

THÈSE

Pour obtenir le grade de

DOCTEUR DE L'UNIVERSITÉ GRENOBLE ALPES

Spécialité : Informatique

Arrêté ministériel : 25 mai 2016

Présentée par

Sunrise WANG

Thèse dirigée par **Nicolas HOLZSCHUCH**, Directeur de recherche,
Université Grenoble Alpes

préparée au sein du **Laboratoire Laboratoire Jean Kuntzmann**
dans **l'École Doctorale Mathématiques, Sciences et**
technologies de l'information, Informatique

Utilisation efficace des échantillons dans le rendu photo-réaliste

Efficient Sample Usage in Photo-Realistic Rendering

Thèse soutenue publiquement le **8 mars 2022**,
devant le jury composé de :

Monsieur NICOLAS HOLZSCHUCH

Directeur de recherche, INRIA CENTRE GRENOBLE-RHONE-ALPES,
Directeur de thèse

Madame VALERIE PERRIER

Professeur des Universités, GRENOBLE INP, Présidente

Madame BEIBEI WANG

Professeur associé, Nanjing University Science & Technology,
Examinatrice

Monsieur DAVID COEURJOLLY

Directeur de recherche, CNRS DELEGATION RHONE AUVERGNE,
Examineur

Monsieur KARTIC SUBR

Professeur associé, The University of Edinburgh, Rapporteur

Monsieur TAMY BOUBEKEUR

Professeur, TELECOM PARIS, Rapporteur



Abstract

Rendering photo-realistic images of a virtual scene requires the simulation of how light interacts with it. This typically requires a large number of samples detailing the received and outgoing radiance between surfaces. As this is an expensive process, it is important to be efficient with this information. This thesis investigates two different areas for this. The first is the Low-Rank Matrix Completion problem, which looks to fill in the missing entries of some partially observed low-rank matrix. We apply this to the Many-Lights problem, a direct lighting problem that involves rendering scenes with large numbers of lights. This problem is particularly suited to Low-Rank Matrix Completion as the matrix formulation of it is shown to be approximately low-rank. We propose a modified version of the Adaptive Matrix Completion algorithm, which we term Boolean Adaptive Matrix Completion, to reduce the number of visibility computations between lights and surfaces, allowing significantly faster renderings of the final images. The second area we investigate is the efficient reuse of Monte-Carlo samples when the probability measure has changed. This is important as naively combining samples taken from different probability distributions can lead to higher variance, or even bias. We propose a hybrid method, which we term Reweighting-Augmentation, which aims to address this. We investigate this method in two different application areas. The first is progressive path tracing with animated cameras, where the sampling distribution changes on glossy surfaces due to a change in the incident direction. The second is the Practical Path Guiding algorithm, where sampling distributions change as the irradiance function of a scene is iteratively learned. We found that our proposed method showed promising results for the former, allowing for reduced variance in the final generated images. Although we did not obtain positive results for the latter, we believe that the problem should still be investigated further and there is reason to remain positive.

Acknowledgements

I would first and foremost like to thank my thesis supervisor Nicolas for his guidance during my doctoral studies. The research was not always smooth, but your patience really helped me pull through.

I would also like to thank the members of jury for their insightful feedback both during my defense, as well as through the reports provided and the discussions afterwards.

I would like to thank both INRIA and UGA for funding my doctoral studies, as well as to the city of Grenoble for being a fantastic host city, providing me with many beautiful mountains to explore.

Much thanks also to my parents, who have provided endless support in all forms during the course of my studies. As well as to all my wonderful friends, who put up with me and helped keep me sane by accompanying me on all sorts of fun activities during my studies.

Finally, I would like to give thanks to the members of the MAVERICK research team, who made me feel at home right from the beginning. It isn't always easy moving to a new job, never mind a new country, and I could not have asked for a better work environment.

Contents

Acknowledgements	iii
List of Figures	xi
List of Tables	xv
Nomenclature	xvii
1 Introduction	1
1.1 Low-Rank Matrix Completion with Instant Radiosity	2
1.1.1 Summary of Hypotheses	3
1.2 Reusing Samples across Probability Measures	3
1.2.1 Summary of Hypotheses	4
1.3 Contributions	4
1.3.1 Low-Rank Matrix Completion with Instant Radiosity	4
1.3.2 Reusing Samples across Probability Measures	5
1.4 Structure	5
2 The Physics of Light and Photo-Realistic Rendering	7
2.1 Physical Models of Light	7
2.1.1 Ray Model of Light	7
2.1.2 Wave Model of Light	9
2.1.3 Photon Model of Light	9
2.2 Radiometry	11
2.2.1 Radiometric Quantities	12
Radiant Flux	12
Irradiance	12
Radiant Intensity	12
Radiance	13
2.2.2 Materials	13
Ideal Lambertian Surfaces	13
The Bidirectional Scattering Distribution Function	14
2.2.3 The Rendering Equation	14
2.2.4 Drawbacks of Radiometric Model of Light	15
2.3 Photo-Realistic Rendering	16
2.3.1 Finite Elements Method and Radiosity	16
2.3.2 Monte-Carlo Methods	17
Monte-Carlo Method for Expected Value	17
Monte Carlo Markov Chains	18
Recent Developments	19
2.3.3 Biased Methods	20
2.3.4 Real-Time Photo-Realistic Rendering	20

I	Matrix Completion with the Many-Lights Problem	21
3	Instant Radiosity and The Many-Lights Problem	23
3.1	Instant Radiosity	23
3.2	Virtual Spherical Lights	25
3.3	Scalable Many-Lights Methods	26
3.3.1	Hierarchical Clustering	27
	Lightcuts	27
	Lightcuts with Multiple Receiving Points	28
	Improving the Accuracy of Lightcuts	28
	Precomputing Lightcuts	29
	Fast Generation of Light Trees	29
3.3.2	Matrix-based Methods	29
	The Many-Lights Matrix and Row Column Matrix Sampling	29
	Localized Clustering for the Many-Lights Matrix	30
	Explicitly Exploiting the Low-Rank Nature of the Many-Lights Matrix	31
	Matrix-Based Methods in Photo-Realistic Rendering	32
3.3.3	VPL Sampling	32
	Accounting for Visibility when Sampling	32
	Progressively Learning the Sampling Distribution	33
3.3.4	Visibility Approximation	33
	Exploiting Light Coherence	33
	Exploiting Receiving Point Coherence	34
	Exploiting Light and Receiving Point Coherence	34
4	Inferring Incomplete Matrices	35
4.1	The Low-Rank Matrix Completion Problem	35
4.1.1	Required number of observations	36
	Rank	37
	Coherence	37
4.2	Algorithms for Completing Matrices	38
4.2.1	Semidefinite Programming	38
4.2.2	Singular Value Thresholding	39
4.2.3	Adaptive Matrix Completion	39
4.3	Compressive Sensing and Low-Rank and Sparse Matrix Separation	41
4.3.1	Compressive Sensing	41
4.3.2	Low-Rank and Sparse Matrix Separation	42
5	Approximating Visibility in the Many-Lights Problem using Adaptive Matrix Completion	43
5.1	Generating receiving points	43
5.2	VPL generation	43
5.3	Dividing receiving points into slices	44
5.4	VPL clustering	45
5.5	The Visibility Matrix	46
5.6	Approximating visibility using Adaptive Matrix Completion	48
5.6.1	Eliminating the pseudo-inverse	48
	Gauss-Jordan Elimination	49
	Boolean Matching	50
5.6.2	Adaptively Sampling the rows	50
5.6.3	Dynamically adjusting per slice row sample-rate	51
5.6.4	Recursive column sampling	52
5.7	Shading	52
5.7.1	Virtual Disc Lights	53

5.8	Summary	53
6	Evaluation of Boolean Adaptive Matrix Completion for the Many-Lights problem	55
6.1	Validation for algorithm design choices	55
6.1.1	Boolean Adaptive Matrix Completion	55
6.1.2	Adaptive Sampling	57
6.1.3	Adapting Row Sample-Rates	57
6.1.4	Recursive Column Sampling	57
6.1.5	Completing visibility with Nuclear Norm Minimization	58
6.2	Comparisons to Other Many-Lights Methods	59
6.2.1	Experimental setup	59
	Selected Many-Lights Methods	59
	Comparison Metric	60
	Parameters	60
	Hardware Details	61
6.2.2	Test scenes	61
	Cornell Box	62
	Hairball	62
	Sponza	62
	San Miguel	62
	Staircase	62
	Living Room	62
	Breakfast Room	62
	Classroom	63
	Modern Hall	63
	Bathroom	63
	Kitchen	63
	Grey & White Room	63
6.2.3	Results	63
	LightSlice	64
	Matrix Separation	64
	IlluminationCut	65
	Full visibility sampling	67
6.3	Limitations	67
6.3.1	Visual Artefacts	67
6.3.2	The rank of visibility	68
6.3.3	Full Column Sampling when Error is Detected	69
6.3.4	Non-Shrinking Row Sample-Rates	69
6.4	Conclusion and Future Work	69
II	Using Samples Across Probability Measures in Monte Carlo Rendering	71
7	Sample Reuse Across Probability Measures	73
7.1	Existing Sample Reuse Methods	73
7.1.1	Rejection Sampling	73
7.1.2	Reweighting	75
	Importance Sampling	76
	Path Guiding	77
	Resampled Importance Sampling	77
7.1.3	Augmentation	77
7.1.4	Rejection-Augmentation	79
7.2	Reweighting-Augmentation	82

7.3	Preliminary Results	83
7.3.1	Experimental Setup	83
7.3.2	Results and Discussion	85
7.3.3	Conclusion	86
8	Example Application: Progressive Path Tracing	87
8.1	Progressive Path Tracing	87
8.1.1	Change of sampling distribution	89
	Higher Variance	89
	Bias	90
8.1.2	Outdated information in Accumulation Buffer	91
	Pixel No Longer Corresponds to Same Surface Area	91
	Changing Rendering Equation	92
8.2	Progressive Path Tracing with Reweighting-Augmentation	93
8.2.1	Overview	93
8.2.2	Initial Samples	93
8.2.3	Change of Scene Camera	94
8.2.4	Backproject Initial Samples	94
8.2.5	Generate New Samples	94
8.2.6	Apply Previous Samples	96
	Updating and Storing Samples	96
	Dealing with Undersampling	97
8.2.7	Combining All Samples	97
8.3	Results and Discussion	98
8.3.1	Experimental Setup	98
	Selected Strategies	98
	Evaluation	98
8.3.2	Comparisons	100
	Reweighting	100
	Discard	101
	Full Sample	101
8.3.3	Adjusting S_- and S_+	102
8.3.4	Selecting number of RIS Candidates	104
8.4	Conclusions, Drawbacks, and Future Work	105
8.4.1	Drawbacks	105
8.4.2	Future Work	106
9	Example Application: Practical Path Guiding	107
9.1	Practical Path Guiding	108
9.1.1	Initialization	109
9.1.2	Check for Termination	110
9.1.3	Generating New Samples	110
9.1.4	Recording Samples	111
	Next Event Estimation	111
	Spatio-Directional Filtering	111
9.1.5	Refining the S-D Tree	112
9.1.6	Rendering the Final Image	113
9.2	Reusing Training Samples in Practical Path Guiding	113
9.2.1	Reusing Training Samples with Reweighting-Augmentation	113
	The Augmented S-D Tree	115
	Computing Integrals	115
	Incorporate Previous Samples	116
	Dealing with Undersampling	117
	Changes to PPG	117

	ix
9.2.2 Combining Renders from Training Iterations	118
<i>ESS</i> as Inverse Variance	118
A Surrogate for <i>ESS</i>	118
Application to PPG	119
A note on average inverse variance and bias	119
9.3 Results and Discussion	120
9.3.1 Experimental Setup	120
Reweighting-Augmentation on Training Samples	120
Using \widehat{ESS} to Recombine Training Renders	121
9.3.2 Results for Reweighting-Augmentation on Training Samples	121
Samples Per Pixel	122
Spatial Tree Depth	123
9.3.3 Results for \widehat{ESS} to Recombine Training Renders	124
9.4 Conclusions and Future Work	125
10 Conclusion & Future Directions	127
10.1 Low-Rank Matrix Completion with Many-Lights Problems	127
10.2 Reusing Samples across Probability Distributions	127
10.2.1 Progressive Path Tracing	128
10.2.2 Practical Path-Guiding	128
10.2.3 Additional Future Work	129
Bibliography	131
A Additional Results for BAMC	139
A.1 BAMC Scalability Results	139
A.2 Full Results for Comparisons Against Other Many-Lights Methods	139
B Additional Results for Progressive Path-Tracing	153
C Additional Results for Practical Path Guiding	159
C.1 Comparisons by Samples Per Pixel	159
C.2 Comparisons by Static Spatial Tree Depth	159
C.3 Intermediate Render Combination Results	159

List of Figures

2.1	Ray Model of Light	8
2.2	Participating Media	8
2.3	Caustics	8
2.4	Rayleigh	9
2.5	Rainbow	9
2.6	Iridescent Soap Bubbles	10
2.7	Examples of Diffraction	10
2.8	The Photoelectric Effect	10
2.9	Lasers	11
2.10	Fluorescence and Phosphorescence	11
2.11	Lambertian Material Model	13
2.12	Example Lambertian Materials	14
2.13	Unifying BRDF and BTDF	15
2.14	Radiosity	16
2.15	Unidirectional Path Tracing	17
2.16	Bidirectional Path Tracing	18
2.17	Monte Carlo Markov Chain	19
2.18	Metropolis Light Transport Path Mutation	19
2.19	Monte-Carlo Markov Chain State Exploration Problem	20
3.1	Stages of IR	24
3.2	IR as Bi-Directional Path Tracing	24
3.3	Bright spot artefacts in IR	25
3.4	VSL model	26
3.5	Lightcuts clustering example	27
3.6	A tree cut	28
3.7	Light and Gather tree product graph	29
3.8	The Many-Lights Matrix	30
3.9	Many-Lights Matrix Singular Values	31
4.1	Low-Rank Matrix Completion	36
4.2	Matrix Rank	36
4.3	Matrix Rank and Coherence	37
4.4	Compressive Sensing Example	41
4.5	Low-Rank and Sparse Matrix Separation Example	42
5.1	BAMC method architecture	44
5.2	Receiver point generation for our AMC Many-Lights method	45
5.3	Dividing our Many-Lights scenes into slices	46
5.4	Visibility and Luminance Ranks	47
5.5	Gauss Jordan Elimination as Pseudo-Inverse	49
5.6	Manifestation of Low-Rank Areas in the Visibility Matrix	50
5.7	Sample Allocation using Adaptive Visibility Sampling	50
5.8	Adaptive Visibility Sampling Selection Probability	51
5.9	An Ideal Slicing Scheme	52

