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# Vers une intégration décisionnelle en planification de production

David Lemoine

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# UNIVERSITE DE NANTES

*Laboratoire des Sciences du Numérique de Nantes (LS2N)  
UMR CNRS 6004*

*ECOLE DOCTORALE MathSTIC*

Dossier présenté en vue de l'obtention du diplôme d'

## HABILITATION A DIRIGER DES RECHERCHES

proposé par

**David LEMOINE**

Soutenue le 09 Septembre 2020 devant le jury composé de :

Pr. Nathalie Bostel	Université de Nantes	Présidente du jury
Pr. Nathalie Sauer	Université de Lorraine	Rapporteur
Pr. Gilles Goncalves	Université d'Artois	Rapporteur
Pr. Jacques Lamothe	IMT Mines Albi-Carmaux	Rapporteur
Pr. Safia Kedad-Sidhoum	CNAM Paris	Examinatrice
Pr. Sylvie Norre	Université Clermont Auvergne	Examinatrice
Pr. Vincent Hovelaque	Université de Rennes 1	Examinateur
Pr. Alexandre Dolgui	IMT Atlantique	Directeur de recherche





*Franz Liszt (1811-1886)*





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Première partie

**Synthèse des activités**



# Curriculum Vitæ

## David LEMOINE

Né le 26 Octobre 1971 à Versailles.

IMT Atlantique Bretagne-Pays de la Loire  
Département Automatique, Productique et Informatique  
Campus de Nantes  
4 rue Alfred Kastler - La Chantrerie  
CS 20722  
44307 Nantes Cedex 3  
France  
Téléphone : (33) 2.51.85.83.35  
Courriel : david.lemoine@imt-atlantique.fr

## SITUATION PROFESSIONNELLE

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**Maitre Assistant de l'Institut Mines-Télécom** au *Département Automatique, Productique et Informatique d'IMT Atlantique Bretagne Pays de Loire* depuis Septembre 2010, 5<sup>e</sup> échelon.

**Membre de l'équipe de recherche** *Systèmes Logistiques et de Production (SLP)* du Laboratoire des Sciences du Numérique de Nantes (LS2N) - UMR CNRS 6004.

**Responsable d'Option** *Management de la Performance et du Risque (MPR)* de la filière ingénieur d'IMT Atlantique.

**Co-Responsable pédagogique** *Transformation digitale des systèmes industriels (FIT)* de la filière ingénieur par apprentissage d'IMT Atlantique.

## FORMATION

---

- 2005-2008**  
*Clermont Ferrand II* **Thèse de Doctorat** de l'Université Blaise Pascal  
sous la direction de M. Michel GOURGAND et Mme Sylvie  
NORRE : *Modèles génériques et méthodes de résolution pour la*  
*planification tactique mono et multi-site* - Date de soutenance :  
4 Décembre 2008, Mention Très Honorable
- 2006**  
*Conservatoire National*  
*des Arts et Metiers* **Diplôme d'ingénieur** Mention STIC  
Spécialité : Informatique, Modélisation, Optimisation - Mention  
Très Bien
- 2004-2005**  
*Clermont Ferrand II* **Master de Recherche** de l'Université Blaise Pascal.  
Spécialité : Modèles Systèmes Intelligence - Option : Recherche  
Opérationnelle et Productique - Mention Assez Bien
- 2004**  
*Conservatoire National*  
*des Arts et Metiers* **Diplôme d'Études Supérieures Techniques**  
Spécialité : Modélisation, Optimisation

## EXPÉRIENCES PROFESSIONNELLES

---

- Depuis Sept. 2010**  
*Mines Nantes*  
*IMT Atlantique* **Maitre Assistant de l'Institut Mines Télécom** au Dépar-  
tement Automatique et Productique et Informatique d'IMT At-  
lantique
- Sept. 2009 - Août 2010**  
*Mines Nantes* **Maitre Assistant Associé** au Département Automatique et  
Productique de Mines Nantes
- Sept. 2008 - Août 2009**  
*Université Jean Monnet* **Attaché Temporaire d'Enseignement et de Recherche** au  
Département Qualité Logistique Industrielle et Organisation de  
l'IUT de Roanne (Demi-Poste)
- 2002-2005**  
**Technicien en Informatique** MOONDESS Informatique.  
En régie chez BULL S.A. pour le compte de la société MICHELIN
- 1998-2002**  
**Technicien en Informatique** IM3 Informatique.  
En régie chez BULL S.A. pour le compte de la société MICHELIN  
et ALCATEL CIT

## RECHERCHE ET VALORISATION

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Mes travaux de recherche se sont principalement focalisés sur les problèmes de planification de production et notamment par le biais de l'optimisation de modèles dits de « taille de lots ». Je me suis particulièrement intéressé aux problématiques de synchronisation de décisions (pouvant engendrées des problèmes d'infaisabilité des plans de production) et à l'intégration de contraintes financières en sus des coûts logistiques traditionnellement utilisés. Ces travaux seront abordés dans la seconde partie de ce document. Dans cette section, j'exposerai les projets et contrats auxquels j'ai pu contribuer (en tant que porteur, chef de projet ou encore participant), l'ensemble de mes publications et mes différentes participations à l'animation scientifique (encadrement de thèses de doctorat, animation d'un groupe de recherche, formation doctorale ou industrielle, etc.).

### Projets et contrats de recherche

#### PROJETS DE RECHERCHE ACADÉMIQUE

---

##### **Prise de décision dans la chaîne logistique en milieu incertain**

2006

*Projet GDR MACS**Recherche*

- *Définition et classification des différents types d'incertitude dans la chaîne logistique*
- L'objectif de ce projet était de recenser les différents types d'incertitudes pouvant être rencontrés dans la chaîne logistique. Dans une seconde phase, une définition de chaque type d'incertitude a été proposée, ainsi qu'une classification s'appuyant sur ces définitions.
- En collaboration avec Sylvie Norre, Michel Gourgand, Séverine Durieux, Jean-Luc Paris, Samir Lamouri, Patrick Génin, Caroline Thierry, Hélène Fargier.
- Partenaires : SUPMECA, IRIT
- Durée : 1 an - Montant 2000,00 €

##### **TASC**

2007

*Projet GDR RO**Recherche*

- *Intégration du transport dans la chaîne logistique*
- Ce projet avait pour but d'étudier différentes manières d'intégrer du transport comme élément de décision dans le management de la chaîne logistique, dans chacun des trois niveaux décisionnels classiques.
- En collaboration avec Sylvie Norre, Michel Gourgand, Laurent Deroussi, Marie-Ange Manier, Hervé Manier
- Partenaires : le SET
- Durée : 1 an - Montant 2000,00 €



**Modèle de planification générique multisite**  
*Projet GDR RO*

2008  
*Recherche*  
*Développement*

- *Modèle de planification générique multisite prenant en compte les problèmes de transport avec capacité*
- Devant l'absence de modèle de planification multisite, l'objectif était de recenser puis de définir les spécifications d'un modèle de connaissance générique prenant en compte les contraintes de transport. Ce modèle a été traduit en un modèle mathématique permettant de retrouver les modèles multisite de la littérature et un modèle de données en XML a été proposé.
- En collaboration avec Sylvie Norre, Michel Gourgand, Laurent Deroussi, Marie-Ange Manier, Hervé Manier
- Partenaires : le SET
- Durée : 1 an - Montant 2000,00 €

**Vers une intelligence particulière**  
*Projet GDR RO*

2011  
*Recherche*

- *Définition de nouvelles topologies pour des méthodes basées sur des essaims de particules*
- L'objectif est de définir de nouvelles topologies dynamiques s'appuyant sur des concepts collaboratifs afin d'améliorer l'efficacité des essaims particuliers.
- En collaboration avec Maurice Leclerc, Patrice Siarry, Laurent Deroussi, Michel Gourgand et Abas El Dor
- Partenaires : le LIMOS et l'UPEC
- Durée : 1 an - Montant 2000,00 €

**FILEAS-FOG**  
*Projet ANR*

Depuis 2018  
***Co-porteur***  
*Coordinateur pour IMT-Atlantique*

- *Integrating Financial Health in Supply Chain Management. Intégration de la dimension Financière dans l'Optimisation des chaînes logistiques*
- Faisant suite à un projet de type Fond Unique Interministériel (FUI), ce projet académique a pour ambition d'étudier l'impact de décisions financières sur les décisions logistiques, et vice versa. Dans ce projet, j'ai pu participer à la définition des problématiques traitées et l'écriture de dossier ANR. Je suis également responsable d'un WorkPackage pour lequel j'ai encadré un Post-Doctorant (18 mois) et je suis également co-encadrant d'une thèse de doctorat (Pooya Hendayatinia) portant sur l'impact des décisions de financement sur le processus de planification tactique de la production.
- Partenaires : l'IGR-IAE de Rennes et le laboratoire du CREM de Rennes
- Durée : 4 ans (2018-2022) - Montant 277 200,00 € pour IMT-Atlantique

**Michelin**  
*Contrat Industriel*

2008  
*Recherche*  
*Développement*

- *Gestion des stocks avec demandes différenciées*
- Dans le cadre de son plan de réduction des niveaux de stocks, Michelin a souhaité expérimenter une méthode de gestion à demandes différenciées, fonction du niveau de contrat de ses clients. Mon rôle a été de développer les méthodes d'optimisation permettant de fixer les différents seuils de réapprovisionnement pour un DRP afin de minimiser les niveaux de stocks tout en assurant le taux de satisfaction contractuel de chaque client. J'ai ensuite développé une application intégrant ces derniers en vue d'un déploiement opérationnel.
- En collaboration avec Sylvie Norre, Michel Gourmand
- Partenaires : Michelin.
- Durée : 10 mois - Montant 10 000,00 €

**GE Healthcare**  
*CIFRE*

2012-2015  
***Co-Responsable***  
*Encadrement*

- *Amélioration des processus de prévision et de gestion des stocks dans le cas d'une chaîne logistique de pièces de rechange*
- Le but de ce projet était d'élaborer des modèles de prévision pour les pièces de rechange de matériels médicaux en intégrant des indicateurs de performance issus de la gestion de stock afin d'améliorer la performance de la chaîne logistique de General Electric. Cette problématique a fait l'objet de la thèse d'Adnane Lazrak.
- En collaboration avec Bruno Castanier.
- Partenaires : LARIS, Université d'Angers, General Electric HealthCare
- Durée : 4 ans - Montant 60000,00 €

**ORLoGES**  
*Plan Investissement d'Avenir*

2012-2016  
***Responsable de lot***  
*Encadrement*  
*Recherche*

- *Optimisation des Réseaux Logistiques en Génie Civil avec les aspects Economiques et Sociétaux*
- Le projet porte sur l'optimisation des transports inhérents aux chantiers de construction de routes et de terrassement. Il adresse des aspects stratégiques (Localisation de plateformes de stockage), tactiques (planification de ressources et de l'approvisionnement) et opérationnels (Tournées de véhicules). J'étais responsable du lot « planification tactique » dont le but était de fournir un moteur d'optimisation permettant de planifier le flux matière inter-plateforme et les ressources de transports adéquates afin d'assurer le transfert de matière. Cette problématique a fait l'objet d'une recherche post-doctorale (Thomas

Vincent).

- En collaboration avec Nathalie Bostel (chef de projet), Olivier Péton, Fabien Lehuédé.
- Partenaires : Luc Durand (porteur), PTV, OPAL Conseil, Kaliterre.
- Durée : 4 ans - Montant 357 000,00 €

### **RCSM**

*Projet FUI 15*

2013-2017

***Chef de projet***

*Encadrement*

- *Risk, Credit Chain & Supply Chain Management*
- Ce projet porte sur l'élaboration d'une plateforme web pour le financement de filières industrielles. Dans ce contexte, notre objectif était de modéliser le Besoin en Fonds de Roulement (BFR) en fonction des plans de production déterminés puis de développer des algorithmes de simulation du BFR puis d'optimisation des plans de production selon cet indicateur de performance. Dans ce projet s'est inscrit le Master de recherche puis la Thèse de doctorat de Yuan Bian ainsi qu'une recherche post-doctorale de 18 mois (Fabrice Gayraud).
- En collaboration avec Thomas Yeung, Nathalie Bostel, Safia Keddad Sidhoum, Vincent Hovelaque, Jean-Laurent Viviani.
- Partenaires : Weave (porteur), Marsh, Kyriba, LIP6, IGR Rennes, Armines/Mines Nantes, INSA Rouen.
- Durée : 4 ans - Montant 260 800,00 € pour Armines/Mines Nantes

### **GE Medical**

*CIFRE*

2016-2020

***Co-Responsable***

*Encadrement*

- *Vers un outil d'aide à la décision pour l'amélioration des flux de retours de pièces de rechange dans une chaîne logistique complexe : application à General Electric Medical System*
- Ce projet porte sur l'élaboration d'un outil d'aide à la décision pour la détermination la charge et et la planification de la capacité d'un atelier de réparation de systèmes médicaux. Cette problématique fait l'objet de la thèse d'Hamza El Garrab.
- En collaboration avec Bruno Castanier.
- Partenaires : LARIS, Université d'Angers, General Electric HealthCare
- Durée : 4 ans - Montant 70000,00 €

## Encadrement

Dans cette section, je présente l'ensemble des encadrements scientifiques que j'ai pu assurer.  
La Table 1 les récapitule.

Type	soutenue	en cours	total
Post-doctorat		0	3
Thèse de Doctorat	4	2	6
Thèse de Master	4	0	4

TABLE 1 – Encadrements

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### ENCADREMENT DE POST-DOCTORANTS

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**Fabrice Gayraud**  
*Projet Fileas Fog*

Octobre 2017 - Mars 2019  
18 mois

- *Planification de production sous contraintes de capacité*
- Optimisation du BFR sous contrainte de capacité de production
- Encadrement scientifique, assistance au développement
- En collaboration avec Guillaume Massonnet

**Fabrice Gayraud**  
*Projet RCSM*

Septembre 2016 - Aout 2017  
12 mois

- *Planification de production à capacité infinie avec optimisation du BFR*
- Optimisation du BFR à capacité infinie pour un processus PIC.
- Encadrement scientifique, assistance au développement
- En collaboration avec Nathalie Bostel

**Thomas Vincent**  
*Projet ORLoGES*

Janvier 2014 - Décembre 2014  
12 mois

- *Planification de ressources de transport et de flux matières*
- Optimisation de l'utilisation des ressources de transport de flux matières entre les plateformes de stockage et de production.
- Encadrement scientifique, assistance au développement

**Pooya Hedayatinia**

Janvier 2019 - en cours

*Projet Fileas Fog*

- *Incorporating the financial dimension in tactical production planning decisions*
- Financement dans le cadre du projet ANR Fileas Fog
- *Directeur et co-encadrants* : Jean Laurent Viviani, Guillaume Massonnet et David Lemoine
- Taux d'encadrement : 30 %

**Hamza El Garrab**

Janvier 2017 - en cours

*Projet GE Medical*

- *Vers un outil d'aide à la décision pour l'amélioration des flux retours de pièces de rechange dans une chaîne logistique complexe : Application à General Electric Medical Systems*
- Financement ANRT (CIFRE)
- *Directeur et co-encadrant* : Bruno Castanier, David Lemoine
- Taux d'encadrement : 50 %

**Hasan Al Hasan**

Septembre 2015 - soutenue le 28 Mai 2019

- *Surgical case scheduling with sterilizing activity constraints*
- 50 % Angers Loire Métropole, 50 % IMT Atlantique
- *Directeur et co-encadrants* : David Rivreau, Christelle Guéret et David Lemoine
- Taux d'encadrement : 30 %
- Position actuelle : A.T.E.R à l'université d'Angers

**Yuan Bian**

Septembre 2014 - soutenue le 19 Décembre 2017

*Projet RCSM*

- *Tactical planning for physical and financial flows for supply chain in a multi-site context*
- Financement dans le cadre du projet RCSM
- *Directeur et co-encadrants* : Nathalie Bostel, David Lemoine et Thomas Yeung
- Taux d'encadrement : 40 %
- Position actuelle : Post Doctorant à l'UCAS (University of Chinese Academy of Sciences)

**Adnane Lazrak**  
Projet GEHC

Janvier 2012 - soutenue le 2 Décembre 2015

- *Amélioration des processus de prévision et de gestion des stocks dans le cas d'une chaîne logistique des pièces de rechange*
- Financement ANRT (CIFRE)
- *Directeur et co-encadrant* : Bruno Castanier, David Lemoine
- Taux d'encadrement : 50 %
- Position actuelle : EMEA Customer Solutions Team Leader chez GE HealthCare

**Martin Géhan**  
Projet GEHC

Octobre 2011 - soutenue le 14 Octobre 2014

- *Contribution à l'intégration des processus de planification de la production et de la maintenance en contexte incertain*
- MESR
- *Directeur et co-encadrant* : Bruno Castanier, David Lemoine
- Taux d'encadrement : 50 %
- Position actuelle : Ingénieur R&D chez Météodyn

#### ENCADREMENT DE MASTERS

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- Arvita Kusumawati : stage de Master « Management and Optimization of Supply Chains and Transport » MOST (IMT Atlantique). *Résolution du Multi-Level Lot-Sizing Problem* (2017, 6 mois)
- Balaji Bandla Chandrababu : stage de Master MOST (IMT Atlantique). *Etat de l'art sur les modèles de planification* (2016, 6 mois)
- Yuan Bian : stage de Master (Université Pierre et Marie Curie). *Modèle mathématique et algorithmes exacts pour la planification tactique avec des contraintes financières du besoin en fonds de roulement* (2014, 6 mois)
- Joseph Shaine : stage de Master « Optimisation en Recherche Opérationnelle » ORO (Université de Nantes). *Planification conjointe de la maintenance et de la production* (2011, 6 mois)

## Publications

La Table 2 synthétise l'ensemble de mes publications

Type	06-08	09	10	11	12	13	14	15	16	17	18	19	20	Total
Revue Internationale	1	1	1	1				1		1	1	1	1	9
Revue Francophone	1			1										2
Chapitre d'ouvrage													2	2
Conférence Internationale	3	1		1		1	1	1	1	3	1	1	1	15
Conférence Francophone	2		1				2		1		1	1		8
Conférence Int. sans actes							1	1	1					3
Congrès Fr. sans actes	4		2	1	1		1		1	2	1	1		14
Colloque sans actes		1				1		1						3
Total	11	3	4	4	1	2	5	4	4	6	4	4	4	56

TABLE 2 – Synthèse des publications

### ARTICLES PUBLIÉS DANS UNE REVUE INTERNATIONALE

1. Yuan Bian, David Lemoine, Nathalie Bostel, and Thomas G. Yeung. *Two-level uncapacitated lot sizing problem considering financial cost of working capital requirement*. *Frontiers of Engineering Management*, 2020. doi: 10.1007/s42524-019-0069-5
2. Hasan Al Hasan, Christelle Gueret, David Lemoine, and David Rivreau. *Surgical case scheduling with sterilising activity constraints*. *International Journal of Production Research*, 57(10):2984–3002, September 2019. doi: 10.1080/00207543.2018.1521015
3. Yuan Bian, David Lemoine, Thomas G. Yeung, Nathalie Bostel, Vincent Hovelaque, Jean-Laurent Viviani, and Fabrice Gayraud. *A dynamic lot-sizing-based profit maximization discounted cash flow model considering working capital requirement financing cost with infinite production capacity*. *International Journal of Production Economics*, 196:319–332, 2018b. doi: 10.1016/j.ijpe.2017.12.002
4. Belgacem Bettayeb, Nadjib Brahimi, and David Lemoine. *Integrated dynamic single item lot-sizing and quality inspection planning*. *International Journal of Production Research*, 56(7):2611–2627, October 2018. doi: 10.1080/00207543.2017.1385869
5. A. El Dor, David Lemoine, M. Clerc, P. Siarry, L. Deroussi, and M. Gourgand. *Dynamic cluster in particle swarm optimization algorithm*. *Natural Computing*, Springer, 14(4): 655–672, 2015. doi: 10.1007/s11047-014-9465-2
6. Laurent Deroussi and David Lemoine. *Discrete Particle Swarm Optimization for the Multi-Level Lot-Sizing Problem*. *International Journal of Applied Metaheuristic Computing*, 2(1):44–57, 2011. doi: 10.4018/jamc.2011010104
7. Michel Gourgand, David Lemoine, and Sylvie Norre. *Metaheuristic for the Capacitated Lot Sizing Problem: a software tool for MPS elaboration*. *International Journal of Mathematics in Operational Research*, 2(6):724–747, 2010a. doi: 10.1504/IJMOR.2010.035496
8. Michael Comelli, Pierre Féliès, David Lemoine, and N. Tchernev. *Tactical planning for optimal cash flow and value sharing in Supply chain*. *International Journal of Logistics Systems and Management*, 5(3-4):323 – 343, 2009. doi: 10.1504/IJLSM.2009.022500
9. Michael Comelli, Michel Gourgand, and David Lemoine. *A review of tactical planning models*. *Journal of Systems Science and Systems Engineering*, 17(2):204–229, 2008. doi: 10.1007/s11518-008-5076-8

## ARTICLES PUBLIÉS DANS UNE REVUE FRANCOPHONE

1. Michel Gourgand, Nathalie Grangeon, David Lemoine, and Sylvie Norre. *Méthode pour une planification robuste : approche par couplage*. Revue d'Ingénierie Numérique collaborative, 3(1-2):Micado, 2011
2. Michael Comelli and David Lemoine. *Optimisation d'un modèle de planification tactique d'une chaîne logistique de type Flow Shop Hybride*. e-revue des Sciences et Technologies de l'Automatique, 5(1):8–13, 2008

## CHAPITRES D'OUVRAGE

1. Pascale Marangé, David Lemoine, Alexis Aubry, Sara Himmiche, Sylvie Norre, Christelle Bloch, and Pétin Jean-François. *Coupling robust optimization and Model-Checking techniques for robust scheduling in the context of Industry 4.0*. In Dolgui Sokolov, Ivanov, editor, Scheduling in Industry 4.0 and Cloud Manufacturing. International Series in Operation Research and Management Science, Springer, In Press
2. Sophie Bretesché and David Lemoine. *Introduction: vous avez dit Transformation?* In Briand Bretesché, editor, Sur les chemins de la transformation digitale. Presses des Mines, In Press

## ARTICLES PUBLIÉS DANS DES ACTES DE CONFÉRENCES INTERNATIONALES

1. Hamza El Garrab, Bruno Castanier, David Lemoine, Adnane Lazrak, and Robert Heidsieck. *Towards Hybrid Machine Learning Models in Decision Support Systems for predicting the Spare Parts Reverse Flow in a Complex Supply Chain*. In 8th International Conference on Information Systems, Logistics and Supply Chain (ILS2020), pages 1 – 10, Austin, USA, April 2020
2. Fabrice Gayraud, David Lemoine, and Guillaume Massonnet. *An Optimal Policy for the Capacitated Lot Sizing Problem with financing of the Working Capital Requirement*. IFAC-PapersOnLine, 52(13):975 – 980, 2019. ISSN 2405-8963. doi: 10.1016/j.ifacol.2019.11.321. 9th IFAC Conference on Manufacturing Modelling, Management and Control MIM 2019
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## Animation et rayonnement

### Animation de la recherche

CO-ANIMATEUR DU GROUPE DE TRAVAIL « BERMUDES »

Depuis 2015

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- Groupe de travail rattaché au GdR Macs depuis 2003 et au GdR RO depuis 2006
- En collaboration avec Sylvie Norre et Christelle Bloch
- Tâches administratives :
  - Participation à la rédaction des bilans annuels et des perspectives d'évolution
  - Refonte et administration du site du GT ([www.gt-bermudes.fr](http://www.gt-bermudes.fr))
- Organisation de sessions :
  - 10 lors des journées STP du GDR MACS depuis 2015
  - 5 lors des congrès de la ROADEF depuis 2015
  - 1 lors de la conférence internationale MIM 2016
  - 2 journées spéciales : à Clermont-Ferrand en 2016 et à Tours en 2017
- Organisation et réalisation de formations doctorales :
  - 1 lors de l'école doctorale MACS 2019 à Bordeaux

### Évaluation de la recherche

ARBITRAGE DE COMMUNICATIONS SCIENTIFIQUES

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- Pour des revues internationales : IJPR, IJPE, C&OR, CIE, RAIRO-OR, JESA.
- Pour des conférences internationales : MOSIM 2014, MIM 2016.
- Pour des congrès francophones : Roadef depuis 2015.

EXPERTISES

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- Évaluation d'un dossier CIFRE pour l'ANRT en 2012

### Formation Doctorale ou à destination des industriels

En 2019, dans le cadre de l'école MACS des Journées Doctorales MACS, j'ai eu la possibilité de participer à l'élaboration d'une session de formation à destination des doctorants. S'appuyant sur le groupe de travail SED (Système à Evenements Discrets) et Bermudes (groupe de travail dont je suis co-animateur), nous avons proposé un module de 2 jours sur les interactions entre modèles de simulation et recherche opérationnelle afin de montrer la complémentarité des techniques issues de ces communautés. Nous avons illustré nos propos avec un cas d'étude portant sur l'ordonnancement robuste d'un atelier de production et proposé une méthodologie permettant d'atteindre le niveau de robustesse souhaité par un décideur.

En 2011, j'ai été contacté par la société Primagaz afin de d'organiser et de dispenser une formation de 15 heures autour de l'effet Forrester. Celle-ci a fait l'objet d'un contrat de 3500 €.

Lors de cette formation, il a fallu, dans un premier temps mettre en évidence ce phénomène au travers d'un jeu pédagogique le « jeu de la bière », puis, dans un second temps, montrer comment nous pouvions tenter de le juguler par l'utilisation d'outils logistiques et en repensant l'organisation de la chaîne logistique.

L'objectif de la formation était de démontrer aux participants l'importance de l'utilisation des outils de la logistique mais également le rôle prépondérant de la communication entre les différents acteurs de la chaîne.

### **Conférences à destination des industriels**

- « *La Supply Chain Finance, Planification tactique de la Supply Chain Financière* ». Les rencontres Appli'Dev, 28 Mars 2019, Le Mans, France.
- « *Planification tactique, état de l'art* ». Salon de l'industrie, 20 Mars 2014, Nantes, France.
- « *Planification tactique, état des lieux et perspectives* ». Journées « Les Chemins de l'Expertise » du Pôle Achat Supply Chain Atlantique (PASCA), 4 Décembre 2009, Saint Nazaire, France.

## RESPONSABILITÉS PÉDAGOGIQUES ET ENSEIGNEMENTS

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L'évaluation de la charge d'un enseignant chercheur à IMT Atlantique diffère des pratiques rencontrées dans les établissements universitaires. En effet, chaque activité liée à l'enseignement (cours, responsabilités, encadrements de projets, etc.) est comptabilisée sous la forme Unité Pédagogique (UP). Cette unité d'œuvre qui permet ainsi de définir la charge portée par l'enseignant chercheur au sein de l'institution, correspond peu ou prou à l'heure équivalent TD du système universitaire. C'est donc cette métrique qui sera utilisée dans la suite de cette partie.

### Responsabilités pédagogiques

Dans cette section, je commencerai par détailler mes responsabilités au sein des formations existantes de l'école. Dans un deuxième temps, je parlerai de la formation d'ingénieur par apprentissage dont j'ai eu en charge l'élaboration pédagogique et la rédaction d'une partie du dossier d'accréditation au titre d'ingénieur. Je finirai par les charges collectives auxquelles j'ai pu participer.

#### Responsabilités dans les formations existantes

Dans cette section, je me propose d'exposer les responsabilités pédagogiques que j'ai eues depuis mon intégration à Mines-Nantes. Dans un premier temps je présenterai la responsabilité d'option et, dans un second temps, la responsabilité d'Unité de Valeur. J'assume ces responsabilités depuis Septembre 2011.

**La responsabilité d'Option** Depuis le passage à un cursus de 3 ans pour la filière ingénieur, l'intégration de chaque étudiant dans son option intervient en début d'année 2. Aussi, le périmètre de responsabilité échu à une personne en charge d'une option s'étend sur les deux dernières années même s'il se concentre plus particulièrement sur l'année terminale. Le rôle du responsable d'option se décompose en trois grandes fonctions :

- L'accompagnement des étudiants,
- La gestion administrative de l'option,
- Le niveau scientifique, la cohérence pédagogique de l'option et son adéquation au monde industriel.

Souvent, la construction du projet professionnel pour un étudiant est un processus long et difficile. Dans ce contexte, le choix de son option est une étape importante. Aussi, l'une des missions du responsable d'option est de le guider dans ce choix à travers un processus se déroulant lors de la première année d'école et comprenant des temps forts tels que :

- les sessions de questions/réponses où les étudiants viennent à la rencontre du responsable d'option pour pouvoir se documenter sur l'option, ses débouchés, etc.
- la journée trajectoire durant laquelle le responsable d'option anime une série de tables rondes pour lesquelles il a invité un certain nombre d'anciens optionnaires afin qu'ils puissent décrire leurs parcours professionnels et répondre aux interrogations des étudiants.

A l'issue de cette année, le responsable d'option reçoit chaque étudiant postulant à l'entrée de son option lors d'un entretien (appelé entretien Plan Personnel de Formation -PPF-) durant lequel l'étudiant expose son projet professionnel et discute de son adéquation avec l'option qu'il

a choisi. Loin d'être une épreuve de sélection, cet entretien permet surtout de s'assurer de la bonne adéquation de l'option avec les attentes de l'élève.

Une fois l'étudiant admis dans l'option, le responsable d'option devient son interlocuteur privilégié jusqu'à la fin de sa scolarité et lui apporte de l'aide quant à une définition plus aboutie de son projet professionnel et de ses éventuels souhaits de parcours à l'étranger (Année de césure, Transfert de crédits, double diplôme, etc.). Il valide également le sujet d'ingénierie proposé à l'étudiant pour son PFE.

Concernant la gestion administrative de l'option, elle se concentre uniquement sur la dernière année et se focalise sur la **planification globale de l'option** (en concertation avec les responsables d'Unité de Valeur -U.V.- concernés), sur la **boucle d'amélioration continue** mise en place par la Direction de la Formation et de la Vie Scolaire (DFVS) pour améliorer la qualité des processus de gestion des options et sur la **participation aux jurys des études** de dernière année.

Le responsable d'option est également chargé de veiller à la **cohérence du programme pédagogique** de l'option et à son **adéquation avec les attentes du marché de l'emploi**. Outre les ajustements de programme qui peuvent intervenir et qui relèvent également du responsable d'U.V. concerné, l'option est actualisée en moyenne tous les 4 ans et le rôle de responsable d'option est de **s'assurer du bon fonctionnement de ce processus** en conduisant la réflexion sur les orientations de l'option, en **collaboration avec un panel d'experts industriels**, puis de sa **déclinaison opérationnelle**.

Ainsi, la responsabilité d'option est une tâche lourde, prenante mais passionnante car le responsable d'option se trouve être au plus proche des élèves tout en étant le relais de l'administration (et notamment de la DFVS), de son département et des anciens de l'option auprès de ceux-ci. Cette place centrale dans la dernière année de formation permet un **suivi et une aide efficace** pour chaque étudiant de son option. Jusqu'en 2014, la charge de responsable d'option était valorisée à hauteur de 20 UPs. Depuis elle l'est à hauteur de 30.

En Septembre 2011, j'ai pris la responsabilité de l'option *Qualité et Sûreté de Fonctionnement* : cela a été une vraie gageure pour moi car les préoccupations de l'option étaient assez éloignées de mon champ de compétences. Aussi cela m'a demandé une vraie immersion dans ce programme pédagogique afin de pouvoir mener à bien cette tâche. Grâce au soutien de l'ancien responsable d'option, j'ai pu la faire évoluer en y intégrant, notamment, toute une partie autour de l'**efficacité opérationnelle**. J'ai pu également mener une **actualisation** qui a abouti à une légère **redéfinition des contours de l'option** devenant ainsi l'option *Management de la Performance et du Risque* (MPR). Depuis 8 ans maintenant que dure cette expérience, je ne peux que me réjouir d'avoir pu tenir ce rôle qui m'a beaucoup appris (que ce soit sur le plan humain, organisationnel ou thématique) et que je pense avoir mené si ce n'est avec succès, du moins avec engagement.

**La responsabilité d'Unité de Valeur** L'unité de Valeur (U.V.) est un ensemble d'activités pédagogiques (cours, TP, TD) d'un volume de 45h (de 60h dans le Master internationale MOST, et de 90h pour la formation ingénieur avant 2012) constituant un ensemble pédagogique cohérent.

En phase de conception, le rôle du responsable d'U.V. est de définir le programme et les objectifs attendus de l'U.V. (en cohérence avec les orientations pédagogiques du cursus auquel l'U.V. appartient) puis de synthétiser le tout dans une fiche descriptive.

Une fois l'U.V. mise en production, il doit s'assurer du recrutement des personnes intervenants



dans celle-ci, de la planification des interventions, du suivi des résultats des étudiants et de leur mise à disposition dans le système d'information. De plus, il doit établir chaque année un bilan de l'U.V. avec les élèves afin d'en assurer l'amélioration, conformément à la politique qualité de l'école.

La Table 3 récapitule le nombre d'U.V. que j'ai encadrées chaque année ainsi que la charge générée. Il est à noter qu'une U.V. de Tronc Commun (donc de promotion complète) ou de 90h sont valorisées à hauteur de 20 UPs, au lieu de 10 UPs.

Année - Type	Tronc Commun	Option (année 2)	Option (année 3)	Master	Charge (UP)
2011			2 (90h)		40
2012	1	1	2 (90h)		70
2013	1	1	4	1	80
2014	1	1	4	1	80
2015	1	1	7	1	110
2016	1	1	7	1	110
2017	1	1	7	1	110
2018	1	2	7		110
Charge totale					770

TABLE 3 – Résumé des charges d'UV par année

### Élaboration de la formation par apprentissage FIT

L'Institut Mines Telecom, membre fondateur de l'Alliance industrie du futur depuis 2015, encourage fortement le développement des formations et des technologies de demain. En mars 2016 a été lancé le projet « Osons l'industrie du futur », mis en œuvre par l'Alliance industrie du futur dans le cadre du projet Nouvelle France industrielle. Ce projet a pour mission d'accompagner les profondes mutations des métiers et des qualifications.

Dans ce contexte, début 2018 et en partenariat avec ITII, l'école a décidé de mettre en place une formation d'ingénieur par apprentissage autour du génie industriel pour l'industrie 4.0 : la Formation d'Ingénieur par apprentissage Transformation digitale des systèmes industriels (FIT). Celle-ci recrutera majoritairement à l'issue d'un cursus à l'IUT (et particulièrement QLIO...). J'ai été *mandaté pour élaborer le programme pédagogique* de cette nouvelle formation, en collaboration avec une professeur en sociologie du département des Sciences Sociales et de Gestion.

Pour mener à bien cette mission, nous avons sollicité un *certain nombre d'industriels* (Airbus, Thalès, Vaillant Group, Aplix, etc.) pour intégrer un comité qui nous a formulé un avis sur les orientations de la formation et exprimé ses besoins et attendus concernant une telle offre de formation. Fort de leurs recommandations, nous avons élaboré d'une part les *objectifs pédagogiques visés* et d'autres part le *programme complet* de cette dernière.

Comme l'illustre la Figure 1, cette formation s'appuie sur un socle important de compétences scientifiques de base (Mathématiques, Physiques, etc.) et en Sciences Sociales et de Gestion. Elle repose sur trois piliers « domaines » fondamentaux que sont :

- Le Génie Industriel,
- L'Informatique,
- Les Sciences Sociales et de Gestion au travers de l'étude des Systèmes Socio-Techniques.

L'énergie et l'environnement sont des problématiques traitées de façon transverse à toutes les disciplines.

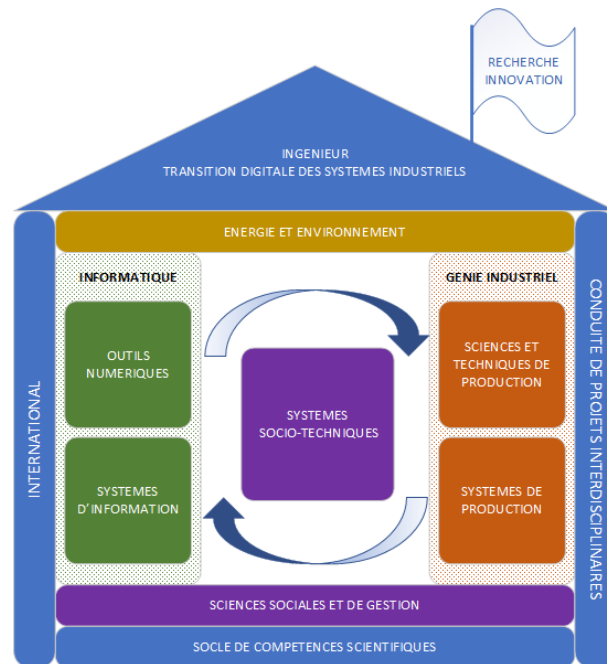


FIGURE 1 – La Formation par Apprentissage FIT

Nous avons également ***fortement contribué à l'écriture du dossier CTI*** en vue de son accréditation pour délivrer le titre d'« ingénieur diplômé ». Celle-ci a été accréditée en Janvier 2020 et sera donc déployée dès Septembre de cette même année. ***J'en prendrai la co-responsabilité.***

## Responsabilités collectives

Dans cette partie, je détaille les charges collectives auxquelles j'ai pu contribuer. Mise à part la présence au concours d'entrée Mines-Télécom, aucune de celles-ci n'est valorisée en termes d'UPs.

**Communication et Recrutement** Lorsque l'école recrutait sur le concours commun Mines-Télécom, j'ai pu participer aux *oraux « professionnels » du concours*. Entre 2010 et 2017, j'ai assuré une présence de 32 jours sur le concours (soit une charge de 96 UPs au total).

J'ai également pu participer à 2 *forums étudiants* (Formatech) ainsi qu'à une journée porte ouverte. Cependant, ce type de communication a été abandonné par l'école en 2012.

L'école est également dotée d'un incubateur d'entreprises. Depuis 2016, je fais partie du *jury de pré-admission et du jury d'admission* dans ce dernier.

**La naissance d'IMT atlantique : le Projet Grand Ouest** Le Projet Grand Ouest (PGO) est le projet dont l'aboutissement a été la création d'IMT Atlantique. Dans ce cadre là, j'ai pu participer à *plusieurs groupes de travail* et notamment :

- le Groupe de Travail Formation qui a eu la charge de définir l'architecture et le contenu disciplinaire des années de formation,
- le Groupe de Travail Système d'Information qui a fourni les préconisations quand au choix du système d'information de l'école.

Depuis la création effective d'IMT Atlantique, j'ai pu aussi m'impliquer dans *divers groupes de travail* dont le but était de définir de façon effective le contenu des Unités d'enseignements, et notamment :

- Le Groupe de Travail Projets qui définit pour chaque semestre, le type de projet qu'aura à mener un élève
- Le Groupe de Travail Système en Réseau qui définit les concepts devant être enseignés autour des réseaux (Logistiques, énergétiques, informatiques, etc.)
- Le Groupe de Travail Thématique d'Approfondissement qui a pour charge de définir les contours des TAFs.

Dans le cadre du Groupe de Travail TAF, j'ai eu la charge de *définir les grandes lignes de la TAF Management de la Performance et du Risque des systèmes industriels complexes (MPR)* qui est une transposition de l'option dont j'ai la charge dans le nouveau cursus IMT Atlantique. La définition précise et la mise en œuvre opérationnelle sont assurées par un autre enseignant chercheur.

Enfin je suis aussi *Maitre d'ouvrage sur le Système d'Information* Compétences/Jury : en effet, la démarche d'évaluation par les compétences étant à ce jour peu répandue, le système d'information de l'école ne sait pas prendre en compte cette spécificité. Aussi un système d'information spécifique doit être élaboré.

**Vie institutionnelle** Je fais partie de plusieurs instances liées à la vie institutionnelle de l'école, en tant qu' élu suppléant :

- depuis 2014 au Comité Technique (Mines Nantes puis IMT Atlantique),
- depuis 2014 au CHSCT (Mines Nantes puis IMT Atlantique),
- depuis 2017 au Comité de l'Enseignement.

Entre 2013 et 2017, j'ai été nommé au Conseil de Discipline. Depuis 2017 et en tant qu' élu suppléant au Comité de l'Enseignement, je suis devenu suppléant à ce même conseil.

## Enseignements

J'ai débuté mes enseignements en 2006, au début de ma thèse de doctorat, en tant que vacataire à *l'IUT d'Allier*. De façon concomitante, j'ai pu également enseigner au *Centre d'Auvergne du Conservatoire National des Arts et Métiers*. En 2008, l'année d'obtention de mon doctorat, j'ai poursuivi mes enseignements à *l'IUT de Roanne* dans le cadre d'un demi-poste d'Attaché Temporaire d'Enseignement et de Recherche (ATER). Cependant, c'est depuis mon recrutement à *Mines Nantes* en Septembre 2009 en tant que Maître Assistant Associé puis en tant que Maître Assistant en 2010, que j'ai dispensé l'essentiel de mes activités d'enseignement. Aussi, je commencerai à présenter ces dernières et j'évoquerai ensuite et plus brièvement mes activités dans les autres établissements.

### Mines Nantes, IMT Atlantique

Dans cette section, je présente les activités que j'ai pu mener depuis mon intégration à Mines Nantes, dans le département Automatique, Productique et Informatique, et directement liées à mes enseignements. Celles-ci se décomposent en trois parties : les cours, les projets et, pour finir, des activités annexes telles que les soutenances ou les innovations pédagogiques.

#### Les Enseignements

Concernant les cours que j'ai pu donner, ils se situent majoritairement dans le champ du Génie Industriel et vont du *niveau Bac+3 au niveau Bac+5*. Mes principales interventions se situent dans le tronc commun de formation ingénieur, dans l'option *Gestion des Opérations en Production et Logistique* (GOPL) et dans le Master International *Management and Optimization of Supply chains and Transport* (MOST).

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#### PRINCIPAUX COURS ET EXERCICES PÉDAGOGIQUES

<b>Planification de la production</b>	2009 - ...
<i>Bac +5, GOPL</i>	15 UP

- Après avoir situé les principales problématiques liées à la planification de la production, ce cours s'attache à dresser un panorama complet des modèles mathématiques de base et des méthodes de résolution y afférents. Un accent est également mis sur la modélisation de situations concrètes que les futurs ingénieurs pourront être amenés à traiter durant leur carrière et sur la notion de capacité de production et son lien avec les décisions plus opérationnelles (ordonnancement, etc.).

<b>Tactical Planning</b>	2009 - ...
<i>Bac +5, MOST</i>	18.75 UP

- *En langue anglaise* et s'adressant à un public dont l'expertise en Recherche Opérationnelle est plus faible que pour la filière ingénieur, ce cours présente les enjeux de la planification tactique et s'attache à donner les propriétés structurelles fondamentales rencontrées dans les situations les plus simples. Quelques algorithmes exacts et approchés

(et notamment l'Algorithme de Wagner-Whitin) sont également proposés aux étudiants, dans le cadre d'une planification à capacité infinie.

### Pilotage Industriel de l'Entreprise étendue

Bac +3, Tronc Commun

2012 - 2019

17 UP

- Ce cours introductif présente les principaux concepts et problèmes rencontrés en logistique. Chaque grande fonction logistique est illustrée par une étude de cas simple et traitée de façon quantitative afin de montrer quelques techniques scientifiques pouvant être mises en place pour pouvoir apporter des réponses efficaces et pertinentes à ces problématiques.

#### SYNTHÈSE DES ENSEIGNEMENTS

Intitulé	Type	Niveau	U.P.	Lieu	Année
<b>INFORMATIQUE</b>					
Algorithmics	CM/TD/TP	BAC+4	15	Mines Nantes	2009-2010
<b>RECHERCHE OPÉRATIONNELLE</b>					
Graphs	CM/TD	BAC+4	10	Mines Nantes	2009
Mixed Integer Programming	CM/TD	BAC+4	10	Mines Nantes	2009
Recherche Opérationnelle : Résolution de Problèmes	TD	BAC+4	30	Mines Nantes	2009
Optimisation : Programmation Linéaire	TD	BAC+3	11,25	Mines Nantes	2009
<b>GÉNIE INDUSTRIEL</b>					
Management de la qualité	TD	BAC+5	3,75	IMT Atlantique	2012-2019
Planification de la production	CM	BAC+5	15	IMT Atlantique	Depuis 2009
Tactical Planning	CM	BAC+5	18,75	IMT Atlantique	Depuis 2009
Tactical Planning	CM	BAC+5	8	Univ. Nantes	2010-2011
Pilotage Industriel de l'Entreprise étendue	TD	BAC+4	17	IMT Atlantique	2012-2019
Introduction aux techniques de l'efficacité opérationnelles	TD	BAC+4	11	IMT Atlantique	2012-2019
Simulation Stochastique	TD	BAC+4	15	Mines Nantes	2012-2015
Management des opérations	TP	BAC+4	7,5	Mines Nantes	2009-2010
Modèles de prévision et Gestion des stocks	TP	BAC+4	7,5	Mines Nantes	2009-2010

TABLE 4 – Cours dispensés à Mines Nantes/IMT Atlantique

#### Encadrements de projets et stages

Comme souligné par la Commission aux Titres d'Ingénieur (CTI) dans son avis relatif à l'accréditation de Mines Nantes en 2014<sup>1</sup>, la pratique d'un enseignement par projets fait partie des points forts de Mines Nantes. Aussi, l'encadrement de projets d'élèves tient une place importante

1. Avis n°2014-05-02

dans l'activité d'un enseignant chercheur de cette école. Pour ma part, j'ai surtout participé à l'encadrement des projets suivants :

- **Mission Courte (M.C.)** : Les objectifs de ce projet sont de permettre à l'élève ingénieur de 2<sup>e</sup> année de « *devenir un acteur du monde industriel en proposant une réponse adaptée à une problématique industrielle* » dans le cadre d'une immersion d'un mois, en groupe, en entreprise. Ce projet est donc mené par un groupe d'environ 4 étudiants en fin de semestre 3 de la formation ingénieur.
- **P.I.S.T** : Le Projet d'Ingénierie Scientifique et Technique (P.I.S.T, ex Projet d'Orientation Scientifique et technique) est le projet d'ingénierie du semestre 4, portant sur des sujets à spectre large et donc demandant aux étudiants une forte autonomie dans l'acquisition des savoirs nécessaires à la résolution de la problématique posée. Basé sur des équipes d'environ 8 personnes, il nécessite un renforcement du volet méthodologique (et notamment la gestion de projet). Il s'agit donc d'un apprentissage collaboratif et coopératif où plusieurs alternatives sont proposées :
  - S'initier / se former par la recherche avec pour objectif principal d'analyser et synthétiser des données scientifiques ;
  - Se placer dans une démarche d'entrepreneuriat ; montage d'une activité ;
  - Résoudre un problème d'ingénieur complexe alliant conception d'un système avec ou sans réalisation technique.
 A l'issue de celui-ci, les élèves doivent devenir autonomes en étant acteurs de leur projet.
- **Projet d'Option (P.O, ex P.C.I.)** : L'objectif du projet de semestre 5 est de développer les capacités des élèves à résoudre un problème d'ingénieur au sein d'une équipe. Chaque élève doit prouver son aptitude à mobiliser ses compétences et celles de ses partenaires en tenant compte de la particularité du contexte de la question posée. La question posée est relative à une problématique industrielle dans un contexte d'innovation dans les méthodes, outils et pratiques du domaine lié à son option. La question doit alors mener à la construction d'une solution spécifique au problème. Dans ce cadre, les élèves devront mettre en place une approche structurée de gestion de projet et mettre en œuvre des compétences techniques fortes.
- **Projet de Fin d'Étude (P.F.E.)** : D'une durée de 6 mois, le Projet de Fin d'Étude présente 2 objectifs distincts et complémentaires :
  - C'est un exercice de validation finale. Le projet confié est représentatif d'une mission d'ingénieur et doit présenter une réelle valeur ajoutée pour l'entreprise. Ainsi il permet d'évaluer la capacité de l'élève à agir avec compétence en situation professionnelle réelle.
  - C'est un exercice pédagogique. De par le spectre couvert par le sujet, de par la nature et la qualité de l'encadrement Entreprise et Ecole, l'élève est amené à identifier, acquérir et mobiliser des compétences, puis à les combiner dans un nouveau contexte. Il apprend à tirer les leçons d'une expérience et à alimenter une réflexion sur son Projet Professionnel.

Hors la mission courte, mon implication dans ces projets *ne se limite pas à l'encadrement* de ceux-là puisque cela englobe également la *proposition du sujet du projet* concernant le projet PIST, la *prospection des entreprises et la définition du sujet pour les projets d'option ainsi que la validation des sujets de stage concernant les PFEs*.

La Table 5 recense le nombre de projets que j'ai pu encadrer depuis 2009 ainsi que la charge (exprimée en UPs) qui leur est associée.

Année	M.C.	P.I.S.T	P.O.	P.F.E.	Total	Nb UPs
2009				3	3	36
2010		0,5	1	5	6,5	108
2011	1	0,5	3,5	4	9	186
2012			2	3	5	111
2013		1	3	5	9	130
2014		1	3	7	11	164
2015	1	1	2	5	8	130
2016	3		2	5	10	98
2017			3	7	10	134
2018			3	5	8	100
Total	4	4	22,5	49	79,5	1197

TABLE 5 – Type et nombre de projets encadrés depuis 2009

### Autres établissements

Entre 2006 et 2008, durant ma thèse de doctorat, j'ai enseigné en tant que *vacataire au sein de l'IUT d'Allier* et notamment dans le département Gestion Logistique et Transport (GLT). Pendant ces trois ans, j'ai pu enseigner dans plusieurs disciplines (Mathématiques, Logistique, Informatique) et à plusieurs niveaux (*IUT et Licence professionnelle*). J'ai également pu m'investir dans des tâches collectives (telles que le suivi des élèves, la promotion du département GLT, etc.). Ces années ont été décisives dans mon parcours professionnel car c'est grâce à cette expérience que j'ai décidé de m'orienter dans le monde de l'enseignement supérieur. Parallèlement, les cours que j'ai pu dispenser au CNAM, en recherche opérationnelle, m'ont permis d'appréhender l'enseignement à un public très hétérogène en termes de parcours et de niveaux ce qui a été particulièrement enrichissant et challengeant.

A la suite de ma thèse j'ai également enseigné en tant qu'*ATER à l'IUT de Roanne* dans le département QLIO. C'est dans ce contexte que j'ai commencé à focaliser mon enseignement sur les disciplines relevant du Génie Industriel : ceci m'a également beaucoup appris thématiquement parlant et c'est durant cette année que j'ai pu appréhender et développer mes connaissances sur l'efficacité opérationnelle (j'ai pu notamment assister à l'université du Lean qui s'est tenue à Sevrier, près d'Annecy du 1<sup>er</sup> au 3 Septembre 2008). C'est fort de cette connaissance là que j'ai pu introduire dans le programme de l'option QSF de Mines Nantes, l'UV sur l'*efficacité opérationnelle*.

Durant les années 2015, 2016 et 2017, j'ai également enseigné les mathématiques dans les UVs Outils logiciels du Département GEII *de l'IUT d'Angers*. Outre les outils mathématiques fondamentaux qui étaient abordés de façon simple (Polynômes, Transformés intégrales, résolutions d'équations différentielles, etc.), l'idée était également d'initier les étudiants à la résolution numérique de problèmes mathématiques (en utilisant Scilab).

De 2010 à 2013 puis depuis 2017, j'interviens en tant que vacataire au département GLT de *l'IUT d'Allier* sur des thématiques liées à la planification de la production (MPR II) au niveau Bac +3 et à la conception et pilotage de la chaîne logistique en DUT (Bac +2).

Je suis également sollicité chaque année pour être membre des *jurys de soutenance de projets de fin d'étude* pour les « Managers de la chaîne logistique (MCL) » (BAC +5) et les « Responsables Opérationnels des Flux (ROF) » (BAC +4) du *CNAM ISERPA d'Angers* ainsi que pour les étudiants en *Master Informatique parcours Optimisation en Recherche Opérationnelle « Optimisation en Recherche Opérationnelle (ORO) » de l'université de Nantes* et qui est co-habilité avec l'école.

### Innovation pédagogique : le projet IceScream

Lors de mon arrivée à Mines Nantes et dans mon cours de « planification de la production » en dernière année de l'option GOPL, j'ai pu mettre en place le « jeu de la bière »<sup>2</sup> afin d'illustrer la dynamique des systèmes et plus précisément l'effet Forrester. Cependant, si ce jeu permet de mettre en évidence ce dernier, il ne permet pas, de manière aisée, la mise en place d'outils visant à tenter de le juguler.

Le but d'IceScream est donc de proposer une suite au jeu de la bière, au travers d'un jeu sérieux dans lequel le joueur évolue dans une entreprise de fabrication de glace, son objectif étant de mettre en place un certain nombre d'outils permettant d'amoinrir l'effet coup de fouet. Actuellement ce jeu est en phase « pilote » et librement utilisable sur <http://icescream.imt-atlantique.fr><sup>3</sup>.

Cette version pilote a été développée conjointement avec IMT Atlantique, IMT Mines-Albi-Carmaux et le CNAM Pays De la Loire. Elle a été financée par :

- La fondation UNIT à hauteur de 30 k€
- Le CNAM à hauteur de 20 k€
- La Grande Ecole Virtuelle (GEV) pour 20 k€
- UNR Auvergne pour 10 k€
- IUT en ligne pour 5 k€

**En tant que porteur du projet**, mon rôle a initialement été de **monter le groupe projet** en sollicitant des partenaires académiques, de rechercher puis de monter les **dossiers de financement et de les défendre devant les financeurs**.

Une fois les financements obtenus (à hauteur de 85 k€), j'ai participé au développement du jeu en tant qu'**expert pédagogique** ainsi qu'aux différentes phases de test avec divers utilisateurs. J'ai également pris en charge son développement algorithmique et notamment le simulateur embarqué dans le jeu. Toutefois, IceScream demande à être développé davantage : des demandes de financement vont être déposées afin de poursuivre l'aventure et de le rendre pleinement utilisable dans le cadre d'une formation.

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2. <https://beergame.org>

3. lien testé le 02/12/2019





Deuxième partie

**Synthèse Scientifique**



# Introduction

Dans cette partie, je présenterai mes activités de recherche et notamment celles effectuées depuis l'obtention de mon doctorat en 2008.

Dans un premier chapitre, je ferai une synthèse des différentes thématiques que j'ai pu abordées, l'intégration des décisions étant un axe fort du développement de ma recherche.

Ensuite, je détaillerai plus précisément deux thématiques liées à l'intégration décisionnelle et que j'ai particulièrement investies :

- L'intégration de la maintenance dans le processus de planification de la production. Il s'agit plus particulièrement d'étudier l'impact de la politique de maintenance dans l'élaboration de plans de production.
- L'intégration des décisions de financement dans le processus de planification de la production. Faisant suite à des travaux que j'avais pu mener lors de ma thèse, cette recherche s'inscrit dans deux contrats de recherche (RCSM et Fileas Fog) et vise à mettre en évidence l'impact de la décision financière dans la décision de production.



# Chapitre 1

## Synthèse des activités de recherche

J'ai débuté ma recherche lors de mon Master en 2005 sur le thème de la planification tactique de production. En partant de la méthode « Materials Requirements Planning (MRP) » d'Orlicky Orlicky et al. (1994) qui planifie produits finis et composants à capacité infinie, j'ai rapidement été amené à m'intéresser à son extension, le modèle MRP 2 « Manufacturing Ressources Planning » Vollmann et al. (1997) dont l'une des avancées majeures est la prise en compte de la capacité des ressources de production. La figure 1.1 schématise le fonctionnement de cette méthodologie.

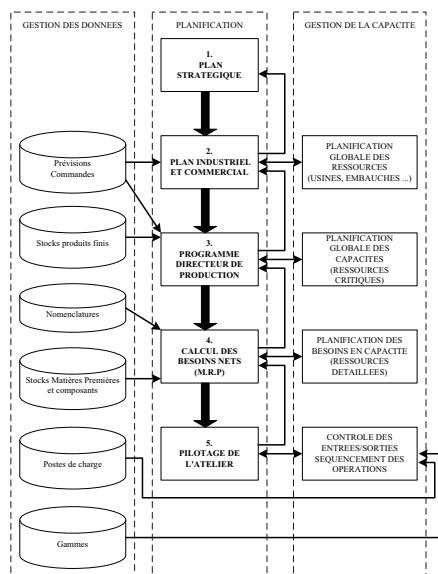


FIGURE 1.1 – La structure hiérarchique de planification MRP II Vollmann et al. (1997)

Ce principe de planification hiérarchisée est toujours très usité dans le monde industriel malgré ses limites essentiellement dues à son manque d'intégration décisionnelle et à une gestion grossière de la capacité, limites qui peuvent conduire au mieux à une gestion peu efficace en terme de coûts et d'occupation de la ressource, au pire à des problèmes d'infaisabilité des plans

de production Bitran and Tirupati (1993); Pochet and Wolsey (2006).

Pour pallier ces difficultés et en particulier celles liées à la minimisation des coûts et à une meilleure gestion de la capacité, des modèles mathématiques dits de « Lot Sizing » ont été développés Drexl and Kimms (1997). Cependant, ces modèles souffrent également de cette même carence à gérer efficacement la ressource car la capacité est estimée de façon très grossière et sa consommation, modélisée linéairement. Or, comme le soulignent Byrne and Bakir (1999), cette consommation n'est généralement pas linéaire car elle dépend entre autres de l'ordonnancement de cette même production et des files d'attente qui ne manqueront pas d'être générées lors de la fabrication : on voit donc que l'utilisation de ces modèles mathématiques ne pallient pas les problématiques d'intégration de la décision (qu'on appelle usuellement l'intégration verticale).

Depuis, on a vu l'émergence des entreprises en réseau et du réseau logistique. Cela a encore complexifié les processus de planification industrielle puisqu'apparaît alors la nécessité de coordonner l'ensemble des activités de ce réseau, comme l'illustre la Figure 1.2.

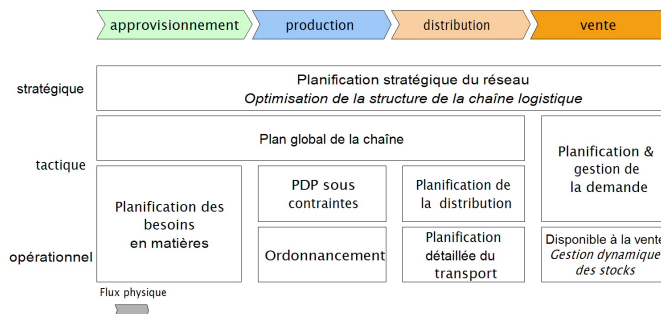


FIGURE 1.2 – Planification de la chaîne logistique Meyr et al. (2008)

Donc au problème d'intégration verticale, s'ajoute le problème de synchronisation horizontale comme l'illustre la Figure 1.3.

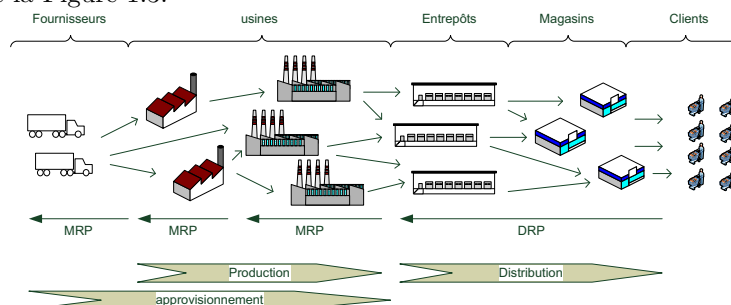


FIGURE 1.3 – L'intégration horizontale

Depuis ma thèse, ces problématiques d'intégration et de synchronisation constituent l'un des deux axes principaux de ma recherche. Le second axe que j'ai également initié durant ma thèse en collaboration avec Michael Comelli et Pierre Fenies Comelli et al. (2009) vise à intégrer les décisions financières dans le processus de planification de la production : en effet, traditionnellement la planification de production vise à minimiser les coûts engendrés par les flux physiques (production, stockage, lancement en production, etc.) mais ne considère pas son impact sur la

planification financière de l'entreprise (par exemple en termes de budgétisation monétaire ou encore en besoin en fond de roulement pour assurer le financement du cycle d'exploitation).

Ainsi, depuis ma thèse de doctorat, c'est sous l'angle de l'intégration des décisions dans le processus de planification de la production que j'ai poursuivi mes travaux de recherche.

La Figure 1.4 schématise la répartition thématique de ma recherche depuis l'année 2005.

## 1.1 L'intégration verticale et horizontale

L'un des objectifs principaux de mes travaux de thèse a été d'élaborer un modèle de connaissance générique pour la planification tactique multi-site et sa traduction en modèle mathématique Lemoine (2008). De type centralisé Thierry et al. (1994), ce modèle « lot sizing » fait apparaître trois types de ressource partageant bon nombre de caractéristiques, notamment la capacité :

- la production,
- le stockage,
- le transport.

Comme l'ont montré Dauzère-Péres and Lasserre (1994), la capacité de production est fortement liée à l'ordonnancement qui découle des plans de production. Dans le projet Orloges Thomas et al. (2014), nous avons pu également montrer que cette constatation se transposait aux ressources de transport et aux problématiques de tournées de véhicule : en effet, le problème à résoudre était de déterminer des quantités à transporter afin de pouvoir approvisionner ou déblayer des chantiers de constructions de route tout en tenant compte des capacités de transport et des politiques d'optimisation du chargement des camions (camions pleins et éviter au maximum les retours à vide), propres à ce type de problématique. Ainsi, nous avons instancié le modèle générique (intégration horizontale) en le particulierisant pour tenir compte des spécificités d'optimisation du chargement, la faisabilité étant testée par un autre module dédié à la planification de tournées de véhicule. Bien que se pose alors la problématique de l'intégration verticale, nous n'avons pas poursuivi, pour le moment, de recherche dans cette direction mais c'est une perspective qui nous paraît particulièrement intéressante.

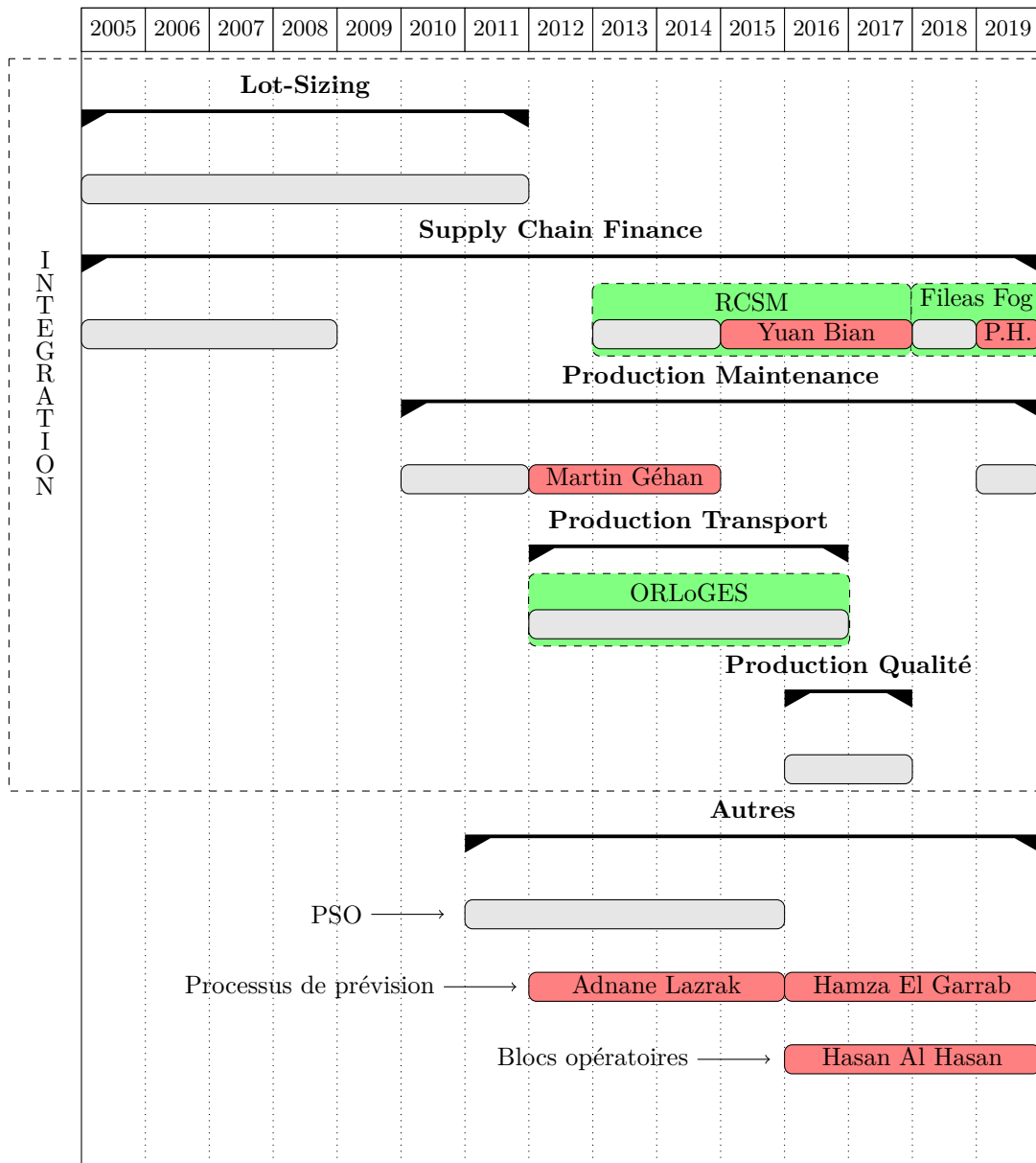
Toujours autour de cette problématique d'intégration verticale, j'ai effectué une action de recherche en collaboration avec Belgacem Bettayeb et Nadjib Brahimy autour de l'intégration du processus d'inspection de la qualité « produit » dans le processus de planification de la production. L'objectif de cette étude était d'en montrer l'intérêt en fournissant un premier modèle de « lot-sizing » mono-produit et à capacité finie permettant :

- de planifier la fabrication d'un type de produit en prenant en compte la capacité consommée par le processus d'inspection et en minimisant les coûts induits (logistiques, de non conformités et d'inspection),
- de mesurer l'impact de la capacité consommée par le processus d'inspection sur la qualité fournie au client.

