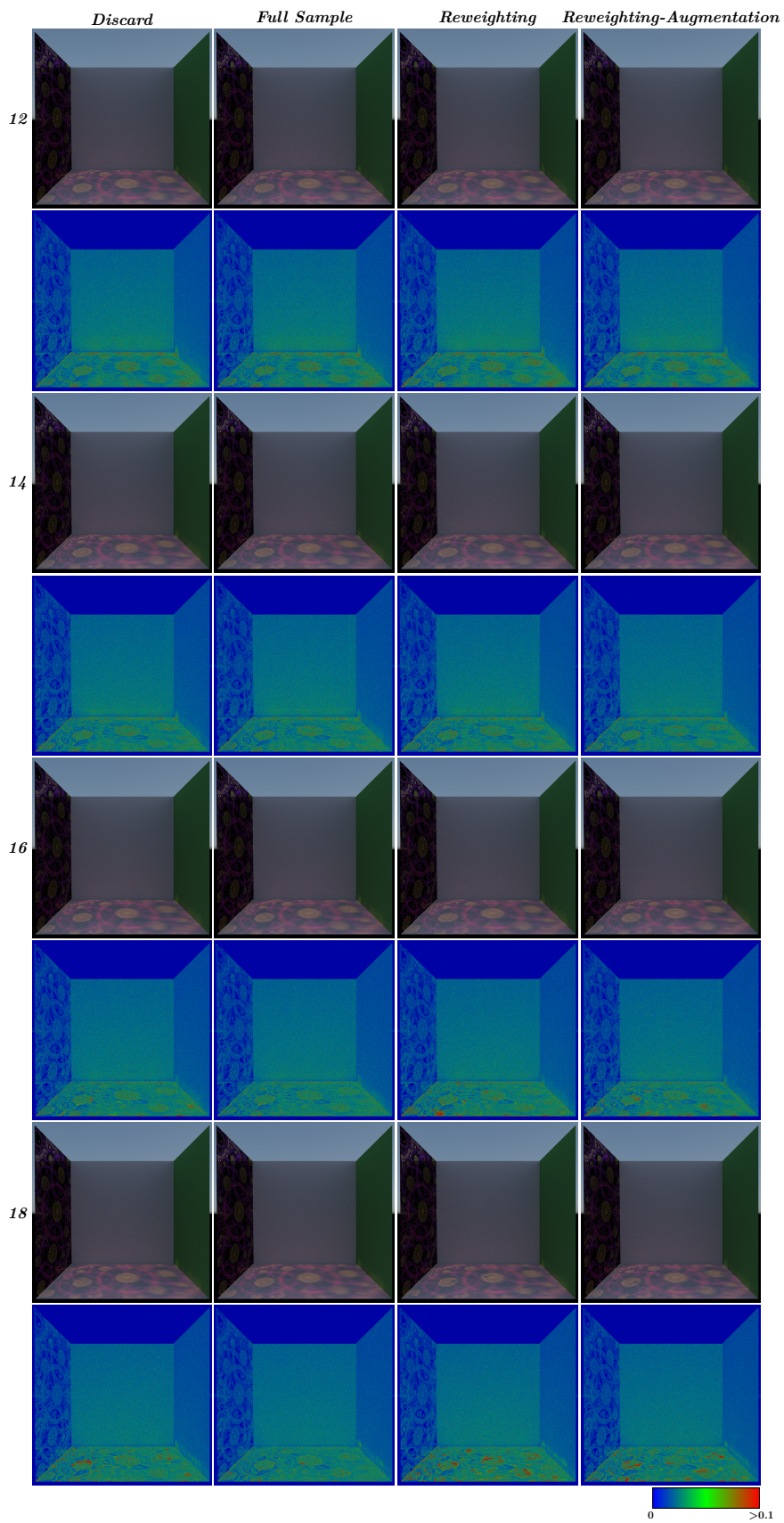


FIGURE C.1: Renders for the different strategies for the Fractal reflectance scene. The error shown is L_1 . The columns represent the different strategies, and the rows represent the number of spatial tree levels.