5.10	Recursive Column Sampling	53
5.11	Virtual Disc Lights	54
6.1	BAMC Performance Scalability	55
6.2	BAMC accuracy	56
6.3	Adaptive Visibility Sampling Plots	57
6.4	Visual Improvement from Adaptive Sampling	58
6.5	Example Failure Case of Adaptive Visibility Sampling	59
6.6	Visual Improvements for Adapting Row Sample-Rates	60
6.7	Singular Value Thresholding Images	61
6.8	Test Scenes	61
6.9	BAMC Kitchen Results	64
6.10	BAMC San Miguel Results	65
6.11	BAMC Sponza Results	66
6.12	BAMC Light Leaks	67
6.13	BAMC Banding	68
6.14	Rank of Visibility	69
7.1	Rejection Sampling	74
7.2	Reweighting	76
7.3	Augmentation	78
7.4	Rejection-Augmentation	80
7.5	Reweighting-Augmentation	82
7.6	One dimensional testing functions	83
7.7	One dimensional testing probability distributions	84
7.8	Plot of majorization constants for augmentation and rejection sampling	84
8.1	Backprojection Artefacts from Outdated Pixel History	88
8.2	Mixture Model from Combining Previous Samples with Current	89
8.3	Change of Sampling Distribution from Changing Camera Positions	90
8.4	Backprojection and Cases Where it Fails	91
8.5	Our Simplified Backprojection Strategy	94
8.6	Illustration of RIS	95
8.7	Illustration of How We Apply the Previous Saved Samples	97
8.8	Progressive Path Tracing Test Scenes	99
8.9	RMSE vs Camera Distance Plots for Progressive Path Tracing	101
8.10	Equal Budget Renders for Moderately Glossy Cornell Box with Large Camera Displacement	102
8.11	RMSE versus c Plot for Adjusting S_+	102
8.12	Augmentation Artefacts From a Large c	103
8.13	Noisiness of Integral Estimates when Varying RIS Candidates	104
8.14	RIS Candidate Samples Plot	105
8.15	Integral Estimates or RIS Candidate Samples	105
9.1	Example scenario of when Incident Radiance Dominates BSDF	108
9.2	PPG S-D Tree Structure	110
9.3	Traversing a Directional Tree in PPG	111
9.4	Product Distribution Versus MIS Distribution	112
9.5	Spatial Filtering in PPG	112
9.6	PPG Framework and Our Contributions	114
9.7	Illustration of Computing an Augmented Tree	115
9.8	Incorporating Previous Training Samples	117
9.9	Ground Truths of Test Scenes	120
9.10	Renders and Error Images for Fractal Reflectance Scene	121
9.11	Renders and Error Images for Kitchen Scene	122

9.12	Training Sample Reuse Plots by SPP	123
9.13	Training Sample Reuse Plots by Spatial Tree Depth	124
A.1	BAMC Cornell Box Equal Time Renders and Error Plot	142
A.2	BAMC Hairball Equal Time Renders and Error Plot	143
A.3	BAMC Sponza Equal Time Renders and Error Plot	144
A.4	BAMC San Miguel Equal Time Renders and Error Plot	145
A.5	BAMC Staircase Equal Time Renders and Error Plot	146
A.6	BAMC Living Room Equal Time Renders and Error Plot	147
A.7	BAMC Breakfast Room Equal Time Renders and Error Plot	148
A.8	BAMC Classroom Equal Time Renders and Error Plot	149
A.9	BAMC Modern Hall Equal Time Renders and Error Plot	149
A.10	BAMC Bathroom Equal Time Renders and Error Plot	150
A.11	BAMC Kitchen Equal Time Renders and Error Plot	150
A.12	BAMC Grey & White Room Equal Time Renders and Error Plot	150
B.1	Equal Budget Renders for Rough Cornell Box	154
B.2	Equal Budget Renders for Moderately Glossy Cornell Box	155
B.3	Equal Budget Renders for Highly Glossy Cornell Box	156
B.4	Equal Budget Renders for Kitchen	157
B.5	Equal Budget Renders for Modern Hall	158
C.1	Fractal Reflectance Renders by SPP	161
C.2	Kitchen Renders by SPP	162
C.3	Fractal Reflectance Renders by Spatial Tree Level	163
C.4	Kitchen Renders by Spatial Tree Level	164
C.5	Fractal Reflectance Renders for Different Combination Strategies	165
C.6	Kitchen Renders for Different Combination Strategies	165

List of Tables

6.1	Adaptive Row Sampling Stats	57
6.2	Recursive Sampling statistics	59
7.1	Preliminary Results for Reweighting-Augmentation	85
9.1	Intermediate Render Combination Results	124
A.1	BAMC scalability results	139
A.2	Full Lightslice Results	140
A.3	Full IlluminationCut Results	141
A.4	Full Matrix Separation Results	141
A.5	Full BAMC and Full Sampling Results	151
C.1	Progressive Path Tracing Comparisons by Samples Per Pixel	159
C.2	Progressive Path Tracing Comparisons by Spatial Tree Depth	160

Nomenclature

IR	Instant Radiosity
VPL	Virtual Point Light
VSL	Virtual Spherical Light
VSL	Virtual Disc Light
SVD	Singular Value Decomposition
LRMC	Low-Rank Matrix Completion
AMC	Adaptive Matrix Completion
BAMC	Boolean Adaptive Matrix Completion
RMSE	Root Mean Squared Error
<i>ESS</i>	Effective Sample Size
\widehat{ESS}	Effective Sample Size surrogate
SMC	Sequential Monte Carlo
RIS	Resampled Importance Sampling
BSDF	Bidirectional Scattering Distribution Function
CDF	Cumulative Distribution Function
PPG	Practical Path Guiding

Part I

Matrix Completion with the Many-Lights Problem

Part II

Using Samples Across Probability Measures in Monte Carlo Rendering

Bibliography

- [ÅUW07] Oskar Åkerlund, Mattias Unger, and Rui Wang. “Precomputed visibility cuts for interactive relighting with dynamic BRDFs”. In: *Proceedings of the 15th Pacific Conference on Computer Graphics and Applications (PG’07)*. IEEE. 2007, pp. 161–170.
- [BARA06] Aner Ben-Artzi, Ravi Ramamoorthi, and Maneesh Agrawala. “Efficient Shadows for Sampled Environment Maps”. In: *Journal of Graphics Tools* 11.1 (2006), 13–36. ISSN: 1086-7651. DOI: [10 . 1080 / 2151237X . 2006 . 10129211](https://doi.org/10.1080/2151237X.2006.10129211). URL: <https://www.tandfonline.com/doi/full/10.1080/2151237X.2006.10129211>.
- [BGH05] David Burke, Abhijeet Ghosh, and Wolfgang Heidrich. “Bidirectional importance sampling for direct illumination”. In: *Proceedings of the Sixteenth Eurographics conference on Rendering Techniques*. 2005, pp. 147–156.
- [Bit16] Benedikt Bitterli. *Rendering resources*. <https://benedikt-bitterli.me/resources/>. 2016.
- [Bit+20] Benedikt Bitterli, Chris Wyman, Matt Pharr, Peter Shirley, Aaron Lefohn, and Wojciech Jarosz. “Spatiotemporal reservoir resampling for real-time ray tracing with dynamic direct lighting”. In: *ACM Transactions on Graphics* 39.4 (2020). ISSN: 0730-0301, 1557-7368. DOI: [10 . 1145 / 3386569 . 3392481](https://doi.org/10.1145/3386569.3392481). URL: <https://dl.acm.org/doi/10.1145/3386569.3392481>.
- [BMB] Norbert Bus, Nabil H Mustafa, and Venceslas Biri. *IlluminationCut source code*. <https://perso.esiee.fr/~mustafan/Software/IlluminationCut/>. Accessed: 2020-04-14.
- [BMB15a] N. Bus, N. H. Mustafa, and V. Biri. “Global Illumination Using Well-Separated Pair Decomposition: Global Illumination Using WSPD”. In: *Computer Graphics Forum* 34.8 (2015), 88–103. ISSN: 01677055. DOI: [10 . 1111 / cgf . 12610](https://doi.org/10.1111/cgf.12610). URL: <https://onlinelibrary.wiley.com/doi/10.1111/cgf.12610>.
- [BMB15b] N. Bus, N. H. Mustafa, and V. Biri. “IlluminationCut”. In: *Computer Graphics Forum* 34.2 (2015), 561–573. ISSN: 01677055. DOI: [10 . 1111 / cgf . 12584](https://doi.org/10.1111/cgf.12584). URL: <https://onlinelibrary.wiley.com/doi/10.1111/cgf.12584>.
- [BN76] James F. Blinn and Martin E. Newell. “Texture and reflection in computer generated images”. en. In: *Communications of the ACM* 19.10 (1976), 542–547. ISSN: 0001-0782, 1557-7317. DOI: [10.1145/360349.360353](https://doi.org/10.1145/360349.360353). URL: <https://dl.acm.org/doi/10.1145/360349.360353>.
- [BSD08] Louis Bavoil, Miguel Sainz, and Rouslan Dimitrov. “Image-space horizon-based ambient occlusion”. en. In: *ACM SIGGRAPH 2008 talks on - SIGGRAPH ’08*. Los Angeles, California: ACM Press, 2008, p. 1. ISBN: 9781605583433. DOI: [10 . 1145 / 1401032 . 1401061](https://doi.org/10.1145/1401032.1401061). URL: <http://portal.acm.org/citation.cfm?doid=1401032.1401061>.
- [CCS10] Jian-Feng Cai, Emmanuel J. Candès, and Zuowei Shen. “A Singular Value Thresholding Algorithm for Matrix Completion”. In: *SIAM Journal on Optimization* 20.4 (2010), 1956–1982. ISSN: 1052-6234, 1095-7189. DOI: [10 . 1137 / 080738970](https://doi.org/10.1137/080738970). URL: <https://epubs.siam.org/doi/10.1137/080738970>.
- [Cdd] *Diffraction Grating*. <http://s-educat.blogspot.com/2010/05/diffraction-grating.html>. Accessed: 2022-05-04.
- [CG85] Michael F. Cohen and Donald P. Greenberg. “The hemi-cube: a radiosity solution for complex environments”. In: *ACM SIGGRAPH Computer Graphics* 19.3 (1985), 31–40.

- ISSN: 0097-8930. DOI: [10.1145/325165.325171](https://doi.org/10.1145/325165.325171). URL: <https://dl.acm.org/doi/10.1145/325165.325171>.
- [Cha+17] Chakravarty R. Alla Chaitanya, Anton S. Kaplanyan, Christoph Schied, Marco Salvi, Aaron Lefohn, Derek Nowrouzezahrai, and Timo Aila. “Interactive reconstruction of Monte Carlo image sequences using a recurrent denoising autoencoder”. In: *ACM Transactions on Graphics* 36.4 (2017). ISSN: 0730-0301, 1557-7368. DOI: [10.1145/3072959.3073601](https://doi.org/10.1145/3072959.3073601). URL: <https://dl.acm.org/doi/10.1145/3072959.3073601>.
- [Che05] Yuguo Chen. “Another look at rejection sampling through importance sampling”. In: *Statistics & Probability Letters* 72.4 (2005), 277–283. ISSN: 01677152. DOI: [10.1016/j.spl.2005.01.002](https://doi.org/10.1016/j.spl.2005.01.002). URL: <https://linkinghub.elsevier.com/retrieve/pii/S0167715205000374>.
- [CR09] Emmanuel J. Candès and Benjamin Recht. “Exact Matrix Completion via Convex Optimization”. In: *Foundations of Computational Mathematics* 9.6 (2009), 717–772. ISSN: 1615-3375, 1615-3383. DOI: [10.1007/s10208-009-9045-5](https://doi.org/10.1007/s10208-009-9045-5). URL: <http://link.springer.com/10.1007/s10208-009-9045-5>.
- [Cra+11] Cyril Crassin, Fabrice Neyret, Miguel Sainz, Simon Green, and Elmar Eisemann. “Interactive Indirect Illumination Using Voxel Cone Tracing”. en. In: *Computer Graphics Forum* 30.7 (2011), 1921–1930. ISSN: 01677055. DOI: [10.1111/j.1467-8659.2011.02063.x](https://doi.org/10.1111/j.1467-8659.2011.02063.x). URL: <https://onlinelibrary.wiley.com/doi/10.1111/j.1467-8659.2011.02063.x>.
- [CRT06] E.J. Candès, J. Romberg, and T. Tao. “Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information”. In: *IEEE Transactions on Information Theory* 52.2 (2006), 489–509. ISSN: 0018-9448. DOI: [10.1109/TIT.2005.862083](https://doi.org/10.1109/TIT.2005.862083). URL: <http://ieeexplore.ieee.org/document/1580791/>.
- [Dif] *Diffraction Spike*. https://www.photographingspace.com/howto-diffraction-spikes/antares_light_60s_800iso_30c_558stdev_20130627-23h16m58s435ms_resized/. Accessed: 2022-05-04.
- [DK16] Ken Dahm and Alexander Keller. “Learning light transport the reinforced way”. In: *Proceedings of the 2016 International Conference on Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing*. Springer. 2016, pp. 181–195.
- [Don+06] M. Donikian, B. Walter, Kavita Bala, S. Fernandez, and D.P. Greenberg. “Accurate direct illumination using iterative adaptive sampling”. In: *IEEE Transactions on Visualization and Computer Graphics* 12.3 (2006), 353–364. ISSN: 1077-2626. DOI: [10.1109/TVCG.2006.41](https://doi.org/10.1109/TVCG.2006.41). URL: <http://ieeexplore.ieee.org/document/1608022/>.
- [EMR18] Víctor Elvira, Luca Martino, and Christian P Robert. “Rethinking the effective sample size”. In: *arXiv preprint arXiv:1809.04129* (2018).
- [FBG02] Sebastian Fernandez, Kavita Bala, and Donald P Greenberg. “Local illumination environments for direct lighting acceleration”. In: *Proceedings of the 13th Eurographics workshop on Rendering*. 2002, pp. 7–14.
- [Flu] *Why Do Scorpions Glow Under Ultraviolet Light*. <https://www.terminix.com/blog/bug-facts/why-do-scorpions-glow-under-ultraviolet-light/>. Accessed: 2022-05-04.
- [Gel+95] Andrew Gelman, John B Carlin, Hal S Stern, and Donald B Rubin. *Bayesian data analysis*. Chapman and Hall/CRC, 1995.
- [Geo+12] Iliyan Georgiev, Jaroslav Křivánek, Stefan Popov, and Philipp Slusallek. “Importance Caching for Complex Illumination”. In: *Computer Graphics Forum* 31.2pt3 (2012), 701–710. ISSN: 01677055. DOI: [10.1111/j.1467-8659.2012.03049.x](https://doi.org/10.1111/j.1467-8659.2012.03049.x). URL: <https://onlinelibrary.wiley.com/doi/10.1111/j.1467-8659.2012.03049.x>.
- [HA90] Paul Haeberli and Kurt Akeley. “The accumulation buffer: hardware support for high-quality rendering”. In: *ACM SIGGRAPH Computer Graphics* 24.4 (1990), 309–318. ISSN: 0097-8930. DOI: [10.1145/97880.97913](https://doi.org/10.1145/97880.97913). URL: <https://dl.acm.org/doi/10.1145/97880.97913>.

