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### **Utilisation efficace des échantillons dans le rendu photo-réaliste**

### **Efficient Sample Usage in Photo-Realistic Rendering**

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## *Abstract*

Rendering photo-realistic images of a virtual scene requires the simulation of how light interacts with it. This typically requires a large number of samples detailing the received and outgoing radiance between surfaces. As this is an expensive process, it is important to be efficient with this information. This thesis investigates two different areas for this. The first is the Low-Rank Matrix Completion problem, which looks to fill in the missing entries of some partially observed low-rank matrix. We apply this to the Many-Lights problem, a direct lighting problem that involves rendering scenes with large numbers of lights. This problem is particularly suited to Low-Rank Matrix Completion as the matrix formulation of it is shown to be approximately low-rank. We propose a modified version of the Adaptive Matrix Completion algorithm, which we term Boolean Adaptive Matrix Completion, to reduce the number of visibility computations between lights and surfaces, allowing significantly faster renderings of the final images. The second area we investigate is the efficient reuse of Monte-Carlo samples when the probability measure has changed. This is important as naively combining samples taken from different probability distributions can lead to higher variance, or even bias. We propose a hybrid method, which we term Reweighting-Augmentation, which aims to address this. We investigate this method in two different application areas. The first is progressive path tracing with animated cameras, where the sampling distribution changes on glossy surfaces due to a change in the incident direction. The second is the Practical Path Guiding algorithm, where sampling distributions change as the irradiance function of a scene is iteratively learned. We found that our proposed method showed promising results for the former, allowing for reduced variance in the final generated images. Although we did not obtain positive results for the latter, we believe that the problem should still be investigated further and there is reason to remain positive.



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# Nomenclature

<b>IR</b>	Instant Radiosity
<b>VPL</b>	Virtual Point Light
<b>VSL</b>	Virtual Spherical Light
<b>VSL</b>	Virtual Disc Light
<b>SVD</b>	Singular Value Decomposition
<b>LRMC</b>	Low-Rank Matrix Completion
<b>AMC</b>	Adaptive Matrix Completion
<b>BAMC</b>	Boolean Adaptive Matrix Completion
<b>RMSE</b>	Root Mean Squared Error
<i>ESS</i>	Effective Sample Size
$\widehat{ESS}$	Effective Sample Size surrogate
<b>SMC</b>	Sequential Monte Carlo
<b>RIS</b>	Resampled Importance Sampling
<b>BSDF</b>	Bidirectional Scattering Distribution Function
<b>CDF</b>	Cumulative Distribution Function
<b>PPG</b>	Practical Path Guiding



## Part I

# Matrix Completion with the Many-Lights Problem



## Part II

# Using Samples Across Probability Measures in Monte Carlo Rendering





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## Appendix C

# Additional Results for Practical Path Guiding

### C.1 Comparisons by Samples Per Pixel

We provide the full results for our Practical Path Guiding Experiments when varying the number of samples per pixel. These are as follows:

- Figure C.1 shows renders for the Fractal Reflectance scene,
- Figure C.2 shows renders for the Kitchen scene,
- Table C.1 shows statistics for our experiments.

		127spp	255spp	511spp	1023spp
Fractal Reflectance	Rew-Aug	0.04347(0.00099)	0.03059(0.00047)	0.02173(0.00041)	0.01591(0.00027)
	Rew	0.04518(0.00074)	0.03178(0.00064)	0.02237(0.00036)	0.01613(0.00023)
	None	0.04222(0.00099)	0.03012(0.00061)	0.02171(0.00039)	0.01609(0.00031)
	Double	0.03747(0.00024)	0.02806(0.00036)	0.02084(0.00024)	0.01558(0.00022)
Kitchen	Rew-Aug	0.10905(0.00203)	0.08234(0.0017)	0.06091(0.00128)	0.04649(0.00047)
	Rew	0.10908(0.00183)	0.08042(0.00266)	0.0607(0.00125)	0.04715(0.00078)
	None	0.10794(0.00129)	0.08125(0.00211)	0.06118(0.00124)	0.04655(0.00126)
	Double	0.10793(0.00298)	0.08067(0.00133)	0.06074(0.00108)	0.04671(0.00081)

TABLE C.1: Statistics for Progressive Path Tracing by samples per pixel across all scenes. Shown in parentheses are the standard deviations.

