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# Thèse de Doctorat

Xiao YANG

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## Green hub location-routing problem for LTL transport

### JURY

Président : **M. Marc SEVAUX**, Professeur, Université de Bretagne Sud  
Rapporteurs : **M<sup>me</sup> Caroline PRODHON**, Maître de conférences HDR, Université de Technologie de Troyes  
**M. Mozart MENEZES**, Professeur, KEDGE Business School  
Directrice de thèse : **M<sup>me</sup> Nathalie BOSTEL**, Professeur, Université de Nantes  
Co-directeur de thèse : **M. Pierre DEJAX**, Professeur, IMT Atlantique  
Co-encadrant de thèse : **M. Marc PAQUET**, Professeur, École de Technologie Supérieure, Montréal



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## **Problem presentation and state of the art**



# Introduction

## 1.1 Background and motivation

In supply chain management and logistics systems, the transportation costs often represent an important part. The design of transportation network offers a great potential to reduce costs, time, the environmental impacts and improve service quality. However, the planning and operation of less-than-truckload (LTL) freight transportation networks are a challenge for transportation corporates because there are many origins (suppliers) and destinations (clients) with small demand to serve in the network. Direct transportation between origins-destinations would require plenty of vehicles which are often not fully loaded. In order to reduce the number of vehicles and fully use their capacity, it is an efficient option to locate one or several facilities called hubs in the network. The hubs collect, sort and consolidate the freight from many origins, then ship it to the destinations or transfer it to other hubs. The Hub Location Problem (HLP) is concerned with the design of a transportation network where suppliers and clients are in direct connection with a designated hub. The field of the HLP has been abundantly researched for more than thirty years with a large amount of works. These works has been classified and synthesized in the review papers of [Campbell \[1994\]](#), [Klincewicz \[1998\]](#), [Bryan and O'Kelly \[1999\]](#), [Klose and Drexl \[2005\]](#), [Alumur and Kara \[2008\]](#), [Campbell and O'Kelly \[2012\]](#), [Farahani et al. \[2013\]](#).

As opposed to the HLP, the Hub Location and Routing Problem (HLRP) corresponds to the design of a hub network system where the collection of goods from suppliers to a given hub and the distribution of goods from a destination hub to clients are organized through vehicle routing. The HLRP encompasses both the strategic and operational decision levels. From a total cost perspective, it includes the strategies of deciding the number of hubs to open and their location. At the operational decision level, it determines the assignment of origins and destinations to the open hubs, the flow transfer between hubs and planning of the pick-up/delivery routes. In the transport of freight, the collection and distribution of goods may be organized separately through distinct collection and delivery routes, or jointly, such as it is the case for postal services. The research on the problem of the hub location-routing is limited. Considering the wide range of applications of this problem offers many opportunities for research.

Recently, the concerns about the environmental impact on freight transport and goods operations have been mounting. It is predicted that over 80 % of the transport companies will be significantly influenced by the global warming, especially the CO<sub>2</sub> emissions, by the year of 2020 ([Piecyk and McKinnon \[2010\]](#)). Such fact indicates the importance of incorporating environmental factors into the logistics-related decisions. The emissions of GHG (Green House Gas) have already been considered in areas such as the Pollution Routing Problem (PRP) ([Barth et al. \[2005\]](#), [Xiao et al. \[2012\]](#), [Demir et al. \[2012\]](#), [Demir et al. \[2014\]](#) and [Kramer](#)



et al. [2015]). In the field of the HLRP, only the work of Mohammadi et al. [2013c] has introduced the environmental effects into the HLRP in the form of a multi-objective mixed integer linear programming model.

## 1.2 Thesis objectives

In this thesis, we consider the Capacitated Single Allocation Hub Location-Routing problem (CSAHLRP) with independent collection and delivery processes. We seek to address the hub location and vehicle routing strategies such that the location of hubs, the allocation of supplier/client nodes to hubs, the routings between nodes allocated to the same hub, as well as the inter-hub freight transportations, in order to achieve an efficient network design system.

The first objective of the thesis is to optimize the total network cost of the HLRP. To reach this objective, we propose a mixed integer linear programming model (MILP) for the CSAHLRP with aims at minimizing the total cost for the less-than-truck load (LTL) transport network. Computational experimentations are conducted with CPLEX solver on the basis of a set of instances of different sizes and characteristics which we have generated. Furthermore, we propose and experiment a memetic algorithm to solve large-size CSAHLRPs.

The second objective is to study and balance the relationship of cost and environmental effect of transport. We extend the single-objective HLRP model into a bi-objective MILP model for minimizing the total cost and CO<sub>2</sub> emissions. Experiments on small instances are conducted with CPLEX by means of the epsilon constraint method. To solve the bi-objective CSAHLRP, a modified memetic algorithm (MA) combined with a fast elitist non-dominated sorting genetic algorithm (NSGAII) and an efficient non-domination level update (ENLU) method is developed to exhibit approximations of the Pareto front.

At last, a two-step procedure is proposed to solve the single-objective HLRP based on a hub location problem (HLP) and two distinct vehicle routing problems for suppliers and clients allocated to each hub by the first step. Our single-objective MILP model is decomposed accordingly and our MA is adapted to solve the HLRP following these two steps.

A data base of instances of different sizes and characteristics has been developed in order to conduct extensive experiments for solving all these problems using the different solution techniques and validate our approaches.

## 1.3 Outline of the thesis

This thesis contains four parts : Part I includes a comprehensive state-of-the-art about the Hub Location Problem (HLP), the Location Routing Problem (LRP), the Hub Location-Routing Problem (HLRP) and also relevant works concerning the environmental factors and especially the consideration of CO<sub>2</sub> emissions of transport (Chapter 2). Part II addresses the single-objective HLRP (Chapters 3 and 5). Part III is devoted to the study on the bi-objective HLRP (Chapters 6 and 7). Finally, the two-phase method is presented and solved in Part IV (Chapters 8 and 9).

Chapter 2 provides a state-of-the-art for the HLP, the LRP and the HLRP, addressing the problem definitions, classifications and mathematical formulations, as well as the solution methods. Furthermore, since we extended the HLRP into a multi-objective problem with environmental considerations, related problems such as the Pollution Routing Problem (PRP) are briefly introduced. Finally, we propose a conclusion and identify several research directions.

Chapter 3 describes a mathematical model of the single-objective Hub-location Routing Problem. The model contains the decisions of the Capacitated Single Allocation Hub Location Problem (CSAHLRP) such as the determination of hub numbers, the decisions of hub location and flows exchange between hub points. It also integrates advanced vehicle routing formulations and decision variables, such as flow variable on vehicle, to schedule local tours for the HLRP.

Chapter 4 proposes a Memetic Algorithm (MA) for the single-objective HLRP, combining a genetic algorithm (GA) and an iterated local search (ILS), to determine location and routing jointly.

Chapter 5 presents in detail the computational experiments which we performed with two solution methods, solving the MILP with the CPLEX solver and the MA. The generation of data sets used for all the experiments is explained, as well as the parameters setting of the CPLEX and the MA. Then, the computational results of both methods are investigated and compared.

Chapter 6 investigates the impacts of CO<sub>2</sub> emissions on transport, both for the collection and delivery routing and for inter-hub transport, and integrated the CO<sub>2</sub> emission formulations into the single-objective model of the HLRP to construct a bi-objective model with aims at minimizing both cost and CO<sub>2</sub> emissions. Furthermore, to solve the bi-objective CSAHLRP, a memetic algorithm (MA) combined with a fast elitist non-dominated sorting genetic algorithm (NSGAI) is developed.

Chapter 7 presents the experimental results of the proposed bi-objective MA, which are compared with the results of the single-objective MA and solving the bi-objective MILP model with Epsilon Constraint (EC) method.

Chapter 8 proposes a two-phase method to solve the single-objective HLRP where the hub location and routes planing are considered sequentially in two phases. Adapted MILP models are proposed and the single-objective MA is adapted to solve the problems.

Chapter 9 analyzes the experimental results obtained by the two-phase MILPs solved with CPLEX and by the MA, and compares the results with the global single-objective HLRP models.

Finally, an overview of the main contributions of the thesis is summarized and some future research directions are proposed in Chapter 10.



## Literature review

Our researched Hub Location-Routing Problem (HLRP) can be considered as an integrated problem of the Hub Location Problem (HLP) and the Vehicle Routing Problem (VRP). It is also similar to the Many-to-Many Location-Routing Problem (MMLRP) which is a variant of the Location-Routing Problem (LRP). Thus it is necessary to give an overview of the two main close problems of the HLP and the LRP before researching the HLRP. This chapter provides the state-of-the-art for the HLP, the LRP and the HLRP in Sections 2.1, 2.2 and 2.3, addressing the problem definitions, classifications and mathematical formulations, as well as the solution methods. Furthermore, since we extended the HLRP into a multi-objective problem with environmental considerations, the related problems such as the Pollution Routing Problem (PRP) and the sustainable Supply Chain Network Design (SCND) are briefly introduced in Section 2.4. Finally, we make a conclusion and suggest several future research proposals in Section 2.5.

### 2.1 The Hub Location Problem

The Hub Location Problem (HLP) tackles the location of hub facilities and the assignment of customers to the hubs in the hub networks where the cost required for establishing hubs and transferring flows between hubs is lower than the cost required for transporting flows directly. The main features of the HLP (Campbell and O'Kelly [2012]) are : first, the demand is presented as the flows between the origin and destination (O-D) pairs instead of individual demands ; second, the flows transported via inter-hub benefit from a discounted cost ; last, the locations of the hubs and the allocation scheme are to be determined. The HLPs are a challenging topic since most of the problems are NP-hard even if the locations of the hubs are known (Alumur and Kara [2008]). The HLPs have been applied mainly to airlines and airport industries, postal delivery systems, supply chain management, freight transportations, telecommunication services and emergency services. More real-life applications of the HLPs can be found in Farahani et al. [2013].

The HLP and its variants have been widely researched since 1987 and several reviews have provided a comprehensive understanding in terms of fundamental definitions, classifications, hub network topologies, mathematical models, solution methods, uncertainty and competition, and so on. Campbell [1994] gave the first survey on the discrete HLPs. Klincewicz [1998] focused on the telecommunication industry and reviewed the facility location problem in the backbone/tributary network. Bryan and O'Kelly [1999] reviewed the Hub-and-Spoke (HS) networks in air transportation. Klose and Drexl [2005] provided a review of the facility location models for distribution system, especially the continuous location models and the network location models. Alumur and Kara [2008] classified the network hub location models and gave a review of the HLPs from the year of 1987 to 2007. In the article of Campbell and O'Kelly [2012], the origins of

the hub location especially in transportation were reflected and the research of the hub location problem for 25 years were presented. They provided an insight into recent research, discussing the shortages and promising directions. [Farahani et al. \[2013\]](#) highlighted the research works that are published after 2007 and summarized the new recent trend of the HLPs to take into account in the future.

In this section, we provide the main criteria for classifying the HLPs. Then some fundamental and important mathematical models are presented describing various types of the HLPs. Finally, the articles of the HLPs after the year 2012 are reviewed as a supplementary of the previously published survey papers.

### 2.1.1 Classification of the HLPs

In order to classify the hub location problems, various criteria can be used. One of the main classifying criteria is the way of determining the hub number. When the number of hubs to open is pre-specified, the problem is called the  $p$ -HLPs, and extremely, if just one hub is established, the problem is defined as the single-Hub Location Problems (single-HLPs). On the other hand, there are problems where the number of open hubs has to be determined as part of the solution. The second important criterion is based on the allocation schemes : the single allocation scheme in which each non-hub node must be allocated to only one selected hub node ; the multiple allocation where the non-hub nodes are allowed to be assigned to more than one hub. Recently, a  $r$ -allocation HLP was proposed ([Yaman \[2011\]](#), [Todosijević et al. \[2017\]](#)), allowing each non-hub node to be allocated to at most  $r$  hubs. Another important criterion takes the center and median objectives into account. The problem seeking to minimize total transportation cost (mini-sum) is the hub median problem while the problem with the objective of minimizing the maximal distance between a hub and the non-hub nodes (mini-max) is named the hub center problem. All the commonly used criteria are given as follows :

- The domain of candidate hub nodes : the discrete domain where the hubs are predetermined nodes ; the continuous domain where the solution domain is a sphere or plane ; the network where the candidate hub can be located at all the nodes of the network ;
- The certainty of the environment : the deterministic problem in which all the parameters are known ; the non-deterministic problem in which at least one parameter (such as travel time or customer demand) is stochastic ;
- The number of hub nodes to be located : pre-specified or not ;
- The allocation scheme : single allocation and multiple allocation ;
- The objective : center objective (mini-max) and median objective (mini-sum) ;
- The hub capacity : uncapacitated and capacitated ;
- The cost of locating hub nodes : fixed cost and no cost ;
- The cost of transportation between non-hub nodes and hub nodes : fixed cost, variable cost and no cost ;

There are also some other hub location problems such as : the dynamic HLP, including multiple time periods with various costs and adding new demand of O-D pairs in each time period ([Correia et al. \[2011\]](#)) ; the competitive HLP, considering the competition between multiple service providers ([Niknamfar et al. \[2017\]](#)) ; and the HLPs with reliability considerations ([Parvaresh et al. \[2013\]](#), [An et al. \[2015\]](#), [Azizi et al. \[2016\]](#)).

## 2.1.2 Mathematical models of the HLPs

### Single-Hub Location Problem

The first mathematical formulation in the research field of the HLP was proposed by O'Kelly [1987b]. It dealt with an Uncapacitated Single Allocation Hub Median Location Problem in the network domain. In this problem, the number of open hubs is specified as only one (Single-HLP). No establishing cost of the hub facility is considered.  $C_{ik}$  stands for the unit transportation cost between node  $i$  and hub  $k$ .  $O_i$  and  $D_i$  are the outgoing and ingoing flows from node  $i$ . The binary decision variable  $Y_{ik}$  is equal to 1 if node  $i$  is allocated to a hub located at node  $k$ , 0 otherwise.  $Y_{kk}$  equals to 1 means that the location of  $k$  is a hub node. The single-HLP is formulated as follows :

$$\min \sum_i \sum_k C_{ik} Y_{ik} (O_i + D_i) \quad (2.1)$$

Subject to

$$\sum_k Y_{kk} = 1 \quad (2.2)$$

$$\sum_k Y_{ik} = 1 \quad \forall i \quad (2.3)$$

$$Y_{ik} \leq Y_{kk} \quad \forall i, k \quad (2.4)$$

$$Y_{ik} \in \{0, 1\} \quad \forall i, k \quad (2.5)$$

The objective function (2.1) minimizes the total transferring cost via the hub. Constraints (2.2) indicate that there is only one hub in the network. Constraints (2.3) are the allocation constraints of the non-hub nodes. Constraints (2.4) ensure that non-hub nodes can only be connected to the hub node. Constraints (2.5) define the decision variables.

### Single Allocation $p$ -Hub Median Location Problem

The model of the Single Allocation  $p$ -hub Location Problem (SA $p$ HLP) is presented as an extension of the single-HLP of O'Kelly [1987b]. Here, the SA $p$ HLP is formulated as a quadratic integer model. The number of hubs to be located in the model is denoted by  $p$ . The demand between two non-hub nodes should be transferred through at least one and at most two hub nodes. The non-hub nodes are allocated to only one hub (single allocation). In addition, a discount parameter  $\alpha$  ( $0 \leq \alpha < 1$ ) denotes the economies of scale due to the inter-hub transfer.  $W_{ij}$  is the amount of flow between two nodes  $i$  and  $j$ . The SA $p$ HLP is formulated as follows :

$$\begin{aligned} & \sum_i \sum_k C_{ik} Y_{ik} \left( \sum_j W_{ij} \right) + \sum_k \sum_i C_{ki} Y_{ik} \left( \sum_j W_{ji} \right) \\ & + \alpha \sum_i \sum_j \sum_k \sum_m W_{ij} C_{km} Y_{ik} Y_{jm} \end{aligned} \quad (2.6)$$

Subject to

$$\sum_k Y_{ik} = 1 \quad \forall i \quad (2.7)$$

$$\sum_k Y_{kk} = p \quad (2.8)$$

$$Y_{ik} \leq Y_{kk} \quad \forall i, k \quad (2.9)$$

$$Y_{ik} \in \{0, 1\} \quad \forall i, k \quad (2.10)$$

Regarding the objective function (2.6), it minimizes the total transportation cost in the hub network. The first term stands for the transportation cost from non-hub node  $i$  to hub node  $k$  if node  $i$  is allocated to hub  $k$ . The second term stands for the transportation cost from hub node  $k$  to non-hub node  $i$  if node  $i$  is allocated to hub  $k$ . Moreover, the third term of the objective function calculates the transferring cost between two hubs  $k$  and  $m$  with a discount factor. Constraints (2.7) ensure the single allocation. Constraints (2.8) stipulate that exactly  $p$  hub nodes are selected. Constraints (2.9) enforce node  $i$  be allocated to a location  $k$  only if a hub is located at node  $k$ . Finally, Constraints (2.10) define the binary decision variables.

### Multiple allocation $p$ -Hub Median Location Problem

Campbell [1991] proposed a linear mathematical formulation for the  $p$ -hub median location problem. This problem is a multiple allocation problem in which every non-hub node could be allocated to more than one hub.  $C_{ij}^{km}$  is defined as the unit transportation cost starting from non-hub node  $i$ , transferring between hub nodes  $k$  and  $m$  and ending at non-hub node  $j$  (Formulation (2.11)).

$$C_{ij}^{km} = C_{ik} + \alpha C_{km} + C_{mj} \quad (2.11)$$

The non-negative allocation variables are denoted as  $X_{ij}^{km}$  ( $X_{ij}^{km} \geq 0$ ). Furthermore, the binary decision variables  $Y_k$  equal to 1 when a hub is located at node  $k$ , otherwise 0. The other assumptions are similar to the  $p$ -HLPs. The Multiple Allocation  $p$ -Hub Median Location Problem (MA $p$ HMLP) is formulated as follows :

$$\min \sum_i \sum_j \sum_k \sum_m C_{ij}^{km} W_{ij} X_{ij}^{km} \quad (2.12)$$

Subject to

$$\sum_k Y_k = p \quad (2.13)$$

$$\sum_k \sum_m X_{ij}^{km} = 1 \quad \forall i, j \quad (2.14)$$

$$X_{ij}^{km} \leq Y_m \quad \forall i, j, k, m \quad (2.15)$$

$$X_{ij}^{km} \leq Y_k \quad \forall i, j, k, m \quad (2.16)$$

$$X_{ij}^{km} \geq 0 \quad \forall i, j, k, m \quad (2.17)$$

$$Y_k \in \{0, 1\} \quad \forall k \quad (2.18)$$

The objective function (2.12) minimizes the total transportation cost. Constraint (2.13) ensure that exactly  $p$  hubs should be located. Constraints (2.14) indicate that each O–D pair  $(i, j)$  is allocated to the hub nodes  $k$  and  $m$ . Constraints (2.15) and (2.16) ensure the consistency between variables  $X_{ij}^{km}$  and  $Y_k$ . Constraints (2.17) and (2.18) define the decision variable types.

Instead of considering unit flow transportation cost between two nodes, Campbell [1994] suggested a

fixed link cost  $g_{ik}$  to connect the non-hub nodes to the hub nodes. The binary variables  $Z_{ik}$  equal to 1 if a link between the non-hub node  $i$  and the hub node  $k$  is built. The cost term (objective function) is provided below :

$$\sum_i \sum_k g_{ik} Z_{ik} \quad (2.19)$$

The constraints are similar to the MA $p$ HMLP (Constraints (2.13) to (2.18)). Besides, the model also introduces the capacity of hubs to limit the flows allocated to each hub. That is, the flows going through each hub must fit the capacity of the hub. The capacity of the hub  $k$  is denoted as  $h_k$ . The constraints (2.20) should be considered in the  $p$ -hub median location problem :

$$\sum_m \sum_i \sum_j W_{ij} X_{ij}^{km} + \sum_s \sum_i \sum_j W_{ij} X_{ij}^{sk} \leq h_k Y_k \quad \forall k \quad (2.20)$$

In Constraints (2.20), the first and second terms represent the total incoming and outgoing flows for the hub  $k$ .

### Continuous $p$ -Hub Location Problem

In the continuous  $p$ -HLPs, hub nodes are located based on a plane or a sphere (O'Kelly [1986], Aykin and Brown [1992]) and each non-hub node must be allocated to exactly one hub (single allocation). The  $p$  hub nodes to be located are fully interconnected. The objective of the model is to minimize the total cost. At least one and at most two hub nodes should be visited for transferring flows of each O-D pair. The hubs are uncapacitated and no fixed establishing hub cost is considered.

The notation of the problem is similar to the SA $p$ HMLPs. Moreover, a vector  $N_i$  stands for the location vector of non-hub node  $i$ ; the decision variable  $P_k$  denotes the location vector of hub node  $k$  ( $k = 1, \dots, p$ );  $d(i, j)$  is considered as the Euclidean distance between two nodes  $i$  and  $j$ . The continuous  $p$ -HLP is formulated as follows :

$$\min \sum_i \sum_j \sum_k \sum_m W_{ij} Y_{ik} Y_{jm} (d(N_i, P_k) + \alpha d(P_k, P_m) + d(N_j, P_m)) \quad (2.21)$$

Subject to

$$\sum_k Y_{ik} = 1 \quad \forall i \quad (2.22)$$

$$P_k = (a_k, b_k) \quad k = 1, \dots, p \quad (2.23)$$

$$Y_{ij} \in \{0, 1\} \quad \forall i, j \quad (2.24)$$

The objective function (2.21) minimizes the total transportation cost in the hub network. The first term is the transportation cost originated from the non-hub node  $i$  to the hub node  $k$ , and the second term is the transportation cost between hub nodes, finally the third term is the transportation cost generated from the hub node  $k$  to the non-hub node  $m$ . Constraints (2.22) indicate the single allocation. Constraints (2.23) and (2.24) define the decision variables.

### $p$ -Hub Center Location Problem

The  $p$ -Hub Center Location Problem aims to minimize the maximum distance (cost) of origin-destination (O-D) pairs. The real-life applications of this problem exist in the emergency facility location problem and the perishable commodities transportation problem (Campbell [1994]). Note that the constraints and notation of the problem are similar to the MA $p$ HMLPs (constraints (2.13) to (2.18)) except for the objective



function which is formulated as follows :

$$\min \max_{i,j,k,m} C_{ij}^{km} W_{ij} X_{ij}^{km} \quad (2.25)$$

The objective function (2.25) minimizes the maximum transportation cost of O-D pairs. For other improved models of the  $p$ -Hub Center Location Problems, the articles of [Kara and Tansel \[2000\]](#), [Yaman and Elloumi \[2012\]](#), [Campbell et al. \[2007\]](#) and [Ernst et al. \[2009\]](#) are recommended.

### Hub Covering Location Problems

The classic Hub Covering Location Problem seeks to find the best locations for hubs so as to ensure each O-D pair is covered by a pair of hub nodes ([Campbell \[1994\]](#)). Each O-D pair is covered if the cost (time, etc.) is lower than or equal to a pre-specified value of  $\gamma_{ij}$  (Constraint (2.26)).

$$C_{ij}^{km} \leq \gamma_{ij} \quad (2.26)$$

The Hub Covering Location Problem can be further developed into three other variants : the  $p$ -Hub Covering Location Problem, the Hub Set Covering Location Problem and the  $p$ -Hub Maximal Covering Location Problems.

Regarding the  $p$ -Hub Covering Location Problem, the only difference compared to the classic ones is the pre-specified number of open hubs.

The Hub Set Covering Location Problem deals with the selection of open hubs which are subjected to a fixed establishing cost, and the allocations of O-D pairs to the hubs. The notation of the problem is given as :  $F_k$  is the fixed cost of selecting node  $k$  as a hub ;  $C_{ij}^{km}$  is the transfer cost from the origin node  $i$  to the destination node  $j$  via hubs located at nodes  $k$  and  $m$  ;  $\gamma_{ij}$  stands for the maximum cost allowed for covering links connecting the nodes  $i$  and  $j$  ;  $Y_k$  is the location variable and  $V_{ij}^{km}$  is equal to 1 if the hubs  $k$  and  $m$  cover the O-D pair  $(i, j)$ , otherwise 0. The Hub Set Covering Location Problem is formulated as follows :

$$\min \sum_k F_k Y_k \quad (2.27)$$

Subject to

$$\sum_k \sum_m V_{ij}^{km} X_{ij}^{km} \geq 1 \quad \forall i, j \quad (2.28)$$

$$X_{ij}^{km} \leq Y_m \quad \forall i, j, k, m \quad (2.29)$$

$$X_{ij}^{km} \leq Y_k \quad \forall i, j, k, m \quad (2.30)$$

$$X_{ij}^{km} \geq 0 \quad \forall k \quad (2.31)$$

$$Y_k \in \{0, 1\} \quad \forall k \quad (2.32)$$

The objective of the model is to minimize the total cost of opening new hubs (Formula (2.27)). Constraints (2.28) impose that an O-D pair is covered by at least one hub pair. Constraints (2.29) and (2.30) ensure that the origin node  $i$  and destination node  $j$  can only be allocated to the nodes of  $k$  and  $m$  that are selected as hubs. Constraints (2.31) and (2.32) define the decision variables.

The  $p$ -Hub Maximal Covering Location Problems is devoted to locate  $p$  hubs to maximize the demands within a coverage distance. The fixed cost of opening hubs is not considered in the model. This problem is formulated as follows :

$$\max \sum_i \sum_j \sum_k \sum_m W_{ij} V_{ij}^{km} X_{ij}^{km} \quad (2.33)$$

The objective function (2.33) maximizes the amount of transportation demand covered. The constraints are the same as in the MA $p$ HMLP from (2.15) to (2.18).

### Multi-objective $p$ -Hub Location Problem

The cost and service, as well as sustainable considerations, are of great importance in hub networks designing. The services regarded in many articles include the service time (da Graça Costa et al. [2008]), the travel time (Mohammadi et al. [2013a], Ebrahimi Zade and Lotfi [2015], Yang et al. [2017]) and the responsiveness and social aspects (Zhalechian et al. [2017a]). The environment aspects in literature mainly include the pollution gas such as CO<sub>2</sub> emissions (Mohammadi et al. [2014], Ghodratinama et al. [2015]).

The multi-objective HLP model presented was proposed by da Graça Costa et al. [2008] with two objectives : minimizing the transportation cost and the maximum service time of the hub nodes. It is an uncapacitated single allocation  $p$ -HLP and the hub nodes are fully connected. At least one and at most two hub nodes should be visited for transferring flows between two non-hub nodes. There is no fixed cost to initiate service at hub nodes. A new notation of  $T_k$  is introduced which stands for the operating time of hub  $k$  to process one unit flow. The multi-objective HLP is formulated as follows :

$$\min \sum_i \sum_j \sum_k \sum_m W_{ij} Y_{ik} Y_{jm} (C_{ik} + \alpha C_{km} + C_{jm}) \quad (2.34)$$

$$\min \max_k \{T_k (\sum_i \sum_j W_{ij} Y_{ik} + \sum_i \sum_j W_{ji} Y_{jm} Y_{ik})\} \quad (2.35)$$

The first objective function (2.34) minimizes the total transportation cost in the hub network. The second objective function (2.35) minimizes the maximum service time of hub nodes. The constraints are the same as in the SA $p$ HMLP (Constraints (2.7) to (2.10)).

### Capacitated Single Allocation Hub Location Problem

Regarding the models of the Capacitated Single Allocation Hub Location Problem (CSAHL), Ernst and Krishnamoorthy [1999] provided an integer linear formulation with an inter-hub flow fraction variable and an allocation variable. Correia et al. [2011] considered the hub capacity as another decision variable with varied fixed installing costs corresponding to relevant hub capacity. They proposed several mixed integer formulations and a set of preprocessing tests to reduce the formulation size. Saiedy et al. [2011] presented an improved model of CSAHL with  $n$ -hub centers with less indices and constraints to get the model faster to solve by CPLEX for small and medium instances. Karimi et al. [2014] imposed the hub capacity constraint on the single hierarchical hub median location problem and studied the effect of hub capacity on total costs based on the Iranian Airport Data (IAD).

Here, we describe the model proposed by Ernst and Krishnamoorthy [1999]. Assume that in the postal delivery network, mail from postal districts (nodes in the network) has to be collected by a mail sorting center (hub in the network) with a limited sorting ability (hub capacity). The model deals with the number and location of hubs and allocations of non-hub nodes to hubs in order to minimize the total network cost.

The CSAHL in a postal delivery network is defined on a complete graph  $G = (N, A)$  in which  $N$  is the set of  $n$  nodes and  $A$  is the set of arcs. The decision variable  $z_{ik} \in \{0, 1\}$  equals to 1 if the non-hub node  $i$  is allocated to a hub  $k$ , and 0 otherwise. If one hub is located at node  $k$ ,  $z_{kk} = 1$ , otherwise 0. Another decision variable  $Y_{kl}^i$  is defined as the total amount of commodity flows from the non-hub  $i$  transferred between the hubs  $k$  and  $l$ . Further,  $\alpha$ ,  $\beta$  and  $\gamma$  are denoted as the unit transporting cost of inter-hub transfers, pick-ups and deliveries, respectively.  $O_i = \sum_{j \in N} q_{ij}$  and  $D_i = \sum_{j \in N} q_{ji}$  represent the demand of the pick-up and the delivery of the node  $i$ .  $d_{ij}$  is the distance from node  $i$  to node  $j$ .  $\Gamma_k$  is the capacity of hub  $k$ . The fixed locating cost is denoted as  $F_k$ . Then the formulation of the CSAHL is given as

$$\min \sum_{i \in N} \sum_{k \in N} d_{ik} z_{ik} (\beta O_i + \gamma D_i) + \sum_{i \in N} \sum_{k \in N} \sum_{l \in N} \alpha d_{kl} Y_{kl}^i + \sum_{k \in N} F_k z_{kk} \quad (2.36)$$

Subject to

$$\sum_{k \in N} z_{ik} = 1 \quad \forall i \in N \quad (2.37)$$

$$z_{ik} \leq z_{kk} \quad \forall i, k \in N \quad (2.38)$$

$$\sum_{i \in N} O_i z_{ik} \leq \Gamma_k z_{kk} \quad \forall k \in N \quad (2.39)$$

$$\sum_{l \in N} Y_{kl}^i - \sum_{l \in N} Y_{lk}^i = O_i z_{ik} - \sum_{j \in N} q_{ij} z_{jk} \quad \forall i, k \in N \quad (2.40)$$

$$z_{ik} \in \{0, 1\} \quad \forall i, k \in N \quad (2.41)$$

$$Y_{kl}^i \geq 0 \quad \forall i, k \in N \quad (2.42)$$

Objective function (2.36) minimizes the collection and delivery cost, the inter-hub transportation cost and the fixed cost of locating hubs. Constraints (2.37) are the single allocation constraint. Constraints (2.38) ensure that the non-hub nodes are allocated to a hub only if it is open. Constraints (2.39) limit total collection demand on each hub. Constraints (2.40) inquire that the supply at the nodes is determined by the allocations  $z_{ik}$ . Constraints (2.41) and (2.42) specify the decision variables.

### 2.1.3 Data sets of the HLPs

In the hub location research, two data sets are commonly used : the AP (Australian Post) data set and the CAB (U.S. Civil Aeronautics Board) data set. They provide valuable benchmarks derived from real-world applications of various scales and have been applied in most of the hub location literature. The CAB data set is based on the airline passenger traffic between 25 US cities in 1970 and was first introduced for hub location in O'Kelly [1987b]. The AP data set (Ernst and Krishnamoorthy [1996]) is derived from the Australian postal delivery network with up to 200 nodes of postal districts in Sydney, Australia. The flow matrix of the CAB is symmetrical while the flow matrix of the AP is not. Another data set used in the literature is the Turkey postal (PTT) data set for ground transportation between 81 cities in Turkey. This data set was introduced in Tan and Kara [2007] to deal with the latest arrival hub covering problem, minimizing the number of hubs within a predetermined time bound. Meyer et al. [2009] created random instances with up to 400 nodes which are called URAND data sets for the single allocation p-hub center problem. Based on the AP data set with 200 nodes, Silva and Cunha [2009] generated four larger instances with 300 and 400 nodes and applied reduction factors (0.9 and 0.8) to the fixed costs of hubs. Table 2.1 shows the details of the data sets and the first reference in the field of the HLP.

TABLE 2.1 : Data sets of the HLPs

Acronym	First reference	Maximum number of nodes	Size <sup>1</sup>	Parameter <sup>2</sup>	Resource
CAB	O'Kelly [1987b]	25	2-4/10-25	$\alpha = \{0.2, 0.4, 0.6, 0.8, 1.0\}$ $\beta = \gamma = 1$	U.S. airline passenger traffic
AP	Ernst and Krishnamoorthy [1996]	200	2-20/10-200	$\alpha = 0.75, \beta = 3$ $\gamma = 2$	Australian postal delivery network
PTT	Tan and Kara [2007]	81	-	$\alpha' = \{0.2, 0.4, 0.6, 0.8, 1.0\}$	Turky national postal service (PTT)
URAND	Meyer et al. [2009]	400	2-5/100-400	$\alpha = 0.75$	Random
Silva	Silva and Cunha [2009]	400	2-5/300-400	$\alpha = 0.75, \beta = 3$ $\gamma = 2$	Based on AP data set with 200 nodes

<sup>1</sup> Structure of the instance size : min-max number of facilities/ min-max number of customers

<sup>2</sup>  $\alpha$  : inter-hub cost coefficient;  $\beta$  : pick-up cost coefficient;  $\gamma$  : delivery cost coefficient;  $\alpha'$  : inter-hub travel time coefficient

### 2.1.4 State-of-the-art of the HLPs in recent five years

In this part, the main research of the HLPs covering the period from 2013 to 2017 are reviewed as a complementary of the previous survey articles.

#### (1) Uncapacitated HLPs

It is seemed that the uncapacitated HLPs are the main focus in the field of HLPs in recent years. [Marić et al. \[2013\]](#) designed a Memetic Algorithm (MA) with two local search heuristics for solving the Uncapacitated Single Allocation Hub Location Problem (USAHLP). The experimental results based on the Civil Aeronautics Board and Australian Postal (CAB/AP) data sets ([Ernst and Krishnamoorthy \[1999\]](#)) showed the superiority of the proposed MA over existing heuristic approaches for solving the USAHLP. [Abyazi-Sani and Ghanbari \[2016\]](#) proposed an efficient Tabu search (TS) with several new tabu rules for the USAHLP. The computational experiments were conducted on all the CAB/AP data sets and also the data set with 300 and 400 nodes created by [Silva and Cunha \[2009\]](#). Compared with recently proposed approaches, the results showed that the proposed TS was able to find all the optimal and best benchmark solutions. More importantly, it also decreased the computational time.

[Kratika \[2013\]](#) dealt with the Uncapacitated Multiple Allocation  $p$ -hub Median Problem (UMAPHMLP) by an Electromagnetism-like method (EM). The EM was able to find excellent results on large-scale instances of the standard AP data set with up to 200 nodes and the instances with up to 400 nodes which was created by [Meyer et al. \[2009\]](#). Later, they extended this method to solve the capacitated version of the problem. [Bailey et al. \[2013\]](#) proposed a new solution method based on a Particle Swarm Optimization (PSO) to solve the UMAPHMLP. To solve the Uncapacitated Single Allocation  $p$ -Hub Location Problem (USAPHLP), [Rasoulinejad et al. \[2013\]](#) introduced a Tabu Search (TS) and applied a clustering method to improve the performance of the TS. Recently, [Meier \[2017\]](#) introduced the integer variables for the number of used vehicles into the Uncapacitated Single Allocation  $p$ -Hub Median Location Problem (USAPHMLP). A mixed integer program formulation with fewer variables but more constraints were proposed to form a more precise model for the problem.

Regarding the Uncapacitated Hub Location Covering Problem, [Peker and Kara \[2015\]](#) extended the definition of coverage and introduced “partial coverage” which changes with distance. They studied the “partial converge” of the single and multiple  $p$ -Hub Maximal Covering Problem by relaxing the definition of the binary coverage. The  $p$ -Hub Maximal Covering Problem aims to locate  $p$  hubs within a coverage distance so as to maximize the demands. [Silva and Cunha \[2017\]](#) developed a TS heuristic for the Uncapacitated Single Allocation  $p$ -Hub Maximal Covering Problem to solve to optimality the instances with up to 50 nodes from the AP data set. Furthermore, the proposed TS was also able to solve all instances of the AP data set with up to 200 nodes in shorter CPU times. [Ebrahimi-Zade et al. \[2016\]](#) proposed a dynamic model with flexible covering radius for the multi-period Hub Set Covering Problem and solved it by a modified Genetic Algorithm (GA) based on a real-world case study.

[Damgacioglu et al. \[2015\]](#) coped with the uncapacitated single allocation Planar Hub Location Problem (PHLP) in which the solution domain of the problem is a plane. A mathematical formulation was proposed for the PHLP and a Genetic Algorithm (PHLGA) was developed to solve the problem.

#### (2) Capacitated HLPs

Compared to its uncapacitated version, capacitated HLPs were not so extensively studied. [Correia et al. \[2014\]](#) considered the multiple products transportation of the capacitated single-allocation hub location problem (CSAHLP). Two cases were considered : (1) the hubs can handle only one product or (2) the hubs can handle all the types of products. Later, [Stanojević et al. \[2015\]](#) proposed a hybrid optimization method, consisting of an Evolutionary Algorithm (EA) and a Branch-and-Bound (B& B) method to solve the CSAHLP and tested it on the standard Australia Post (AP) hub data sets with up to 300 nodes.

#### (3) Variant HLPs

Some researches extend the HLPs with more realistic variants in the real-world such as the unpredictable hub disruptions ([Parvaresh et al. \[2013\]](#), [An et al. \[2015\]](#), [Azizi et al. \[2016\]](#)), the multiple products ([Correia et al. \[2014\]](#)), the multimodal hubs ([SteadieSeifi et al. \[2014\]](#), [Serper and Alumur \[2016\]](#)), the hubs which are not fully linked ([Martins de Sá et al. \[2015a\]](#), [Martins de Sá et al. \[2015b\]](#)), the serving orders ([Puerto](#)

et al. [2011], Puerto et al. [2013])) and so on.

One of the interest of the variant HLPs in recent years is the HLP with unpredictable hub disruptions. This problem considers the hub disruptions which may be caused by natural disasters, labour dispute and weather conditions. Parvaresh et al. [2013] formulated a bi-level game model for the Multiple Allocation  $p$ -Hub Median Problem under intentional disruptions. Two algorithms based on the Simulated Annealing (SA) were defined to solve the problem. An et al. [2015] proposed a set of reliable Hub-and-Spoke (H&S) network design models for both single and multiple allocation schemes considering disruptions and alternative routes. A Lagrangian relaxation (LR) algorithm with variable fixing and a Branch-and-Bound (B&B) method was implemented to solve the CAB instances. Azizi et al. [2016] assumed that if a hub was disrupted, the entire demand initially served by this hub was handled by a backup facility. They proposed a mathematical formulation to minimize the regular and hub fail transportation cost. A genetic algorithm (GA) was implemented to test large instances of the CAB and Turkish Postal System.

Saboury et al. [2013] proposed an advanced mathematical programming formulation for a specific application of the HLP, namely the Hub Location Problem with fully interconnected backbone and access networks. Two hybrid heuristics incorporating a Variable Neighborhood Search (VNS) into the SA and TS were developed to obtain the optimal solutions of 24 small instances from the literature. Furthermore, some newly generated medium and large instances were solved efficiently in a quite short CPU time.

In the articles of Martins de Sá et al. [2015a]a and Martins de Sá et al. [2015b]b, a hub network where not all the hub facilities were fully interconnected was considered. The problem was called the Hub Line Location Problem (HLLP) in which the locations of all the selected hubs were linked together by means of a path or a line. The problem coped with the minimization of the total travel time for O-P pairs. A Benders-Branch-and-Cut algorithm and several heuristic algorithms are applied to solve the problem.

Puerto et al. [2016] focused on the Capacitated Single Allocation Ordered Median Hub Location problem (CSA-OMHLP) which was originally from Puerto et al. [2011] and Puerto et al. [2013]. The Ordered Median Hub Location problem introduced the ordinal information by applying rank dependent compensation factors on routings from the origin nodes to the hubs. They presented a new formulation with two preprocessing phases and a Branch-and-Bound-and-Cut (B&B&Cut) based algorithm to solve the model. The tests on AP data set proved that the proposed method gave good solutions in competitive running times.

Serper and Alumur [2016] considered the intermodal hub networks by introducing alternative transportation modes and vehicle types. Furthermore, they incorporated decisions on designing the inter-hub network that the hubs are not fully interconnected. A variable neighbourhood search algorithm was developed and tested on the Turkish network and CAB data sets. Such study is seemed to be so far the first hub location study in the literature determining the optimal number of various types of vehicles to operate the intermodal hub network.

### (3) The HLPs with uncertainty

Recently, there is a new trend in the research of the HLPs taking into account the presence of uncertainty. Uncertainty can be regarded as an unforeseen and unpredictable situations of a system due to the flaws of human knowledge (Guzmán et al. [2016]). The uncertainty in the hub networks may occur in flows, costs, times and hub operations.

Yang et al. [2013b] presented a  $p$ -Hub Center Problem with fuzzy travel times. They adopted the Value-at-Risk (VaR) criterion in the formulation and developed a hybrid algorithm of Genetic Algorithm (GA) and local search. The numerical experiments showed that the proposed algorithm outperformed the LINGO solver and the standard GA. Mohammadi et al. [2016] studied a bi-objective Single Allocation  $p$ -Hub Center-Median Problem (BS $p$ HCMP) and assumed that the flows, costs, times and hub operations were under uncertainty. A fuzzy-queuing approach was used to model the uncertainties and an Evolutionary Algorithm (EA) based on game theory and Invasive Weed Optimization (IWO) algorithm was developed. The overview of the literature on the fuzzy models and resolution methods was given by Guzmán et al. [2016].

Talbi and Todosijević [2017] focused on the uncertainty in the flow for the Uncapacitated Multiple Allocation  $p$ -Hub Median Problem (UMAP $p$ HMP). They formulated a deterministic UMAP $p$ HMP model and then introduced uncertainty into the model by assuming that the flows to be transferred are subject to

uncertainty. A new robustness measure was proposed in order to build a solution which is robust for any realization of the flow. Furthermore, a Variable Neighborhood Search (VNS) was developed to evaluate the performance of the robustness measures on benchmark  $p$ -hub instances.

The studies of the HLPs are presented in Table 2.3 and 2.4. Table 2.3 gives a review of the HLPs without uncertainty while Table 2.4 shows the HLP references taking into account the uncertainties. To introduce various kinds of HLPs properly, The meanings of different notation are explained in Table 2.2.

TABLE 2.2 : Notations for different HLPs

Capacity of hubs	Allocation scheme	Type of HLPs	Number of hubs
Capacitated (C)	Single allocation (SA)	Median (M)	Single (1)
Uncapacitated (U)	Multiple allocation (MA)	Center (T)	More than one ( $p$ )
	r-allocation (RA)	Covering (V)	
		Set covering (SV)	
		Maximum covering (MV)	
		Line location (HLLP)	
		Order Median (OM)	
		Sustainable (SHLP)	

### 2.1.5 Conclusion

In this section, we include a summary of the main classifications of the hub location problems. The fundamental mathematical formulations for various hub location problems are also presented. Furthermore, we provide a concise overview of the main developments and most recent trends in hub location problems by reviewing the researches in recent five years including exact methods, heuristic and metaheuristic solution methods. The described research shows that the field of hub location is moving towards new directions in terms of considering various realistic problem variants.

## 2.2 The Location-routing Problems

The objective of the Location-Routing Problem (LRP) is to serve customers at minimum total cost by jointly determining the locations of candidate facilities and constructing an associated set of vehicle routes. It encompasses two NP-hard problems : the classic Facility Location Problem (FLP) and the Vehicle Routing Problem (VRP). In special cases, if all customers are served by a depot through direct transportation, the LRP becomes a standard location problem. On the other hand, if the locations of the facilities are pre-specified, the LRP reduces to a VRP (Nagy and Salhi [2007]). Wide real-life applications of the LRPs have been studied in terms of locating capacitated urban distribution centers, consumer goods distributions, postal or parcel deliveries and so on (Drexler and Schneider [2015]). For example, Menezes et al. [2016] developed an easily implementable rough-cut approach and implemented it to solve two real life cases : the supermarket chain of Casino Group in southeast France and the household material recycling network in Calgary of Canada. This methodology was based on a pragmatic transformation of the distances between the candidate locations and the demand nodes. It divided the LRP in two sub-problems : locating a collection of regional distribution centres (RDCs) to minimize the total travelled distance ; finding the optimal routes for the configuration obtained in the first stage (VRP).