- [Haš+09] Miloš Hašan, Jaroslav Křivánek, Bruce Walter, and Kavita Bala. “Virtual spherical lights for many-light rendering of glossy scenes”. In: *ACM Transactions on Graphics* 28.5 (2009). ISSN: 0730-0301, 1557-7368. DOI: [10.1145/1618452.1618489](https://doi.org/10.1145/1618452.1618489). URL: <https://dl.acm.org/doi/10.1145/1618452.1618489>.
- [Has+20] J. Hasselgren, J. Munkberg, M. Salvi, A. Patney, and A. Lefohn. “Neural Temporal Adaptive Sampling and Denoising”. In: *Computer Graphics Forum* 39.2 (2020), 147–155. ISSN: 0167-7055, 1467-8659. DOI: [10.1111/cgf.13919](https://onlinelibrary.wiley.com/doi/10.1111/cgf.13919). URL: <https://onlinelibrary.wiley.com/doi/10.1111/cgf.13919>.
- [HDG99] David Hart, Philip Dutré, and Donald P. Greenberg. “Direct illumination with lazy visibility evaluation”. In: *Proceedings of the 26th annual conference on Computer graphics and interactive techniques - SIGGRAPH '99*. ACM Press, 1999, 147–154. ISBN: 9780201485608. DOI: [10.1145/311535.311551](http://portal.acm.org/citation.cfm?doid=311535.311551). URL: <http://portal.acm.org/citation.cfm?doid=311535.311551>.
- [HMT11] N. Halko, P. G. Martinsson, and J. A. Tropp. “Finding Structure with Randomness: Probabilistic Algorithms for Constructing Approximate Matrix Decompositions”. In: *SIAM Review* 53.2 (2011), 217–288. ISSN: 0036-1445, 1095-7200. DOI: [10.1137/090771806](http://epubs.siam.org/doi/10.1137/090771806). URL: <http://epubs.siam.org/doi/10.1137/090771806>.
- [HP02] Heinrich Hey and Werner Purgathofer. “Importance sampling with hemispherical particle footprints”. In: *Proceedings of the 18th spring conference on Computer graphics - SCCG '02*. ACM Press, 2002, p. 107. ISBN: 9781581136081. DOI: [10.1145/584458.584476](http://portal.acm.org/citation.cfm?doid=584458.584476). URL: <http://portal.acm.org/citation.cfm?doid=584458.584476>.
- [HPB07] Miloš Hašan, Fabio Pellacini, and Kavita Bala. “Matrix row-column sampling for the many-light problem”. In: *ACM Transactions on Graphics* 26.3 (2007), p. 26. ISSN: 0730-0301, 1557-7368. DOI: [10.1145/1276377.1276410](https://dl.acm.org/doi/10.1145/1276377.1276410). URL: <https://dl.acm.org/doi/10.1145/1276377.1276410>.
- [HR10] Fu-Chung Huang and Ravi Ramamoorthi. “Sparsely Precomputing The Light Transport Matrix for Real-Time Rendering”. In: *Computer Graphics Forum* 29.4 (2010), 1335–1345. ISSN: 01677055. DOI: [10.1111/j.1467-8659.2010.01729.x](https://onlinelibrary.wiley.com/doi/10.1111/j.1467-8659.2010.01729.x). URL: <https://onlinelibrary.wiley.com/doi/10.1111/j.1467-8659.2010.01729.x>.
- [Huo+16a] Yuchi Huo, Rui Wang, Tianlei Hu, Wei Hua, and Hujun Bao. “Adaptive matrix column sampling and completion for rendering participating media”. In: *ACM Transactions on Graphics* 35.6 (2016). ISSN: 0730-0301, 1557-7368. DOI: [10.1145/2980179.2980244](https://dl.acm.org/doi/10.1145/2980179.2980244). URL: <https://dl.acm.org/doi/10.1145/2980179.2980244>.
- [Huo+16b] Yuchi Huo, Rui Wang, Tianlei Hu, Wei Hua, and Hujun Bao. “Adaptive matrix column sampling and completion for rendering participating media”. In: *ACM Transactions on Graphics (TOG)* 35.6 (2016).
- [ICG86] David S. Immel, Michael F. Cohen, and Donald P. Greenberg. “A radiosity method for non-diffuse environments”. In: *ACM SIGGRAPH Computer Graphics* 20.4 (1986), 133–142. ISSN: 0097-8930. DOI: [10.1145/15886.15901](https://dl.acm.org/doi/10.1145/15886.15901). URL: <https://dl.acm.org/doi/10.1145/15886.15901>.
- [Jak10] Wenzel Jakob. *Mitsuba renderer*. <http://www.mitsuba-renderer.org>. 2010.
- [Jen95] Henrik Wann Jensen. “Importance driven path tracing using the photon map”. In: *Proceedings of the 1995 Eurographics Workshop on Rendering Techniques*. Springer, 1995, pp. 326–335.
- [Kaj86] James T. Kajiya. “The rendering equation”. In: *ACM SIGGRAPH Computer Graphics* 20.4 (1986), 143–150. ISSN: 0097-8930. DOI: [10.1145/15886.15902](https://dl.acm.org/doi/10.1145/15886.15902). URL: <https://dl.acm.org/doi/10.1145/15886.15902>.
- [Kel97] Alexander Keller. “Instant Radiosity”. In: *Proceedings of the 24th Annual Conference on Computer Graphics and Interactive Techniques*. SIGGRAPH '97. 1997, pp. 49–56. DOI: [10.1145/258734.258769](https://doi.org/10.1145/258734.258769).
- [Kon92] Augustine Kong. “A note on importance sampling using standardized weights”. In: *University of Chicago, Dept. of Statistics, Tech. Rep* 348 (1992).

- [Kos+19] Matias Koskela, Kalle Immonen, Markku Mäkitalo, Alessandro Foi, Timo Viitanen, Pekka Jääskeläinen, Heikki Kultala, and Jarmo Takala. “Blockwise Multi-Order Feature Regression for Real-Time Path-Tracing Reconstruction”. In: *ACM Transactions on Graphics* 38.5 (2019). ISSN: 0730-0301, 1557-7368. DOI: [10.1145/3269978](https://doi.org/10.1145/3269978). URL: <https://dl.acm.org/doi/10.1145/3269978>.
- [KS13] Akshay Krishnamurthy and Aarti Singh. “Low-rank matrix and tensor completion via adaptive sampling”. In: *arXiv preprint arXiv:1304.4672* (2013).
- [KS14] Akshay Krishnamurthy and Aarti Singh. “On the power of adaptivity in matrix completion and approximation”. In: *arXiv preprint arXiv:1407.3619* (2014).
- [Lar] Rasmus Munk Larsen. *PROPACK—Software for large and sparse SVD calculations*. <http://sun.stanford.edu/~rmunk/PROPACK/>. Accessed: 2021-09-01.
- [Las] *Lasers*. <https://commons.wikimedia.org/wiki/File:Lasers.jpeg>. Accessed: 2022-05-04.
- [LW93] Eric P Lafortune and Yves Willems. “Bi-directional path tracing”. In: *Proceedings of Compugraphics’ 93*. 1993, pp. 145–153.
- [LY20] Daqi Lin and Cem Yuksel. “Real-Time Stochastic Lightcuts”. In: *Proceedings of the ACM on Computer Graphics and Interactive Techniques* 3.1 (2020). ISSN: 2577-6193. DOI: [10.1145/3384543](https://doi.org/10.1145/3384543). URL: <https://dl.acm.org/doi/10.1145/3384543>.
- [M19] Thomas Müller. “Practical Path Guiding in Production”. In: *ACM SIGGRAPH Courses: Path Guiding in Production, Chapter 10*. SIGGRAPH ’19. Los Angeles, California: ACM, 2019, 18:35–18:48. DOI: [10.1145/3305366.3328091](https://doi.org/10.1145/3305366.3328091).
- [Mar+17] Michael Mara, Morgan McGuire, Benedikt Bitterli, and Wojciech Jarosz. “An efficient denoising algorithm for global illumination”. In: *Proceedings of High Performance Graphics 2017*. ACM, 2017. ISBN: 9781450351010. DOI: [10.1145/3105762.3105774](https://doi.org/10.1145/3105762.3105774). URL: <https://dl.acm.org/doi/10.1145/3105762.3105774>.
- [Mar+18] M. Maria, N. Mustafa, T. Bardoux, J. Defaye, and V. Biri. “Visibility based WSPD for Global Illumination.” in: *Proceedings of the 13th International Joint Conference on Computer Vision, Imaging and Computer Graphics Theory and Applications*. SCITEPRESS - Science and Technology Publications, 2018, 81–90. ISBN: 9789897582875. DOI: [10.5220/0006601000810090](https://doi.org/10.5220/0006601000810090). URL: <http://www.scitepress.org/DigitalLibrary/Link.aspx?doi=10.5220/0006601000810090>.
- [McG17] Morgan McGuire. *Computer Graphics Archive*. <https://casual-effects.com/data>. July 2017. URL: <https://casual-effects.com/data>.
- [Men+20] Xiaoxu Meng, Quan Zheng, Amitabh Varshney, Gurprit Singh, and Matthias Zwicker. “Real-time Monte Carlo Denoising with the Neural Bilateral Grid”. In: *31st Eurographics Symposium on Rendering*. The Eurographics Association. 2020.
- [Met+53] Nicholas Metropolis, Arianna W. Rosenbluth, Marshall N. Rosenbluth, Augusta H. Teller, and Edward Teller. “Equation of State Calculations by Fast Computing Machines”. en. In: *The Journal of Chemical Physics* 21.6 (1953), 1087–1092. ISSN: 0021-9606, 1089-7690. DOI: [10.1063/1.1699114](https://doi.org/10.1063/1.1699114). URL: <http://aip.scitation.org/doi/10.1063/1.1699114>.
- [MGN17] Thomas Müller, Markus Gross, and Jan Novák. “Practical Path Guiding for Efficient Light-Transport Simulation”. In: *Computer Graphics Forum* 36.4 (2017), 91–100. ISSN: 01677055. DOI: [10.1111/cgf.13227](https://doi.org/10.1111/cgf.13227). URL: <https://onlinelibrary.wiley.com/doi/10.1111/cgf.13227>.
- [Mü+19] Thomas Müller, Brian McWilliams, Fabrice Rousselle, Markus Gross, and Jan Novák. “Neural Importance Sampling”. In: *ACM Transactions on Graphics* 38.5 (2019). ISSN: 0730-0301, 1557-7368. DOI: [10.1145/3341156](https://doi.org/10.1145/3341156). URL: <https://dl.acm.org/doi/10.1145/3341156>.
- [Neh+07] Diego Nehab, Pedro V Sander, Jason Lawrence, Natalya Tatarchuk, and John R Isidoro. “Accelerating real-time shading with reverse reprojection caching”. In: *Graphics hardware*. Vol. 41. 2007, pp. 61–62.

