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# Low-level modeling for trains routing and scheduling in railway stations

Quoc Khanh Dang

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**Centrale Lille**

**THÈSE**

Présentée en vue d'obtenir le grade de

**DOCTEUR**

en

**Spécialité : AUTOMATIQUE, GÉNIE INFORMATIQUE,  
TRAITEMENT DU SIGNAL ET DES IMAGES**

par

**Quoc Khanh DANG**

**DOCTORAT DELIVRE PAR CENTRALE LILLE**

Titre de la thèse

**Low-Level Modelling for Trains Routing and Scheduling  
in Railway Stations**

**Modélisation de bas niveau pour le routage et  
l'ordonnancement des trains dans les gares ferroviaires**

Soutenue le 18 Novembre 2021 devant le jury d'examen:

<b>Président</b>	M. Ouajdi KORBAA	Professeur des Universités, Université de Sousse (Tunisie)
<b>Rapporteur</b>	M. Walter SCHÖN	Professeur des Universités, Université de Technologie de Compiègne
<b>Rapporteur</b>	M. Pierre-Alain YVARS	Professeur des Universités, Institut Supérieur de Mécanique de Paris ISAE-SupMéca
<b>Examinateur</b>	M. Ouajdi KORBAA	Professeur des Universités, Université de Sousse (Tunisie)
<b>Examinatrice</b>	Mme. Olga BATTAL	Docteure, HDR, Kedge Business School (Campus de Bordeaux)
<b>Examinatrice</b>	Mme. Sana JABRI	Docteure, Responsable FDMS (Fiabilité, Disponibilité, Maintenabilité et Sécurité), Thales Ground Transportation Systems
<b>Ininvité</b>	M. Martin PRIETO	Ingénieur gestion des moyens, SNCF Réseau
<b>Directeur de thèse</b>	M. Khaled MESGHOUNI	Maître de Conférences, HDR, Centrale Lille
<b>Co-Directeur de thèse</b>	M. Armand TOGUYÉNI	Professeur des Universités, Centrale Lille
<b>Co-Encadrant</b>	M. Thomas BOURDEAUD'HUY	Maître de Conférences, Centrale Lille

Thèse préparée au Centre de Recherche en Informatique, Signal et Automatique de  
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## RÉSUMÉ SUBSTANTIEL

Le réseau ferroviaire en Europe ainsi que dans plusieurs régions du monde a connu des grandes évolutions: le développement de l'interopérabilité des systèmes ferroviaires, le développement des trains à grande vitesse, l'expansion des infrastructures ferroviaires ainsi que les politiques publiques favorisant incitant et imposant la réduction des moyens de transport polluants comme les voitures et les camions, ont conduit à une progression importante de la demande de transport ferroviaire. Cette dernière a des conséquences directes sur la gestion et l'optimisation du trafic ferroviaire qui deviennent de plus en plus complexes.

Durant cette thèse, nous avons eu l'opportunité de nous intéresser au problème de la planification, de l'ordonnancement et de l'allocation des ressources ferroviaires. Ces problèmes doivent être solutionnés pour augmenter la capacité ferroviaire et résoudre les problèmes de saturation dans un réseau ferroviaire. En effet, le développement du transport ferroviaire a mené à l'apparition des problèmes de saturation de capacité. Dans ce cas, le but de nos travaux de recherche est de développer des outils permettant d'accroître la capacité opérationnelle des systèmes ferroviaires.

Ces travaux doivent permettre la mise en relation des outils d'optimisation basés sur la recherche opérationnelle (modélisation mathématique et résolution de problèmes par des approches de programmation linéaire et heuristiques) avec des nouveaux systèmes de contrôle et de signalisation ferroviaire. Les systèmes de contrôle et de signalisation ont pour but principal de garantir la sécurité des circulations en évitant les risques d'accident tels que le rattrapage, le nez-à-nez, le cisaillement, le déraillement, etc. Pour cela, les voies sont principalement découpées en *sections* qui peuvent être sur une ligne ferroviaire, on parle aussi de *cantons*, ou dans une gare. Afin d'éviter les collisions, les systèmes de contrôle et de signalisation doivent garantir que deux trains ne soient pas simultanément dans la même section.

La gare ferroviaire est un élément important dans la gestion de la capacité ferroviaire en raison des contraintes liées à ses quais et aux voies permettant aux trains d'y entrer. La gare ferroviaire est un nœud ferroviaire qui interconnecte plusieurs lignes ferroviaires. L'allocation des ressources ferroviaires pour les trains en gare est cruciale pour gérer les situations de saturation et ainsi optimiser la capacité ferroviaire.

Ces travaux de thèse portent sur les problèmes de routage et

d'ordonnement des trains dans les gares ferroviaires avec une forte densité de circulations. L'objectif est de développer un modèle de gestion des ressources ferroviaires favorisant la fluidification du trafic à ressources constantes.

La méthode traditionnelle qui est généralement acceptée et utilisée dans les gares ferroviaires est l'*interlocking de route* (Le Bliguet and Kjær (2008), Vanit-Anunchai (2009)). Une route est une combinaison de sections et d'aiguillages adjacents par lesquels les trains passent pendant leur opérations dans les gares ferroviaires. Cette méthode alloue toutes les ressources pour réserver une route pour un train jusqu'à ce que cette route soit libérée. Pour assurer la sécurité, le système d'interlocking vérifie par un tableau des routes de train que la route ne cause pas de conflit avec les autres routes précédemment allouées. Une route ne peut pas être allouée si les autres routes en conflit ont été allouées et n'ont pas encore été libérées. Cette technique est mise en œuvre par plusieurs études de gestion opérationnelle des circulations dans les gares ferroviaires (Kroon (1997), Zwaneveld and Kroon (1997) et Bai et al. (2014)) et de vérification de modèles des systèmes de contrôle et signalisation ferroviaire (Mark (1998), Kanso et al. (2009) et Busard et al. (2015)). Elle garantit que des trains peuvent circuler en toute sécurité sans interruptions dans les gares ferroviaires mais elle limite la possibilité d'augmenter la capacité ferroviaire et de résoudre ainsi les problèmes de saturation. En effet, Bai et al. (2015) dans une étude récente utilisant la réservation de route, a développé un modèle de prise de décision avec un algorithme innovant qui peut construire un tableau d'horaires sans conflit pour les circulations des trains (environ 250 trains) dans la gare Bordeaux pendant une journée. Toutefois, les résultats montrent que 37 trains doivent être retardés pour un total de 182 minutes de retard.

Contrairement à cette approche, ces travaux de recherche se concentrent sur la réservation progressive des ressources utilisées par les trains tout au long de leur route dans une gare ferroviaire. Cette approche permet à un plus grand nombre de trains de circuler simultanément dans une gare ferroviaire et d'augmenter ainsi la capacité ferroviaire. Elle exige un système de contrôle et de signalisation ferroviaire solide et fiable pour assurer la sécurité. Une petite défaillance des équipements du système ferroviaire pourrait causer des conséquences graves. Cette approche ne garantit pas la circulation continue des trains durant leur route. Un train pourrait attendre dans une section de sa route si la section suivante est réservée par un autre train. Ainsi, il faut prendre en compte des fonctions objectives qui minimisent ces temps d'attente pour réduire la consommation d'énergie.

Nous nous sommes focalisés dans nos travaux de recherche sur la modélisation mathématique utilisant la programmation linéaire mixte en nombre entiers.

Cette modélisation mathématique consiste à transformer des systèmes ferroviaires en modèles de programmation linéaire. Nous avons souhaité construire le modèle le plus général possible afin de pouvoir étudier les problématiques ferroviaires selon plusieurs points de vue. Ainsi, la modélisation construit un modèle abstrait en se basant sur des éléments du système ferroviaire: *section* et *connecteur*. Ce modèle abstrait est capable de s'adapter à tous les réseaux ferroviaires. Une *section* peut représenter une section dans la gare ferroviaire ou un canton entre deux gares ferroviaires. Dans le contexte d'une gare ferroviaire, nous proposons de considérer qu'un *connecteur* est un aiguillage, un point d'entrée, un point de départ ou simplement un point de séparation de deux sections dans les gares ferroviaires. Toute *section* dans la gare ferroviaire est délimitée par deux *connecteurs*.

Nous donnons des règles de construction permettant de construire des topologies abstraites permettant de mener des études formelles, tout en préservant les règles de circulation réelles afin que les résultats obtenus soient fidèles aux possibilités qui pourraient être mises en œuvre dans la réalité. Ensuite, les problèmes de routage et d'ordonnancement des trains dans les gares ferroviaires sont formalisés en tenant compte des activités de trains (notion de routage et notion d'ordonnancement), de la gestion des circulations et des contraintes de sécurité.

Les problèmes de routage et d'ordonnancement considérant la réservation progressive des sections sont formalisés en tant que problèmes linéaires en nombres entiers (Dang et al. (2019)). Ce modèle contient des contraintes de temps, des contraintes de sécurité, des temps de référence et des contraintes de ressources. Il peut aussi concerner des mécanismes de couplage et de découplage (Dang et al. (2020)). Des fonctions objectives basées sur des critères pertinents sont définies et différents modèles mathématiques basés sur le management du circulations sont construits. En effet, trois modèles avec différentes techniques de gestion sont donnés: déviation des circulations techniques, déviation des circulations techniques et commerciales ou annulations des trains.

Le problème que nous étudions est un problème de grande taille et très

complexe. En effet, en plus de l'évitement des collisions, on doit éviter les blocages dans les sections en raison de la réservation progressive des ressources. Les problèmes de routage et d'ordonnancement des trains dans les gares ferroviaires sont divisés en sous-problèmes résolus séquentiellement par une heuristique basée sur la technique d'horizon glissant (*rolling horizon*). Cette technique se compose de trois types de sous-groupes: le sous-groupe gelé, le sous-groupe de tampon et le nouveau sous-groupe.

La première période de cette technique se compose de deux sous-groupes. Le sous-groupe des premiers trains est le sous-groupe de tampon, le sous-groupe des derniers trains est le nouveau sous-groupe. Après avoir résolu cette période, la solution du sous-groupe de tampon de cette période est fixée pour la prochaine période et devient le sous-groupe gelé. Le nouveau sous-groupe de cette période devient le sous-groupe de tampon de la prochaine période. Des nouveaux trains sont considérés (le nouveau sous-groupe) pour la prochaine période. A partir de la seconde période, les trois sous-groupes sont donc considérés. Cette procédure est répétée jusqu'à la fin de l'horizon de planification. La démonstration de cette procédure est présentée sur la Figure 1 (30 trains pour chaque période).

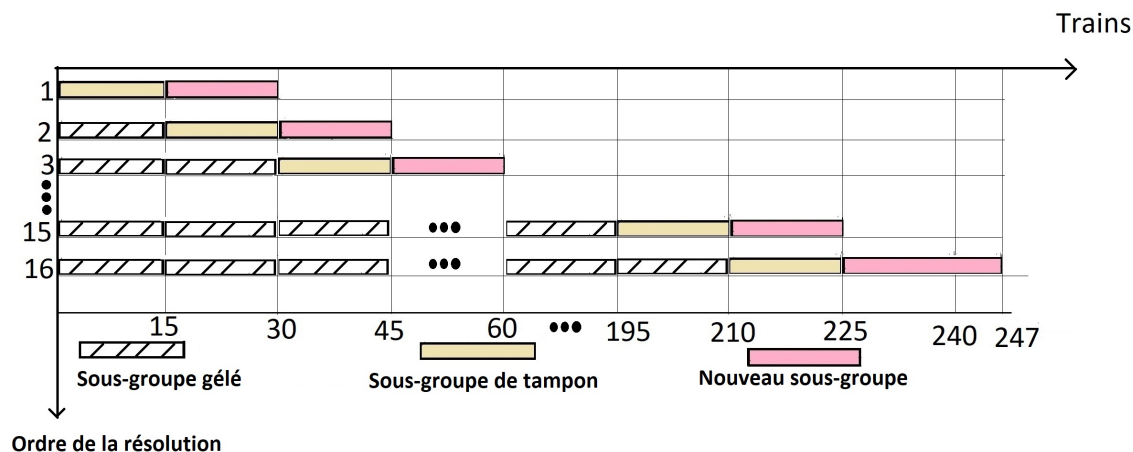


Fig. 1 Technique d'horizon glissant.

Les modèles mathématiques ont été implémentés à l'aide de l'outil CPLEX. Nous avons réalisé des expériences numériques sur les données de la gare de Bordeaux pour les circulations de toute une journée, problème déjà étudié dans [Bai et al. \(2015\)](#).



Les résultats montrent la bonne performance et l'efficacité de nos modèles. Les tableaux horaires sont visualisés par trois diagrammes de Gantt distincts correspondant aux sections, aux connecteurs et aux trains.

Notre proposition de considérer un paradigme de réservation des sections à la volée (interlocking de sections) plutôt que pour des routes complètes (interlocking de routes) est prometteur, et justifie le développement d'études ultérieures concernant les systèmes de sécurité permettant de mettre en œuvre ces mécanismes de gestion en gare même dans les situations où des circuits de voie ne sont pas présents pour détecter la présence des trains dans les sections considérées.

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# List of abbreviations

ATP	Automatic Train Protection
ATC	Automatic Train Control
ATO	Automatic Train Operation
ATS	Automatic Train Supervision
CBTC	Communications-Based Train Control
ERTMS	European Rail Traffic Management System
EVC	European Vital Computer
ETCS	European Train Control System
GoA	Grade of Automation
GSM-R	Global System for Mobile Communications for Railway
KVB	<i>Contrôle de Vitesse par Balises</i>
LEU	Lineside Encoder Unit
RBC	Radio Block Center
RER	<i>Réseau Express Régional</i>
TER	<i>Transport express régional</i>
TGV	<i>Train à Grande Vitesse</i>



# List of symbols

$[\alpha_s^l, \beta_s^l]$	Actual time interval of occupation of section $s$ by circulation $l$ .
$[v_c^l, \omega_c^l]$	Actual time interval of occupation of connector $c$ by circulation $l$ .
$W_s^l$	Time taken for circulation $l$ remaining stopped at section $s$ .
$\alpha_s^l$	Actual starting time of occupation of section $s$ by circulation $l$ .
$\beta_s^l$	Actual ending time of occupation of section $s$ by circulation $l$ .
$v_c^l$	Actual starting time of occupation of connector $c$ by circulation $l$ .
$\omega_c^l$	Actual ending time of occupation of connector $c$ by circulation $l$ .
$L$	Maximum permissible deviation time for technical circulations.
$\Psi$	Maximum permissible deviation time for commercial circulations.
$\xi^l$	Deviation time of circulation $l$ at platform.
$\mathbb{T}$	Set of all trains considered.
$\mathbb{T}_{11}$	Set of trains that have one entering circulation and one leaving circulation.
$\mathbb{T}_{21}$	Set of trains that have two entering circulations and one leaving circulation.
$\mathbb{T}_{12}$	Set of trains that have one entering circulation and two leaving circulations.
$\mathbb{L}$	Set of all circulations considered.
$\mathbb{L}_{ent}$	Set of entering circulations.
$\mathbb{L}_{leav}$	Set of leaving circulations.
$\mathbb{L}_{cross}$	Set of crossing circulations.
$\mathbb{L}^t$	Set of ordered circulations of train $t$ .
$S_s^l$	Passage of circulation $l$ going through section $s$ . If circulation $l$ passes through section $s$ , $S_s^l = 1$ , otherwise $S_s^l = 0$ .
$C_c^l$	Passage of circulation $l$ going through connector $c$ . If circulation $l$ passes through connector $c$ , $C_c^l = 1$ , otherwise $C_c^l = 0$ .
$X_c^{ll'}$	Chronological order of two circulations $l, l'$ with routes containing a common connector $c$ . If circulation $l$ passes through connector $c$ before circulation $l'$ , $X_c^{ll'} = 1$ , otherwise $X_c^{ll'} = 0$ .
$Y_s^{ll'}$	Chronological order of two circulations $l, l'$ with routes containing a common section $s$ . If circulation $l$ passes through section $s$ before circulation $l'$ , $Y_s^{ll'} = 1$ , otherwise $Y_s^{ll'} = 0$ .
$Z_{ss'}^l$	Passage from section $s$ to section $s'$ in the route of circulation $l$ . If circulation $l$ travels from section $s$ to section $s'$ , $Z_{ss'}^l = 1$ , otherwise $Z_{ss'}^l = 0$ .
$P_p^l$	Stopping platform of circulation $l$ . If platform $p$ is allocated to circulation $l$ as a stopping platform, $P_p^l = 1$ , otherwise $P_p^l = 0$ .



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$\Delta_s^l$	Time taken to traverse section $s$ by circulation $l$ .
$\Theta^l$	Time taken for a circulation $l$ going through a connector.
$\Gamma^t$	Time taken for a coupling system or decoupling system at platform of train $t$ .
$A^l$	Preferred arrival time to the platform of train $t$ where $l \in \mathbb{L}_{ent}^t$ .
$B^l$	Preferred departure time from the platform of train $t$ where $l \in \mathbb{L}_{leav}^t$ .
$D^l$	Preferred arrival time of train $t$ at railway station where $l \in \mathbb{L}_{cross}^t$ .
$E^l$	Preferred arrival time to the railway station of train $t$ where $l \in \mathbb{L}_{ent}^t$ .

# Introduction

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## 1.1 Background

Nowadays, the railway network in Europe and also most areas in the world must deal with capacity problems, the demand for transport by rail increases without stopping. It requires a constantly increasing capacity.

A railway system consists of three elements: infrastructure, rolling stock and railway operation rules. Railway infrastructure includes railway tracks, stations and other structures, buildings and signalling equipments. Rolling stock refers to railway vehicles including both power or un-powered vehicles, for example locomotives, cars, etc.

Railway operation requires very complex decision making process. It is a wide research area. Some important issues are scheduling, rescheduling, timetable robustness, safety, power consumption and assessment of railway capacity.

Railway stations are the focal points of a railway infrastructure. Train regularly stop to load or unload passengers or freight or both. Railway stations can be a bottleneck within an existing or future railway infrastructure. Train routing and scheduling problem is usually considered as the construction of timetable for trains at railway stations taking account all concrete station infrastructures, safety constraints and user requirements.

In this context, many railway stations request better performance in the train routing and scheduling process. Models and approaches for train routing and scheduling problem in railway stations are primarily attempted to obtain feasible solution of some conflicts (conflict between trains), then furthermore to adapt solutions to the complex conditions (more trains, heavier traffic, more requirements, etc.) in order to increase railway capacity. Currently, decision support tools for these problems are lacking, the problems are usually solved manually with simple automated support only. It is necessary to implement better modelling and solutions in order to improve railway capacity.

## 1.2 Motivation and aim of the thesis

The train routing and scheduling in railway stations is a complex task. The solutions of previous models and approaches for train routing and scheduling in railway stations are not yet effective enough. Due to the fast growing demand for capacity, a decision support tool with an optimization of the utilization of railway resources must be developed.

The development of such system must respect a certain number of constraints and achieve the following objectives:

1. The main objective is to design, implement and evaluate an off-line management tool for helping railway station managers in everyday task of managing timetable. The tool must handle a large problem (full-day timetable) within a short computation time.
2. An innovative model for railway station management is needed in order to solve the scheduling and routing problem based on the limitation of current state-of-art.
3. The development and implementation of effective algorithms for optimal train routing and scheduling for railway station in order to increase the capacity of railway stations should be addressed. The objective is to find an optimized timetable within a short computation time.

The achievement of the first objective clearly requires the resolution of the others two objectives. The development of an off-line management tool for a complex railway station must include suitable models and algorithms for scheduling and routing of trains passing through the railway station.

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## 1.3 Thesis contribution

An innovative contribution is presented in this thesis and realises the combination of research objectives given in previous section. Then, we briefly introduce the main achievements.

A decision model for railway station management is developed to generate an off-line timetable which deals with the routing and scheduling problem. The resources of railway stations which must be allocated are platforms, switches, etc. The timetables can be displayed by time-space charts (Gantt chart), for instance: time-platform chart and time-switch chart. It is easy to verify the conflict-free timetables and to implement routing and scheduling solutions by railway station managers.

The train scheduling and routing problem considering the reservation of each section independently is formalized as a mixed integer linear programming model in [Dang et al. \(2019\)](#). Unlike traditional methods (route interlocking) that allocate all resources to setup a route for a train until the route is released, this model focuses on the progressive reservation of resources (section interlocking) used by trains along their route on the railway station. This model contains time constraints, safety constraints, reference times, resources constraints such as compatibility of platform and switches. It is also concerned with coupling and decoupling mechanism in [Dang et al. \(2020\)](#). The exact method is solved by CPLEX to validate the decision model.

The computation time to solve the optimal train scheduling and routing problem is too big when dealing with full day problem. In this case, a math-heuristic approach based on rolling horizon methodology which divides the large problem into many sub-problems is designed and implemented to solve the problem. The model can find solutions which should be near-optimal within a reasonable time.

In this thesis, the experiment data is established on real data of a French railway station which was also studied by [Bai et al. \(2015\)](#). For these real world data, computational experiments proves that our decision support tool with the reservation of each section independently makes a better use of railway resources and it increases the capacity of railway stations.

## 1.4 Thesis outline

This section gives a short introduction to each chapter.

Chapter 2 provides a global view of rail ecosystem. Firstly, some knowledge about railway infrastructure is presented. Then, the railway signalling and control systems are introduced. After that, the method to assess the railway capacity of railway network is introduced.

Chapter 3 provides the state-of-the-art in railway traffic management to position our problematic. Recent contributions with various models for timetable design are compared and classified with regards to train routing and scheduling problems in railway stations. The literature reviews about rescheduling problem and the robustness in railway management are also discussed.

In chapter 4, we describe the problem of routing and scheduling in railway stations in many parts: railway topology, trains' activities, reference data and constraints for safety.

In chapter 5, a mixed-integer linear programming model is described in terms of parameters, decision variables and constraints. Furthermore, three linear programming models for resolution are also introduced: model allowing deviation time for technical circulations, model allowing deviation time for commercial circulations and model allowing cancellation of trains.

In chapter 6, numerical experiments with exact method tested on real cases in a French railway station are presented to point out the high computation time of this approach. Then, we introduce a math-heuristic approach based on rolling horizon methodology to solve the full-day problem. The heuristic method is tested to find a conflict-free timetable within a short computation time.

The main results obtained in this thesis are summarized in chapter 7. Further research is also recommended.

# Railway ecosystem

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## 2.1 Introduction

Railway network consists of three key elements: infrastructure, train control system (or *railway signalling and control system*) and traffic management. Railways use a dedicated infrastructure. The objective of train control

system and methods for traffic management, which are implemented in this infrastructure, is to perform a railway transport planning which maximises the railway capacity while ensuring the safety of rail transport.

The typical functional structure of a railway management system is displayed in the Figure 2.1:

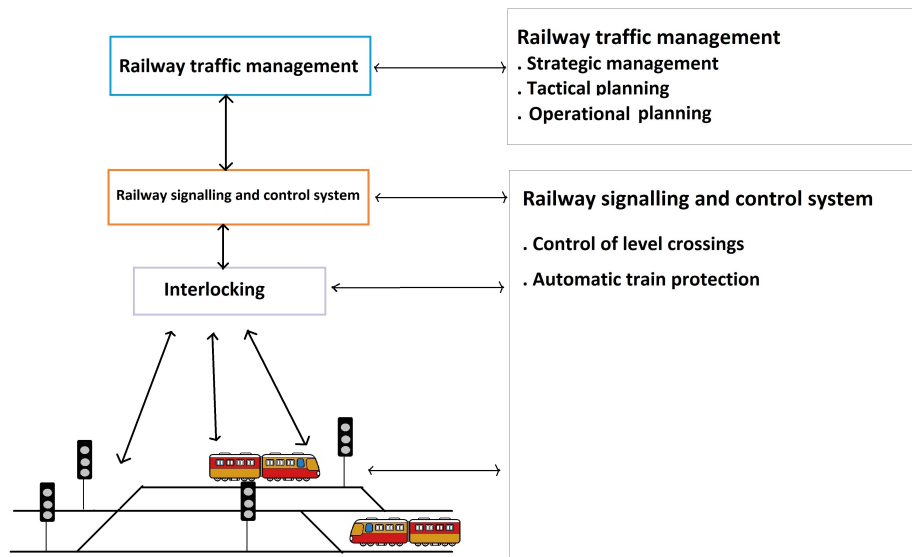


Fig. 2.1 Functional structure of a railway management system.

In this chapter, we first present a description of railway infrastructure. Then we introduce the railway signalling and control system as well as the different types of railway signalling and control systems, particularly in Europe. Finally, we introduce the method to assess the capacity of railway network.

## 2.2 Railway infrastructure

Railway infrastructure includes railway lines, equipment and other structures to support the railway lines. That support includes the management of train traffic, passenger or freight transport, and the maintenance of the infrastructure manager's property for these purposes.

### 2.2.1 Railway line

**Railway line** is a railroad (or *railway track*) that has a start, an end, and connects two railway nodes, see Figure 2.2. It consists of the rails, fasteners, railroad ties and ballast. It enables trains to move by providing a surface for their wheels to roll upon.

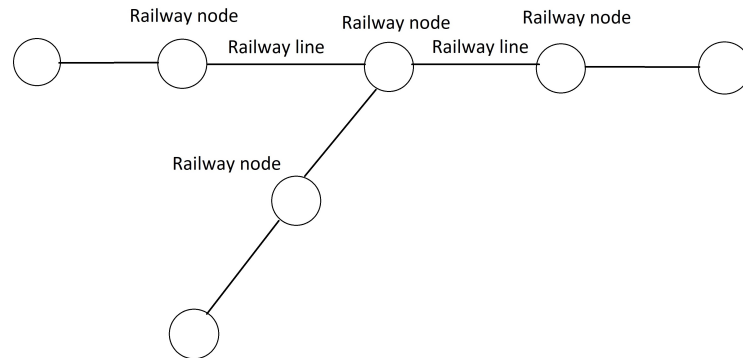


Fig. 2.2 Railway network: railway lines and railway nodes.

There are two types of *railway track* in railway line: *single-track* and *double-track*.

- A single-track railway is a railway track where trains circulating in both directions share the same track.
- Double-track railway usually contains two tracks to separate trains with opposite direction of travel. Trains run one track in each direction.

Single-track railway is cheaper to build and maintain than double-track railway, but has operational and safety disadvantages.

A railway line is composed of different components such as track section, signal and block.

**A track section** is a part of railway line that is bounded by two end points. From the point of view of railway management, a track section is a minimal element of railway line which can detect whether or not a train is circulating on it.

**A railway signal** or *signal* is an instruction or a warning information to transmit to train driver. The driver interprets the signal's indication and acts



accordingly. The form of railway signal can be a colour light signal which shows different colours to authorise train movements. For high speed railway lines, modern signalling systems provide the necessary information to the driver on displays in the train cab because it is more practical for train drivers.

A **block** is a section of railway line delimited by railway signals. The principle of block system will be introduced later in Section 2.3.

According to functionality, a railway line can be classified into three types:

- **Urban railway:** provides the main mode of transport for passengers on a daily basis between city centers or nearby towns (for example, metro, tramway, light rail). The metros and tramways consist of independent railway lines. These railway lines connect many stations which do not need to have the possibilities of changing railway lines. The distances between these stations are typically short.

- **Suburban railway:** operates within a metropolitan area, connecting suburban area to a central city (for example, RER, *Réseau Express Régional* for Paris area).

- **Mainline railway:** provides connections between cities. Mainline railway in many countries have at least a double-track railway and often contain many parallel tracks. Mainline railway is operated at higher speed than suburban and urban railway and may also be operated under shared access by many railways companies.

In the mainline railway, there are often **passing sidings** (or *passing loop*) where trains travelling in the same direction can overtake, see figure 2.3. The first train arriving must stop or move slowly in the passing siding, while the second train may pass at speed. Trains travelling in opposite directions can also pass each other by passing sidings.

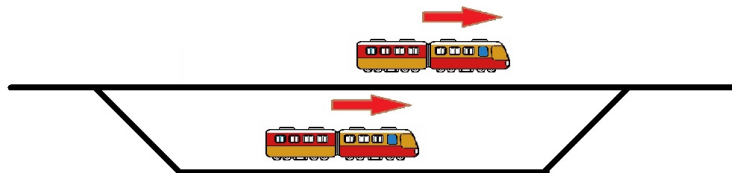


Fig. 2.3 Train overtaking.

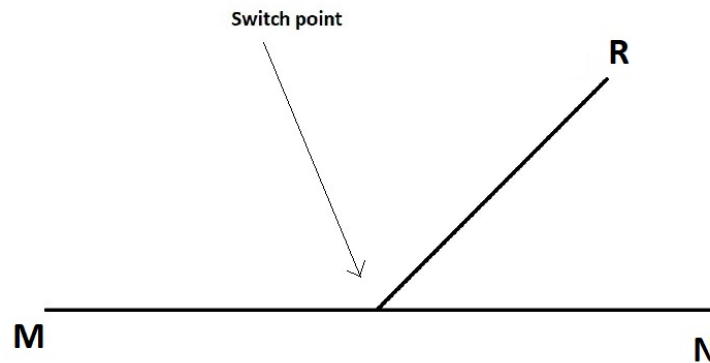


Fig. 2.4 An example of simple switch.

**Switch** or *turnout* or *point* is a mechanical installation enabling trains to be guided from one track to another.

An example of simple switch is described in Figure 2.4.

Figure 2.4 shows four possibilities for a train to travel through the railway infrastructure: Main (M) to Normal (N), Main (M) to Reverse (R), Normal(N) to Main( M) and Reverse (R) to Main (M). Therefore, it is necessary to have a switch in order to set the right route for a train. Some signals sending to the train driver are required to let the driver know there is no coming train from other side and the route has been prepared for this train. These switches are driven with a help of mechanical mechanism or electrically.

### 2.2.2 Railway node

**Railway node** is a node in railway network, which connects different railway lines. It offers the possibilities for trains to change railway lines according to their itineraries. A railway node can be a large railway station to connect several railway lines. But a railway node may not be a railway station, it can be a railway junction which gives only services for technical operation and does not have any platform.

**A railway station** or train station is a railway facility where trains regularly stop during travel. It has at least one platform. In the railway station, passengers can board or unboard from trains. The loading and unloading goods for freight trains are also provided in railway station.

**A junction** is a place at which two or more railway lines converge or

diverge. This implies a physical connection between railway lines, provided by switches and railway signals. If three or more railway lines are passing a railway station, then this railway station is called a junction.

An UML class diagram displays the relation between all components of railway infrastructure presented in this section is given in figure 2.5.

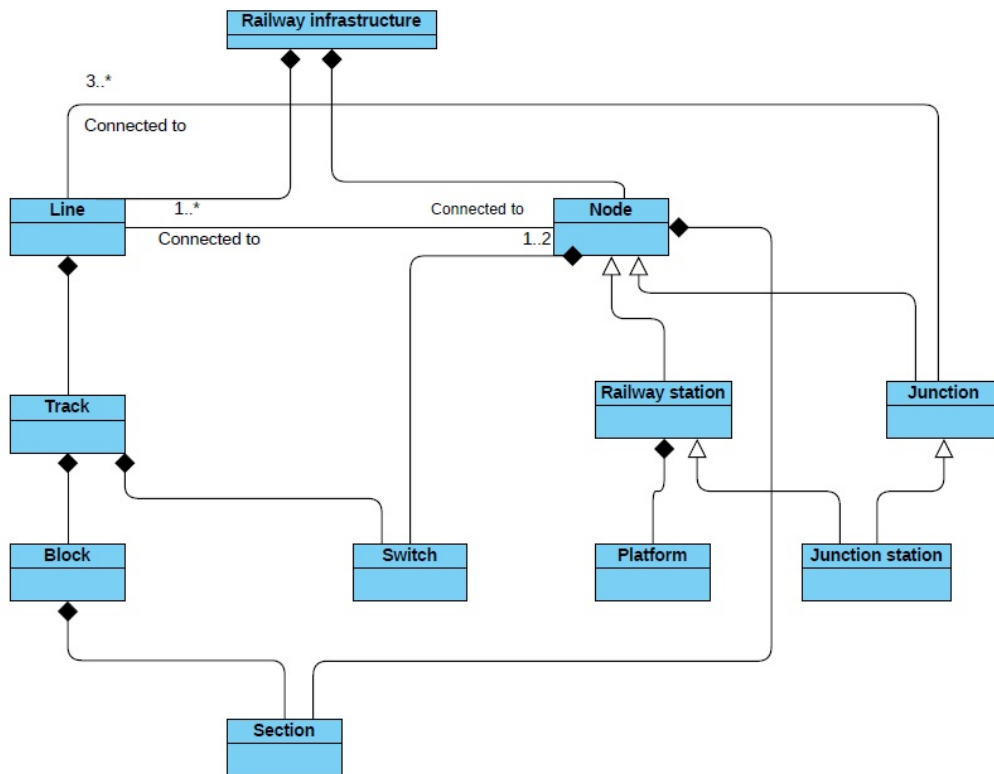


Fig. 2.5 UML class diagram of railway infrastructure.

Some remarks concerning this UML class diagram:

- A railway line connects two railway nodes (displays by **1..2**).
- A railway node can connect one or many railway lines (displays by **1..\***).
- A railway station is a railway node which consists of at least one platform.
- A junction is a railway node which connects at least three railway lines (displays by **3..\***).

- A junction station is a junction and also a railway station. In others words, it is a junction which consists of at least one platform.

In many papers (De Luca Cardillo and Mione (1998), Carey and Carville (2003), Cornelsen and Di Stefano (2007), Bai et al. (2014), etc.), a junction station is usually simply called a *railway station* because the problems most frequently encountered in junction are the problems in railway stations which connect many railway lines.

## 2.3 Railway signalling and control systems

### 2.3.1 Objective of railway signalling and control systems

A detailed objective of railway signalling and control systems is presented in Health and Safety commission (HSC) (2006). The primary objectives of a railway signalling and control system are to:

- give movement authorities (permission for a train to move to a specific location) which do not conflict with other trains.
- prevent collision between trains.
- prevent derailment of trains due to excessive speed.
- protect *level crossings* (an intersection where a railway line crosses a road).

A railway signalling and control system may be further enhanced to:

- give indications that enable the safe maximum speed relative to railway line condition and geometry, distances to signals or obstructions to be known.
- detect and protect against damage to railway infrastructure.
- detect and protect against unauthorised entry.
- provide protection to people working on the railway line.

In general, according to Poré and Moëns (2015) railway signalling and control systems have two main aspects: safety and exploitation.

### 2.3.1.1 Safety

A railway accident can cause severe consequences. Railway signalling and control systems have to ensure the safety of traffic: safety of passengers and also the goods transported. Five risks of accidents can be presented as follows.

**Catch-up collision:** A faster train may catch a slower train circulating in the same direction. If it happens, the head of the faster train will collide with the tail of the slower train, see Figure 2.6.



Fig. 2.6 Catch-up collision.

**Front to front collision:** It is forbidden to place two trains in opposite directions approaching each other on a single-track railway line. Otherwise, the front to front collision is inevitable, see Figure 2.7.



Fig. 2.7 Front to front collision.

**Side collision:** It takes place in a junction where two railway lines converge. A train circulating in a railway line collides with the second train circulating in convergent railway line, see Figure 2.8.

**Derailment:** It occurs when a train runs off its rails. The derailment of a train can be caused by various reasons: excessive speed, collision with another object, mechanical failure of railway line or wheels. For example, there is a derailment of a suburban train from Manresa to Barcelona which crashed into a rockslide in November 20, 2018.

**Collision with a non-rail obstacle:** it is the collision between a train with a non-rail obstacle such as road vehicle (in case of level crossing, see Figure 2.9), animal...

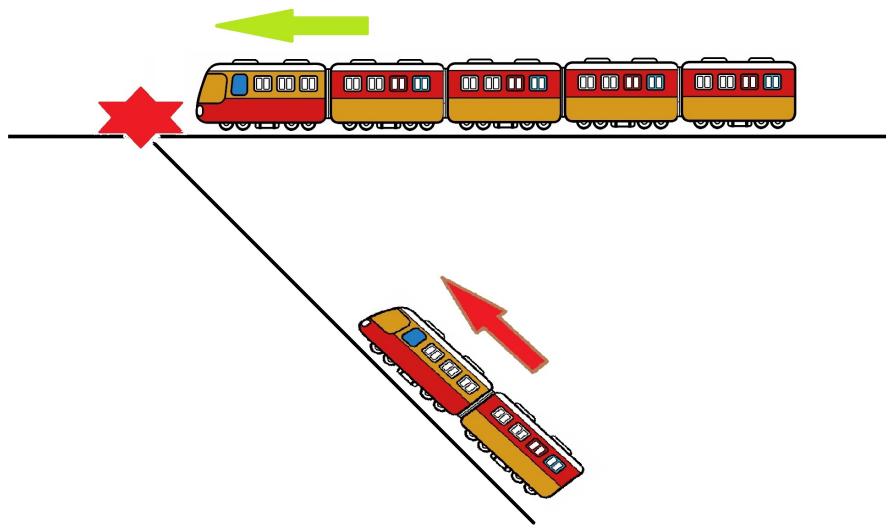


Fig. 2.8 Side collision.

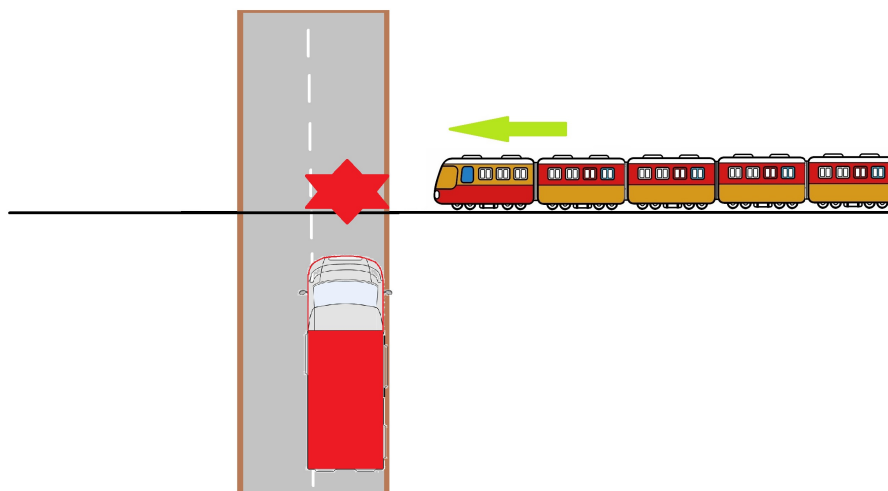


Fig. 2.9 Level crossing accident.

### 2.3.1.2 Exploitation

As described above, the railway signalling and control system is developed to prevent many accident risks. Over time, due to the development of technologies, the railway signalling and control systems have evolved to improve the exploitation of railway network.

### **Traffic flow and traffic control improvement**

In a railway network, many railway traffics access the same railway resources such as railway stations, lines, etc. The key role of the exploitation is to manage the reservation of these resources when they are shared among different traffics.

From commercial point of view, the final objectives are to:

- guarantee the commercial times given in the plan of transport.
- increase the passage frequency for stop stations.

### **Infrastructure cost efficiency**

The global cost of a railway network consists of the acquisition cost (including the cost of infrastructure construction, rolling stocks, equipments, etc.), the cost of operation and the cost of maintenance.

The implementation of some railway signalling and control systems can make cost savings in railway services and infrastructure managements.

For example, the implementation of a modern railway signalling and control system such as European Railway Traffic Management System (ERTMS) level 2 not only increases the railway capacity but also reduces the maintenance costs through the possibility of removing trackside signals.

## **2.3.2 Railway interlocking systems**

Within a railway signalling and control system, the interlocking is the logic architecture which ensures that the routing of trains and the signals given to trains provide safe operation in railway network. Trains are routed through the railway infrastructure using switches. Trains are controlled and regulated through the railway infrastructure using signals. The location of trains is detected using track circuits or axle counters.

### **2.3.2.1 Train detection equipments**

**Track circuit** is used in most railway lines to determine which track sections are occupied by trains. This equipment is fairly simple in design and is in

use since 1872.

Track circuits work by running a circuit using the rails to connect a power source at one end point of track section with a relay at the other end point. The basic principle of track circuit lies in the connection of the two rails by the wheels and axle of trains to short out an electrical circuit. When no train is present, the power flow from the power source through one rail, through the relay, and returns to the power source through the other rail. The relay will be energised and show an "unoccupied" signal, see Figure 2.10.

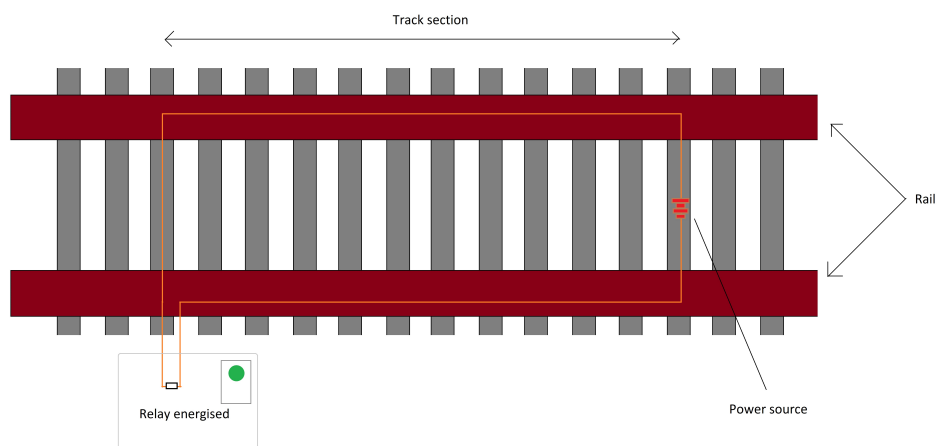


Fig. 2.10 Unoccupied track circuit.

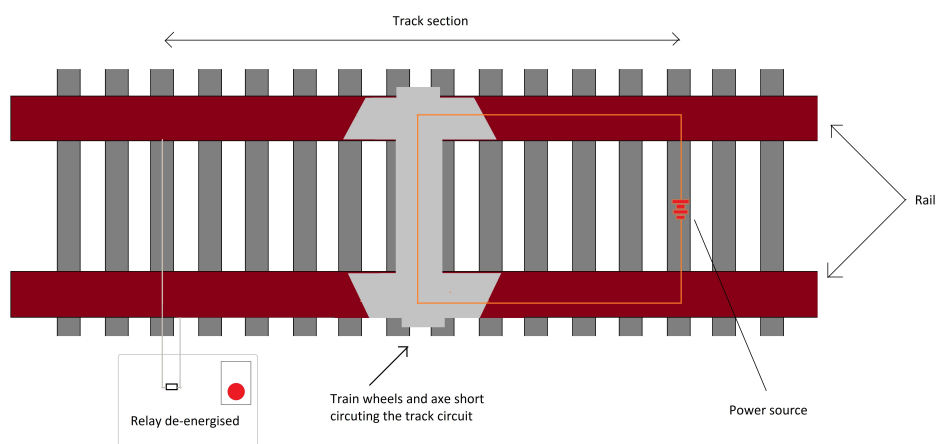


Fig. 2.11 Occupied track circuit.

When a train is present, the metal wheels and axle conduct the circuit as a short cut which bypasses the relay. This de-energises the relay and an



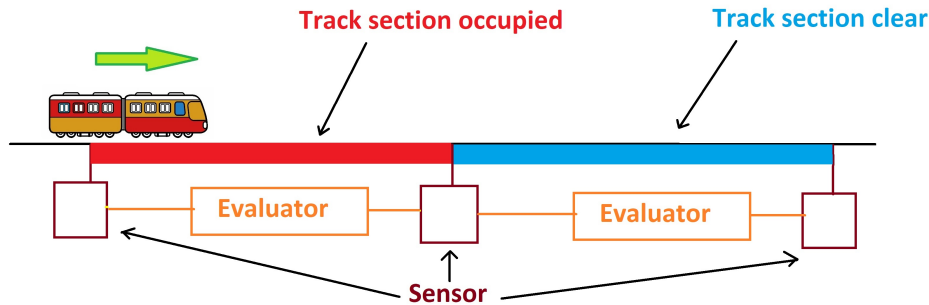


Fig. 2.12 Train detection by axle counters.

"occupied" signal is displayed. If the circuit is broken in case of broken rail or failed power source, the relay will be also de-energised and shows the "occupied" signal, see Figure 2.11.

**Axle counter** is an alternative method for train detection. It is a device that detects the passing of trains between two points on a track section. This device consists of a wheel sensor for each end point of the track section and an evaluation unit for counting the axles of the train in and out of the track section, see Figure 2.12. If the two counts are the same, the track section will be clear for another train.