The LRPs have been studied for decades since 1998 with a significant amount of literature on the general LRPs and their variant problems. Several survey papers of the LRPs have given an overview of the literature from different perspectives. It seems that Balakrishnan et al. [1987] gave the first survey on the iterated facility location and the vehicle routing problems. Other early survey papers can be found in Laporte et al. [1988], Laporte [1989], Berman et al. [1995], Min et al. [1998] and Nagy and Salhi [2007]. Recently, Lopes

TABLE 2.3 : Reviews of the HLPs without uncertainty

Problem	Article	Exact solution algorithms	Heuristics/metaheuristics
U-RA-M- $p$ -HLP	<a href="#">Todosjjević et al. [2017]</a>		Variable Neighborhood Search
U-SA-V-HLP	<a href="#">Dukkanci and Kara [2017]</a>		Heuristic based on subgradient approach
U-SA-MV- $p$ -HLP	<a href="#">Silva and Cunha [2017]</a>		Tabu Search
U-SA-M- $p$ -HLP	<a href="#">Meier [2017]</a> <a href="#">Mesgari and Barzinpour [2016]</a>	MIP formulations	Variable Neighborhood Search &&Social network centrality measure
U-SA-T- $p$ -HLP	<a href="#">Brimberg et al. [2017]</a>		Nested Variable Neighborhood Descent Strategy
U-SA-M- $p$ -HLP	<a href="#">Azizi et al. [2016]</a>		Genetic Algorithm
U-SA-SV-HLP	<a href="#">Ebrahimi-Zade et al. [2016]</a>		Genetic Algorithm
U-SA-HLP	<a href="#">Damgacioglu et al. [2015]</a> <a href="#">An et al. [2015]</a> <a href="#">Saboury et al. [2013]</a> <a href="#">Marić et al. [2013]</a> <a href="#">Bailey et al. [2013]</a> <a href="#">Rasoulinejad et al. [2013]</a>	Lagrangian relaxation && Branch-and-Bound	Genetic Algorithm  Variable Neighborhood Search Memetic Algorithm Particle Swarm Optimization Simulated Annealing
U-SA-OM-HLP	<a href="#">Puerto et al. [2013]</a>	Branch-and-Bound-and-Cut	
U-MA-T- $p$ -HLP	<a href="#">Miskovic [2017]</a>		Memetic Algorithm
U-MA-HLLP	<a href="#">Martins de Sá et al. [2015b]</a>	Benders-Branch-and-Cut	Heuristic algorithms
U-MA-M- $p$ -HLP	<a href="#">Mahmutogullari and Kara [2016]</a> <a href="#">Parvaresh et al. [2013]</a> <a href="#">Kratka [2013]</a>		Enumeration-based algorithms Tabu Search Electromagnetism-like Method
U-MA-HLP	<a href="#">He et al. [2015]</a>		MIP heuristic
U-HLLP	<a href="#">Martins de Sá et al. [2015a]</a>	Benders Decomposition Algorithm	
C-SA-HLP	<a href="#">Hoff et al. [2017]</a> <a href="#">Tanash et al. [2017]</a> <a href="#">Serper and Alumur [2016]</a> <a href="#">Stanojević et al. [2015]</a> <a href="#">Niknamfar et al. [2017]</a>	Branch-and-Bound	Memory structures &&Local Search  Variable Neighborhood Search Evolutionary Algorithm && Branch-and-Bound Multi-objective Biogeography-based Optimization && Opposition NSGA-II
C-SA-OM-HLP	<a href="#">Puerto et al. [2016]</a>	Branch-and-Bound-and-Cut	
MV- $p$ -HLP	<a href="#">Peker and Kara [2015]</a>	MILP formulations	

TABLE 2.4 : Reviews of the HLPs with uncertainty

Single-objective HLPs			
Problem	Article	Exact solution algorithms	Heuristic /metaheuristic solution algorithms
C-MA-HLP	<a href="#">Meraklı and Yaman [2017]</a>	Benders decomposition algorithm	
U-SA-T-p-HLP	<a href="#">Gao and Qin [2016]</a> <a href="#">Yang et al. [2013b]</a> <a href="#">Yang et al. [2013a]</a>	Chance-constrained programming	Intelligent algorithm Genetic Algorithm && Local search Particle Swarm Optimization
U-SA-IH&S	<a href="#">Yang et al. [2016]</a>	Fuzzy random programming	Multi-start simulated annealing
U-MA-M-p-HLP	<a href="#">Meraklı and Yaman [2016]</a>	Benders decomposition algorithm	
U-MA-HLP	<a href="#">Zhai et al. [2016]</a>		Variable neighbourhood search && Genetic Algorithm
U-RA-M-p-HLP	<a href="#">Peiró et al. [2014]</a>		Heuristic based on GRASP
U-HLP	<a href="#">Shahabi and Unnikrishnan [2014]</a> <a href="#">Shahabi and Unnikrishnan [2014]</a>	Conic integer programming Conic integer programming	
Multi-objective HLPs			
Problem	Article	Exact solution algorithms	Heuristic /metaheuristic solution algorithms
U-SA-HLP	<a href="#">Yang et al. [2017]</a>	Two-phase approach	
C-SA-HLP	<a href="#">Zhalechian et al. [2017b]</a>	Two-phase solution method	
C-SA-p-HLP	<a href="#">Zhalechian et al. [2017c]</a>	Me-based possibilistic programming	Hybrid differential evolution&& imperialist competitive algorithm
U-SA-T-M-p-HLP	<a href="#">Mohammadi et al. [2016]</a>	Fuzzy-queuing approach	Evolutionary algorithm based on game theory && invasive weed optimization
C-SA-V-p-HLP	<a href="#">Ghodratnama et al. [2015]</a> <a href="#">Mohammadi et al. [2013a]</a>	Fuzzy goal programming &&Torabi and Hassini's method	Imperialist Competitive Algorithm
C-SA-SHLP	<a href="#">Mohammadi et al. [2014]</a>	Mixed possibilistic– stochastic programming	Simulated annealing && Imperialist Competitive Algorithm



et al. (2013) [Lopes et al. \[2013\]](#) presented a taxonomy taking into account the structure characteristics, solution methods and objectives of the LRPs up to 2013. [Prodhon and Prins \[2014\]](#) analysed the recent literature of the standard and new extensions of the LRP since the survey of [Nagy and Salhi \[2007\]](#). [Drexl and Schneider \[2015\]](#) focused on the survey of variants and extensions of the LRP. [Albareda-Sambola \[2015\]](#) provided an overview of the most relevant contributions on the so-called classical location-routing problems. Later in 2017, [Schneider and Drexl \[2017\]](#) reviewed the literature of standard LRP between 2006 and spring 2016 by presenting detailed descriptions of the individual articles.

In this section, we classified the LRP and its variant problems by several criteria that most of the literature used in Section 2.2.1. The benchmark instances are also given in Section 2.2.2. In Section 2.2.3, we focus on the standard Capacitated Location-routing Problem (CLRP), describing the definition of the standard CLRP, the mathematical models and afterwards, the solution methods to solve the standard CLRP are reviewed from the year of 2002 to 2017. Section 2.2.4 gives a brief introduction of the multi-objective LRPs and highlights the most relevant articles to our problems. Finally, the conclusion of the main contribution is in the Section 2.2.5.

### 2.2.1 Classification of the LRPs

Since there are numerous types or variants of LRPs, a summary of the important classifying criteria helps to capture the main characteristics of the problems and get an overview of the research field of the LRPs. The basic and main criteria of the LRPs are shown below :

- The solution domain : the discrete domain where the potential locations for opening facilities are particular nodes of a graph; the continuous domain where the facilities are allowed to be located anywhere in a sphere or plane; the network where the candidate locations may be chosen to locate facilities at any vertex or anywhere on a link (edge, arc) of a network ;
- The nature of data : the LRPs in which all the data are pre-specified (deterministic data); the LRPs that some of the data are given with a probability distributions (stochastic data); and the LRPs that some of the data are in the form of fuzzy numbers (fuzzy data). One can refer to the survey paper of [Berman et al. \[1995\]](#) regarding the LRPs with uncertainty ;
- The planning period : the static problems compromise only one determined period ; the periodic LRPs (PLRP) include multiple planning periods with deterministic data ; the dynamic LRPs also consider multiple planning periods but some information such as customer demand are unknown at first and become available over time ;
- The vertex routing and arc routing : the vertex routing problems serve customers at vertices while the Location-arc routing problems (LARP) consider demands along the arcs of the network ;
- The echelon of distribution network : in the single echelon problems, customers are directly served by depots ; in the multiple echelon problems, routes are added to supply the depots from several main facilities or plants before serving the customers. For each echelon, some pre-specified vehicles are only allowed to visit the facilities defining the echelon. There are load transfers among different echelons. In particular, many papers of the two-echelon LRPs (2E-LRP) have appeared in the last few years and one survey paper of the 2E-LRP is found given by [Cuda et al. \[2015\]](#) ;
- The number of objective functions : single-objective problems usually consider the minimization of the total cost while multi-objective problems deal with several objective functions such as cost, service quality and environment affection ;

Other main variant LRPs are the Prize-collecting LRPs (PCLRPs), the Location-inventory-routing problem, the LRP with Simultaneous Pickup and Delivery (LRPSPD) and the Many-to-many LRPs. PCLRPs

allow some customers not to be visited by any tour but a penalty called outsourcing cost are applied to these customers (Ahn et al. [2012]). Location-inventory-routing problems take the inventory into account and tackle with the decisions of time and amount of demand to order from the manufacturer (Zhang et al. [2008]). The LRPSD not only consider the delivery scheme but also include the pick-up demand of customers in the network. Goods are delivered to one customer from one selected depot. At the same time, the goods are picked up from the customer. The simultaneous deliveries and pick-ups for one customer happen in the same vehicle (Karaoglan et al. [2011]). The many-to-many LRPs (MMLRPs), which is very similar to our researched problem, consider both the simultaneous or separate pick-ups and deliveries with pre-determined O-D demands. The decisions of load transfer between the facilities are also part of the solutions (Nagy and Salhi [1998]). The concrete reviews of the MMLRPs are presented in Section 2.3.

Recently, a new variant of LRPs called the Latency Location-Routing Problem (LLRP) is introduced consisting of optimally determining both the locations of depots and the routes of vehicles to minimize the waiting time of recipients (Moshref-Javadi and Lee [2016]). Other variant LRPs were discussed in the article of Drexl and Schneider [2015].

### 2.2.2 Benchmark instances of the LRPs

In order to evaluate the efficiency of the developed solution methods and the solution quality, some standardized benchmark instances have been extensively used in recent literature. The list of the main benchmark instances are show in Table 2.5 (Drexl and Schneider [2015], Schneider and Drexl [2017]). The benchmarks for the LRP are also widely used in many papers of LRP variants to assess the performance of their proposed methods. Table 2.5 presents the reference which was the first to introduce the instances, the numbers and size of the instances, as well as the internet link of getting access to those instances. For simplicity, the following sections use the acronyms of the benchmark instances to discuss the articles that are mentioned.

TABLE 2.5 : Benchmark instances of the LRPs

Acronym	First reference	Number of Instances	Size <sup>1</sup>	Link
Perl	Ph.D. dissertation of Perl (1983)	5	2–15/12–318	<a href="http://sweet.ua.pt/sbarreto/_private/SergioBarretoHomePage.htm">sweet.ua.pt/sbarreto/_private/SergioBarretoHomePage.htm</a>
TB	<a href="#">Tuzun and Burke [1999]</a>	36	10–20/100–200	<a href="http://prodhonc.free.fr/homepage">prodhonc.free.fr/homepage</a>
B	Ph.D. dissertation of Barreto (2003) and used in <a href="#">Barreto et al. [2007]</a>	13	5–14/21–150	<a href="http://sweet.ua.pt/sbarreto/_private/SergioBarretoHomePage.htm">sweet.ua.pt/sbarreto/_private/SergioBarretoHomePage.htm</a>
ADF	<a href="#">Albareda-Sambola et al. [2005]</a>	15	5–10/10–30	Not on the Internet
PPW	<a href="#">Prins et al. [2006b]</a>	30	5–10/20–200	<a href="http://prodhonc.free.fr/homepage">prodhonc.free.fr/homepage</a>
ABR	<a href="#">Akca et al. [2009]</a>	12	5/30–40	<a href="http://claudio.contardo.org/instances">claudio.contardo.org/instances</a>
BMW	<a href="#">Baldacci et al. [2011]</a>	4	14/150–199	<a href="http://claudio.contardo.org/instances">claudio.contardo.org/instances</a>
HKM	<a href="#">Harks et al. [2013]</a>	27	100–1000/1000–10,000	<a href="http://www.coga.tu-berlin.de/v-menuue/download_media/clrlib">www.coga.tu-berlin.de/v-menuue/download_media/clrlib</a>
<b>2E-LRP<sup>2</sup></b>				
GPTV	<a href="#">Gonzalez-Feliu et al. [2008]</a>	105	1/2–4/12–50	<a href="http://people.brunel.ac.uk/?mastjib/-jeb/orlib/vrp2einfo.html">people.brunel.ac.uk/?mastjib/-jeb/orlib/vrp2einfo.html</a>
CPMT	<a href="#">Crainic et al. [2010]</a>	132	1/2–10/50–250	<a href="http://people.brunel.ac.uk/?mastjib/-jeb/orlib/vrp2einfo.html">people.brunel.ac.uk/?mastjib/-jeb/orlib/vrp2einfo.html</a>
NPP-N	<a href="#">Nguyen et al. [2010]</a>	24	1/5–10/20–200	<a href="http://prodhonc.free.fr/homepage">prodhonc.free.fr/homepage</a>
NPP-P	<a href="#">Nguyen et al. [2010]</a> (PPW modified by adding one level-0 facility)	30	1/5–10/20–200	<a href="http://prodhonc.free.fr/homepage">prodhonc.free.fr/homepage</a>
S	<a href="#">Sterle [2008]</a>	93	2–5/3–20/8–200	Not on the Internet
<b>Other LRPs</b>				
Prodhon	PLRP <sup>3</sup> , <a href="#">Prodhon and Prins [2008]</a>	30	5–10/20–200	<a href="http://prodhonc.free.fr/homepage">prodhonc.free.fr/homepage</a>
KAKD	LRPSPD <sup>4</sup> , <a href="#">Karaoglan et al. [2011]</a>	37	2–10/8–100	Not on the Internet
LPFS	LARP <sup>5</sup> , <a href="#">Lopes et al. [2014]</a>	30	11–140	<a href="http://lore.web.ua.pt">lore.web.ua.pt</a>

<sup>1</sup> Structure of the instance size : min–max number of facilities/ min–max number of customers

<sup>2</sup> 2E-LRP : two echelon location-routing problem

<sup>3</sup> PLRP : periodic location-routing problem

<sup>4</sup> LRPSPD : location-routing problem with simultaneous pick-up and delivery

<sup>5</sup> LARP : location-arc routing problem

### 2.2.3 State-of-the-art of the standard CLRP in fifteen years

This section addresses the standard CLRP which we define as a discrete, static, deterministic, single-echelon and single-objective location-routing problem. No inventory decisions are considered. In the problem, the following constraints must be satisfied :

- (1) Both the depots and vehicles are capacitated and subject to a fixed cost when they are utilized;
- (2) The set of potential depots is finite;
- (3) The total demand of customers assigned to one depot must fit its capacity;
- (4) Each customer is served by one single vehicle and assigned to only one depot;
- (5) Each route begins and ends at the same hub;
- (6) Each vehicle performs at most one trip and the total demand of the customers it visits must not exceed its capacity.

In the following parts of this section, we introduce the mathematical models of the standard CLRP, furthermore, a wide overview on the literature researching the standard CLRP is given.

#### Mathematical model for the standard CLRP

The initial studies of the LRPs focused on the version with either uncapacitated depots or uncapacitated vehicles (Laporte et al. [1986], Laporte et al. [1988]). In 2002, Wu et al. [2002] proposed a mathematical formulation for the standard CLRP considering an unlimited homogeneous fleet. Later, Belenguer et al. [2011] added a new variable into the CLRP model and developed a two-index vehicle-flow formulation.

Prins et al. [2006b] proposed a mixed integer linear programming (MILP) model with only binary decision variables. According to Prins et al. [2006b], the CLRP can be defined on a complete and undirected graph  $G(N, A, C)$ , in which  $N$  is the set of nodes consisting of the potential depot subset  $H$  and customer subset  $I$ ;  $A$  is the set of arcs and each arc  $a \in A$  satisfies the triangle inequality;  $C$  is the distance-based cost matrix of traversing each arc. A set of unlimited homogeneous vehicles  $J$  are shared by all the hubs and subject to a capacity constraint. The notations used in the model of CSALRP are shown in Table 2.6.

TABLE 2.6 : Notations used in the model of the CLRP

$F_k$	Opening cost of hub $k$
$\Gamma_k$	Capacity of hub $k$
$D_i$	Demand of customer $i$
$Q$	Capacity of a vehicle
$VF$	Fixed cost of using a vehicle
$C_a$	Cost of traversing the arc $a$
$Y_k$	Binary variable : if the hub is located at node $k$ or not
$z_{ik}$	Binary variable : if customer $i$ is assigned to hub $k$ or not
$f_{aj}$	Binary variable : if arc $a$ is traversed by vehicle $j$

The model of the standard CLRP can be formulated as

$$\min \sum_{k \in H} F_k Y_k + \sum_{a \in A} \sum_{j \in J} C_a f_{aj} + \sum_{j \in J} \sum_{a \in \delta^+(J)} VF f_{aj} \quad (2.43)$$

Subject to

$$\sum_{j \in J} \sum_{a \in \delta^-(i)} f_{aj} = 1 \quad \forall i \in I \quad (2.44)$$

$$\sum_{i \in I} \sum_{a \in \delta^-(k)} D_i f_{aj} \leq Q \quad \forall j \in J \quad (2.45)$$

$$\sum_{a \in \delta^+(k)} f_{aj} - \sum_{a \in \delta^-(k)} f_{aj} = 0 \quad \forall j \in J, \forall k \in N \quad (2.46)$$

$$\sum_{a \in \delta^+(k)} f_{aj} \leq 1 \quad \forall j \in J \quad (2.47)$$

$$\sum_{a \in L(S)} f_{aj} \leq |S| - 1 \quad \forall S \subseteq I, \forall j \in J \quad (2.48)$$

$$\sum_{a \in \delta^+(k) \cap \delta^-(I)} f_{aj} + \sum_{a \in \delta^-(i)} f_{aj} \leq 1 + z_{ik} \quad \forall i \in I, \forall j \in J, \forall k \in K \quad (2.49)$$

$$\sum_{i \in I} D_i z_{ik} \leq \Gamma_k Y_k \quad \forall k \in K \quad (2.50)$$

$$f_{aj}, z_{ik}, Y_i \in \{0, 1\} \quad \forall a \in A, \forall k \in K, \forall i \in I, \forall j \in J \quad (2.51)$$

The objective function (2.43) minimizes the total costs which include the hub opening costs, arc traversing costs and costs of using vehicles. Constraints (2.44) ensure that each customer is served by a single route. Constraints (2.45) guarantee that the total demand each vehicle visits must respect the vehicle capacity. Constraints (2.46) and (2.47) guarantee the continuity of each route and that each route starts and ends at the same hub. Constraints (2.48) eliminate illegal subtours. Constraints (2.49) show that a customer can be assigned to a hub only if there are routes connecting the hub. Constraints (2.50) are the hub capacity constraint. Constraints (2.51) define the decision variables.

## The solution methods for the standard CLRP

This part provides an overview of the literature on the standard CLRP from the year 2002 to 2017. As stated, the CLRP includes two problems known to be NP-hard. The exact methods proposed to solve the standard CLRP in the literature are rare compared to the heuristic and metaheuristic methods which are usually much easier and more flexible to implement and modify. The total articles found for the standard CLRP are listed in Table 2.7 presenting the solution methods, the benchmark instances and programme platforms each article used. The relevant abbreviations for the solution methods are summarized in Table 2.8.

### (1) Exact solution methods

There are five works found which proposed exact algorithms to solve the standard CLRP. Akca et al. [2009] presented a branch-and-price algorithm (B&P) based on the set-partitioning formulation and proposed five classes of valid inequalities. An elementary shortest path problem with resource constraints (ESPPRC), which was well-studied as a subproblem in many different routing problems, was solved by means of the exact and heuristic variants of the label setting algorithms. For computational testing, they implemented four variants of the B&P algorithms based on the different pricing schemes. Computational experiments are performed with the Perl and B instances. Furthermore, they created the ABR instances

to test the algorithm. The largest instances with 5 depots and 40 customers were solved to optimality and good upper bounds could be quickly obtained. Later, [Baldacci et al. \[2011\]](#) applied the formulation of [Akca et al. \[2009\]](#) and described a new exact algorithm. In the method, a bounding procedure based on dynamic programming (DP) and dual ascent (DA) methods decomposed the LRP into a multi-capacitated depot vehicle-routing problems (MCDVRPs). The computational experiments were conducted with the Perl, TB, B, ABR, PPW and BMW instances. 50 out of the 60 instances including the largest instance of 14 depots and 199 customers were solved to optimality. Furthermore, 25 of those instances were solved to optimality for the first time and 25 best-known solutions were improved.

[Belenguer et al. \[2011\]](#) proposed a zero-one linear model which was strengthened by new families of valid inequalities. A Branch-and-Cut (B&C) approach was presented and evaluated by testing on the Perl, B, ABR and PPW instances. The computational results showed that 26 instances with up to 5 depots and 50 customers were solved to optimality. [Contardo et al. \[2013a\]](#) extended the two-index formulation of [Belenguer et al. \[2011\]](#) and introduced three new formulations based on vehicle flows and commodity flows. The results showed that the proposed two-index vehicle-flow formulation produced tighter gaps and was able to solve large instances with up to 100 customers. In the article of [Contardo et al. \[2013a\]](#), numerous additional inequalities are added to the arc-variable formulation of [Belenguer et al. \[2011\]](#). An exact algorithm based on Cut-and-Column Generation was proposed and showed that the algorithm outperformed the results obtained by [Belenguer et al. \[2011\]](#) and [Contardo et al. \[2013a\]](#).

### (2) Heuristic solution methods

[Barreto et al. \[2007\]](#) proposed a cluster analysis based on the sequential heuristic of distribution-first, location-second. Moreover, four grouping techniques and six proximity measures were used to obtain different versions of the heuristic. The evaluation of the proposed heuristic proved its good performance with an average small gap.

[Prins et al. \[2007\]](#) combined the Lagrangian relaxation (LR) and the granular tabu search (GTS) to form an iterative two-stage heuristic. The initial feasible solution was generated by a greedy heuristic. In the first stage, each route of the current solution was aggregated into a super-customer and a repair procedure disaggregated the super-customers into a new LRP solution. Then in the second stage, the solution was improved by a GTS heuristic. The experiments showed that the procedure was able to find the same or even better solutions in short time compared to the methods of [Tuzun and Burke \[1999\]](#), [Prins et al. \[2006b\]](#) and [Barreto et al. \[2007\]](#). [Chen and Ting \[2007\]](#) proposed another two-stage heuristic. The first stage was similar to [Prins et al. \[2007\]](#) while the second stage applied a SA to improve the solution. Computational tests were conducted using the Perl and B instances. The proposed heuristic was capable of improving the results with less computing time. A different two-phase hybrid heuristic algorithm was developed by [Escobar et al. \[2013\]](#). The construction phase applied a Lin–Kernighan–Helsgaun heuristic and the second phase used a modified GTS with different diversification strategies to improve the solutions. Computational experiments showed that the proposed algorithm was able to find the highest number of best known solutions and even produced several new best known solutions in short computing time.

[Lopes et al. \[2008\]](#) developed a decision-support tool (DST) to implement the CLRP with two levels (depots and customers). The solution procedure embedded was a sequential route-first location-second heuristic. It enabled access to online geographic data through web map servers (WMS). Computational experiments were conducted using the B instances. The average gap between the heuristic solution and the lower bound obtained by a two-index flow formulation was 4.8%. [Lam and Mittenthal \[2013\]](#) proposed a three-stage heuristic by applying a hierarchical clustering approach, a Lin–Kernighan–Helsgaun heuristic (Helsgaun) and an LS with string-relocate and string-exchange moves respectively.

### (3) Metaheuristic solution methods

[Wu et al. \[2002\]](#) divided the Multi-depot Location-routing problem (MDLRP) into two sub-problems, the location-allocation problem and the general vehicle routing problem, respectively. A simulated annealing (SA) based decomposition approach was developed to solve each sub-problem in a sequential manner. [Bouhafs et al. \[2006\]](#) iteratively applied the SA for selecting depots and the Ant Colony Optimization for determining routes.

Prins et al. [2006a] presented a Greedy randomized adaptive search procedure (GRASP) complemented by a learning process and a path relinking (PR). In the GRASP, a randomized extended Clarke and Wright algorithm (RECWA) was used to generate solutions and construct an elite set. In the post-optimization step, they used the PR to improve new solutions from the elite set. Computational results indicated that compared to the methods of Tuzun and Burke [1999] and Barreto et al. [2007] the proposed algorithm improved the solution quality but the method of Tuzun and Burke (1999) was faster on the tests with TB instances. For the same problem, Prins et al. [2006b] described another method called the Memetic Algorithm with Population Management (MA|PM). A small population was firstly initialized by the RECWA (Prins et al. [2006a]) and a randomized nearest-neighbor-based (NN) method. Then the solutions were improved by applying Local Search (LS) with given probabilities. To keep the diversity of the population, a distance measure proposed by Prins et al. [2006a] was applied. Such method was able to obtain the best solutions for all instance sets of TB, B, and PPW but with higher computing time.

Duhamel et al. [2008] proposed a different MA by applying a giant-tour without route or depot delimiters to present a complete solution. A splitting procedure with a dynamic-programming based labeling algorithm was used to evaluate the fitness. In numerical tests on the TB and PPW instances, the quality of the proposed MA was between the quality of Prins et al. [2006b] and Prins et al. (Prins et al. [2006a], Prins et al. [2007]). However, the MA required significantly higher computing time. Later in 2010, Duhamel et al. [2010] described a hybrid GRASP with an Evolutionary Local Search (ELS) to solve the CLRP. The initial solutions were constructed by the RECWA of Prins et al. [2006b] and transformed into a giant tour representation. A tabu list with forbidden facilities was modified by a number of ELS iterations. The algorithm stopped after a given number of GRASP iterations. Such method improved a large number of best solutions compared to the methods of Prins et al. [2006a] and Prins et al. [2007] but still required higher computing time.

Marinakis and Marinaki [2008b] provided a bilevel genetic algorithm (GA). The GRASP and Expanding Neighborhood search (ENS) procedure described in Marinakis and Marinaki [2008a] were applied to generate the initial population. A case study of a Greece company was presented. No results comparison with Prins et al. [2006b], Prins et al. [2006a] and Prins et al. [2007] was discussed.

In the article of Yu et al. (2010), an SA based solution approach was presented. The experiment of the proposed method on the TB, B, and PPW instances showed that the solution quality of the SA dominated all the methods of Tuzun and Burke [1999], Bouhafs et al. [2006], Prins et al. [2006b], Prins et al. [2006a], Prins et al. [2007], Barreto et al. [2007] and Duhamel et al. [2010] with a sacrifice of higher computing time which was, however, considered to be acceptable for a strategic problem.

Derbel et al. [2011] investigated many heuristic solution approaches to solve the LRP and proposed a variable neighbourhood search (VNS) algorithm. Such algorithm embedded a variable neighbourhood descent (VND) into the LS process. The performance of the proposed method was tested on the B instances. Competitive results were obtained compared with existing solution methods. No computing time was reported. Jabal-Ameli et al. [2011] proposed another VNS with facility reduction mechanism and obtained competitive solutions. Ting and Chen [2013] developed an ACO algorithm which iteratively applied three ant colonies for facility location, customer assignment and routing. The ACO was able to improve the solution quality of Yu et al. [2010] on the B, TB, PPW instances in reasonable time.

Escobar et al. [2014] applied a VNS to select the neighbourhood structure for the GTS. The algorithm was able to achieve better solution quality within a shorter time. Contardo et al. [2014] presented a matheuristic using a GRASP and heuristic column generation. The results were compared with numerous previous authors and proved to be capable of yielding similar or tighter average gaps for all instance sets. Recently, Schneider and Löffler [2015] introduced a tree-based search algorithm (TBSA) to tackle large-scale instances. The proposed TBSA combined a tree-like search on depot configurations and a GTS for route improvement. Computational experiments showed the good solution quality on newly generated instances with up to 600 customers and 30 depots.

TABLE 2.7 : State-of-the-art of the standard CLRP

Author	Problem	Exact Algorithm	Heuristics/ Metaheuristics	Program	Benchmark instances
Akca et al. [2009]	CLRP	B&P		Linux based worksation	Perl/B/ABR
Belenguer et al. [2011]	CLRP	B&C		Visual C++	Perl/B/ABR/PPW
Baldacci et al. [2011]	CLRP	DP&DA		Visual CPLEX 11.0	Perl/TB/B/ABR/ PPW/BMW
Contardo et al. [2013a]	CLRP	B&C		Visual CPLEX 12.2	Perl/TB/B/ABR/PPW
Contardo et al. [2013b]	CLRP	C&CG		Visual C++ /CPLEX 12.2	Perl/TB/B/ABR/ PPW/BMW
Wu et al. [2002]	MDLRP		SA based composition	LINGO/Visual C++	Perl/Created
Bouhafaf et al. [2006]	CLRP		SA+ACS	–	Perl/B
Prins et al. [2006a]	CLRP		GRASP+PR	Visual C++	TB/B/PPW
Prins et al. [2006b]	CLRP		MA PM	Visual C++	TB/B/PPW
Barreto et al. [2007]	CLRP		Cluster analysis based heuristic	–	B
Prins et al. [2007]	CLRP		LR+GTS	Visual C++	TB/B/PPW
Chen and Ting [2007]	MDLRP		LH+SA	Visual C++	Perl/B
Lopes et al. [2008]	CLRP		DST	Visual Basic 6.0	B
Duhamel et al. [2008]	CLRP		MA	Borland C++ 6.0	TB/B/PPW
Marinakakis and Marinaki [2008a]	CLRP		HybPSO	Fortran 90	Perl/B
Marinakakis and Marinaki [2008b]	CLRP		Bilevel GA	Fortran 90	Perl/B /Greek company
Duhamel et al. [2010]	CLRP		GRASP+ELS	Borland C++ 6.0	TB/B/PPW
Sodsoon [2010]	CLRP		MMAS	Visual C++ 6.0	Perl
Yu et al. [2010]	CLRP		SA	C language	TB/B/PPW
Jokar and Sahraeian [2011]	CLRP		Two-phase heuristic	MATLAB R2009b	B/PPW
Nadizadeh et al. [2011]	CLRP		GCM	MATLAB 7.0.4	B
Derbel et al. [2011]	CLRP		VND	C language	B
Jabal-Ameli et al. [2011]	CLRP		VND	MATLAB R2007b	TB/B
Jokar and Sahraeian [2012]	CLRP		SA	MATLAB	B
Ting and Chen [2013]	CLRP		MACO	Borland C++ 5.0	Perl/TB/B/PPW
Lam and Mittenthal [2013]	MDLRP		Clustering- based heuristic	–	TB/B
Escobar et al. [2013]	CLRP		Two-phase HGTS	Visual C++ /CPLEX 12.1	TB/B/PPW
Contardo et al. [2014]	CLRP		GRASP + LS	Visual C++ /CPLEX12.5	ABR/TB/B/BMW
Escobar et al. [2014]	CLRP		GVTNS	C++	TB/B/PPW
Rybičková et al. [2016]	CLRP		GA	language Julia	Created
Kechmane et al. [2016]	CLRP		MA	C language	PPW
Lopes et al. [2016]	CLRP		hybrid GA	C#	TB/B/PPW
Quintero-Araujo et al. [2017]	CLRP		BR	Java	PPW/B/ABR
Schneider and Löffler [2015]	CLRP		TBSA	C++	TB/PPW/Created



TABLE 2.8 : Summary of abbreviations of the solution methods

B&P	Branch-and-Price algorithm
B&C	Branch-and-Cut algorithm
DP&DA	Dynamic programming and dual ascent methods
C&CG	Cut-and-column generation
SA	Simulated Annealing
ACS	Ant Colony System
GRASP	Greedy randomized adaptive search procedure
PR	Rath relinking
MA PM	Memetic Algorithm with Population Management
LR	Lagrangean relaxation
GTS	Granular Tabu Search
LH	Lagrangean heuristic
DST	decision-support tool
MA	Memetic Algorithm
HybPSO	Hybrid Particle Swarm Optimization
GA	Genetic Algorithm
ELS	Evolutionary Local Search
MMAS	MAX-MIN Ant System
GCM	Greedy Clustering Method
VND	Variable Neighborhood Descent based heuristic
MACO	Multiple Ant Colony Optimization
2-Phase HGTS	Two-phase Hybrid Granular Tabu Search
LS	Local Search
GVTNS	Granular Variable Tabu Neighborhood Search
BR	Biased-randomization
TBSA	Tree-based Search Algorithm

#### 2.2.4 The multi-objective LRPs

There has been a significant amount of literature published on the general LRP and its multiobjective variants. Among those problems, the hazardous waste location-routing problem is one of the hub location problems focusing on multiple objectives. Further discussions can refer to the article of [Lopes et al. \[2013\]](#). Two recent articles from [Ardjmand et al. \[2015\]](#) and [Asgari et al. \[2017\]](#) proposed a Genetic Algorithm (GA) to solve the hazardous waste LRP. In consideration to the total cost and risk in the Hazardous Materials Management (HAZMAT) LRP network, [Ardjmand et al. \[2015\]](#) formulated a bi-objective model and proposed a GA to find high quality solutions in a short time. [Asgari et al. \[2017\]](#) developed a multi-objective model for the obnoxious waste LRP with three objective functions of minimizing the undesirability of the treatment and disposal facility, the total cost, and the risk of transporting untreated wastes. A Memetic Algorithm was proposed applying a Tabu Search (TS) algorithm as the local search and was experimented on the HAZMAT system in Singapore.

Another focus of the multi-objective LRP lies in reducing the environmental effect in addition to the minimization of total cost. [Govindan et al. \[2014\]](#) introduced a two-echelon location-routing problem with time-windows (2E-LRPTW) in a perishable food supply chain network (SCN). They proposed a bi-objective optimization model by considering the cost and environmental effect at the same time. A hybrid multi-objective metaheuristic algorithm combining the adapted Multi-Objective Particle Swarm Optimization (MOPSO) and the Adapted Multi-Objective Variable Neighborhood Search (AMOVNS) algorithm was proposed. More recently, [Tricoire and Parragh \[2017\]](#) focused on a Green City Hub Location-Routing Problem (GCHLRP) with fleet-size-and-mix for a long-term horizon. They built a bi-objective model in

which only single allocation was allowed and routing length was restricted. The demand of customers to be served differed on different days within the horizon. Two objectives were intended to be obtained simultaneously : to minimize the fixed cost which was caused by opening hubs and using vehicles ; and to minimize the CO<sub>2</sub> emissions which were incurred by daily operational routing. The CO<sub>2</sub> emissions were dependent on different arcs and vehicle loads. After modeling, a decomposition approach was used in a separate set covering model and was tested on the instances from the industrial partners to generate approximate fronts. Farrokhi-Asl et al. [2017] proposed and compared the Non-Dominated Sorting Genetic Algorithm (NSGA-II) and Multi-Objective Particle Swarm Optimization (MOPSO) for a bi-objective location-routing-problem in waste collection management. The results showed that the NSGA-II outperforms the MOPSO in solving the target problems. Toro et al. [2017] proposed a bi-objective Capacitated Location-Routing (CLRP) model with the purposes of minimizing the operational costs and the environmental effect (CO<sub>2</sub> emissions). Based on the forces acting on each used vehicle, they developed a fuel consumption formulation. A classic  $\epsilon$ -constraint method was implemented on eight test scenarios, up to 100 customers, and a sensitivity analysis was conducted to reveal the trade-offs between the two objectives.

### 2.2.5 Conclusion

The main contribution of this section lies in giving a large overview of the standard CLRP. The definitions and models are presented and the main literature of 15 years up to 2017 has been reviewed by detailed descriptions of the solution methods to solve the standard CLRP.

## 2.3 The Hub Location-Routing Problem

The Hub Location-Routing Problem (HLRP) is similar to the Many-to-Many Location-Routing Problem (MMLRP) introduced by Nagy and Salhi [1998], which deals with terminal locations, plans vehicle tours and designs inter-hub flow exchanges to satisfy customer demands in order to minimize the total cost.

Table 2.9 shows the main characteristics of the HLP, the LRP and the HLRP. The symbol "×" means that the problem inherits the features. The flow exchanged between origin-destination pairs (O-D) and the direct links between hubs constitute two main features of the HLRP (de Camargo et al. [2013]). Figure 2.1 gives a simple illustration of the different network structures of the classic HLP, the standard LRP and the HLRP. The HLP emphasizes the importance of the inter-hub connection and flow transfer while consider only direct transportation between hubs and customers. On contrary, the LRP addresses the necessity of routing customers to reduce the pick-up and/or delivery cost, however, this problem ignores the connection among hubs. The HLRP integrates all the considerations.

TABLE 2.9 : The differences between the HLP, the LRP and the HLRP

Problem	Hub location	Non-hub nodes allocation	Local tour	O-D demand
HLP	×	×		×
LRP	×	×	×	
HLRP	×	×	×	×

Although the Hub Location Problem (HLP), the Location-Routing Problem (LRP) and the Vehicle Routing Problem (VRP) have aroused much attention and have been widely investigated separately, previous research in the area of HLRP is limited.

As mentioned above, the concept of the Many-to-Many Location-Routing Problem (MMLRP) was first introduced by Nagy and Salhi in 1998 (Nagy and Salhi [1998]). They considered the demands to satisfy each pair of customers (suppliers and clients) as well as the capacity restrictions of terminals. The pick-up and delivery processes might or might not be made simultaneously within the same tours but local tours were subjected to a routing length constraint. Wasner and Zäpfel [2004] addressed the Austria postal service

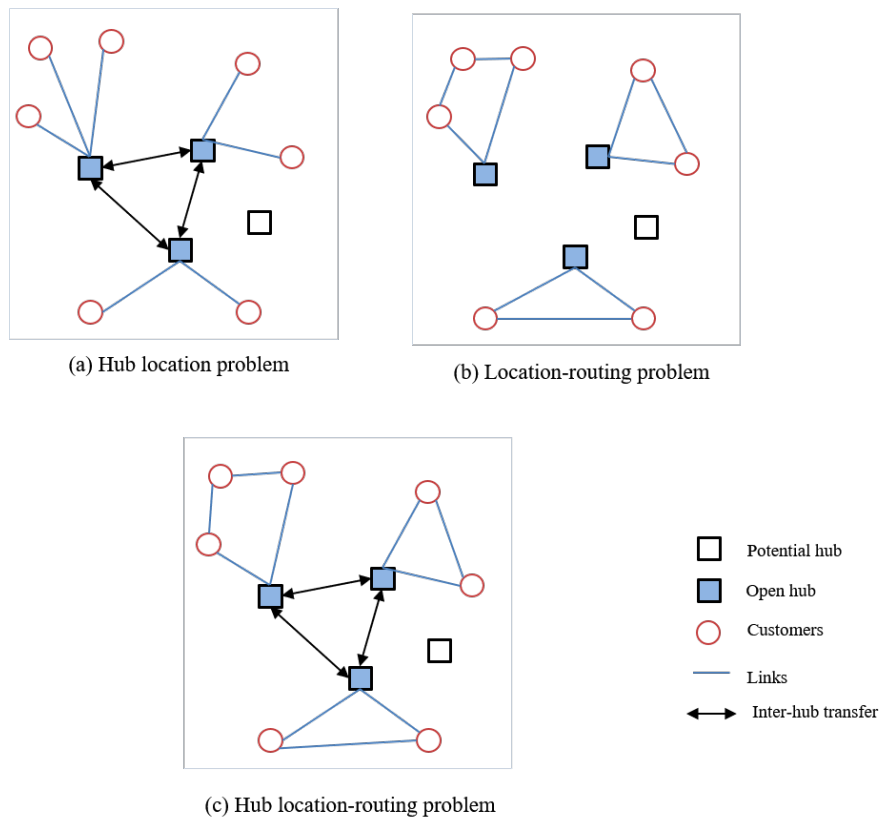


FIGURE 2.1 : Illustration of the related problem networks

problem as a case study and investigated the possibility of direct connections between non-hub pairs for the capacitated single allocation MMLRP. They developed a hierarchical method due to the difficulty in solving the problem by standard solvers. Only one instance was solved in both these articles.

Catanzaro et al. [2011] proposed a Partitioning-Hub-Location-Routing Problem (PHLRP), partitioning the target network into several sub-networks. For each sub-network, hub locations, multiple allocations and traffic routings were scheduled with an LTL system. They presented an Integer Programming (IP) model and explored some valid inequalities to strengthen it.

de Camargo et al. [2013] provided a new formulation and solved the MMLRP by means of a specially tailored Benders decomposition algorithm. In their assumptions, the hubs and vehicles were subjected to a fixed installation cost and each customer had to be visited once and was subjected to a charge, called the handling cost, for each assignment. A discount was applied to lower the transportation cost if the freight was transferred through hubs because of the full truckload transportation assumption between hubs. Furthermore, they allowed pick-ups and deliveries to occur simultaneously and imposed a maximum tour duration. Their formulation led to a decomposition into two sub-problems, leading to the application of an efficient Bender decomposition. Computational experiments were carried out based on the Australian Post (AP) standard data set and the approach succeeded in solving problems with up to 100 nodes. In the study of Setak et al. [2013], a comprehensive and flexible model of the HLRP was proposed. The aim was to build a general hub network topology to minimize the total cost, establishing hub nodes and inter-hub links, and connecting hub nodes and non-hub nodes. Paths containing only one hub node or a direct link from origin to destination were allowed. In their experiments, data from the Australian Post (AP) and Civil Aeronautics Board (CAB) were used. Mokhtari and Abbasi [2014] used a Variable Neighborhood Particle Swarm Optimization Algorithm (VNPSO) to solve the many-to-many HLRP with uncapacitated hubs. The efficiency of the proposed algorithm was compared to a Bender's decomposition algorithm using numerous samples created randomly. Numerical results indicated that it performed better for large-scale problems. Zhang et al. [2014] proposed a model for the CSAHLRP and developed a branch-and-cut algorithm, conducting experiments using the AP data set. Bostel et al. [2015] studied the HLRP applied to the special features of postal

service systems in which collections and deliveries may occur simultaneously within the same routes. A memetic algorithm based on a genetic algorithm and a local search was developed to solve the problem. To the best of our knowledge, only one article may be found for the study of the multi-objective HLRP. [Mohammadi et al. \[2013b\]](#) focused on a stochastic green HLRP (SGHLRP) and tried to minimize the total cost and the environmental effect. They proposed a multi-objective mixed integer linear programming model and applied a Multi-Objective Invasive Weed Optimization (MOIWO) algorithm to identify Pareto optimal solutions.

