

# Optimal sensor placement and density estimation in large-scale traffic networks

Thesis defense

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2021/03/29



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3. Dynamic density estimation
4. Average density estimation
5. GTL-Ville: Experimental platform

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# 1. Introduction

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3. Dynamic density estimation

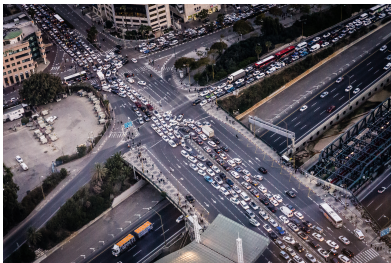
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# Introduction

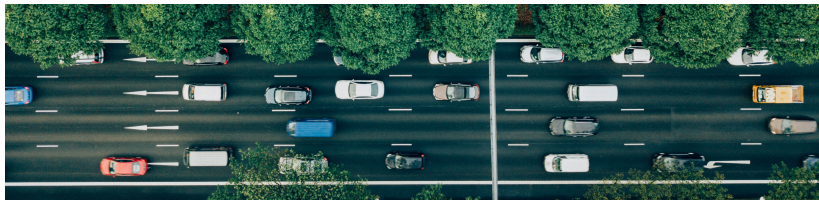


Modern Intelligent transportation systems deal with the development of control strategies to reduce congestion.

Traffic monitoring is one major component, and deals with the use of limited data to estimate the state of a large traffic system.



# Traffic variables



- ▶ Density  $\rho$  : vehicles per unit length (veh/km)
- ▶ Flow  $\varphi$  : vehicles traversing a location per unit time (veh/h)
- ▶ Road speed  $v$  : average of instantaneous vehicle speeds (km/h)

# Problem definition

We deal with the problem of **Traffic State Estimation**.



## Subproblems:

- ▶ **Network sensor location problem:** identify the minimum number of sensors and optimal locations.
- ▶ **Estimation:** use of heterogeneous data sources to calculate flow and density in roads.



# Contributions

- ▶ Sensor location problem for two technologies: flow and turning ratio sensors. Application in steady-state and dynamical networks.
- ▶ Flow and density estimation approaches using data from flow sensors, turning ratio sensors and Floating Car Data. Use of data-based models for the case of dynamical networks.
- ▶ Estimation of the average density of a region of a traffic network using a low-dimensional observer using boundary flow data.
- ▶ Construction of experimental platform GTL-Ville to validate the results using real data.



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# Previous works

Steady-state networks have been widely used to develop flow sensor location methods [Gentili and Mirchandani, 2012].

## ▶ No turning ratio information

- ▶ (Ng, 2012) used node models to calculate the number of sensors to estimate flow.
- ▶ (He, 2013) proposed a graph-based method to find sensor locations.
- ▶ (Rinaldi and Viti, 2017) used O/D route information to reduce number of sensors.

## ▶ Turning ratio information everywhere

- ▶ (Shao et al., 2016) used flow constraints at nodes to find sensor locations.
- ▶ (Lovisari et al., 2016) used virtual variances to find sensor locations in order to reduce estimation noise.

No work considers the location of **flow** and **turning ratio** sensors.



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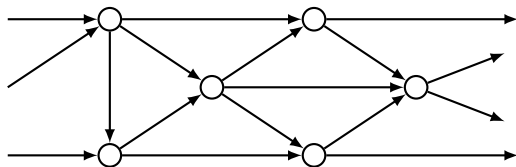
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# Traffic networks

A traffic network is a collection of roads connected via intersections. They are modeled as a directed graph  $\mathcal{G} = \{\mathcal{N}, \mathcal{E}\}$ :

- ▶  $\mathcal{N}$  are the nodes which represent intersections.
- ▶  $\mathcal{E}$  are the edges which represent roads.

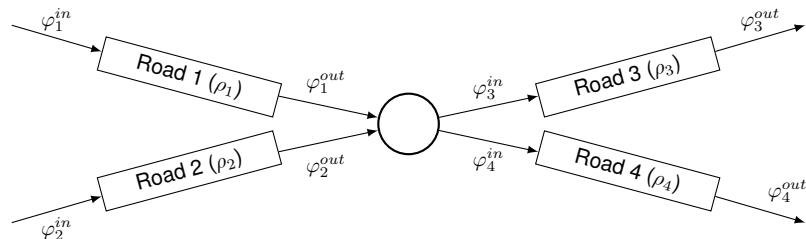


## Data sources:

- ▶ Flow sensors are located in  $\mathcal{S} \subset \mathcal{E}$ .
- ▶ Turning ratio sensors are located in  $\mathcal{R} \subset \mathcal{N}$ .
- ▶ Road speeds are available for all roads  $\mathcal{E}$ , using FCD.

# Flow conservation

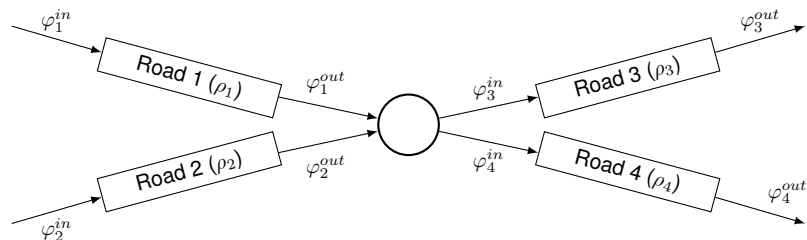
Each road has a density  $\rho_i$ , incoming flow  $\varphi^{in}$  and outgoing flow  $\varphi^{out}$ .



The mass conservation at each intersections requires

$$\varphi_1^{out} + \varphi_2^{out} = \varphi_3^{in} + \varphi_4^{in} \quad (1)$$

# Flow conservation - Turning ratios



**Turning ratios** (TR) model the average vehicle route choices.

$r_{i,j}$  : proportion of vehicles in road  $i$  turning to  $j$ .

$$\varphi_3^{in} = r_{1,3}\varphi_1^{out} + r_{2,3}\varphi_2^{out} \quad (2)$$

$$\varphi_4^{in} = r_{1,4}\varphi_1^{out} + r_{2,4}\varphi_2^{out} \quad (3)$$

# Flow conservation constraints

As we consider steady-state networks, for each road  $\varphi^{in} = \varphi^{out} = \varphi$ . Flow conservation implies the following linear constraints

- ▶ If turning ratios are measured ( $\mathcal{R}$ ):

$$\varphi_i - \sum_j r_{i,j} \varphi_j = 0 \quad \implies \quad A(\mathcal{R})\varphi = \mathbf{0}$$

- ▶ If turning ratios are not measured ( $\mathcal{U} = \mathcal{N} \setminus \mathcal{R}$ ):

$$\sum_{i \in \text{In}} \varphi_i - \sum_{j \in \text{Out}} \varphi_j = 0 \quad \implies \quad B(\mathcal{U})\varphi = \mathbf{0}$$

- ▶ Flow sensor data

$$C(\mathcal{S})\varphi = \varphi^{\text{mea}}$$

Overall

$$\begin{bmatrix} L(\mathcal{R}) \\ C(\mathcal{S}) \end{bmatrix} \varphi = \begin{bmatrix} \mathbf{0} \\ \varphi^{\text{mea}} \end{bmatrix} \quad \text{where} \quad L(\mathcal{R}) = \begin{bmatrix} A(\mathcal{R}) \\ B(\mathcal{U}) \end{bmatrix}$$



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# Network sensor location problem

Given  $n_{\mathcal{R}}$  TR sensors, we want to uniquely estimate the flow of each road.

- ▶ What is the minimum number of flow sensors?
- ▶ Where to locate flow and TR sensors?

Unique solutions are obtained  $\iff L(\mathcal{R})$  and  $C(\mathcal{S})$  form a full row rank matrix.

$$\begin{aligned} (\mathcal{S}^*, \mathcal{R}^*) = \underset{\mathcal{S}, \mathcal{R}}{\operatorname{argmin}} \quad & |\mathcal{S}| \\ \text{subject to} \quad & \operatorname{rank} \left( \begin{bmatrix} L(\mathcal{R}) \\ C(\mathcal{S}) \end{bmatrix} \right) = n_{\mathcal{E}} \end{aligned} \quad (4)$$

This formulation is NP-complete (Bianco et al., 2006). Can efficient solutions be found?