### 2.3.2.2 Railway interlocking types

Railway interlocking can be categorised as manual, mechanical or electrical interlocking.

#### Manual interlocking

The first railway signalling and control system was very simple. The signalling logic relied on station agents who observe the position of train and give indications by setting the colour of traffic lights or using red flag and green flag (green for proceed signal and red for stop signal). The manual interlocking is associated with the manual block system which will be represented in Section 2.3.3.

The important step in the interlocking process is to check what authority is given to the train which approaches points (switch rails). Points can be set either for main line or branch line. In the manual interlocking, the station

agent must physically relocate a lever associated with the switch. Once the switches are in the position required and locked properly, the station agent gives a proceed signal (green signal).

### Mechanical interlocking

The mechanical interlocking improves the form of interlocking compared to the manual interlocking. It gives greater safety and less manpower.

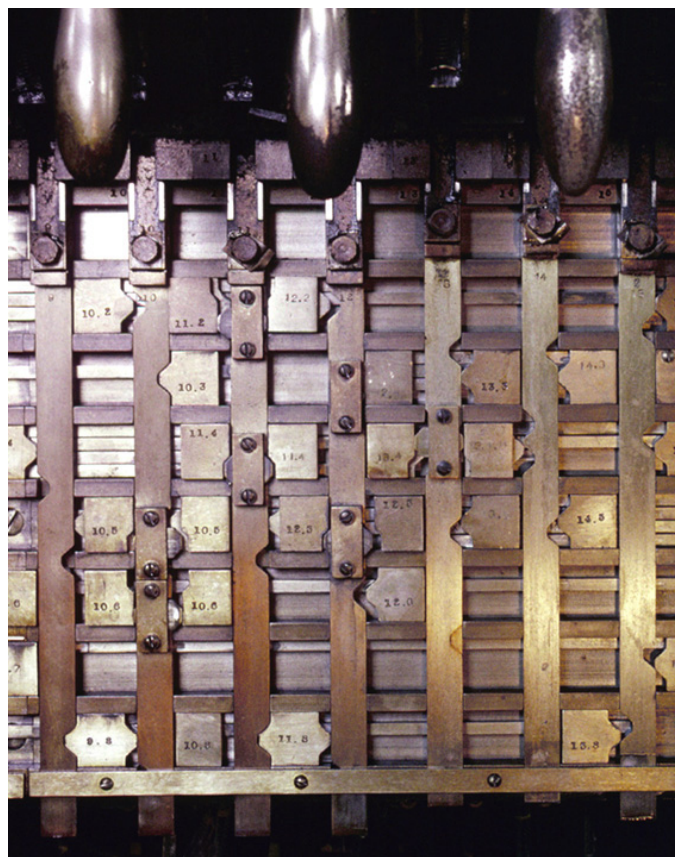


Fig. 2.13 The mechanical interlocking. (source: Wikipedia)

In the mechanical interlocking, the operator controls the switches by a *locking bed* consisting of steel bars forming a grid. There is a physical connection between levers which are arranged in a row in the frame, see Figure 2.13. A flat bar (called a tappet) is attached to each lever and has notches in it. Tie bars are placed at right angles to tappets and are provided with suitable shaped pieces which exactly fit in notches. When a point lever is pulled, it causes the tappet which it is connected to move. The motion gets

transferred to the other tappets which are connected by tie rod. Some tappets gets pushed in, some pushed out as a result. In case a tappet is free and pushed into the notch, it locks the point lever connected to that tappet. If it is already locked, it will come out of notch and the point lever becomes free.

### Different cases of mechanical interlocking:

#### One lever locking another

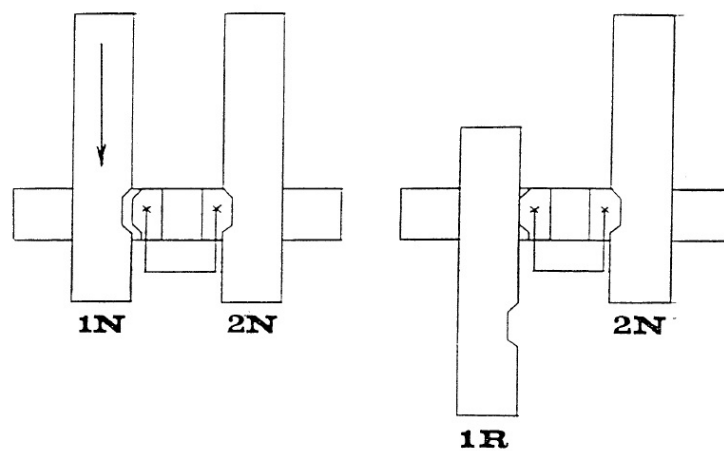


Fig. 2.14 One lever locking another: 1 locks 2. Source: **BRITISH RAILWAYS BOARD** Signal & Telecommunications Engineering Department (1991)

In the left side of Figure 2.14, when lever 1 is pulled, the tappet will move in the direction of the arrow. The positions of the tappets will be as in the right side of Figure 2.14.

This is *1 locks 2*. Lever 1 is now reverse and lever 2 is locked because the tappet of lever 2 cannot be moved. It is being held firmly by the tie bar and the tappet of lever 1. Lever 2 will only be free to be pulled when lever 1 returns to normal position.

Similarly when lever 2 is pulled, the tappet of lever 1 will be held by the bar and the tappet of lever 2. Therefore, the lever 2 locks lever 1 in this case.

#### One lever released by another

In the left side of Figure 2.15, the lever 1 cannot be pulled because the bar

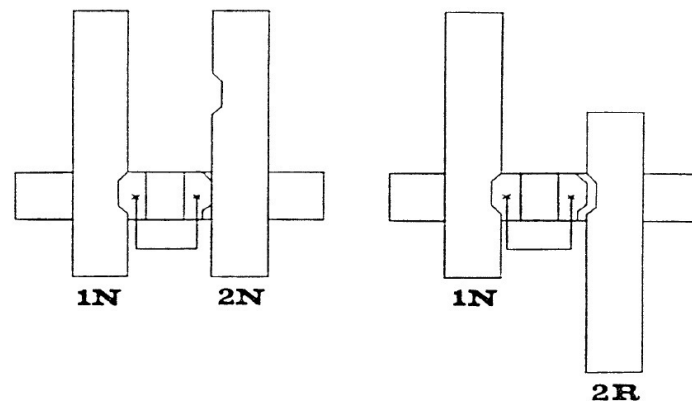


Fig. 2.15 One lever released by another: 1 released by 2. Source: **BRITISH RAILWAYS BOARD Signal & Telecommunications Engineering Department (1991)**

cannot move to the right as the lock at the tappet of lever 2. In the right side of Figure 2.15, when the lever 2 is pulled, the tappet of lever 1 can move the bar to the right, the lever 2 is free to be pulled. It means that *1 is released by 2*.

#### One lever locking another in either position

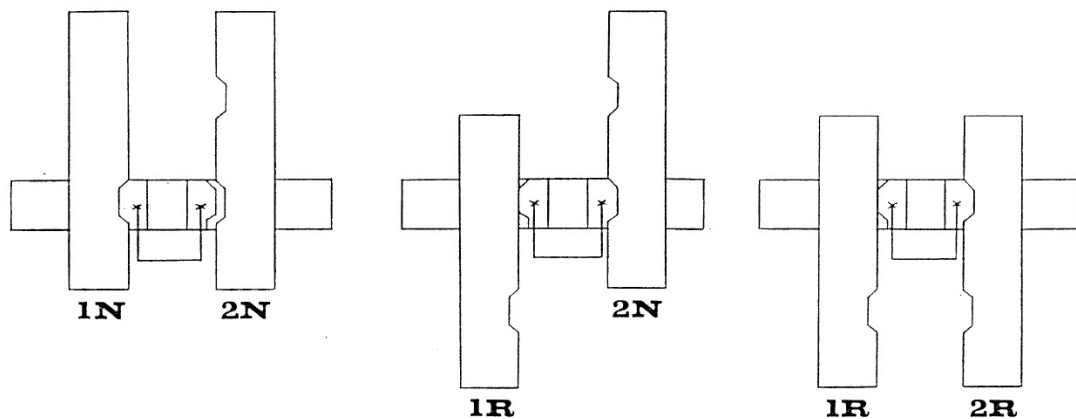


Fig. 2.16 One lever locking another in either position: 1 locks 2 in both position. Source: **BRITISH RAILWAYS BOARD Signal & Telecommunications Engineering Department (1991)**

In the Figure 2.16, lever 1 locks lever 2 in both normal position and reverse position. This is done by cutting two notches in the tappet of lever 2, one in the normal position and one in the reversed position.

### Conditional locking

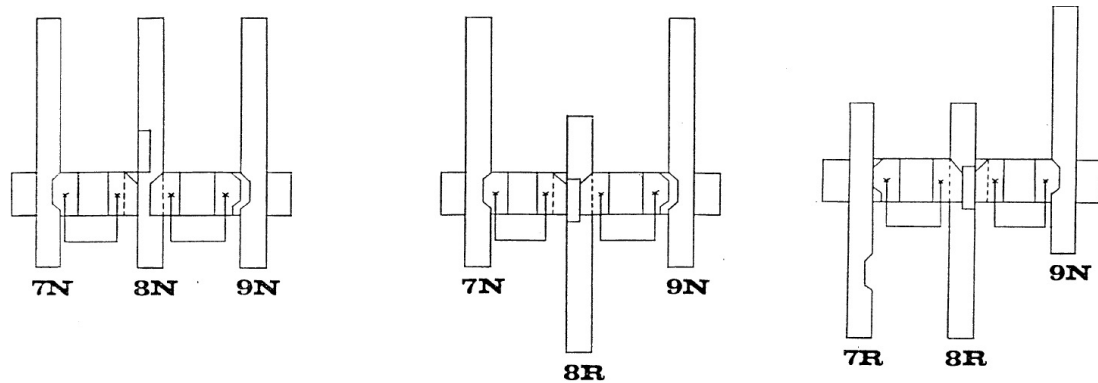


Fig. 2.17 Conditional lock: 7 locks 9 with 8 reverse . Source: **BRITISH RAILWAYS BOARD Signal & Telecommunications Engineering Department (1991)**

In the Figure 2.17, if lever 8 is pulled reverse, the tappet of lever 8 will come between the two locks. When lever 7 is pulled reverse, it drives the lock to the right. Because of the block of the tappet of lever 8, the bar will move to the right. This makes the tappet of lever 9 is locked. We have *7 locks 9 with 8 reverse*.

### Electrically powered interlocking

The need for railway capacity and development of electricity evolves railway signalling and control to a next level. The switches are controlled electrically by point machines and the interlocking is implemented in relay technology (electrical switches which are also applied in track circuit in Section 2.3.2.1). These interlockings consists of complex circuitry made up of relays in an arrangement of relay logic that ascertain the state or position of each signal appliance. When appliances are operated, their change of positions opens some circuits that lock out other appliances that would conflict with the new position. Similarly, other circuits are closed when the appliances they control become safe to operate. All points and signals are operated electrically from a central location, the mechanic levers are replaced by buttons or toggles on a panel or a video interface.

The modern interlockings are computer based interlockings where the wired networks of relays are replaced by software logic to control signal appliances. They have been implemented in many different configurations

taking under consideration of latest computer developments.

The computer based interlocking consists of computer hardware (microprocessor, memory, input cards, output cards, etc.) and computer software (firmware, operational system and control programme). The interlocking functions and logics associated with the inputs and outputs of the hardware are defined in a control programme. They are kept in a microprocessor module. This is a list of instructions designed specifically for a signalling and control system layout and rules of operations.

For the point machines, the computer based interlocking is not capable to provide power to directly control them. Relays are still used to perform this functionality.

### 2.3.2.3 Interlocking of route

The traditional methods, which are usually accepted and used in railway stations by railway companies, are the interlocking of route. These methods allocate all resources to setup a route for a train until the route is released. For the interlocking of route, there are two important concepts called *train route* and *train route table* (Le Bliguet and Kjær (2008), Vanit-Anunchai (2009)). A train can be allowed to follow a certain route called a *train route*. The safety rules to let a train follow a specific route are described by a *train route table*.

#### 2.3.2.3.1 Train route

**Route** or *path* is a combination of sequential track sections, which are connected by points or switches, where trains must pass through railway stations during their operations. Each train route has a starting point, an ending point and some track sections which consist of track circuits that connect the starting point and the ending point.

An *entrance signal* is linked to the route. A train is only allowed to enter a route if the state of a given signal allows it.

The principle of locking route and unlocking route is described in the following way:

- 1. Setting the points:** An operator has to put the points (switches) in the correct position.

**2. Locking route:** If the points are in the correct positions, the operator will push a specific button to lock the route. In this situation, the points cannot be switched until the release of the route.

**3. Setting the entrance signal of the route to proceed signal:** When the route is locked, the signal will be switched to a proceed signal to allow a train to enter the route.

**4. Setting the entrance signal to stop signal:** When a train occupies the first track section of the route, the entrance signal of the route is switched to a stop signal to prevent other trains from entering the route.

**5. Unlocking route:** When a train reaches the ending point of the train route, the route is released and the points in the route will be unlocked.

The routes can be divided into three types:

- *Entering routes:* to enable trains to enter the railway station.
- *Leaving routes:* to enable trains to exit the railway station.
- *Crossing routes:* to enable trains to pass through the railway station without any stop at platforms.

Routes are said to be *conflicting* if they are not allowed to be locked at the same time (for example if they have physical resources in common).

#### 2.3.2.3.2 Train route table

For a given railway station, all the train routes and the concrete safety rules associated with these routes are defined in the *train route table* of the railway station.

Table 2.1 is the train route table for the Panthong station of which signalling layout is shown in Figure 2.18.

The route identifications are labelled by entering point where the entrance signal takes place (in the first column "from"). For example, the route from entering point 3 to leaving point 23 is called 3(1), the route from entering

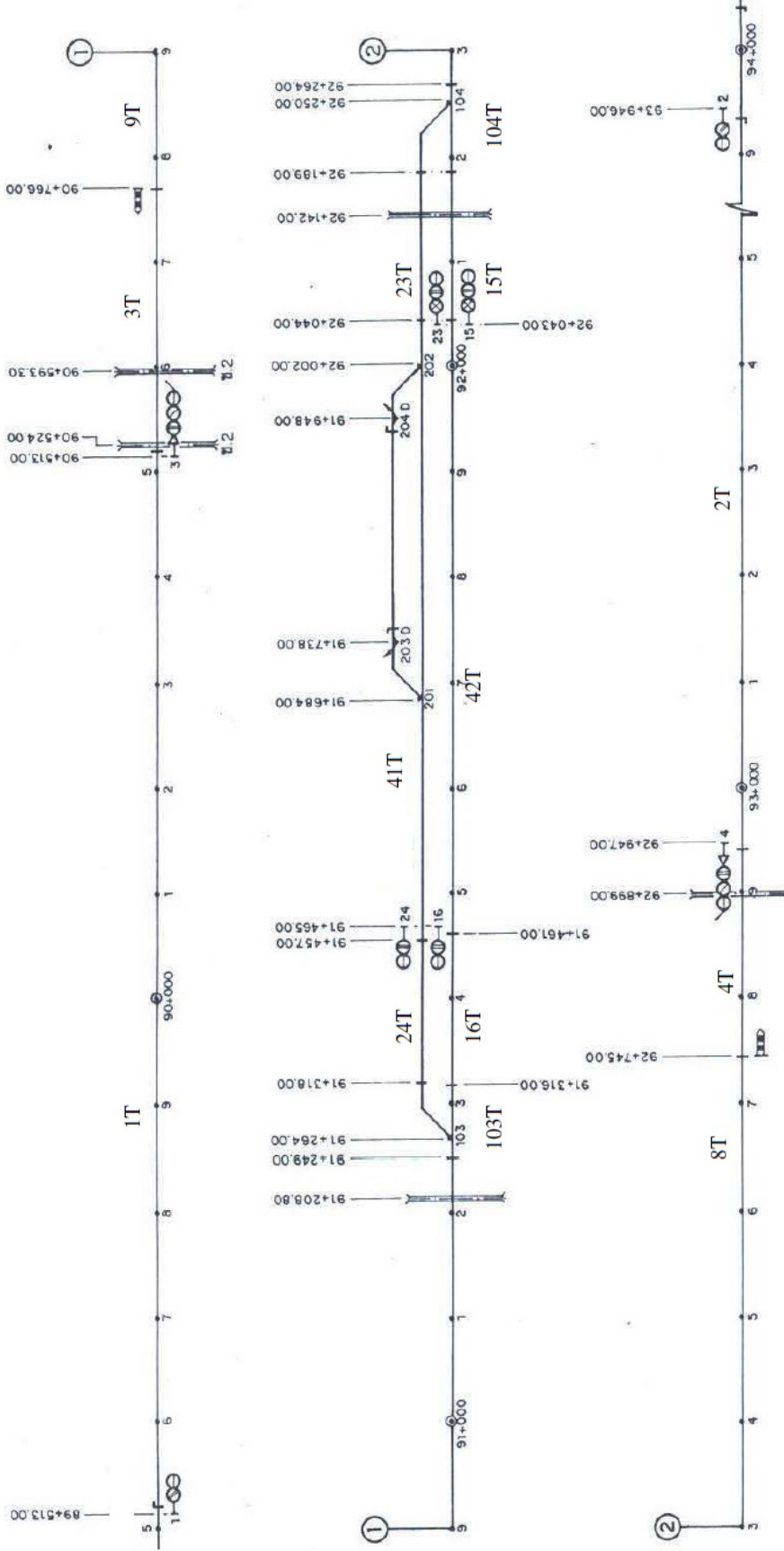


Fig. 2.18 Signalling layout of the Panthong station. Source: Vanit-Amunchai (2009)



TABLE 2.1 A train route table for Panthong station. Source: Vanit-Anunchai (2009)

ROUTE		INTERLOCKING				CONTROL					
		REQUIRES	SET & LOCKS POINTS		REQUIRES	ASPECT	SIGNAL AHEAD	REQUIRES TC			
From	To		ROUTE NORMAL	NORMAL	REVERSE			NORMAL	KEYLOCK	CLEAR	AT TIME OF CLEARING ONLY
								TC CLEAR	OCC FOR		TIME
1	3						Y G	3 AT R# 3 AT Y# OR G#			
2	4						Y G	4 AT R# 4 AT Y# OR G#			
3(1)	23	16,24,4(1),4(2),3(2)		103,104	201,202, 203,204		Y+Jl	23 AT R#	3T,9T,103T,24T,41T, 23T,104T,8T,4T	42	42 FOR 60 sec
3(2)	15	16,24,4(1),4(2),3(1)	103,104				Y G	15 AT R# 15 AT G#	3T,9T,103T,16T,42T, 15T,104T,8T,4T	41	41 FOR 60 sec
4(1)	16	15,23,3(1),3(2),4(2)	104,103				Y G	16 AT R# 16 AT G#	4T,8T,104T,15T,42T, 16T,103T,9T,3T	41	41 FOR 60 sec
4(2)	24	15,23,3(1),3(2),4(1)		104,103	201,202, 203,204		Y+Jl	24 AT R#	4T,8T,104T,23T,41T, 24T,103T,9T,3T	42	42 FOR 60 sec
15	UP BLOCK SECTION	23,4(1),4(2)	104,103				G		15T,104T,8T,4T,2T,TOL		
23	UP BLOCK SECTION	15,4(1),4(2)		104,103			G		23T,104T,8T,4T,2T,TOL		
16	DOWN BLOCK SECTION	24,3(1),3(2)	103,104				G		16T,103T,9T,3T,1T,TOL		
24	DOWN BLOCK SECTION	16,3(1),3(2)		103,104			G		24T,103T,9T,3T,1T,TOL		

point 3 to leaving point 15 is called 3(2).

Each row in the table represents the safety rules how to lock and unlock each route. For example, the route 3(2) requires that the track sections (which are track circuits in this case) 3T, 9T, 103T, 16T, 42T, 15T, 104T, 8T, 4T are *cleared* or unoccupied (in the column "clear") and the points 103 and 104 are in normal position. Then the entrance signal at starting point 3 can be set to *yellow-Y* (warning signal) or *green-G* (proceed signal) at the column ("aspect") depending on the signal at ending point 15 of the route (in the column "Signal ahead"). If the signal 15 shows *red-R*, the signal 3 shows *yellow-Y*. If the signal 15 shows *green-G* and the route 3(2) is locking, the signal 3 will show *green-G*.

The column "Requires Route Normal" shows conflict routes. For example, The conflict routes of route 3(2) are 16, 24, 4(1), 4(2), 3(1).

### 2.3.2.3.3 Conclusion

The interlocking of route is implemented for many studies of management in railway stations (Kroon (1997), Zwaneveld and Kroon (1997) and Bai et al. (2014)) and model checking of railway signalling and control systems (Mark (1998), Kanso et al. (2009) and Busard et al. (2015)).

This approach guarantees that trains can circulate safely without interruptions in railway stations but it limits the possibility of increasing

the railway capacity and solving the saturation problems. Indeed, in a recent study using the reservation of complete route Bai et al. (2015), a decision model is developed with an innovative algorithm. It constructs a train timetable conflict-free for the circulations of trains (about 250 trains) for a day in the Bordeaux station. However, the results show that there are 37 trains which have 182 minutes of delays in total.

### 2.3.3 The concept of block

In order to avoid collisions between trains, the principle used in most country is to define sections of railway line which are called blocks. Two trains must not be in the same block at the same time. The presence of a train in a block triggers the signal which prevents others trains from entering this block. It even guarantees that a certain number of empty blocks separates two successive trains to allow safe braking.

Different block systems are used to inform the driver about the presence of another train in next block. The block systems can be classified into two categories: manual block and automatic block.

**Manual block:** The position of train and traffic signals are not determined automatically. The station agents communicate each other by phone to transmit the position of train and set the colour of traffic lights. This system is used for the railway lines with very low traffic and the length of a block can be very long: 20-30 kilometres.

**Automatic block:** The automatic block systems detect automatically whether blocks are occupied (by track circuits, axle counters...) in order to give indications to approaching trains. In France, there are two types of automatic blocks:

- **Automatic light block (*Block Automatique Lumineux* BAL):** This system is generally used for the railway lines with very high traffic. The length of blocks is generally 1.5 kilometres in France with a maximal length of train of 800 meters. When a train occupies a block, the signal in the end point of the predecessor block is changed to red (stop signal) and the signal of its previous block is changed to yellow (warning signal), see Figure 2.19.

- **Automatic block with restrained permissiveness (*Block Automatique à Permissivité Restreinte* BAPR):** This system is implemented for the railway lines with moderate traffic where BAL is too

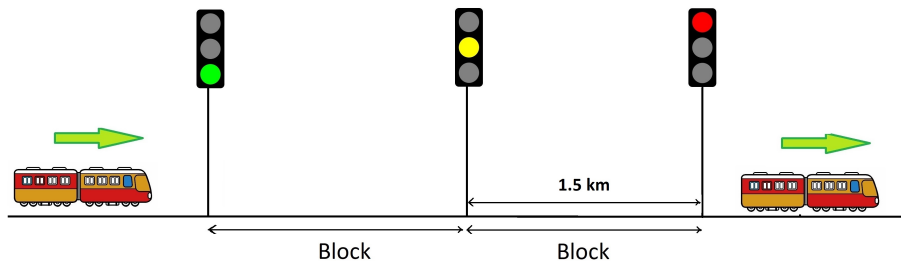


Fig. 2.19 Automatic light block.

expensive. The length of blocks is between 6 and 15 kilometres. This system is similar to BAL but the signals at the end point of the predecessor block has only two states: stop signal (red) and clear signal (green). The warning signal is placed before the traffic light of the beginning point for each block and the distance is around 1.5 kilometres to cover braking time, see Figure 2.20.

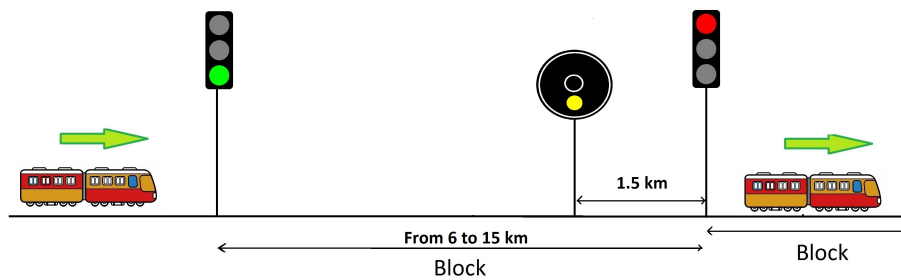


Fig. 2.20 Automatic block with restrained permissiveness.

### 2.3.4 Automatic Train Control system

The automation of train movements was developed from the need to enforce the railway traffic control, see [Trentesaux et al. \(2018\)](#). **Automatic train control (ATC)** is a train control system for railway network that involves a speed control mechanism to ensure the safety of railway. It usually has three functions: Automatic Train Protection (ATP), Automatic Train Operation (ATO) and Automatic Train Supervision (ATS).

**Automatic Train Protection (ATP):** It is a system which safely monitors the driver and automatically brakes the train if it passes beyond their permitted speed.

The principle of ATP is based on four informations:

- Conditions of track (permitted speed) and signalling (the state of signals).
- Informations of train (speed and location).
- Driver behaviour.
- Indications to the driver.

The ATP system calculates the permitted speed based on the informations of train and the conditions of track and signalling. The permitted speed is checked against the actual speed of train. If the permitted speed is exceeded, warning indications are sent to the driver or there is an emergency braking depending on the amplitude of the excess, see Figure 2.21.

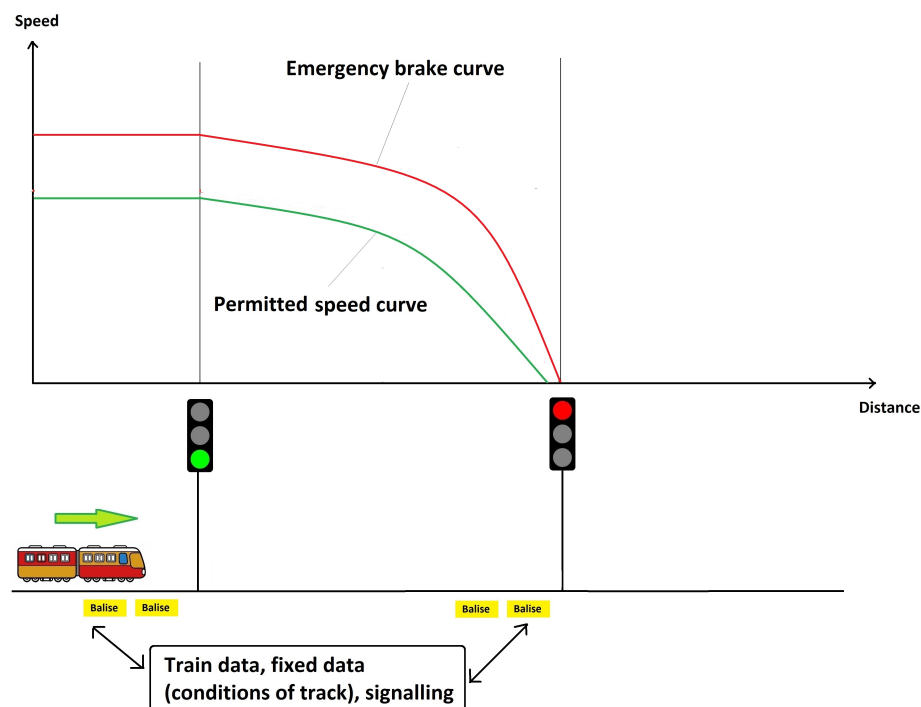


Fig. 2.21 Automatic Train Protection.

In Figure 2.21, the red curve is the ATP emergency brake curve, the green curve is the permitted speed curve. In this way, the ATP system provides

safety margins. If the speed of train exceeds the speed of green curve, the ATP system will send warning indications to the driver or automatically reduce the speed of train. If the speed of train reaches the red curve, the ATP system will automatically brake the train urgently.

**Automatic Train Operation (ATO):** This system implements partial or complete automatic train driving and performs the functions of a driver depending on the Grades of Automation, which are defined by the standard IEC 62290-1 (IEC 2009), as shown in Table 2.2.

A system can be called ATO when it corresponds at least to GoA 2. Modern

TABLE 2.2  
GRADE OF AUTOMATION.

Grade of Automation	Description	Speed supervision	Acceleration and braking	Door closure	Operation in case of disruption
GoA 1	Non-Auto	Partial	Driver	Driver	Driver
GoA 2	Semi-Auto	Auto	Auto	Driver	Driver
GoA 3	Driverless	Auto	Auto	Partial	Driver
GoA 4	Unattended	Auto	Auto	Auto	Auto

systems may implement a complete automatic train control system which is called *driverless* (GoA 3). However, in most cases, there is always a driver onboard to react in case of emergency situations or technical failures.

**Automatic Train Supervision (ATS):** is a system which manages the traffic and schedules of trains in real-time according to traffic monitoring information. The main functions of ATS are to monitor train and update train status, maintain schedule and reschedule in case of disturbances.

### 2.3.5 Communication systems

The communication systems between train and trackside (or *lineside*) ensure that the driver is always attentive and respects the indications of railway signals. The railway signals can be the signals installed along the railway line using sound/luminous indications or cab signals that display indications and informations inside the locomotive cab. Cab signals are mandatory for high speed line such as the TGV (French: Train à Grande Vitesse) in France.

There are two types of transmission:

**Spot transmission:** The transmission of information is performed only at certain points that are equipped with balises.

**Continuous transmission:** The transmission of information is permanent regardless of the position of trains.

### 2.3.5.1 Systems with spot transmission

The important equipment of systems with spot transmission is called a *balise*.



Fig. 2.22 Balise between the rails. (source: Wikipedia)

A **balise** is an electronic beacon or transponder placed between the rails, see Figure 2.22. The balise can send *telegrams* to an onboard system of trains passing over it. The informations a balise sends to trains can be the maximal speed permitted, the distance to next signal, the state of signal, etc. The onboard system of trains determines the curve of control (speed diagram during the route of train) and ensures that trains will not exceed maximal permitted speed. Because the system uses spot transmission, trains must travel over the balise to obtain the movement authority. **KVB** (*Contrôle*

*de Vitesse par Balises*) in France and **ERTMS (European Rail Traffic Management System, introduced later in Section 2.3.7) level 1** are two examples of systems with spot transmission.

### 2.3.5.2 Systems with continuous transmission

The key part of systems with continuous transmission is *cab signalling*.

**Cab signalling** is a railway system that communicates the informations and status of railway lines to the locomotive cab of driver. All cab signalling systems must have a continuous in-cab indication to inform the driver of railway line condition ahead.

Currently, the systems with continuous transmission are **ERTMS level 2 and level 3**.

## 2.3.6 Communications-Based Train Control (CBTC) and moving block

The development of **Communications-Based Train Control (CBTC)** technology is a specific control system for metro systems. It is a “continuous, automatic train control system using high-resolution train location determination, independent of track circuits; continuous, high-capacity, bidirectional train-to-wayside data communications; and trainborne and wayside processors capable of implementing Automatic Train Protection (ATP) functions, as well as optional Automatic Train Operation (ATO) and Automatic Train Supervision (ATS) functions” as defined in IEEE 1474 standards for CBTC, see [IEEE \(2005\)](#).

Across many developing cities, the railway signalling and control systems need to evolve to meet the growing needs of transport. Therefore, the operation managers focus on maximising railway capacity. The main objective of CBTC is to increase railway capacity by safely reducing the headway (the time or the distance interval) between trains circulating along the railway lines.

### Moving block

In a traditional block (fixed block) (presented in Section 2.3.3), the headway is related to the train speed and the length of blocks. This distance is calculated based on the most rigorous case (train with highest speed and

poorest braking system). Therefore, it significantly reduces the capacity of railway lines with various types of trains.

Unlike the fixed block systems, in the modern moving block CBTC systems, see Figure 2.23, the movement authority of trains is not statically defined by the railway infrastructure. In fact, trains are continuously communicating their exact position to the trackside in railway line to define the blocks in real-time by control centres. Therefore the moving block allows reducing the headway (the distance between trains) while maintaining the safety distance between trains, thus it can increase the railway capacity.

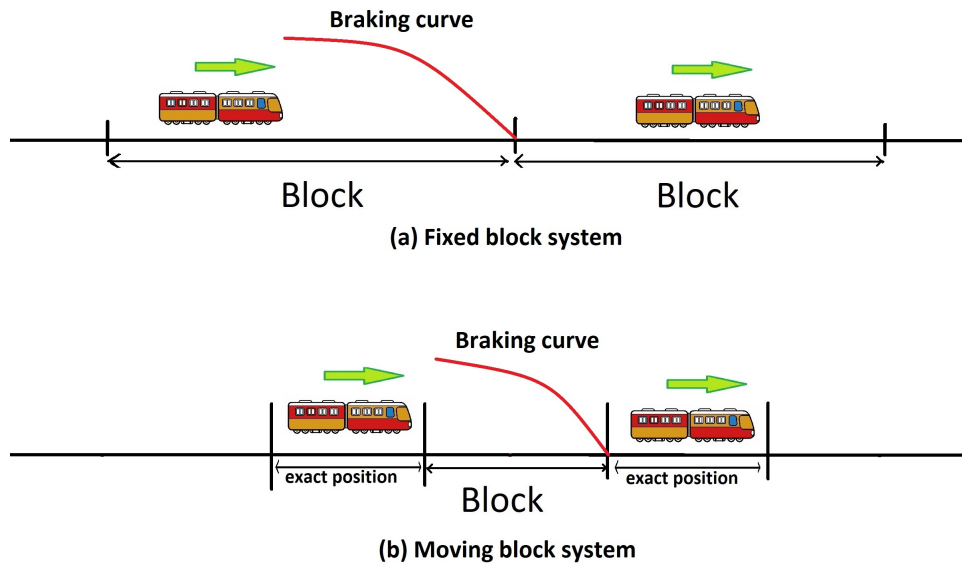


Fig. 2.23 Moving block system.

The moving block system is deployed as Communications-Based Train Control for some urban railways such as metro systems but it is still considered for the next generation of ERTMS (level 3).

### 2.3.7 European Rail Traffic Management System (ERTMS) (<https://www.ertms.net>)

#### 2.3.7.1 The incompatibility of railway signalling and control systems

Railway companies have developed specific systems for national railway operator. Today various railway signalling and control systems exist across Europe (some systems are mentioned in previous sections such as TVM and



KVB in France, LZB in Germany and Australia.) All of these systems are incompatible one with each other. For example, Thalys trains, which connect Paris, Brussels, Cologne and Amsterdam, must be equipped of 8 different systems. This complexity leads to additional costs and many inconveniences such as lower reliability rates, problems of communication, etc. It also make the driver's work more complicated. He must be able to interpret the signals of each systems. This has caused sometimes accidents. For example, the Paddington rail crash at Ladbroke Grove in London, England in 1999 was caused by the driver who misinterpreted a red signal.

European Rail Traffic Management System (ERTMS) is a standard railway signalling and control system created for the interoperability of trains in European railway network. It has been developed to replace the different national railway signalling and control systems in Europe. ERTMS brings considerable advantages: increased capacity, higher reliability rates, open supply market and improved safety. The replacement from national railway signalling and control systems to ERTMS is a huge challenge which is already in progress. Actually in Europe, it is mandatory to equip all new main lines with ERTMS.

### 2.3.7.2 ERTMS composition

ERTMS has two main components:

- **ETCS (European Train Control System)** is the part of ERTMS for controlling trains. It calculates a maximum speed permitted for each train and transmits it to the driver and it also continuously monitor the driver's response to this information. An onboard computer called EVC (European Vital Computer) compares the speed of train with the maximum speed permitted and automatically applies emergency braking if it exceeds the maximum speed limit. The EVC implements an ATP function.

- **GSM-R (Global System for Mobile Communications-Railways)** is a radio system used for the communication between train and control centres. It is based on the GSM mobile telephony standard but it is adapted to the high speed that trains can reach (up to 500 km/h). It provides some advanced communication functions.

### 2.3.7.3 ERTMS levels

In order to enable its progressive adaptation in all European countries, the implementation of ERTMS is divided into different functional levels. The

levels of ERTMS basically depends on how the railway line is equipped and the way the informations are transmitted between trains and trackside. According to UNIFE (2014) and Barger et al. (2009), there are three main levels of ERTMS .

### ERTMS Level 1

ERTMS level 1 is designed as an add-on to or overlays the existing railway signalling and control system already equipped with trackside signals and train detectors. Communication between trains and trackside is ensured by *Eurobalises*, dedicated balises (presented in Section 2.3.5.1) of ERTMS. Eurobalises pick up signals from the trackside signals via signal adapters and telegram *LEU Encoders* and transmit them to the train as a movement authority. The movement authority gives to the train informations such as its limit of speed and the goal distance that are authorised at the moment depending on the location of the previous train. Based on these informations, the EVC of train calculates the maximum speed permitted of train and the next braking point if needed. The speed of train is continuously monitored by the onboard system. Because of the spot transmission, the train must travel over the Eurobalise to obtain the next movement authority, see Figure 2.24.

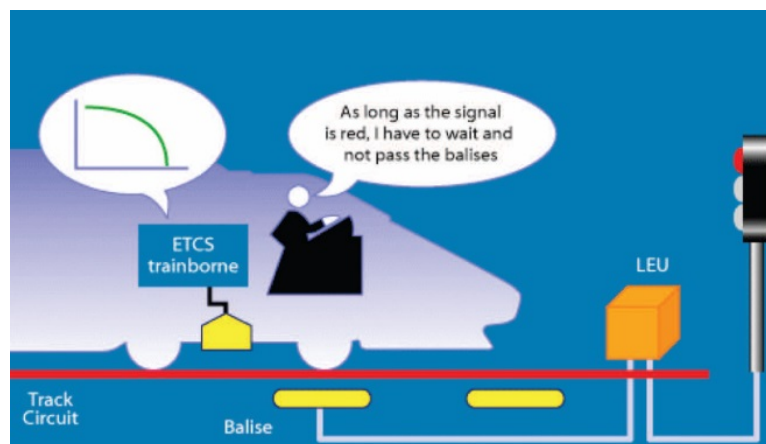


Fig. 2.24 ERTMS level 1. (source: UNIFE-The European Rail Industry)

### ERTMS level 2

As opposed to level 1, ERTMS level 2 does not need trackside signals. The ETCS can receive a movement authority at any time from a Radio Block Center (RBC) via GSM-R system, see Qiu et al. (2014). The Eurobalises are only used to transmit the informations of route such as train position. The

driver receives a continuous transmission of railway line specific data and state of signals on the route ahead, this allows the train to reach its optimal speed but still maintain a safe braking distance, see Figure 2.25.

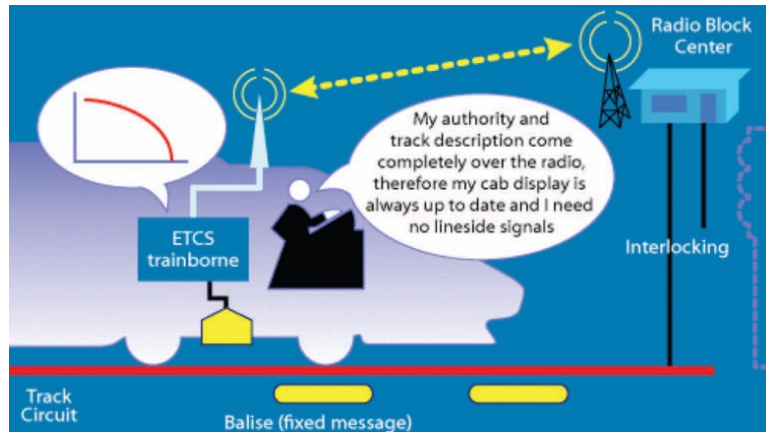


Fig. 2.25 ERTMS level 2. (source: UNIFE-The European Rail Industry)

ERTMS level 2 offers the reduction of maintenance costs through the possibility of removing trackside signals and also increases the railway capacity by reducing the distance between trains while still maintaining a safe braking distance.

### ERTMS level 3

ERTMS level 3 involves the *moving block* technology which is presented in Section 2.3.6. It is also a signalling and control system that provides continuous train supervision with continuous communication between the train and trackside. The difference with level 2 is that the train location and integrity are managed by the ERTMS system. Thus, there is no need for train detection systems on the trackside. Train integrity (train must be complete and not have been accidentally split) is supervised by the train. This makes it possible to optimise the railway capacity and to further reduce trackside equipment, see Figure 2.26.

ERTMS level 3 is still in its conceptual phase and has not yet been developed because of the main challenge: the current technology of GSM-R system is not sufficiently reliable to determine the exact position of trains, see Biagi et al. (2017).

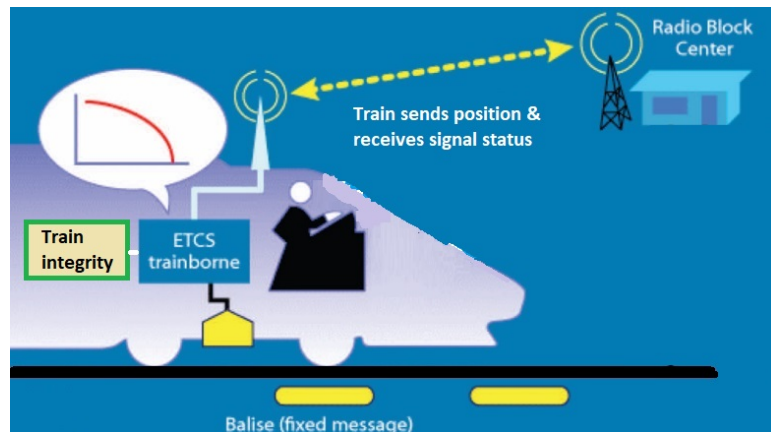


Fig. 2.26 ERTMS level 3. (Figure adapted from UNIFE-The European Rail Industry)

The Table 2.3 below compares the differences between ERTMS level 1, 2 and 3:

TABLE 2.3  
ERTMS LEVELS.

	Level 1	Level 2	Level 3
Communication between train and control center	Eurobalises	Radio Block Centre (via GSM-R)	Radio Block Centre (via GSM-R)
Block	Fixed block	Fixed block	Moving block
Trackside train detection equipment	Yes	Yes	No
Railway signal	Colour traffic lights signals	Cab signalling	Cab signalling
Train integrity onboard	No	No	Yes
Continuous communication	No (spot transmission)	Yes	Yes

## 2.4 Railway capacity

### 2.4.1 Notions of capacity

The capacity of a railway can be defined in many ways. In general, the definition of capacity is the maximum number of rolling stocks that can be able to operate on a railway infrastructure during a specific time interval. The

capacity depends on the schedule and the railway infrastructure. According to [Abril et al. \(2008\)](#), different notions of capacity are used in the railway systems:

- **Theoretical capacity:** Maximum number of rolling stocks that could run over a route in ideal conditions during a given time interval.

- **Practical capacity:** The capacity of railway system can be offered under normal operating conditions at a reasonable level of reliability. The practical capacity represents a more realistic measure. It is usually around 60-75% of the theoretical capacity, which has been introduced in [UIC \(1983\)](#).

- **Used capacity:** The actual traffic volume over the network, usually lower than the practical capacity.

- **Available capacity:** It is the difference between the Used capacity and the Practical capacity and provides an indication of extra trains that can be handled by the railway network.

According to these definitions, it is possible to introduce different methods to evaluate the capacity consumption. While the theoretical capacity can be calculated by using analytical formulas, the calculation of the practical capacity requires the timetable and the reliability required of railway network.

## 2.4.2 Methods to evaluate capacity

Various approaches have been developed to evaluate railway capacity. The most relevant methods can be classified in two levels: analytical methods and optimisation methods.

### 2.4.2.1 Analytical method for a railway line

This method calculate the railway capacity by using mathematical formulas or algebraic expressions. These formulas usually obtain theoretical capacities and determine practical capacities as a percentage in comparison. Some literature exist for these analytical methods: [Petersen \(1974\)](#), [Chen and Harker \(1990\)](#), [Burdett and Kozan \(2006\)](#).

### Capacity with homogeneous traffic

The capacity available to circulate in an infrastructure can be represented by "*sillon*" (in France). In the graphic timetable, a *sillon* is a line which represents a rolling stock. If all rolling stocks circulate at the same speed and have the same itinerary, the capacity can be easily calculated. In graphic timetable, all *sillons* are parallel, see Figure 2.27. This figure presents the traffic in a railway line which connects two stations A and B. All trains operating in this railway line have the same speed and the same itinerary (travelling non-stop from station A to station B).

In this case, the capacity depends on the time interval between successive trains, which is denoted by  $d$  in the figure.