All the articles mentioned above concern problems in which the number of hubs to open needs to be decided. Other articles research similar problems but with predetermined open hub numbers. Such problems are called Many-to-Many  $p$  Hub-Location Routing Problems (MM $p$ HLRP). [Çetiner et al. \[2010\]](#) developed an iterative two-stage solution procedure for locating hubs and planning local tours. In their assumptions, they allowed multiple allocations of non-hub to hub nodes and simultaneous pick-ups and deliveries, while imposing a maximum tour length constraint. They applied their procedure to a randomly-generated data set and studied a case from the Turkish postal service. [Sun \[2013\]](#) decomposed the HLRP into two sub-problems : the HLP in the first stage and the multi-depot VRP in the second. An Ant Colony Optimization (ACO) algorithm was applied to the two stages. [Rieck et al. \[2014\]](#) designed a many-to-many network structure with three layers : the supplier layer, the potential hub layer and the delivery layer. A mixed-integer linear model was proposed and applied to solve 15-node networks to optimality. [Rodríguez-Martín et al. \[2014\]](#) introduced a Hub Location and Routing Problem that is very close to the Single Allocation  $p$ -Hub Median Problem (SA $p$ HMP). This aimed to position  $p$  hubs, allocate customer nodes to a single hub and connect within the same tour customer nodes that had the same hub allocation. At most  $q$  nodes could be allocated to one hub and just one travelling tour was allowed for each hub. A branch-and-cut algorithm was developed to solve the problem. More recently, a solution method based on an Endosymbiotic Evolutionary Algorithm (EEA) was proposed by [Sun \[2015\]](#) to deal with the hub location and vehicle routing problems simultaneously. [Zameni and Razmi \[2015\]](#) presented a mixed-integer formulation for a multimodal  $p$ -hub location-routing problem with simultaneous pick-ups and deliveries. They considered a multimodal transportation system, which consisted of different transport modes with different characteristics.

The articles on the HLRP described in this section are listed and their main characteristics compared in Table 2.10. The notations used are explained in Table 2.11. These tables illustrate the wide variety of the characteristics of problems addressed and solution techniques used.

## 2.4 Environmental considerations

With the increasing concern for sustainable factors, environmental considerations have become important to consider in logistics and transport planning models. Many factors may be considered, such as are discussed in [Chardine-Baumann and Botta-Genoulaz \[2014\]](#), among which the use of resources, especially energy consumption, and pollution emitted, among which greenhouse gas (GHG) are recognized as a major pollution factor in industrial and transport activities. Among those, the CO<sub>2</sub> emissions are considered as a representative factor of GHG. In this research, we will consider the minimization of CO<sub>2</sub> emissions of transport as well the minimization of costs as a second objective.

Many researches have been devoted to the estimation and analytical formulation of CO<sub>2</sub> emissions. Generally speaking, the amount of CO<sub>2</sub> emissions and other pollution gas emitted by a vehicle mainly depends on the vehicle load, speed and travelling distance, among other factors ([Bektaş and Laporte \[2011\]](#)). In order to calculate CO<sub>2</sub> emissions, two estimation methods are used : the fuel-based and the distance based methodologies ([Ubeda et al. \[2011\]](#)).

In the fuel-based method, the CO<sub>2</sub> emissions are based on the fuel's consumption ([Barth et al. \[2005\]](#)). By means of catching temporal CO<sub>2</sub> emissions based on the vehicle load, the speed and the travelling distance, this methodology can be applied to estimate CO<sub>2</sub> emissions for road freight transportation problems such as the Pollution Routing Problem (PRP) ([Barth et al. \[2005\]](#), [Xiao et al. \[2012\]](#), [Demir et al. \[2012\]](#),

TABLE 2.10 : Recent literature of the HLRP

Authors	Hub capacity	Number of hubs	Hub location	Non-Hub allocation	Routing constraints	Exact method	Heuristics	Application /Data	Problem size	Pick-up /delivery
<b>Our research</b>	<b>Yes</b>	<b>Unfixed</b>	<b>Yes</b>	<b>Single</b>	<b>Capacitated</b>	<b>CPLEX</b>	<b>MA<sup>1</sup></b>	<b>AP</b>	<b>100</b>	<b>Distinct</b>
Nagy and Salhi [1998]	Yes	Unfixed	Yes	Single	Length	-	Hierarchical	One instance	249	Simultaneous
Liu and Lee [2003]	No	One	No	Single + direct	Capacitated	-	Heuristic	Random instances	25	Distinct
Wasner and Zäpfel [2004]	Yes	Unfixed	Yes	Multiple + direct	Capacitated	-	Hierarchical	Austrian parcel	10	Simultaneous
Çetiner et al. [2010]	No	$p$ hubs	Yes	Multiple	Length	-	Two-stage heuristic	Turkish postal	81	Simultaneous
Catanzaro et al. [2011]	No	Unfixed	Yes	Multiple	Node number	B&C <sup>2</sup>	-	Random instances	450	Distinct
de Camargo et al. [2013]	No	Unfixed	Yes	Single	Time	BDA <sup>3</sup>	-	AP <sup>4</sup>	100	Simultaneous
Setak et al. [2013]	No	Unfixed	Yes	Single + direct	No	CPLEX	-	AP & CAB <sup>5</sup>	20	Distinct
Mohammadi et al. [2013c]	Yes	Unfixed	Yes	Single	Capacitated ; Arrival time	-	MOIWO <sup>6</sup>	Random instances	100	Simultaneous
Mokhtari and Abbasi [2014]	No	Unfixed	Yes	Single	Time	-	VNPSO <sup>7</sup>	Random instances	300	Simultaneous
Rodríguez-Martín et al. [2014]	No	$p$ hubs	Yes	Single	Node number	B&C	-	AP & CAB	50	Distinct
Rieck et al. [2014]	No	$p$ hubs	Yes	Single + direct	Capacitated	-	Multi-start + GA <sup>8</sup>	Timbertrade industry	140	Distinct
Zhang et al. [2014]	Yes	Unfixed	Yes	Single	Capacitated	CPLEX-B&C	-	AP	100	Distinct
Zameni and Razmi [2015]	No	$p$ hubs	Yes	Single	Time	GAMS	GA	AP & CAB	50	Simultaneous
Sun [2013]	Yes	$p$ hubs	Yes	Single	Capacitated	-	Two-stage ACO <sup>9</sup>	Random instances	200	Simultaneous
Sun [2015]	Yes	$p$ hubs	Yes	Single	Capacitated	-	EEA <sup>10</sup>	Random instances	200	Distinct
Bostel et al. [2015]	Yes	Unfixed	Yes	Single	Capacitated	CPLEX	MA	AP postal	100	Simultaneous

TABLE 2.11 : Solution method notation of the HLRP

<sup>1</sup> MA	Memetic Algorithm	<sup>2</sup> B&C	Branch-and-Cut algorithm
<sup>3</sup> BDA	Benders Decomposition Algorithm	<sup>4</sup> AP	Australian Post standard data set
<sup>5</sup> CAB	Civil Aeronautics Board data set	<sup>6</sup> MOIWO	Multi-Objective Invasive Weed Optimization
<sup>7</sup> VNPSO	Variable Neighborhood Particle Swarm Optimization algorithm	<sup>8</sup> GA	Genetic Algorithm
<sup>9</sup> ACO	Ant Colony Optimization algorithm	<sup>10</sup> EEA	Endosymbiotic Evolutionary Algorithm

Demir et al. [2014] and Kramer et al. [2015]).

Alternatively, the distance based methodologies use emission factors based on travelled distance to estimate CO<sub>2</sub> emissions (Ubeda et al. [2011]). This method is based on the distance the vehicle travels and the distance-based emission factors which are based on the fuel type. Instead of the real vehicle load used by the first method, it considers fixed load coefficients to generate the distance-based emission factor, such as the emission factor of fully loaded, 50%-loaded, empty truck, etc. Therefore this second method may evaluate the CO<sub>2</sub> emissions only when the percentages of vehicle load and relevant emission factors are assumed known.

Both the Commission of the European Communities (DGVII) and the Department for Environment, Food and Rural Affairs of United Kingdom (DEFRA) apply the distance-based method to estimate general

CO<sub>2</sub> emissions for global transportation. The report of DGVII (MEET, [Hickman et al. \[1999\]](#)) classifies vehicles into several categories according to their gross weights. A formulation is developed to calculate CO<sub>2</sub> emission factors corresponding to unloaded and fully loaded vehicles. The DEFRA directly presents CO<sub>2</sub> emission factors corresponding to different vehicle loads based on estimated data of UK vehicles ([DEFRA \[2005\]](#)).

In this research we will use both methods for estimated emissions for collection or distribution tours, where vehicle loads are varying, or for inter-hub transport where we assume full vehicle loads.

## 2.5 Conclusions and research proposals

In this chapter, we reviewed recent literature on the HLP and the LRP which have attracted many researchers, as well as the literature on the HLRP. We shed light on basic classification, mathematic models and solution approaches (algorithms) for both the classic and variant problems. By means of surveying the mathematical models and solutions methods, we can inspire from the efficient formulations and methods to determine our research problem and build the problem models.

It is obvious that the research on the HLRP in LTL transport network is very limited. Most of the papers on the HLRP fail to take into account vehicle capacity constraints and the determination of the open hub numbers at the same time. Almost all the researches of the HLRP consider the demand structure that the collection and delivery of products happen simultaneously. Thus the development of a general model for the HLRP that the collections and deliveries are performed separately in capacitated vehicles is a promising research direction. It also can be concluded from the literature of the HLP, the LRP and the HLRP that although some heuristics or exact solution methods have been proposed, only small and medium problems can be solved. The development of the hybrid metaheuristic methods is the trend to solve the large-size problem. Furthermore, although the requirement of reducing the effect of transport on environment is mounting, the environment aspect in the HLRP literature is ignored. There is a need to develop the models taking both the economical and environmental aspects into consideration. Inspired by the literature of the Pollution Vehicle Routing Problem (PRP) and sustainable Supply Chain Network Design (SCND), the environment aspect can be considered in the HLRP.

Our work, developed in the next chapters, is devoted to the CSAHLRP with distinct pick-ups and deliveries for general many-to-many LTL transport. We propose a new MILP model by integrating the efficient flow variables and constraints of the VRP into the classic HLP formulations. Solutions are obtained by solving the model with an MILP solver, and developing a Memetic Algorithm (MA) for instances of different sizes based on the AP data set. Further, we extend the single-objective HLRP into a multi-objective problem by considering both the total cost and the CO<sub>2</sub> emissions caused by transportation (freight collection, delivery and inter-hub transfer). A hybrid MA is developed to generate the approximated Pareto Front of the non-dominated cost/CO<sub>2</sub> solutions. At last, the HLRP model is decomposed into the HLP and VRP models. The proposed single-objective is adapted to deal with the HLRP by solving the HLP first and then the VRP on the separate collections and deliveries.





## Single-objective HLRP for minimizing cost





## Mathematical model for the single-objective HLRP

In this chapter, we present a single-objective Hub Location-Routing mathematical formulation. The model contains decisions of the determination of hub numbers, the decisions of hub location and product flows exchange between hub points. It also integrates vehicle routing formulations and decision variables, such as the flow variables on vehicle, to schedule local tours for the HLRP. The formulations of hub location and allocation are inspired from the classic HLP model (O'Kelly [1987a], Ernst and Krishnamoorthy [1999], Correia et al. [2011], Saiedy et al. [2011], Karimi et al. [2014]). The other important routing and vehicle load (flow) variables and formulations are motivated by the works of Single Capacitated Vehicle Routing Problem (SCVRP) (Kara et al. [2007], Toth and Vigo [2014]) and Pollution Vehicle Routing problem (PRP) (Demir et al. [2012], Xiao et al. [2012], Demir et al. [2014] and Kramer et al. [2015]).

This chapter is organized as follows. Section 3.1 introduces the problem definition of the CSAHLRP and corresponding assumptions. Section 3.2 presents notations for parameters and decision variables. With these notations, a Mixed Integer Linear Programming (MILP) HLRP model is proposed with comprehensive explanations.

### 3.1 Problem definition

The CSAHLRP studied here is defined on a complete directed graph  $G = (N, A)$  containing a set of vertices  $N$  and a set of arcs  $A$  where  $N = H \cup I \cup J$ .  $H = \{k \mid k = 1, 2, \dots, h\}$  presents a set of potential hubs. Each potential hub is capacitated and subjected to a fixed cost  $F_k$  once selected open.  $I = \{i \mid i = h+1, h+2, \dots, h+m\}$  and  $J = \{j \mid j = h+m+1, h+m+2, \dots, h+m+n\}$  stand for the sets of customers and clients who should be served. The numbers and locations of potential hubs, suppliers and clients are known data. Each pair of  $i$  and  $j$  ( $i \in I, j \in J$ ) is associated with a given amount of freight flow  $q_{ij}$  to be shipped between them. The total supply  $O_i = \sum_{j \in J} q_{ij}$  of supplier  $i$  and demand  $D_j = \sum_{i \in I} q_{ij}$  of client  $j$  should be satisfied. The set  $A = A_1 \cup A_2 \cup A_3$  includes the collection arc set  $A_1 = \{(i, j) \mid i, j \in I \cup H\}$ , the delivery arc set  $A_2 = \{(i, j) \mid i, j \in J \cup H\}$  and the inter-hub transfer arc set  $A_3 = \{(l, k) \mid l, k \in H\}$ . A fleet of identical vehicles available for collections and deliveries is denoted as set  $V$  with a fixed capacity  $Q$ . Once a vehicle travels from non-hub node  $i$  to non-hub node  $j$ , a routing cost is incurred that is dependent on the distance  $d_{ij}$ . The transportation costs between two hubs are determined by travelling distances and transferred flows and inter-hub transport is not subject to capacity restrictions. To model the collection and

delivery routes, a flow variable  $f_{ij}$  is used (see Karaoglan and Altıparnak (2105)), representing the vehicle load on each arc  $(i, j)$ . Figure 3.1 illustrates the general network of the HLRP.

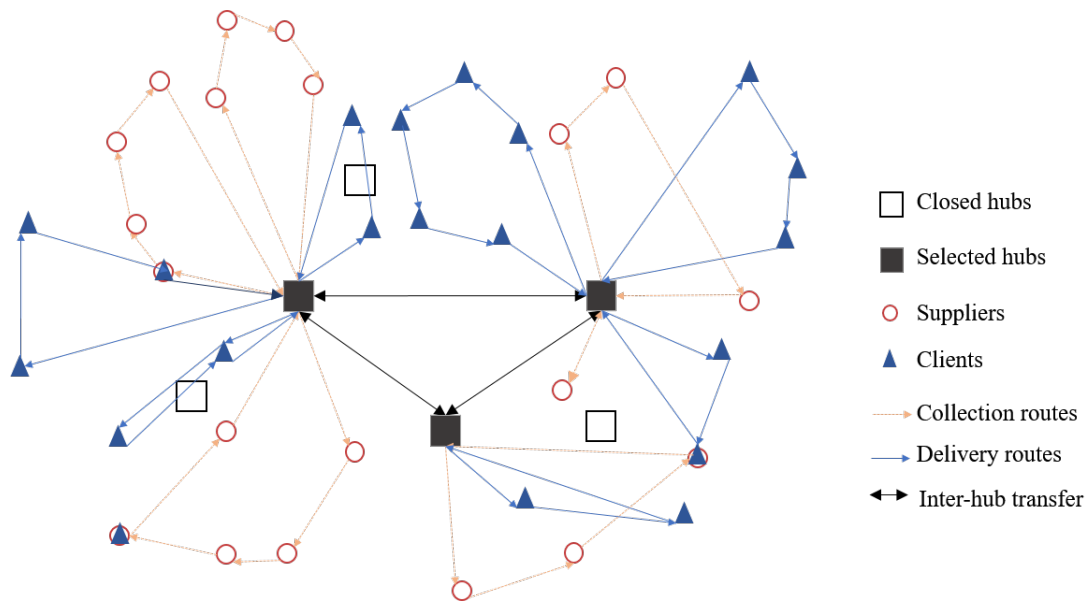


FIGURE 3.1 : General network of the HLRP

The following other constraints must hold :

- The capacity of each hub and collection/delivery vehicle must not be exceeded.
- The collection and delivery processes are independent. Collections are executed preceding long distances transfers of inter-hub, while deliveries are performed after the inter-hub transport processes.
- Each route must start and end at the same selected hub.
- During collections and deliveries, suppliers /clients on the same route must be assigned to the same selected hub.
- The demand of each non-hub node has to be consolidated in the allocated hub, which means that the direct transportations for supplier-client pairs are forbidden.
- Suppliers or clients on the same route must be assigned to only one selected hub.
- Each route must be visited by only one vehicle and each vehicle must contain only one type of flow (collected or delivered).
- The offers of suppliers can be transferred at most two hubs before delivering to clients.
- In inter-hub transportations, vehicles connect hubs directly to one another and there are no tours

The problem is to determine simultaneously the location of the hubs, the allocation of the suppliers and clients to hubs, the collection and delivery routing processes between nodes allocated to the same hub, as well as the inter-hub freight transportation. The optimization goal is to minimize the total fixed and variable costs.

TABLE 3.1 : Notation used in the model of the HLRP

<i>Sets</i>	<i>Description</i>
$H$	Set of hub nodes, $H = \{k \mid k = 1, 2, \dots, h\}$
$I$	Set of supplier nodes, $I = \{i \mid i = h + 1, h + 2, \dots, h + m\}$
$J$	Set of client nodes, $J = \{j \mid j = h + m + 1, h + m + 2, \dots, h + m + n\}$
$N$	Set of all nodes, $N = H \cup I \cup J$
$A_1$	Set of arcs in collection routing, $A_1 = \{(i, j) : i, j \in I \cup H\}$
$A_2$	Set of arcs in delivery routing, $A_2 = \{(i, j) : i, j \in J \cup H\}$
$A_3$	Set of arcs in inter-hub transfer, $A_3 = \{(l, k) : l, k \in H\}$
$A$	Set of all arcs, $A = \{(i, j) : i, j \in N\}$
<i>Parameters</i>	
$F_k$	Fixed cost of operating hub $k$
$\Gamma_k$	Capacity of hub $k$
$C$	Fixed cost of a vehicle
$c_k$	Handling cost to operate one unit product in hub $k$ , $k \in H$
$Q$	Capacity of a vehicle
$q_{ij}$	Flow quantity from supplier $i \in I$ to client $j \in J$
$d_{ij}$	Distance between two nodes $i$ and $j$ , arc $(i, j) \in A$
$\alpha$	Unit cost parameter for the inter-hub transport
$\beta$	Unit cost parameter for the collection tour
$\gamma$	Unit cost parameter for the delivery tour
$O_i$	Total quantity of flow originating at supplier $i$ , $O_i = \sum_{j \in J} q_{ij}$
$D_j$	Total quantity of flow for client $j$ , $D_j = \sum_{i \in I} q_{ij}$
<i>Decision variables</i>	
$Y_{kl}^i$	The fraction of flow shipped from supplier $i$ via hubs $k$ to $l$ , $k, l \in H$ , and $i \in I$
$z_{ik}$	The allocation variable of a node $i$ to a hub $k$ . It is equal to 1 if the node $i$ is allocated to the hub $k$ , 0 otherwise; especially, $z_{kk} = 1$ if the hub $k$ is selected to be open, $i \in N, k \in H$
$x_{ij}$	Equal to 1 if a vehicle traverses arc $(i, j)$ , and 0 otherwise
$f_{ij}$	Vehicle load on arc $(i, j)$ if a vehicle travels directly from node $i$ to node $j$ , otherwise 0, $(i, j) \in A_1 \cup A_2$

### 3.2 A mathematical model for the single-objective HLRP

All the notions used in developing the corresponding HLRP model are summarized in Table (3.1). Using these notation, the HLRP model for minimizing total costs can be formulated as follows :

$$\begin{aligned}
 \text{HLRP-COST} \quad \min \quad & \sum_{k \in H} F_k z_{kk} + \sum_{i \in I} \sum_{(k,l) \in A_3} \alpha d_{kl} O_i Y_{kl}^i + \sum_{(i,j) \in A_1} \beta d_{ij} x_{ij} + \sum_{(i,j) \in A_2} \gamma d_{ij} x_{ij} \\
 & + \sum_{i \in I} \sum_{k \in H} c_k O_i z_{ik} + \sum_{i \in I} \sum_{(k,l) \in A_3} c_l O_i Y_{kl}^i + \sum_{k \in H} \sum_{i \in I \cup J} C x_{ki}
 \end{aligned} \tag{3.1}$$

Subject to

— hub location constraints :

$$z_{ik} \leq z_{kk} \quad \forall i \in N, \forall k \in H \quad (3.2)$$

$$\sum_{k \in H} z_{ik} = 1 \quad \forall i \in I \cup J \quad (3.3)$$

$$\sum_{i \in I} O_i z_{ik} \leq \Gamma_k z_{kk} \quad \forall k \in H \quad (3.4)$$

$$\sum_{j \in J} D_j z_{jl} \leq \Gamma_l z_{ll} \quad \forall l \in H \quad (3.5)$$

$$\sum_{l \in H} Y_{kl}^i = z_{ik} \quad \forall i \in I, \forall k \in H \quad (3.6)$$

$$\sum_{k \in H} Y_{kl}^i O_i = \sum_{j \in J} q_{ij} z_{jl} \quad \forall i \in I, \forall l \in H \quad (3.7)$$

— collection routing constraints :

$$\sum_{j \in I \cup H} x_{ij} = 1 \quad \forall i \in I \quad (3.8)$$

$$\sum_{i \in I \cup H} x_{ij} - \sum_{i \in I \cup H} x_{ji} = 0 \quad \forall j \in I \cup H \quad (3.9)$$

$$x_{ki} \leq z_{ik} \quad \forall i \in I, k \in H \quad (3.10)$$

$$x_{ik} \leq z_{ik} \quad \forall i \in I, k \in H \quad (3.11)$$

$$x_{ij} + z_{ik} + z_{jl} \leq 2 \quad \forall i, j \in I, i \neq j, \forall k, l \in H, k \neq l \quad (3.12)$$

$$\sum_{j \in I \cup H} f_{ij} - \sum_{j \in I \cup H} f_{ji} = O_i \quad \forall i \in I \quad (3.13)$$

$$\sum_{i \in I} f_{ik} = \sum_{i \in I} z_{ik} O_i \quad \forall k \in H \quad (3.14)$$

$$f_{ij} \leq (Q - O_j) x_{ij} \quad \forall i \in I \cup H, \forall j \in I \quad (3.15)$$

$$O_i x_{ij} \leq f_{ij} \quad \forall i \in I, \forall j \in I \cup H \quad (3.16)$$

$$\sum_{i \in I} f_{ki} = 0 \quad \forall k \in H \quad (3.17)$$

$$f_{ik} \leq Q x_{ik} \quad \forall i \in I, k \in H \quad (3.18)$$

— delivery routing constraints :

$$\sum_{j \in J \cup H} x_{ij} = 1 \quad \forall i \in J \quad (3.19)$$

$$\sum_{i \in J \cup H} x_{ij} - \sum_{i \in J \cup H} x_{ji} = 0 \quad \forall j \in J \cup H \quad (3.20)$$

$$x_{jk} \leq z_{jk} \quad \forall j \in J, k \in H \quad (3.21)$$

$$x_{kj} \leq z_{jk} \quad \forall j \in J, k \in H \quad (3.22)$$

$$x_{ij} + z_{ik} + z_{jl} \leq 2 \quad \forall i, j \in J, i \neq j, \forall k, l \in H, k \neq l \quad (3.23)$$

$$\sum_{i \in J \cup H} f_{ij} - \sum_{i \in J \cup H} f_{ji} = D_j \quad \forall j \in J \quad (3.24)$$

$$\sum_{j \in J} f_{kj} = \sum_{j \in J} z_{jk} D_j \quad \forall k \in H \quad (3.25)$$

$$f_{ij} \leq (Q - D_i) x_{ij} \quad \forall i \in J, \forall j \in J \cup H \quad (3.26)$$

$$D_j x_{ij} \leq f_{ij} \quad \forall i \in J \cup H, \forall j \in J \quad (3.27)$$

$$\sum_{j \in J} f_{jk} = 0 \quad \forall k \in H \quad (3.28)$$

$$f_{kj} \leq Q x_{kj} \quad \forall j \in J, k \in H \quad (3.29)$$

— domain of variables :

$$z_{ik} \in \{0, 1\} \quad \forall i \in N, \forall k \in H \quad (3.30)$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in N, \forall j \in N \quad (3.31)$$

$$0 \leq Y_{kl}^i \leq 1 \quad \forall i \in I, \forall k, l \in H \quad (3.32)$$

$$f_{ij} \geq 0 \quad \forall (i, j) \in A_1 \cup A_2 \quad (3.33)$$

— valid inequalities :

$$x_{ij} + x_{ji} \leq 1 \quad \forall i, j \in I \quad (3.34)$$

$$x_{ij} + x_{ji} \leq 1 \quad \forall i, j \in J \quad (3.35)$$

$$\sum_{k \in H} z_{kk} \geq \left\lceil \frac{\sum_{i \in I} \sum_{j \in J} q_{ij}}{\Gamma_k} \right\rceil \quad (3.36)$$

$$\sum_{k \in H} \sum_{i \in I} x_{ki} \geq \left\lceil \frac{\sum_{i \in I} O_i}{Q} \right\rceil \quad (3.37)$$

$$\sum_{k \in H} \sum_{j \in J} x_{kj} \geq \left\lceil \frac{\sum_{j \in J} D_j}{Q} \right\rceil \quad (3.38)$$

In addition, routing variables  $x_{ij}$  would be ignored in a preprocessing step whenever  $O_i + O_j > Q, \forall i, j \in I, i \neq j$  or  $D_i + D_j > Q, \forall i, j \in J, i \neq j$  (Karaoglan and Altıparmak [2015]).

The objective function (3.1) minimizes the total fixed and variable costs of the CSAHLRP network. More precisely, it includes the fixed cost for opening hubs, the transportation cost between hubs, local collection and delivery routing costs, the freight operating cost in hubs, and the fixed cost of routing vehicles once used. Constraints (3.2) ensure that non-hub nodes can be allocated to a hub only if the hub is open. Constraints (3.3) force each non-hub node to be assigned to only one hub (single allocation). Hub capacity constraints (3.4) and (3.5) limit the total collection and delivery load on hubs. Constraints (3.6) and (3.7) are flow conservation equations. They impose the demand of each supplier or client to be served by the allocated hub (Ernst and Krishnamoorthy [1999]). Constraints (3.8) to (3.18) assure a reasonable collection process. Constraints (3.8) guarantee that each supplier is visited just once. Constraints (3.9) guarantee an equal number of incoming and outgoing arcs. Constraints (3.10) - (3.12) eliminate illegal routes that do not start and end at the same hub. Constraints (3.13) are the flow conservation constraints for collections. Each time the vehicle serves a supplier, it must load all of its demand. Constraints (3.14) ensure that the total collection load entering each open hub equals the total demand of the suppliers who are allocated to the hub. Constraints (3.15) and (3.16) provide an upper and lower bound for the collection flows. Constraints (3.17) guarantee that the load on each vehicle is zero when leaving one open hub for collecting (Karaoglan and Altıparmak [2015], Yu and Lin [2016]). Constraints (3.18) impose that if there is no arc between a supplier node and a hub node, the flow should be zero. Constraints (3.19)-(3.29) relative to the delivery processes represent conditions similar to those for the collections. Constraints (3.30)-(3.33) specify the variables  $z_{ik}$ ,  $x_{ij}$ ,  $Y_{kl}^i$  and  $f_{ij}$ , respectively. Constraints (3.34) to (3.38) are valid inequalities : constraints

(3.34) and (3.35) are sub-tour elimination; constraint (3.36) restricts the minimum number of open hubs (Bostel et al. [2015]); finally, constraints (3.37) and (3.38) provide a lower bound of the total number of vehicles required in any feasible solution.

### 3.3 Conclusion

In this Chapter, we focus on the CSAHLRP with distinct pick-ups and deliveries for general many-to-many LTL transport and define the researched problem in details. The characteristic of considering the predetermined flow for pairs of origin-destination is the main difference between the proposed HLRP and the LRP. We propose a new MILP model by integrating the efficient flow variables and constraints of the VRP into the classic HLP formulations.

# A memetic algorithm for the single-objective HLRP

Since the HLRP integrates a hub location problem (HLP) and a vehicle routing problem (VRP), which are both NP-hard optimization problems, commercial solvers cannot solve large instances and the number of exact solution methods of the HLP and the VRP is limited. Thus, heuristic and metaheuristic algorithms have been developed in order to obtain good quality solutions in a reasonable computing time. In this chapter, we propose a Memetic Algorithm (MA), combining a genetic algorithm (GA) and an iterated local search (ILS), to determine location and routing jointly. This algorithm is inspired from [Derbel et al. \[2012\]](#).

This chapter is organized as follows. Section 4.1 gives a global overview of the general MA. Section 4.2 depicts the sketch and specific process of the proposed MA. Section 4.2 presents the systematic view of the MA process. Section 4.2.1 to 4.2.4 describe the GA operation in terms of the solution representation, the initial population, the selection, crossover and mutation operators. Section 4.2.5 gives the way to develop the ILS on local collection/delivery and hub location parts independently. Finally, a conclusion is presented in Section 4.3.

## 4.1 An overview of the memetic algorithm

Heuristic methods are specific problem-dependent techniques which are often trapped by local optimum in search process, while the metaheuristic methods guide a series of heuristics by incorporating them in an iterative framework for a better evolution and for the acceleration of convergence in search space ([Glover and Kochenberger \[2006\]](#)). The research of metaheuristics has been growing rapidly in the last few years. Various classifications of metaheuristics can be found in articles of [Blum and Roli \[2003\]](#), [Lozano and García-Martínez \[2010\]](#) and [Blum et al. \[2011\]](#). Among these metaheuristics, the evolutionary algorithms (EA), especially the hybrid EAs, have attracted great attentions because of their flexibility, simplicity and adaptability in a changing environment ([Blum and Roli \[2003\]](#)). According to the control strategy, the hybrid EAs are grouped into three types : the collaborative EAs, the integrative EAs and the metaheuristics with evolutionary intensification and diversity (I&D) components ([Lozano and García-Martínez \[2010\]](#)). In collaborative hybrid EAs, different EAs are applied to exchange information such as solutions and parameters in a parallel or in a pipeline way. The integrative hybrid EAs, such as memetic algorithm (MA), integrate one algorithm (subordinate) as a component into the EA (master). The third type of the hybrid EAs replaces main I&D components with evolutionary ones to improve the performance.

The word "memetic" of memetic algorithms is inspired from the interpretation of human culture being



able to be composed into simple units, namely memes (see [Bateson \[1978\]](#)). Strong and useful memes propagate in human communities and form new memes by being modified and combined, which therefore generates the human knowledge, at the same time, the weak and uninteresting ones die away. Inspired by the diffusion way of the memes, the memetic algorithm (MA) hybrid an Evolutionary Algorithm (EA) which is composed of an integrated sequence of existing operators and a set of local searches which is used to search an optimal (or quasi-optimal) solution ([Neri and Cotta \[2012\]](#)). As mentioned, the MA belongs to the class of integrative hybrid metaheuristics. The basic scheme of the MA is that a population-based EA provides a global framework to obtain solutions and a local search procedure is executed within it to improve the qualities of theses solutions.

The algorithmic structures of the MA offer a general guideline to solve complex practical problems :

- (1) Initializing procedure. The first set of solutions with required numbers (population size) is produced. The producing process can be random as for the traditional EA. In the other way, sophisticated mechanisms such as the heuristics can be used at the beginning of MA in attempt to inject high-quality initial solutions.
- (2) Parents selection and combination. The selection procedure is responsible for determining which candidate individuals can survive for reproduction. The performance of each individual is typically related to its fitness which can be measured by the objective function or other guiding functions based on the objective function. Selected parents are therefore combined to created new promising offspring into the following generation.
- (3) Local improvement. It starts from an individual and searches the neighbourhood by means of modifying the configuration of the individual at each step. The modification is composed of many moves such as changing the value of one variable, swapping two variables, replacing one variable with the other and so on, with aims to reach the local optimum ([Moscato and Cotta \[2003\]](#)). The newly generated offspring with preferable structure are accepted to the next searching round, otherwise, the current individual is kept. A termination criterion should be set to stop the process. The stopping criteria can be the maximum number of iterations, the maximum consecutive iterations without improvement and other mechanisms ([Deb \[2001\]](#)) depending on specific characteristics of the problem.
- (4) New generation. In this step, decisions have to be made whether an offspring should survive in the next generation. Considerations of both the quality and diversity of one offspring are the basic rules.

Due to their nature-inspired characteristics, the memetic algorithms are capable of exploring and identifying more promising areas of the search spaces during genetic operations. Furthermore, to overcome the low efficiency of the GA in locating optimum solutions, the local searches resort to determine local optimum solutions in specific regions ([Blum et al. \[2011\]](#)). The MA has been utilized to handle numerous real-world problems in fields of machine learning and robotics, electronics and engineering, molecular optimization, transportation problems an so on. They have shown a remarkable success especially to cope with NP hard problems. Many variants of the MA have been applied to deal with the LRP and HLRP and some important works can be seen in [Table 4.1](#).

[Prins et al. \[2006a\]](#) and [Prodhon and Prins \[2008\]](#) solved the LRP by a Memetic Algorithm with Population Management (MA|PM) which was first proposed in [Sörensen and Sevaux \[2006\]](#). The algorithm of MA|PM starts on a small population and improves solutions by local search. Subsequently, a dynamic distance-based population management technique is utilized to replace the traditional mutation operator. [Prins et al. \[2006a\]](#) focused on the Capacitated Location-Routing Problem (CLRP) and conducted the experiments on three sets of randomly generated Euclidean instances. The results indicated a good performance compared to other metaheuristics. Later, [Prodhon and Prins \[2008\]](#) applied the MA|PM on the periodic location-routing problem (PLRP). [Prodhon \[2011\]](#) hybridized an Evolutionary Local Search (ELS) with a heuristic based on the Randomized Extended Clarke and Wright Algorithm (RECWA) to solve large instances of the PLRP. This method outperformed the previous methods for solving the PLRP. [Karaoglan and](#)

TABLE 4.1 : The application of MA on the LRP and the HLRPs

Problem	Article	method	Data size)
CLRP	Prins et al. [2006a]	MA   PM	200
	Derbel et al. [2012]	GA+ILS	30
PLRP	Prodhon and Prins [2008]	MA   PM	200
	Prodhon [2011]	ELS+RECWA	200
LRSPD	Karaoglan and Altiparmak [2010]	GA+SA	200
LRIP	Forouzanfar and Tavakkoli-Moghaddam [2012]	GA	60
		GA+LS	50
TLRP	Martínez-Salazar et al. [2014]	GA+TS	50
		GA+SA+IPF	100
CLRPMB	Karaoglan and Altiparmak [2015]	GA+SA+IPF	100
USApHLRP	Rieck et al. [2014]	Multi-start+GA	146
USApHLRP	Zameni and Razmi [2015]	GA	50 (AP& CAB)
CSAHLRP	Zhang et al. [2014]	GA+ILS	100 (AP)
CSAHLRP	<b>Our research</b>	<b>GA+ILS</b>	<b>100 (AP)</b>

Altiparmak [2010] proposed a hybrid metaheuristic based on a GA and a Simulated Annealing (SA) method to cope with a Location-Routing Problem with simultaneous pickup and delivery (LRPSPD). Experimental results were compared to the upper bounds generated by the MIP and indicated its capability in finding optimal or very good quality solutions. Derbel et al. [2012] introduced a hybrid metaheuristic that a GA is combined with an ILS using four neighbourhood structures to solve the CLRP. After being tested on five benchmark problem sets, it showed that the results were better than the best previously ones obtained by a Tabu Search (TS) heuristic. Forouzanfar and Tavakkoli-Moghaddam [2012] applied a GA to solve the Location-Routing-Inventory Problems (LRIP). Martínez-Salazar et al. [2014] presented and compared two MAs. One replace the mutation process with a simple local search, while the other utilized a TS as the mutation operator. Computational results of these two algorithms indicated that the second one was better. Recently, Karaoglan and Altiparmak [2015] considered a Capacitated Location-Routing Problem with Mixed Backhauls (CLRPMB) by a MA and compared the results with the lower bounds obtained by the Branch-and-Cut (B & C) algorithm. The proposed MA was a combination of a GA, a SA and a integer programming formulation (IPF). The instances of the CLRP and the CLRPMB derived from the literature were tested by the proposed MA. As a result, it proved that MA improved some best known feasible solutions and obtained good quality solutions.

The application of MAs also can be found in solving the HLRPs in literature. Rieck et al. [2014] presented a metaheuristic algorithm for an Uncapacitated Single Allocation  $p$ -Hub Location-Routing Problem (USApHLRP). This algorithm embedded a multi-start procedure which was based on a fix-and-optimize scheme into a GA. The computational experiments were conducted on randomly generated instances. Zameni and Razmi [2015] focused on the USApHLRP and solved the small and medium scales by GAMS software while a GA was developed to solve the large instances. Zhang et al. [2014] integrated a set of ILS into a GA for solving a Capacitated Single Allocation Hub Location-Routing Problem (CSAHLRP). Other articles of MAs and hybrid GAs that are related to the researched HLRP are listed in Table 4.2.

## 4.2 Algorithmic design of the MA for the HLRP

In this section, the algorithmic design of the MA for the HLRP is presented, aiming at minimizing the total network cost of the HLRP. First of all, the generic framework and pseudo-code of the proposed MA are introduced in Figure 4.1 and **Algorithm 1**. The initial population is generated using both heuristic

TABLE 4.2 : Literatures of MA/hybrid GA on related problems

Problem	Articles
VRP	Berger and Barkaoui [2003], Baker and Ayechev [2003], Prins [2004], Ombuki et al. [2006], Potvin [2007], Ho et al. [2008], Prins [2009], Nagata et al. [2010], Vidal et al. [2012], Mohammed et al. [2012], Barkaoui and Gendreau [2013], Nalepa and Czech [2014], Cattaruzza et al. [2014], Liu et al. [2014]
HLP	Abdinnour-Helm [1998], Topcuoglu et al. [2005], Kratica et al. [2011], Kratica et al. [2012], Cunha and Silva [2007], Stanimirović [2007], Stanimirović [2008], Stanimirović [2012], Takano and Arai [2009], Naeem and Ombuki-Berman [2010], Mohammadi et al. [2010], Sun and Park [2012], Marić et al. [2013], Bashiri et al. [2013]
LRP	Prins et al. [2006a], Derbel et al. [2012], Prodhon and Prins [2008], Prodhon [2011], Karaoglan and Altiparmak [2010], Forouzanfar and Tavakkoli-Moghaddam [2012], Martínez-Salazar et al. [2014], Karaoglan and Altiparmak [2015]
HLRP	Rieck et al. [2014], Zhang et al. [2014], Zameni and Razmi [2015]

methods to accelerate the convergence of the algorithm and randomized solutions to keep its diversity. Each individual is evaluated by a fitness function corresponding to the objective function plus a penalty cost based on the capacity violation. Pairs of parent solutions are selected by a unique fitness binary tournament selection (line 6), and a one-point crossover with a probability is applied on the selected parents pairs, followed by a mutation procedure (line 8 and line 10) (Section 4.2.4). Next, local searches are implemented iteratively on the vehicle routing and hub location parts of the chromosomes to create new offsprings (line 11) (Section 4.2.5). When the new offspring improves the current best solution, it is automatically saved as the current best (line 13). Finally, the newly-generated offspring is added to the current generation and the worst individual is eliminated (line 17). The whole process stops when the maximum number of iterations of the algorithm or the number of successive iterations without improvement is reached (line 5). The following sections describe the main phases of the MA in detail.

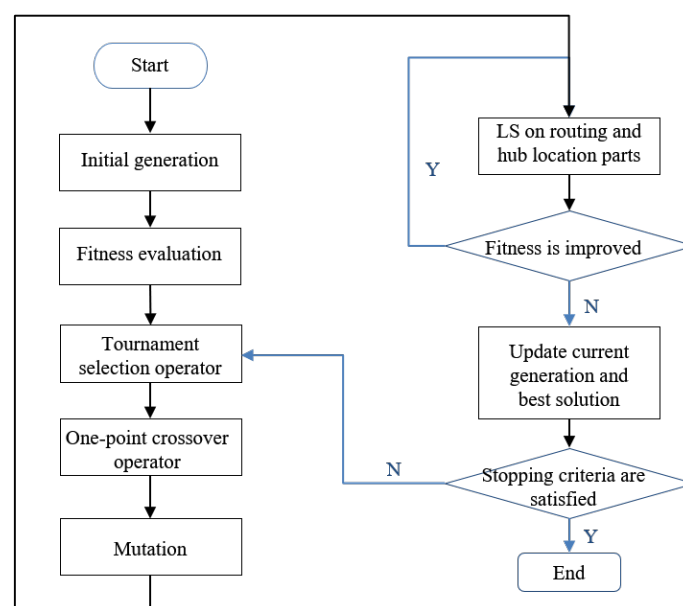


FIGURE 4.1 : Generic framework of the MA

**Algorithm 1** The MA for the HLRP

**Input**  $Npop$  : population size;  $P_c$  : probability of crossover;  $P_m$  : probability of mutation;  $MaxI$  : maximum iterations;  $NoImp$  : maximum iterations without improvement

**Output** Best solution  $x_{best}$

```

1:  $n \leftarrow 0$ ;  $m \leftarrow 0$ ;  $S \leftarrow \emptyset$ 
2: Generate initial generation with  $Npop$  chromosomes
3: Calculate fitness  $F_{eva}(x)$  for each chromosome  $x$ 
4: Find current best solution  $x_{best}$ ,  $F_{eva}(x_{best}) \leftarrow \text{Min } F_{eva}(x)$ 
5: while ( $n < MaxI$ ) and ( $m < NoImp$ ) do
6:   Select pairs of parent chromosomes based on a tournament unique fitness selection operator and stored in a solution set of  $S$ 
7:   for each pair of parents in the set of  $S$  do
8:     Apply a 1-point crossover operator with a probability  $P_c$  to the two selected parents to obtain the offspring  $x_{new}$ 
9:   end for
10:   Apply mutation to  $x_{new}$  with a probability  $P_m$ 
11:   Apply iterated local search (ILS) to  $x_{new}$  and obtain new solution  $x'_{new}$ 
12:   if  $F_{eva}(x'_{new}) < F_{eva}(x_{best})$  then
13:     Update current best solution  $x_{best} \leftarrow x'_{new}$ 
14:   else
15:      $m \leftarrow m + 1$ 
16:   end if
17:   Update current generation
18:    $n \leftarrow n + 1$ 
19: end while
return  $x_{best}$ 

```

### 4.2.1 Solution representation and evaluation

A fundamental issue in designing a GA is to represent individuals as a set of chromosomes. Different genetic representations are proposed depending on the problem and can affect the performance of the GA (Ardjmand et al. [2015], Deng et al. [2016]). In our approach, each chromosome  $P(x)$  stands for one solution  $x$  and contains two vectors : the selected hub vector  $H(x)$  and the non-hub nodes routing vector  $A(x)$ , according to the encoding scheme proposed by Prins et al. [2006a]. Vector  $H(x)$  contains the selected hubs and their assignment configuration. Vector  $A(x)$  records the permutation of suppliers and clients according to their served sequences on a route. The positions of the open hubs in vector  $H(x)$  are connected with their allocated suppliers and clients in vector  $A(x)$ . As one supplier/client must be served by only one hub, the vector  $A(x)$  does not contain duplicates, that is, each supplier/client must appear only once in the vector  $A(x)$ . More specifically, consider a chromosome  $P(x)$  and its two vectors  $H(x) = \{h_1, h_2, \dots, h_n\}$  and  $A(x) = \{a_1, a_2, \dots, a_n\}$ . For every  $i = \{1, 2, \dots, n\}$ , if  $a_i = j$ ,  $h_i = k$ , the non-hub node  $j$  is assigned to hub  $k$ . The tours for collections and deliveries can be deduced following the allocation scheme. According to the fixed sequence in vector  $A(x)$ , one vehicle starts from the first open hub and visits suppliers following the allocation scheme of the route until the capacity of the vehicle is reached. Then a new tour begins and the process continues until all the suppliers are routed. The same procedure takes place for the delivery routes.

Take an example with 3 hubs, 5 suppliers and 5 clients. Figure 4.2 illustrates one representation of the chromosome  $P(x)$  and its corresponding network sketch. The hubs, suppliers and clients are numbered in natural numbers 1, 2, ..., 13. Nodes 1, 2 and 3 represent the hubs. Nodes 4, 5, ..., 8 stand for the suppliers (numbers in gray in Figure 4.2) and the remaining nodes from 9 to 13 are the clients. Hubs 1 and 3 are selected open. The arrows between vectors  $H(x)$  and  $A(x)$  in the first part of the figure indicate the allocation scheme. For instance, the position of the supplier nodes 7, 6 and 8 correspond to hub 3 in vector  $H(x)$ , which means that these suppliers are allocated to hub 3. If the total demand of the three suppliers can be served by one vehicle (fit vehicle capacity), the route from hub 3 starts with supplier 7, then goes to

6 and 8 and ends back at hub 3. Similarly, clients 9, 13 and 12 are assigned to hub 1. However, if there is insufficient vehicle space after serving client 9, a new route is created beginning with client 13, to continue the delivery process. This method of representation is simple and fast at simultaneously capturing locations and routings, which enables us to build feasible solutions and apply GA operators efficiently.