# Sensor location

To solve (4), we propose the following sequential problems

1. Independent location of TR sensors

$$\mathcal{R}^* = \operatorname{argmax}_{\mathcal{R}} \sum_{k \in \mathcal{R}} \operatorname{outdegree}(k) \quad \text{s.t. } |\mathcal{R}| = n_{\mathcal{R}} \quad (5)$$

2. Location of flow sensors using  $\mathcal{R}^*$ .

$$\begin{aligned} & \text{find } \mathcal{S}^* \subset \mathcal{E} \\ & \text{such that } |\mathcal{S}^*| = n_{\mathcal{E}} - \operatorname{rank} L(\mathcal{R}^*), \\ & C(\mathcal{S}^*) \in \ker L(\mathcal{R}^*). \end{aligned} \quad (6)$$

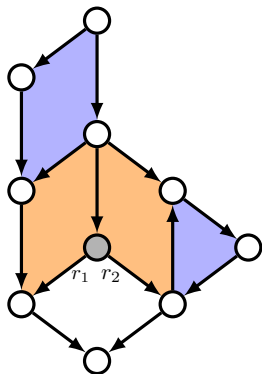
## Theorem

Solving problems (5) and (6) is equivalent to solving problem (4).



# Location of flow sensors

The cycle space of the network graph corresponds to the kernel of  $L(\mathcal{R})$ .

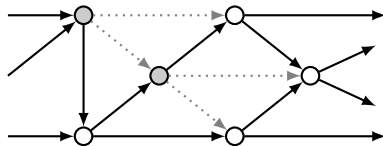


$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & -r_1 \\ 0 & 0 & -r_2 \\ 0 & 0 & r_1 + r_2 \\ 0 & 1 & 0 \\ 0 & 1 & r_2 \\ 0 & 0 & r_2 \\ 0 & 0 & -r_1 \\ 0 & 0 & r_1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in \ker L(\mathcal{R})$$

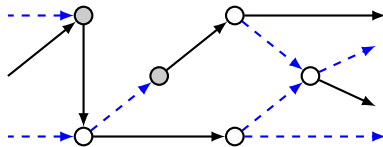


# Sensor location algorithm

1. Remove of all but one of the outgoing links from nodes  $\mathcal{R}$ .



2. Construct a spanning tree.

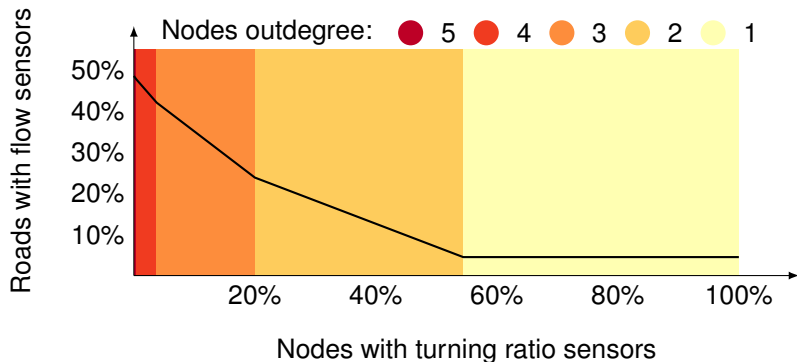


3. Locate sensors  $\mathcal{S}$  in the links not contained in the tree.

# Trade-off between sensor types

The minimum number of sensors  $n_{\mathcal{S}}^*$  is

$$n_{\mathcal{S}}^* = n_{\mathcal{E}} - n_{\mathcal{N}} - \sum_{k \in \mathcal{R}^*} (\text{outdegree}(k) - 1)$$

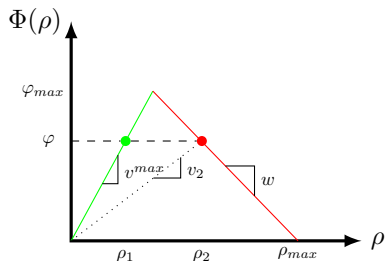


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# Flow and density estimation

The fundamental diagram is an empirical relation between flow and density (Greenshields, 1934). Speed from FCD is used to discriminate between free-flow and congestion (Daganzo, 1995).



$$\varphi = F(\mathbf{v})\rho + d(\mathbf{v})$$

$$F(\mathbf{v})_{i,i} = \begin{cases} v^{max} & \text{if } v_i \geq v^{max} \\ -w & \text{if } v_i < v^{max} \end{cases}$$

$$d(\mathbf{v})_i = \begin{cases} 0 & \text{if } v_i \geq v^{max} \\ w\rho_{max} & \text{if } v_i < v^{max} \end{cases}$$

State estimation is postulated as a quadratic optimization problem

$$\min_{\hat{\varphi}, \hat{\rho}} \left\| \begin{bmatrix} I & -F(\mathbf{v}) \\ C(\mathcal{S}) & 0 \end{bmatrix} \begin{bmatrix} \hat{\varphi} \\ \hat{\rho} \end{bmatrix} - \begin{bmatrix} d(\mathbf{v}) \\ \varphi_m \end{bmatrix} \right\|^2$$

$$\text{s.t. } L(\mathcal{R})\hat{\varphi} = 0$$

$$0 \leq \hat{\varphi} \leq \varphi_{max} \quad , \quad 0 \leq \hat{\rho} \leq \rho_{max}$$



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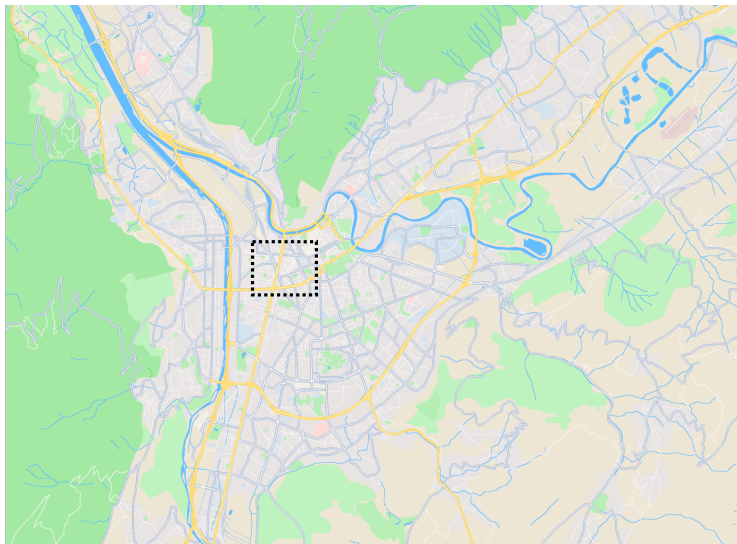
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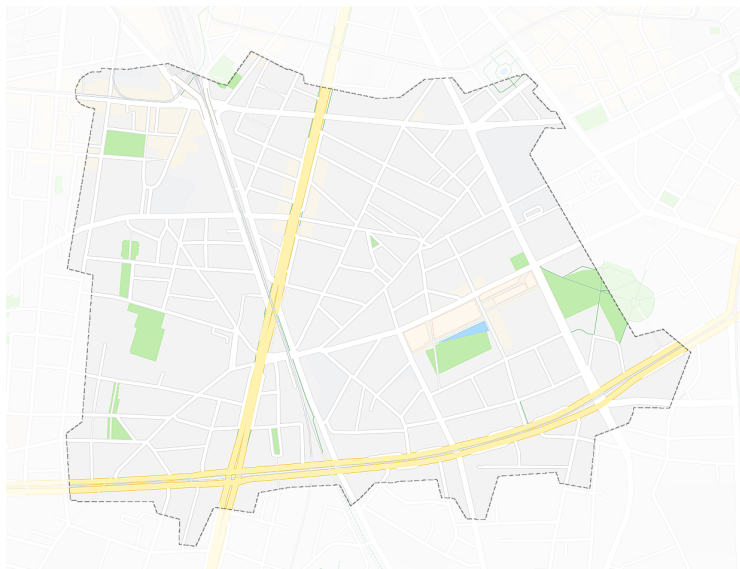