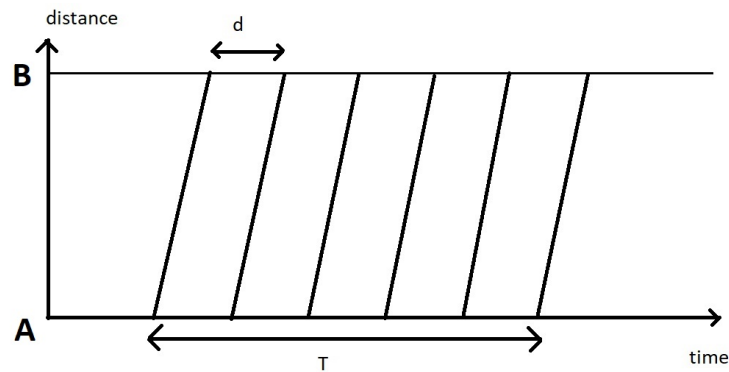


Fig. 2.27 Space-time diagram for homogeneous traffic.

The theoretical capacity during an interval time  $T$  is calculated by the formula:

$$\text{Theoretical Capacity} = \frac{T}{d} \quad (2.1)$$

For example, an Automatic light block (BAL) system has blocks with 2 km length. The speed of each train is 100 km/h.

In order to calculate the minimal delay between two consecutive trains in railway line, it is necessary to evaluate the safety distance that the driver can perceive the railway signals.

The safety distance between two successive trains in rigorous case are equal to the sum of the length of 2 blocks, the length of train and the

distance necessary covering the time that the driver perceives the railway signals (operating time):

Headway distance =  $2 \times$  length of block + length of train + operating time

We assume this value is equal to 5 km.

Therefore, the delay between two successive trains are:

$$d = \frac{5 \text{ km}}{100 \text{ km.h}^{-1}} = 0.05 \text{ h} = 3 \text{ mins}$$

The capacity in this railway network during 1 hour ( $T$ ) is:

$$\text{Theoretical Capacity} = \frac{1}{0.05} = 20 \text{ trains}$$

The railway lines of metro and also some busy railway lines of RER in big cities are railway lines with homogeneous traffic. This configuration can maximize the capacity of railway network.

### Capacity with mixed traffic

The previous case is a special case. In fact, most of railway infrastructures have mixed traffic with various type of rolling stocks and different routes. Therefore, their circulations does not have the same speed. In the graphic timetable, the *sillons* are no longer parallel, see Figure 2.28.

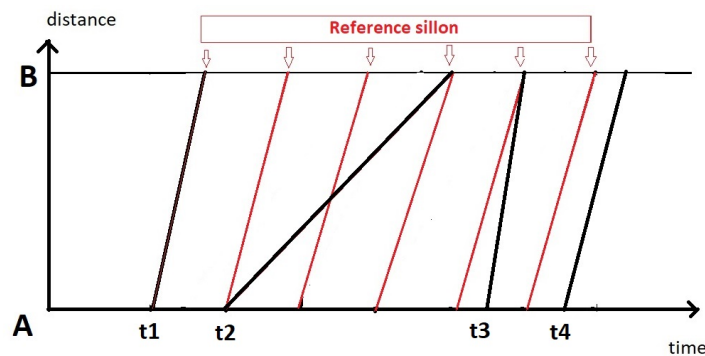


Fig. 2.28 Space-time diagram for mixed traffic.

The mixed traffic reduces the capacity. The slow train and the fast train could consume many reference sillons. For example, in the Figure 2.28, train

t1 is a reference sillon. Train t2 is a slower train which consumes three reference sillons.

Train t3 is a faster train. It must depart late to keep the minimum distance from train t2 during their operations. In consequence, train t4, which is a reference sillon, must depart late to keep the minimum distance from train t3. As a result, train t3 and train t4 consume more than two reference sillons.

In this case, the capacity depends on the speed of trains and the order of trains operating in this line. This order can vary depending on different operating decisions. Therefore, it is no longer possible to make a formula to calculate exactly the capacity. The construction of timetables is the only way to calculate exactly the capacity. The Figure 2.29 displays two different timetables with different capacities for the same traffic in a railway line.

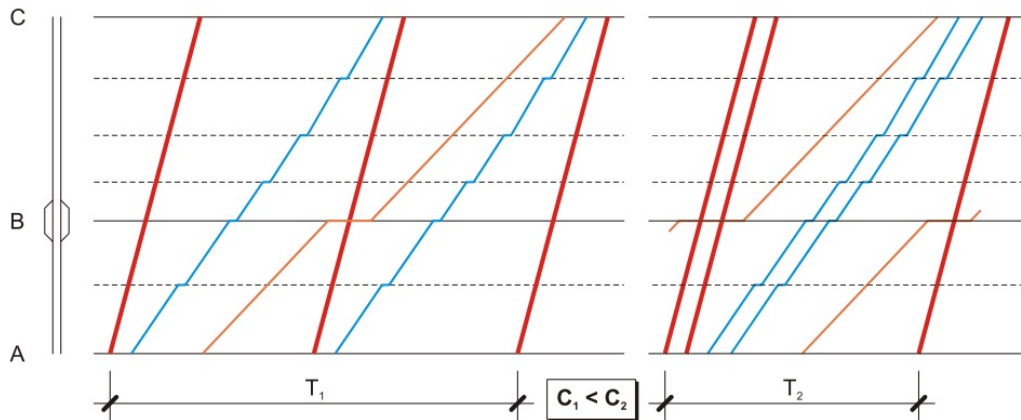


Fig. 2.29 Two space-time diagrams with different order of trains for mixed traffic. Source: Buri (2008)

According to Réseau Ferré de France (RFF) (2005) the theoretical capacity can be calculated by the formula 2.1 using the speed of reference trains which are the most frequent train circulating in the railway line considered. A formula calculating the capacity consumption for each train based on this theoretical capacity is presented in this article and recalled by equation 2.2.

$$C_i = \frac{|T_i - T_R|}{d} \times \omega + 1 \quad (2.2)$$



Where:

- $C_i$  is the capacity consumption of train  $i$
- $T_i$  is the time that train  $i$  passes through railway line considered
- $T_R$  is the reference time that train passes through railway line considered
- $d$  is the time interval between successive reference trains
- $\omega$  is a coefficient which represents the possibility of optimizing railway capacity (the optimisation of railway capacity consists of the possibility of regrouping trains with same speed or avoid inserting a fast train in the middle of slow trains). This coefficient is the same for all trains in the railway line but it is different depending on the time horizon considered.

The term "+1" in the formula allows to verify the fact that a reference train consumes a unit capacity (a *sillon*).

#### 2.4.2.2 Optimisation methods to evaluate a railway line capacity

The optimisation methods provide more strategic methods to evaluate the railway capacity problem compared to analytical methods.

Optimisation methods are based on obtaining optimal saturated timetables. These optimal timetables are usually obtained by using mathematical programming techniques such as Mixed Integer Linear Programming Formulations, graph algorithms. These methods calculate the railway capacity by scheduling a maximum number of possible trains in a timetable (starting from an empty timetable or with an initial timetable).

The literature focusing on optimal timetables will be introduced in Section 3.3 on page 55.

#### 2.4.3 Capacity of a railway station

The evaluation of capacity for a railway station is similar to the methods for a railway network in general which are presented previously. However, in railway stations, the capacity depends not only on the conflicts between successive trains but also the converging or diverging traffics in many

junctions.

In a railway station, each railway line may interact with other railway lines. Therefore, the routes of many trains must contain some common sections to go through railway junction. These parts of railway infrastructure are shared resources which have a high impact on the capacity of a railway station.

In order to ensure the safety, it is necessary to reserve shared resources for a train before entering in its route and forbid other trains from reserving the same shared resources. When the first train releases the shared resources, the second train can reserve these shared resources in its turn.

In fact, the reservation duration of shared resources for each train is greater than their effective occupation duration. Moreover, the release of shared resources can be delayed due to the technical issues in switch areas. Clearly, the reservation of infrastructure resources for the routes of trains have a high impact on the capacity of railway stations. There are three different reservations: reservation of a complete route (Kroon (1997), Zwaneveld and Kroon (1997), Bai et al. (2014)), reservation of a complete route but release of resource successively (Pellegrini et al. (2014)) and reservation of resources independently (which has not been researched yet). These route reservation procedures will be introduced in Section 3.5 on page 70.

**A complex railway station** is a part of railway network characterised by a variable configuration due to the presence of elements as turnouts or crossings in a very limited area. The evaluation of carrying capacity of complex railway stations is a typical problem in metropolitan areas where the same infrastructures are used for different services (national, regional, urban, passenger, freight, etc.). The frequency of these services is usually high during specific periods of the day. In these cases, the main problems to be considered include the identification of the infrastructure critical resources and also the definition of the most effective actions for the full exploitation of the railway capacity while respecting the safety.

#### 2.4.4 Strategies to increase capacity

This section introduces some strategies to increase railway capacity. These strategies can or cannot be implemented depending on the railway infrastructure and the view of railway manager.

#### 2.4.4.1 Building new infrastructure

The simplest solution to increase capacity is to increase the number of railway tracks. This solution is expensive but it gives flexible ways to increase capacity. In fact, due to the building of new tracks, groups of trains which circulate in the same direction can be placed in the same line. Therefore, it can easily remove the constraints with trains which circulate in opposite direction. Another way to increase capacity in this new infrastructure is to place fastest trains in a same line and slowest trains in another line.

In reality, this solution generally cannot be implemented because of various reasons, mainly financial issues. Moreover, the increase of tracks may create an overcapacity and it is very difficult to get a payback for the investment.

#### 2.4.4.2 Increase relatively homogeneous traffic

In a saturation line, without building new track, the solution to increase the number of trains is to make different *sillons* more homogeneous. An example of this solution is the removal of some stops of slowest trains or the increase of the number of stops of fastest trains. Due to these modifications, the *sillons* in the graphic timetable will become nearly parallel.

This solution is practicable for local railways. In fact, in case of urban traffics, the speed of passenger trains is generally faster than the speed of freight trains, see [U.S. Transportation report \(2019\)](#). Therefore, it is feasible to make homogeneous traffics by adding some stops for passenger trains, see [Buri \(2008\)](#).

Clearly, an other way to increase homogeneous traffic is to increase the speed of slowest trains. It reduces the difference of speed between *sillons* and gives the possibility of adding another train behind these trains, see [Figure 2.30](#).

#### 2.4.4.3 Performance of rolling stocks

The rolling stock is an essential component of a railway system. Its performances and characteristics have a high impact on capacity. In fact:

- The maximum speed, the braking system have an influence on the commercial speed of trains. Therefore, it influences the capacity by the

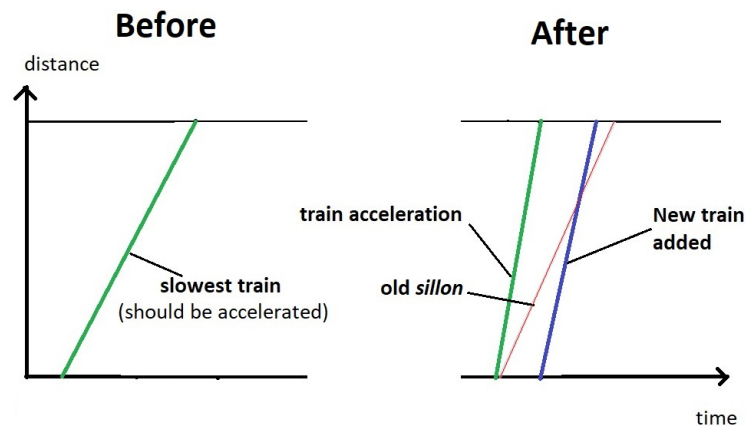


Fig. 2.30 The impact on capacity of the acceleration of slowest train.

influence on the number of *sillons*.

- The length of trains and the type of trains (one or two floors) have impact on the capacity that corresponds to the number of transported passengers or transported goods.

The increase of train length or the implementation of trains with two floors do not only give more seats on trains (increase capacity) but also increase the comfort for passengers. This solution still needs some investment for the innovation of rolling stocks and some works on infrastructure like increasing the length of platforms or adapting the height of new rolling stocks. For example, the length of blocks should be at least two times the length of train.

#### 2.4.4.4 Railway traffic management

The construction of new infrastructure takes a long time and a lot of money to design and build. Thus, it is highly recommended to optimise the utilisation of existing infrastructure resources to increase capacity. The capacity of a railway network depends on the railway signalling and control system as described above in Section 2.3. For example, a railway signalling and control system with moving block rather than fixed block will improve the railway capacity. On the other hand, railway traffic management allows to deliver optimisation planning models which also improve performance, increase capacity and manage disturbance.

## 2.5 Conclusion

This chapter introduces the railway ecosystem with some key elements to help the reader better understand our research context and the problematic. We introduce the railway ecosystem in three sections: railway infrastructure, railway signalling and control system and railway capacity.

A key question addressed by train operating companies is how to improve quality of railway services and increase capacity in railway networks. Currently they have the constraint of investments in railway infrastructure and railway signalling and control system due to extremely high investment costs. We are interested in the problems of railway traffic management due to its economic and strategic importance. Each problem should have an adequate topological representation of railway network based on railway infrastructure and its elements. The modelling of problems of railway traffic management should also take account of the current technologies and requirements of railway signalling and control system. The modelling of problems of railway traffic management will be presented in the next chapter.

# Railway exploitation for traffic fluidification: steps, issues and approaches

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## 3.1 Introduction

After having studied the railway ecosystem in the previous chapter, in this one, we will introduce the problems of railway traffic management along the chronology of the construction of transport plan and its management over the course of time. Firstly, we introduce different levels of representation from a high granularity with macroscopic vision to a fine granularity with microscopic vision for the structure of railway network. Then we address the problems in railway traffic management with an elaboration of plan of transport. After the

general overview of the railway traffic management, we focus on the problem of routing and scheduling of trains inside railway stations because it has a high impact on the fluidification of railway traffic. After that, we also summarise the state of the art about train rescheduling problem and the robustness in railway management. After that, we present our orientation of research; we propose a model which considers the reservation of sections independently in order to deal with the routing and scheduling problem in complex railway stations.

## **3.2 Railway traffic management**

Many countries in Europe as well as in the world face increasing demand for railway transportation. The development of railway networks increases the complexity of the problems related to railway traffic management. Therefore, it is necessary to develop new decision support tools for these problems.

### **3.2.1 Representation of railway network**

There are many researches on optimisation and decision support for railway traffic management due to its economic and strategic importance. For this, it is essential to have an adequate representation of the structure of a railway network and its constituent elements. This adequate representation should be related to the problem considered. In fact, different representations of railway networks are classified from a high granularity with a macroscopic vision (for example, the track allocation problem for main lines needs a macroscopic representation due to the large area considered) to a fine granularity with a microscopic vision (for example, the routing and scheduling problem for a railway station establish train timetables by a microscopic representation with a fine granularity).

Below we present four different levels of representation, from high level with low precision to low level with fine granularity:

#### **First level of representation (macroscopic representation): lines and nodes**

In this representation, a railway network is modelled as a graph. Railway lines are modelled by the graph edges and railway nodes are its vertices. An example of this level of representation is displayed in Figure 2.2 in Section 2.2.1 on page 7. The capacity and the travel time in these

railway nodes can be estimated. Due to its low accuracy, this representation is mainly used for strategic management problems and to have a global vision.

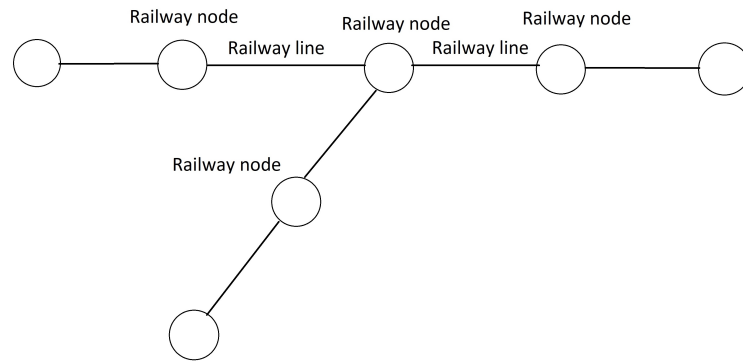


Fig. 2.2 Representation of railway network: lines and nodes. (repeated from page 7)

### Second level of representation (less macroscopic representation): tracks and routes

This representation gives the details for each railway node and railway line, see Figure 3.1.

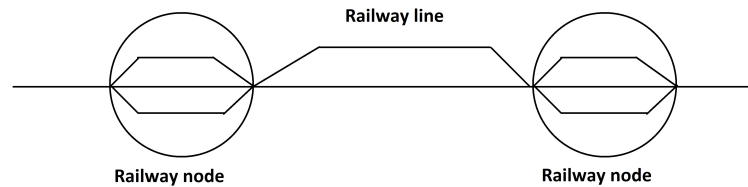


Fig. 3.1 Representation of railway network: tracks and routes.

This representation provides a good trade-off between the size of problem and the quality of representation.

### Third level of representation (less macroscopic representation): blocks

In this representation, each line is divided into many blocks which are related to the management of railway signalling and control system. The concept of block is introduced in Section 2.3.3 on page 25. This representation



is usually used for problems in a small part of a railway network which requires a precise evaluation of capacity.

**Fourth level of representation (microscopic representation): sections**

This level has the most fine granularity. In this representation, each block is divided into many track sections. Each section consists of at least one track circuit for the train detection mechanism (which is presented in Section 2.3.2.1 on page 14). This representation can determine the exact position of trains in the railway station. It is mainly used for the problems in a complex railway station or the management in real-time.

On the other hand, it is not strictly mandatory to use one level of representation at a study. Many levels of representation can be used in the same time to facilitate the study.

A summary table of the four levels is presented in Table 3.1:

TABLE 3.1  
SUMMARY TABLE OF THE FOUR LEVELS.

Level	Structure	Description	Previous studies using the representation
1 (macro)	Lines and nodes	A station is a node in the network	Caprara et al. (2002), Giglio and Sacco (2016)
2	Tracks and routes	Each station is detailed as many tracks	Carey (1994), Burggraeve and Vansteenwegen (2017)
3	Blocks	Each line is divided into many blocks	Oliveira (2001)
4 (micro)	Sections	Each block is divided to many sections	Delorme et al. (2001), Rodriguez (2007), Pellegrini et al. (2014)

### 3.2.2 Classification of problems

This section introduces the elaboration of transport plans in railway traffic with different problems from very early stages with the assessment of long

term demands to last stages with demands at the last moment and the management of the inevitable disturbances in real time.

The railway companies have to distribute and attribute railway capacity (*sillon* in French used in France) to all parts of railway network while ensuring the best use of railway infrastructure as well as the quality of railway services. The attribution of capacity should accommodate the clients' demands, however, it must respect the disposition of railway infrastructure and the requirement of safety.

The main stages of the elaboration of transport plan in railway traffic are introduced in Figure 3.2. Each stage is the continuity and the progression of the previous stage.

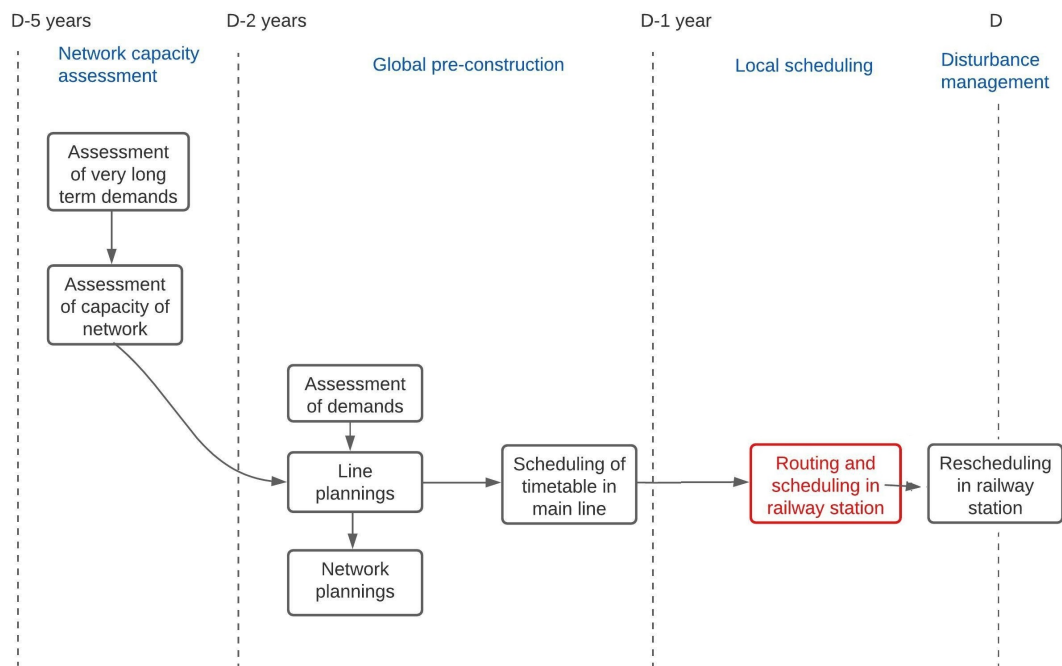


Fig. 3.2 Chronology of problems of railway traffic.

### 3.2.2.1 Network capacity assessment

Between D-5 years and D-2 years, before the formal demand of *sillons*, **the assessment of very long term demands** estimate the demand of clients (activities of transportation of passengers and goods) which is related to the construction of infrastructure and the pre-construction of plan of *sillons*. **The assessment of capacity of network** allows to determine the available capacity based on the current infrastructure and to study the construction of new infrastructure in order to satisfy the need of clients.

### 3.2.2.2 Global pre-construction

In the pre-construction phase (between D-2 years and D-1 year), firstly, the expressions of need of clients are transmitted to railway companies. In **the assessment of demands** stage, the railway companies exchange with clients about their needs in order to confirm and specify their needs taking account the constraints of capacity and their knowledge of the market.

After that, **the line planning** stage is the phase of construction of *sillons*. In this stage, the itineraries of trains, which consist of consecutive railway lines and railway nodes in the railway network for the operation of trains, are defined. This stage must specify the departure time of trains at origin station and the arrival time of trains at destination station in their itineraries, as well as the frequencies of *sillons*.

**The network planning** is the stage to understand how the plan of *sillons* delivers services to passengers and freight users, how it works with local level, the regions and the train operators.

With a good knowledge of railway activities through **the network planning** stage, **the scheduling of timetable in main line** is the last stage of pre-construction phase which generates a timetable for itinerary of trains. It must specify the departure time and arrival time of trains at each station in their itinerary.

Due to the limited capacity of railway infrastructure and the different needs of clients, the construction of *sillons* and scheduling timetables in main line are generally associated with a problem of optimisation which can be solved by mathematical methods. For this, the topology and layout of railway network must be structured. Clearly, the microscopic models are not necessary in this case. Most approaches of previous researches consider

macroscopic models to develop the decision support tool for this problem. In such models, the topology and layout of railway stations are simplified to nodes with a limited capacity. This capacity defines the maximal number of trains that can operate at the same time in railway stations. Consecutive blocks are aggregated to railway lines connecting the railway stations. [Lusby et al. \(2011b\)](#) provide a survey on various model formulations for this problem.

[Caprara et al. \(2002\)](#) consider the problem of scheduling train in the Italian railway network. They prove the considered problem is NP-hard and they propose a graph theoretic formulation for the problem using a directed multigraph  $G=(V, A)$  in which nodes in  $V$  correspond to departures/arrivals at a certain station at a given time instant. Each arc in  $A$  represents the duration between two events, for example the travel time between stations or the dwell time at stations (stopping time), see [Figure 3.3](#) taken from [Caprara et al. \(2002\)](#) for an illustration. The node set has the form  $\{\sigma, \tau\} \cup (U^2 \cup \dots \cup U^s) \cup (W^1 \cup \dots \cup W^{s-1})$ , where  $\sigma$  and  $\tau$  are a *source node* and *terminal node*, sets  $U^i$  and  $W^i$  represent the set of arrival time and departure time in station, respectively.  $p^1, p^2, p^3$  in this figure are three examples of routes (or path) for a graph with 6 nodes ( $W^1, U^2, W^2, U^3, W^3, U^4$ ) which includes a source node ( $\sigma$ ), a terminal node ( $\tau$ ) and 4 stations. The authors propose a Lagrangian relaxation solution approach (see [Lemaréchal \(2001\)](#)) that relaxes a large number of constraints in this formulation. The approach is tested on real instances in Italy containing up to 73 stations and 500 trains during a time horizon of one day. Most cases of all problems could be solved within small percentage gap (less than 2%) which is the percentage between the value of the best heuristic solution and the best upper bound. However, on other cases particularly highly congested cases, the optimality gap can be as high as 20%. [Caprara et al. \(2006\)](#) extend the formulation to include several additional constraints that arise in real world applications: station capacities, maintenance operations and prescribed timetable for certain trains.

[Cacchiani et al. \(2008\)](#) consider the same problem in [Caprara et al. \(2002\)](#), [Caprara et al. \(2006\)](#). They describe an exact branch-and-cut-and-price algorithm and two heuristic approaches to solve this problem. Both heuristics construct feasible solutions by fixing some itineraries of train. Several local search strategies are proposed to improve the solutions constructed by the heuristics. In these local search strategies, for each train which has positive shift, they remove the associated itinerary from the solution and find the optimal itinerary for this train with the itineraries of the other trains fixed. Once all trains have been considered, the trains that are currently cancelled are considered. After processing all these trains, if the solution has been

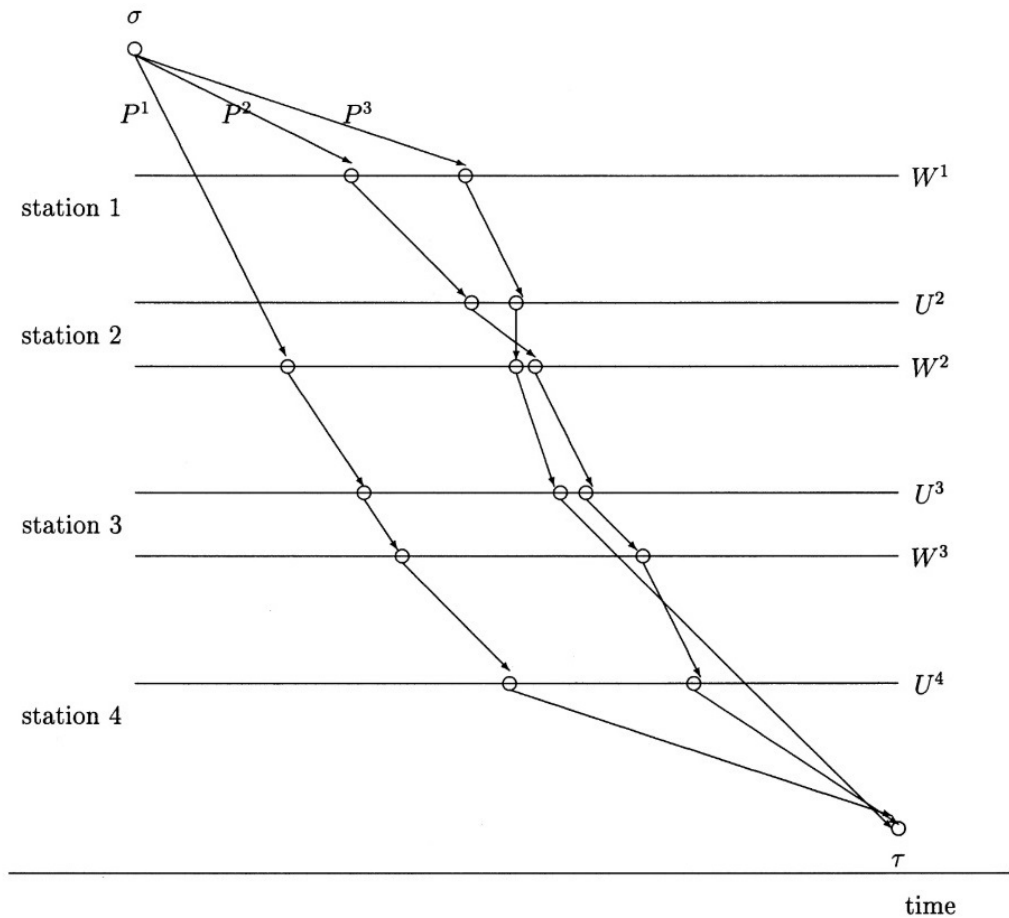


Fig. 3.3 An example of train itineraries.

improved, the procedure is re-applied. The computational experiments considered 11 real instances at Italian railway network to compare with the Lagrangian method of [Caprara et al. \(2006\)](#). The results show that the first heuristic obtains a better solution than the solution of Lagrangian approach. But the computation time is significantly longer. The second heuristic obtains similar quality of solutions to Lagrangian approach. The exact approach is able to solve 3 of the 11 instances in less than 28 hours.

[Borndörfer et al. \(2006\)](#) propose an auction based approach (see [Vries de and Vohra \(1998\)](#) for combinatorial auctions) for the optimal track allocation problem in railway network. Railway network, train characteristics and safety requirements are described by a simplified but still complex macroscopic model. In this model, *slots*, which represent the available capacity of railway network for a particular time, are modelled as combinations of a sequence of blocks. To request a slot, train operating companies must specify the

sequence of stations where trains will pass through, time specifications and the type of trains. The model is tested in the Hannover-Fulda-Kassel area of the German railway network which contains 31 station nodes. The base case considers 946 train requests and all trains have their origin station and terminal station. Extra test cases are generated by allowing flexibility (from 1 minute to 5 minutes) on train departure times. This flexibility increases the search space of the optimisation problem. The computation times increase from 6s without flexibility to 3 days in case of 5 minutes flexibility.

### 3.2.2.3 Local scheduling

#### 3.2.2.3.1 Routing and scheduling in railway station

This is the stage to assign a route for each train and the construction of corresponding timetable in railway nodes. In railway stations, this problem can include the platforming problem which assigns the platform for each train passing through a railway station.

From D-1 year, the railway companies constructs the timetable of trains in railway stations based on the plan of *sillons* in the pre-construction phase and the requirements of commercial time, see [Sncf RESEAU \(2020\)](#) .

The objective of train routing and scheduling problem faced by railway station managers is to generate a conflict-free timetable which consists of two types of circulations. The first set corresponds to the commercial circulations through a study carried out by administrative levels (national, regional, freight) in the previous phase. The second one consists of the technical circulations added by the railway station managers for preparation or reparation operation, hence, the origin or destination of these circulations is a depot inside the railway station.

The timetable must ensure that there are no conflict of trains over routes and platforms while allowing the coupling/decoupling of trains, respecting their reference times at platform and other requirements such as the compatibility of train length with platforms.

A railway station consists of external sections, a large number of internal sections and platforms. A train can enter a railway station from a number of external sections which connect with the railway network outside, and it can leave the railway station also through external sections. The entering

section and leaving section are given for each train and each external section corresponds to a direction of travel. The routing and scheduling problem is to assign each train to a complete route including external sections, internal sections and platform. In order to ensure the safety, the route of one train depends on the routes of the others.

When trains arrive at platform, a customer service time must be given for passengers to board or unboard trains. Moreover, the reference platforms corresponding to train lengths and the constraints of coupling/decoupling mechanisms must also be taken into account.

In complex railway stations which receive high demands of *sillons*, the construction of a free-conflict timetable is very complex. This problem is generally solved by optimisation methods which will be presented in Section 3.3.

#### **3.2.2.3.2 Phase of adaptation**

A phase of adaptation is sometimes needed. The construction of timetables must be made due to demands of need of clients at the last moment.

Different demands of clients at the last moment are described below:

- Demand of cancellation of *sillons* is made by clients as soon as possible. The cancellation must be carried out by railway companies before 24 hours.
- Demand of creation of *sillons* is made by clients due to the new need of transport not later than D-8 days.
- Demand of modification of *sillons* is made by clients due to the evolution of their need not later than D-8 dates. The modification may have no impact on the timetable such as the change of type of trains, length of train, etc. or it may affect the existing timetable with a demand of *sillons* delay.

Once the demands of clients in the phase of adaptation are not compatible with the existed timetable, the railway companies must modify this timetable.

### 3.2.2.4 Disturbance management

The railway networks are susceptible to disturbances caused by failures in the infrastructure and trains which lead to delays and inconvenience for passengers. In this case, the reconstruction of timetable in **rescheduling in railway station** stage must be carried out in real time (operation day). The processing to deal with this problem will be presented in Section 3.4.

## 3.3 Routing and scheduling in railway station

### 3.3.1 Definition of train routing and scheduling problem in railway station

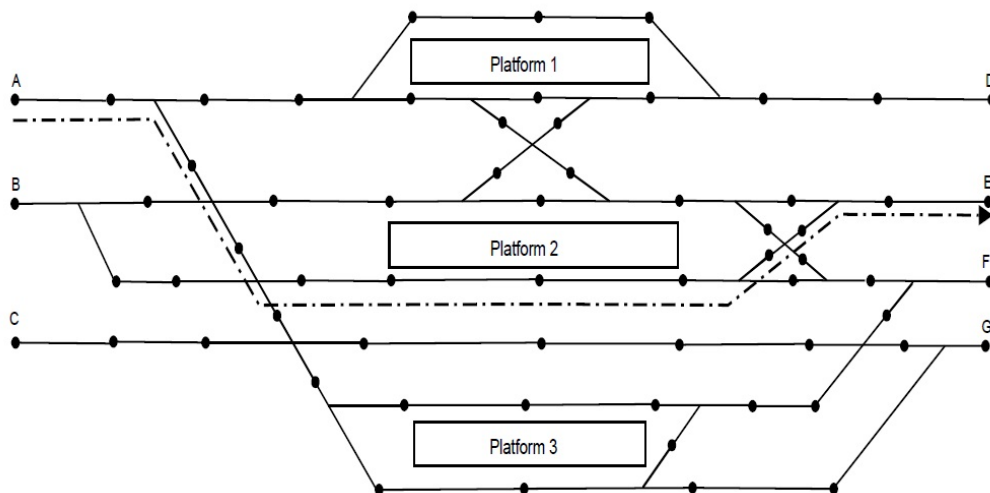


Fig. 3.4 An example of railway junction showing possible routes. Source: Lusby et al. (2011a)

Railway stations have a limited physical capacity that is often insufficient to smoothly accommodate all traffics. The insufficient capacity appears in terms of conflicts: many trains request a same part of infrastructure resources. In case of conflicts, trains must be waiting for their turn for the allocation of resources. Railway interlocking is designed to prevent trains from colliding because of these conflicts of infrastructure resources. *Junctions* are the physical locations on which conflicts are most likely to occur. The network of junctions can be divided into a number of *sections*. These are essentially segments on which there can be only one train at most at a time for safety reasons. Some examples include switches, crossings and platforms. A junction consists of a number of entering points and leaving points. Thus, a route of



trains through the junction is a sequence of sections connecting an entering point to a leaving point. The train may or may not stop at a platform inside the railway station. Depending on the number of these points and also the number of switches in the junction, there may be a lot of possible routes. An example of railway junction is described in Figure 3.4 taken from [Lusby et al. \(2011a\)](#). This railway junction consists of many sections (the dots mark the boundaries of sections). Six *bordering* points A, B, C, D, E, F can be entering points and leaving points depending on the direction of routes. Three platforms are placed alongside some sections providing access to trains. The dash dot line in this figure illustrates a possible route from entering point A to leaving point E.

A formal definition of the routing and scheduling problem for railway station can be described as follow: given the layout of a railway station, the arrival and departure times, as well as the arrival and departure directions of a set of trains, what is the maximum number of trains that can be assigned a route through the railway station? This simple objective function can lead to many other studies:

- If the solution to this problem is less than the number of train given, thus there is no feasible timetable for all trains given. In this case, let us look for if there is a feasible timetable for all trains within the allowable time deviations of trains?
- If the solution to this problem is equal to the number of trains given, the timetable of the solution may or may not be saturated. How many trains can we add to this railway station?

### **3.3.2 Literature review**

The traditional process to generate timetable for a railway station is divided into many stages [Watson \(2001\)](#). Firstly, a provisional timetable is generated by railway operation managers based on the volume of traffic and the traffic frequencies requested. Then, operation managers need to check whether the timetable is feasible while satisfying the capacity, safety and customer service of railway station. At the same time, this timetable must take account of all the required technical operations of railway station (for example: maintenance). Moreover, as a bottleneck problem, the routing and scheduling problems in busy railway stations are very complex with respect to a limited period of time. We classify the contributions by the model proposed and the

approach dealing with these problems.

### 3.3.2.1 Conflict graph approaches

These approaches use conflict graph methodology to model the train routing and scheduling problem. The conflict graph approaches represent each route as a node. Any two nodes (representing two routes) in conflict are connected by an edge. The conflict graph approaches find a conflict free set of routes by selecting nodes which are not directly connected.

Figure 3.5 taken from Lusby et al. (2011b) illustrates an example conflict graph with ten train routes. In this example, the edge between  $\nu_1$  and  $\nu_2$  indicates that the corresponding train routes cannot be assigned simultaneously.

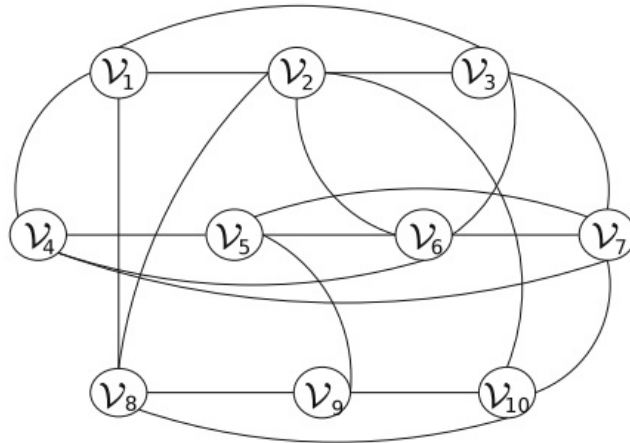


Fig. 3.5 An example of conflict graph. Source: Lusby et al. (2011b)

In this methodology, there are two different approaches which have been used to model the train routing and scheduling problem: node packing formulation and graph colouring approach.

**Node packing formulation:** Let us define a node of the graph for each allowable train-route as  $(t, r)$ ,  $F_t$  is the set allowable routes for train  $t \in \mathbb{T}$ ,  $F_{tt'}$  is a set of pair of compatible routes for two trains  $t$  and  $t'$  (no conflict between two routes). A generic integer programming formulation of the node packing problem in the context of train routing and scheduling problem is given below:

$$\text{Max } \sum_{t \in T} \sum_{r \in F_t} W_{tr} x_{tr}$$

Where:

- $x_{tr}$  is a boolean decision variable for the selection of the particular train  $t$ -route  $r$ .

- $W_{tr}$  is the weight of the decision  $x_{tr}$ , it represents the benefit received in assigning the corresponding train route.

Subject to:

$$\forall t \in T, r, r' \in F_t \quad x_{tr} + x_{tr'} \leq 1 \quad (3.1)$$

$$\forall t \neq t' \in T, r \in F_t, r' \in F_{t'}, r, r' \notin F_{tt'} \quad x_{tr} + x_{t'r'} \leq 1 \quad (3.2)$$

Constraint (3.1) means that there is only one selection route for each train and constraint (3.2) ensures that there is no conflict between any pair of selected routes for all trains.

The first attempt of this model to solve the train routing and scheduling problem is [Zwaneveld and Kroon \(1997\)](#) and [Kroon \(1997\)](#). They consider the reservation of a complete route which guarantees that each train can travel without interruption along the reserved route. They also include shunting decisions, which are the move of a train to a depot track from a platform in the station (and inversely), and small deviations for preferred arrival time and departure time of trains. They prove that if each train has at most two routing possibilities, a solution can be computed in polynomial time.

[Zwaneveld et al. \(2001\)](#) improves the model of [Zwaneveld and Kroon \(1997\)](#) and [Kroon \(1997\)](#). The problem is formulated as a weighted node packing model by making some assumptions about shunting decisions, preferences of trains for platforms and routes. A shunting movement refers to the process of driving a train to (or from) a depot from (or to) a platform in the station. [Zwaneveld et al. \(2001\)](#) also includes preprocessing and reduction techniques in the solution process. In order to reduce the size of problem, three techniques separately called node-dominance, set-dominance and iterating set-dominance are proposed. But the deviations of the arrival and departure times are not taken into account. The computational results are presented for three railway

stations in the Netherlands. The largest railway station consists of 264 sections and 40 platforms. For a problem of 79 trains passing through this railway station, 75 routed trains are found in the optimal solution of their model.

[Delorme \(2003\)](#) develops two approaches for the proposed node packing formulation. The first approach is similar to that of [Zwaneveld et al. \(2001\)](#) and involves extensive preprocessing through variable dominance. They test this approach in a node of Pierrefitte-Gonesse with random instances and specific instances. The specific instances have up to 200 trains with 3720 variables and 482887 constraints. The preprocessing technique is very successful in reducing problem size. On average in case of specific instances, 16% of the variables and 90% of the constraints are removed. However, the exact solution is not efficient. The gap between the best known solution and the solution found can be 400% for the random instances and 25% for specific instances after 14 hours of computation time. The second approach presents the GRASP (Greedy Randomized Adaptive Search Procedure) metaheuristic. This method consists of two phases. Firstly, they implement a greedy heuristic to find an initial feasible solution. Secondly, they improve the solution through a local search heuristic. This approach is presented in the paper [Delorme et al. \(2004\)](#). The average results obtained of this approach are only 2.2% less than the optimal solutions (best known solutions).

[Caprara et al. \(2011\)](#) deals with train platforming problem (which is the routing problem that finds an assignment of trains to platforms in a railway station). They consider a general quadratic objective function and minimise it by involving deviation from preferred platforms and deviation from platform times. There are two types of platform: regular platforms corresponding to platforms which are used in priority and dummy platforms which are used only in case necessary to find feasible solution. They test their approach in three stations in Italy. The instances of the largest station has 237 trains passing through railway station. In 3 out of 11 cases of results obtained, the solution is optimal. In others 3 cases, the gap between the solution found and the optimal solution is less than 1%. In the remaining 5 cases, the gap is ranging from about 15% to a huge value.

**Graph colouring approach:** This approach is also based on the conflict graph. However, unlike the node packing formulation, the aim is not to select nodes of the conflict graph but to assign a colour to each node such that no two adjacent nodes have the same colour.

[De Luca Cardillo and Mione \(1998\)](#) explains that the routing and

scheduling problem in railway station is a graph colouring problem. It can represent every train as a node in the conflict graph, and then connect any two nodes for which the corresponding trains cannot be assigned the same platform. In this case, each platform is denoted as a colour. They use an heuristic algorithm combined with reduction techniques to solve this problem. Their experiments for a railway station involving 242 trains and 16 platforms cannot find solutions in reasonable time.

[Billionnet \(2003\)](#) considers the problem of assigning trains to the available sections at a railway station. The author uses an integer-programming to solve the same problem of [De Luca Cardillo and Mione \(1998\)](#) which is formulated as a graph-colouring problem. This model could easily solve the problem for large stations involving up to 200 trains and 14 platforms. In the work of both [Billionnet \(2003\)](#) and [De Luca Cardillo and Mione \(1998\)](#), no deviation times of arrival and departure time of trains at platform are permitted.

[Cornelsen and Di Stefano \(2007\)](#) uses also graph colouring approach for the routing and scheduling problem in a railway station. They similarly fix the arrival and departure time of trains. However, their topology of railway station is less complex. They consider some variations of the problem with linear and cyclic timetables and show how to solve them as a graph colouring approach. However, they do not provide any result of experiments in this paper.

### **3.3.2.2 Constraint programming approach**

The constraint programming approach considers train routing and scheduling problem in railway station as a job shop scheduling: trains (jobs) use several sections (machines) to complete their route, an operation is associated with the occupation of a section. A sequence of operations is a complete route of trains passing through railway station.

[Carey \(1994\)](#) proposes a constraint programming approach for the routing and scheduling problem in railway station. The route of train is concerned with assigning trains and train times to sections, platform, so as to avoid train conflicts while minimizing costs and satisfying travel demands. The numerical example in his paper has 10 connectors, 28 sections, 10 trains and requires less than one minute to be solved. The scheduling strategy is to find the route of trains one by one until all trains are routed and if necessary, the

route of trains can be rescheduled until a feasible solution is found.

[Delorme et al. \(2001\)](#) compares the constraint programming approach with the node packing formulation GRASP (Greedy Randomised Adaptive Search Procedure). The constraint programming formulation is solved using constraint propagation. The instances of experiments are generated in Pierrefitte-Gonesse node. The constraint programming formulation with the limitations of routes has a big number of decision variables and constraints. However, the best results have been obtained with the constraint programming approach. It performs better when choosing a suitable start time of trains.