In order to compare and select the chromosome solutions, every individual is evaluated in terms of its fitness value, determined by a fitness evaluation function  $F_{eva}(x)$ . The fitness function  $F_{eva}(x)$  of a solution  $x$  is defined as :

$$F_{eva}(x) = ObjValue(x) + Penalty(x) \tag{4.1}$$

$$Penalty(x) = \sigma \sum_{k \in H} \max\{0, \sum_{i \in I} z_{ik} O_i - \Gamma_k\} + \sigma \sum_{k \in H} \max\{0, \sum_{j \in J} z_{jk} D_j - \Gamma_k\} \tag{4.2}$$

where  $ObjValue(x)$  denotes the objective value of solution  $x$  calculated by Equation (3.1) (see Section 3.2).  $\sigma$  is the penalty parameter with a large value,  $\sum_{i \in I} z_{ik} O_i$  and  $\sum_{j \in J} z_{jk} D_j$  represent the total demand of suppliers and clients allocated to an open hub  $k$ ,  $\Gamma_k$  is the capacity of hub  $k$ , for loading or unloading operations. Thus the sum of the collection and delivery quantities violating the capacity of all open hubs is multiplied by the penalty parameter  $\sigma$  to calculate the penalty cost  $Penalty(x)$  for solution  $x$  (Equation (4.2)).

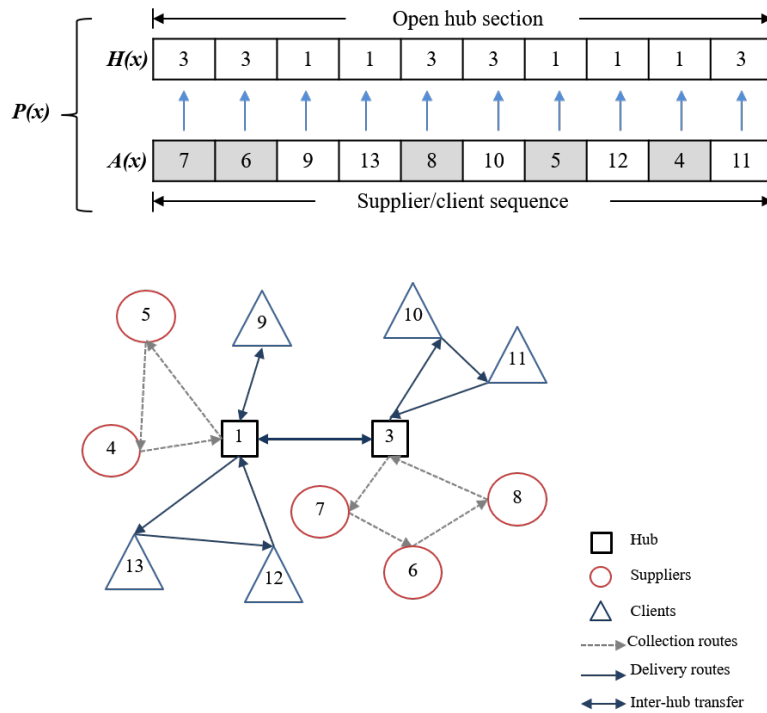


FIGURE 4.2 : Representation of an HLRP solution

### 4.2.2 Initialization of a population

To initialize the MA, an initial population must be generated. As mentioned in the global framework of the MA, the set of initial solutions consists of heuristic and random ones. Figure 4.3 presents the three main steps of the generation procedure : hub locating, non-hub allocating and local vehicle routing.

More specifically, in **Algorithm 2**, the minimum number  $MinH$  of hubs to open is first calculated in order to satisfy the total demand. Then, the subsets  $H_1, H_2, \dots, H_n$  of the potential hub set  $H$  are generated.

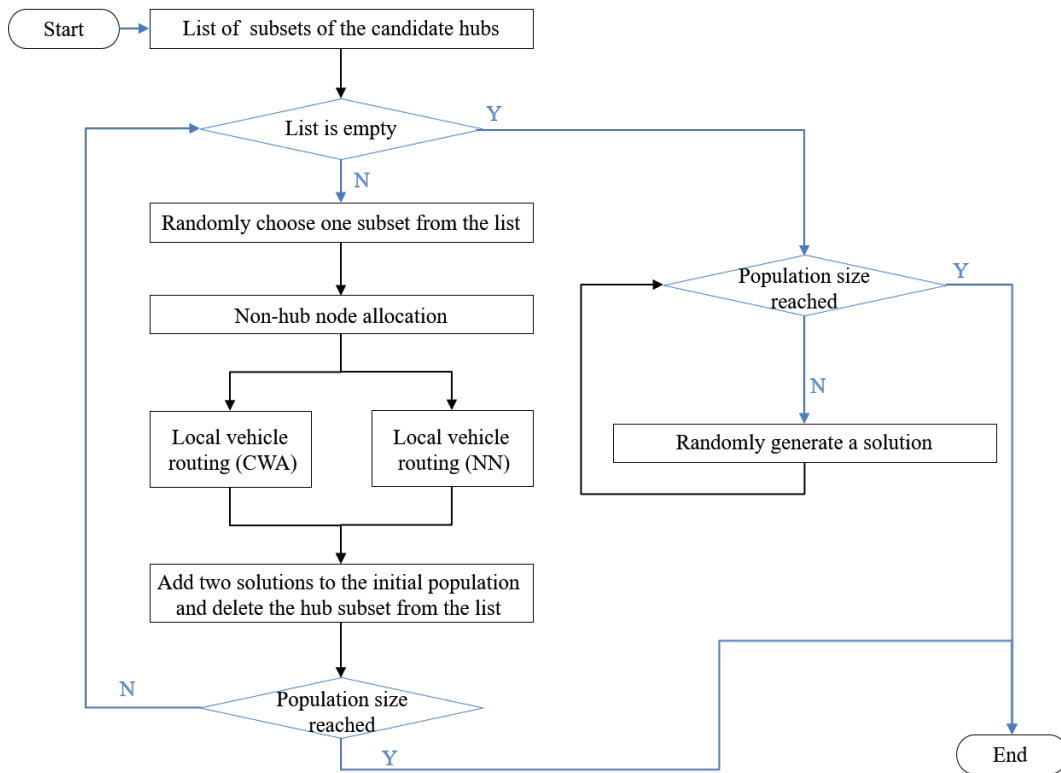


FIGURE 4.3 : Generation of an initial population

Each hub subset  $H_i$  must meet the minimum hub number requirement such as  $MinH \leq |H_i| \leq |H|$  (line 3). Each time, one of the hub subsets is selected non-repetitively as the current open hub configuration (line 5). Non-hub nodes are allocated to these open hubs by the allocation procedure (line 6). Under such an allocation scheme, two algorithms (NN and CWA) are applied to create two different local collection and delivery routing schemes (line 7). This means that two feasible solutions are generated at the same time with the same hub locations and allocations but different local tours. The process iterates until the required population size is reached or all the hub location subsets have been considered. Because of the limited number of hub subsets, the heuristic procedure may fail to reach the required population size. The remaining solutions are randomly produced.

### Allocation process

The allocation process is inspired from the allocation part of the Extended Clarke and Wright Algorithm (ECWA) proposed by [Karaoglan and Altıparmak \[2015\]](#). The main reason for allocating nodes to hubs is to determine for each non-hub node, the first and second nearest hubs and calculate the saving (difference in costs/distances) between the two allocations (**Algorithm 3** : line 2 to line 5). Then, the non-hub nodes are classified in non-increasing order of their savings. The non-hub nodes with the largest saving are allocated first, then the available hub capacity is updated and the process continues. When some nodes cannot be allocated to their nearest hub, their saving is recalculated. Furthermore, random new hubs may be opened if there are still suppliers/clients left because the remaining capacity of the open hubs is not enough to serve them (lines 16 to 18). Depots without customers are closed (line 22). Assignment procedures for collection and delivery are independent. **Algorithm 3** shows the allocation procedure for suppliers, which is similar for clients.

---

**Algorithm 2** Generating initial solutions
 

---

**Input**  $H$  : potential hub set;  $\Gamma$  : hub capacity;  $D$  : total demand;  $Npop$  : population size; other HLRP data

**Output** Generation  $Gen$

```

1:  $count \leftarrow 0, i \leftarrow 1$ 
2:  $MinH \leftarrow \left\lceil \frac{D}{\Gamma} \right\rceil$  // Minimum hub number
3: Generate non-repetitive hub subsets  $H_1, H_2, \dots, H_n$  of  $H$  that  $MinH \leq |H_i| \leq |H|, i = 1, 2, \dots, n$ 
4: repeat
5:   Select a subset  $H_i$  as the open hub set
6:   Non-hub node allocation (Algorithm 3)
7:   Local routing for collections and deliveries (NN & CWA)
8:    $count \leftarrow count + 2; i \leftarrow i + 1$ 
9: until  $count = Npop$  or  $i = n$ 
10: if  $count < Npop$  then
11:   repeat
12:     Randomly generate one individual
13:      $count \leftarrow count + 1$ 
14:   until  $count = Npop$ 
15: end if
   return  $Gen$ 

```

---

**Algorithm 3** Supplier allocation procedure

---

**Input**  $H^*$  : open hub set;  $I$  : supplier set;  $\Gamma_k$  : hub capacity for hub  $k$ ,  $k \in H^*$ ;  
 $d_{ij}$  : distance between two nodes  $i$  and  $j$ ,  $i, j \in H^* \cup I$

**Output** Assignment scheme for each supplier node

- 1:  $SI \leftarrow I$ ;  $CapH_k \leftarrow \Gamma_k$ ;  $z_{ik} \leftarrow 0$ ;  $S_i \leftarrow 0$ ,  $i \in \{1, 2, \dots, |SI|\}$  // Initialization
- 2: **for** each supplier  $i$ ,  $i \in SI$  **do**
- 3:    $k'(i) \leftarrow \arg \min\{d_{ik} \mid O_i \leq CapH_k\}$ ,  $k \in H^*$  // Find the nearest hub for supplier  $i$
- 4:    $l'(i) \leftarrow \arg \min\{d_{il} \mid O_i \leq CapH_l\}$ ,  $l \in (H^* \setminus k'(i))$  // Find the second nearest hub for supplier  $i$
- 5:    $S_i \leftarrow d_{i,l'(i)} - d_{i,k'(i)}$  // Savings for supplier  $i$
- 6: **end for**
- 7: **for all** suppliers  $i$ ,  $i \in SI$  **do**
- 8:   Find a supplier  $i^*$  with the largest saving :  $i^* \leftarrow \arg \max\{S_i\}$ ,  $i \in SI$
- 9: **end for**
- 10: **if**  $O_{i^*} \leq CapH_{k'(i^*)}$  **then**
- 11:    $z_{i^*,k'(i^*)} \leftarrow 1$  // Assign supplier  $i^*$  to hub  $k'(i^*)$
- 12:    $CapH_{k'(i^*)} \leftarrow CapH_{k'(i^*)} - O_{i^*}$  // Update the current hub capacity
- 13:    $SI \leftarrow SI \setminus i^*$  // Remove the supplier node that has been allocated from List  $SI$
- 14: **end if**
- 15: **if**  $SI \neq \emptyset$  **then**
- 16:   **if** (there is at least one supplier  $i \in SI$ , for all  $k \in H^*$ ,  $O_i > CapH_k$ ) **then**
- 17:     Open a new hub  $h$ ,  $h \in H$  and  $h \notin H^*$
- 18:      $H^* \leftarrow h$
- 19:   **end if**
- 20:   **go to** Step 2
- 21: **end if**
- 22: Close the hubs with no assigned supplier nodes
- return** supplier assignment plans

---

**Routing process**

For each hub, the local routing problem, for the non-hub nodes allocated to it, is solved by two different heuristics (NN and CWA) and separately for suppliers and clients. The classic NN approach is based on the simple idea of inserting the nearest neighbor of the last inserted node into the route until the capacity of the vehicle is exhausted. New routes start from the hub until all the allocated non-hub nodes of a given hub are inserted.

An improved version of the CWA, proposed by [Caccetta et al. \[2013\]](#), generates the second routing solution. It calculates all the savings between non-hub nodes, and creates a savings list in a non-increasing order. The pairs of nodes are successively considered according to the savings list to build the routes. Three situations may occur : (i) if neither of the two nodes has been assigned to a route, a new route is built between them; (ii) if one of the two nodes has been included in the existing route without violating the vehicle capacity, the two nodes are connected and added to the same route; (iii) if both nodes have already been inserted into two different routes, the two nodes are not connected. The process repeats until all the nodes are inserted into a route.

**4.2.3 Selecting parents for crossover**

Since higher quality solutions can be captured with high-quality neighbourhood, good parents should be selected for the crossover process to form the new generation. Many selection methods have been researched in the literature, such as tournament selection, roulette wheel selection, ranking selection and so on ([Reeves \[2003\]](#)). After preliminary experiments, we adopt a unique fitness binary tournament selection ([Fortin and Parizeau \[2013\]](#)) to keep the diversity of the selected parents.



The unique fitness binary tournament selection defines a unique fitness set  $\mathcal{F}$ ,  $\mathcal{F} = \{f_{eva}(i) | i \in |\mathcal{F}|\}$ , including the unique fitness of the individuals from the current generation without repeating values (Fortin and Parizeau [2013]). Here,  $f_{eva}(i)$  stands for the value of the  $i$ th fitness in  $\mathcal{F}$ . Take a population with 5 individuals as a simple example. Assume that the fitness values of the five individuals are  $F_{eva}(x_1) = e$ ,  $F_{eva}(x_2) = f$ ,  $F_{eva}(x_3) = e$ ,  $F_{eva}(x_4) = b$  and  $F_{eva}(x_5) = e$ , respectively. Thus, the unique fitness set  $\mathcal{F} = \{e, f, b\}$ . If the fitness value  $e$  is selected during the selection procedure, one of the solutions sharing the same fitness value (individual  $x_1, x_3$  or  $x_5$ ) is randomly included in a selection set  $S$ . The advantage lies in limiting the chances of individuals with the same fitness reproducing so that the preservation of diversity is stimulated.

**Algorithm 4** illustrates the selection process.  $\tau$  is denoted as the number of fitnesses to select in each iteration of the selection procedure and a solution set  $S$  is used to store the selected individuals. The value of  $\tau$  equals the minimum value between the number of unique fitnesses and double the number of solutions left to select (line 3) (Fortin and Parizeau [2013]). Random  $\tau$  unique fitnesses from fitness set  $\mathcal{F}$  are stored in a list  $G$  (line 4). Next, the fitnesses in list  $G$  are compared with each other and the one with the smallest value is assigned to  $\mathbf{p}$  (line 6), after which, one of the solutions with the same fitness of  $\mathbf{p}$  is added to the final selection set  $S$  (lines 7 and 8). If the number of selected solutions  $|S|$  is not equal to the size of the population, the whole procedure continues. Finally, the individuals in the set of  $S$  are randomly arranged in pairs with the purpose of applying the crossover operator in the next phase (line 11).

---

#### Algorithm 4 Unique fitness selection

---

**Input**  $Gen$  : current generation ;  $Npop$  : population size ;  $\mathcal{F}$  : unique fitness set of the individuals in  $Gen$

**Output** Selected parents set  $S$

```

1:  $\tau \leftarrow 0, S \leftarrow \emptyset, G \leftarrow \emptyset$ 
2: while  $|S| \neq Npop$  do // Tournament selection
3:    $\tau \leftarrow \min\{2(Npop - |S|), |\mathcal{F}|\}$  // Generate the number of fitnesses to be compared
4:   Select random  $\tau$  fitnesses from  $\mathcal{F}$  and store in list  $G$ 
5:   for ( $j \leftarrow 1, 3, 5, \dots, |G| - 1$ ) do
6:      $\mathbf{p} \leftarrow \arg \min\{G(j), G(i + 1)\}$  // Compare two fitnesses and select the better one
7:     Randomly select one individual  $x$  so that  $F_{eva}(x) = \mathbf{p}, x \in Gen$ 
8:      $S \leftarrow S \cup x$  // Put the selected individual in the set of  $S$ 
9:   end for
10: end while
11: The individuals in the set of  $S$  are arranged randomly in pairs // Pairs of parents for the crossover
return  $S$ 

```

---

### 4.2.4 Crossover and mutation

The selection procedure is followed by a one-point crossover operation on both the hub location vector  $H(x)$  and the routing vector  $A(x)$  with a probability  $P_c$ , simultaneously swapping nodes on selected parent pairs to form new offspring. Figure 4.4 illustrates the crossover procedure on chromosomes. Two crossover points  $P_L$  and  $P_R$  are defined for the location part and the routing part, respectively. The hub location vector  $H(x)$  of a new offspring combines the code of Parent 1 before crossover point  $P_L$  and the code of Parent 2 after  $P_L$ . On the routing vector  $A(x)$ , the new offspring takes the code of Parent 1 before  $P_R$  as the first part. The second part sequentially copies the code of  $A(x)$  of Parent 2 except for the nodes that have been copied from Parent 1.

The mutation operator mutates the chromosome by using two different methods on the two sections  $A(x)$  and  $H(x)$ . An example is shown in Figure 4.5. In the location section  $H(x)$ , the hub assignment is modified by randomly choosing hubs to be replaced by others. Such a procedure makes it possible to open a new hub or close a hub. In the routing section  $A(x)$ , the random locations of two points are exchanged.

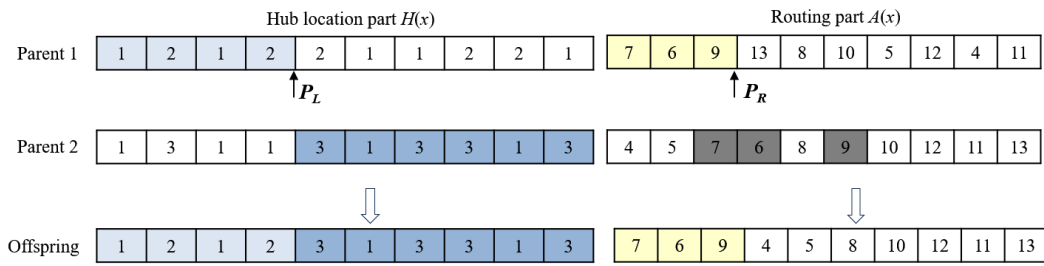


FIGURE 4.4 : The crossover operator

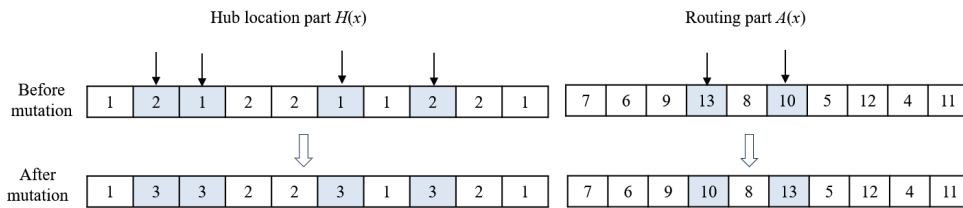


FIGURE 4.5 : Illustration of the mutation : hub 3 is opened to replace some locations of current open hubs ; the positions of two non-hub nodes are exchanged

### 4.2.5 Local search method

The iterated local search (ILS) aims to improve further the newly generated offspring. Inspired by the method of [Manzour-al Ajdad et al. \[2012\]](#), this section describes the iterated local search (ILS) heuristic that is implemented to seek a local optimal solution by exploring the neighbourhood of offspring. As illustrated in [Algorithms 5](#) and [6](#), first the local search is applied to the routing part of the chromosome with swap and insertion operators (line 1). Then, four operators are applied sequentially on the hub section : hub replacement, hub closing, hub opening and hub swapping operators (lines 2 to 8). Operations on the routing part restart after each operation on the hub location part (line 7). All the operators are applied on offspring chromosomes before they are decoded into real HLRP solutions.

#### ILS on routing

The ILS procedure on the routing part of a selected chromosome uses the following two neighbourhoods (Figure 4.6) :

- (1) Swap : Two non-hub nodes are selected to exchange their positions while other nodes remain unchanged.
- (2) Insertion : One non-hub node is shifted from its position and inserted into a random position in the routing vector. The operation is conducted on all the non-hub nodes. The insertion for each node is applied to all the positions to select the best one.

The two operators on the chromosome provide two types of possible changes to the real routing schemes : changing the node sequence in the same route and conserving the same hub allocation or reassigning nodes to a new route, which alters not only the routing schedule but also the allocation scheme.

**Algorithm 5** Iterated local search**Input** Current best solution  $x_{best}$ ; newly generated chromosome  $P(x_{new})$ **Output** Updated best solution  $x_{best}$ 

- 1: Local search on routing part of  $P(x_{new})$  (**Algorithm 6**)
  - 2: **for** each local search operator on hubs **do**
  - 3:   Local search on the hub part of  $P(x_{new})$  to obtain new offspring  $P(x'_{new})$
  - 4:   **if**  $x'_{new} < x_{best}$  **then**
  - 5:      $x_{best} \leftarrow x'_{new}$ ; // Update the current best solution
  - 6:   **end if**
  - 7:   Local search on the routing part of  $P(x'_{new})$  (**Algorithm 6**)
  - 8: **end for**
- return**  $x_{best}$

**Algorithm 6** Local search on routing**Input** Current best solution  $x_{best}$ ; the chromosome to be operated :  $P(x_{new})$ **Output** Updated best solution  $x_{best}$ ;

- 1: **for** each local search operator on routing **do**
  - 2:   Apply local search operator to the routing part of chromosome  $P(x_{new})$  to generate new offspring  $P(x'_{new})$
  - 3:   **if**  $x'_{new} < x_{best}$  **then**
  - 4:      $x_{best} \leftarrow x'_{new}$  // Update the current best solution
  - 5:   **end if**
  - 6:    $P(x_{new}) \leftarrow P(x'_{new})$
  - 7: **end for**
- return**  $x_{best}$ ;

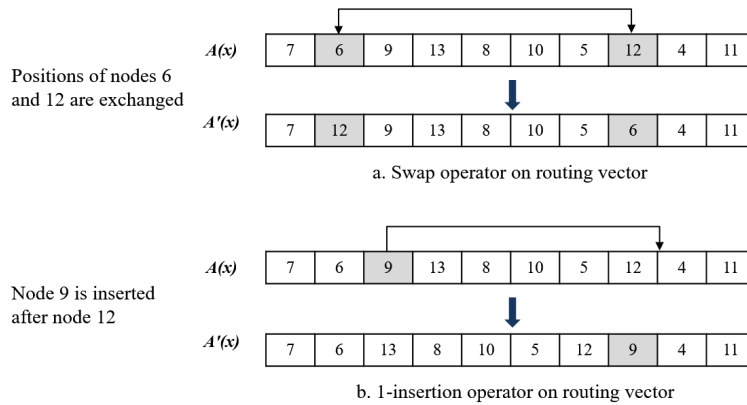


FIGURE 4.6 : Local search on the routing

**ILS on hub location and allocation**

During the ILS on the hub location part, four local search neighbourhoods are used sequentially in order to explore better the hub location and non-hub allocation solutions (Figure 4.7).

- (1) Replacement. If not all the potential hubs have been selected to be opened in the hub location vector, one random open hub is chosen and replaced by a hub previously closed.
- (2) Closing. One open hub is randomly selected to be closed and is replaced by another currently open hub.
- (3) Opening. Several positions are randomly chosen and a new potential hub is opened and placed on the selected positions. If all the potential hubs are open, the operation will be skipped and other operations will continue.
- (4) Swap. Two positions are randomly selected and the corresponding hubs are exchanged while other nodes remain unchanged.

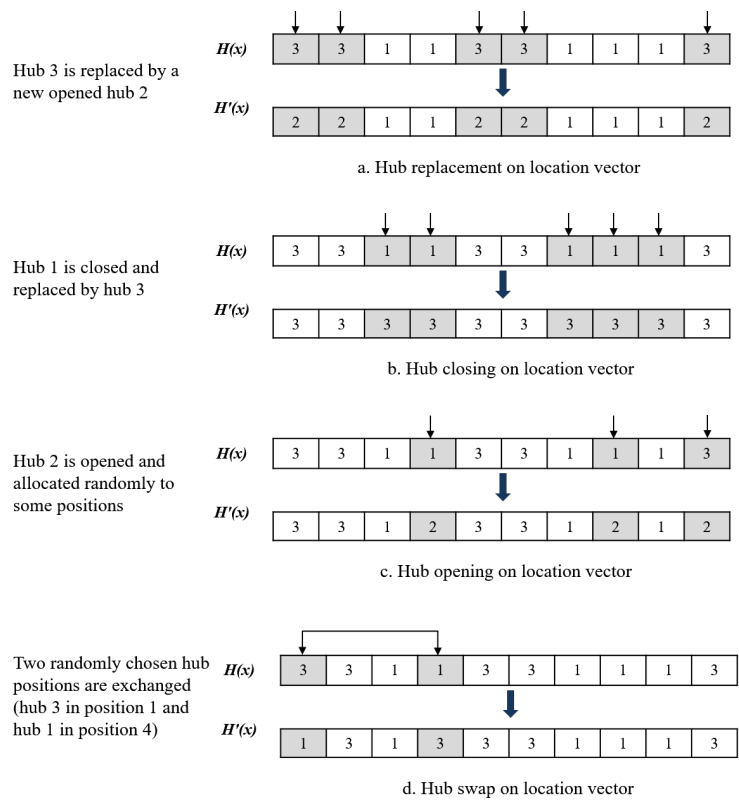


FIGURE 4.7 : Local search on the hub location and allocation

**4.3 Conclusion**

In conclusion to this chapter, we present the algorithmic structures of MA to the considered HLRP. In the MA framework, the initial population includes heuristic and random individuals. Three different simple heuristics are applied : the algorithm inspired by the ECWA is used to allocate non-hub nodes to opened hub facilities, the NN and CWA take the hub location and allocation solutions of the ECWA as the input and are responsible for scheduling local collection and delivery tours. Random individuals are also allowed when the amount of the heuristic ones are not able to satisfy the required population size. Individuals

from the initial generation are selected by calling a tournament selection operator based on their unique fitnesses to generate new offspring by an one-point crossover and a mutation operator. Afterwards, the ILS is implemented to search for the best neighbourhood individuals by means of modifying the genes on the chromosome. In order to evaluate the performance of the proposed MA, in the following chapter, the computational experiments are conducted on both the model of the HLRP proposed in Chapter 3 and the proposed MA.

## Computational experiments for the single-objective HLRP

In this chapter, we show details of the computational experiments which we performed with two solution methods : the CPLEX solver and the MA we proposed. The generation of data sets used for all the experiments are explained, as well as the parameters setting of the CPLEX and the MA. Then, we present and investigate the computational results of both methods.

we first describe the data sets and parameters used for all the experiments in Section 5.1. Then, the implementation and assessment of the MILP model is presented in Section 5.2, where we discuss the tuning of CPLEX parameters and the efficiency of the valid inequalities of the MILP model. The assessment of the memetic algorithm is discussed in Section 5.3, where we describe the parameters and variants of the implementation of the algorithm. Computational results are presented in Section 5.4, where we investigate the results obtained with CPLEX and the MA in terms of non-hub node allocation, hub location and collection/delivery routes depending on the sizes and parameters of instances. Lastly, some sensitivity analysis questions are discussed in Section 5.5 regarding the stability of the MA and the influence of fixed costs on the solutions.

### 5.1 Data and parameters

Since there are no published benchmark instances for the HLRP, we have generated the problem instance networks on the basis of the AP data set (Ernst and Krishnamoorthy [1999]). According to our hypothesis, it is assumed that the processes of picking up the freight from the suppliers and delivering it to the clients are distinct, and that the demand flows between each supplier-client pair are predetermined. Hubs may be located at the same geographical position as suppliers or clients. The distances between two nodes have been extracted from the AP data set. The flows (in tons) for each supplier-client pair have been generated uniformly within the interval  $[0.15, 1.0]$  in order to be consistent with the capacity of the vehicles (15 tons, see Table 5.3) and the possibility of loading the freight of several suppliers (resp. clients) within the same collection (resp. delivery) routes.

We have generated 27 instances, each with up to 100 non-hub nodes and 10 potential hub nodes. Instances with 10 or 15 supplier and client nodes are referred to as “small instances”; those with 20 to 30 supplier and client nodes as “medium instances”, and those with 35 to 50 supplier and client nodes as “large instances”. The detail utilized data sets are shown in Table 5.1.

In addition to different configurations of potential hub numbers, we consider three scenarios for each instance, with different hub capacities. By doing so, we can explore the influences of hub capacities on the solutions. The names of the instances are denoted as  $|H|-|I|-|J|-|\Gamma|$ .  $H$  stands for the candidate hub number set,  $|H| \in \{3, 6, 10\}$ .  $I$  and  $J$  are sets of supplier and client nodes :  $|I|=|J| \in \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$ .  $\Gamma$  refers to the set of hub types with integer capacities corresponding to  $1/3$ ,  $1/2$  or  $1$  of the total demand, which depends of the instance size in the AP data set.

Tables 5.2 and 5.3 show the values of parameters for hubs and vehicles. The parameters concerning vehicles (fixed costs and unit transportation cost) are based on logistics data from the French Comité National Routier CNR<sup>1</sup> data base. The unit cost of handling unit flow in hubs was communicated by a French logistic company.

TABLE 5.1 : Data structures of the HLRP

Size	Notation $ H - I - J $	Hub capacity $ \Gamma $
Small	3/6/10-10-10	10/15/30
	3/6/10-15-15	30/45/90
Medium	3/6/10-20-20	45/60/120
	3/6/10-25-25	45/60/120
	3/6/10-30-30	60/90/165
	3/6/10-35-35	60/90/180
Large	3/6/10-40-40	75/105/210
	3/6/10-45-45	75/105/195
	3/6/10-50-50	75/120/225

TABLE 5.2 : Parameter values for hubs

Name	Value	Name	Value
Fixed cost $F_k$ (€)	1000	Handling cost $c_k$ (€/t)	1.78

TABLE 5.3 : Cost parameter values for vehicles

Name	Value	Name	Value
Load capacity $Q$ (ton)	15	Fixed cost for tour $C$ (€)	100
Unit transfer cost $\alpha$ (€/km.t)	0.057	Unit collection cost $\beta$ (€/km)	0.8
Unit delivery cost $\gamma$ (€/km)	0.8		

The proposed MILP model is coded in Visual studio C++ 2012 and solved with CPLEX 12.6.1., while the MA approach is implemented in Visual studio C++ 2012 using PCs with 3.07 GHz and 8 GB RAM memory. The general notation used in the tables is explained below :

- $UB$  : best objective value found by CPLEX in 3 hours for each instance ;
- $LB$  : lower bound found by CPLEX within three hours ;
- $\%Gap$  : deviation in % between the best objective found by CPLEX and the lower bound found by CPLEX for each instance. Here,  $\%Gap = \frac{UB - LB}{UB} \times 100\%$  ;

<sup>1</sup><http://www.cnr.fr/en>

- $Z_{best}$  : best objective value found by the MA in 10 runs for each instance ;
- CPU time (s) : total CPU time of CPLEX in seconds to obtain the best objective ;
- $\%GapMA$  : deviation in % between the best objective found by MA and the lower bound found by CPLEX for each instance. Here,  $\%GapMA = \frac{Z_{best} - LB}{Z_{best}} \times 100\%$  ;
- $T_{total}$  (s) : total CPU time of 10 runs of the MA ;
- Open hub : hub location scheme of the best solution ;
- Route numbers : total number of collection and delivery routes of the best solution.

## 5.2 CPLEX assessments

### 5.2.1 CPLEX parameter tuning

Since the solving strategy of CPLEX is controlled by a variety of parameters, tuning them for a given model and instance set is an efficient way of improving the performance of the solution process. Preliminary experiments were conducted using the tuning tool of CPLEX to analyze the HLRP model and determine the values of the parameters that might provide the best performance as a possible alternative to the default parameter setting. All the experiments of CPLEX in the article are conducted with a computing time limitation of 3 hours. In addition, to avoid a failure due to running out of memory for some instances, we set a tree memory limit of 1500 megabytes.

The parameter values and meanings for each CPLEX parameter are shown in Tables 5.4, 5.5 and 5.6, together with the default value. The first and most critical CPLEX parameter is the "MIPEmphasis", which controls trade-offs between feasibility, optimality and speed in MILP solving (Table 5.4). The second important CPLEX parameter, "Probe", sets the extent of probing to be performed on variables before MILP branching (Table 5.5). Another CPLEX parameter, "NodeSel", sets the rules for selecting the next node to process when backtracking (Table 5.6).

TABLE 5.4 : Description of the "MIPEmphasis" parameter

Parameter value	Meaning
0	Balance optimality and feasibility (default value)
1	Emphasize feasibility over optimality
2	Emphasize optimality over feasibility
3	Emphasize moving best bound
4	Emphasize finding hidden feasible solutions

TABLE 5.5 : Description of the "Probe" parameter

Parameter value	Meaning
-1	No probing
0	Automatic : let CPLEX choose (default value)
1	Moderate probing level
2	Aggressive probing level
3	Very aggressive probing level



TABLE 5.6 : Description of the “NodeSel” parameter

Parameter value	Meaning
0	Depth-first search
1	Best-bound search (default value)
2	Best-estimate search
3	Alternative best-estimate search

TABLE 5.7 : Computational results with various values of the “MIPEmphasis” parameter

Instance $ H - I - J $	Value of MIPEmphasis	Hub capacity	$UB$	$LB$	$\%LB$	CPU time (s)	Open hub
3-15-15	0	45	7462.16	7461.42	0.00	5708.81	1,3
	1	45	7462.16	7358.38	1.39	10800.00	1,3
	<b>2</b>	45	<b>7462.16</b>	<b>7461.42</b>	<b>0.00</b>	1654.27	1,3
	3	45	7610.30	6683.56	12.17	10800.10	1, 3
	4	45	7462.16	7461.42	0.00	4868.48	1,3
6-15-15	0	45	7941.40	6945.99	12.53	6683.29	3, 5
	1	45	7462.16	6771.94	9.25	10800.10	1,3
	<b>2</b>	45	<b>7462.16</b>	<b>7365.29</b>	<b>1.30</b>	10758.80	1,3
	3	45	8283.96	6303.38	23.91	10800.62	4, 5
	4	45	7641.28	7319.08	4.22	10800.20	1,3
10-15-15	0	45	7520.19	6887.13	8.42	10800.20	3, 7
	1	45	7558.49	6750.89	10.68	5620.59	3, 7
	2	45	7520.19	6866.90	8.69	10800.10	3, 7
	3	45	9948.98	6073.97	38.95	10800.39	1, 5, 6
	<b>4</b>	45	<b>7324.31</b>	<b>7062.16</b>	<b>3.58</b>	10800.17	3, 7
3-20-20	0	60	8834.60	8089.80	8.43	5695.94	2, 3
	1	60	8887.65	7795.58	12.29	5788.13	2, 3
	<b>2</b>	60	<b>8818.09</b>	<b>8218.53</b>	<b>6.80</b>	7210.90	2, 3
	3	60	9737.83	7237.44	25.68	10830.57	2, 3
	4	60	8985.71	7971.49	11.29	1798.52	2, 3
6-20-20	0	60	8688.73	7450.89	14.25	5416.45	2, 5
	1	60	<b>8344.64</b>	7349.01	11.93	10800.50	4, 5
	<b>2</b>	60	8743.26	<b>7694.95</b>	11.99	8509.99	2, 5
	3	60	17397.65	6983.38	59.86	10800.32	1, 2, 3, 4, 5, 6
	4	60	8384.87	7473.72	<b>10.87</b>	5483.81	4, 5
10-20-20	0	60	8887.31	<b>7067.59</b>	20.48	10800.40	2, 10
	1	60	<b>8771.65</b>	7026.70	<b>19.89</b>	7044.94	4, 10
	<b>2</b>	60	8920.17	<b>7067.59</b>	20.77	10800.20	2, 10
	3	60	15164.34	6615.81	56.37	10800.45	1, 3, 8
	4	60	8930.93	6987.30	21.76	10800.20	2, 10

TABLE 5.8 : Computational results with various values of the “Probe” (MIPEmphasis=2)

Instance $ H - I - J $	Value of Probe	Hub capacity	$UB$	$LB$	$\%LB$	CPU time (s)	Open hub
3-15-15	-1	45	7686.60	6811.45	11.39	10800.1	1,3
	0	45	7462.16	7461.42	0.00	1654.27	1,3
	1	45	7563.56	6810.52	9.96	10800.4	1,3
	2	45	7920.67	6795.13	14.21	10800.2	1,3
	<b>3</b>	45	<b>7462.16</b>	<b>7461.42</b>	<b>0.00</b>	2867.89	1,3
6-15-15	-1	45	8535.16	6203.83	27.31	10800.51	1, 6
	<b>0</b>	45	<b>7462.16</b>	<b>7365.29</b>	<b>1.30</b>	10758.80	1,3
	1	45	9528.43	6143.42	35.53	10800.06	2, 4, 6
	2	45	8205.48	6186.77	24.60	10800.09	3, 4
	3	45	7777.28	7332.20	5.72	10800.20	1,3
10-15-15	-1	45	10137.55	5681.88	43.95	10800.03	4, 10
	0	45	7520.19	6866.90	8.69	10800.10	3, 7
	1	45	9332.45	5692.86	39.00	10800.26	7, 10
	2	45	9332.45	5692.86	39.00	10800.50	7, 10
	<b>3</b>	45	<b>7419.54</b>	<b>7091.31</b>	<b>4.42</b>	10800.20	3, 7
3-20-20	-1	60	11380.61	7072.62	37.85	10800.07	1, 2
	0	60	8818.09	8218.53	6.80	7210.90	2, 3
	1	60	9848.75	7134.95	27.55	10800.37	2, 3
	2	60	9423.12	7146.59	24.16	10800.45	2, 3
	<b>3</b>	60	<b>8647.53</b>	<b>8289.40</b>	<b>4.14</b>	10801.30	2, 3
6-20-20	-1	60	9198.84	6295.61	31.56	10800.04	5, 6
	<b>0</b>	60	<b>8743.26</b>	<b>7694.95</b>	<b>11.99</b>	8509.99	2, 5
	1	60	9095.09	6204.51	31.78	10800.18	2, 5
	2	60	11049.25	6231.52	33.03	10800.14	1, 5
	3	60	9018.76	7686.20	14.78	7818.56	5, 6
10-20-20	-1	60	9680.66	5941.86	38.62	10800.78	6, 10
	0	60	8920.17	7067.59	20.77	10800.20	2, 10
	1	60	<b>8836.65</b>	6462.38	26.87	10800.31	6, 10
	2	60	<b>8836.65</b>	6462.38	26.87	10800.26	6, 10
	<b>3</b>	60	8892.90	<b>7137.98</b>	<b>19.73</b>	10800.10	2, 8

TABLE 5.9 : Computational results with various values of the “NodeSel” (MIPEmphasis=2)

Instance $ H - I - J $	Value of NodeSel	Hub capacity	UB	LB	%LB	CPU time (s)	Open hub
3-15-15	0	45	8081.53	6590.66	18.44	10800.00	1,3
	<b>1</b>	45	<b>7462.16</b>	<b>7461.42</b>	<b>0.00</b>	1654.27	1,3
	2	45	<b>7462.16</b>	<b>7461.42</b>	<b>0.00</b>	4440.55	1,3
	3	45	7462.16	6679.13	10.49	10800.30	1,3
6-15-15	0	45	8130.35	6088.56	25.11	10800.46	1, 6
	<b>1</b>	45	<b>7462.16</b>	<b>7365.29</b>	<b>1.30</b>	10758.80	1,3
	2	45	7641.28	7237.06	3.02	10800.20	1,3
	3	45	7893.69	6197.77	21.48	10800.16	3, 6
10-15-15	0	45	11377.64	5854.74	48.54	10800.07	1, 2, 3, 4, 7, 8
	<b>1</b>	45	<b>7520.19</b>	<b>6866.90</b>	<b>8.69</b>	10800.10	3, 7
	2	45	7920.32	6792.19	14.24	10800.20	3, 7
	3	45	8659.80	5864.17	32.28	10800.14	3, 6
3-20-20	0	60	14870.28	7064.44	52.49	10800.07	1, 2, 3
	<b>1</b>	60	<b>8818.09</b>	<b>8218.53</b>	<b>6.80</b>	7210.90	2, 3
	2	60	9264.33	7805.41	15.75	1729.06	2, 3
	3	60	11172.95	7030.78	37.07	10800.20	1, 2, 3
6-20-20	0	60	17909.40	6853.64	61.73	10800.12	1, 2, 4, 6
	<b>1</b>	60	8743.26	<b>7694.95</b>	<b>11.99</b>	8509.99	2, 5
	2	60	<b>8581.63</b>	7328.60	14.60	5008.45	4, 5
	3	60	9273.86	6158.03	33.60	10800.33	5, 6
10-20-20	0	60	20125.48	5888.42	70.74	10800.14	1, 2, 3, 5, 6, 7, 8, 9, 10
	<b>1</b>	60	<b>8920.17</b>	<b>7067.59</b>	<b>20.77</b>	10800.20	2, 10
	2	60	9431.47	6987.30	25.84	10800.30	6, 10
	3	60	10935.84	5922.43	45.84	10800.38	1, 7, 10

TABLE 5.10 : Comparison of the CPLEX parameter settings

Instance $ H - I - J $	CPLEX parameter	Hub capacity	UB	LB	%LB	CPU time (s)	Open hub
3-15-15	M2	45	7462.16	7461.42	0.00	1654.27	1,3
	M2&P3	45	7462.16	7461.42	0.00	2867.89	1,3
6-15-15	M2	45	<b>7462.16</b>	<b>7365.29</b>	<b>1.30</b>	10758.80	1,3
	M2&P3	45	7777.28	7332.20	5.72	10800.20	1,3
10-15-15	M2	45	7520.19	6866.90	8.69	10800.10	3, 7
	M2&P3	45	<b>7419.54</b>	<b>7091.31</b>	<b>4.42</b>	10800.20	3, 7
3-20-20	M2	60	8818.09	8218.53	6.80	7210.90	2, 3
	M2&P3	60	<b>8647.53</b>	<b>8289.40</b>	<b>4.14</b>	10801.30	2, 3
6-20-20	M2	60	<b>8743.26</b>	<b>7694.95</b>	<b>11.99</b>	8509.99	2, 5
	M2&P3	60	9018.76	7686.20	14.78	7818.56	5, 6
10-20-20	M2	60	8920.17	7067.59	20.77	10800.20	2, 10
	M2&P3	60	<b>8892.90</b>	<b>7137.98</b>	<b>19.73</b>	10800.10	2, 8

The results of our experiments for tuning the CPLEX parameters with data sets of different sizes are presented in the Tables of 5.7, 5.8 5.9 and 5.10. The experiments with the “MIPEmphasis” parameter while keeping the other CPLEX parameters as default are presented in Table 5.7. In most cases, setting “MIPEmphasis” to 2 (emphasizing optimality over feasibility) yields competitive upper bounds and lower bounds with smaller gaps. Regarding the experiments with the “Probe” parameter, “MIPEmphasis” was pre-set to 2 and the other CPLEX parameters to their default value. The very aggressive probing level (parameter value 3) improved the solutions for most of the instances, especially by obtaining competitive lower bounds (Tables 5.8 and 5.10). Similar experiments with “NodeSel” parameter, setting “MIPEmphasis” to 2 and the other CPLEX parameters to default, showed that the default value of the “NodeSel” parameter (1) (best-bound search) outperformed other “NodeSel” values (Table 5.9). These tuned CPLEX parameter values were retained in our further experiments. Note from the above tables that different pre-set CPLEX parameters may affect the solutions, generating different location, allocation and routing plans for the solutions of our problems.

## 5.2.2 Efficiency of valid inequalities in the MILP model

To analyze the efficiency of valid inequalities that we proposed in our MILP model, some tests were conducted to compare the results obtained with and without implementing these valid inequalities. Table 5.11 compares the results obtained for different sizes of instances. They show that including the valid inequalities in the proposed model provides a good performance in terms of improving lower bounds and decreasing gaps under the same time limitations (3 hours). It also decreases the upper bounds for most of the instance tests by up to more than 10%. The resulting solutions may consequently be changed in terms of opened hubs and the number of routes, which are generally reduced.