# Grenoble traffic network

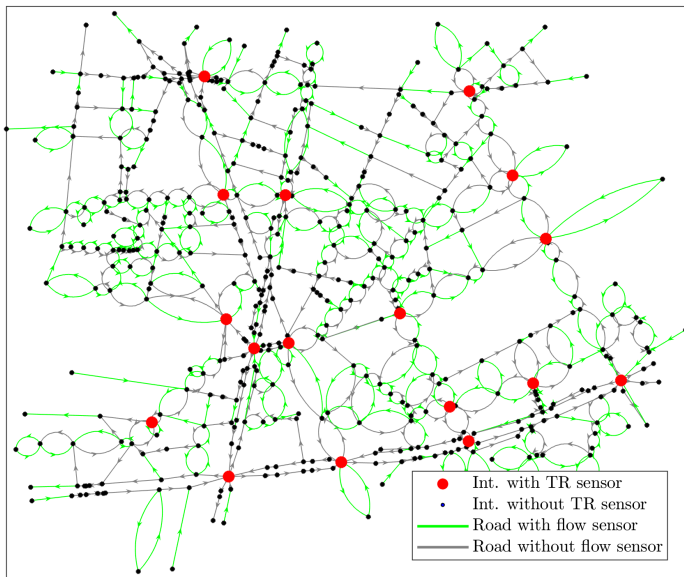




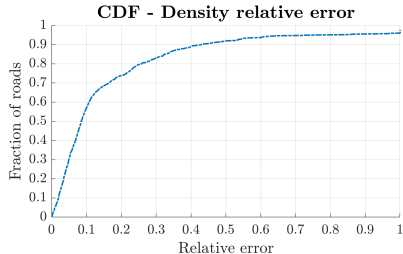
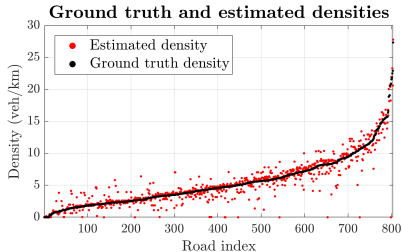
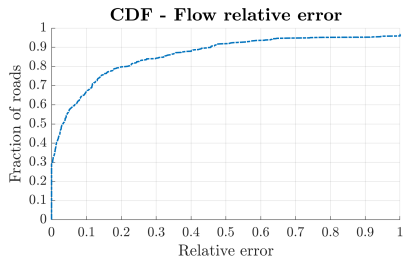
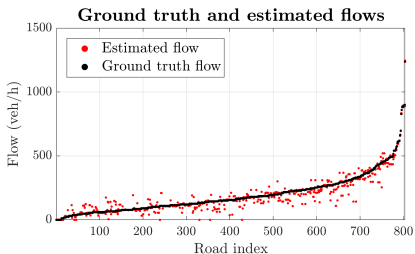
# Zone under study



# Sensor location



# Estimation error



# Section conclusions

- ▶ The trade-off between TR and flow sensors is considered. Reduction of the overall number of sensors and cost.
- ▶ Density and flow are estimated by combining data from both sensors, and FCD, using the FD.
- ▶ The number of sensors can still be too large for certain applications.
- ▶ The steady-state assumption is useful, but can induce errors during transient periods.



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## Previous works

Dynamic TSE approaches have mostly targeted highways (Ferrara et al., 2018), and few works have considered the urban case (Seo et al., 2017).

Works considering the urban case include:

- ▶ (Lovisari et al., 2016) uses the FCD and the FD to provide density measurements, in junction with a Luenberg-like observer.
- ▶ (Gu et al. 2017) uses Kalman filtering using a linearized CTM (Daganzo, 1995) and the FD.

However, current models do not effectively describe vehicle decelerations at intersections (Jabari, 2016) and (Liou et al., 2017).

(Rostami-Shahrbabaki, 2020) proposes a data-based method using connected vehicles. This data might not be readily available.

**Our contribution:** data-based method using speed data from FCD, and TR/flow sensors.



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# Dynamic model

Traffic dynamics are governed by the conservation equation.

$$\frac{d}{dt}\boldsymbol{\rho}(t) = L^{-1}(\boldsymbol{\varphi}^{\text{in}}(t) - \boldsymbol{\varphi}^{\text{out}}(t)) \quad (7)$$

where  $L$  is diagonal matrix of road lengths.

$$\boldsymbol{\varphi}^{\text{in}}(t) = R^{\top} \boldsymbol{\varphi}^{\text{out}} + \boldsymbol{\varphi}^{\text{ext}}(t) \quad (8)$$

where  $\boldsymbol{\varphi}^{\text{ext}}$  are external incoming flows at the network boundaries,  $R$  TR matrix. From these equations,

$$\frac{d}{dt}\boldsymbol{\rho}(t) = L^{-1}(R^{\top} - \mathbb{I})\boldsymbol{\varphi}^{\text{out}}(t) + L^{-1}B\mathbf{u}(t) \quad (9)$$





# Dynamic model

Consider the hydrodynamic relation

$$\varphi^{\text{out}}(t) \approx V(t)\rho(t) \quad (10)$$

where  $V(t)$  is the average vehicle speeds.

**Assumption:** The speed and density throughout a road section do not vary significantly in the spatial domain.

Thus, the estimator is

$$\frac{d}{dt}\hat{\rho}(t) = L^{-1}(R^{\top} - \mathbb{I})V(t)\hat{\rho}(t) + L^{-1}B\mathbf{u}(t) \quad (11)$$

**Required data:** flows, road speeds, and **turning ratios**.



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# Turning ratios

TR are required for all intersections. Hard in practice.

**Simplification:** use of empirical methods to estimate TR using network properties (Furth, 1990)<sup>1</sup>.

**Our method:** use of road importance to determine most likely turns. Calibration possible using flow data.


However, these methods introduce uncertainty,

$$\mathbf{e} = \lim_{t \rightarrow \infty} (\boldsymbol{\rho}(t) - \hat{\boldsymbol{\rho}}(t)) = (M^{-1} - \hat{M}^{-1})B\mathbf{u}$$

where  $M = (\mathbb{I} - R^T)V$ , and  $\hat{M} = (\mathbb{I} - \hat{R}^T)V$ .

**Sensor location:** Few TR are available. Find intersections that have the larger effect on  $\mathbf{e}$ . Calculate intersection sensitivity weight.

---

<sup>1</sup>Used road density, presence of detours, and road angles. 



# Sensitivity weight

Let  $\Xi = \hat{R} - R$ . For each intersection  $n$  and incoming road  $i$  we calculate the error energy due to TR deviations;

1. Calculate the Jacobian

$$J_i = \frac{\partial \mathbf{e}}{\partial \Xi_i} \quad (12)$$

2. Calculate the error energy using the Frobenius norm  $\|J_i\|_F^2$
3. Calculate the total intersection energy

$$w_n = \sum_{i \in \mathcal{I}(n)} \|J_i\|_F^2 \quad (13)$$

4. Locate available sensors in intersections with the highest  $w_n$ .



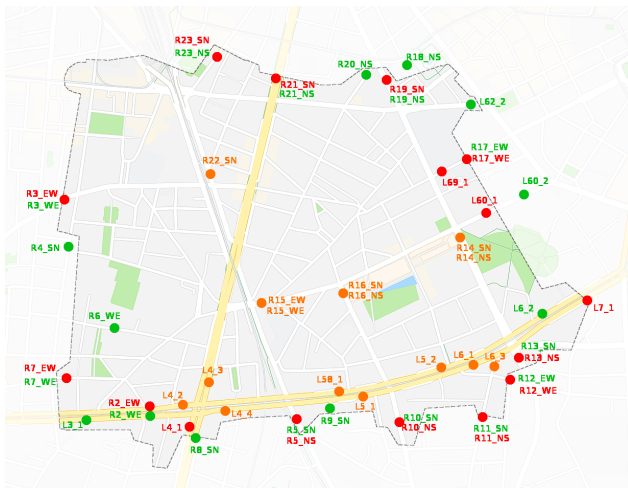
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# Turning ratio sensors