Cette recherche a fait l'objet d'une publication en conférence internationale Bettayeb et al. (2016) et d'une extension en revue Bettayeb et al. (2018).

A mon arrivée à Mines Nantes, j'ai eu l'opportunité de débiter une collaboration avec Bruno Castanier dont le cœur d'expertise se trouve être la maintenance et notamment celle des systèmes industriels. Partant du constat que peu de travaux avaient été menés sur l'intégration de la maintenance dans le processus de planification de la production alors que son impact sur la capacité de production est loin d'être négligeable (les pannes machine engendrant des temps d'arrêt et de la maintenance curative ou encore la capacité consommée par la maintenance préventive par exemple), nous avons commencé par proposer une approche itérative visant à assurer la faisabilité des plans de production sous une certaine probabilité fixée par le décideur





Légende : ■ Projet  
■ Thèse ■ Hors thèse

FIGURE 1.4 – Répartition de mes thématiques de recherche depuis 2005

Castanier and Lemoine (2011a). Les résultats obtenus étant prometteurs, cette problématique a fait l'objet d'une première thèse qui a été soutenue en 2014 par Martin Géhan (2014). Cette recherche autour de la planification de la production et de la maintenance fera l'objet d'un développement spécifique au chapitre 2 de cette partie puisqu'elle correspond à la dynamique de

recherche initiée depuis mon intégration dans l'équipe Système Logistique et de Production du LS2N.

## 1.2 L'intégration financière

C'est durant ma thèse et grâce à une collaboration avec Michael Comelli et Pierre Fenies que j'ai pu commencer une recherche autour de l'impact de la planification de la production sur le management financier de l'entreprise, voire de la chaîne logistique.

Traditionnellement, le critère d'évaluation des plans de production se base sur une évaluation des coûts logistiques générés. Or il a été souligné l'intérêt de manager par les critères financiers Vickery et al. (2003). Cependant, ceux-ci (qui sont essentiellement d'ordre budgétaires) sont usuellement calculés en prenant comme données d'entrée le résultat du processus de planification. De plus, et au delà de l'estimation budgétaire pouvant être faite sur la base des plans de production générés, il est également nécessaire sur un horizon moyen terme d'évaluer les fluctuations de trésorerie afin d'anticiper les périodes de trésorerie négative pouvant entraîner une cessation de paiement et les périodes de trésorerie positive pouvant être optimisées par un processus de Cash Management Badell et al. (2005). Aussi, s'appuyant sur les liens forts unissant les flux physiques et financiers, nous avons proposé une fonction générique financière permettant d'évaluer le Cash-Flow généré par les plans de production d'une chaîne logistique. Cette fonction a été implantée dans un modèle de planification multi-site permettant ainsi d'optimiser le processus de planification en maximisant le Cash Flow Généré par la chaîne. Ensuite, en nous appuyant sur un modèle de coûts ABC Cooper and Kaplan (1991), nous avons proposé un modèle de partage de la valeur générée entre les différentes entités constitutives de la chaîne logistique, afin d'assurer un niveau minimum de trésorerie à chacune d'elles et de garantir la survie de la chaîne logistique. Ces travaux ont donné lieu à une publication en conférence internationale Comelli et al. (2006b) et à un article en revue Comelli et al. (2009).

En 2013, c'est par le biais du projet FUI 15 *Risk, Credit Chain & Supply Chain Management (RCSM)*, que j'ai poursuivi ma recherche sur l'intégration de la finance dans le processus de planification. Dans ce projet à vocation industrielle, notre objectif était de proposer une fonction financière permettant d'évaluer le besoin en financement d'une entreprise lui permettant d'assurer son cycle d'exploitation. Couplée à un modèle de « lot-sizing », elle devait permettre de minimiser ce besoin en fond de roulement afin de permettre à l'entreprise de se financer au meilleur coût (et à moindre risque pour les financeurs) auprès d'acteurs de sa filière industrielle via une plateforme web de financement. Ces travaux ont fait l'objet d'une thèse de doctorat qui a été soutenue en 2018 par Yuan Bian Bian (2017) et se poursuivent désormais grâce au projet ANR *Integrating Financial Health in Supply Chain Management Intégration de la dimension Financière dans l'Optimisation des chaînes logistiques (FILEAS-FOG)* dont je suis co-porteur avec Vincent Hovelaque du CREM de l'université de Rennes. Je détaillerai de manière plus approfondie cette axe de recherche dans le troisième chapitre de cette partie.

## 1.3 Autres travaux

Dans cette section, je présente rapidement les autres travaux que j'ai pu mener et qui se trouvent un peu à la marge de ma dynamique de recherche.

J'ai continué de travailler sur les problèmes d'optimisation de la production et notamment sur le problème du *Multi Level Lot Sizing Problem (MLLP)* qui est un problème de planification multi-niveau à capacité infinie en collaboration avec Laurent Deroussi. En nous appuyant

sur le principe d'heuristique mono-niveau à modification de coûts Blackburn and Millen (1982); Bookbinder and Koch (1990), nous avons développé une méthode d'optimisation couplant une approche par Essaim Particulaire Eberhart and Kennedy (1995) et l'algorithme de Wagner-Whitin Wagner and Whitin (2004) qui planifie la production niveau par niveau. L'intérêt de cette approche était d'utiliser une Particle Swarm Optimization (PSO) dans un environnement continu (l'espace des coûts) contrairement à l'approche discrète proposée par Han et al. (2009). Nous avons montré que, par ce biais, une solution optimale du problème était accessible et nous avons testé cette méthode sur le jeu d'instances de Han et al. (2009) sur lequel nous avons surpassé les résultats obtenus par sa PSO discrète. Cette recherche a fait l'objet d'une conférence internationale Deroussi and Lemoine (2009) et d'une publication en revue Deroussi and Lemoine (2011).

Dans le cadre du projet GDR-RO *Vers une intelligence particulière*, j'ai pu travailler en collaboration avec Abbas El Dor, Laurent Deroussi, Patrick Siarry, Maurice Leclerc et Michel Gourgand autour de l'élaboration d'une nouvelle topologie pour la PSO. Dans le cadre de cette méthode, la topologie définit la façon de communiquer entre les différentes particules de l'essaim. Aussi, la performance de la méthode est fortement corrélée au design de la topologie utilisée Kennedy (1999). L'idée était donc d'en proposer une, adaptative et basée sur une interconnection de clusters de particules Mendes et al. (2004) constitués de manière dynamique en fonction des performances de chaque particule. Puis celle-ci a été testée sur un ensemble de fonctions de test issu de la littérature et de problèmes réels C. Cagnina et al. (2008); Suganthan et al. (2005) : les résultats obtenus montrent la pertinence d'une telle topologie. Cette recherche a fait l'objet d'une publication en revue Dor et al. (2015).

Courant 2015, j'ai été sollicité par Christelle Guéret et David Rivreau pour travailler avec eux sur un problème de planification de blocs opératoires prenant en compte les contraintes liées aux activités de stérilisation des matériels utilisés au cours des opérations (des kits opératoires). Cette problématique à laquelle le CHU d'Angers est confronté, a donné lieu à une thèse soutenue en 2019 par Hasan Al Hasan Al Hasan (2019) co-financée par Angers-Loire-Métropole et IMT Atlantique. S'appuyant sur l'unité de soin orthopédique du CHU comme champ expérimental, Hasan Al Hasan a proposé un certain nombre d'outils d'aide à la décision permettant d'optimiser la planification des opérations chirurgicales tout en tenant compte des contraintes liées à la stérilisation des kits opératoires. Partant d'un cas déterministe, il a proposé un modèle mathématique en programmation linéaire mixte qu'il a ensuite pu robustifier de façon à pouvoir tenir compte des incertitudes liées aux données opératoires et permettre ainsi une planification plus réaliste au regard de la réalité opérationnelle de l'unité de soin. Ensuite, il a proposé une technique d'horizon roulant afin de permettre une déclinaison opérationnelle de sa recherche. Ses résultats ont été présentés aux praticiens hospitaliers intervenants dans l'unité orthopédique et ont montré une amélioration plus que sensible du processus de planification. Ces travaux ont fait l'objet d'une communication en conférence internationale Al Hasan et al. (2018b) et d'une publication en revue Al Hasan et al. (2019).

Depuis 2012 et en compagnie de Bruno Castanier, nous avons initié une collaboration avec General Electric HealthCare (GEHC) autour de la gestion de la supply chain de pièces de rechange. Cette collaboration a donné lieu à deux thèses CIFRE dont l'une a été soutenue en 2015 par Adnane Lazrak Lazrak (2015) et l'autre est actuellement en cours. Dans la première thèse, Adnane Lazrak a travaillé sur la fiabilisation du processus de prévision de la demande de pièces de rechange dans le cadre de la chaîne logistique de GEHC. Dans ce contexte, il a proposé de nouvelles approches de sélection de méthodes de prévision fonction d'une segmentation par profil de la demande et du niveau de maturité des pièces de rechange et se basant sur des critères de

Supply Chain Management tel que la priorisation en service ou en inventaire. Puis il a proposé une intégration des processus de prévision et de gestion des stocks permettant également une différenciation client basée sur des niveaux de service contractualisés. Les résultats de ses travaux ont été présentés en conférence francophone Lazrak et al. (2014) et, puisque c'est aussi le but d'une thèse CIFRE, ont été déployés chez GEHC : ils ont contribué à des améliorations significatives en termes de niveau de service et d'inventaire chez GEHC. A l'issue de cette thèse et devant les résultats probants obtenus industriellement, nous avons enchaîné en 2016 sur une deuxième thèse CIFRE qui s'intéresse à un processus aval de la prévision de la demande : le processus de réparation des pièces détachées défectueuses et, notamment, la prévision de la charge générée par les demandes de réparations. En effet, GEHC a beaucoup de difficulté à estimer correctement la charge générée par les demandes de réparation malgré une amélioration de la prévision de la demande en pièces détachées. Le but de cette thèse est donc de modéliser le processus de génération de la charge (qui est un compromis entre des règles de gestion implantées dans le système d'information et la connaissance d'experts métiers) par l'utilisation de procédés de type « machine learning » puis d'optimiser la répartition de la charge sur l'ensemble des ateliers de réparation.



## Chapitre 2

# Planification de la production et de la maintenance : vers un modèle intégré

Dans ce chapitre, je vais exposer les travaux que j'ai pu mener en collaboration avec Bruno Castanier depuis mon arrivée à Mines Nantes. Ceux-ci portent sur la prise en compte des politiques de maintenance dans le processus de planification, ces activités étant bien souvent totalement décorréliées Meyr et al. (2008) alors que l'impact de la maintenance sur la production est loin d'être négligeable puisqu'elle en affecte les capacités.

Lors du démarrage de ces travaux, peu de recherche avaient été menés sur la prise en compte de la maintenance dans un contexte de planification de demande dynamique à capacité finie. On peut citer essentiellement Aghezzaf et al. (2007); Najid et al. (2011); Fitouhi and Nourelfath (2014). Dans Aghezzaf et al. (2007), les auteurs considèrent un système de production dont le taux de défaillance était indexé sur un état de dégradation du système de production, cet état de dégradation étant uniquement fonction du temps écoulé depuis la dernière maintenance préventive (et non sur la charge subie par le système de production). La politique de maintenance est périodique et une maintenance préventive aura toujours lieu en début de période. Dans Najid et al. (2011), les auteurs étendent les travaux précédents en cassant la périodicité de la politique de maintenance préventive en définissant des fenêtres de temps (en nombre de périodes) durant laquelle une maintenance préventive doit avoir lieu (elle sera toujours placée en début de période). Cette contrainte de fenêtre de temps est levée dans Fitouhi and Nourelfath (2014) et ils montrent une forte amélioration des résultats obtenus (bien qu'ayant préservé le placement de la maintenance préventive en tout début de période). Dans ces recherches, les plans obtenus sont systématiquement faisables en moyenne.

Aussi, nous nous sommes fixés comme objectif d'étudier l'intégration de différentes politiques de maintenance au processus de planification en levant plusieurs de ces hypothèses fortes et notamment en indexant les modèles de dégradation de la performance du système de production à sa charge et en permettant au décideur de paramétrer le niveau de robustesse de la solution proposée. L'objectif de ce chapitre est donc d'en exposer les principaux résultats.

Dans ce qui suit, nous ferons les hypothèses suivantes :

- Chaque panne est cataleptique : pour redémarrer une maintenance corrective est exigée et elle sera minimale (c'est à dire qu'elle ne modifiera pas l'état du système). Cependant

elle aura un coût et elle consommera une certaine capacité.

- Une maintenance préventive sera toujours parfaite. Ainsi, le système de production sera supposé remis à neuf (c'est-à-dire sans dégradation). Elle aura également un coût et consommera également de la capacité de production.

## 2.1 Prise en compte du processus de maintenance dans la planification

Le but de cette recherche menée lors de la thèse de Martin Géhan Géhan (2014) était d'étudier l'impact du processus de maintenance sur la planification de la production. Plus précisément, il s'agissait de développer une approche par réservation de capacité afin de pouvoir prendre en compte la consommation de cette dernière par les opérations de maintenance, qu'elles soient correctives ou préventives. Les objectifs étaient donc les suivants :

- Comment évaluer au mieux la capacité à réserver pour les actions de maintenance ?
- Quelle politique décisionnelle déployer opérationnellement afin de maximiser la faisabilité du plan de production lors de sa mise en œuvre ?

Dans cette étude, nous avons considéré le problème de planification mono-niveau à capacité finie et avec prise en compte des temps de configuration du système de production, connu sous la nomenclature *LS – C – ST* Pochet and Wolsey (2006). Contrairement Aghezzaf et al. (2007), nous avons considéré la durée de fonctionnement  $a$  du système depuis la dernière maintenance préventive (et non le temps écoulé depuis cette dernière) c'est à dire la charge induite (capacité utilisée) par l'ensemble des produits fabriqués depuis la dernière maintenance préventive et nous avons modélisé son taux de défaillance associé par le taux de défaillance instantanée d'une loi de

Weibull :  $\lambda(a) = \frac{\beta}{\eta} \left(\frac{a}{\eta}\right)^{\beta-1}$  où  $\beta$  désigne son paramètre de forme et  $\eta$  son paramètre d'échelle.

Pour commencer, nous avons considéré la version mono-produit (ce qui permet de modéliser la durée de fonctionnement comme un nombre de produits fabriqués depuis la dernière maintenance préventive). Le problème de production et de planification conjointe associée a été formulé sous forme d'un processus de décision Markovien. Cette modélisation a permis de mettre en place une méthode de résolution par programmation dynamique stochastique Géhan et al. (2013) et de déduire, par analyse des séquences de production optimales obtenues par cette dernière, un certain nombre de propriétés structurelles liées à l'éventuelle positionnement de maintenances préventives dans une période de production :

- sous nos hypothèses, il y a au plus une maintenance préventive par période (ce qui paraît raisonnable au vu de la taille d'une période dans un processus de planification tactique),
- les maintenances préventives sont positionnées de préférence en début ou en fin de période (c'est à dire avant ou après toute production, ce qui permet d'éviter des coûts de reconfiguration dûs à l'opération de maintenance),
- la charge de production entre deux maintenances préventives correspond approximativement à la charge  $a_1^*$  minimisant le coût moyen de maintenance (préventive et corrective).

Ensuite, nous avons déterminé la charge  $a_2^*$  permettant de réduire la capacité moyenne consommée par unité fabriquée en tenant compte du temps de reconfiguration, de la capacité consommée par une maintenance préventive et par les nombres moyens de maintenances correctives induites par cette charge. Par ce calcul, nous avons pu déterminer une capacité maximale  $C_t^{max}(a_2^*)$  pouvant être allouée à la production pour chaque période  $t$ , la capacité résiduelle étant dédiée aux opérations de maintenance.

Ainsi, dans une première phase et en utilisant *LS – C – ST* muni de ces capacités maximales,

nous avons pu construire un plan de production. Dans une seconde phase, celui-là a été évalué opérationnellement par application de la programmation dynamique stochastique précédente en la limitant à la production prévue pour chaque période (les unités ne pouvant pas être effectivement produites étant comptées et pénalisées comme de la demande perdue). Sur l'ensemble des instances testées, le coût moyen obtenu est à 4.33% du coût moyen optimal obtenu par la programmation dynamique stochastique.

Bien que répondant aux deux questions posées en début de section, cette technique qui utilise la programmation dynamique stochastique dans sa seconde phase s'est vite trouvée limitée par la taille des instances pouvant être résolues. Aussi, on a substitué à cette dernière une heuristique se basant sur les propriétés structurelles précédemment décrites afin de simplifier l'évaluation opérationnelle du plan de production obtenu. Se basant sur  $a_1^*$  et sur le fait qu'une maintenance préventive se situe préférentiellement en début ou en fin de période, l'heuristique répond aux deux questions suivantes :

- Est-il judicieux d'effectuer une maintenance préventive en début de période ?
- Si tout le plan de production prévu pour une période  $t$  a pu être réalisé, est-il judicieux de placer une maintenance préventive à la fin de celle-ci ?

L'utilisation de cette heuristique permet de s'affranchir du grand nombre d'états générés par la programmation dynamique stochastique et donc de ne plus être confronté aux limitations précédentes. Les performances moyennes obtenues se situent alors autour de 1.44% du coût moyen obtenu par la méthode précédente, ce qui représente une dégradation très raisonnable au regard de ses apports.

Cette approche a été ensuite étendue au cas multi-produits. Ici, une modélisation par processus de décision Markovien aurait obligé à intégrer des informations supplémentaires dans chaque état et, au vue des limitations rencontrées dans le cas mono-produit et du nombre d'états présentement générés, l'utilisation de la programmation dynamique stochastique étaient vouée à l'échec. Aussi, nous avons modélisé le problème de production et de maintenance comme un processus aléatoire non Markovien qui nous a permis de reprendre, en l'adaptant, l'heuristique mono-produit et d'utiliser cette modélisation pour évaluer le coût moyen généré par le plan lors de la seconde phase de cette dernière.

L'extension au cas multi-produits a fait clairement apparaître l'impact que peut avoir le séquençement des lots de production et, par extension, la nécessité de prendre en compte l'ordonnancement si on veut pouvoir encore affiner la détermination de politiques décisionnelles efficaces opérationnellement : il est donc clair qu'on ne pourra pas se limiter à terme à une intégration décisionnelle maintenance-production sans la prolonger par une intégration verticale de l'ordonnancement. En corollaire, on voit donc que le principe de réservation de la capacité peut s'avérer très conservatif et que s'en affranchir constituera sans doute un gain de performance non négligeable.

## 2.2 Intégration d'un critère de performance de la fonction maintenance dans la planification de la production

Au vu des conclusions sur l'évaluation opérationnelle des plans de production multi-produits, nous nous sommes d'une part recentrés sur une approche plus tactique de l'intégration de la production et de la maintenance et d'autre part affranchis du partitionnement de la capacité pour en réserver une partie aux opérations de maintenance. De plus, nous avons fait l'hypothèse d'au plus une maintenance préventive par période et nous avons conservé la modélisation du taux de défaillance via une loi de Weibull.



Pour prendre en compte l'impact de la capacité consommée par les opérations de maintenance, nous avons intégré dans le processus de planification un indicateur de performance classique en maintenance qui est la disponibilité asymptotique du système de production. Celui-ci représente le ratio entre la capacité allouée à la production effective entre deux maintenances préventives consécutives et la capacité consommée au total (en prenant donc en compte les temps moyens de maintenance) depuis la dernière maintenance préventive jusqu'à la prochaine non-incluse Cocozza-Thivent (1997). Il s'écrit :

$$D(a) = \frac{a}{a + C_{prev} + \Lambda(0, a) C_{corr}}, \quad \forall a \in [0, +\infty[ \quad (2.1)$$

où :

- $a$  représente la charge entre deux maintenances préventives consécutives,
- $C_{prev}$  la capacité consommée pour une maintenance préventive,
- $C_{corr}$  la capacité consommée pour une maintenance corrective,
- $\Lambda(0, a)$  le nombre moyen de pannes pendant l'exécution de la charge  $a$ .

**Définition 2.1** (Plan de production  $\delta$ -disponible)

On dira qu'un plan de production est  $\delta$ -disponible si et seulement si toutes ses disponibilités asymptotiques sont supérieures ou égales à  $\delta$ .

Un tel seuil  $\delta$  peut être interprété comme un niveau de fonctionnement contractualisé entre les services de production et de maintenance, traduisant une gestion adéquate des opérations de maintenance. Il permet également, comme l'illustre la Figure 2.1, de déterminer analytiquement une fenêtre de charge  $[a_{min}, a_{max}]$  dans laquelle placer une maintenance préventive permettant de respecter **en moyenne** la  $\delta$ -disponibilité du plan de production.

Ainsi, en s'appuyant sur le problème de planification de la production  $LS - C - ST$  et en y intégrant des contraintes linéaires de calcul de charge et de fenêtre de charge pour la maintenance préventive ainsi que les coûts liés à la maintenance, nous avons pu formuler ce problème sous forme d'un programme linéaire en nombres entiers. Cependant, l'intégration du calcul des coûts moyens engendrés par la maintenance corrective induisait une telle complexité (due aux grands nombres de variables nécessaires pour ce calcul) que sa résolution par solveur n'était pas envisageable en termes de temps de calcul. Aussi, nous avons résolu heuristiquement le problème de planification en ôtant ces coûts de la fonction objectif puis en les rajoutant a posteriori Géhan et al. (2013).

Une étude de sensibilité autour du paramètre  $\delta$  a également été menée et a mis en évidence l'impact du niveau de ce seuil sur les coûts de production en montrant que minimiser les coûts de production et satisfaire la contrainte de disponibilité étaient des objectifs antagonistes. Aussi, il sera donc nécessaire au décideur d'effectuer un compromis entre niveau de disponibilité du système et coûts de production.

## 2.3 Planification conjointe de la production et de la maintenance : des approches robustes

Depuis le début de ce chapitre, nous avons implicitement lié l'acceptation d'un plan de production par un décideur à sa faisabilité en moyenne. Or, ce dernier peut avoir des exigences différentes en termes de robustesse. C'est pour cela que nous avons continué nos recherches en prenant en compte cette contrainte. Pour cela, nous avons défini le concept d' $\varepsilon$ -faisabilité.

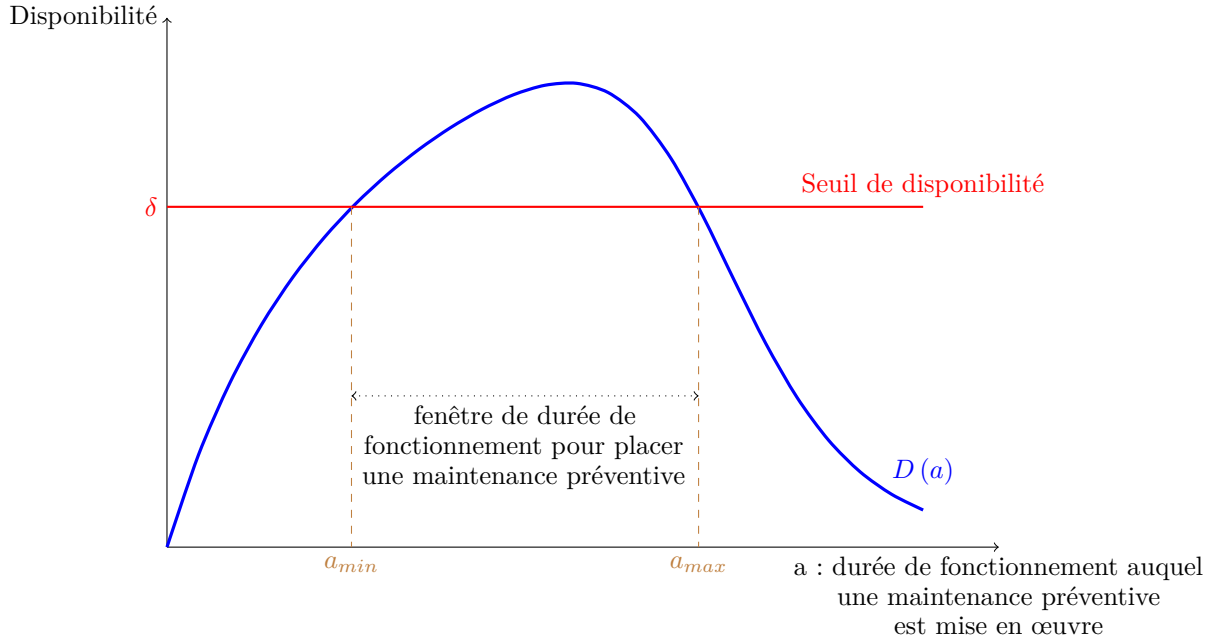


FIGURE 2.1 – Détermination de la fenêtre de maintenance préventive

**Définition 2.2** (Période  $\varepsilon$ -faisable)

On dira qu'une période  $t \in [1, T]$  est  $\varepsilon$ -faisable si et seulement si la probabilité de ne pas dépasser la capacité de production de celle-ci (en prenant en compte l'intégralité de la consommation de capacité, maintenances comprises) est supérieure ou égale à  $\varepsilon$ .

En conservant l'hypothèse selon laquelle au plus une maintenance préventive peut être planifier par période et qu'elle ne peut l'être qu'en début ou en fin de production, une traduction de cette définition est donnée par la contrainte probabiliste suivante :

$$\mathbb{P} \left\{ \left[ \sum_{i=1}^N (p_i Q_{it} + \tau_i X_{it}) + C_{prev} M_t + C_{corr} N_f \left( a_t^{fin} - \sum_{i=1}^N p_i Q_{it}, a_t^{fin} \right) \right] \leq C_t \right\} \geq \varepsilon, \quad t \in [1, T] \quad (2.2)$$

où :

- $N$  correspond aux nombres de types de produit à planifier,
- $T$  est la taille de l'horizon de planification, exprimée en nombre de périodes,
- $p_i$  est la capacité unitaire consommée pour la fabrication d'un produit de type  $i$ ,  $i \in [1, N]$ ,
- $\tau_i$  est la capacité consommée pour la configuration du système de production en vue de la fabrication d'un lot de produits de type  $i$ ,  $i \in [1, N]$ ,
- $C_{prev}$  représente la capacité consommée pour une maintenance préventive,
- $C_{corr}$  représente la capacité consommée pour une maintenance corrective,
- $N_f(x, y)$  donne le nombre de défaillances observées entre les durées de fonctionnement  $x$  et  $y$ ,

- $Q_{it}$  donne la quantité de produits  $i$  fabriquée durant la période  $t$ ,  $\forall (i, t) \in [1, N] \times [1, T]$ ,
- $X_{it}$  indique si une configuration du système de production a eu lieu en vue de fabriquer un lot de produits  $i$  durant la période  $t$ ,  $\forall (i, t) \in [1, N] \times [1, T]$ ,
- $M_t$  indique si une maintenance préventive a été planifié durant la période  $t$ ,  $t \in [1, T]$ ,
- $a_t^{fin}$  représente la durée de fonctionnement du système de production à la fin de la période  $t$ ,  $t \in [1, T]$ .

**Définition 2.3** (Plan de production  $\varepsilon$ -faisable)

On dira qu'un plan de production est  $\varepsilon$ -faisable si et seulement si chaque période de production est  $\sqrt[T]{\varepsilon}$ -faisable.

Dans un premier temps nous nous sommes intéressés à l'extension de  $LS - C - ST$  prenant en compte cette contrainte probabiliste et à sa résolution par une méthode heuristique Géhan et al. (2014a,b).

### 2.3.1 $LS - C - ST$ avec contraintes d' $\varepsilon$ -faisabilité

En conservant les hypothèses prises précédemment (au plus une maintenance préventive par période et modélisation du taux de défaillance via une loi de Weibull), la première idée a été d'intégrer cette contrainte d' $\varepsilon$ -faisabilité en la linéarisant. Cependant, il s'est avéré impossible de résoudre un tel modèle car le nombre de variables générées était si important que celui-ci ne pouvait pas être chargé en mémoire (pour ordre d'idée, sur une instance de 10 périodes avec une capacité constante de 100, le nombre de variables générées pour la linéarisation est de l'ordre du milliard). Aussi l'idée du développement d'une heuristique de résolution s'est naturellement imposée. Cette heuristique itérative se décompose en trois modules imbriqués et utilise la notion de durée de fonctionnement maximum pour une maintenance préventive qui avait émergé lors de nos travaux précédents :

- Module 1 : Il s'agit de ici de résoudre le modèle linéaire  $LS - C - ST$  en y intégrant uniquement la détermination des périodes où apparaissent les maintenances préventives (ici, on ne prend pas en compte les maintenances correctives) en fonction d'une durée de fonctionnement maximale  $a_{max}$ .
- Module 2 : L' $\varepsilon$ -faisabilité du plan déterminé par le Module 1 est testé. Si ce n'est pas le cas, les capacités du système de production pour les périodes non  $\sqrt[T]{\varepsilon}$ -faisables sont décrémentées afin de réserver de la capacité pour des opérations de maintenance et le Module 1 est relancé.
- Module 3 : Si le plan obtenu par le Module 1 est  $\varepsilon$ -faisable, alors une procédure ad hoc de recalcul de  $a_{max}$  est utilisé afin de tenter de minimiser les coûts de maintenance et le Module 1 est alors relancé.

L'heuristique s'arrête au bout d'un certain nombre d'itérations pré-défini. La Figure 2.2 schématise le fonctionnement de cette dernière.

Les tests menés ont permis de montrer les résultats suivants :

- En cas de nécessité de réajustement de la capacité, le ré-ajustement de la première période non  $\sqrt[T]{\varepsilon}$ -faisable uniquement et par une seule unité de capacité donne de meilleurs résultats

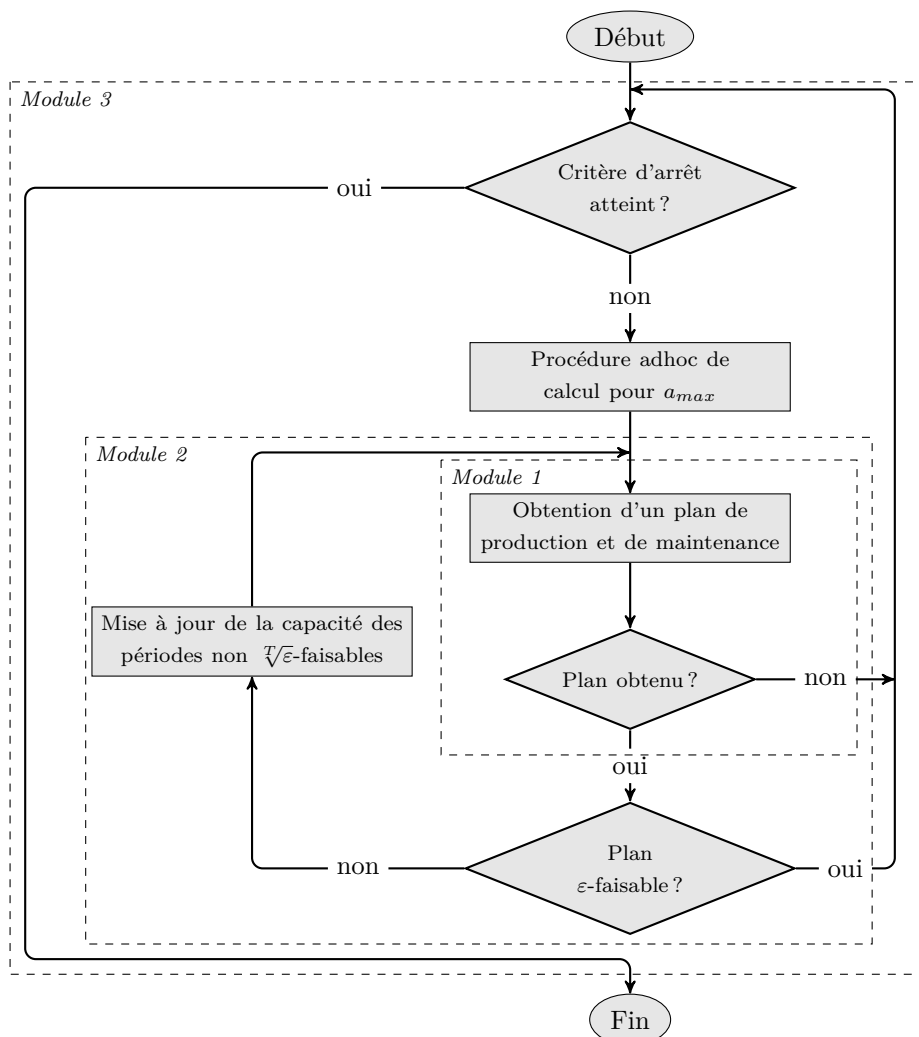


FIGURE 2.2 – Représentation de l’algorithme de l’heuristique

en terme d’obtention de plans  $\varepsilon$ -faisable. Cela est dû à une diminution moins drastique de la capacité globale par l’heuristique.

- L’augmentation du seuil de faisabilité  $\varepsilon$  a un gros impact sur la faisabilité du problème et sur le coût total d’une solution obtenue. Ce qui n’est pas étonnant car plus la valeur est haute, plus le plan est robuste donc plus de capacité est réservée pour les opérations de maintenance.

Dans cette recherche, le taux de défaillance du système (modélisé par une loi de Weibull) ne dépend que de la charge qu’a subi le système depuis sa dernière maintenance préventive et non pas de sa dégradation, de son usure. Or, la dégradation d’un système de production entraîne une détérioration de la performance de ce dernier et donc peut avoir une conséquence directe sur le temps de fabrication (la capacité unitaire) d’un produit. Et, si on considère que la détérioration du système peut être plus ou moins forte en fonction du type de produit fabriqué,

on voit également qu'il est nécessaire de tenir compte du séquençement de la production. C'est donc dans cette direction que nous avons poursuivi nos recherches.

### 2.3.2 Intégration de la dégradation du système et maintenances conditionnelles

Nous nous replaçons dans le contexte du problème de planification de production  $LS-C-ST$  et nous nous intéressons à un système de production dont le taux de performance se dégrade au fur et à mesure de son usage. Nous notons  $x_t$  le taux de performance effectif au niveau de charge  $t$  depuis la dernière maintenance préventive, taux que nous supposons mesurable pour tout  $t$ . Nous supposons également que la dégradation de ce taux est un processus stochastique croissant, continu et fonction de la charge et que son espérance est fonction de la charge induite par le produit fabriqué (son temps unitaire de traitement).

Plus formellement, nous notons  $\{X_t, t > 0\}$  le processus stochastique modélisant l'évolution de la dégradation du taux performance du système production non maintenu, où  $t$  représente la charge déjà traitée par le système de production. Nous supposons qu'à l'état neuf ( $X_0$ ) le taux est égal à 1 et que le processus est strictement décroissant jusqu'à 0 (état dans lequel la machine est tellement dégradée qu'elle n'est plus apte à produire). On suppose que sur un intervalle  $[t, t + \Delta t]$ , la perte de performance est modélisée par une variable aléatoire qui suit une loi Gamma  $\Gamma(\alpha\Delta t, \beta)$  où  $\alpha$  et  $\beta$  ont été préalablement estimés. Cette modélisation sous forme d'un processus gamma homogène est classique en maintenance van Noortwijk (2009). Cependant, les temps opératoires étant particulièrement faibles au regard de l'horizon de planification, nous supposons dans notre modélisation que le taux de performance est constant durant la fabrication d'une unité de produit et que son évolution n'intervient qu'en fin de production de cette unité.

Comme dit précédemment, nous prenons également en compte l'allongement de la durée des temps opératoires en fonction du niveau du taux de performance du système. Plus précisément, si  $x_t$  représente le taux de performance du système de production au niveau de charge  $t$ , nous notons  $p_i(x_t)$  la capacité unitaire de production d'un produit  $i$  associé à ce taux et nous aurons donc, pour un système non maintenu :

- $x_{t+p_i(x_t)} < x_t$ ,
- $p_i(x_{t+p_i(x_t)}) \geq p_i(x_t)$ .

Nous prenons également en compte les défaillances (pannes) du système de production, défaillances que nous supposons toujours catalectiques et nécessitant une maintenance corrective pour remettre le système en marche. Nous supposons ces défaillances distribuées en fonction d'un taux de défaillance  $\lambda(x)$  lui même dépendant du taux de performance de ce système. De plus, nous supposons ce taux décroissant en  $x$ . Ainsi, plus le taux de performance du système sera faible, plus la probabilité de pannes sera grande. Puisque nous avons fait l'hypothèse que le taux de performance était constant lors de la fabrication d'un produit, il en sera de même pour le taux de défaillance et si la fabrication d'un produit  $i$  débute au niveau de charge  $t$ , alors le nombre espéré de pannes durant sa fabrication sera  $\lambda(x_t)p_i(x_t)$ .

La Figure 2.3 illustre la modélisation du taux de performance pour une période de production déterminée, en considérant le système neuf en début de période.

On peut voir au travers de cette modélisation que l'intégration de la dégradation du système sous-entend la prise en compte du séquençement des lots de production.

Dans Sloan (2004), l'auteur a montré l'optimalité des politiques de maintenance préventive basées sur l'atteinte d'un seuil maximal de dégradation dans le cas mono-produit. C'est pour cela que nous avons décidé d'intégrer une telle politique dans le processus de planification.

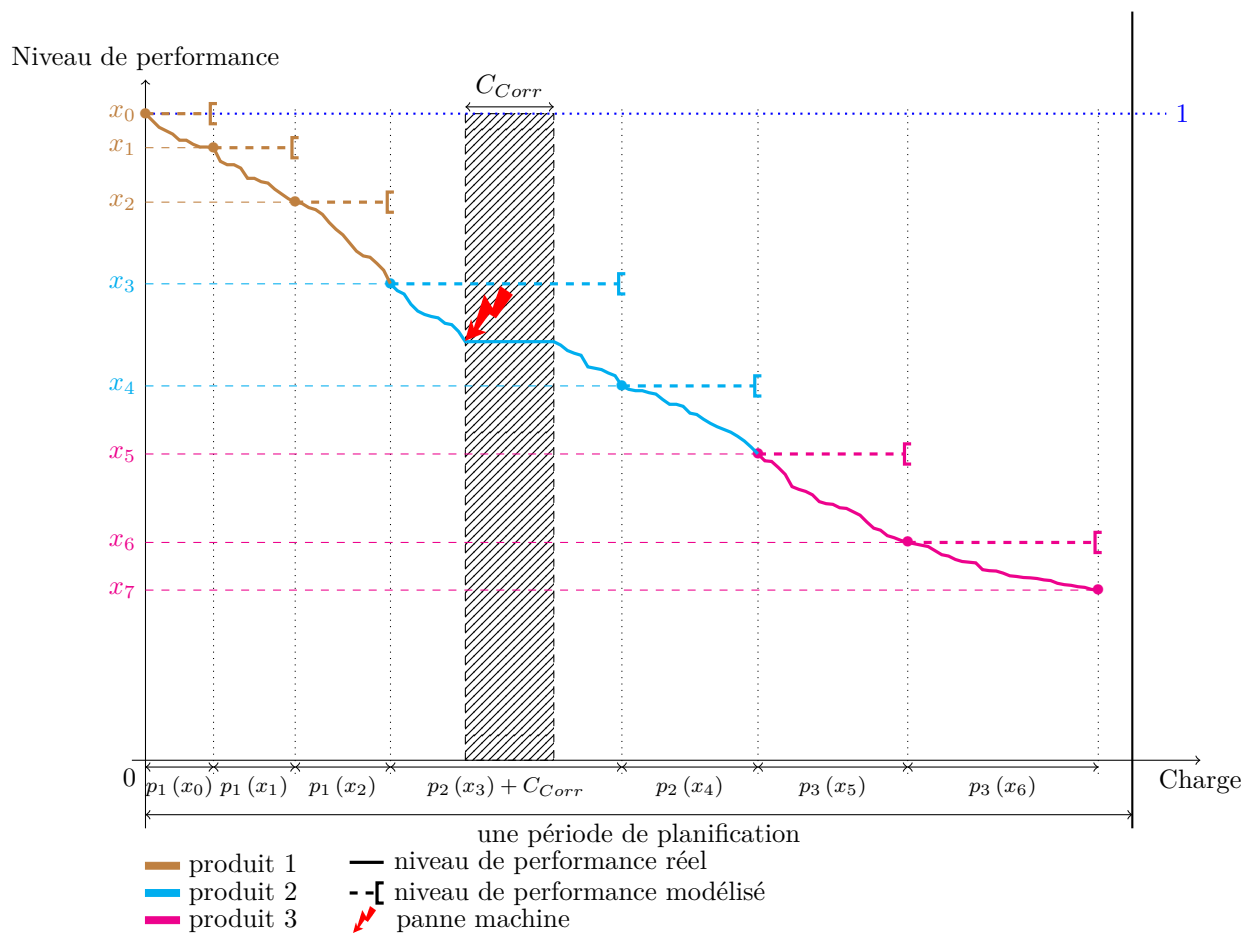


FIGURE 2.3 – Exemple de scénario de la réalisation d'un plan de production en fonction de l'évolution de la dégradation de la performance

### Intégration d'une politique de maintenance basée sur le niveau de performance du système de production

L'objectif de cette recherche Castanier and Lemoine (2011a,b) est donc de déterminer un plan  $\varepsilon$ -faisable minimisant les coûts de production et de maintenance, en s'appuyant sur une maintenance conditionnelle basée sur un niveau minimal de taux de performance : plus précisément, dès que le système aura atteint un niveau de performance déterminée  $x_{min}$ , une maintenance préventive sera faite. Cette idée était déjà sous-jacente dans l'heuristique précédente Géhan et al. (2014a,b) puisqu'elle déterminait  $a_{max}$  la durée de fonctionnement maximale du système pouvant être atteint avant le déclenchement d'une maintenance préventive. Cependant, dans le cas d'une politique de maintenance conditionnelle de ce type, la maintenance préventive n'aura pas nécessairement lieu en début de période. Aussi, nous avons repris et étendu le principe de fonctionnement de cette dernière.

Puisqu'il n'est pas possible d'intégrer directement la notion de dégradation du système et ses conséquences dans  $LS - C - ST$ , nous avons repris le principe de la ré-estimation de la capacité utilisée dans l'heuristique et nous avons développé une méthode basée sur les 3 phases suivantes.

Phase 1 : En utilisant les capacités du système, un plan de production est généré. Dans ce modèle, les coûts de maintenance ne sont pas pris en compte.

Phase 2 : Dans la seconde phase, l' $\varepsilon$ -faisabilité du plan est testée par le biais d'un modèle de simulation intégrant le modèle de dégradation du système de production, diverses règles de séquençement de lot de production et la politique de maintenance conditionnelle basée sur un seuil de dégradation minimal  $x_{min}$ . Les coûts moyens de maintenance sont intégrés à l'issue de la simulation. Si le plan est  $\varepsilon$ -faisable, alors l'algorithme passe à la phase 3, sinon les capacités des périodes en défaut sont ré-estimées et la phase 1 est relancée.

Phase 3 : Les coûts de maintenance étant liées à la valeur de  $x_{min}$ , nous faisons varier ce paramètre de façon à les minimiser tout en préservant l' $\varepsilon$ -faisabilité du plan obtenu.

Concernant la phase 3, les simulations numériques nous ont montré que l'évolution des coûts de maintenance en fonction de la valeur de  $x_{min}$  aboutissait à une fonction mono-modale telle que représentée par la Figure 2.4.

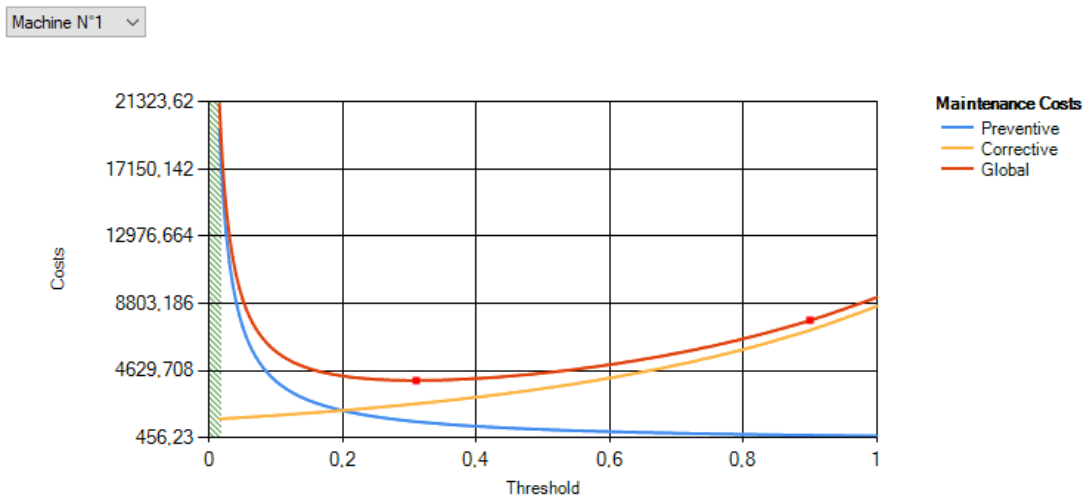


FIGURE 2.4 – Evolution des coûts de maintenance en fonction du seuil  $x_{min}$

Aussi, la recherche du  $x_{min}^*$  minimisant les coûts de maintenance a pu être faite efficacement par l'utilisation d'un principe dichotomique basé sur la suite de Fibonacci.

Puisque les phases 1 et 2 dépendent de la valeur de  $x_{min}$ , nous réinjectons donc la valeur de  $x_{min}^*$  au niveau de la phase 1 et nous réitérons le processus en le redémarrant avec les capacités initiales. Ainsi nous construisons une suite de plans  $\varepsilon$ -faisables de coûts décroissants et l'heuristique s'arrête dès qu'il n'y a plus d'amélioration de ceux-là.

La Figure 2.5 schématise la méthode mise en œuvre.

Les tests menés sur des instances issues de la littérature Eppen and Martin (1987) ont montré une grande efficacité de ce schéma d'optimisation de production et de la maintenance. Cette recherche a été intégrée dans une plateforme de démonstration à vocation industrielle.

Cependant, on peut s'interroger sur le critère  $\varepsilon$ -faisabilité telle que nous l'avons défini pour un plan de production : l' $\varepsilon$ -faisabilité peut facilement générer une infaisabilité du processus de planification de production car elle requiert que toutes les périodes sont  $\sqrt[\mathcal{T}]{\varepsilon}$ -faisables. Or, cela élimine d'office tous les plans de production où pourraient apparaître des « phénomènes de

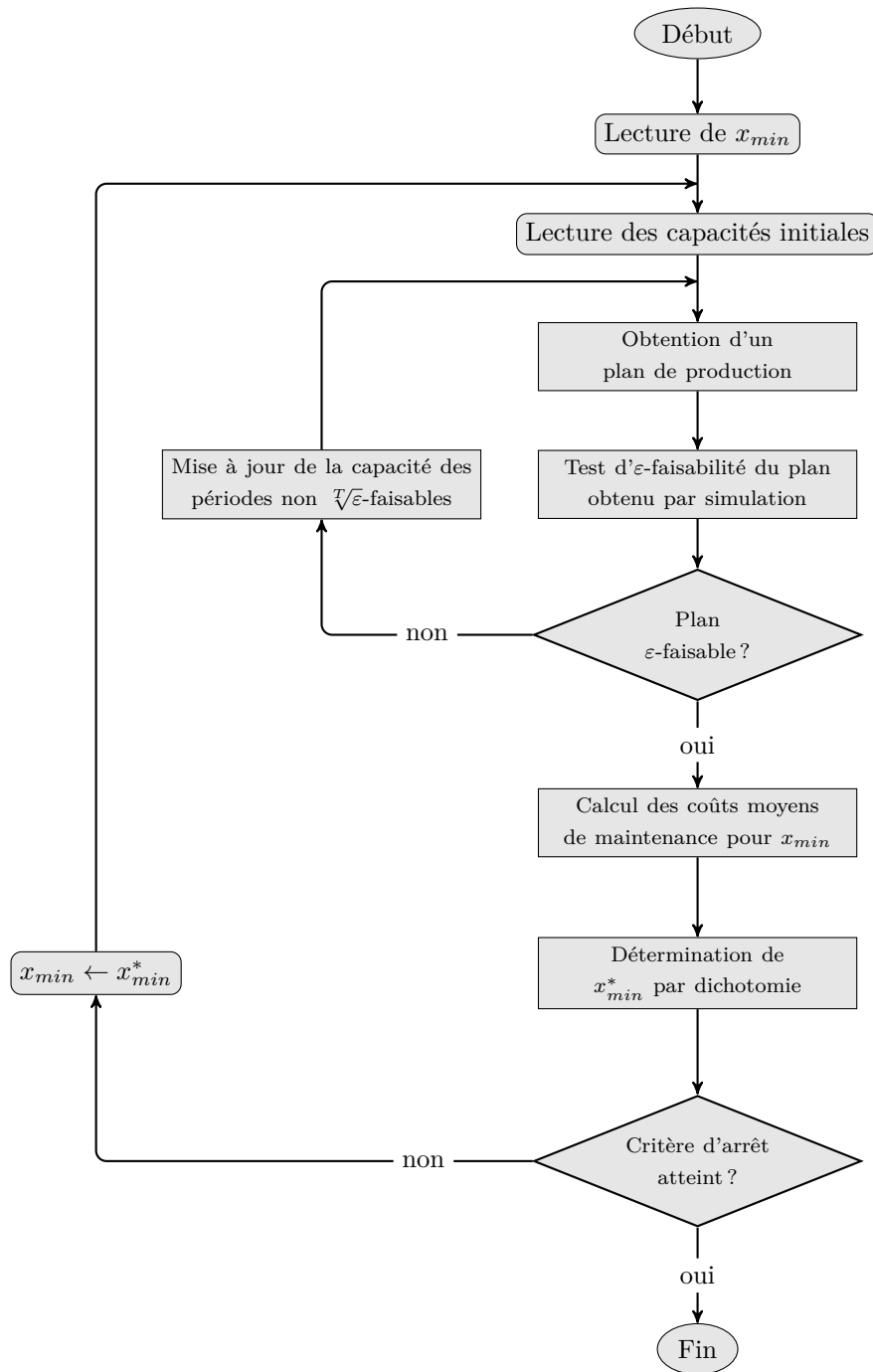


FIGURE 2.5 – Représentation de la méthode

compensation ». En effet, considérons un plan de production tel que chacune de ses périodes  $t$



soit  $\varepsilon_t$ -faisable : un tel plan ne serait pas moins acceptable qu'un plan  $\varepsilon$ -faisable si  $\prod_{t=1}^T \varepsilon_t \geq \varepsilon$ , du point de vue d'un décideur. En considérant ces nouveaux plans, on voit que la solution obtenue sera au moins d'aussi bonne qualité (si ce n'est qu'elle a toutes les chances d'être meilleure). Aussi, nous définissons l' $\varepsilon$ -fiabilité par :

**Définition 2.4** (Plan de production  $\varepsilon$ -fiable)

On dira qu'un plan de production est  $\varepsilon$ -fiable si et seulement si chaque période de production  $t \in [1, T]$  est  $\varepsilon_t$ -faisable et  $\prod_{t=1}^T \varepsilon_t \geq \varepsilon$ .

Cependant, l'utilisation d'un seuil unique de niveau de performance minimal dans un contexte  $\varepsilon$ -fiabilité est peu adapté car chaque période  $t$  a son propre niveau d'acceptabilité contrairement à  $\varepsilon$ -faisabilité. Aussi, nous allons mettre en place une nouvelle politique de maintenance conditionnelle s'appuyant sur cette spécificité.

### Intégration d'une politique de maintenance prédictive et $\varepsilon$ -fiabilité

Les approches de maintenance conditionnelle comme celle décrite précédemment se basent sur des indicateurs d'état (typiquement le niveau de performance du système) ou de pronostic comme la Durée de Vie Résiduelle (RUL). Ceux-ci permettent alors de définir des structures d'optimalité de politiques décisionnelles de maintenance Lee et al. (2006). L'utilisation de la Durée de Vie Résiduelle sert clairement de socle pour la maintenance prédictive<sup>1</sup> dont l'importance ne fait que croître dans le contexte de l'Industrie 4.0.

Aussi, nous définissons une politique de maintenance prédictive s'appuyant sur le concept d' $\varepsilon$ -fiabilité et reposant donc sur l'estimation de l'aptitude du système de production à réaliser son plan de production pour chaque période. Pour ce faire, nous allons utiliser un principe d'allocation de la faisabilité qui définira, pour chaque période  $t$ , le niveau d' $\varepsilon_t$ -faisabilité à atteindre. Si, pour une période  $t$  ce niveau n'est pas atteint, alors une unique maintenance préventive sera mise en place en début de période (cette règle s'appuie sur les propriétés structurelles mises en évidence par la programmation dynamique stochastique décrite précédemment et se trouve être globalement conforme à ce qui se pratique opérationnellement) Lemoine and Castanier (2019).

Le principe d'allocation de faisabilité utilisé est le suivant :

1. Selon la norme NF EN 13306 X 60-319, une maintenance prédictive est « maintenance conditionnelle exécutée en suivant les prévisions extrapolées de l'analyse et de l'évaluation de paramètres significatifs de la dégradation du bien »

**Principe** (Allocation de la faisabilité)

Soit à déterminer un plan de production  $\varepsilon$ -fiable sur un horizon de  $T$  périodes.  
 Pour chaque période  $t \in [1, T]$ , on pose

$$\varepsilon_t = \sqrt[t-(t-1)]{\varepsilon} \prod_{k=1}^t FR_k(x_k^d) \quad (2.3)$$

où  $FR_k(x_k^d)$  est la probabilité de réalisation du plan de production à la période  $k$  connaissant le niveau de performance  $x_k^d$  du système de production en début de fabrication à cette période.

On voit donc qu'un plan vérifiant que  $\forall t \in [1, T], FR_t(x_t^d) \geq \varepsilon_t$  est  $\varepsilon$ -fiable.  
 Le schéma d'optimisation proposé est représenté par la Figure 2.6.

Nous avons repris les tests en prenant les mêmes instances qu'avec la précédente politique de maintenance conditionnelle. Il s'avère que les résultats obtenus par la politique de maintenance prédictive sont meilleurs en termes de coûts. Cela s'explique assez aisément par le fait que la politique de maintenance prédictive considérée est bien moins agressive pour les capacités du système de production. En effet, l' $\varepsilon$ -faisabilité oblige toutes les périodes à respecter  $\sqrt[t]{\varepsilon}$ -faisabilité alors que  $\varepsilon$ -faisabilité laisse beaucoup plus de possibilités en terme de répartition de la faisabilité. Ainsi, on peut avoir une période avec une  $\varepsilon$ -faisabilité assez faible par rapport aux autres et donc, ainsi, éviter des ré-estimations de capacité drastiques.

Cependant, on observe que si le système de production est soumis à des phénomènes de dégradation rapides de son niveau de performance, alors l'approche prédictive n'arrive plus à fournir de solutions réalisables contrairement à l'approche basée sur le niveau de performance minimal. Cela provient du fait que dans la première approche conditionnelle présentée, la maintenance préventive n'est plus automatiquement placée en début de période et que cela permet d'avoir un pilotage plus fin du niveau de performance du système de production et que, de plus, nous ne sommes plus limités à une seule maintenance préventive par période. Ce schéma a été intégré dans une plateforme de démonstration à vocation industrielle.

On voit donc que l'utilisation de l' $\varepsilon$ -faisabilité permet d'obtenir des résultats de meilleures factures. Cependant, il faudra sans doute lever l'hypothèse de la maintenance préventive en début de période : une hybridation des deux approches pourrait sans doute apporter un début de réponse.

## 2.4 Conclusion et perspectives

Depuis mon arrivée à l'École des Mines de Nantes, j'ai pu commencer à explorer un certain nombre de pistes concernant l'intégration des politiques de maintenance dans le processus de planification.

Dans la thèse de Martin Géhan, nous avons commencé par indexer le niveau de dégradation du système sur sa durée de fonctionnement et nous avons proposé plusieurs méthodes de résolution s'appuyant sur des propriétés structurelles déduites par programmation dynamique stochastique. Pour ce faire, nous avons utilisé un principe classique de partitionnement de la capacité. Nous nous sommes ensuite libérés de cette réservation de la capacité par l'utilisation d'un indicateur de performance usité dans le monde de la maintenance en définissant la  $\delta$ -disponibilité du plan.

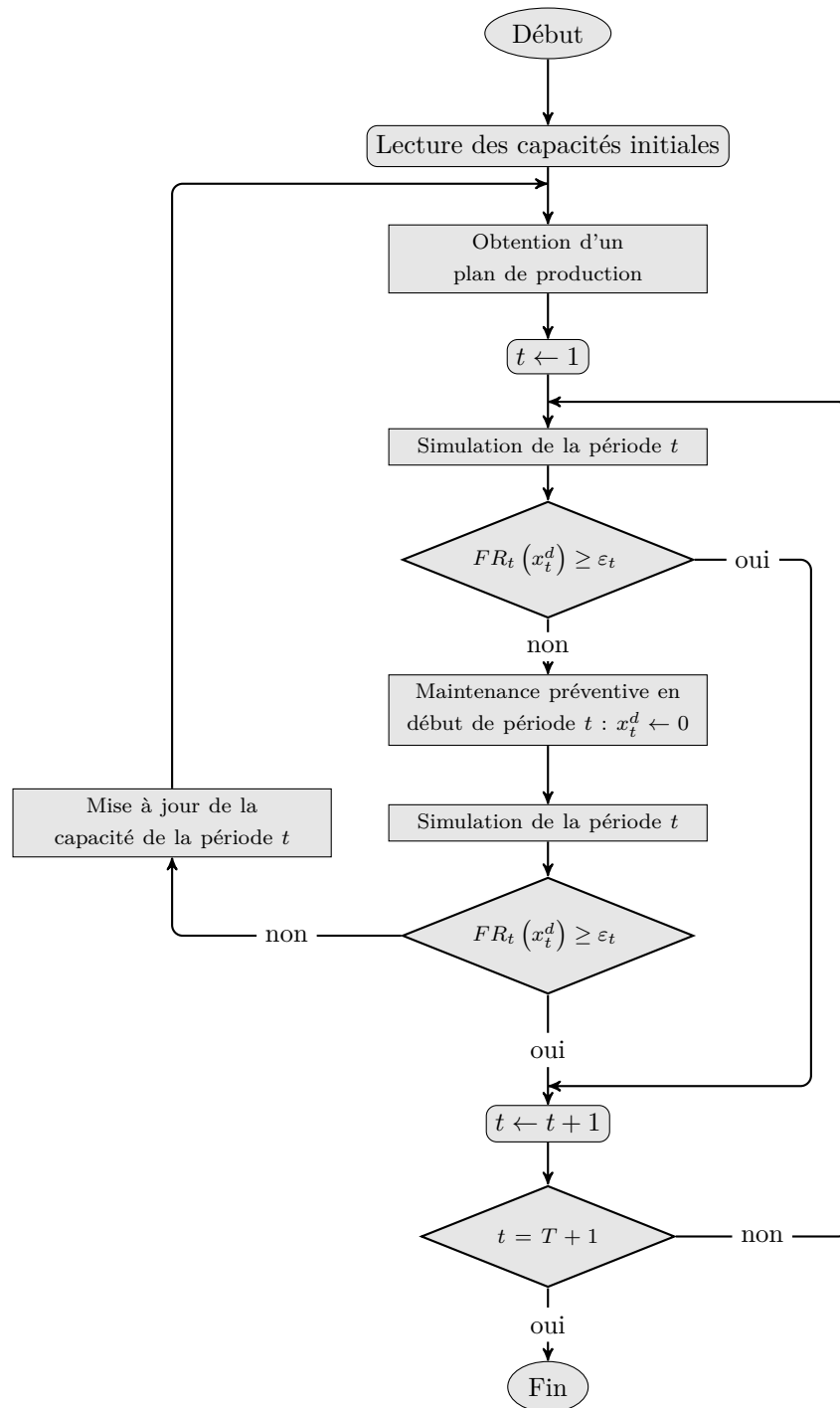


FIGURE 2.6 – Schéma d'optimisation pour la politique de maintenance prédictive considérée

Pour permettre au décideur de paramétrer le niveau de robustesse qu'il souhaite obtenir, nous avons proposé de robustifier les solutions obtenues en définissant l' $\varepsilon$ -faisabilité des plans et en proposant un schéma itératif basé sur de la ré-estimation des capacités de production. Nous avons ensuite étendu ce schéma en considérant une modélisation de la dégradation basée sur un processus stochastique (une loi Gamma) : l'utilisation d'un modèle de simulation nous a permis de prendre en compte son impact sur les temps de production et d'utiliser des règles de gestion définissant le séquençement de la fabrication des différents lots de produits. Nous avons implanté une politique de maintenance conditionnelle basée sur un seuil minimal de niveau de performance du système.

L' $\varepsilon$ -faisabilité des plans de production induisant une consommation drastique des capacités par la maintenance, nous avons affaibli ce critère en définissant l' $\varepsilon$ -fiabilité des plans de production, nous permettant ainsi d'avoir une plus grande flexibilité dans les probabilités de faisabilité de chaque période des plans de production. Ce nouveau critère nous a permis de mettre en place une politique de maintenance prédictive dans le schéma précédent améliorant sensiblement les résultats obtenus.

Cependant, beaucoup reste à faire. En effet, avec l'avènement de l'Industrie 4.0, nous nous orientons de plus en plus vers ces approches intégrées. La mise à disposition de plus en plus massive de données permet la calibration et l'utilisation de modèles considérés avant comme purement théoriques (par exemple les modèles de dégradation) ouvrant ainsi la porte à l'utilisation de politiques de maintenance de plus en plus fines (maintenance conditionnelle, prédictive) qu'il faudra prendre en compte dans les modèles classiques de production. De plus et en corollaire de l'utilisation de telles modélisations, nous avons souligné l'impact du séquençement de la production : il faudra donc étendre cette intégration maintenance-production de façon verticale par le biais de l'ordonnancement. Bien que nous ayons commencé à toucher du doigt cette réalité (notamment par la simulation et les règles de gestion), il faudra sans doute aller vers une intégration de l'ordonnancement dans le modèle de planification Dauzère-Péres and Lasserre (1994), ordonnancement prenant en compte les aspects liés à la dégradation et à la maintenance.



## Chapitre 3

# Planification Financière

Dans ce chapitre, nous résumons les travaux relatifs de la planification financière que nous avons menés depuis 2012 au travers du projet FUI 15 *Risk, Credit Chain & Supply Chain Management (RCSM)* et de sa suite, le projet ANR *Integrating Financial Health in Supply Chain Management Intégration de la dimension Financière dans l'Optimisation des chaînes logistiques (FILEAS-FOG)*.

Le but du projet *RCSM* était de proposer une plateforme technologique et un ensemble d'outils d'aide à la décision permettant à une filière industrielle de financer ses acteurs tout en maîtrisant, entre autres, les risques de défaillance financière des emprunteurs. La Figure 3.1 schématise le principe de fonctionnement de cette dernière.