TABLE 5.11 : Efficiency assessment of the valid inequalities

Instance  H - I - J - Γ	Model without valid inequalities					Model with valid inequalities				
	UB	LB	%Gap	Open hub	Number of routes	%UB <sup>1</sup>	%LB <sup>2</sup>	%Gap	Open hub	Number of routes
3-15-15-30	8828.59	8390.40	4.96	1, 2, 3	11	1.79	<b>0.91</b>	5.78	1, 2, 3	11
6-15-15-30	8707.18	7295.05	16.22	2, 4, 5	12	<b>-3.04</b>	<b>5.16</b>	<b>9.14</b>	1, 5, 6	13
10-15-15-30	8524.27	6680.63	21.63	5, 7, 10	12	1.32	<b>7.89</b>	<b>16.55</b>	1, 4, 5	12
3-20-20-45	10200.29	9593.82	5.95	1, 2, 3	18	<b>-2.25</b>	<b>0.23</b>	<b>3.56</b>	1, 2, 3	17
6-20-20-45	10435.48	8418.60	19.33	2, 3, 5	18	<b>-0.74</b>	<b>7.30</b>	<b>12.79</b>	3, 4, 5	17
10-20-20-45	10395.17	7901.05	23.99	2, 5, 10	19	<b>-4.40</b>	<b>7.70</b>	<b>14.37</b>	2, 8, 9	18
3-25-25-45	11811.05	10154.53	14.03	1, 2, 3	18	<b>-2.62</b>	-0.15	<b>11.85</b>	1, 2, 3	18
6-25-25-45	13502.91	8963.01	33.62	1, 2, 3	19	<b>-10.13</b>	<b>4.18</b>	<b>23.05</b>	1, 2, 3	17

Note :  $UB'$  and  $LB'$  are denoted as the upper bound and lower bound obtained by CPLEX with valid inequalities.

<sup>1</sup> Decreased/increased level of the upper bound (objective value) :  $\%UB = (UB' - UB) / UB \times 100\%$

<sup>2</sup> Decreased/increased level of the lower bound :  $\%LB = (LB' - LB) / LB \times 100\%$

## 5.3 MA assessments

### 5.3.1 Parameter settings for the MA

The parameters of the proposed MA were tuned in terms of population size, maximum number of iterations and probabilities used for crossover and mutation. Preliminary experiments show that when the population size increases to 200, the overall performance of the MA is improved, but larger population sizes lead to no further improvement. Furthermore, setting the probability of crossover and mutation to 0.8 and 0.7, respectively, is the best combination. In order to obtain high quality solutions, the MA was run 10 times for each instance (this policy will be evaluated later on in regards to the experimental results). Below are some justifications to support these parameter settings.

We iterated the MA process up to 1000 iterations and recorded the results every 100 iterations for 10 runs each, i.e. for iteration number  $\{100, 200, 300, \dots, 1000\}$ . The results show that the solutions of small and medium instances remain unchanged after 100 iterations in all 10 runs (such as instance 6-25-25) of the MA. On the other hand, the MA can improve solutions of large instances until the iterations reach 200 (such as 10-40-40).

In Tables 5.12 and 5.13, the column “Run rank” refers to the rank of the run of the MA from 1 to 10. The column “Initial generation” indicates the best solution in the initial population. The next three columns refer to the best solution found after every 100 iterations of the MA. In the column “Run rank” of Table 5.12, the notation “1-10” indicates that all the solutions obtained with a run number of 1 to 10 of the MA are identical. The small instance solutions (e.g. instance 6-25-25 for different hub capacities) were not improved after 100 iterations. Table 5.13 provides the solutions of the large instance 10-40-40 with a hub capacity of 75 for 10 runs while Figure 5.1 plots the average solutions of 10 runs for successive iterations of the MA. The solutions are improved continuously until the generation of 140. More iterations lead to no more improvement. Therefore, two stopping criteria were set for each run of the MA : the iterations were stopped whenever no improvement was obtained every 100 iterations, with a maximum number of 200.

### 5.3.2 Implementation of the MA

In order to find the most efficient way to combine the operators of the MA for the allowed number of iterations (200), several variants were tested, especially the following two, denoted MA-1 and MA-2. Finally MA-2 was retained for our experiments with our complete set of instances.

TABLE 5.12 : Results of the MA up to 1000 iterations for medium instances

Instance $ H - I - J - \Gamma $	Run rank	Initial generation	The 100th iteration	The 200th iteration	Iterations 300-1000
6-25-25-45	1-10	11204.20	11003.34	11003.34	11003.34
6-25-25-60	1-10	10657.20	10228.00	10228.00	10228.00
6-25-25-120	1	9251.17	9158.65	9158.65	9158.65
	2	9251.17	9158.65	9158.65	9158.65
	3	9251.17	9071.24	9071.24	9071.24
	4	9251.17	9156.66	9156.66	9156.66
	5	9251.17	9158.65	9158.65	9158.65
	6	9251.17	9158.65	9158.65	9158.65
	7	9251.17	9062.72	9062.72	9062.72
	8	9251.17	9062.72	9062.72	9062.72
	9	9251.17	9158.65	9158.65	9158.65
	10	9251.17	9158.65	9158.65	9158.65

TABLE 5.13 : Results of the MA up to 1000 iterations for large instances

Instance $ H - I - J - \Gamma $	Run rank	Initial generation	100 iterations	200 iterations	Iterations 300-1000
10-40-40-75	1	15493.40	14902.80	14902.80	14902.80
	2	15404.10	14797.90	14797.90	14797.90
	3	15420.40	14980.30	14765.90	14765.90
	4	15516.70	14662.60	14662.60	14662.60
	5	15261.50	14678.00	14678.00	14678.00
	6	15149.10	14731.50	14731.50	14731.50
	7	15420.40	14726.20	14714.70	14714.70
	8	15391.70	14945.70	14945.70	14945.70
	9	15240.60	14462.00	14462.00	14462.00
	10	15798.50	14891.40	14882.70	14882.70

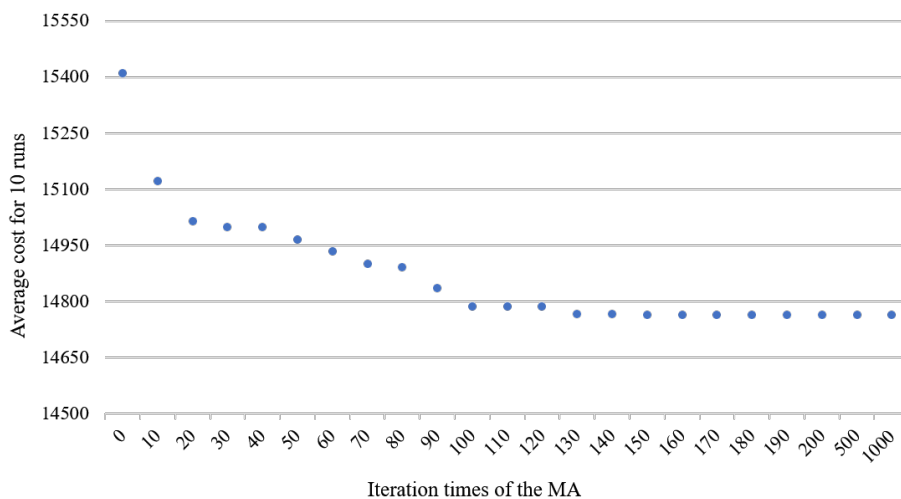


FIGURE 5.1 : Average results of the MA in 10 runs up to 1000 iterations (Instance 10-40-40-75)

For each iteration of the algorithm, MA-1 actually ignores the mutation process and executes systematically the full local search procedure on every newly generated offspring of the population after the crossover. This variant of the MA implementation yields good quality solutions but needs a very long computing time for large instances. In fact, the CPU running time of the LS phase consumes more than 95% of the total time of the algorithm. Alternatively, MA-2 fully executes the mutation step of the genetic part of

the algorithm with a probability after the crossover process, but the LS is called only after every 10 iterations of the genetic part, with the goal of reducing the computing time. Furthermore, the LS step is applied on three offspring only, which are randomly selected from the ten best offspring of the current generation. In addition, for each operator of the LS, once the target of the current offspring is improved, the procedure jumps to the next operator without considering the other offspring.

TABLE 5.14 : Computational tests on different ways of implementing the MA

Instance $ H - I - J $	Hub capacity	MA-1			MA-2				
		$Z_{best}$	CPU time(s)	Open hub	$Z_{best}$	CPU time(s)	Open hub	%Time	%Cost
3-10-10	10	5750.33	22.01	1, 2, 3	<b>5746.53</b>	<b>16.52</b>	1, 2, 3	<b>-24.94%</b>	<b>-0.07%</b>
	15	<b>4269.15</b>	23.49	1, 2	<b>4269.15</b>	<b>12.60</b>	1, 2	<b>-46.36%</b>	0.00
	30	<b>3277.36</b>	42.73	2	<b>3277.36</b>	<b>14.36</b>	2	<b>-66.39%</b>	0.00
3-25-25	45	<b>11626.37</b>	295.02	1, 2, 3	11676.80	<b>83.38</b>	1, 2, 3	<b>-71.74%</b>	0.40%
	60	10568.01	255.67	1, 3	<b>10557.60</b>	<b>97.25</b>	1, 3	<b>-61.97%</b>	<b>-0.10%</b>
	120	9938.67	372.607	1	<b>9834.14</b>	<b>98.24</b>	1	<b>-73.64%</b>	<b>-1.05%</b>
6-30-30	60	13114.97	1143.10	2, 4, 6	<b>13052.10</b>	<b>306.87</b>	4, 5, 6	<b>-73.15%</b>	<b>-0.48%</b>
	90	11976.11	1224.82	4, 6	<b>11973.20</b>	<b>411.62</b>	2, 6	<b>-66.39%</b>	<b>-0.02%</b>
	165	11408.26	1631.38	2	<b>11401.80</b>	<b>338.96</b>	2	<b>-78.93%</b>	<b>-0.06%</b>
10-45-45	75	<b>13209.28</b>	$\geq 4h$	3, 5, 9	13486.70	<b>2630.10</b>	3, 5, 9	<b>-83.18%</b>	2.10%
	105	<b>12501.18</b>	$\geq 5h$	8, 9	12557.00	<b>2471.17</b>	8, 9	<b>-88.41%</b>	0.45%
	195	<b>12033.95</b>	$\geq 8h$	8	12134.90	<b>3025.74</b>	1	<b>-89.50%</b>	0.84%
10-50-50	75	<b>16284.53</b>	$\geq 4h$	2, 3, 5, 10	16441.56	<b>2953.06</b>	2, 5, 10	<b>-85.98%</b>	0.96%
	120	<b>15242.27</b>	$\geq 8h$	2, 5	15471.72	<b>5645.03</b>	8, 10	<b>-82.51%</b>	1.51%
	225	<b>14185.63</b>	$\geq 13h$	8	14712.64	<b>4960.14</b>	8	<b>-89.41%</b>	3.72%

Table 5.14 presents the MA results of some small, medium and large instances running 10 times. Item %Time indicates the reduced percentage of the computing time when comparing MA-2 and MA-1.  $\%Time = (T_2 - T_1) / T_1 \times 100\%$  where  $T_1$  and  $T_2$  are the CPU time of MA-1 and MA-2, respectively. Item %Cost indicates the increased or decreased percentage of the total cost when comparing MA-2 to MA-1.  $\%Cost = (Z_2 - Z_1) / Z_1 \times 100\%$  where  $Z_1$  and  $Z_2$  are the total cost of MA-1 and MA-2. For small and medium instances (e.g. instances 3-10-10, 3-25-25 and 6-30-30), the performance of MA-2 is comparable to that of MA-1 in terms of solution quality while MA-2 greatly reduces the total computing time compared to MA-1. For large instances (e.g. instances 10-45-45 and 10-50-50), the solutions found by MA-2 are inferior to those of MA-1 with an increased cost percentage of up to 3.72%. However, MA-2 reduces by almost 90% the computing time of MA-1. Thus, in order to achieve a good overall performance in terms of solution quality and computing time, we finally retained MA-2 for the experiments on our complete instance set. The results are presented in the next section.

## 5.4 Analysis of computational results

Tables 5.15 and 5.16 present the problem solutions obtained by CPLEX and the MA. Table 5.15 reveals that the CPLEX solver can find optimal solutions for the smallest instances only. It has difficulties in solving medium-sized problems, obtaining low quality solutions with gaps of up to 39.91% (instance 10-30-30-165). The computing time with CPLEX reaches 3 hours for most of the instances, even many small ones. The CPLEX solutions for large instances are not presented, as their gaps reach as high as 70%. The proposed MA finds solutions for all instance sets and is capable of solving some small-sized problems to optimality. Moreover, for medium-sized problems, it reduces the biggest gap of CPLEX from 39.91% to 13.64%. Most importantly, the MA dramatically reduces the computing time for small and medium problems. Computing times for 10 runs for each instance lie between 16 and 850 seconds for small- to medium-sized instances. Large-sized instances are solved in no more than 6000 seconds for the largest of them (Instance 10-50-50-

120). In terms of logistics solutions, the numbers of open hubs are usually identical for CPLEX and the MA but they may not be the same hubs. The numbers of routes are comparable.

TABLE 5.15 : Computational results for small- and medium-sized instances

Instance $ H - I - J $	Hub capacity	CPLEX						MA				
		$UB$	$LB$	$\%Gap$	CPU time(s)	Open hub	Number of routes	$Z_{best}$	$\%Gap$ MA	$T_{total}$ (s)	Open hub	Number of routes
3-10-10	10	<b>5666.52</b>	5665.96	<b>0.00</b>	593.79	1, 2, 3	6	5746.53	1.40	<b>16.52</b>	1, 2, 3	6
	15	<b>4269.15</b>	4268.74	<b>0.00</b>	217.22	1, 2	4	<b>4269.15</b>	<b>0.00</b>	<b>12.60</b>	1, 2	4
	30	<b>3277.36</b>	3277.06	<b>0.00</b>	18.80	2	4	<b>3277.36</b>	<b>0.00</b>	<b>14.36</b>	2	4
6-10-10	10	5666.52	5379.10	5.07	10800.10	1, 2, 3	6	<b>5659.78</b>	<b>4.96</b>	<b>58.57</b>	3, 4, 6	6
	15	<b>4269.15</b>	4180.61	<b>2.07</b>	10800.00	1, 2	4	<b>4269.15</b>	<b>2.07</b>	<b>59.04</b>	1, 2	4
	30	<b>3272.23</b>	3271.92	<b>0.00</b>	157.76	4	4	<b>3272.23</b>	<b>0.00</b>	<b>73.36</b>	4	4
10-10-10	10	5792.80	5054.14	12.75	9192.72	1, 2, 8	6	<b>5659.78</b>	<b>10.70</b>	<b>91.94</b>	3, 4, 6	6
	15	4363.80	4169.43	4.45	10800.20	8, 10	4	<b>4258.45</b>	<b>2.09</b>	<b>82.25</b>	2, 10	4
	30	<b>3245.62</b>	3245.35	<b>0.00</b>	591.82	10	4	<b>3245.62</b>	<b>0.00</b>	<b>87.76</b>	10	4
3-15-15	30	<b>8986.43</b>	8467.11	<b>5.78</b>	10754.40	1, 2, 3	11	9015.79	6.09	<b>22.89</b>	1, 2, 3	11
	45	<b>7584.04</b>	7298.24	<b>3.15</b>	10536.69	1, 3	12	7638.35	4.45	<b>34.19</b>	1, 3	12
	90	<b>6484.38</b>	6279.90	<b>3.15</b>	10800.06	3	11	6539.41	3.97	<b>60.63</b>	3	11
6-15-15	30	8442.86	7671.57	9.14	10471.90	1, 5, 6	13	<b>8127.86</b>	<b>5.61</b>	<b>32.68</b>	1, 3, 5	12
	45	<b>7107.65</b>	6854.66	<b>3.56</b>	10800.15	3, 5	12	<b>7107.65</b>	<b>3.56</b>	<b>40.88</b>	3, 5	12
	90	6247.91	5977.35	4.33	10454.62	5	12	<b>6179.78</b>	<b>3.28</b>	<b>89.62</b>	5	11
10-15-15	30	8637.04	7207.55	16.55	10800.30	1, 4, 5	12	<b>7863.46</b>	<b>8.34</b>	<b>114.30</b>	5, 7, 10	12
	45	7013.07	6331.47	9.72	10800.11	7, 10	12	<b>6861.88</b>	<b>7.73</b>	<b>131.45</b>	7, 10	12
	90	<b>6199.35</b>	5974.59	<b>3.63</b>	10800.14	5	11	<b>6199.35</b>	<b>3.63</b>	<b>145.86</b>	5	11
3-20-20	45	<b>9970.62</b>	9615.48	<b>3.56</b>	10800.10	1, 2, 3	17	10024.00	4.08	<b>65.34</b>	1, 2, 3	16
	60	<b>8825.23</b>	8671.00	<b>1.75</b>	10800.09	2, 3	16	9048.58	4.17	<b>82.64</b>	2, 3	17
	120	<b>8046.43</b>	7749.60	<b>3.69</b>	10314.67	3	16	8057.46	3.82	<b>83.86</b>	3	16
6-20-20	45	10357.90	9033.56	12.79	10800.10	3, 4, 5	17	<b>9806.10</b>	<b>7.88</b>	<b>113.05</b>	2, 4, 5	17
	60	9257.22	8183.36	11.6	10800.23	4, 5	16	<b>9022.21</b>	<b>9.30</b>	<b>116.44</b>	3, 4	17
	120	8046.43	7684.09	4.50	10800.53	3	16	<b>8041.56</b>	<b>4.45</b>	<b>128.07</b>	3	16
10-20-20	45	9938.18	8509.61	14.37	10800.10	2, 8, 9	18	<b>9632.53</b>	<b>11.66</b>	<b>280.64</b>	8, 9, 10	18
	60	9520.10	7618.43	19.98	10800.30	5, 9	17	<b>8725.87</b>	<b>12.69</b>	<b>244.04</b>	7, 8	17
	120	7982.97	7017.70	12.09	10800.30	10	16	<b>7885.86</b>	<b>11.01</b>	<b>257.96</b>	7	16
3-25-25	45	<b>11502.00</b>	10139.00	<b>11.85</b>	9145.94	1, 2, 3	18	11676.80	13.17	<b>83.38</b>	1, 2, 3	18
	60	10602.79	9488.80	10.51	9369.65	1, 2, 3	18	<b>10557.60</b>	<b>10.12</b>	<b>97.25</b>	1, 3	16
	120	9865.98	9154.83	7.23	7024.90	1	16	<b>9834.14</b>	<b>6.91</b>	<b>98.24</b>	1	16
6-25-25	45	12135.20	9337.74	23.05	10800.30	1, 2, 3	17	<b>11125.50</b>	<b>16.07</b>	<b>154.74</b>	1, 2, 5	16
	60	12148.32	8963.21	26.22	10800.20	2, 5	19	<b>10252.00</b>	<b>12.57</b>	<b>205.04</b>	1, 5	16
	120	<b>9036.62</b>	8376.68	<b>7.30</b>	10800.09	5	16	9062.72	7.57	<b>181.49</b>	5	16
10-25-25	45	13209.30	8868.87	32.86	10800.20	1, 2, 3	18	<b>10856.20</b>	<b>18.31</b>	<b>378.57</b>	1, 8, 10	17
	60	13154.29	8043.80	38.85	10800.40	1, 3	24	<b>10159.40</b>	<b>20.82</b>	<b>367.40</b>	1, 8	16
	120	10281.30	7478.24	27.26	10800.19	5, 6	17	<b>9056.17</b>	<b>17.42</b>	<b>406.98</b>	5	16
3-30-30	60	15402.67	12812.91	16.82	10800.10	1, 2, 3	27	<b>14732.00</b>	<b>13.03</b>	<b>124.47</b>	1, 2, 3	25
	90	<b>12945.41</b>	11688.06	<b>9.71</b>	10800.20	1, 2	24	12994.00	10.05	<b>164.00</b>	1, 2	24
	165	11498.92	10928.82	4.96	10800.10	2	24	<b>11401.80</b>	<b>4.15</b>	<b>147.97</b>	2	23
6-30-30	60	14296.41	11439.38	19.98	10800.20	2, 4, 5	27	<b>13052.10</b>	<b>12.36</b>	<b>306.87</b>	4, 5, 6	24
	90	13838.82	10461.64	24.40	10800.90	2, 6	27	<b>11973.20</b>	<b>12.62</b>	<b>411.62</b>	2, 6	24
	165	12246.40	10531.57	14.00	10800.20	4	25	<b>11401.80</b>	<b>7.63</b>	<b>338.96</b>	2	23
10-30-30	60	17617.62	11161.86	36.64	10800.70	1, 5, 7	33	<b>12952.60</b>	<b>13.83</b>	<b>653.84</b>	2, 6, 7	23
	90	15289.92	10301.50	32.63	10800.70	4, 5, 8	27	<b>11987.10</b>	<b>14.06</b>	<b>847.83</b>	4, 6	22
	165	16362.83	9831.77	39.91	10800.90	1, 7, 8	34	<b>11384.40</b>	<b>13.64</b>	<b>773.67</b>	8	23

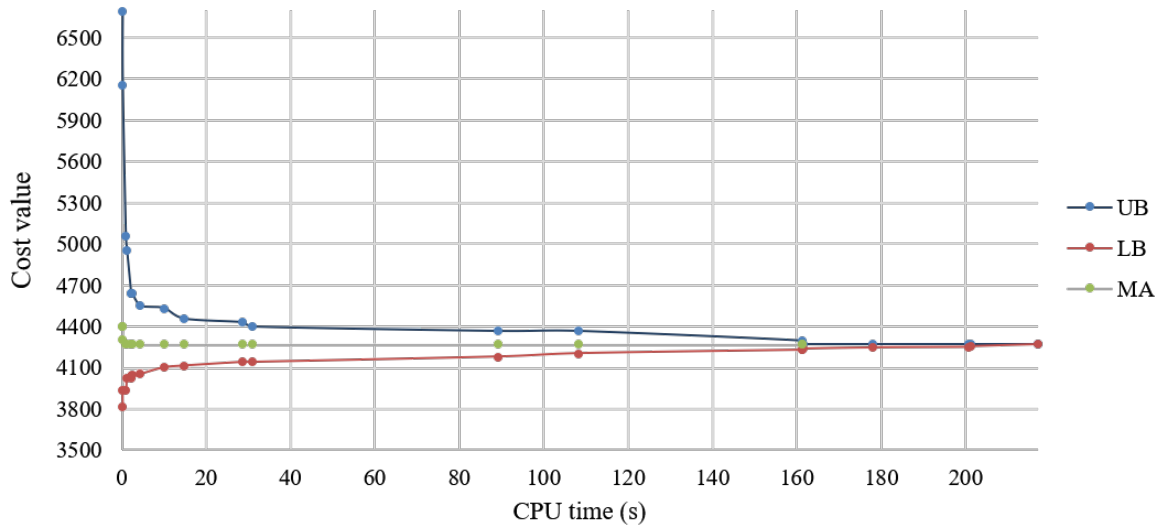


TABLE 5.16 : Computational results of the MA for large sized instances

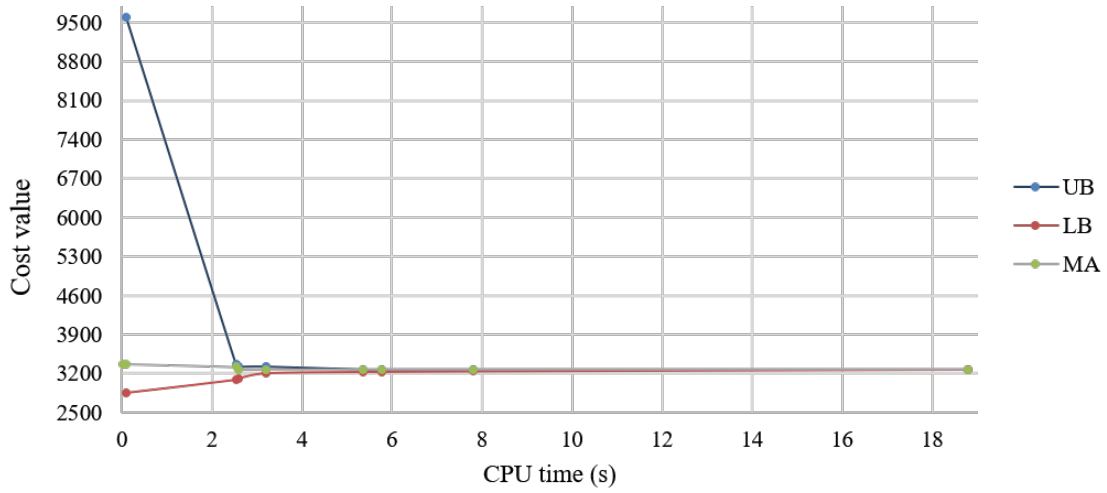
Instance $ H - I - J $	Hub capacity	$Z_{best}$	$T_{total}$ (s)	Open hub	Number of routes	Instance $ H - I - J $	Hub capacity	$Z_{best}$	$T_{total}$ (s)	Open hub	Number of routes
3-35-35	60	15303.30	97.10	1, 2, 3	27	3-40-40	75	16172.60	200.44	1, 2, 3	30
	90	12704.90	178.36	1, 2	26		105	15233.90	295.92	2, 3	29
	180	11895.80	209.94	1	24		210	13442.10	280.27	2	28
6-35-35	60	13486.80	364.61	1, 4, 6	27	6-40-40	75	15257.80	324.14	3, 4, 5	31
	90	12747.70	301.76	1, 6	26		105	14317.30	507.60	4, 5	30
	180	11997.80	500.38	6	25		210	13296.30	736.69	5	28
10-35-35	60	13579.80	552.54	1, 4, 6	27	10-40-40	75	14586.30	1355.50	2, 7, 10	30
	90	12655.50	1116.98	1, 2	26		105	13838.00	2591.37	2, 10	29
	180	11975.80	1246.83	6	25		210	13267.60	2021.55	4	28
3-45-45	75	14364.90	698.43	1, 2, 3	26	3-50-50	75	17242.60	219.578	1, 2, 3	33
	105	13282.50	582.82	1, 3	25		120	16324.10	723.49	1, 2	32
	195	12165.40	483.29	1	25		225	14848.40	375.73	2	32
6-45-45	75	13808.20	839.39	1, 2, 5	27	6-50-50	75	16669.30	2175.56	2, 3, 5	34
	105	12808.70	1153.75	1, 5	26		120	15564.30	2558.65	2, 5	32
	195	12158.20	1067.82	1	25		225	14719.20	1231.40	2	30
10-45-45	75	13486.70	2630.10	3, 5, 9	28	10-50-50	75	16441.56	2953.06	2, 5, 10	33
	105	12557.00	2471.17	8, 9	26		120	15471.72	5645.03	8, 10	32
	195	12134.90	3025.74	1	25		225	14712.64	4960.14	8	32

Figure 5.2 gives an insight into how the solutions are found by CPLEX and the MA within the optimization process in terms of increasing time for a small instance set (3-10-10) with two hub capacities (the X scales are limited to the first seconds of the optimization process). The time and relevant objective value of the MA refer here to the average values of the 10 runs.  $UB$  and  $LB$  indicate the upper bound and lower bound found by CPLEX. The initial solution found by the MA, which is very close to the optimal one, is much smaller than that found by CPLEX. We observe that the MA can find the optimal solutions in a few seconds. Similar results are observed with larger instances. The initial solution found by the MA is much smaller than that found by CPLEX and the solution is improved continuously and quickly to obtain a good quality solution whose cost value is much smaller than that of the solution found by CPLEX in 3 hours.

Finally, and logically, the solutions are improved, reaching lower costs, when the hub capacity increases, looking at the MA results for instances with 3 candidate hubs as an example in Figure 5.3. Node numbers in the graph indicate the total number of hubs, suppliers and clients. The graphs show that for a given node number, increasing the hub capacity results in an equal or decreasing cost. However, as shown in Figure 5.4, the instance sets with larger node numbers are more difficult to solve, causing increased gaps. Besides, the solver can find better solutions if there is more choice of potential hubs (Figure 5.5).



a. Instance 3-10-10 with hub capacity of 15



b. Instance 3-10-10 with hub capacity of 30

FIGURE 5.2 : Solution evolution with CPLEX and the MA for instance 3-10-10 (two hub capacities)

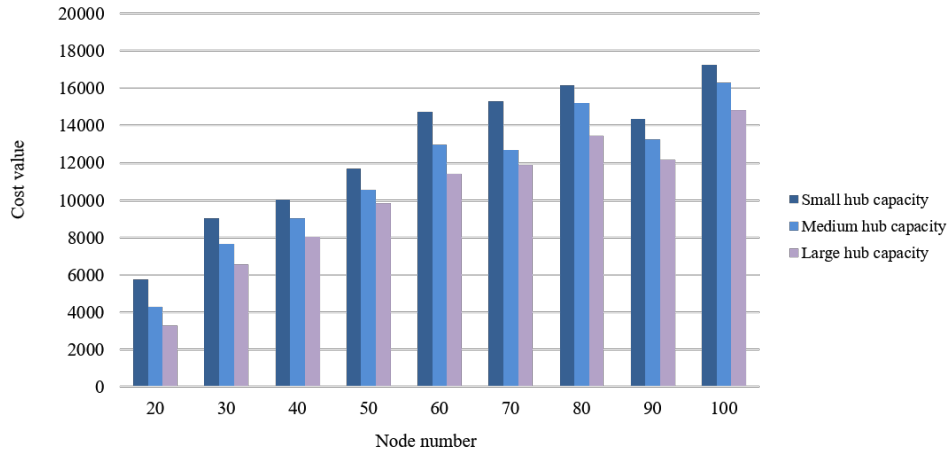


FIGURE 5.3 : MA results with different hub capacities (3 potential hubs) : when the capacity of hubs increases, the cost decreases

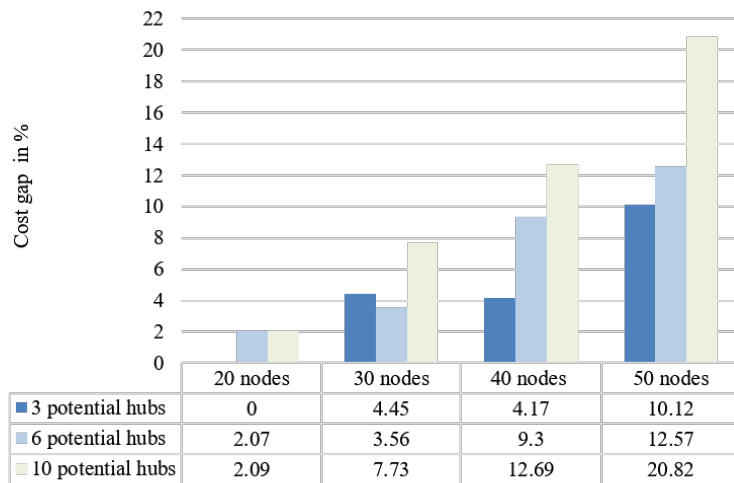


FIGURE 5.4 : Solution gaps with different potential hub numbers obtained by the MA : the gap increases when the number of nodes and the number of potential hubs increase

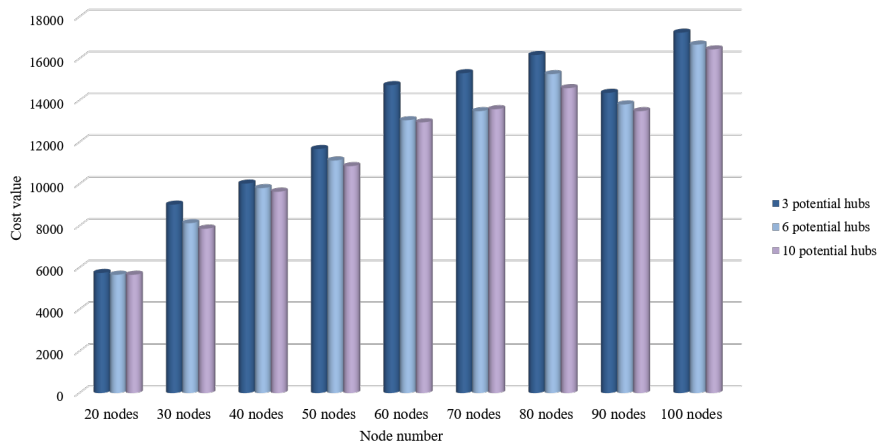


FIGURE 5.5 : MA results with different potential hub numbers (small hub capacity)

## 5.5 Sensitivity analysis

### 5.5.1 Stability assessment of the MA

This section assesses the performance of the proposed MA in terms of the stability of the solutions obtained by the ten runs for each iteration regarding solving the instance sets. For the ten runs, we computed the coefficient of relative standard deviation (RSD),  $RSD = \frac{SD}{\bar{Z}} \times 100\%$ . This indicator shows the variance in % of the objective values of ten runs compared to the average value. Here,  $SD$  is denoted as the standard deviation between the average objective value  $\bar{Z}$  in 10 runs and the best value  $Z_i$  found by the MA for the  $i$ th run,  $SD = \sqrt{\frac{\sum_{i=1}^{10} (\bar{Z} - Z_i)^2}{10}}$ . Table 5.17 gives the details of the solution results  $Z_{best}$ , the average objective values  $\bar{Z}$  and  $RSD$  values. Most of the latter are below 1%, and the average  $RSD$  is 0.86 % for all the instances. Figure 5.6 shows the average  $RSD$  value for different problem scales. Regarding the instances with different potential hub values, the average  $RSD$  values show relatively smooth curves without significant fluctuations. The stability of the MA proves its robustness and usefulness for decision-making. The small values attained by the  $RSD$  also suggest that just one or a few runs could be performed instead of ten to determine meaningful solutions of real-life problems in a dramatically reduced computing time. This would make it possible to test more alternative scenarios and sensitivity analyses, which is essential for decision-making.

TABLE 5.17 : The values of the performance indicators of the MA

Instance $ H - I - J $	Hub capacity	$Z_{best}$	$\bar{Z}$	$RSD$ (%)	Instance $ H - I - J $	Hub capacity	$Z_{best}$	$\bar{Z}$	$RSD$ (%)
3-10-10	10	5746.53	5875.03	1.02	3-15-15	30	9015.79	9197.77	0.93
	15	4269.15	4346.48	1.45		45	7638.35	7648.21	0.15
	30	3277.36	3305.91	0.89		90	6539.41	6577.51	0.61
6-10-10	10	5659.78	5685.16	0.72	6-15-15	30	8127.86	8149.25	0.46
	15	4269.15	4273.32	0.16		45	7107.65	7107.65	0.00
	30	3272.23	3278.57	0.36		90	6179.78	6204.06	0.16
10-10-10	10	5659.78	5701.61	0.39	10-15-15	30	7863.46	7985.20	1.44
	15	4258.45	4356.21	1.43		45	6861.88	7058.62	1.84
	30	3245.62	3270.34	1.19		90	6199.35	6239.15	0.55
3-20-20	45	10024.00	10070.54	0.27	3-25-25	45	11676.8	11708.41	0.16
	60	9048.58	9172.50	0.53		60	10557.6	10592.83	0.24
	120	8057.46	8112.80	0.36		120	9834.14	9949.95	0.47
6-20-20	45	9806.10	9929.67	0.84	6-25-25	45	11125.50	11185.62	0.20
	60	9022.21	9027.58	0.18		60	10252.00	10359.96	0.54
	120	8041.56	8070.13	0.35		120	9062.72	9129.23	0.42
10-20-20	45	9632.53	9689.02	0.46	10-25-25	45	10856.20	10995.55	1.05
	60	8725.87	8932.15	2.62		60	10159.40	10281.32	0.69
	120	7885.86	7946.51	0.58		120	9056.17	9142.30	0.80
3-30-30	60	14732.00	14790.89	0.31	3-35-35	60	15303.30	15332.62	0.21
	90	12994.00	13303.53	2.23		90	12704.90	12825.50	0.55
	165	11401.80	11475.92	0.56		180	11895.80	12096.61	0.75
6-30-30	60	13052.10	13086.44	0.24	6-35-35	60	13486.80	13515.46	0.28
	90	11973.20	11989.19	0.11		90	12747.70	12780.62	0.13
	165	11401.80	11498.66	0.45		180	11997.80	12114.40	0.42
10-30-30	60	12952.60	13165.92	1.55	10-35-35	60	13579.80	13862.39	1.90
	90	11987.1	12282.38	2.01		90	12655.50	12997.95	2.27
	165	11384.4	11492.18	0.72		180	11975.80	12191.16	1.48
3-40-40	75	16172.60	16232.34	0.28	3-45-45	75	14364.90	14485.52	0.59
	105	15233.90	15488.97	1.13		105	13282.50	13399.95	0.51
	210	13442.10	13615.45	0.87		195	12165.40	12252.92	0.58
6-40-40	75	15257.80	15339.58	0.22	6-45-45	75	13808.20	13964.20	0.85
	105	14317.30	14361.29	0.30		105	12808.70	13234.49	2.08
	210	13296.30	13395.88	0.76		195	12158.20	12261.14	0.69
10-40-40	75	14586.3	14941.4	2.12	10-45-45	75	13486.7	13838.82	1.81
	105	13838	14369.58	2.53		105	12557.00	12846.71	1.84
	210	13267.6	13507.67	1.69		195	12134.90	12345.26	1.27
3-50-50	75	17242.6	17304.01	0.26	6-50-50	75	16669.30	16865.87	0.46
	120	16324.10	16464.26	0.48		120	15564.30	15733.77	0.85
	225	14848.40	15007.79	0.52		225	14719.20	14967.48	0.91
10-50-50	75	16441.56	16811.56	1.24					
	120	15471.72	15883.81	1.86					
	225	14712.64	15021.89	2.26					

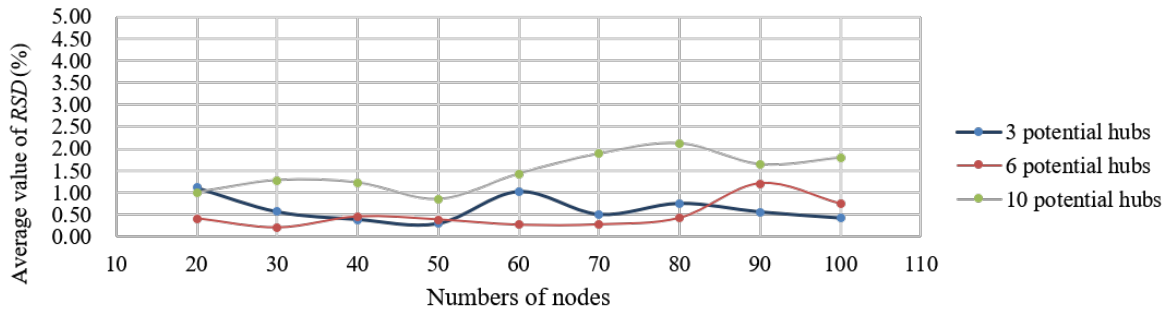


FIGURE 5.6 : Average values of the RSD indicators of the single-objective MA

### 5.5.2 Influence of the hub fixed cost

In the analysis presented above, a value of 1000 was used for the hub fixed costs. However for real-life applications, the fixed costs of logistics facilities may be highly dependent on the particular case. In this section, we report a sensitivity analysis conducted on a possible range of fixed costs  $F_k = \{1000, 500, 200, 100, 0\}$ . All the results presented in Table 5.18 were obtained with the MA. In this table, the  $F_k$  column indicates the fixed cost value of opening one potential hub. The Fix\_hub column represents the total fixed cost of the opened hubs. Column **Cost'** records the total cost of the solutions excluding the fixed hub cost,  $\mathbf{Cost}' = Z_{best} - \text{Fix\_hub}$ . The following five columns correspond to **Cost'** and stand for the collection cost, the delivery cost, the inter-hub transfer cost, the handling cost and the fixed cost of using the vehicles, respectively. It can be observed in Table 5.18 that, for a given hub capacity, the fixed hub cost has no obvious influence on the solutions of small instances (e.g. instance 6-20-20) until it is set to a small value of 100 or even 0, which leads logically to opening more hubs and a reduced value of **Cost'**. For large instances (e.g. instance 10-35-35), when the fixed hub cost decreases, the number of open hubs and the inter-hub transfer cost increase while the collection cost, delivery cost and **Cost'** decrease. When there is no fixed hub cost, all the potential hubs are open and the best solution (the lowest **Cost'**) is obtained. This confirms the fact that a larger number of open hubs yields a lower value of the total cost (excluding fixed hub costs). The total cost on an HLRP network is reduced by increasing the volume of inter-hub transportations, which is a fundamental hypothesis of Hub Location Problems.

TABLE 5.18 : Sensitivity analysis on the hub fixed cost

Instance $ H - I - J $	Hub capacity	$F_k$	Open hub	$Z_{best}$	Fix_hub	Cost'	Collect cost	Deliv_cost	Inter-hub cost	Handle cost	Fix_V
6-20-20	45	1000	2, 3, 5	9786.87	3000	6786.87	2527.13	1789.44	548.92	321.38	1600
		500	2, 3, 5	8286.87	1500	6786.87	2527.13	1789.44	548.92	321.38	1600
		200	2, 3, 5	7386.87	600	6786.87	2527.13	1789.44	548.92	321.38	1600
		100	2, 3, 4, 6	7094.27	400	6694.27	2431.23	1571.51	762.95	328.59	1600
		0	<b>1,2,3,4,5,6</b>	6593.05	0	<b>6593.05</b>	2380.97	1489.97	782.03	340.09	1600
	60	1000	3, 4	9022.21	2000	7022.21	2745.16	1835.18	458.48	283.39	1700
		500	3, 4	8022.21	1000	7022.21	2745.16	1835.18	458.48	283.39	1700
		200	3, 4	7422.21	400	7022.21	2745.16	1835.18	458.48	283.39	1700
		100	3, 4, 6	7120.05	300	6820.05	2745.16	1659.98	528.57	286.35	1600
		0	<b>1,2,3,4,5,6</b>	6585.17	0	<b>6585.17</b>	2380.97	1493.44	767.86	342.90	1600
	120	1000	3	8118.95	1000	7118.95	3014.13	2320.09	0.00	184.73	1600
		500	3	7618.95	500	7118.95	3014.13	2320.09	0.00	184.73	1600
		200	3	7334.59	200	7118.95	3029.77	2320.09	0.00	184.73	1600
		100	2, 3, 6	7185.71	300	6885.71	2737.47	1700.26	558.20	289.78	1600
		0	<b>1,2,3,4,5,6</b>	6585.17	0	<b>6585.17</b>	2380.97	1493.44	767.86	342.90	1600
10-35-35	60	1000	1, 4, 6	13579.80	3000	10579.80	3184.48	3073.48	1102.38	519.44	2700
		500	1, 2, 4, 6	12096.50	2000	10096.50	2901.26	2727.64	1127.09	540.52	2800
		200	1, 2, 4, 5, 6, 8	10791.40	1200	9691.40	2473.98	2367.77	1495.23	554.38	2700
		100	1, 2, 3, 4, 6, 8	10375.90	600	9775.90	2359.16	2265.70	1597.45	553.55	3000
		0	<b>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</b>	9544.05	0	<b>9544.05</b>	1993.95	2166.77	1717.42	565.92	3100
	90	1000	1, 2	12655.50	2000	10665.50	3664.28	3270.35	672.05	448.86	2600
		500	1, 2, 7	11690.00	1500	10190.00	2981.73	3063.52	1060.79	484.00	2600
		200	1,2,4,6,8	10767.40	1000	9767.40	2474.04	2419.48	1425.76	548.10	2900
		100	1,2,4,6,8	10228.00	500	9728.00	2481.83	2474.24	1424.43	547.46	2800
		0	<b>1, 2, 4, 5, 6, 7, 8</b>	9477.82	0	<b>9477.82</b>	2157.88	2237.03	1619.83	563.07	2900
	180	1000	6	11975.80	1000	10975.80	4011.31	4162.94	0.00	301.55	2500
		500	2, 6	11579.80	1000	10571.80	3423.21	3304.53	803.78	448.33	2600
		200	1,2,4,6,8	10748.70	1000	9748.70	2596.53	2387.85	1423.35	541.00	2800
		100	1, 2, 4, 5, 6, 8	10211.90	600	9611.90	2429.35	2305.50	1520.47	556.85	2800
		0	<b>1,2,3,4,5, 6, 7, 8, 9, 10</b>	9526.86	0	<b>9526.86</b>	1952.32	2103.72	1800.78	570.04	3100

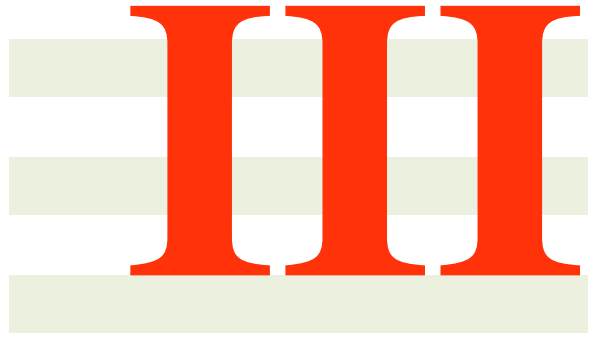
## 5.6 Conclusion

In this chapter, we conducted extensive comparative experiments to solve the problem and model with the CPLEX solver and with the memetic metaheuristic, with the goal of determining solutions of least total cost in a reasonable computing time. For the purpose of experiments, we generated a benchmark of instances of different sizes and characteristics, ranging from 3-10-10 to 10-50-50 potential hubs, suppliers and clients respectively, with a wide range of hub capacities. In order to achieve significant results, we tuned the main parameter of CPLEX and compared different implementations of the memetic algorithm.