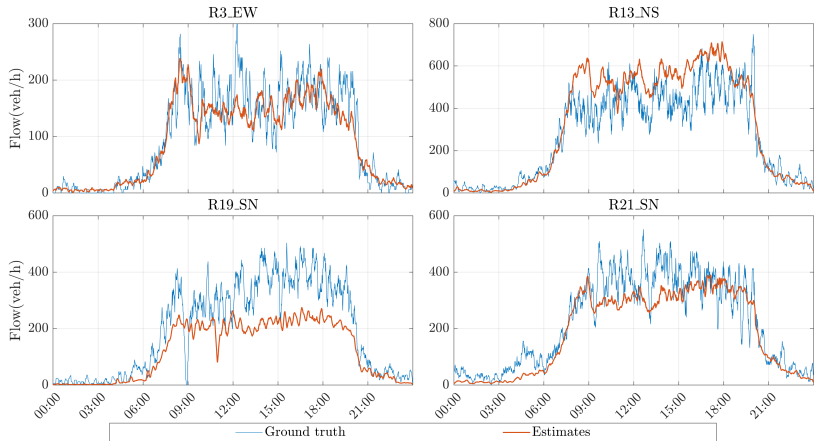
# Flow sensors



Sensors: **Boundary inflows**, **Boundary outflows**, **Validation sensors**

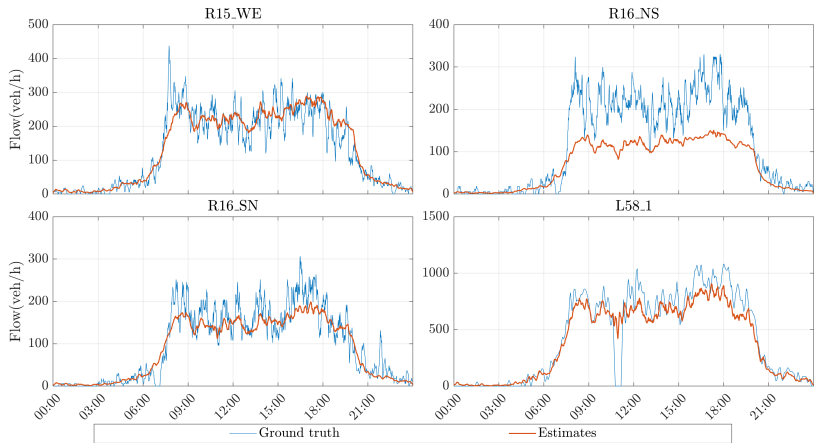


# Results - Flow time series





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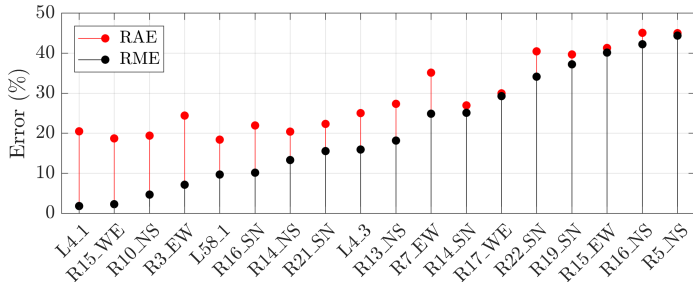
# Results - Relative mean error

**Relative Mean Error:**

$$\text{RME} = \frac{\int (\mathbf{x}(t) - \hat{\mathbf{x}}(t)) dt}{\int \mathbf{x}(t) dt}$$

**Relative Absolute Error:**

$$\text{RAE} = \frac{\int |\mathbf{x}(t) - \hat{\mathbf{x}}(t)| dt}{\int \mathbf{x}(t) dt}$$



# Section conclusions

- ▶ A data-based density estimation method is proposed. This is based on the conservation equation, and the use of FCD to estimate road outflows.
- ▶ A method to locate few TR sensors is proposed. Measure of intersection sensitivity.
- ▶ Validation with real data shows accurate flow reconstruction for most locations.
- ▶ However, mismatches are found, which are due in part to errors in the TR estimation.
- ▶ Systemic flow underestimation is evidenced. Measurement of few inflows is missing.

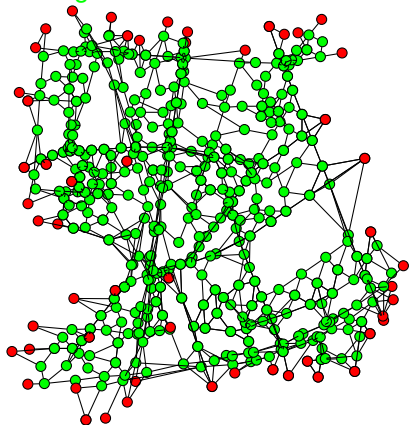


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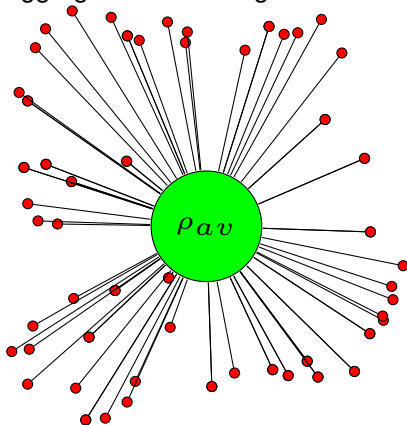


# Zone aggregation

Red nodes are at the boundaries, while green nodes are internal.



Internal road sections are aggregated into a single node.



# Motivation

Such aggregations are useful to:

- ▶ Reduce the high dimensions of large networks (Huang, 2020).
- ▶ Limit the computational cost of network processing.
- ▶ Ignore individual states of unimportant nodes in some applications.

In traffic, region-based methods have received increasing attention

- ▶ (Geroliminis et al., 2008) showed the existence of the Macroscopic Fundamental Diagram (MFD). Convex relation between the average density and total outflow of a region.
- ▶ (Haddad and Zheng, 2020) makes use of the MFD to implement perimeter control strategies.

However, there are few methods to estimate the average density of a region. (Aboudolas and Geroliminis, 2013) and (Saedi et al., 2020) used few density samples as proxy for the entire region.



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# Traffic model

Density dynamics are given by the non-linear model

$$\dot{\rho}(t) = L^{-1}(R^{\top} - \mathbb{I})V(t)\rho(t) + B\mathbf{u}(t) \quad (14)$$

Current theory is only applicable to the case of linear systems, (Fernando et al., 2010) and (Niazi et al., 2019).

**Assumption:** vehicle speeds are constant,  $V(t) = V$ , as in free-flow regime. Thus,

$$\dot{\rho}(t) = A\rho(t) + L^{-1}B\mathbf{u}(t) \quad (15)$$

where  $A = L^{-1}(R^{\top} - \mathbb{I})V$ .

Results are analyzed during congestion as well.





# Average density dynamics

We split the state into **internal nodes**  $\rho_1 \in \mathbb{R}^m$  and **boundary nodes**  $\rho_2 \in \mathbb{R}^s$ . Matrices  $A$  and  $B$  are split accordingly,

$$\rho = \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}, \quad A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}. \quad (16)$$

We are interested in the estimation of the average density of the unmeasured nodes,  $\rho_{av}(t) = \frac{1}{m} \mathbf{1}^\top \rho_1(t)$ , which evolves according to

$$\dot{\rho}_{av}(t) = \frac{1}{m} \mathbf{1}^\top A_{11} \mathbf{1} \rho_{av}(t) + \frac{1}{m} \mathbf{1}^\top A_{12} \rho_2(t) + \frac{1}{m} \mathbf{1}^\top A_{11} \sigma(t) \quad (17)$$

where  $\sigma(t) = \rho_1(t) - \mathbf{1} \rho_{av}(t)$ .