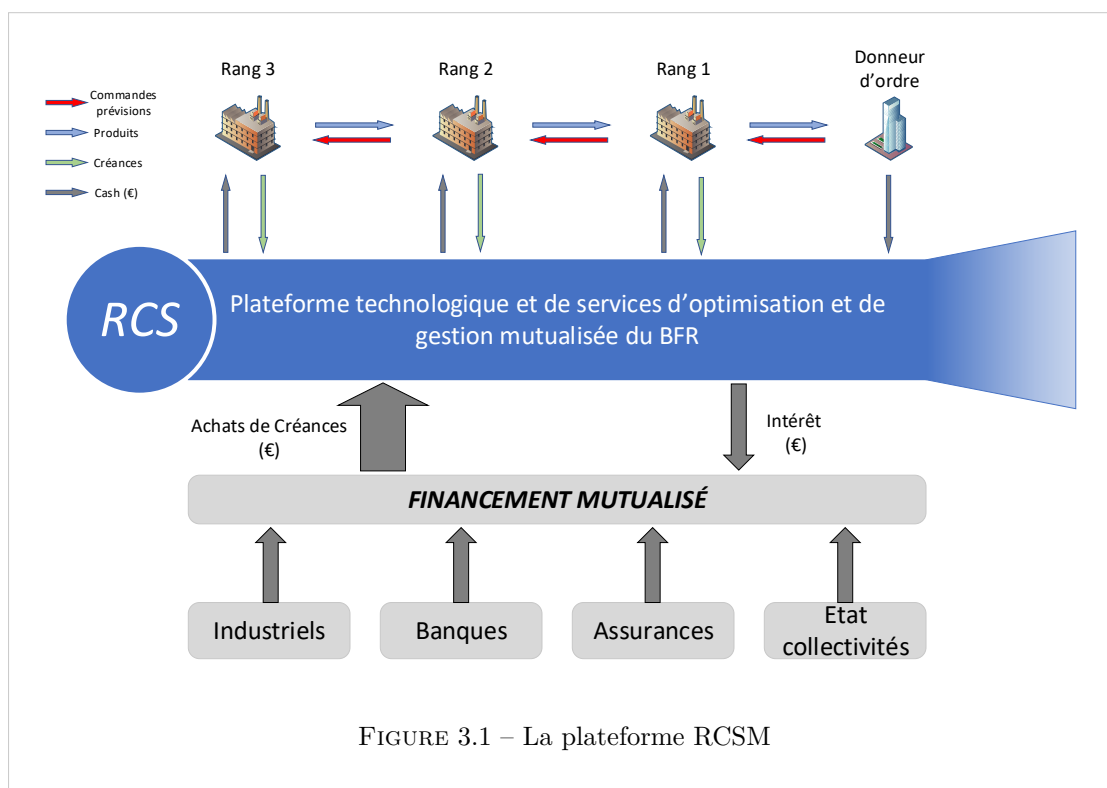


FIGURE 3.1 – La plateforme RCSM

Bien que la séparation des décisions opérationnelles et financières ait été théorisée par Modigliani et Miller Modigliani and Miller (1958) (en utilisant des hypothèses très restrictives), les travaux de Babich and Sobel (2004) ont permis de montrer que les problématiques financières et opérationnelles étaient fortement imbriquées. De plus, dans Chen et al. (2014), les auteurs ont montré qu'optimiser conjointement ces deux dimensions permettait d'augmenter sensiblement la performance globale de l'entreprise. Or dans Vickery et al. (2003), la pertinence de manager les flux physiques par des critères financiers a été prouvée. Aussi, cela justifie de choisir un indicateur financier pouvant d'une part traduire la performance de l'entreprise et d'autre part, être relié à la gestion du flux physique généré par la fabrication de l'ensemble des produits de l'entreprise. Le Besoin en Fonds de Roulement répond particulièrement bien à ces exigences. En effet, dans Timme and Williams-Timme (2000), les auteurs ont montré que ce dernier était un indicateur important permettant de mesurer la performance de la chaîne logistique. De plus, son influence sur la performance financière et la profitabilité de l'entreprise a été largement mise en évidence Ding et al. (2013); Enqvist et al. (2014).

Aussi, les objectifs du projet RCSM<sup>1</sup> étaient de :

1. Réduire le Besoin en Fonds de Roulement (BFR) excédentaire (stocks et délais de paiement) au niveau cluster / filière par une meilleure collaboration entre les Donneurs d'Ordre, leurs réseaux de fournisseurs PME / PMI multi-rangs, et les organismes de financement
2. Libérer des capacités de financement (et donc libérer un potentiel de croissance) pour les PME / PMI, via la mutualisation et le lissage des besoins et des excédents de trésorerie entre les acteurs de la Supply Chain
3. Réduire le coût de financement et le coût des risques (opérationnels, financiers) pour tous les acteurs de la Supply Chain, et améliorer ainsi la compétitivité des filières industrielles
4. Accélérer le « cash-flow » sur la filière, en synchronisant les flux de paiement et les flux physiques sur toute la Supply Chain et en optimisant la gestion des crédits interentreprises
5. Sécuriser et améliorer la performance de la Supply Chain financière grâce à la mutualisation des risques et leur supervision

Cependant, l'atteinte de tels objectifs nécessitent d'être capable :

- d'évaluer le BFR généré par un plan de production (en l'occurrence, celui de l'emprunteur)
- en se basant sur les paramètres d'entrée de l'emprunteur (coûts, demande, etc.), de proposer un plan de production « cible » répondant à la demande de ses clients et minimisant son besoin en financement.

En effet, par ce biais, la plateforme sera capable de proposer à l'emprunteur un ensemble de préconisations visant à diminuer son besoin en fonds de roulement (et donc son besoin en financement). Cela aura pour conséquence de limiter les risques encourus par les financeurs et donc de diminuer le taux d'intérêt de son emprunt.

Ainsi, nous présenterons dans une première section la modélisation du BFR que nous avons adoptée, puis nous montrerons les impacts de cette fonction financière sur certains modèles de « lot sizing ».

### 3.1 Modélisation du BFR

Traditionnellement, l'analyse financière décompose le BFR en deux composantes<sup>2</sup> :

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1. Dossier de demande de financement de RCSM, FUI 15  
 2. <https://www.mazars.fr/Accueil/Expertises/Financial-Advisory-Services/Glossaire-Definition/B/Besoin-en-fonds-de-roulement-BFR>

- le BFR d'exploitation qui correspond aux créances clients et la valeur des stocks desquels on soustrait les dettes fournisseurs
- le BFR hors exploitation (dettes envers les fournisseurs d'immobilisations, les impôts et taxes ou encore les dettes envers le personnel, etc.).

Le BFR hors exploitation n'ayant peu ou pas d'impact sur la planification du flux physique, nous considérerons par la suite uniquement le BFR d'exploitation que nous appellerons, par simplification, le BFR.

Pour modéliser ce dernier, nous avons adopté la méthodologie présentée dans Theodore Faris and Hutchison (2002); Hofmann and Kotzab (2010) qui précise que le BFR généré par la fabrication d'un article dépend :

- des sommes engagées dans les opérations liées à sa production (achat, configuration des machines, production, stockage, etc.)
- du temps à devoir financer celles-ci jusqu'à ce qu'intervienne le paiement du client lié à la vente de ce même article.

La figure 3.2 illustre ce principe de modélisation du BFR généré par un article dans le cas où le délai de paiement du fournisseur et le délai de paiement du client sont tous deux égaux à une période. Elle permet également de mettre en exergue l'impact des délais de paiement client et fournisseur ainsi que l'aspect cumulatif généré par le coût de stockage par période.

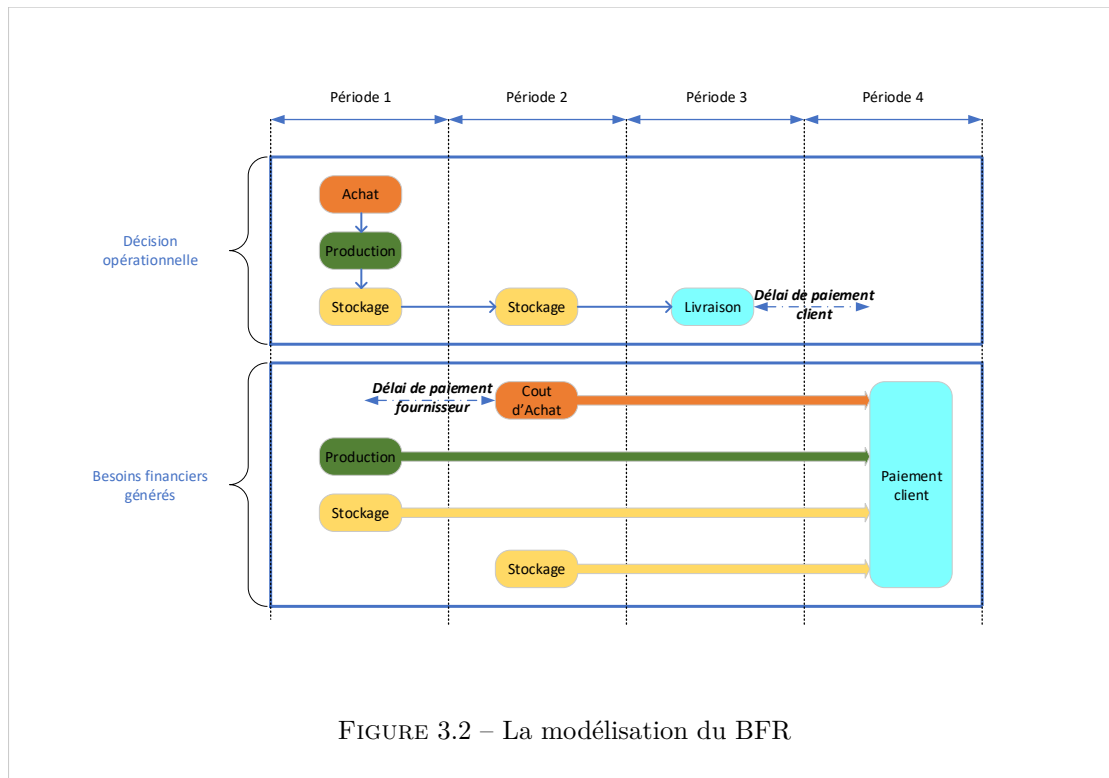


FIGURE 3.2 – La modélisation du BFR

Cependant, cette modélisation induit une difficulté liée au coût fixe de configuration de machine. En effet, ce coût est compté à chaque lot de production : à supposer que la taille du lot ne soit pas unitaire et que les articles du lot ne soient pas livrés au même moment (qu'ils soient imputés à des demandes de périodes différentes par exemple), cela implique que ce coût soit réparti uniformément sur chaque article du lot (on aura donc une non-linéarité de la modélisation



mathématique sous-jacente).

Ainsi, le BFR calculé sera la somme des BFRs engendrés par chaque coût logistique (Achat, Production, Stock, Configuration, etc.)

## 3.2 Intégration du financement du BFR dans les modèles de planification

Le but de cette recherche est de montrer l'impact de la prise en compte de critères financiers tels que l'optimisation du financement du BFR dans le processus de planification tactique. Les problèmes de planification de production peuvent être modélisés par le biais de modèles mathématiques appelés modèles de « lot sizing » Drexl and Kimms (1997). Aussi, nous allons montrer ici les premiers résultats obtenus en intégrant le besoin de financement du BFR dans certains de ces modèles. Dans un premier temps, nous présenterons le modèle s'appuyant sur les hypothèses de la formule de Wilson Harris (1990), puis nous basculerons sur des modèles discrets à capacité infinie puis finie et pour finir par un premier modèle multi-niveau.

### 3.2.1 Le modèle de Wilson

Ce modèle est le plus simple que nous pouvons rencontrer en planification de la production. Aussi connu sous le nom de « Quantité économique », il utilise les hypothèses suivantes :

- la demande est constante et répartie uniformément dans le temps,
- la capacité de production n'est pas prise en compte (on parle alors de capacité infinie),
- les délais de production sont nuls,
- la demande est servie à temps (pas de backlog),
- les coûts logistiques sont constants sur l'horizon de planification.

Une solution optimale de ce modèle est donnée par une formule analytique simple Harris (1990) et réalise un parfait compromis entre les coûts de configuration machine et les coûts de stockage : ce compromis est par ailleurs à la base de plusieurs heuristiques proposées dans des configurations plus complexes.

Nous nous plaçons sous les mêmes hypothèses et nous allons regarder l'impact du financement du BFR. La Table 3.1 donne les paramètres utilisés dans la modélisation proposée.

Paramètres	Définition
$T$	Taille de l'horizon de planification
$D$	Demande totale sur l'horizon
$h$	coût de stockage par unité de temps
$p$	coût de production par produit
$s$	coût de configuration machine pour un lot de production
$a$	coût d'approvisionnement par unité de produit
$r_c$	délai de paiement pour le client
$r_f$	délai de paiement pour le fournisseur
$\alpha$	taux d'intérêt pour l'emprunt finançant le BFR

TABLE 3.1 – Paramètres du modèle dans le cadre de la formule de Wilson

La Table 3.2 explicite le calcul du BFR pour chaque coût logistique.

Composante BFR	Représentation Graphique	Formulation mathématique
Achat	<p>Coût</p> <p><math>aQ</math></p> <p>0</p> <p><math>r_f</math> <math>r_c</math> <math>\frac{QT}{D}</math> <math>\frac{QT}{D} + r_c</math></p> <p>Horizon</p> <p>■ BFR généré par le coût d'achat</p>	$a \left[ Q (r_c - r_f) + \frac{Q^2 T}{2D} \right]$
Production	<p>Coût</p> <p><math>pQ</math></p> <p>0</p> <p><math>r_c</math> <math>\frac{QT}{D}</math> <math>\frac{QT}{D} + r_c</math></p> <p>Horizon</p> <p>■ BFR généré par le coût de production</p>	$p \left( Q r_c + \frac{Q^2 T}{2D} \right)$
Configuration	<p>Coût</p> <p><math>s</math></p> <p>0</p> <p><math>r_c</math> <math>\frac{QT}{D}</math> <math>\frac{QT}{D} + r_c</math></p> <p>Horizon</p> <p>■ BFR généré par le coût de configuration</p>	$s \left( r_c + \frac{QT}{2D} \right)$
Stock	<p>Coût</p> <p><math>hQ</math></p> <p>0</p> <p><math>\tau</math> <math>\tau + r_c</math> <math>\frac{QT}{D}</math> <math>\frac{QT}{D} + r_c</math></p> <p>Horizon</p> <p>■ BFR généré par le stock restant à l'instant <math>\tau</math></p> <p><math>BFR_\tau, \tau \in \left[ 0, \frac{QT}{D} \right]</math></p>	$\int_0^{\frac{QT}{D}} BFR_\tau d\tau = h \left( \frac{Q^3 T^2}{6D^2} + \frac{r_c Q^2 T}{2D} \right)$

TABLE 3.2 – Modélisation du BFR en fonction des coûts logistiques

La fonction de coût à optimiser est la somme des coûts logistiques et de financement du BFR

généralisé par la fabrication de lots de  $Q$  unités, soit :

$$f(Q) = \underbrace{(a+p)D + hT\frac{Q}{2} + s\frac{D}{Q}}_{\text{coûts logistiques}} + \underbrace{\alpha \times BFR}_{\text{Financement du BFR}} \quad (3.1)$$

La convexité de la fonction de coût permet de trouver aisément une solution optimale  $Q^*$  du problème :

**Proposition 3.1**

Sous les hypothèses et notations précédentes et en posant

$$\Delta = \frac{3\alpha^2 h^2 T^4}{s^2 (1 + \alpha r_c)^2 D^4} - \frac{[h + \alpha (hr_c + a + p)]^3 T^3}{2s^3 (1 + \alpha r_c)^3 D^3}$$

- si  $\Delta > 0$ ,

$$Q^* = \frac{1}{\sqrt[3]{\frac{hT^2\alpha}{6D^2s(1+\alpha r_c)}} \left\{ \sqrt[3]{1 + \sqrt{1 - \frac{D[h(1+\alpha r_c) + \alpha(a+p)]^3}{6h^2s(1+\alpha r_c)\alpha^2 T}}} + \sqrt[3]{1 - \sqrt{1 - \frac{D[h(1+\alpha r_c) + \alpha(a+p)]^3}{6h^2s(1+\alpha r_c)\alpha^2 T}}} \right\}}$$

- si  $\Delta = 0$ ,

$$Q^* = \frac{D[h + \alpha (hr_c + a + p)]}{2\alpha h T}$$

- si  $\Delta < 0$ ,

$$Q^* = \frac{1}{2\sqrt{\frac{[h(1+\alpha r_c) + \alpha(a+p)]T}{6Ds(1+\alpha r_c)}} \cos \left\{ \frac{1}{3} \arccos \left[ \sqrt{\frac{6h^2s(1+\alpha r_c)\alpha^2 T}{[h(1+\alpha r_c) + \alpha(a+p)]^3 D}} \right] \right\}}$$

Cette résolution permet de faire ressortir un certain nombre d'éléments intéressants :

1. Cette formule est bien une extension de la formule de Wilson (il suffit de prendre  $\alpha = 0$ )
2. Le résultat ne dépend pas du délai de paiement du fournisseur
3. Le compromis obtenu n'est plus le même qu'avec la formule de Wilson. En effet, il ne relève plus uniquement du compromis entre les coûts de configuration et les coûts de stockage, mais intègre désormais les différents paramètres financiers inhérents au calcul du BFR. Plus précisément, il s'écrit

$$\frac{D}{Q}s(1 + \alpha r_c) = \frac{Q}{2}T[h + \alpha (hr_c + a + p)] + \alpha \frac{hT^2}{3D}Q^2 \quad (3.2)$$

4. La taille de lot déterminée par cette formule est toujours inférieure à la taille donnée par la formule de Wilson. En d'autres termes, le plan de production obtenu possède un nombre de lançements de production supérieur, ce qui implique un abaissement du niveau de stock (ce qui n'est pas étonnant vu le poids quadratique du coût de stockage dans le nouveau compromis donné par la formule 3.2)
5. La taille de lot  $Q$  est croissante en  $s$  et  $r_c$ , décroissante pour les autres paramètres. On retrouve, de manière étendue, le comportement de la taille de lot donnée par la formule

de Wilson (plus le coût de configuration est élevé, plus la taille de lot est grand : on fait du stock ; plus le coût de stockage est élevé, plus la taille de lot est faible : on baisse le niveau de stock)

### 3.2.2 Le modèle mono-niveau à capacité infinie LS-U

Le modèle mono-niveau à capacité infinie (LS-U dans la nomenclature définie par Pochet and Wolsey (2006)) est le pendant du modèle de Wilson mais lorsque la demande est variable dans le temps. Il s'obtient par discrétisation de l'horizon en périodes et agrégation de la demande sur celles-ci. Il constitue, en quelque sorte, le modèle de base de la planification à demande variable.

Sa résolution s'appuie sur une propriété structurelle forte : « La production à stock nul (Zero Inventory Order - ZIO) » qui dit qu'il existe une solution optimale dans laquelle on ne produit que si le stock est nul. Cette propriété permet de reformuler aisément le problème sous la forme d'un problème de plus court chemin acyclique et se résout alors en temps polynomial par programmation dynamique Wagner and Whitin (2004). A l'instar du cas précédent, nous reprenons les mêmes hypothèses et nous nous intéressons à l'impact de l'intégration du financement du BFR. La Table 3.3 donne les paramètres du modèle.

$T$	: Nombres de périodes formant l'horizon de planification
$D_t$	: Demande à servir à la fin de la période $t$ , $\forall t \in [1, T]$
$h$	: Coût unitaire de stockage par période
$a$	: Coût d'approvisionnement par unité de produit
$p$	: Coût unitaire de production
$s$	: Coût fixe de configuration de machine
$r_c$	: délai de paiement (en nombre de périodes) pour le client
$r_f$	: délai de paiement (en nombre de périodes) pour le fournisseur
$\alpha$	: Taux d'intérêt pour l'emprunt finançant le BFR

TABLE 3.3 – paramètres du modèle LS-U<sub>BFR</sub>

Conformément à la méthodologie utilisée pour modéliser le BFR qui impose une traçabilité de la production, nous nous basons sur une formulation étendue de LS-U qui permet de spécifier précisément à quelle demande est attribuée quelle production : c'est la formulation « Facility Location Extended Formulation » donnée par Krarup and Bilde (1977). La Table 3.4 donne les variables de décision du modèle.

$X_{t,k}$	: Quantité produite à la période $t$ pour satisfaire une partie de la demande à la période $k$ , $\forall (t, k) \in [1, T]^2$ avec $t \leq k$ .
$Y_t$	: Variable binaire indiquant si une configuration machine est nécessaire à la période $t$ $\forall t \in [1, T]$ .

TABLE 3.4 – Variables utilisées dans le modèle LS-U<sub>BFR</sub>

La Table 3.5 explicite le calcul du BFR pour chaque coût logistique.

Composante BFR	Représentation Graphique	Formulation mathématique
Achat	<p>Coût</p> <p><math>aX_{tk}</math></p> <p>durée de financement</p> <p>Horizon</p> <p><math>t</math> <math>t+r_f</math> <math>k</math> <math>k+r_c-1</math></p> <p>■ BFR généré par le coût d'achat pour <math>X_{tk}</math></p>	$aX_{tk}(k-t+r_c-r_f)$
Production	<p>Coût</p> <p><math>pX_{tk}</math></p> <p>durée de financement</p> <p>Horizon</p> <p><math>t</math> <math>t+1</math> <math>k</math> <math>k+r_c-1</math></p> <p>■ BFR généré par le coût de production de <math>X_{tk}</math></p>	$pX_{tk}(k-t+r_c)$
Configuration	<p>Coût</p> <p><math>s</math></p> <p><math>\hat{s} = \frac{s}{\sum_{j \geq t} X_{tj}}</math></p> <p><math>\hat{s}X_{tk}</math></p> <p>durée de financement</p> <p>Horizon</p> <p><math>t</math> <math>t+1</math> <math>k</math> <math>k+r_c-1</math></p> <p>■ BFR généré par le coût de setup ramené à <math>X_{tk}</math></p>	$\frac{sX_{tk}}{\sum_{j \geq t} X_{tj}}(k-t+r_c) \quad (3.3)$
Stock	<p>Coût</p> <p><math>hX_{tk}</math></p> <p>durée de financement</p> <p>Horizon</p> <p><math>t</math> <math>t+1</math> <math>k</math> <math>k+r_c-1</math></p> <p>■ BFR généré par la mise en stock restant de <math>X_{tk}</math></p>	$hX_{tk} \frac{k-t+2r_c+1}{2}(k-t)$

TABLE 3.5 – Modélisation du BFR en fonction des coûts logistiques

La fonction de coût à optimiser est la somme des coûts logistiques et de financement du BFR généré par le plan de production, soit :

$$\underbrace{\sum_{t=1}^T \left( a \sum_{k=t-r_f}^T X_{(t-r_f)k} + p \sum_{k=t}^T X_{tk} + sY_t + h \sum_{l=1}^t \sum_{k=t+1}^T X_{lk} \right)}_{\text{coûts logistiques}} + \underbrace{\alpha \times BFR}_{\text{Financement du BFR}} \quad (3.4)$$

Le modèle peut donc s'écrire sous la forme :

$$\text{Minimiser (les coûts logistiques et le financement du BFR)} \quad (3.5)$$

sous contraintes:

$$\sum_{t=1}^k X_{tk} = D_k \quad \forall k \in [1, T] \quad (3.6)$$

$$X_{tk} \leq D_k \times Y_t \quad \forall (k, t) \in [1, t] \times [1, T] \quad (3.7)$$

$$X_{tk} \geq 0, Y_t \in [0, 1] \quad \forall t \in [1, T], \forall k \in [1, T] \quad (3.8)$$

$$Y_t \in \{0, 1\} \quad \forall t \in [1, T] \quad (3.9)$$

La contrainte (3.6) permet de s'assurer que la demande est exactement couverte pour chaque période. L'équation (3.7) est la contrainte de configuration qui s'assure que la machine est configurée s'il y a une production de prévue. (3.8) et (3.9) sont les contraintes de positivité et d'intégrité.

Ce modèle n'est pas linéaire en raison de l'intégration du financement du BFR dans la fonction objectif et, plus précisément, par le financement du coût de configuration (3.3). Nous avons étendu cette formulation en y intégrant la possibilité de prendre en compte la problématique d'actualisation de la valeur au cours du temps Helber (1998). Ainsi, nous avons défini les nouveaux paramètres suivants :

- $\beta$  : le taux d'actualisation de la valeur pour chaque période
- $h_t$  : la valeur du coût de stockage actualisé à la période  $t$ .  $h_t = h \frac{1}{(1+\beta)^t} \quad \forall t \in [1, T]$
- $a_t$  : la valeur du coût d'approvisionnement actualisé à la période  $t$ .  $a_t = a \frac{1}{(1+\beta)^t} \quad \forall t \in [1, T]$
- $p_t$  : la valeur du coût de production actualisé à la période  $t$ .  $p_t = p \frac{1}{(1+\beta)^t} \quad \forall t \in [1, T]$
- $s_t$  : la valeur du coût de configuration actualisé à la période  $t$ .  $s_t = s \frac{1}{(1+\beta)^t} \quad \forall t \in [1, T]$

Puis, nous avons montré que la propriété ZIO restait dominante pour cette modélisation Bian et al. (2018b) :

**Théorème 3.1** (Propriété ZIO pour  $LS - U_{BFR}$ )

Sous les hypothèses et les notations précédentes, il existe un plan de production optimal qui vérifie

$$\left( \sum_{k=t}^T X_{tk} \right) \left( \sum_{l=1}^{t-1} \sum_{k=t}^T X_{lk} \right) = 0 \quad \forall t \in [2, T] \quad (3.10)$$

Cette propriété structurelle forte nous permet donc de reformuler le problème sous la forme d'un plus court chemin comme représenté par la Figure 3.3.

La valuation de chaque arc est donnée par la formule :

$$E_{tk} = s + \sum_{l=t}^{k-1} D_l \left( a_{t+r_f} + p_t + \sum_{i=t}^{l-1} h_t \right) + \alpha \sum_{l=t}^{k-1} D_l \left[ \sum_{j=t}^T (a_{j+r_f} - a_{j+r_c}) + \sum_{j=t}^{l+r_c-1} \left( p_j + \frac{s_j}{\sum_{i=t}^{k-1} D_i} \right) + \sum_{i=t}^{l-1} \sum_{j=i}^{l+r_c-1} h_j \right] \quad (3.11)$$

Cette reformulation permet d'en déduire un algorithme de programmation dynamique Wagner and Whitin (2004) en  $O(T^4)$  prouvant que ce problème est polynomial.

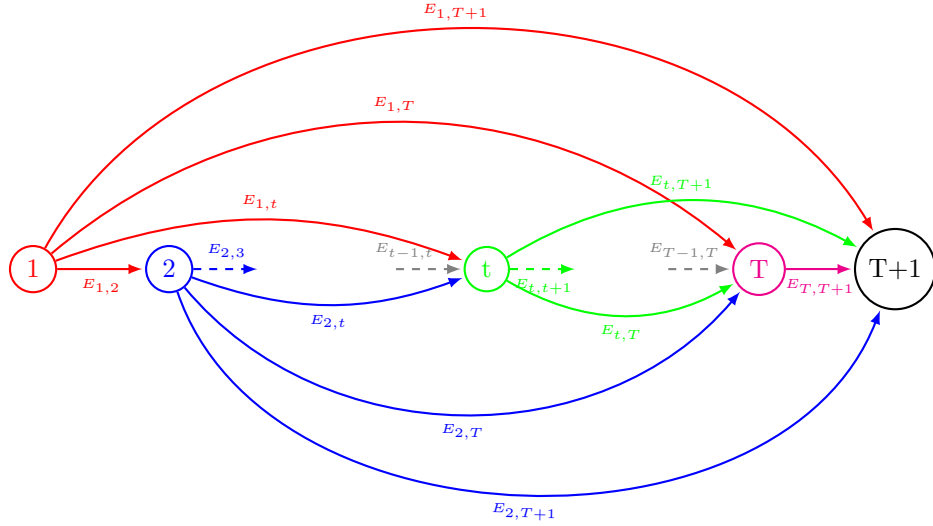


FIGURE 3.3 – Représentation comme un problème de plus court chemin

### 3.2.3 Le modèle mono-niveau à capacité finie et constante WW-CC

Dans Gayraud et al. (2019), nous avons étendu le cas précédent en ajoutant une capacité finie au système de production et nous avons également considéré des coûts variables.

Contrairement à LS-U, le problème de planification classique (LS-C) ainsi obtenu est NP-Difficile dans le cas général Bitran and Yanasse (1982). Cependant, dans le cas où la capacité est constante sur l'horizon (LS-CC), Florian and Klein (1971) ont montré qu'on pouvait le résoudre de façon polynomiale par un algorithme de programmation dynamique. Aussi, nous avons commencé par étudier ce cas là.

La Table 3.6 donne les paramètres du modèle.

$T$	: Nombres de périodes formant l'horizon de planification
$D_t$	: Demande à servir à la fin de la période $t$ , $t \in [1, T]$
$h_t$	: Coût unitaire de stockage à la période $t$ , $t \in [1, T]$
$a_t$	: Coût unitaire d'approvisionnement à la période $t$ , $t \in [1, T]$
$p_t$	: Coût unitaire de production à la période $t$ , $t \in [1, T]$
$s_t$	: Coût fixe de configuration de machine à la période $t$ , $t \in [1, T]$
$r_c$	: Délai de paiement (en nombre de périodes) pour le client
$r_f$	: Délai de paiement (en nombre de périodes) pour le fournisseur
$C$	: capacité de production à chaque période
$\alpha$	: Taux d'intérêt pour l'emprunt finançant le BFR

TABLE 3.6 – paramètres du modèle LS-CC<sub>BFR</sub>

La modélisation du BFR étant identique au cas précédent, la formulation du modèle est donc la suivante :

Minimiser (les coûts logistiques et le financement du BFR) (3.12)

sous contraintes:

$$\sum_{t=1}^k X_{tk} = D_k \quad \forall k \in [1, T] \quad (3.13)$$

$$\sum_{k=t}^T X_{tk} \leq CY_t \quad \forall t \in [1, T] \quad (3.14)$$

$$X_{tk} \geq 0, Y_t \in [0, 1] \quad \forall t \in [1, T], \forall k \in [1, T] \quad (3.15)$$

$$Y_t \in \{0, 1\} \quad \forall t \in [1, T] \quad (3.16)$$

La contrainte (3.13) permet de s'assurer que la demande est exactement couverte pour chaque période. L'équation (3.14) est la contrainte de capacité et de configuration : elle s'assure que la machine est configurée s'il y a une production de prévue et que sa capacité n'est pas dépassée en cas de production. (3.15) et (3.16) sont les contraintes de positivité et d'intégrité.

Dans la suite, nous avons besoin des deux définitions suivantes :

**Définition 3.5** (Problème de Wagner-Whitin et coûts non spéculatifs Pochet and Wolsey (2006))

En utilisant les notations précédentes, un problème de « lot sizing » classique est dit de la forme Wagner-Whitin (WW) si ses coûts de production et de stockage vérifient

$$p_{t+1} \leq p_t + h_t \quad (3.17)$$

Dans ce cas, on dit que la structure de coût est non spéculative.



**Définition 3.6** (Intervalle de régénération Pochet and Wolsey (2006))

Un intervalle  $[k, l]$  est appelé intervalle de régénération si les niveaux de stock à la fin de la période  $k - 1$  et  $l$  sont nuls mais que tous les niveaux de stocks intermédiaires sont strictement positifs.

Nous avons étendu la définition d'un problème de type WW en y intégrant le coût d'achat :

$$p_{t+1} + \left[ 1 - \frac{\alpha r_f}{1 + \alpha(1 + r_c)} \right] a_{t+1} \leq p_t + h_t + \left[ 1 - \frac{\alpha r_f}{1 + \alpha(1 + r_c)} \right] a_t \quad (3.18)$$

Et nous avons montré que sous cette hypothèse de structure de coûts et de faisabilité de la planification, alors la propriété structurelle ZIO restait dominante Dixon and Silver (1981) :

**Théorème 3.2** (Propriété ZIO pour  $WW - CC_{BFR}$ )

Sous les hypothèses et les notations précédentes, il existe un plan de production optimal qui vérifie

$$\left( C - \sum_{k=t}^T X_{tk} \right) \left( \sum_{l=1}^{t-1} \sum_{k=t}^T X_{lk} \right) \sum_{k=t}^T X_{tk} = 0 \quad \forall t \in [1, T] \quad (3.19)$$

Ce théorème qui généralise le cas à capacité infinie  $LS - U_{BFR}$  à cette structure de coûts (3.18) en étendant ainsi les travaux de Bian et al. (2018b), nous dit que :

- si le stock à la période  $t - 1$  est non nul, alors la production à la période  $t$  est nulle ou à pleine capacité
- si le stock à la période  $t - 1$  est nul, alors on peut produire une quantité quelconque (mais inférieure à la capacité à la période  $t$ ).

Or, d'après le principe de décomposition des inventaires Florian and Klein (1971), une solution optimale est une suite d'intervalles de régénération consécutifs. Si on considère un intervalle de régénération  $[k, l]$ , seuls les stocks à la fin de la période  $k - 1$  et à la fin de la période  $l$  sont nuls. Le théorème (3.2) nous dit que sur chaque période composant l'intervalle de régénération, nous ne rencontrerons que deux cas : soit une production à pleine capacité soit aucune production, à l'exception éventuelle de la période  $k$  où la production  $Q_k$  pourra ne pas être à pleine capacité (on remarquera qu'elle ne pourra pas être nulle sinon le stock à la fin de la période  $k + 1$  resterait nul ce qui contredit le fait que  $[k, l]$  soit un intervalle de régénération).

La capacité étant supposée constante, si  $Q_k \neq C$  alors nécessairement

$$Q_k = \left( \sum_{t=k}^l D_t \right) \text{ mod } C \quad (3.20)$$

Ce résultat montre que dans ce cas, l'algorithme de Florian and Klein (1971) s'applique (en modifiant le calcul des coûts de façon à prendre en compte le financement du BFR) et que le problème est donc polynomial (plus précisément, l'algorithme est en  $O(T^6)$ ).

### 3.2.4 Vers les modèles multi-niveau

Depuis le début de ce chapitre, nous nous sommes intéressés aux problématiques de planification mono-niveau mono-produit (dans le cas à capacité finie) : dans ce cadre là, les flux physique et financier générés sont particulièrement simples et la traçabilité du produit est aisée comme l'illustre la Figure 3.4.

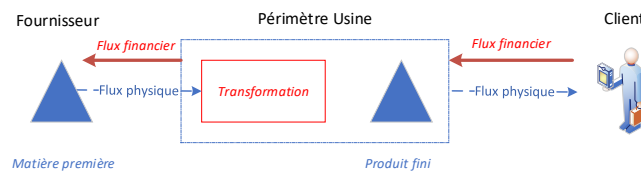


FIGURE 3.4 – Exemple de flux générés par une modélisation mono-niveau

En effet, une unité de matière première achetée donne lieu, par une opération de transformation, à une unité de produit fini qui sera stockée puis délivrée au client final : pour résumer, nous sommes donc dans un processus de fabrication de type un pour un. Dans un contexte multi-niveau, on s'intéresse à la production d'un ou plusieurs produits finis mais aussi à tout ou partie des composants intervenants dans le processus de fabrication. En effet, à l'instar du processus MRP Orlicky et al. (1994), on prend en compte d'une part les nomenclatures produits, le stock de chaque article planifié, les délais d'obtention, etc. Cette complexité est illustrée par la Figure 3.5.

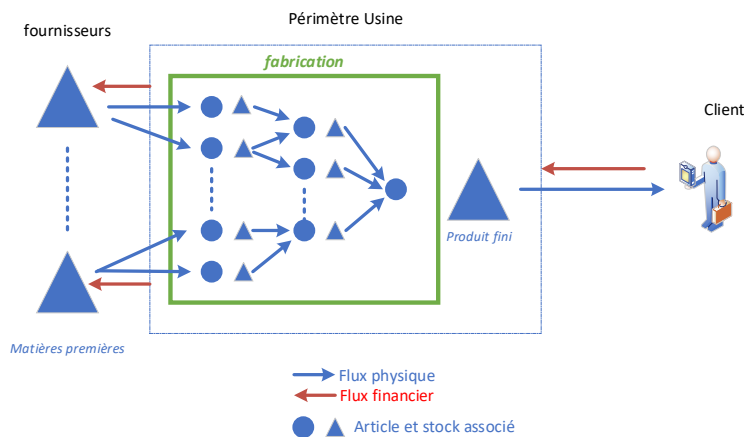


FIGURE 3.5 – Exemple de flux générés par une modélisation multi-niveau

Aussi, outre la complexité algorithmique engendrée par ce type de modèle de planification, on voit apparaître une complexité systémique nouvelle, engendrée par les processus de fabrication et la difficulté de tracer efficacement chaque produit. Mais celle-ci peut encore s'accroître si on considère des transactions financières internes comme dans le cas d'un réseau logistique interne, comme le montre la Figure 3.6.

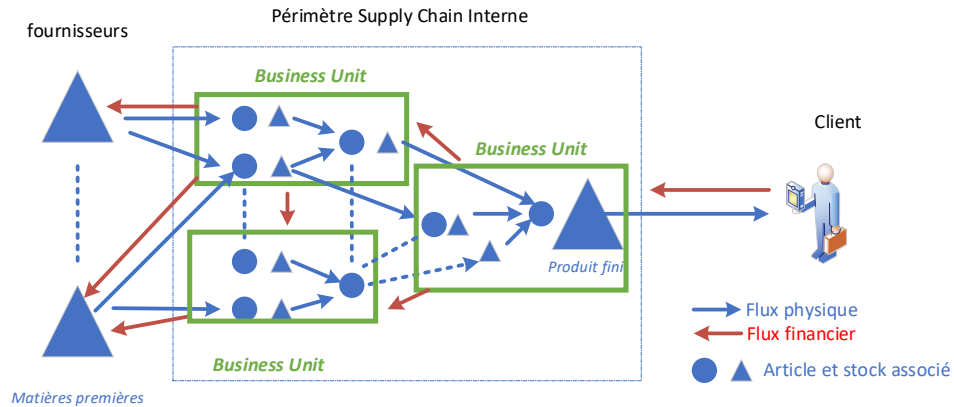


FIGURE 3.6 – Exemple de flux générés par une modélisation multi-niveau sur une supply chain interne

On voit donc que la modélisation du BFR devient particulièrement délicate dans le cas multi-niveau et pose un certain nombre de questions sur la structure organisationnelle de l'entreprise, même dans le cadre de centralisation de la décision de planification Thierry et al. (1994).

Dans Bian et al. (2020), nous nous sommes intéressés à la problématique multi-niveau la plus simple : le Multi-Level Lot Sizing Problem à nomenclature série et à capacité infinie (ML-S/LS-U). Zangwill (1969) a pu résoudre ce problème de « Lot Sizing » de façon exacte par programmation dynamique en  $O(NT^4)$  où  $N$  représente la profondeur de la nomenclature série considérée.

Dans Zangwill (1969), l'auteur considère la supply chain représentée par la Figure 3.7 : c'est une succession de sites de transformation (d'où la nomenclature série en 1 pour 1). De plus, il prend comme hypothèse des coûts constants.

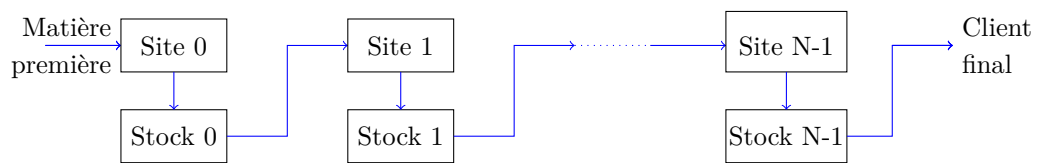
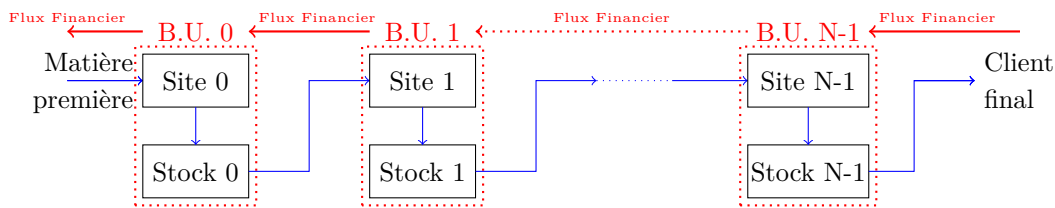


FIGURE 3.7 – La chaîne logistique ML-S/LS-U Zangwill (1969)

Nous avons donc pris la même structure de chaîne en considérant des flux financiers entre chaque site comme le montre la Figure 3.8.

Dans ce cas précis, on voit que les produits fabriqués sont facilement traçables et que les flux financiers sont simples à modéliser entre les sites en utilisant nos hypothèses de modélisation. Nous avons montré que, sous les hypothèses prises par Zangwill (1969), la propriété ZIO restait valide et qu'on pouvait résoudre le problème par programmation dynamique en  $O(NT^6)$

FIGURE 3.8 – La chaîne logistique interne ML-S/LS-U<sub>BFR</sub> Bian (2017)

### 3.3 Conclusion et perspectives

Comme l'ont montré Vickery et al. (2003); Chen et al. (2014), l'intégration de considérations financières dans le processus de planification tactique constitue un enjeu énorme en termes de performance industrielle. Aussi, dans le cadre des projets RCSM et FILEAS FOG, nous avons commencé par poser les bases méthodologiques pour modéliser le Besoin en Fonds de Roulement en fonction des coûts logistiques générés par les plans de production. Puis nous avons intégré cette fonction financière au sein de certains modèles de « Lot-Sizing » que nous avons pu résoudre efficacement à l'optimalité. La Table 3.7 synthétise les résultats numériques obtenus.

Modèle	Méthode de résolution	Complexité algorithmique
Wilson <sub>BFR</sub>	Analytique	N.A.
WW-U <sub>BFR</sub>	Programmation dynamique	$O(T^4)$
WW-CC <sub>BFR</sub>	Programmation dynamique	$O(T^6)$
ML-S/LS-U <sub>BFR</sub>	Programmation dynamique	$O(NT^6)$

TABLE 3.7 – Problèmes résolus

Cependant, nous ne sommes qu'au début de cette recherche. En effet, la littérature autour des modèles de base du « Lot Sizing » intégrant le besoin en financement reste à construire (Drexler and Kimms (1997); Comelli et al. (2008)). Outre la complexité algorithmique à laquelle il faudra se confronter (notamment pour les modèles mono-niveau), c'est bien sur les modèles multi-niveau que les efforts de modélisation devront désormais porter : la complexité systémique qu'ils induisent ouvre un champ de recherche tout à fait considérable et passionnant qu'il faudra défricher avant que de pouvoir songer à y apporter une solution numérique satisfaisante.

Au delà de ces modèles de base, l'avènement de l'entreprise multi-site, nous portera aussi à considérer les problématiques de transport et des coûts qu'ils génèrent. A terme, cela me permettra sans doute d'intégrer ces considérations dans le modèle de planification multi-site générique que j'ai pu proposer dans ma thèse Lemoine (2008). Mais le chemin sera encore bien long.



## Chapitre 4

# Bilan et Perspectives

Durant ma thèse de doctorat, je me suis focalisé sur les problématiques de planification de production et notamment sur la résolution de modèle de « Lot-Sizing ». J'avais déjà pu commencer à m'interroger sur la notion de capacité dans les systèmes de production, notion faisant apparaître la problématique d'intégration verticale de la décision. J'avais commencé à réfléchir sur l'intégration horizontale de la décision dans le cadre d'une chaîne logistique interne avec l'élaboration d'un modèle générique multi-site intégrant les problématiques liées aux transports entre les différents sites de production, de stockage etc. J'avais également entamé l'étude de l'impact de la décision financière sur la planification de production, par le biais d'une collaboration avec Michael Comelli et Pierre Fenies, tout deux doctorants en même temps que moi.

Depuis mon arrivée à l'École des Mines de Nantes, j'ai continué mes recherches sur ces différents axes. En effet, mon intégration au sein de l'équipe de recherche Systèmes Logistiques et de Production (SLP) dont le spectre d'activités de recherche est particulièrement large m'a permis de développer cette thématique d'intégration avec notamment la prise en compte des problématiques de maintenance ou de contrôle qualité.

Mon implication dans le projet RCSM m'a également permis de poursuivre mes travaux autour de la Supply Chain Finance. Alors qu'initialement ma recherche dans ce domaine s'était plus focalisée sur les problèmes d'optimisation de Cash-Flow et de partage de la valeur, ce projet à vocation industrielle a réorienté mon approche vers la prise en compte des besoins en financement. L'expression de ce besoin industriel d'optimisation du Besoin en Fonds de Roulement a fait émerger une nouvelle thématique de recherche en « Lot-Sizing » dans laquelle tout reste encore à construire. La richesse disciplinaire du consortium de ce projet a permis de mettre en place une vraie synergie avec des chercheurs venant du monde de la finance, synergie qui trouve sa parfaite illustration dans le cadre du projet ANR Fileas Fog dont j'ai été un des initiateurs et dont je suis également co-porteur.

Depuis ma prise de fonction à l'École des Mines de Nantes en 2009, j'ai pu co-encadrer 6 thèses dont 4 ont déjà été soutenues. J'ai également encadré 4 Masters et 3 Post-Doctorants. Outre mes participations à divers projets de recherche, j'ai également pu prendre un certain nombre de responsabilités dans certains d'entre eux : porteur de projet pour mon institution pour le FUI RCSM (260 800 €), co-porteur du projet ANR Fileas Fog (277 200 €) et responsable de Work-Package pour le projet PIA ORLoGES. En termes de publication, je suis entre-autres co-auteur de 9 publications en revue internationale et 2 publications en revue francophone et de 17 communications en conférences internationales avec comité de lecture et actes.

Lors de ces projets, j'ai pu initié ou être impliqué dans un certain nombre de collaborations académiques et/ou industrielles. On peut citer par exemple la collaboration avec le CREM et l'Université Rennes 1 qui a mené au dépôt de l'ANR Fileas Fog, celle avec le LARIS sur les problématiques de planification des blocs opératoires, mais encore avec des entreprises comme Luc Durant (pour le projet Orloges), Marsh ou Weave (RCSM). Nous avons également mis en place une collaboration forte avec General Electric Healthcare qui s'est déjà traduite par 2 thèses CIFRE, deux projets de fin d'études pour des ingénieurs de la filière MPR que je dirige, un apprenti pour la filière ingénierie logicielle de l'école et par une forte implication de leur part dans le montage de la formation par apprentissage FIT.

Depuis 2014, je suis co-animateur du groupe de travail Bermudes affilié au GdR MACS et au GdR RO pour lequel j'ai pu participer à l'organisation de sessions dans plusieurs conférences (MOSIM, MIM etc.), congrès (Roadef) ou journées du pôle Sciences et Techniques de Production GdR MACS. Outre les collaborations que j'ai pu mener ou initier lors des projets de recherche et que j'ai précédemment citées, j'ai pu également participer au montage d'une collaboration entre les Groupes de Travail Bermudes et Systèmes à Événement discret (SED) qui s'est concrétisé par l'élaboration d'une formation doctorale lors de l'école doctorale MACS de Juin 2019. A ce jour, un chapitre de livre autour des interactions entre Optimisation Discrète et Systèmes à Événements Discrets est par ailleurs en phase d'écriture sur cette thématique.

À la lumière de mon parcours depuis mon arrivée dans le monde de la recherche, je souhaite poursuivre mes activités en les développant plus précisément autour des axes suivants :

- *Les modèles de Lot-Sizing* : Au cœur de ma recherche doctorale, j'ai depuis utilisé ces modèles pour étudier les effets d'une meilleure estimation de la capacité de production par le biais de l'intégration de la décision ou de l'impact des décisions financières sur les plans de production. Ainsi, d'objets de recherche ils sont au cours du temps devenus contextes de recherche. Cependant, avec l'avènement de l'industrie 4.0, la digitalisation des systèmes industriels et la montée en puissance de l'informatique décisionnelle, la résolution de modèles de plus en plus intégrés va s'avérer nécessaire. Aussi, je souhaite reprendre mes travaux sur ce domaine et notamment autour :
  - des modèles de planification multi-niveau. Si la résolution des modèles multi-niveau à capacité infinie semble désormais bien maîtrisée (on peut citer, par exemple les travaux de Xiao et al. (2014)), des méthodes d'optimisation efficaces pour leurs pendant à capacité finie semblent encore devoir être développées.
  - des modèles de planification multi-site. Durant ma thèse, j'ai pu développer un modèle de planification multi-site générique. Celui-ci permet, par instanciation, de retrouver les modèles de planification de la littérature. Cependant, il se heurte à l'écueil de la centralisation de la décision Thierry et al. (1994) puisqu'elle en est l'une des hypothèses fondamentales. Buer et al. (2013) a ouvert la voie à la prise en compte d'une décision décentralisée dans la résolution de modèles multi-niveau (et donc, du coup, multi-site même si les aspects transports ne sont pas pris en compte dans ces travaux). Cependant, la décentralisation de la décision dans une supply chain pose un certain nombre de questions comme par exemple celles de la négociation, de la contractualisation, de la prise de décision avec asymétrie d'informations. Aussi, la prise en compte de ces aspects représente un défi sur lequel je souhaiterais me pencher. La thèse de Pooya Hedayatinia que je co-encadre actuellement, prend en compte une partie de ces problématiques dans le cadre du problème du News Vendor. Cela nous permettra de mieux appréhender ces difficultés.
- *L'intégration des décisions* : De nombreuses perspectives ont déjà été présentées dans les

précédents chapitres. Cependant, dans un contexte multi-site, on voit bien que l'intégration du transport devient cruciale. Aussi, outre les aspects liés à la maintenance et à la qualité que je souhaite continuer à développer, une meilleure intégration du transport et de ses problématiques est un axe d'investigation que j'aimerais aussi approfondir. Le projet ORLoGES m'a permis de commencer à défricher un peu cette problématique en la reliant au modèle multi-site que j'ai développé durant ma thèse.

- *La supply chain Finance* : Comme dit précédemment, la prise en compte du besoin en financement dans l'optimisation des plans de production a ouvert des perspectives de recherche très larges puisqu'elle implique la reconstruction de la théorie développée pour les modèles de Lot-Sizing. Aussi, que ce soit autour de la formalisation de ses nouveaux modèles et de leurs résolutions, tout reste à faire. De plus, l'intégration des décisions et des coûts qui sont engendrés par les activités prises en compte (Transport, maintenance, inspections etc.) seront d'autant plus pertinents dans cette problématique. Je souhaite donc poursuivre dans cette voie qui me semble plus que prometteuse si je m'en réfère à l'intérêt industriel qu'elle génère. Je souhaiterais également poursuivre les travaux initiés avec Michael Comelli et Pierre Fenies autour de l'optimisation du Cash-Flow et du partage de la valeur dans la chaîne logistique, cette thématique semblant particulièrement se prêter aux aspects multi-site. Des liens semblant se dessiner assez naturellement entre besoin de financement et partage de la valeur (par le biais des niveaux de trésorerie), il sera sans doute important d'étudier l'intégration ou du moins les impacts de ces deux approches l'une sur l'autre.

L'expérience que j'ai pu accumuler durant ces années lors de ces diverses collaborations et projets m'incite à poursuivre dans la voie de la recherche collaborative et interdisciplinaire :

- Collaborative car le projet RCSM a montré l'intérêt des interactions « Académique-Industriel ». La complémentarité de ce projet avec Fileas Fog (un ANR recherche) montre que l'Académique et l'Industriel peuvent (et ont tout à intérêt à) tisser des liens forts au travers de leurs problématiques même si les échelles temporelles sont évidemment différentes.
- Interdisciplinaire car c'est grâce à ces différentes rencontres, à ces différentes cultures scientifiques (maintenance, finance etc.) que j'ai pu développer des thématiques de recherche sur l'intégration de décisions prenant mieux en compte les problématiques rencontrées dans les disciplines concernées.

L'avènement de la digitalisation industrielle (l'industrie 4.0), de l'interconnexion des systèmes d'information et de la mise à disposition de masses de données toujours plus considérables alliée à une puissance de calcul toujours plus grande me confortent dans l'idée qu'un certain nombre d'enjeux majeurs en logistique et notamment en planification se trouvent désormais dans l'intégration des décisions en production et la prise en compte de critères autres que purement logistiques (par exemple financiers) et donc, en corollaire, dans la recherche collaborative et interdisciplinaire.

Cependant, la réussite à collaborer avec le monde industriel passe beaucoup par notre capacité à communiquer autour de notre recherche, à la vulgariser et à l'illustrer. Deux moyens efficaces résident d'une part dans l'élaboration de plateformes de démonstration à vocation industrielle et d'autre part dans la formation de nos étudiants, futurs cadres dans l'industrie, aux techniques avancées, à l'innovation et aux interactions avec les laboratoires de recherche. C'est pour répondre au premier point que j'ai entrepris le développement d'une plateforme centralisant l'ensemble de mes recherches et illustrant les résultats que nous pouvons obtenir. Quant



au second point, la formation par apprentissage FIT et son fil rouge autour de l'innovation, son Unité d'Enseignement dédiée à la recherche ainsi que les projets interdisciplinaires que les étudiants auront à mener tout au long de leur cursus seront autant d'éléments favorisant ces futures interactions et collaborations.

Troisième partie  
Sélection d'articles



# Sélection d'articles

Les articles suivants sont annexés à la suite ce manuscrit :

1. Michel Gourgand, David Lemoine, and Sylvie Norre. *Metaheuristic for the Capacitated Lot Sizing Problem: a software tool for MPS elaboration*. International Journal of Mathematics in Operational Research, 2(6):724–747, 2010a. doi: 10.1504/IJMOR.2010.035496
2. A. El Dor, David Lemoine, M. Clerc, P. Siarry, L. Deroussi, and M. Gourgand. *Dynamic cluster in particle swarm optimization algorithm*. Natural Computing, Springer, 14(4): 655–672, 2015. doi: 10.1007/s11047-014-9465-2
3. Bruno Castanier and David Lemoine. *A Preliminary Integrated Model for Optimizing Tactical Production Planning and Condition-Based Maintenance*. In Proceedings of 2011 International Conference on Industrial Engineering and Systems Management (IESM), pages 998–1007, Metz, France, May 2011a. ISBN 978-2-9600532-3-4
4. Belgacem Bettayeb, Nadjib Brahimi, and David Lemoine. *Integrated dynamic single item lot-sizing and quality inspection planning*. International Journal of Production Research, 56(7):2611–2627, October 2018. doi: 10.1080/00207543.2017.1385869
5. Yuan Bian, David Lemoine, Thomas G. Yeung, Nathalie Bostel, Vincent Hovelaque, Jean-Laurent Viviani, and Fabrice Gayraud. *A dynamic lot-sizing-based profit maximization discounted cash flow model considering working capital requirement financing cost with infinite production capacity*. International Journal of Production Economics, 196:319–332, 2018b. doi: 10.1016/j.ijpe.2017.12.002
6. Hasan Al Hasan, Christelle Gueret, David Lemoine, and David Rivreau. *Surgical case scheduling with sterilising activity constraints*. International Journal of Production Research, 57(10):2984–3002, September 2019. doi: 10.1080/00207543.2018.1521015

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## Metaheuristic for the Capacitated Lot Sizing Problem: a software tool for MPS elaboration

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Michel Gourgand, David Lemoine\* and Sylvie Norre

LIMOS, UMR 6158, CNRS  
Blaise Pascal University, Clermont Ferrand, France  
E-mail: gourgand@isima.fr  
E-mail: lemoine@moniut.univ-bpclermont.fr  
E-mail: norre@moniut.univ-bpclermont.fr

\*Corresponding author

**Abstract:**

The Master Production Schedule (MPS) elaboration plays a major part in tactical planning. Among mathematical models (called 'lot sizing problem') which deal with the tactical planning, a particular one is dedicated to this problem: the Capacitated Lot Sizing Problem (CLSP). The literature about the resolution of this problem is huge, nevertheless few metaheuristics have been developed in order to solve it. In this paper, we propose to use optimization methods based on a Simulated Annealing. The data encoding which is needed for such an approach is based on a production planning matrix. We defined a neighborhood system which is made up of several kinds of possible moves which consist in shifting or swapping quantities which have been produced in order to minimize logistic costs (setup or holding costs). We also proposed a bi-objective function which integrates logistic costs as well as an evaluation of the degree of the capacities' violation. Such a function allows the neighborhood system to temporarily leave the set of feasible solutions for the CLSP in order to escape from local minimas. We have tested our optimization methods on benchmarks from literature and we outperform the best results given for these instances. Finally, we present a software tool which elaborates MPS thanks to several optimization methods such as our metaheuristics or some heuristics from literature, with an user friendly interface.

**Keywords:** CLSP, metaheuristic, neighborhood system, bi-objective function, software tool, MPS.

**Reference** to this paper should be made as follows: Gourgand, M., Lemoine, D. and Norre, S. (xxxx) 'Metaheuristic for the Capacitated Lot Sizing Problem: a software tool for MPS elaboration', *Int. J. of Mathematics in Operational Research*, Vol. x, No. x, pp.xxx-xxx.

**Biographical notes:** Michel Gourgand is Professor of computer sciences in the University of Blaise Pascal (Clermont-Ferrand, France) and manages the research team Modelling and Decision Aid of the LIMOS CNRS UMR 6158 (Laboratoire d'Informatique, de Modélisation et d'Optimisation des Systèmes). His research and teaching interests include manufacturing system modelling, scheduling problems, supply

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chain management and metaheuristics.

David Lemoine received his PhD in Computer Science in LIMOS at the University of Blaise Pascal in France. His research interests include issues related to tactical planning optimization (lot-sizing models ...): mathematical programming (mathematical models, lagrangean relaxations ...), approximated methods (heuristics, metaheuristics ...).

Sylvie Norre is Professor of computer sciences in the University of Blaise Pascal (Clermont-Ferrand, France) and is a member of the research team Modelling and Decision Aid of the LIMOS CNRS UMR 6158. Her research and teaching interests include operational research, especially approximated methods and performance evaluation (heuristics, metaheuristics) for scheduling and planning problems.

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## 1 Introduction

Production management deals with a set of decisions in order to ensure company continuity. These decisions have to allow the production of customers demand, at the right due date and with the lowest costs. Thus, production management must find a compromise between the satisfaction of customers demand and production criteria such as logistic costs, production times and products quality. Usually, this set is divided into three decisional levels (Giard (2003)): the strategic level (long-term: more than 18 months), the tactical level (midterm: from 6 to 18 months) and finally the operational level (short-term: from daily to weekly). Each level can be characterized on the one hand by the horizon on which decisions are taken and, on the other hand, by the costs that these decisions can bring about. It defines a framework for the next lower level.

Transversely to these decisional degrees, a major problem can be underlined in production management: the industrial planning. Thus, this problem can be also divided into three hierarchical levels: the strategical, tactical and operational plannings.

1. strategical planning (long-term) deals with, for instance, the elaboration of the Supply Chain Network (facilities or warehouses locations etc.) and provides a framework for the tactical planning.
2. tactical planning (midterm) consists in the elaboration of production plans which minimize logistic costs (for instance production, holding or setup costs). It means that the aim of tactical planning is to determine, for each item, quantities to be manufactured per period in order to meet the customers' request at a lower cost.
3. operational planning (short-term) aim is to schedule the production defined at tactical planning level on the production system.

In this paper, we focus on tactical planning. Referring to hierarchical planning system proposed by Vollmann et al. (1997), tactical planning is composed of three

planning levels:

- the Sales and Operation Planning (S&OP),
- the Master Production Schedule (MPS),
- the Material Requirement Planning (MRP).

The S&OP consists in finding a balance between sales and production level on a midterm horizon (around 1 year). On this horizon, production capacity is not considered as fixed but can vary thanks to the use of overtime, sub contracting, hiring, firing and so on.

The MPS is defined by Genin (2003) as a problem for which the first goal is to find an optimal production plan which meets the customers' requests and provides release dates and amounts for all these products. The objective function lies in minimizing production, holding and setup costs.

Elaborating internal component planning is the main goal of MRP. By using bills of materials, these plans are deduced from the MPS.

Usually, tactical production planning are modelled thanks to mathematical models called 'lot-sizing problems'. A huge number of models have been proposed in the literature (Drexel and Kimms (1997), Rizk and Martel (1997)). Since our contribution deals with MPS elaboration, we focus on the 'Capacitated Lot Sizing Problem' (CLSP) which is known to be the lot-sizing mathematical model dedicated to our problem (Drexel and Kimms (1997)). The latter has been proved to be NP-Hard (Bitran and Yanasse (1982)). Therefore, its resolution has inspired a lot of research and more specifically on approximated methods (Comelli et al. (2008), Karimi et al. (2003), Jans and Degraeve (2007)). Nevertheless, Brahim (2004) points out that few metaheuristics have been used to solve it, despite their efficiency for the resolution of NP-Hard Problems. Thus, the aim of our contribution is to provide effective optimization methods based on a simulated annealing in order to elaborate MPS for large size benchmarks. A software tool which incorporates this method is also designed.

Our paper is organised as follows: in Section 2, we present the CLSP, its extensions as well as a brief survey of the optimization methods which can be found in the literature in order to solve it. The next section deals with our approach: we describe the data encoding as well as an initial neighborhood system which are used in our metaheuristic. Due to the problems to access at each feasible solution, some new moves have been added to the initial neighborhood system and a bi-objective function whose aim is on the one hand to find a feasible solution for the CLSP and on the other hand to allow the moves in solutions' set has been designed. In section 4, some results on benchmarks from literature are given. Section 5 is dedicated to the software tool and finally, conclusion and perspectives are given.

## 2 The Capacitated Lot Sizing Problem (CLSP)

In this section, we present the CLSP, its main extensions as well as a brief survey of optimization methods which can be found in the literature in order to solve it.

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### 2.1 CLSP : the mathematical model and its extensions

The CLSP has been designed in order to elaborate the MPS. Thus, only end-items are taken into account. Such a model is called ‘mono-level lot-sizing model’, contrary to ‘multi-level lot-sizing model’ which can also take into account components production planning (MRP). According to Drexel and Kimms (1997), the CLSP can be considered as the model dedicated to the MPS.

*Parameters for the CLSP:*

$T$	: number of periods,	
$N$	: number of items,	
$M$	: a very large number,	
$C_t$	: available capacity of the machines at period $t$ ,	$t \in [1, T]$
$d_{it}$	: external demand for item $i$ at period $t$ ,	$i \in [1, N], t \in [1, T]$
$I_{i0}$	: initial inventory for item $i$ ,	$i \in [1, N]$
$p_i$	: capacity request to produce one unit of item $i$ ,	$i \in [1, N]$
$h_i$	: non-negative holding costs for item $i$ for one period,	$i \in [1, N]$
$s_i$	: non-negative setup costs for item $i$ for one period,	$i \in [1, N]$
$b_i$	: non-negative production costs for item $i$ for one period,	$i \in [1, N]$

*Decision variables for the CLSP:*

$Q_{it}$	: production quantity of item $i$ at period $t$ ,	$i \in [1, N], t \in [1, T]$
$I_{it}$	: inventory of item $i$ at the end of period $t$	$i \in [1, N], t \in [1, T]$
$X_{it}$	: binary variable which indicates whether a setup for item $i$ occurs at period $t$ ( $X_{it} = 1$ ) or not ( $X_{it} = 0$ )	$i \in [1, N], t \in [1, T]$

Using this notation, the CLSP can formally be couched as a mixed-integer programming model:

$$\text{Minimize } \sum_{t=1}^T \left[ \sum_{i=1}^N (s_i X_{it} + b_i Q_{it} + h_i I_{it}) \right] \quad (1)$$

Subject to:

$$I_{it} = I_{i(t-1)} + Q_{it} - d_{it} \quad \forall (i, t) \in [1, N] \times [1, T] \quad (2)$$

$$p_i Q_{it} \leq M X_{it} \quad \forall (i, t) \in [1, N] \times [1, T] \quad (3)$$

$$\sum_{i=1}^N p_i Q_{it} \leq C_t \quad \forall t \in [1, T] \quad (4)$$

$$X_{it} \in \{0, 1\} \quad \forall (i, t) \in [1, N] \times [1, T] \quad (5)$$

$$I_{it}, Q_{it} \geq 0 \quad \forall (i, t) \in [1, N] \times [1, T] \quad (6)$$

(1) is the objective function, equal to the sum of setup, holding and production costs that we seek to minimize. Equation (2) represents inventory balances. Inequality (3) represents the setup constraints: production of an item can only take



place if the machine is set up for that particular item. Inequality (4) represents capacity constraints. Inequality (5) is the non negativity conditions and setup variables are defined to be binary (6).

For instance, in such a model, an acceptable solution will necessary meet the customers request without delay, as a result the flexibility of the production system could be very weak (backlogging is not allowed ...). To deal with this kind of drawbacks, many extensions have been proposed in the literature. Here are the main ones:

- Backlogging (Zangwill (1969)): this extension allows production backlogs. In order to do that, new decision variables  $I_{it}^-$  (the backlog quantity of item  $i$  at the end of period  $t$ ) are introduced. Their aim is to count the number of item  $i$  which are late at the end of period  $t$ . Of course, a backlogging cost  $h_i^-$  is charged to the objective function. Thus, the latter becomes:

$$\text{Minimize } \sum_{t=1}^T \left[ \sum_{i=1}^N (s_i X_{it} + b_i Q_{it} + h_i I_{it} + h_i^- I_{it}^-) \right] \quad (7)$$

and the new inventory balances equation is:

$$I_{it} + I_{it}^- = I_{i(t-1)} + I_{i(t-1)}^- + Q_{it} - d_{it} \quad \forall (i, t) \in [1, N] \times [1, T] \quad (8)$$

- Overtime (Diaby et al. (1992)): instead of allowing backlogs in production, another choice can be to temporarily increase the capacities of the production system, by resorting to overtime for instance. This possibility can be modelled by modifying the capacity constraint (4) in the following way:

$$\sum_{i=1}^N p_i Q_{it} \leq C_t + O_t \quad \forall t \in [1, T] \quad (9)$$

where  $O_t$  is a decision variable which represents the increase of the capacity at period  $t$ . As well as backlogging, using this additional capacity at period  $t$  entails a new logistic cost  $m_t$  (the overtime cost) in the objective function as follows:

$$\text{Minimize } \sum_{t=1}^T \left[ \sum_{i=1}^N (s_i X_{it} + b_i Q_{it} + h_i I_{it}) + m_t O_t \right] \quad (10)$$

- Setup time (Trigeiro et al. (1989)): the resource reconfiguration time modeling can be useful to make model more realistic. Indeed, such a setup can take too much time for being neglected and can be translated into a capacity consumption. Thus, capacity constraint (4) becomes:

$$\sum_{i=1}^N (p_i Q_{it} + \tau_{it} X_{it}) \leq C_t \quad \forall t \in [1, T] \quad (11)$$

where  $\tau_{it}$  represents the capacity consumption due to the ressource configuration for producing item  $i$  at period  $t$ . Trigeiro et al. (1989) have shown that using this extension makes the feasibility problem NP-Complete.