- [Net] Netflix. *Netflix Prize*. <https://www.netflixprize.com/>. Accessed: 2020-03-22.
- [NKS19] Luong Trung Nguyen, Junhan Kim, and Byonghyo Shim. “Low-Rank Matrix Completion: A Contemporary Survey”. In: *IEEE Access* 7 (2019), 94215–94237. ISSN: 2169-3536. DOI: 10.1109/ACCESS.2019.2928130. URL: <https://ieeexplore.ieee.org/document/8759045/>.
- [OP] Jiawei Ou and Fabio Pellacini. *Lightslice source code*. <https://jiawei.ooo/lightslice>. Accessed: 2020-03-23.
- [OP11] Jiawei Ou and Fabio Pellacini. “LightSlice: matrix slice sampling for the many-lights problem”. In: *ACM Transactions on graphics (TOG)* 30.6 (2011), p. 179.
- [Pho] *Phosphorescence*. <https://fr.wikipedia.org/wiki/Phosphorescence>. Accessed: 2022-05-04.
- [Pix] Pixabay. *Pixabay*. <https://pixabay.com/illustrations/fractal-neon-rings-geometric-1937572/>. Accessed: 2021-11-23.
- [Rai] *rainbow*. <https://en.wiktionary.org/wiki/rainbow>. Accessed: 2022-05-03.
- [RD16] Hauke Rehfeld and Carsten Dachsbacher. “Lightcut interpolation”. In: *Proceedings of High Performance Graphics*. 2016, pp. 99–108.
- [Rit+07] Tobias Ritschel, Thorsten Grosch, Jan Kautz, and Stefan Müller. “Interactive illumination with coherent shadow maps”. In: *Proceedings of the 18th Eurographics conference on Rendering Techniques*. Eurographics Association. 2007, pp. 61–72.
- [Rit+12] Tobias Ritschel, Carsten Dachsbacher, Thorsten Grosch, and Jan Kautz. “The State of the Art in Interactive Global Illumination”. In: *Computer Graphics Forum* 31.1 (2012), 160–188. ISSN: 01677055. DOI: 10.1111/j.1467-8659.2012.02093.x. URL: <https://onlinelibrary.wiley.com/doi/10.1111/j.1467-8659.2012.02093.x>.
- [Sch+17] Christoph Schied, Anton Kaplanyan, Chris Wyman, Anjul Patney, Chakravarty R. Alla Chaitanya, John Burgess, Shiqiu Liu, Carsten Dachsbacher, Aaron Lefohn, and Marco Salvi. “Spatiotemporal variance-guided filtering: real-time reconstruction for path-traced global illumination”. In: *Proceedings of High Performance Graphics 2017*. ACM, 2017. ISBN: 9781450351010. DOI: 10.1145/3105762.3105770. URL: <https://dl.acm.org/doi/10.1145/3105762.3105770>.
- [SD11] P Sen and S Darabi. “Compressive Rendering: A Rendering Application of Compressed Sensing”. In: *IEEE Transactions on Visualization and Computer Graphics* 17.4 (2011), 487–499. ISSN: 1077-2626. DOI: 10.1109/TVCG.2010.46. URL: <http://ieeexplore.ieee.org/document/5432169/>.
- [SG92] A. F. M. Smith and A. E. Gelfand. “Bayesian Statistics without Tears: A Sampling-Resampling Perspective”. In: *The American Statistician* 46.2 (1992), p. 84. ISSN: 00031305. DOI: 10.2307/2684170. URL: <https://www.jstor.org/stable/2684170?origin=crossref>.
- [SL06] Joshua Steinhurst and Anselmo Lastra. “Global importance sampling of glossy surfaces using the photon map”. In: *Proceedings of the 2006 IEEE Symposium on Interactive Ray Tracing*. IEEE. 2006, pp. 133–138.
- [Soa] *Soap Bubble*. https://fr.m.wikipedia.org/wiki/Fichier:Soap_Bubble_-_foliage_background_-_iridescent_colours_-_Traquair_040801.jpg. Accessed: 2022-05-04.
- [SPD18] Christoph Schied, Christoph Peters, and Carsten Dachsbacher. “Gradient Estimation for Real-time Adaptive Temporal Filtering”. In: *Proceedings of the ACM on Computer Graphics and Interactive Techniques* 1.2 (2018). ISSN: 2577-6193. DOI: 10.1145/3233301. URL: <https://dl.acm.org/doi/10.1145/3233301>.
- [SWZ96] Peter Shirley, Changyaw Wang, and Kurt Zimmerman. “Monte Carlo techniques for direct lighting calculations”. In: *ACM Transactions on Graphics* 15.1 (1996). ISSN: 0730-0301, 1557-7368. DOI: 10.1145/226150.226151. URL: <https://dl.acm.org/doi/10.1145/226150.226151>.

- [Tan93] Martin A. Tanner. *Tools for Statistical Inference*. Springer Series in Statistics. Springer US, 1993. ISBN: 9781468401943. DOI: [10.1007/978-1-4684-0192-9](https://doi.org/10.1007/978-1-4684-0192-9). URL: <http://link.springer.com/10.1007/978-1-4684-0192-9>.
- [Tat+14] Natasha Tatarchuk, Brian Karis, Michal Drobot, Nicolas Schulz, Jerome Charles, and Theodor Mader. “Advances in real-time rendering in games, part I (full text not available)”. en. In: *ACM SIGGRAPH 2014 Courses on - SIGGRAPH '14*. Vancouver, Canada: ACM Press, 2014, 1–1. ISBN: 9781450329620. DOI: [10.1145/2614028.2615455](https://doi.org/10.1145/2614028.2615455). URL: <http://dl.acm.org/citation.cfm?doid=2614028.2615455>.
- [TCE05] Justin F Talbot, David Cline, and Parris Egbert. “Importance resampling for global illumination”. In: *Proceedings of the 16th Eurographics conference on Rendering Techniques*. 2005, pp. 139–146.
- [TTT99] Kim-Chuan Toh, Michael J Todd, and Reha H Tütüncü. “SDPT3—a MATLAB software package for semidefinite programming, version 1.3”. In: *Optimization methods and software* 11.1-4 (1999), pp. 545–581.
- [Tul] Andrew Tulloch. *Fast Randomized SVD*. <https://research.fb.com/blog/2014/09/fast-randomized-svd/>. Accessed: 2020-03-22.
- [Vea98] Eric Veach. *Robust Monte Carlo methods for light transport simulation*. Stanford University, 1998.
- [VG95] Eric Veach and Leonidas J. Guibas. “Optimally combining sampling techniques for Monte Carlo rendering”. In: *Proceedings of the 22nd annual conference on Computer graphics and interactive techniques - SIGGRAPH '95*. ACM Press, 1995, 419–428. ISBN: 9780897917018. DOI: [10.1145/218380.218498](https://doi.org/10.1145/218380.218498). URL: <http://portal.acm.org/citation.cfm?doid=218380.218498>.
- [VG97] Eric Veach and Leonidas J. Guibas. “Metropolis light transport”. In: *Proceedings of the 24th annual conference on Computer graphics and interactive techniques - SIGGRAPH '97*. ACM Press, 1997, 65–76. ISBN: 9780897918961. DOI: [10.1145/258734.258775](https://doi.org/10.1145/258734.258775). URL: <http://portal.acm.org/citation.cfm?doid=258734.258775>.
- [VKK18] Petr Vévoda, Ivo Kondapaneni, and Jaroslav Křivánek. “Bayesian online regression for adaptive direct illumination sampling”. In: *ACM Transactions on Graphics* 37.4 (2018). ISSN: 0730-0301, 1557-7368. DOI: [10.1145/3197517.3201340](https://doi.org/10.1145/3197517.3201340). URL: <https://dl.acm.org/doi/10.1145/3197517.3201340>.
- [Vor+14] Jiří Vorba, Ondřej Karlík, Martin Šik, Tobias Ritschel, and Jaroslav Křivánek. “On-line learning of parametric mixture models for light transport simulation”. In: *ACM Transactions on Graphics* 33.4 (2014). ISSN: 0730-0301, 1557-7368. DOI: [10.1145/2601097.2601203](https://doi.org/10.1145/2601097.2601203). URL: <https://dl.acm.org/doi/10.1145/2601097.2601203>.
- [WÅ09] Rui Wang and Oskar Åkerlund. “Bidirectional Importance Sampling for Unstructured Direct Illumination”. In: *Computer Graphics Forum* 28.2 (2009), 269–278. ISSN: 01677055, 14678659. DOI: [10.1111/j.1467-8659.2009.01366.x](https://doi.org/10.1111/j.1467-8659.2009.01366.x). URL: <https://onlinelibrary.wiley.com/doi/10.1111/j.1467-8659.2009.01366.x>.
- [Wal+05] Bruce Walter, Sebastian Fernandez, Adam Arbree, Kavita Bala, Michael Donikian, and Donald P. Greenberg. “Lightcuts: a scalable approach to illumination”. In: *ACM Transactions on Graphics* 24.3 (2005), 1098–1107. ISSN: 0730-0301, 1557-7368. DOI: [10.1145/1073204.1073318](https://doi.org/10.1145/1073204.1073318). URL: <https://dl.acm.org/doi/10.1145/1073204.1073318>.
- [Wal+06] Bruce Walter, Adam Arbree, Kavita Bala, and Donald P. Greenberg. “Multidimensional lightcuts”. In: *ACM Transactions on Graphics* 25.3 (2006), 1081–1088. ISSN: 0730-0301, 1557-7368. DOI: [10.1145/1141911.1141997](https://doi.org/10.1145/1141911.1141997). URL: <https://dl.acm.org/doi/10.1145/1141911.1141997>.
- [Wal+07] Bruce Walter, Stephen R Marschner, Hongsong Li, and Kenneth E Torrance. “Microfacet Models for Refraction through Rough Surfaces”. In: *Rendering Techniques 2007* (2007), 18th.

- [Wal+08] Bruce Walter, Kavita Bala, Milind Kulkarni, and Keshav Pingali. “Fast agglomerative clustering for rendering”. In: *Proceedings of the 2008 IEEE Symposium on Interactive Ray Tracing*. IEEE. 2008, pp. 81–86.
- [Wan+09] Jiaping Wang, Yue Dong, Xin Tong, Zhouchen Lin, and Baining Guo. “Kernel Nyström method for light transport”. In: *ACM Transactions on Graphics* 28.3 (2009). ISSN: 0730-0301, 1557-7368. DOI: [10.1145/1531326.1531335](https://doi.org/10.1145/1531326.1531335). URL: <https://dl.acm.org/doi/10.1145/1531326.1531335>.
- [WC13] Yu-Ting Wu and Yung-Yu Chuang. “VisibilityCluster: Average Directional Visibility for Many-Light Rendering”. In: *IEEE Transactions on Visualization and Computer Graphics* 19.9 (2013), 1566–1578. ISSN: 1077-2626. DOI: [10.1109/TVCG.2013.21](https://doi.org/10.1109/TVCG.2013.21). URL: <http://ieeexplore.ieee.org/document/6464264/>.
- [WDP99] Bruce Walter, George Drettakis, and Steven Parker. “Interactive rendering using the render cache”. In: *Eurographics Workshop on Rendering Techniques*. Springer. 1999, pp. 19–30.
- [Wil78] Lance Williams. “Casting curved shadows on curved surfaces”. en. In: *ACM SIGGRAPH Computer Graphics* 12.3 (1978), 270–274. ISSN: 0097-8930. DOI: [10.1145/965139.807402](https://doi.org/10.1145/965139.807402). URL: <https://dl.acm.org/doi/10.1145/965139.807402>.
- [WKB12a] Bruce Walter, Pramook Khungurn, and Kavita Bala. “Bidirectional lightcuts”. In: *ACM Transactions on Graphics* 31.4 (2012), 1–11. ISSN: 0730-0301, 1557-7368. DOI: [10.1145/2185520.2185555](https://doi.org/10.1145/2185520.2185555). URL: <https://dl.acm.org/doi/10.1145/2185520.2185555>.
- [WKB12b] Bruce Walter, Pramook Khungurn, and Kavita Bala. “Bidirectional lightcuts”. In: *ACM Transactions on Graphics (TOG)* 31.4 (2012), p. 59.
- [WRC88] Gregory J. Ward, Francis M. Rubinstein, and Robert D. Clear. “A ray tracing solution for diffuse interreflection”. en. In: *ACM SIGGRAPH Computer Graphics* 22.4 (1988), 85–92. ISSN: 0097-8930. DOI: [10.1145/378456.378490](https://doi.org/10.1145/378456.378490). URL: <https://dl.acm.org/doi/10.1145/378456.378490>.
- [Yuk19] Cem Yuksel. “Stochastic Lightcuts”. In: *Proceedings of High-Performance Graphics 2019*. Strasbourg, France: The Eurographics Association, 2019, pp. 27–32. ISBN: 978-3-03868-092-5. DOI: [10.2312/hpg.20191192](https://doi.org/10.2312/hpg.20191192).
- [Zen+21] Zheng Zeng, Shiqiu Liu, Jinglei Yang, Lu Wang, and Ling-Qi Yan. “Temporally Reliable Motion Vectors for Real-time Ray Tracing”. In: *Computer Graphics Forum* 40.2 (2021), 79–90. ISSN: 0167-7055, 1467-8659. DOI: [10.1111/cgf.142616](https://doi.org/10.1111/cgf.142616). URL: <https://onlinelibrary.wiley.com/doi/10.1111/cgf.142616>.
- [Zim+15] Henning Zimmer, Fabrice Rousselle, Wenzel Jakob, Oliver Wang, David Adler, Wojciech Jarosz, Olga Sorkine-Hornung, and Alexander Sorkine-Hornung. “Path-space Motion Estimation and Decomposition for Robust Animation Filtering”. In: *Computer Graphics Forum* 34.4 (2015), 131–142. ISSN: 0167-7055, 1467-8659. DOI: [10.1111/cgf.12685](https://doi.org/10.1111/cgf.12685). URL: <https://onlinelibrary.wiley.com/doi/10.1111/cgf.12685>.
- [ZS19] Jiaxin Zhang and Michael D. Shields. “Efficient Monte Carlo resampling for probability measure changes from Bayesian updating”. In: *Probabilistic Engineering Mechanics* 55 (2019), 54–66. ISSN: 02668920. DOI: [10.1016/j.probengmech.2018.10.002](https://doi.org/10.1016/j.probengmech.2018.10.002). URL: <https://linkinghub.elsevier.com/retrieve/pii/S0266892018300535>.
- [ZZ19] Quan Zheng and Matthias Zwicker. “Learning to Importance Sample in Primary Sample Space”. In: *Computer Graphics Forum* 38.2 (2019), 169–179. ISSN: 0167-7055, 1467-8659. DOI: [10.1111/cgf.13628](https://doi.org/10.1111/cgf.13628). URL: <https://onlinelibrary.wiley.com/doi/10.1111/cgf.13628>.