### 3.3.2.3 Heuristic approaches

[Carey and Carville \(2003\)](#) proposes a linear model. Heuristic methods are developed according to train planners' objectives. They allow the deviations on the arrival time and departure times of trains. Their algorithm schedules each train one by one. For each train, they check feasible platforms and for each of these platforms, they check if there are any conflicts with other trains that are already scheduled. If there are conflicts, the arrival time and departure time of train are changed to resolve conflicts. The experiment example has 12 main platforms (with 34 sub-platforms) and 491 trains with 900 arrivals and departures. The computation times can be from a few seconds to several hours depending on the heuristic method and the train planners' objectives.

[Bai et al. \(2014\)](#) proposes a mixed-integer program dealing with the routing and scheduling problem for railway stations. Due to the computational complexity of this integer programming method, [Bai et al. \(2015\)](#) applies a hybrid algorithm combining branch-and-bound and Math-Heuristic algorithms to solve the problem with large size. The set of trains is divided into small sub-problems to optimise in time sequence. The experiment example has 250 trains divided in sub-groups, the biggest group has about 60 trains. The computation time is a few minutes with 182 minutes deviation of departure times of 37 trains.

TABLE 3.2  
ROUTING AND SCHEDULING PROBLEM IN RAILWAY STATION LITERATURE REVIEW.

Paper	Model	Solution approach	Route reservation
Carey (1994)	Constraint programming	Heuristic	Complete route
Zwaneveld and Kroon (1997)	Node packing	Branch and Cut	Complete route
Kroon (1997)	Node packing	Branch and Cut	Complete route
De Luca Cardillo and Mione (1998)	Graph colouring	Heuristic	Complete route
Delorme et al. (2001)	Constraint programming	Constraint propagation	Complete route
Zwaneveld et al. (2001)	Node packing	Branch and Cut	Complete route
Carey and Carville (2003)	–	Heuristic	Complete route
Delorme (2003)	Node packing	Metaheuristic	Complete route
Billionnet (2003)	Graph colouring	Branch and Bound	Complete route
Cornelsen and Di Stefano (2007)	Graph colouring	Graph colouring algorithms	Complete route
Caprara et al. (2011)	Node packing	Heuristic	Complete route
Bai et al. (2014)	Mixed-integer programming	Branch and Bound	Complete route
Bai et al. (2015)	Mixed-integer programming	Heuristic	Complete route

### 3.3.3 Conclusion of the literature review

The Table 3.2 summarises all the reviewed contributions of the routing and scheduling problem in railway station.

In this section, we have reviewed literature contributions to routing and scheduling problem in railway station and discussed various approaches that have been proposed to solve this problem. In current state of art, the

reservation of complete route is the only method to model the routing and scheduling problem in railway station. At present, the lower level reservation method is implemented for the rescheduling problem. For example, the Dutch railway companies enforce the *route locking and sectional release system*. In this method, when train arrives at an entering point of railway station, it must reserve all sections it is going to use. Trains successively release each reserved section after traversing them. Some buffer time is usually incorporated into the release time of sections to build some robustness. This method of reservation is implemented in Pellegrini et al. (2014) for rescheduling problem in railway stations. They manage traffic disturbances and compares the performance of this method with the reservation of complete route (called *route lock route release* in their paper). Their results show that the model with route lock sectional release system gives better optimal solution quality. In other word, the solution chosen with the reservation of complete route implies longer delay in trains. This difference is statistically significant.

## 3.4 Railway disturbances management

The disturbances such as delays, interruptions, failures, etc. can occur in the railway traffic management. While initial delays cannot be prevented themselves, running time supplements and buffer times can be included into the timetable to reduce propagation of delays by absorbing minor disturbances. This is called the *robustness* of timetable. However, no timetable can be made resistant enough against major disturbances which affect deeply circulations of train traffic. The increasing extra *robustness* in the timetable for major disturbances can only be done by increasing substantially time reserves, therefore it will decrease at the same time the railway capacity.

Therefore, an alternative strategy is to realise online adjustments during operations to restore the feasibility of timetables and minimise the negative effects of the disturbances. This is called *rescheduling*.

### 3.4.1 Robustness in railway management

#### 3.4.1.1 Definition

In the point of view of railway manager, after the construction of conflict-free timetables, it is important to determine robust timetables that perform well under disturbances, avoiding delay propagation as much as possible.



The railway managers need a stable solution. The timetable should not be reconstructed for small perturbations. Moreover, the service quality, which minimises the delays and cancellations, is an important factor for railway companies.

Robustness in railway management can be called reliability or quality of service. Since train operations are not perfect, some minor disturbances or failures may occur in the real management of trains during the operation. Some buffer times must be taken into account in order to design a robust timetable. For example, we can give buffer times between successive trains passing through a same section. In case of minor disturbances, a train needs more time to pass through this section. The buffer times will reduce the propagation of delays for successive trains.

A robust timetable can tolerate a certain level of disturbances or failures during operations. It ensures that the service can recover quickly when trains are delayed. Another definition of robustness is the recover of service without significant modifications.

#### **3.4.1.2 Literature review**

Recently, robustness has received attention in the literature and real applications for the optimisation of railway management. A recent survey on robustness in railway management is introduced in [Lusby et al. \(2018\)](#).

[Cacchiani et al. \(2012\)](#) proposes *bi-criteria approaches* which are related to bi-objective approach for robust timetabling [Schöbel and Kratz \(2009\)](#). The objective function consists of two terms that appear to be in contrast: the first goal is to maximise the capacity of solutions and the other goal is to maximise its robustness. They propose a heuristic algorithm based on Lagrangian optimisation. The Lagrangian robustness method consists of inserting buffer times corresponding to longer stops of trains at the railway stations. Therefore, it reduces delay propagation. They make the experiments on real-world instances provided by the Italian Railways. The railway network of Modane-Milan (54 stations) is experimented with instances of 100, 200, 300 and 400 trains. The railway Chiasso-Milan (16 stations) is experimented with 194 trains and Chiasso-Rome (102 stations) is experimented with 41 trains. The results show that their approach often obtains robust solutions of good quality.

Liebchen et al. (2009) introduces *recoverable robustness*. The idea is that recovery actions can be used to recover (make feasible) a timetable through limited changes. The recovery actions consists of delaying events or cancellation. The recoverable robust approaches are proposed in Cicerone et al. (2009), D'Angelo et al. (2011) and Caprara et al. (2014). In Caprara et al. (2014), the recovery action consists of propagating the delay over the given railway network. It is applied to the train platforming problem that considers a railway station and a set of trains passing through this railway station. They make the experiments in the station Palermo Centrale. The experiments consider seven time intervals during the day at the station Palermo Centrale. The results show that averaged over all instances, the delay reduction is 25 percent. The reduction is almost 50 percent in only one case.

Fischetti and Monaci (2009), Fischetti et al. (2009) propose a heuristic method for robustness which is called *light robustness*. The idea is to improve the robustness of the timetable while relaxing some feasibility constraints instead of requiring feasibility of the solution in all the considered scenarios. They makes the experiments on Knapsack Problem and a real-world case of the Italian railways operator to compare the light robustness with other robust approach. The results that the light robustness is often able to produce solutions whose quality is comparable with other robust approach, though it requires much less effort in terms of model formulation and computation time.

Kroon et al. (2008) improve the robustness of timetable by a stochastic programming method, see Birge and Louveaux (2011). They propose a two-stage stochastic model. The first stage of model consists of the timetabling part, the second stage consists of a simulation part for evaluating the robustness of the timetable. The idea is to minimise the sum of delays in the first stage and to generate a set of delay scenarios in the second stage. Some additional time variables are introduced in the first stage to absorb disturbances in the second stage. Another application of stochastic programming is applied in Meng and Zhou (2011). The stochastic programming with recourse framework is considered. The proposed model optimises schedules for a relatively long rolling horizon in which a stochastic program combined with a multi layer branching strategy is used to select a robust horizon plan. The experiments focus on a 138km single track network connecting 18 stations in China. The results show that the robust solutions are better than the expected value by a range of 10-30%.

In view of the above, the application of optimisation techniques to

robustness in railway management is still very limited. In general, buffer times are added to the travel time of trains in the construction of timetables. This is a basic robust measure against delay propagation. However, the railway companies focus mainly on the income of the companies which are directly related to railway capacity.

## 3.4.2 Train rescheduling problems

### 3.4.2.1 Definition

The usual method how railways manage their traffic in the long term is through a designed plan of operations defining itineraries, orders and timing for all trains. This process is called *off-line management*. In practice, disturbances may influence the off-line plan, causing delays or temporary unavailability of some routes. If the *robust timetables* are not able to recover, the rolling stocks schedules from the off-line management must be *rescheduled* in order to adapt to the disrupted situation and minimize the negative effects of the disturbances. This process is called *online management* (or real-time management). The possible modification situations include changing stopping times at scheduled stops, changing train speeds, or adjusting orders of trains. The major modifications can be considered such as changing train routes or cancelling (removing) one or many trains in the plan of operations.

Online railway management (rescheduling problem) has been widely investigated by researchers. Several heuristic and meta-heuristic algorithms are developed and applied on rescheduling problem.

Comparing with offline management(scheduling problem), there is only limited time available for creating a new timetable. There maybe have a cost associated with the deviation of the existing timetable. Deviations from the timetable require rescheduling of some resources. Some deviations are more serious than others. In practice, there is a distinction between minor incidents (called disturbances) and major incidents (called disruptions), see [Cacchiani et al. \(2014\)](#).

A disturbance is a small perturbation in a small part of the railway operations (the delay of departure time of a train for example). Disturbances are usually absorbed by the robustness of railway system or can be handled by small changes to the resource allocation.

More serious perturbations are known as disruptions. Disruptions cause

serious timetable changes where trains are cancelled or rerouted.

The different types of rescheduling decisions can be divided as follows:

- changing arrival and departure times of trains including modification of stopping schedules.
- reordering of trains.
- rerouting of trains.
- cancellations of trains.

According to the description above, the rescheduling problem in a railway network can be defined as follows: considering the affected trains in a railway network and the original timetable for these trains including the assignment of sections, platforms, orders and timing, etc, rescheduling is to find a new feasible timetable by satisfying the objective functions such as minimising the delays, costs and the commercial constraints (arrival time, departure time of trains at platform).

#### 3.4.2.2 Literature review

Some surveys on models and solution approaches to rescheduling in railway network are introduced in [Törnquist \(2007\)](#) and [Fang et al. \(2015\)](#).

[Higgins et al. \(1996\)](#) proposes a model for the rescheduling trains in real time on a single track line. Their model mainly addresses the operational problem of trains in real time but can apply at the strategic level to evaluate the impacts of timetable changes in case of train delays. They use a lower bound estimate of the remaining crossing and delay to reduce the search space in the Branch-and-bound tree to find the solution. The algorithm is tested on a rail corridor containing 14 sections. 31 trains are scheduled on the busiest day of the week. Their algorithm only required 46s to determine the optimal solution.

[Norio et al. \(2005\)](#) proposes an idea to use passengers' dissatisfaction as the objective criteria of train rescheduling problem. Hence, the authors consider train rescheduling problem as a constraint optimisation problem in which dissatisfaction should be minimised. They introduce an algorithm combining PERT (Program Evaluation and Review Technique) and simulated

annealing. The experimental results show that this model works quite fast and it support versatile methods of rescheduling including cancellation, change of train-set operation schedule, change of track, etc.

Mazzarello and Ottaviani (2007) proposes a three steps heuristic algorithm which are from small modification actions to major modification actions: the first step allows only the change in orders of trains, the second step allows route modification option and the third step allow new routes selected. The algorithm is tested in Schiphol station, in Netherlands. The results show that it is possible to manage the requested 27 trains per hour timetable through the Schiphol bottleneck with reasonable performance.

D'Ariano et al. (2007) proposes a branch-and-bound algorithm for solving the scheduling problem with fixed routes. The implementation of a real-time traffic management system called ROMA (Railway traffic Optimization by Means of Alternative graphs) is presented in D'Ariano et al. (2008) to automatically recover disturbances. ROMA is able to automatically control traffic, evaluating the detailed effects of train reordering and rerouting actions, while taking into account minimum distance headways between consecutive trains. The computational experiments carried out on a heavily congested area of the Dutch railway network. It shows very promising performance of the algorithm in finding near optimal solutions to practical size instances within short computation times.

Rodriguez (2007) presents a constraint programming model for real-time train scheduling in a railway junction. The author defines delays of trains as the time spent waiting on a section, the time decelerating and accelerating of train. The objective of this constraint programming formulation is to minimise the sum of delays. The author makes the experiments on the Pierrefitte-Gonesse node, in North of Paris. Test instances have between 6 and 24 trains. The results show that the solution obtained has a significant improvement in performance with a relative wasted time is reduced by 62-95 percent. The computation time is less than the time required to apply the solutions in real conditions.

Pellegrini et al. (2014) proposes a track-circuit based model dealing with perturbations. In their paper, all track-circuits belonging to a block must be reserved for a train, they develop two models with different granularities of representation. In the first model, all track-circuit in a block are released only when the train exits each block. In the second model, each track-circuit is unlocked after the train has exited it. The results show that the solution

of the first model has more delays than the second one and this difference decreases slowly when increasing the time horizon considered.

[Sato et al. \(2013\)](#) proposes a rescheduling model including train reordering and adjustments of time that improves the passenger services (minimizing of waiting time, travelling time, etc.) In this model, they minimise further inconvenience to passengers instead of consecutive delays caused by the disruption. The model first solves the arrival delay minimisation problem and then optimises the timetable in terms of the inconvenience based on the delay-minimised timetable and the timetable flexibility parameter. The experimental results have proved the trade-off between the inconvenience minimisation and the delay minimisation.

[Caimi et al. \(2012\)](#) proposes a model that allows rerouting and changes in the platform assignment in a complex station. The model is based on discrete-time model predictive control. They propose a closed-loop discrete-time control system, an approach commonly used in model predictive control. A forecast module computes a forecast of the evolution of the railway system based on the current system state and indeed commercial offer. The forecast is then used to detect and resolve potential resource conflicts by proposing new disposition schedules. These disposition schedules can be combined with forecasted time-distance graphs to visualise their effects onto the system for the dispatcher. The control loop is finally closed by forwarding disposition decisions to the infrastructure and trains. The model is tested in the station area Berne, in Switzerland. This station has 13 platforms and it is one of the most challenging train rescheduling areas in Switzerland. Their model successfully managed railway traffic for an operational day with small computation times.

[Piconese et al. \(2014\)](#) proposes a mixed-integer programming model for a regional railway network. They analyse four alternative objective functions in order to find the optimal solution which is a good compromise between total delay, number of rescheduled trains and computation times. The model is tested in the railway network connecting Mungivacca and Putignano stations in Italy. During a day, 24 even trains and 22 odd train run on the railway line in opposed direction. The results show that the model carries out the rescheduling process in a short computation time.

Some algorithms dealing with rescheduling problem can be referred to the solution of train routing and scheduling problem in railway station to deal with big size problems.

## 3.5 Conclusion

This chapter introduces the literature review on the fluidification problems for railway traffic. Firstly, we introduced the different problems of railway traffic management with an elaboration of plan of transport from macro to micro on the point of view of railway companies. Then the literature review about the routing and scheduling problem in railway station is summarised and discussed on current state-of-the-art limitations. Finally, the railway disturbances management is presented with two main problems: *robustness* and *rescheduling*.

In our study, we are particularly interested in the routing and scheduling problem of railway station. There are various types of trains (TGV, TER, Freight) which arrive at railway stations. Due to very high capacity demand of railway stations, a tentative timetable is produced, the traffic feasibility in railway stations is not always guaranteed. In addition, the timetable also needs to allocate infrastructure resources in time and space for all regular passenger trains including extra traffic operation for short-term requests which may be delivered few days before operation. A new feasible timetable must be produced and it is followed in real time to manage disturbances. The timetable specifies the routing and scheduling of trains through railway stations at a precision of minutes including the platform usage, the order of trains, the departure time and arrival time of each train to each location of its routes. Traditionally, timetables have been generated manually by drawing trains on the time-distance diagram, where train schedules are manually adjusted so that all constraints are fulfilled. This process can take a long time and it usually stops when a feasible timetable is found.

As mentioned in the Section 2.3.2.3.3 Page 24, we have introduced the limitation of the interlocking of route in railway stations. The current state of the art cannot properly solve the saturation problem in complex railway stations. Indeed, the literature in Table 3.2 show that the reservation of complete route is currently the only method which is used to deal with the routing and scheduling problem in railway stations. In the reservation of complete route, trains must reserve all physical resources in their route before using them. This method guarantees that trains can circulate safely without interruptions. The end of the reservation of physical resources depends on the system of interlocking of route applied:

- *Route lock route release (transit rigide* in French): All physical resources of a train route are released when the train finishes its route. It means that a

conflicting route (with this train route) of another train can be reserved only after the complete release of this train route.

- *Route lock sectional release (transit souple in French)*: A physical resource of a train route is released when the train leaves it. Thus, as soon as the physical resources in conflict with a conflicting route of another train are released, this conflicting route can be reserved.

In our study, we want to assess the interest and performance of a model considering the reservation of physical resources independently (*sectional lock sectional release*). This implies low-level modeling considerations with respect to the speed and length of trains. A physical resource can be reserved when a train arrives and it can be released after the train leaves it. This technique allows a larger number of trains to operate simultaneously in the railway station, so that the use of available resources can be more efficient. It allows the full exploitation of the capacity of railway stations.

The reservation of sections independently or *sectional lock sectional release* model implements the progressive reservation of resources used by trains along their route on the railway station. The *sectional lock sectional release* model offers a better exploitation of the available capacity but requires more equipment liability to guarantee safety. Currently, this model has not been researched or implemented in railway systems because of two main reasons. The first reason is the safety. This model needs a good reliable systems to check the occupation of each section. A small failure may cause serious consequences. The second reason is that this model does not guarantee the continuous circulation of trains. Train may stop at some sections in the middle of its route if the next section is occupied by another train. This interruption (train stops and restarts) consumes a lot of energy.

Our problem is positioned in context and in theory. Train routing and scheduling problem is a complement sub-problem in railway traffic management. Our research focuses in dense area, typically in interchange central stations. The objective is to construct a timetable which ensures the safety (no conflict of trains in the routes) while respecting reference platforms and allowing coupling/decoupling trains. A detailed description of our problem will be introduced in the section below.





# Problem formalization

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## 4.1 Introduction

In the previous chapter, we have introduced several related works on train routing and scheduling problem. Our research focuses in complex railway stations where the frequency of traffic is usually high during specific periods of the day. We are interested in the routing and scheduling problem in railway stations because currently it requires very high capacity. The objective is to develop a model to generate a timetable which has to fit the existing infrastructure while respecting safety and commercial schedules.

In this chapter, we describe the problem of routing and scheduling in railway stations in many parts: railway topology, trains' activities, reference data and constraints for safety. The Section 4.2 carries out a formalization

of railway topology to consider the railway infrastructure. The allocated resources are defined as two components: *sections* and *connectors*.

We want to construct an abstract model as general as possible in order to be able to study the problems of railway management from several points of view. In this abstract model, a *section* can represent a track section or a block depending on the level of representation in railway network and the point of view of railway management. A *connector* can represent a switch, an entering point, a leaving point in a junction or just a simple separate point between sections.

For example, in the context of a railway line between two stations, a section is a block in the railway line, a connector is a boundary point (where the railway signals take place) which separates two successive blocks.

In the context of a railway station, we propose to consider that a section is a track section of the railway station, a connector is a switch, an entering point or a leaving point in the railway station. This implies a low-level modelling for routing and scheduling trains in railway stations. This model allows studying the problems very finely, especially at the temporal level. This level of detail is rarely studied in literature. In some studies, it is implemented to deal with the rescheduling problem in complex railway stations such as [Pellegrini et al. \(2014\)](#) where a section represents a track circuit.

We introduce the principles of construction which allow establishing abstract topologies. The abstract topologies allow to conduct formal studies while preserving the rules of circulation of railway systems in order to obtain the realistic results which can be implemented in reality.

Trains' activities are formalized in Section 4.3. We focus more precisely on activities in railway stations. In this section, we analyse scheduling notations, routing notations and present types of circulations and coupling/decoupling mechanism.

## 4.2 Topology of a railway station

We propose to study a topology based on two types of generic components: "section" and "connector".

### 4.2.1 Section

A **section** is a segment of railway station that can contain only one train at a time.

The set of sections in a railway station is denoted by  $S = \{s_1, s_2, \dots, s_S\}$  where  $S$  is the cardinal number of  $S$ .

In a railway station, we can partition the set of *sections* into three subsets. A section can be an *external section* where trains can enter or leave railway station, an *internal section* which is bounded by two points (switches) or a *platform* providing convenient access to trains.

An **external section** is a section surrounding the railway station, located at the entrance of the railway station, represented by a line which connects from a connector to the outside of the railway station where trains can enter or leave railway station. The set of external sections in a railway station is denoted by  $E = \{e_1, e_2, \dots, e_E\}$  where  $E$  is the cardinal number of  $E$ . Obviously,  $E \subset S$ .

An **internal section** is a section inside railway station where trains can pass through. The internal sections are not platforms. The set of internal sections in a railway station is denoted by  $I = \{i_1, i_2, \dots, i_I\}$  where  $I$  is the cardinal number of  $I$ . Thus,  $I \subset S$ ,  $I \cap E = \emptyset$ .

A **platform** is a section from which passengers can await, board or unboard from trains. Train can usually stop long-time in platforms.

The set of platforms in a railway station is denoted by  $P = \{p_1, p_2, \dots, p_P\}$  where  $P$  is the cardinal number of  $P$ . Thus,  $P \subset S$ ,  $P \cap E = \emptyset$ ,  $P \cap I = \emptyset$  and  $S = E \cup I \cup P$ .

**Length of platform.** The length of platforms must be taken into account to be compatible with the trains to meet the requirements of customer services. The length of platforms can be categorized as  $\{long, medium, short\}$ . Obviously, if a train is compatible with short platforms, it will also be compatible with medium platforms and long platforms. Therefore, we have three possible subsets of platforms  $P_{ref}^{long}$ ,  $P_{ref}^{medium}$ ,  $P_{ref}^{short}$  with  $P_{ref}^{long} \subset P_{ref}^{medium} \subset P_{ref}^{short} = P$ .

### 4.2.2 Connector

A **connector** is a point which connects several sections.

The set of connectors in a railway station is denoted by  $\mathbb{C} = \{c_1, c_2, \dots, c_C\}$  where  $C$  is the cardinal number of  $\mathbb{C}$ .

In a railway station, a connector can be a switch which is connected from platform to internal section, from internal section to another internal section or from internal section to external section. It can be also an aggregation of several physical switches which will be described in Section 4.2.3.

A **bordering connector** is a connector surrounding the railway station, connected to an external station where trains can enter or leave the railway station.

The set of bordering connectors in a railway station is denoted by  $\mathbb{B} = \{b_1, b_2, \dots, b_B\}$  where  $B$  is the cardinal number of  $\mathbb{B}$ . Obviously,  $\mathbb{B} \subset \mathbb{C}$ .

#### Relations between sections and connectors

The topology we consider corresponds to a sequence of sections and connectors. Each section is bounded by only two connectors.

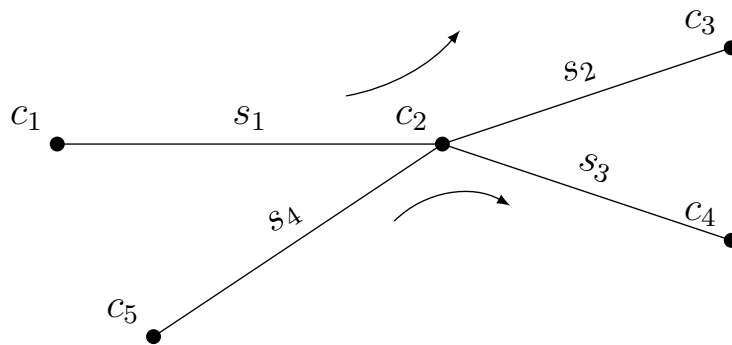


Fig. 4.1 Relations between sections and connectors

For every  $s \in \mathbb{S}$ , we denote the set of connectors connected with section  $s$  by  $\mathbb{C}_s$ . In the Figure 4.1, connectors  $c_1, c_2$  are connected with section  $s_1$ . We have  $\mathbb{C}_{s_1} = \{c_1, c_2\}$ .

For every  $c \in \mathbb{C}$ , we denote the set of sections connected with connector  $c$  by  $\mathbb{S}_c$ . In the Figure 4.1, sections  $s_1, s_2, s_3, s_4$  are connected with connector  $c_2$ .

We have  $\mathbb{S}_{c_2} = \{s_1, s_2, s_3, s_4\}$ .

It is possible to limit the direction of circulation from a section to another section in the railway station by prohibiting some travel directions. For this purpose, we introduce the notion "doublet" to identify the travel directions allowed.

**Sections doublet.**  $(s_i, s_j)$  is a doublet of connector  $c$  when  $s_i, s_j \in \mathbb{S}_c$  and trains can traverse from section  $s_i$  to section  $s_j$  by connector  $c$ .

The set of doublets of a connector  $c$  is denoted by  $\mathbb{K}_c = \{(s_1, s'_1), (s_2, s'_2), \dots, (s_K, s'_K)\}$  where  $K$  is the cardinal number of  $\mathbb{K}_c$ .

We must remark that a doublet of connectors represents only one travel direction. For example, a doublet  $(s_1, s_2)$  of connector  $c_1$  ( $(s_1, s_2) \in \mathbb{K}_{c_1}$ ) represents the travel direction from section  $s_1$  to section  $s_2$  by connector  $c_1$  (the travel direction is represented by an arrow in the figure). The reverse exists only in case where we have another doublet  $(s_2, s_1)$  for connector  $c_1$  ( $(s_2, s_1) \in \mathbb{K}_{c_1}$ ).

For example, in the Figure 4.1, if trains can traverse from left to right, then  $\mathbb{K}_{c_2} = \{(s_1, s_3), (s_1, s_2), (s_4, s_2), (s_4, s_3)\}$ .

For every  $s \in \mathbb{S}$ , the set of reachable sections from section  $s$  is denoted by  $\mathbb{S}_s$ . In the Figure 4.1,  $\mathbb{S}_{s_1} = \{s_2, s_3\}$ .

For every  $s \in \mathbb{S}$ , the set of sections which have section  $s$  as a reachable section is denoted by  $\hat{\mathbb{S}}_s$ . In the Figure 4.1,  $\hat{\mathbb{S}}_{s_2} = \{s_1, s_4\}$ .

For every  $s \in \mathbb{S}$ , for every  $s' \in \mathbb{S}_s$ , it exists only one connector denoted as  $c_{ss'}$  between these two reachable sections. In the Figure 4.1,  $c_{s_1 s_3}$  is  $c_2$ .

An example of the topology of a railway station is represented in the Figure 4.2.

We can clearly see that sections  $s_2, s_7$  are platforms, sections  $s_1, s_3, s_4, s_5, s_6, s_8$  are internal sections and sections  $s_9, s_{10}, s_{11}, s_{12}$  are external sections.

In this railway station,  $c_1, c_2, c_3, c_4, b_1, b_2, b_3, b_4$  are connectors while  $b_1, b_2, b_3, b_4$  are bordering connectors.

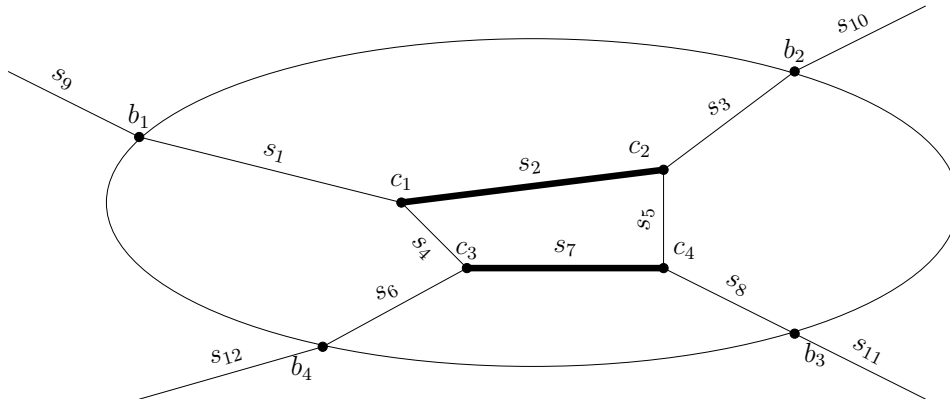


Fig. 4.2 An example of railway station

### 4.2.3 Principles of construction

**Infeasibility route.** A train cannot pass along some specific routes. For example, in the Figure 4.3, the route with an acute angle  $[c_5, s_4, c_1, s_5, c_6]$  is infeasible. The elimination of infeasible routes is carried out by defining the set of doublets (introduced previously) which can define the feasible routes in the railway station.

**Superfluous connectors.** If all the paths including connector  $c_1$  contain also the connector  $c_2$  in common, the conflict on the connector  $c_1$  can be represented by  $c_2$ . Then connector  $c_1$  is a superfluous connector and it is not included in the set of connectors of the abstract topology.

In the Figure 4.3, all paths containing the connector  $c_1$  pass also the connector  $c_2$ . Therefore, the physical connector  $c_1$  can be represented by the abstract connector  $c_2$ .

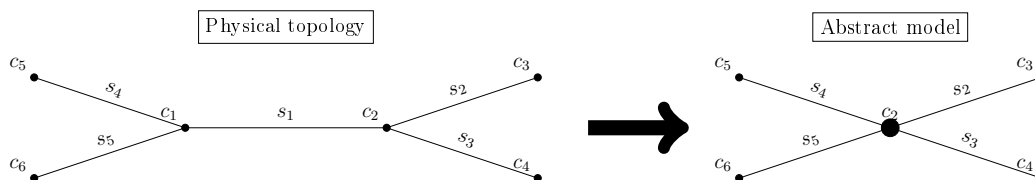


Fig. 4.3 Superfluous connectors.

Note that in order to retain the realism of this presentation, these abstractions are only applied when the length of section  $s_1$  is very

short (typically less than the length of train).

## 4.3 Trains' activities

An instance of our problem corresponds to a scenario which takes place in a given time horizon. The time horizon is defined by the starting time and the ending time of the scenario. During this scenario, we deal with a set of trains with their references and parameters.

### 4.3.1 Management

The train routing and scheduling problem faced by railway station managers is to generate a conflict-free timetable which consists of two categories of circulations: commercial and technical circulations (mentioned in Section 3.2.2.3.1 Page 53). The commercial circulations are given by several administrative levels (national, regional, freight) over a large time horizon. The technical circulations are added by the railway station managers to prepare or repair trains.

We consider a set of trains during a period of time to be routed and scheduled at a railway station. Trains have corresponding arrival and departure times at platform. Each train consists of many circulations. While the commercial circulations must respect strictly the arrival and departure times given, the technical circulations can be scheduled within the allowable deviations.

Trains enter the railway station at their entering track section and leave the railway station at their leaving track section. For each train, feasible paths can be determined by the entering track section, the leaving track section and possibly other aspects such as a set of allowed or preferred platforms.

At railway stations, there is often a choice of two or more *platforms* at which a train can stop for passengers to board or unboard. Although these choices can give managers some flexibility, they also introduce potential conflicts between trains. The conflicts can occur due to congestion, restriction on intersections, preferred arrival or departure times, etc.



## 4.3.2 Trains and Circulations

### 4.3.2.1 Trains

The traffic in the railway infrastructure is defined by a set of trains  $\mathbb{T} = \{t_1, t_2, \dots, t_T\}$  where  $T$  is the number of trains.

Every train  $t \in \mathbb{T}$  consists of a set of ordered *circulations*  $\mathbb{L}^t = \{l_1^t, l_2^t, \dots, l_{L^t}^t\}$  where  $L^t$  is the cardinal number of  $\mathbb{L}^t$ .

**Note:** In this study, a train could have it most three circulations.

**Train platform.** If a train must stop at a platform, we must allocate one and only one platform to train  $t$ , denoted as  $p_t \in \mathbb{P}$ . For each train passing through the railway station, the route of all circulations must be determined with the condition that all circulations arrives at or departs from the same platform  $p_t$ .

In this thesis, the platforms will be represented in the figures as bold lines.

**Length of train.** The length of trains must be taken into account as a criterion to chose a platform. Three kinds of train' lengths are defined as  $\{short, medium, long\}$  as well as three kinds of platforms' lengths.

Depending on the compatibility of train with the length of platforms, each train has a list of reference platforms to be chosen denoted by  $\mathbb{P}_{ref}^t$ .

The chosen platform must respect the reference list  $\mathbb{P}_{ref}^t$  for the train  $t$ :  $p_t \in \mathbb{P}_{ref}^t$

In this model, depending on the length of trains, three sets of reference platforms are considered:  $\mathbb{P}_{ref}^{long}$ ,  $\mathbb{P}_{ref}^{medium}$  and  $\mathbb{P}_{ref}^{short}$  with  $\mathbb{P}_{ref}^{long} \subset \mathbb{P}_{ref}^{medium} \subset \mathbb{P}_{ref}^{short}$  (introduced in Page 75).

### 4.3.2.2 Circulations

This study considers three types of circulations: entering circulation, leaving circulation and crossing circulation; and two categories: commercial circulation and technical circulation. Based on types and categories, several classes of circulations are defined.

## 4.3.2.2.1 Commercial entering circulation

An **entering circulation** is a circulation of a train which travels from an external section to a platform, see the Figure 4.4. The set of entering circulations is denoted by  $\mathbb{L}_{ent}$ .

The trains are represented in the figures as blue rectangles. The figures demonstrate the position of trains at different moments.

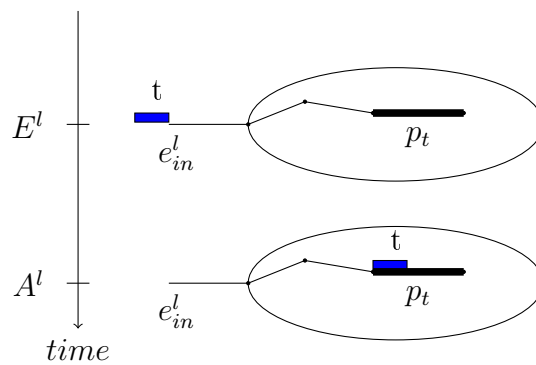


Fig. 4.4 Entering circulation

**Reference time  $A^l$ .** An entering circulation  $l \in \mathbb{L}_{ent}^t$  is associated to a reference time  $A^l$ . This reference time  $A^l$  is the preferred arrival time to the platform by train  $t$ .

**Reference time  $E^l$ .** An entering circulation  $l \in \mathbb{L}_{ent}^t$  can be associated to a reference time  $E^l$ . This reference time  $E^l$  is the preferred arrival time to the entry of the railway station by train  $t$ .

The entering circulation with passengers is called commercial entering circulation and the train must arrive at platform strictly in time  $A^l$  (which is communicated with passengers before).

We consider that the entering circulation could arrive to the entry of the railway station in time or later than  $E^l$ .

Note that we have the constraint  $E^l < A^l$  and the difference of these two reference times ( $A^l - E^l$ ) must be large enough to allow the entering circulation to use different possible routes.

#### 4.3.2.2.2 Commercial leaving circulation

A **leaving circulation** is a circulation of a train which travels from a platform to an external section, see the Figure 4.5. The set of leaving circulations is denoted by  $\mathbb{L}_{leav}$ .

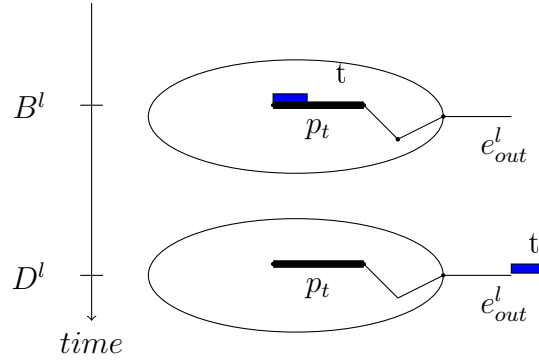


Fig. 4.5 Leaving circulation

**Reference time  $B^l$ .** A leaving circulation  $l \in \mathbb{L}_{leav}^t$  is associated to a reference time  $B^l$ . This reference time  $B^l$  is the preferred departure time of train  $t$  on the platform.

**Reference time  $D^l$ .** A leaving circulation  $l \in \mathbb{L}_{leav}^t$  can be associated to a reference time  $D^l$ . This reference time  $D^l$  is the preferred leaving time from the railway station by train  $t$ .

The leaving circulation with passengers is called commercial leaving circulation and the train must depart from platform strictly in time  $B^l$  (which is communicated with passengers before).

We consider that the leaving circulation could leave the railway station in time or earlier than  $D^l$ .

Note that we have the constraint  $B^l < D^l$  and the difference of these two reference times ( $D^l - B^l$ ) must be large enough to allow the leaving circulation to use different possible routes.

#### 4.3.2.2.3 Technical entering circulation

If the passengers have not yet boarded the train during the entering circulation, it is a technical entering circulation.

A technical entering circulations also has the reference times  $A^l$  and  $E^l$  as a commercial entering circulation.

**Reference time  $A^l$ .** An entering circulation  $l \in \mathbb{L}_{ent}^t$  is associated to a reference time  $A^l$ . This reference time  $A^l$  is the preferred arrival time to the platform by train  $t$ .

**Reference time  $E^l$ .** An entering circulation  $l \in \mathbb{L}_{ent}^t$  can be associated to a reference time  $E^l$ . This reference time  $E^l$  is the preferred arrival time to the railway station by train  $t$ .

However, a technical circulation does not have to arrive at platform in time. It can arrive early, then the arrival time at platform can be adjusted by a deviation time which must not exceed a permissible deviation time  $L$  before the reference arrival time  $A^l$  to ensure the service time on the platform. In this case, the arrival time at platform of technical entering circulation is between  $A^l - L$  and  $A^l$ .

#### 4.3.2.2.4 Technical leaving circulation

In technical leaving circulation, passengers have already left the train during the leaving circulation.

A technical leaving circulations also has the reference times  $B^l$  and  $D^l$  as a commercial leaving circulation.

**Reference time  $B^l$ .** A leaving circulation  $l \in \mathbb{L}_{leav}^t$  is associated to a reference time  $B^l$ . This reference time  $B^l$  is the preferred departure time of train  $t$  on the platform.

**Reference time  $D^l$ .** A leaving circulation  $l \in \mathbb{L}_{leav}^t$  can be associated to a reference time  $D^l$ . This reference time  $D^l$  is the preferred departure time from the railway station by train  $t$ .

However, the departure time of a technical leaving circulation from platform can be adjusted by a deviation time which must not exceed a permissible deviation time  $L$  after the reference departure time  $B^l$  for some technical activities such as maintenance operation, cleaning services, replacing of accessories, etc. In this case, the departure time from platform

of the technical leaving circulation is between  $B^l$  and  $B^l + L$ .

#### 4.3.2.2.5 Crossing circulation

A **crossing circulation** is a circulation of a train which passes through the railway station from an external section to another external section and does not stop at any platform, see the Figure 4.6. The set of crossing circulations is denoted by  $\mathbb{L}_{cross}$ .

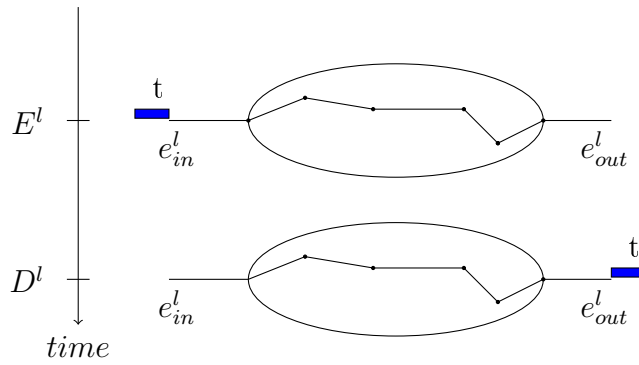


Fig. 4.6 Crossing circulation of a passing train

A crossing circulations has two reference times  $E^l$  and  $D^l$ .

**Reference time  $E^l$ .** An crossing circulation  $l \in \mathbb{L}_{cross}^t$  can be associated to a reference time  $E^l$ . This reference time  $E^l$  is the preferred arrival time to the railway station by train  $t$ .

**Reference time  $D^l$ .** A crossing circulation  $l \in \mathbb{L}_{cross}^t$  can be associated to a reference time  $D^l$ . This reference time  $D^l$  is the preferred departure time from the railway station by train  $t$ .

A crossing circulation  $l \in \mathbb{L}_{cross}^t$  is associated to the arrival time of train  $t$  at railway station  $E^l$  and to the departure time at railway station  $D^l$ . It means that the arrival time of train  $t$  at its first section of railway station (obviously an external section) is equal or greater than  $E^l$  and the departure time of train  $t$  at its last section of railway station is equal or less than  $D^l$ .

A train which does not stop at any platform while travelling through the railway station has only one circulation and it is a crossing circulation.

Note that all types of trains can stop at only one platform but they are allowed to traverse other platforms.

#### 4.3.2.2.6 Management of circulations

As a first step, the manager of a railway station tries to find a solution which schedules a non-conflict timetable for all trains passing through the railway station and respecting their commercial times. In this case, the deviation of reference times for technical circulations is allowed.

If no solution which respects all commercial times can be found, the deviation of reference times is allowed not only for technical circulations but also for commercial circulations in order to get a feasible solution.

In a complex railway station, it could be difficult to get a feasible solution on the peak time. To ensure the feasibility, the possibility of cancellation of trains should be considered. These trains cancelled may be reinserted in later process.

#### 4.3.2.2.7 Coupling and decoupling mechanism

If trains have to be coupled at the railway station, the coupling trains must have two entering circulations and one leaving circulation, see Figure 4.7.

The first entering circulation, which stops at the platform, can be called "leading train" and the second entering circulation has to be coupled onto the first entering circulation at the same platform. The time taken for a coupling system or decoupling system at platform is denoted by  $\Gamma^t$ . The leaving circulation begins when the coupling mechanism ends. Thus, we have the constraints  $A^{l_1} + \Gamma^t \leq B^{l_3}$  and  $A^{l_2} + \Gamma^t \leq B^{l_3}$ . The order of entering circulations are defined by the reference arrival time  $A^l$  of each entering circulation given from data. In the Figure 4.7, the circulation 1 arrives before the circulation 2.

Similarly, if trains have to be decoupled into two parts at the railway station, the decoupling trains must have one entering circulation and two leaving circulations, see Fig. 4.8. The entering circulation arrives at a platform to be decoupled, the two leaving circulations are assigned to two external sections starting from the same platform. The order of two leaving

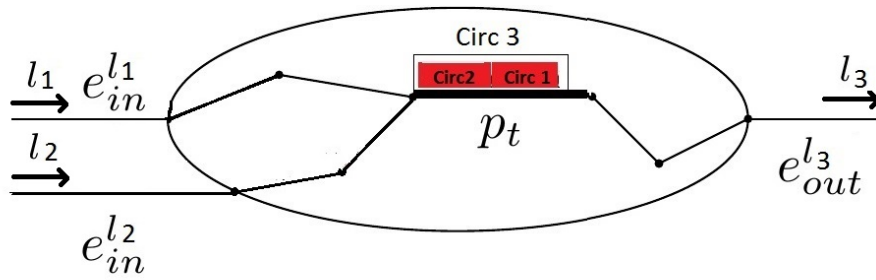


Fig. 4.7 Coupling system of trains.

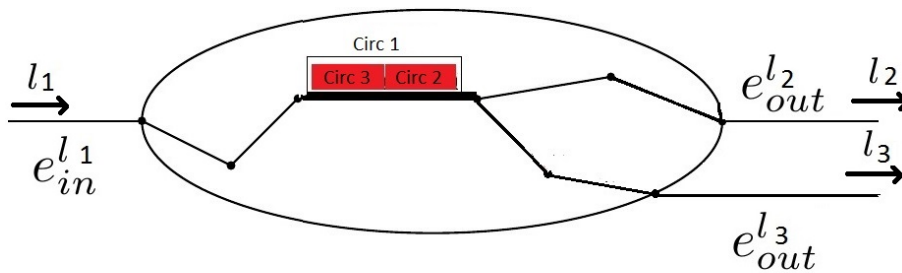


Fig. 4.8 Decoupling system of trains.

circulations must be given from data.

There are three types of stopping trains in this study:

- Stopping trains that have only one entering circulation and one leaving circulation. The set of this type of train is denoted by  $\mathbb{T}_{11}$ .
- Stopping trains that have two entering circulations and one leaving circulation (train coupling). The set of this type of train is denoted by  $\mathbb{T}_{21}$ . We consider  $A^{l_1} < A^{l_2}$ , it means that circulation  $l_1$  enters platform before circulation  $l_2$ .
- Stopping trains that have one entering circulation and two leaving circulations (train decoupling). The set of this type of train is denoted by  $\mathbb{T}_{12}$ .