### C.2 Comparisons by Static Spatial Tree Depth

### C.3 Intermediate Render Combination Results

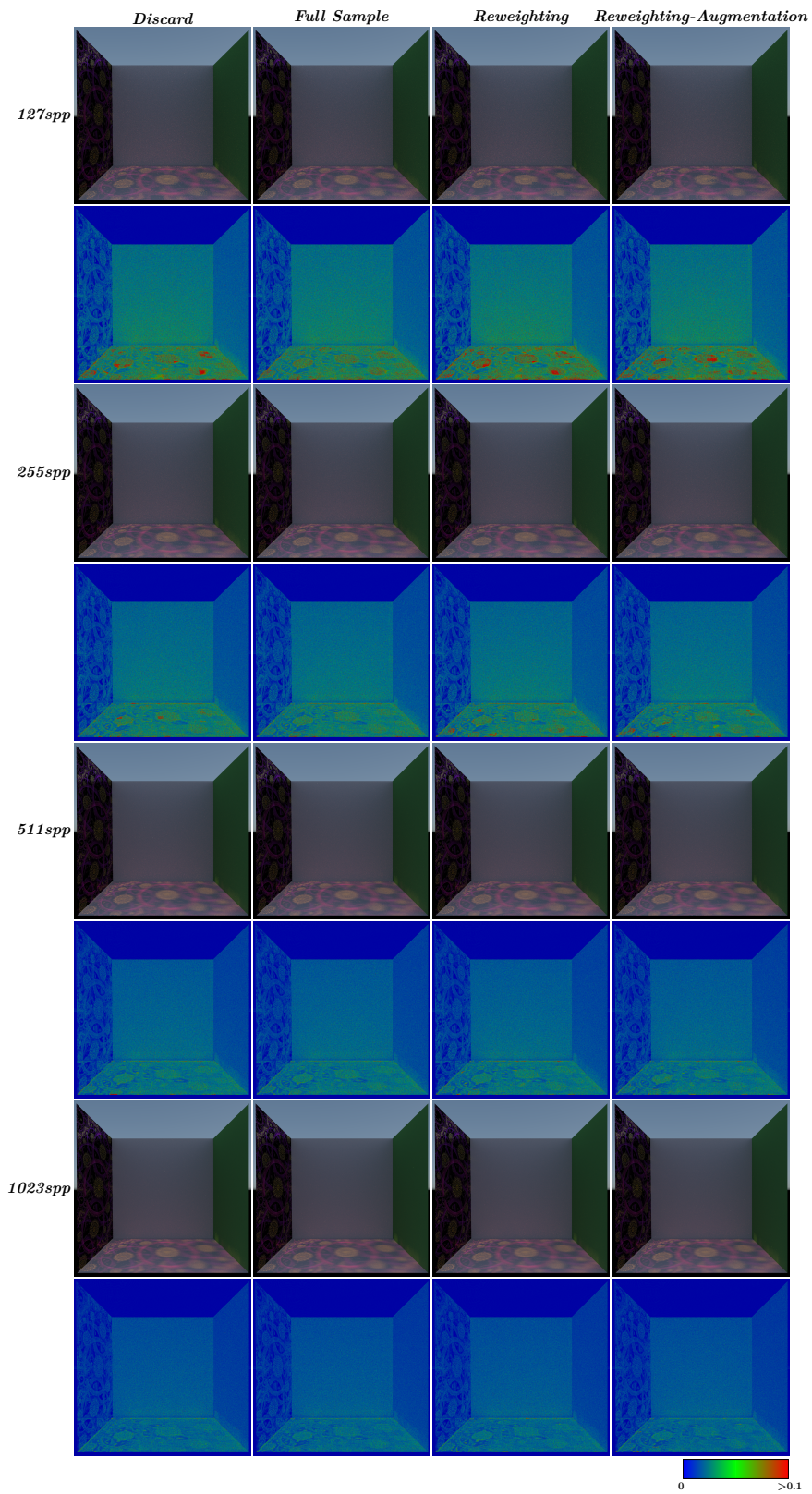


FIGURE C.1: Renders for the different strategies for the Fractal reflectance scene. The error shown is  $L_1$ . The columns represent the different strategies, and the rows represent the number of samples per pixel.