Appendix A

Additional Results for BAMC

A.1 BAMC Scalability Results

		San Miguel			Classroom			Kitchen		
		1280 x 720 @ 4, 100k VPLs			1280 x 720 @ 4, 100k VPLs			1280 x 720 @ 4, 100k VPLs		
		Time	RMSE	Samples	Time	RMSE	Samples	Time	RMSE	Samples
AMC	$\alpha=10\%$ cols=1000	88.10s	0.0254	15.36%	73.37s	0.0221	17.17%	81.68s	0.0904	15.85%
	$\alpha=12.5\%$ cols=1000	121.75s	0.0265	18.15%	108.82s	0.0201	19.93%	118.69s	0.0905	18.53%
	$\alpha=15\%$ cols=1000	189.23s	0.0246	20.70%	206.63s	0.0194	22.58%	201.55s	0.0899	21.18%
	$\alpha=17.5\%$ cols=1000	350.70s	0.0204	23.31%	427.37s	0.0197	25.13%	372.51s	0.0918	23.71%
	$\alpha=20\%$ cols=1000	568.07s	0.0198	25.84%	828.32s	0.0198	27.69%	654.42s	0.0900	26.22%
	$\alpha=10\%$ cols=1500	127.15s	0.0267	14.58%	110.91s	0.0204	16.22%	120.04s	0.0819	14.87%
	$\alpha=10\%$ cols=2000	164.23s	0.0276	13.99%	146.70s	0.0210	15.52%	156.29s	0.0743	14.27%
	$\alpha=10\%$ cols=2500	197.10s	0.0284	13.49%	180.95s	0.0211	14.90%	189.98s	0.0687	13.82%
	$\alpha=10\%$ cols=3000	230.15s	0.0297	13.15%	207.66s	0.0241	14.38%	224.51s	0.0633	13.50%
BAMC	$\alpha=10\%$ cols=1000	65.42s	0.0235	17.30%	52.16s	0.0199	19.65%	52.31s	0.0894	17.13%
	$\alpha=12.5\%$ cols=1000	72.60s	0.0207	19.86%	57.57s	0.0192	22.21%	58.01s	0.0909	19.68%
	$\alpha=15\%$ cols=1000	79.86s	0.0198	22.33%	62.93s	0.0195	24.66%	62.91s	0.0910	22.19%
	$\alpha=17.5\%$ cols=1000	86.17s	0.0203	24.82%	67.72s	0.0180	27.00%	67.88s	0.0900	24.59%
	$\alpha=20\%$ cols=1000	92.49s	0.0194	27.17%	71.61s	0.0195	29.34%	72.36s	0.0894	26.98%
	$\alpha=10\%$ cols=1500	95.49s	0.0221	16.70%	76.70s	0.0182	19.28%	75.63s	0.0792	16.38%
	$\alpha=10\%$ cols=2000	125.32s	0.0231	16.33%	101.31s	0.0191	18.96%	98.02s	0.0747	15.91%
	$\alpha=10\%$ cols=2500	154.20s	0.0240	15.95%	124.92s	0.0192	18.63%	120.60s	0.0696	15.57%
	$\alpha=10\%$ cols=3000	183.14s	0.0252	15.68%	152.80s	0.0197	18.25%	143.31s	0.0624	15.30%
BAMC (GE)	$\alpha=10\%$ cols=1000	157.68s	0.0201	18.60%	167.77s	0.0191	21.73%	101.51s	0.0914	19.64%
	$\alpha=12.5\%$ cols=1000	165.92s	0.0183	20.97%	172.93s	0.0190	23.81%	104.02s	0.0899	21.59%
	$\alpha=15\%$ cols=1000	172.00s	0.0187	23.17%	176.22s	0.0195	26.00%	106.66s	0.0904	23.57%
	$\alpha=17.5\%$ cols=1000	178.27s	0.0184	25.52%	180.57	0.0189	28.13%	110.81s	0.0909	25.76%
	$\alpha=20\%$ cols=1000	184.20s	0.0183	27.75%	183.62s	0.0189	30.28%	113.52s	0.0915	27.90%
	$\alpha=10\%$ cols=1500	330.97s	0.0192	18.43%	402.28s	0.0203	21.92%	186.77s	0.0824	19.74%
	$\alpha=10\%$ cols=2000	528.14s	0.0228	18.14%	621.92s	0.0258	21.79%	312.96s	0.0749	20.38%
	$\alpha=10\%$ cols=2500	767.49s	0.0276	17.96%	832.50s	0.0278	21.56%	476.72s	0.0700	20.74%
	$\alpha=10\%$ cols=3000	1043.99s	0.0322	17.77%	1039.84s	0.0448	21.27%	669.25s	0.0657	21.05%

TABLE A.1: Boolean Adaptive Matrix Completion statistics when compared to the original algorithm and the Gaussian elimination variant in the Classroom, San Miguel, and Kitchen scenes.

Table A.1 provides full results on the scalability and accuracy of BAMC.

A.2 Full Results for Comparisons Against Other Many-Lights Methods

We provide equal time images along with their error images and their time versus error plots. These are as follows:

- Cornell Box in Figure A.1
- Hairball in Figure A.2
- Sponza in Figure A.3

Columns		250	500	1000	2000	4000
Bathroom 100k VSLs 1024x1024 @ 4	Time	182.39s	234.11s	366.67s	662.31s	1261.27s
	Error (RMSE)	0.176	0.146	0.123	0.104	0.094
Breakfast room 100k VPLs 1280x720 @ 4	Time	100.92s	154.01s	273.36s	515.86s	998.68s
	Error (RMSE)	0.019	0.017	0.015	0.014	0.012
Cornell box 100k VPLs 800x600 @ 4	Time	26.09s	48.95s	96.36s	191.89s	378.43s
	Error (RMSE)	0.0038	0.0027	0.0019	0.0014	0.001
Classroom 100k VPLs 1280x720 @ 4	Time	122.82s	172.86s	289.17s	534.38s	1049.91s
	Error (RMSE)	0.097	0.078	0.063	0.06	0.056
Hairball 100k VPLs 800x600 @ 4	Time	80.94s	118.34s	194.89s	348.77s	658.16s
	Error (RMSE)	0.013	0.01	0.0087	0.0079	0.0065
Living room 100k VPLs 1280x720 @ 4	Time	124.67s	175.59s	297.67s	553.17s	1070.74s
	Error (RMSE)	0.019	0.015	0.012	0.0104	0.0102
Kitchen 100k VSLs 1280x720 @ 4	Time	67.53s	132.24s	264.66s	527.48s	1051.77s
	Error (RMSE)	0.125	0.115	0.098	0.082	0.069
San-Miguel 100k VPLs 1280x720 @ 4	Time	156.38s	220.4s	353.02s	624.1s	1175.71s
	Error (RMSE)	0.026	0.023	0.019	0.02	0.018
Sponza 100k VPLs 1280x720 @ 4	Time	79.45s	146.24s	273.11s	538.55s	1052.02s
	Error (RMSE)	0.017	0.013	0.011	0.009	0.0092
Staircase 100k VPLs 720x1280 @ 4	Time	130.03s	179.01s	297.74s	556.01s	1077.31s
	Error (RMSE)	0.027	0.022	0.018	0.019	0.015
Modern Hall 100k VPLs 1024x1024 @ 4	Time	169.95s	232.95s	374.96s	672.75s	1264.93s
	Error (RMSE)	0.0097	0.069	0.052	0.042	0.037
Grey & White Room 100k VPLs 1280x720 @ 4	Time	113.03s	166.71s	285.74s	531.94s	1038.54s
	Error (RMSE)	0.095	0.072	0.059	0.049	0.048

TABLE A.2: Statistics for our LightSlice across all scenes.

- San Miguel in Figure [A.4](#)
- Staircase in Figure [A.5](#)
- Living Room in Figure [A.6](#)
- Breakfast Room in Figure [A.7](#)
- Classroom in Figure [A.8](#)
- Modern Hall in Figure [A.9](#)
- Bathroom in Figure [A.10](#)
- Kitchen in Figure [A.11](#)
- Grey & White Room in Figure [A.12](#)

We also provide tables detailing the full set of data obtained. Table [A.2](#) shows the full results for Lightslice, Table [A.3](#) shows the full results for IlluminationCut, Table [A.4](#) details the full results for Matrix Separation, and Table [A.5](#) shows the full results for our method, which includes using BAMC and full visibility sampling.

Error threshold		10%	5%	2%	1%	0.5%
Breakfast room 100k VPLs 1280x720 @ 4	Time	101.95s	150.14s	262.91s	440.74s	745.58s
	Error (RMSE)	0.09	0.061	0.039	0.027	0.019
	Visibility samples	51.06%	47.36%	44.55%	47.55%	50.05%
Cornell box 100k VPLs 800x600 @ 4	Time	41.76s	59.21s	102.33s	161.01s	250.67s
	Error (RMSE)	0.014	0.0077	0.0047	0.0033	0.0026
	Visibility samples	1.87%	2.43%	2.58%	3.13%	3.76%
Classroom 100k VPLs 1280x720 @ 4	Time	164.12s	224.24s	343.13s	486.74s	775.85s
	Error (RMSE)	0.088	0.064	0.046	0.034	0.029
	Visibility samples	20.96%	26.44%	35.75%	44.06%	54.85%
Hairball 100k VPLs 800x600 @ 4	Time	53.68s	79.52s	122.83s	188.41s	304.46s
	Error (RMSE)	0.021	0.016	0.011	0.0083	0.0067
	Visibility samples	19.65%	20.72%	20.61%	20.61%	21.44%
Living room 100k VPLs 1280x720 @ 4	Time	122.65s	179.08s	303.48s	493.86s	772.62s
	Error (RMSE)	0.063	0.044	0.037	0.03	0.027
	Visibility samples	4.05%	3.9%	3.81%	3.9%	3.9%
San-Miguel 100k VPLs 1280x720 @ 4	Time	105.5s	146.51s	260.18s	410.68s	717.16s
	Error (RMSE)	0.074	0.059	0.054	0.042	0.034
	Visibility samples	40.3%	38.79%	38.75%	41.16%	46.48%
Sponza 100k VPLs 1280x720 @ 4	Time	153.81s	230.32s	404.11s	614.11s	975.07s
	Error (RMSE)	0.117	0.075	0.056	0.047	0.035
	Visibility samples	18.42%	15.53%	16.66%	18.94%	22.62%
Staircase 100k VPLs 720x1280 @ 4	Time	95.51s	138.49s	261.03s	436.58s	772.84s
	Error (RMSE)	0.056	0.049	0.036	0.029	0.026
	Visibility samples	3.3%	3.4%	3.56%	3.6%	3.61%

TABLE A.3: Statistics for IlluminationCut across all scenes.

Max clusters		500	1000	2000	4000	8000
Breakfast room 100k VPLs 1280x720 @ 4	Time	30.46s	74.44s	173.18s	464.6s	1182.892s
	Error (RMSE)	0.035	0.034	0.034	0.034	0.033
	Visibility samples	50.48%	39.33%	30.17%	30.51%	29.67%
Cornell box 100k VPLs 800x600 @ 4	Time	7.83s	17.33s	36.16s	73.56s	135.44s
	Error (RMSE)	0.01	0.0094	0.0094	0.0094	0.0094
	Visibility samples	82.56%	82.46%	82.56%	81.77%	73.58%
Classroom 100k VPLs 1280x720 @ 4	Time	36.63s	76.03s	162.33s	353.43s	742.95s
	Error (RMSE)	0.032	0.029	0.027	0.026	0.026
	Visibility samples	38.41%	29.46%	23.37%	18.76%	14.65%
Hairball 100k VPLs 800x600 @ 4	Time	28.19s	65.52s	152.11s	344.92s	781.92s
	Error (RMSE)	0.02	0.02	0.02	0.02	0.019
	Visibility samples	45.2%	36.9%	31.81%	29.4%	51.39%
Living room 100k VPLs 1280x720 @ 4	Time	30.45s	68.69s	155.7s	350.38s	760.42s
	Error (RMSE)	0.014	0.013	0.012	0.014	0.014
	Visibility samples	65.09%	60.11%	60.39%	62.88%	65.12%
San-Miguel 100k VPLs 1280x720 @ 4	Time	28.23s	57.32s	125.23s	277.77s	618.08s
	Error (RMSE)	0.041	0.038	0.036	0.035	0.034
	Visibility samples	26.16%	16.9%	12.05%	9.42%	7.62%
Sponza 100k VPLs 1280x720 @ 4	Time	25.93s	60.65s	136.47s	299.73s	629.65s
	Error (RMSE)	0.029	0.026	0.026	0.026	0.025
	Visibility samples	50.7%	46.7%	45.43%	44.72%	43.82%
Staircase 100k VPLs 720x1280 @ 4	Time	38.26s	86.55s	182.42s	394.77s	835.47s
	Error (RMSE)	0.018	0.016	0.015	0.015	0.015
	Visibility samples	71.86%	67.47%	60.26%	57.73%	55.63%

TABLE A.4: Statistics for Matrix Separation across all scenes.

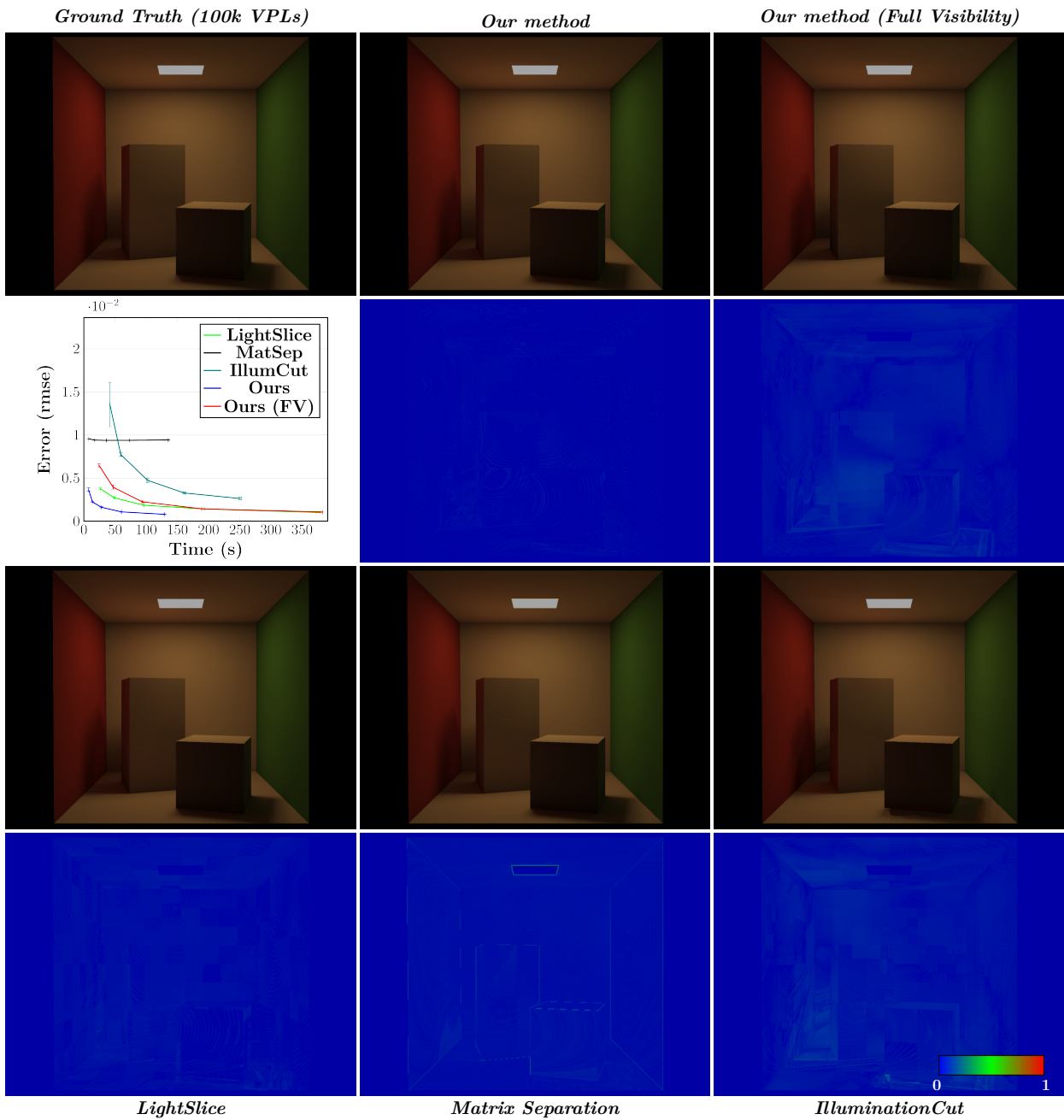


FIGURE A.1: Equal time renders (~ 30 s) and time vs error plot for Cornell box scene. Error images shows L_1 error.