## 4.4 Routing and Scheduling

This model uses the fourth level of representation (microscopic representation) which is presented in Section 3.2.1 Page 46. In this representation, each

section in the model consists of at least one track circuit to detect trains. It can determine the exact position of each train in the railway station during its operation. In case that track circuits are not yet deployed in railway stations, we assume that there are some other ways to detect a train in section. For example, the position of a train is transmitted by station agents.

#### 4.4.1 Study hypotheses

**Railway station.** A railway station  $\mathcal{U} = (\mathcal{S}, \mathcal{C})$  is defined by a set of sections  $\mathcal{S}$  and a set of connectors  $\mathcal{C}$  on which trains follow a route  $r \in \mathbb{R}$  which are defined using sections and connectors in the set  $\mathcal{S}$  and  $\mathcal{C}$ .

In the railway station, to solve routing and scheduling problem, the routes of train are to be considered. A route is constructed by combinations of external sections, internal sections and platforms. Therefore, a train may have several possible routes.

To describe accurately the railway station, we need to firstly choose the elementary resources of a route. The tracks and the switches of the railway station are represented by sections and connectors in our model. If two trains are on the conflicting routes, there are two possible cases shown in the Figure 4.9.

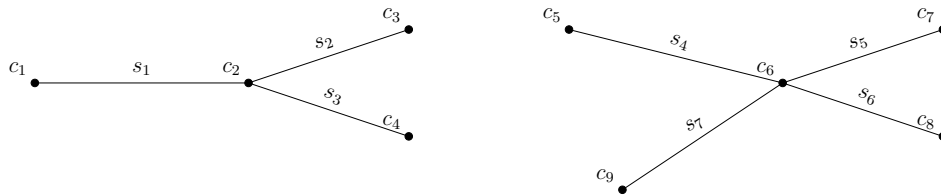


Fig. 4.9 Two cases of conflicting routes.

The two pairs of conflicting routes in the Figure 4.9 can be represented in the Table 4.1.

If a route is represented by a list of sections, the conflict of two routes in the first case is represented by the section  $s_1$  in common, but we cannot see the conflict between the two routes in the second case although these two routes are in conflict.



TABLE 4.1  
COMPARISON BETWEEN TWO CASES OF CONFLICTING ROUTES.

Route definition	Case 1	Case 2
Sections	$\{s_1, s_2\}$ $\{s_1, s_3\}$	$\{s_4, s_6\}$ $\{s_7, s_5\}$
Connectors	$\{c_1, c_2, c_3\}$ $\{c_1, c_2, c_4\}$	$\{c_5, c_6, c_8\}$ $\{c_9, c_6, c_7\}$

If a route is represented by a list of connectors, the conflict of two routes in the second case is clearly detected by the connector  $c_6$  in common.

Therefore, in order to detect correctly the routing conflicts, the list of connectors must be included to describe the route and the reservation times of this type of elementary resource must be considered to ensure that no conflict exists.

Depending on the type of route reservation procedures, the description of a route includes whether or not the list of sections. If the reservation of a complete route in railway stations is considered, it is not necessary to include the list of sections in the route. The representation by a list of successive connectors is enough to detect the conflict between routes.

If the *route locking and sectional release system* or the reservation of sections independently are considered, both the list of sections and connectors must be included in the route. For example, the first route of second case in Figure 4.9 is described by  $\{c_5, s_4, c_6, s_6, c_8\}$ . Indeed, in these cases, the availability of each elementary resource (section, connector) is independently checked and updated during the operation of trains.

As we mentioned in the previous chapter, we are interested in the progressive reservation of resources. The hypotheses used in this model are described below:

- **Hypothesis 1 (H1):** In this model, we consider the reservation of sections independently. Unlike traditional methods that allocate all resources to setup a route for a train until the route is released, our model allows the progressive reservation of sections and connectors used by trains along their routes on the railway station. The trains reserve the sections independently. A section is reserved when a train arrives and it can be released after the train

leaves it. Note that in our model, each train consists of a set of circulations. The resources are reserved by its circulations.

- **Hypothesis 2 (H2):** All trains operating in the railway station have the same speed which does not change during the operations. The time taken by circulation  $l$  to run from the beginning to the end of section  $s$  is denoted by  $\Delta_s^l$ , see Figure 4.11.

It depends on the length of each section and the speed of trains. It is given as:

$$\Delta_s^l = \frac{\text{length of section } s - \text{length of train}}{\text{speed of circulation } l} \quad (4.1)$$

- **Hypothesis 3 (H3):** Circulations do not pass a section or a connector twice. This assumption is made to prevent circulations going around in cycle. Certain previous studies deal with the cyclic scheduling, see [Korbaa et al. \(2003\)](#) and [Bourdeaud'huy and Korbaa \(2006\)](#) but we do not consider it in our model.

The time taken for a circulation  $l$  going through a connector is denoted by  $\Theta^l$ , see Figure 4.11. It depends on the length and the speed of train. It is calculated as:

$$\Theta^l = \frac{\text{length of train}}{\text{speed of circulation } l} \quad (4.2)$$

We simply consider a connector is a point. Therefore, the times taken for a circulation going through each connector have the same value for all connectors.

- **Hypothesis 4 (H4):** Circulations can stop at any section during their operations.

Due to the reservation of sections independently, there may exist some cases where the next section that a circulation must pass through is reserved by another circulation. Therefore, this circulation must stop and wait at current section until the next section is released before operating.

## 4.4.2 Parameters

### 4.4.2.1 Parameters of given data

Without loss of generality, this study considers that every train  $t$  consists of a maximum of two entering circulations and one leaving circulation (or one entering circulation and two leaving circulations).

In this study, the depots, where trains are housed when not being used, repaired or maintained, are not considered in the railway station. For all circulations arriving and leaving at a platform, the reference arrival time  $A^l$  of entering circulations at the platform and the reference departure time  $B^l$  of leaving circulations at platform are known.

A train passing through the railway station has circulations which are given external sections and need to be assigned to a route. The external sections of circulations of the train  $t$  are denoted by  $e_{in}^{l_1}, e_{in}^{l_2}, \dots, e_{out}^{l_3}, e_{out}^{l_4}, \dots \in \mathbb{E}$ . A normal train without coupling or decoupling mechanism has two external sections. On the other hand, train using coupling or decoupling mechanism consists of three circulations. Therefore, it has three external sections.

Every train  $t \in \mathbb{T}$  is given some parameters, see Table 4.2:

TABLE 4.2 PARAMETERS OF TRAINS' ACTIVITIES

Type	Circulations	External sections	Reference times	Reference platforms
Stopping train	$l_1, l_2 \in \mathbb{L}_{ent} \quad l_3 \in \mathbb{L}_{leav}$	$e_{in}^{l_1}, e_{in}^{l_2}, e_{out}^{l_3}$	$A^{l_1}, A^{l_2}, B^{l_3}, E^{l_1}, E^{l_2}, D^{l_3}$	$\mathbb{P}_{ref}$
	$l_1 \in \mathbb{L}_{ent} \quad l_2 \in \mathbb{L}_{leav}$	$e_{in}^{l_1}, e_{out}^{l_2}$	$A^{l_1}, B^{l_2}, E^{l_1}, D^{l_2}$	
	$l_1 \in \mathbb{L}_{ent} \quad l_2, l_3 \in \mathbb{L}_{leav}$	$e_{in}^{l_1}, e_{out}^{l_2}, e_{out}^{l_3}$	$A^{l_1}, B^{l_2}, B^{l_3}, E^{l_1}, D^{l_2}, D^{l_3}$	
Crossing train	$l \in \mathbb{L}_{cross}$	$e_{in}^l, e_{out}^l$	$E^l, D^l$	

### 4.4.2.2 Safety

**Allocation of resources.** The interactions of all elementary resources: sections (consisting of external sections, internal sections and platforms) and connectors (including bordering connectors) are considered in the railway station. Each resource cannot be reserved by more than one circulation during the same time interval. The circulation reserves each resource only once during its operation.

In the Figure 4.10, if a circulation  $l$  travels from section  $s$  to  $s'$  by connector  $c$ , we consider that the section  $s'$  is reserved when connector  $c$

is occupied by circulation  $l$  and the section  $s$  is released when the circulation  $l$  leaves connector  $c$ .

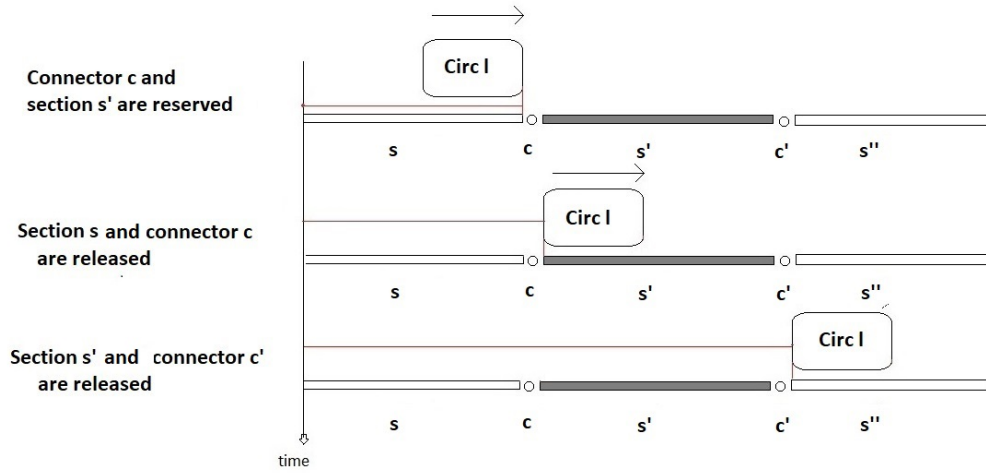


Fig. 4.10 Allocation of sections and connectors.

**Headway.** The train headway is a measurement of the distance or the time between consecutive trains in railway system. It is very important for the safety of railway.

In order to reinforce the safety of railway station, we denote by  $\sigma$  the safety delay time to separate two circulations using the same section or connector. This delay time must occur between the end of the first circulation and the start of the other one on the same section or connector.

### 4.4.3 Effective Occupation Times

In this section, train scheduling notations are described in terms of circulations. In order to go through railway station, the circulations of a train travel from section to section by passing through connectors.

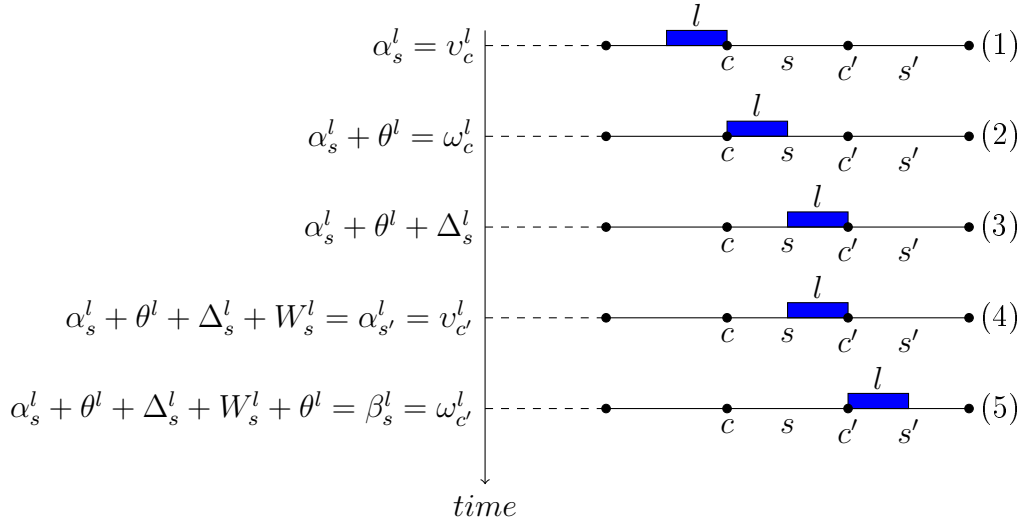
Due to the reservation of sections independently, we must define the starting time of occupation of a section and the ending time of occupation of a section by circulations.

**Actual time interval of occupation of section:**  $[\alpha_s^l, \beta_s^l]$  are defined as the actual time interval of occupation of section  $s$  by circulation  $l$  with

$\alpha_s^l, \beta_s^l \in \mathbb{N}$  and  $\alpha_s^l < \beta_s^l$ , see Figure 4.11.

**Actual time interval of occupation of connector:**  $[v_c^l, \omega_c^l]$  are defined as the actual time interval of occupation of connector  $c$  by circulation  $l$  with  $v_c^l, \omega_c^l \in \mathbb{N}$  and  $v_c^l < \omega_c^l$ , see Figure 4.11.

**Waiting time:** The time taken for circulation  $l$  remaining stopped at section  $s$  is defined by  $W_s^l$  in case the next section is not available.



- (1): Start of occupation of section  $s$  and connector  $c$
- (2): Circulation has passed through inbound connector, end of reservation of connector  $c$
- (3): Circulation has run from the begin to the end of section  $s$
- (4): Circulation has remained at section  $s$  for a while, start of occupation of section  $s'$  and connector  $c'$
- (5): End of occupation of section  $s$  and connector  $c'$  by circulation  $l$   
 ( $s$  and  $c'$  are not yet free to be reserved due to the constraints of safety  $\sigma$ .  
 Section  $s$  and connector  $c'$  can be reserved by another circulation from  $\beta_s^l + \sigma$ )

Fig. 4.11 Occupation of section  $s$  by circulation  $l$ .

### Time occupations of coupling and decoupling trains

The Figure 4.12 indicates all sections that circulations of a coupling train pass through.

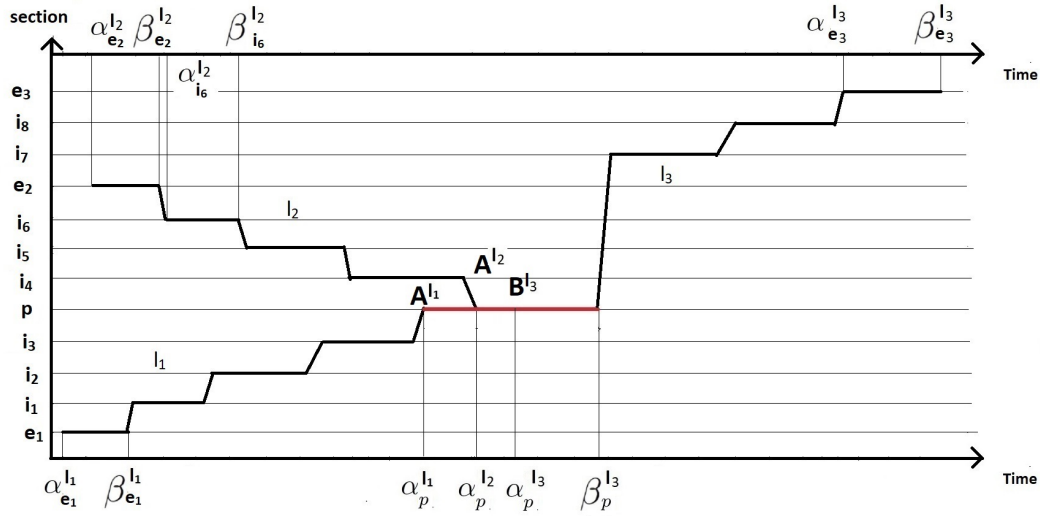


Fig. 4.12 Train with two entering circulations and one leaving circulation.

The first entering circulation enters the railway station from external section  $e_1$  with the corresponding time occupation  $[\alpha_{e_1}^{l_1}, \beta_{e_1}^{l_1}]$ . It passes through internal sections  $i_1, i_2, i_3$  and starts to occupy platform  $p$  at  $\alpha_p^{l_1}$  (corresponding to the reference arrival time  $A^{l_1}$ ).

The second entering circulation enters the railway station from external section  $e_2$  with the corresponding time occupation  $[\alpha_{e_2}^{l_2}, \beta_{e_2}^{l_2}]$ . It passes through internal sections  $i_6, i_5, i_4$  and starts to occupy platform  $p$  at  $\alpha_p^{l_2}$  (corresponding to the reference arrival time  $A^{l_2}$ ).

The coupling process of these two entering circulation occurs after the arrival of the second entering circulation at platform. After the coupling process and the passenger boarding or unboarding at platform, the leaving circulation which represents the coupled train starts to leave the platform at  $\alpha_p^{l_3}$  (corresponding to the reference departure time  $B^{l_3}$ ). It passes through internal sections  $i_7, i_8$  and leaves the railway station by external section  $e_{out}^{l_3}$ .

## 4.5 Conclusion

Our train routing and scheduling problem in railway stations is formalised in many aspects: railway topology, trains' activities, routing and scheduling notation. The topology of railway station is formalized by two key components: *sections* and *connectors*. The principles of construction for

abstract topology are also introduced to conduct formal studies while preserving the rules of circulations of railway systems. In order to validate this formalisation, commercial times, time parameters, decision times of occupation are defined corresponding to our study hypotheses.

# Mathematical models

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## 5.1 Introduction

In this section, we propose a mathematical model as a mixed-integer linear program with the parameters and hypotheses presented in previous section for our problem which consists of routing and scheduling problem. The decision model is described in two steps: decision variables and constraints. Three models for resolution are proposed at the end of section.



## 5.2 Decision Variables

Hereafter, the function  $\delta(Q)$  is such that  $\delta(Q) = 1$  if the condition  $Q$  is valid, otherwise 0. The time variables are represented by Greek letters and the boolean decision variables are represented by capital letters. A sufficiently large constant is denoted by  $H$ .

The decision variables of our model are introduced as follow:

- $S_s^l$ : boolean variable, represents the passage of circulation  $l$  going through section  $s$ .  $S_s^l = \delta(\text{circulation } l \text{ passes through section } s)$ .

- $C_c^l$ : boolean variable, represents the passage of circulation  $l$  going through connector  $c$ .  $C_c^l = \delta(\text{circulation } l \text{ passes through connector } c)$ .

- $Y_s^{ll'}$ : boolean variable, represents the chronological order of traversing of section  $s$  for two circulations  $l, l'$ .  $Y_s^{ll'} = \delta(\text{circulation } l \text{ passes through section } s \text{ before circulation } l')$ .

- $X_c^{ll'}$ : boolean variable, represents the chronological order of traversing connector  $c$  for two circulations  $l, l'$ .  $X_c^{ll'} = \delta(\text{circulation } l \text{ passes through connector } c \text{ before circulation } l')$ .

- $Z_{ss'}^l$ : boolean variable, represents the passage from section  $s$  to section  $s'$  in the route of circulation  $l$ .  $Z_{ss'}^l = \delta(\text{circulation } l \text{ travels from section } s \text{ to section } s')$ .

- $[\alpha_s^l, \beta_s^l]$ : integer variables, the actual time interval of occupation of section  $s$  by circulation  $l$ .

- $[\nu_c^l, \omega_c^l]$ : integer variables, the actual time interval of occupation of connector  $c$  by circulation  $l$ .

- $W_s^l$ : integer variable, the time taken for circulation  $l$  remaining stopped at section  $s$ .

- $P_p^l$ : boolean variables, represents the stopping platform of circulation  $l$ .  $P_p^l = \delta(\text{platform } p \text{ is allocated to circulation } l \text{ as a stopping platform})$ .

## 5.3 Constraints

### 5.3.1 Routing constraints

This section presents constraints which ensure that circulations can travel from their origin to their destination. Circulations must respect the topology of railway station to pass from a section to another section.

#### Constraint of topology:

- If the doublet  $(s, s')$  does not exist, it means that section  $s'$  is not reachable from  $s$ . Thus,  $Z_{ss'}^l$  is equal to 0:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \notin \mathbb{S}_s \quad Z_{ss'}^l = 0 \quad (5.1)$$

#### Route of circulation:

- We consider that circulations are not allowed to pass through a connector more than one time in this model in order to prevent circulations going around in cycle.

If a circulation  $l$  passes through a connector  $c$ , there must be two sections, connected to this connector, which are in the route of circulation  $l$ :

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad C_c^l = 1 \Rightarrow \sum_{s \in \mathbb{S}_c} S_s^l = 2$$

This constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad \begin{cases} \sum_{s \in \mathbb{S}_c} S_s^l - 2 \leq H \cdot (1 - C_c^l) \\ 2 - \sum_{s \in \mathbb{S}_c} S_s^l \leq H \cdot (1 - C_c^l) \end{cases} \quad (5.2)$$

**Note:** If  $C_c^l = 1$ , the inequation (5.2) implies that  $\sum_{s \in \mathbb{S}_c} S_s^l - 2 \leq 0$  and  $2 - \sum_{s \in \mathbb{S}_c} S_s^l \leq 0$ , it means that  $\sum_{s \in \mathbb{S}_c} S_s^l = 2$ . If  $C_c^l = 0$ , the inequation is always true because  $H$  is a sufficiently large constant.

**Note:** We remind that  $\mathbb{S}_c$  is a set of sections connected with connector  $c$ .

- If a circulation passes from section  $s$  to  $s'$ , it cannot pass from section  $s'$  to  $s$ :

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l + Z_{s's}^l \leq 1 \quad (5.3)$$

• If a circulation enters a section, this circulation must pass through this section:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad \sum_{s' \in \hat{\mathbb{S}}_s} Z_{s's}^l = 1 \Rightarrow S_s^l = 1$$

This constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad S_s^l \geq \sum_{s' \in \hat{\mathbb{S}}_s} Z_{s's}^l \quad (5.4)$$

• If a circulation leaves a section, this circulation must pass through this section:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad \sum_{s' \in \mathbb{S}_s} Z_{ss'}^l = 1 \Rightarrow S_s^l = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad S_s^l \geq \sum_{s' \in \mathbb{S}_s} Z_{ss'}^l \quad (5.5)$$

**Note:** In constraints (5.4) and (5.5) the equality represents the case of internal section. The inequality represents the case of external sections and platforms. For example, a circulation can pass through an external section ( $S_s^l = 1$ ) but it cannot enter this external section ( $\sum_{s' \in \mathbb{S}_s} Z_{ss'}^l = 0$ ) in case that this external section is the first section in the route of this circulation.

• If a circulation travels from section  $s$  to section  $s'$ , it must use the connector  $c_{ss'}$  between these two sections:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l = 1 \Rightarrow C_{c_{ss'}}^l = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l \leq C_{c_{ss'}}^l \quad (5.6)$$

### Constraints of external sections:

• Entering circulation  $l$  must pass through and leave the external section given  $e_{in}^l$ :

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \quad S_{e_{in}}^l = 1 \quad (5.7)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \quad \sum_{s' \in \mathbb{S}_{e_{in}^l}} Z_{e_{in}^l s'}^l = 1 \quad (5.8)$$

- This entering circulation  $l$  must not pass through others external sections:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall s \in \mathbb{E} \setminus \{e_{in}^l\} \quad S_s^l = 0 \quad (5.9)$$

- Leaving circulation  $l$  must enter and pass through the external section given  $e_{out}^l$ :

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t \quad S_{e_{out}^l}^l = 1 \quad (5.10)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t \quad \sum_{s' \in \hat{\mathbb{S}}_{e_{out}^l}} Z_{s' e_{out}^l}^l = 1 \quad (5.11)$$

- This leaving circulation  $l$  must not pass through others external sections:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall s \in \mathbb{E} \setminus \{e_{out}^l\} \quad S_s^l = 0 \quad (5.12)$$

- Crossing circulation  $l$  must pass through and leave the external section given  $e_{in}^l$  and it must enter and pass through the external section given  $e_{out}^l$ :

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad S_{e_{in}^l}^l = 1 \quad (5.13)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{s' \in \mathbb{S}_{e_{in}^l}} Z_{e_{in}^l s'}^l = 1 \quad (5.14)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad S_{e_{out}^l}^l = 1 \quad (5.15)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{s' \in \hat{\mathbb{S}}_{e_{out}^l}} Z_{s' e_{out}^l}^l = 1 \quad (5.16)$$

- This crossing circulation  $l$  must not pass through others external sections:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t, \forall s \in \mathbb{E} \setminus \{e_{in}^l, e_{out}^l\} \quad S_s^l = 0 \quad (5.17)$$

- A crossing circulation or an entering circulation  $l$  must arrive in time or later than reference time  $E^l$  at the external section given  $e_{in}^l$ :

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \cup \mathbb{L}_{ent}^t \quad \alpha_{e_{in}^l}^l \geq E^l \quad (5.18)$$

- A crossing circulation or a leaving circulation  $l$  must leave in time or earlier than reference time  $D^l$  from the external section given  $e_{out}^l$ :

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \cup \mathbb{L}_{leav}^t \quad \beta_{e_{out}^l}^l \leq D^l \quad (5.19)$$

**Constraints of internal sections:** If a circulation enters an internal section, it must leave this internal section. Conversely, if this circulation leaves this internal section, it must enter this internal section.

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \sum_{s' \in \hat{\mathbb{S}}_s} Z_{s't}^l = \sum_{s'' \in \mathbb{S}_s} Z_{ss''}^l \quad (5.20)$$

**Constraints of no stopping platforms:** We consider that trains can pass through some platforms but does not stop at these platforms. If a circulation enters a no stopping platform, it must leave this platform. Conversely, if this circulation leaves this platform, it must enter this platform:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \quad P_p^l = 0 \Rightarrow \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l = \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l$$

These constraints are expressed using the linear constraints below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \quad \begin{cases} \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l - \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l \leq H \cdot P_p^l \\ \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l - \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l \leq H \cdot P_p^l \end{cases} \quad (5.21)$$

$H$  is a sufficiently large constant.

**Note:** If  $P_p^l = 0$ , the inequation (5.21) implies that  $\sum_{s' \in \mathbb{S}_p} Z_{ps'}^l - \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l \leq 0$  and  $\sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l - \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l \leq 0$ , it means that  $\sum_{s' \in \mathbb{S}_p} Z_{ps'}^l = \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l$ . If  $P_p^l = 1$ , the inequation is always true because  $H$  is a sufficiently large constant.

### 5.3.2 Constraints of stopping platforms

#### Train platform:

- There is only one stopping platform for entering circulation and leaving circulation:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t \quad \sum_{p \in \mathbb{P}} P_p^l = 1 \quad (5.22)$$

- The chosen platform must respect the reference list  $\mathbb{P}_{ref}^t$  for the train  $t$ :

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t \quad \sum_{p \in \mathbb{P}_{ref}^t} P_p^l = 1 \quad (5.23)$$

- There is no stopping platform for crossing circulation:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{p \in \mathbb{P}} P_p^l = 0 \quad (5.24)$$

- Entering circulations and leaving circulations of the same train must have the same platform:

$$\forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad P_p^l = P_p^{l'} \quad (5.25)$$

- An entering circulation must enter the stopping platform:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad P_p^l = 1 \Rightarrow \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l \geq P_p^l \quad (5.26)$$

- An entering circulation must not leave the stopping platform:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad P_p^l = 1 \Rightarrow \sum_{s \in \mathbb{S}_p} Z_{ps}^l = 0$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \mathbb{S}_p} Z_{ps}^l \leq (1 - P_p^l) \quad (5.27)$$

- In a similar way, a leaving circulation must leave a stopping platform:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad P_p^l = 1 \Rightarrow \sum_{s \in \mathbb{S}_p} Z_{ps}^l = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \mathbb{S}_p} Z_{ps}^l \geq P_p^l \quad (5.28)$$

- A leaving circulation must not enter the stopping platform:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad P_p^l = 1 \Rightarrow \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l = 0$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l \leq (1 - P_p^l) \quad (5.29)$$

#### Reference times at platform:

We remind reference times of circulation which are defined previously as follow:

$A^l$  is the preferred arrival time to the platform of circulation  $l$ .

$B^l$  is the preferred departure time from the platform of circulation  $l$ .

The actual time interval of occupation of platform  $p \in \mathbb{P}$  by a circulation  $l$  is defined by  $[\alpha_p^l, \beta_p^l]$ .

The starting time of occupation of a platform is represented in Figure 5.1, we assume that the entering circulation of a train arrives at a stopping platform when the circulation leaves the connector connected with the platform. Hence, the entering circulation allows passengers to board or unboard the train. After that, the leaving circulation of this train will pass through and leaves the platform.

The time interval of occupation of a stopping platform of an entering circulation must respect the preferred arrival time of platform  $A^l$ :

$$\forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ P_p^{l_1} = 1 \Rightarrow \alpha_p^{l_1} + \Theta^{l_1} = A^{l_1}$$

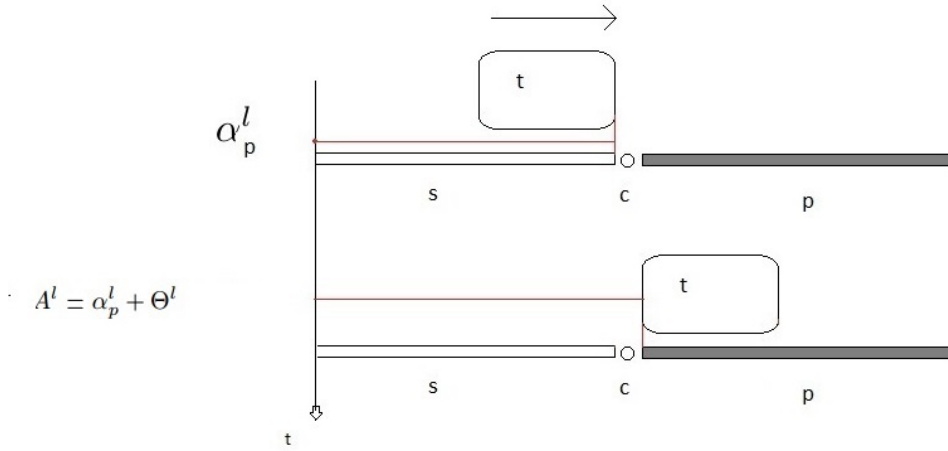


Fig. 5.1 The starting time of occupation of a platform

This constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \begin{cases} A^{l_1} - \alpha_p^{l_1} - \Theta^{l_1} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \quad (5.30)$$

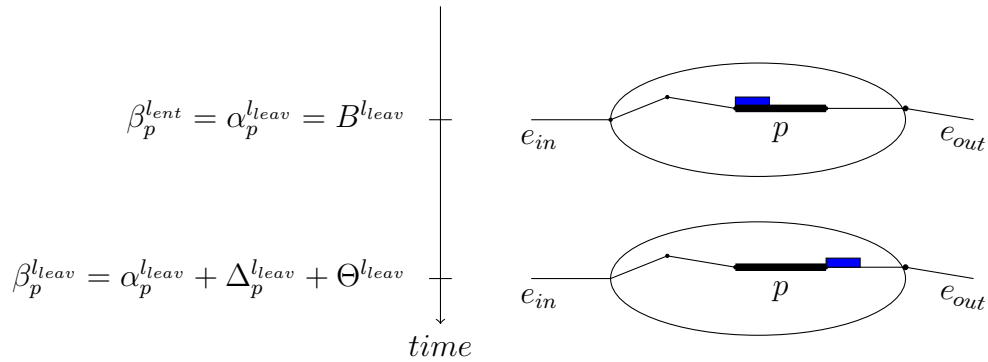


Fig. 5.2 Time interval of occupation of platform for the entering circulation and the leaving circulation of a train

- The time interval of occupation of stopping platform of an entering circulation must respect the starting time of leaving circulation at platform. The leaving circulation starts to reserve the stopping platform when the entering circulation leaves this platform, see Figure 5.2. We have the constraint below:

$$\forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad P_p^{l_1} = 1 \Rightarrow \beta_p^{l_1} = \alpha_p^{l_2}$$



This constraint is expressed using the linear constraint below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \beta_p^{l_1} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_2} - \beta_p^{l_1} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.31)$$

The time interval of occupation of a stopping platform of a leaving circulation must respect the preferred departure time of platform  $B^l$ :

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ P_p^{l_2} = 1 \Rightarrow \alpha_p^{l_2} = B^{l_2} \end{aligned}$$

This constraint is expressed using the linear constraint below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.32)$$

We consider that the entering circulation is ended after the boarding or unboarding of passengers. Then the leaving circulation starts and it must traverse the platform, see Figure 5.2. In case that train has one leaving circulation, the constraint of the time interval of occupation of stopping platform of a leaving circulation is expressed below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ P_p^{l_2} = 1 \Rightarrow \beta_p^{l_2} = \alpha_p^{l_2} + \Delta_p^{l_2} + \Theta^{l_2} \end{aligned}$$

This constraint is expressed using the linear constraint below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^{l_2} + \Delta_p^{l_2} + \Theta^{l_2} + W_p^{l_2} - \beta_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \beta_p^{l_2} - \alpha_p^{l_2} - \Delta_p^{l_2} - \Theta^{l_2} - W_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.33)$$

In case of train having two leaving circulations, we consider that the second leaving circulation starts to occupy the stopping platform when the first leaving circulation leaves this platform, we have constraints below:

$$\forall t \in \mathbb{T}, l_2, l_3 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad P_p^{l_2} = 1 \Rightarrow \beta_p^{l_2} = \alpha_p^{l_3}$$

This constraint is expressed using the linear constraint below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_2, l_3 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \beta_p^{l_2} - \alpha_p^{l_3} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_3} - \beta_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.34)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_3 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ P_p^{l_3} = 1 \Rightarrow \beta_p^{l_3} = \alpha_p^{l_3} + \Delta_p^{l_3} + \Theta^{l_3} \end{aligned}$$

This constraint is expressed using the linear constraint below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_3 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^{l_3} + \Delta_p^{l_3} + \Theta^{l_3} + W_p^{l_3} - \beta_p^{l_3} \leq H \cdot (1 - P_p^{l_3}) \\ \beta_p^{l_3} - \alpha_p^{l_3} - \Delta_p^{l_3} - \Theta^{l_3} - W_p^{l_3} \leq H \cdot (1 - P_p^{l_3}) \end{cases} \end{aligned} \quad (5.35)$$

### 5.3.3 Constraints of occupation times

#### Actual time interval of occupation:

We remind time decision variables and time parameters for the occupation of section and connector as follows:

The actual time interval of occupation of a section  $s \in \mathbb{S}$  by a circulation  $l$  is defined by  $[\alpha_s^l, \beta_s^l]$  and the time taken for circulation  $l$  remaining stopped at section  $s$  is defined by variables  $W_s^l$ .

The actual time interval of occupation of a connector  $c \in \mathbb{C}$  by a circulation  $l$  is defined by  $[v_c^l, \omega_c^l]$ .

$\Delta_s^l$  is the time taken to run from the begin to the end of section  $s$  by circulation  $l$ .

$\Theta^l$  is the time taken for a circulation  $l$  going through a connector.

- The actual time intervals of occupations of sections and connectors are represented in Figure 5.3. The time taken for a circulation  $l$  going through a connector is  $\Theta^l$ . Therefore, the constraint of occupation of connector by circulation is expressed as follows:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad \omega_c^l = v_c^l + \Theta^l \quad (5.36)$$

For internal section, as Figure 5.3 illustrates, the ending time of an internal section  $s$ :  $\beta_s^l = \omega_c^l + \Delta_s^l + 2\Theta^l + W_s^l$ . We have  $\omega_c^l = v_c^l + \Theta^l$  as the constraint 5.36 and consider that the section  $s$  is reserved when connector  $c$  is occupied by

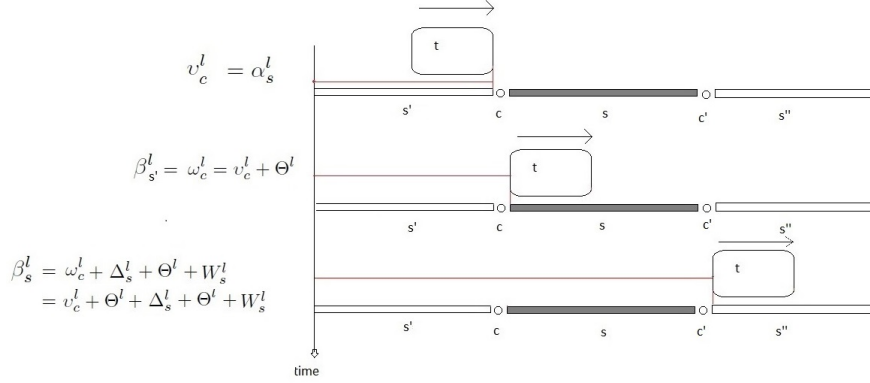


Fig. 5.3 The actual time intervals of occupation of sections and connectors

circulation  $l$ :  $v_c^l = \alpha_s^l$ . Therefore, the constraint of the actual time interval of occupations of sections for internal section is expressed as follows:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \beta_s^l = \alpha_s^l + \Delta_s^l + 2\Theta^l + W_s^l \quad (5.37)$$

For external section, the circulation passes through only one connector. We have the constraint as follows:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{E} \quad \beta_s^l = \alpha_s^l + \Delta_s^l + \Theta^l + W_s^l \quad (5.38)$$

A no stopping platform, where circulations pass through and do not stop at it, can be considered as an internal section. We have the constraint as follows:

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ P_p^l = 0 \Rightarrow \beta_p^l = \alpha_p^l + \Delta_p^l + 2\Theta^l + W_p^l \end{aligned}$$

This constraint is expressed using the linear constraints below:

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^l + \Delta_p^l + 2\Theta^l + W_p^l - \beta_p^l \leq H \cdot P_p^l \\ \beta_p^l - \alpha_p^l - \Delta_p^l - 2\Theta^l - W_p^l \leq H \cdot P_p^l \end{cases} \end{aligned} \quad (5.39)$$

**Note:** If  $P_p^l = 0$ , the inequation (5.39) implies that  $\alpha_p^l + \Delta_p^l + 2\Theta^l + W_p^l - \beta_p^l \leq 0$  and  $\beta_p^l - \alpha_p^l - \Delta_p^l - 2\Theta^l - W_p^l \leq 0$ , it means that  $\beta_p^l = \alpha_p^l + \Delta_p^l + 2\Theta^l + W_p^l$ . If  $P_p^l = 1$ , the inequation is always true because  $H$  is a sufficiently large constant.

To pass through an internal section or a platform, trains pass through two connectors (constraints (5.37) and (5.39)). To pass through an external section, trains pass through only one bordering connector (constraint (5.38)), see figure 5.4.

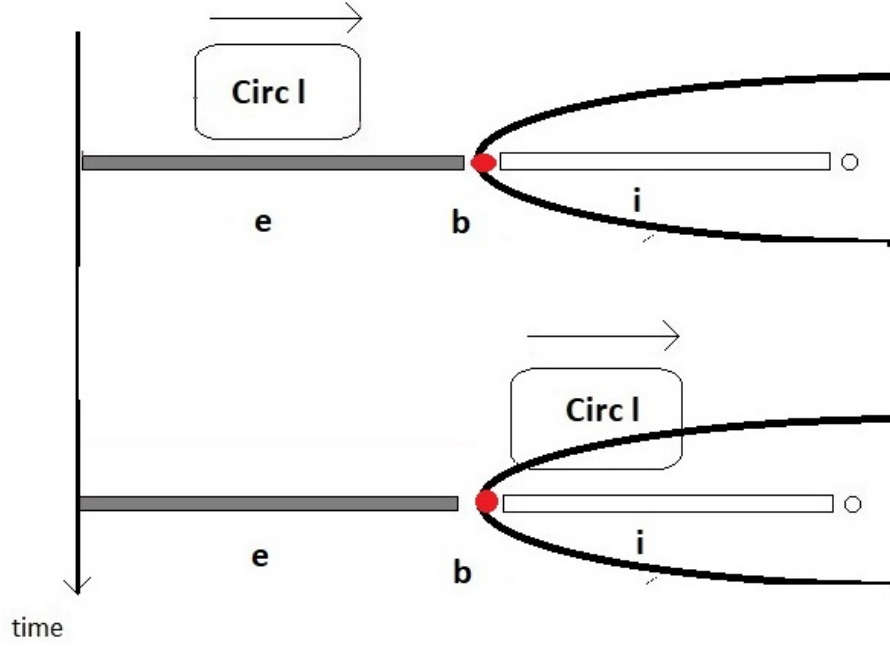


Fig. 5.4 Allocation of external sections and bordering connectors.

### Succession of sections:

The actual time intervals of occupations of two consecutive sections are represented in Fig. 5.5.

- If circulation  $l$  travels from section  $s$  to section  $s'$  by connector  $c$ , we consider that the section  $s'$  is reserved when connector  $c$  is occupied by circulation  $l$ . The constraint for the actual time intervals of occupations of two consecutive sections is expressed below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l = 1 \Rightarrow v_{c_{ss'}}^l = \alpha_{s'}^l$$

This constraint is expressed using the linear constraints below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad \begin{cases} v_{c_{ss'}}^l - \alpha_{s'}^l \leq H \cdot (1 - Z_{ss'}^l) \\ \alpha_{s'}^l - v_{c_{ss'}}^l \leq H \cdot (1 - Z_{ss'}^l) \end{cases} \quad (5.40)$$

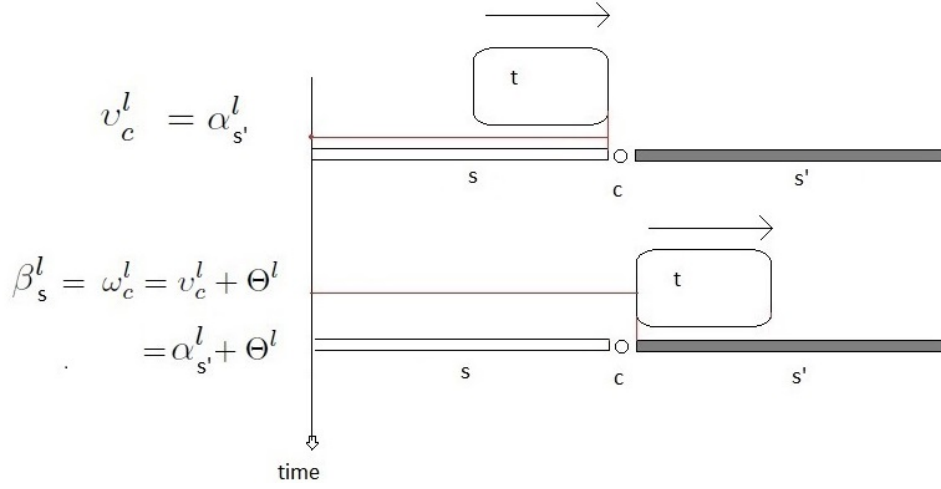


Fig. 5.5 The actual time intervals of occupation of two consecutive sections.

**Note:** If  $Z_{ss'}^l = 1$ , the inequation (5.40) implies that  $v_{c_{ss'}}^l - \alpha_{s'}^l \leq 0$  and  $\alpha_{s'}^l - v_{c_{ss'}}^l \leq 0$ , it means that  $v_{c_{ss'}}^l = \alpha_{s'}^l$ . If  $Z_{ss'}^l = 0$ , the inequation is always true because  $H$  is a sufficiently large constant.

- The corresponding occupation times of two sections must respect the constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l = 1 \Rightarrow \beta_s^l = \alpha_{s'}^l + \Theta^l$$

This constraint is expressed using the linear constraints below:

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \\ & \begin{cases} \alpha_{s'}^l + \Theta^l - \beta_s^l \leq H \cdot (1 - Z_{ss'}^l) \\ \beta_s^l - \alpha_{s'}^l - \Theta^l \leq H \cdot (1 - Z_{ss'}^l) \end{cases} \end{aligned} \quad (5.41)$$

**Note:** If  $Z_{ss'}^l = 1$ , the inequation (5.41) implies that  $\alpha_{s'}^l + \Theta^l - \beta_s^l \leq 0$  and  $\beta_s^l - \alpha_{s'}^l - \Theta^l \leq 0$ , it means that  $\beta_s^l = \alpha_{s'}^l + \Theta^l$ . If  $Z_{ss'}^l = 0$ , the inequation is always true because  $H$  is a sufficiently large constant.

### 5.3.4 Safety constraints

The safety constraints ensure that two circulations cannot pass the same section or the same connector at the same time. We use the actual time interval variables and ordering variables defined previously to express these

constraints.

### Occupation of sections:

We remind that the chronological order of two circulations  $l, l'$  passing through a common section  $s$  is denoted by  $Y_s^{ll'}$ .

- When two circulations use the same section, one circulation must be scheduled before the other:

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall s \in \mathbb{S} \quad Y_s^{ll'} + Y_s^{l'l} = 1 \quad (5.42)$$

Two circulations passing through a common section cannot be scheduled during the same time interval.

During the coupling or decoupling operation, the stopping platform of a train can be reserved in the same time by two circulations of this train.