# Average detectability

Consider the one-dimensional open-loop observer

$$\dot{\hat{\rho}}_{av}(t) = \frac{1}{m}[\mathbf{1}^\top A_{11} \mathbf{1} \hat{\rho}_{av}(t) + \mathbf{1}^\top A_{12} \mathbf{y}(t)] \quad (18)$$

## Definition: Average detectability

A system is called average detectable if the open loop observer (18) converges, i.e.,  $\hat{\rho}_{av}(t) \rightarrow \rho_{av}(t)$  as  $t \rightarrow \infty$ .

## Theorem: Average detectability (Niazi et al., 2019)

A system is average detectable if and only if  $\mathbf{1}^\top A_{11} = -\gamma \mathbf{1}$ , with  $\gamma > 0$ .



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# Virtual graph

In general, traffic networks are not average detectable. **Is it possible to obtain an average detectable graph by splitting roads into cells?**

## Definition: virtual graph

Given  $\mathcal{G}$  and  $\mathbf{n} \in \mathbb{N}^m$ , a virtual graph  $\mathcal{G}^{(\mathbf{n})}$  is such that:

- ▶ Road  $i$  is divided into  $n_i$  cells.
- ▶ The length of the  $k$ -th of road  $i$  is  $\delta_i^{(k)}$ .
- ▶ The total length of each road is conserved (sum of cell lengths).
- ▶ Road speeds and turning ratios are unmodified.

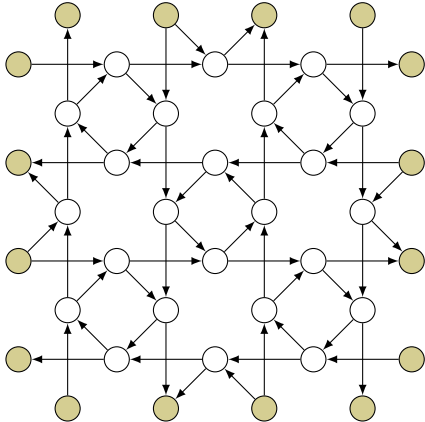
## Problem

Find  $\mathbf{n}$ ,  $\delta^{(\mathbf{n})}$ , and constant  $\gamma > 0$ , such that  $\mathcal{G}^{(\mathbf{n})}$  is average detectable.

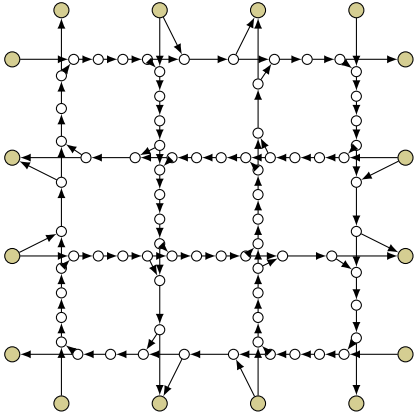


# Example - Manhattan grid network

Not average detectable



Average detectable



# Error analysis

The average densities  $\rho_{av}$  for  $\mathcal{G}$  and  $\rho_{av}^{(\mathbf{n})}$  for  $\mathcal{G}^{(\mathbf{n})}$  need not be equal.

$$\rho_{av} = \frac{1}{m} \sum_i \rho_i \quad , \quad \rho_{av}^{(\mathbf{n})} = \frac{1}{\mathbf{n}^\top \mathbf{1}} \sum_i \sum_k \rho_i^{(k)}$$

If the inputs vary slowly,

$$\rho_{av}^{(\mathbf{n})} - \rho_{av} \approx \frac{\mathbf{n}^\top \boldsymbol{\sigma}}{\mathbf{n}^\top \mathbf{1}} \quad \text{with} \quad \boldsymbol{\sigma} = \boldsymbol{\rho}_1 - \rho_{av} \mathbf{1} \quad (19)$$

## Proposition

For large networks, the probability of  $\mathbf{n}$  and  $\boldsymbol{\sigma}$  being orthogonal is asymptotically 1. Thus,

$$\lim_{m \rightarrow \infty} P(|\rho_{av}^{(\mathbf{n})} - \rho_{av}| < \epsilon) = 1 \quad (20)$$



# Virtual division for general networks

## Theorem 2

An virtual graph  $\mathcal{G}^{(\mathbf{n})}$  is average detectable if and only if

$$\delta_i^{(k)} = \frac{1}{(\mathbf{d}_i^\top \mathbf{n} + k)\gamma} \quad (21)$$

where  $\mathbf{d}_i^\top$  depends only on the TR and road speeds.

Cell lengths are directly computed from  $\mathbf{n}$  and  $\gamma$ . Therefore, the problem only depends on these variables.



# Approximate solutions

## Theorem 3

Let  $\mathbf{n}$  and  $\gamma$  satisfy

$$[K(\gamma) - V(\mathbb{I} - R_{11})^{-1}V_1^{-1}]\mathbf{n} = \frac{1}{2}\mathbf{1}, \quad (22)$$

where  $K(\gamma)_{i,i} = \frac{e^{\gamma l_i/v_i}}{e^{\gamma l_i/v_i} - 1}$ . Then,

$$\left| l_i - \sum_k \delta_i^{(k)} \right| \sim O\left(\frac{1}{v_i \mathbf{d}_i^\top \mathbf{n} + 1}\right) \quad (23)$$

## Theorem 4

There is  $\gamma_{max}$  such that if  $0 < \gamma < \gamma_{max}$ , the solution to (22) is positive. Moreover, as  $\gamma \rightarrow \gamma_{max}$ , then  $\|\mathbf{n}\| \rightarrow \infty$  and (23) goes to 0.





# Algorithm

Using Theorems 3 and 4,  $\mathbf{n}$  and  $\gamma$  can be calculated by iteratively approximating  $\gamma_{max}$ . Consider the bisection algorithm:

**Initialization:**  $\gamma_1 = 0$ , and  $\gamma_2 = \max_i(-\log(\lambda(R_{11})v_i/\ell_i))^2$ .

1. Set  $\gamma \leftarrow (\gamma_1 + \gamma_2)/2$ .
2.  $\mathbf{n} = [K(\gamma) - V(\mathbb{I} - R_{11})^{-1}V_1^{-1}]^{-1}\mathbf{1}/2$ .
3. If  $n_i < 0$ , set  $\gamma_2 \leftarrow \gamma$ . Go to Step 1.
4. If  $|\ell_i - \sum_k \delta_i^{(k)}| > \epsilon$ , set  $\gamma_1 \leftarrow \gamma$ . Go to Step 1.
5. Return  $(\mathbf{n}, \gamma)$ .

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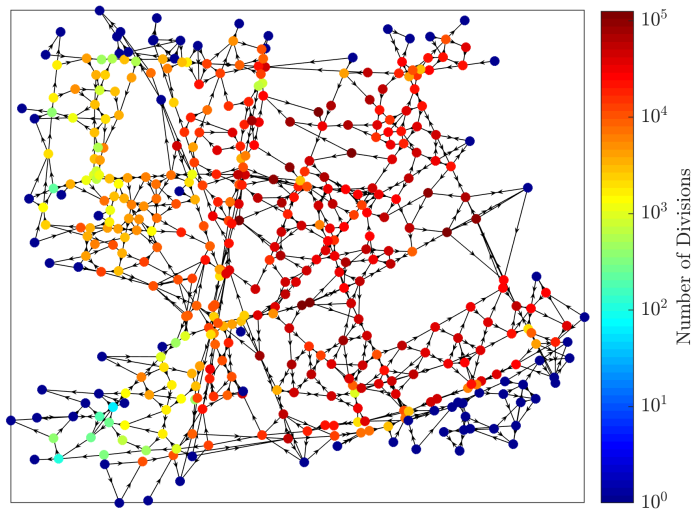
<sup>2</sup>This value of  $\gamma_2$  is proven to be greater than  $\gamma_{max}$