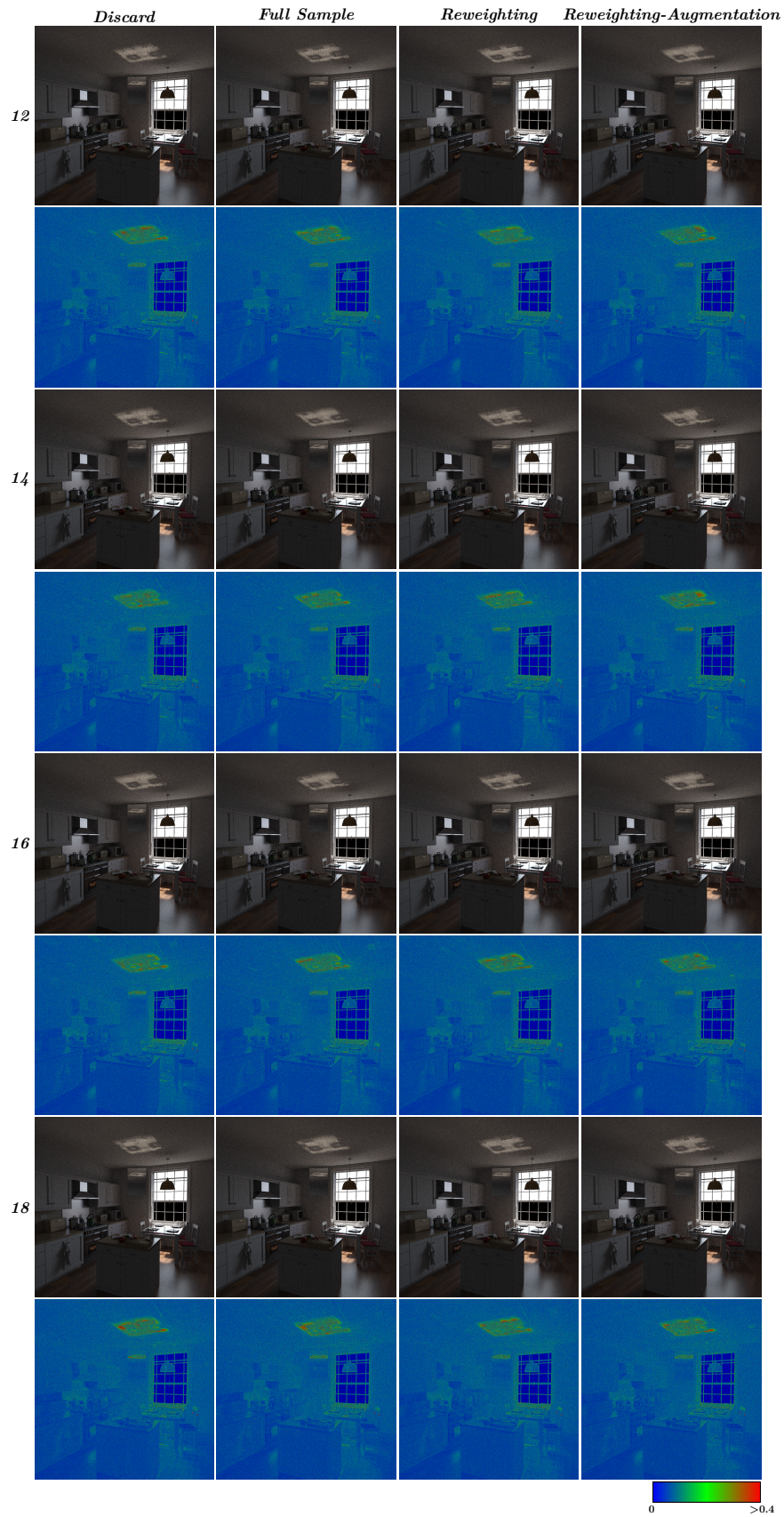


FIGURE C.2: Renders for the different strategies for the Kitchen scene. The error shown is L_1 . The columns represent the different strategies, and the rows represent the number of spatial tree levels.