- In case of two circulations of two different trains, the constraint is expressed below:

$$\begin{aligned} & \forall t, t' \in \mathbb{T}, t \neq t', \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall s \in \mathbb{S} \\ & \begin{cases} \beta_s^l + \sigma \leq \alpha_s^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'}) \\ \beta_s^{l'} + \sigma \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l}) \end{cases} \end{aligned} \quad (5.43)$$

**Note:** If section  $s$  is in the route of both circulations  $l$  and  $l'$ , so that  $S_s^l = 1, S_s^{l'} = 1$  and either  $Y_s^{ll'} = 1$  or  $Y_s^{l'l} = 1$ . It means that  $3 - S_s^l - S_s^{l'} - Y_s^{ll'} = 0$  or  $3 - S_s^l - S_s^{l'} - Y_s^{l'l} = 0$ . In the first case, we have  $\beta_s^l + \sigma \leq \alpha_s^{l'}$ , it means that circulation  $l$  leaves section  $s$  before the arriving of circulation  $l'$  at section  $s$ . The second constraint is trivially verified ( $Y_s^{l'l} = 0$ ). In the other case, we have  $\beta_s^{l'} + \sigma \leq \alpha_s^l$ , it means that circulation  $l'$  leaves section  $s$  before the arriving of circulation  $l$  at section  $s$ .

- In case of two circulations of a train, the constraint is expressed below for all sections which are not a platform:

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall s \in \mathbb{S} \setminus \mathbb{P} \\ & \begin{cases} \beta_s^l + \sigma \leq \alpha_s^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'}) \\ \beta_s^{l'} + \sigma \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l}) \end{cases} \end{aligned} \quad (5.44)$$

• In case that section is a no stopping platform, the constraint is expressed below:

$$\forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall p \in \mathbb{P} \quad \begin{cases} \beta_p^l + \sigma \leq \alpha_p^{l'} + H \cdot (3 - S_p^l - S_p^{l'} - Y_p^{ll'} + P_p^l) \\ \beta_p^{l'} + \sigma \leq \alpha_p^l + H \cdot (3 - S_p^l - S_p^{l'} - Y_p^{ll'} + P_p^l) \end{cases} \quad (5.45)$$

**Note:** The constraint of occupation of sections for the stopping platform is expressed in the constraints (5.30)-(5.35) in the previous part of this section.

### Occupation of connectors:

We remind that the chronological order of two circulations  $l, l'$  passing through a common connector  $c$  is denoted by  $X_c^{ll'}$ .

• When two circulations use the same connector, one circulation must be scheduled before the other:

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall c \in \mathbb{C} \quad X_c^{ll'} + X_c^{l'l} = 1 \quad (5.46)$$

• Two circulations passing through a common connector cannot be scheduled during the same time interval:

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall c \in \mathbb{C} \quad \begin{cases} \omega_c^l + \sigma \leq v_c^{l'} + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{ll'}) \\ \omega_c^{l'} + \sigma \leq v_c^l + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{l'l}) \end{cases} \quad (5.47)$$

**Note:** If connector  $c$  is in the route of both circulations  $l$  and  $l'$ , so that  $C_c^l = 1, C_c^{l'} = 1$  and either  $X_c^{ll'} = 1$  or  $X_c^{l'l} = 1$ . It means that  $3 - C_c^l - C_c^{l'} - X_c^{ll'} = 0$  or  $3 - C_c^l - C_c^{l'} - X_c^{l'l} = 0$ . In the first case, we have  $\omega_c^l + \sigma \leq v_c^{l'}$ , it means that circulation  $l$  leaves connector  $c$  before the arriving of circulation  $l'$  at connector  $c$ . The second constraint is trivially verified ( $X_c^{l'l} = 0$ ). In the other case, we have  $\omega_c^{l'} + \sigma \leq v_c^l$ , it means that circulation  $l'$  leaves connector  $c$  before the arriving of circulation  $l$  at section  $s$ .

### 5.3.5 Constraints for the management of technical and commercial circulations

As introduced in Section 4.3.2.2.6 in Page 85, corresponding to the management policy of technical and commercial circulations, we propose to study three different models.

### 5.3.5.1 Deviation of technical circulations

The deviation of reference times for technical circulations is allowed in order to find a non-conflict timetable.

We remind that the maximum permissible deviation time is denoted by  $L$ .

The constraints (5.30) and (5.32) for technical circulations are replaced by the constraints below:

For technical entering circulation:

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ P_p^{l_1} = 1 \Rightarrow A^{l_1} - L \leq \alpha_p^{l_1} + \Theta^{l_1} \leq A^{l_1} \end{aligned}$$

This constraint is expressed using the linear constraint below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ \begin{cases} A^{l_1} - L - \alpha_p^{l_1} - \Theta^{l_1} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.48)$$

For technical leaving circulation:

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ P_p^{l_2} = 1 \Rightarrow B^{l_2} \leq \alpha_p^{l_2} \leq B^{l_2} + L \end{aligned}$$

This constraint is expressed using the linear constraint below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - L \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.49)$$

### 5.3.5.2 Model with time relaxation

We consider a model which allows deviation of reference times not only for technical circulations but also for commercial circulations.

The maximum permissible deviation time for commercial circulations is denoted by  $\Psi$ .

The constraints (5.30) and (5.32) are replaced by the constraints (5.48), (5.49) above for technical circulations and two constraints for commercial circulations below:



For commercial entering circulation:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ & P_p^{l_1} = 1 \Rightarrow A^{l_1} - \Psi \leq \alpha_p^{l_1} + \Theta^{l_1} \leq A^{l_1} \end{aligned}$$

This constraint is expressed using the linear constraint below:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ & \begin{cases} A^{l_1} - \Psi - \alpha_p^{l_1} - \Theta^{l_1} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.50)$$

For commercial leaving circulation:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ & P_p^{l_2} = 1 \Rightarrow B^{l_2} \leq \alpha_p^{l_2} \leq B^{l_2} + \Psi \end{aligned}$$

This constraint is expressed using the linear constraint below:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ & \begin{cases} B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - \Psi \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.51)$$

### 5.3.5.3 Model with cancellation of trains

In case of difficulty to find a feasible solution, a model allowing cancellation of trains should be considered to guarantee the existence of solutions within reasonable time.

An additional boolean variable is added to the model to represent the cancellation of trains:  $F_t$ .

$F_t=1$  if the train  $t$  is cancelled. Otherwise,  $F_t=0$ .

When a train is cancelled, all conflict of occupation resources of this train must be removed.

In this case, the constraints (5.43), (5.44), (5.45), (5.47) are replaced by the constraints below:

If two circulations are not in the same train, the constraint is expressed below:

$$\begin{aligned} & \forall t, t' \in \mathbb{T}, t \neq t', \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall s \in \mathbb{S} \\ & \begin{cases} \beta_s^l + \sigma \leq \alpha_s^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'} + F_t + F_{t'}) \\ \beta_s^{l'} + \sigma \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l} + F_t + F_{t'}) \end{cases} \end{aligned} \quad (5.52)$$

**Note:** If the train  $t$  or  $t'$  is cancelled,  $F_t + F_{t'} \geq 1$ . We have  $3 - S_s^l - S_s^{l'} - Y_s^{ll'} + F_t + F_{t'} \geq 1$ .  $H$  is a big enough constant, so that the constrain (5.52) is relaxed.

Similarly, we have the other constraints below:

In case that two circulations are in the same train, the constraint is expressed below for all sections which are not a platform:

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall s \in \mathbb{S} \setminus \mathbb{P} \\ & \begin{cases} \beta_s^l + \sigma \leq \alpha_s^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'} + F_t) \\ \beta_s^{l'} + \sigma \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l} + F_t) \end{cases} \end{aligned} \quad (5.53)$$

In case that section is a no stopping platform, the constraint is expressed below:

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ & \begin{cases} \beta_p^l + \sigma \leq \alpha_p^{l'} + H \cdot (3 - S_p^l - S_p^{l'} - Y_p^{ll'} + P_p^l + F_t) \\ \beta_p^{l'} + \sigma \leq \alpha_p^l + H \cdot (3 - S_p^l - S_p^{l'} - Y_p^{l'l} + P_p^l + F_t) \end{cases} \end{aligned} \quad (5.54)$$

**Occupation of connectors:** Two circulations passing through a common connector cannot be scheduled during the same time interval:

$$\begin{aligned} & \forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall c \in \mathbb{C} \\ & \begin{cases} \omega_c^l + \sigma \leq v_c^{l'} + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{ll'} + F_t + F_{t'}) \\ \omega_c^{l'} + \sigma \leq v_c^l + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{l'l} + F_t + F_{t'}) \end{cases} \end{aligned} \quad (5.55)$$

## 5.4 Objective functions

Depending on the model which is used to solve the optimisation problem, some objective functions defined as follows:

### 5.4.1 Minimising of the totals of deviation time of technical circulations

**Objective function 1 (Obj1):** Minimise the totals of deviation time of technical circulations.

The schedule of trains should respect the reference times of technical circulations as close as possible to ensure the best fit for human resource management.

A new decision variable is introduced to define this objective function:

$\xi^l$ : integer variable, represents the deviation time of circulation  $l$  at platform.

The constraint corresponding to this variable for entering circulation is expressed below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ P_p^{l_1} = 1 \Rightarrow \xi^l = A^{l_1} - (\alpha_p^{l_1} + \Theta^{l_1}) \end{aligned}$$

This constraint is expressed using the linear constraint below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ \begin{cases} A^{l_1} - \alpha_p^{l_1} - \Theta^{l_1} - \xi^l \leq H \cdot (1 - P_p^{l_1}) \\ \xi^l + \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.56)$$

The constraint corresponding to this variable for leaving circulation is expressed below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ P_p^{l_2} = 1 \Rightarrow \xi^l = \alpha_p^{l_2} - B^{l_2} \end{aligned}$$

This constraint is expressed using the linear constraint below:

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \xi^l + B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - \xi^l \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.57)$$

Clearly, for all commercial circulation  $l$  without permission for deviation:  
 $\xi^l = 0$

The objective function 1 is expressed as follows:

$$\text{Obj1: } \min \sum_{l \in \mathbb{L}} \xi^l$$

### 5.4.2 Minimising of the number of interruptions

**Objective function 2 (Obj2):** minimise the number of interruptions.

In reality, an interruption (train stops and restarts) consumes a lot of energy. The numbers of interruptions of trains should be minimised to reduce this consumption.

In order to define this objective function, a new decision variable is defined:

$R_s^l$ : boolean variable, represents the interruption of circulation  $l$  at section  $s$ .

$$R_s^l = \delta(\text{circulation } l \text{ waits at section } s).$$

We remind that  $W_s^l$  is the waiting time of circulation  $l$  at section  $s$ .

The constraint corresponding to this variable is expressed below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad \begin{cases} W_s^l = 0 \Rightarrow R_s^l = 0 \\ W_s^l > 0 \Rightarrow R_s^l = 1 \end{cases}$$

This constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad \begin{cases} R_s^l \leq W_s^l \\ W_s^l \leq R_s^l \cdot H \end{cases} \quad (5.58)$$

**Note:**

If  $W_s^l = 0$ , the first inequation implies that  $R_s^l \leq 0$ . It implies  $R_s^l = 0$ . The other inequation is trivially verified.

If  $W_s^l > 0$ , the second inequation implies that  $0 < R_s^l \cdot H$ . It implies  $R_s^l = 1$ . The other inequation is trivially verified.

The objective function 2 is expressed as follows:

$$\text{Obj2: } \min \sum_{l \in \mathbb{L}} \sum_{s \in \mathbb{S}} R_s^l$$

### 5.4.3 Minimising of the maximum number of occupation of sections

**Objective function 3 (Obj3):** minimise the maximum number of occupation of sections.

A good distribution of circulations throughout sections can guarantee the robustness of timetables. It is better to avoid the allocation of a section for many circulations which operate during a short period of time. If the section is blocked due to an accident or a technical problem, circulations using the route which contains this section are delayed. Minimising the maximum number of occupation of sections can help increase the independence of routes. Then it will reduce the negative impact of disturbances by minimising the number of circulations delayed.

A new decision variable is defined to determine this objective function:

$\chi_s$ : integer variable, represents the total number of occupation of section  $s$  by circulations.

We remind that  $S_s^l$  represents the passage of circulation  $l$  going through section  $s$ .

The constraint corresponding to this new decision variable is expressed below:

$$\forall s \in \mathbb{S} \quad \chi_s = \sum_{l \in \mathbb{L}} S_s^l \quad (5.59)$$

To determine the maximum number of occupation of sections, another decision variable must also be defined as below:

$\Lambda$ : integer variable, represents the maximum number of occupation of sections.

The constraint corresponding to this variable is expressed below:

$$\forall s \in \mathcal{S} \quad \chi_s \leq \Lambda \quad (5.60)$$

The objective function 3 is expressed as follows:

$$\text{Obj3: } \min \Lambda$$

#### 5.4.4 Minimising of the number of cancelled train

**Objective function 4 (Obj4):** minimises the number of cancelled train.

In the model with cancellation of trains, we defined a boolean variable  $F_t$  to represent the cancellation of train  $t$ . Therefore, the objective function 4 is expressed as follows:

$$\text{Obj4: } \min \sum_{t \in \mathcal{T}} F_t$$

### 5.5 Mathematical models

We select different characteristics presented above to construct different models for our numerical experiments.

**Model 1:** This is the basic model to find a feasible solution for our routing and scheduling problem in railway stations.

In this model, only technical circulations are allowed to associate with the deviation time  $\xi^l$ . The deviation of commercial circulations and the cancellation of trains are not allowed. The interruptions of all circulations are also disallowed ( $\forall l \in \mathcal{L}, \forall s \in \mathcal{S} \quad W_s^l = 0$ ).

The objective functions for this model are to:

- minimise the total of deviation time of technical circulations (*Obj1*)

- minimise the maximum number of occupations of sections (*Obj3*)

**Note:** These objective functions in our models are not optimised simultaneously.

The *model 1* is defined by:

$$Obj1: \min \sum_{l \in \mathbb{L}} \xi^l$$

$$Obj3: \min \Lambda$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \notin \mathbb{S}_s \quad Z_{ss'}^l = 0 \quad (5.1)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad \begin{cases} \sum_{s \in \mathbb{S}_c} S_s^l - 2 \leq H \cdot (1 - C_c^l) \\ 2 - \sum_{s \in \mathbb{S}_c} S_s^l \leq H \cdot (1 - C_c^l) \end{cases} \quad (5.2)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l + Z_{s's}^l \leq 1 \quad (5.3)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad S_s^l \geq \sum_{s' \in \mathbb{S}_s} Z_{s's}^l \quad (5.4)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad S_s^l \geq \sum_{s' \in \mathbb{S}_s} Z_{ss'}^l \quad (5.5)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l \leq C_{c_{ss'}}^l \quad (5.6)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \quad S_{e_{in}^l}^l = 1 \quad (5.7)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \quad \sum_{s' \in \mathbb{S}_{e_{in}^l}^l} Z_{e_{in}^l s'}^l = 1 \quad (5.8)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall s \in \mathbb{E} \setminus \{e_{in}^l\} \quad S_s^l = 0 \quad (5.9)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t \quad S_{e_{out}^l}^l = 1 \quad (5.10)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t \quad \sum_{s' \in \hat{\mathbb{S}}_{e_{out}^l}^l} Z_{s'e_{out}^l}^l = 1 \quad (5.11)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall s \in \mathbb{E} \setminus \{e_{out}^l\} \quad S_s^l = 0 \quad (5.12)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad S_{e_{in}^l}^l = 1 \quad (5.13)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{s' \in \mathbb{S}_{e_{in}^l}^l} Z_{e_{in}^l s'}^l = 1 \quad (5.14)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad S_{e_{out}^l}^l = 1 \quad (5.15)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{s' \in \hat{\mathbb{S}}_{e_{out}^l}^l} Z_{s'e_{out}^l}^l = 1 \quad (5.16)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t, \forall s \in \mathbb{E} \setminus \{e_{in}^l, e_{out}^l\} \quad S_s^l = 0 \quad (5.17)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \cup \mathbb{L}_{ent}^t \quad \alpha_{e_{in}^l}^l \geq E^l \quad (5.18)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \cup \mathbb{L}_{leav}^t \quad \beta_{e_{out}^l}^l \leq D^l \quad (5.19)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \sum_{s' \in \hat{\mathbb{S}}_s} Z_{s's}^l = \sum_{s'' \in \mathbb{S}_s} Z_{ss''}^l \quad (5.20)$$



$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ & \begin{cases} \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l - \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l \leq H \cdot P_p^l \\ \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l - \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l \leq H \cdot P_p^l \end{cases} \end{aligned} \quad (5.21)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t \quad \sum_{p \in \mathbb{P}} P_p^l = 1 \quad (5.22)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t \quad \sum_{p \in \mathbb{P}_{ref}^t} P_p^l = 1 \quad (5.23)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{p \in \mathbb{P}} P_p^l = 0 \quad (5.24)$$

$$\forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad P_p^l = P_p^{l'} \quad (5.25)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l \geq P_p^l \quad (5.26)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \mathbb{S}_p} Z_{ps}^l \leq (1 - P_p^l) \quad (5.27)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \mathbb{S}_p} Z_{ps}^l \geq P_p^l \quad (5.28)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l \leq (1 - P_p^l) \quad (5.29)$$

For commercial entering circulation:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ & \begin{cases} A^{l_1} - \alpha_p^{l_1} - \Theta^{l_1} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.30 \text{ for only commercial circulations})$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \beta_p^{l_1} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^l) \\ \alpha_p^{l_2} - \beta_p^{l_1} \leq H \cdot (1 - P_p^l) \end{cases} \end{aligned} \quad (5.31)$$

For commercial leaving circulation:

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad (5.32 \text{ for only commercial circulations}) \\ \begin{cases} B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned}$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^{l_2} + \Delta_p^{l_2} + \Theta^{l_2} - \beta_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \beta_p^{l_2} - \alpha_p^{l_2} - \Delta_p^{l_2} - \Theta^{l_2} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.33 \text{ where } W_p^{l_2} = 0)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_2, l_3 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \beta_p^{l_2} - \alpha_p^{l_3} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_3} - \beta_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.34)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_3 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^{l_3} + \Delta_p^{l_3} + \Theta^{l_3} - \beta_p^{l_3} \leq H \cdot (1 - P_p^{l_3}) \\ \beta_p^{l_3} - \alpha_p^{l_3} - \Delta_p^{l_3} - \Theta^{l_3} \leq H \cdot (1 - P_p^{l_3}) \end{cases} \end{aligned} \quad (5.35 \text{ where } W_p^{l_3} = 0)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad \omega_c^l = v_c^l + \Theta^l \quad (5.36)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \beta_s^l = \alpha_s^l + \Delta_s^l + 2\Theta^l \quad (5.37 \text{ where } W_s^l = 0)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{E} \quad \beta_s^l = \alpha_s^l + \Delta_s^l + \Theta^l \quad (5.38 \text{ where } W_s^l = 0)$$

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^l + \Delta_p^l + 2\Theta^l - \beta_p^l \leq H \cdot P_p^l \\ \beta_p^l - \alpha_p^l - \Delta_p^l - 2\Theta^l \leq H \cdot P_p^l \end{cases} \end{aligned} \quad (5.39 \text{ where } W_p^l = 0)$$

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \\ \begin{cases} v_{c_{ss'}}^l - \alpha_{s'}^l \leq H \cdot (1 - Z_{ss'}^l) \\ \alpha_{s'}^l - v_{c_{ss'}}^l \leq H \cdot (1 - Z_{ss'}^l) \end{cases} \end{aligned} \quad (5.40)$$

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \\ \begin{cases} \alpha_{s'}^l + \Theta^l - \beta_s^l \leq H \cdot (1 - Z_{ss'}^l) \\ \beta_s^l - \alpha_{s'}^l - \Theta^l \leq H \cdot (1 - Z_{ss'}^l) \end{cases} \end{aligned} \quad (5.41)$$

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall s \in \mathbb{S} \quad Y_s^{ll'} + Y_s^{l'l} = 1 \quad (5.42)$$

$$\begin{aligned} \forall t, t' \in \mathbb{T}, t \neq t', \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall s \in \mathbb{S} \\ \begin{cases} \beta_s^l + \sigma \leq \alpha_s^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'}) \\ \beta_s^{l'} + \sigma \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l}) \end{cases} \end{aligned} \quad (5.43)$$

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall s \in \mathbb{S} \setminus \mathbb{P} \\ \begin{cases} \beta_s^l + \sigma \leq \alpha_s^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'}) \\ \beta_s^{l'} + \sigma \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l}) \end{cases} \end{aligned} \quad (5.44)$$

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ \begin{cases} \beta_p^l + \sigma \leq \alpha_p^{l'} + H \cdot (3 - S_p^l - S_p^{l'} - Y_p^{ll'} + P_p^l) \\ \beta_p^{l'} + \sigma \leq \alpha_p^l + H \cdot (3 - S_p^l - S_p^{l'} - Y_p^{l'l} + P_p^l) \end{cases} \end{aligned} \quad (5.45)$$

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall c \in \mathbb{C} \quad X_c^{ll'} + X_c^{l'l} = 1 \quad (5.46)$$

$$\begin{aligned} \forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall c \in \mathbb{C} \\ \begin{cases} \omega_c^l + \sigma \leq v_c^{l'} + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{ll'}) \\ \omega_c^{l'} + \sigma \leq v_c^l + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{l'l}) \end{cases} \end{aligned} \quad (5.47)$$

For technical entering circulation:

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ \begin{cases} A^{l_1} - L - \alpha_p^{l_1} - \Theta^{l_1} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.48)$$

For technical leaving circulation:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ & \begin{cases} B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - L \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.49)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ & \begin{cases} A^{l_1} - \alpha_p^{l_1} - \Theta^{l_1} - \xi^l \leq H \cdot (1 - P_p^{l_1}) \\ \xi^l + \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.56)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ & \begin{cases} \xi^l + B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - \xi^l \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.57)$$

$$\forall s \in \mathbb{S} \quad \chi_s = \sum_{l \in \mathbb{L}} S_s^l \quad (5.59)$$

$$\forall s \in \mathbb{S} \quad \chi_s \leq \Lambda \quad (5.60)$$

**Model 2:** If the problem cannot be solved by *model 1*. We consider a model in which not only the technical circulations but also the commercial circulations are also allowed to associate with the deviation time  $\xi^l$ . The interruptions of all circulations are allowed.

The objective functions for this model are to:

- minimise the totals of deviation time of all circulations (*Obj1* including commercial circulations)
- minimise the number of interruptions (*Obj2*)

The *model 2* is defined by:

$$Obj1: \min \sum_{l \in \mathbb{L}} \xi^l$$

$$Obj2: \min \sum_{l \in \mathbb{L}} \sum_{s \in \mathbb{S}} R_s^l$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \notin \mathbb{S}_s \quad Z_{ss'}^l = 0 \quad (5.1)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad \begin{cases} \sum_{s \in \mathbb{S}_c} S_s^l - 2 \leq H \cdot (1 - C_c^l) \\ 2 - \sum_{s \in \mathbb{S}_c} S_s^l \leq H \cdot (1 - C_c^l) \end{cases} \quad (5.2)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l + Z_{s's}^l \leq 1 \quad (5.3)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad S_s^l \geq \sum_{s' \in \mathbb{S}_s} Z_{s's}^l \quad (5.4)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad S_s^l \geq \sum_{s' \in \mathbb{S}_s} Z_{ss'}^l \quad (5.5)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l \leq C_{c_{ss'}}^l \quad (5.6)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \quad S_{c_{in}}^l = 1 \quad (5.7)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \quad \sum_{s' \in \mathbb{S}_{e_{in}^l}} Z_{e_{in}^l s'}^l = 1 \quad (5.8)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall s \in \mathbb{E} \setminus \{e_{in}^l\} \quad S_s^l = 0 \quad (5.9)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t \quad S_{e_{out}^l}^l = 1 \quad (5.10)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t \quad \sum_{s' \in \mathbb{S}_{e_{out}^l}^l} Z_{s' e_{out}^l}^l = 1 \quad (5.11)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall s \in \mathbb{E} \setminus \{e_{out}^l\} \quad S_s^l = 0 \quad (5.12)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad S_{e_{in}^l}^l = 1 \quad (5.13)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{s' \in \mathbb{S}_{e_{in}^l}} Z_{e_{in}^l s'}^l = 1 \quad (5.14)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad S_{e_{out}^l}^l = 1 \quad (5.15)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{s' \in \mathbb{S}_{e_{out}^l}^l} Z_{s' e_{out}^l}^l = 1 \quad (5.16)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t, \forall s \in \mathbb{E} \setminus \{e_{in}^l, e_{out}^l\} \quad S_s^l = 0 \quad (5.17)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \cup \mathbb{L}_{ent}^t \quad \alpha_{e_{in}^l}^l \geq E^l \quad (5.18)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \cup \mathbb{L}_{leav}^t \quad \beta_{e_{out}^l}^l \leq D^l \quad (5.19)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \sum_{s' \in \hat{\mathbb{S}}_s} Z_{s's}^l = \sum_{s'' \in \mathbb{S}_s} Z_{ss''}^l \quad (5.20)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ & \left\{ \begin{array}{l} \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l - \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l \leq H \cdot P_p^l \\ \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l - \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l \leq H \cdot P_p^l \end{array} \right. \quad (5.21) \end{aligned}$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t \quad \sum_{p \in \mathbb{P}} P_p^l = 1 \quad (5.22)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t \quad \sum_{p \in \mathbb{P}_{ref}^t} P_p^l = 1 \quad (5.23)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{p \in \mathbb{P}} P_p^l = 0 \quad (5.24)$$

$$\forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad P_p^l = P_p^{l'} \quad (5.25)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l \geq P_p^l \quad (5.26)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \mathbb{S}_p} Z_{ps}^l \leq (1 - P_p^l) \quad (5.27)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \mathbb{S}_p} Z_{ps}^l \geq P_p^l \quad (5.28)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l \leq (1 - P_p^l) \quad (5.29)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \beta_p^{l_1} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^l) \\ \alpha_p^{l_2} - \beta_p^{l_1} \leq H \cdot (1 - P_p^l) \end{cases} \end{aligned} \quad (5.31)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^{l_2} + \Delta_p^{l_2} + \Theta^{l_2} + W_p^{l_2} - \beta_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \beta_p^{l_2} - \alpha_p^{l_2} - \Delta_p^{l_2} - \Theta^{l_2} - W_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.33)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_2, l_3 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \beta_p^{l_2} - \alpha_p^{l_3} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_3} - \beta_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.34)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_3 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^{l_3} + \Delta_p^{l_3} + \Theta^{l_3} + W_p^{l_3} - \beta_p^{l_3} \leq H \cdot (1 - P_p^{l_3}) \\ \beta_p^{l_3} - \alpha_p^{l_3} - \Delta_p^{l_3} - \Theta^{l_3} - W_p^{l_3} \leq H \cdot (1 - P_p^{l_3}) \end{cases} \end{aligned} \quad (5.35)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad \omega_c^l = v_c^l + \Theta^l \quad (5.36)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \beta_s^l = \alpha_s^l + \Delta_s^l + 2\Theta^l + W_s^l \quad (5.37)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{E} \quad \beta_s^l = \alpha_s^l + \Delta_s^l + \Theta^l + W_s^l \quad (5.38)$$

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^l + \Delta_p^l + 2\Theta^l + W_p^l - \beta_p^l \leq H \cdot P_p^l \\ \beta_p^l - \alpha_p^l - \Delta_p^l - 2\Theta^l - W_p^l \leq H \cdot P_p^l \end{cases} \end{aligned} \quad (5.39)$$

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \\ \begin{cases} v_{c_{ss'}}^l - \alpha_{s'}^l \leq H \cdot (1 - Z_{ss'}^l) \\ \alpha_{s'}^l - v_{c_{ss'}}^l \leq H \cdot (1 - Z_{ss'}^l) \end{cases} \end{aligned} \quad (5.40)$$



$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \\ & \begin{cases} \alpha_{s'}^l + \Theta^l - \beta_s^l \leq H \cdot (1 - Z_{ss'}^l) \\ \beta_s^l - \alpha_{s'}^l - \Theta^l \leq H \cdot (1 - Z_{ss'}^l) \end{cases} \end{aligned} \quad (5.41)$$

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall s \in \mathbb{S} \quad Y_s^{ll'} + Y_s^{l'l} = 1 \quad (5.42)$$

$$\begin{aligned} & \forall t, t' \in \mathbb{T}, t \neq t', \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall s \in \mathbb{S} \\ & \begin{cases} \beta_s^l + \sigma \leq \alpha_{s'}^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'}) \\ \beta_{s'}^{l'} + \sigma \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l}) \end{cases} \end{aligned} \quad (5.43)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall s \in \mathbb{S} \setminus \mathbb{P} \\ & \begin{cases} \beta_s^l + \sigma \leq \alpha_{s'}^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'}) \\ \beta_{s'}^{l'} + \sigma \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l}) \end{cases} \end{aligned} \quad (5.44)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ & \begin{cases} \beta_p^l + \sigma \leq \alpha_p^{l'} + H \cdot (3 - S_p^l - S_p^{l'} - Y_p^{ll'} + P_p^l) \\ \beta_p^{l'} + \sigma \leq \alpha_p^l + H \cdot (3 - S_p^l - S_p^{l'} - Y_p^{l'l} + P_p^l) \end{cases} \end{aligned} \quad (5.45)$$

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall c \in \mathbb{C} \quad X_c^{ll'} + X_c^{l'l} = 1 \quad (5.46)$$

$$\begin{aligned} & \forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall c \in \mathbb{C} \\ & \begin{cases} \omega_c^l + \sigma \leq v_c^{l'} + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{ll'}) \\ \omega_c^{l'} + \sigma \leq v_c^l + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{l'l}) \end{cases} \end{aligned} \quad (5.47)$$

For technical entering circulation:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ & \begin{cases} A^{l_1} - L - \alpha_p^{l_1} - \Theta^{l_1} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.48)$$

For technical leaving circulation:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ & \begin{cases} B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - L \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.49)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ \begin{cases} A^{l_1} - \alpha_p^{l_1} - \Theta^{l_1} - \xi^l \leq H \cdot (1 - P_p^{l_1}) \\ \xi^l + \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.56)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \xi^l + B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - \xi^l \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.57)$$

For commercial entering circulation:

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ \begin{cases} A^{l_1} - \Psi - \alpha_p^{l_1} - \Theta^{l_1} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.50)$$

For commercial leaving circulation:

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - \Psi \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.51)$$

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \\ \begin{cases} R_s^l \leq W_s^l \\ W_s^l \leq R_s^l \cdot H \end{cases} \end{aligned} \quad (5.58)$$

**Model 3:** If the problem still cannot be solved by *model 2*. We consider a model which has the same characteristics of *model 2* but where cancellation of trains is allowed.

The objective functions for this model are to:

- minimise the totals of deviation time of commercial circulations (*Obj1* including commercial circulations)
- minimise the number of train cancelled (*Obj4*)

The *model 3* is defined by:

$$Obj1: \min \sum_{l \in \mathbb{L}} \xi^l$$

$$Obj4: \min \sum_{t \in \mathbb{T}} F_t$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \notin \mathbb{S}_s \quad Z_{ss'}^l = 0 \quad (5.1)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad \begin{cases} \sum_{s \in \mathbb{S}_c} S_s^l - 2 \leq H \cdot (1 - C_c^l) \\ 2 - \sum_{s \in \mathbb{S}_c} S_s^l \leq H \cdot (1 - C_c^l) \end{cases} \quad (5.2)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l + Z_{s's}^l \leq 1 \quad (5.3)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad S_s^l \geq \sum_{s' \in \mathbb{S}_s} Z_{s's}^l \quad (5.4)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad S_s^l \geq \sum_{s' \in \mathbb{S}_s} Z_{ss'}^l \quad (5.5)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l \leq C_{c_{ss'}}^l \quad (5.6)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \quad S_{c_{in}}^l = 1 \quad (5.7)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \quad \sum_{s' \in \mathbb{S}_{e_{in}^l}} Z_{e_{in}^l s'}^l = 1 \quad (5.8)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall s \in \mathbb{E} \setminus \{e_{in}^l\} \quad S_s^l = 0 \quad (5.9)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t \quad S_{e_{out}^l}^l = 1 \quad (5.10)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t \quad \sum_{s' \in \mathbb{S}_{e_{out}^l}^l} Z_{s' e_{out}^l}^l = 1 \quad (5.11)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall s \in \mathbb{E} \setminus \{e_{out}^l\} \quad S_s^l = 0 \quad (5.12)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad S_{e_{in}^l}^l = 1 \quad (5.13)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{s' \in \mathbb{S}_{e_{in}^l}} Z_{e_{in}^l s'}^l = 1 \quad (5.14)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad S_{e_{out}^l}^l = 1 \quad (5.15)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{s' \in \mathbb{S}_{e_{out}^l}^l} Z_{s' e_{out}^l}^l = 1 \quad (5.16)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t, \forall s \in \mathbb{E} \setminus \{e_{in}^l, e_{out}^l\} \quad S_s^l = 0 \quad (5.17)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \cup \mathbb{L}_{ent}^t \quad \alpha_{e_{in}^l}^l \geq E^l \quad (5.18)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \cup \mathbb{L}_{leav}^t \quad \beta_{e_{out}^l}^l \leq D^l \quad (5.19)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \sum_{s' \in \hat{\mathbb{S}}_s} Z_{s's}^l = \sum_{s'' \in \mathbb{S}_s} Z_{ss''}^l \quad (5.20)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ & \left\{ \begin{array}{l} \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l - \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l \leq H \cdot P_p^l \\ \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l - \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l \leq H \cdot P_p^l \end{array} \right. \quad (5.21) \end{aligned}$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t \quad \sum_{p \in \mathbb{P}} P_p^l = 1 \quad (5.22)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t \quad \sum_{p \in \mathbb{P}_{ref}^t} P_p^l = 1 \quad (5.23)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{cross}^t \quad \sum_{p \in \mathbb{P}} P_p^l = 0 \quad (5.24)$$

$$\forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}_{ent}^t \cup \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad P_p^l = P_p^{l'} \quad (5.25)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l \geq P_p^l \quad (5.26)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \mathbb{S}_p} Z_{ps}^l \leq (1 - P_p^l) \quad (5.27)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \mathbb{S}_p} Z_{ps}^l \geq P_p^l \quad (5.28)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l \leq (1 - P_p^l) \quad (5.29)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \beta_p^{l_1} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^l) \\ \alpha_p^{l_2} - \beta_p^{l_1} \leq H \cdot (1 - P_p^l) \end{cases} \end{aligned} \quad (5.31)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^{l_2} + \Delta_p^{l_2} + \Theta^{l_2} + W_p^{l_2} - \beta_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \beta_p^{l_2} - \alpha_p^{l_2} - \Delta_p^{l_2} - \Theta^{l_2} - W_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.33)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_2, l_3 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \beta_p^{l_2} - \alpha_p^{l_3} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_3} - \beta_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.34)$$

$$\begin{aligned} \forall t \in \mathbb{T}, l_3 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^{l_3} + \Delta_p^{l_3} + \Theta^{l_3} + W_p^{l_3} - \beta_p^{l_3} \leq H \cdot (1 - P_p^{l_3}) \\ \beta_p^{l_3} - \alpha_p^{l_3} - \Delta_p^{l_3} - \Theta^{l_3} - W_p^{l_3} \leq H \cdot (1 - P_p^{l_3}) \end{cases} \end{aligned} \quad (5.35)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad \omega_c^l = v_c^l + \Theta^l \quad (5.36)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \beta_s^l = \alpha_s^l + \Delta_s^l + 2\Theta^l + W_s^l \quad (5.37)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{E} \quad \beta_s^l = \alpha_s^l + \Delta_s^l + \Theta^l + W_s^l \quad (5.38)$$

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ \begin{cases} \alpha_p^l + \Delta_p^l + 2\Theta^l + W_p^l - \beta_p^l \leq H \cdot P_p^l \\ \beta_p^l - \alpha_p^l - \Delta_p^l - 2\Theta^l - W_p^l \leq H \cdot P_p^l \end{cases} \end{aligned} \quad (5.39)$$

$$\begin{aligned} \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \\ \begin{cases} v_{c_{ss'}}^l - \alpha_{s'}^l \leq H \cdot (1 - Z_{ss'}^l) \\ \alpha_{s'}^l - v_{c_{ss'}}^l \leq H \cdot (1 - Z_{ss'}^l) \end{cases} \end{aligned} \quad (5.40)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \\ & \begin{cases} \alpha_{s'}^l + \Theta^l - \beta_s^l \leq H \cdot (1 - Z_{ss'}^l) \\ \beta_s^l - \alpha_{s'}^l - \Theta^l \leq H \cdot (1 - Z_{ss'}^l) \end{cases} \end{aligned} \quad (5.41)$$

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall s \in \mathbb{S} \quad Y_s^{ll'} + Y_s^{l'l} = 1 \quad (5.42)$$

$$\begin{aligned} & \forall t, t' \in \mathbb{T}, t \neq t', \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall s \in \mathbb{S} \\ & \begin{cases} \beta_s^l + \sigma \leq \alpha_s^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'} + F_t + F_{t'}) \\ \beta_s^{l'} + \sigma \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l} + F_t + F_{t'}) \end{cases} \end{aligned} \quad (5.52)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall s \in \mathbb{S} \setminus \mathbb{P} \\ & \begin{cases} \beta_s^l + \sigma \leq \alpha_s^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'} + F_t) \\ \beta_s^{l'} + \sigma \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l} + F_t) \end{cases} \end{aligned} \quad (5.53)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall p \in \mathbb{P} \\ & \begin{cases} \beta_p^l + \sigma \leq \alpha_p^{l'} + H \cdot (3 - S_p^l - S_p^{l'} - Y_p^{ll'} + P_p^l + F_t) \\ \beta_p^{l'} + \sigma \leq \alpha_p^l + H \cdot (3 - S_p^l - S_p^{l'} - Y_p^{l'l} + P_p^l + F_t) \end{cases} \end{aligned} \quad (5.54)$$

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall c \in \mathbb{C} \quad X_c^{ll'} + X_c^{l'l} = 1 \quad (5.46)$$

$$\begin{aligned} & \forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall c \in \mathbb{C} \\ & \begin{cases} \omega_c^l + \sigma \leq v_c^{l'} + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{ll'} + F_t + F_{t'}) \\ \omega_c^{l'} + \sigma \leq v_c^l + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{l'l} + F_t + F_{t'}) \end{cases} \end{aligned} \quad (5.55)$$

For technical entering circulation:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ & \begin{cases} A^{l_1} - L - \alpha_p^{l_1} - \Theta^{l_1} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.48)$$

For technical leaving circulation:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ & \begin{cases} B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - L \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.49)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ & \begin{cases} A^{l_1} - \alpha_p^{l_1} - \Theta^{l_1} - \xi^l \leq H \cdot (1 - P_p^{l_1}) \\ \xi^l + \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.56)$$

$$\begin{aligned} & \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ & \begin{cases} \xi^l + B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - \xi^l \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.57)$$

For commercial entering circulation:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \\ & \begin{cases} A^{l_1} - \Psi - \alpha_p^{l_1} - \Theta^{l_1} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_1} + \Theta^{l_1} - A^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \end{aligned} \quad (5.50)$$

For commercial leaving circulation:

$$\begin{aligned} & \forall t \in \mathbb{T}, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \\ & \begin{cases} B^{l_2} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_2}) \\ \alpha_p^{l_2} - B^{l_2} - \Psi \leq H \cdot (1 - P_p^{l_2}) \end{cases} \end{aligned} \quad (5.51)$$

## 5.6 Continuous-time model

Based on two types of time presentation, the formulation of timing constraints can be classified into two main categories: discrete-time models and continuous-time models. A comparison of two type of time models is studied in [Floudas and Lin \(2004\)](#).

In the discrete-time model, the time horizon is divided into a number of time intervals of uniform durations and events. To achieve a suitable approximation of the original problem, it is needed to use a time interval that is sufficiently small. In our problem, the duration of this time interval is one second. This leads to very large combinatorial problems of intractable size, especially for real-case problems.

Due to the limitations of the discrete-time approach, the continuous-time model is developed by many researchers. In this model, events can take place at any point in the continuous domain of time. Because of the possibility of



eliminating a major fraction of the inactive event-time interval assignments using the continuous-time approach, the mathematical programmings usually have much smaller sizes and require less computational efforts for their solution. However, due to the variable nature of the timings of the events, the mathematical models with the continuous-time representation could have more complicated structures compared to the discrete-time representation.

In our model, we want to use the continuous-time model to reduce the computation time. But it will be meaningless if we get a rational number as timing variable, for example 435.24 seconds. Fortunately, based on the studies of the mathematical model, we can prove that all timing variables will get integer values.

Some theorems about integral property are presented in [Hoffman and Kruskal \(2010\)](#):

Define the polyhedra:

$$P(b) = \{x \mid A \cdot x \leq b\}$$

where  $b$  is integral and  $A$  is a fixed integral matrix.

The paper proved that  $P(b)$  has the integral property if and only if every minor of  $A$  equals to 0, +1 or -1. The matrix which satisfies this condition is said to have the unimodular property.

We focus on the formulation of timing constraints in our model. The equations (5.36), (5.37) and (5.38) are formulated as equation:

$$A \cdot x = b$$

This equation can be rewritten as:

$$\begin{aligned} A \cdot x &\leq b \\ A \cdot x \geq b &\iff (-1)A \cdot x \leq -b \end{aligned}$$

Clearly, if matrix  $A$  is unimodular, matrix  $(-1)A$  is also unimodular.

The equations (5.35) to (5.39) and equations (5.43), (5.44), (5.45), (5.47) are formulated as  $A \cdot x \leq b$ .

Thus, all timing constraints in our model are formulated as the polyhedra of Hoffman and Kruskal (2010). The integer value of timing decision variables is guaranteed.

We propose to solve our problem by the continuous-time approach. The decision variables  $[\alpha_s^l, \beta_s^l]$ ,  $[v_c^l, \omega_c^l]$ ,  $W_s^l$  can be defined as positive real variables. In this case, the continuous-time model not only satisfies our computation requirement of the integer decision variables, but also reduces the computation time.

## 5.7 Conclusion

In this chapter, we succeed to develop the mathematical models which aim to find conflict-free timetables for railway stations. The model is described with all basic constraints in terms of decision variables, constraints and objective functions. Then we introduce three mathematical models that we consider for our numerical experiments. The first model only allows deviation time for technical circulations. The second model allows deviation time for both technical and commercial circulations. Finally, the cancellation of trains processing is integrated into the last mathematical model. In order to reduce computation time, these models can be considered as continuous-time models by defining time variables as positive real variables.

In the next chapter, we will apply our mathematical models in a real case study of the railway station of Bordeaux to assess the performance and effectiveness of our models.



CHAPTER 6  
Case study

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## 6.1 Introduction

We have already formulated train routing and scheduling problem in railway stations, designed three mathematical models to generate free-conflict timetables allowing deviation time of technical circulations, allowing deviation time of both technical and commercial circulations and allowing the cancellation of trains.

In this chapter, in order to assess our method, we consider the real case study of the railway station of Bordeaux. We need to handle the full day

problem of this railway station. Due to the high complexity of our models, we propose a subgroup partitioning strategy based on rolling horizon approach and apply three models in order to generate a free-conflict timetable for full day problem.

## 6.2 Topology

We study the railway station of Bordeaux which is the case study of [Bai et al. \(2015\)](#), see [Figure 6.1](#). As the southern part of the railway station receives always the overload of trains' activities, we focus only on the southern part to reduce the size of the problem and do not treat the northern part. The topology of the northern part is replaced by an external section *North* and a connector which can be occupied by more than one train at the same time. Conflicts on the northern part are not taken into account. However, our model could apply to the full layout of railway station including the northern part if needed.

The principles of construction of our topology are applied to this railway station in order to minimise the number of connectors (by removing superfluous connectors with a rule which is introduced in [Section 4.2.3](#) page 78) without the elimination of feasible routes to reduce the size of problem.

For example, the [Figure 6.2](#) shows the elimination of superfluous connector  $c2'$  because all routes containing connector  $c2'$  pass also connector  $c2$ .

The abstract layout of the railway station with 15 platforms, 52 sections and 18 connectors is given in [Figure 6.3](#).

In our model, we use the progressive reservation of resources (sections and connectors). In France, the average length of train is about 400 meters. The speed of trains operating in railway stations is usually limited at 50-90 km/h. Applying the formulas [\(4.1\)](#) and [\(4.2\)](#) with an average length of section of 1000 metres and the speed of train of 50 km/h, we have  $\Delta$  (time taken to traverse section  $s$  by circulation) is about 44 seconds and  $\Theta$  (time taken for a circulation going through a connector) is about 29 seconds.