The results show that CPLEX can solve the model to optimality only for small instances and finds feasible solutions for larger instances only with a gap, whose size grows with that of the instances. Solution times reach 3 hours even for medium-sized instances and convergence is slow. This prevents solving large realistic instances in this fashion. Alternatively, our memetic algorithm can find feasible solutions to the problem for all types of instances in a reasonable computing time and with limited gaps compared to the lower bounds of CPLEX. Convergence of the memetic algorithm to good solutions is fast and the solutions are very stable. We also conducted sensitivity analyses essential in a decision-making process, such as regarding the results in terms of the fixed cost of opening hubs, potential hub numbers and capacities. Fewer potential hubs and a larger capacity of hubs will logically reduce the complexity of the problem; more potential hubs and a larger capacity of hubs result in a decrease in the total network costs.







## **Bi-objective HLRP**



## A mathematical model and a MA for the bi-objective HLRP

With the concerns about sustainable development, the environmental impact of freight transport and goods operations have been an increasing matter of interest. It is indeed predicted that over 80 % of the transport companies will be significantly influenced by the global warming, especially relatively to the CO<sub>2</sub> emissions, by the year of 2020 (Piecnyk and McKinnon [2010]). Such facts suggests the importance of incorporating environmental factors into logistics-related decisions.

In this chapter, we first investigate the impacts of CO<sub>2</sub> emissions due to transport, on the Hub Location-Routing Problem (HLRP). By including the costs and CO<sub>2</sub> emission functions, we then propose a bi-objective mathematical model of the green HLRP, considering both economic and environmental factors. Second, we describe a bi-objective Memetic Algorithm (MA). This bi-objective MA associates a previously proposed single-objective MA to a Non-dominated Sorting Genetic Algorithm (NSGAI) and an Efficient Non-domination Level Update (ENLU) method, as well as a two-dimensional local search. This procedure is intended to generate an approximation of the Pareto front of non dominated solutions of the problem.

This chapter is organized as follows. Section 6.1 gives the problem definition of the proposed bi-objective model and describes the formulations of calculating CO<sub>2</sub> emissions. The bi-objective model of the HLRP is then presented. Section 6.2 develops a bi-objective MA which is inspired by the single-objective MA proposed in Chapter 4 to generate the approximate Pareto Front.

### 6.1 Problem definition and mathematical formulations

#### 6.1.1 General features

We propose a bi-objective model for the CSAHLRP with separate collection and delivery tours which is an extension of the single objective model introduced in Chapter 3. For the sake of clarity, we describe the complete model below. The model is defined on a complete directed graph  $G = (N, A)$  containing a set of vertices  $N$  and a set of arcs  $A$  where  $N = H \cup I \cup J$ .  $H = \{k \mid k = 1, 2, \dots, h\}$  presents a set of potential hubs. Each potential hub is capacitated and subjected to a fixed cost  $F_k$  once selected open.  $I = \{i \mid i = h+1, h+2, \dots, h+m\}$  and  $J = \{j \mid j = h+m+1, h+m+2, \dots, h+m+n\}$  stand for the sets of customers and clients who should be served. The numbers and locations of potential hubs, suppliers and clients are known data. Each pair of  $i$  and  $j$  ( $i \in I, j \in J$ ) is associated with a given amount of freight flow  $q_{ij}$  to be shipped between them. The total supply  $O_i = \sum_{j \in J} q_{ij}$  of supplier  $i$  and demand  $D_j = \sum_{i \in I} q_{ij}$  of client

$j$  should be satisfied. The set  $A = A_1 \cup A_2 \cup A_3$  includes the collection arc set  $A_1 = \{(i, j) : i, j \in I \cup H\}$ , the delivery arc set  $A_2 = \{(i, j) : i, j \in J \cup H\}$  and the inter-hub transfer arc set  $A_3 = \{(l, k) : l, k \in H\}$ . A fleet of identical vehicles available for collections and deliveries is denoted as set  $V$  with a fixed capacity  $Q$ . Once a vehicle travels from non-hub node  $i$  to non-hub node  $j$ , a routing cost is incurred that is dependent on the distance  $d_{ij}$ . The transportation costs between two hubs are determined by travelling distances and transferred flows and inter-hub transport is not subject to capacity restrictions. To model the collection and delivery routes, a flow variable  $f_{ij}$  is used (see [Karaoglan and Altıparmak \[2015\]](#)), representing the vehicle load on each arc  $(i, j)$ . The following other constraints must hold :

- The capacity of each hub and collection/delivery vehicle must not be exceeded.
- The demand of each non-hub node has to be consolidated in the allocated hub. The direct transportations for supplier-client pairs are forbidden.
- The demand of suppliers can be transferred at most two hubs before delivering to clients.
- Suppliers or clients on the same route must be assigned to the same and only one selected hub (single allocation).
- Each route must start and end at the same selected hub.
- The collection and delivery processes are independent. Collections are executed preceding the long-distance transfers among the open hubs while deliveries are performed after the inter-hub transport processes.
- Each route must be visited by only one vehicle and each vehicle must contain only one type of flow (collected or delivered).
- In inter-hub transportations, vehicles connect hubs directly to one another and there are no tours.

The goal of the model is to minimize the total cost and CO<sub>2</sub> emissions of transport by locating the hubs, allocating the suppliers/clients to the open hubs, planning the collection/delivery routes between the nodes and the inter-hub freight transportations.

## 6.1.2 The CO<sub>2</sub> emission formulations

In this section, we consider the CO<sub>2</sub> emissions caused by transport. Three types of transportation emissions are considered : the CO<sub>2</sub> emissions during the collections, the inter-hub transportations and the deliveries. Regarding the collection and delivery activities, the vehicle loads to and from the hubs are unknown in advance, variable and change from one node to another depending on the collections or deliveries. So we will use the fuel-based method which is more appropriate for estimating the emissions in this case, according to Section 2.4 of Chapter 2. To the contrary, we assume that regarding inter-hub transport, trucks are always fully loaded. Therefore we will use the distance-based method to estimate the emissions originating from inter-hub transport.

The CO<sub>2</sub> emission formulation for collection and delivery transportations is therefore based on the fuel consumption model described by [Barth et al. \[2005\]](#), [Demir et al. \[2012\]](#) and [Demir et al. \[2014\]](#), which estimates the fuel consumptions of each vehicle instantaneously depending of its load (see Section 2.4, Chapter 2). As a result, the CO<sub>2</sub> emissions emitted along the arcs of a collection or delivery tour for a given type of vehicle may be estimated by the following formula :

$$e(d, f) = (w_1 + w_2 f_{ij})d_{ij} \quad (6.1)$$

where  $e(d, f)$  represent the CO<sub>2</sub> emissions along an arc of length  $d_{ij}$  with a vehicle load  $f_{ij}$  and  $w_1$  and  $w_2$  are fixed coefficients.

Concerning the inter-hub transportation, assumed to be corresponded to a full-loaded transport process, the CO<sub>2</sub> emissions of which are estimated by a distance-based formulation (MEET, Hickman et al. [1999]). This formulation combines a hot emission  $E_{hot}$  and a cold-start emission  $E_{start}$  (see Section 2.4, Chapter 2).  $E_{hot}$  stands for the CO<sub>2</sub> emissions mainly caused by fuel consuming.  $E_{start}$  is a fixed emission according to the type of vehicle. It results that for a given type of vehicle travelling along a distance between hubs  $k$  and  $l$ , the CO<sub>2</sub> emissions may be estimated by :

$$E(d_{kl}) = e_{hot} \times d_{kl} + E_{start} \quad (6.2)$$

### 6.1.3 Bi-objective model for the HLRP

The model of the bi-objective HLRP is an extension of the single-objective model of the HLRP. We formulate the CO<sub>2</sub> emission objective function **HLRP-CO<sub>2</sub>** and integrate it into the single-objective model **HLRP-COST** in Chapter 3, which aims at minimizing total cost, to form the bi-objective one. For sake of clarity, we present the whole model in this section. The bi-objective HLRP model for minimizing total costs and CO<sub>2</sub> emissions can be formulated as follows :

#### Sets

- $H$  – Set of hub nodes,  $H = \{k \mid k = 1, 2, \dots, h\}$ ;
- $I$  – Set of supplier nodes,  $I = \{i \mid i = h + 1, h + 2, \dots, h + m\}$ ;
- $J$  – Set of client nodes,  $J = \{j \mid j = h + m + 1, h + m + 2, \dots, h + m + n\}$ ;
- $N$  – Set of all nodes,  $N = H \cup I \cup J$ ;
- $A_1$  – Set of arcs in collection routing,  $A_1 = \{(i, j) : i, j \in I \cup H\}$ ;
- $A_2$  – Set of arcs in delivery routing,  $A_2 = \{(i, j) : i, j \in J \cup H\}$ ;
- $A_3$  – Set of arcs in inter-hub transfer,  $A_3 = \{(l, k) : l, k \in H\}$ ;
- $A$  – Set of all arcs,  $A = \{(i, j) : i, j \in N\}$ ;

#### Parameters

- $F_k$  – Fixed cost of operating hub  $k$ ;
- $\Gamma_k$  – Capacity of hub  $k$ ;
- $c_k$  – Handling cost to operate one unit product in hub  $k$ ,  $k \in H$ ;
- $C$  – Fixed cost of a vehicle;
- $Q$  – Capacity of a vehicle;
- $q_{ij}$  – Flow quantity from supplier  $i \in I$  to client  $j \in J$ ;
- $d_{ij}$  – Distance between two nodes  $i$  and  $j$ , arc  $(i, j) \in A$ ;
- $\alpha$  – Unit cost parameter for the inter-hub transport;
- $\beta$  – Unit cost parameter for the collection tour;
- $\gamma$  – Unit cost parameter for the delivery tour;
- $O_i$  – Total quantity of flow originating at supplier  $i$ ,  $O_i = \sum_{j \in J} q_{ij}$ ;
- $D_j$  – Total quantity of flow for client  $j$ ,  $D_j = \sum_{i \in I} q_{ij}$ ;
- $E(d_{kl})$  – CO<sub>2</sub> emission for inter-hub transfer from hub  $k$  to  $l$  when the vehicle is full-loaded,  $l, k \in H$ ;
- $w_1, w_2$  – CO<sub>2</sub> emission constants of transportation.

### Decision variables

$Y_{kl}^i$  – the fraction of flow shipped from supplier  $i$  via hubs  $k$  to  $l$ ,  $l \in H$ ,  $k \neq l$  and  $i \in I$ ;

$z_{ik}$  – the allocation variable of a node  $i$  to a hub  $k$ . It is equal to 1 if the node  $i$  is allocated to the hub  $k$ , 0 otherwise; especially,  $z_{kk} = 1$  if the hub  $k$  is selected to be open,  $i \in N$ ,  $k \in H$ ;

$x_{ij}$  – is equal to 1 if a vehicle traverses arc  $(i, j)$ , and 0 otherwise.

$f_{ij}$  – vehicle load on arc  $(i, j)$  if a vehicle travels directly from node  $i$  to node  $j$ , otherwise 0.  $(i, j) \in A_1 \cup A_2$ ;

With respect to those notations, the bi-objective HLRP model for minimizing total costs and CO<sub>2</sub> emissions can be formulated as follows :

#### HLRP-COST

$$\begin{aligned} \min \sum_{k \in H} F_k z_{kk} + \sum_{i \in I} \sum_{(k,l) \in A_3} \alpha d_{kl} O_i Y_{kl}^i + \sum_{(i,j) \in A_1} \beta d_{ij} x_{ij} + \sum_{(i,j) \in A_2} \gamma d_{ij} x_{ij} \\ + \sum_{i \in I} \sum_{k \in H} c_k O_i z_{ik} + \sum_{i \in I} \sum_{(k,l) \in A_3} c_l O_i Y_{kl}^i + \sum_{k \in H} \sum_{i \in I \cup J} C x_{ki} \end{aligned} \quad (6.3)$$

#### HLRP-CO<sub>2</sub>

$$\min \sum_{(i,j) \in A_1} (w_1 x_{ij} + w_2 f_{ij}) d_{ij} + \sum_{(i,j) \in A_2} (w_1 x_{ij} + w_2 f_{ij}) d_{ij} + \sum_{i \in I} \sum_{(k,l) \in A_3, k \neq l} E(d_{kl}) O_i Y_{kl}^i \quad (6.4)$$

Subject to

— hub location constraints :

$$z_{ik} \leq z_{kk} \quad \forall i \in N, \forall k \in H \quad (6.5)$$

$$\sum_{k \in H} z_{ik} = 1 \quad \forall i \in I \cup J \quad (6.6)$$

$$\sum_{i \in I} O_i z_{ik} \leq \Gamma_k z_{kk} \quad \forall k \in H \quad (6.7)$$

$$\sum_{j \in J} D_j z_{jl} \leq \Gamma_l z_{ll} \quad \forall l \in H \quad (6.8)$$

$$\sum_{l \in H} Y_{kl}^i = z_{ik} \quad \forall i \in I, \forall k \in H \quad (6.9)$$

$$\sum_{l \in H} Y_{lk}^i O_i = \sum_{j \in J} q_{ij} z_{jk} \quad \forall i \in I, \forall k \in H \quad (6.10)$$

— collection routing constraints :

$$\sum_{j \in I \cup H} x_{ij} = 1 \quad \forall i \in I \quad (6.11)$$

$$\sum_{i \in I \cup H} x_{ij} - \sum_{i \in I \cup H} x_{ji} = 0 \quad \forall j \in I \cup H \quad (6.12)$$

$$x_{ki} \leq z_{ik} \quad \forall i \in I, k \in H \quad (6.13)$$

$$x_{ik} \leq z_{ik} \quad \forall i \in I, k \in H \quad (6.14)$$

$$x_{ij} + z_{ik} + z_{jl} \leq 2 \quad \forall i, j \in I, i \neq j, \forall k, l \in H, k \neq l \quad (6.15)$$

$$\sum_{j \in I \cup H} f_{ij} - \sum_{j \in I \cup H} f_{ji} = O_i \quad \forall i \in I \quad (6.16)$$

$$\sum_{i \in I} f_{ik} = \sum_{i \in I} z_{ik} O_i \quad \forall k \in H \quad (6.17)$$

$$f_{ij} \leq (Q - O_j)x_{ij} \quad \forall i \in I \cup H, \forall j \in I \quad (6.18)$$

$$O_i x_{ij} \leq f_{ij} \quad \forall i \in I, \forall j \in I \cup H \quad (6.19)$$

$$\sum_{i \in I} f_{ki} = 0 \quad \forall k \in H \quad (6.20)$$

$$f_{ik} \leq Q x_{ik} \quad \forall i \in I, k \in H \quad (6.21)$$

— delivery routing constraints :

$$\sum_{j \in J \cup H} x_{ij} = 1 \quad \forall i \in J \quad (6.22)$$

$$\sum_{i \in J \cup H} x_{ij} - \sum_{i \in J \cup H} x_{ji} = 0 \quad \forall j \in J \cup H \quad (6.23)$$

$$x_{jk} \leq z_{jk} \quad \forall j \in J, k \in H \quad (6.24)$$

$$x_{kj} \leq z_{jk} \quad \forall j \in J, k \in H \quad (6.25)$$

$$x_{ij} + z_{ik} + z_{jl} \leq 2 \quad \forall i, j \in J, i \neq j, \forall k, l \in H, k \neq l \quad (6.26)$$

$$\sum_{i \in J \cup H} f_{ij} - \sum_{i \in J \cup H} f_{ji} = D_j \quad \forall j \in J \quad (6.27)$$

$$\sum_{j \in J} f_{kj} = \sum_{j \in J} z_{jk} D_j \quad \forall k \in H \quad (6.28)$$

$$f_{ij} \leq (Q - D_i)x_{ij} \quad \forall i \in J, \forall j \in J \cup H \quad (6.29)$$

$$D_j x_{ij} \leq f_{ij} \quad \forall i \in J \cup H, \forall j \in J \quad (6.30)$$

$$\sum_{j \in J} f_{jk} = 0 \quad \forall k \in H \quad (6.31)$$

$$f_{kj} \leq Q x_{kj} \quad \forall j \in J, k \in H \quad (6.32)$$

— domain of decision variables :

$$z_{ik} \in \{0, 1\} \quad \forall i \in N, \forall k \in H \quad (6.33)$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in N, \forall j \in N \quad (6.34)$$

$$0 \leq Y_{kl}^i \leq 1 \quad \forall i \in I, \forall k, l \in H \quad (6.35)$$

$$f_{ij} \geq 0 \quad \forall (i, j) \in A_1 \cup A_2 \quad (6.36)$$



— valid inequalities :

$$x_{ij} + x_{ji} \leq 1 \quad \forall i, j \in I \quad (6.37)$$

$$x_{ij} + x_{ji} \leq 1 \quad \forall i, j \in J \quad (6.38)$$

$$\sum_{k \in H} z_{kk} \geq \left\lceil \frac{\sum_{i \in I} \sum_{j \in J} q_{ij}}{\Gamma_k} \right\rceil \quad (6.39)$$

$$\sum_{k \in H} \sum_{i \in I} x_{ki} \geq \left\lceil \frac{\sum_{i \in I} O_i}{Q} \right\rceil \quad (6.40)$$

$$\sum_{k \in H} \sum_{j \in J} x_{kj} \geq \left\lceil \frac{\sum_{j \in J} D_j}{Q} \right\rceil \quad (6.41)$$

In addition, routing variables  $x_{ij}$  would be ignored in a preprocessing step whenever  $O_i + O_j > Q, \forall i, j \in I, i \neq j$  or  $D_i + D_j > Q, \forall i, j \in J, i \neq j$  (Karaoglan and Altiparmak [2015]).

The objective function (6.3) minimizes the total fixed and variable costs of the CSAHLRP network including the fixed cost for opening hubs, the transportation cost between hubs, local collection and delivery routing costs, the freight operating cost in hubs, and the fixed cost of using vehicles. The objective function (6.4) intends to minimize the total transportation CO<sub>2</sub> emissions. The first and second term denote the transportation emissions during collections and deliveries which is determined by both the travelling distance and vehicle load. The third term computes the CO<sub>2</sub> emissions of long distance transportation between hubs depending on the distances (Jamshidi et al. [2012]).

Constraints (6.5) to (6.10) are classic constraints of the HLP. Constraint (6.5) ensure that non-hub nodes can be allocated to a hub only if the hub is open. Constraint (6.6) force each non-hub node  $I$  to be assigned to only one hub (single allocation). Hub capacity constraints (6.7) and (6.8) limit the total collection and delivery load on hubs. Constraints (6.9) and (6.10) are flow conservation equations. They impose the demand of each supplier or client to be served by the allocated hub (Ernst and Krishnamoorthy [1999]).

Constraints (6.11) to (6.21) assure a reasonable collection process. Constraint (6.11) make sure that each supplier is visited just once. Constraint (6.12) guarantee an equal number of incoming and outgoing arcs. Constraints (6.13) - (6.15) eliminate illegal routes that do not start and end at the same hub. Constraints (6.16) are the flow conservation constraints for collections. Each time the vehicle serves a supplier, it must load all of its demand. Constraint (6.17) ensure that the total collection load entering each open hub equals the total demand of the suppliers who are allocated to the hub. Constraints (6.18) and (6.19) provide an upper and lower bound for the collection flows. Constraint (6.20) guarantee that the load on each vehicle is zero when leaving one open hub for collecting (Karaoglan and Altiparmak [2015], Yu and Lin [2016]). Constraints (6.21) impose that when there is no arc between a supplier node and a hub node, the flow should be zero. Constraints (6.22)-(6.32) relative to the delivery processes represent conditions similar to those for the collections.

Constraints (6.33)-(6.36) specify the variables  $z_{ik}, x_{ij}, Y_{kl}^i$  and  $f_{ij}$ , respectively. Constraints (6.37) to (6.41) are valid inequalities : constraints (6.37) and (6.38) are sub-tour elimination (Karaoglan and Altiparmak [2015]); constraint (6.39) restricts the minimum number of open hubs (Bostel et al. [2015]); finally, constraints (6.40) and (6.41) provide a lower bound of the total number of vehicles required in any feasible solution.

## 6.2 Memetic algorithm for the bi-objective HLRP

In this section, we adapt the memetic algorithm developed for the mono-objective case (Chapter 4) to solve the bi-objective HLRP minimizing the cost and CO<sub>2</sub> emissions. It applies the general framework of the MA and is combined with a fast elitist Non-dominated Sorting Genetic Algorithm (NSGAI). The following subsections describe the algorithmic design of the bi-objective MA in terms of solution representation,

global framework, generation of initial population, fast non-dominating sorting, genetic operators (selection, crossover and mutation), a non-dominance level update sorting method and the two-dimensional local searches.

### 6.2.1 Solution representation

As initially proposed by Derbel et al. [2012], a candidate solution  $x$  is represented by a chromosome  $P(x)$  containing two vectors : the selected hub vector  $H(x)$  and the non-hub nodes routing vector  $A(x)$  (Figure 6.1). Vector  $H(x)$  keeps a set of selected hubs and their assignment configuration. Vector  $A(x)$  records the non-repeated permutation of suppliers and clients conforming to their served sequences on a route (routing level). Each supplier/client is allocated to the open hub in the corresponding position in vector  $H(x)$ . For a chromosome  $P(x)$ , its two vectors are enumerated as  $H(x) = \{h_1, h_2, \dots, h_n\}$  and  $A(x) = \{a_1, a_2, \dots, a_n\}$ . For every position  $i = \{1, 2, \dots, n\}$ , if  $h_i = k, a_i = j$ , the non-hub node  $j$  is assigned to hub  $k$ . The tours for collections and deliveries can be deduced based on the feasible allocation configurations. According to the fixed sequence in vector  $A(x)$ , one vehicle starts from the first open hub and adds suppliers with the same allocating scheme into the route until the capacity of the vehicle is reached and a new tour begins. Such process continues until all the suppliers are routed. The same procedure happens to the delivery routes. More details can be found in Chapter 4.

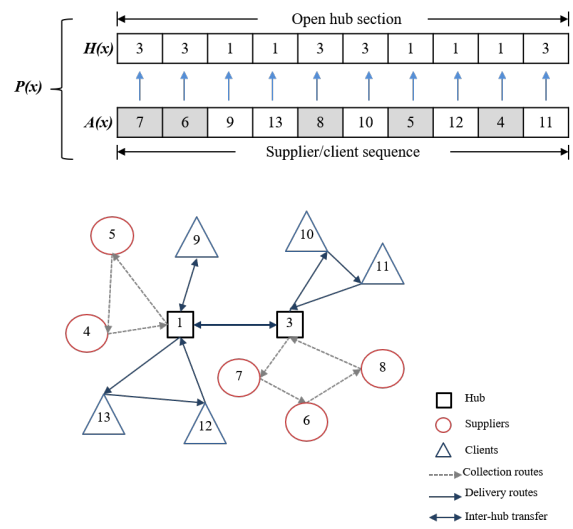


FIGURE 6.1 : Solution representation of the HLRP

### 6.2.2 Global framework of the bi-objective MA

In the MA, the initial population is originally ranked and sorted in different levels based on the NSGAI (Deb et al. [2002]). The density of the solutions in the population is calculated by a crowded-comparison operator (fitness) to guide the selection process at the various stages of the algorithm. When an offspring is generated, the Non-Dominance Level (NDL) of all solutions is updated. Instead of sorting all the current individuals each time, an Efficient Non-dominance Level Update (ENLU) method (Li et al. [2016]) just updates the NDL of some solutions that need to be changed, which reduces the time complexity from  $O(mN^2)$  (NSGAI procedure) to  $O(mN\sqrt{N})$ . The main framework is described as follows and showed in Figure 6.2. More details of each step will be specifically introduced in the following sections.

Step 1 : Initialization and classification

Initial population consists in heuristic and random solutions. Initial solutions are classified into feasible and infeasible solution archives.

*Step 2* : Fast non-dominated sorting (NSGAI)

We define  $F_i$  as the solution set in which all the solutions ranked in NDL  $i$  are stored. The set  $F, F = \{F_1, F_2 \dots F_n\}$ , represents the NDL structure of the current generation. The solutions in lower NDLs dominate the solutions in higher NDLs. The solutions in  $F_1$  dominate all the solutions in the other NDLs.

**(Step 3 to Step 6 repeat until stopping criterion is reached)***Step 3* : Fitness

The fitness for each feasible individual is calculated by the crowded-comparison operator, while the fitnesses of the infeasible individuals are zero.

*Step 4* : Selection, crossover and mutation

A unique fitness set is obtained to guide the binary tournament selection procedure. The approach selects the best fitnesses and the individuals sharing the best fitness are randomly chosen to survive. Each time two individuals are randomly compared in the selection process. Three cases are considered : If the two solutions are feasible, the solution in lower NDL is selected ; otherwise, if they are in the same NDL, the solution with the biggest fitness (crowding-distance operator) survives ; In the second case in which one feasible and one infeasible solutions are compared, the feasible one survives ; the third case compares two infeasible solutions and selects a random solution. After the selection procedure, a one-point crossover operator with a crossover probability is applied to a pair of selected parents to generate new offsprings, which are then mutated with a mutation probability.

*Step 5* : Non-dominance level update sorting

New generated offsprings are compared with the current generation and the NDLs are updated.

*Step 6* : Local search

A two-dimensional iterated local search on the non-dominated individuals on the NDL 1 is performed. The local search explores the solution space first on the cost objective value and then on the CO<sub>2</sub> objective value to find new non-dominated solutions.

### 6.2.3 Initial population

The initial population includes both heuristic and randomized solutions. Three fast heuristics are implemented to generate good initial solutions : a Nearest Neighborhood Algorithm (NN), a Clarke and Wright algorithm (CWA) (Caccetta et al. [2013]) and an Extended Clarke and Wright Algorithm (ECWA) (Karaoğlan and Altıparmak [2015]). The thorough idea of generating the heuristic solutions is to consider all the combinations of hubs (all the non-repetitive open hub subsets) which must meet the minimum hub number requirement. For each open hub subset, the non-hub nodes are allocated to the open hubs by the algorithm ECWA. Following this allocation scheme, two feasible solutions are generated at the same time by two algorithms (NN and CWA) which create two different local collection and delivery routing schemes. Since the number of generated individuals is dependent on the amount of hub subsets and may fail to reach the required population size, the remaining solutions are randomly produced. Generated solutions are classified into feasible and infeasible sets by means of checking hub capacity violation.

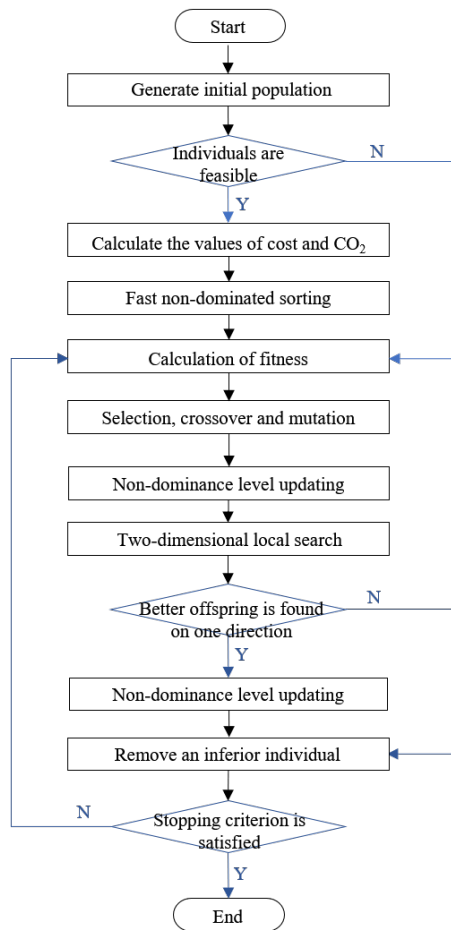


FIGURE 6.2 : Generic framework of the proposed MA for the bi-objective HLRP

### 6.2.4 Fast non-dominated sorting

The fast elitist Non-dominated Sorting Genetic Algorithm (NSGAII) is proposed by [Deb et al. \[2002\]](#) and has proved its high performance in terms of convergence and spread of solutions. It can be observed in **Algorithm 7**, we associate to each feasible solution  $\mathbf{p}$ ,  $n_p$  the number of solutions which dominate solution  $\mathbf{p}$  and the set  $S_p$  including the solutions dominated by solution  $\mathbf{p}$ . At the beginning, the individuals in the feasible solution set  $P$  are compared with each other (lines 2 to 9) to obtain  $n_p$  and  $S_p$ . The individuals which are non-dominated by none of the others, corresponding to  $n_p = 0$ , are ranked in NDL 1 and included in set  $F_1$  (lines 10 and 11). For each individual  $\mathbf{p}$  in  $F_1$ , we consider the set  $S_p$  of solutions dominated by  $\mathbf{p}$  and for each individual  $\mathbf{q}$  in  $S_p$ , we reduce its  $n_q$  by one. If  $n_q$  of a solution  $\mathbf{q}$  is reduced to 0, solution  $\mathbf{q}$  is added to the second non-dominated level  $F_2$  (line 25). The process repeats until all the individuals have been ranked in a NDL.

### 6.2.5 Fitness function

In the proposed MA, the fitness of a feasible individual is associated to its crowding-distance with other solutions in the same NDL ([Deb et al. \[2002\]](#)). The crowding distance estimates solution density surrounding a specific solution by calculating the size of the largest cuboid enclosing the solution point without including any other point (Figure 6.3). In **Algorithm 8**,  $F_i$  denotes a set of  $N_i$  non-dominated solutions in NDL  $i$  (line 1).  $L[j]_{dist}$  is the fitness of solution  $j$  which equals the sum of the normalized crowding-distances of all the objective values of solution  $j$ . The set  $L_{dist}(i)$  stores the fitnesses of all the individuals with the same NDL  $i$ . Firstly,  $L_{dist}(i)$  and  $L[j]_{dist}$  are initialized (lines 2 to 5). Regarding each objective  $m$ , the individuals are sorted in ascending order based on the value of objective  $m$  (line 7). Let  $L[j]_m$  refer to the  $m$ -th objective

**Algorithm 7 NSGAI****Input** Initial population  $Gen$ ; Feasible solution set  $P, P \subset Gen$ **Output** Non-dominance level structure  $F = \{F_1, F_2 \dots F_i\}$ 


---

```

1:  $S_p \leftarrow \emptyset, n_p \leftarrow 0, i \leftarrow 1$  // Initialisation
2: for each individual  $p, p \in P$  do
3:   for each individual  $q, q \in P, q \neq p$  do
4:     if ( $p \prec q$ ) then
5:        $S_p \leftarrow S_p \cup q$  // Include  $q$  into  $p$ 's dominating set  $S_p$ 
6:     else if ( $q \prec p$ ) then
7:        $n_p \leftarrow n_p + 1$  // The number of solutions dominating solution  $p$  increases
8:     end if
9:   end for
10:  if ( $n_p = 0$ ) then
11:     $F_1 \leftarrow F_1 \cup p$  //members of the first NDL
12:  end if
13: end for
14: while  $F_i \neq \emptyset$  do
15:    $H \leftarrow \emptyset$ 
16:   for each  $p \in F_i$  do
17:     for each  $q \in S_p$  do
18:        $n_q \leftarrow n_q - 1$ 
19:       if ( $n_q = 0$ ) then // There is no solution dominating  $q$ 
20:          $H \leftarrow H \cup q$ 
21:       end if
22:     end for
23:   end for
24:    $i \leftarrow i + 1$ 
25:    $F_i \leftarrow H$  // Current front is formed with all members
26: end while
   return  $F = \{F_1, F_2 \dots F_i\}$ 

```

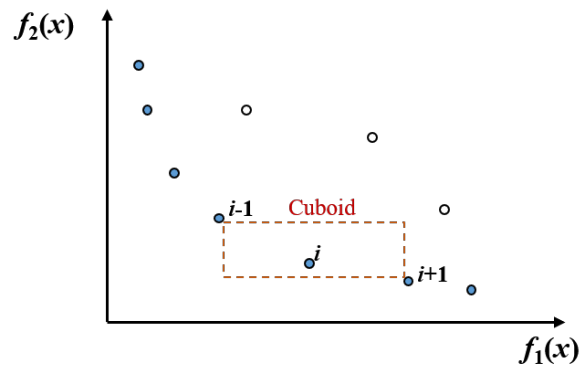
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value of the  $j$ -th individual. The “extreme solutions” (solution 1 and solution  $N_i$ ), with the smallest and largest objective value, are assigned an infinite crowding distance (line 8). For each intermediary solution, the procedure computes the normalized difference between the following and preceding individuals of current objective  $m$ , and sums it to the individual crowding distance (line 9 to 10). Here parameters  $f_m^{max}$  and  $f_m^{min}$  are the minimum and maximum values of objective  $m$  found in  $F_i$ . In the same NDL, solutions with bigger crowding-distance value (loose density in the objective space) have a better chance to be selected. The fitness of an infeasible solution is set to 0. Equations (6.42) show the fitness function  $F_{eva}(\mathbf{x})$  of individual  $\mathbf{x}$ :

$$F_{eva}(\mathbf{x}) = \begin{cases} 0 & \text{if solution } \mathbf{x} \text{ is infeasible} \\ L[\mathbf{x}]_{dist} & \text{if solution } \mathbf{x} \text{ is feasible} \end{cases} \quad (6.42)$$

## 6.2.6 Selection

The selection procedure select unique fitness of parent solutions by a binary tournament selection (Fortin and Parizeau [2013]). A unique fitness set  $\mathcal{F}_P$  (without repeating values) for the feasible solutions is built to limit the recurrence of solutions with the same fitness. Each fitness is marked with a level corresponding to the NDL of its corresponding individuals. The fitnesses of the infeasible solutions are stored in a set  $\mathcal{F}_O$  and marked with a NDL that is as large as infinity. Algorithm 9 presents the selection process.  $\tau$  represents the number of fitnesses to select at each iteration of the selection procedure and a solution set  $S$  is used to store

FIGURE 6.3 : The crowding-distance of solution  $i$ **Algorithm 8 Crowding-distance operator****Input** Non-dominated solution set  $F_i$  with a NDL of  $i$ **Output** Crowding distance set  $L_{dist}(i)$ 

```

1:  $N_i \leftarrow |F_i|$  // Number of individuals
2:  $L_{dist}(i) \leftarrow \emptyset$  // Set of crowding distances
3: for  $j \leftarrow 1$  to  $N_i$  do // Initialization
4:    $L[j]_{dist} \leftarrow 0$ 
5: end for
6: for  $m \leftarrow 1$  to 2 do // Two objectives : cost and CO2 emissions
7:   Sort solutions in  $F_i$  in ascending order according to the value of  $m$ 
8:    $L[1]_{dist} \leftarrow L[N_i]_{dist} \leftarrow \infty$  // Boundary solutions
9:   for  $j \leftarrow 2$  to  $N_i - 1$  do
10:     $L[j]_{dist} \leftarrow L[j]_{dist} + \frac{L[j+1]_m - L[j-1]_m}{f_m^{max} - f_m^{min}}$  // The normalized crowding distance
11:   end for
12:    $L_{dist}(i) \leftarrow L_{dist}(i) \cup L[j]_{dist}$ 
13: end for
   return  $L_{dist}(i)$ 

```

the selected individuals. The value of  $\tau$  equals the minimum value between the number of unique fitnesses and twice the number of individuals remaining to be selected (line 4).  $\tau$  unique fitnesses selected randomly from the fitness set  $\mathcal{F}$  are stored in a list  $G$  (line 5). Then, the fitnesses in list  $G$  are compared between any two and the best ones (half of  $\tau$  fitnesses) are selected based on the selection criteria below (**SelectRoule** in line 7) :

**Selection criteria**

- (1) If the NDLs of two fitnesses are different, the fitness with the smallest rank is selected;
- (2) If the NDLs of two fitnesses are equal and the fitness values are bigger than 0, the largest fitness survives;
- (3) If both fitnesses are equal to 0, one random individual survives.

Each time one fitness is selected, one random individual sharing the same fitness is added to the final selection set  $S$  (lines 8 and 9). The whole procedure continues until the number of selected solutions  $|S|$  reaches the population size. Finally, the individuals in the set  $S$  are randomly arranged in pairs with the purpose of applying the crossover operator in the next phase (line 12).

**Algorithm 9 Selection operator**


---

**Input**  $Gen$  : current generation;  $N_{pop}$  : population size;  $\mathcal{F}_P$  : unique fitness set of the feasible individuals in  $P$ ;  $\mathcal{F}_O$  : fitness set of the infeasible individuals in  $O$ ,  $P \cup O = Gen$

**Output** Selected parents set  $S$

- 1:  $\tau \leftarrow 0, S \leftarrow \emptyset, G \leftarrow \emptyset$
- 2:  $\mathcal{F} \leftarrow \mathcal{F}_P \cup \mathcal{F}_O$
- 3: **while**  $|S| \neq N_{pop}$  **do**
- 4:    $\tau \leftarrow \min\{2(N_{pop} - |S|), |\mathcal{F}|\}$  // Generate the number of fitnesses to be compared
- 5:   Select random  $\tau$  fitnesses from  $\mathcal{F}$  and store in list  $G$
- 6:   **for**  $(j \leftarrow 1, 3, 5, \dots, |G| - 1)$  **do**
- 7:      $p \leftarrow \text{SelectRule}\{G(j), G(i + 1)\}$  // Compare two fitnesses and select one according to the selection criteria
- 8:     Randomly select one individual  $x$  so that  $F_{eva}(\mathbf{x}) = p, \mathbf{x} \in Gen$
- 9:      $S \leftarrow S \cup \mathbf{x}$  // Put the selected individual in the set of  $S$
- 10:   **end for**
- 11: **end while**
- 12: The individuals in the set of  $S$  are arranged randomly in pairs // Pairs of parents for the crossover

**return**  $S$

---

**6.2.7 Crossover and mutation**

The selection procedure is followed by a one-point crossover operation simultaneously swapping nodes on selected parent pairs to form new offsprings. The hub location vector  $H(x)$  of a new offspring combines the code of the first parent before crossover point  $P_H$  and the code of the second parent after  $P_H$ . The first part of the routing vector  $A(x)$  of the offspring takes the same code as the first parent before the crossover point  $P_A$ . The second part of  $A(x)$  is made of the code of the second parent, deleting the nodes which have been placed in the first part and sequentially placing the remaining nodes. The mutation operator mutates the chromosome by using two different methods on sections  $A(x)$  and  $H(x)$ . In the location section  $H(x)$ , the hub assignment is modified by randomly choosing hubs to be replaced by others. With this procedure, it is possible to open a new hub or to close a hub. In the routing section  $A(x)$ , the locations of two randomly selected points are exchanged.

**6.2.8 Non-dominance level update sorting**

This section intends to explore an efficient way to add new offspring into the current generation and update the solutions whose NDL requires to be changed (Li et al. [2016]). Assume that the current NDL sets are  $\{F_1, F_2, \dots, F_i, \dots, F_n\}$ . The total number of solutions in NDL set  $F_i$  is  $|F_i|$  and  $F_i(j)$  represents the  $j$ th solution in set  $F_i$ . According to the property of the non-dominance level, the solutions in the lower level cannot be dominated by any solution of a higher level and each solution in  $F_i$  dominates at least one solution in  $F_{i+1}$  ( $1 \leq i < n$ ). Therefore, the solutions can move to at most the next non-dominance level during the updating.

Four circumstances are taken into account to rank a new offspring  $x'$  with a NDL and to update the NDLS of other solutions in current generation (**Algorithm 10**): (1) the new offspring  $x'$  which is dominated by a solution in  $F_i$  is moved to be compared with the next NDLS  $F_{i+1}$  (lines 15 and 16). If  $F_i$  is the last NDLS ( $i = n$ ),  $x'$  is added into a newly created NDLS  $F_{n+1}$ ; (2) the new offspring  $x'$  which dominates all the solutions in  $F_i$  is added into  $F_i$  and all the other solutions in  $F_i$  are moved to  $F_{i+1}$  (lines 17 to 19); (3) the new offspring  $x'$  which is non-dominated by the other solutions in  $F_i$  is added into  $F_i$  (lines 20 to 22); (4) the last but the most complex case is when the new offspring dominates some solutions and is non-dominated by the remaining ones in  $F_i$  (line 24). **Algorithm 11** presents the pseudo-code for the fourth case. Offspring  $x'$  is added into  $F_i$ , and the solutions in  $F_i$  which are dominated by  $x'$  are moved from  $F_i$  to a set  $S$ . In a special case when  $i = n$ , the solutions in  $S$  are assigned a newly generated NDLS  $F_{n+1}$  and the process

stops. Otherwise, each solution  $\mathbf{x}$  in  $S$  is compared with all the solutions in the next NDL  $F_j$ . If some of the solutions in  $F_j$  are dominated by the solutions in  $S$ , these dominated solutions are moved into a temporary archive  $T$  and similar operations of **Algorithm 11** are conducted to re-rank the solutions in  $T$  in a higher NDL ( $F_{j+1}$  or more).

---

**Algorithm 10 Efficient Non-dominance Level Update (ENLU)**


---

**Input** Non-dominance level structure  $F = \{F_1, F_2 \dots F_n\}$ , offspring solution  $x'$

**Output** Non-dominance level structure  $F$

```

1:  $case1 \leftarrow 0, case2 \leftarrow 0, case3 \leftarrow 0$  // Initialisation of three cases
2: for  $i \leftarrow 1$  to  $n$  do // For each current NDL
3:    $S \leftarrow \emptyset$ 
4:   for  $j \leftarrow 1$  to  $|F_i|$  do
5:     if ( $x'$  is non-dominated with  $F_i(j)$ ) then
6:        $case1 \leftarrow 1$ 
7:     else if ( $x'$  is dominated by  $F_i(j)$ ) then
8:        $case2 \leftarrow 1$ 
9:       break
10:    else
11:       $case3 \leftarrow 1$ 
12:       $S \leftarrow S \cup F_i(j)$ 
13:    end if
14:  end for
15:  if ( $cas2 = 1$ ) then // the offspring is dominated
16:    break
17:  else if ( $cas1 = 0$ ) and ( $cas3 = 1$ ) then // the offspring dominates all the solutions in current level
18:    move all solutions originally in  $F_i$  to  $F_{i+1}$ 
19:    break
20:  else if ( $cas1 = 1$ ) and ( $cas3 = 0$ ) then // the offspring is non-dominated with all the solutions in current
    level
21:    Add  $x'$  in  $F_i$ 
22:    break
23:  else
24:    LevelUpdate( $S, i + 1$ ) (Algorithm 11) // the offspring dominates some solutions in current level
25:    break
26:  end if
27: end for
28: Delete the solution with the worst fitness in the maximum NDL
    return  $F$ 

```

---

After assigning an offspring with a NDL, one inferior individual of current generation is removed (line 28, **Algorithm 10**): when there still exist some infeasible solutions in the population, a random infeasible individual is eliminated; when all the solutions are feasible, the algorithm gets rid of the solution with the worst fitness in the last level  $F_n$ .