1. Introduction
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  3. Dynamic density estimation
  4. Average density estimation
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    - Virtual network definition
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# Number of cell divisions



# Estimator deployment

The number of divisions can be very high. Fortunately, the virtual graph is not needed to estimate  $\rho_{av}^{(\mathbf{n})}$ .

$$\dot{\hat{\rho}}_{av}^{(\mathbf{n})} = -\gamma \hat{\rho}_{av}^{(\mathbf{n})} + \frac{\gamma}{\mathbf{n}^\top \mathbf{1}} \mathbf{n}^\top V_1^{-1} (\mathbb{I} - R_{11}^\top)^{-1} R_{21}^\top V_2 \mathbf{y}(t) \quad (24)$$

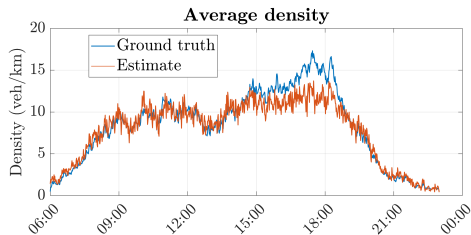
We require:

- ▶  $(\mathbf{n}, \gamma)$  provided by the previous algorithm.
- ▶ Turning ratios and average speeds, graph properties.
- ▶ Measurements of external inflows  $\mathbf{y}(t)$ .

The estimator is efficiently deployed even for very large dimensions.

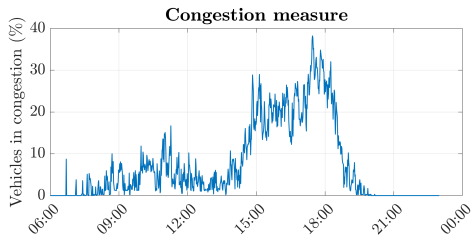


# Average density estimation



**Total error:**  
12%

**Error during  
congestion:**  
18%.



**Error outside  
congestion:**  
8%.

# Section conclusions

- ▶ An open-loop average density estimator is proposed for large traffic networks.
- ▶ This estimator is shown to converge for virtual networks. Division of roads into cells of specific length.
- ▶ Although not exact, the estimation error is bounded. Reduction of error for large networks, and large number of divisions.
- ▶ The linear regime assumption can limit the scope of the method.
- ▶ Nevertheless, for not too large congestions, the estimator provides satisfactory results.



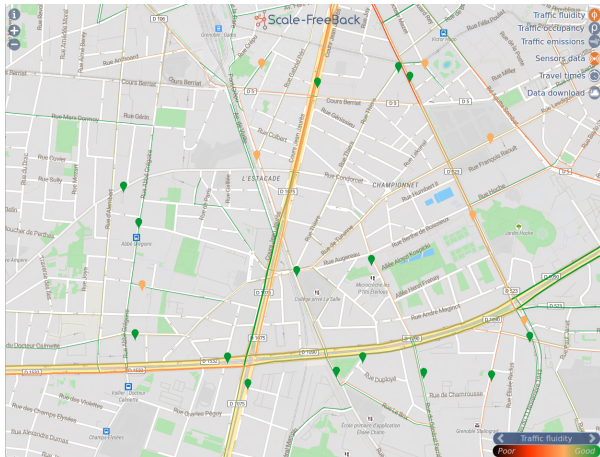
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# Experimental platform

GTL-Ville (<http://gtlville.inrialpes.fr/>)

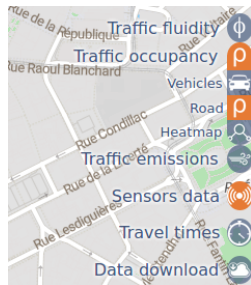




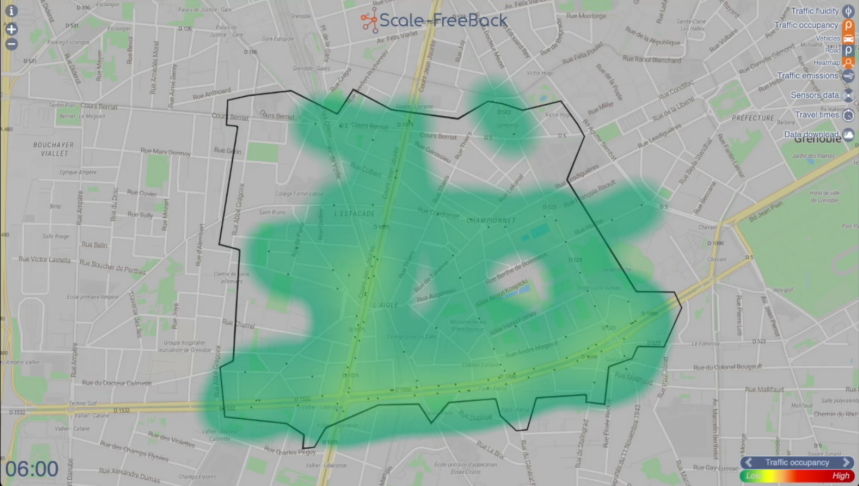


# Functionality - Density estimation

Input data is processed, filtered, and missing samples are imputed.  
The described estimation methods are applied.



# Density estimation - Demonstration



2D density via Gaussian Kernel Estimation (Mollier et al., 2018).



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## Conclusions



## Journal papers

- ▶ M. Rodriguez-Vega, C. Canudas-de-Wit, H. Fourati. "Average density estimation for urban traffic networks: application to the Grenoble network". *Transp. Res. B.* 2021. (Under review)
- ▶ M. Rodriguez-Vega, C. Canudas-de-Wit, H. Fourati. "Location of turning ratio and flow sensors for flow reconstruction in large traffic networks". *Transp. Res. B.* 2021, 21-40.

## Conference papers

- ▶ M. Rodriguez-Vega, C. Canudas-de-Wit, H. Fourati. "Urban network traffic state estimation using a data-based approach." CTS, Lille, June 2021.
- ▶ M. Rodriguez-Vega, C. Canudas-de-Wit, H. Fourati. "Average density detectability in traffic networks using virtual road divisions". IFAC World Congress, Berlin, July 2020.
- ▶ M. Rodriguez-Vega, C. Canudas-de-Wit, H. Fourati. "Joint flow and density reconstruction in large traffic networks using partial turning ratio information". CDC, Miami, December 2018.
- ▶ A. Ladino, C. Canudas-de-Wit, A. Kibangou, H. Fourati, M. Rodriguez-Vega. "Density and flow reconstruction in urban traffic networks using heterogeneous data sources". ECC, Limassol, June 2018.



# Sensor location

1. In steady state, we analyzed the trade-off between flow and turning ratio sensors. This method requires few additional parameters, but may require a high number of sensors.
2. In dynamical conditions, TRs are required. We proposed an approach in which heuristic values can be used. The estimation error is reduced by detecting the most critical locations to locate few TR sensors.



# Flow and density estimation

We considered three different scenarios:

1. In stationary networks, sensor data and the FD with FCD speed data are used to jointly estimate both variables via an optimization problem.
2. In dynamical networks, we propose a data-based approach that computes road outflows using FCD speed data and previous density estimates.
3. A method to estimate the average density of a zone using a 1D estimator. This estimator is shown to converge for a network where roads are divided in virtual cells.



# Conclusions

Validation using real data showed that the described methods provide encouraging results.

Some open questions and improvements remain for future works:

- ▶ Improvement of TR estimates.
- ▶ Expansion of the average-density estimation method to better capture congestion.
- ▶ Estimation of energy consumption and vehicle emissions.