In the work of [Bai et al. \(2014\)](#) and [Bai et al. \(2015\)](#), the railway station has 247 trains and about 500 circulations per day. They do not treat the northern part of Bordeaux railway station. They use the reservation of

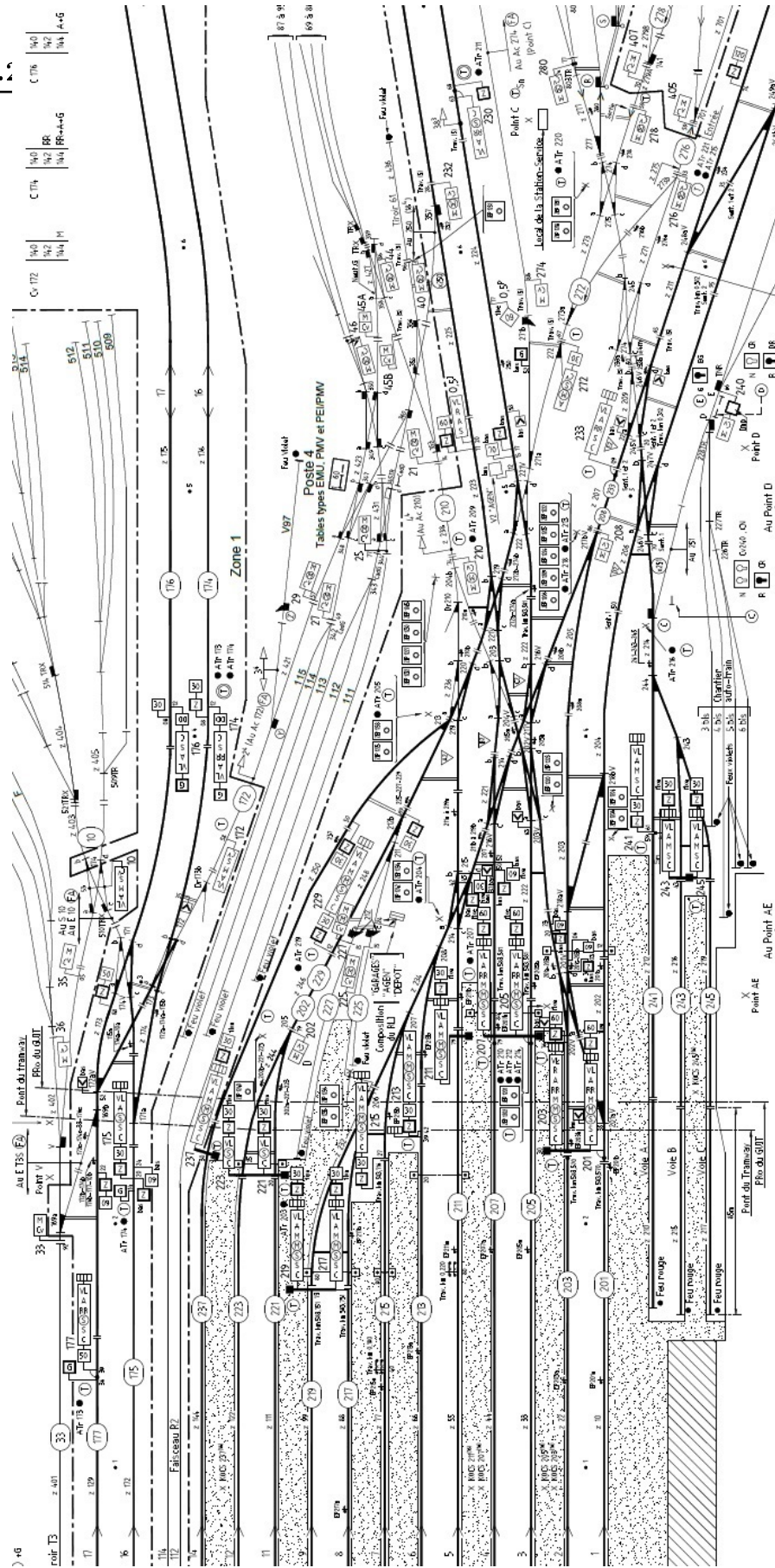


Fig. 6.1 The southern part of Bordeaux station.

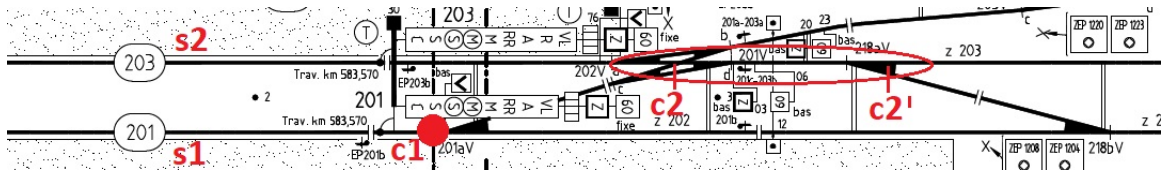


Fig. 6.2 The construction of connector 2.

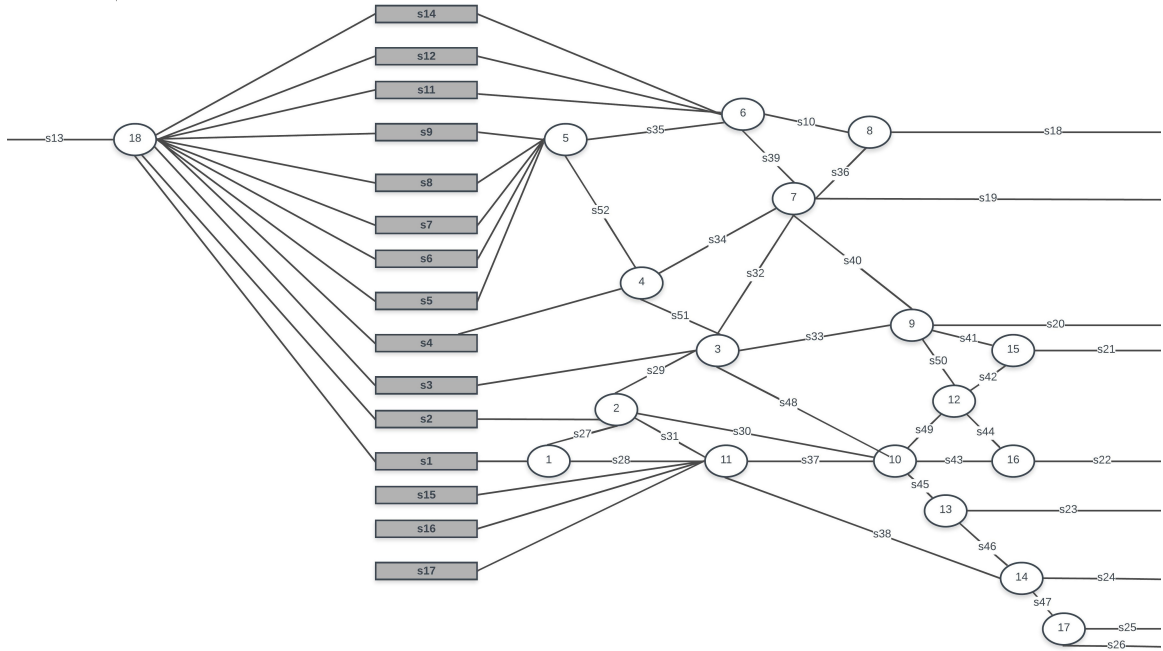


Fig. 6.3 Studied railway station.

complete route for their model. The number of feasible routes they consider is 75 routes with different number of connectors which vary from 2 connectors (and 3 sections) to 7 connectors (and 8 sections). The traversing time of these routes corresponding our parameters ( $\Delta = 44$  seconds and  $\Theta = 29$  seconds) is resumed in Table 6.1.

There are in total 6 routes consisting of 2 connectors and 3 sections, 19 routes with 3 connectors, 22 routes with 4 connectors, 18 routes with 5 connectors, 9 routes with 6 connectors and 1 route with 7 connectors. In results, the average traversing time of a route is about 343.79 seconds.

In the work of Bai et al. (2014) and Bai et al. (2015), they consider that the time allocated to each circulation to operate in its route is always 300 seconds which is approximately the same value of our average traversing time. Therefore, the effectiveness of our approach can be compared to the

TABLE 6.1  
THE TRAVERSING TIME OF ROUTES WITH  $\Delta = 44s$  AND  $\Theta = 29s$

Number of connectors and sections	Number of routes	Traversing time (s)
2 connectors and 3 sections	6	190
3 connectors and 4 sections	19	263
4 connectors and 5 sections	22	336
5 connectors and 6 sections	18	409
6 connectors and 7 sections	9	482
7 connectors and 8 sections	1	555
Average traversing time of route: 343.79 seconds		

approach of Bai et al. (2015) by the experimental results of these test cases.

The following parameters are used:

- The maximum permissible deviation ( $L$ ) for technical circulations is 600 seconds which is the lowest value in the experimentations of Bai et al. (2014). In Bai et al. (2015), in order to solve the full day problem, they increase this parameter to 3600 seconds (60 minutes) to absorb all train cancellations.

- The maximum permissible deviation ( $\Psi$ ) for commercial circulations is 600 seconds which is the same value as in the work of Bai et al. (2015) to solve the full day problem.

- The duration to traverse connector by circulation ( $\Theta$ ) is 29 seconds for all connectors and the duration to traverse section by circulation ( $\Delta$ ) is 44 seconds for all except section *North* 13 where its duration is 300 seconds. As explained above, we focus only on the southern part of railway station to reduce the size of the problem. The layout of the northern part is replaced by the external section 13. It represents the beginning of the northern lines leaving the station. Consequently, we assume that this section 13 and the connector 18 (connected to section 13) can be occupied by more than one train. This will remove the safety constraints for this section and this connector which are expressed in equations 5.43 and 5.47.

- The safety delay time between two circulations using the same section or connector ( $\sigma$ ) is 5 seconds.



### 6.3 Data visualization

In order to easily examine the results, we express the results by Gantt Chart, see Figure 6.4.

In the Gantt Chart, the horizontal axis is real-time in one day and the vertical axis is the list of trains in chronological order. Trains are represented by many rectangles filled in colours. They represent the occupation of sections and connectors where trains pass through. The section and connector number can be found on the top left of the rectangle. Length of the horizontal edge stands for the duration of standstill on sections and connectors. The occupation time of sections and connectors are marked on the beginning and the end of the rectangle. A big rectangle is tied around each circulation to represent the possible deviation of circulation. A normal rectangle represents a commercial circulation and a dashed rectangle represents a technical circulation.

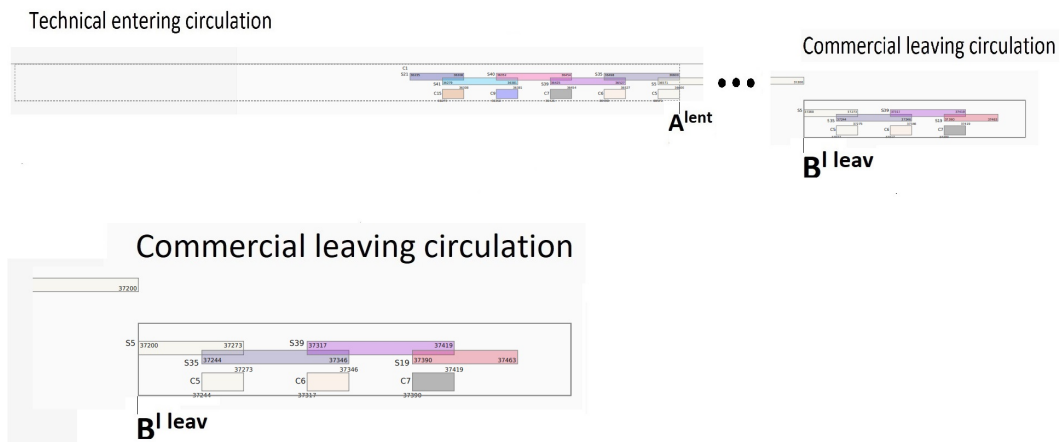


Fig. 6.4 Visualization of a train for *model 1* with  $L = 600s$  and  $\Psi = 0$ .

The visualizations of all computational results is reachable by <https://github.com/khanhvn06/visual>.

### 6.4 Numerical experiments

The computation study is conducted under C++ in Visual Studio 2017 and CPLEX version 12.8. The computer hardware runs Windows 64-bit operating system with AMD Ryzen 5 1600 CPU at 3.2 GHz and 16GB memory of RAM. The time limit used to solve each instance is 1800 seconds (30 minutes) which

is higher than 500 seconds time limit in the work of Bai et al. (2015).

In order to assess the limitation of computer with our model, the first 240 trains of the problem are divided into small size problems in chronological order where trains in each sub-problem should pass through the railway station in a same period of time. Thus, we have 24 groups of 10 trains, 12 groups of 20 trains, 8 groups of 30 trains, 6 groups of 40 trains or 5 groups of 45 trains. The results of *model 1* (allowing deviation of technical circulations) with *obj1* (minimising of deviation time) are presented in Table 6.2. The first column consists of the number of trains of each group. Columns two, three and four are the minimum, the average and the maximum computation time needed to solve our mathematical model. Columns five and six count the number of cases solved and the number of cases that cannot be solved within 30 minutes.

We can see that when the number of trains in each group increases, the computation time is increased remarkably, see Figure 6.5.

TABLE 6.2  
INITIAL RESULTS OBTAINED BY SOLVING *model 1*

Trains per group	Min solving time(s)	Avg solving time(s)	Max solving time(s)	Group solved	Group unsolved after 30 mins
10	1.67	2.30	2.94	24	0
20	9.50	116.63	690.55	12	0
30	230.31	496.59	826.17	8	0
40	924.11	1350.75	1652.56	3	3
45	-	-	-	0	5

**Note:** The resolution of each group of trains does not take into account the conflicts of resources (sections and connectors) with other group of trains. Mostly the conflicts occur in between two consecutive groups in chronological order but also in more distant groups when the number of trains in each group is small. The results for 8 groups of 30 trains show that that there are many conflicts of occupations of sections and connectors between trains in different groups of trains. For example, the Figure 6.6 displays the conflict of occupation of section 10 between train 56 (in second group) and train 72 (in third group). The occupations of sections (by second) are marked on the begin and the end of rectangles. The second circulation of train 56 occupies section 10 at the time interval [29144, 29246] and the first circulation of train 72 occupies section 10 at the time interval [29225, 29327]. Clearly, there is a

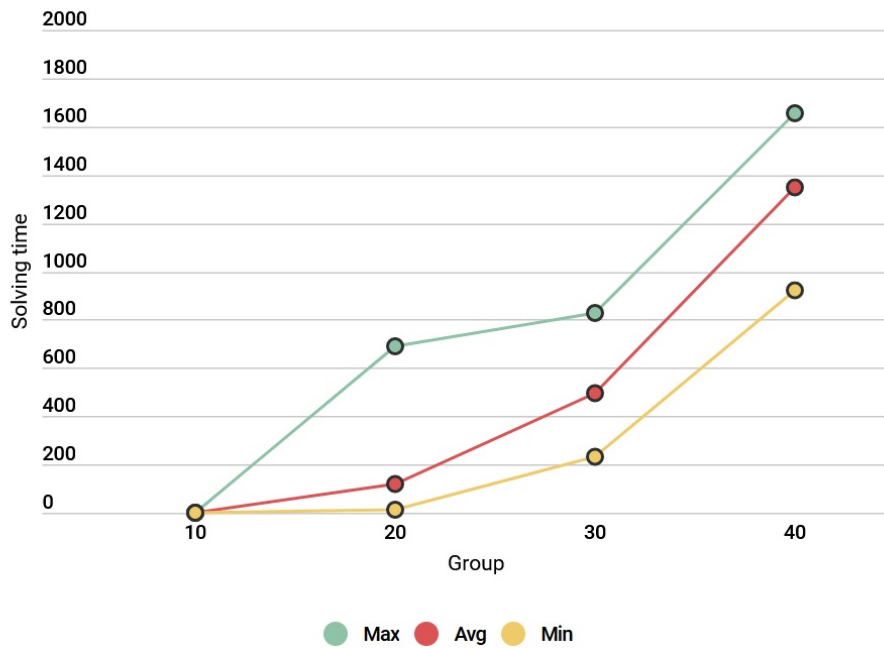


Fig. 6.5 The diagram of solving times.

time intersection between these two rectangles ([29225, 29246]). It means that section 10 is occupied by two trains 56 and 72 at the same time at [29225, 29246].

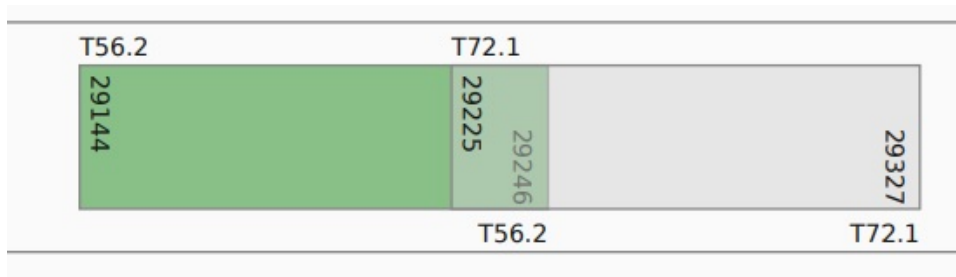


Fig. 6.6 The conflict of occupation of s10 between t56 and t72.

The Figure 6.7 displays the conflict of occupation of connector 13 between train 28 (in first group) and train 36 (in second group).

The Gantt Charts of the occupations of sections and connectors for 8 groups of 30 trains are displayed as links [https://github.com/khanhvn06/visual/blob/master/group30\\_sections.pdf](https://github.com/khanhvn06/visual/blob/master/group30_sections.pdf) and [https://github.com/khanhvn06/visual/blob/master/group30\\_connecteurs.pdf](https://github.com/khanhvn06/visual/blob/master/group30_connecteurs.pdf).

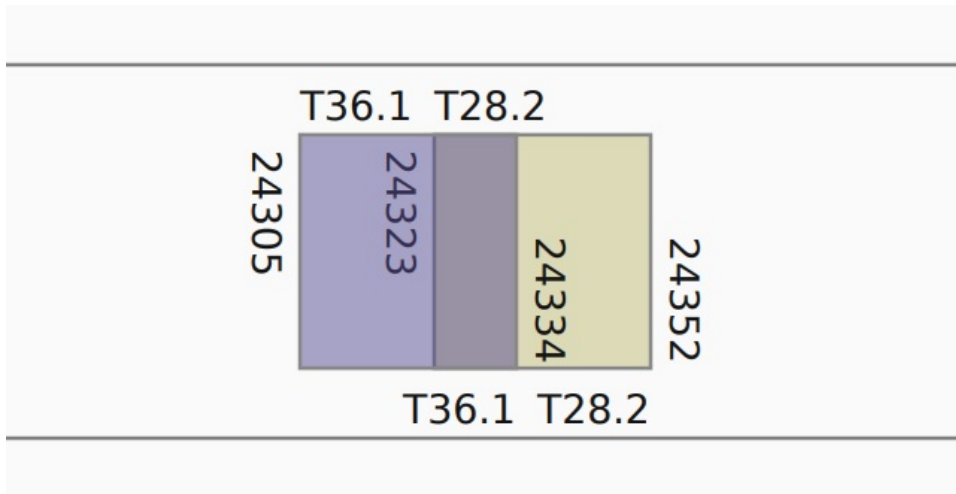


Fig. 6.7 The conflict of occupation of c13 between t28 and t36.

For the moment, we can solve up to 30 trains. Indeed, only 3 of the 6 problems of 40 trains can be solved and all the problem of 45 trains cannot be solved within 30 minutes of time limit due to the high complexity of our model. Thus, we propose a subgroup partitioning strategy based on rolling horizon approach which can divide the problem into small size problems but still take into account the conflicts of between groups of trains for the resolution of each group in order to generate a feasible timetable for the full day problem.

## 6.5 Subgroups partitioning strategies based on rolling horizon approach

### 6.5.1 Introduction

In order to solve a complex problem with larger size, a suitable Math-Heuristic approach should be considered to solve it. The model for routing and scheduling problem in railway station considers a time interval called the rolling horizon in which all trains must arrive and leave the railway station. Some works dealing with train scheduling problem propose Math-Heuristic methods based on rolling horizon approach.

### 6.5.2 State of art in rolling horizon approach

Wang et al. (2014) proposes an approach to solve simultaneously the optimal routing and scheduling problems for multiple trains. The solving is based

on a greedy method. In the greedy approach, the problem of the first train is solved first, then the scheduling problem of the following train is solved based on the first solution obtained, and the process is repeated. The simultaneous approach solves the scheduling problem of the leading train and following trains at the same time. The simulation results show that the greedy approach has a lower performance than the simultaneous approach but it has a lower less computation time: the computation times for greedy approach are from around 1 second to 250 seconds and from around 2 seconds to 420 seconds for simultaneous approach.

Nielsen et al. (2012) propose a rolling horizon approach to solve the rescheduling problem. In this approach, they consider only those trips that are within a certain time horizon at which rescheduling takes place. Whenever new information on timetable is available, the circulation is rescheduled for further time horizon. They tested with values for the horizon parameter (represents the number of trains in each instance) between 2 and 5 hours. The results show that a longer horizon gives better solutions with the cost of computation time (from 2 seconds to 50 seconds for each instance). They tested the instances for the update parameter (the fixed output from the previously solution) from 30 to 120 minutes with 4 hours of horizon parameter. The results show that updating more often (smaller value of update parameter or less fixed trains) gives better solution.

Bai et al. (2015) applied a rolling horizon algorithm to solve routing and scheduling problem for railway station. In this approach, each instance consists of three sub-groups. The first sub-group composes the fixed output from the previous solution (valid constraints). The second sub-group ensures conflict-free between the first sub-group and the third sub-group. The last sub-group represents its new sub-group. They solve the full-day problem which is divided into 8 instances with 30 trains for each sub-group. The computation time for each instance varies from 6 seconds to 500 seconds.

### 6.5.3 Solving principles

In order to deal with the routing and scheduling problem of railway station for a full day, we apply subgroups partitioning strategies to divide all trains of a day into many small subgroups in chronological order.

Due to the size of the whole problem, the problem is solved step by step using a rolling horizon approach controlled by a parameter  $N \in \mathbb{N}$  (horizon width) corresponding to the number of trains of each period. This parameter

expresses how far ahead (the number of next trains in chronological order) we wish to take into account for each period.

At each period, the mathematical model is solved for  $N$  trains. The first  $F$  trains ( $F < N$ ) solved will be stored to belong the final full solution, whereas the remainder last  $N-F$  trains are relaxed to be considered again in the next period. At each period, the first  $F$  trains are denoted as the "buffer" sub-group, the remainder last  $N-F$  trains are denoted as "new" sub-group. The new sub-group of a period becomes the "buffer sub-group" of the next one which acts as a conflicts holding area between these two periods. The solutions of the buffer sub-group of a period will be fixed at next period and will become the fixed sub-group.

The value of  $N$  and  $F$  can vary for each period but the following principles must be respected:

- **Principle 1:** The solution of all trains of fixed sub-groups corresponds to the result of the previous solution and will become the valid constraints in the next solution.

- **Principle 2:** There is no time intersection between fixed sub-group and new sub-group in the same period to avoid the primary infeasibility between the data given of the new sub-group and the results of fixed sub-group. It means that when the first train of new sub-group arrive at railway station, all trains of fixed sub-group must have already left.

- **Principle 3:** The computation time for the resolution of each period should adapt to a time limit. Thus, the size of trains group must not be too big.

An example of the subgroup partitioning strategy with  $F=N/2$  for all periods can be described as follows: the solution corresponding to the first  $N/2$  trains is stored to belong to the final full solution and becomes the fixed sub-group of the next period, whereas the remainder last  $N/2$  trains are relaxed to be considered as buffer sub-group again in the next period. Thus,  $T$  trains would be divided into  $2 \times T/N$  groups of  $N$  trains in chronological sequence :  $[1, N]$ ,  $[N/2+1, 3N/2]$ ,  $[N + 1, 2N]$  ...,  $[(T/(N/2)-1) \cdot N/2+1, T]$ .

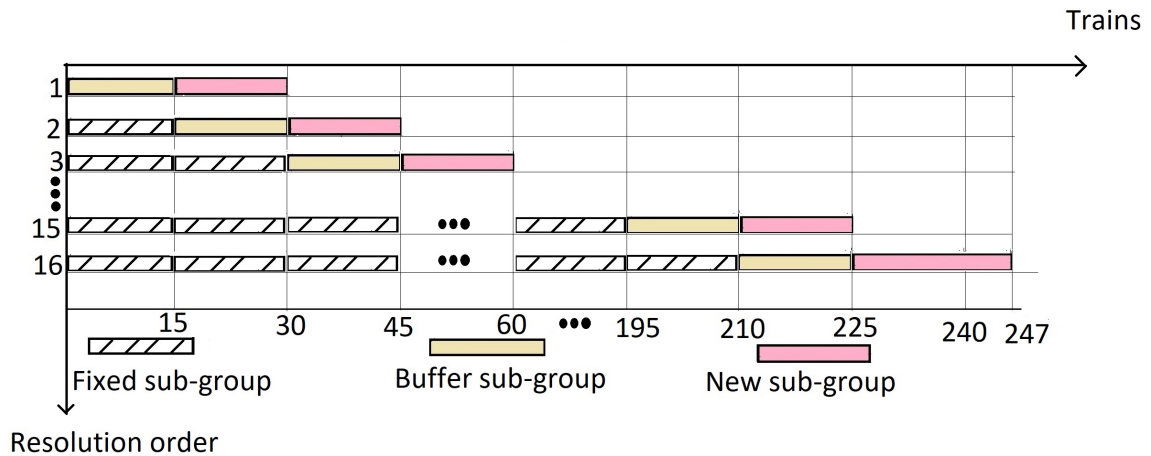


Fig. 6.8 Subgroup partitioning strategy.

An illustrative example is given in Figure 6.8 for  $N=30$ .

The parameter  $N$  (the number of trains for each period) chosen must guarantee that for each period, the  $N/2$  trains of fixed sub-group have no possible timing intersection with the  $N/2$  trains of new sub-group. Thus, no primary infeasibility may be produced by fixing the  $N/2$  first trains of each period.

## 6.6 Computational results

The results of problem with exact method in Table 6.2 show that our mathematical model can solve problems with 30 trains at maximum. Bigger groups of trains do not guarantee a solution can be found within 30 minutes of computation time. In our experiment, the circulations of full day problem are divided into 15 groups of trains with the number of trains in each group  $N=30$ , and each sub-group has 15 trains. The constraints of **principle 2** are verified by the visualisation of trains for the solution of 8 groups of 30 trains in the initial results (Table 6.2) which is displayed in [https://github.com/khanhvn06/visual/blob/master/group30\\_trains.pdf](https://github.com/khanhvn06/visual/blob/master/group30_trains.pdf)

The verification of no time intersection for group of 30 trains (sub-group of 15 trains) are presented in Table 6.3. Note that the arrival time of trains at external section 13 and the departure time of trains from external section 13 are not taken into account (because we do not consider the safety constraints

for this section as mentioned in Section 6.2 Page 143).

TABLE 6.3  
VERIFICATION OF NO TIME INTERSECTION

Subgroup	Arrival time at the railway station of first train(s)	Departure time from the railway station of last train(s)
1-15	1808	18443
16-30	16534	24396
31-45	21201	26869
46-60	26202	29890
61-75	28101	32616
76-90	31771	37202
91-105	35682	40583
106-120	37748	46009
121-135	46114	50903
136-150	49461	56256
151-165	54021	60323
166-180	58928	63623
181-195	62134	67943
196-210	65408	70763
211-225	68661	72970
226-240	71922	79750

For example, the departure time from the railway station of the last train of the first subgroup 1-15, which is the fixed subgroup of second period (described in Figure 6.8 above), is 18443 seconds. The arrival time at the railway station of the first train of the third group 31-45, which is the new subgroup of second period, is 21201 seconds. We can clearly verify that there is no time intersection between these two subgroups. Therefore it guarantees that there is no primary infeasibility in the resolution of second period.

### 6.6.1 Results on full day timetable (*model 1*)

As a first step, we try to solve the full day problem by *model 1* with the possibility of deviation of technical solutions. We consider that the preferred arrival time to the railway station of entering circulation  $E^l$  and the preferred departure time from the railway station of leaving circulation  $D^l$  are related



to the length of the longest possible routes for each circulation (from external section to platform).

In the first instance, trains can operate at all possible routes corresponding to their external sections. In this case, the permissible traversing time of each circulation corresponds to its longest possible route. In the next instances, we decrease the permissible traversing time of each circulation by a percentage of the longest possible route. This limits the number of possible routes for each circulation and therefore increase the potential conflict of resources between circulations. It allow us to vary the complexity of our test cases.

On the first numerical experiment, the values of  $E^l$  and  $D^l$  are associated to the length of longest possible route for each circulation. These values depend on the external section which is given to the circulation. The possible routes corresponding with each external section are resumed in Table 6.4.

**Note:** The route of leaving circulation includes the traversing time of platform. The traversing time of platform is not included in the route of entering circulation, as described in Section 4.3.2.2 Page 80. We do not consider the conflicts in external section 13 and connector 18, therefore it is not necessary to define the values of  $E^l$  and  $D^l$  corresponding to external section 13.

TABLE 6.4  
THE POSSIBLE ROUTES CORRESPONDING WITH EACH EXTERNAL SECTION

External section	Entering circulation				Leaving circulation				Number of possible routes	Corresponding platform
	Longest route		Shortest route		Longest route		Shortest route			
	Connectors and sections	Travel time(s)	Connectors and sections	Travel time(s)	Connectors and sections	Travel time(s)	Connectors and sections	Travel time(s)		
18	5 connectors 5 sections	365	2 connectors 2 sections	146	5 connectors 5 sections 1 platform	409	2 connectors 2 sections 1 platform	190	17	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
19	4 connectors 4 sections	292	2 connectors 2 sections	146	4 connectors 4 sections 1 platform	336	2 connectors 2 sections 1 platform	190	17	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
20	4 connectors 4 sections	292	2 connectors 2 sections	146	4 connectors 4 sections 1 platform	336	2 connectors 2 sections 1 platform	190	23	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
21	6 connectors 6 sections	438	3 connectors 3 sections	219	6 connectors 6 sections 1 platform	482	3 connector 3 sections 1 platform	263	38	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14, s15, s16, s17
22	6 connectors 6 sections	438	3 connectors 3 sections	219	6 connectors 6 sections 1 platform	482	3 connector 3 sections 1 platform	263	38	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14, s15, s16, s17
23	5 connectors 5 sections	365	3 connectors 3 sections	219	5 connectors 5 sections 1 platform	409	3 connector 3 sections 1 platform	263	15	s1, s2, s3, s4, s5, s6, s7, s8, s9, s15, s16, s17
24	6 connectors 6 sections	438	2 connectors 2 sections	146	6 connectors 6 sections 1 platform	482	2 connector 2 sections 1 platform	190	22	s1, s2, s3, s4, s5, s6, s7, s8, s9, s15, s16, s17
25	7 connectors 7 sections	511	3 connectors 3 sections	219	7 connectors 7 sections 1 platform	555	3 connectors 3 sections 1 platform	263	22	s1, s2, s3, s4, s5, s6, s7, s8, s9, s15, s16, s17
26	7 connectors 7 sections	511	3 connectors 3 sections	219	7 connectors 7 sections 1 platform	555	3 connectors 3 sections 1 platform	263	22	s1, s2, s3, s4, s5, s6, s7, s8, s9, s15, s16, s17

TABLE 6.5  
COMPUTATION TIMES OF *model 1* MINIMIZING THE TOTALS OF DEVIATION TIME OF TECHNICAL CIRCULATIONS (OBJ1) WITH  $E^l$  AND  $D^l$   
CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF THE LENGTH OF LONGEST ROUTE.

Train group	Fixed subgroup	Buffer subgroup	New subgroup	Variables before presolve	Constraints before presolve	Variables after presolve	Constraints after presolve	Solving time	obj1	Number of deviations of technical circulations
1-30 [0h-7h]	-	1-15	16-30	430920	1426986	47046	96900	826.52s (13.78 mins)	0	0
16-45 [4h30-7h30]	1-15	16-30	31-45	835380	2697161	42180	86993	267.73s (4.46 mins)	0	0
31-60 [6h-8h]	16-30	31-45	46-60	851032	2745853	38969	80210	88.25s (1.47 mins)	0	0
46-75 [7h30-9h]	31-45	46-60	61-75	851032	2745837	47666	98005	1182.39s (19.71 mins)	0	0
61-90 [8h-10h]	46-60	61-75	76-90	851032	2745859	39202	80490	803.81s (13.40 mins)	0	0
76-105 [8h30-10h30]	61-75	76-90	91-105	851032	2745858	47763	97550	606.20s (10.10 mins)	0	0
91-120 [10h-12h30]	76-90	91-105	106-120	851032	2745855	42264	86548	502.48s (8.37 mins)	0	0
106-135 [10h30-14h]	91-105	106-120	121-135	851032	2745853	41111	84961	152.31s (2.54 mins)	0	0
121-150 [12h30-15h]	106-120	121-135	136-150	835380	2697093	35722	73913	190.55s (3.18 mins)	0	0
136-165 [14h-16h30]	121-135	136-150	151-165	866824	2794930	38614	79315	668.44s (11.14 mins)	0	0
151-180 [15h-17h30]	136-150	151-165	166-180	866824	2794864	43536	89198	30 mins	31s	3 ( <b>t158</b> (21s), <b>t162</b> (5s), <b>t164</b> (5s))
166-195 [16h30-18h30]	151-165	166-180	181-195	866824	2795011	35984	74127	145.97s (2.43 mins)	31s	3 ( <b>t158</b> (21s), <b>t162</b> (5s), <b>t164</b> (5s))
181-210 [17h30-19h]	166-180	181-195	196-210	835380	2697080	37386	76701	139.25s (2.32 mins)	0	0
196-225 [18h30-20h]	181-195	196-210	211-225	835380	2697104	34544	71474	112.84s (1.88 mins)	0	0
211-247 [19h-24h]	196-210	211-225	226-247	1067248	3416491	55394	113469	479.67s (7.99 mins)	0	0

The longest route and shortest route corresponding with external section 25 are displayed in Figure 6.9.

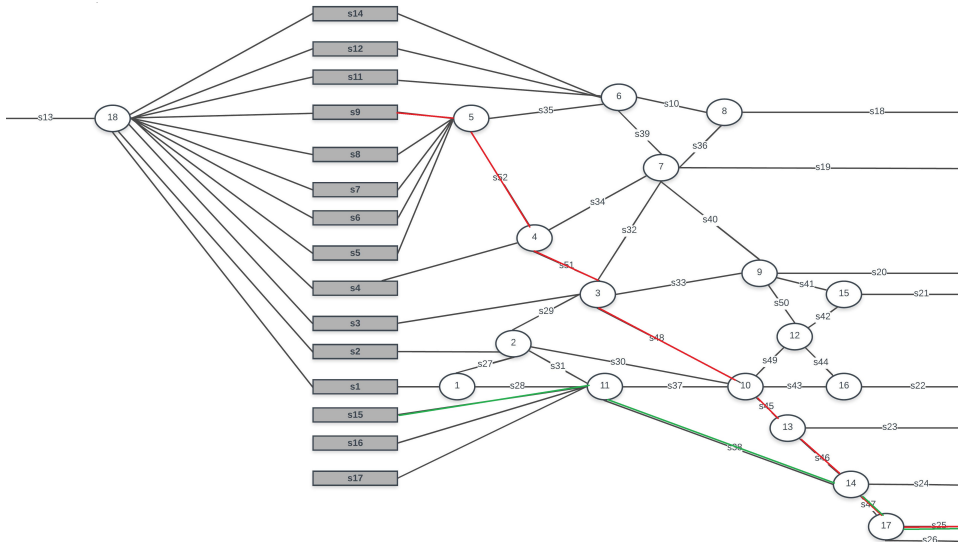


Fig. 6.9 The longest route and shortest route corresponding with external section 25.

The longest route corresponding with external section 25 is displayed in red ( $\{s25, c17, s47, c14, s46, c13, s45, c10, s48, c3, s51, c4, s52, c5, s9\}$ ) in the figure and it corresponds with the platform s9. There are also 4 other longest possible routes corresponding with platforms s8, s7, s6 and s5.

The shortest route corresponding with external section 25 is displayed in green ( $\{s25, c17, s47, c14, s38, c11, s15\}$ ) in the figure and it corresponds with the platform s15. There are also 2 other shortest possible routes corresponding with platforms s16 and s17.

Note that corresponding with each platform there can be more than one possible route. For example, there is a longer possible route corresponding with platform s15 ( $\{s25, c17, s47, c14, s46, c13, s45, c10, s37, c11, s15\}$ ). In spite of its longer traversing time, it is still considered as a possible route. In some cases, this alternate route to platform s15 can be assigned to a circulation if the section s38 is too busy.

The results of *model 1* with *obj1* are presented in Table 6.5. The full day problem with 247 trains is divided into 15 train groups with  $N=30$  shown in the first column. Each train group is divided into three subgroups: fixed subgroup, buffer subgroup and new subgroup which are described in the

2nd, 3th and 4th columns. Once variables and constraints shown in 5th and 6th columns are sent to the solver, CPLEX presolve eliminates redundant constraints and variables according to valid constraints. The reduced problem is described in 7th and 8th columns. Note that the number of variables before presolve correspond with the total number of circulations. Therefore it can vary depending on the train group. Clearly, train groups consisting of trains with 3 circulations have more variables than train groups consisting of only trains with 2 circulations despite having the same number of trains. The computation times of solutions and the objective values within 30 minutes of running time limit are given in the 9th and 10th columns. The last column shows the number of deviations and trains with deviation. The trains in fixed subgroup and buffer subgroup are noted in bold. It means that these deviations will be included in the final solution of full day problem.

The first results in Table 6.5 show that in the first 10 groups of trains, there is no deviation of trains. In the next group of trains 151-180, the best solution which can be found within 30 minutes of time limit has 3 deviations of technical circulations in 3 trains (t158, t162, t164) for a total of 31 seconds of deviation time. In group of trains 166-195, 3 trains with deviations are inherited from the previous group. There is no deviation in the last 3 groups of trains. The full day problem is solved with 31 seconds of deviation time of technical circulations in 3 trains. The total of computation time is 132.77 minutes (2 hours 13 minutes).

This instance can be comparable to the problem studied of Bai et al. (2015). In the solution of the study of Bai et al. (2015), there are 182 minutes of deviation of commercial circulations of 37 trains: 3 trains more than 10 minutes, 8 trains more than 6 minutes and 29 trains less than 5 minutes. We found a solution with 31 seconds of deviation time of technical circulations in 3 trains and there is no deviation of commercial circulations. Our model with the reservation of each section independently, which is more precise, makes a better use of railway resources and therefore increases the capacity of railway stations.

For further experimentations, the values of  $E^l$  and  $D^l$  are adjusted in order to assess the performance of our model. In the next numerical experiments, the length of route to calculate  $E^l$  and  $D^l$  for the short trains (which can assign any platform of 15 platforms in the railway station) is decreased by 5% of the length of longest route for each experimentation. We do not decrease the permissible traversing time for the circulations of medium trains and long

trains which can only assign platforms from  $p_1(s_1)$  to  $p_7(s_7)$  and from  $p_1(s_1)$  to  $p_6(s_6)$  respectively. A small decrease of the permissible traversing time of route removes several preferred platforms of trains. It increases significantly the difficulty to get a feasible solution.

The possible routes remaining for each external section corresponding to the permissible traversing time of 95%, 90% and 85% of the length of longest route are presented in Tables 6.6, 6.7 and 6.8 respectively. A decrease of 5% from 100% to 95% of the length of longest route reduces the number of possible routes for all external section. From 95% to 85%, the number of possible routes for each external section is not changed except s25, s26 in case of 85%. Clearly, the permissible travel time decreases from 95% to 85%. Although the number of possible routes is unchanged, the permissible travel time will limit the permissible deviation time of circulations.

The results with  $E^l$  and  $D^l$  corresponding to the length of 95%, 90% and 85% of longest route are respectively presented in three Tables 6.9, 6.10 and 6.11.

In the second results in Table 6.9, there is no deviation of trains in the first 5 groups of trains. In the next group 76-105, the best solution which can be found within 30 minutes of time limit has 2 deviations of technical circulations in 2 trains (t94 and t98) for a total of 123 seconds of deviation time. In group 91-120, the optimal solution has 1 deviation (29 seconds) of technical circulation in train t98. It means that the best solution in the resolution of group 76-105 is not the optimal solution. The valid constraints of fixed subgroup 76-90 for the resolution of group 91-120 help reducing the computation time to find an optimal solution. In group 106-135, the deviation of train t98 is inherited from the previous group. There are no deviation in the next 2 groups of trains. In group 151-180 and 166-195, there is only 1 deviation of technical circulations in train t162 which is lower than the results of these two groups in the first results in Table 6.5. The difference is due to the different fixed subgroups 136-150 in group 151-180 which depends on the results of all previous groups of trains (from group 1-30 to group 136-165). There is no deviation in the last 3 groups. The full day problem is solved with 34 seconds of deviation time of technical circulations in 2 trains. The total of computation time is 116.48 minutes (1 hour 57 minutes).

In the third results in Table 6.10, there is no deviation of trains in the first 3 groups of trains. In the next group 46-75 there are 3 deviations of technical circulations in 3 trains (t57, t58 and t64) for a total of 76 seconds

of deviation time. In group 61-90, the deviations of 2 trains t57 and t58 are inherited from the previous group. The resolution of this group still has the deviation of t64 which is the same as the result of previous group. In group 76-105, the deviation of train t64 is inherited from the previous group 61-90. The next 3 groups have no deviation. In group 136-165, there are 2 deviations of technical circulations in 2 trains t158 and t162. In group 151-180, there are 2 deviations of technical circulations in 2 trains t162 and t178. The train t162 is in fixed subgroup of group 151-180. Therefore, the deviation of train t162 will be included in the final solution. In group 166-195, the deviation of train 162 is inherited from the previous group. The full day problem is solved with 81 seconds of deviation of technical circulations in 4 trains. The total of computation time is 133.17 minutes (2 hours 14 minutes).

In the fourth result in Table 6.11, there is no deviation of trains in the first 3 groups of trains. In the next group 46-75, no feasible solution can be found for 30 minutes of running time limit. This occurs because of the limit of deviation time corresponding to the permissible traversing time of circulations. In the previous result (case of 90% of the length of longest route), t64 has a deviation of 21 seconds of its entering circulation which enters the railway station from external section s21 and arrives at platform s6. The reference arrival time at platform ( $A^l$ ) of this circulation is 29220 seconds. The route chosen is the longest route corresponding to external section s21 {s21, c15, s41, c9, s33, c3, s51, c4, s52, c5, s6}. The travel time of this route is 365 seconds. A deviation time of 21 seconds means that the circulation arrives at platform earlier than 21 seconds. Thus, the arrival time at platform of this circulation is equal to 29199 seconds ( $\alpha_p^l + \theta^l = A^l - 21 = 29220 - 21$ ) and the arrival time at railway station of this circulation is equal to 28834 seconds ( $\alpha_{s21}^l = \alpha_p^l + \theta^l - 365 = 29199 - 365$ ) which respects the reference arrival time at the railway station for entering circulation from s21 in case of 90% (equal or greater than  $E^l = A^l - 394 = 28826$ , see Constraint 5.18) but do not satisfy the constraint in case of 85% ( $E^l = A^l - 372 = 28848$ ).

The results show that the total of computation time to solve the full day problem is about 2 hours. When we decrease the permissible traversing time of route for each circulation (from 100% to 85% of the length of longest route), the deviation time is increased. Most of the deviations occur in the peak time of day: [8h-10h] and [15h-17h]. In case corresponding to the length of 85% of longest route, no feasible solution can be found in train group of 46-75t for 30 minutes of running time limit. Therefore, we will use the *model 2* with the possibility of deviation of commercial circulations in the next section to solve this case.