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- Infinite capacity (Wagner and Whitin (1958)): in this case, the capacities of the production system are supposed to be infinite (in literature, this extension is also called ‘without capacity’). Wagner and Whitin (1958) have proved that an optimal solution for this problem necessarily verifies the ‘zero-switch’ property (also called ‘Wagner-Whitin property’): production for an item  $i$  at period  $t$  can take place if, and only if, inventory of item  $i$  at the end of period  $t-1$  is equal to 0 ( $Q_{it} \times I_{i(t-1)} = 0, \forall (i, t) \in [1, N] \times [1, T]$ ). According to this property, they conceived a polynomial time dynamic programming algorithm based on a shortest path algorithm which is able to solve this problem.

## 2.2 CLSP : state of the art

In this brief survey, we focus on the optimization methods which have been designed to solve the CLSP in its basic version. These methods can be divided into two sets: mathematical programming and approximated methods. Aim of mathematical programming is to find optimal solutions despite computing time which can be prohibitive, whereas approximated methods try to quickly find good solutions by sacrificing this optimality. Since CLSP is NP-Hard (Bitran and Yanasse (1982)), research has rather focussed on the elaboration of approximated methods.

Despite their great efficiency, few mathematical programming methods have been designed, due to huge computing time for solving real industrial cases. Judging from literature, we can distinguish three kinds of mathematical programming methods which have been used to solve CLSP:

- Branch & Bound based methods (Gelders et al. (1986), Hindi (1995))
- Branch & Cut based methods (Leung et al. (1989), Pochet and Wolsey (1991))
- methods which used a shortest path reformulation of the CLSP (Eppen and Martin (1987))

If we focus on approximated methods, we can divide them into three sets: common sense heuristics, mathematical programming heuristics and metaheuristics:

- common sense heuristics are usually greedy algorithms. They tackle the CLSP problem according to three axes: period per period (Dixon and Silver (1981)), item per item (Kirca and Kökten (1994)) or by improvement (Dogramaci et al. (1981)). The two first ones are construction heuristics whereas the last one tries to improve a non necessary feasible plan.
- mathematical programming heuristics are based on lagrangean and linear relaxation, on column generation or Dantzig-Wolfe algorithm procedure (Huisman et al. (2003)) by using the Manne’s approximated reformulation of the CLSP model (Manne (1958)).
- few works have been done in order to solve CLSP by using metaheuristics (Brahimi (2004)). Among them, we can highlight Salomon et al. (1993) which have used a simulated annealing and a tabu search methods, Haase et al. (1996) which have developed genetic algorithm and Hindi (1996) which has designed a tabu search algorithm.

For more exhaustive state of the art, readers can refer to Comelli et al. (2008) or Karimi et al. (2003).

### 3 Metaheuristics for the CLSP

The aim of this paper is to present a metaheuristic based on Simulated Annealing which can provide rather good solution for large instances (20 items over 100 periods). In order to use such an approach, we have to define (Michalewicz and Fogel (2002)):

- a data encoding which is able to represent the problem solutions,
- a neighborhood system (a procedure which allows a move from a solution to an other one),
- a function which is able to evaluate a solution.

A necessary (but not sufficient) condition for a smooth running of such a method is to keep every solution accessible and to ensure the reversibility of the neighborhood system (Hajek (1988)):

- accessibility can be seen as a path connectedness property: for each couple of solutions, we can move from one to the other by applying one or more times the neighborhood system.
- reversibility of the neighborhood system is its ability to be symmetrical: if a solution  $x$  can be obtained from a solution  $y$  by applying one time the neighborhood system, then it will be possible to obtain the solution  $y$  from  $x$  by also applying one time the neighborhood system.

In this section, we propose an initial neighborhood system. Nevertheless, it does not meet accessibility condition. Therefore, we extend it with two new moves and we design a bi-objective function: their aim is to allow the optimization method to take out from feasible solutions sets.

#### 3.1 The data encoding

In the CLSP, we can find three kinds of decision variables:

- the production quantity of item  $i$  at period  $t$ ,  $Q_{it}$ ,
- the inventory of item  $i$  at the end of period  $t$ ,  $I_{it}$ ,
- the binary variable  $X_{it}$  which indicates whether a setup for item  $i$  occurs at period  $t$  ( $X_{it} = 1$ ) or not ( $X_{it} = 0$ ).

In order to encode a solution for the CLSP, we should take into account the set of all decision variables. Nevertheless, inventory balances (Equation (2)) show the bijection between inventory and production variables. Indeed, Equation (2) can be also written in the following manner:

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$$Q_{it} = I_{it} - I_{i(t-1)} + d_{it} \quad \forall (i, t) \in [1, N] \times [1, T] \quad (12)$$

This equality proves that production quantities can be deduced from inventory decision variables in a unique way. Reciprocally, by appropriately adding the inventory balances equations, we obtain:

$$I_{it} = I_{i0} + \sum_{k=1}^t (Q_{ik} - d_{ik}) \quad \forall (i, t) \in [1, N] \times [1, T] \quad (13)$$

It shows that inventory decision variables can be deduced from production decision variables in a unique way. Thus, taking into account inventory variables in addition to production variables into our data encoding will be redundant.

With regards to setup decisions variables  $X_{it}$ , they clearly depend on the production quantities. More precisely,

$$X_{it} = \chi_{]0, +\infty[}(Q_{it}) \quad \forall (i, t) \in [1, N] \times [1, T] \quad (14)$$

where  $\chi_{]0, +\infty[}$  is characteristic function of the subset  $]0, +\infty[$ .

As such a function is not a bijection (in fact, it is a surjective function), we can conclude that it is possible to deduce the setup variables from production quantities in a unique way. Nevertheless, the opposite may be generally wrong: indeed, if we know the periods where production take place, it is generally impossible to deduce production quantities in a unique way. Salomon et al. (1993) have designed their data encoding by using the setup decisions variables. They have computed the production quantities for each item  $i$  thanks to a greedy algorithm. Nevertheless, they cannot ensure that the determined production quantities were the best solution among all the production plans which verify the setup values given by their data encoding. Thus, by restricting the search space of their simulated annealing in this manner, they cannot insure that optimal solutions will be in this restriction.

All these observations lead us to only represent production decision variables in our data encoding. Thus, a solution will be encoded by using a matrix  $Q \in \mathcal{M}_{N \times T}(\mathbb{N})$  where  $\forall (i, t) \in [1, N] \times [1, T]$ ,  $Q_{it}$  represents the production quantity for item  $i$  at period  $t$ .

### 3.2 The initial neighborhood system

The neighborhood system is a procedure which has to allow the move from a feasible solution to a set of other feasible solutions. For the CLSP, a solution is feasible if, and only if, production quantities meet the customers request without delay and respect the production system capacities. The first condition can be expressed by the Equation (6) (inventory decision variables are always non negative) and the second one by the capacity constraint (4).

Let  $Q \in \mathcal{M}_{N \times T}(\mathbb{N})$  an feasible solution for the CLSP. From the latter, we wish to obtain a new feasible solution  $Q^*$ . In order to do that, we have to move production quantities from a period to an other one for at least one item. We suppose

that we have to move a quantity  $K$  of item  $i$  which has been produced at period  $t$ , to the period  $t'$ . We have to characterize such a quantity  $K$ .

Firstly, it is obvious that we cannot shift more quantity than the production quantity of item  $i$  at period  $t$ . Therefore  $K \leq Q_{it}$ . Moreover, once the quantity  $K$  will have been shifted, the capacity of the production system shall not have to be exceeded at period  $t'$ . This can be expressed by the constraint (4):

$$\sum_{j=1}^N p_j Q_{jt'} + p_i K \leq C_{t'}$$

As  $p_i$  is supposed to be not equal to 0 (otherwise, the problem for item  $i$  can be solved with a polynomial algorithm, see Wagner and Whitin (1958)) and  $K$  is an integer number, it comes:

$$K \leq \left\lfloor \frac{C_{t'} - \sum_{i=1}^N p_i Q_{it'}}{p_i} \right\rfloor$$

Thus,  $K$  have to verify:

$$0 \leq K \leq \min \left( \left\lfloor \frac{C_{t'} - \sum_{i=1}^N p_i Q_{it'}}{p_i} \right\rfloor, Q_{it} \right) \tag{15}$$

Even if inequality (15) ensures the respect of the capacity constraint (4), it does not mean that inventory variables will be non negative after such a shift. With regards to inventory decision variables, we can distinguish two cases:

Case 1:  $t' < t$

As illustrated by Figure 1, in this case, positivity of inventory variables will always be preserved. Indeed, since solution  $Q$  meets the customer requests, this is necessarily identical for solution  $Q^*$ : in fact, the production is only anticipated.

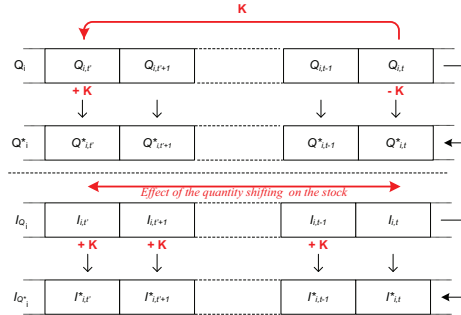
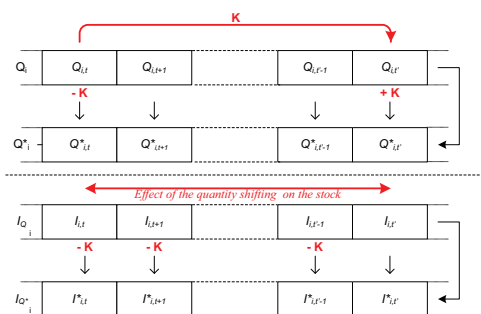


Figure 1 Effect of the quantity shifting (case 1)

Case 2:  $t < t'$

As shown on Figure 2, the fact of shifting the production quantity  $K$  of item  $i$  from period  $t$  to period  $t'$  implies the subtraction of this value to all the inventory decision variables for item  $i$  between periods  $t$  and  $t' - 1$ . Thus, in order to keep these variables non negative, quantity  $K$  has to verify:

$$0 \leq K \leq \min_{t \leq l < t'} (I_{il}) \quad (16)$$



**Figure 2** Effect of the quantity shifting (case 2)

The conjunction between inequalities (15) and (16) allows us to express the following proposition:

**Proposition 1** *Let  $Q$  a feasible solution for the CLSP. In order to obtain a new feasible solution  $Q^*$  from  $Q$  by shifting a production quantity  $K$  of item  $i$  from period  $t$  to period  $t'$ ,  $K$  has to verify:*

- if  $t' \leq t$ , then

$$0 \leq K \leq \min \left( \left\lfloor \frac{C_{t'} - \sum_{i=1}^N p_i Q_{it'}}{p_i} \right\rfloor, Q_{it} \right)$$

- if  $t' > t$ , then

$$0 \leq K \leq \min \left( \left\lfloor \frac{C_{t'} - \sum_{i=1}^N p_i Q_{it'}}{p_i} \right\rfloor, Q_{it}, \min_{t \leq l < t'} (I_{il}) \right)$$

The neighborhood system which we propose, largely uses this proposition. In fact, it is made up of four kinds of possible moves which act on the logistic costs: the holding and setup costs.

## 3.2.1 The first move: shifting without guide

The first move we propose consists in shifting a quantity  $K$  which respects the conditions expressed in proposition 1. The aim of this move is twofold:

- try to remove a maximum of setup time,
- try to reduce the quantities in stock.

In order to remove a setup for an item  $i$  at period  $t$ , we have to shift the whole production of this item at this period to an other one  $t'$ . To favour this, we choose  $K$  as big as possible, according to proposition 1 (we called this policy the  $K_{max}$  rule). Nevertheless, systematically choosing the  $K_{max}$  rule is generally not appropriated to reduce the quantities in stock. Indeed, in this case, the quantities which have to be shifted could be less than  $K_{max}$ . That's why we allow to choose  $K$  at most equal to  $K_{max}$ . Thus,  $K$  will be determined thanks to these two kinds of policies which will be chosen according to a fixed probability. After many tests, we choose to apply  $K_{max}$  rule with a probability of  $\frac{2}{3}$ . Figure 3 illustrates the principle of the first move.

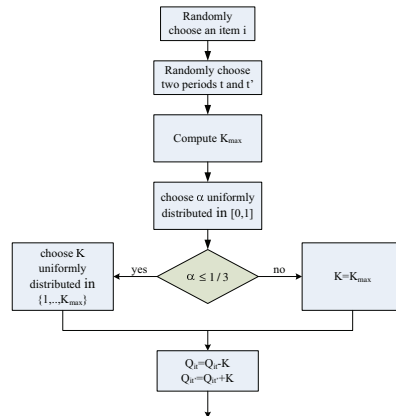


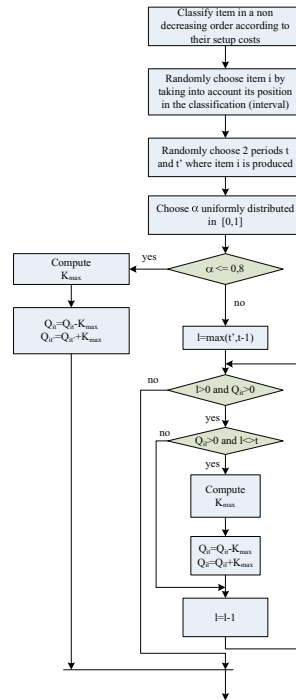
Figure 3 The first move algorithm

## 3.2.2 The second move: shifting in order to remove setup time

The aim of this second move is to favour the removal of setups which are the most expensive among the setup costs. Thus, items are classified in a non-decreasing order according to their setup costs. The item will be chosen randomly by using this classification. However, we have to favour the choice of the most expensive

items: we also allocate an integer interval to each item. The size of this interval will be function of the position of its item in the classification. Thus, if the position of item  $i$  in the classification is  $j$ , the allocated integer interval will be  $\left[1 + \sum_{k=1}^{j-1} k, \sum_{k=1}^j k\right]$  (we can notice that the size of this interval is equal to the position of its item in the classification). Then, the item  $i$  on which the second move will act is selected as follows: a number  $\beta$  is chosen uniformly distributed in the integer interval  $\left[1, \frac{N \times (N+1)}{2}\right]$ . Thus,  $\beta$  belongs to only one interval of an item  $i$ : by the way, we determine the item  $i$  which is selected.

Figure 4 illustrates the principle of the second move.



**Figure 4** The second move algorithm

As soon as item  $i$  was selected, the second move proceeds as follows: firstly, two periods  $t$  and  $t'$  where item  $i$  is produced are chosen. Then, two strategies are proposed:



- the first one is similar to the  $K_{max}$  rule described in the first move.
- in the second one, we try to shift the entire production of item  $i$  at period  $t$  by smoothing it into other periods where there is also production for that item. This smoothing is done backward from the period  $l = \max(t', t - 1)$  to period 1. For a period  $p$  between 1 and  $l$ , if there is production for item  $i$ , we use the  $K_{max}$  rule for this period. If the production for item  $i$  at period  $t$  is still not null we go backwards to the period  $p - 1$ , otherwise the move is stopped: the whole production of item  $i$  at period  $t$  has been shifted. As soon as  $p$  is equal to 0, we leave the remaining production of item  $i$  at period  $t$  and the move is stopped.

The two strategies will be chosen thanks to a probability: after many tests, we choose to apply the first one with a probability of 0,8.

### 3.2.3 The third move: shifting in order to reduce holding costs

The aim of this move is to favour the reducing of stocks which are the most expensive among the holding costs. In fact, its principle is almost similar to the second one. The differences are the following two points: firstly, the items are classified in a non-decreasing order according to their holding costs (instead of their setup costs). Secondly, when the period  $t'$  is chosen, we impose that  $t' \geq t$ . Then, the smoothing procedure is applied from  $t'$  to period  $t$  (Indeed, if the end period of the smoothing procedure is before  $t$ , the holding costs could increase).

### 3.2.4 The fourth move: the swap between two items

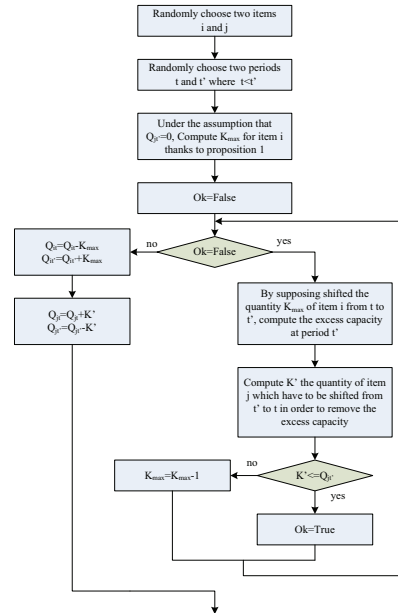
This move allows to swap production quantity between two kinds of items  $i$  and  $j$  and two periods  $t$  and  $t'$ . At the beginning, we have to choose randomly two items  $i$  and  $j$  and two periods  $t$  and  $t'$  under the assumption that  $t < t'$ . Then, we suppose that item  $j$  is not produced at period  $t'$ .

According to this assumption, we compute  $K_{max}$  the maximal quantity of item  $i$  that we can shift from period  $t$  to period  $t'$  thanks to the proposition 1.

Next, we suppose that we have shifted this quantity to period  $t'$  and we determine if the capacity of the production system has been exceeded by taking into account the production of item  $j$  at period  $t'$ . If necessary, we try to remove the excess capacity by shifting the smallest quantity of item  $j$  from period  $t'$  to period  $t$ .

If we cannot remove the whole excess capacity, we decrease  $K_{max}$  of one unit of item  $i$  and we repeat the procedure until we find the maximum quantity of item  $i$  which can be shifted from period  $t$  to period  $t'$  and whose excess capacity that has been generated can be balanced by a shift of item  $j$  from period  $t'$  to period  $t$ . Then, the swap is done.

Figure 5 illustrates the principle of the fourth move.



**Figure 5** The fourth move algorithm

The first neighborhood system that we propose is made of the four moves which have been described. Each move can be selected under the hypothesis of equiprobability.

### 3.3 The bi-objective function

For the CLSP, a solution is feasible if, and only if, production quantities meet the customers request without delay and respect the production system capacities. The first neighborhood system allows the move from a feasible solution to a set of other feasible solutions. Nevertheless, it can be confronted to an accessibility problem. This could arrive particularly if the remaining capacity is scarce (for instance, the capacity of the production system is hardly sufficient to ensure the production required to meet the customers' request). To deal with this problem, it would be interesting to relax capacity constraints (4), like it has been done with overtime extension (9). Thus, we allow the neighborhood system to leave the set of feasible solutions for the CLSP. Afterwards, we call solution any non feasible solution which

meets the customers' request (it does not respect the capacity constraints). Thus, feasible solutions for the CLSP is a subset of the solutions.

Since the aim is to find the best feasible solution for the CLSP, we have to favour its obtaining. Therefore, we have to elaborate an objective function which penalizes the violations of capacity constraints. To do that, we propose to define a bi-objective function  $f$  as follows: for each solution  $Q$ ,  $f(Q) = (Cap(Q), Cost(Q))$  where

- $Cap(Q)$  is a function which evaluate the degree of the violation of the capacity ( $Cap(Q) = 0$  if all the capacity constraints (4) are respected, otherwise  $Cap(Q) > 0$ ),
- $Cost(Q)$  is the cost of the solution, given by the objective function of the CLSP (1)

In order to favour the obtaining of a feasible solution, we define a total order of the solution set:

**Definition 1** *Let  $Q$  and  $Q'$  two solutions.  $Q$  is called 'better solution' than  $Q'$  (and we note  $Q < Q'$ ) if, and only if,*

- either  $Cap(Q) < Cap(Q')$
- or  $Cap(Q) = Cap(Q')$  and  $Cost(Q) < Cost(Q')$

Thus, according to this order, a feasible solution for the CLSP will be necessarily better than any non feasible solution for the CLSP. Nevertheless, we have to define the  $Cap$  function. It has to verify:

- $Cap(Q) = 0$  if, and only if,  $Q$  is a feasible solution for the CLSP,
- $Cap(Q) > 0$  otherwise.

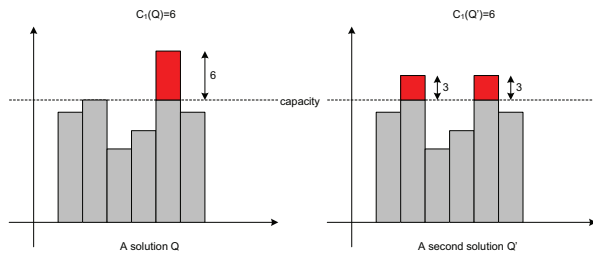
To be feasible, a solution  $Q$  has to verify the capacities constraints (4), that is equivalent to:

$$\sum_{t=1}^T \text{Max} \left[ 0, \sum_{i=1}^N (p_i Q_{it}) - C_t \right] = 0$$

Consequently, our first idea was to define the following function  $C_1$  as the  $Cap$  function:

$$C_1(Q) = \sum_{t=1}^T \text{Max} \left[ 0, \sum_{i=1}^N (p_i Q_{it}) - C_t \right]$$

This idea is almost similar to the objective function of overtime extension (10) in which the coefficient  $m_t$  would be greater than the cost of each feasible solution for CLSP. Nevertheless, in order to find a feasible solution, it would be suitable that this function can smooth the production which exceeds the production capacities. Indeed, as illustrated in Figure 6, the function  $C_1$  cannot distinguish two solutions which are very different in terms of smoothing.

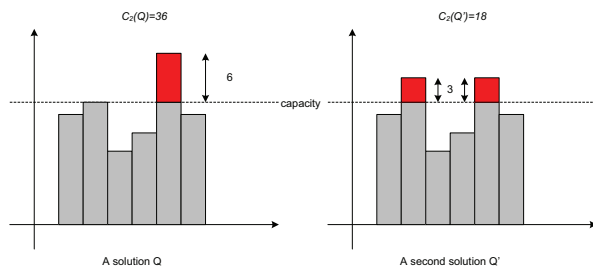


**Figure 6** Two solutions which are equivalent for function  $C_1$

In order to take into account this aspect, we propose to define the function  $C_2$ :

$$C_2(Q) = \sum_{t=1}^T \text{Max} \left[ 0, \sum_{i=1}^N (p_i Q_{it}) - C_t \right]^2$$

This quadratic function favours the smoothing load as shown of Figure 7.



**Figure 7** The evaluation of the same solutions thanks to function  $C_2$

Thus, we define  $Cap(Q) = C_2(Q)$ .

### 3.4 The final neighborhood system

In order to fully exploit this bi-objective function by allowing the neighborhood system to take out from feasible solutions sets, two new moves are added to the first neighborhood system. These moves use the proposition 2, a version of proposition 1 which does not take into account the capacity constraints.

**Proposition 2** Let  $Q$  a solution for the CLSP. In order to obtain a new solution  $Q^*$  from  $Q$  by shifting a production quantity  $K$  of item  $i$  from period  $t$  to period  $t'$ ,  $K$  has to verify:

- if  $t' \leq t$ , then

$$0 \leq K \leq Q_{it}$$

- if  $t' > t$ , then

$$0 \leq K \leq \min \left( Q_{it}, \min_{t \leq l < t'} (I_{il}) \right)$$

3.4.1 The fifth move: shifting without guide and with infinite capacity

This move is almost similar to move 1. Nevertheless, it does not take into account the capacity of the production system at periode  $t'$ . Indeed,  $K_{max}$  is now computed thanks to proposition 2.

3.4.2 The sixth move: swap with infinite capacity

This move is a simplified version of the move 4. In fact,  $K$  (resp.  $K'$ ) the quantity of item  $i$  (resp. item  $j$ ) to be shifted from period  $t$  to period  $t'$  (resp. from period  $t'$  to period  $t$ ) is chosen uniformly distributed in the integer interval  $[1, K_{max}]$  (resp.  $[1, K'_{max}]$ ) where  $K_{max}$  (resp.  $K'_{max}$ ) is computed thanks to proposition 2

The final neighborhood system that we propose is made of the six moves which have been described. Each move can be selected under the hypothesis of equiprobability.

3.5 Our optimization scheme

Figure 8 illustrates the principle of our optimization method.

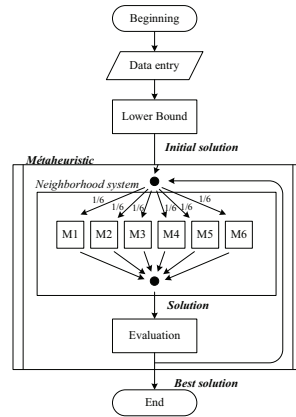


Figure 8 Our optimization scheme

In order to evaluate the quality of the best solutions obtained by the metaheuristic, we have designed a lower bound computation based on the lagrangean relaxation of the capacity constraints (4). Chen and Thizzy (1990) have proved that with a lagrangean relaxation, the best lower bounds are obtained by relaxing these constraints. This relaxation has been combined with subgradient optimization according to Held and Karp (1970) scheme which has given rather good results with the TSP problem. By using this method, we obtain on the one hand rather good lower bounds and on the other hand an initial solution for our optimization method (Lemoine (2008)). Indeed, only the capacity constraints have been relaxed. Therefore the solution given by the lagrangean relaxation meets the customer requests. Thus, the latter is a solution which can be used by our metaheuristic.

#### 4 Our results

We have tested our metaheuristic on the Eppen and Martin (1987) instances. Indeed, they can be considered as reference ones for the CLSP (Comelli et al. (2008)). These benchmarks is a set of 17 instances which have been generated by their authors by using instances from Dixon and Silver (1981) and Thizy and Van Wassenhove (1983).

The table 1 gives the results obtained with our metaheuristic after 5 replications with 20 000 000 iterations for each of them (less than 30' of computing time). We can find the name of the instance as well as its size (number of items and number of periods). Best solution, Worst solution, Mean solution as well as Standard deviation are mentioned. Each solution  $Q$  is described thanks its value given by the bi-objective function:  $(Cap(Q)||Cost(Q))$ . N.A. means Non Applicable. Results which have been proved as optimal in literature are emphasized in bold and italic.

Name	Size	Best Sol.	Worst Sol.	Mean Sol.	Std
EM1	8 × 8	(0   <b><i>8430</i></b> )	(0  8870)	(0  8695,4)	47,2759
EM2	8 × 8	(0  7918)	(0  7952)	(0  7940)	34,1872
EM3	8 × 8	(0   <b><i>7610</i></b> )	(0  7620)	(0  7616)	12,5524
EM4	8 × 8	(0   <b><i>7520</i></b> )	(0  7560)	(0  7528)	41,2349
EM5	50 × 8	(0  48591)	(0  48831)	(0  48741,2)	86,9157
EM6	100 × 8	(0  94234)	(0  94500)	(0  94380,2)	80,9489
EM7	150 × 8	(0  137537)	(0  138234)	(0  137906,8)	151,8985
EM8	200 × 8	(0  187700)	(0  187860)	(0  187786)	33,2172
EM9	20 × 13	(0  5860,1)	(0  6058,3)	(0  5934,356)	196,3156
EM10	100 × 10	(0  20913,2)	(0  20945)	(0  20923,16)	17,4515
EM11	100 × 10	(0  20956,4)	(0  20958,2)	(0  20957,68)	1,0264
EM12	100 × 10	(0  21076,2)	(0  21136,7)	(0  21098,892)	32,2888
EM13	100 × 10	(0  21294)	(0  21375,2)	(0  21341,54)	50,5072
EM14	100 × 10	(0  21311,2)	(0  21420,4)	(0  21355,64)	58,1618
EM15	100 × 10	(0  21404)	(0  21504,6)	(0  21467,18)	58,231
EM16	100 × 10	(89,87 22567,82)	(89,87 22588,7)	N.A.	N.A.
EM17	200 × 10	(0  43812,54)	(0  43978,10)	(0  43914,24)	65,7409

**Table 1** Our metaheuristic's results

These results, and particularly the obtained standard deviations, show that our metaheuristic is rather stable. We can notice that our optimization have not found a feasible solution for EM16 (in fact, this instance hasn't got any feasible solution (Eppen and Martin (1987))).

Table 2 compares our results with the lower bound computed thanks to the lagrangean relaxation. Gap are computed thanks to the following formula:

$$GAP = \frac{\text{Our result} - \text{Lower Bound}}{\text{Lower Bound}} \%$$

Name	Size	Lower Bound	Best Solution	Gap
EM1	8 × 8	7837,59	(0  <b>8430</b> )	7,56 %
EM2	8 × 8	7722,27	(0 7918)	2,53 %
EM3	8 × 8	7534,16	(0  <b>7610</b> )	1,01 %
EM4	8 × 8	7464,16	(0  <b>7520</b> )	0,75 %
EM5	50 × 8	48472,64	(0 48591)	0,25 %
EM6	100 × 8	93991,21	(0 94234)	0,26 %
EM7	150 × 8	137361,14	(0 137537)	0,13 %
EM8	200 × 8	187400,31	(0 187700)	0,16 %
EM9	20 × 13	5668,36	(0 5860,1)	3,38 %
EM10	100 × 10	20899,27	(0 20913,2)	0,07 %
EM11	100 × 10	20950,39	(0 20956,4)	0,03 %
EM12	100 × 10	21033,47	(0 21076,2)	0,20 %
EM13	100 × 10	21192,63	(0 21294)	0,48 %
EM14	100 × 10	21236,63	(0 21311,2)	0,35 %
EM15	100 × 10	21288,78	(0 21404)	0,54 %
EM16	100 × 10	22916,95	(89,87 22567,82)	N.A.
EM17	200 × 10	43631,94	(0 43812,54)	0,41 %

**Table 2** The gap between the computed lower bound and our results

We can see that the gap between our results and the lower bound are quite small. We can notice that, for the smallest instances, the Gap is more important. This is due to the quality of the lower bound. Indeed, for these instances, optimal solutions have been obtained. Chen and Thizzy (1990) have already pointed out the fact that when the size of the instance is rather small then, the use of a lagrangean relaxation of the capacity constraints does not provide good lower bound.

Name	Epp	H.D.W.	H.OSL	H.T.	B. Known	B.Sol	Gap
EM1	<b>8430</b>	<b>8430</b>	<b>8430</b>	8730	<b>8430</b>	<b>8430</b>	0
EM2	<b>7910</b>	<b>7910</b>	<b>7910</b>	8250	<b>7910</b>	7918	+0,11
EM3	<b>7610</b>	<b>7610</b>	<b>7610</b>	7690	<b>7610</b>	<b>7610</b>	0
EM4	<b>7520</b>	<b>7520</b>	<b>7520</b>	<b>7520</b>	<b>7520</b>	<b>7520</b>	0
EM5	49129,5	48686,2	48690	49001,9	48686,2	<b>48591</b>	-0,19
EM6	94379,7	94318,4	97130	95681,3	94318,4	<b>94234</b>	-0,08
EM7	137819	137829,1	138772,3	139755,2	137819	<b>137537</b>	-0,2
EM8	188122	188296,7	189646,3	191274,4	188122	<b>187700</b>	-0,22
EM9	<b>5807,66</b>	5814,52	5812	5860,31	<b>5807,66</b>	5860,1	+0,91
EM10	20923	20929,3	<b>20911,2</b>	20929,3	<b>20911,2</b>	20913,2	+0,01
EM11	20962	21015,89	20990,07	21069,85	20962	<b>20956,4</b>	-0,02
EM12	21082,1	21121,83	<b>21059,59</b>	21267,65	<b>21059,59</b>	21076,2	-0,02
EM13	21336,2	21295,1	<b>21257</b>	21319,82	<b>21257</b>	21294	+0,18
EM14	21381,43	21338,84	21320,11	21368,47	21338,84	<b>21311,2</b>	-0,12
EM15	21402,1	21400,92	<b>21369,86</b>	21463,34	<b>21369,86</b>	21404	+0,16
EM16	Inf.	Inf.	Inf.	Inf.	Inf.	Inf.	NA
EM17	43710,2	43775,2	<b>43650</b>	43809,36	<b>43650</b>	43812,54	+0,38

**Table 3** literature results versus ours

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Table 3 gives a comparison between the most significant results which can be found in the literature and ours. In this table, we can find:

- Epp. which are the results given by Eppen and Martin (1987),
- H.D.W. which are the results given by the Dantzig-Wolfe approach given by Hindi (1995),
- H.OSL which are the results given by a solver heuristic approach using OSL solver, given by Hindi (1995),
- H.T. are the results given by the Taboo Search developed in Hindi (1996),
- B. Known are the best known solution in the litterature,
- B. sol are our best result for the considered instance,
- Gap is the Gap between our result and the best known solution. Gap is computed thanks to the following formula

$$GAP = \frac{\text{Our result} - \text{Best Known}}{\text{Best Known}} \%$$

N.A. means Non Applicable, Inf. Infeasible. For each instance, the best Results are emphasized in bold.

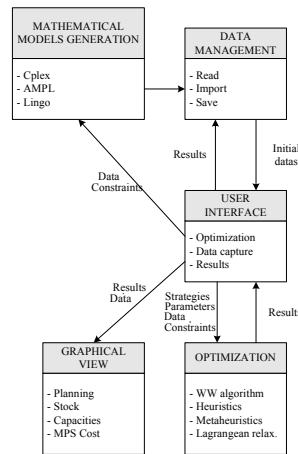
Our results are rather good. Indeed, for 9 instances among 16, best solution is obtained by our method. Moreover, for 6 instances, we improve the best know solution. We can also notice that, except for the last instance (EM17), our meta-heuristic outperforms the Taboo Search proposed by Hindi (1996). Finally, we can highlight that, even if for some instances our method does not improve the best known solution, the Gap between the latter and our results is rather small (less than one percent). Thus, we can conclude to the efficiency of our approach.

## 5 Software tool for the MPS elaboration

According to Drexl and Kimms (1997), the CLSP can be considered as the model dedicated to the MPS elaboration. We have proposed an efficient method to solve this problem. Thus, we have included it into a software tool which has an user friendly interface and whose aim is to elaborate MPS in an industrial context. Due to the quickness of our optimization method, this software can be used, for instance, in small and medium sized businesses in order to elaborate their production plans according to different scenarios relating to capacities of production and customers' demands. Figure 9 gives the functional scheme of our software tool.

This application is maked up of five units whose each of them is dedicated to a particular functionality of the program.





**Figure 9** The functional scheme of the software tool

We can find:

- the 'Data Management' unit which is dedicated to the management of files and data. Thanks to this unit, user can save or load his own benchmarks and can even import instances from literature.
- the 'Mathematical Generations Models' unit is able to generate CLSP mathematical models for several solvers such as Cplex or Lingo.
- the 'Optimization' unit is dedicated to the optimization methods which could be used in order to elaborate the MPS. We can find some heuristics found in the literature (Dixon and Silver (1981), Dogramaci et al. (1981), Trigeiro et al. (1989)). The algorithm of Wagner and Whitin (1958) which provides an optimal solution when capacity are considered as infinite, has also been implemented. Of course, metaheuristics which use our neighborhood system and the bi-objective function (Kangaroo, Hill Climbing, Simulated annealing ...) as well as lagrangean relaxation described before have been incorporated.
- the 'User interface' unit allows the user to specify the parameters which describe the production system (capacity etc.), customer's request for each item at each period as well as logistic costs (holding, production and setup costs). Optimization method can also be parametrized (user can define the time dedicated to the optimization process, the Gap which has to be obtained, the optimization method which has to be chosen, etc.). Figure 10 shows the user interface for optimization methods.
- the aims of 'Graphical View' unit are twofold: firstly it allows the user to follow the optimization method's current results (Figure 10) and secondly,

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final results (inventory position (Figure 11), production planning (Figure 12) etc.) are represented thanks to some charts.

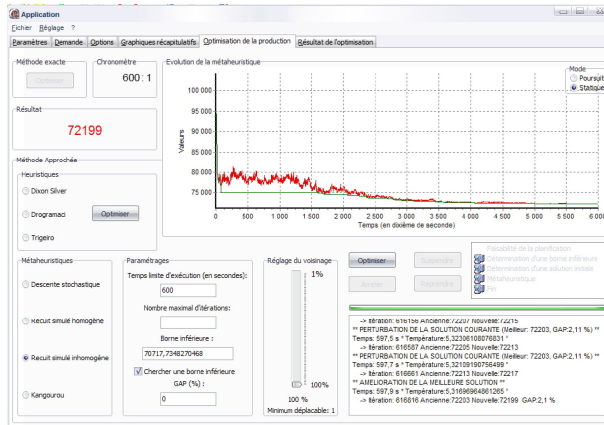


Figure 10 The user interface for the optimization method

In Figure 11, we can find the holding planning associated to the production planning elaborated by our optimization method. User can also check the holding position through the 'holding position chart', which represents the inventory level, for each product, over the whole horizon.

	période 1	période 2	période 3	période 4	période 5	période 6	période 7	période 8	période 9	période 10	période 11	période 12	période 13	période 14	période 15
produit 1	0	91	91	0	107	0	124	2	188	37					
produit 2	0	0	0	91	0	95	0	0	0	93					
produit 3	0	0	0	0	107	0	186	91	0	105					
produit 4	0	0	0	0	0	91	0	99	0	0					
produit 5	200	105	1	101	0	104	97	1	96	2					
produit 6	0	0	0	0	0	0	0	0	0	0					

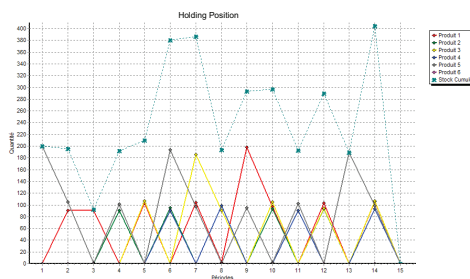


Figure 11 Graphical view of the inventory position

As shown in Figure 12, the production planning elaborated by our optimization method is given. Moreover, user can take a look at this production planning thanks to scrolling charts which represent cumulative production plan as well as detailed production plan for each item.

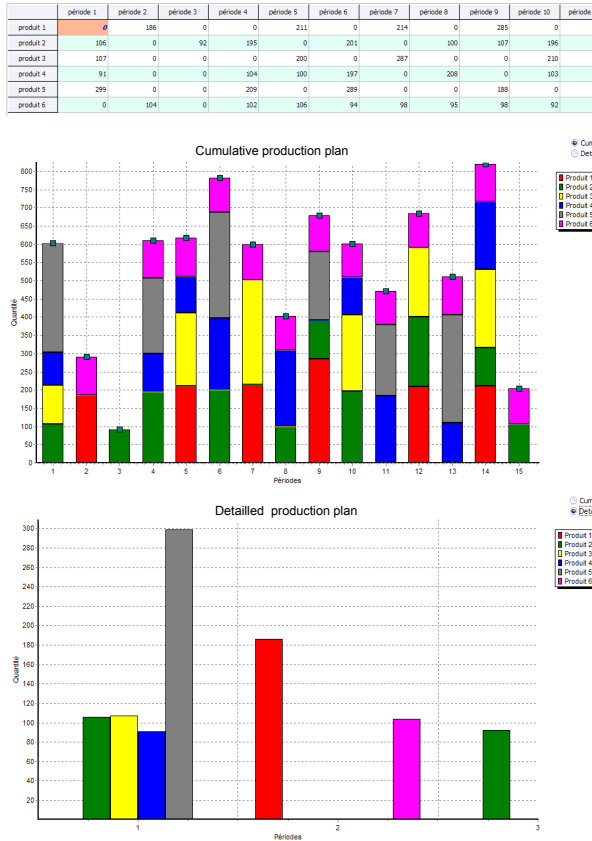


Figure 12 Graphical view of the production planning

## 6 Conclusion

In this paper we have presented a new optimization methods based on a Simulated Annealing and a bi-objective function which is able to elaborate a Master Production Schedule by taking into account the capacities of the production system. A lagrangean relaxation of capacity constraints has been used in order to compute lower bound. The efficiency of our approach has been shown: indeed, some best known solutions have been improved and the stability of our method has been highlighted. A software tool for MPS elaboration has been designed: the proposed methods have been implemented and an user friendly interface has been designed. At the moment, we are working on the adaptation of our optimization method on the 'CLSP with setup time' whose feasibility is NP-Complete. Nevertheless, the first results are very promising: indeed, we have already outperformed Miller et al. (2000) results on the instances given by Trigeiro et al. (1989). This is due to the properties of the bi-objective function. But, we still have to improve the neighborhood system in order to be competitive in front of the best methods which have been developped on this problem ((Pochet and Wolsey (1991)), Miller et al. (2000)). Nevertheless, we can emphasize the genericity of our approach: this optimization method can be easily adapt to other CLSP extensions and this kind of multi-objective function can be adapted to a lot of problems.

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## Dynamic cluster in particle swarm optimization algorithm

Abbas El Dor · David Lemoine · Maurice Clerc ·  
Patrick Siarry · Laurent Deroussi · Michel Gourgand

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**Abstract** Particle swarm optimization is an optimization method based on a simulated social behavior displayed by artificial particles in a swarm, inspired from bird flocks and fish schools. An underlying component that influences the exchange of information between particles in a swarm, is its topological structure. Therefore, this property has a great influence on the comportment of the optimization method. In this study, we propose *DCluster*: a dynamic topology, based on a combination of two well-known topologies viz. *Four-cluster* and *Fitness*. The proposed topology is analyzed, and compared to six other topologies used in the standard PSO algorithm using a set of benchmark test functions and several well-known constrained and unconstrained engineering design problems. Our comparisons demonstrate that *DCluster* outperforms the other tested topologies and leads to satisfactory performance while avoiding the problem of premature convergence.

**Keywords** Particle swarm optimization · Neighborhood topologies · Social networks · Global best · Local best · Dynamic cluster

A. El Dor · M. Clerc · P. Siarry (✉)  
Université de Paris-Est Créteil, LiSSi (E.A. 3956), 61 Avenue du  
Général de Gaulle, 94010 Créteil, France  
e-mail: siarry@u-pec.fr

D. Lemoine  
Ecole des Mines de Nantes, IRCCyN (CNRS UMR 6597), 4 Rue  
A. Kastler, 44307 Nantes, France

L. Deroussi · M. Gourgand  
Université Blaise Pascal, LIMOS (CNRS UMR 6158), Campus  
des Cézeaux, 63173 Aubière, France

### 1 Introduction

Particle swarm optimization, introduced by Eberhart and Kennedy (1995), inspired from bird flocking behavior, is a class of population-based global optimization algorithms for solving complex problems. It uses coordinated particles to explore the search space where each particle position corresponds to a point of this space. Each particle can fly through the search space with a certain velocity. Towards this purpose, each particle is initialized with a random position and a random velocity. Later, its position and its velocity are updated, as part of the search process intelligently, based on the particle's learning experience and information exchanged with its neighbors.

Since its inception in 1995, PSO has attracted a great deal of attention from researchers because of its many advantages; it is simple, fast and can be implemented in few lines of code. The PSO optimization paradigm has become significantly popular among swarm intelligence-based algorithms (Engelbrecht 2006). Many ideas were introduced thereafter for trying to improve the original version, known to show a fast convergence that may lead the algorithm to be trapped in a local optimum. Kennedy (1999) demonstrated that the choice of neighbors for each particle has a predominant influence on the efficiency of PSO. A lot of research has been conducted in the design of such sets of neighborhood particles and has led to the definition of a variety of topologies for Particle Swarm Optimization. In this paper, we will focus on the concept of neighborhood (topology) in PSO and we propose an efficient topology called *DCluster*. We test *DCluster* on a set of well-known benchmarks in literature and we compare our results with those from popular topologies for Particle swarm optimization.

This article is organized as follows: the particle swarm optimization paradigm is briefly presented in Sect. 2. In Sect. 3, the concept of topology for PSO is introduced and the commonly used topologies are discussed. In Sect. 4, the proposed topology is described. An evaluation of seven different topologies (including *DCluster*) and their comparison are reported in Sect. 5 and finally, we conclude the study in Sect. 6.

## 2 Particle swarm optimization

Particle swarm optimization (PSO) is a swarm intelligent algorithm which has been motivated by the flocking behavior of birds and fish schools. This population-based metaheuristic had been initially designed to solve continuous optimization problems. Although PSO has been applied to discrete optimization, the present application deals with a continuous PSO version. Therefore, we consider a continuous optimization problem that can be written according to the following manner  $\text{Optimize } f(x)$  where  $\Omega$  is a convex set of  $\mathbb{R}^n$  and  $f$  is a function from  $\Omega$  to  $\mathbb{R}$ . In the present PSO framework, we consider a minimization problem, without loss of generality.

In PSO, there is a set of particles  $S = \{P_i\}_{i=1,\dots,n}$  called swarm. A particle is a simple software agent which flies in the search space  $\Omega$ . It may exchange information with other particles in the swarm to find an optimal solution. Thus, at each iteration  $t$ , a particle  $P_i$  is characterized by four components:

- its position  $x_{it} \in \Omega$  which is a candidate solution to the optimization problem,
- its velocity  $\vec{v}_{it}$  which represents the flight direction of the particle,
- its best position  $pbest_i$  according to the objective function  $f$ ,
- a set of particles  $N_{it} \subset S$  with which it can communicate. This set is usually called the neighborhood of  $P_i$  and comprises the concerned particle too. Each particle of this neighborhood is also called a neighbor of  $P_i$ .

For each particle and at each iteration  $t + 1$  of the algorithm, a new velocity is computed based on a combination of the following three properties, which define the paradigm of the PSO:

- the physical property. The particle  $P_i$  tends to keep its previous direction of displacement  $\vec{v}_{it}$ ,
- the cognitive property. The particle  $P_i$  tends to move towards its best position  $pbest_i$ ,
- the social property. The particle uses information obtained from its current neighbors in  $N_{it+1}$ .

We can notice that, usually, the information given by the neighborhood of the particle is the best  $pbest$  position of all the particles in the neighborhood and is denoted by  $lbest_i$ :

$$lbest_i = \left\{ pbest_k \text{ where } P_k \in \underset{P_j \in N_{it}}{\text{argmin}} [f(pbest_j)] \right\} \quad (1)$$

Thus the new velocity  $\vec{v}_{it+1}$  is computed based on the Eq. 2:

$$\vec{v}_{it+1} = \vec{\mathcal{L}}(\vec{v}_{it}, pbest_i - x_{it}, lbest_i - x_{it}) \quad (2)$$

where  $\vec{\mathcal{L}}$  is a vectorial function of  $\mathbb{R}^n$  usually defined by:

$$\begin{aligned} \vec{\mathcal{L}}(\vec{v}_{it}, pbest_i - x_{it}, lbest_i - x_{it}) \\ = \vec{v}_{it} + c_1 r_{1t} \cdot [pbest_i - x_{it}] + c_2 r_{2t} \cdot [lbest_i - x_{it}] \end{aligned} \quad (3)$$

where

- $r_{1t}$  and  $r_{2t}$  are two random vectors of  $\mathbb{R}^n$  uniformly chosen in  $[0, 1]^n$  and called *acceleration coefficients*,
- $c_1$  and  $c_2$  are two real constants called *cognitive* and *social* coefficients, respectively (and, therefore, which are fixed throughout the optimization algorithm).
- $\cdot$  is the product between two vectors, element by element.

In order to control the influence of the velocity of the particle in the determination of its new position, Shi and Eberhart (1999) have extended this formulation by introducing an *inertia weight*  $w \in \mathbb{R}$ . Thus, by using the same notation, the vectorial function becomes:

$$\begin{aligned} \vec{\mathcal{L}}(\vec{v}_{it}, pbest_i - x_{it}, lbest_i - x_{it}) \\ = w \vec{v}_{it} + c_1 r_{1t} \cdot [pbest_i - x_{it}] + c_2 r_{2t} \cdot [lbest_i - x_{it}] \end{aligned} \quad (4)$$

Finally, the new position of the particle  $P_i$  is calculated according to the following expression:

$$x_{it+1} = x_{it} + \vec{v}_{it+1} \quad (5)$$

The PSO algorithm in its synchronous version, which is used in this paper, is summarized in Fig. 1.

As pointed out by Kennedy (1999), the design of the neighborhood of each particle has a great influence on the performance of PSO. Therefore, several kinds of neighborhood designs have been proposed. In the next section, we present previously reported work on neighborhood topological structures for PSO.

## 3 Topologies in particle swarm optimization

As described in the previous section, a particle  $P_i$  of the swarm  $S$  can exchange some information with other particles in the swarm. This property can be modeled using a graph in which the vertices are the particles, and the arcs are the communication links between two particles. If the



## Algorithm 1

---

```

Initialization:
for each  $P_i \in \mathcal{S}$  do
  randomly choose the position  $x_{i0} \in \Omega$ , the velocity  $\vec{v}_{i0}$  and set  $pbest_i = x_{i0}$ 
  evaluate  $f(x_{i0})$ 
end
set iteration  $t = 0$ .
Algorithm:
while the stopping criterion is not satisfied do
  for each  $P_i \in \mathcal{S}$  do
    determine the set  $N_{it}$  of the neighbors of  $P_i$ 
    update  $lbest_i$  according to equation (1)
    compute  $\vec{v}_{i,t+1}$  using relation (2)
    update position  $x_{i,t+1}$  using equation (5)
  end
  for each  $P_i \in \mathcal{S}$  do
    evaluate  $f(x_{i,t+1})$  and update  $pbest_i$  if necessary
  end
  set  $t = t + 1$ 
end

```

---

Fig. 1 Pseudo-code of synchronous PSO

communication is unidirectional, the graph is directed, otherwise it is undirected. Sometimes, a particle may communicate with itself, thus the corresponding vertex may have a loop.

Here, we focus on bidirectional communication, thus the considered graph is undirected and denoted by  $G = (S, E)$  where  $E$  is the set of the edges. Thus, the neighborhood of a particle  $P_i$  is the set of particles which are adjacent to  $P_i$  in the graph  $G$ . The set of edges between each particle  $P_i$  and its neighbors  $N_{it}$  forms the communication graph, also called the topology.

In addition, while the PSO algorithm progresses, the communication links between the particles may also change. Therefore, at each iteration, the topology can evolve. Accordingly, there exist two kinds of topologies:

- static topologies. Communications between particles are fixed at the beginning of the algorithm. Thus the topology (and, consequently the neighborhood of each particle) will not change. Therefore, we can denote the neighborhood of each particle  $P_i$  by  $N_i$  instead of  $N_{it}$ .
- dynamic topologies. The communication between particles may change at each iteration and the topology will keep getting dynamically updated.

We present in a first subsection, some of the main static topologies used in literature (Bastos-Filho et al. 2008), and some of the main dynamic topologies in a second subsection.

### 3.1 The static topologies

In Kennedy and Mendes (2002), the authors focused on static topologies for the PSO. They used metrics defined by

Watts (1999) and Watts and Strogatz (1998) to classify topologies according to their effectiveness and based on some properties of the objective function such as modality. These metrics are:

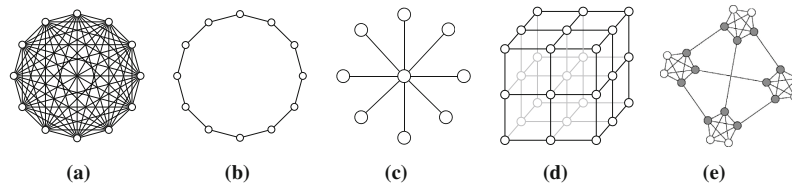
- $\mathcal{K}$  the cardinal number of the neighborhood of the particles,
- the amount of clustering  $\mathcal{C}$  which is the average number of neighbors-in-common for the particles.

Using a huge number of benchmarks, they showed that random graphs can be efficient for PSO when  $\mathcal{K} = 5$ , but they also noticed that regular structures as in standard PSO work well too. Some of the commonly used static topologies suggested in previous literature for standard PSO include:

- *Gbest*. In the original paper of PSO, Eberhart and Kennedy (1995) used a topology called *Gbest* in which each particle can communicate with all the other ones. Therefore, in this case, the graph  $G = (S, E)$  is complete (for each particle  $P_i$ ,  $N_i = S$ ). Consequently, all the particles  $P_i$  have the same  $lbest_i$  defined by (1): this common position is also called *Gbest* (for Global best position) and the Eq. (3) becomes:

$$\begin{aligned} \vec{L}(\vec{v}_{it}, pbest_i - x_{it}, Gbest - x_{it}) \\ = \vec{v}_{it} + c_1 r_{1t} \cdot [pbest_i - x_{it}] + c_2 r_{2t} \cdot [Gbest - x_{it}] \end{aligned} \quad (6)$$

As opposed to the *Gbest* topology, the other ones are called *Lbest* (local best position): indeed, in these cases, particles use the best position of their neighbors and not the best position in the whole swarm.



**Fig. 2** Static topologies: (a) *Gbest* topology, (b) *Ring* topology, (c) *Wheel* topology, (d) *von Neumann* topology and (e) *Four-clusters* topology

- *Ring*. In Mendes et al. (2004), a ring is defined as a graph in which every vertex is connected to two others.
- *Wheel* also called *Focal* or *Star*. In Kennedy and Mendes (2002), this topology is defined as follows: one central node influences, and is influenced by, all other members of the population.
- *Von Neumann*. In Mendes et al. (2003), Von Neumann graph is defined as a square lattice whose extremities connect as a torus.
- *Four-clusters*. According to Mendes et al. (2004), the topology is defined as a graph with four cliques connected among themselves by gateways.

These static topologies are represented in Fig. 2.

In Kennedy and Mendes (2002), the authors conclude that no static topology can be efficient for all the continuous optimization problems and the efficiency of such topology largely depends on the landscape of the objective function. For instance, *Gbest* which instantiates the most immediate communication possible, is usually not a good topology for complex function landscape (this is due to its propensity to converge too quickly to a local optimum), whereas *Ring* topology which converges very slowly, usually exhibits good performances in complicated landscapes. *Von Neumann* and *Four-clusters* seem to be good compromises between quality of the obtained solution and speed of convergence. Therefore, they might be the most efficient standard static topologies.

As mentioned before, a static topology can be efficient for one kind of optimization problems (for instance *Gbest* for unimodal functions) but can be quite the opposite on a large set of other types of landscapes. This is due to its inability to evolve during the discovery of the optimization landscape. Dynamic topologies were consequently designed, to overcome this limitation.

### 3.2 The dynamic topologies

In this section, we present some dynamic topologies. They can be classified into two groups based on the regularity of the structure used, to define the topology.

#### 3.2.1 Dynamic topologies without regular structure

In Clerc (2006), the author proposed a randomized topology, named here R2006. According to the distribution probability chosen in order to determine which particle is informed by whom, the size of neighborhood for each particle follows a Binomial distribution where the mean value of this distribution is just greater than  $\mathcal{K}$ , the preferred size of particle's neighborhood. If the current iteration does not improve the objective function, the topology is redesigned according to the same scheme at the next iteration. This dynamic topology has been used in several versions of PSO, as in SPSO 2006, SPSO 2007 and SPSO 2011 (PSC 2012).

#### 3.2.2 Dynamic topologies with regular structure

One of the most natural ways to define such dynamic topology is to allow a particle to communicate with its  $\mathcal{K}$ th closest particles in the search space  $\Omega$  according to the Euclidean distance. Hence, the underlying graph is a  $\mathcal{K}$ -regular Graph. In this paper, this topology is called Geometric. This approach has been generalized by using spatial subdivisions of  $\Omega$  by using for instance Delaunay triangulation (Lane et al. 2008) or Voronoi diagram (Safavieh et al. 2009). Such topologies are called *Geographical topologies* as opposed to *Social topologies* which do not use these geographical considerations but usually define static topologies.

In Janson and Middendorf (2005), the authors propose a dynamic topology, named here Hierarchy. This topology is defined by using a regular tree as the communication graph. This tree is characterized by its branching degree  $\mathcal{K} - 2$  and its depth  $h$  (therefore neighborhoods of particles which are not placed on a leaf or on the root of the tree have all the same size  $\mathcal{K}$ ). The position of each particle evolves according to the following rule: using a breadth-first search, particle on a node is compared to all particles which are on its child nodes according to their best solutions. If a particle on a child node outperforms the particle on the parent node, then they swap their positions in the tree. As the authors noticed the great influence of the

branching degree, they proposed a procedure to dynamically change it (by decreasing it).

Richards and Ventura (2003) propose a dynamic topology which tries to exploit the idea that the *ring* topology is more adapted to multimodal landscape whereas *Gbest* topology converges quickly in a monomodal context. Thus, beginning with a *ring* topology, new edges are periodically added in order to obtain a *Gbest* topology at 80 % of the maximum number of iterations. Therefore, at the beginning, the topology allows a survey of the search space whereas particles are focusing on the most promising area when topology converges to *Gbest*.

In Wang and Xiang (2008a), the authors propose the *Fitness* topology. In *Fitness*, the authors also use a *ring* topology but on this ring, particles are positioned in a non decreasing order of their personal best values, in a clockwise direction. Then, they only consider monodirectional communication links in an anticlockwise direction (of course, the authors consider an undirected one). Therefore, a particle can only receive informations from a better particle except the best one which is connected to the worst particle. In order to improve this topology, they designed a rolling window on the ring: a particular set of contiguous particles (the window) is assigned to each particle and when we move from a particle to the next one on the ring, the window is shifted forward one particle. If for one particle, the corresponding window is composed with at most one better particle, then a “Learn Far From Better Ones” strategy (Wang and Xiang 2008b) is applied, otherwise a “centroid of mass” strategy inspired from Kennedy (2000) is used.

In Liang and Suganthan (2005), the authors proposed the Dynamic Multi-Swarm Particle Swarm Optimizer (DMS-PSO), which is based on the local version of PSO with a new neighborhood topology. In DMS-PSO, the sub-swarms are dynamic and the sub-swarms' size is very small. The whole population is divided into a large number of sub-swarms, these sub-swarms are regrouped frequently by using various regrouping schedules and information is exchanged among the particles in the whole swarm. Zhao, Liang, Suganthan, Nasir and Li proposed also other sophisticated topologies in Zhao et al. (2011), Liang and Suganthan (2006), Nasir et al. (2012) and Li et al. (2011).

Dynamic topologies are more adapted to complicated landscapes thanks to their ability to take into account parts of search space already discovered. Then, a static topology may not be suitable for all situations. Therefore, we proposed a new dynamic topology called *DCluster* which combines *Four-clusters* and *Fitness* topologies in order to exploit the salient properties of their underlying topologies. *DCluster* topology is explained in the next section.

#### 4 The proposed topology

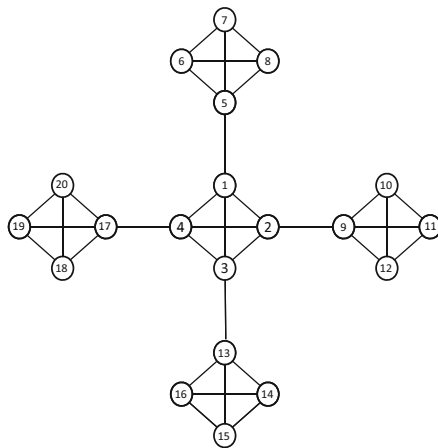
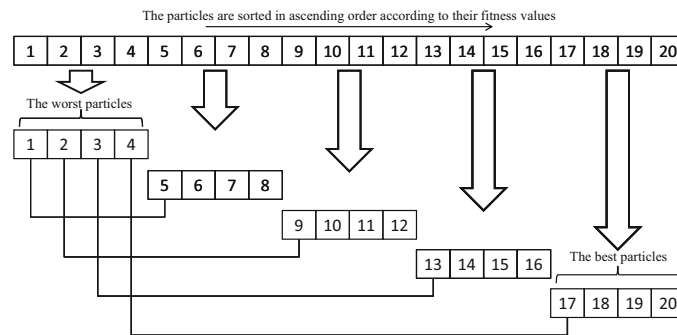
As described earlier, static topologies may suffer from many limitations, which affect the convergence of the standard PSO algorithm. Considering the fact that a dynamic topology may play a significant role in complex optimization problems, we propose *DCluster* - a swarm that is segmented into several sub-swarms having the same size (called Clusters), based on the following rule: the particles will be grouped according to the values of their current solutions in order to create level clusters. Thus, the algorithm tends to employ better clusters to aid clusters having comparatively poorer solutions. Such a topology may be named as *DCluster* for Dynamic Cluster.

At each iteration  $t$ , the design of the topology follows four steps:

1. For each particle, evaluate its position according to the objective function and sort particles according to their current values in a non decreasing order, as in *Fitness* topology.
2. Divide the sorted set of particles into several subsets (with the same size) of particles with respect of the non decreasing order. The number of subsets has to be chosen in order to be equal to the number of particles in a subset plus one.
3. In each subset, connect all the particles together (the obtained graph for each subset is complete). Note that each particle is also connected to itself. At the end of this step, the clusters are created and can be sorted in a non decreasing order according to the sum of the values of their own particles. Therefore, the cluster no. 1 will be composed of the worst particles of the swarm, the cluster no. 2 with the worst particles by removing particles of cluster no. 1 from the swarm and so on. Therefore, the last cluster is composed of the best particles. The three steps are summarized in Fig. 3.
4. Cluster no. 1 is placed in the center of the topology (and is called “central cluster”). By sorting particles of this cluster in a non decreasing order, its first particle is linked to the worst particle of Cluster no. 2, its second particle to the worst particle of Cluster no. 3, and so on. At the end, each particle of the central cluster is linked to the worst particle of another cluster. Hence, all clusters in this topology, including the central cluster, have a fully connected neighborhood, as shown in Fig. 4

The reason why the central cluster is linked to each cluster by only one gateway and with the worst particle of the latter is to avoid a premature convergence to a local optimum by slowing down the propagation of the informations in the whole swarm. Another point to notice is that

**Fig. 3** The partitioning of the table into clusters



**Fig. 4** The structure of *DCluster* topology

the use of such topology imposes a particular size of the swarm. Indeed, due to the step 2 of the design of the topology, the size of the swarm has to be computed according to the Eq. (7):

$$|S| = N \times (N + 1) \tag{7}$$

where  $N$  is the size of each cluster.

Using previous notations, pseudocode of PSO using *DCluster* topology is given in Fig. 5

In the following section, we present results obtained with our topology and we compare them with results obtained with other topologies found in the literature and known to be efficient.

### 5 Experiments and discussion

In this section, we present a comparison between *DCluster* topology and six other topologies taken from the literature of PSO. In order to achieve this comparison, sets of benchmark functions and real-life problems [twelve functions from CEC2005 (Suganthan et al. 2005), and twelve real-life problems] are used in our experiments that are detailed in el Dor et al. (2012), Ali et al. (2010), Clerc (2006), El-Saleh et al. (2009), Liu et al. (2010) and Cagnina et al. (2008). In Shifted functions, the coordinates were shifted by using an offset vector  $O = (o_1, o_2, \dots, o_n)$ . Thus the global optimum is translated, and it is expected that this operation makes the problem more difficult to solve. In Rotated functions, the Salomon's method (Salomon 1996) is used to rotate the functions. This method generates an orthogonal matrix  $M$  to get a new variable  $y (y = M \times x)$ , where  $x$  is the original variable. The used offset vectors and orthogonal matrices are taken from the proceedings of the Conference CEC 2005. These functions and the real-life problems are presented in the Appendix and are used to compare the results obtained by all PSO versions with different topologies.

#### 5.1 Experimental setting

The seven different topologies including the proposed one [R2006 (Clerc 2006), *DCluster*, *Fitness* (Wang and Xiang 2008a), *Four-clusters* (Mendes et al. 2004), Geometric, Hierarchy (Janson and Middendorf 2005), and *Gbest*], were implemented in the standard version of PSO SPSO2006<sup>1</sup> (PSC 2012), and their performances were compared. The *cognitive* and *social* coefficients  $c_1$  and  $c_2$  are chosen similar ( $c_1 = c_2 = 1.19$ ) and the inertia weight  $w$  is fixed to 0.72 for each algorithm [the default values used in the *Standard* PSO 2007 (PSC 2012) and in Hsieh et al. (2009)].

<sup>1</sup> SPSO2006 uses the random topology R2006.