# Appendices



## Sketch of proof

The column sum for an internal cell  $k > 1$  of road  $i$  must satisfy

$$-\frac{v_i}{\delta_i^{(k)}} + \frac{v_i}{\delta_i^{(k-1)}} = -\gamma.$$

At intersections (ending cells), the column sums satisfy

$$-\frac{v_i}{\delta_i^{(1)}} + \sum_{j=1}^m \frac{r_{ij}v_i}{\delta_j^{(n_j)}} = -\gamma. \quad (25)$$

Joining all the columns, we obtain a system of equations

$$(\mathbb{I} - R_{11})\boldsymbol{\delta}_{(1)}^{-1} = \gamma[R_{11}V_1^{-1}\mathbf{n} + (\mathbb{I} - R_{11})V_1^{-1}\mathbf{1}] \quad (26)$$

This system is always solvable, giving

$$\frac{1}{\delta_i^{(1)}} = \gamma \left( \mathbf{d}_i^\top \mathbf{n} + \frac{1}{v_i} \right) \quad (27)$$



## Sketch of proof for Theorem 3

The finite fraction sums can be approximated using the natural logarithm,

$$\sum_{k=1}^n \frac{1}{z+k} = \ln \left( z + n + \frac{1}{2} \right) - \ln \left( z + \frac{1}{2} \right) + O(n^{-2})$$

Thus, we rewrite

$$f_i(\mathbf{n}, \gamma) = \ell_i - \frac{v_i}{\gamma} \left[ \ln \left( v_i \mathbf{d}_i^\top \mathbf{n} + n_i + \frac{1}{2} \right) - \ln \left( v_i \mathbf{d}_i^\top \mathbf{n} + \frac{1}{2} \right) + O(\mathbf{n}^{-2}) \right]$$

From the Taylor series of logarithms, we note that  $\ln(\mathbf{a}^\top \mathbf{n} + c) - \ln(\mathbf{a}^\top \mathbf{x} + c) \sim O((\mathbf{a}^\top \mathbf{n} + 1)^{-1})$

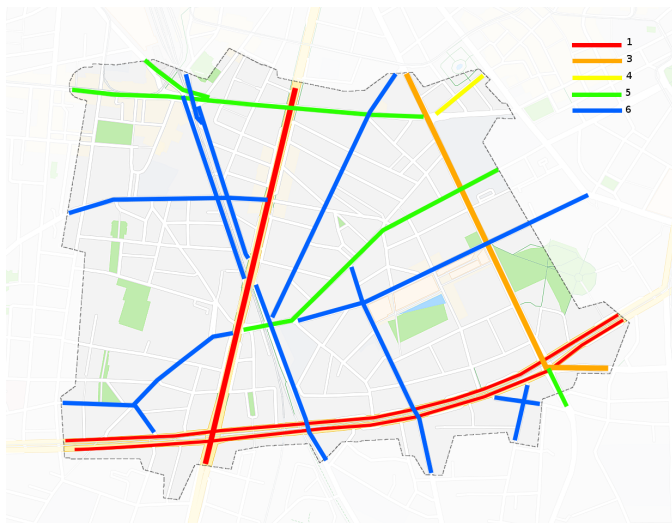
Using logarithm properties, we note that

$$0 = \ell_i - \frac{v_i}{\gamma} \left[ \ln \left( v_i \mathbf{d}_i^\top \mathbf{x} + x_i + \frac{1}{2} \right) - \ln \left( v_i \mathbf{d}_i^\top \mathbf{x} + \frac{1}{2} \right) \right]$$

if and only if  $[(K_\gamma - \mathbb{I})^{-1} K_\gamma - V(\mathbb{I} - R_{11})^{-1} V_1^{-1}] \mathbf{x} = \frac{1}{2} \mathbf{1}$ .



# Functional Road Classification



# Functional Road Classification

**Table:** Description of the road classes provided by TomTom.

Class	Short description
0	Highways, Motorways
1	Major roads of high importance
2	Other major roads
3	Secondary roads
4	Local connecting roads
5	Local roads of high importance
6	Local roads
7	Local roads of minor importance
8	Other roads



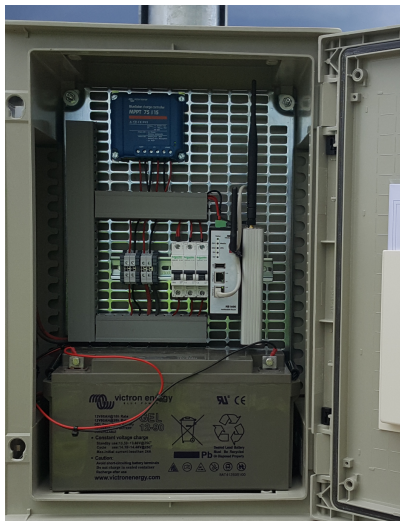
# Floating car data - Average speed

Aggregated vehicle trajectories of a fraction of vehicles in the network are used to provide the average road speed of a subset of roads. Due to technical limitations, data is provided not for each road. Instead, roads are divided into partitions, such that each partition contains adjacent roads of the same FRC, going in the same direction.

The average length of each partition is of 200 m.  
Only FRC above 7 are given speed values.



# Bluetooth sensors





# Bluetooth sensors



# Turning ratio estimation

For unmeasured intersections, turning ratios are estimated using the FRC of the outgoing roads, which is used as a proxy of road importance.

For each class  $c$ , define a weight  $\theta(c)$ . We assume that TRs are proportional to the weights of the outgoing roads, as

$$\frac{r_{i,j}}{r_{i,k}} = \frac{\theta(c_j)}{\theta(c_k)} \quad (28)$$

where  $c_k$  is the class of road  $k$ .



# Turning ratio estimation

The FRC weight values are calculated by solving a flow-matching optimization problem

$$\min_{\theta} \quad \|\mathbf{y} - \hat{\mathbf{y}}(\mathbf{u}, \theta)\|^2 \quad (29)$$

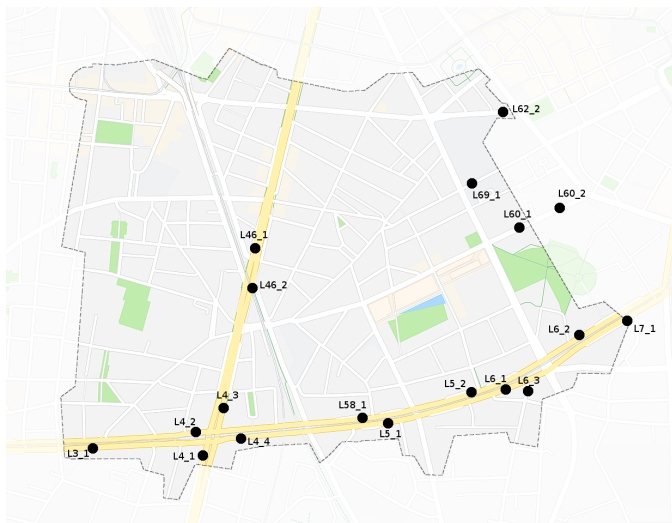
where  $\theta$  is the class-weight vector,  $\mathbf{y}$  and  $\mathbf{u}$  are the daily averages of boundary outflows and inflow, respectively, from sensors, and

$$\hat{\mathbf{y}}(\mathbf{u}, \theta) = (\mathbb{I} - R(\theta)^\top)^{-1} B\mathbf{u} \quad (30)$$

are the estimated steady-state flows.