TABLE 6.6  
THE POSSIBLE ROUTES CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF 95% OF THE LENGTH OF LONGEST ROUTE

External section	Entering circulation			Leaving circulation			Number of possible routes	Corresponding platform
	Longest route		Permissible traversing time(s)	Longest route		Permissible traversing time(s)		
	Connectors and sections	Travel time(s)		Connectors and sections	Travel time(s)			
18	4 connectors 4 sections	292	346	4 connectors 4 sections 1 platform	336	388	16	s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
19	3 connectors 3 sections	219	277	3 connectors 3 sections 1 platform	263	319	16	s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
20	3 connectors 3 sections	219	277	3 connectors 3 sections 1 platform	263	319	7	s2, s3, s4, s11, s12, s14
21	5 connectors 5 sections	365	416	5 connectors 5 sections 1 platform	409	457	32	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14, s15, s16, s17
22	5 connectors 5 sections	365	416	5 connectors 5 sections 1 platform	409	457	22	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14, s15, s16, s17
23	4 connectors 4 sections	292	346	4 connectors 4 sections 1 platform	336	388	9	s1, s2, s3, s4, s15, s16, s17
24	5 connectors 5 sections	365	416	5 connectors 5 sections 1 platform	409	457	16	s1, s2, s3, s4, s15, s16, s17
25	6 connectors 6 sections	438	485	6 connectors 6 sections 1 platform	482	527	16	s1, s2, s3, s4, s15, s16, s17
26	6 connectors 6 sections	438	485	6 connectors 6 sections 1 platform	482	527	16	s1, s2, s3, s4, s15, s16, s17



TABLE 6.7  
THE POSSIBLE ROUTES CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF 90% OF THE LENGTH OF LONGEST ROUTE

External section	Entering circulation			Leaving circulation			Number of possible routes	Corresponding platform
	Longest route		Permissible traversing time(s)	Longest route		Permissible traversing time(s)		
	Connectors and sections	Travel time(s)		Connectors and sections	Travel time(s)			
18	4 connectors 4 sections	292	328	4 connectors 4 sections 1 platform	336	368	16	s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
19	3 connectors 3 sections	219	262	3 connectors 3 sections 1 platform	263	302	16	s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
20	3 connectors 3 sections	219	262	3 connectors 3 sections 1 platform	263	302	7	s2, s3, s4, s11, s12, s14
21	5 connectors 5 sections	365	394	5 connectors 5 sections 1 platform	409	433	32	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14, s15, s16, s17
22	5 connectors 5 sections	365	394	5 connectors 5 sections 1 platform	409	433	22	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14, s15, s16, s17
23	4 connectors 4 sections	292	328	4 connectors 4 sections 1 platform	336	368	9	s1, s2, s3, s4, s15, s16, s17
24	5 connectors 5 sections	365	394	5 connectors 5 sections 1 platform	409	433	16	s1, s2, s3, s4, s15, s16, s17
25	6 connectors 6 sections	438	459	6 connectors 6 sections 1 platform	482	499	16	s1, s2, s3, s4, s15, s16, s17
26	6 connectors 6 sections	438	459	6 connectors 6 sections 1 platform	482	499	16	s1, s2, s3, s4, s15, s16, s17

TABLE 6.8  
THE POSSIBLE ROUTES CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF 85% OF THE LENGTH OF LONGEST ROUTE

External section	Entering circulation			Leaving circulation			Number of possible routes	Corresponding platform
	Longest route		Permissible traversing time(s)	Longest route		Permissible traversing time(s)		
	Connectors and sections	Travel time(s)		Connectors and sections	Travel time(s)			
18	4 connectors 4 sections	292	310	4 connectors 4 sections 1 platform	336	347	16	s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
19	3 connectors 3 sections	219	248	3 connectors 3 sections 1 platform	263	285	16	s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
20	3 connectors 3 sections	219	248	3 connectors 3 sections 1 platform	263	285	7	s2, s3, s4, s11, s12, s14
21	5 connectors 5 sections	365	372	5 connectors 5 sections 1 platform	409	409	32	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14, s15, s16, s17
22	5 connectors 5 sections	365	372	5 connectors 5 sections 1 platform	409	409	22	s1, s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14, s15, s16, s17
23	4 connectors 4 sections	292	310	4 connectors 4 sections 1 platform	336	347	9	s1, s2, s3, s4, s15, s16, s17
24	5 connectors 5 sections	365	372	5 connectors 5 sections 1 platform	409	409	16	s1, s2, s3, s4, s15, s16, s17
25	5 connectors 5 sections	365	434	5 connectors 5 sections 1 platform	409	471	11	s1, s2, s3, s15, s16, s17
26	5 connectors 5 sections	365	434	5 connectors 5 sections 1 platform	409	471	11	s1, s2, s3, s15, s16, s17

TABLE 6.9

COMPUTATION TIMES OF *model 1* MINIMIZING THE TOTALS OF DEVIATION TIME OF TECHNICAL CIRCULATIONS (OBJ1) WITH  $E^l$  AND  $D^l$  CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF 95% OF THE LENGTH OF LONGEST ROUTE.

Train group	Fixed subgroup	Buffer subgroup	New subgroup	Variable before presolve	Constraint before presolve	Variable after presolve	Constraint after presolve	Solve time	obj1	Number of deviations of technical circulations
1-30 [0h-7h]	-	1-15	16-30	430920	1426997	36094	74767	191.16s (3.19 mins)	0	0
16-45 [4h30-7h30]	1-15	16-30	31-45	835380	2697137	30732	63749	89.55s (1.49 mins)	0	0
31-60 [6h-8h]	16-30	31-45	46-60	851032	2745834	26208	54704	174.00s (2.90 mins)	0	0
46-75 [7h30-9h]	31-45	46-60	61-75	851032	2745781	32235	66501	234.95s (3.91 mins)	0	0
61-90 [8h-10h]	46-60	61-75	76-90	851032	2745839	28763	59136	238.22s (3.97 mins)	0	0
76-105 [8h30-10h30]	61-75	76-90	91-105	851032	2745814	42892	87629	30 mins	123s	2 (t94(21s), t98(102s))
91-120 [10h-12h30]	76-90	91-105	106-120	851032	2745835	33036	68015	1083.92s (18.07 mins)	29s	1 ( <b>t98</b> )(29s)
106-135 [10h30-14h]	91-105	106-120	121-135	851032	2745821	31548	65429	153.92s (2.57 mins)	29s	1 ( <b>t98</b> (29s))
121-150 [12h30-15h]	106-120	121-135	136-150	835380	2697065	30662	63655	143.44s (2.39 mins)	0	0
136-165 [14h-16h30]	121-135	136-150	151-165	866824	2794894	29133	60071	533.17s (8.89 mins)	0	0
151-180 [15h-17h30]	136-150	151-165	166-180	866824	2794860	33505	68840	30 mins	5s	1 ( <b>t162</b> (5s))
166-195 [16h30-18h30]	151-165	166-180	181-195	866824	2794979	31755	65414	185.64s (3.09 mins)	5s	1 ( <b>t162</b> (5s))
181-210 [17h30-19h]	166-180	181-195	196-210	835380	2697064	24893	51533	107.75s (1.80 mins)	0	0
196-225 [18h30-20h]	181-195	196-210	211-225	835380	2697064	25944	53783	166.50s (2.78 mins)	0	0
211-247 [19h-24h]	196-210	211-225	226-247	1067248	3416459	42237	86800	86.28s (1.44 mins)	0	0

TABLE 6.10  
COMPUTATION TIMES OF *model 1* MINIMIZING THE TOTALS OF DEVIATION TIME OF TECHNICAL CIRCULATIONS (OBJ1) WITH  $E^l$  AND  $D^l$   
CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF 90% OF THE LENGTH OF LONGEST ROUTE.

Train group	Fixed subgroup	Buffer subgroup	New subgroup	Variable before presolve	Constraint before presolve	Variable after presolve	Constraint after presolve	Solve time	obj1	Number of deviations of technical circulations
1-30 [0h-7h]	-	1-15	16-30	430920	1426986	38182	78920	428.42s (7.14 mins)	0	0
16-45 [4h30-7h30]	1-15	16-30	31-45	835380	2697149	31755	65744	109.70s (1.83 mins)	0	0
31-60 [6h-8h]	16-30	31-45	46-60	851032	2745801	28760	59379	240.94s (4.02 mins)	0	0
46-75 [7h30-9h]	31-45	46-60	61-75	851032	2745773	30523	63051	30 mins	76s	3 (t57(34s), t58(21s), t64(21s))
61-90 [8h-10h]	46-60	61-75	76-90	851032	2745847	27445	56435	120.36s (2.01 mins)	76s	3 (t57(34s), t58(21s), t64(21s))
76-105 [8h30-10h30]	61-75	76-90	91-105	851032	2745846	43300	88406	424.61s (7.08 mins)	21s	1 (t64(21s))
91-120 [10h-12h30]	76-90	91-105	106-120	851032	2745855	34929	71666	147s (2.45 mins)	0	0
106-135 [10h30-14h]	91-105	106-120	121-135	851032	2745853	30653	63639	105.01s (1.75 mins)	0	0
121-150 [12h30-15h]	106-120	121-135	136-150	835380	2697101	30707	63758	153.97s (2.56 mins)	0	0
136-165 [14h-16h30]	121-135	136-150	151-165	866824	2794886	29194	60175	30 mins	34s	2 (t158(29s), t162(5s))
151-180 [15h-17h30]	136-150	151-165	166-180	866824	2794848	33911	69679	30 mins	26s	2 (t162(5s), t178(21s))
166-195 [16h30-18h30]	151-165	166-180	181-195	866824	2794987	29854	61557	71.25s (1.19 mins)	5s	1 (t162(5s))
181-210 [17h30-19h]	166-180	181-195	196-210	835380	2697060	26368	54188	131.64s (2.19 mins)	0	0
196-225 [18h30-20h]	181-195	196-210	211-225	835380	2697100	26937	55760	500.58s (8.34 mins)	0	0
211-247 [19h-24h]	196-210	211-225	226-247	1067248	3416471	41350	84639	156.67s (2.61 mins)	0	0

TABLE 6.11  
 COMPUTATION TIMES OF *model 1* MINIMIZING THE TOTALS OF DEVIATION TIME OF TECHNICAL CIRCULATIONS (OBJ1) WITH  $E^l$  AND  $D^l$   
 CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF 85% OF THE LENGTH OF LONGEST ROUTE.

Train group	Fixed subgroup	Buffer subgroup	New subgroup	Variable before presolve	Constraint before presolve	Variable after presolve	Constraint after presolve	Solve time	obj1	Number of deviations of technical circulations
1-30 [0h-7h]	-	1-15	16-30	430920	1426986	38181	78919	197.64s (3.29 mins)	0	0
16-45 [4h30-7h30]	1-15	16-30	31-45	835380	2697141	32073	66395	93.27s (1.55 mins)	0	0
31-60 [6h-8h]	16-30	31-45	46-60	851032	2745785	29144	60789	175.06s (2.92 mins)	0	0
46-75 [7h30-9h]	31-45	46-60	61-75	851032	2745785	32057	66135	30 mins	-	-

### 6.6.2 Results of model with relaxation of commercial trains (*model 2*)

The results of *model 2, obj1* with  $E^l$  and  $D^l$  corresponding to the permissible traversing time of 85% of the length of longest route are presented in Table 6.12. The best solution that can be found in group 46-75 has 1 deviation of technical circulation and 3 deviations of commercial circulations. The final solution obtained by this model has 21 seconds deviation of technical circulations which involves 2 trains (t58 and t72) and 111 seconds deviation of commercial circulations which involves 4 trains (t54, t60, t94 and t128). The total of computation time is 169.87 minutes (2 hours 50 minutes).

We continue our experimentations for *model 2* by decreasing the permissible traversing time of route for each circulation. The possible routes remained for each external section corresponding to the permissible traversing time of 80% of the length of longest route are presented in Table 6.13. In this case, the number of possible routes are remarkably decreased for circulations corresponding to external sections 18, 21, 22, 23 and 24.

The results corresponding to 80% of the length of longest route are presented in Table 6.14. The results show that there is no deviation in the first 2 groups of trains. In the next group 31-60t, no feasible solution can be found for 30 minutes of running time limit.

Therefore, the problem cannot be solved by *model 2* in case corresponding to the permissible traversing time of 80% of the length of longest route due to the limit of possible routes for circulations. The *model 3* with the possibility of cancellation of trains will be applied in the next section in order to solve the problem.

TABLE 6.12

COMPUTATION TIMES OF *model 2* MINIMIZING THE TOTALS OF DEVIATION TIME OF TECHNICAL CIRCULATIONS AND COMMERCIAL CIRCULATIONS (OBJ1) WITH  $E^l$  AND  $D^l$  CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF 85% OF THE LENGTH OF LONGEST ROUTE.

Train group	Fixed subgroup	Buffer subgroup	New subgroup	Variable before presolve	Constraint before presolve	Variable after presolve	Constraint after presolve	Solve time	obj1	Deviations of technical circulations	Deviations of commercial circulations
1-30 [0h-7h]	-	1-15	16-30	430920	1426986	38181	78919	197.64s (3.29 mins)	0	0	0
16-45 [4h30-7h30]	1-15	16-30	31-45	835380	2697141	32073	66395	93.27s (1.55 mins)	0	0	0
31-60 [6h-8h]	16-30	31-45	46-60	851032	2745785	29144	60789	175.06s (2.92 mins)	0	0	0
46-75 [7h30-9h]	31-45	46-60	61-75	851032	2745785	32105	66773	30 mins	56s	1 ( <b>t58</b> (3s))	3 ( <b>t54</b> (1s), <b>t60</b> (34s), t71(18s))
61-90 [8h-10h]	46-60	61-75	76-90	851032	2745819	27365	56703	150.05s (2.50 mins)	56s	2 ( <b>t58</b> (3s), <b>t72</b> (18s))	2 ( <b>t54</b> (1s), <b>t60</b> (34s))
76-105 [8h30-10h30]	61-75	76-90	91-105	851032	2745818	43022	88409	30 mins	99s	1 ( <b>t72</b> (18s))	1 (t91(81s))
91-120 [10h-12h30]	76-90	91-105	106-120	851032	2745835	37511	77221	30 mins	55s	0	1 (t94(55s))
106-135 [10h30-14h]	91-105	106-120	121-135	851032	2745837	30570	64109	367.24s (6.12 mins)	55s	0	1 ( <b>t94</b> (55s))
121-150 [12h30-15h]	106-120	121-135	136-150	835380	2697101	30496	63870	30 mins	97s	2 (t137(11s), t143(65s))	1 ( <b>t128</b> (21s))
136-165 [14h-16h30]	121-135	136-150	151-165	866824	2794918	29404	61104	363.25s (6.05 mins)	21s	0	1 ( <b>t128</b> (21s))
151-180 [15h-17h30]	136-150	151-165	166-180	866824	2794860	32692	67816	1189.89s (19.83 mins)	0	0	0
166-195 [16h30-18h30]	151-165	166-180	181-195	866824	2794995	32094	66668	225.53s (3.76 mins)	0	0	0
181-210 [17h30-19h]	166-180	181-195	196-210	835380	2697072	24795	51916	86.19s (1.44 mins)	0	0	0
196-225 [18h30-20h]	181-195	196-210	211-225	835380	2697080	26527	55959	61.17s (1.02 mins)	0	0	0
211-247 [19h-24h]	196-210	211-225	226-247	1067248	3416443	43183	89433	82.97s (1.38 mins)	0	0	0

TABLE 6.13  
THE POSSIBLE ROUTES CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF 80% OF THE LENGTH OF LONGEST ROUTE

External section	Entering circulation			Leaving circulation			Number of possible routes	Corresponding platform
	Longest route		Permissible traversing time(s)	Longest route		Permissible traversing time(s)		
	Connectors and sections	Travel time(s)		Connectors and sections	Travel time(s)			
18	4 connectors 4 sections	292	292	3 connectors 3 sections 1 platform	263	327	16(ent)/10(leav)	s2(ent), s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
19	3 connectors 3 sections	219	233	3 connectors 3 sections 1 platform	263	268	16	s2, s3, s4, s5, s6, s7, s8, s9, s11, s12, s14
20	3 connectors 3 sections	219	233	3 connectors 3 sections 1 platform	263	268	7	s2, s3, s4, s11, s12, s14
21	4 connectors 4 sections	292	350	4 connectors 4 sections 1 platform	336	385	12	s2, s3, s4, s11, s12, s14, s15, s16, s17
22	4 connectors 4 sections	292	350	4 connectors 4 sections 1 platform	336	385	10	s1, s2, s3, s4, s15, s16, s17
23	4 connectors 4 sections	292	292	3 connectors 3 sections 1 platform	263	327	9(ent)/5(leav)	s1(ent), s2, s3, s4(ent), s15, s16, s17
24	4 connectors 4 sections	292	350	4 connectors 4 sections 1 platform	336	385	11	s1, s2, s3, s15, s16, s17
25	5 connectors 5 sections	365	408	5 connectors 5 sections 1 platform	409	444	11	s1, s2, s3, s15, s16, s17
26	5 connectors 5 sections	365	408	5 connectors 5 sections 1 platform	409	444	11	s1, s2, s3, s15, s16, s17



TABLE 6.14

COMPUTATION TIMES OF *model 2* MINIMIZING THE TOTALS OF DEVIATION TIME OF TECHNICAL CIRCULATIONS AND COMMERCIAL CIRCULATIONS (OBJ1) WITH  $E^l$  AND  $D^l$  CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF 80% OF THE LENGTH OF LONGEST ROUTE.

Train group	Fixed subgroup	Buffer subgroup	New subgroup	Variable before presolve	Constraint before presolve	Variable after presolve	Constraint after presolve	Solve time	obj1	Deviations of technical circulations	Deviations of commercial circulations
1-30 [0h-7h]	-	1-15	16-30	430920	1426986	33103	68243	310.14s (5.17 mins)	0	0	0
16-45 [4h30-7h30]	1-15	16-30	31-45	835380	2697145	26977	55948	126.33s (2.11 mins)	0	0	0
31-60 [6h-8h]	16-30	31-45	46-60	851032	2745785	29144	60789	30 mins	-	-	-

### 6.6.3 Results of model with cancellation of trains (*model 3*)

The results of *model 3, obj4* with  $E^l$  and  $D^l$  corresponding to the permissible traversing time of 80% of the length of longest route are presented in Table 6.16.

The results show that there are 5 trains cancelled in the full day problem. The trains cancelled are t60, t71, t81, t86 and t156 which mostly operate in the peak time. The total of computation time is 182.15 minutes (3 hours 3 minutes).

The external sections and the stopping time at platform corresponding to these trains cancelled are resumed at Table 6.15. The external sections, which are noted in blue, are the ones having less possible routes for circulations due to the decrease of the permissible traversing time from 85% to 80%. The table shows that all these trains have at least one circulation which has less possible routes to be assigned except train t86 which has a high stopping time at platform. Therefore, these trains are cancelled due to the lack of resources (connectors and sections including platforms).

TABLE 6.15  
THE DATA OF TRAINS CANCELLED

Train	External section of entering circulation	External section of leaving circulation	Stopping time at platform (s)
t60	24	18, 23	600
t71	24	18	600
t81	20	22	600
t86	13	13	4200
t156	24	19	1140

TABLE 6.16  
COMPUTATION TIMES OF *model 3* MINIMIZING THE NUMBER OF TRAIN CANCELLED (*obj4*) WITH  $E^l$  AND  $D^l$  CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF 80% OF THE LENGTH OF LONGEST ROUTE.

Train group	Fixed subgroup	Buffer subgroup	New subgroup	Variable before presolve	Constraint before presolve	Variable after presolve	Constraint after presolve	Solve time	obj4	Train cancelled
1-30 [0h-7h]	-	1-15	16-30	430920	1426986	33103	68243	310.14s (5.17 mins)	0	0
16-45 [4h30-7h30]	1-15	16-30	31-45	835380	2697145	26977	55948	126.33s (2.11 mins)	0	0
31-60 [6h-8h]	16-30	31-45	46-60	851077	2745785	25454	53133	30 mins	1	t46
46-75 [7h30-9h]	31-45	46-60	61-75	851077	2745781	24138	50368	30 mins	2	<b>t60</b> , t71
61-90 [8h-10h]	46-60	61-75	76-90	851077	2745831	29174	60281	30 mins	3	<b>t60</b> , <b>t71</b> , t79
76-105 [8h30-10h30]	61-75	76-90	91-105	851077	2745798	38917	79673	30 mins	3	<b>t71</b> , <b>t81</b> , <b>t86</b>
91-120 [10h-12h30]	76-90	91-105	106-120	851077	2745805	33792	69606	161.92s (2.70 mins)	2	<b>t81</b> , <b>t86</b>
106-135 [10h30-14h]	91-105	106-120	121-135	851077	2745837	27106	56615	288.80s (4.81 mins)	0	0
121-150 [12h30-15h]	106-120	121-135	136-150	835425	2697101	27561	57596	150.61s (2.51 mins)	0	0
136-165 [14h-16h30]	121-135	136-150	151-165	866869	2794878	25848	53617	86.05s (1.43 mins)	0	0
151-180 [15h-17h30]	136-150	151-165	166-180	866869	2794860	28723	59381	30 mins	1	<b>t156</b>
166-195 [16h30-18h30]	151-165	166-180	181-195	866869	2794956	30251	62742	151.32s (2.52 mins)	1	<b>t156</b>
181-210 [17h30-19h]	166-180	181-195	196-210	835425	2697072	25623	53497	244.95s (4.08 mins)	0	0
196-225 [18h30-20h]	181-195	196-210	211-225	835425	2697060	23784	50233	249.39s (4.16 mins)	0	0
211-247 [19h-24h]	196-210	211-225	226-247	1067300	3416427	37208	77172	159.36s (2.66 mins)	0	0

TABLE 6.17  
COMPUTATION TIMES OF *model 1* MINIMIZING THE MAXIMUM NUMBER OF OCCUPATION OF SECTIONS (OBJ3) WITH  $E^l$  AND  $D^l$   
CORRESPONDING TO THE PERMISSIBLE TRAVERSING TIME OF THE LENGTH OF LONGEST ROUTE.

Train group	Fixed subgroup	Buffer subgroup	New subgroup	Variable before presolve	Constraint before presolve	Variable after presolve	Constraint after presolve	Solve time	obj3	Most used sections
1-30 [0h-7h]	-	1-15	16-30	430921	1427028	47047	96938	154.16s (2.57 mins)	8	s1, s2, s4, s29, s39, s41, s52
16-45 [4h30-7h30]	1-15	16-30	31-45	835381	2697215	44175	91065	30 mins	10	s2, s3, s4, s32, s39, s45, s49, s51, s52
31-60 [6h-8h]	16-30	31-45	46-60	851033	2745867	40132	82540	30 mins	11	s45, s48, s51
46-75 [7h30-9h]	31-45	46-60	61-75	851033	2745819	47168	97042	30 mins	13	s45
61-90 [8h-10h]	46-60	61-75	76-90	851033	2745893	37295	76677	512.16s (8.54 mins)	14	s45
76-105 [8h30-10h30]	61-75	76-90	91-105	851033	2745844	47239	96569	30 mins	12	s2
91-120 [10h-12h30]	76-90	91-105	106-120	851033	2745877	44082	90171	1390.73s (23.18 mins)	11	s2, s3, s33, s39, s40, s48, s51, s52
106-135 [10h30-14h]	91-105	106-120	121-135	851033	2745903	39974	82605	30 mins	10	s1, s2, s4, s6, s29, s35, s48, s51, s52
121-150 [12h30-15h]	106-120	121-135	136-150	835381	2697143	36156	74657	660.58s (11.01 mins)	11	s5, s10, s36, s52
136-165 [14h-16h30]	121-135	136-150	151-165	866825	2794956	39841	81720	78.66s (1.31 mins)	12	s1, s2, s36, s45
151-180 [15h-17h30]	136-150	151-165	166-180	866825	2794922	44933	92026	554.33s (9.23 mins)	12	s10, s35, s45, s52
166-195 [16h30-18h30]	151-165	166-180	181-195	866825	2795049	36942	75984	323.50s (5.39 mins)	16	s45
181-210 [17h30-19h]	166-180	181-195	196-210	835381	2697118	34201	70431	79.11s (1.32 mins)	11	s35, s39, s45
196-225 [18h30-20h]	181-195	196-210	211-225	835381	2697118	35123	72711	169.27s (2.82 mins)	11	s34, s45
211-247 [19h-24h]	196-210	211-225	226-247	1067249	3416493	55128	112968	30 mins	11	s45, s52

### 6.6.4 Results on robust timetables

In order to reinforce the robustness of timetables, we run the numerical experiments on our models. The objective function of these models is to minimise the maximum number of occupation of sections (*obj3*).

**Note:** The occupations of external sections by circulations are pre-defined before the construction of timetables. Therefore, we do not take into account the external sections for our experimentations.

The results of *model 1* corresponding to the permissible traversing time of the length of longest route is presented in Table 6.17.

The results of the robust timetable show that in the most groups, the section 45 is the most used section by circulations of trains in the group. For example, in group 166-195, section s45 is used by 16 circulations. It means that if section 45 is blocked due to an accident or a technical problem, 16 circulations will be delayed (primary delay).

The total of computation time is 245.38 minutes (4 hours 5 minutes) which is almost double the computation time of *obj1*. Therefore, our models need a lot more time to find robust timetables than timetables with the minimising of deviation times .

## 6.7 Conclusion

In this chapter, we apply the three different mathematical models which are presented in the previous chapter to solve a real case study in the railway station of Bordeaux. Due to the high complexity of these models and the big size of the full day problem, the problem cannot be solved by the exact method. We propose a Math-Heuristic approach to be able solve the full day problem. The subgroup partitioning strategy based on rolling horizon approach is described and applied in the full day problem of Bordeaux station. In the first experimentation, the full day can be solved with a small deviation time of technical circulations (31 seconds). When we reduce the permissible traversing time of circulations, the number of possible routes for each circulation is decreased. Thus, the deviation time of technical circulations is increased until no feasible solution can be found without the deviation of commercial circulations. We use the second model (*model 2*) with the possibility of deviation of commercial circulations to continue

our experimentations. The problem which cannot be solved by the first model (*model 1*) can now be solved by model 2 with 21 seconds deviation of technical circulations and 11 seconds deviation of commercial circulations. We continue reducing the permissible traversing time of circulations for the next numerical experiment. The study case cannot be solved by *model 2*, thus the *model 3* with the possibility of cancellations of trains is applied. The results show that 5 trains must be cancelled due to the lack of resources in order to generate a timetable no conflict. At last, we make experimentations on *model 1* with the objective for the construction of robust timetables which have less circulations to reschedule if a disturbance occurs on the railway station.

In the first instance, by applying our first model (*model 1*) we found a solution which respects all railway requirement with a small deviation (31 seconds) of technical circulations. In the solution of the work in Bai et al. (2015), there are 182 minutes of deviation of commercial circulations of 37 trains: 3 trains more than 10 minutes, 8 trains more than 6 minutes and 29 trains less than 5 minutes. Our model with the reservation of each section independently, which is more precise, makes a better use of railway resources to find a solution suitable with railway requirement and it increases the capacity of railway stations.

In the further instances, we increase the potential conflict of resources by limiting the number of possible routes for each circulation. Our second and third models (*model 2 and 3*) are able to solve the problem if the previous model cannot solve it.



# Conclusions and future research

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## 7.1 Conclusions

Railway networks are operated nearly at capacity margins due to the growing demand in railway traffic. Railway station, which is the bottleneck of railway networks, needs a decision support system to exploit its railway capacity. The traffic management in railway station requires a scheduling timetable which fits to the existing railway infrastructure, while avoiding conflicts between large number of trains passing through railway station, and it must also respect the safety and commercial policy and objectives.

Our research combines railway signalling and control systems knowledges and operations research methods in order to study the routing and scheduling problem in railway station and to address the development of an efficient decision support system. The result timetable must ensure no conflicts between trains, while allowing the coupling and decoupling of trains at platform and respecting their reference times of platforms.



Due to the limit of investment in railway infrastructure, the demands for an efficient allocation of resources are highly increased. The state of the art of the problems in railway station can handle only low traffic densities within a reasonable computation time. The train routing and scheduling problem in railway station with high traffic densities is not yet properly solved due to large formulations that require significant times of processing. We develop a decision support model for railway station considering the reservation of sections independently to generate a conflict-free timetable which consists of two types of circulations. The first set corresponds to the commercial circulations given by administrative levels and the second one consists of the technical circulations added by the railway station managers for preparation or reparation operation. A permissible deviation time  $L$  is associated to the technical circulations. But the model allowing deviation time for technical circulations is not the first choice to solve the problem. In the first time, we find a solution for the problem respecting all reference times with no deviation time. In case no solution is found, we propose to study a model allowing deviation time for technical circulations. The model allowing deviation time for commercial circulations can be considered to find a conflict-free timetable. However, this model may cause delay propagation in railway networks. In a complex railway station, it could be difficult to find a feasible solution on the peak time. The model allowing cancellation of trains should be considered to guarantee a conflict-free timetable.

In order to solve a full day timetable in railway station with larger size problem, trains are divided into many subgroups in chronological order. The subgroups partition strategies based on rolling horizon approach solves the large problem step by step within a reasonable computation time. This algorithm is tested on a full day problem in railway station of Bordeaux. The results show the good performance and effectiveness of our model. The resulting timetables are displayed and verified by three Gantt Chart separately corresponding to sections, connectors and trains.

Our proposal to consider the reservation of section independently (interlocking of section) rather than the reservation of complete route (interlocking of route) is promising, and justifies the development of further studies towards systems of security which implement these management mechanisms in railway stations even in situations where track circuits are not implemented to detect the presence of trains in the sections considered.

## 7.2 Future research

In terms of the improvement of this work, we would like to point out the following perspectives:

### 7.2.1 Train routing and scheduling support tool

In our model, the topology of railway station and trains' activities are imported in the form designed. Thus, our model can be easily applied on other railway stations. The efficiency of our model need to be tested on more real cases in different railway stations. The development of an application with an interface is required to transmit the data and results with railway station managers in future works.

### 7.2.2 Types of trains

The case studied in railway station of Bordeaux does not have any crossing trains which pass through the railway station without stopping at platform. We could make further experimentations on this case study with adding some crossing trains into the full day problem. We could consider some depots inside the railway station. Trains can enter the railway station and stop at the depot or leave the railway station from the depot to outside the railway station. In this case, we could consider trains with only one circulation.

### 7.2.3 Switches manipulation

In our models, we do not consider the time delay of manipulation of switches (put the switches in the correct position depending on the route of trains). It can be implemented to make our models even more realistic. In this case, we must consider a variable corresponding to the current position of switches. If a train arrives at a switch which is not in the correct position for the route of train, the time delay must be added to set the switch in the right direction.

### 7.2.4 Modalities for cutting of subgroups partitioning strategy

The subgroup partitioning strategy is applied in our models with the number of trains of group  $N=30$  and the number of trains of buffer sub-group  $F=N/2$  for all groups (periods). The number of trains of group  $N$  can be less than 30 such as 20, 15, etc. However, we must guarantee that there is no time intersection between  $N/2$  trains of fixed sub-group and  $N/2$  trains of new

sub-group in each period. This gives insight about impact of size group on the quality of solution of approach. Another possibility for applying this approach is to make the experimentations with dynamic group and sub-group sizes where  $N$  and  $F$  can vary for each period. For example, we could regroup trains passing through the railway station in the peak times (which are more difficult to allocate resources) into small periods and trains in the off-peak times (which are easier to allocate resources) should be regrouped into bigger periods.

### 7.2.5 Resolution order of subgroups partitioning strategy

The subgroups partitioning strategy divides the full day problem into many groups of trains in chronological order. We made the experimentations with 30 trains for each group and the full day problem is solved from left group to right group. It means that the first group solving is the first group of trains arriving at the railway station and the last group solving is the last group of train arriving at the railway station. Another possibility is to solve the full day problem from right group to left group. It means that the first group solving is the last group of trains arriving at the railway station and the last group solving is the first group of trains arriving at the railway station. A third option is to solve the full day problem from left and right group to middle group. It means that the first two groups solving are the first group and the last group arriving at the railway station, and the last group solving is the middle group of the list. In this case, the last group solving must have two buffer sub-groups in left and right. The new sub-group is in the middle of these two buffer sub-groups.

### 7.2.6 Improving robustness

The robustness of our models can be improved by constructing the timetables with the capability of absorbing small disturbances or reducing the negative impact of disturbances. A way to do that is to give time margins during the planning process of timetables, see [Kroon et al. \(2008\)](#) and [Meng et al. \(2019\)](#). Due to the capacity constraints of the railway station, a limited amount of time margins can be added. The time margins can be the running time supplements of trains (additional time of  $\Delta$ ,  $\theta$ ) or the additional time of the safety delay time between of two circulations using the same section or connector (additional time of  $\sigma$ ).

### 7.2.7 Rescheduling in real time

In our research, we solved the off-line routing and scheduling problem. However, our models can be pursued to solve the on-line routing and scheduling problem. We can study our models with the implementations of some delays of trains and/or the temporary block passage of certain sections.

The new free-conflict timetables must be reconstructed by adjusting the existing timetables from the previous results without perturbations. If it is difficult to reconstruct the new free-conflict timetables, the model with the possibility of cancellation of trains (*model 3*) should be applied to cancel the trains with serious perturbations which cause many secondary delays of other trains.

A disturbance corresponds to the fact that certain trains last longer than expected in some sections in the timetable. As a consequence, trains may depart from and/or arrive at a section later than planned. The delay of these trains is called a *primary* delay. This delay may easily have negative impact on other trains. In that case, the delay of second train is called a *secondary* delay. When a disturbance has occurred, the initial section allocation may not be possible to guarantee the conflict-free for all access requests. New start and end times of occupation of section for each affected trains may need to be computed. A modification can be to assign a train to another section than initially planned for, or to give a train priority to a section over another train that initially was planned to traverse it first, see [Törnquist \(2007\)](#).

Railway stations frequently face unforeseen events like accidents, malfunctioning infrastructure, etc. As a consequence, part of the railway infrastructure (sections in our model) is temporarily unavailable. Several strategies may be applied to reschedule the trains and resolve the conflicts such as cancelling, delaying or rerouting a number of trains, see [Cacchiani et al. \(2014\)](#).

As the perturbations happen stochastically, there is a limited time for decision making. Rescheduling approaches must be very quick. Otherwise, it may cause further delays in railway stations. Our models have high complexity due to the consideration of progressive reservation of resources. In order to prevent a large increase in computation time, we take into account only trains in the period of disturbance for our models. If no solution can be found in this period, we will extend the period until a solution is found. In case that there are many disturbances in different periods, we should locally

solve the problem for each period.

# Appendix



# Simulation methods

The simulation methods present the operation of a railway system from state to state. Simulation models provide a model which is close to reality. It can validate a given timetable and evaluate the effect of many factors in railway capacity. Some simulation tools have been developed for use in railway companies, consultancies and universities: OpenTrack (OpenTrack Railway Technology), MultiRail (Multimodal Applied Systems), RailSys, Rail Traffic Controller.

OpenTrack (<http://opentrack.ch>) began in 1990 as a research project at the Swiss Federal Institute of Technology and is now developed by OpenTrack Railway Technology. It is used by railways, the railway supply industry, consultancies and universities in different countries. The advantage of OpenTrack compared to other simulation tools is the flexibility in describing signalling systems and the network's topology in special graphs called double vertex graphs. A user can create and manage objects for edges and vertices, and also signals, switches, stations and routes, see Figure 7.1. OpenTrack provides a few standard signals whose characteristics can be customized by the user. Therefore, many signalling systems in the world can be represented.

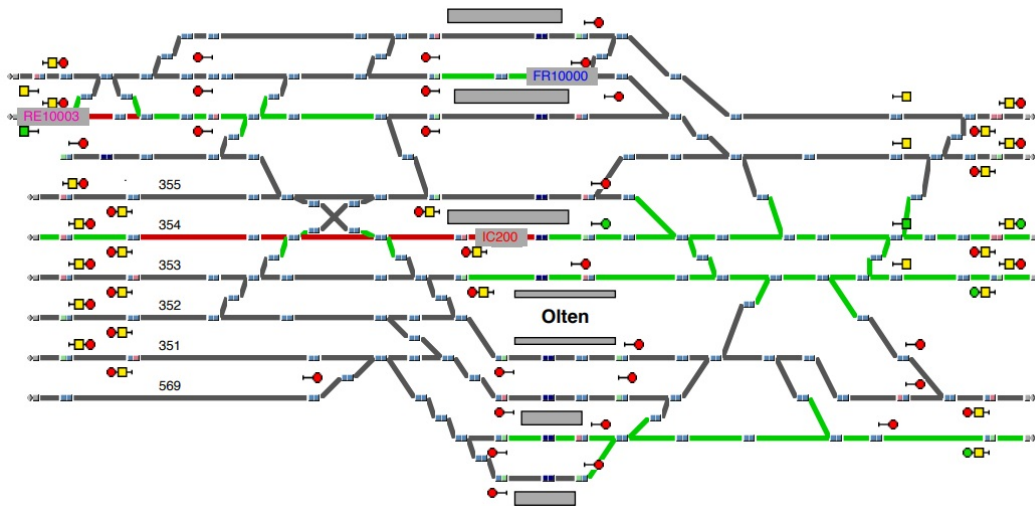


Fig. 7.1 Example of a station (Olten, Switzerland). Source: opentrack.ch

RailSys ([rmcon-int.de](http://rmcon-int.de)) is a simulation tool which manage railway infrastructure, construction and verification of timetables. RailSys allows a



very efficient timetable construction and offers a lot of possibilities for future improvement in timetable construction and operational simulation [Demitz et al. \(2010\)](#).

Rail Traffic Controller ([berkeleysimulation.com](http://berkeleysimulation.com)) is developed by Berkeley Simulation Software. The difference between Rail Traffic Controller and other tools is that it dispatches trains at the much larger network level: greater distances and longer periods of times. Therefore, it is used by most railway companies in the US for estimating the capacity of networks.

# Available tools for railway management

The problems of railway management have been addressed by various tools which are based on:

**Traffic patterns** assess the capacity of a specific traffic pattern (a repeated path that is prescribed for the operations of some trains), see [Flier et al. \(2009\)](#).

**Single-track analytical models** is an optimisation model requiring analytic expressions for the travelling time (including delay times) of trains on a single-track railway with traffic in both directions, see [Petersen \(1974\)](#),

**Algebraic approaches:** the problem of railway management is modelled into an algebraic problem. According to this model, the problem can be solved by calculating in a polynomial time. This approach is not suitable for large railway network because these calculations usually take a long time, see [Egmond R. van \(1999\)](#).

Many railway companies are also working on similar tools:

- DEMIURGE, see [LEPATRE et al. \(2011\)](#), is a tool to assess the capacity of railway network. It is used by SNCF to:

- evaluate the network capacity to absorb additional traffic
- locate the bottlenecks
- assist in decision support for infrastructure investments
- optimise current and future timetables
- calculate the residual capacity of timetables

The optimisation problem is solved by mixed integer linear program. The solution can be exact or approached by dividing the problem into many interdependent sub-problems.

- SISYFE which is developed by SNCF see [Fontaine and Gauyacq \(2001\)](#), is used for the simulation of the evolution of a railway network in normal or disturbances situations. It offers to:

- evaluation of the robustness of railway traffic
- model of the railway network with different: signalling systems, types of train, driver behaviour, etc
- evaluation of investments needed to upgrade line capacity
- valid the railway management system

SISYFE runs the simulation by discrete events which represent the functionalities of railway signals and communications.

- VIRIATO (see <https://www.sma-partner.com/fr/software/viriato>) is mainly used for long-term planning. It can adapt infrastructure to future service concepts and coordinating several operators or products that share the same infrastructure. It can:

- give a graphic timetable to assess the feasibility of a plan, such as insufficient headways between services or conflicting movements over a line simply by looking at graphic timetable.

- provide flexible filter possibilities and layout settings that permit users to make a customer timetable and easily display the desired information.

This support tool leads to various results for the same railway line, depending on the length of the section considered.

- CAPRES is a module of [Lucchini et al. \(2001\)](#) for the elaboration and saturation of timetable variants. By iteration, this tool determines all available extra train itineraries, given all the constraints and interconnections between railway lines. It offers many actions:

- evaluate the unused capacity for railway network.
- detect the bottlenecks in capacity.

- compare the capacity of different structure of timetables.
- assess the impact of adding new lines in network.
- assess the feasibility of a timetable for railway network.

The disadvantage of this method is that the railway network and operational data have to be completed with the information about where, when and how the railway capacity must be used.

• MOM is designed by Barber et al. (2006) to provide efficient and reactive management of railway infrastructures. MOM offers:

- generation of optimised timetables.
- informations on railway capacity and on timetable robustness to help managers make better decisions.
- provide several analytical and empirical methods to obtain conclusions about railway capacity.

This support tool provides several analytical methods such as solving optimised train schedules (Barber et al. (2004)), calculating the railway capacity (Abril et al. (2005)) and dealing with robustness problem in timetabling (Salido et al. (2008)) to obtain conclusions about the railway capacity and to support the process of adapting the railway infrastructure to traffic needs.

The comparison of these tools is presented in Table 7.1:

TABLE 7.1  
COMPARISON OF RAILWAY TRAFFIC MANAGEMENT TOOLS.

Tool	Simulation	Timetable Optimisation	Timetable Management	Capacity analysis	Infrastructure Management	Robustness Analysis
DEMIURGE	No	Yes	No	Yes	Yes	No
SISYFE	Yes	No	Yes	No	Yes	Yes
VIRIATO	No	No	Yes	Limited	Yes	No
CAPRES	No	Yes	No	Yes	Yes	No
MOM	No	Yes	Yes	Yes	Yes	Yes



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**Titre:** Modélisation en bas niveau des problèmes de routage et d'ordonnement des trains dans les gares ferroviaires

**Résumé:** Cette thèse porte sur les problèmes de routage et d'ordonnement des trains dans les gares ferroviaires. L'objectif est de développer un modèle de prise de décision pour aider les managers de la gare à générer un tableau des horaires sans conflits. Le tableau des horaires devrait s'adapter aux infrastructures existantes en respectant la sécurité et les horaires commerciaux.

Contrairement aux méthodes traditionnelles qui allouent toutes les ressources pour réserver une route pour un train jusqu'à ce que cette route soit libérée, notre recherche se concentre sur la réservation progressive des ressources utilisées par les trains le long de leur route dans la gare ferroviaire. Cette technique permet à un plus grand nombre de trains de circuler simultanément dans un nœud ferroviaire et d'augmenter ainsi la capacité ferroviaire.

Nous nous sommes focalisés dans ces travaux de recherche sur la modélisation mathématique utilisant la programmation linéaire mixte en nombre entiers. Cette modélisation mathématique consiste à transformer des systèmes ferroviaires en modèles de programmation linéaire. Nous avons souhaité construire le modèle le plus général possible afin de pouvoir étudier les problématiques ferroviaires selon plusieurs points de vue. Ainsi, la modélisation construit un modèle abstrait en se basant sur des éléments du système ferroviaire: *section* et *connecteur*. Ce modèle abstrait est capable de s'adapter à tous les réseaux ferroviaires.

Une stratégie de partitionnement en sous-groupes basée sur une méthodologie d'horizon glissant est proposée. Cette approche de type maths-heuristique divise le problème en plusieurs sous-problèmes afin de réduire l'explosion combinatoire. Les modèles mathématiques ont été implémentés à l'aide de l'outil CPLEX. Nous avons réalisé des expériences numériques sur les données de la gare de Bordeaux pour les circulations de toute une journée. Les résultats montrent la bonne performance et l'efficacité de nos modèles. Les tableaux horaires sont visualisés par trois diagrammes de Gantt distincts correspondant aux sections, aux connecteurs et aux trains.

**Mots clés:** Programmation mixte en nombres entiers, gestion opérationnelle du trafic ferroviaire, routage des trains, ordonnancement des trains, couplage/découplage des trains.

**Title:** Low-Level Modelling for Trains Routing and Scheduling in Railway Stations

**Abstract:** This thesis focuses on the trains routing and scheduling problem faced by railway station managers. The objective is to develop a decision model for railway station managers to generate a conflict-free timetable. The timetable has to fit to the existing infrastructure while respecting the safety and the commercial schedules.

Unlike traditional methods that allocate all resources to setup a route for a train until the route is released, our work focuses on the progressive reservation of resources used by trains along their route on the railway station. This technique allows a larger number of trains to be routed simultaneously in a railway node and thus increases railway capacity.

In this research work, we focus on mathematical model using mixed-integer linear programming. This mathematical model transforms railway systems into linear programming models. We wanted to build the most general model possible in order to be able to study the railway traffic management problem from several points of view. Thus, the model builds an abstract model based on elements of the railway system: *section* and *connector*. This abstract model is able to adapt to all railway networks.

A subgroup partitioning strategy based on rolling horizon methodology is proposed. This Math-Heuristic approach divides the problem into many sub-problems in order to reduce the combinatorial explosion. The mathematical models were implemented using the CPLEX tool. We run numerical experiments on the data of the railway station of Bordeaux for full day timetable. The results show the good performance and effectiveness of our models. The resulting timetables are displayed and verified by three Gantt Chart separately corresponding to sections, connectors and trains.

**Keywords:** Mixed-integer linear programming, offline railway station management, train routing, train scheduling, train coupling/decoupling.