---

Algorithm 2

---

```

Initialization:
for each  $P_i \in S$  do
  randomly choose the position  $x_{i0} \in \Omega$ , the velocity  $\vec{v}_{i0}$  and set  $pbest_i = x_{i0}$ 
  evaluate  $f(x_{i0})$ 
end
set iteration  $t = 0$ .
Algorithm:
while the stopping criterion is not satisfied do
  /* Design of the topology */
  /* Step 1 */
  Sort the set of particles according to their current values in a non decreasing order: let
   $\{P_{\sigma[1]}, \dots, P_{\sigma[N \times (N+1)]}\}$  this ordered set.
  /* Step 2 */
  for  $i = 1$  to  $N + 1$  do
    Set  $Cluster(i) = \emptyset$ 
    for  $j = 1$  to  $N$  do
       $Cluster(i) = Cluster(i) \cup \{P_{\sigma[(i-1) \times N + j]}\}$ 
    end
  end
  /* Step 3 */
  for  $i = 1$  to  $N + 1$  do
    Connect all the particles in  $Cluster(i)$  together
  end
  /* Step 4 */
  for  $j = 1$  to  $N$  do
    Connect  $P_{\sigma[j]}$  to  $P_{\sigma[j \times N + 1]}$ 
  end
  /* End of the design */
  for each  $P_i \in S$  do
    determine the set  $N_{it}$  of the neighbors of  $P_i$ 
    update  $lbest_i$  according to equation (1)
    compute  $\vec{v}_{i,t+1}$  using relation (2)
    update position  $x_{i,t+1}$  using equation (5)
  end
  for each  $P_i \in S$  do
    evaluate  $f(x_{i,t+1})$  and update  $pbest_i$  if necessary
  end
  set  $t = t + 1$ 
end

```

---

**Fig. 5** Pseudo-code of the *DCluster* PSO

The population size is set to 20 for the seven versions of PSO. The size of the neighborhood for *Fitness* and Geometric is set to five, where it is variable between one and 20 for R2006, with a high probability to be close to five. For the Hierarchy topology, the depth  $h$  is set to four and the branching degree  $\mathcal{K} - 2$  is set to two. In these experiments, *DCluster* uses a regular clustering form as in Fig. 4, where the number of clusters is equal to five and the number of particles in each cluster  $N$  is set to four. The maximum number of function evaluations (FEs) is set to 50,000 in 10-D, for the twelve benchmark functions  $(f_1, f_2, \dots, f_{12})$  given in the section 'Benchmark functions' in the Appendix, and is set to 150,000 in 30-D. For the twelve real problems presented in the section 'Real life problems' in the Appendix, the maximum number of function evaluations (FEs) for each problem is given in Table 3. The KISS (Keep it Simple Stupid) Random Number Generator (RNG) (Marsaglia and Zaman 1993) is used for the seven

topologies. The experiments are performed using a computer equipped with a 64-bit processor.

## 5.2 Experimental results

The simulation results in 10-D and 30-D, on the twelve benchmark functions presented in the section 'Benchmark functions' in the Appendix, are reported in Tables 1 and 2, respectively. More specifically, the mean best value and the standard deviation obtained in 100 experiments for each algorithm and each function using 10 and 30 dimensions are reported in these tables. We added a column showing the  $P$  value (which expresses the probability for a hypothesis to be true) given by the Friedman statistical test, in order to confirm that the results are significantly different (the 100 best found solutions of each algorithm). The best results among those obtained by the seven algorithms are shown in bold. From these experiments, we can notice

**Table 1** Results on the 10-D problems

$f$	R2006	<i>DCluster</i>	<i>Fitness</i>	<i>Four-clusters</i>	Geometric	Hierarchy	<i>Gbest</i>	$P$ value
$f_1$	2.81e+004*	<b>4.99e+000</b>	1.29e+001	44e+000	8.12e+003	2.25e+001	1.55e+005	1.92e-30
	2.79e+005	<b>1.55e+001</b>	4.77e+001	3.10e+001	8.07e+004	6.27e+001	1.29e+006	
$f_2$	1.16e-002	1.79e-014	1.16e-002	1.14e-001	1.62e+000	<b>1.70e-014</b>	8.23e-001	1.67e-50
	1.15e-001	1.54e-014	1.15e-001	3.67e-001	1.48e+000	<b>1.50e-014</b>	2.10e+000	
$f_3$	5.92e-002	<b>2.71e-002</b>	3.78e-002	7.15e-002	1.03e-001	4.84e-002	1.75e-001	2.35e-27
	3.38e-002	<b>2.08e-002</b>	2.84e-002	5.02e-002	6.89e-002	4.11e-002	3.75e-001	
$f_4$	6.06e+000	<b>2.81e+000</b>	3.83e+000	9.49e+000	1.57e+001	4.08e+000	1.76e+001	1.31e-59
	3.07e+000	1.62e+000	1.76e+000	5.31e+000	6.94e+000	1.77e+000	8.45e+000	
$f_5$	<b>3.55e+002</b>	7.62e+002	1.53e+003	5.16e+002	2.51e+003	2.56e+003	5.053e+002	3.37e-14
	<b>1.28e+003</b>	2.63e+003	4.14e+003	1.90e+003	1.69e+004	5.02e+003	1.96e+003	
$f_6$	2.33e+001	<b>2.28e+001</b>	2.31e+001	2.30e+001	2.29e+001	2.33e+001	5.93e+003	1.06e-14
	8.50e-002	<b>7.93e-002</b>	1.05e-001	8.93e-002	8.21e-002	1.06e-001	3.02e+001	
$f_7$	1.12e-001	<b>1.04e-002</b>	8.78e-002	1.87e-001	6.86e-001	5.27e-002	3.18e-001	1.89e-56
	7.68e-002	<b>8.86e-002</b>	7.79e-002	1.17e-001	7.02e-001	3.61e-002	2.71e-001	
$f_8$	1.14e+001	<b>4.96e+000</b>	7.87e+000	1.35e+001	1.23e+001	5.61e+000	1.99e+001	3.86e-35
	5.81e+000	<b>3.40e+000</b>	4.00e+000	4.99e+000	3.78e+000	4.29e+000	8.79e+000	
$f_9$	2.02e-001	<b>1.49e-001</b>	4.35e-001	2.01e-001	3.57e+000	8.17e-001	3.68e-001	1.26e-37
	8.70e-001	<b>2.86e-001</b>	1.04e+000	8.70e-001	1.12e+000	1.55e+000	1.14e+000	
$f_{10}$	3.96e-002	<b>3.66e-015</b>	4.19e-015	5.11e-002	1.14e+000	3.73e-015	6.90e-001	3.21e-84
	2.29e-001	<b>1.23e-015</b>	1.55e-015	2.54e-001	1.21e+000	1.22e-015	8.57e-001	
$f_{11}$	5.02e-002	<b>2.48e-002</b>	3.04e-002	6.45e-002	9.24e-002	4.11e-002	1.22e-001	5.56e-35
	3.02e-002	<b>2.00e-002</b>	2.28e-002	3.78e-002	5.95e-002	2.92e-002	7.25e-002	
$f_{12}$	6.00e+000	<b>2.64e+000</b>	3.65e+000	8.07e+000	1.13e+001	4.08e+000	1.39e+001	9.86e-45
	2.52e+000	<b>1.82e+000</b>	2.15e+000	4.39e+000	4.51e+000	2.10e+000	7.10e+000	

\* The reason of the bad average is due to a single run whose best found solution is 2.80e+006, knowing that the average of the other 99 runs is 7.87e+000

that the *Gbest* topology yields the worst results on most functions. For the other topologies, the results are mitigated with better ones for the *DCluster* topology, thus outperforming the other topologies for the majority of the tested functions. From the Tables 1 and 2, one can notice that the obtained  $P$ -value is less than the significance level  $\alpha$  (threshold chosen to reject the hypothesis checked by the statistical test), which is often 0.05, for all tested functions. Therefore, we can conclude that the obtained results by each algorithm are significantly different from the others, with a confidence level of 95 % ( $P$  value  $\leq 0.05$ ).

In Table 1, the algorithm using *DCluster* topology performs better than the others for  $f_1, f_3, f_4, f_6, f_7, f_8, f_9, f_{10}, f_{11}$  and  $f_{12}$ , but not for function  $f_2$  and  $f_5$ . *DCluster* and Hierarchy show noticeable results for  $f_2$  with Hierarchy having the best result of this function. Indeed, *DCluster* outperforms the other topologies for ten functions out of twelve, and it is outperformed by Hierarchy on one function and by R2006 on one other. The simulation results for 10-D Shifted Ackley, 10-D Shifted Griewank, 10-D Shifted Rastrigin and 10-D Shifted Rosenbrock are shown in Fig. 6. As we can see in this figure, *DCluster* does not lead

to the best solution for each problem compared to other topologies. However, there is no premature convergence, and the best solution found still improves at the end of the execution of PSO. The good convergence of *DCluster* can be explained by the use of five sets of particles dynamically linked. Hence, each set can explore a different region of the search space, that can accelerate the convergence to the global optimum, and prevent the algorithm from being trapped in a local optimum. Figure 6 demonstrates also the fast converging ability of PSO using Geometric and *Four-clusters* topologies in comparison with the others.

Our experimental results for the 30-D functions are shown in Table 2. We compare the mean best values and the standard deviations over 100 runs of the seven PSO algorithms given in Table 2. Despite the fact that the 30-D functions are more difficult than the 10-D ones, PSO using *DCluster* topology still obtains good performances and scales up well with the problem dimension. It is noticeable that the results obtained by *DCluster* in 30-D are indeed not significantly different than those obtained in 10-D, where *DCluster* topology outperforms the others in eight functions out of twelve. PSO using *DCluster* topology performs

Dynamic cluster in particle swarm optimization algorithm

**Table 2** Results on the 30-D problems

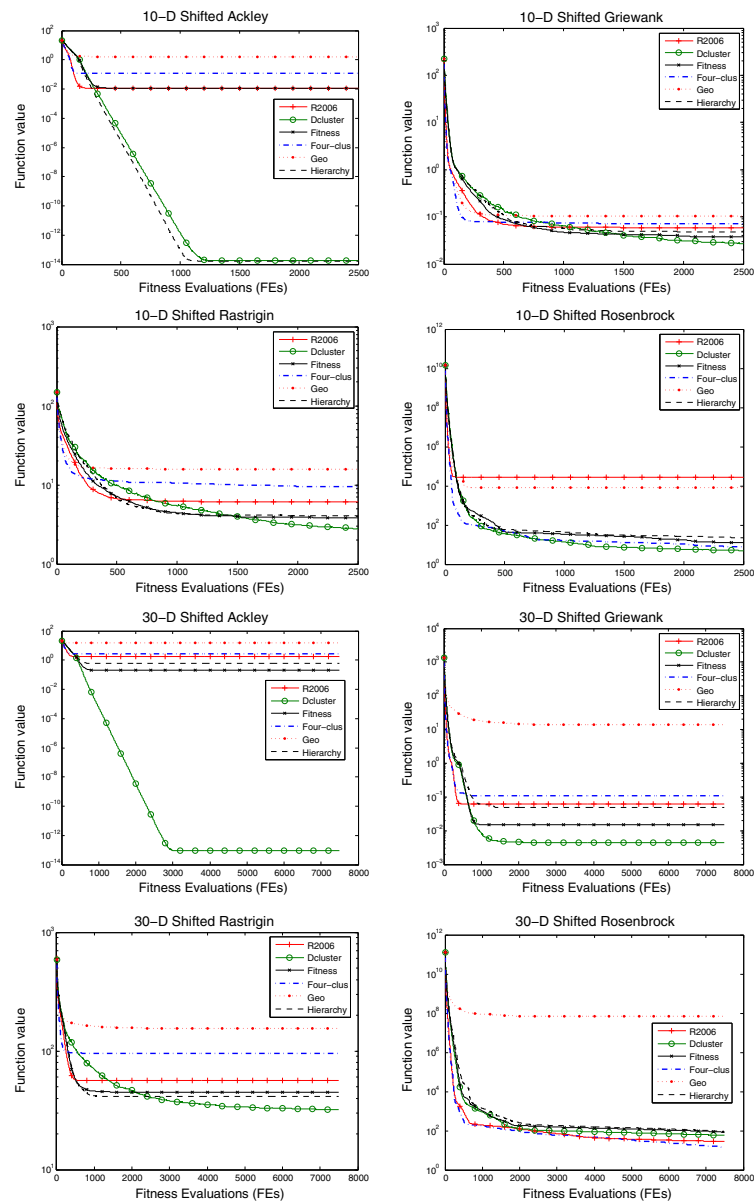
$f$	R2006	<i>DCluster</i>	<i>Fitness</i>	<i>Four-clusters</i>	Geometric	Hierarchy	Gbest	$P$ value
$f_1$	2.83e+001	5.72e+001	8.21e+001	<b>1.50e+001</b>	6.70e+007	9.10e+001	3.99e+006	6.00e-64
	6.66e+001	9.32e+001	1.37e+002	<b>3.32e+001</b>	9.17e+007	1.65e+002	3.07e+007	
$f_2$	1.73e+000	<b>9.00e-014</b>	2.00e-001	2.66e+000	1.47e+001	5.98e-001	8.42e+000	1.10e-86
	1.03e+000	<b>2.45e-014</b>	4.92e-001	1.64e+000	2.26e+000	1.30e+000	2.46e+000	
$f_3$	6.26e-002	<b>4.43e-003</b>	1.50e-002	1.10e-001	1.39e-001	4.87e-002	4.05e+000	1.03e-57
	3.14e-001	<b>9.38e-003</b>	2.18e-002	3.44e-001	1.12e+001	8.43e-002	9.18e+000	
$f_4$	5.65e+001	<b>3.20e+001</b>	4.51e+001	9.49e+001	1.55e+002	4.12e+001	1.27e+002	2.49e-78
	1.85e+001	<b>1.14e-001</b>	1.57e+001	2.53e+001	3.15e+001	1.59e+001	4.28e+001	
$f_5$	1.87e+003	2.13e+003	1.82e+005	<b>1.32e+003</b>	5.67e+008	3.07e+003	9.35e+002	3.89e-46
	4.62e+003	4.72e+003	1.78e+006	<b>3.88e+003</b>	5.69e+008	6.36e+003	2.35e+003	
$f_6$	2.92e+001	<b>2.83e-001</b>	2.91e+001	2.90e+001	2.90e+001	2.92e+001	2.09e+001	8.49e-9
	7.09e-002	<b>5.98e-002</b>	7.85e-002	6.92e-002	8.23e-002	6.85e-002	9.93e-002	
$f_7$	2.11e-002	<b>1.04e-002</b>	1.63e-002	2.37e-002	2.50e+001	1.95e-002	2.37e-002	1.79e-46
	1.65e-002	<b>1.49e-002</b>	1.36e-002	2.20e-002	1.50e+001	1.89e-002	2.11e-002	
$f_8$	7.82e+001	1.03e+002	5.66e+001	7.21e+001	1.09e+002	<b>3.63e+001</b>	1.06e+002	3.21e-54
	4.69e+001	2.08e-001	3.80e+001	2.01e+001	2.44e+001	<b>2.53e+001</b>	2.93e+001	
$f_9$	3.96e+000	1.94e+000	1.90e+001	<b>8.84e-001</b>	6.04e+001	1.72e+001	3.26e+000	5.44e-19
	1.91e+000	1.54e+000	1.09e+001	<b>1.57e+000</b>	2.73e+001	1.66e+001	3.91e+000	
$f_{10}$	1.47e+000	<b>2.08e-002</b>	1.92e-001	2.45e+000	1.07e+001	4.84e-001	6.10e+000	5.87e-55
	9.92e-001	<b>1.47e-001</b>	4.85e-001	1.09e+000	1.86e+000	7.82e-001	1.98e+000	
$f_{11}$	2.71e-002	<b>3.56e-003</b>	9.83e-003	2.94e-002	2.91e+000	1.05e-002	1.35e-001	8.42e-25
	4.99e-002	<b>7.30e-003</b>	1.45e-002	3.80e-002	2.30e+000	1.35e-002	2.48e-001	
$f_{12}$	5.13e+001	<b>3.22e+001</b>	5.08e+001	6.32e+001	7.93e+001	4.61e+001	7.54e+001	1.58e-33
	1.41e+001	<b>1.28e+001</b>	1.62e+001	1.67e+001	1.63e+001	1.62e+001	2.01e+001	

significantly better than the other algorithms for  $f_2$ . PSO using *Four-cluster* topology performs better for  $f_1, f_5$  and  $f_9$  (*Shifted Rosenbrock*, *Rotated Rosenbrock* and *Rosenbrock*), in which the global minimum is inside a long, narrow, parabolic shaped flat valley, and it is difficult to find it. *DCluster* and the other topologies are outperformed by Hierarchy topology on  $f_8$ . In the last column in Table 2, the  $P$  values are reported to ensure that the obtained results of the seven topologies are significantly different. Figure 6 illustrates a representative set of experiments for the same four functions as discussed above in 10-D. The graphs show that PSO using *DCluster* topology achieves better results and avoids the premature convergence. The convergence speed of *DCluster* topology is close to the convergence speed of the other topologies, except for the Geometric topology, in 30-D shifted Rosenbrock function. Geometric topology is trapped in a local optimum early during the search for all functions. Reader can also notice that in the Shifted Ackley function case, *DCluster* significantly outperforms the other topologies: this is probably due to the special characteristic of this function. Indeed, the later includes many local optima, and their number rapidly increases with the dimension  $D$ . For  $D = 10$ , the Hierarchy

topology still can cope with them, but this does not remain true when the dimension increases ( $D = 30$ ).

To ensure the performance of *DCluster* topology, we evaluate the six topologies (*R2006*, *DCluster*, *Fitness*, *Four-clusters*, *Geometric*, *Hierarchy*) in twelve real-life problems (presented in the section 'Real life problems' in the Appendix) having different numbers of dimensions. The results in Table 4 are calculated for these problems in the same way as in 10-D and 30-D on 100 runs. In this table, we added, for each problem, a column showing the number of dimensions and another showing the maximum number of fitness evaluations (Nb. Evals.). The  $P$  values given by the Friedman statistical test are also reported in this table.

In order to preserve feasibility of the solutions for the constrained problems used here, the update of the best positions of the particles is performed using a static penalty approach (Homaifar et al. 1994), in which the penalty factors do not depend on the current generation number, and therefore, remain constant during the entire search process. More specifically, the best position of a particle is updated only if the new candidate best position is feasible, otherwise, it remains unchanged. In this approach, the user



**Fig. 6** Algorithm convergence for the selected problems. Results are averaged over 100 runs



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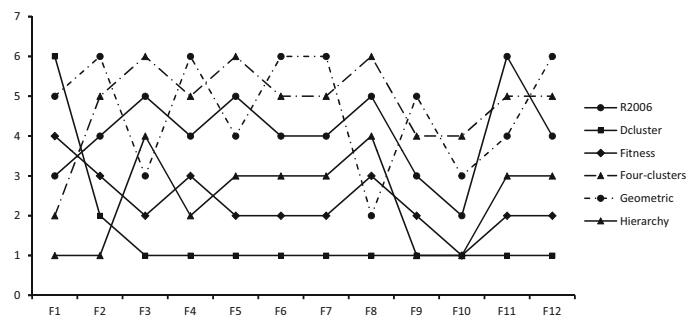
**Table 3** *D* and *FES* for the twelve real-life problems

	$F_1$	$F_2$	$F_3$	$F_4$	$F_5$	$F_6$	$F_7$	$F_8$	$F_9$	$F_{10}$	$F_{11}$	$F_{12}$
<i>D</i>	27	4	6	10	3	5	42	4	4	7	13	10
<i>FES</i>	150,000	50,000	150,000	240,000	50,000	50,000	150,000	300,000	50,000	50,000	15,000	100,000

**Table 4** Results on the real-life problems

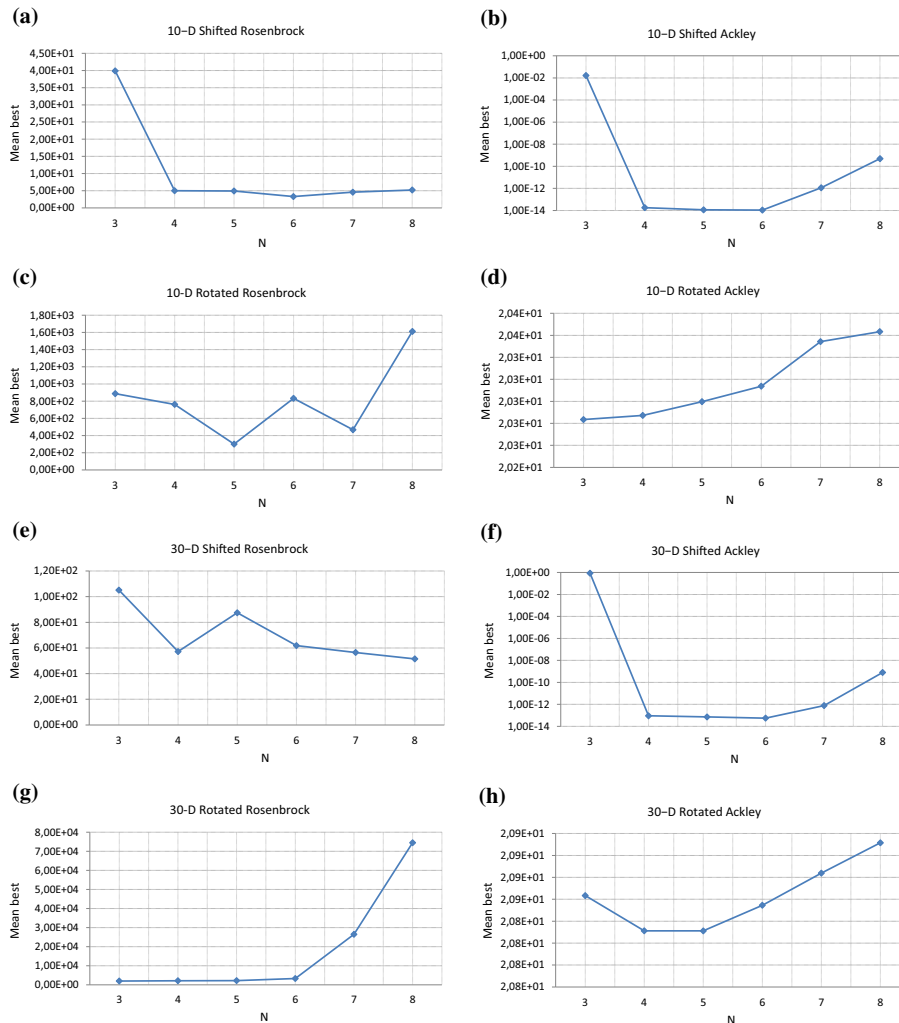
F	R2006	DCluster	Fitness	Four-cluster	Geometric	Hierarchy	<i>P</i> value
$F_1$	1.26e+000	6.63e+000	1.49e+000	1.08e+000	5.74e+000	<b>1.00e+000</b>	1.99e-71
	1.26e+000	1.83e+000	1.57e+000	6.98e-001	1.97e+000	<b>6.19e-001</b>	
$F_2$	3.61e-010	2.89e-010	3.17e-010	9.20e-010	4.37e-009	<b>2.56e-010</b>	8.50e-34
	7.79e-010	6.27e-010	6.01e-010	2.87e-009	7.69e-009	<b>4.91e-010</b>	
$F_3$	8.89e+000	<b>2.21e+000</b>	4.50e+000	1.14e+001	5.74e+000	6.53e+000	2.40e-22
	6.36e+000	<b>3.98e+000</b>	4.97e+000	6.73e+000	1.97e+000	6.14e+000	
$F_4$	4.69e-001	<b>3.90e-001</b>	4.23e-001	6.34e-001	7.21e-001	4.17e-001	1.07e-48
	1.64e-001	<b>1.22e-0001</b>	1.41e-001	1.26e-001	9.03e-002	1.62e-001	
$F_5$	8.54e-003	<b>6.53e-005</b>	2.67e-004	2.21e-002	5.29e-003	2.34e-003	1.04e-8
	2.99e-002	<b>6.50e-004</b>	1.28e-003	5.36e-002	3.23e-002	1.19e-002	
$F_6$	5.29e+002	<b>1.14e+002</b>	1.62e+002	5.99e+002	6.69e+002	1.98e+002	2.04e-34
	5.03e+002	<b>1.96e+002</b>	2.41e+002	5.49e+002	5.66e+002	2.92e+002	
$F_7$	1.04e+002	<b>9.06e+001</b>	9.49e+001	1.09e+002	1.12e+002	9.79e+001	3.68e-31
	1.37e+001	<b>1.02e+001</b>	1.23e+001	1.81e+001	2.90e+001	9.65e+000	
$F_8$	6.68e+000	<b>3.59e-001</b>	3.03e+000	9.92e+000	3.86e-001	5.37e+000	1.00e-7
	2.68e+001	<b>3.58e+000</b>	1.90e+001	4.25e+001	3.84e-001	2.30e+001	
$F_9$	1.15e-002	<b>3.08e-007</b>	5.51e-004	6.98e-002	1.00e-001	<b>3.08e-007</b>	1.97e-83
	5.52e-002	<b>6.48e-017</b>	5.25e-003	1.30e-001	1.33e-001	<b>3.79e-017</b>	
$F_{10}$	3.92e-001	<b>0.00e+00</b>	<b>0.00e+00</b>	1.08e+000	3.92e-001	<b>0.00e+00</b>	1.36e-10
	3.91e+000	<b>0.00e+00</b>	<b>0.00e+00</b>	5.71e+000	3.91e+000	<b>0.00e+00</b>	
$F_{11}$	6.17e+000	<b>2.00e-002</b>	3.57e-001	6.06e+000	5.06e+000	5.97e+001	2.76e-84
	2.28e+000	<b>1.99e-001</b>	8.29e-001	2.55e+000	2.15e+000	9.68e-001	
$F_{12}$	7.06e-001	<b>3.26e-001</b>	4.11e-001	1.21e+000	7.43e+000	5.93e-001	4.39e-52
	6.83e-001	<b>2.92e-001</b>	3.86e-001	1.12e+000	8.97e+000	5.69e-001	

**Fig. 7** The classification of the six algorithms on twelve real problems



defines several levels of violation, and a penalty coefficient is chosen for each in such a way that the penalty coefficient increases as we reach higher levels of violation. This

approach starts with a random population of individuals (feasible or infeasible). An individual is evaluated using:



**Fig. 8** Sensitivity analysis of  $DCluster$  topology, the evolution of the parameter  $N$  for: (a) 10-D shifted rosenbrock, (b) 10-D shifted Ackley, (c) 10-D rotated Rosenbrock, (d) 10-D rotated Ackley,

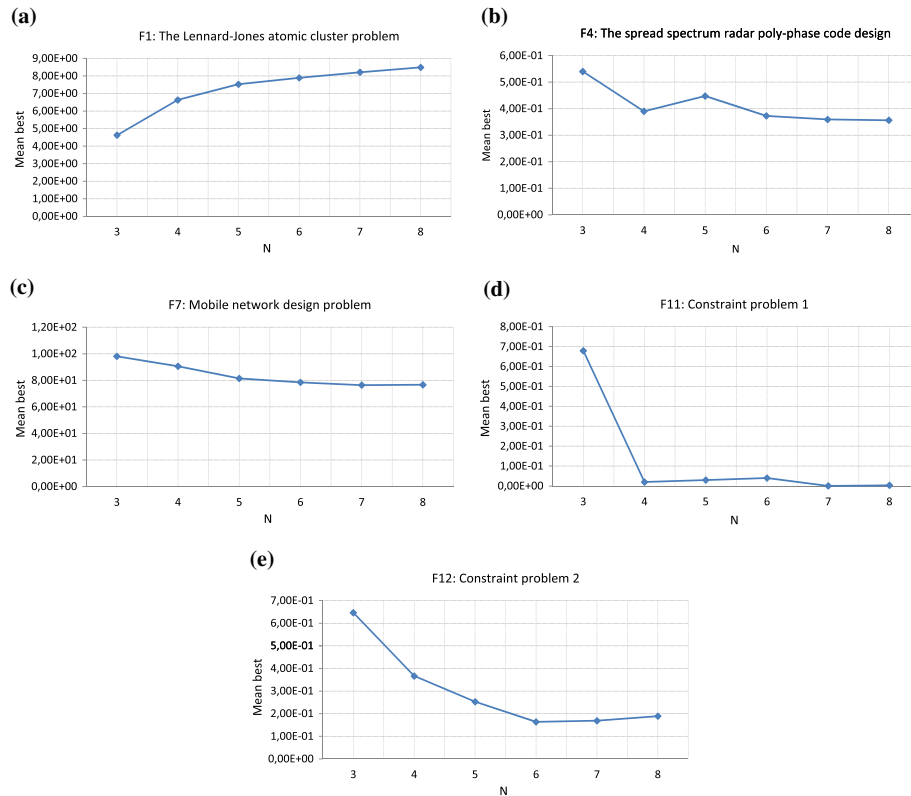
(e) 30-D shifted Rosenbrock, (f) 30-D shifted Ackley, (g) 30-D rotated Rosenbrock and (h) 30-D rotated Ackley

$$fitness(\mathbf{x}) = f(\mathbf{x}) + \sum_{i=1}^m (R_{k,i} \times \max[0, g_i(\mathbf{x})]^2) \quad (8)$$

where  $R_{k,i}$  are the penalty coefficients used,  $m$  is the total number of constraints,  $f(\mathbf{x})$  is the unpenalized objective

function, and  $k = 1, 2, \dots, l$ , where  $l$  is the number of levels of violation defined by the user.

According to our experiments, the best results are obtained when PSO uses  $DCluster$  topology on most problems, except  $F_1$  and  $F_2$ . The results obtained using the other



**Fig. 9** Sensitivity analysis of *DCluster* topology, the evolution of the parameter  $N$  for: (a) F1: the Lennard-Jones atomic cluster problem, (b) F4: the spread spectrum radar poly-phase code design, (c) F7:

mobile network design problem, (d) F11: constraint problem 1, (e) F12: constraint problem 2

topologies are mitigated and we cannot determine which topology performs better than the others. The Hierarchy topology outperforms our proposed topology on  $F_1$  and  $F_2$  and it obtains the same results on  $F_9$  and  $F_{10}$ .  $F_1$  is well known to be very discriminant: indeed, this function has not only many local optima but also, and it is the most important point, an attraction basin of the global one which is quite small. However, for fair comparison we have used the same population size (20) for all topologies, but in this precise case it may be not enough with *DCluster*. Thus, Finding a way to automatically adapt the population size to the difficulty of the problem will be one of the topics of our future work.

Figure 7 clearly shows the classification of the six algorithms on the twelve problems. One can notice that our proposed topology outperforms the other topologies on

most tested problems, where it was ranked at the first place for 10 problems among 12. From this figure, we can observe that the other topologies are unstable, i.e. their results are mitigated.

### 5.3 Sensitivity analysis of *DCluster* topology

The sensitivity of *DCluster* topology was studied by varying the value of the number of particles in each cluster  $N$  and hence the number of clusters which is equal to  $N + 1$ , while the other parameters were left fixed to their default values as in Sect. 5.1. The parameter  $N$  was studied accordingly, by applying PSO using *DCluster* topology on four benchmark functions [Shifted Rosenbrock( $f_1$ ), Shifted

Ackley( $f_2$ ), Rotated Rosenbrock( $f_5$ ) and Rotated Ackley( $f_6$ ) in 10-D and 30-D, and on five real problems ( $F1$ ,  $F4$ ,  $F7$ ,  $F11$  and  $F12$ , see the section ‘Real life problems’ in the Appendix ). For these test problems, the Mean best was computed from 100 runs of the algorithm.

The parameter  $N$  varies between three and eight. Figures 8 and 9 show the value of the best solution found in  $y$ -axis, for each value of the parameter  $N$  in  $x$ -axis. As it can be seen, the choice of the parameter  $N$  can influence significantly the performance of the algorithm. Indeed,  $N$  has to be fitted according to the problem at hand. We can notice that the results obtained, using different values of  $N$ , are mitigated and thus it cannot be determined for which value of  $N$ ,  $DCluster$  achieves the best overall performance. However, the analyzed functions achieve the worst results with small or large values of  $N$  ( $N = 3$  and  $N = 8$ ). Therefore, setting  $N$  to four or five is a compromise to obtain reasonable results in all functions, taking into consideration the fixed numbers of evaluations. In fact, increasing the value of  $N$  demands an increase in the number of evaluations, according to the convergence graphs that show the slow convergence of  $DCluster$ .

### 6 Conclusion

As mentioned by Kennedy (1999), the design of the neighborhood of each particle, and consequently of the topology used, has a great influence on the performance of the PSO. In this paper, we present a new dynamic topology called  $DCluster$ , based on a combination between the  $Four-cluster$  and  $Fitness$  topologies.  $DCluster$  has achieved good results on literature benchmarks (benchmark functions, unconstrained and constrained real-life problems) in comparison to the commonly used topologies found in literature. Indeed, the experiments showed that none of the tested topologies completely dominates the others, but the proposed topology proved to have the best performances for most of the tested problems. In addition, we showed that  $DCluster$  gives a good convergence graph for most of the tested problems, for premature convergence of PSO is less probable.

### Appendix

See Table 5.

**Table 5** Benchmark functions

$f$	Function name	Test function	Search space
$f_1$	Shifted Rosenbrock	$f_1(x) = \sum_{i=1}^{n-1} (100(x_i^2 - z_{i+1})^2 + (z_i - 1)^2) + f_{bias_1}$ $z_i = x_i - o_{2i}, f^* = 390$	$[-100, 100]^N$
$f_2$	Shifted Ackley	$f_2(x) = -20 \exp(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n z_i^2}) - \exp(\frac{1}{n} \sum_{i=1}^n \cos(2\pi z_i)) + 20 + e + f_{bias_2}$ $z_i = x_i - o_{3i}, f^* = -140$	$[-32, 32]^N$
$f_3$	Shifted Griewank	$f_3(x) = \sum_{i=1}^n \frac{x_i^2}{4000} - \prod_{i=1}^n \cos(\frac{x_i}{\sqrt{i}}) + 1 + f_{bias_3}$ $z_i = x_i - o_{4i}, f^* = -180$	$[-600, 600]^N$
$f_4$	Shifted Rastrigin	$f_4(x) = \sum_{i=1}^n (z_i^2 - 10 \cos(2\pi z_i) + 10) + f_{bias_4}$ $z_i = x_i - o_{5i}, f^* = -330$	$[-5, 5]^N$
$f_5$	Rotated Rosenbrock	$f_5(x) = \sum_{i=1}^{n-1} (100(y_i^2 - y_{i+1})^2 + (y_i - 1)^2)$ $y = M_1 * x, f^* = 390$	$[-100, 100]^N$
$f_6$	Rotated Ackley	$f_6(x) = -20 \exp(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n y_i^2}) - \exp(\frac{1}{n} \sum_{i=1}^n \cos(2\pi y_i)) + 20 + e$ $y = M_2 * x, f^* = -140$	$[-32, 32]^N$
$f_7$	Rotated Griewank	$f_7(x) = \sum_{i=1}^n \frac{y_i^2}{4000} - \prod_{i=1}^n \cos(\frac{y_i}{\sqrt{i}}) + 1$ $y = M_3 * x, f^* = -180$	$[-600, 600]^N$
$f_8$	Rotated Rastrigin	$f_8(x) = \sum_{i=1}^n (y_i^2 - 10 \cos(2\pi y_i) + 10)$ $y = M_4 * x, f^* = -330$	$[-5, 5]^N$
$f_9$	Rosenbrock	$f_9(x) = \sum_{i=1}^{n-1} (100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2)$	$[-100, 100]^N$
$f_{10}$	Ackley	$f_{10}(x) = -20 \exp(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}) - \exp(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)) + 20 + e$	$[-32, 32]^N$
$f_{11}$	Griewank	$f_{11}(x) = \sum_{i=1}^n \frac{x_i^2}{4000} - \prod_{i=1}^n \cos(\frac{x_i}{\sqrt{i}}) + 1$	$[-600, 600]^N$
$f_{12}$	Rastrigin	$f_{12}(x) = \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i) + 10)$	$[-5, 5]^N$

## Real life problems

## F1: The Lennard-Jones atomic cluster problem

The Lennard-Jones (LJ) problem is the simplest form of the molecular conformation problem, which consists of finding a configuration of atoms in a cluster or molecule whose potential energy is minimum.

We present the LJ problem as follows: we denote by  $K$  the number of clusters of atoms,  $x^i = (x_1^i, x_2^i, x_3^i)$  represents the coordinates of atom  $i$  in the three-dimensional space, and let be  $X = ((x^1), \dots, (x^K))$ . The LJ potential energy function  $v(r_{ij})$  of a pair of atoms  $(i, j)$  is given by  $v(r_{ij}) = \frac{1}{r_{ij}^{12}} - \frac{1}{r_{ij}^6}$ ,  $1 \leq i, j \leq K$ , where  $r_{ij} = \|x^i - x^j\|$ .

We can notice that, with a single pair of neutral atoms, the LJ potential function is classified as a simple unimodal function. In a cluster with a high number of atoms, the interaction between atoms increases. Then we need to calculate the LJ potential function for each pair of atoms in a cluster. Many local minima are found in the resulting rugged energy landscape. The function to be minimized in this application is as follows:

$$\begin{aligned} \text{Min } V_k(x) &= \sum_{i < j} v(\|x_i - x_j\|) \\ &= \sum_{i=1}^{K-1} \sum_{j=i+1}^K \left( \frac{1}{\|x_i - x_j\|^{12}} - \frac{1}{\|x_i - x_j\|^6} \right). \end{aligned} \quad (9)$$

In this work, we use nine atoms for this problem, hence the search space is  $[-2, 2]^{27}$ . The value of the global optimum is  $f^* = -24.113360$ .

## F2: Gear train design problem

A gear train is a set or system of gears arranged to transfer rotational torque from one part of a mechanical system to another. The problem of gear train is a common problem in the field of mechanical engineering, to be designed such that the gear ratio is as close as possible to  $1/6.931$ . For each gear the number of teeth must be between 12 and 60. Since the number of teeth is to be an integer, the variables must be integers. The mathematical model of gear train design is given by:

$$\text{Min } f(x) = \left\{ \frac{1}{6.931} - \frac{T_d T_b}{T_a T_f} \right\}^2 = \left\{ \frac{1}{6.931} - \frac{x_1 x_2}{x_3 x_4} \right\}^2. \quad (10)$$

Bounds:  $12 \leq x_i \leq 60$ ,  $i = 1, 2, 3, 4$  and  $x_i$  should be integers. Where  $T_a, T_b, T_d$  and  $T_f$  are the numbers of teeth on gears A, B, D and F respectively.

## F3: Frequency modulation sound parameter identification

The frequency modulation sound parameter identification is a complex multimodal optimization problem. The

problem is to satisfy six parameters  $a_1, w_1, a_2, w_2, a_3$  and  $w_3$  of the frequency modulation sound model given below:

$$y(t) = a_1 \times \sin(w_1 \times t \times \theta + a_2 \times \sin(w_2 \times t \times \theta + a_3 \times \sin(w_3 \times t \times \theta))) \quad (11)$$

with  $\theta = (2\pi/100)$ . The fitness function is defined as the sum of square errors between the evolved data and the model data, as follows:

$$f(a_1, w_1, a_2, w_2, a_3, w_3) = \sum_{t=0}^{100} (y(t) - y_0(t))^2$$

where the model data are given by the following equation:

$$y_0(t) = 1.0 \times \sin(5.0 \times t \times \theta + 1.5 \times \sin(4.8 \times t \times \theta + 2.0 \times \sin(4.9 \times t \times \theta))). \quad (12)$$

Bounds:  $-6.4 \leq a_i, w_i \leq 6.35$ ,  $i = 1, 2, 3$ .

## F4: The spread spectrum radar polyphase code design problem

A famous problem of optimal design arises in the field of spread spectrum radar poly-phase codes. Such a problem is very well suited for the application of global optimization algorithms. The problem under consideration is modeled as a min-max nonlinear non-convex optimization problem in continuous variables and with numerous local optima. It can be expressed as follows:

$$\text{Min } f(X) = \max\{f_1(X), \dots, f_{2m}(X)\}. \quad (13)$$

where:

$$X = \{(x_1, \dots, x_n) \in R^n \mid 0 \leq x_j \leq 2\pi, j = 1, 2, \dots, n\} \text{ and } m = 2n - 1,$$

with:

$$f_{2i-1}(x) = \sum_{j=i}^n \cos \left( \sum_{k=|2i-j-1|+1}^j x_k \right) \quad i = 1, 2, \dots, n;$$

$$f_{2i}(x) = 0.5 + \sum_{j=i+1}^n \cos \left( \sum_{k=|2i-j|+1}^j x_k \right) \quad i = 1, 2, \dots, n-1;$$

$$f_{m+i}(X) = -f_i(X), i = 1, 2, \dots, m.$$

Here the objective is to minimize the module of the biggest among the samples of the so-called autocorrelation function which is related to the complex envelope of the compressed radar pulse at the optimal receiver output, while the variables  $x_k$  represent symmetrized phase differences. This problem belongs to the class of continuous min-max global optimization problems. They are characterized by the fact that the objective function is piecewise smooth.

**F5: Compression spring design problem**

This is a simplified version of a more difficult problem (see Clerc 2006). The function to minimize is:

$$\text{Min}f(x) = \pi^2 \frac{x_2 x_3^2 (x_1 + 1)}{4}, \tag{14}$$

where  $x_1 \in \{1, \dots, 70\}$  with a granularity 1,  $x_2 \in [0.6, 3.0]$  and  $x_3 \in \{0.207, \dots, 0.5\}$  with a granularity 0.0001. The best known solution is (7, 1.386599591, 0.292) which gives the fitness value  $f^* = 2.6254214578$ . This problem has five constraints (Clerc 2006), and a penalty method is used to take them into account.

**F6: Perm function**

The function to be minimized is (PSC 2012):

$$\text{Min}f(x) = \sum_{k=1}^5 \left[ \sum_{i=1}^5 (i^k + \beta) \left\{ \frac{x_i}{i} - 1 \right\} \right]^2. \tag{15}$$

In the domain  $x \in [-5, 5]$ , the global optimum is (1, 2, 3, 4, 5) which gives the fitness value  $f^* = 0$ . The value of  $\beta$  is fixed to 10.

**F7: Mobile network design problem**

In mobile network design, the challenge is to efficiently determine the locations of base control stations, mobile switching centers, and their connecting links for given locations of base transceiver stations. This problem is complex to describe, and the reader can refer to El-Saleh et al. (2009) for more information. The search space is made of binary variables (38 variables) and of continuous ones (four variables). Hence, the problem has 42 dimensions.

**F8: Pressure vessel design problem**

This problem was proposed by Kannan and Kramer (1994). The objective is to minimize the total cost, including the cost of the material, forming and welding. The mathematical model of pressure vessel optimization problem can be described as follows (Clerc 2006):

$$\text{Min}f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3. \tag{16}$$

Subject to:

$$g_1(x) = 0.0193x_3 - x_1 \leq 0, \quad g_2(x) = 0.00954x_3 - x_2 \leq 0$$

$$g_3(x) = 750 \times 1728 - \pi x_3^2 \left( x_4 + \frac{4}{3}x_3 \right) \leq 0$$

where  $x_1 \in \{0.0625, \dots, 12.5\}$  with a granularity 0.0625,  $x_2 \in \{0.625, \dots, 12.5\}$  with a granularity 0.0625,  $x_3 \in$

$]0, 240]$  and  $x_4 \in ]0, 240]$ . The best solution is (1.125, 0.625, 58.2901554, 43.6926562) which gives the fitness value  $f^* = 7,197.72893$ .

**F9: Welded beam design problem**

The problem is to design a welded beam for minimum cost, subject to some constraints (Ragsdell and Phillips 1976). The problem can be mathematically formulated as follows:

$$\text{Min}f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2). \tag{17}$$

Subject to:

$$g_1(x) = \tau(x) - 13000 \leq 0 \quad g_2(x) = \sigma(x) - 30000 \leq 0$$

$$g_3(x) = x_1 - x_4 \leq 0$$

$$g_4(x) = 6000 - P_c(x) \leq 0 \quad g_5(x) = 0.125 - x_1 \leq 0$$

$$g_6(x) = \delta(x) - 0.25 \leq 0$$

$$g_7(x) = 0.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leq 0$$

where:

$$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2} \quad M = 6000 \left( 14 + \frac{x_2}{2} \right)$$

$$J = 2 \left\{ \sqrt{2}x_1x_2 \left[ \frac{x_2^2}{12} + \left( \frac{x_1 + x_3}{2} \right)^2 \right] \right\}$$

$$R = \sqrt{\frac{x_2^2}{4} + \left( \frac{x_1 + x_3}{2} \right)^2} \quad \sigma(x) = \frac{504000}{x_4x_3^3}$$

$$\delta(x) = \frac{2.1952}{x_3^3x_4} \quad \tau' = \frac{6000}{\sqrt{2}x_1x_2}$$

$$\tau'' = \frac{MR}{J} \quad P_c(x) = \frac{4.013(30 \times 10^6)\sqrt{\frac{x_3x_4}{36}}}{196}$$

$$\left( 1 - \frac{x_3\sqrt{\frac{30 \times 10^6}{4(12 \times 10^6)}}}{28} \right).$$

With:  $x_1, x_4 \in [0.1, 2.0]$  and  $x_2, x_3 \in [0.1, 10.0]$ .

Best solution:  $x^* = (0.205730, 3.470489, 9.036624, 0.205729)$  where  $f^* = 1.724852$ .

**F10: Speed reducer design problem**

This problem was modeled by Golinski (1973) as a single-level optimization. The objective is to minimize the speed reducer weight while satisfying a number of constraints imposed by gear and shaft design practices. Mathematically, the problem is specified as follows:

$$\text{Min}f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2). \tag{18}$$

Dynamic cluster in particle swarm optimization algorithm

Subject to:

$$g_1(x) = \frac{27}{x_1 x_2^2 x_3} \leq 0 \quad g_2(x) = \frac{397.5}{x_1 x_2^2 x_3^2} \leq 0$$

$$g_3(x) = \frac{1.93x_3^3}{x_2 x_3 x_6^4} \leq 0 \quad g_4(x) = \frac{1.93x_3^3}{x_2 x_3 x_7^4} \leq 0$$

$$g_5(x) = \frac{1.0}{110x_6^3} \sqrt{\left(\frac{745x_4}{x_2 x_3}\right)^2} + 16.9 \times 10^6 - 1 \leq 0$$

$$g_6(x) = \frac{x_2 x_3}{40} - 1 \leq 0 \quad g_7(x) = \frac{5x_2}{x_1} - 1 \leq 0$$

$$g_8(x) = \frac{1.0}{85x_7^3} \sqrt{\left(\frac{745x_5}{x_2 x_3}\right)^2} + 157.5 \times 10^6 - 1 \leq 0$$

$$g_9(x) = \frac{x_1}{12x_2} - 1 \leq 0$$

$$g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0$$

$$g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0.$$

With:  $2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 7.3 \leq x_4 \leq 8.3, 7.8 \leq x_5 \leq 8.3, 2.9 \leq x_6 \leq 3.9,$  and  $5.0 \leq x_7 \leq 5.5.$

Best solution:  $x^* = (3.5, 0.7, 17, 7.3, 7.8, 3.350214, 5.286683)$  where  $f^* = 2,996.348165.$

*F11: Constraint problem 1*

The function to be minimized is:

$$\text{Min}f(x) = 5 \sum_{i=1}^4 x_i - 5 \sum_{i=1}^4 x_i^2 - \sum_{i=1}^{13} x_i. \quad (19)$$

Subject to:

$$\begin{aligned} g_1 &= 2x_1 + 2x_2 + x_10 + x_11 - 10 \leq 0 & g_4 &= -8x_1 + x_{10} \leq 0 & g_7 &= -2x_4 - x_5 + x_{10} \leq 0 \\ g_2 &= 2x_1 + 2x_3 + x_{10} + x_{12} - 10 \leq 0 & g_5 &= -8x_2 + x_{11} \leq 0 & g_8 &= -2x_6 - x_7 + x_{11} \leq 0 \\ g_3 &= 2x_2 + 2x_3 + x_{11} + x_{12} - 10 \leq 0 & g_6 &= -8x_3 + x_{12} \leq 0 & g_9 &= -2x_8 - x_9 + x_{12} \leq 0 \end{aligned}$$

Bounds:  $0 \leq x_i \leq 1$  ( $i = 1, \dots, 9, 13$ ), and  $0 \leq x_i \leq 100$  ( $i = 10, 11, 12$ ). The global optimum is  $(1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 3, 3, 1)$ , where  $f^* = -15.$

*F12: Constraint problem 2*

The function to be minimized is:

$$\begin{aligned} \text{Min}f(x) &= x_1^2 + x_2^2 + x_1 x_2 - 14x_1 - 16x_2 + (x_3 - 10)^2 \\ &\quad + 4(x_4 - 5)^2 + (x_5 - 3)^2 + 2(x_6 - 1)^2 \\ &\quad + 5x_7^2 + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 \\ &\quad + (x_{10} - 7)^2 + 45. \end{aligned} \quad (20)$$

Subject to:

$$g_1 = -105 + 4x_1 + 5x_2 - 3x_7 + 9x_8 \leq 0$$

$$g_4 = -3x_1 + 6x_2 + 12(x_9 - 8)^2 - 7x_{10} \leq 0$$

$$g_2 = 10x_1 - 8x_2 - 17x_7 + 2x_8 \leq 0 \quad g_5 = 5x_1^2 + 8x_2$$

$$+ (x_3 - 6)^2 - 2x_4 - 40 \leq 0$$

$$g_3 = -8x_1 + 2x_2 + 5x_9 - 2x_{10} - 12 \leq 0$$

$$g_6 = x_1^2 + 2(x_2 - 2)^2 - 2x_1 x_2 + 14x_5 - 6x_6 \leq 0$$

$$g_7 = 3(x_1 - 2)^2 + 4(x_2 - 3)^2 + 2x_3^2 - 7x_4 - 120 \leq 0$$

$$g_8 = 0.5(x_1 - 8)^2 + 2(x_2 - 4)^2 + 3x_5^2 - x_6 - 30 \leq 0$$

Bounds:  $-10 \leq x_i \leq 10$  ( $i = 1, 2, \dots, 10$ ). The optimum solution is :  $x^* = (2.171996, 2.363683, 8.773926, 5.095984, 0.9906548, 1.430574, 1.321644, 9.828726, 8.280092, 8.375927)$ , where  $f(x^*) = 24.3062091.$

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## A Preliminary Integrated Model for Optimizing Tactical Production Planning and Condition-Based Maintenance\*

Bruno CASTANIER<sup>a</sup>, David LEMOINE<sup>a</sup>

<sup>a</sup> *IRCCyN, Ecole des Mines de Nantes*

*4 rue Alfred Kastler*

*B.P. 20722 - F-44307 NANTES Cedex 3, France*

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### Abstract

The inclusion of production and maintenance objectives in an overall decision-making approach remains as well an academic challenge as a response to a real industrial need. Based on the observation of mutual dependence between production planning and the stochastic variability of the system production performance due to degradation, we propose in this paper a strategy for optimizing both tactical production planning and maintenance under feasibility constraint. The strategy is driven by an iterative two-unit algorithm. The first unit performs the Master Production Schedule optimization whereas the second unit provides an estimation of the feasibility indicators via a stochastic simulation-based approach. Mathematical framework and the proposed iterative algorithm are directly integrated in friendly-user interface software for optimizing the methodology usability in an industrial context.

*Key words:* Tactical planning, Maintenance, Performance Threshold, Stochastic Simulation base approach, Software tool

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### 1 Introduction

Production management deals with a set of decisions in order to ensure company continuity. These decisions have to allow the production of customers demand, at the right due date and with the lowest costs. Thus, production management must find a compromise between the satisfaction of customers demand and production criteria such as logistic costs, production times and products quality. Usually, this set is divided into three decisional levels [8]: the strategic level (long-term: more than 18 months), the tactical level (midterm: from 6 to 18 months) and finally the operational level (short-term: from daily to weekly). Each level can be characterized on the one hand by the horizon on which decisions are taken and, on the other hand, by the costs that these decisions can bring about. It defines a framework for the next lower level.

Transversely to these decisional degrees, a major problem can be underlined in production management: the industrial planning. Thus, this problem can be also divided into three hierarchical levels: the strategical, tactical and operational plannings.

1. Strategical planning (long-term) deals with, for instance, the elaboration of the Supply Chain Network (facilities or warehouses locations etc.) and provides a framework for the tactical planning.

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\* This paper was not presented at any other revue. Corresponding author D. Lemoine  
*Email addresses:* bruno.castanier@mines-nantes.fr, david.lemoine@mines-nantes.fr.

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2. Tactical planning (midterm) consists in the elaboration of production plans which minimize logistic costs (for instance production, holding or setup costs). It means that the aim of tactical planning is to determine, for each item, quantities to be manufactured per period in order to meet the customers' request at a lower cost.
3. Operational planning (short-term) aim is to schedule the production defined at tactical planning level on the production system.

In this paper, we focus on tactical planning. Referring to hierarchical planning system proposed by [12], tactical planning is composed of three planning levels:

- the Sales and Operation Planning (S&OP),
- the Master Production Schedule (MPS),
- the Material Requirement Planning (MRP).

The S&OP consists in finding a balance between sales and production level on a midterm horizon (around 1 year). On this horizon, production capacity is not considered as fixed but can vary thanks to the use of overtime, sub-contracting, hiring, firing and so on. The MPS is defined by [7] as a problem for which the first goal is to find an optimal production plan which meets the customers' requests and provides release dates and amounts for all these products. The objective function lies in minimizing production, holding and setup costs. Elaborating internal component planning is the main goal of MRP. By using bills of materials, these plans are deduced from the MPS.

Usually, tactical production planning is modelled thanks to mathematical models called 'lot-sizing models'. A classification of the main ones can be found into [3]. In these ones, the specificities of the production system are modelled thanks to its estimated production capacities. However, as pointed out by many authors [2,4], these estimations are often far from the real capacities: indeed, they do not take into account the complex use of the resources of a real system (internal complexity of the system, non-linearity of the workload, random events such as breakdowns ...). Thus, the main reasons highlighted in literature for these wrong estimations are bound to the lack of taking into account the scheduling and breakdowns into their computations. Another point which can be considered in order to be more confident on these estimations is the ageing of the production system. Indeed, more than the classical relationship between ageing and breakdowns which is well-tackled in Maintenance Optimization, we can easily imagine that the *intrinsic* productivity measured in terms of processing time of each item and quality ratio is better for a new production system than an older one. Some models have been developed for taking into account the impact of non-quality due to ageing onto production planning [11]. [11] proves the optimality of condition-based control-limit policies for a one-item production system. In the condition-based maintenance framework, the decision rule is based on the observation of the current system deterioration level. Nevertheless, the one-item production system assumption does not allow highlighting the dependence of the production scheduling on the ageing process and, by transitivity, on the capacity estimation. As far as we know, such issue has never been taken into account in the estimation of capacities during the tactical planning process.

The objective of this study is to investigate the combined production planning and maintenance optimization topic by taking into account the dual reciprocity of both, production scheduling, intrinsic system productivity and breakdowns. More specifically, we propose to study, in an integrated planning framework, the impact of a condition-based maintenance policy for a stochastic deteriorating production system where its productivity is decreasing in usage. Hence, on one side, the maintenance decision rule has to be optimized to provide the balance between the positive impacts onto both the improvement in the system productivity and the reduction of the potential breakdowns unavailability, and the negative impacts onto costs and loss of production capacity due to preventive maintenance actions. On the other side, the goal of the tactical planning process is to find production plans which minimize logistic costs (such as holding, production or setup costs) by taking into account the capacities of the production system which are strongly dependent of the stochastic behaviour of the production system.

The reminder of the paper is organised as follows. Section 2 is devoted on the problem description, the definition of the tactical production planning process with the description of the maintenance decision framework. Assumptions and notations are introduced in this section. In Section 3, we present the overall performance evaluation framework based on an iterative algorithm for both the re-estimation of the capacities and the optimization of the associated production planning. This approach has been implemented into software with a friendly user interface which is presented in section 4. Finally, Section 5 presents the conclusions of this work by highlighting both the contributions and the different potential improvements of the current model for the definition of our future work.

## 2 Problem statement

As mentioned in the introduction, the tactical production planning requires some good estimates of the production capacities. Indeed, these capacities cannot be considered as constant during the overall planning horizon because of the stochastic productivity decrease due to the production system degradation and maintenance downtime. We proposed to illustrate this motivation based on a classical tactical planning problem described in Subsection 2.1. The modelling of the production system state in terms of degradation is detailed in the next subsection as well as its impact of the latter onto the production plan.

### 2.1 The tactical planning process

The aim of tactical planning is to determine the quantities of products to be manufactured over a midterm horizon in order to meet the customers' request and to find a compromise between different logistic costs such as setup or holding costs. Thus, let's consider we have  $N$  types of item whose production has to be planned over a horizon  $H$  which is discretised into  $T$  periods. For each item  $i$ , we suppose that the customers' request  $D_{i\tau}$  is known for each period  $\tau$ .

In order to produce such quantities, a production system is used. The latter can be characterized by  $C_\tau$  its available capacity for producing at period  $\tau$  (for instance, such capacity can be an opening time) and the capacity requested to produce one unit of item  $i$  at period  $\tau$  (which can be modelled as a processing time). This last parameter depends on the performance rate  $x$  of the production system and, therefore, of its degradation level (which depends on the sum of processing times since the last preventive maintenance). In this paper, we assume that  $x \in [0,1]$  and  $x = 1$  means that there is no degradation whereas  $x = 0$  implies that the production system is completely damaged. Thus, we will denote  $p_i(x)$  the capacity requested to produce one unit of item  $i$  at period  $\tau$  when the performance rate of the production system is equal to  $x$ , and  $p_i$  the theoretical capacity associated ( $p_i = p_i(1)$ : there is no degradation). Clearly,  $x \mapsto p_i(x)$  is a non-increasing function: it models that the higher the performance rate is, the lower the processing time is large. Obviously, the production system cannot produce more than its capacity. It means that the capacity which is used to manufacture the planned items at period  $\tau$  cannot exceed the capacity of the production system for this period.

Producing the customer's request leads to logistic costs. The latter usually are the holding cost  $h_i$  for an item  $i$  during one period, the setup cost  $s_i$  for configuring the production system in order to manufacture items  $i$  during one period.

### 2.2 Stochastic modelling of the production capacities due to system degradation

In this paper, we deal with a production system which gradually deteriorates. As mentioned in the last section, this system deterioration leads to reduce the real production capacity (by increasing the processing times). This deterioration is assumed to be time-continuously measurable through a time-variable  $x_t$  which represents the effective performance rate of the system.

Let assume that the deterioration is an increasing time-continuous stochastic process and the expected deterioration growth is a function of the current manufactured item. For instance, suppose that, at time  $t$ , the performance rate of the production system is equal to  $x_t$ : producing an item  $i$  requires the capacity  $p_i(x_t)$ . During the manufacturing of this item, the production system degrades and reaches its new performance rate  $x_{t+p_i(x_t)} < x_t$ . Thus, if no maintenance process occurs, manufacturing a new item  $i$  at this moment, will consume  $p_i(x_{t+p_i(x_t)})$  (which is higher than  $p_i(x_t)$ ). As a consequence, we can notice that, according to their processing times, each type of items has a different impact onto the performance level of the production system, some of them can be more "aggressive" than others.

More formally, note  $\{X_t, t > 0\}$  the stochastic process which models the random behaviour of the non-maintained production system performance rate. This process is assumed to be strictly decreasing from the new state with  $X_t = 1$  to 0 (in case of  $X_t = 0$ , the processing time for manufacturing one item is considered as infinite). Denote  $\Delta(t, t + s)$ , the system performance loss during the production time interval  $[t, t + s]$ . The associated probability distribution of such a random decrement is assumed to be the Gamma distribution with a shape parameter  $\alpha \cdot s$  and a scale parameter  $\beta$  where  $\alpha$  and  $\beta$  are two positive constants that can be estimated with field data or expert judgement. Such processes are called stationary Gamma processes and well-known in maintenance optimization for continuously-deteriorating systems (for review, the reader can refer to [10]). The probability density function of a  $(\alpha, s, \beta)$ -Gamma-distributed variable is, for  $x \in [0,1]$ :

$$f_X(x) = \frac{\beta^{\alpha s}}{\Gamma(\alpha, s)} x^{\alpha s - 1} e^{-\beta x} \tag{1}$$

where  $\Gamma(u) = \int_0^{+\infty} t^{u-1} e^{-t} dt$  is the classical Euler function.

In our contribution, we deal with tactical planning which is a midterm horizon process. Therefore, a one-item processing time is very small compared to the length of a tactical production period. Thus, we assume that the degradation level (and, consequently, the performance rate) is constant over the processing time while manufacturing an item: the deterioration decrement is taken into account at the end of the latter.

However, the impact of the degradation of production system cannot be summarized in an extension of processing times. Indeed, the higher the level of degradation of the production system, the higher it is likely to fail. Moreover, the system failures lead to unavailability duration (the repair time) in the production planning.

Assume the failure rate depends on the system deterioration level and, consequently, is a function of the performance level of the production system (a highly-deteriorated system is more prone to failure than a new or a maintained system). Note  $\lambda(x)$  the system failure rate given the performance level  $x$ . Under the assumption of the one-item production duration compared with a production period, the failure rate is assumed to be constant on each one-item production interval. So, the expected number of failures for producing one item  $i$  given the system performance level  $x \in [0,1]$  is  $\lambda(x) \cdot p_i(x)$ . Note that since the production is stopped in case of failure, the system is no longer deteriorated during the repair time.

To summarize, the system deterioration has some impacts on the production according to the two following aspects:

- the “intrinsic” performance of the production system and, consequently, the introduction of a random variability in the production capacities,
- the system unavailability which acts on the consumption of resources’ capacity.

The modelling of performance rate decreasing for a given production period is illustrated in Fig. 1.

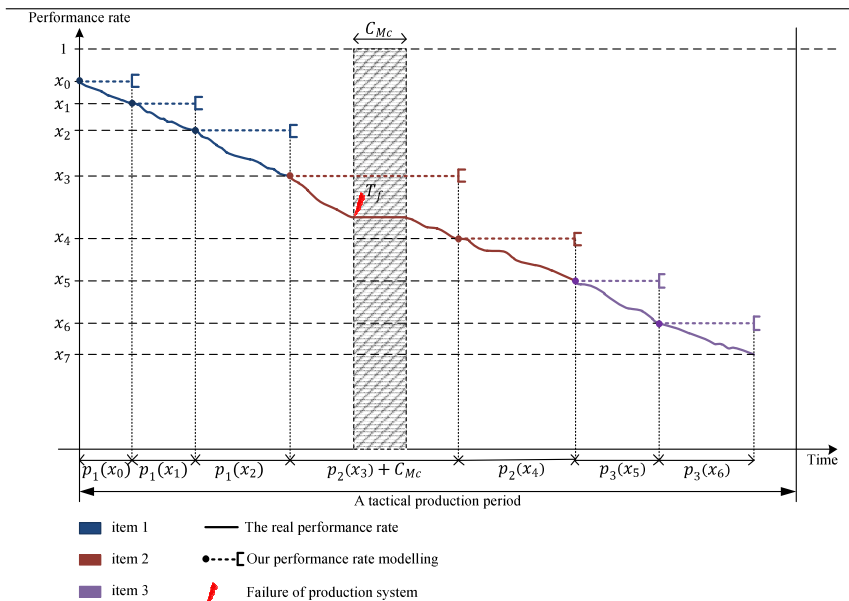


Fig. 1: Illustration of the decrease in the productivity because of the ageing and failures in one production period.

In this example, three types of item are taken into account and we suppose that we deal with the period  $\tau$  of the tactical planning horizon. During this period, the decision made by the tactical planning process is to manufacture 3 items of type 1, 2 items of type 2 and 4 items of type 3. According to the previous notations,  $p_i(x)$  ( $i = 1 \dots 3$ ) are the capacities requested for manufacturing each type of item with the performance rate  $x$  and  $C_\tau$  is the available capacity for producing at period  $\tau$ . We suppose that, at the beginning of the period  $\tau$ , the performance rate is equal to  $x_0$ . As we can notice it, there is an increase of capacities requested for manufacturing items according to the performance rate  $x$  of the production system. In this example, a failure occurs at time  $T_f$ , during the manufacturing of the first item of type 2. Therefore, the production system remains unavailable during the repair time  $C_{Mc}$  and the production resumes after this maintenance. Nevertheless, we can see that only the two first items of type 3 can be produced during the period  $\tau$ , due to the lack of remaining capacity of the production system. Indeed, in our example,

$$\sum_{i=0}^2 p_1(x_i) + \sum_{i=3}^4 p_2(x_i) + \sum_{i=5}^6 p_3(x_i) + C_{Mc} < C_\tau < \sum_{i=0}^2 p_1(x_i) + \sum_{i=3}^4 p_2(x_i) + \sum_{i=5}^7 p_3(x_i) + C_{Mc}$$

### 3 Performance criterion : definition and evaluation

In this section, we present an iterative approach to take into account the impacts of the production system degradation and its maintenance onto the tactical planning process, and more specifically onto the MPS elaboration. The aim of this latter is to provide a feasible tactical plan by a better estimation of the production system's capacity, the feasibility of a tactical plan defining as a given threshold for the probability of success of the tactical plans. In the first subsection, we present the general scheme for our approach. The second subsection is dedicated to the mathematical model for solving the MPS elaboration problem. Next, a stochastic simulation-based algorithm is presented: for updating the estimates of the different feasibility indicators as functions of the system degradation and the associated maintenance actions. Finally, the principle of the reappraising of capacities and the feasibility ratio is explained.

#### 3.1 The iterative optimization process

The proposed approach, illustrated in Fig. 2, is based on an iterative algorithm with the two respective units for the tactical planning process given the capacities' estimates and for the estimates updates by the use of the simulation of the system degradation during the given production plans as well as its maintenance. Such kind of approaches (a combination between tactical planning and simulation based units) has already been developed by [2] or [9], but in a deterministic scheduling context. In this preliminary study, we adapt this approach to the stochastic context of maintenance and degradation of the production system. The algorithm can be described as follows:

Step 1: Customers' request, initial capacities of the production system, theoretical capacities requested for the manufacturing of each type of item as well as logistics costs are given by the user. These parameters are sent to the tactical planning unit.