**Algorithm 11 Non-dominance level update : LevelUpdate( $S, j$ )**

**Input** Non-dominance level structure  $F = \{F_1, F_2 \dots F_n\}$ ;  $S$  : the set of solutions removed from  $F_i, i + 1 = j, i \in \{1, 2, \dots, n\}$

**Output** Update non-dominance level structure  $F$

```

1: Add all solutions in  $S$  to  $F_j, j = i + 1$ 
2: if ( $j \neq n + 1$ ) then
3:    $T \leftarrow \emptyset$ 
4:   for  $j \leftarrow 1$  to  $|S|$  do
5:     for  $k \leftarrow 1$  to  $|F_j|$  do
6:       if ( $S_j \preceq F_j(k)$ ) then
7:          $T \leftarrow T \cup F_j(k)$ 
8:       end if
9:     end for
10:  end for
11:  if ( $T \neq \emptyset$ ) then
12:    LevelUpdate( $T, j + 1$ ) (Algorithm 11)
13:  end if
14: end if
    return  $F$ 

```

**6.2.9 Two-dimensional local search**

The key idea of the two-dimensional local search method lies in applying single-objective local searches on both objectives simultaneously to generate a set of non-dominated solutions during the searches (Tricoire [2012]). Figure 6.4.(a) and 6.4.(b) illustrate the neighbourhood of a given solution in half shaded circles. Figure 6.4.(c) shows the relevant portions of solution space for a given solution in two directions. Given a set of non-dominated solutions, the local search operators explore first the neighbourhood on the direction of minimizing CO<sub>2</sub> emissions (Figure 6.4.(a)), then on the direction of minimizing cost (Figure 6.4.(b)). Algorithm 12 gives an outline of the two-dimensional local search of the bi-objective HLRP. During the LS procedure, if the solution  $x$  is improved in terms of at least one objective, the improved solution  $x'$  is added in the current generation. The ENLU procedure (Algorithm 10) updates the NDL and eliminates the worst solution.

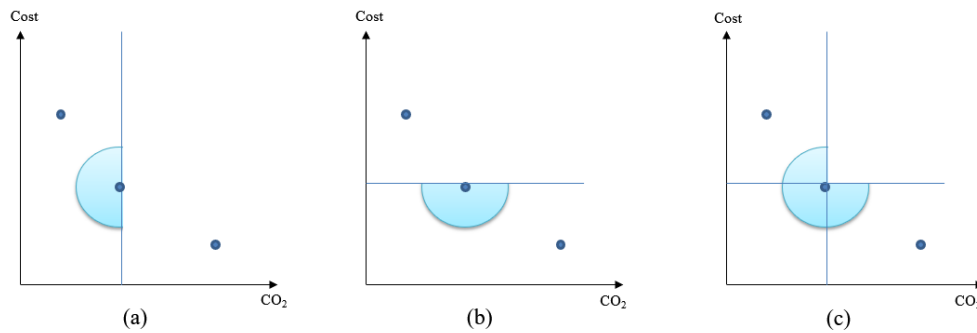


FIGURE 6.4 : Portions of solution space for the bi-objective HLRP : (a) Search space on direction 1 : minimization of cost; (b) Search space on direction 2 : minimization of CO<sub>2</sub>; (c) Overall search spaces

---

**Algorithm 12 Two-directional Local Search**


---

**Input**  $P(x)$  : the offspring chromosome to be operated;  $V_m(x)$  : the value of the  $m$ -th objective of the solution  $x$ ;  
 $Gen$  : current generation

**Output** Updated non-dominance levels;

```

1: for  $m \leftarrow 1$  to 2 do // Local search on two directions : the cost and the CO2
2:   for each local search operator do
3:     Local search on  $P(x)$  to generate feasible offspring  $x'$ 
4:     if  $V_m(x') < V_m(x)$  then
5:        $Gen \leftarrow Gen \cup x'$ 
6:       Non-dominance level update (Algorithm 10)
7:       Delete one inferior solution  $x''$  from  $Gen$ 
8:     end if
9:   end for
10: end for
    return Updated non-dominance levels

```

---

The local search operators on the single-objective HLRP proposed by in Chapter 4 are also applicable to the bi-objective HLRP because the searches on each direction only consider one objective. The local search operators are utilized on the routing part and hub location part of the chromosome : swap and insertion operations on the routing part ; hub replacement, hub closing, hub opening and hub swapping operators on the hub location part.

Figure 6.5 shows how the LS are implemented to find new non-dominated solutions. For each objective (direction) of one solution in NDL 1 (Figure 6.5.(a)), the LS is applied and two new offsprings are generated (Figure 6.5.(b)). A new set of non-dominated solutions is formed by adding the new offsprings into the current generation through the ENLU procedure (Figure 6.5.(c)).

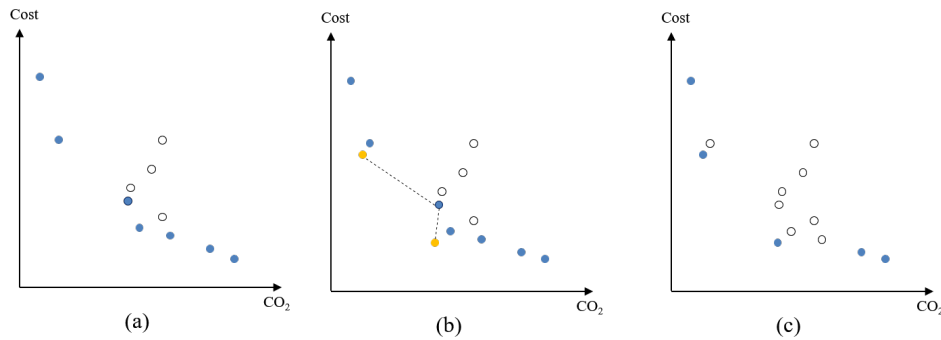


FIGURE 6.5 : Local search on the non-dominated solutions of NDL 1

## 6.3 Conclusion

In this chapter, the environmental aspect is taken into account to the HLRP. Many emission formulations are explored and implemented to generate the green HLRP model for minimizing cost and CO<sub>2</sub> emissions. Furthermore, a bi-objective memetic algorithm is developed to solve the CSAHLRP with respect to minimization of cost and CO<sub>2</sub> emissions. A fast elitist Non-dominated Sorting Genetic Algorithm (NSGAI) is integrated into the process of the MA to sort the initial population in different non-dominance level. After the GA operators, a two-dimensional Local Search is implemented to generate better offsprings. Each time a offspring is generated, an Efficient Non-dominance Level Update (ENLU) method is employed to update the non-dominance levels. The approach builds approximated Pareto Front to exhibit the trade-off relationship between cost and CO<sub>2</sub> emissions. The computational results of the bi-objective MA are presented and investigated in the next chapter.



# Computational experiments for the bi-objective HLRP

In this chapter, the experiment results of the proposed memetic algorithm for the bi-objective HLRP are analyzed, which are compared with the results of the single-objective MA and Epsilon Constraint (EC) method. In Section 7.1, the data set and the global parameters are introduced. The EC method and the results are shown in Section 7.2. Sections 7.3 and 7.4 introduce the parameter settings of the proposed MA and investigate the experiment results. The proposed MA is coded in Visual studio C++ 2012. Both of the MA and the EC method are experimented on PCs with 3.07 GHz and 8 GB RAM memory.

## 7.1 Data and parameters

The data set used to test the bi-objective HLRP takes the same data set were generated to solve the single-objective HLRP. The data set contains 27 instances with up to 100 non-hub nodes and 10 hub nodes. Each location the of supplier/client node point is randomly selected from AP data set and distances between those nodes are extracted (Ernst and Krishnamoorthy [1999]). The flows (in tons) for each supplier-client pair have been generated uniformly within the interval  $[0.15, 1.0]$  in order to be consistent with the capacity of the vehicles (15 tons) and the possibility of loading the freight of several suppliers (resp. clients) within the same collection (resp. delivery) routes. The names of the instances are denoted as  $|H|-|I|-|J|-|\Gamma|$ .  $H$  stands for the candidate hub number set,  $|H| \in \{3, 6, 10\}$ .  $I$  and  $J$  are sets of supplier and client nodes :  $|I|=|J| \in \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$ .  $\Gamma$  refers to the set of hub types with integer capacities corresponding to  $1/3$ ,  $1/2$  or  $1$  of the total demand, which depends of the instance size in the AP data set. Instances with 10 or 15 supplier and client nodes are referred to as “small instances”; those with 20 to 30 supplier and client nodes as “medium instances”, and those with 35 to 50 supplier and client nodes as “large instances”.

Tables 7.1 and 7.2 show the values of parameters for hubs and vehicles. The parameters concerning vehicles (fixed costs and unit transportation cost) are based on logistics data from the French Comité National Routier CNR<sup>1</sup> data base. The unit cost of handling unit flow in hubs was communicated by a French logistic company. The parameter values of CO<sub>2</sub> emissions can be found in Table 7.3.

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<sup>1</sup><http://www.cnr.fr/en>

TABLE 7.1 : Parameter values for hubs

Name	Value	Name	Value
Fixed cost $F_k$ (€)	1000	Handling cost $c_k$ (€/t)	1.78

TABLE 7.2 : Cost parameter values for vehicles

Name	Value	Name	Value
Load capacity $Q$ (ton)	15	Fixed cost for tour $C$ (€)	100
Unit transfer cost $\alpha$ (€ /km.t)	0.057	Unit collection cost $\beta$ (€ /km)	0.8
Unit delivery cost $\gamma$ (€ /km)	0.8		

TABLE 7.3 : Emission parameter values for vehicles (per unit flow)

Emission parameter		Hot emission factor (kg/km)	Cold express emission (kg/cold start)
$w_1$	$w_2$	$E_{hot}$	$E_{start}$
0.510100	0.022220	0.026667	0.033333

## 7.2 Epsilon constraint method

Based on the bi-objective HLRP model stated above, we generated a number of non-dominated solutions by means of the Augmented Epsilon Constraint (AUGMECON) method proposed by Mavrotas [2009]. In order to generate points on the Pareto Front, first we solve the single-objective problems individually with just one objective function and all the constraints of the model, which will generate the minimum and maximum range of two objective values. We define  $f_1(x)$  standing for the objective function of cost (Equation (6.3), Chapter 6) and  $f_2(x)$  the objective function of CO<sub>2</sub> emissions (Equation (6.4), Chapter 6). Let  $X^c$  denote the best solution of the cost model and  $X^e$  the best solution of the CO<sub>2</sub> model.  $z_1(X^c)$  and  $z_2(X^c)$  represent the cost and CO<sub>2</sub> values of solution  $X^c$ . Accordingly,  $z_1(X^e)$  and  $z_2(X^e)$  represent the cost and CO<sub>2</sub> values of solution  $X^e$ . A payoff table will be constructed as

TABLE 7.4 : Payoff table of epsilon constraint method

	Cost value	CO <sub>2</sub> value
$f_1(\mathbf{x})$ (HLRP-COST)	$z_1(X^c)$	$z_2(X^c)$
$f_2(\mathbf{x})$ (HLRP-CO <sub>2</sub> )	$z_1(X^e)$	$z_2(X^e)$

The ranges of the cost and CO<sub>2</sub> values are  $z_1(X^c) \leq f_1(\mathbf{x}) \leq z_1(X^e)$ ,  $z_2(X^e) \leq f_2(\mathbf{x}) \leq z_2(X^c)$ , respectively. We have observed that the single-objective model for minimizing CO<sub>2</sub> is solved much faster in obtaining optimal or good feasible solutions in many experiments. Thus the cost values were posed as constraints and the CO<sub>2</sub> emissions were minimized with the AUGMECON method :

$$\begin{aligned}
 & \min \quad f_2(\mathbf{x}) - eps * s / (z_1(X^e) - z_1(X^c)) \\
 & \mathbf{st} \\
 & \mathbf{x} \in S \\
 & f_1(\mathbf{x}) + s = \epsilon \\
 & \epsilon = z_1(X^e) - h/g * (z_1(X^e) - (z_1(X^c)))
 \end{aligned} \tag{7.1}$$

In the AUGMECON model (7.1),  $eps$  is an adequately small number (usually between  $10^{-3}$  and  $10^{-6}$ ),  $s$  is the slack or surplus variable for the  $\epsilon$  constraint,  $S$  is the feasible region, and  $g$  is the number of the grid points,  $h = 0, 1, \dots, g$ . Here we set  $eps$  to  $10^{-5}$  and  $g$  to 10 which means that 11  $\epsilon$  values will be tested for each problem instance. The objective is to minimize the total CO<sub>2</sub> emissions under a sequence of cost constraints.

The model is coded in Visual studio C++ 2012 and solved with CPLEX 12.6.1 with a maximum CPU time of 3 hours and a 1500MB's limitation of the tree memory for all the problem instances. Due to the large computing time and the difficulty of solving the HLRP, we only conducted the experiments on some small and medium sized instances. The results are shown and compared with the results of the proposed MA in Table 7.13 in the next section. Here, as an example, Table 7.5 shows the payoff table of Instance 10-10-10-10, the results of which are from the single-objective HLRP model. Table 7.6 presents the solutions found by the AUGMECON model. In the second column of Table 7.6, the range of cost values are divided into 10 intervals (11 grid points) according to the payoff table. Under each cost constraint, the model seeks to minimize the objective function of the model (7.1). We obtained 10 solutions : 8 non-dominated and 6 optimal. Since the 4th and 6th solutions are dominated by the solution 3 and 7, respectively, they are not considered in the following discussion. Notice that CPLEX stopped less than 3 hours in some cases because of the limitations of the tree memory.

TABLE 7.5 : The payoff table of Instance 10-10-10-10

	Cost value	CO <sub>2</sub> value
$f_1(\mathbf{x})$ (HLRP-COST)	5792.80	1544.49
$f_2(\mathbf{x})$ (HLRP-CO <sub>2</sub> )	12720.00	735.68

TABLE 7.6 : The results of the AUGMECON model of Instance 10-10-10-10

Number of solutions	$\epsilon$ value (cost constraint)	UB(CO <sub>2</sub> )	LB	Gap (%)	CPU time(s)	Cost value
1	12720.00	735.684	735.68	0.00	8.92	12620.00
2	12027.30	785.03	785.03	0.00	355.19	11707.30
3	11334.60	840.67	840.67	0.00	3130.98	10768.70
4	10641.80	840.67	840.67	0.00	3443.97	10568.70
5	9949.13	896.87	896.87	0.00	7528.55	9754.69
6	9256.41	983.01	894.40	9.01	6973.86	8687.81
7	8563.69	952.51	952.51	0.00	10695.80	8526.67
8	7870.97	1028.98	982.42	4.52	10800.30	7625.44
9	7178.25	1273.94	992.51	22.09	10800.20	6976.49
10	6485.53	1609.60	1017.06	36.81	5830.20	6089.47
11	5792.80	-	1036.01	-	10800.00	-
Total time (s)					70367.95	

### 7.3 Parameter settings for the bi-objective MA

The tuning of several parameters of the MA lies in the probability of crossover and mutation, the size of population and the iteration of the MA (the stopping criterion). The first parameter is the crossover and mutation probability. Since we found that the same values used in the single-objective MA are also efficient in the bi-objective problem, we keep the probability of crossover and mutation as 0.8 and 0.7. The second parameter is the population size. Various population sizes of  $\{50, 100, 200\}$  were tested. The results of population size of 50 and 100 converge quickly with no improvement. Thus the population size of 200 is selected in consideration of the performance of the generation.

The last parameter is the iteration times of the MA. In order to stop the total algorithm, a number of iterations of the MA was predefined as the stopping criterion. We selected three instances with small, medium and large sizes and ran the MA for 1000 iterations. The non-dominated solutions found are recorded during each 100 iterations. In Table 7.7, the column of Iterations indicates the number of iterations (generations) of the MA. The following two columns display the total number of the non-dominated solutions of current iteration and the newly generated non-dominated solutions compared to the ones of the last 100 iterations. The last column records the computing time in seconds. It shows that more solutions can be generated by increasing the iterations of the MA and when the iteration reaches to 300 and 400, the most new non-dominated solutions were yielded. However, it is more time-consuming with more iterations. Figure 7.1 revealed the evolutions of the computing time of Instance 10-50-50-75, indicating that after 400 iterations, the computing time increased dramatically. The non-dominated individuals generated are also investigated and shown in Figures 7.2 to 7.4, which present the non-dominated solutions found in the generation of initial, 100, 200, 300, 400 and 1000 for Instances 6-10-10-15, 10-25-25-45 and 10-50-50-75. Solutions obtained after 400 iterations just improve slightly the approximate Pareto Front. Thus it can be concluded that although more iterations of the MA generate more non-dominated solutions, the approximate Pareto Front is not improved after 400 iterations, at the same time, more iterations costs more computing time. To keep a balance between the computing time and the quality of the approximate Pareto Front, we stop the algorithm after 400 iterations for all the instances. Moreover, for each problem instance, we obtain the final Pareto front as a result of five independent runs of the MA. More runs of the MA does not improve the final front.

Alternatively, the MA fully executes the mutation step of the genetic part of the algorithm with a probability after the crossover process, but the LS is called only after every 10 iterations of the genetic part. After the mutation, the solutions in NDL 1 are arranged in descending order based on the crowding distances. The top 5 of the solutions are applied by the local search operators (if the number of the solutions in NDL 1 is less than 5, all these solutions are applied by the LS).

TABLE 7.7 : The number of non-dominated solutions after each 100 iterations of the MA

Iterations (generation)	Instance 6-10-10-15			Instance 10-25-25-45			Instance 10-50-50-75		
	Total solutions	New solutions	Time (s)	Total solutions	New solutions	Time (s)	Total solutions	New solutions	Time (s)
Initial	7	7	0.05	14	14	0.31	9	9	0.83
100	15	15	6.86	39	39	124.10	26	26	345.48
200	15	4	14.55	42	41	335.69	40	40	1029.40
300	12	9	22.10	72	46	585.97	69	57	2390.12
400	13	8	29.63	88	30	851.97	75	46	4046.30
500	14	3	39.66	95	17	1127.98	79	38	5891.55
600	15	2	62.05	97	8	1405.44	87	28	7934.00
700	15	1	84.24	99	3	1700.66	94	26	9967.79
800	15	0	106.88	99	0	1990.33	98	12	12108.30
900	15	0	129.28	101	2	2277.59	99	9	14273.90
1000	15	0	151.62	101	0	2569.22	103	13	16346.90

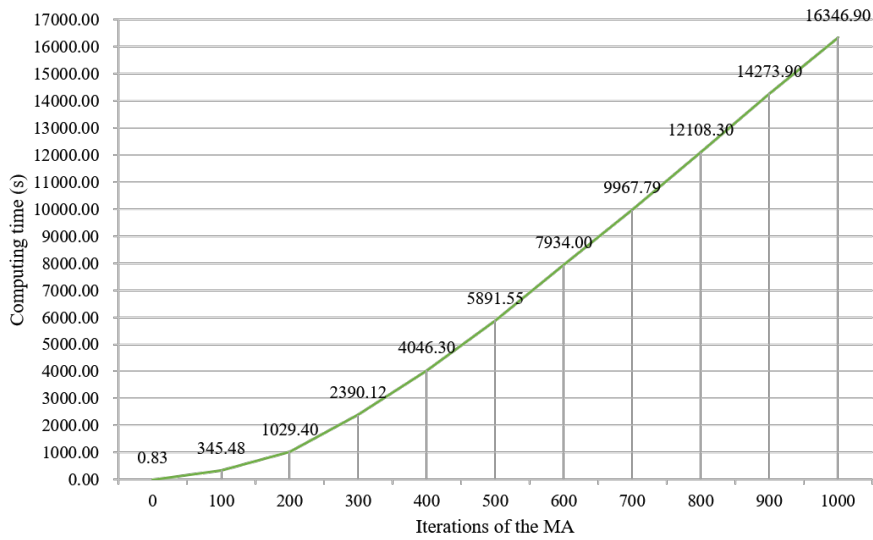


FIGURE 7.1 : Computing time evolution of the bi-objective MA (Instance 10-50-50-75)

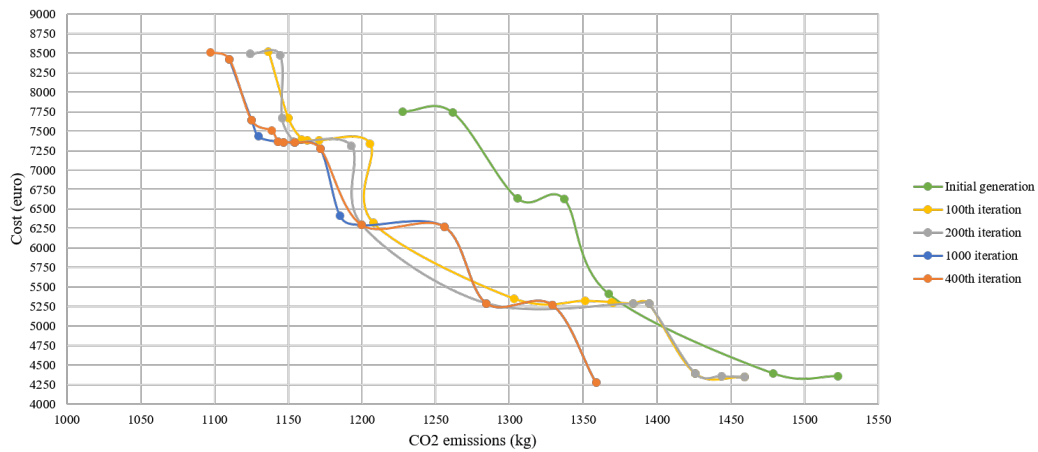


FIGURE 7.2 : Non-dominated solutions with different MA iterations for small instance (6-10-10-15)

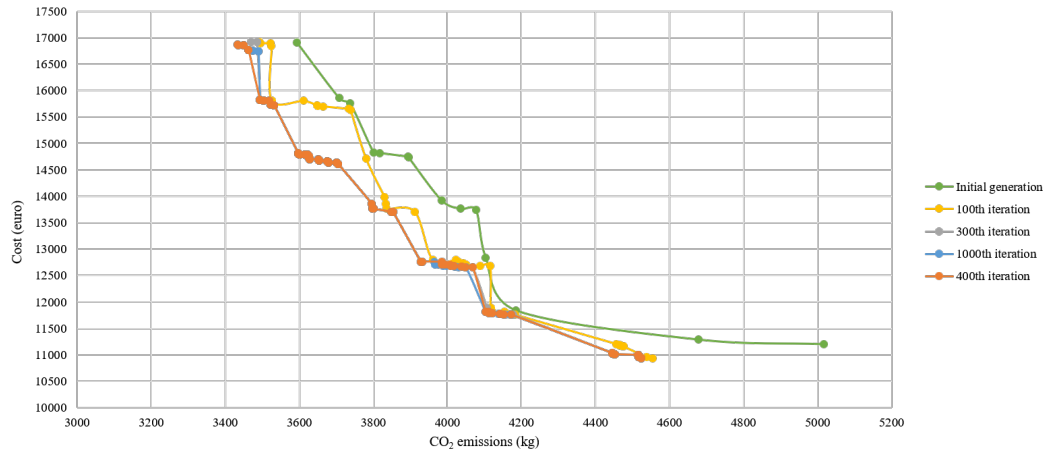


FIGURE 7.3 : Non-dominated solutions with different MA iterations for medium instance (10-25-25-45)



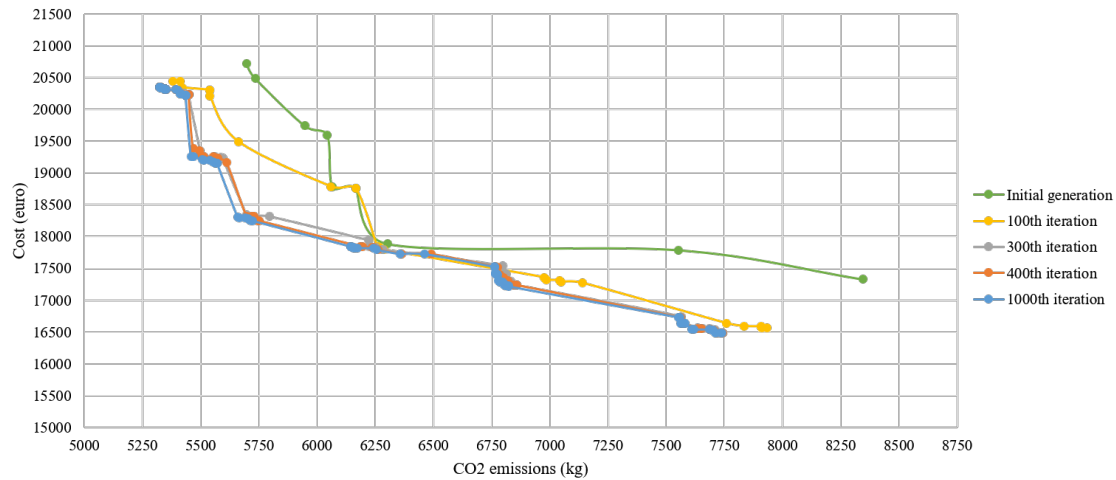


FIGURE 7.4 : Non-dominated solutions with different MA iterations for large instance (10-50-50-75)

### 7.4 Results analysis

Figure 7.5 shows the initial and the 50th generation of the proposed MA for Instance 10-50-50-75, respectively. It can be observed that the solutions in the initial generation are scattered in the solution space. After only 50 generations, the population members move closer to each other and move towards the points with smaller objective values. Figure 7.6 presents the improvement of the generation from 100 to 400 iterations of the MA. The solutions are continuously improved until the MA reaches the 400th generation.

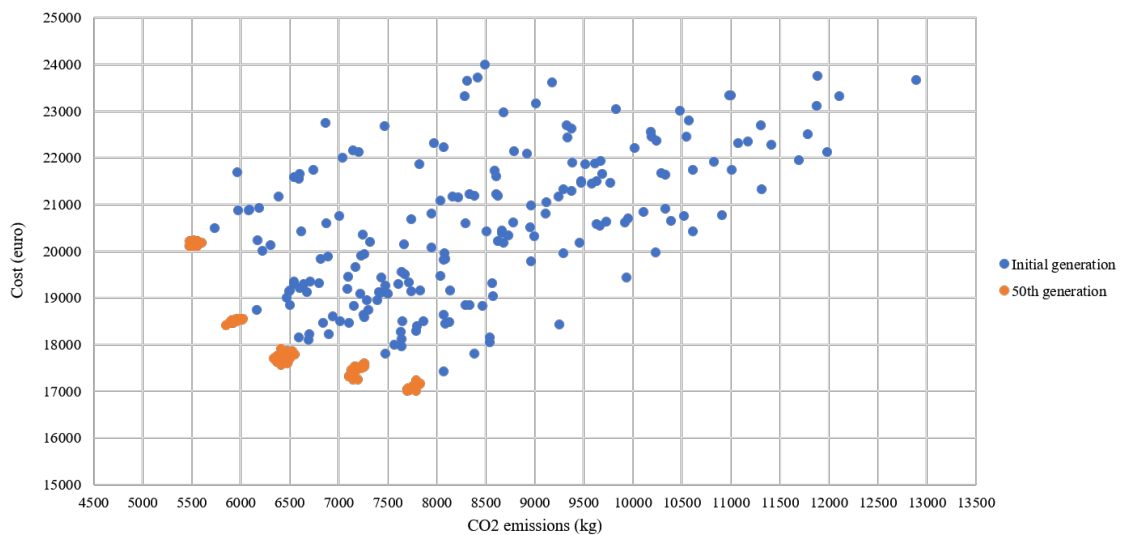


FIGURE 7.5 : Illustration of the convergence of the MA after 50 iterations (Instance 10-50-50-75)

Figures 7.7 and 7.8 explain how the approximate Pareto Front is obtained. For each run of the MA, the non-dominated solutions are generated in Figure 7.7. These solutions are then sorted by the NSGAI, getting rid of the dominated individuals. Finally, the non-dominated solutions are retained in Figure 7.8. It is obvious that non-dominated solutions of Instance 10-50-50-75 are not evenly spread across the approximate Pareto front due to the limited number of candidate hubs and suppliers/clients. More specifically, Figure 7.8 presents a typical trade-off front of hub locations with the black boxes showing which hubs are opened. Different number of open hubs yield the solutions with big differences (the values of cost and CO<sub>2</sub>

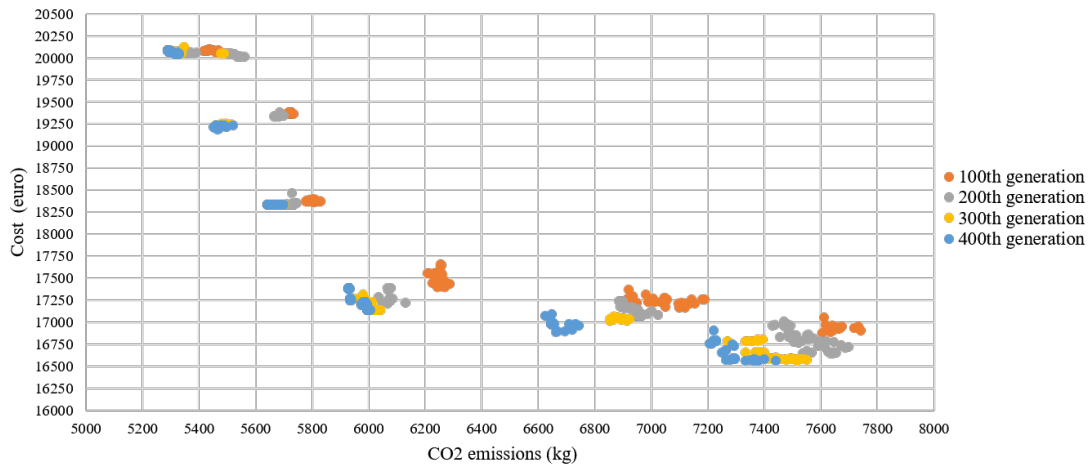


FIGURE 7.6 : Generation evolution from 100 to 400 iterations of the MA (Instance 10-50-50-75)

emissions). Opening three hubs (hubs 2, 5 and 10) generate the solutions with the smallest cost and large CO<sub>2</sub> emissions ; on the contrary, when all the 10 hubs are opened, the value of CO<sub>2</sub> emissions is small while the cost is large. We define the solution with the smallest cost or CO<sub>2</sub> emissions an “extreme solution” of an approximate Pareto front. The similar characters of the approximate Pareto front can be seen for other instances.

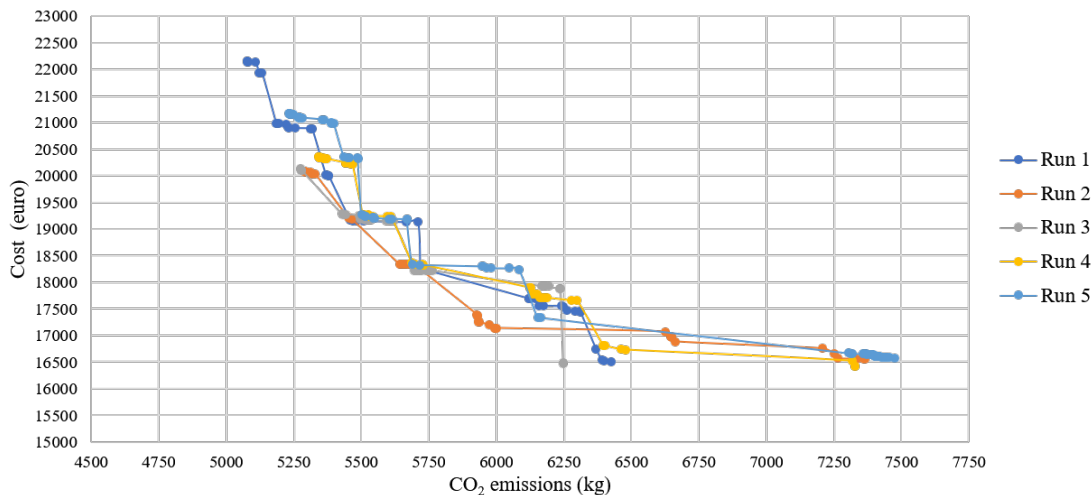


FIGURE 7.7 : Non-dominated solutions for each run of the MA (Instance 10-50-50-75)

To ensure good quality solutions be found, the “extreme solutions” of the proposed MA are compared with the results of the single-objective HLRP solved by CPLEX and the MA (5). Tables 7.8 and Table 7.9 record the number of “extreme solutions” dominating the solutions of the single-objective methods. The columns of “Total” are the total number of the small, medium and large instances tested. The columns of “Dominating” present the number of the “extreme solutions”, both the cost and CO<sub>2</sub> values of which are smaller. When compared to the single-objective model of minimizing cost, the bi-objective MA is able to obtain 34 higher quality “extreme solutions” out of 45 small and medium instances. On the other hand, for the single-objective model of minimizing CO<sub>2</sub> emissions, CPLEX is slightly better than the bi-objective MA (only 15 solutions out of 45 are improved) since most of the small and medium instances are solved to optimality by CPLEX. Regarding the single-objective MA, the bi-objective MA tends to find the “extreme solutions” which are slightly better than the solutions of the single-objective MA. More than half of the single-objective solutions are improved. It seems that the two-dimensional local search is able to explore

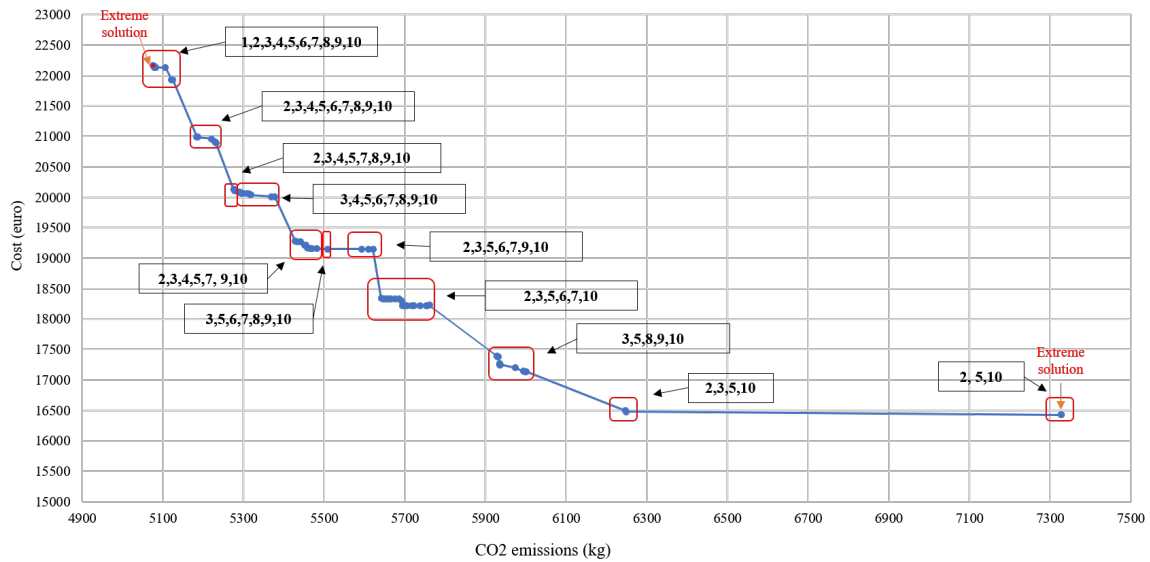


FIGURE 7.8 : The hierarchical nature of the approximate Pareto Front (Instance 10-50-50-75)

more neighbors that the single directional search can not find. Therefore, in most cases, the range of the approximate Pareto Front obtained by the proposed MA covers or even improves the best solutions of single-objective methods. More details of the results are presented in Tables 7.14 to 7.21. The solution in bold indicate that it is better than the solution obtained by the other methods in terms of both cost and CO<sub>2</sub> emissions.

Tables 7.10 to 7.12 show the results by the proposed bi-objective MA : the number of the non-dominated solutions, the “extreme solutions” and the total computing time for 5 runs of the MA. More non-dominated solutions can be found as the problem scale increases. Instances with more potential hubs enlarge the range of the Pareto Front and moves the Front towards left and downward (Figure 7.9), improving the Pareto Front. Solutions with small CO<sub>2</sub> tends to open all the potential hubs, causing a large cost of opening hubs ; solutions with small cost consider the balance of the fixed hub cost and the transportation cost to select hubs to open which increase the CO<sub>2</sub>. In addition, with the same hub location scheme, the cost and CO<sub>2</sub> emissions also show the trade-offs in a limited range.

TABLE 7.8 : The number of “extreme solutions” dominating the solutions of the single-objective model

Comparison with the single-objective model of minimizing cost						
No. of potential hubs	No. of small instances		No. of medium instances		Total number	
	Total	Dominating	Total	Dominating	Total	Dominating
3	6	3	9	4	15	7
6	6	5	9	7	15	12
10	6	6	9	9	15	15
<b>Total number</b>	<b>18</b>	<b>14</b>	<b>27</b>	<b>20</b>	<b>45</b>	<b>34</b>
Comparison with the single-objective model of minimizing CO <sub>2</sub>						
No. of potential hubs	No. of small instances		No. of medium instances		Total number	
	Total	Dominating	Total	Dominating	Total	Dominating
3	6	2	9	2	15	4
6	6	3	9	4	15	7
10	6	0	9	4	15	4
<b>Total number</b>	<b>18</b>	<b>5</b>	<b>27</b>	<b>10</b>	<b>45</b>	<b>15</b>

TABLE 7.9 : The number of “extreme solutions” dominating the solutions of the single-objective MA

Comparison with the single-objective MA of minimizing cost								
No. of potential hubs	No. of small instances		No. of medium instances		No. of large instances		Total number	
	Total	Dominating	Total	Dominating	Total	Dominating	Total	Dominating
3	6	4	9	5	12	10	27	19
6	6	4	9	6	12	6	27	16
10	6	4	9	5	12	9	27	18
<b>Total number</b>	<b>18</b>	<b>12</b>	<b>27</b>	<b>16</b>	<b>36</b>	<b>25</b>	<b>81</b>	<b>53</b>

Comparison with the single-objective MA of minimizing CO <sub>2</sub>								
No. of potential hubs	No. of small instances		No. of medium instances		No. of large instances		Total number	
	Total	Dominating	Total	Dominating	Total	Dominating	Total	Dominating
3	6	5	9	5	12	11	27	21
6	6	3	9	4	12	5	27	12
10	6	4	9	3	12	2	27	9
<b>Total number</b>	<b>18</b>	<b>12</b>	<b>27</b>	<b>12</b>	<b>36</b>	<b>18</b>	<b>81</b>	<b>42</b>

TABLE 7.10 : Results of the small instances by the bi-objective MA

Instance $ H - I - J $	Hub capacity	Number of solutions	Extreme solutions						$T_{total}$ (s) for 5 runs
			Minimum cost solution			Minimum CO <sub>2</sub> solution			
			Cost	CO <sub>2</sub> (kg)	Open hub	Cost	CO <sub>2</sub> (kg)	Open hub	
3-10-10	10	3	5831.25	1505.73	3 (1,2,3)	5844.76	1498.74	3 (1,2,3)	128.60
	15	3	4269.15	1356.04	2 (1,2)	5458.71	1274.68	3 (1,2,3)	100.00
	30	12	3292.09	1447.46	1 (2)	5421.24	1262.78	3 (1,2,3)	469.05
6-10-10	10	5	5674.96	1398.79	3 (3,4,6)	8714.84	1103.36	6 (1,2,3,4,5,6)	361.46
	15	14	4269.15	1352.20	2 (1,2)	7560.33	1082.03	5 (2,3,4,5,6)	488.27
	30	23	3272.23	1415.67	1 (4)	8704.95	1098.55	6 (1,2,3,4,5,6)	647.18
10-10-10	10	26	5653.86	1368.19	3 (2,4,8)	12636.4	813.627	10 (1,2,3,4,5,6,7,8,9,10)	743.98
	15	21	4270.88	1415.75	2 (2,10)	12544.50	752.61	10 (1,2,3,4,5,6,7,8,9,10)	901.93
	30	37	3245.62	1432.28	1 (10)	12520.5	751.689	10 (1,2,3,4,5,6,7,8,9,10)	1085.86
3-15-15	30	11	8903.94	3490.18	3 (1,2,3)	9139.52	3417.68	3 (1,2,3)	234.71
	45	6	7638.35	3175.21	2 (1,3)	8578.73	3087.26	3 (1,2,3)	500.24
	90	14	6484.38	3384.42	1 (3)	8364.55	2939.81	3 (1,2,3)	753.31
6-15-15	30	5	8103.17	2674.95	3 (1,4,5)	10891.20	2362.88	6 (1,2,3,4,5,6)	226.26
	45	9	7107.65	2871.19	2 (3,5)	10891.20	2362.88	6 (1,2,3,4,5,6)	396.99
	90	15	6195.44	3127.02	1 (5)	10891.20	2362.88	6 (1,2,3,4,5,6)	653.52
10-15-15	30	21	7748.22	2496.67	3 (5,9,10)	14990.6	2104.68	10 (1,2,3,4,5,6,7,8,9,10)	1200.26
	45	17	6977.86	2727.92	2 (7,10)	15153.9	2142.01	10 (1,2,3,4,5,6,7,8,9,10)	1314.55
	90	28	6179.78	3143.7	1 (5)	13843.9	2095.77	9 (1,2,4,5,6,7,8,9,10)	1660.11

TABLE 7.11 : Results of the medium instances by the bi-objective MA

Instance $ H - I - J $	Hub capacity	Number of solutions	Extreme solutions						$T_{total}$ (s) for 5 runs
			Minimum cost solution			Minimum CO <sub>2</sub> solution			
			Cost	CO <sub>2</sub> (kg)	Open hub	Cost	CO <sub>2</sub> (kg)	Open hub	
3-20-20	45	2	9995.20	3785.73	3 (1,2,3)	10066.50	3754.40	3 (1,2,3)	356.51
	60	10	8869.83	3751.21	2 (2,3)	10086.30	3674.49	3 (1,2,3)	994.91
	120	12	8025.93	4270.36	1 (3)	10072.50	3689.35	3 (1,2,3)	1510.81
6-20-20	45	13	9806.10	3608.92	3 (2,4,5)	11892.20	3291.17	5 (1,2,4,5,6)	716.77
	60	10	9022.21	3817.45	2 (3,4)	10891.80	3371.53	4 (2,4,5,6)	850.54
	120	21	8168.26	4220.88	1 (3)	11892.20	3291.17	5 (1,2,4,5,6)	1331.47
10-20-20	45	45	9659.79	3421.93	3 (8,9,10)	15481.50	2786.51	9 (2,3,4,5,6,7,8,9,10)	2117.74
	60	30	8772.36	3859.54	2 (9,10)	15510.90	2743.64	9 (1,2,4,5,6,7,8,9,10)	2477.12
	120	46	7885.86	4103.71	1 (7)	16416.00	2733.03	10 (1,2,3,4,5,6,7,8,9,10)	3267.60
3-25-25	45	6	11670.30	4799.12	3 (1,2,3)	11734.90	4770.93	3 (1,2,3)	111.31
	60	13	10568.00	5009.01	2 (1,3)	11288.80	4653.72	3 (1,2,3)	309.58
	120	23	9918.14	5772.74	1 (3)	11227.70	4570.93	3 (1,2,3)	819.69
6-25-25	45	28	11003.30	4706.25	3 (1,2,5)	13656.30	3834.22	6 (1,2,3,4,5,6)	1323.36
	60	59	10329.80	4995.49	2 (1,5)	13710.60	3854.10	6 (1,2,3,4,5,6)	1686.92
	120	26	9040.91	5045.22	1 (5)	13710.80	3854.80	6 (1,2,3,4,5,6)	3380.92
10-25-25	45	97	10771.20	4415.79	3 (1,8,10)	17647.40	3362.46	10 (1,2,3,4,5,6,7,8,9,10)	4071.30
	60	148	10160.10	4791.64	2 (5,10)	16829.10	3423.05	9 (1,2,3,5,6,7,8,9,10)	5063.18
	120	157	9031.86	5087.97	1 (5)	17661.50	3392.94	10 (1,2,3,4,5,6,7,8,9,10)	6393.38
3-30-30	60	8	14586.00	6362.02	3 (1,2,3)	14611.40	6240.27	3 (1,2,3)	123.57
	90	3	13727.30	5629.61	3 (1,2,3)	13749.40	5618.24	3 (1,2,3)	226.73
	165	7	11528.30	6314.13	1 (2)	13065.30	5372.74	3 (1,2,3)	1839.07
6-30-30	60	38	12878.50	5271.05	3 (4,5,6)	15588.60	4647.83	6 (1,2,3,4,5,6)	1798.98
	90	27	11965.40	5576.79	2 (4,6)	15720.50	4640.85	6 (1,2,3,4,5,6)	2838.03
	165	31	11528.30	6314.13	1 (2)	15729.70	4638.17	6 (1,2,3,4,5,6)	4647.24
10-30-30	60	70	12852.50	5344.25	3 (4,6,8)	18435.00	4088.57	9 (1,2,4,5,6,7,8,9,10)	5604.37
	90	63	11957.70	5522.70	2 (2,6)	19487.80	4064.15	10 (1,2,3,4,5,6,7,8,9,10)	8033.56
	165	66	11391.40	6154.92	1 (8)	18427.60	4069.25	9 (1,2,4,5,6,7,8,9,10)	12915.20