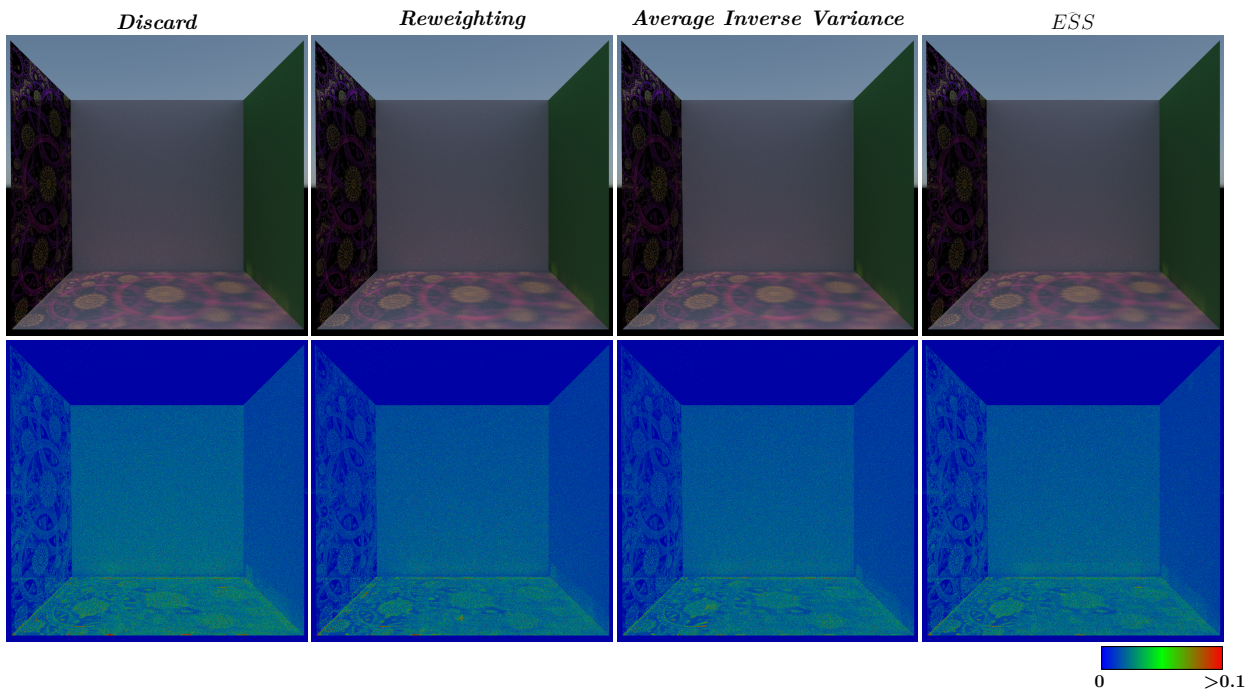


FIGURE C.3: Renders and error images for the different intermediate render combination strategies for the Fractal Reflectance scene. Error shown is L_1 .

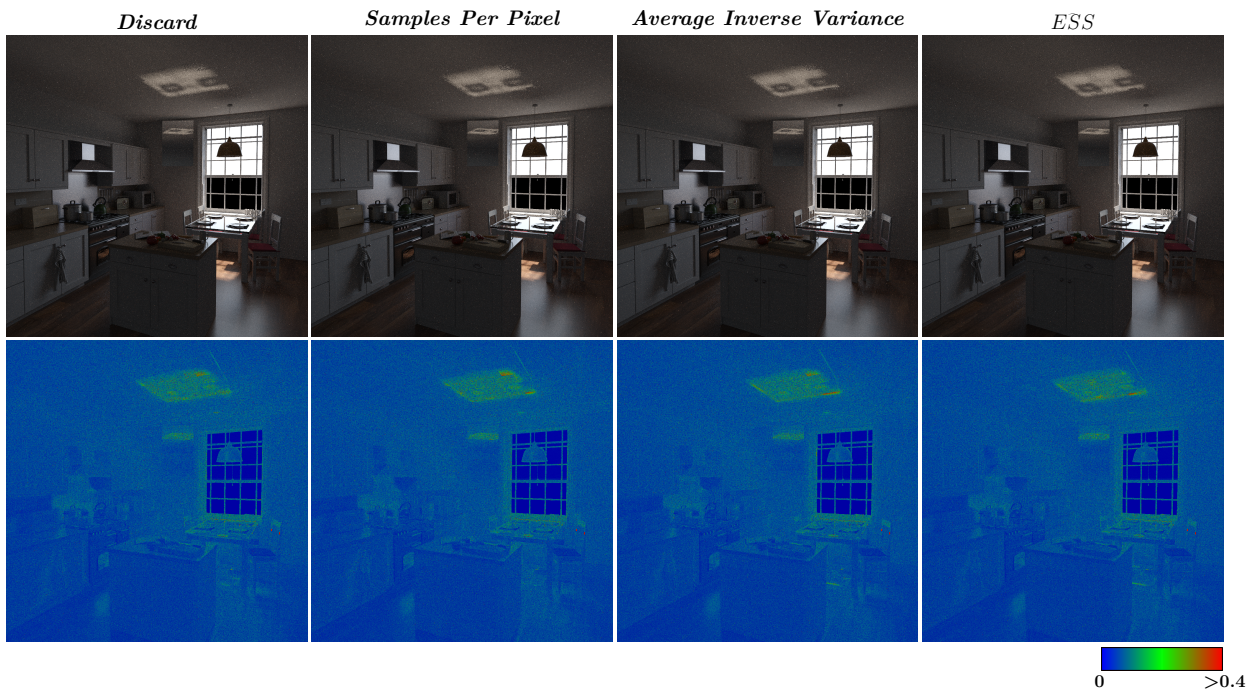


FIGURE C.4: Renders and error images for the different intermediate render combination strategies for the Kitchen scene. Error shown is L_1 .