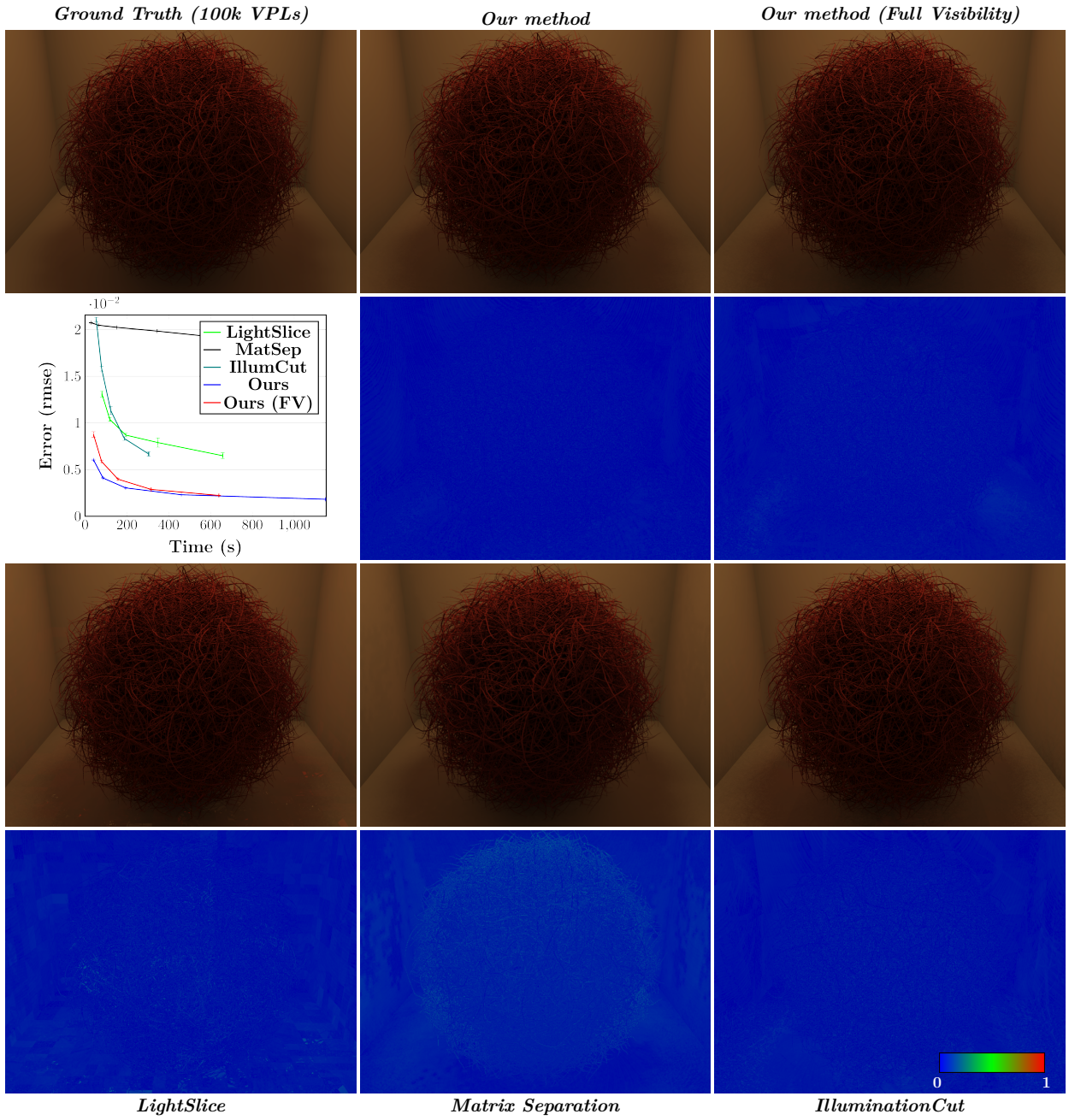


FIGURE A.2: Equal time renders (~80s) and time vs error plot for hairball scene. Error images shows L_1 error.

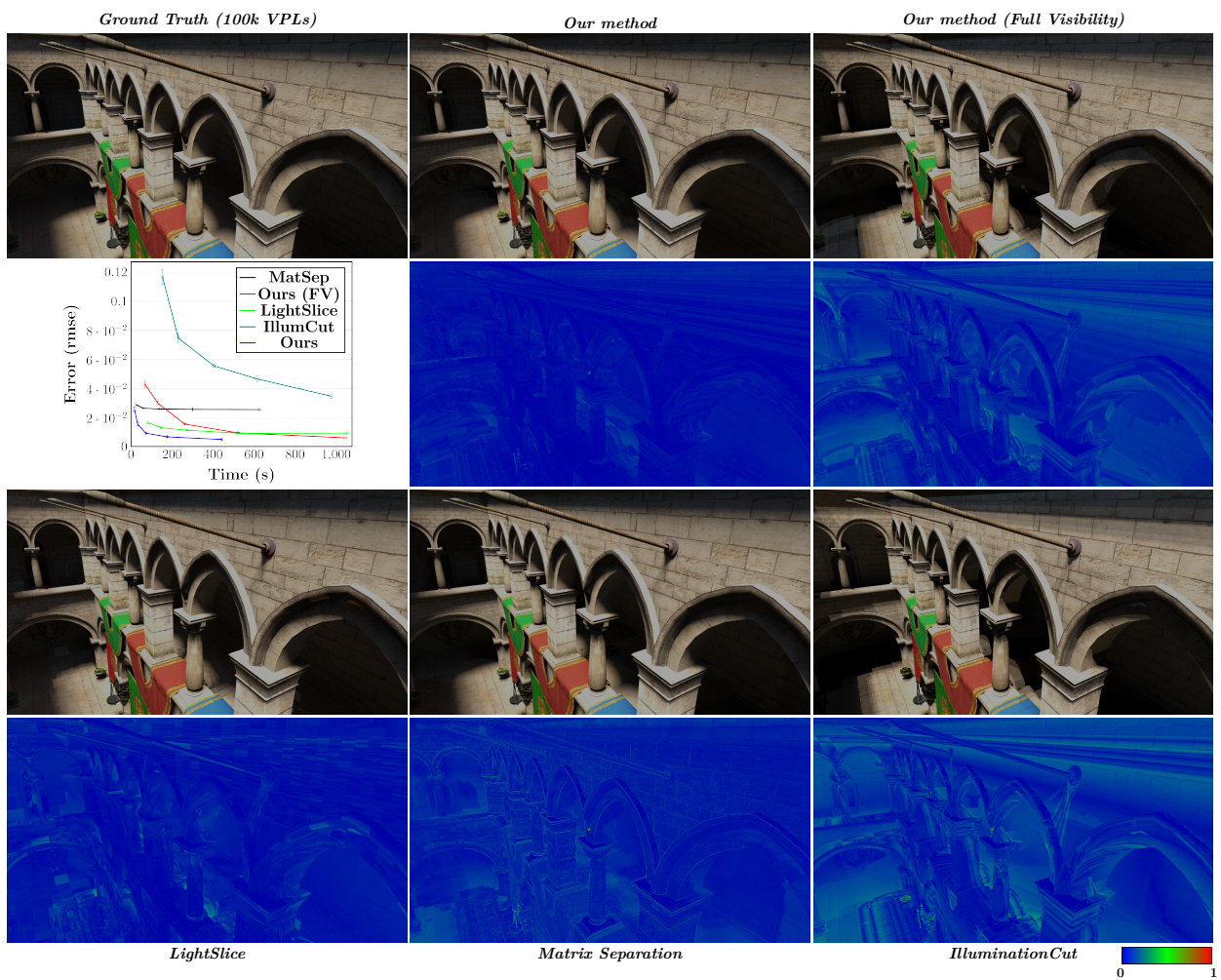


FIGURE A.3: Equal time renders (~ 140 s) and time vs error plot for Sponza scene. Error images show L_1 error.

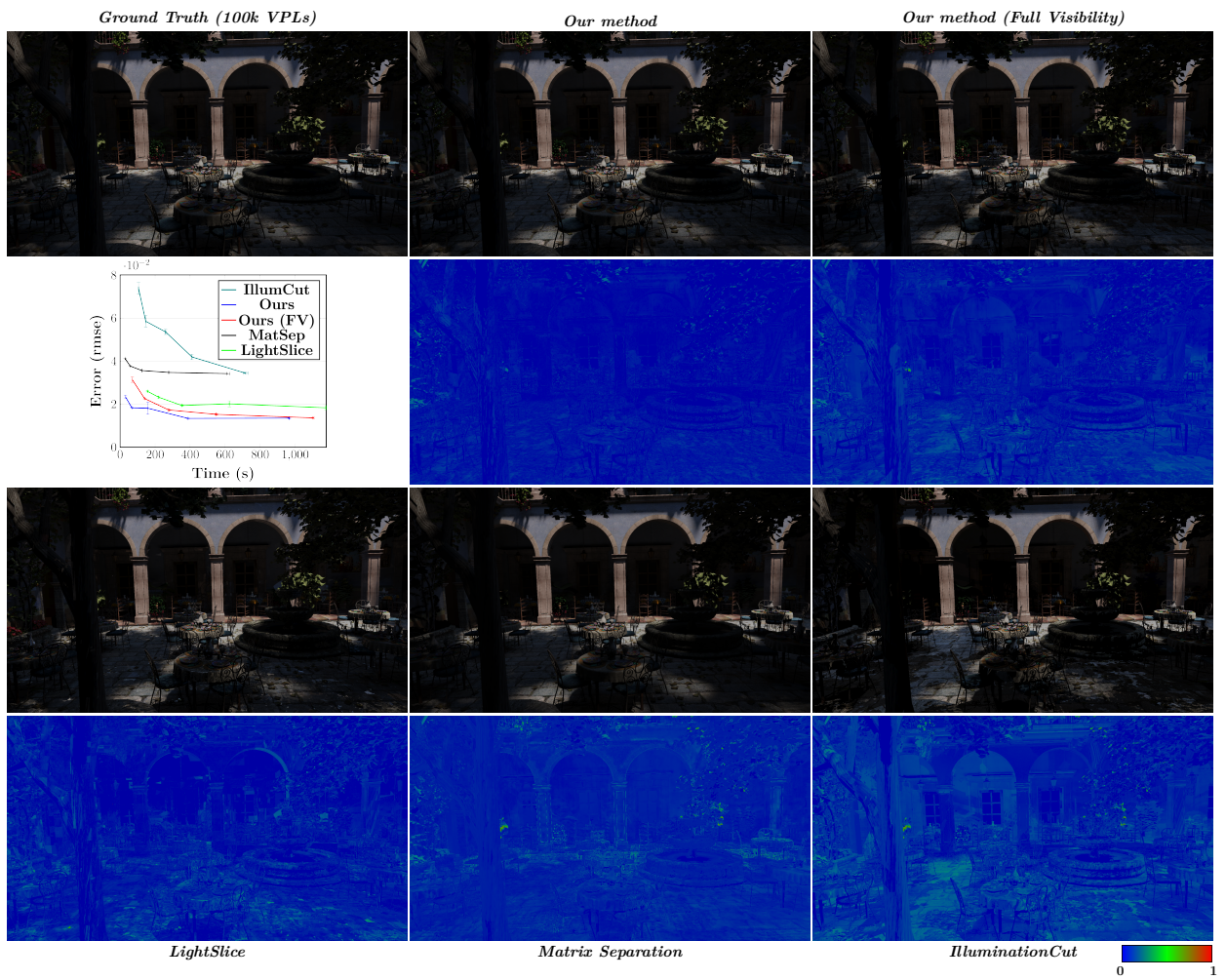


FIGURE A.4: Equal time renders (~125s) and time vs error plot for San Miguel scene. Error images shows L_1 error.