Step 2: According to Customer's request, theoretical capacities requested for the manufacturing of each item, logistic costs and current capacities of the production system, the tactical planning unit elaborates a Master Production Schedule. This latter is sent to the simulation unit.

Step 3: In the simulation unit, the MPS feasibility is estimated based on Monte Carlo techniques. A lot of runs of production process based on the stochastic simulation of the production system degradation and the associated maintenance actions according to the MPS given by the tactical planning unit. At the end of this process, for each period a feasibility rate and an expected capacity requested for manufacturing the entire planned items are given. If, for each period, the feasibility rate is higher than an acceptance rate  $AR$  given by the user, the tactical plan is supposed to be feasible and thus, the process stops with success else, go to step 4.

Step 4: Here, capacities of the system of production are reappraising for each period where the feasibility rate is lower than the acceptance rate, according to the expected capacity requested for ensuring the MPS. Then, go back to Step 2.

### 3.2 The tactical planning unit

In this section, we deal with the MPS elaboration which is a tactical planning plan. In order to elaborate such plan, mathematical models have been developed [3]. These models are called “Lot-Sizing Problem”. Among them, a particular one is dedicated to this problematic: the Capacitated Lot-Sizing Problem (CLSP) [5]. Using notations defined in section 2.1, in its basic version CLSP can formally be couched as a mixed-programming model:

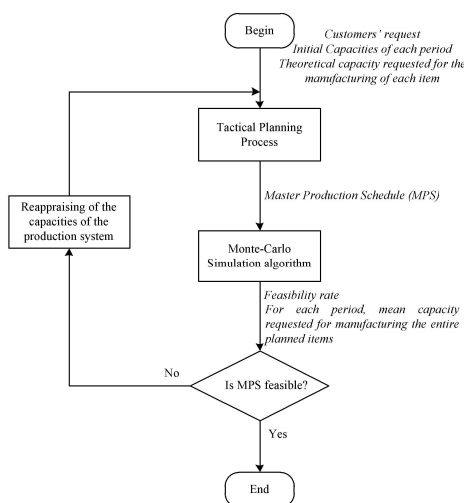


Fig. 2: the general scheme of the iterative process

$$\text{Min} \sum_{t=1}^T \left[ \sum_{i=1}^N (s_i X_{it} + h_i I_{it}) \right] \quad (2)$$

Subject to :

$$I_{i\tau} = I_{i(\tau-1)} + Q_{i\tau} - D_{i\tau} \quad \forall (i, \tau) \in \llbracket 1, N \rrbracket \times \llbracket 1, T \rrbracket \quad (3)$$

$$\sum_{i=1}^N p_i Q_{i\tau} \leq C_\tau \quad (4)$$

$$Q_{i\tau} \leq C_\tau X_{i\tau} \quad (5)$$

$$I_{i\tau}, Q_{i\tau} \in \mathbb{N} \quad (6)$$

$$X_{i\tau} \in \{0,1\} \quad (7)$$

Where  $Q_{it}$  is the production quantity of item  $i$  at period  $\tau$ ,  $I_{i\tau}$  is the inventory of item  $i$  at the end of period  $\tau$  and  $X_{i\tau}$  is the binary variable which indicates whether a setup for item occurs at period ( $X_{i\tau} = 1$ ) or not ( $X_{i\tau} = 0$ ).

The objective function, Eq. (2), is defined by the sum of the setup, the holding and the production costs. The respective constraints are inventory balances, Eq. (3), the capacity constraints, Eq. (4), the setup constraint: due to these previous restrictions, production of an item can only take place if the machine is set up for that particular item, Eq. (5), the non-negativity conditions, Eq. (6) and, finally, the binary definition of the setup variables given by Eq. (7).

In its basic formulation, the CLSP can fail in finding a solution and especially if the capacities of production system are too scarce. To avoid this problem, we integrate in the tactical planning process backlogs [13] or some shortage [1] into the meeting the customers' demand. Backlogging cost  $b_i$  for an item  $i$  during one period as

well as the shortage cost  $sh_i$  for an item  $i$  are defined. We also introduce two new variables:  $I_{i\tau}^-$  the number of item  $i$  which are in backlog at period  $\tau$  and  $SH_{i\tau}$  the number of item  $i$  which are in shortage at period  $\tau$ . Finally, the objective function (2) becomes

$$\text{Min} \sum_{\tau=1}^T \left[ \sum_{i=1}^N (s_i X_{i\tau} + h_i I_{i\tau} + b_i I_{i\tau}^- + sh_i SH_{i\tau}) \right] \quad (8)$$

In (8), backlogging and shortage costs are taken into account. The inventory balance (3) becomes:

$$I_{i\tau} - (I_{i\tau}^- + SH_{i\tau}) = I_{i(\tau-1)} - I_{i(\tau-1)}^- + Q_{i\tau} - D_{i\tau} \quad \forall (i, \tau) \in \llbracket 1, N \rrbracket \times \llbracket 1, T \rrbracket \quad (9)$$

And, finally, the non-negativity constraint (6) is replaced by:

$$I_{i\tau}, I_{i\tau}^-, SH_{i\tau}, Q_{i\tau} \in \mathbb{N} \quad (10)$$

Using this model, we can be sure that there is always an admissible solution to the tactical planning process (in the worst case, the MPS could be empty).

### 3.3 The Monte-Carlo simulation unit

As presented in section 2.2, the system degradation due to usage is assumed to be observable through the reduction in the system performance rate and so, in the respective increase in the production capacities using for manufacturing each type of item. Moreover, the failure frequency is assumed to be dependent on this system degradation. Hence, to ensure an acceptable performance rate and to limit the failure downtimes, we propose to implement a condition-based maintenance policy: the maintenance decision is no longer defined on time- or age-based parameters but on the real system state measured after the production of each item. Although more difficult to implement in an operational context than classical maintenance policies, these condition-based approaches have proven their efficiency in terms of overall performance in several engineering areas [6].

In our study, two kind of maintenance are taken into account:

- The replacement policy which consists in replacing preventively or correctively the system by a “new” system with the same production and deterioration characteristics. In our case, we propose to use a classical control-limit decision rule: a preventive replacement is performed if the system state observed after the production of an item is lower than a preventive maintenance threshold  $\delta$ . Such a replacement incurs a given cost  $P_{Mp}$  and consumes a fixed capacity  $C_{Mp}$ . Nevertheless, a replacement can be performed only if the remaining time before the end of the production at period  $\tau$  is sufficiently large, i.e. greater than  $C_{Mp}$ . If this condition is not met, the production continues and the replacement will be done at the beginning of the next production period. After a replacement, the capacity requires for manufacturing an item of type  $i$  is supposed to be  $p_i(1)$  (the performance rate  $x$  becomes equal to 1).
- The repair which consists in restoring the failed system in a functioning state without considering the “as good as new” condition. Here, only the minimal repair which restores the system in the “as bad as old” state (i.e. the deterioration level after the repair remains identical than before the failure) is considered. A minimal repair is done immediately in case of the system failure. We introduce a unitary minimal repair cost  $P_{Mc}$  and the respective consumed capacity  $C_{Mc}$ .

The flowchart in Fig. 3, sketches the production scenario and the associated maintenance decisions given the MPS when the system degrades. This partially illustrates the proposed Monte-Carlo simulation algorithm for providing different feasibility information for each period  $\tau$  such as:

- The expected feasibility rate  $FR_\tau$  which is the probability of the success of the manufacturing of all the quantities given by the MPS
- The expected capacity  $EC_\tau$  required for manufacturing the whole quantities planned in the MPS without any capacity constraint.

### 3.4 The feasibility criterion and the reappraising phase

**Definition:** A MPS is said to be feasible if, and only if, the lowest expected feasibility ratio of each period  $FR_\tau$  is higher than the given acceptance threshold  $AR$ , i.e.

$$\text{MPS is feasible} \Leftrightarrow \min_{\tau \in \llbracket 1, T \rrbracket} FR_\tau \geq AR \quad (11)$$

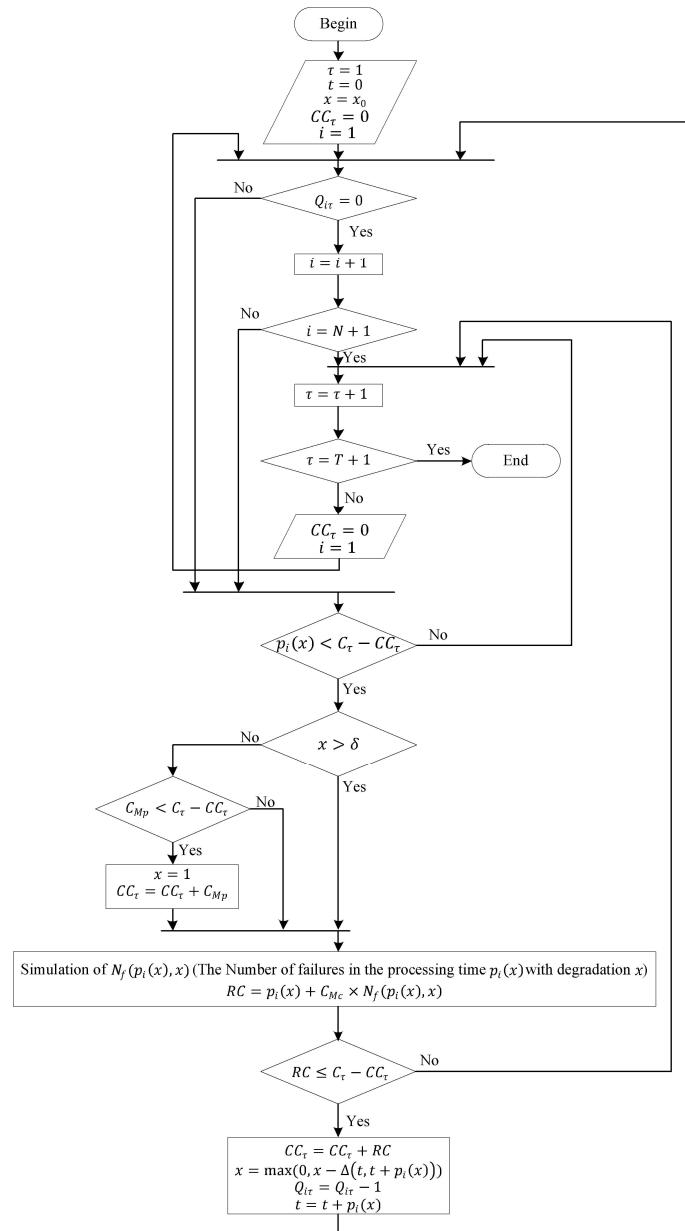


Fig. 3: The Monte Carlo Simulation Algorithm



In case of non-feasibility, the MPS has to be re-optimized with new one-period capacities. The reappraising phase allows to update those capacities for each period  $\tau$  based on the one-period expected capacities  $MC_\tau$  provided by the simulation algorithm. Therefore, we have 2 cases:

- Case 1:  $EC_\tau \leq C_\tau$ . The capacity of the production system is enough to manufacture all the planned items. Therefore, the latter remains the same.
- Case 2:  $EC_\tau > C_\tau$ . The capacity required to manufacture the entire quantities planned by the MPS exceeds the capacity of the production system. Thus, the capacity of the production system needs to be reappraised we introduce a capacity reduction factor  $\gamma_\tau$ :

$$\gamma_\tau = \left\lfloor \frac{C_\tau}{EC_\tau} \times 10^3 \right\rfloor \times 10^{-3} \quad (12)$$

Thus,  $C_\tau$  becomes  $C_\tau \gamma_\tau$

It is very easy to prove that our approach converges in a finite number of iteration by noticing that we have defined for each period a non-increasing positive sequence  $C_\tau$ . Therefore, we can see that this latter converges toward a limit  $L_\tau$ . Moreover, since  $C_\tau$  can take only a finite number of values (at most, all the numbers which have at the most three decimal place between initial production capacity at period  $\tau$  and 0), the Cauchy criterion insures that the convergence takes place at a finite rank.

#### 4 Implementation of the approach in a software tool.

For improving the applicability of our approach in an industrial context or at least to get their feedback, the algorithm is directly implemented in a friendly-user interface software. Final numerical results are numerically and graphically provided (Fig. 4).



Fig. 4: The software tool for our approach

This software is divided into four units:

1. Data management unit for loading, saving or importing data from an external database;
2. Tactical planning process unit for the construction and optimization of the instance-related mathematical model using Cplex Solver;
3. Simulation unit for estimating the feasibility production plans using Scilab software;
4. Graphical interface, Fig. 4, for the problem specification with all the parameters (capacities, costs, degradation, etc.), as well the presentation of the intermediate results obtained during the optimization process and the final solution through different charts summarizing the associated indicators. Method's current results and the final results are represented thanks to some charts.

#### 5 Conclusions and future work

The motivation for this communication is to contribute to the reduction of the gap between scientific research and industrial needs in the production planning topic. In order to meet such a goal, we choose to study the

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feasibility problems of production plans by taking into account the progressive degradation of the production system while seeking to minimize the impact of the associated maintenance. Even if the proposed work has to be seen as a preliminary work, major contributions on both modeling and optimization can be highlighted.

For the modeling contribution, advanced results in Maintenance Optimization have been integrated in a Production Planning framework in case multi-items manufacturing. Hence, our model extends classical combined production-maintenance approaches which essentially focus on maintenance downtimes, with a greater contribution in the interaction between degradation and production system performance. The introduction of such degradation information is easier to relate to the system performance than the classical failure rate, and so more comprehensive and so consequently more applicable for the users.

For the optimization contribution, we extend the algorithm previously developed by [2] in a deterministic context for taking into account the stochastic behavior of the system degradation and failures. A stochastic simulation-based approach is proposed for circumventing the numerical evaluation problems.

The proposed model offers some great opportunities for both optimization and modeling future work. From the optimization perspectives, first of all, a preventive maintenance threshold optimization algorithm has to be integrated in the general framework to ensure the optimality of the system performance in the planning horizon. Then, different other preventive maintenance strategies such as opportunistic condition-based maintenance policy should be studied to favor preventive maintenance during low production times. From the tactical planning point of view, the considered optimization problems are generally NP-Hard, so optimization methods should be developed in order to replace Cplex Solver and to provide good solutions to the planning process in a minimal time. Another point to investigate is the formula (12) (the reappraising coefficient) which can be considered as too drastic (even if it ensures the convergence of the approach): some further works must be done in order to refine it. The replacement of the capacity of the production system by scheduling constraints (such as in the [4] approach) with the taking into account of stochastic variables for maintenance process could be also a great improvement way. Another improvement axis for the applicability of our approach in an industrial context can be the study of more classical system performance metrics such as machine capability or the more explicit integration of non-quality issues.

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## Integrated dynamic single item lot-sizing and quality inspection planning

Belgacem Bettayeb<sup>a\*</sup>, Nadjib Brahim<sup>b</sup> and David Lemoine<sup>a</sup>

<sup>a</sup>SLP/LS2N UMR CNRS 6004, IMT Atlantique, Nantes, France; <sup>b</sup>Rennes School of Business, Rennes, France

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This paper proposes an integrated model for single item dynamic lot-sizing problem and Quality Inspection Planning (QIP). The objective is to provide a model of production planning that takes into account a targeted level of outgoing quality or an Acceptable quality level (AQL) when the manufacturing system inherently generates a proportion of defectives that increases significantly when the system switches from the in-control state to the out-of-control state. The average outgoing quality of each period of time of the planning horizon is bounded as a function of the inspection capacity. The effects of integrating QIP are analysed and discussed through several experiments representing different quality control system's parameters, i.e. inspection capacity, inspection cost and AQL. The simulation results show that it is very important to take into account the inspection process into production planning decisions. This study will help the decision-makers to negotiate service levels or react properly to given customer quality requirements based on cost and lead time parameters in addition to their process characteristics in terms of capability and stability.

**Keywords:** lot-sizing; quality assurance; production planning; inspection capacity; average outgoing quality

### 1. Introduction

One of the main objectives in production planning is to find a trade-off between cost, lead time and quality. However, most production planning mathematical models (lot-sizing models), are limited to the consideration of the trade-off between cost and lead time only. This applies particularly to discrete time lot-sizing models, while there has been a considerable interest of integration of continuous-time lot-sizing models with quality. In fact, in traditional production planning models the main objective is to minimise costs or maximise profit while satisfying customer demand and/or meeting system restrictions such as production capacity. Many simplifying assumptions are made with most production planning models. One of the assumptions consists of assuming that the production system is perfect and generates no defects. However, in practice production systems are imperfect and there is usually an inherent proportion of defective units produced. It was also shown through surveys and practice experience that improving quality helps companies to increase their market share (See Keller and Noori 1988). Hence, it is not relevant to ignore quality aspects in production processes. This is why we propose a model which integrates quality issues and inspection decisions.

Quality control activities have a direct link with other activities in the company such as inventory control. Still, research on integrated decisions including quality control and inventory control are quite recent and just started in the mid-1980s with the publications of Lee and Rosenblatt (1985), Rosenblatt and Lee (1986) and Porteus (1986). These publications and others showed the importance of considering quality issues and process imperfection in inventory control policies/decisions to meet different objectives such as meeting quality and cycle time expectations (Colledani and Tolio 2009), preventing excessive scrap production (Bettayeb, Bassetto, and Sahnoun 2014) or enhance productivity (Kim and Gershwin 2005). The recent surveys (Wright and Mehrez 1998; Khan et al. 2011; Rahim and Ben-Daya 2012) about extended inventory control models with imperfect quality witness the increasing interest, especially in the last two decades (Andriolo et al. 2014), in this topic and its applications such as wafer fabrication (Lee 1992) and paper production (Moisio and Virolainen 1993).

It seems that Lee and Rosenblatt (1985), Rosenblatt and Lee (1986) and Porteus (1986) are among the first to publish on importance of the relationship between quality and lot sizing. Porteus (1986) developed a deterministic model to demonstrate the importance relationship between quality and lot size. They present a closed form solution to the optimal lot-size quantity given that the production process can go out of control with a certain probability and that there is a cost associated with repairing each defective unit of the product. The paper concluded that the optimal strategy consists of a combination in quality improvement investment and set-up cost reduction (to reduce lot sizes). Keller and Noori (1988) extended Porteus work by considering that demand during lead time is probabilistic. They proposed a four-step solution procedure to solve a lot-size reorder-point model.

\*Corresponding author. Email: [belgacem.bettayeb@mines-nantes.fr](mailto:belgacem.bettayeb@mines-nantes.fr)

Porteus (1986) and Rosenblatt and Lee (1986) consider that the process is checked in the beginning of any new lot to ensure that it starts in an 'in-control' state. If the process is in-control when it starts processing an item, it will either switch to out-of-control or continue in the in-control state after the production of the item. Once the process switches to out-of-control state, it will stay in that state and keep producing defective items till all items in the lot are produced. Khouja (2005) and later Jaber (2006) reformulated the model of Porteus (1986) to allow process adjustment (interruption) within a production cycle (lot). The idea of Khoudja's formulation might be motivated by the 'andon' system in JIT philosophy which empowers operators to stop the process if something goes wrong (Jaber 2006).

Porteus (1986) considers that all items produced in the in-control state are conform and all those produced in the out-of-control state are non-conform. Djamaludin, Murthy, and Wilson (1994) assumes that only a fraction of products are non-conform in each state. There are two types of non-conforming items: Type-A and Type-B. While, items of type A are non-operational, items of type B are operational but have performance characteristics which are inferior to those of conforming items (See Djamaludin, Murthy, and Wilson 1994). While type-A items must be fixed or replaced, items of type B can be sold to the customer as they are at the risk of having to replace them later for the customer as part of a free replacement warranty policy (Djamaludin, Murthy, and Wilson 1994). Djamaludin, Murthy, and Wilson (1994) and Yeh and Chen (2006) consider items of type B nonconformity.

The majority of publications integrating quality control with production planning and inventory control decisions are limited to static demand models. However, most of industrial applications consider dynamic demand, consequently static models are not applicable in these cases. Thus, our objective in this paper is to contribute to research in this field through a deep analysis of the interaction between inspection decisions and different cost and process characteristics in the context of dynamic demand production planning decisions. We consider a single item lot-sizing model (Brahimi et al. 2006, 2017) with production, set-up and inventory holding costs in addition to inspection costs and inventory holding cost of sampled and inspected units. When a product lot is received by the customer, it is inspected and if its level of quality (outgoing quality) is higher than an agreed-upon level, than the lot is accepted, otherwise, it is returned back to the supplier. We determine the optimal production, sample and inspection levels to balance between COQ and the other 'logistics' costs. We are also limiting our study to class A problems only, that is, non-conform items are non-operational. Finally, replacement of non-conform items is assumed to be instantaneous. Furthermore, a comparison is made between the proposed model and a decomposed model in which there is no contract on the quality level between the supplier and the customer, however, all defective units detected by the customer are returned to the supplier who has the responsibility of replacing them with non-defective ones. This paper is an extension of a research work presented in the 8th IFAC Conference on Manufacturing Modelling, Management and Control (Bettayeb, Brahimi, and Lemoine 2016)

In the literature, the closest models to ours are those presented in Teunter, Bayindir, and Van Den Heuvel (2006), Goerler and Voß (2016) and Fakher, Nourelfath, and Gendreau (2016). Teunter, Bayindir, and Van Den Heuvel (2006) propose a model where returns are allowed in the production system. That is, the finished product is either made from manufacturing or remanufacturing. The objective was to minimise total cost composed of holding cost for returns and finished goods in addition to set-up costs. An exact dynamic programming algorithm and some heuristics are proposed to solve two cases of the problem: joint or separate set-ups for manufacturing and remanufacturing. Here, the quantity of returns is an input to the model and does not depend on the process characteristics. Similarly to Teunter, Bayindir, and Van Den Heuvel (2006), there are two inputs to the production system in Goerler and Voß (2016): materials coming from storage and defectives to be reworked. This model is closer to ours in the sense that the quantities to be reworked depend on the characteristics of the process and the amount of defectives is generated. However, the model of Goerler and Voß (2016) does not propose any link with the customers and the possibility that they return a product lot if it does not meet quality requirements. Finally, the model proposed by Fakher, Nourelfath, and Gendreau (2016) is very comprehensive and integrates both inspection (quality) and maintenance decisions with a multi-product, multi-machine production system. Due to the complexity of the model, the authors proposed two metaheuristics (a genetic algorithm and a tabu search) to solve it. Compared to Fakher, Nourelfath, and Gendreau (2016), our model focuses on the analysis of the interaction between production planning decisions and inspection decisions based on different costs and process characteristics. We believe that an analysis of our model based on optimal solutions obtained through a MILP formulation will give a better insight into such interactions. It will make it easier to guide the decision-makers on how to negotiate service levels or react properly to given customer quality requirements based on cost and lead time parameters in addition to their process characteristics in terms of capability and stability.

Table 1 summarises the characteristics of previously cited research works compared to our contributions.

The contributions of this paper can be summarised in the following:

- (1) An integrated quality inspection and production planning model.
- (2) Analysis of the model based on some performance indicators with respect to several process characteristics and costs.
- (3) Consideration of non-conform items both in in-control and out-of-control states at different probabilities.

Table 1. Summary of the most relevant related work and the characteristics of the proposed models.

	Time		Demand		Products		System's States		Defectives/Retu-rns Management				
	Continuous	Discrete	Deterministic	Stochastic	Mono	Multi	Mono	Multi	Reinspect	Rework/repair	Retind/replace	Penalty/Scrap	Inspection error
Fakher, Nourfath, and Gendreau (2016)		x	x			x		x		x			x
Ben-Daya and Rahim (2003)	x		x		x			x					x
Colledani and Tollo (2009)	x			x	x			x		x			x
Chan, Ibrahim, and Lochert (2003)	x		x		x			x				x	
Djamatudin, Murthy, and Wilson (1994)		x	x			x		x		x			
Goerler and Voß (2016)		x	x			x		x		x			
Jaber (2006)	x		x					x		x			
Kang et al. (2017)	x		x					x					
Keller and Noori (1988)	x			x						x			
Khoulja (2005)	x		x		x			x		x			
Lee (1992)	x		x		x			x		x			
Porteus (1986)	x		x		x			x		x			
Rosenblatt and Lee (1986)	x		x		x			x		x			
Teunter, Bayindir, and Van Den Heuvel (2006)		x	x					x		x			
Yeh and Chen (2006)	x		x					x		x			
Wang and Sheu (2003)	x		x					x		x			
Zhang and Gerchak (1990)	x		x					x		x			
Our work		x	x					x		x			

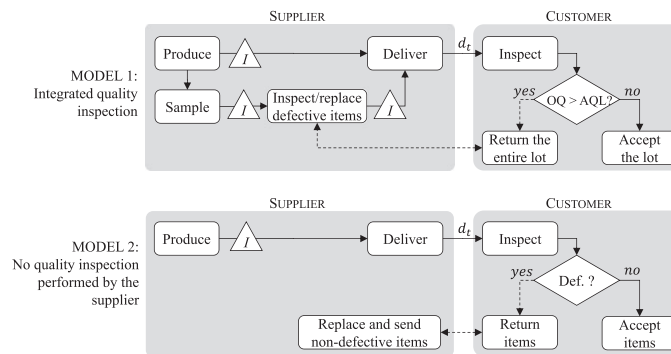


Figure 1. The two compared quality assurance policies.

(4) Comparison with a decomposed model with a basic contract between the supplier and customer.

As mentioned previously, this paper is an extension of the conference paper of Bettayeb, Brahimi, and Lemoine (2016). The extensions consisted of improving certain assumptions such as:

- Taking into account the inspection lead time. This is particularly important in certain industries such as the pharmaceutical and semi-conductor industries.
- Fine tuning of the *Outgoing Quality* model (see Section 2.1). In particular, the new formulation considers the quantity delivered without inspection instead of a rough estimation based on customer demand.

Furthermore, extensive experiments were carried out on new instances and very detailed analysis is provided for several indicators.

The remainder of the paper is organised as follows. Section 2 will present the model and the linear programming formulation. Section 3 presents numerical tests and discussions of the obtained results. Finally, a conclusion and some research perspectives are presented in Section 4.

**2. Problem description and formulation**

The Problem under consideration in this work concerns the joint planning of production and quality inspection. The aim of this study is to highlight the interaction of these two activities at the tactical/operational decision levels of a production system.

Figure 1 illustrates two different models. Our integrated model is denoted as MODEL 1. In this model, there is an agreement between the supplier and customer with respect to minimum level of accepted quality or the average quality level (AQL). Once a lot is delivered, it undergoes the inspection process of the customer. If the latter finds that the percentage of defectives exceeds AQL, then the whole lot is returned to the supplier. Otherwise, the customer accepts the whole lot including any defectives it might contain. MODEL 1 in Figure 1, is a reference model in which the supplier makes the product and delivers it to the customer. The customer will return any defective units he/she finds in the delivered lot. The supplier must make the same amount of returned units with no defects.

MODEL 1 and MODEL 2 are formulated as mixed integer linear programmes (MILPs) and their corresponding network models are represented in Figures 2 and 3, respectively.

Before presenting the MILP formulations and the network models, the following notation is used:

**Parameters**

- $T$  number of periods in the planning horizon
- $t$  index of planning periods,  $t \in \{1, 2, \dots, T\}$
- $c_t^p$  unit production cost at period  $t$

- $c_t^s$  setup cost at period  $t$
- $c_t^h$  unit holding cost per time period at period  $t$
- $c_t^i$  unit inspection cost at period  $t$
- $d_t$  demand at period  $t$
- $D$  maximum demand over all periods, i.e.  $\max_{t=1, \dots, T} d_t$
- $L$  inspection lead time
- $p_0$  proportion of defectives when the process is in-control
- $p_1$  proportion of defectives when the process is out-of-control, with  $p_1 = \delta p_0$ , where  $\delta$  is the drift magnitude
- AQL acceptable quality level (AQL), i.e. proportion of defectives beyond which the delivery's quality is considered unsatisfactory
- $M_t$  an upper bound on the production quantity

**Decision variables**

- $X_t$  quantity to produce at period  $t$
- $X_t'$  quantity to sample at period  $t$
- $X_t''$  quantity to deliver without inspection at period  $t$
- $B_{t,x}''$  binary decision variable = 1 if  $X_t'' = x - 1$ ; 0 otherwise, where  $x$  is integer
- $Y_t$  binary decision variable = 1 if production is to be set up at period  $t$ ; 0 otherwise.
- $Z_t$  quantity to inspect at period  $t$
- $I_t$  inventory level of non-inspected items at the end of period  $t$
- $I_t'$  inventory level of sampled products at the end of period  $t$
- $I_t''$  inventory level of inspected products at the end of period  $t$
- $R_t$  expected quantity of replaced items that are produced at period  $t$
- $W_t$  state of requirement satisfaction at period  $t$ , i.e.  $W_t = 1$  if  $OQ_t > AQL$ ; 0 otherwise
- $OQ_t$  expected proportion of defectives delivered at period  $t$

**Time to drift distribution**

- $f_X$  Probability mass function of random variable  $X$
- $T_t^\delta$  Random variable associated to the time to drift since the beginning of period  $t$ , expressed by the number of items processed until drift, of period  $t$

**2.1 Calculating the outgoing quality  $OQ_t$**

As shown in the list of decision variables above, the outgoing quality variable ( $OQ_t$ ) represents the expected proportion of defectives delivered at period  $t$ . It is expressed as:

$$OQ_t = \frac{ENDD_t}{d_t} \quad \forall t \tag{1}$$

where  $ENDD_t$  is the expected number of defectives within uninspected quantity. To compute  $ENDD_t$ , we must express the random variable  $NDD_t$  associated to the number of defectives delivered at period  $t$ . Assuming that (in the worst case) all of the non-inspected products' quantity  $X_t''$  has been processed successively during the current period  $t$ ,  $NDD_t$  will depend on the quantity processed and the random variable associated to the time to drift  $T_t^\delta$  (with  $f$  its probability mass function).

It can be expressed as follows:

$$NDD_t = p_0 \cdot X_t'' + (p_1 - p_0) \cdot (X_t'' - T_t^\delta) \cdot \mathbb{1}_{T_t^\delta \leq X_t''}$$

Where:

$$\mathbb{1}_{T_t^\delta \leq X_t''} = 1 \quad \text{if } T_t^\delta \leq X_t'' \quad \text{and } 0 \quad \text{otherwise.}$$

Then

$$ENDD_t = E[p_0 \cdot X_t'' + (p_1 - p_0) \cdot (X_t'' - T_t^\delta) \cdot \mathbb{1}_{T_t^\delta \leq X_t''}] \tag{2}$$

PROPOSITION For all  $t = 1 \dots T$ ,  $OQ_t$  can be expressed as:

$$OQ_t = \frac{ENDD_t}{d_t} = \frac{1}{d_t} \left( X_t'' \cdot p_0 + (p_1 - p_0) \cdot \sum_{x=2}^D B_{t,x}'' \sum_{y=1}^{x-1} (x - 1 - y) \cdot f_{T_t^\delta}(y) \right) \tag{3}$$

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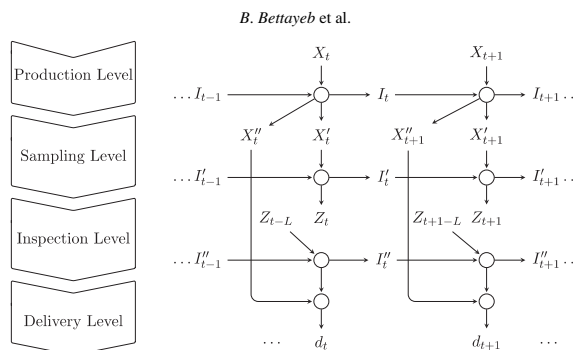


Figure 2. MODEL 2 network flow model and decision variables.

*Proof.* We transform ENDD<sub>t</sub> as follows:

$$\begin{aligned}
 \text{ENDD}_t &= E[p_0 \cdot X''_t + (p_1 - p_0) \cdot (X''_t - T_t^\delta) \cdot \mathbb{1}_{T_t^\delta \leq X''_t}] \\
 &= p_0 \cdot X''_t + (p_1 - p_0) \cdot \left( X''_t \cdot E[\mathbb{1}_{T_t^\delta \leq X''_t}] - E[T_t^\delta \cdot \mathbb{1}_{T_t^\delta \leq X''_t}] \right) \\
 &= p_0 \cdot X''_t + (p_1 - p_0) \cdot X''_t \cdot \left( \text{Pr}(T_t^\delta \leq X''_t) - E[T_t^\delta \cdot \mathbb{1}_{T_t^\delta \leq X''_t}] \right) \\
 &= p_0 \cdot X''_t + (p_1 - p_0) \cdot \left( X''_t \cdot \sum_{x=1}^{X''_t} f(y) - \sum_{x=1}^{X''_t} x \cdot f(y) \right) \\
 &= p_0 \cdot X''_t + (p_1 - p_0) \cdot \sum_{x=2}^{X''_t} (X''_t - x) \cdot f(y) \\
 &= p_0 \cdot X''_t + (p_1 - p_0) \cdot \sum_{x=2}^{X''_t+1} B''_{t,x} \sum_{y=1}^{x-1} (x-1-y) \cdot f(y)
 \end{aligned}$$

where:  $B''_{t,x} = 1$  if  $X''_t = x - 1$  and 0 otherwise.

In other words, the proportion of defectives is expressed as the proportion of defectives when the process is in-control ( $p_0$ ) plus the proportion of defectives increment during out-of-control state multiplied by the expected duration of drift during the period.

Hence,

$$OQ_t = \frac{\text{ENDD}_t}{d_t} = \frac{1}{d_t} \left( X''_t \cdot p_0 + (p_1 - p_0) \cdot \sum_{x=2}^{X''_t+1} B''_{t,x} \sum_{y=1}^{x-1} (x-1-y) \cdot f_{T_t^\delta}(y) \right) \tag{4}$$

□

### 2.2 Integrated model formulation

The network flow model and the associated decision variables are sketched out in Figure 2. We distinguish three different types of decisions. The first type corresponds to decisions related to production, i.e. the quantities to produce and quantities to stock. In the second type, the decisions concern the sampling of products to be inspected in the current period or later, that is why this model assumes the existence of a stock of sampled products. In the inspection decisions, the quantities of products to inspect and to stock are determined. This model proposes the possibility to take into account an inspection lead time  $L$ .

The model presented in Figure 2 is used to evaluate and to analyse a common quality assurance strategy. This strategy is based on a specific agreement between the customer and manufacturer which specifies that each lot delivered to customer with a proportion of defectives that exceeds an AQL is returned to manufacturer, who inspects the entire lot, replaces defectives and redelivers a 100% inspected lot (defect free) to the customer. The customer assumes all defectives of the lot if his input quality control system reveals a proportion of defectives less than or equal to AQL. In order to optimise the average unit cost (AUC), the manufacturer's production planning should balance between the COQ control and the risk of lots' return and their induced costs. Based on the model of Figure 2, the integrated planning of production and quality optimisation problem can be formalised as follows:



$$\text{Minimise } TC = \sum_{t=1}^T \left[ c_t^p \cdot (X_t + R_t) + c_t^s \cdot Y_t + c_t^h \cdot (I_t + I_t' + I_t'') + c_t^i \cdot (Z_t + W_t \cdot d_t) \right] \quad (5)$$

subject to

$$\begin{aligned} I_{t-1} + X_t - I_t - X_t' - X_t'' &= 0 & \forall t = 1 \dots T & \quad (6) \\ I_{t-1}' + X_t' - I_t' - Z_t &= 0 & \forall t = 1 \dots T & \quad (7) \\ I_{t-1}'' + Z_{t-L} + X_t'' - I_t'' &= d_t & \forall t = 1 \dots T & \quad (8) \\ X_t - M_t \cdot Y_t &\leq 0 & \forall t = 1 \dots T & \quad (9) \\ OQ_t - W_t &\leq \text{AQL} & \forall t = 1 \dots T & \quad (10) \\ d_t (OQ_t - \text{AQL} \cdot (1 - W_t)) - R_t &\leq 0 & \forall t = 1 \dots T & \quad (11) \\ X_t &\leq \text{PCAPA}_t & \forall t = 1 \dots T & \quad (12) \\ Z_t &\leq \text{ICAPA}_t & \forall t = 1 \dots T & \quad (13) \\ X_t, X_t', X_t'', I_t, I_t', I_t'', R_t, Z_t, OQ_t &\geq 0 & \forall t = 1 \dots T & \quad (14) \\ B_{t,x}, Y_t, W_t &\in \{0, 1\} & \forall t = 1 \dots T & \quad (15) \end{aligned}$$

where,  $\forall t = 1 \dots T$ :  $OQ_t$  is defined by (4), and

$$f_{T_t}^h(x) = \begin{cases} \frac{0.07}{0.25 \cdot \text{PCAPA}_t} & \text{if } 0 < x \leq 0.25 \cdot \text{PCAPA}_t \\ \frac{0.1}{0.25 \cdot \text{PCAPA}_t} & \text{if } 0.25 \cdot \text{PCAPA}_t < x \leq 0.5 \cdot \text{PCAPA}_t \\ \frac{0.3}{0.25 \cdot \text{PCAPA}_t} & \text{if } 0.50 \cdot \text{PCAPA}_t < x \leq 0.75 \cdot \text{PCAPA}_t \\ \frac{0.5}{0.25 \cdot \text{PCAPA}_t} & \text{if } 0.75 \cdot \text{PCAPA}_t < x \leq \text{PCAPA}_t \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

The objective function in (5) is composed of production costs, set-up costs, inventory holding costs and inspection costs, respectively. The production cost of each period includes those of replaced items when a lot is returned from customer. The holding costs are induced by the stocks of products that are delivered directly to customer, the stocks of products sampled that are to be inspected in the subsequent periods and the stocks of products inspected (defect free) that are to be delivered in the subsequent periods. The inspection costs of each period include those of the quantity delivered if the lot is rejected by the customer. Constraints (6)–(8) correspond to flow conservation constraints for production, sampling and inspection steps, respectively. The binary decision variables corresponding to set-up and lot rejection by the customer are related to quantity produced and the outgoing quality through constraints represented by Constraints (9) and Constraints (10), respectively. Constraints (11) determines the minimum quantity to be replaced. Inequalities (12)–(13) correspond, respectively, to production and inspection capacity constraints. The outgoing quality of each period is estimated using Equation (4). The probability mass function of the drift time (expressed in number of items produced) is defined empirically in Equation (16). In this paper, this function has been set so that it corresponds to an increasing drift rate, where the more the production machine is loaded, the more is the number of defective items produced. Nevertheless, any other valid probability mass function can be used. Finally, non-negativity and integrality conditions are defined by Constraints (14) and (15).

**2.3 Reference model formulation**

The reference model (MODEL 2, in Figure 1) is represented by the network in Figure 3. This model is similar to the representation of Wagner–Whitin model (Wagner and Whitin 1958) proposed in Zangwill (1969). It is further extended with the consideration of production capacity constraints and an extra cost associated with the reproduction of non-conform products that were returned by the customer.

The MILP formulation of the model is as follows:

$$\text{Minimise } TC^0 = \sum_{t=1}^T \left[ c_t^p \cdot (X_t + R_t) + c_t^s \cdot Y_t + c_t^h \cdot I_t \right] \quad (17)$$

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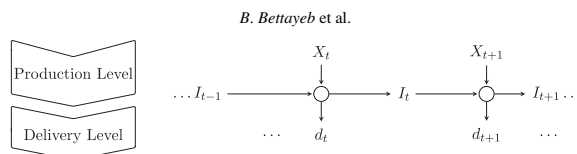


Figure 3. MODEL 1 network flow model and decision variables.

Subject to

$$I_{t-1} + X_t - I_t = d_t \quad \forall t = 1 \dots T \tag{18}$$

$$X_t - M_t \cdot Y_t \leq 0 \quad \forall t = 1 \dots T \tag{19}$$

$$X_t \leq PCAPA_t \quad \forall t = 1 \dots T \tag{20}$$

$$X_t, I_t, R_t \geq 0 \quad \forall t = 1 \dots T \tag{21}$$

$$Y_t \in \{0, 1\} \quad \forall t = 1 \dots T \tag{22}$$

where,  $\forall t = 1 \dots T$ :

$$R_t = OQ_t \cdot d_t = ENDD_t = d_t \cdot p_0 + (p_1 - p_0) \cdot \sum_{x=1}^{d_t} (d_t - x) \cdot f_{T^{\beta}} \left( \frac{x}{PCAPA_t} \right) \tag{23}$$

In this model, the objective function is composed of the set-up costs, the production costs, the inventory holding costs and the cost of replaced items. The latter correspond to the defective items delivered, and then returned by the customer at each time period,  $R_t$ , which is calculated using Equation (23). Actually,  $R_t$  and  $OQ_t$  are not decision variables in this model and can be calculated beforehand. Constraints (18) are the inventory balance equations. The link between binary set-up variables  $Y_t$  and production variables  $X_t$  is set in Constraints (19). Inequalities (20) are capacity constraints. Finally, Constraints (21) and (22) are non-negativity and integrality constraints. Notice that Constraints (19) and (20) are usually replaced with Constraints  $X_t \leq \min(M_t, PCAPA_t) \cdot Y_t$ .

It is easy to show that these two problems are NP-hard at least in the weak sense. It has as a special case the capacitated single item lot-sizing problem which known to be NP-hard in the weak sense in general. It might be interesting to think about efficient algorithms to solve them. However, our objective in this paper, as mentioned before, is to show the interaction between quality inspection decisions and different cost and process characteristics in the context of dynamic demand production planning decisions.

### 3. Experiments and discussions

The models were implemented and solved using FICO Xpress solver (FICO 2016) and all numerical tests were carried out on a personal computer with Intel Core i5-2.6 Ghz CPU with 8 GB RAM.

#### 3.1 Experiments setting

We will see the correlation between average outgoing quality (AOQ) and different parameters such as level of customer quality requirements, inspection capacity and inspection lead time. We will also check when the integrated approach becomes more interesting than the non-inspection policy with respect to the same previously mentioned parameters.

Our datasets are modified from a subset of instances from Brahim and Dauzère-Pères (2015) where only instances with  $T = 24$  periods were used.

The complete list of parameters that were used in the experiments is presented in Table 2. Each parameter is drawn from a set of values and has a default value shown in bold. For each combination of parameter values, we calculate the average over five randomly generated instances.

In Table 2, production capacity was chosen to be tight ( $1.05 \times$  total demand), normal and loose ( $2.0 \times$  total demand). Inspection capacity is chosen to be equal to 10, 50, 70 and 100% of production capacity. This is in order to represent a variety of types of processes as it is well known that inspection capacities can vary considerably from one industry to another. In the

Table 2. Ranges of the parameters.

ID	Parameter name	Range
PCapaR	Production Capacity Ratio (relative to demand)	{1.05, <b>1.2</b> , 2}
ICapaR	Inspection Capacity Ratio (relative to production capacity)	{0.1, 0.3, 0.5, 0.7, <b>1</b> }
ICostR	Inspection Cost Ratio (relative to production cost)	{0.1, 0.5, <b>1</b> , 3}
$p_0$	Proportion of defectives when process is in-control	{0.005, <b>0.03</b> , 0.15}
$p_1$	Proportion of defectives when process is out-of-control	{ <b>2</b> $p_0$ , 5 $p_0$ , 1}
AQL	AQL	{0.005, 0.01, 0.025, <b>0.03</b> , 0.05, 0.06, 0.07, 0.1, 0.15, 0.3, 0.75, 1}

same way, inspection cost varies and depends on the type and technique of inspection (e.g. destructive vs. non-destructive inspections). The choice of values for  $p_0$  reflects different levels of a process capability, where  $p_0 = 0.005$  corresponds to a highly capable process, while  $p_0 = 0.15$  means that the process is not capable in the sense that either it has a design problem or it is operating in its early learning phase. The values of  $p_1$  are used to represent stability levels of the process from a stable state ( $p_1 = 2p_0$ ) to an unstable state ( $p_1 = 1$ ). It is proportional to the magnitude of drift from in-control to out-of-control states. This drift is usually due to what is called assignable causes of failure. Finally, we evaluated the model with a large spectrum of values for customer quality requirement (AQL). Though we know that, in practice, AQL = 0.3 is already a fairly large value, we have tested larger values (AQL = 0.75 and AQL = 1.0).

The other parameters are production costs which are drawn from a uniform distribution in [16, 20], inventory holding costs drawn from [3, 4] and production set-up costs which are equal to 200 times the inventory holding costs.

### 3.2 Performance indicators

As mentioned in the previous section, we are interested in analysing the performance of the integrated model based on three indicators: AOQ, COQ and AUC. This performance is considered from the manufacturer's point of view and it compares our integrated approach with a no-inspection policy as a base case. The latter policy supposes that no inspection is made by the manufacturer and that the customer returns all defective items in each delivered lot which are replaced by the manufacturer who assumes the induced production cost.

### 3.3 Average outgoing quality

The average outgoing quality level AOQ(%) is defined as:

$$AOQ(\%) = 100 \times \frac{1}{T} \sum_{t=1}^T OQ_t \quad (24)$$

where  $OQ_t$  is the outgoing quality level at period  $t$  associated with a given solution, in other words, the proportion of non-conform units in the delivery of a given period. The AOQ is a well-known indicator that represents the average level of quality delivered to customers.

Figure 4 shows the evolution of AOQ according to inspection capacity rate (ICapR). The three upper curves show this evolution for production capacity rates equal to 1.05, 1.2 and 2.0 times the demand. The lower curve corresponds to the AQL, i.e. the maximum AOQ acceptable for the customer. We can see that AOQ is correlated to the capacity allocated to the inspection operations. This result is not surprising if we consider that the more inspection capacity is allocated, the more inspection operations can be done and, consequently, the higher is the outgoing Quality.

Figure 4 can be used to determine the minimum ICapR required for the supplier to ensure that the AQL is met. This corresponds to intersection point with AQL curve. For example, when PCapaR = 2, the inspection capacity must be at least 20% of the production capacity.

Figure 5 represents, with respect to process stability and capability, the impact of AQL on AOQ while minimising the costs in the MILP formulation. For example, in Figure 5(a), we plot three curves of AOQ corresponding to three process capability levels ( $p_0 = 0.005$ ,  $p_0 = 0.03$ ,  $p_0 = 0.15$ ), in the case of a highly stable process. The straight line (AOQ = AQL) corresponds to the level required by the customer. In other words, being below the straight line means that customer quality requirements are met or exceeded, otherwise, quality requirements are not met. Figure 5(b) is a zoom on lower values of AQL. A first result of this experiment is that if AQL is too low (below 1%, in our case), from an economic point of view, it

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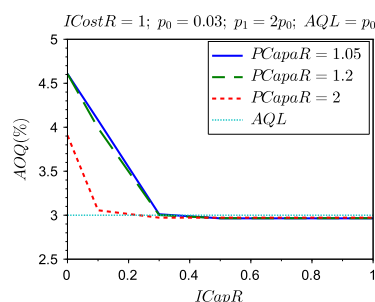


Figure 4. Effects of production and inspection capacities on AOQ.

is no more interesting to reach this customer requirement. Actually, it becomes more interesting for the supplier to assume the cost of returns rather than investing in more inspection capacity.

For AOQ curve segments overlapping with the straight line ( $AOQ = AQL$ ), the optimal cost allows us to meet customer requirements. Once we are below the  $AOQ = AQL$  line, it means that no more inspection (consequently, no inspection costs) are needed and the capability of the process is enough to meet the AQL of the customer. The breakpoint at which no more inspection is required depends both on process capability ( $p_0$ ) and its stability (represented by  $p_1$ ) as shown on Figure 5(c)–(f). The higher the capability of the process ( $p_0$  low), the lower the value of the breakpoint. Also, the breakpoint is higher with lower stability processes (Figure 5(e) and (f)) compared to higher ones.

### 3.4 Average unit cost

The AUC is obtained by dividing the total cost over the total demand  $AUC = TC / \sum_{t=1}^T d_t$ . Notice that the AUC is a well-known indicator in production planning and is the basis of the heuristic Least Unit Cost in lot sizing (Silver, Pyke, and Thomas 2017). In our case, quality costs are also included in the AUC together with logistics costs.

The indicator analysed in this section is the relative variation of the optimised AUC  $\Delta AUC(\%)$  compared to a ‘No-Inspection’ policy. It is given by:

$$\Delta AUC(\%) = 100 \times \frac{AUC - AUC^0}{AUC^0} \quad (25)$$

where  $AUC^0$  is the optimum AUC in ‘No-Inspection’ policy. It is worth noting that  $\Delta AUC$  mainly translates the cost of inspections. Indeed, the total production, set-up and inventory holding costs are approximately the same for the two policies. However, the no-inspection policy induces returns costs which are just correlated to the capability and stability of the process. The integrated policy considers the trade-off between inspection cost and returns cost given the value of AQL. That is, if quality requirement is much lower than process capability, then even if there is no inspection, the delivered lots contain inherent non-conform units but at a level which is still acceptable for the customer (based on the service contract negotiated with the supplier).

Figure 6 shows the impact of the level of production process control on the trade-off between the AQL and the AUC. The gap  $\Delta AUC(\%)$  between the AUCs of inspection and no-inspection strategies is plotted as a function of the average required quality level (AQL). The analysis is done for highly stable processes ( $p_1 = 2p_0$  and  $p_1 \leq 0.3$ ) on Figure 6(a), for fairly stable processes ( $p_1 = 5p_0$  and  $p_1 \leq 0.75$ ) on Figure 6(b), and unstable processes ( $p_1 = 1.0$ ) on Figure 6(c).

A first remark on these three figures is the AUC of the inspection policy decreases with AQL and becomes smaller than or equal to the no-inspection policy cost after a certain value of AQL.

Focusing on Figure 6(a), we notice that the lower the capability of the process, the higher the breakpoint at which inspection and no-inspection policies become economically equivalent. For example, in the case of a highly capable process ( $p_0 = 0.005$ ), the breakpoint is for  $AQL \approx 1\%$  while it corresponds to  $AQL \approx 17\%$  for a process with lower capability. In the capable process, the difference between costs of no-inspection and integrated policies after the breakpoint is much smaller than for lower capability processes. This is due to the fact that in a capable process, there are very few returns (conseq. return

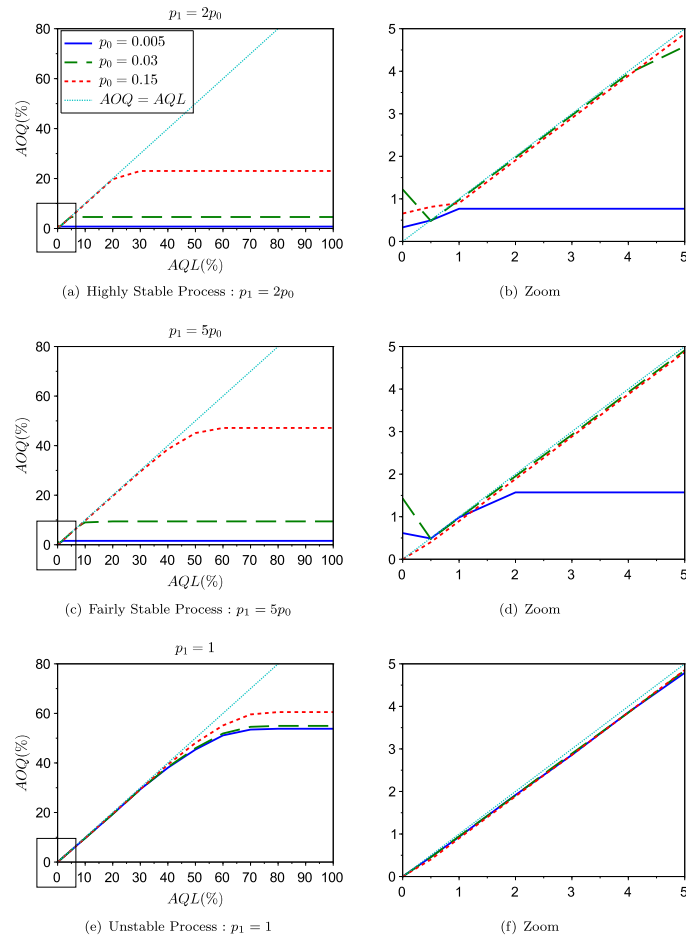


Figure 5. Effects of AQL and Process Stability on the AOQ.

costs) even with the no-inspection policy. And when the customer requirements are very high ( $AQL \leq 1\%$ ), the inspection costs become very high and hence a no-inspection policy would be less expensive in this case because of the very low level of returns.

In the case of a low capability process, once the breakpoint is reached, very few inspections (if any) are required while there will be high levels of returns from the customers in the case of a no-inspection policy. This explains the large gap between the costs of the two policies.

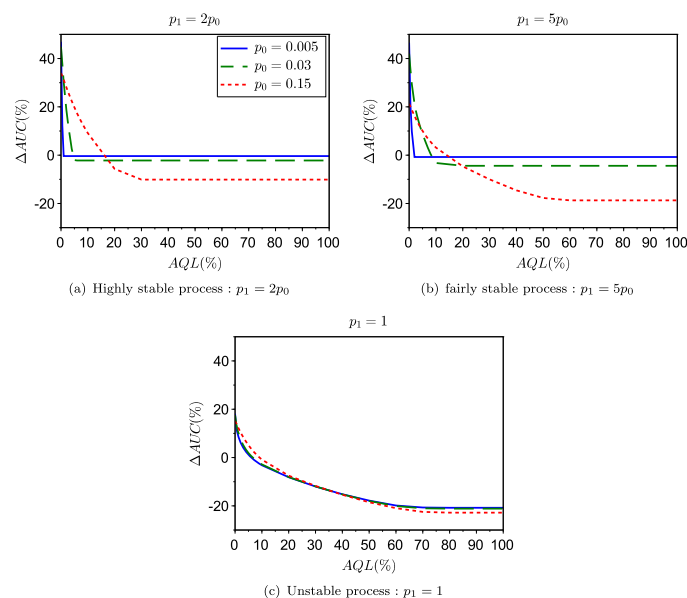


Figure 6. Effects of AQL and inspection cost on  $\Delta AUC(\%)$ .

With respect to stability, we compare the three Figure 6(a), (b) and (c). We notice that as stability decreases, the gap between inspection and no-inspection policies becomes higher for larger values of AQL. Furthermore, when the process is unstable, the curves of the three capability levels almost overlap. This is explained as follows. In a no-inspection policy with an unstable process, the amount of non-conform units and returns is very high. Moreover, in an unstable process, the inspection cost is close or higher than the cost of returns when AQL is low (high customer requirements), which means that, in the inspection policy, it is more interesting to avoid inspection costs at all even for low AQL values.

This analysis complements the analysis on the AOQ in the previous section. It was mentioned that it is economically not interesting to meet customer requirements when they are too high and the process is highly stable and capable. On the other hand, there is no difference in processes of different capabilities if they are unstable, in the sense that it is economically interesting to meet customer requirements even if they are very high.

In Figure 7, we analyse the effect of AQL on the AUC for different values of inspection cost (inspection cost ratio, ICOSTR) and for high, medium and low capability process levels on Figure 7(a), (b) and (c), respectively. In terms of stability, the process is supposed to evolve from an unstable state to a stable state ( $p_1 = 1 \rightarrow p_1 = 5p_0 \rightarrow p_1 = 2p_0$ ). It is also assumed that the process stays the same amount of time in each of these states.

Regardless of capability level, one can notice on the Figure 7(a), (b) and (c) that, while unit inspection cost decreases as customers requirements decrease, the pace of this decrease is much higher when process capability is high (Figure 7(a)) compared to medium and low capabilities (Figure 7(b) and (c)). It is also interesting to notice that, in general, decreasing customer quality requirements has very small impact on AUC when the inspection cost is small ( $ICostR = 0.5$  and  $ICostR = 0.1$ ). This is explained by the fact that AUC is highly dependent on the unit inspection cost.

For each inspection cost, one can identify the minimum AQL value ( $AQL^{\min}$ ) beyond which it becomes profitable to use the integrated inspection policy. For example, For a highly capable process,  $AQL^{\min}$  is approximately equal to 0, 1%, 4%, and larger than 30%, for inspection cost rates of 0.1, 0.5, 1 and 3, respectively.

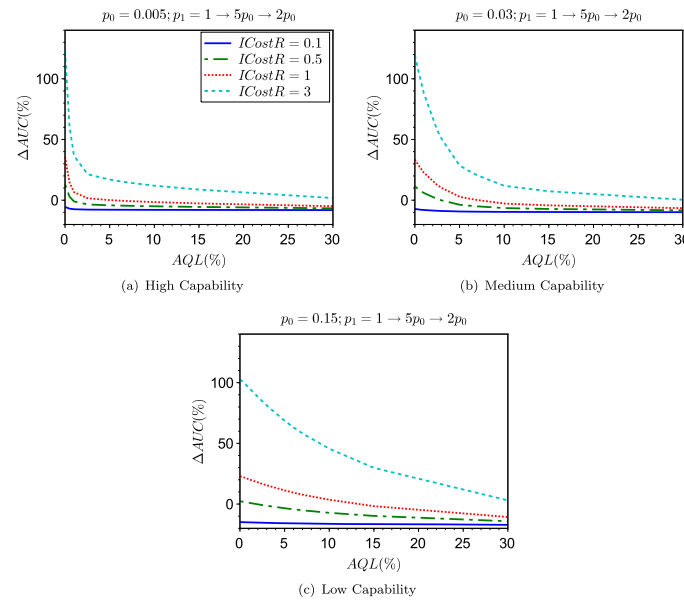


Figure 7. Effects of AQL and inspection cost on  $\Delta AUC(\%)$ .

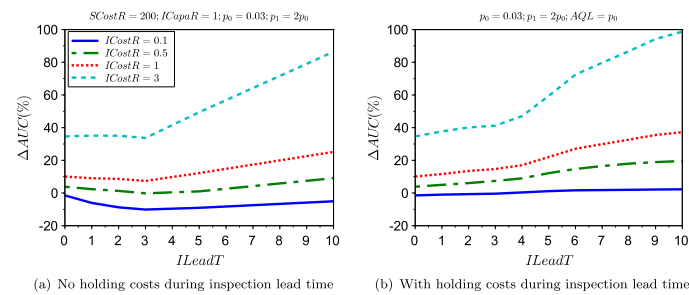


Figure 8. Effects of inspection lead time and inspection cost on the AUC ( $\Delta AUC(\%)$ ).

Figure 8(a) and (b) show the impact of lead time on the AUC. Figure 8(a) was built in the beginning of the numerical tests. One can see that  $\Delta AUC$  decreases and then increases with the inspection lead time. This phenomenon can be explained by two reasons: firstly, no inventory holding costs were considered for sampled units during lead time, which is not realistic. Secondly, the demand has a particular structure in the used instances. In fact, the demands of first early periods in the planning horizon were set to zero (Brahimi and Dauzère-Pères 2015) as it is the case in most lot-sizing instances. Hence, the early periods are used in this case to stock products 'no cost'.

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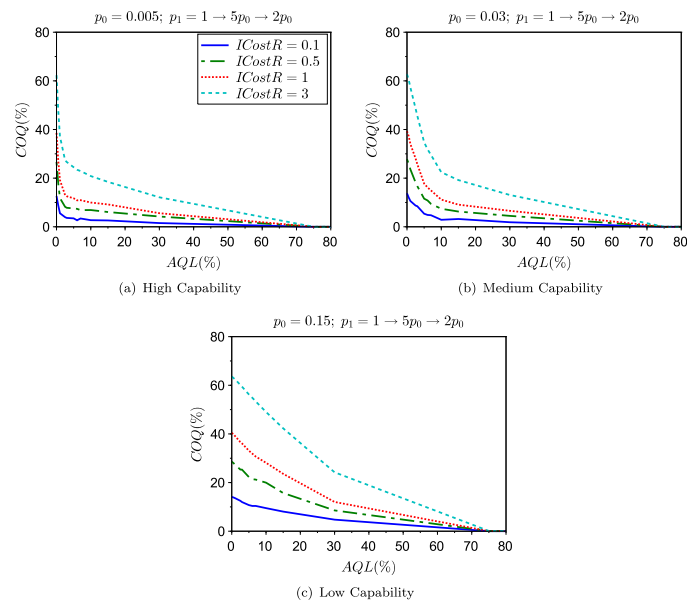


Figure 9. Effects of AQL and inspection cost on the COQ%.

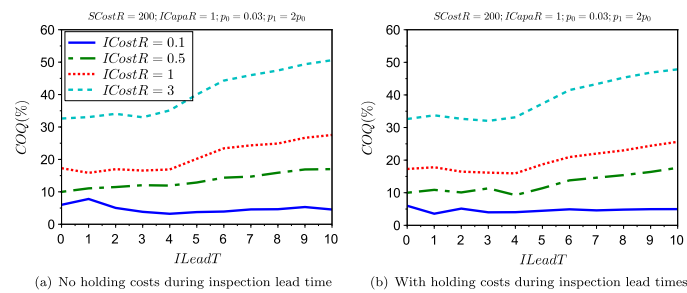


Figure 10. Effects of AQL and inspection cost on the COQ%.

New experiments were carried out with consideration of holding cost of sampled units during lead time (Figure 8(b)). These results are more realistic where increased lead time implies higher costs. This is more visible when the inspection costs are much higher than production costs (ICostR = 3) and thus, even higher than the inventory holding cost.



### 3.5 Cost of quality

We define COQ the proportion of quality cost in the AUC. It includes inspection cost and inventory holding cost of sampled and inspected products.

Figure 9 shows the COQ vs. customer requirement (AQL) and for different levels of process capabilities. Similar to the experimental results in Figure 7, different values of inspection cost (inspection cost ratio, ICosTR) and for high, medium and low capability process levels on Figure 7(a), (b) and (c), respectively. In terms of stability, the process is supposed to evolve from an unstable state to a stable state ( $p_1 = 1 \rightarrow p_1 = 5p_0 \rightarrow p_1 = 2p_0$ ). It is also assumed that the process stays the same amount of time in each of these states.

Figure 9 supports the results presented on Figure 7. It shows that the trend of AUC is mostly due to the COQ.

It is worth noting that inspection takes place even for very high values of AQL. This is due to the fact that the results are averaged over the three levels of stability.

In the same way, we analysed  $\Delta AUC$  as a function of lead time in Section 3.4, we can see on Figure 10 that the variation of COQ as a function of lead time has the same behaviour as  $\Delta AUC$ , showing again that the total cost is mostly explained by the COQ.

## 4. Conclusion and perspectives

The objective in production planning is to find the trade-off between cost, customer service level and quality. However, production planning models (lot-sizing models) are limited to the consideration of the trade-off between cost and service level. In this paper, we proposed a new model whose aim is to integrate quality consideration into a production planning problem, the single item lot-sizing problem with finite capacity. This model has been tested on several instances and the obtained results show that it is very important to take into account the inspection process into production planning decisions. Through this study, we proposed several managerial insight including in particular:

- Even if inspection cost is negligible, more inspections is no more profitable (over-quality) starting from a given inspection capacity threshold which depends only on other production parameters (production capacity, set-up costs, etc.)
- When quality requirements are extremely high, it can be economically more interesting for the supplier to assume the cost of some returns rather than investing in more inspection capacity.
- It could be more profitable for supplier to invest in process improvement to make it more capable and stable instead of investing in more inspection capacity.
- In particular cases, customer required quality levels are so costly that it becomes irrelevant to satisfy them.

Another conclusion from this study is that process capability and stability are other parameters, besides process capacity, that should be taken into account in the production planning decisions.

The role of the capability of the production process is highlighted in order to meet the quality required by customers. Our results show the relevance of such integration in order to take into account the non-quality effect due to the drift of non-well mastered production operations, in the production planning process.

In their future work, authors project to enhance the proposed model by addressing the following issues. Firstly, as the current model assumes that the returned lots are inspected and the defectives are replaced instantaneously without consuming production capacity, nor inspection capacity, it will be more realistic, and probably more challenging, to take into account this feedback which leads to a model allowing backlogs and deliveries' lead times. Secondly, it is also projected to focus on the case with variable inspection lead time and its effects on quality control (outgoing quality) and on total costs. In fact, in some manufacturing systems the inspection stations are, not only of limited capacity, but also associated with queuing systems which delay the inspection output and therefore the detection of eventual out-of-control state. Finally, it will be interesting to study the case with multiple products with different industrial life cycle phases (ramp-up, maturity and decline) where the manufacturing system's maturity regarding each product evolves during time, dependently on the inspection intensity. These phenomena are of great importance in high-mix and low-volume production like in semi-conductor industry.

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**Disclosure statement**

No potential conflict of interest was reported by the authors.