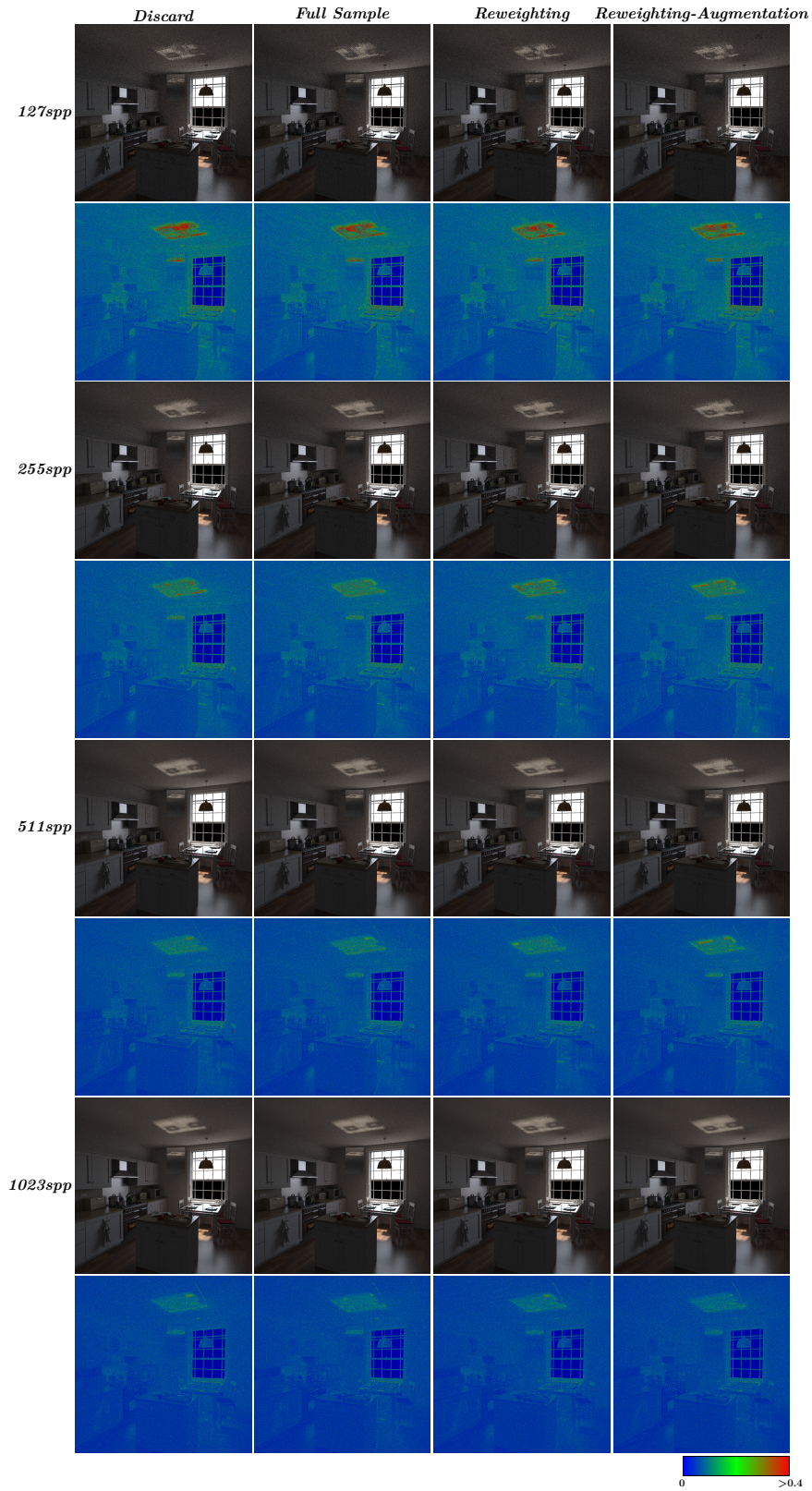


FIGURE C.2: Renders for the different strategies for the Kitchen scene. The error shown is  $L_1$ . The columns represent the different strategies, and the rows represent the number of samples per pixel.