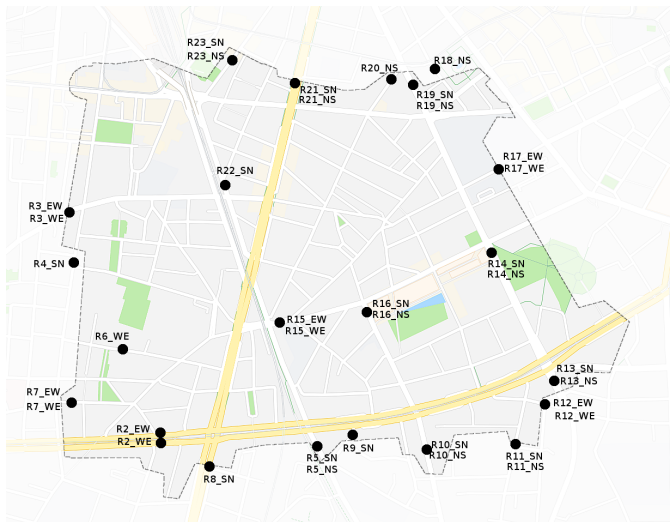
# Induction loops



# Microwave radars



# Microwave radars



# Scenario 1

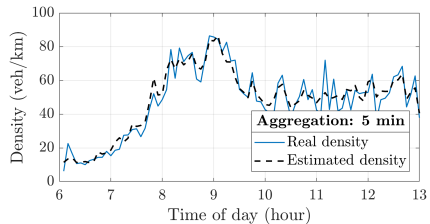
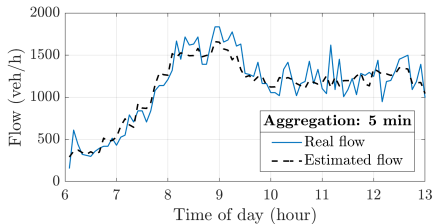
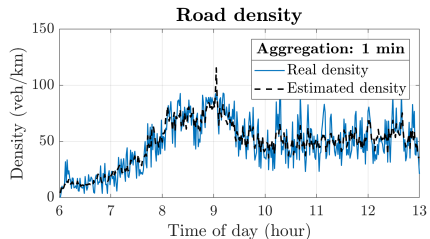
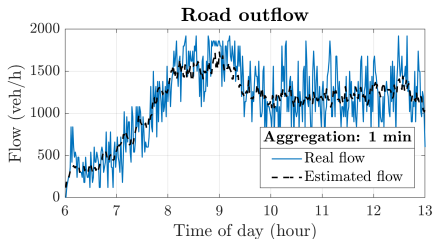
We evaluate the estimator using the microscopic simulator Aimsun in the city of Grenoble.

**Scenario 1:** Turning ratios are known everywhere. Boundary inflows follow a time-varying profile.

The boundary inflows are measured and used as inputs to the estimator. The space-mean speed of each road is measured using floating car data.

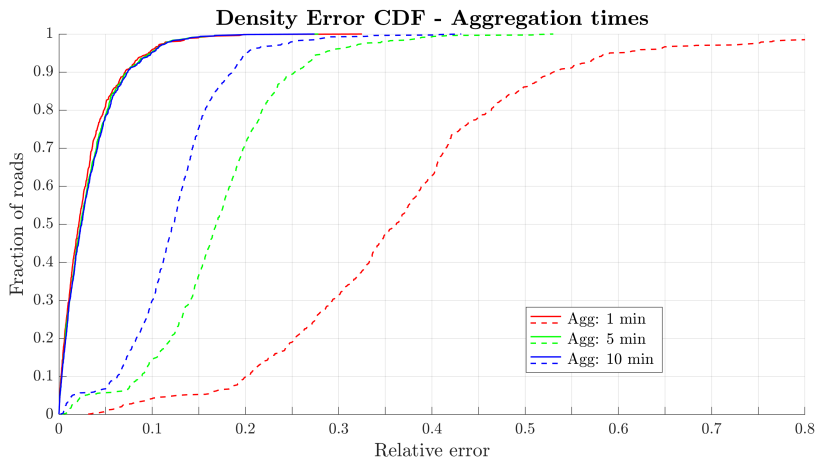


# Flow and density time series for a single road





# Scenario 1: Estimation error



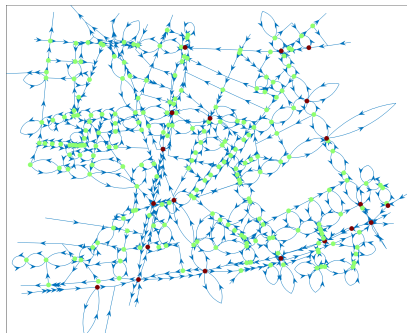
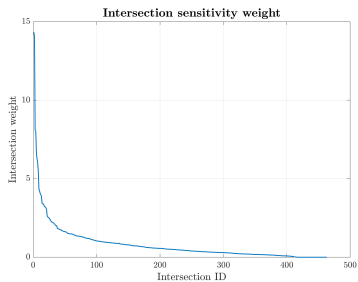
## Scenario 2

**Scenario 2:** Turning ratios everywhere are added a uniform 10% noise to represent uncertainty.

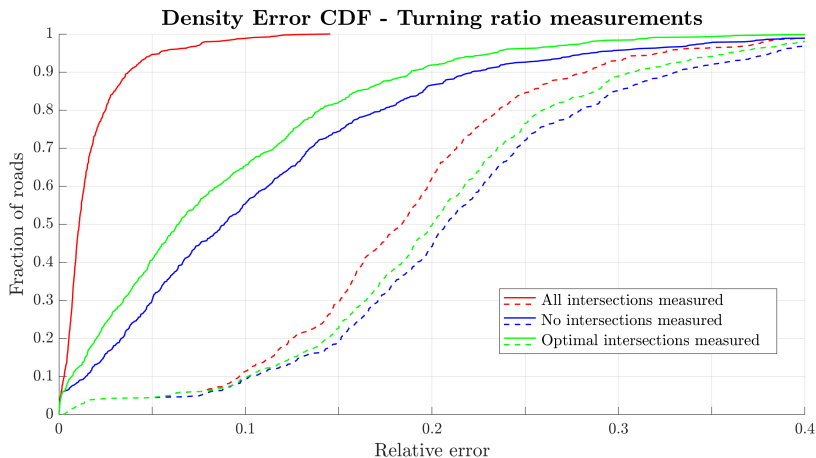
The sensor location method is used to select 20 intersections. For these places, the proportion of vehicles taking each direction is measured directly.



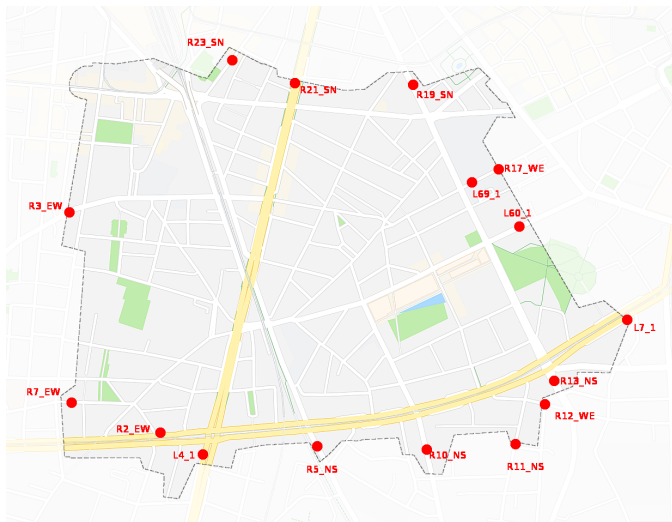
# Scenario 2: Turning ratio sensor location



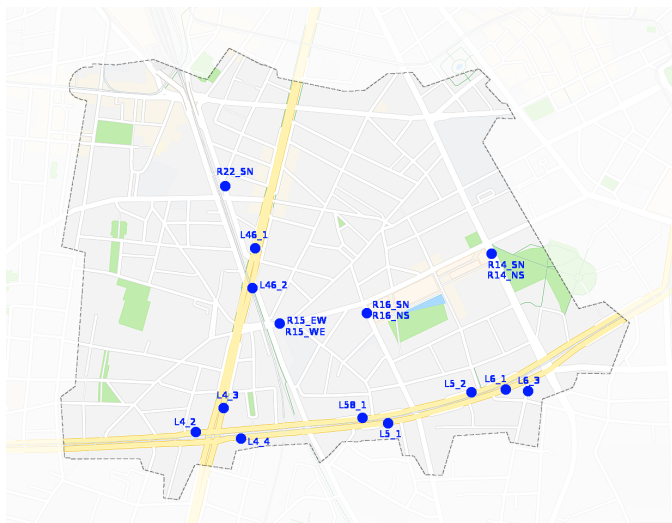
# Scenario 1: Estimation error



# Boundary outflow sensors



# Internal validation sensors (Outflows)



# Boundary inflow sensors