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## A dynamic lot-sizing-based profit maximization discounted cash flow model considering working capital requirement financing cost with infinite production capacity



Yuan Bian<sup>a,\*</sup>, David Lemoine<sup>a</sup>, Thomas G. Yeung<sup>a</sup>, Nathalie Bostel<sup>b</sup>, Vincent Hovelaque<sup>c</sup>, Jean-Laurent Viviani<sup>c</sup>, Fabrice Gayraud<sup>a</sup>

<sup>a</sup> LS2N, UMR CNRS 6004, IMT Atlantique, 4, Rue Alfred Kastler, B.P. 20722, 44307 Nantes, France

<sup>b</sup> LS2N, UMR CNRS 6004, University of Nantes, 58 rue Michel Ange, B.P. 420, 44606 Saint-Nazaire, France

<sup>c</sup> Univ Rennes, CNRS, CREM-UMR 6211, F-35000 Rennes, France

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### ABSTRACT

In times of crisis, companies need free cash flow to efficiently react against all uncertainty to ensure solvency. However, classical dynamic lot-sizing models only consider the physical flow of goods. In this paper, we introduce a first link between dynamic lot-sizing and the financial aspects of working capital requirements (WCR). We propose a new generic WCR model which allows us to evaluate the company's financial situation throughout the planning horizon. Moreover, a dynamic lot-sizing-based, discounted cash flow model is established for single-site, single-level, single-product and infinite capacity cases. It is shown that the zero-inventory ordering property holds for this case and thus a polynomial-time algorithm may be utilized. Numerical tests are presented in order to show the relevance of our approach compared with the traditional dynamic lot-sizing model.

### 1. Introduction

According to the Ernst & Young annual Working Capital Management (WCM) report of 2012 devoted to the leading 1000 US companies in year 2011, on average, \$330 billion dollars of Net Working Capital (NWC) is unnecessarily immobilized (see in Ernst (2012)). This range of cash opportunity corresponds to, respectively, between 3% and 6% of their aggregate sales. Buchmann et al. (2008) also stress that savings on Working capital Requirement (WCR) as a potential source of cash to fund growth is often neglected by companies. Suboptimal WCM not only reduces potential gains, it also raises companies' risk. A company needs to carefully manage its WCR in order to ensure financial liquidity or reduce insolvency risk. Additionally, optimal WCM can unlock internal capital and provide financial resources to financially constrained firms. Especially during the last financial crisis, bank loans were extremely difficult to obtain by companies, especially those in the development phase (see in Wu et al. (2016)). Many companies suffer from lack of credit and insufficient working capital. Small suppliers are forced to accept unfavorable payment terms from their customers, exacerbating their financial situation. Moreover, tight or unavailable bank credit reduces the working capital level of these companies. As a result, some companies must

suspend their operations which can disrupt the whole supply chain as reported in Benito et al. (2010) and Ernst (2010).

In spite of its importance for firm (and more generally the supply chain) performance and survival, WCM has been neglected by the literature on production planning. More generally, as stated by Birge (2014), "Operations management models typically only consider the level and organization of a firm's transformation activities without considering the financial implications of those activities". The separation of the operational decision (minimizing operational costs or maximizing operational profits) and financial decision (minimizing financial costs or maximizing financial profit) was theoretically grounded on the famous Modigliani and Miller (1958) Theorem and practically by the functional organization of companies. However, in recent years, the literature on operations and supply chain management, following the original work of Babich and Sobel (2004), became aware of the fact that financing and operational problems are imbricated and that optimizing the two dimensions jointly could improve the global performance of the company as shown in Chen et al. (2014). A recent review of this literature based on a risk management framework has been proposed by Zhao and Huchzermeier (2015). Nevertheless, in the production planning field, the majority of models typically ignore the financial consequences of production planning

\* Corresponding author.

E-mail address: [yuan.bian@vip.163.com](mailto:yuan.bian@vip.163.com) (Y. Bian).

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decisions. In an attempt to fill this gap, the aim of the paper is to explicitly introduce the financial dimension in a tactical production planning model.

Traditionally, tactical production planning aims at determining the production quantity for each period of the planning horizon. It is often known for its strong influence in terms of customer service quality and production related operation costs. Two main classes of models distinguish the problem based on continuous/discrete time considerations: for example, with infinite capacity, the Economic Order Quantity (EOQ) model of [Harris \(1913\)](#) for constant demand over continuous time and the Uncapacitated Lot-Sizing model (ULS) of [Wagner and Whitin \(1958\)](#) for time-varying demand over discrete time. Readers can refer to the literature review of [Brahimi et al. \(2017\)](#) for single item cases and of [Comelli et al. \(2006\)](#) for model classification. However, production planning is determined following the well-known Manufacturing Resource Planning (MRP II) logic for a medium-term objective. As indicated by [Helber \(1998\)](#), this decision does not reflect difference in Net Present Value (NPV) of cash flow due to planned operations in real world market system. Moreover, financial aspects are rarely considered in dynamic lot-sizing literature.

In order to better take into account the financial dimension in a production planning context, we first replace the total operational cost minimization objective by the overall financial objective of the firm, which is the wealth maximization objective. The total wealth represents the sum of the net present values of all future cash flows. This remains the main stream objective in corporate finance as stated in [Jensen \(2001\)](#). The importance of this objective has also been recognized in the production planning literature (see [Serrano et al. \(2017\)](#)).

Second, we introduce a joint management of both physical and financial flows within a discounted cash flow-based dynamic LSP model. We consider both classical operational costs (e.g., purchasing, setup, production) and financial costs. More specifically, financial costs are generated by the need to finance the Working Capital Requirement (WCR) induced by operational decisions. In practice, the WCR is known as a key indicator to monitor and control a company's financial situation. The WCR is mainly caused by the timing mismatch between the cash inflows and the cash outflows associated with the total production and commercial cycle as presented in [Theodore Farris and Hutchison \(2002\)](#). According to the cash conversion cycle methodology proposed by these authors, the free cash flow level can be approximately measured as the difference between the working capital and WCR. As a consequence, a lower operating WCR gives the company better liquidity and helps to ensure solvency. By this direct link with cash flow, the WCR increasingly draws the attention of finance departments in a financial crisis context. By introducing the financing cost of the WCR, the traditional trade-off between set-up costs and inventory costs is therefore modified. If the company orders lots of bigger size, it will not only increase inventory costs, but also the WCR, leading to higher financial costs.

The main contributions of this paper are the following:

- First, we propose a WCR model adapted to the production planning context. This allows us to precisely measure the WCR during the planning horizon;
- Second, we demonstrate that the Zero-Inventory-Ordering property remains valid in this problem. This ensures a strong structural property of the optimal solution and its exact resolution in polynomial time. Moreover, it shows no additional computational complexity by introducing the financial cost into the optimization objective;
- Finally, we show significant differences between optimal production planning obtained by the classic dynamic lot-sizing model and our proposed model.

This remainder of this paper is organized as follows: In Section 2, related works that consider a financial aspect in tactical production planning and profit maximization models are reviewed. In Section 3, we propose a generic model of operating WCR in a tactical production

planning context. We then develop a profit maximization model based on a classic dynamic LSP model by using the OWCR model for the single-product with infinite capacity case. In Section 4, the Zero-Inventory-Ordering property is proven and a dynamic programming algorithm is consequently established. In Section 5, we analyze the differences of both the optimal production planning of the proposed model and the classical dynamic lot-sizing model in terms of production planning. We end with a short conclusion and show insights for further research.

## 2. Literature review

The management of financial flow is an important part of supply chain management (SCM) as shown in [Cooper et al. \(1997\)](#) and [Vidal and Goetschalckx \(2001\)](#). However, formal analytical models have only recently become available in [Raghavan and Mishra \(2011\)](#) and [Liu and Cruz \(2012\)](#). They aim to understand how the planning, management and control of financial flows in supply chains affect the chain profitability. In addition, inventory management models have been reformulated to take into account financial considerations such as trade credit, delay in payment or financial risks, as in [Aggarwal and Jaggi \(1995\)](#), [Berling and Rosling \(2005\)](#) and [Gupta and Wang \(2009\)](#). Moreover, [Protopappa-Sieke and Seifert \(2010\)](#) elaborate on the interest of considering decision making in both operational and financial levels for the physical and financial supply chain by determining the optimal purchase order quantity. In addition, [Zeballos et al. \(2013\)](#) explain the evolution of the Modigliani and Miller Theorem in the field of inventory management and propose a single-product, finite horizon model that considers the financial aspects of working capital constraints, payment delays and short-term debts. These studies highlight the fact that in supply chain management, physical flow and financial information must be managed jointly.

We consider WCR in tactical production planning because working capital management (WCM) has been identified as an important indicator for measuring supply chain performance by [Timme and Williams-Timme \(2000\)](#). The influence of WCM on a company's performance and profitability have been confirmed by several national studies by [Ding et al. \(2013\)](#), [Enqvist et al. \(2014\)](#), [Vahid et al. \(2012\)](#) and [Bei and Wijewardana \(2012\)](#) and in different industrial sectors, e.g., the restaurant and automotive industry in [Lind et al. \(2012\)](#) and [Mun and Jang \(2015\)](#), respectively. In addition, [Kieschnick et al. \(2013\)](#) report that for the average firm the incremental dollar invested in net working capital is worth less than the incremental dollar held in cash. Since free cash flow level is the difference between working capital level and the WCR level, if the working capital remains constant, the minimum level of WCR represents the maximum level of free cash flow. According to the accounting formula of WCR in finance, delays in payment both from the customer and to the supplier significantly influence the level of WCR. In practice, such a delay is often accepted by the supplier to the customer in the form of trade credit.

Furthermore, as a financial consideration, the time value of money has drawn significant attention for building cash flow analysis models, namely the Discounted Cash Flow (DCF) model. The Net Present Value (NPV) metric is adopted to measure the present value of future cash flow in order to calculate the profitability of a projected investment or project. Since the work of [Trippi and Lewin \(1974\)](#), the time value of money aspect has been largely addressed based on the classic EOQ model. More related works are discussed in reviews of [Taleizadeh and Nematollahi \(2014\)](#) and [Martínez-Costa et al. \(2014\)](#). In the single-site dynamic lot-sizing literature, the DCF models can also be found since the work of [Helber \(1998\)](#) in various industry contexts. The author develops a cash-flow oriented multi-level lot-sizing model in MRP II (Manufacturing Resource Planning) system. Recently, [Lim et al. \(2013\)](#) establish a mathematical model in order to obtain the optimal product portfolio and the processing route of an integrated, resource-efficient rice mill complex with a profit maximization objective. [Mitra et al. \(2014\)](#) propose a Mixed Integer Linear Programming (MILP) formulation that combines the

operational and strategic decisions for continuous power-intensive processes under time-sensitive electricity prices.

In addition, the aspect of working capital has often been considered in term of capital or budget constraints in inventory management literature. Recently, Wang et al. (2007a) address the acquisition and allocation of LCD (Liquid-Crystal Display) manufacturing industries which requires a high amount of capital investments. In this problem, the authors included not only constraints of operational aspect, e.g., resource capacity constraints and backorder, but also financial aspects of finite budget and the time value of capital and asset. Wang et al. (2007b) propose a mathematical programming model for a generalized resource portfolio problem in a context of the semiconductor testing industry considering the financial resource constraints and the time value of capital. Chao et al. (2008) study the influence of loan limits on the operation decision in the EOQ context. Thomas and Bollapragada (2010) develop an analytical decision tool based on a capacity planning model to estimate product costs and demands in order to maximize the NPV of profit for General Electric. Lusa et al. (2012) develop a MILP formulation of an integral planning model with a maximum level of working capital constraints.

In this paper, therefore, we are interested in proposing a new generic operating WCR (OWCR) model in a tactical planning framework for a dynamic LSP model. For this reason, we propose a DCF-based profit maximization model considering the upstream and downstream payment delays and the financing cost of OWCR. A comparison between the aforementioned single-site dynamic lot-sizing-based models and our research is presented in Table 1. *MinC* stands for cost minimization objective and *MaxP* stands for profit maximization objective.

### 3. Discounted cash flow based profit maximization model

#### 3.1. OWCR modeling

The discounted cash flows method explicitly takes into account the time difference between the cash flows but it does not take into account the financial borrowing costs because these costs, in traditional models, are often supposed not to affect the optimal operational decision. As we challenge this assumption, we want to integrate only the financial costs linked to operational decisions in the cash flows (i.e., financing cost of OWCR). To do so, we propose a generic OWCR formulation in a tactical planning context. In this formulation, both the delay in payment to supplier and from client are taken into account and the time value of money is considered in order to calculate present values of incoming and outgoing cash flows. Furthermore, it should be possible to measure the level of inventory precisely over the planning horizon with this OWCR formulation as inventory holding costs make up a substantial portion of the OWCR.

We therefore adopt the cash to cash cycle methodology presented in Theodore Farris and Hutchison (2002) and Hofmann and Kotzab (2010) where the OWCR for producing a unit of product depends on the amount invested in the related operations (e.g., purchasing, setup, production and inventory holding) and the number of periods to finance these investments before recovering from corresponding sales. However, we first consider that the profit portion of the sales revenue is not available to cover the OWCR. This is because the company's profit margin does not correspond to the definition of working capital requirement since it has never been required. Furthermore, this assumption supposes that the company does not prioritize its profit allocation to WCR financing. Profit can be allocated to any of a number of objectives for the firm including debt reduction, internal or external investment, or dividend payments. Our model is a partial model of the company and we do not assume that allocation of profit to WCR financing rather than to other objectives (e.g., new investments) is optimal. Moreover, following the Modigliani Miller Theorem, a company should not allocate a specific financial resource (in this case, profit) to a specific object (in this case, WCR financing). In consequence, we adopt the scheme that the WCR generated by producing a product is only effectively recovered when that product is sold.

**Table 1**  
Comparison with single-site models in the dynamic lot-sizing literature.

Authors	Finite budget	Time value of money	Delay of payment	WCR	Objective
Helber (1998)		✓			MinC
Wang et al. (2007a)	✓	✓			MaxP
Wang et al. (2007b)	✓	✓			MaxP
Thomas and Bollapragada (2010)	✓	✓			MaxP
Lusa et al. (2012)	✓	✓			MaxP
Lim et al. (2013)		✓			MaxP
Mitra et al. (2014)		✓			MinC
This work		✓	✓	✓	MaxP

This assumption complicates the calculation of the financing duration of setup costs because it is paid in its entirety at a single point in time for the entire production lot. Thus, we must progressively and uniformly recover the setup cost from the sales revenue of all products in the same production lot. These products therefore share the fixed setup cost equally as the amount invested and the number of periods to finance is the time difference between the period when the setup occurs and the period of sale of each product.

#### 3.2. Assumptions

Since this model is built in the context of the ULS (Uncapacitated Lot-Sizing) problem, the OWCR formulation and the corresponding profit maximization model proposed in this paper are based on the following assumptions:

- Production:
  - No replenishment and production delays as they are negligible compared with period duration;
  - Demand should be met on time (no backlogging);
  - Only inventory of final product is considered;
  - Initial and final stock are defined as zero;
  - One unit of finished product is manufactured using one unit of raw material.
- Financial:
  - All logistic and financial costs are paid at the beginning of periods;
  - The profit from selling products is not be used for financing the OWCR;
  - The fixed setup cost of a given lot is progressively and uniformly recovered over the different periods during which the products of the corresponding lot are sold;
  - All holding costs paid for a product for all periods it remains in inventory are not recovered until it's sale.

#### 3.3. Physical and financial flow illustration

With the aforementioned assumptions, we express in detail in Fig. 1, the physical and financial flows generated by a production in period 1 for the demand of period 3. In this case, we assume the delays in payment from client and to supplier are both one period.

- Physical flows are presented in the top block. They are generated by decisions made in the operating cycle, such as purchase, production, inventory holding and delivery. Therefore, the physical flows link all decisions in the graphic with the single line arrow.
- The financial flows are displayed in the bottom block. First, the logistic costs are immediately generated by the physical activities presented by small squares in the financial block. Meanwhile, the WCR financing costs are also to be paid from the same period of corresponding logistic costs until the reception of revenue. These financial costs are presented with the double line arrow.

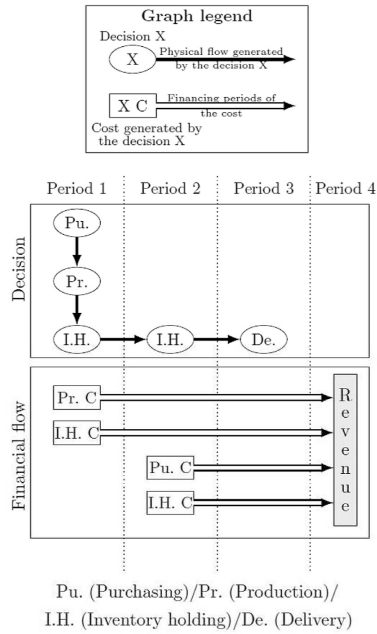


Fig. 1. Physical and financial flows illustration.

3.4. Parameters and decision variables

In this model, parameters and decision variables are adopted for the single-site, single-level, single-product with infinite capacity case. Their notations are given in Tables 2 and 3 respectively. We assume that these parameters are all nonnegative. We choose the decision variables of the FAL (Facility Location) formulation of the ULS. These disaggregated variables allow us to measure the exact amount of incoming and outgoing payments for all production related operations considered for calculating the OWCR.

3.5. Mathematical formulation

In our problem, OWCR includes all financial needs for the operations of purchasing, setup, production and inventory holding for satisfying a known, but variable demand in an operation cycle. The financial need of

Table 2 Parameters for OWCR modeling.

Parameter	Definition
$T$	Horizon length
$d_t$	Demand in period $t$
$v$	Unit product price
$h$	Unit inventory cost per period
$p$	Unit production cost
$s$	Fixed setup cost
$a$	Unit raw material cost
$r_c$	Delay in payment from client
$r_f$	Delay in payment to supplier
$\alpha$	Discount rate per period
$\beta$	Interest rate for financing OWCR per period

Table 3 Decision variables for OWCR modeling.

Variable	Definition
$Q_t$	Total production quantity in period $t$
$X_{k,t}$	Production quantity in period $t$ for satisfying (a part of) demand in period $k$
$I_t$	Product quantity held in inventory in period $t$
$Y_t$	Binary variable which indicates whether a setup occurs in period $t$ or not

each  $X_{k,t}$  is modeled as the product of the corresponding operation cost and the number of periods to finance.

- The operation cost is a function of the operation unit cost and the value of  $X_{k,t}$ . All unit costs of purchasing raw material, production and inventory holding are given as parameters. However, all products in an  $X_{k,t}$  share the setup cost. The setup unit cost per product is thus  $\frac{s}{Q_t}$ . However, when  $Q_t = 0$ , there is a problem of singularity. For this reason, it must be further reformulated as  $\frac{sY_t}{Q_t + (1 - Y_t)}$  to prevent this issue. Consequently, if  $Q_t = 0$ ,  $Y_t$  will then be zero. The denominator will be one. If  $Q_t > 0$ , then  $Y_t = 1$  which leads to  $\frac{sY_t}{Q_t + (1 - Y_t)} = \frac{s}{Q_t}$ . Nevertheless, due to this non-linear formulation, we should check that the Zero-Inventory-Ordering (ZIO) property of this model still holds in order to establish an efficient solution algorithm.
- The number of periods to finance depends firstly on the difference between  $k$  and  $t$ . Furthermore, the mismatch between payment to the supplier and payment from the client may extend the number of periods of the horizon beyond the last demand for financing the production related costs. The payment delays (i.e.,  $r_c$  and  $r_f$ ) are introduced as parameters in order to quantify this mismatch. Using these parameters, we define the start period to finance the corresponding production related costs as well as the end period, as presented in Table 4.

Although the costs of purchasing, set-up and production are all one-time payments for each production lot, the inventory holding cost is paid regularly in all periods where the stock is held in inventory. For this reason, there is a cumulative effect of financing the inventory holding costs. Assuming that a production lot covers all demands from period  $t$  to period  $k$  means that the production occurs in period  $t$  at the latest. A unit of product of the demand in period  $k$  will be sold in period  $k$  and the client's payment will arrive in period  $k + r_c$ . The inventory holding cost for this item should be paid consequently from period  $t$  until period  $k - 1$  (noted as "OWCR<sub>inv</sub> series"). The first of these payments occurs in period  $t$  and must be financed until period  $k + r_c - 1$  (before the arrival of the client's payment, so a total of  $k + r_c - t$  periods must be financed). The second is paid in period  $t + 1$  until the same period of  $k + r_c - 1$  (in total,  $k + r_c - (t + 1)$  periods to finance.), etc. Consequently, for each  $X_{k,t}$ , the financing period is  $\frac{1+(k-t)}{2}(k-t) + r_c(k-t)$  periods including all periods to finance before and after the demand delivery.

Following this formulation of OWCR, the OWCR at period  $t$ , written as  $O_t$ , can be expressed in the following:

$$O_t = a \sum_{j=1}^{t-r_f} \sum_{l=t-r_c+1}^T X_{j,l} + p \sum_{j=1}^t \sum_{l=t-r_c+1}^T X_{j,l} + s \sum_{j=1}^t \sum_{l=t-r_c+1}^T \frac{Y_j}{Q_j + (1 - Y_j)} X_{j,l} + h \times \sum_{j=1}^t \sum_{l=t-r_c+1}^T X_{j,l} \cdot (\min\{l, t+1\} - j) \tag{1}$$

The formulation  $\sum_{j=1}^{t-r_f} \sum_{l=t-r_c+1}^T X_{j,l}$  represents all products whose production related operations still have to be financed at period  $t$  considering the payment delays. Especially for measuring the total amount of inventory holding cost to finance at period  $t$ , we need to count the number of completed payments in the OWCR<sub>inv</sub> series until period  $t$  for each  $X_{j,l}$ . However, we must consider whether or not we have sold the

**Table 4**  
Formulation of OWCR components.

Operations	Operation cost		Periods to finance				
	Unit cost	×	Quantity	from	(the period of outgoing payment)	until	(the period of incoming payment)
Purchasing	a	×	$X_{jk}$	from	$(t + r_j)$	until	$(k + r_c - 1)$
Setup	$\frac{sY_t}{Q_t - (1-\alpha)Y_t}$	×	$X_{jk}$	from	t	until	$(k + r_c - 1)$
Production	p	×	$X_{jk}$	from	t	until	$(k + r_c - 1)$
Inventory holding	h	×	$X_{jk}$	Financing periods (FP)			

products of  $X_{j,l}$  before reception of the client's payment (in other words, whether or not  $X_{jk}$  is still in inventory).  $\min\{l, t + 1\} - j$  precisely represents the total number of all payments for these two possible cases. The difference between these two cases as presented in formulation (1) is:

- If we have sold at period  $t$  (i.e.  $l \leq t < l + r_c$ ), then the total amount to finance for inventory holding will not change during the periods before arrival of the client's payment. This is because we no longer need to pay the inventory holding cost for  $X_{jk}$  in these periods. Thus, no additional inventory hold cost must be financed which means the total amount is a function of  $l - j$ .
- On the contrary, if the  $X_{jk}$  is still in inventory (i.e.  $j \leq t < l$ ), then the total amount of inventory holding costs to finance depends on  $t + 1 - j$ .

This formulation of OWCR allows us to follow the variation of OWCR over time in the planning horizon. It will be used in the following section for calculating the financing cost of OWCR.

3.6. Objective function

The objective of this problem is to maximize the NPV of the profit (denoted by *Profit*) by satisfying variable demand. To simplify the problem, the NPV of profit is defined as the difference between the NPV of revenue and the NPV of expenses. In our model, the revenue per period is a function of constant unit sales prices and demand quantity. The expenses include the production costs and the OWCR financing costs. The production cost is the sum of the costs for raw material purchasing, machine setup, production and inventory holding. The OWCR financing cost depends on the interest rate and the OWCR for satisfying all the demand on-time. Considering the payment delays, the planning horizon must be extended to the period when the last incoming or outgoing cash flow occurred. Mathematically, this accounting horizon is presented as  $T + \max\{r_j, r_c\}$ . For simplification purposes,  $T$  denotes the accounting horizon in this paper. Since the time value of money is considered in this model, both cash inflow (i.e., revenues) and outflow (i.e., expenses) are presented in NPV as follows:

- **NPV of revenue in period  $t$ ,  $R_t$ :** due to the delay of the client's payment, we only receive the client's payment for the demand of period  $t - r_c$  in period  $t$ . Thus,  $R_t = \frac{1}{(1+\alpha)^t} \sum_{k=t-r_c}^T V D_{t-r_c}$ .
- **NPV of production cost in period  $t$ ,  $LC_t$ :** all expenses of production related operations must be paid immediately when they occur except for the purchasing cost which can be delayed  $r_j$  periods according to the supplier agreement. Therefore,  $LC_t = \frac{1}{(1+\alpha)^t} (\alpha Q_{t-r_j} + pQ_t + sY_t + hI_t)$  with  $Q_t = \sum_{k=t}^T X_{jk}$  and  $I_t = \sum_{j=1}^t \sum_{k=t+1}^T X_{jk}$ .
- **NPV of OWCR financing cost in period  $t$ ,  $FC_t$ :** The OWCR financing cost depends only on the interest rate per period and the amount of financial need (i.e.,  $O_t$ ) to cover in period  $t$ .  $FC_t = \frac{1}{(1+\alpha)^t} \beta O_t$ .

By summing over all periods of the accounting horizon, the objective function is formulated as follows:

$$Profit = \sum_{t=1}^T (R_t - LC_t - FC_t)$$

3.7.  $ULSP_{(WCR)}$  model

A mixed-integer model of the profit maximization problem,  $ULSP_{(WCR)}$ , is formulated as follows:

$$\begin{aligned}
 &Max \quad Profit \\
 s.t. \quad &Q_t = \sum_{k=t}^T X_{jk} \quad \forall t \\
 &d_k Y_t - X_{jk} \geq 0 \quad \forall t, k \\
 &Y_t - Q_t \leq 0 \quad \forall t \\
 &\sum_{t=1}^k X_{jk} = d_k \quad \forall k \\
 &X_{jk} \geq 0, Y_t \in \{0, 1\} \quad \forall t, k
 \end{aligned} \tag{1.1} \quad -(1.6)$$

The objective function is to maximize the NPV of total profit. The constraints are similar to those of the FAL formulation of the ULS. Constraints (1.2) indicate that  $Q_t$  is the total quantity of production that occurs in period  $t$ . Constraints (1.3) ensure that a setup is executed before production. Constraints (1.4) prevent a setup from occurring in a period with no production. Finally, constraints (1.5) ensure that all demands are satisfied.

4. Solution method

In order to solve the  $ULSP_{(WCR)}$  problem efficiently, we propose an algorithm which avoids the difficulty of the non-linear formulation of the setup cost in the OWCR calculation. This algorithm is similar to the algorithm which is based on the *Zero-Inventory-Ordering (ZIO)* property established for the ULS problem with infinite capacity by Wagner and Whitin (1958).

4.1. Zero-Inventory-Ordering property

In a ZIO-type planning, production is only planned when no products remain in the inventory. We demonstrate that an optimal solution can always be found within all possible plans that respect the ZIO property. Moreover, since only ZIO type plans need to be investigated to find the optimal solution, the number of feasible solutions to investigate is significantly reduced. The ZIO property can be expressed as follows:

**Theorem.** *There exists an optimal production plan in which  $I_{t-1} \times Q_t = 0$  for all  $t$  (where  $I_t$  is the inventory at the end of the period  $t$ ) in  $ULSP_{(BFR)}$  problem.*

The proof is presented in Appendix A. The main idea is to demonstrate that we can always improve the objective value by eliminating violation(s) of the property by transforming a production plan which does not comply with the ZIO property to a ZIO-type plan. Eventually, we can prove the ZIO-type production program provides the optimal objective value.



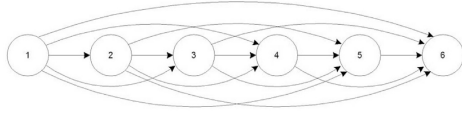


Fig. 2. Reformulation as a shortest path problem.

4.2. Algorithm description

Since the demand, the delay in payment from the customers, and the discounted rate are all given, the NPV of revenue is fixed for each instance. Thus, we only need to minimize the NPV of all costs. Then, like the classical shortest path problem, the  $ULS_{P(BFR)}$  problem is formulated with an acyclic oriented graph  $ULS_{P(BFR)}$  (see Fig. 2). The nodes  $V_t$  represent the  $T$  periods in the planning horizon including a dummy node at the end (node 6 in Fig. 2). Moreover, determining the production quantity benefits from the ZIO property which means that the production quantity can only be the sum of demands in the following periods. We thus avoid the difficulty caused by the nonlinear formulation of setup cost to finance. Consequently, an arc  $E_{ik}$  denotes a production planned in period  $t$  for all demands between periods  $t$  and  $k - 1$ . With this concept and the ZIO property, we determine the precise quantity and period of a production as well as the delivery periods. We are thus able to deduce the period of client's payment for financial cost calculation. Finding a shortest path therefore allows us to obtain the optimal production program with the minimum total NPV of all costs generated. Furthermore, with the dummy node, we are able to express all types of production lots in this acyclic graph including "make-to-order" type production.

In the objective function of  $ULS_{P(WCR)}$ , an aggregate OWCR formulation is given. However, a  $X_{ik}$ -based formulation of NPV of OWCR is required for arc value calculations. In the former formulation, the OWCR of an operation is the product of the corresponding unit cost,  $X_{ik}$ , and the number of periods to finance. Considering the NPV of OWCR, the discount rates are different (decreasing) during the periods to finance. Thereby, the discounted OWCR depends on the period where the payment occurred.  $WCR_{ik}^{pur}$ ,  $WCR_{ik}^{set}$ ,  $WCR_{ik}^{prod}$  and  $WCR_{ik}^{inv}$  respectively stand for the OWCR generated by operations of purchasing, set-up, production and inventory holding for producing the  $X_{ik}$ . For example, in purchasing raw material for the production of  $X_{ik}$ , the payment to supplier is made in period  $t + r_f$  and the client's payment is received in period  $k + r_c$ . Therefore, the corresponding discounted OWCR represents all financial needs to be covered between  $t + r_f$  and  $k + r_c$ . However, we are not able to ensure the timing order between the payment to supplier and the collection of clients payment. For this reason, we consider that we continue to finance the payment to the supplier until the end of the planning horizon as well as the payment from customer. The mathematical formulation can be written as

$$WCR_{ik}^{pur} = aX_{ik} \left[ \sum_{j=t+r_f}^T \frac{1}{(1+a)^j} - \sum_{j=k+r_c}^T \frac{1}{(1+a)^j} \right]. \quad (\text{By considering the financing cost of OWCR, the horizon is extended because some payments may be received from the client or sent to the supplier after the last period of the planning horizon}).$$

This formula allows us to correctly cover the two cases whether the payment to supplier or client's payment occurs first. In the first case, a negative OWCR in purchasing will be obtained which can be considered as an additional refund for the OWCR of other operations. In the second case,  $WCR_{ik}^{pur}$  is a positive financial need to finance.

Following the same concept, the OWCR in other operations can be formulated as

$$\bullet WCR_{ik}^{set} = \frac{sV_t}{Q_t(1-V_t)} X_{ik} \sum_{j=t}^{k+r_c-1} \frac{1}{(1+a)^j},$$

$$\bullet WCR_{ik}^{prod} = pX_{ik} \sum_{j=t}^{k+r_c-1} \frac{1}{(1+a)^j},$$

$$\bullet \text{ and } WCR_{ik}^{inv} = hX_{ik} \sum_{l=t}^{k-1} \sum_{j=l}^{k+r_c-1} \frac{1}{(1+a)^j}.$$

For WCR of inventory holding, if  $k = t$  (i.e.,  $X_{ik}$  is produced for demand in the same period), then there is no inventory holding cost. Combining all these components, the total OWCR for all  $X_{ik}$ , denoted as  $WCR_{total}$ , can be written as follows:

$$WCR_{total} = \sum_{i=1}^T \sum_{k=1}^T (WCR_{ik}^{pur} + WCR_{ik}^{set} + WCR_{ik}^{prod} + WCR_{ik}^{inv})$$

To simplify the NPV formulation, note that  $a'_t = a \frac{1}{(1+a)^t}$ ,  $s'_t = s \frac{1}{(1+a)^t}$ ,  $p'_t = p \frac{1}{(1+a)^t}$  and  $h'_t = h \frac{1}{(1+a)^t}$ . With the ZIO property, the arc values are presented as follows (with  $LC_{ik}$  representing all the production costs):

$$E_{ik} = LC_{ik} + \beta \sum_{l=t}^{k-1} (WCR_{il}^{pur} + WCR_{il}^{set} + WCR_{il}^{prod} + WCR_{il}^{inv})$$

$$= \left( a'_{t+r_f} + p'_t \right) \sum_{l=t}^{k-1} d_l + s'_t + \sum_{l=t}^{k-1} d_l \sum_{i=l}^{l-1} h'_i + \beta \sum_{l=t}^{k-1} d_l \left[ \left( \sum_{j=l}^T a'_{j+r_f} - \sum_{j=l}^T a'_{j+r_c} \right) + \sum_{j=l}^{l+r_c-1} p'_j + \sum_{j=l}^{l+r_c-1} \frac{s'_j}{\sum_{i=l}^{i-1} d_i} + \sum_{j=l}^{l-1} \sum_{i=j}^{l+r_c-1} h'_i \right] \quad (2)$$

The computation of  $E_{ik}$  is done in  $O(T^2)$ . The objective of this algorithm is to find the shortest path to the last node. To do so, we need to progressively locate the shortest path to other nodes. We therefore examine all possible paths to each node from all its predecessors through the arc in between. This can be carried out by the recursive function

$$Cost[t] = \min_{j \in [1, t-1]} \{ Cost[j] + E_{jt} \}$$

with  $Cost[t]$  the minimum total cost to finance to satisfy all demands until period  $t$ . Thus, a dynamic programming algorithm is established as presented in Algorithm 1. The complexity of the algorithm is  $O(T^4)$ , thus the  $ULS_{P(WCR)}$  problem can be solved in polynomial time.

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**Algorithm 1** Solving  $ULS_{P(WCR)}$

---

**Require:** All parameter values

**for**  $t = 2$  to  $T-1$  **do**

**for**  $j = 1$  to  $t-1$  **do**

**if**  $Cost[j] > Cost[t] + E_{jt}$  **then**

$Cost[t] = Cost[j] + E_{jt}$

**end if**

**end for**

**end for**

---

5. Numerical examples

Through the following numerical tests, we reveal differences between optimal production plans for profit maximization and costs minimization objectives. These tests illustrate the influence of considering the financing cost of OWCR on an optimal production program. With the proven ZIO property, the optimal program of profit maximization problem is calculated and denoted as  $P_{WCR}^{opt}$ . Moreover, the optimal program of the traditional  $ULS$  model is denoted as  $P_{ilk}^{opt}$ . The tests are organized as follows:

- Firstly, a comparison between  $P_{WCR}^{opt}$  and  $P_{ilk}^{opt}$  with the same set of parameter values is provided to show the differences in the production programs;
- Secondly, the evolution of a production program following the variation of discount and interest rates is presented as well as the decrease of average inventory level;

**Table 5**  
Numerical example parameter values.

Parameter	Value
Purchasing unit cost, $a$	0
Setup cost, $s$	600
Production unit cost, $p$	0
Inventory holding unit cost per period, $h$	1
Delay in payment from customer, $r_c$	5
Delay in payment to supplier, $r_f$	0
Unit sales price	not considered

- Thirdly, we compare the objective value of *ULS* and *ULS<sub>P(WCR)</sub>* model by applying respectively  $P_{uls}^{opt}$  and  $P_{wcr}^{opt}$  in both models.

For the following tests, we use the demand of the instance of [Trigeiro et al. \(1989\)](#) (G-72, demand 7). Values of other parameters are set to adapt the *ULS* concept which considers only setup and inventory holding costs (see [Table 5](#)):

- Since the purchasing and production cost are not considered in the traditional *ULS* model, these unit costs are given as zero in order to compare the optimal programs under the same conditions. Accordingly, the delay in payment to the supplier involved only in purchasing is irrelevant in determining the production planning. Thus,  $a = p = r_f = 0$ ;
- The setup cost,  $s$ , is given as 600 and the unit cost of inventory holding per period,  $h$ , is fixed at 1;
- The delay in payments from customer,  $r_c$ , is set to 5 periods;
- The unit selling price will not be considered as it is a constant parameter and will not impact the production program. However, it should be taken into account for the periodic *OWCR* maximization objective.

5.1. Production program comparisons

We compare optimal programs separately calculated by these two models with  $\alpha = 0.08$  and  $\beta = 0.1$ .  $P_{wcr}^{opt}$  and  $P_{uls}^{opt}$  are illustrated in [Fig. 3](#). We observe that

- The production programs are different. The *ULS* model cannot always provide the optimal solution for maximizing the profit;
- The production lots of  $P_{wcr}^{opt}$  are smaller with a profit maximization objective compared with  $P_{uls}^{opt}$  which leads to more setups and lower inventory levels on average;

The reason for this difference in production plans is the cumulative effect of financing the inventory holding costs of a product for all periods until sold. Considering the financial cost, more holding related cost is generated for each product held in inventory. In this case, fewer products are held in inventory during the horizon even though production and

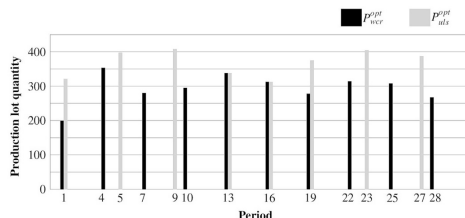


Fig. 3. Optimal production program comparison with fixed parameter values.

financial costs for setup costs are higher.

5.2. Production program evolution with different rates of discount and interest

In order to illustrate the influences of the two financial aspects (time value of money and financing cost of *OWCR*) on a production program, the discount and interest rates are respectively varied from 0.01 to 0.1 in steps of 0.01. The result of the comparison is presented in [Fig. 4](#). The bold grid represents the numbers of setups in optimal programs proposed by the *ULS<sub>P(WCR)</sub>* model and the light grid shows the number of setups in optimal programs in the *ULS* case (which remain unchanged). In this figure, observations can be summarized as follows:

- When the rates are very small, we obtain the same production program as the optimal *ULS* program;
- In other cases, the  $P_{wcr}^{opt}$  are different from  $P_{uls}^{opt}$ . Two more production lots may be planned for the financial model when the rates are relatively large.

Consequently, we first deduce from the second observation that considering the financial aspects radically changes the production program from the traditional *ULS* program which only optimizes logistic costs. Next, since  $P_{wcr}^{opt}$  generally proposes more setups, fewer products are held in inventory during the planning horizon compared with the *ULS* case. This decrease of average inventory level is presented numerically in [Table 6](#), which compares the average inventory level with  $P_{uls}^{opt}$ . For each rate combination, the difference in number of setups (#Setup) and average level of inventory in percentage is given by  $\Delta Avg.Stock = \frac{Inv_{wcr} - Inv_{uls}}{Inv_{uls}} \times 100\%$ , as presented in [Table 6](#).

The difference between optimal programs becomes more significant when the financial rates increase. A large decrease of 26% of average inventory level can be reached with relatively high discount and interest rate according to these tests. Essentially, the financing cost of inventory holding cost is quickly increased due to the cumulative effect of a higher inventory level. As a consequence, it is undesirable to hold too much

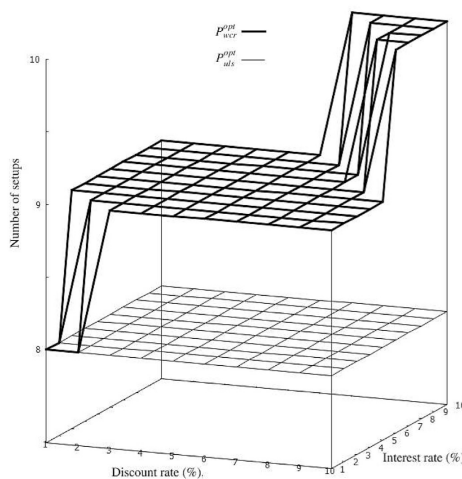


Fig. 4. Comparison of number of setups between optimal plans with different interest rates.

**Table 6**  
Differences in number of setups and average inventory level between optimal programs of  $ULSP(WCR)$  and  $ULS$ .

Parameter	$\beta = 0.01$	$\beta = 0.02$	$\beta = 0.08$	$\beta = 0.1$
$\alpha = 0.01$	#Setup =	=	+1	+1
	$\Delta$ Avg.Stock	0	-14.2%	-14.2%
$\alpha = 0.02$	#Setup =	+1	+1	+1
	$\Delta$ Avg.Stock	0	-14.2%	-14.2%
$\alpha = 0.08$	#Setup +1	+1	+1	+2
	$\Delta$ Avg.Stock	-14.08%	-14.08%	-26.7%
$\alpha = 0.1$	#Setup +1	+1	+2	+2
	$\Delta$ Avg.Stock	-14.08%	-14.08%	-26.32%

inventory when seeking to maximize the NPV of profit i.e., more frequent setups are preferred.

5.3. Production and financial cost comparisons

Within these tests, we find three cases where  $P_{uls}^{opt}$  and  $P_{wcr}^{opt}$  are identical when the rates are small. For other cases, we compare the NPV of total costs including the financing cost of OWCR generated by  $P_{uls}^{opt}$  and  $P_{wcr}^{opt}$ , denoted respectively as  $NPV_{uls}$  and  $NPV_{wcr}$ . Since  $P_{wcr}^{opt}$  generates a minimum NPV of total cost, the increase of NPV in applying the  $P_{uls}^{opt}$  is computed as  $\Delta_{NPV}ULS = \frac{NPV_{uls} - NPV_{wcr}}{NPV_{wcr}} \times 100\%$ . The result is presented in Fig. 5 ( $\Delta_{NPV}ULSP(WCR)$ ) represents the minimum NPV of total cost with  $P_{wcr}^{opt}$ .  $P_{uls}^{opt}$  generates a lower NPV of total cost by at most 3% compared with the NPV obtained by  $P_{uls}^{opt}$ .

However, because the production cost generated by  $ULS$  model, denoted as  $Log_{uls}$  is already optimal, the increase of production cost generated by applying  $P_{wcr}^{opt}$  (denoted as  $Log_{wcr}$ ) is logically higher. It is calculated as  $\Delta_{LOG}ULSP(WCR) = \frac{Log_{wcr} - Log_{uls}}{Log_{uls}} \times 100\%$  and presented in Fig. 6.

5.4. Program evaluation with different purchasing costs and delays in payment to supplier

Considering the delay in payment to supplier only affects the purchasing cost so we combine the tests of these two parameters as in Fig. 7. In these tests, we vary the purchasing unit cost from 0 to 90 in steps of 10. Moreover, the delay in payment to the supplier varies from 0 to 20 in steps of 2, and the rates are fixed at 3%. First, we observe that following

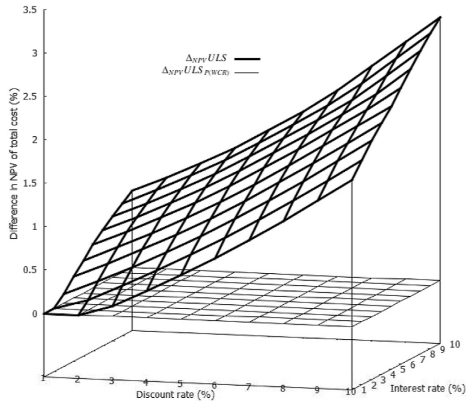


Fig. 5. Difference in NPV of total cost including the financing cost.

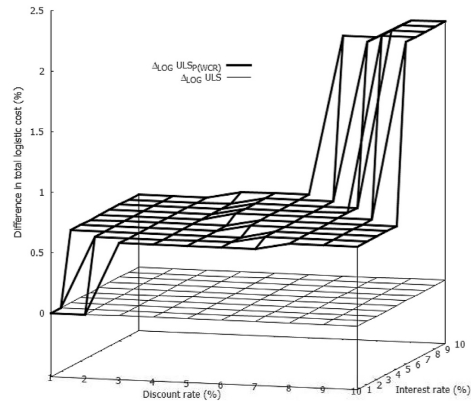


Fig. 6. Difference in logistic cost with different rates.

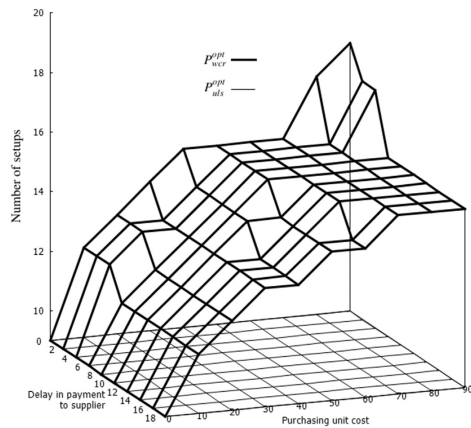


Fig. 7. Production plan comparison with varying purchasing unit cost and downstream delay.

the increase of purchasing unit cost with a fixed delay, more production lots are launched which means that fewer products will be held in inventory. The reason is that when the purchasing cost is more expensive, it is preferred to launch production later considering the discount effect which make early activities more costly in term of NPV. Second, with a fixed purchasing unit cost and an increasing delay to the supplier, the production size will generally increase to keep more NPV of inventory value in the system and to return cash back to the supplier as late as possible.

5.5. Program evaluation with different production unit costs

Based on the parameter values given in Table 5, we set the production unit cost at 5 and the rates at 3%. The result is presented in Fig. 8. The production lot size will decrease with an increasing production unit cost which means the productions are generally postponed. This is because that the NPV of the production costs is higher in an early period due to

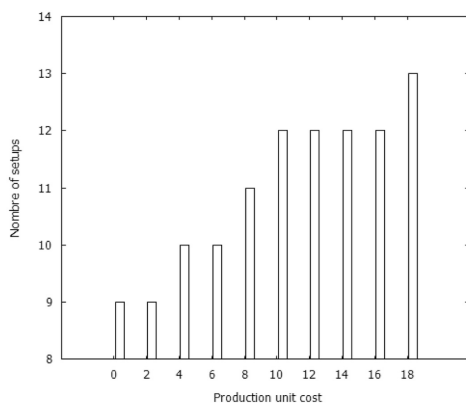


Fig. 8. Variation of number of setups with varying production unit cost.

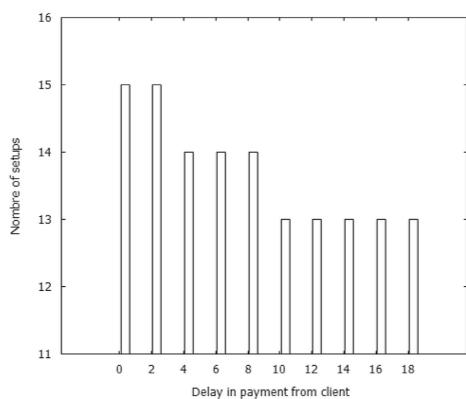


Fig. 9. Production plan comparison with varying delay in payment from client.

the discount effect and the number of periods to finance is greater if the production occurs in an early period.

**Appendix A. Proof of Theorem**

Firstly, we denote that:

- $X^{ULS_{(wcr)}}$  represents all feasible solution in term of  $X_{ik}$ ;
- $P$  is a production plan;
- $Profit(P)$  stands for the objective value in the proposed model with  $P$ .

**Definition 1.** A production planning is complied with the ZIO property if and only if the production is only planned when the inventory level drops to zero. With our notation, it can be expressed mathematically as

$$Q_t \times \left[ \sum_{k=1}^{t-1} \left( \sum_{i=t}^T X_{ik} \right) \right] = 0 \quad \forall t \in [2, T]$$

**5.6. Program evaluation with different delays in payment from the client**

Similar tests are performed with the rates set at 5%. Moreover, the purchasing and production unit cost are set equal to 10. We then vary the delay in payment from client from 0 to 20 in steps of 2. The result is shown in Fig. 9. We observe that the increasing delay in payment from the client causes a larger production lot size. This is due to postponing the periods for manufacturing the demand and paying the associated costs in later periods.

**6. Conclusions**

In this paper, firstly, we propose a new generic model of operations-related working capital requirements. This type of model allows us to measure the evolution of OWCR over the planning horizon. Secondly, a dynamic lot-sizing-based profit maximization model is developed considering both the financing cost of OWCR and the time value of money. This model is established for the single-site, single-level, single-production and infinite production capacity case. After proving the Zero-Inventory-Ordering property of this problem, an exact method is built based on dynamic programming. With the polynomial-time algorithm, we are able to not only reach the optimal plan which maximizes the net present value of profit, but also evaluate the profitability of satisfying a series of demands. Numerical tests are provided to show the interest of applying our OWCR model compared with the classical ULS model. We find that we have less interest in holding products in inventory due to their dramatic amplification of financial costs. On the other hand, authors are nevertheless aware of the limits of this paper as the first study which theoretically considers the WCR cost in a classic dynamic lot-sizing problem. Immediate application for real-world problems remains a challenge, however, we believe this study contributes significantly to promoting further studies in more complex and realistic cases.

For future research may focus on reducing the complexity of the proposed algorithm by considering the approaches presented in Wagelmans et al. (1992), Aggarwal and Park (1993) and Federgruen and Tzur (1991) since this algorithm is close to that of Wagner-Whitin. Moreover, future research can be undertaken to consider time-varying cost parameters. In order to establish a global profitability evaluation, a two-level (customer-factory) profit maximization problem could also be addressed. Furthermore, the production capacity and risks in supply chain management should be taken into account among other extensions.

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In other words, it can be written as:

$$\begin{cases} Q_t > 0 & \Rightarrow X_{k,t} = 0 \quad \forall (k, t) \in [1, t-1] \times [t, T] \\ X_{k,t} > 0 & \Rightarrow Q_t = 0 \quad \forall t \in [k+1, T] \end{cases}$$

**Conjecture Appendix A.1.** *There is a plan  $P^* \in X^{(ULS_{r,brn})}$  which verifies the ZIO property while*

$$Profit(P^*) = \max_{P \in X^{(ULS_{r,brn})}} Profit(P)$$

Let  $(P) \in X^{(ULS_{r,brn})}$  be a planning which does not comply with the ZIO property. Note that  $(t, m, n) \in [1, T]^3$  is a triplet with  $m < t \leq n$  and a violation of the property can be presented as  $Q_t \times X_{m,n} > 0$ . Then, we assume that several violations can be found in  $P$ .

$$\begin{aligned} X'_{k,t} &= X_{k,t} \quad \forall (k, t) \in [1, T]^2 - \{(m, n), (t, n)\} \\ X'_{m,n} &= 0 \quad X'_{t,n} = X_{t,n} + X_{m,n} \end{aligned}$$

According to the above-mentioned definition, we find one less violation of the property in  $P'$  than in  $P$ . We now need to prove that  $Profit(P) - Profit(P') \leq 0$ . To do so, we go through each term of the objective function. To simplify the NPV formulation, note that  $a'_t = a \frac{1}{(1+\alpha)^t}$ ,  $s'_t = s \frac{1}{(1+\alpha)^t}$ ,  $p'_t = p \frac{1}{(1+\alpha)^t}$ ,  $h'_t = h \frac{1}{(1+\alpha)^t}$  and  $v'_t = v \frac{1}{(1+\alpha)^t}$ . Essentially, these cost parameters decrease over time.

In the following proof, we examine the difference of all terms in the objective functions,  $P$  and  $P'$ , respectively the revenue, the logistic cost and the OWCR financing cost. At first, there is no difference in NPV of Revenue between these two plans. This is because the demand and unit selling price (as well as discounted unit selling price.) are identical over planning horizon. Thus,  $R(P) - R(P') = 0$ . Then, the difference in logistic cost can therefore be written as:

$$LC(P) - LC(P') = \sum_{j=1}^T [a'_{j-\tau_j} (Q_{j-\tau_j} - Q'_{j-\tau_j}) + p'_j (Q_j - Q'_j) + s'_j (Y_j - Y'_j) + h'_j (I_j - I'_j)] = X_{m,n} (a'_{m-\tau_j} - a'_{t-\tau_j}) + X_{m,n} (p'_m - p'_t) + s'_t (Y_m - Y'_m) + X_{m,n} \sum_{j=m}^{t-1} h'_j$$

Since  $m < t$  and  $Y_m \geq Y'_m$ , we obtain:

$$LC(P) - LC(P') > 0$$

Next, for OWCR financing cost, we examine its components one by one:

Firstly, since we apply constant unit costs to calculate purchasing, production and inventory holding costs, comparison on these operations are relatively direct as follows:

$$\begin{aligned} WCR^{pur}(P) - WCR^{pur}(P') &= \sum_{t=1}^T \sum_{k=t}^T aX_{k,t} \left[ \sum_{j=t}^t \frac{1}{(1+\alpha)^{j+\tau_j}} - \sum_{j=k}^t \frac{1}{(1+\alpha)^{j+\tau_j}} \right] - \sum_{t=1}^T \sum_{k=t}^T aX'_{k,t} \left[ \sum_{j=t}^t \frac{1}{(1+\alpha)^{j+\tau_j}} - \sum_{j=k}^t \frac{1}{(1+\alpha)^{j+\tau_j}} \right] = X_{m,n} \left[ \sum_{j=m}^t a_j - \sum_{j=t}^t a_j \right] \\ &= X_{m,n} \sum_{j=m}^{t-1} a_j > 0 \end{aligned}$$

$$WCR^{prod}(P) - WCR^{prod}(P') = \sum_{t=1}^T \sum_{k=t}^T pX_{k,t} \sum_{j=t}^{k+\tau_j-1} \frac{1}{(1+\alpha)^j} - \sum_{t=1}^T \sum_{k=t}^T pX'_{k,t} \sum_{j=t}^{k+\tau_j-1} \frac{1}{(1+\alpha)^j} = X_{m,n} \left[ \sum_{j=m}^{k+\tau_j-1} p'_j - \sum_{j=t}^{k+\tau_j-1} p'_j \right] = X_{m,n} \sum_{j=m}^{t-1} p'_j > 0$$

$$\begin{aligned} WCR^{inv}(P) - WCR^{inv}(P') &= \sum_{t=1}^T \sum_{k=t}^T hX_{k,t} \sum_{l=t}^{k+\tau_j-1} \frac{1}{(1+\alpha)^l} - \sum_{t=1}^T \sum_{k=t}^T hX'_{k,t} \sum_{l=t}^{k+\tau_j-1} \frac{1}{(1+\alpha)^l} = X_{m,n} \left( \sum_{l=m}^{n-1} \sum_{j=l}^{n+\tau_j-1} h'_j - \sum_{l=t}^{n-1} \sum_{j=l}^{n+\tau_j-1} h'_j \right) \\ &= X_{m,n} \left( \sum_{l=m}^{m+n-1} \sum_{k=0}^{m-1} \left( h'_k \frac{1}{(1+\alpha)^l} \right) + \sum_{l=m+n}^{n-1} \sum_{j=l}^{n+\tau_j-1} h'_j \right) > 0 \end{aligned}$$

Since the cost parameters are all positive, we confirm that the OWCR of these three operations is greater with  $P$  than the OWCR with  $P'$ .

We now focus on the difference in OWCR of set-up. The difficulty comes from the fact that the shared set-up cost per product (in period  $m$  and  $t$ ) changes with the quantity of  $X_{k,t}$ :

$$\begin{aligned} WCR^{set}(P) - WCR^{set}(P') &= \sum_{t=1}^T \sum_{k=t}^T \frac{sY_t X_{k,t}}{Q_t + (1-Y_t)} X_{k,t} \sum_{j=t}^{k+\tau_j-1} \frac{1}{(1+\alpha)^j} - \sum_{t=1}^T \sum_{k=t}^T \frac{sY_t X'_{k,t}}{Q_t + (1-Y_t)} X_{k,t} \sum_{j=t}^{k+\tau_j-1} \frac{1}{(1+\alpha)^j} \\ &= \sum_{k=m}^T X_{m,k} \sum_{j=m}^{k+\tau_j-1} \frac{s'_j Y_m}{Q_m + (1-Y_m)} + \sum_{k=t}^T X_{k,t} \sum_{j=t}^{k+\tau_j-1} \frac{s'_j Y_t}{Q_t + (1-Y_t)} - \sum_{k=m}^T X'_{m,k} \sum_{j=m}^{k+\tau_j-1} \frac{s'_j Y'_m}{Q'_m + (1-Y'_m)} - \sum_{k=t}^T X'_{k,t} \sum_{j=t}^{k+\tau_j-1} \frac{s'_j Y'_t}{Q'_t + (1-Y'_t)} \\ &= \underbrace{\sum_{k=m}^T \left( X_{m,k} \sum_{j=m}^{k+\tau_j-1} \frac{s'_j Y_m}{Q_m + (1-Y_m)} - X'_{m,k} \sum_{j=m}^{k+\tau_j-1} \frac{s'_j Y'_m}{Q'_m + (1-Y'_m)} \right)}_{Part1} + \underbrace{\sum_{k=t}^T \left( X_{k,t} \sum_{j=t}^{k+\tau_j-1} \frac{s'_j Y_t}{Q_t + (1-Y_t)} - X'_{k,t} \sum_{j=t}^{k+\tau_j-1} \frac{s'_j Y'_t}{Q'_t + (1-Y'_t)} \right)}_{Part2} \end{aligned}$$

Next, we evaluate part 1. Two possible cases are discussed in following paragraphs:

Case 1:  $Q'_m = 0$ . In this case,

$$\text{Part 1} = \sum_{k=m}^T X_{m,k} \sum_{j=m}^{k+r-1} \frac{s'_j Y_m}{Q_m + (1 - Y_m)} = \sum_{k=m}^T X_{m,k} \sum_{j=m}^{k+r-1} \frac{s'_j}{Q_m} > 0$$

Case 2:  $Q'_m > 0$ . In this case,

$$\begin{aligned} \text{Part 1} &= \sum_{k=m}^T \left( X_{m,k} \sum_{j=m}^{k+r-1} \frac{s'_j Y_m}{Q_m + (1 - Y_m)} - X'_{m,k} \sum_{j=m}^{k+r-1} \frac{s'_j Y'_m}{Q'_m + (1 - Y'_m)} \right) \\ &= \sum_{k=m}^T \left( X_{m,k} \sum_{j=m}^{k+r-1} \frac{s'_j}{Q_m} - X'_{m,k} \sum_{j=m}^{k+r-1} \frac{s'_j}{Q'_m} \right) \\ &= \frac{1}{Q_m Q'_m} \sum_{k=m}^T \sum_{j=m}^{k+r-1} s'_j (X_{m,k} Q'_m - X'_{m,k} Q_m) \\ &= \frac{1}{Q_m Q'_m} \left[ \sum_{j=m}^{n+r-1} s'_j X_{m,n} Q'_m + \sum_{k=m, k \neq n}^T \sum_{j=m}^{k+r-1} s'_j (X'_{m,k} Q'_m - X'_{m,k} Q_m) \right] \\ &= \frac{1}{Q_m Q'_m} \left[ \sum_{j=m}^{n+r-1} s'_j X_{m,n} Q'_m + \sum_{k=m, k \neq n}^T \sum_{j=m}^{k+r-1} s'_j (X'_{m,k} Q'_m - X'_{m,k} (Q'_m + X_{m,n})) \right] \\ &= \frac{1}{Q_m Q'_m} \left[ \sum_{j=m}^{n+r-1} s'_j X_{m,n} Q'_m - \sum_{k=m, k \neq n}^T \sum_{j=m}^{k+r-1} s'_j X'_{m,k} X_{m,n} \right] \\ &= \frac{X_{m,n}}{Q_m Q'_m} \left[ \sum_{j=m}^{n+r-1} s'_j Q'_m - \sum_{k=m, k \neq n}^T \sum_{j=m}^{k+r-1} s'_j X'_{m,k} \right] \\ &= \frac{X_{m,n}}{Q_m Q'_m} \left[ \sum_{j=m}^{n+r-1} s'_j \sum_{k=m}^{i-1} X_{m,k} - \sum_{k=m}^{i-1} \sum_{j=m}^{k+r-1} s'_j X'_{m,k} \right] \\ &= \frac{X_{m,n}}{Q_m Q'_m} \sum_{k=m}^{i-1} X_{m,k} \left( \sum_{j=m}^{n+r-1} s'_j - \sum_{j=m}^{k+r-1} s'_j \right) \\ &= \frac{X_{m,n}}{Q_m Q'_m} \sum_{k=m}^{i-1} X_{m,k} \sum_{j=k+r-1}^{n+r-1} s'_j > 0 \end{aligned}$$

Thus, part 1 is positive in all cases. We then go through the part 2:

$$\begin{aligned} \text{Part 2} &= \sum_{k=i}^T \left( X_{k,i} \sum_{j=i}^{k+r-1} \frac{s'_j Y_i}{Q_i + (1 - Y_i)} - X'_{k,i} \sum_{j=i}^{k+r-1} \frac{s'_j Y'_i}{Q'_i + (1 - Y'_i)} \right) \\ &= \sum_{k=i}^T \left( \frac{X_{k,i}}{Q_i} \sum_{j=i}^{k+r-1} s'_j - \frac{X'_{k,i}}{Q'_i} \sum_{j=i}^{k+r-1} s'_j \right) \\ &= \frac{1}{Q_i Q'_i} \sum_{k=i}^T \left( X_{k,i} Q'_i \sum_{j=i}^{k+r-1} s'_j - X'_{k,i} Q_i \sum_{j=i}^{k+r-1} s'_j \right) \\ &= \frac{1}{Q_i Q'_i} \sum_{k=i}^T \left( X_{k,i} (Q_i + X_{m,n}) \sum_{j=i}^{k+r-1} s'_j - X'_{k,i} Q_i \sum_{j=i}^{k+r-1} s'_j \right) \\ &= \frac{1}{Q_i Q'_i} \sum_{k=i}^T \left( X_{k,i} X_{m,n} \sum_{j=i}^{k+r-1} s'_j - Q_i (X_{k,i} - X'_{k,i}) \sum_{j=i}^{k+r-1} s'_j \right) \\ &= \frac{1}{Q_i Q'_i} \left( \sum_{k=i}^T X_{k,i} X_{m,n} \sum_{j=i}^{k+r-1} s'_j - Q_i X_{m,n} \sum_{j=i}^{n+r-1} s'_j \right) \end{aligned}$$

In order to determine the sign of part2, two following cases must be carefully examined:

Case A:  $Q'_i \geq Q_m$ . We have

$$\frac{X_{m,n}}{Q'_i} \sum_{j=n}^{n+r-1} s'_j < \frac{X_{m,n}}{Q_m} \sum_{j=n}^{n+r-1} s'_j$$

Then, we attempt to compare this element with part 1:

$$\begin{aligned}
 \text{Part 1} &= \sum_{k=m}^T \frac{X_{m,k}}{Q_m} \sum_{j=m}^{k+r_c-1} s'_j \\
 &= \sum_{k=m, k \neq n}^T \frac{X_{m,k}}{Q_m} \sum_{j=m}^{k+r_c-1} s'_j + \frac{X_{m,n}}{Q_m} \sum_{j=n}^{n+r_c-1} s'_j \\
 &\geq \frac{X_{m,n}}{Q_m} \sum_{j=n}^{n+r_c-1} s'_j \\
 &\geq \frac{X_{m,n}}{Q_m} \sum_{j=n}^{n+r_c-1} s'_j
 \end{aligned}$$

Case A-1:  $Q'_m = 0$ . In this case,

$$\text{Then, Part 1} + \text{Part 2} \geq \frac{X_{m,n}}{Q_m} \sum_{j=n}^{n+r_c-1} s'_j - \frac{X_{m,n}}{Q'_m} \sum_{j=n}^{n+r_c-1} s'_j \geq \left( \frac{X_{m,n}}{Q_m} - \frac{X_{m,n}}{Q'_m} \right) \sum_{j=n}^{n+r_c-1} s'_j > 0 \text{ with } Q'_m \geq Q_m$$

Case A-2:  $Q'_m > 0$ . In this case,

$$\begin{aligned}
 \text{Part 1} &= \frac{X_{m,n}}{Q_m Q'_m} \sum_{k=m}^{t-1} X_{m,k} \sum_{j=k+r_c-1}^{n+r_c-1} s'_j \\
 &= \frac{X_{m,n}}{Q_m Q'_m} \sum_{k=m}^{t-1} X_{m,k} \left( \sum_{j=m}^{n+r_c-1} s'_j - \sum_{j=m}^{k+r_c-1} s'_j \right) \\
 &\geq \frac{X_{m,n}}{Q_m Q'_m} \sum_{k=m}^{t-1} X_{m,k} \left( \sum_{j=m}^{n+r_c-1} s'_j - \sum_{j=m}^{t+r_c-1} s'_j \right) \\
 &\geq \frac{X_{m,n}}{Q_m} \left( \sum_{j=m}^{n+r_c-1} s'_j - \sum_{j=m}^{t+r_c-1} s'_j \right) \\
 &\geq \frac{X_{m,n}}{Q'_m} \left( \sum_{j=m}^{n+r_c-1} s'_j - \sum_{j=m}^{t+r_c-1} s'_j \right) \\
 &= \frac{X_{m,n}}{Q'_m} \sum_{j=t+r_c-1}^{n+r_c-1} s'_j
 \end{aligned} \tag{A.1}$$

Moreover, we have:

$$\begin{aligned}
 &\frac{1}{Q_t Q'_t} \sum_{k=t}^T X_{t,k} X_{m,n} \sum_{j=t}^{k+r_c-1} s'_j \geq \\
 &\frac{X_{m,n}}{Q_t Q'_t} \sum_{k=t}^T X_{t,k} \sum_{j=t}^{k+r_c-1} s'_j \geq \\
 &\frac{X_{m,n}}{Q_t Q'_t} \sum_{k=t}^T X_{t,k} \sum_{j=t}^{t+r_c-1} s'_j \geq \\
 &\frac{X_{m,n}}{Q_t Q'_t} \sum_{j=t}^{t+r_c-1} s'_j \sum_{k=t}^T X_{t,k} \geq \\
 &\frac{X_{m,n}}{Q'_t} \sum_{j=t}^{t+r_c-1} s'_j
 \end{aligned} \tag{A.2}$$

Thus, with (A.1) and (A.2),

$$\text{Part1} + \text{Part2} \geq \frac{X_{m,n}}{Q'_t} \sum_{j=t+r_c-1}^{n+r_c-1} s'_j + \frac{X_{m,n}}{Q'_t} \sum_{j=t}^{t+r_c-1} s'_j - \frac{X_{m,n}}{Q'_t} \sum_{j=t}^{n+r_c-1} s'_j \geq 0$$

In this case, the sum of part 1 and part 2 is also positive.