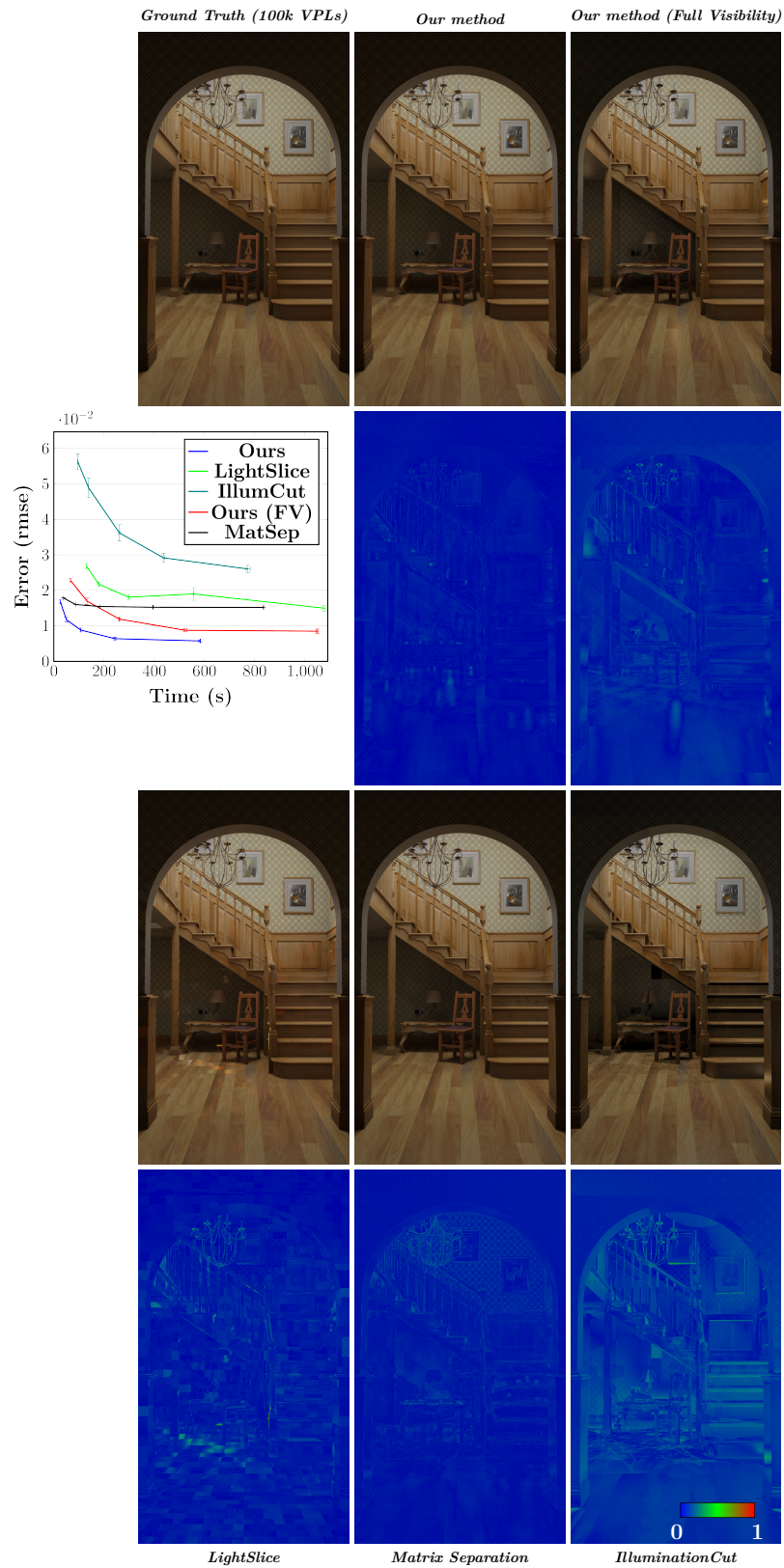


FIGURE A.5: Equal time renders (~ 110 s) and time vs error plot for staircase scene. Error images shows L_1 error.

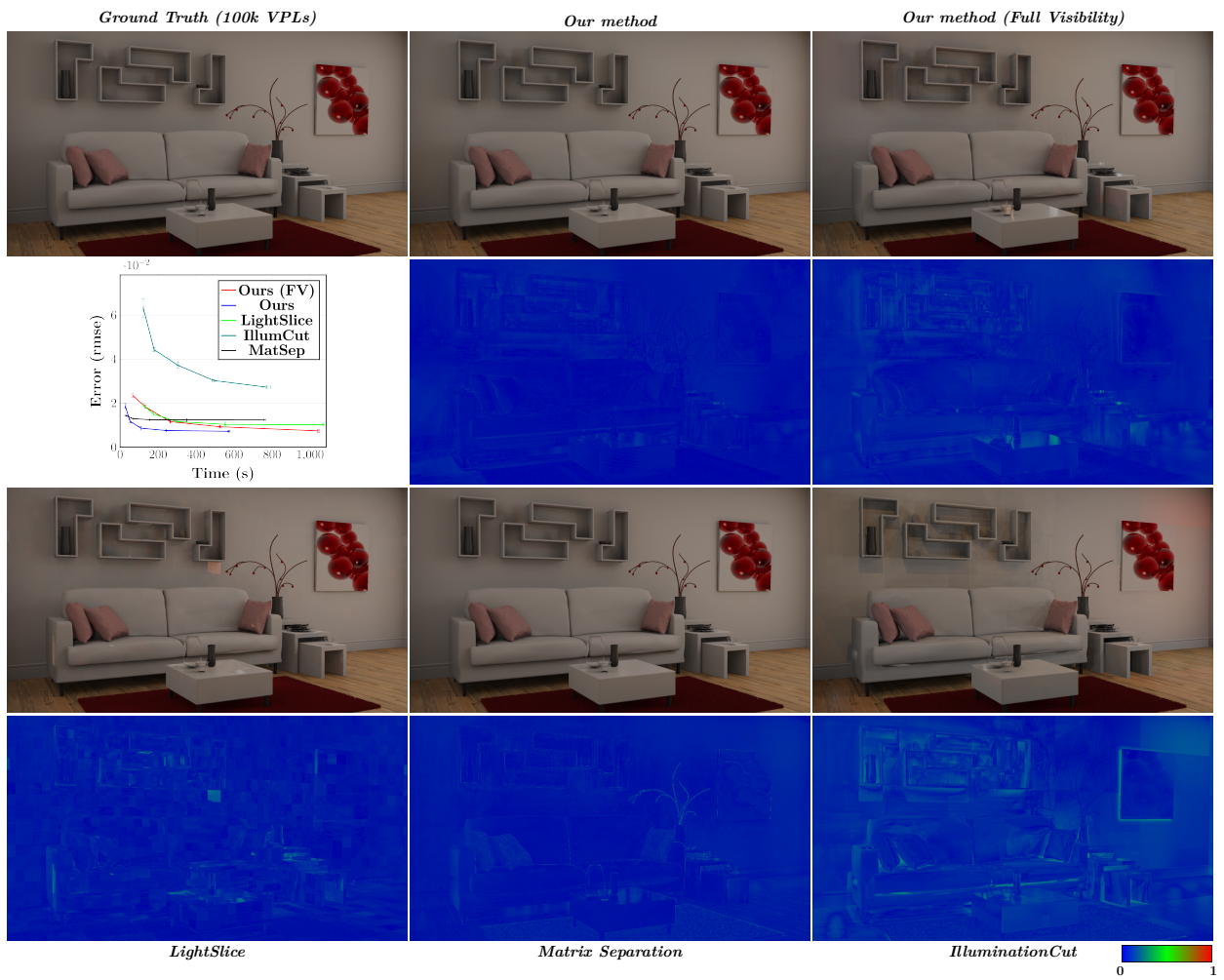


FIGURE A.6: Equal time renders (~110s) and time vs error for living room scene. Error images shows L_1 error.

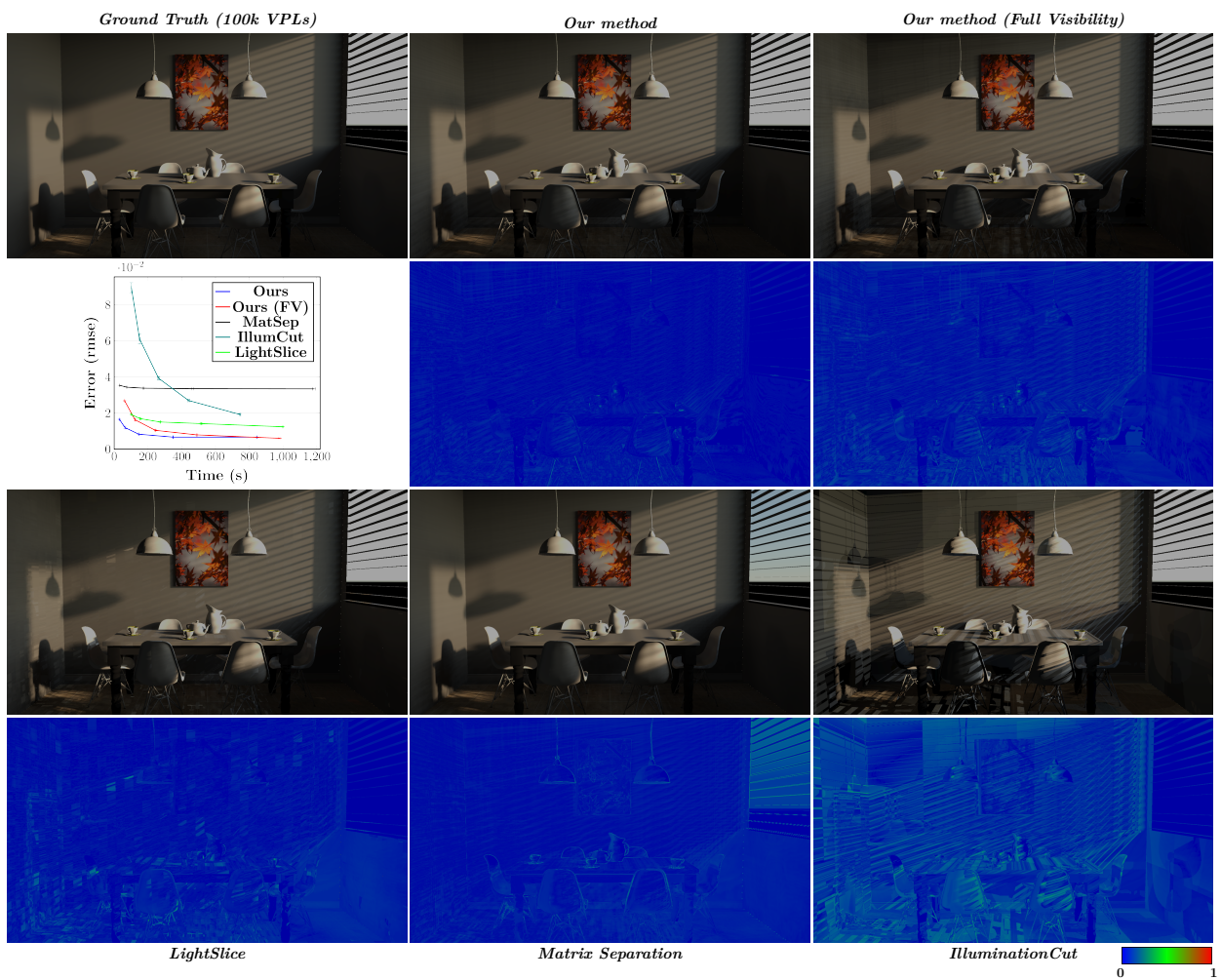


FIGURE A.7: Equal time renders (~ 80 s) and time vs error plot for breakfast room scene. Error images shows L_1 error.

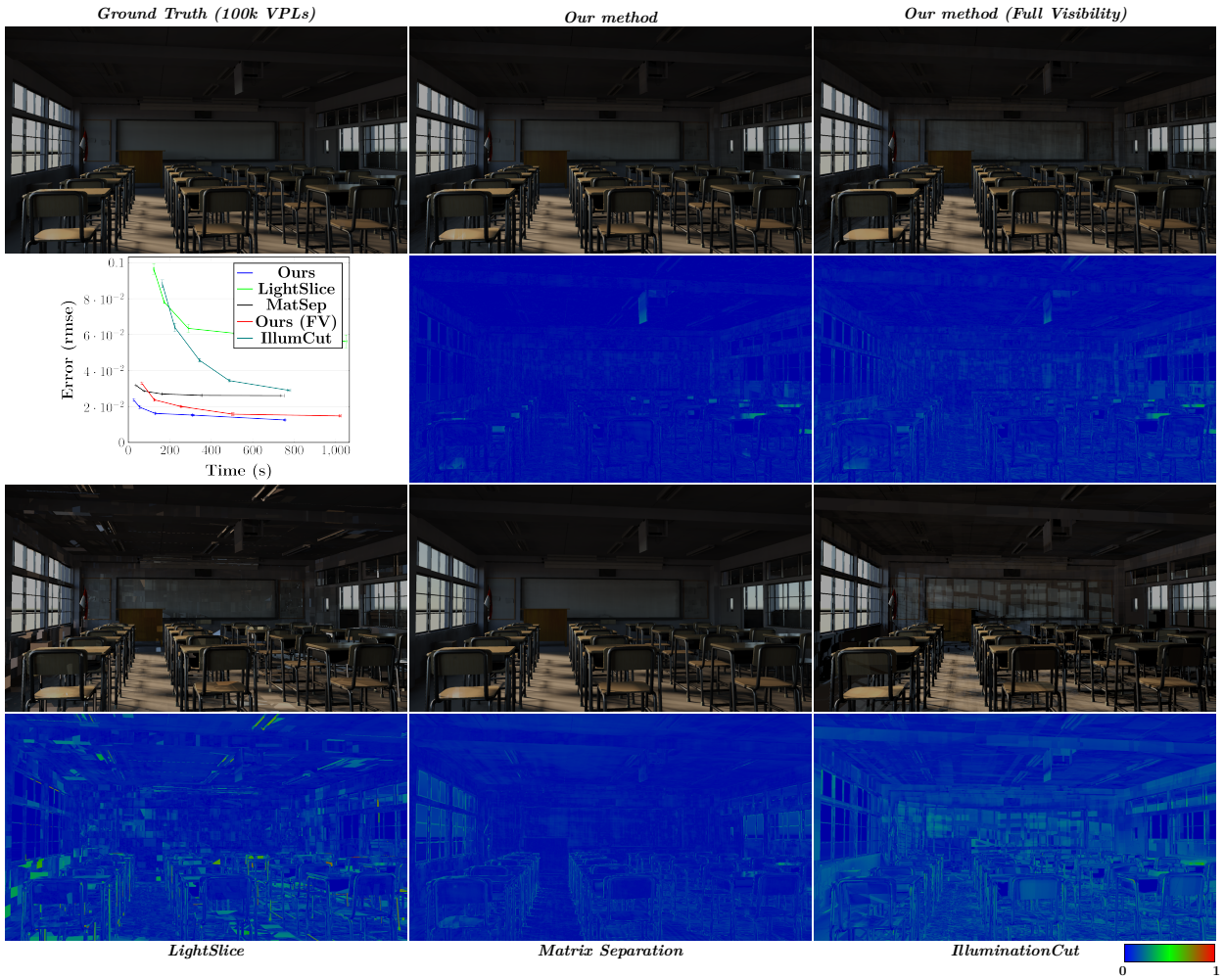


FIGURE A.8: Equal time renders (~55s) and time vs error plot for classroom scene. Error images shows L_1 error.