**Case B:**  $Q'_t < Q_m$ .

We distinguish again both cases of part 1:

Case B-1:  $Q'_m = 0$ . Impossible because  $Q'_t \geq Q_m$ .

Case B-2:  $Q'_m > 0$ . Part 1 can be transformed as follows:

$$\begin{aligned} \frac{X_{m,n}}{Q_m Q'_m} \sum_{k=m}^{t-1} X_{m,k} \sum_{j=k+r_t-1}^{n+r_t-1} s'_j &\geq \\ \frac{X_{m,n}}{Q_m Q'_m} \sum_{j=t+r_t-2}^{n+r_t-1} s'_j \sum_{k=m}^{t-1} X_{m,k} &\geq \\ \frac{X_{m,n}}{Q_m Q'_m} \sum_{j=t+r_t-2}^{n+r_t-1} s'_j Q'_m &\geq \\ \frac{X_{m,n}}{Q'_m} \sum_{j=t+r_t-2}^{n+r_t-1} s'_j & \end{aligned}$$

Part 2 can be formulated as:

$$\begin{aligned} \frac{1}{Q_t Q'_t} \left( \sum_{k=t}^T X_{ik} X_{m,n} \sum_{j=t}^{k+r_t-1} s'_j - Q_t X_{m,n} \sum_{j=t}^{n+r_t-1} s'_j \right) &= \\ \frac{X_{m,n}}{Q_t Q'_t} \left( \sum_{k=t}^T X_{ik} \sum_{j=t}^{k+r_t-1} s'_j - Q_t \sum_{j=t}^{n+r_t-1} s'_j \right) &\geq \\ \frac{X_{m,n}}{Q_t Q'_t} \left( \sum_{j=t}^{t+r_t-1} s'_j \left( \sum_{k=t}^T X_{ik} \right) - Q_t \sum_{j=t}^{n+r_t-1} s'_j \right) &\geq \\ \frac{X_{m,n}}{Q_t Q'_t} \left( \sum_{j=t}^{t+r_t-1} s'_j Q'_t - Q_t \sum_{j=t}^{n+r_t-1} s'_j \right) &\geq \\ -\frac{X_{m,n}}{Q'_t} \sum_{j=t+r_t-1}^{n+r_t-1} s'_j &\geq \\ -\frac{X_{m,n}}{Q'_m} \sum_{j=t+r_t-1}^{n+r_t-1} s'_j & \end{aligned}$$

In consequence,

$$\text{Part 1} + \text{Part 2} = \frac{X_{m,n}}{Q_m} \sum_{j=t+r_t-2}^{n+r_t-1} s'_j - \frac{X_{m,n}}{Q'_m} \sum_{j=t+r_t-1}^{n+r_t-1} s'_j \geq \frac{X_{m,n}}{Q_m} s'_{t+r_t-2} \geq 0$$

To summarize, the sum of part 1 and part 2 is positive in all cases which means  $P$  generates a bigger OWCR financing cost than  $P'$ . To conclude, considering all above-mentioned comparisons, the NPV of revenue is equal with these two plans, but the NPV of the total cost is greater with  $P$ . Therefore, we favor the  $P'$  which has one less violation of ZIO property. In general, we are able to deduce that the optimal planning is the ZIO type planning with no violations at all.

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## Surgical case scheduling with sterilising activity constraints

Hasan Al Hasan<sup>a,b,c\*</sup>, Christelle Guéret<sup>a</sup>, David Lemoine<sup>b</sup> and David Rivreau<sup>a,c</sup>

<sup>a</sup>LARIS, Université d'Angers, Angers, France; <sup>b</sup>LS2N, IMT Atlantique, Nantes, France; <sup>c</sup>LARIS, Université Catholique de l'Ouest, Angers, France

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In this paper, we study a real scheduling problem which consists in scheduling a set of elective surgical cases requiring surgical instruments and tools in several operating rooms. The objectives are to minimise the overtime of the surgical unit staff, the number of operating rooms used and the number of instruments processed in emergency in the sterilising unit while respecting the current level of service represented by the total number of patients operated per month at the orthopaedic surgery unit. This research was performed in collaboration with the University Hospital of Angers in France (CHU Angers), which has also provided historical data for the experiments. We propose a mixed integer linear programming model for the problem which is solved in a lexicographic fashion. We also propose a robust formulation to deal with the uncertain surgery durations. Both the deterministic and robust formulations are then compared over a set of instances provided by the CHU. The solutions obtained are competitive in terms of number of operating rooms and significantly improve those implemented operationally at the CHU in terms of overtime and emergencies at the sterilising unit.

**Keywords:** operating rooms in health services; elective surgeries planning; sterilisation unit; scheduling; mixed-integer linear programming; robust optimisation

### 1. Introduction

Surgery is one of the most important activities in hospitals. It has been recognised as a main source of income as it generates approximately two thirds of hospital revenues (Jackson 2002). In addition, it counts for approximately 40% of hospital costs (Macario et al. 1995) through the use of facilities (operating rooms, intensive care beds, etc.) and the personnel costs (surgeons, nurses, anaesthetists, etc.). For these reasons, numerous researchers studied the operating room (OR) planning and scheduling problems in both elective (non-urgent) and non-elective (urgent) contexts (Cardoen, Demeulemeester, and Beliën 2010; Guerriero and Guido 2011).

In the elective case, the OR planning and scheduling problems can be viewed as being made up of three phases corresponding to three decision levels (Testi, Tanfani, and Torre 2007). The first phase (strategic level) corresponds to the problem of determining the number, type and working hours of the available ORs and of assigning these ORs to surgical specialities (Zhang et al. 2009; Dexter et al. 2002; Van Berkel and Blake 2007; Dexter et al. 1999).

The second phase (tactical level) corresponds to the problem of creating a timetable, often referred to as the 'master surgical schedule' (MSS) on a medium-term horizon. This timetable defines the specific assignment of ORs to surgeons. A new MSS must be generated whenever the total amount of OR time changes or when the make-up of some specialities changes. This can occur not only as a response to long-term changes in the overall OR capacity or fluctuations in staffing, but also in response to seasonal fluctuations in demand (Blake, Dexter, and Donald 2002; Blake and Donald 2002; Beliën and Demeulemeester 2007; van Oostrum et al. 2008). Traditionally, the MSS is a cyclic timetable. However, some researchers investigated the adoption of an MSS that dynamically adapts to the changes in surgery load (Agnētis et al. 2012, 2014).

The last phase (operational level), which is called 'surgical case scheduling', is generally separated into two sub-problems referred to as 'advance scheduling' and 'allocation scheduling' (Magerlein and Martin 1978).

Advance scheduling is the process of assigning a surgery date to a patient. This problem determines the date only and does not deal with the order of the surgeries in each day nor the starting time of each surgery. Several research studies address this problem with various performance criteria. For instance, in (Fei et al. 2008), the authors developed a decomposition-based branch-and-price algorithm to solve the assignment problem with an objective of minimising total operating costs.

\*Corresponding author. Email: [alhasan@uco.fr](mailto:alhasan@uco.fr)

Furthermore, in (Hans et al. 2008), the authors proposed constructive and local search heuristics for the assignment problem with the objective of freeing ORs and capacity while taking into account the uncertainty that may occur in this problem (e.g. patient cancellation). In (Velásquez, Melo, and Küfer 2008), the authors proposed an approach for the dynamic advance scheduling problem, where the objective is to find appointments for surgeries in real time. Their approach is based on a bin packing problem which takes into account the limited resource availability.

Given this advance schedule, the second sub-problem then determines the precise sequence of surgical procedures and the allocation of resources for each OR time block in order to implement it as efficiently as possible. Similar to the advance scheduling sub-problem, many researchers tackled the allocation scheduling problem with various performance criteria. For instance, Cardoen, Demeulemeester, and Beliën (2009a) developed several mixed integer linear programming solution approaches, which are either exact or heuristic, and consider multiple objectives including minimising the starting time for children and priority patients and minimising the stay in recovery after the closure of the day-care centre. In (Herring and Herrmann 2012), the authors presented a single-day scheduling problem formulation and proposed several threshold-based heuristics to solve the sequencing problem. Their threshold identifies how much open time to preserve for future primary cases. In (Dexter, Macario, and Traub 1999), the authors used computer simulations to compare 10 different variable-sized bin packing algorithms when solving the sequencing problem with the objective of maximising OR use. Furthermore in (Cardoen, Demeulemeester, and Beliën 2009b), the authors used a branch-and-price methodology with the objective of minimising the peak use of recovery beds, the occurrence of recovery overtime and the violation of various patient and surgeon preferences.

Some researchers proposed approaches that solve the advance and allocation scheduling sub-problems at the same time. For example, in (Roland et al. 2010), the authors proposed a meta-heuristic that integrates both the planning and scheduling phases in a single procedure with the objective of minimising the costs induced by the use of the operating rooms and by overtime pay. In (Guinet and Chaabane 2003), they proposed a two-step approach to solve the advance scheduling and the sequencing problems in order to minimise the fixed patient intervention costs (ORs overtime and patient waiting time). Firstly, the patient interventions are assigned to ORs on a medium-term horizon. Secondly, the patient interventions are rescheduled every day to integrate more characteristics (recovery room, cleaning operations, etc.) for human and material resource synchronisation.

One of the major challenges faced when building accurate surgical case schedules is the large uncertainty accompanying the surgical service that corresponds to the difference between the planned and actual durations for the surgical activities. In addition to the work done on the deterministic version of the problem which ignores such uncertainties, many researchers implemented stochastic and robust approaches in order to incorporate these uncertainties when solving the advance and allocation scheduling problems.

Starting with the advance scheduling problem, the authors in (Saadoui et al. 2015) considered the problem of assigning surgeries to ORs from a waiting list while taking into account the recovery phase for each patient in one of the limited recovery beds. They propose an approach in which a slack is given to each surgery in the waiting list and then a knapsack model chooses the surgeries for a given day from the list. Finally, the assignment of these surgeries to the ORs is done using a mixed integer programming model with the objective of using the OR time more efficiently and minimising the makespan. Furthermore, the authors in (Min and Yih 2010) proposed a sample average approximation method to obtain an optimal surgery schedule while minimising the total number of patients and overtime costs. In contrast, the authors in (Neyshabouri and Berg 2017) proposed a two-stage robust optimisation model to address the uncertainty in surgery durations and length of stay in the intensive care unit after the surgery. They propose an adapted column-and-constraint generation method to obtain exact solutions for the proposed formulations. Likewise, the authors in (Addis, Carello, and Tánfani 2014) proposed a robust formulation based on the formulation presented in (Bertsimas and Sim 2004). Their objective was to minimise the costs associated with patients tardiness and waiting time while allowing overtime when needed. Another approach was presented in (Denton et al. 2010), where the authors proposed two models to solve the problem of assigning surgical cases to ORs on a given day of surgery with the objective of minimising the total costs represented by the sum of fixed OR opening costs and the variable cost of overtime. Their first model was a two-stage stochastic linear program with binary decisions in the first stage and simple resource in the second stage. The second model was the robust counterpart. They then compare the results of the two models and show that the robust methods perform much faster than the stochastic model and have the benefit of limiting the worst-case outcome of the resource problem. In contrast, the authors in (Hans et al. 2008) proposed constructive and local search heuristics for the robust surgery loading problem which concerns assigning surgeries and sufficient planned slack to OR days. Their objectives were to maximise the capacity use by minimising the total planned slack and to minimise the risk of overtime, and thus cancelled patients.

Concerning the allocation sub-problem, the authors in (Denton, Viapiano, and Vogl 2007) proposed a stochastic optimisation model and some practical heuristics for computing OR surgery sequences. They focused on the effects of sequencing surgeries and scheduling start times.

Finally, some researchers tackled both advance and allocation scheduling sub-problems while dealing with surgery duration uncertainties. The authors in (Marques and Eugénia 2017) proposed three mixed integer linear programming models from the perspective of the various stakeholders. They compare the results of the deterministic versions with a robust method that they developed based on the objective functions presented in each version of the models.

Although the surgical case scheduling problem has been studied widely in the literature, very few researchers consider the sterilisation activities. In (Beroule et al. 2016), the authors propose several metaheuristics to solve an advance and allocation scheduling problem including the medical device sterilisation step with the objective of minimising the number of medical devices required at the same time to respect a schedule. A branch-and-price technique is applied in (Cardoen, Demeulemeester, and Beliën 2009c) to solve an allocation scheduling problem (sequencing problem) in a day care center with the objective of minimising the peak use of recovery beds, the occurrence of recovery overtime and the violation of various patient and surgeon preferences while including the medical device sterilisation constraints.

In this study, we investigate the real advance and allocation scheduling problem of the university hospital of Angers (CHU Angers) while considering sterilisation constraints. More specifically, we focus on the orthopaedic surgery unit (OSU) which consists of three ORs that deal exclusively with elective surgeries. The main problems faced at the CHU are financial. Therefore, we must firstly ensure good control over the costs induced by the use of overtime, secondly reduce the costs of personnel and resources by closing ORs when possible, and thirdly reduce the costs induced by the use of expensive surgical instruments, which are sometimes rented directly from the manufacturers and which can only be available at the surgery unit in limited quantities. These limitations require precise planning of surgeries to ensure the fluidity of the necessary process to sterilise the surgical instruments, carried out by an independent sterilisation unit (SU).

In this article, we focus on the problem of scheduling surgeries to ORs while minimising the overtime, minimising the number of rooms opened, and minimising the number of kits being treated urgently in the SU.

The remainder of this paper is organised as follows: Section 2 specifies the problem and explains the work process at the Angers CHU. In Section 3, a deterministic mathematical formulation for the problem is proposed, while its robust counterpart is presented in Section 4. The experimental results are presented in Section 5, and we conclude in Section 6.

## 2. Description of the problem

The Angers CHU includes several units and an SU which centralises all the sterilising operations. As said before, we focus on the orthopaedic surgery unit activities and the way in which this unit interacts with the SU. In our problem, the master surgical schedule is taken from the hospital human resource management policy, and so we are given the shifts of surgeons over the planning horizon (one month).

There are three ORs at the OSU. Table 1 shows the OR opening hours. Note that some rooms can be closed on some days for technical maintenance reasons and due to the large number of surgeries to perform, an OR may stay open exceptionally after closing hours for up to 3 h. Due to the substantial fixed costs induced by opening an OR, the CHU imposes a minimal occupancy rate of 80% on average for the OSU.

Between 10 and 14 surgeons share these rooms according to the predefined timetable. More precisely, this schedule indicates the days when the surgeons can operate, and the list of ORs that each surgeon can use each day. We know for each surgeon the list of surgeries he/she has to perform over the horizon, and we assume that each surgeon can use up to one room per day.

At the OSU, surgeries can be divided into two classes: ambulatory and non-ambulatory. The first class groups surgeries for patients who are hospitalised for less than 24 h with no overnight stay (outpatients), while the second class represents surgeries for patients who receive longer care (inpatients). The main impact of these two classes on the process is that all ambulatory surgeries must finish before 3 p.m. while the non-ambulatory surgeries can stay till the closing hours of the OR. The reason behind this difference is that outpatients need to spend at least 2 h in the recovery room after surgery before being discharged, and the recovery room closes at 5 p.m., while inpatients are transferred after surgery to the intensive care unit which is open 24 h a day.

Each surgery is also characterised by an estimated duration time which is calculated using 10 months historical data by the CHU as the average duration of the surgeries of the same type, and a list of surgical instruments. The instruments are

Table 1. Opening hours of the operating rooms.

Room	Days per week	Opening time	Closing time
1 & 2	M, T, W, Th, F	8:15	17:00
3	M, T, W, F	8:15	14:30

organised and packed in small boxes called kits. Each kit contains tools that are generally used together for specific types of surgery. There are 212 kit types with between 1 and 15 boxes for each type.

Typically, a surgery uses more than one kit, and each kit can be used for more than one type of surgery. Moreover, all the kits reserved for a surgery are not systematically used. After each surgery, the kits that were not used are sent to the OSU store room, while the others are kept in water for 30 min for pre-disinfection before being collected by the SU.

The SU opens every working day at 7 a.m. and closes at 8.30 p.m. The sterilising process is performed in several steps : the instruments are first cleaned by automatic washers, then reassigned in their corresponding kits before being processed through sterilising machines. Finally, the kits are kept at the SU to cool off before being returned to their unit where they are stored before being used for another surgery. On average, the whole kit sterilisation process takes approximately four and half hours. If for some reason a unit needs a kit urgently (mainly because of the surgery schedule), an emergency call is made to the SU. This kit is then treated as first priority throughout the whole process at the SU. Figure 1 shows the movement cycle of the OSU kits.

Due to the logistic organisation of the CHU, the times for collecting and delivering kits to each unit are fixed (see Table 2) and the CHU does not want to modify them at this stage of the study. In particular, an on demand pick-up policy is not an option according to the high number of surgeries carried out in the different ORs of all the units of the CHU every day.

From the average kit processing time at the SU and the pick-up and delivery hours (Table 2), we have the following situations:

- (1) A kit collected at 11.30 a.m., 1 p.m. or 2.30 p.m. on day ( $t$ ) can be used in the morning on day ( $t + 1$ ) if it is treated as a priority at the SU (*priority kit*, case 1). If it is not treated as a priority, its process may finish during the morning of day  $t + 1$ . In this case, it will be delivered to the OSU by the 2.30 p.m. shuttle of day  $t + 1$  and it is then considered that it cannot be used before 2.30 p.m. on day ( $t + 1$ ).
- (2) A kit collected at 4 p.m. on day ( $t$ ) can be used in the morning on day ( $t + 1$ ) if it is treated urgently at the SU (*urgent kit*). If it is not treated urgently, its process will be finished during the morning of day  $t + 1$  and it will be delivered to the OSU by the 2.30 p.m. shuttle of day  $t + 1$ . In this case, it can be used from 2.30 p.m. on day ( $t + 1$ ).
- (3) A kit collected at 5.30 p.m., 6.30 p.m. on day ( $t$ ) or 7 a.m. on day ( $t + 1$ ) can be used on day ( $t + 1$ ) from 2.30 p.m. if it is treated as a priority at the SU (*priority kit*, case 2). If it is not treated as a priority, it will be available on day ( $t + 1$ ) from 5.30 p.m.

As explained before, the pick-up/delivery hours are fixed, thus, when a late surgery ends after 6.30 p.m., the used kits stay at the OSU till the next pick-up the following morning.

In the current work flow at the CHU, each surgeon defines the date of the surgery during the consultation based on his/her shifts, and the condition and availability of the patient, without considering the schedule of the other surgeons. Then, at the end of each week, a nurse checks that there are no kit constraint violations in the next week's schedule. The main check consists in verifying that the surgeries scheduled each day are compatible with the number of kits owned by the OSU. Despite the effort of increasing the working hours of the SU, the number of urgent and priority kits to process remains high. This is particularly critical for the urgent kits as the start of the machines of the first step may be delayed in order to wait for the arrival of the urgent kits at the SU, thus generating a substantial loss of productivity. As for the priority kits, they are

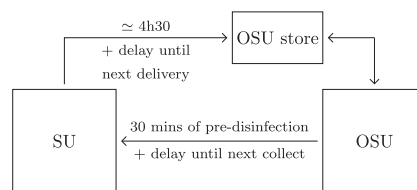


Figure 1. Process followed by the kits.

Table 2. SU pickups and deliveries to the OSU.

Pick-up	07:00	11:30	13:00	14:30	16:00	17:30	18:30
Delivery	07:00	—	—	14:30	—	17:30	—

Table 3. Problems faced by the CHU from Sept. 2014 to June 2015.

Month	Violated constraints		Overtime		ORs			Kits	
	#late amb	#violated kits	total (min)	max	# opened rooms	Min Occ. rate	Avg. Occ. rate	#urgent	#priority
Sept.	14	5	1382	190	59	29.9%	79.2%	3	39
Oct.	14	5	1692	227	59	12.9%	76%	3	44
Nov.	6	10	1322	113	48	19%	79.3%	8	49
Dec.	10	8	1800	98	48	44%	82.9%	14	64
Jan.	14	5	2370	130	59	29.7%	81.9%	6	58
Feb.	8	4	2140	201	52	50.8%	79%	1	53
March	6	13	1454	170	59	33.1%	80.6%	28	72
April	9	2	1584	100	49	23.7%	81.5%	4	46
May	10	7	1500	126	46	12.3%	77.3%	8	42
June	10	3	1930	326	60	9.9%	70.7%	4	27
Average	10.1	6.2	1717.4		53.9		78.8%	7.9	49.4

taken into account upon arrival by moving them to the start of the queue to be treated first once a new batch of kits is going in (even if normally, the SU works on a first in first out queue policy). Ultimately, both urgent and priority kits delay other kits from other units and induce considerable pressure on the SU staff.

The main problem currently faced by the CHU is the lack of global planning as surgeries at the OSU are scheduled independently of each other without fully taking into account the kit sterilising time constraints. This generates violated constraints (late ambulatory surgeries, kits not ready on time), overtime, variable occupancy rate of the OR, and urgent and priority kits. Table 3 shows, for example, the number of late ambulatory surgeries (supposed to finish before 3 p.m.), the number of kits that were required for a surgery but not available on time, the total and maximum overtime in minutes, the number of rooms opened, with their minimum and average occupancy rates, and the number of urgent and priority kits for the period from September 2014 to June 2015.

Considering the situation, the objective of this study is to schedule all surgeries at the OSU while minimising the operating costs and the inconvenience in the SU. The costs considered are represented by the overtime at the OSU and the fixed costs for opening ORs. In the actual context of budgetary restriction and of hard working conditions for the hospital staff, the first objective for the CHU is to schedule all surgeries in order to minimise the total overtime of the OSU staff members, thus reducing costs while taking into account the staff quality of life at work. The second objective is to minimise the number of ORs used, since opening an OR generates substantial fixed costs. Finally, the third objective is to keep the number of urgent and priority kits as low as possible. These objectives have to be minimised while maintaining the same current level of service by respecting the total number of surgeries performed per month at the OSU. Note that unlike many other surgery units, the bed capacity is not a problem for the OSU: the number of beds in the ward was in fact set 18 years ago and has never been changed. But, over the past few years, outpatient surgery has increased considerably while inpatient surgery has decreased.

The following section presents a mathematical model for solving this problem.

### 3. Mathematical formulation

Formally, the problem consists in scheduling  $O$  surgical operations using  $K$  kit types performed by  $S$  surgeons in  $R$  ORs on a horizon of  $T$  days. The activity at the OSU is split into  $J = 4$  consecutive time intervals for each day in order to take into account the reuse kit conditions induced by the SU process.

In this problem, each surgeon  $s$  must perform a set  $\lambda_s$  of surgeries and is assigned to work on specific days in specific rooms (with parameter  $\delta_{sr} = 1$  if surgeon  $s$  can use room  $r$  on day  $t$ , and 0 otherwise). For each surgery  $i$  we know its duration  $p_i$  in minutes, its type (we set parameter  $a_i = 1$  if surgery  $i$  is ambulatory, and 0 otherwise). Each ambulatory surgery must finish before a given daily limit  $A_{max}$ . The required quantity  $q_{ik}$  for surgery  $i$  of each kit type  $k$  is given. For each kit type  $k$ , we know the number  $Q_k$  of kits available for this type.

Clearly, starting from the various kit reuse conditions listed above, we see that some of the pick-up hours, although they allow the load to be distributed over the SU, do not have any impact on the surgery schedule. For example, all kits collected at 11.30 a.m., 1 p.m. and 2.30 p.m. will be reusable from the same hour the next day. Thus, we can consider only the 2.30 p.m. pick-up shuttle. We can also ignore the pick-up shuttle of 5.30 p.m. and 6.30 p.m. as the kits collected at these hours will be reusable at the same hour as those collected at 7 a.m. the next day. Furthermore, since used kits must be pre-disinfected for 30 min at the OSU after a surgery, a kit can be collected by a shuttle if the surgery that used it ends at

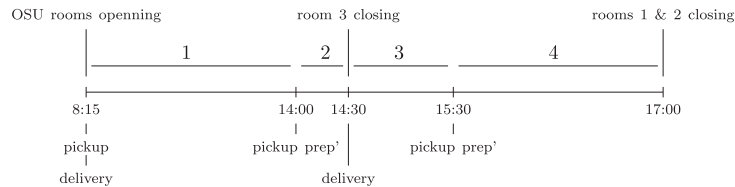


Figure 2. The four periods of an OR.

Table 4. Reusing conditions for the kits.

End of kit utilisation on day $t$	Next possible reuse on day $t + 1$			
	Period 1	Period 2	Period 3	Period 4
Period 1	Priority kit (case 1)	Priority kit (case 1)	Normal	Normal
Period 2 or 3	Urgent kit	Urgent kit	Normal	Normal
Period 4	–	–	Priority kit (case 2)	Priority kit (case 2)

least 30 min before the shuttle time. This leads us to consider the pick-up ‘preparation limits’ at 2 p.m. and 3.30 p.m. instead of the actual pick-up times of 2.30 p.m. and 4 p.m.

Similarly, for the delivery shuttles, the 5.30 p.m. shuttle can be ignored since no surgery is supposed to start after 5.30 p.m. We can also assume that the first delivery shuttle arrives at 8.15 a.m. (room opening) instead of 7 a.m.

From these key times, we can restrict our attention of the OSU activity, with respect to the kit requirements over 4 periods, as shown in Figure 2.

Finally, the interaction between period of surgeries and kit reuse can be summarised as shown in Table 4.

For each room  $r$  and each day  $t$ , the length of period  $j$  in minutes is denoted by  $d_{rt}^j$  (0 if the room is closed that day and cannot be used). For added convenience, the parameter  $d_{rt}^{b,f} = \sum_{j=b}^f d_{rt}^j$  will specify the total duration from period  $b$  to period  $f$  of day  $t$  for room  $r$ . Finally, for each room at each day, the overtime cannot exceed a given duration  $\epsilon_{max}$ .

Priority kits and urgent kits are penalised through respective  $cp$  and  $cu$  unit costs.

From the problem description, we have the following constraints:

- All surgical operations must be scheduled.
- Each surgeon can only perform surgeries in a room if he/she is allowed to use the room on that day.
- Each day, a surgeon can use at most one room.
- Each day, the overtime in each room cannot exceed  $\epsilon_{max}$ .
- Ambulatory surgeries must finish before time limit  $A_{max}$ .
- Numbers of resources (kits) must be respected, taking into account the reuse conditions given in Table 4.

The objective is to schedule all the surgical operations in order to minimise the total overtime, the total number of rooms opened, and the total penalties of emergencies and priorities in the SU.

We will solve the problem in three steps:

- (1) In the first step, we seek to minimise the overtime (in minutes) in order to be able to schedule all the surgeries on the planning horizon.
- (2) We then schedule all surgeries while minimising the total number of rooms opened, allowing the minimum overtime found in step 1.
- (3) Finally, we schedule all surgeries while minimising the total penalties of urgent and priority kits at the SU without exceeding the number of rooms opened found in step 2 nor the total overtime from step 1.

The parameters, variables, and constraints of the mixed-integer linear program (MILP) we propose are as follows.

We start with a summary of the parameters:

- $T$  total number of days in the horizon
- $J$  number of periods in day
- $K$  total number of kit types

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$O$	total number of surgeries
$R$	total number of operating rooms
$S$	total number of surgeons
$p_i$	duration of surgery $i$
$\lambda_s$	set of surgeries for surgeon $s$
$q_{ik}$	required quantity of kits of type $k$ for surgery $i$
$Q_k$	total quantity of kits of type $k$ owned by the block
$cu$	urgent kit penalty
$cp$	priority kit penalty
$d_{rt}^j$	duration of period $j$ of day $t$ for room $r$ (0 if the room is closed that day)
$d_{rt}^{bf}$	operational duration from period $b$ to period $f$ ( $\geq b$ ) of day $t$ for room $r$
$\delta_{srt}$	binary parameter equal to 1 if surgeon $s$ can use room $r$ on day $t$ , 0 otherwise
$a_i$	binary parameter equal to 1 if surgery $i$ is ambulatory, 0 otherwise
$A_{max}$	latest time for ambulatory surgeries to be performed at
$\varepsilon_{max}$	maximum allowed overtime for any room on any given day
$u_\gamma$	binary parameter equal to 1 if $\gamma = J$ and 0 otherwise

We introduce the following decision variables:

$w_{itr}$	binary variable equal to 1 if surgery $i$ is scheduled at day $t$ in room $r$
$x_{itr}^{bf}$	binary variable equal to 1 if surgery $i$ begins at period $b$ and finishes at $f$ , on day $t$ , in room $r$
$\varepsilon_{tr}$	integer variable representing the total overtime in room $r$ at day $t$
$L_{tr}$	binary variable equal to 1 if room $r$ is used at day $t$
$U_{srt}$	binary variable equal to 1 if surgeon $s$ is using room $r$ at day $t$
$E_{tk}$	integer variable representing the total urgent kits of type $k$ at day $t$
$Y_{tk}^1$	integer variable representing the total priority kits (case 1) of type $k$ at day $t$
$Y_{tk}^2$	integer variable representing the total priority kits (case 2) of type $k$ at day $t$

Constraints (1) ensure that all surgeries are scheduled and constraints (2) state that each surgery must start and finish on the day it is scheduled.

$$\sum_{t=1}^T \sum_{r=1}^R w_{itr} = 1, \quad \forall i \in \{1, \dots, O\} \quad (1)$$

$$\sum_{b=1}^J \sum_{f=b}^J x_{itr}^{bf} = w_{itr}, \quad \forall i \in \{1, \dots, O\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\} \quad (2)$$

Constraints (3) state that each day in each room no more than one surgery can be scheduled over the time  $\gamma$  which separates two periods (surgeries with  $b \leq \gamma$  and  $f > \gamma$ ). In other words, they prevent the case where more than one surgery start in one period and extend into other periods.

$$\sum_{i=1}^O \sum_{b=1}^{\gamma} \sum_{f=\gamma+1}^J x_{itr}^{bf} \leq 1, \quad \forall \gamma \in \{1, \dots, J-1\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\} \quad (3)$$

Constraints (4) control the workload of surgeries for each interval of the day and each room. The first term of constraints (4) calculates the total duration of surgeries that start and finish between  $\beta$  and  $\gamma$ . The second term computes the sum, over all surgeries starting before  $\beta$  and finishing after  $\gamma$ , of the part of the surgeries that overlap periods  $\beta$  to  $\gamma$ . The sum of the two terms must be less than or equal to the total duration of periods from  $\beta$  to  $\gamma$  if the room is open (plus overtime  $\varepsilon_{tr}$  if  $\gamma$  is the last period).

$$\sum_{i=1}^O \sum_{b=\beta}^{\gamma} \sum_{f=b}^{\gamma} p_i \cdot x_{itr}^{bf} + \sum_{i=1}^O \sum_{b=1}^{\beta-1} \sum_{f=\gamma+1}^J d_{tr}^{\beta\gamma} \cdot x_{itr}^{bf} \leq d_{tr}^{\beta\gamma} \cdot L_{tr} + u_\gamma \cdot \varepsilon_{tr}, \quad (4)$$

$$\forall \beta \in \{1, \dots, J\}, \quad \forall \gamma \in \{\beta, \dots, J\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\}$$

Constraints (5) state that a surgeon is using a room if there is at least one surgery scheduled for him/her in that room on that day, while constraints (6) state that a room is considered used on a day if there is at least one surgery scheduled in the



room for that day.

$$w_{itr} \leq U_{srt}, \quad \forall s \in \{1, \dots, S\}, \quad \forall i \in \lambda_s, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\} \quad (5)$$

$$w_{itr} \leq L_{tr}, \quad \forall i \in \{1, \dots, O\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\} \quad (6)$$

Constraints (7) indicate that a surgeon is only allowed to use rooms that he/she is assigned to, while constraints (8) ensure that each surgeon does not use more than one room each day.

$$U_{srt} \leq \delta_{srt}, \quad \forall s \in \{1, \dots, S\}, \quad \forall r \in \{1, \dots, R\}, \quad \forall t \in \{1, \dots, T\} \quad (7)$$

$$\sum_{r=1}^R U_{srt} \leq 1, \quad \forall s \in \{1, \dots, S\}, \quad \forall t \in \{1, \dots, T\} \quad (8)$$

Constraints (9)–(10) ensure that there is enough time to sterilise each kit between two uses at the OSU. Constraints (9) state that the number of kits used per day must respect the available number of these kits. Constraints (10) ensure that the kits used at the last period of day  $t$  are not used at the first and second periods of the next day  $t + 1$ .

$$\sum_{i=1}^O \sum_{r=1}^R q_{ik} \cdot w_{itr} \leq Q_k, \quad \forall t \in \{1, \dots, T\}, \quad \forall k \in \{1, \dots, K\} \quad (9)$$

$$\sum_{i=1}^O \sum_{r=1}^R q_{ik} \cdot \left( \sum_{b=1}^J x_{itr}^{bf} + \sum_{b=1}^2 \sum_{f=b}^J x_{i(t+1)r}^{bf} \right) \leq Q_k, \quad (10)$$

$$\forall t \in \{1, \dots, T - 1\}, \quad \forall k \in \{1, \dots, K\}$$

The expression of urgent kits is stated in constraints (11) and the expressions of priority kits (case 1 and 2) are stated in constraints (12)–(13).

$$\sum_{i=1}^O \sum_{r=1}^R q_{ik} \cdot \left( \sum_{f=2}^J \sum_{b=1}^f x_{itr}^{bf} + \sum_{b=1}^2 \sum_{f=b}^J x_{i(t+1)r}^{bf} \right) - Q_k \leq E_{ik}, \quad (11)$$

$$\forall t \in \{1, \dots, T\}, \quad \forall k \in \{1, \dots, K\}$$

$$\sum_{i=1}^O \sum_{r=1}^R q_{ik} \cdot \left( \sum_{f=1}^J \sum_{b=1}^f x_{itr}^{bf} + \sum_{b=1}^2 \sum_{f=b}^J x_{i(t+1)r}^{bf} \right) - Q_k - E_{ik} \leq Y_{ik}^1, \quad (12)$$

$$\forall t \in \{1, \dots, T\}, \quad \forall k \in \{1, \dots, K\}$$

$$\sum_{i=1}^O \sum_{r=1}^R q_{ik} \cdot \left( \sum_{f=2}^J \sum_{b=1}^f x_{itr}^{bf} + \sum_{b=1}^J \sum_{f=b}^J x_{i(t+1)r}^{bf} \right) - Q_k - E_{ik} \leq Y_{ik}^2, \quad (13)$$

$$\forall t \in \{1, \dots, T\}, \quad \forall k \in \{1, \dots, K\}$$

Next, constraints (14) ensure that all ambulatory surgeries finish before  $A_{max}$ . The first part of the left term of these constraints calculates the total duration of all surgeries that are assigned to finish in the first and second periods, and the second part adds the total duration of only the ambulatory surgeries that finish in the third period. This sum must be less than or equal

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to  $A_{max}$ . This is due to the fact that the third period ends after  $A_{max}$  so we consider that ambulatory surgeries finish before the others in that period.

$$\sum_{i=1}^O \sum_{b=\beta}^2 \sum_{f=b}^2 p_i \cdot x_{ir}^{bf} + \sum_{i=1}^O \sum_{b=\beta}^3 a_i \cdot p_i \cdot x_{ir}^{b3} \leq A_{max} - \sum_{j=1}^{\beta-1} d_{ir}^j, \quad (14)$$

$$\forall \beta \in \{1, \dots, 3\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\}$$

Constraints (15) states that no ambulatory surgeries finish at the fourth period.

$$\sum_{b=1}^J a_i \cdot x_{ir}^{bj} = 0, \quad \forall i \in \{1, \dots, O\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\} \quad (15)$$

Finally, constraints (16) impose that the overtime at each room at any day does not exceed  $\varepsilon_{max}$ .

$$\varepsilon_{ir} \leq \varepsilon_{max}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\} \quad (16)$$

The multiple objective functions (17) are taken into account by using a lexicographic method.

First, we minimise  $f_1$  the total overtime required to schedule all surgeries. We then add the resulting value of  $(\sum_{t=1}^T \sum_{r=1}^R \varepsilon_{ir})$  as an upper bound in our model and minimise  $f_2$  the total number of rooms opened without exceeding the overtime found for  $f_1$ . Finally, we set the resulting  $(\sum_{t=1}^T \sum_{r=1}^R L_{ir})$  as an upper bound in our model and minimise  $f_3$  the total penalty cost of urgent and priority kits without exceeding the total overtime found for objective  $f_1$  and the total number of rooms opened found for objective  $f_2$ .

$$\text{Minimize } Lex \left( f_1 : \sum_{i=1}^T \sum_{r=1}^R \varepsilon_{ir}; f_2 : \sum_{i=1}^T \sum_{r=1}^R L_{ir}; f_3 : \sum_{i=1}^T \sum_{k=1}^K [cu \cdot E_{ik} + cp \cdot (Y_{ik}^1 + Y_{ik}^2)] \right) \quad (17)$$

#### 4. Robust formulation

In the previous section, we presented a deterministic model to solve the problem. However, when comparing the difference between the achieved and planned durations for all the surgeries, only 987 surgeries out of the total 2069 surgeries (approximately 47.7%) had a real duration that differed from the planned duration by less than 20 minutes, while 939 surgeries (approximately 45.4%) had a difference between 20 min and 1 h. For the rest of the surgeries, the difference can reach up to 3 h. To deal with these large uncertainties in surgery durations, we propose to improve our approach with robust optimisation techniques.

Robust optimisation is a field of optimisation theory that deals with optimisation problems in which a certain measure of robustness is sought against uncertainty in the values of the problem parameters. Three main robust optimisation formulations were presented in the literature (Bertsimas and Sim 2004). The first is the formulation of Soyster (Soyster 1973), in which, the solution must be feasible for every possible scenario of the uncertain data. This formulation yields the highest protection 'Robustness' but at the expense of being extremely conservative. The second formulation (Ben-Tal and Nemirovski 2000) is less conservative than the formulation of Soyster but its main drawback is that it is a non-linear model which makes it unattractive for solving robust discrete optimisation models. The third and final formulation is that of Bertsimas and Sim (Bertsimas and Sim 2004). The main idea behind this formulation is to introduce a parameter that controls the level of 'robustness' against the level of conservatism of the solution based on the assumption that it is unlikely that all the uncertain parameters will take their maximum values.

The random variables that we will consider in our case represent the stochastic duration  $\tilde{p}_i$  of each surgery  $i$ . The value of  $\tilde{p}_i$  may vary in a given interval  $[\hat{p}_i - \hat{\sigma}_i, \hat{p}_i + \hat{\sigma}_i]$ , where,  $\hat{\sigma}_i$  represents the standard deviation parameter of the normal distribution function  $F(\hat{p}_i)$  for each surgery type and  $\hat{p}_i$  represents the central value. These values were calculated using 10 months' historical data for each surgery type and applied to all the surgeries in this type. Our goal is to protect against the maximum value of this random variable expressed as  $\hat{p}_i + \hat{\sigma}_i$ .

In our case, the formulation of Bertsimas and Sim guarantees that any solution is feasible for all the constraints if at most  $\Gamma$  surgeries reach their maximum duration in each room  $r$  and each day  $t$  over a full length day (8h45), assuming that the other surgeries take their central value. These surgeries are chosen so that they have the worst impact on the total duration of surgeries assigned to room  $r$  on day  $t$ , and our solution is then forced to be feasible for this realisation.

We use a ratio  $\Gamma^{bf}$  of this  $\Gamma$  for each periods pair  $(b,f)$ ,  $b \in \{1, \dots, J\}$ ,  $f \in \{b, \dots, J\}$  such that:

$$\Gamma^{bf} = (d_{bf} / d_{1J}) \cdot \Gamma \tag{18}$$

where:

$$d_{bf} = \max(d_{rt}^{bf}), \quad \forall r \in \{1, \dots, R\}, \quad \forall t \in \{1, \dots, T\} \tag{19}$$

In our deterministic model presented in Section 3, the two constraints concerned with the uncertainties are constraints (4) and (14). As explained before, we will force the feasibility in any solution given that at most  $\Gamma$  surgeries take their maximum duration each full day in each room by replacing these constraints respectively by:

$$\sum_{i=1}^O \sum_{b=\beta}^{\gamma} \sum_{f=b}^{\gamma} \bar{p}_i \cdot x_{ir}^{bf} + \hat{p}_{ir}^{\beta\gamma} + \sum_{i=1}^O \sum_{b=1}^{\beta-1} \sum_{f=\gamma+1}^J d_{ir}^{\beta\gamma} \cdot x_{ir}^{bf} \leq d_{ir}^{\beta\gamma} \cdot L_r + u_{\gamma} \cdot \varepsilon_r, \tag{20}$$

$$\forall \beta \in \{1, \dots, J\}, \quad \forall \gamma \in \{\beta, \dots, J\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\}$$

and:

$$\sum_{i=1}^O \sum_{b=\beta}^2 \sum_{f=b}^2 \bar{p}_i \cdot x_{ir}^{bf} + \hat{p}_{ir}^{\beta 2} + \sum_{i=1}^O \sum_{b=\beta}^3 a_i \cdot \bar{p}_i \cdot x_{ir}^{b3} + \hat{p}_{ir}^{\beta 3} \leq A_{max} - \sum_{j=1}^{\beta-1} d_{ir}^j, \tag{21}$$

$$\forall \beta \in \{1, \dots, 3\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\}$$

Where for each day  $t$ , each room  $r$ , each starting period  $\beta$  and finishing period  $\gamma$ ,  $\hat{p}_{ir}^{\beta\gamma}$  is the optimal value of the following linear program  $(\Omega_{ir}^{\beta\gamma})$ :

$$(\Omega_{ir}^{\beta\gamma}) = \text{maximise} \sum_{i=1}^O \left( \sum_{b=\beta}^{\gamma} \sum_{f=b}^{\gamma} \hat{p}_i \cdot x_{ir}^{bf} \right) \cdot z_i \tag{22}$$

$$\sum_{i=1}^O z_i \leq \Gamma^{\beta\gamma} \tag{23}$$

$$0 \leq z_i \leq 1, \quad \forall i \in \{1, \dots, O\} \tag{24}$$

We then consider the dual of problem  $(\Omega_{ir}^{\beta\gamma})$ :

$$\text{Minimize} \sum_{i=1}^O \eta_{ir}^{\beta\gamma} + \Gamma^{\beta\gamma} z_{ir}^{\beta\gamma} \tag{25}$$

$$\text{Subject to : } \eta_{ir}^{\beta\gamma} + z_{ir}^{\beta\gamma} \geq \sum_{b=\beta}^{\gamma} \sum_{f=b}^{\gamma} \hat{p}_i \cdot x_{ir}^{bf}, \quad \forall i \in \{1, \dots, O\} \tag{26}$$

$$\eta_{ir}^{\beta\gamma} \geq 0, \quad \forall i \in \{1, \dots, O\} \tag{27}$$

$$z_{ir}^{\beta\gamma} \geq 0 \tag{28}$$

Since problem  $(\Omega_{ir}^{\beta\gamma})$  is feasible and bounded for all  $\Gamma^{\beta\gamma} \in [0, |O|]$ , then by strong duality, the dual problem in (25) is also feasible and bounded and their objective values coincide. From this, the complete robust model formulation is obtained

from the deterministic model by replacing constraint (4) by:

$$\sum_{i=1}^O \sum_{b=\beta}^{\gamma} \sum_{f=b}^{\gamma} \bar{p}_i \cdot x_{ir}^{bf} + \sum_{i=1}^O \eta_{ir}^{\beta\gamma} + \Gamma^{\beta\gamma} z_{ir}^{\beta\gamma} + \sum_{i=1}^O \sum_{b=1}^{\beta-1} \sum_{f=\gamma+1}^J d_{ir}^{\beta\gamma} \cdot x_{ir}^{bf} \leq d_{ir}^{\beta\gamma} \cdot L_{ir} + u_{\gamma} \cdot \varepsilon_{ir}, \quad (29)$$

$$\forall \beta \in \{1, \dots, J\}, \quad \forall \gamma \in \{\beta, \dots, J\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\}$$

constraint (14) by:

$$\sum_{i=1}^O \sum_{b=\beta}^2 \sum_{f=b}^2 \bar{p}_i \cdot x_{ir}^{bf} + \sum_{i=1}^O \eta_{ir}^{\beta 2} + \Gamma^{\beta 2} z_{ir}^{\beta 2} + \sum_{i=1}^O \sum_{b=\beta}^3 a_i \cdot \bar{p}_i \cdot x_{ir}^{b3} + \sum_{i=1}^O a_i \cdot \eta_{ir}^{\beta 3} + \Gamma^{\beta 3} z_{ir}^{\beta 3}$$

$$\leq A_{max} - \sum_{j=1}^{\beta-1} d_{ir}^j, \quad \forall \beta \in \{1, \dots, 3\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\}$$
(30)

and by adding the following three constraints:

$$\eta_{ir}^{\beta\gamma} + z_{ir}^{\beta\gamma} \geq \sum_{b=\beta}^{\gamma} \sum_{f=b}^{\gamma} \bar{p}_i \cdot x_{ir}^{bf}, \quad (31)$$

$$\forall i \in \{1, \dots, O\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\}, \quad \forall \beta \in \{1, \dots, J\}, \quad \forall \gamma \in \{\beta, \dots, J\}$$

$$\eta_{ir}^{\beta\gamma} \geq 0, \quad \forall i \in \{1, \dots, O\}, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\}, \quad \forall \beta \in \{1, \dots, J\}, \quad \forall \gamma \in \{\beta, \dots, J\} \quad (32)$$

$$z_{ir}^{\beta\gamma} \geq 0, \quad \forall t \in \{1, \dots, T\}, \quad \forall r \in \{1, \dots, R\}, \quad \forall \beta \in \{1, \dots, J\}, \quad \forall \gamma \in \{\beta, \dots, J\} \quad (33)$$

### 5. Experimental results

#### Test data

In order to test our method, we used 10 test instances provided by Angers CHU. Each instance corresponds to the activity of the OSU during one month. In all instances, the number of rooms is 3 and the number of kit types is 212. Table 5 describes the other characteristics of the instances.

The values of the parameters are as follows:

- Latest time  $A_{max}$  for ambulatory surgeries is 3 p.m.
- Maximum overtime  $\varepsilon_{max}$  allowed each day is 3 h
- Penalty cost  $cu$  of urgent kits is equal to 5
- Penalty cost  $cp$  of priority kits is equal to 1

Table 5. Characteristics of the instances.

Instance	$T$	$O$	$S$
1	22	220	14
2	23	216	13
3	20	190	14
4	23	191	13
5	21	222	13
6	20	210	11
7	22	227	12
8	21	192	12
9	17	173	10
10	22	228	13

Table 6. Periods duration for the ORs.

$d_{rt}^j$	Rooms 1 & 2	Room 3
$d_{rt}^1$	5h45	5h45
$d_{rt}^2$	0h30	0h30
$d_{rt}^3$	1h00	0h00
$d_{rt}^4$	2h30	0h00

Note that the urgent and priority penalties have been evaluated by the CHU according to the disturbance and stress perceived at the SU.

In addition, Table 6 shows the duration of each period in the three ORs for day  $t$  when the rooms are open (0 when a room is closed).

The original data received from the CHU included the planned schedule (with planned surgery durations, i.e. the durations estimated by the surgeons at the consultation date), and the achieved schedule (with real surgery durations). In these schedules, some OSU surgeries were performed outside the unit. We, however, had to consider these surgeries due to their impact on the numbers of kits. In order to do so, we created a fourth room that is exclusive to these surgeries performed outside the unit and set the dates and times for these surgeries as in the original schedule in this new room. Furthermore, some surgeries from other units were performed at the OSU. We considered these surgeries by decreasing the duration of the corresponding periods at the surgery dates by the duration of each of these surgeries.

Finally, to use a process similar to that of the CHU, we first solve the problem starting with the planned durations of the surgeries and then in a second step we simulate the work process of the CHU by replacing the planned durations with the real durations. These two phases allow a fair comparison between our results and the real schedule of the CHU.

Our testing environment is:

- Intel® Core™ i3-2120 @ 3.30 GHz
- 8 GB of RAM
- IBM® ILOG® CPLEX® 12.5

#### Comparison of planned schedules

In order to test the performance of our models, we ran the experiments on the instances described in Table 5. We used the planned surgery durations in order to compare our planned schedules with the planned schedules of the CHU. For the robust model, we studied the data from the CHU to determine the value of  $\Gamma$  and found that on average approximately one surgery takes the worst duration scenario ( $\bar{p}_i + \bar{p}_i$ ) in each room during a full day. We, therefore, ran our experiments using the value  $\Gamma = 1$ . The time limits for these experiments were set to 1 h per objective (3 h per instance) for the deterministic model and 2 h per objective (6 h per instance) for the robust model.

Table 7 contains the number of violated constraints in the CHU planned schedule. In this table, the '#late amb's' column represents the number of ambulatory surgeries that end after 3 p.m. and the '#violated kits' column represents the total number of kits that do not respect the reuse conditions defined in Table 4. In the current work process of the CHU, this case may in fact occur because the staff members check that the number of medical devices used on that day does not exceed the number owned by the CHU, but they do not consider the surgeries assigned the day before. Even if this situation has to be strictly avoided, it still can occur as kits used after 3.30 p.m. on day  $t$ , may be required before 2.30 p.m. on day  $t + 1$  by sending a special shuttle to the SU outside the collection hours just to collect these kits. These constraints were violated regularly in the original data with an average of 7.6 late ambulatories and 10.2 violated kits per month, which have a negative impact on the performance of the CHU units and the SU in general, in addition to the delays in surgery times while waiting for the violated kits. In our solutions, no such constraints are violated as they are considered as hard constraints in the mathematical models.

Following the order of the multiple objectives, the tables below compare the three objective functions between the deterministic and robust planned schedules and that of the CHU. In these tables, the (\*) symbol represents an optimal solution found using the MILP.

Table 8 compares the total overtime between the three schedules. On average, the overtime in the schedules obtained by the deterministic method is 215.5 min (3 h 35 min) per month, while it is 804.1 min (13 h 24 min) per month in the original data. The average difference between these two schedules is 9 h 48 min per month, representing a decrease of approximately 73.2%. In the case of the robust planned schedule, the 'MILP value' column represents the overtime reported by the solver

Table 7. Violated constraints.

Instance	CHU planned schedule	
	#late amb	#violated kits
1	11	5
2	9	7
3	5	10
4	9	18
5	12	12
6	7	5
7	4	25
8	5	9
9	8	7
10	6	4
Average	7.6	10.2

Table 8. Overtime (Objective 1).

Instance	CHU planned schedule		Deterministic MILP planned schedule				Robust MILP planned schedule				
	overtime (in min)	max	LB	overtime (in min)	max	CPU (s)	LB	MILP value	overtime (in min)	max	CPU (s)
1	590	131	0	0*	0	16	0	153	87	17	7200
2	714	176	368	384	90	3600	369	517	313	131	7200
3	502	81	89	89*	23	1089	0	412	297	16	7200
4	656	77	314	318	94	3600	0	913	655	151	7200
5	1159	120	312	312*	123	58	98.9	1084	701	109	7200
6	1278	183	347	350	84	3600	0	491	232	53	7200
7	641	109	225	232	130	3600	0	430	199	48	7200
8	561	65	334	334*	106	1619	57.3	455	317	67	7200
9	870	105	136	136*	37	1031	0	157	62	19	7200
10	1070	266	0	0*	0	781	0	19	0	0	7200
Average	804.1			215.5					286.3		

which includes the added time that the MILP uses to ensure the robustness of the solution and the 'overtime' column represents the actual overtime without the added time by the solver. The average overtime in the robust planned schedules is 286.3 min (4 h 46 min) per month. The average difference between the robust schedules and those of the CHU is 8 h 37 min per month, representing a decrease of approximately 64.4%. In addition, the 'max' column shows the maximum overtime in minutes found in a day during the month considered. In six instances, the deterministic solution provided lower values than the CHU's solution, while the robust solutions provided lower values than the CHU's solution in eight instances. The 'LB' column shows the lower bound found by the MILP. These bounds represent the inevitable overtime caused by the fact that sometimes, the total durations of the surgeries assigned to some surgeons exceed the total duration of their shifts. We see that six solutions are optimal in the deterministic schedules, and the others are very close to the lower bound (only few more minutes). The same cannot be said about the robust approach with no optimal solutions found. This is due to the added complexity of the robust MILP which also affects the lower bound calculations as the model needs way more time to report more accurate bounds. Finally, the 'CPU' column represents the execution time for the models. The large improvements in the deterministic solutions happen in the first 20 min and very small improvements are found in the remaining time, while the robust model takes more time to find good solutions.

Table 9 shows the number of ORs opened with the deterministic and robust methods and in the original data and also compares the room occupancy rates in the three schedules. In the deterministic and robust approaches, we were able to reduce the number of rooms opened in 7 instances out of 10 compared with the schedule of the CHU. We were able to close approximately 1.9 ORs on average each month in the deterministic schedule while the conservative nature of the robust model can be seen in the lower number (1.6) of rooms closed on average each month. These freed rooms can be used to perform more surgeries. The 'LB' column shows the lower bound found by the solver concerning the rooms opened. The 'CPU' column represents the execution time for both models concerning only the rooms opened objective. In the seven instances where the time limit was reached in the deterministic MILP, the solver found the best solution in the first 20 min and was unable to find a better solution nor prove the optimality of the best found solution. We tried to increase this time

Table 9. Number of opened rooms (Objective 2).

Instance	CHU planned schedule			Deterministic MILP planned schedule					Robust MILP planned schedule				
	#opened rooms	Min Occ. rate	Avg. Occ. rate	LB	# opened rooms	CPU (s)	Min Occ. rate	Avg. Occ. rate	LB	# opened rooms	CPU (s)	Min Occ. rate	Avg. Occ. rate
1	59	27.6%	80.2%	55.1	58	3600	48.6%	81.7%	48.8	58	7200	64.6%	81.3%
2	59	13.8%	77.5%	52.3	55	3600	51.6%	82.4%	46.3	56	7200	19%	79.2%
3	48	18.3%	80.7%	44.6	46	3600	55.2%	85.4%	39.6	46	7200	43.7%	83.7%
4	48	42.4%	86.5%	46.8	48	3600	40.7%	87.4%	42.3	48	7200	75.6%	86.7%
5	59	28.8%	85.1%	55.2	58	3600	59.8%	83.9%	49.4	58	7200	71.9%	86%
6	52	51.2%	83.3%	52	52*	23	51.6%	83.2%	44.5	52	7200	55.2%	82.9%
7	59	34.3%	81.5%	55.3	59	3600	47.9%	84.1%	48.8	59	7200	39.6%	81.6%
8	49	26.1%	82.9%	48	48*	13	63.7%	85.5%	41.6	48	7200	70.4%	83.7%
9	46	11.6%	79.4%	43	43*	14	58.2%	86%	37.6	44	7200	64.1%	82.6%
10	60	9.5%	72.3%	50.1	53	3600	49.3%	84.5%	45.6	54	7200	55.4%	77.6%
Average	53.9		80.9%		52			84.4%		52.3			82.5%

Table 10. Number of urgent and priority kits (Objective 3).

Instance	CHU planned schedule		Deterministic MILP planned schedule			Robust MILP planned schedule		
	# urgent	# priorities	# urgent	# priorities	CPU (s)	# urgent	# priorities	CPU (s)
1	2	51	0*	0*	1199	1	7	7200
2	2	56	0	2	3600	0	5	7200
3	8	68	0	7	3600	3	8	7200
4	13	82	0	5	3600	1	6	7200
5	6	68	0	9	3600	4	2	7200
6	2	59	0	11	3600	1	4	7200
7	28	98	0*	0*	1165	0	11	7200
8	0	56	0*	0*	497	0	2	7200
9	2	53	0	1	3600	0	12	7200
10	3	35	0	2	3600	1	1	7200
Average	6.6	62.6	0	3.7		1.1	5.8	

limit to 12 h without any improvements. Consequently, setting the time limit for this objective for the deterministic approach to 30 min appears to be a good option. The robust MILP, however, could not prove optimality for any of the solutions after 3 h.

When comparing the occupancy rates, we observe approximately 84.4% on average in the deterministic solutions, 82.5% in the robust solutions and 80.9% for the CHU. In addition, the 'Min Occ. rate' column represents the minimum occupancy rate found in each schedule. In the deterministic schedule, 9 instances had a minimum occupancy rate greater than 45%, compared with seven instances in the robust schedule. Finally, in the case of the CHU, only one instance had a minimum occupancy rate greater than 45% with four instances below 20%. These values show that both solutions have better room load distribution than those of the CHU.

Finally, Table 10 shows the total number of urgent and priority kits obtained with our methods and those in the original data. In the original data, the average number of urgent kits is 6.6 per month and the average number of priority kits is 62.6 with the average number of problem kits (urgent + priority) approximately 69.2 per month. The deterministic model was able to eliminate all the urgent kits and provide an average of 3.7 priority kits per month, with a decrease of 94.65% from the CHU's solution in the total number of problem kits. In the case of the robust solution, the average number of urgent kits is 1.1 per month and the average number of priority kits is 5.8, with a decrease of 90.03% from the CHU's solution in the total problem kits number. The 'CPU' column represents the execution times for both model concerning only the urgent and priority kits objective. In the 3 instances where the deterministic model found an optimal solution, the average execution time is 953 s. Following the same behaviour as in the previous objectives, the deterministic model finds the large improvements in the solutions in the first 20 min and very small improvements during the remaining 40 min, while the robust model takes more time to find good solutions.

#### Comparison of achieved schedules

As the name suggests, the planned schedule relies on the estimated durations before the surgeries take place. In order to obtain the effective schedule based on the real durations, we developed an algorithm that simulates the work process of the OSU to create the corresponding achieved schedule from the planned schedule. Essentially, it consists in replacing the planned duration of each surgery with the real duration and inserting a break of 20 min between each two consecutive surgeries to clean the room (this break is already included in the planned durations). This algorithm provides a means of keeping track of the current available number of each kit type and the current state of each room. Note that the solution from our model indicates the date, room, and the period for each surgery without specifying the exact order of the surgeries within each period. After several tests, we found that there were no significant differences between different orders of the surgeries within each period. This is due to the small number of surgeries that start and end in the same period, leaving little possibility for improvement, so the default surgery order (from the original data) was kept.

Table 11 shows the achieved schedule of the CHU which represents what actually happened at the CHU, while Table 12 shows the achieved schedule obtained using the deterministic MILP and Table 13 shows the achieved schedule obtained using the robust MILP. By comparing these results we see that the robust model decreased the number of late ambulatories by approximately 45% compared with the deterministic model and by approximately 78.22% compared with the CHU's solution. Likewise, it managed to decrease the total overtime by approximately 45.95% compared with the deterministic model and by approximately 61.26% compared with the CHU's used overtime (savings of approximately 17 h 32 min



Table 11. CHU achieved schedule.

Instance	Violated constraints		Overtime		ORs			Kits	
	#late amb	#violated kits	total	max	#opened	Min Occ. rate	Avg. Occ. rate	#urgent	#priorities
1	14	5	1382	190	59	29.9%	79.2%	3	39
2	14	5	1692	227	59	12.9%	76%	3	44
3	6	10	1322	113	48	19%	79.3%	8	49
4	10	8	1800	98	48	44%	82.9%	14	64
5	14	5	2370	130	59	29.7%	81.9%	6	58
6	8	4	2140	201	52	50.8%	79%	1	53
7	6	13	1454	170	59	33.1%	80.6%	28	72
8	9	2	1584	100	49	23.7%	81.5%	4	46
9	10	7	1500	126	46	12.3%	77.3%	8	42
10	10	3	1930	326	60	9.9%	70.7%	4	27
Average	10.1	6.2	1717.4		53.9		78.8%	7.9	49.4

Table 12. Deterministic MILP achieved schedule.

Instance	Violated constraints		Overtime		ORs			Kits	
	#late amb	#violated kits	total	max	#opened	Min Occ. rate	Avg. Occ. rate	#urgent	#priorities
1	4	0	979	135	58	65.1%	87.8%	0	11
2	4	0	1268	189	55	31.4%	89.7%	0	24
3	3	0	1305	156	46	55.0%	89.8%	0	14
4	9	0	1745	130	48	53.9%	90.7%	1	17
5	5	0	1990	200	58	68.3%	89.1%	0	44
6	3	0	1034	206	52	71.4%	89.4%	0	34
7	1	0	965	125	59	48.1%	89.5%	5	31
8	3	0	1014	103	48	66.3%	91.4%	0	1
9	6	0	950	137	43	75.7%	88.8%	0	7
10	2	0	1060	240	53	54.3%	87.0%	0	6
Average	4	0	1231		52		89.3%	0.6	18.9

Table 13. Robust MILP achieved schedule ( $\Gamma = 1$ ).

Instance	Violated constraints		Overtime		ORs			Kits	
	#late amb	#violated kits	total	max	# opened rooms	Min Occ. rate	Avg. Occ. rate	#urgent	#priority
1	1	0	418	59	58	58.3%	88.0%	1	5
2	1	0	690	189	56	15.4%	86.2%	1	2
3	3	0	777	75	46	54.9%	89.9%	2	9
4	2	0	1041	133	48	64.3%	90.9%	0	4
5	7	0	1265	137	58	40.8%	88.8%	3	3
6	1	0	523	86	52	59.7%	89.4%	0	5
7	0	0	546	98	59	57.2%	89.7%	0	8
8	3	0	671	143	48	66.1%	90.5%	0	3
9	3	0	494	98	44	58.2%	87.0%	0	9
10	1	0	228	62	54	38.5%	83.1%	0	2
Average	2.2	0	665.3		52.3		88.4%	0.7	5

overtime compared with the 8 h 6 min savings obtained by the deterministic model). In addition, the 'max' column shows the maximum overtime found in a day during the month considered. In seven instances, the robust model managed to provide lower values than the deterministic model and the CHU's solution.

Moving forward, the number of rooms opened does not change since our method only uses rooms opened in the planned solution. Thus, the robust schedules still save 1.6 ORs on average each month and the deterministic schedules save 1.9 ORs. Next, the robust model managed to achieve an average occupancy rate for the rooms opened of approximately 88.4% which is less than the average in the deterministic solution (89.3%) since the deterministic solution uses fewer rooms and the rooms are more packed but better than the average for the CHU (78.8%). Similarly, the minimum occupancy rate found

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in the robust schedule in seven instances is greater than 50% (compared with 8 for the deterministic model and 1 for the CHU).

Concerning the last criterion, the robust model used approximately 0.7 urgent and 5 priority kits per month (5.7 problem kits per month), while in the deterministic results, 0.6 urgent and 18.9 priority kits were used on average (19.5 problem kits per month) and in the CHU's schedule, 7.9 urgent and 49.4 priority kits were used (57.3 problem kits per month). Thus, the robust model was able to decrease the number of urgent and priority kits by 70.8% compared with the deterministic model and approximately 90.1% compared with the CHU's solution.

If we compare the difference between achieved and planned schedules in both solutions, we see that the total overtime for all the months in the deterministic solutions increased from 2155 min in the planned schedule to 12,310 min in the achieved schedule (+471.2%), and from 2863 min to 6653 minutes (+132.3%) in the robust solutions. Similarly, the total number of problem kits (urgent + priorities) increased from 37 to 195 (+427%) in the deterministic solutions and decreased from 69 to 57 (-17.4%) in the robust solutions. Finally, in the robust achieved schedule, we found out that 22 ambulatory surgeries were scheduled after 3 p.m. (late ambulatories), while this number in the initial robust planned schedule was 0. At the same time, the change in the number of late ambulatory surgeries between planned and achieved schedules in the deterministic solutions increased from 0 to 40. Given these results, the robust model shows far more robustness against the uncertainties found in surgery durations than the deterministic model and it provides significant improvements compared with the actual planning of the CHU.

## 6. Conclusion and perspectives

This study focuses on a real surgical case scheduling problem including sterilising activity constraints, and three objective functions. We propose a deterministic mixed integer linear programming formulation and its robust counterpart which are solved in a lexicographic fashion. First, we solved the problem with our proposed approaches considering estimated surgery durations in order to create the planned schedule. Our results represent a significant improvement over those of the CHU in terms of overtime and urgent kits at the SU. We then proposed a method to simulate the OSU work process in day to day operations in order to generate the achieved schedule using real surgery durations in place of the estimated (planned) durations. Our deterministic achieved schedule still showed better results than those in the achieved schedule of the CHU. Despite the good results compared with the achieved schedules, there was a significant degradation between our planned and achieved schedules. This is due to the uncertainty in the real duration of the surgeries, caused by numerous factors (*e.g.* surgery complications). The robust model is far less sensitive to these uncertainties however. Finally, our results showed a decrease of 61.3% of the total used overtime, 90.1% of the total problem kits used (urgent and priority), and 78.2% of the total number of late ambulatory surgeries (performed after 3 p.m.) than the solutions implemented at the CHU. The next step is to jointly address the surgical case scheduling and the master surgical scheduling problems in order to improve the use and management of resources.

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