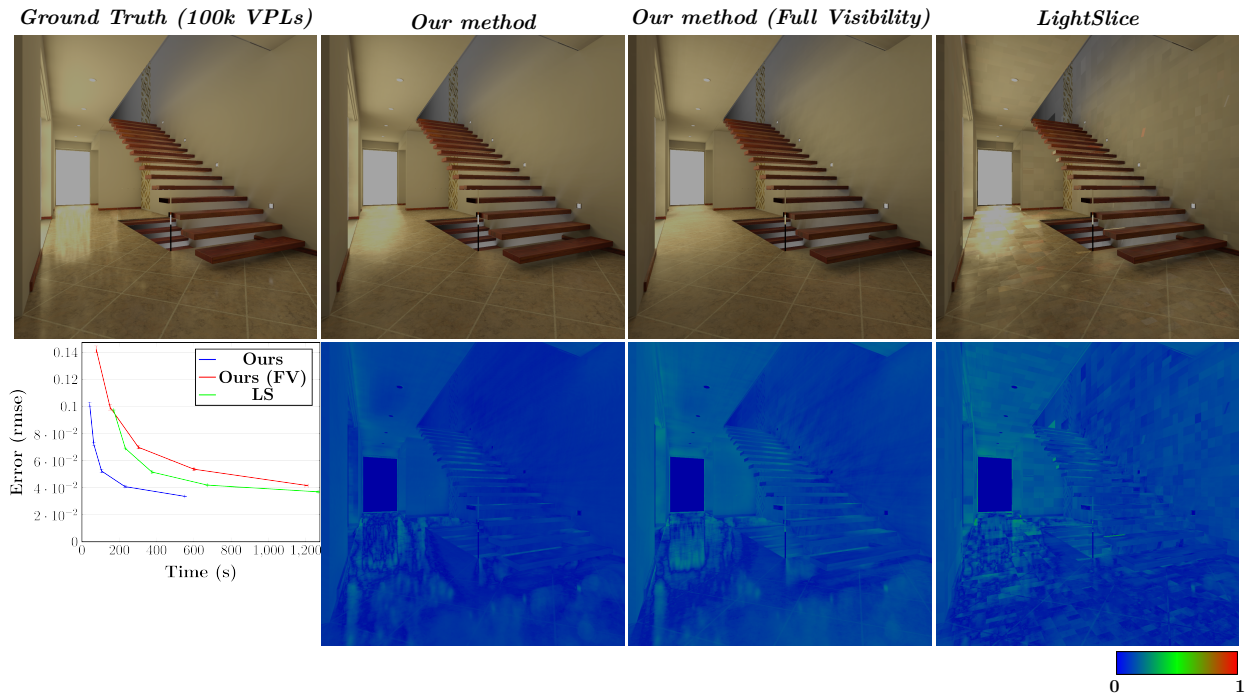


FIGURE A.9: Equal time renders (~230s) and time vs error for Modern Hall scene. Error images shows L_1 error.

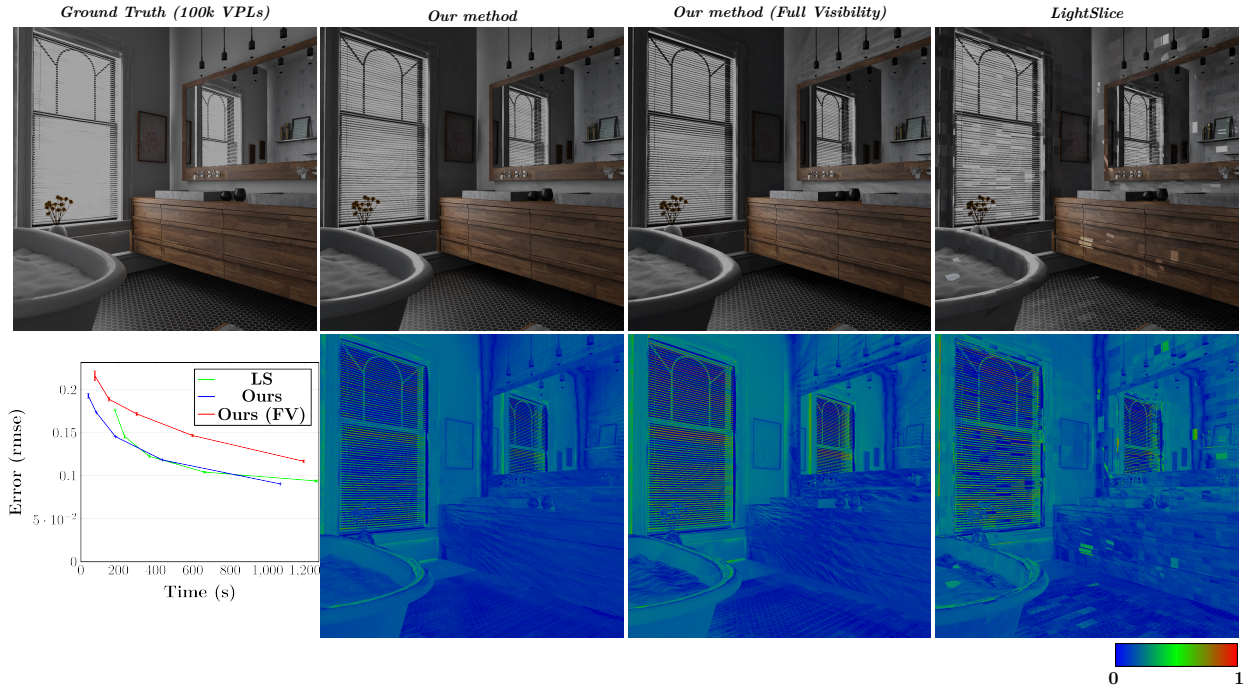


FIGURE A.10: Equal time renders (~185s) and time vs error plot for bathroom scene. Error images shows L_1 error.

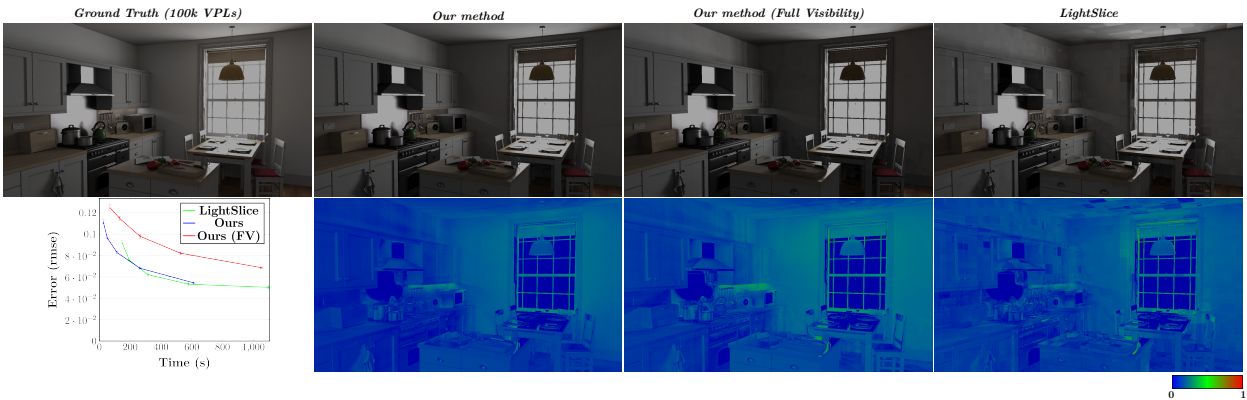


FIGURE A.11: Equal time renders (~60s) and time vs error for kitchen scene. Error images shows L_1 error.

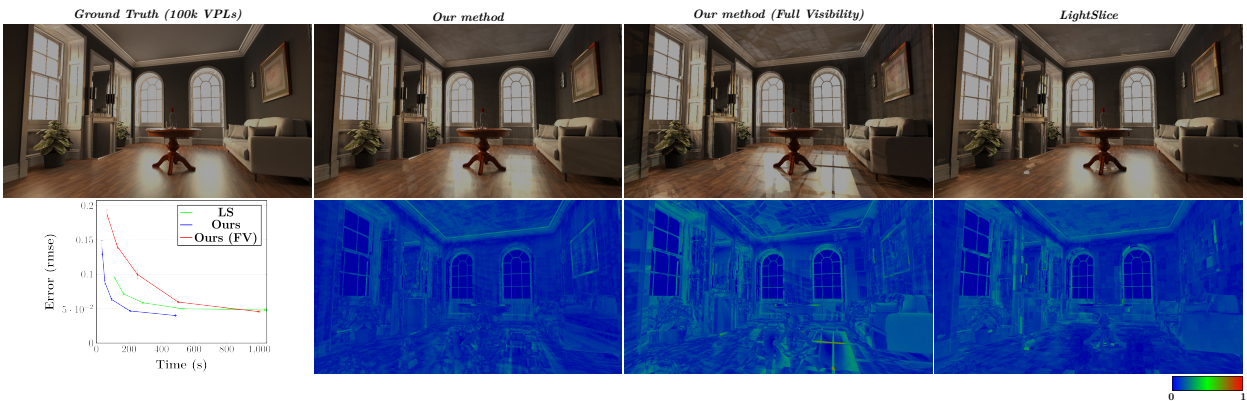


FIGURE A.12: Equal time renders (~220s) and time vs error for Grey & White Room scene. Error images shows L_1 error.

Clusters per slice		250	500	1000	2000	4000	8000
Bathroom 100k VSLs 1024x1024 @ 4	Time		39.42s	81.98s	185.48s	436.16s	1068.98s
	Time (only clustering)	74.97s	150.26s	299.33s	597.96s	1194.73s	
	Error (RMSE)		0.193	0.173	0.146	0.119	0.091
	Error (RMSE, only clustering)	0.216	0.189	0.172	0.149	0.119	
	Visibility samples		19.92%	21.16%	23.52%	26.48%	29.4%
Breakfast room 100k VPLs 1280x720 @ 4	Time		30.89s	63.82s	145.78s	348.78s	845.09s
	Time (only clustering)	61.74s	123.12s	243.96s	489.2s	980.99s	
	Error (RMSE)		0.017	0.012	0.0083	0.0067	0.0067
	Error (RMSE, only clustering)	0.027	0.016	0.0105	0.008	0.0061	
	Visibility samples		18.9%	19.95%	22.76%	27.36%	31.99%
Cornell box 100k VPLs 800x600 @ 4	Time		7.51s	13.83s	28.19s	60.33s	129.12s
	Time (only clustering)	24.28s	46.95s	93.72s	189.95s	381.05s	
	Error (RMSE)		0.0037	0.0022	0.0016	0.0011	0.0008
	Error (RMSE, only clustering)	0.0065	0.0039	0.0023	0.0014	0.0011	
	Visibility samples		9.2%	9.26%	9.89%	10.99%	12.68%
Classroom 100k VPLs 1280x720 @ 4	Time		26.38s	54.99s	129.243s	310.51s	755.81s
	Time (only clustering)	65s	125.44s	253.82s	504.44s	1022.6s	
	Error (RMSE)		0.024	0.0196	0.0162	0.0152	0.0125
	Error (RMSE, only clustering)	0.033	0.024	0.02	0.016	0.015	
	Visibility samples		15.71%	16.83%	19.96%	23.77%	27.33%
Hairball 100k VPLs 800x600 @ 4	Time		40.56s	86.11s	194.23s	460.43s	1150.02s
	Time (only clustering)	40.53s	79.35s	157.71s	316.36s	639.95s	
	Error (RMSE)		0.006	0.0041	0.003	0.0023	0.0019
	Error (RMSE, only clustering)	0.0088	0.0059	0.004	0.0029	0.0022	
	Visibility samples		36.71%	37.98%	40.19%	43.11%	46.45%
Living room 100k VPLs 1280x720 @ 4	Time		27.09s	52.07s	109.36s	244.1s	574.04s
	Time (only clustering)	66.06s	132.77s	263.48s	526.1s	1043.42s	
	Error (RMSE)		0.019	0.012	0.0086	0.0076	0.0072
	Error (RMSE, only clustering)	0.024	0.018	0.012	0.0093	0.0074	
	Visibility samples		14.35%	13.46%	13.9%	15.53%	18.45%
Kitchen 100k VSLs 1280x720 @ 4	Time		25.2s	51.49s	114.15s	262.41s	612.66s
	Time (only clustering)	67.53s	132.24s	264.66s	527.48s	1051.77s	
	Error (RMSE)		0.112	0.097	0.083	0.068	0.054
	Error (RMSE, only clustering)	0.125	0.115	0.098	0.082	0.069	
	Visibility samples		15.56%	15.03%	16.89%	19.6%	22.57%
San-Miguel 100k VPLs 1280x720 @ 4	Time		30.77s	67.33s	158.75s	388.64s	966.41s
	Time (only clustering)	69.92s	139.83s	278.03s	549.3s	1103.81s	
	Error (RMSE)		0.023	0.0183	0.0181	0.0134	0.0135
	Error (RMSE, only clustering)	0.031	0.023	0.017	0.015	0.014	
	Visibility samples		12.97%	14.38%	16.64%	19.96%	23.15%
Sponza 100k VPLs 1280x720 @ 4	Time		16.3s	32.6s	71.76s	175.59s	442.49s
	Time (only clustering)	66.99s	131.17s	261.89s	521.69s	1043.18s	
	Error (RMSE)		0.026	0.015	0.0092	0.0067	0.0048
	Error (RMSE, only clustering)	0.043	0.03	0.015	0.0094	0.006	
	Visibility samples		7.16%	7.5%	8.93%	11.81%	16.16%
Staircase 100k VPLs 720x1280 @ 4	Time		25.58s	50.05s	106.82s	243.37s	583.3s
	Time (only clustering)	65s	132.14s	261.02s	522.5s	1048.93s	
	Error (RMSE)		0.017	0.012	0.0088	0.0064	0.0057
	Error (RMSE, only clustering)	0.023	0.017	0.012	0.0088	0.0085	
	Visibility samples		14.26%	14.01%	14.96%	17%	20.23%
Modern Hall 100k VPLs 1024x1024 @ 4	Time		41.52s	63.33s	105.38s	230.39s	551.69s
	Time (only clustering)	76.43s	151.05s	302.72s	600.54s	1204.17s	
	Error (RMSE)		0.102	0.072	0.052	0.041	0.034
	Error (RMSE, only clustering)	0.142	0.099	0.07	0.054	0.042	
	Visibility samples		10.16%	12.58%	13.65%	15.97%	19.3%
Grey & White Room 100k VPLs 1280x720 @ 4	Time		34.13s	52.1s	93.82s	206.88s	485.78s
	Time (only clustering)	64.9s	127.84s	251.43s	502.33s	998.86s	
	Error (RMSE)		0.139	0.089	0.063	0.047	0.04
	Error (RMSE, only clustering)	0.188	0.141	0.01	0.06	0.046	
	Visibility samples		13.89%	13.65%	14.92%	17.6%	21.68%

TABLE A.5: Statistics for our method across all scenes.