TABLE 7.12 : Results of the large instances by the bi-objective MA

Instance $ H - I - J $	Hub capacity	Number of solutions	Extreme solutions						$T_{total}$ (s) for five runs
			Minimum cost solution			Minimum CO <sub>2</sub> solution			
			Cost	CO <sub>2</sub> (kg)	Open hub	Cost	CO <sub>2</sub> (kg)	Open hub	
3-35-35	60	3	14587.10	6246.41	3 (1,2,3)	14603.90	6241.53	3 (1,2,3)	212.60
	90	33	12623.80	5952.05	2 (1,2)	13765.50	5603.89	3 (1,2,3)	1358.98
	180	30	12168.90	6612.14	1 (1)	13435.40	5608.56	3 (1,2,3)	3641.68
6-35-35	60	56	13392.30	5446.99	3 (1,4,6)	16111.90	4654.08	6 (1,2,3,4,5,6)	3604.13
	90	78	12625.00	5879.15	2 (2,6)	16149.10	4653.46	6 (1,2,3,4,5,6)	4711.90
	180	64	12071.10	6533.25	1 (6)	16149.10	4655.60	6 (1,2,3,4,5,6)	7952.50
10-35-35	60	78	13506.30	5715.40	3 (1,2,6)	18725.50	4111.02	9 (1,2,3,4,5,6,7,8,10)	11788.30
	90	76	12705.90	5732.97	2 (1,4)	19779.10	4051.28	10 (1,2,3,4,5,6,7,8,9,10)	15813.90
	180	65	12051.80	6639.90	1 (6)	18740.70	4111.79	9 (1,2,3,4,5,6,7,8,10)	24082.00
3-40-40	75	13	16066.00	6962.21	3 (1,2,3)	16197.00	6853.72	3 (1,2,3)	325.58
	105	38	15206.70	7435.66	2 (1,2)	15637.50	6553.53	3 (1,2,3)	982.89
	210	97	13417.50	7428.76	1 (2)	15724.40	6670.78	3 (1,2,3)	3048.44
6-40-40	75	67	15328.20	6344.15	3 (3,4,5)	17472.10	5316.80	6 (1,2,3,4,5,6)	4782.40
	105	67	13885.90	6649.84	2 (4,5)	17529.10	5340.61	6 (1,2,3,4,5,6)	6902.36
	210	83	13282.20	7321.81	1 (5)	17663.90	5353.70	6 (1,2,3,4,5,6)	11451.80
10-40-40	75	73	14612.70	6014.34	3 (2,5,10)	20007.70	4612.24	9 (1,3,4,5,6,7,8,9,10)	18576.50
	105	51	13687.30	6431.53	2 (2,10)	19987.80	4588.83	9 (1,3,4,5,6,7,8,9,10)	23834.90
	210	100	13165.70	7405.01	1 (5)	20178.70	4584.12	9 (1,3,4,5,6,7,8,9,10)	37141.50
3-45-45	75	63	14294.10	6336.42	3 (1,2,3)	14519.60	6204.08	3 (1,2,3)	626.66
	105	25	13353.00	6380.85	2 (1,2)	14600.00	6319.74	3 (1,2,3)	2414.79
	195	66	12047.10	6744.57	1 (1)	14083.10	6176.04	3 (1,2,3)	4985.50
6-45-45	75	188	13925.80	5912.93	3 (2,3,5)	16828.20	5481.92	6 (1,2,3,4,5,6)	8130.60
	105	81	12823.70	6076.64	2 (1,5)	15853.30	5513.80	5 (1,2,4,5,6)	5910.32
	195	105	12162.70	6803.82	1 (1)	16856.80	5511.05	6 (1,2,3,4,5,6)	7837.90
10-45-45	75	102	13209.30	5519.39	3 (3,5,9)	20297.90	4711.60	10 (1,2,3,4,5,6,7,8,9,10)	31991.30
	105	113	12428.40	5746.30	2 (8,9)	19317.10	4665.70	9 (2,3,4,5,6,7,8,9,10)	38676.10
	195	150	12111.30	6815.60	1 (1)	20410.00	4669.68	10 (1,2,3,4,5,6,7,8,9,10)	50243.80
3-50-50	75	36	17043.40	7642.11	3 (1,2,3)	17157.50	7588.11	3 (1,2,3)	926.05
	120	38	16099.70	7984.76	2 (1,2)	16542.90	6961.95	3 (1,2,3)	3074.31
	225	87	14777.90	8283.47	1 (2)	16097.20	6691.41	3 (1,2,3)	8429.92
6-50-50	75	36	16556.2	6450.02	4 (1,2,3,5)	18670.4	5953.01	6 (1,2,3,4,5,6)	5293.64
	120	116	15501.00	7796.98	2 (2,5)	18675.6	6068.97	6 (1,2,3,4,5,6)	8532.99
	225	61	14826.80	8292.34	1 (2)	17647.40	6097.73	5 (1,2,3,4,5)	5209.28
10-50-50	75	74	16423.40	7327.94	3 (2,5,10)	22155.30	5076.42	10 (1,2,3,4,5,6,7,8,9,10)	19480.90
	120	70	15272.70	7456.37	2 (8,10)	20960.60	5117.58	9 (2,3,4,5,6,7,8,9,10)	29069.90
	225	71	14291.30	7954.63	1 (8)	21208.50	5136.84	9 (1,2,3,4,5,7,8,9,10)	44201.00

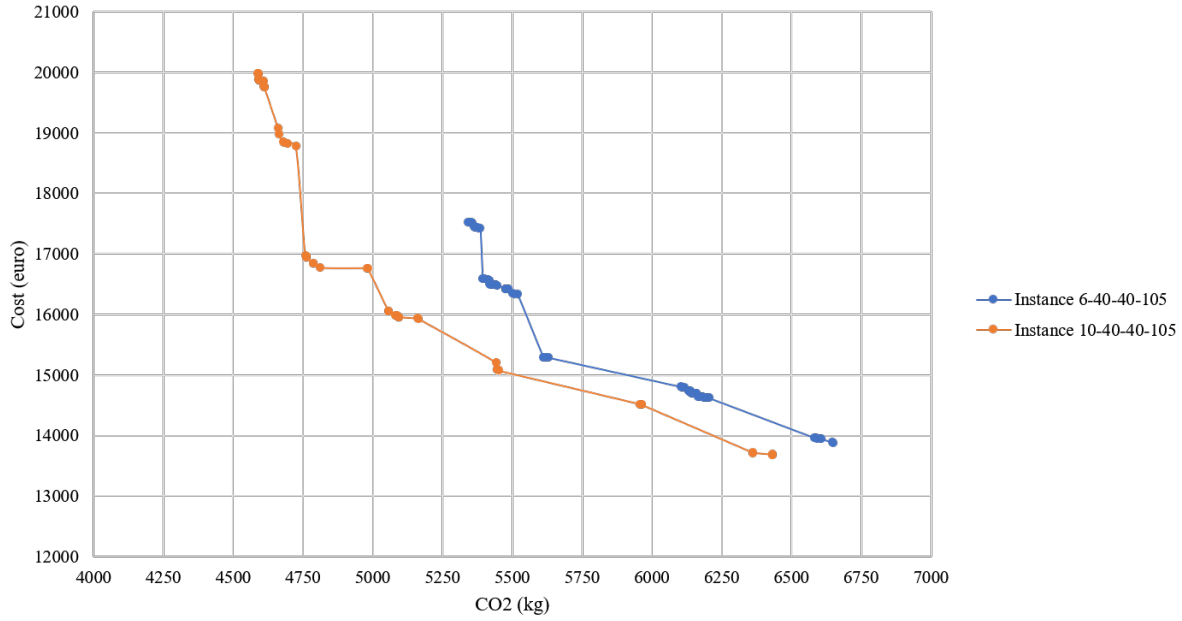


FIGURE 7.9 : The approximate Pareto Front of instances with 80 nodes (medium hub capacity) : the instance with 10 potential hubs obtain superior front than the instance with 6 potential hubs

## 7.5 Performance assessment of the bi-objective MA

Since the outcome of our method is an approximate Pareto front, to evaluate the quality of this approximation Pareto front is important. Various measures have been developed to assess the performance of the multi-objective optimizers. Considering that each measure only provides one aspect of the respective front, we use three measures to evaluate our method in different aspects.

The first evaluating indicator is the unary epsilon indicator  $I_\epsilon(A, B)$  introduced by Zitzler et al. [2003], indicating how far two approximate Pareto Fronts  $A$  and  $B$  are from each other. For a bi-objective minimization problem, a non-dominated solution with objective vector  $z^1 = (z_1^1, z_2^1)$  is called  $\epsilon$ -dominating another non-dominated solution with objective vector  $z^2 = (z_1^2, z_2^2)$  if and only if  $\forall i = 1, 2, z_i^1 < \epsilon \times z_i^2$  for a given  $\epsilon$ . In practice, the binary  $\epsilon$ -indicator  $I_\epsilon(A, B)$  can be calculated as follows :

$$\epsilon_{z^1, z^2} = \max_{1 \leq i \leq 2} z_i^1 / z_i^2 \quad \forall z^1 \in A, \forall z^2 \in B \quad (7.2)$$

$$\epsilon_{z^2} = \min_{z^1 \in A} \epsilon_{z^1, z^2} \quad \forall z^2 \in B \quad (7.3)$$

$$I_\epsilon(A, B) = \max_{z^2 \in B} \epsilon_{z^2} \quad (7.4)$$

The smallest value of the indicator is 1 and the Pareto Front with a smaller indicator value is better.

The second indicator is the Ratio of approximated Pareto Front, introduced by Altıparmak et al. [2006]. It calculates the percentage of solution numbers from a approximated Pareto front  $A$  that are not dominated by all the solutions from another approximated Pareto front  $B$ . To compute the Ratio of approximated Pareto front  $A$  :

$$RP_A = (|A| - |\{X \in A \mid \exists Y \in A \cup B : Y \succeq X\}|) / |A| \quad (7.5)$$

in which  $Y \succeq X$  means that  $X$  is dominated by solution  $Y$ . A big value of  $RP_A$  means more number of

solutions from the approximate Pareto set  $A$  are not dominated by any member of the set  $A \cup B$ .

The last indicator is called the hypervolume indicator (Zitzler et al. [2003])  $H$ , representing the scale of the space covered by a set of non-dominated solutions respect to a reference point (Equation (7.6)). We take the nadir point as the reference point. A set with a larger hypervolume is better.

$$H = (H_A - H_B)/H_B \times 100\% \quad (7.6)$$

In order to assess the approximate Pareto Front generated by the proposed MA, we first generated another set of Pareto approximation by means of the AUGMECON method (Mavrotas [2009]), then a reference set is generated consisting of the set of all non-dominated solutions provided by both AUGMECON method and the MA. The values of the unary epsilon indicator and the ratio indicator are calculated by comparing solutions of the AUGMECON method and the MA to the reference set.

Table 7.13 reports the number of non-dominated solutions found by the bi-objective MA and the AUGMECON method, the three indicator values and also the CPU time for small and medium instances. The last row shows the average value of these items. The results show that the MA is able to generate more non-dominated solutions for small and medium instances with the average CPU time of 1462.58 seconds, whilst AUGMECON method can only find average 6.12 points in a large computing time (the average value is 76234.66 seconds). Regarding the performance assessment indicators, the MA is superior to the AUGMECON method in both the unary and hypervolume indicators mainly due to its capability to find good extreme solutions and more numbers of non-dominated solutions. In terms of the R indicator, the MA gives a slightly better value in average. For small instances, the AUGMECON method performs better in finding optimal solutions, which leads to a big R indicator value. When the problem scales increase, the R values of the AUGMECON method decrease and are inferior to which of the MA.

TABLE 7.13 : Values of unary epsilon, Ratio (R), Hypervolume (Hyper) and CPU Time

Instance  H - I - J - Γ	Number of non-dominated points			Unary $\epsilon$ -indicator		R-indicator		Hyper	CPU Time (s)	
	MA	AUGMECON		MA	AUGMECON	MA	AUGMECON	(%)	MA	AUGMECON
		Total solutions	Optimal solutions							
3-10-10-15	3	3	3	1	1.02	0.67	1	1.01	100.00	1361.11
3-10-10-30	12	5	5	1.01	1.04	0.83	1	1.22	469.05	713.86
6-10-10-10	5	6	3	1.04	1.02	0.6	0.83	-3	361.46	6583.97
6-10-10-15	14	5	5	1.03	1.02	0.71	1	-5.02	488.27	58537.09
6-10-10-30	23	7	7	1.05	1.02	0.61	1	-6.24	647.18	24100.89
10-10-10-10	26	8	5	1.11	1.18	0.38	0.75	6.63	743.98	70367.95
10-10-10-15	21	10	6	1.05	1.11	0.76	0.9	1.2	901.93	54142.97
10-10-10-30	37	10	8	1.05	1.06	0.92	0.9	2.1	1085.86	51255.46
3-15-15-30	11	2	0	1.01	1.01	0.91	1	5.11	234.71	118800.30
3-15-15-45	6	2	2	1.01	1.01	0.33	1	-10.44	500.24	63338.30
3-15-15-90	14	3	3	1.02	1.02	0.43	1	-4.44	753.31	31032.85
6-15-15-30	5	4	3	1	1.03	0.8	0.75	7.8	226.26	28120.36
6-15-15-45	9	6	5	1.01	1.05	1	0.67	6.59	396.99	38831.41
6-15-15-90	15	7	7	1.01	1.01	1	0.43	0.27	653.52	46261.07
10-15-15-30	21	10	5	1.06	1.12	0.14	0.8	2.35	1200.26	80637.01
10-15-15-45	17	10	5	1.08	1.05	0.71	0.8	-3.93	1314.55	83871.48
10-15-15-90	28	9	3	1.06	1.14	0.54	0.78	5.74	1660.11	93277.61
6-20-20-45	13	6	1	1.02	1.12	1	0.33	39.99	716.77	117213.70
6-20-20-60	10	5	0	1.04	1.1	1	0.8	18.08	850.54	118801.00
6-20-20-120	21	6	0	1.02	1.13	1	0.5	9.92	1331.47	118801.80
10-20-20-45	45	6	1	1.03	1.21	0.89	0.83	24.39	2117.74	113155.80
10-20-20-60	30	7	1	1.01	1.22	0.97	0.29	26.01	2477.12	114792.80
10-20-20-120	46	10	1	1.01	1.17	0.91	0.5	19.18	3267.60	111700.00
10-25-25-45	97	5	0	1	1.45	1	0	185.97	4071.30	118801.30
10-25-25-60	148	4	0	1	1.53	1	0	162.68	5063.18	118802.60
10-25-25-120	157	3	0	1	1.76	1	0	201.91	6393.38	118800.50
Average	<b>32.08</b>	6.12		<b>1.02</b>	1.14	<b>0.77</b>	0.69	<b>26.73</b>	<b>1462.58</b>	76234.66



## 7.6 Conclusion

In this chapter, computational experiments of the bi-objective MA were conducted and the results were favourably compared with those obtained by solving our bi-objective MILP model with an epsilon constraint method. Firstly, the parameters of the MA are tuned in terms of the population size and the iteration times. Then the process of generating and improving approximated Pareto Front is investigated.

The computational results showed that our method is efficient in providing high quality solutions for small instances in reasonable time. Epsilon constraint method takes a large computing time and is only able to yield fronts for the small and medium instances. The bi-objective MA produces competitive solutions with the epsilon constraint method in a fast computing time. This method could find approximate Pareto front for large instances while the epsilon constraint method could not obtain solutions in 3 hours. We also investigated the “extreme solutions”, the solutions with the smallest cost or smallest CO<sub>2</sub> emissions, with the best solutions obtained by the single-objective method. Many “extreme solutions” dominate the solutions of the single-objective model and MA. It seems that the bi-objective MA tends to find better “extreme solutions” than the single-objective one but with a longer computing time. The bi-objective MA captures the trade-off between minimizing total cost and CO<sub>2</sub> emissions and exhibits approximations of the Pareto front in a reasonable computing time.

TABLE 7.14 : Results comparison between the single-objective model of minimizing cost and the bi-objective MA (small instances)

Instance $ H - I - J $	Hub capacity	Single-objective model-minimizing cost (CPLEX)					Bi-objective MA-extreme solution-smallest cost			
		UB (Cost)	Gap (%)	CO <sub>2</sub> (kg)	Open hub	CPU Time (s)	Cost	CO <sub>2</sub> (kg)	Open hub	$T_{total}$ (s) for 5 runs
3-10-10	10	<b>5666.52</b>	0.00	<b>1471.70</b>	1,2,3	593.79	5831.25	1505.73	1,2,3	128.60
	15	4269.15	0.00	1502.29	1,2	217.22	<b>4269.15</b>	<b>1356.04</b>	1,2	100.00
	30	3277.36	0.00	1530.22	2	18.80	3292.09	1447.46	2	469.05
6-10-10	10	5666.52	5.07	1444.11	1,2,3	10800.10	5674.96	1398.79	3,4,6	361.46
	15	4269.15	2.07	1378.58	1,2	10800.00	<b>4269.15</b>	<b>1352.20</b>	1,2	488.27
	30	3272.23	0.00	1457.81	4	157.76	<b>3272.23</b>	<b>1415.67</b>	4	647.18
10-10-10	10	5792.80	12.75	1544.49	1,2,8	9192.72	<b>5653.86</b>	<b>1368.19</b>	2,4,8	743.98
	15	4363.80	4.45	1493.60	8,10	10800.20	<b>4270.88</b>	<b>1415.75</b>	2,10	901.93
	30	3245.62	0.00	1491.81	10	591.82	<b>3245.62</b>	<b>1432.28</b>	10	1085.86
3-15-15	30	8986.43	5.78	3678.36	1,2,3	10754.40	<b>8903.94</b>	<b>3490.18</b>	1,2,3	234.71
	45	7584.04	3.15	3350.93	1,3	10536.69	7638.35	3175.21	1,3	500.24
	90	6484.38	3.15	3461.76	3	10800.06	<b>6484.38</b>	<b>3384.42</b>	3	753.31
6-15-15	30	8442.86	9.14	2997.28	1,5,6	10471.90	<b>8103.17</b>	<b>2674.95</b>	1,4,5	226.26
	45	7107.65	3.56	3072.90	3,5	10800.15	<b>7107.65</b>	<b>2871.19</b>	3,5	396.99
	90	6247.91	4.33	3269.20	5	10454.62	<b>6195.44</b>	<b>3127.02</b>	5	653.52
10-15-15	30	8637.04	16.55	3286.11	1,4,5	10800.30	<b>7748.22</b>	<b>2496.67</b>	5,9,10	1200.26
	45	7013.07	9.72	2872.48	7,10	10800.11	<b>6977.86</b>	<b>2727.92</b>	7,10	1314.55
	90	6199.35	3.63	3310.07	5	10800.14	<b>6179.78</b>	<b>3143.70</b>	5	1660.11

TABLE 7.15 : Results comparison between the single-objective model of minimizing cost and the bi-objective MA (medium instances)

Instance $ H - I - J $	Hub capacity	Single-objective model-minimizing cost (CPLEX)					Bi-objective MA-extreme solution-smallest cost			
		UB (Cost)	Gap (%)	CO <sub>2</sub> (kg)	Open hub	CPU Time (s)	Cost	CO <sub>2</sub> (kg)	Open hub	$T_{total}$ (s) for 5 runs
3-20-20	45	9970.62	3.56	3884.89	1,2,3	10800.10	9995.20	3785.73	1,2,3	356.51
	60	8825.23	1.75	3985.89	2,3	10800.09	8869.83	3751.21	2,3	994.91
	120	8046.43	3.69	4388.88	3	10314.67	<b>8025.93</b>	<b>4270.36</b>	3	1510.81
6-20-20	45	10357.90	12.79	4204.20	3,4,5	10800.10	<b>9806.10</b>	<b>3608.92</b>	2,4,5	716.77
	60	9257.22	11.60	4256.21	4,5	10800.23	<b>9022.21</b>	<b>3817.45</b>	3,4	850.54
	120	8046.43	4.50	4420.06	3	10800.53	8168.26	4220.88	3	1331.47
10-20-20	45	9938.18	14.37	3736.80	2,8,9	10800.10	<b>9659.79</b>	<b>3421.93</b>	8,9,10	2117.74
	60	9520.10	19.98	4395.74	5,9	10800.30	<b>8772.36</b>	<b>3859.54</b>	9,10	2477.12
	120	7982.97	12.09	4286.43	10	10800.30	<b>7885.86</b>	<b>4103.71</b>	7	3267.60
3-25-25	45	11502.00	11.85	4892.32	1,2,3	9145.94	11670.30	4799.12	1,2,3	111.31
	60	10602.80	10.51	5277.23	1,2,3	9369.65	<b>10568.00</b>	<b>5009.01</b>	1,3	309.58
	120	9865.98	7.23	5889.18	1	7024.90	<b>9918.14</b>	<b>5772.74</b>	3	819.69
6-25-25	45	12135.20	23.05	5551.65	1,2,3	10800.30	<b>11003.30</b>	<b>4706.25</b>	1,2,5	1323.36
	60	12148.30	26.22	6339.43	2,5	10800.20	<b>10329.80</b>	<b>4995.49</b>	1,5	1686.92
	120	9036.62	7.30	5116.88	5	10800.09	9040.91	5045.22	5	3380.92
10-25-25	45	13209.60	32.86	5670.53	1,2,3	10800.20	<b>10771.20</b>	<b>4415.79</b>	1,8,10	4071.30
	60	13154.30	38.85	6477.13	1,3	10800.40	<b>10160.10</b>	<b>4791.64</b>	5,10	5063.18
	120	10281.30	27.26	5079.90	5,6	10800.19	<b>9031.86</b>	<b>5087.97</b>	5	6393.38
3-30-30	60	15402.70	16.82	6910.90	1,2,3	10800.10	<b>14586.00</b>	<b>6362.02</b>	1,2,3	123.57
	90	12945.40	9.71	6356.45	1,2	10800.20	13727.30	5629.61	1,2,3	226.73
	165	11498.90	4.96	6462.61	2	10800.10	11528.30	6314.13	2	1839.07
6-30-30	60	14296.40	19.98	6345.32	2,4,5	10800.20	<b>12878.50</b>	<b>5271.05</b>	4,5,6	1798.98
	90	13838.80	24.40	6874.12	2,6	10800.90	<b>11965.40</b>	<b>5576.79</b>	4,6	2838.03
	165	12246.40	14.00	6880.91	4	10800.20	<b>11528.30</b>	<b>6314.13</b>	2	4647.24
10-30-30	60	17617.60	36.64	8204.67	1,5,7	10800.70	<b>12852.50</b>	<b>5344.25</b>	4,6,8	5604.37
	90	15289.90	32.63	7333.51	4,5,8	10800.70	<b>11957.70</b>	<b>5522.70</b>	2,6	8033.56
	165	16362.80	39.91	7140.5	1,7,8	10800.90	<b>11391.40</b>	<b>6154.92</b>	8	12915.20

TABLE 7.16 : Results comparison between the single-objective model of minimizing CO<sub>2</sub> and the bi-objective MA (small instances)

Instance $ H - I - J $	Hub capacity	Single-objective model-minimizing CO <sub>2</sub> (CPLEX)					Bi-objective MA-extreme solution-smallest CO <sub>2</sub>			
		UB (CO <sub>2</sub> )	Gap (%)	Open hub	Cost	CPU Time (s)	CO <sub>2</sub> (kg)	Cost	Open hub	$T_{total}$ (s) for 5 runs
3-10-10	10	<b>1413.86</b>	0.00	1,2,3	<b>5807.03</b>	474.19	1498.74	5844.76	1,2,3	128.60
	15	1274.68	0.00	1,2,3	5558.71	123.76	<b>1274.68</b>	<b>5458.71</b>	1,2,3	100.00
	30	<b>1262.78</b>	0.00	1,2,3	5621.24	31.14	<b>1262.78</b>	<b>5421.24</b>	1,2,3	469.05
6-10-10	10	1063.32	0.00	1,2,3,4,5,6	8750.77	478.83	1103.36	8714.84	1,2,3,4,5,6	361.46
	15	1050.58	0.00	1,2,3,4,5,6	8585.96	47.19	1082.03	7560.33	2,3,4,5,6	488.27
	30	<b>1050.58</b>	0.00	1,2,3,4,5,6	<b>8585.96</b>	113.07	1098.55	8704.95	1,2,3,4,5,6	647.18
10-10-10	10	735.68	0.00	1,2,3,4,5,6,7,8,9,10	12720.00	6.75	813.63	12636.40	1,2,3,4,5,6,7,8,9,10	743.98
	15	723.66	0.00	1,2,3,4,5,6,7,8,9,10	12803.02	3.37	752.61	12544.50	1,2,3,4,5,6,7,8,9,10	901.93
	30	723.66	0.00	1,2,3,4,5,6,7,8,9,10	12703.02	4.15	751.69	12520.50	1,2,3,4,5,6,7,8,9,10	1085.86
3-15-15	30	3423.17	3.43	1,2,3	9098.48	10800.11	3417.68	9139.52	1,2,3	234.71
	45	3045.93	0.00	1,2,3	8628.05	1578.82	3087.26	8578.73	1,2,3	500.24
	90	2884.41	0.00	1,2,3	8479.69	241.99	2939.81	8364.55	1,2,3	753.31
6-15-15	30	2362.88	0.00	1,2,3,4,5,6	11191.18	42.42	<b>2362.88</b>	<b>10891.20</b>	1,2,3,4,5,6	226.26
	45	2362.88	0.00	1,2,3,4,5,6	11191.18	45.35	<b>2362.88</b>	<b>10891.20</b>	1,2,3,4,5,6	396.99
	90	2362.88	0.00	1,2,3,4,5,6	11291.18	42.76	<b>2362.88</b>	<b>10891.20</b>	1,2,3,4,5,6	653.52
10-15-15	30	1984.98	0.00	1,2,3,4,5,6,7,8,9,10	15316.61	108.50	2104.68	14990.60	1,2,3,4,5,6,7,8,9,10	1200.26
	45	1979.26	0.00	1,2,3,4,5,6,7,8,9,10	15174.80	176.72	2142.01	15153.90	1,2,3,4,5,6,7,8,9,10	1314.55
	90	1979.26	0.00	1,2,3,4,5,6,7,8,9,10	15274.80	91.10	2095.77	13843.90	1,2,4,5,6,7,8,9,10	1660.11

TABLE 7.17 : Results comparison between the single-objective model of minimizing CO<sub>2</sub> and the bi-objective MA (medium instances)

Instance $ H - I - J $	Hub capacity	Single-objective model-minimizing CO <sub>2</sub> (CPLEX)					Bi-objective MA-extreme solution-smallest CO <sub>2</sub>				
		UB (CO <sub>2</sub> )	Gap (%)	Open hub	Cost	CPU Time (s)	CO <sub>2</sub> (kg)	Cost	Open hub	$T_{total}$ (s) for 5 runs	
3-20-20	45	3711.13	0.00	1,2,3	10111.65	1677.59	3754.40	10066.50	1,2,3	356.51	
	60	<b>3635.14</b>	0.00	1,2,3	<b>9984.84</b>	952.81	3674.49	10086.30	1,2,3	994.91	
	120	3639.57	0.00	1,2,3	10118.83	473.52	3689.35	10072.50	1,2,3	1510.81	
6-20-20	45	3244.70	0.56	1,2,3,4,5,6	13197.59	10800.12	3291.17	11892.20	1,2,4,5,6	716.77	
	60	3247.20	0.44	1,2,3,4,5,6	13013.35	10800.04	3371.53	10891.80	2,4,5,6	850.54	
	120	3241.37	0.57	1,2,3,4,5,6	13180.27	10800.39	3291.17	11892.20	1,2,4,5,6	1331.47	
10-20-20	45	2708.59	0.00	1,2,3,4,5, 6,7,8,9,10	16729.12	7993.32	2786.51	15481.50	2,3,4,5, 6,7,8,9,10	2117.74	
	60	2708.59	0.00	1,2,3,4,5, 6,7,8,9,10	16729.12	6733.28	2743.64	15510.90	1,2,4,5, 6,7,8,9,10	2477.12	
	120	2708.59	0.00	1,2,3,4,5, 6,7,8,9,10	16829.12	3172.20	2733.03	16416.00	1,2,3,4,5, 6,7,8,9,10	3267.60	
3-25-25	45	4548.75	5.53	1,2,3	11764.50	10800.11	4770.93	11734.90	1,2,3	111.31	
	60	4766.25	11.10	1,2,3	12131.95	10800.06	<b>4653.72</b>	<b>11288.80</b>	1,2,3	309.58	
	120	4524.67	6.08	1,2,3	11606.15	10800.03	4570.93	11227.70	1,2,3	819.69	
6-25-25	45	3835.51	9.40	1,2,3,4,5,6	14236.98	10800.07	<b>3834.22</b>	<b>13656.30</b>	1,2,3,4,5,6	1323.36	
	60	3982.59	12.64	1,2,3,4,5,6	14837.02	10800.32	<b>3854.10</b>	<b>13710.60</b>	1,2,3,4,5,6	1686.92	
	120	3939.65	11.34	1,2,3,4,5,6	14527.92	10800.12	<b>3854.80</b>	<b>13710.80</b>	1,2,3,4,5,6	3380.92	
10-25-25	45	3511.84	13.13	1,2,3,4,5, 6,7,8,9,10	18231.49	10800.26	<b>3362.46</b>	<b>17647.40</b>	1,2,3,4,5, 6,7,8,9,10	4071.30	
	60	3438.99	11.28	1,2,3,4,5, 6,7,8,9,10	18422.60	10800.26	<b>3423.05</b>	<b>16829.10</b>	1,2,3,5, 6,7,8,9,10	5063.18	
	120	3386.73	9.60	1,2,3,4,5, 6,7,8,9,10	18187.72	10800.39	3392.94	17661.50	1,2,3,4,5, 6,7,8,9,10	6393.38	
3-30-30	60	6391.98	8.34	1,2,3	15475.18	10800.04	<b>6240.27</b>	<b>14611.40</b>	1,2,3	123.57	
	90	5539.11	4.86	1,2,3	13788.01	10800.03	5618.24	13749.40	1,2,3	226.73	
	165	5344.72	2.02	1,2,3	13369.74	10800.09	5372.74	13065.30	1,2,3	1839.07	
6-30-30	60	4619.08	4.39	1,2,3,4,5,6	15805.87	10800.07	4647.83	15588.60	1,2,3,4,5,6	1798.98	
	90	4654.22	5.03	1,2,3,4,5,6	16081.93	10800.09	<b>4640.85</b>	<b>15720.50</b>	1,2,3,4,5,6	2838.03	
	165	4575.33	3.33	1,2,3,4,5,6	16096.30	10800.40	4638.17	15729.70	1,2,3,4,5,6	4647.24	
10-30-30	60	4150.95	10.13	1,2,3,4,5, 6,7,8,9,10	20183.27	10800.20	<b>4088.57</b>	<b>18435.00</b>	1,2,4,5, 6,7,8,9,10	5604.37	
	90	4278.89	12.80	1,2,3,4,5, 6,7,8,9,10	20239.49	10800.23	<b>4064.15</b>	<b>19487.80</b>	1,2,3,4,5, 6,7,8,9,10	8033.56	
	165	3994.54	6.35	1,2,3,4,5, 6,7,8,9,10	20065.92	10800.15	4069.25	18427.60	1,2,4,5, 6,7,8,9,10	12915.20	

TABLE 7.18 : Results comparison between the single-objective MA of minimizing cost and the bi-objective MA (small and medium instances)

Instance $ H - I - J $	Hub capacity	Single-objective MA-minimum cost				Bi-objective MA-smallest cost			
		$Z_{best}$ (cost)	CO <sub>2</sub> (kg)	Open hub	$T_{total}$ (s)	$Z_{best}$ (cost)	CO <sub>2</sub> (kg)	Open hub	$T_{total}$ (s)
3-10-10	10	5746.53	1517.57	1,2,3	16.52	5831.25	1505.73	1,2,3	128.60
	15	4269.15	1415.65	1,2	12.60	<b>4269.15</b>	<b>1356.04</b>	1,2	100.00
	30	3277.36	1492.34	2	14.36	3292.09	1447.46	2	469.05
6-10-10	10	5659.78	1482.07	3,4,6	58.57	5674.96	1398.79	3,4,6	361.46
	15	4269.15	1411.81	1,2	59.04	<b>4269.15</b>	<b>1352.20</b>	1,2	488.27
	30	3272.23	1488.00	4	73.36	<b>3272.23</b>	<b>1415.67</b>	4	647.18
10-10-10	10	5659.78	1424.80	3,4,6	91.94	<b>5653.86</b>	<b>1368.19</b>	2,4,8	743.98
	15	4258.45	1461.08	2,10	82.25	4270.88	1415.75	2,10	901.93
	30	3245.62	1505.29	10	87.76	<b>3245.62</b>	<b>1432.28</b>	10	1085.86
3-15-15	30	9015.79	3719.58	1,2,3	22.89	<b>8903.94</b>	<b>3490.18</b>	1,2,3	234.71
	45	7638.35	3408.77	1,3	34.19	<b>7638.35</b>	<b>3175.21</b>	1,3	500.24
	90	6539.41	3627.33	3	60.63	<b>6484.38</b>	<b>3384.42</b>	3	753.31
6-15-15	30	8127.86	2955.39	1,3,5	32.68	<b>8103.17</b>	<b>2674.95</b>	1,4,5	226.26
	45	7107.65	3046.81	3,5	40.88	<b>7107.65</b>	<b>2871.19</b>	3,5	396.99
	90	6179.78	3345.48	5	89.62	6195.44	3127.02	5	653.52
10-15-15	30	7863.46	2912.96	5,7,10	114.30	<b>7748.22</b>	<b>2496.67</b>	5,9,10	1200.26
	45	6861.88	2768.84	7,10	131.45	6977.86	2727.92	7,10	1314.55
	90	6199.35	3344.24	5	145.86	<b>6179.78</b>	<b>3143.70</b>	5	1660.11
3-20-20	45	10024.00	3963.97	1,2,3	65.34	<b>9995.20</b>	<b>3785.73</b>	1,2,3	356.51
	60	9048.58	3889.14	2,3	82.64	<b>8869.83</b>	<b>3751.21</b>	2,3	994.91
	120	8057.46	4301.58	3	83.86	<b>8025.93</b>	<b>4270.36</b>	3	1510.81
6-20-20	45	9806.10	3669.61	2,4,5	113.05	<b>9806.10</b>	<b>3608.92</b>	2,4,5	716.77
	60	9022.21	3861.62	3,4	116.44	<b>9022.21</b>	<b>3817.45</b>	3,4	850.54
	120	8041.56	4337.30	3	128.07	8168.26	4220.88	3	1331.47
10-20-20	45	9632.53	3527.65	8,9,10	280.64	9659.79	3421.93	8,9,10	2117.74
	60	<b>8725.87</b>	<b>3699.73</b>	7,8	244.04	8772.36	3859.54	9,10	2477.12
	120	7885.86	4180.06	7	257.96	<b>7885.86</b>	<b>4103.71</b>	7	3267.60
3-25-25	45	11676.80	4926.71	1,2,3	83.38	<b>11670.30</b>	<b>4799.12</b>	1,2,3	111.31
	60	10557.60	5282.91	1,3	97.25	10568.00	5009.01	1,3	309.58
	120	9834.14	5817.00	1	98.24	9918.14	5772.74	3	819.69
6-25-25	45	11125.50	4953.9	1,2,5	154.74	<b>11003.30</b>	<b>4706.25</b>	1,2,5	1323.36
	60	10252.00	5137.41	1,5	205.04	10329.80	4995.49	1,5	1686.92
	120	9062.72	5192.18	5	181.49	<b>9040.91</b>	<b>5045.22</b>	5	3380.92
10-25-25	45	10856.20	4481.20	1,8,10	378.57	<b>10771.20</b>	<b>4415.79</b>	1,8,10	4071.30
	60	10159.40	4805.06	1,8	367.40	10160.10	4791.64	5,10	5063.18
	120	9056.17	5103.81	5	406.98	<b>9031.86</b>	<b>5087.97</b>	5	6393.38
3-30-30	60	14732.00	6635.12	1,2,3	124.47	<b>14586.00</b>	<b>6362.02</b>	1,2,3	123.57
	90	12994.00	6380.28	1,2	164.00	13727.30	5629.61	1,2,3	226.73
	165	11401.80	6457.16	2	147.97	11528.30	6314.13	2	1839.07
6-30-30	60	13052.10	5633.39	4,5,6	306.87	<b>12878.50</b>	<b>5271.05</b>	4,5,6	1798.98
	90	11973.20	5797.83	2,6	411.62	<b>11965.40</b>	<b>5576.79</b>	4,6	2838.03
	165	11401.80	6448.21	2	338.96	11528.30	6314.13	2	4647.24
10-30-30	60	12952.60	5589.60	2,6,7	653.84	<b>12852.50</b>	<b>5344.25</b>	4,6,8	5604.37
	90	11987.10	5903.78	4,6	847.83	<b>11957.70</b>	<b>5522.70</b>	2,6	8033.56
	165	11384.40	6366.22	8	773.67	11391.40	6154.92	8	12915.20

TABLE 7.19 : Results comparison between the single-objective MA of minimizing cost and the bi-objective MA (large instances)

Instance $ H - I - J $	Hub capacity	Single-objective MA-minimum cost				Bi-objective MA-smallest cost			
		$Z_{best}$ (cost)	CO <sub>2</sub> (kg)	Open hub	$T_{total}$ (s)	$Z_{best}$ (cost)	CO <sub>2</sub> (kg)	Open hub	$T_{total}$ (s)
3-35-35	60	15303.30	6952.16	1,2,3	97.10	<b>14587.10</b>	<b>6246.41</b>	1,2,3	212.60
	90	12704.90	6057.36	1,2	178.36	<b>12623.80</b>	<b>5952.05</b>	1,2	1358.98
	180	11895.80	6798.79	1	209.94	12168.90	6612.14	1	3641.68
6-35-35	60	13486.80	5661.56	1,4,6	364.61	<b>13392.30</b>	<b>5446.99</b>	1,4,6	3604.13
	90	12747.70	6138.95	1,6	301.76	<b>12625.00</b>	<b>5879.15</b>	2,6	4711.90
	180	11997.80	6659.17	6	500.38	12071.10	6533.25	6	7952.50
10-35-35	60	13579.80	5662.71	1,4,6	552.54	13506.30	5715.40	1,2,6	11788.30
	90	12655.50	5966.00	1,2	1116.98	12705.90	5732.97	1,4	15813.90
	180	11975.80	6809.31	6	1246.83	12051.80	6639.90	6	24082.00
3-40-40	75	16172.60	7255.45	1,2,3	200.44	<b>16066.00</b>	<b>6962.21</b>	1,2,3	325.58
	105	15233.90	7556.42	2,3	295.92	<b>15206.70</b>	<b>7435.66</b>	1,2	982.89
	210	13442.10	7730.36	2	280.27	<b>13417.50</b>	<b>7428.76</b>	2	3048.44
6-40-40	75	15257.80	6521.68	3,4,5	324.14	15328.20	6344.15	3,4,5	4782.40
	105	14317.30	7015.38	4,5	507.60	<b>13885.90</b>	<b>6649.84</b>	4,5	6902.36
	210	13296.30	7500.93	5	736.69	<b>13282.20</b>	<b>7321.81</b>	5	11451.80
10-40-40	75	14586.30	6310.51	2,7,10	1355.50	14612.70	6014.34	2,5,10	18576.50
	105	13838.00	6755.55	2,10	2591.37	<b>13687.30</b>	<b>6431.53</b>	2,10	23834.90
	210	13267.60	7639.78	4	2021.55	<b>13165.70</b>	<b>7405.01</b>	5	37141.50
3-45-45	75	14364.90	6573.51	1,2,3	698.43	<b>14294.10</b>	<b>6336.42</b>	1,2,3	626.66
	105	13282.50	6951.06	1,3	582.82	13353.00	6380.85	1,3	2414.79
	195	12165.40	7044.28	1	483.29	<b>12047.10</b>	<b>6744.57</b>	1	4985.50
6-45-45	75	13808.20	5923.65	1,2,5	839.39	13925.80	5912.93	2,3,5	8130.60
	105	12808.70	6224.91	1,5	1153.75	12823.70	6076.64	1,5	5910.32
	195	<b>12158.20</b>	<b>7017.83</b>	1	1067.82	12162.70	6803.82	1	7837.90
10-45-45	75	13486.70	5637.81	3,5,9	2630.10	<b>13209.30</b>	<b>5519.39</b>	3,5,9	31991.30
	105	12557.00	5908.19	8,9	2471.17	<b>12428.40</b>	<b>5746.30</b>	8,9	38676.10
	195	12134.90	6999.14	1	3025.74	<b>12111.30</b>	<b>6815.60</b>	1	50243.80
3-50-50	75	17242.60	8032.10	1,2,3	219.58	<b>17043.40</b>	<b>7642.11</b>	1,2,3	926.05
	120	16324.10	8377.90	1,2	723.49	<b>16099.70</b>	<b>7984.76</b>	1,2	3074.31
	225	14848.40	8559.91	2	375.73	<b>14777.90</b>	<b>8283.47</b>	2	8429.92
6-50-50	75	16669.30	7408.02	2,3,5	2175.56	<b>16556.20</b>	<b>6450.02</b>	1,2,3,5	5293.64
	120	15564.30	7932.25	2,5	2558.65	<b>15501.00</b>	<b>7796.98</b>	2,5	8532.99
	225	14719.20	8547.17	2	1231.40	14826.80	8292.34	2	5209.28
10-50-50	75	16441.56	7607.38	2,5,10	2953.06	<b>16423.40</b>	<b>7327.94</b>	2,5,10	19480.90
	120	15471.72	7814.64	8,10	5645.03	<b>15272.70</b>	<b>7456.37</b>	8,10	29069.90
	225	14712.64	8363.55	8	4960.14	<b>14291.30</b>	<b>7954.63</b>	8	44201.00

TABLE 7.20 : Results comparison between the single-objective MA of minimizing CO<sub>2</sub> and the bi-objective MA (small and medium instances)

Instance  H - I - J	Hub capacity	Single-objective MA-minimum CO <sub>2</sub>				Bi-objective MA-smallest CO <sub>2</sub>			
		Z <sub>best</sub> (CO <sub>2</sub> )	Cost	Open hub	T <sub>total</sub> (s)	Z <sub>best</sub> (CO <sub>2</sub> )	Cost	Open hub	T <sub>total</sub> (s)
3-10-10	10	1537.20	5878.51	1,2,3	23.61	<b>1498.74</b>	<b>5844.76</b>	1,2,3	128.60
	15	1287.37	5480.75	1,2,3	27.99	<b>1274.68</b>	<b>5458.71</b>	1,2,3	100.00
	30	1295.05	5517.07	1,2,3	56.61	<b>1262.78</b>	<b>5421.24</b>	1,2,3	469.05
6-10-10	10	<b>1074.18</b>	<b>8656.48</b>	1,2,3,4,5,6	28.82	1103.36	8714.84	1,2,3,4,5,6	361.46
	15	1058.97	8503.08	1,2,3,4,5,6	49.97	1082.03	7560.33	2,3,4,5,6	488.27
	30	<b>1063.32</b>	<b>8650.77</b>	1,2,3,4,5,6	85.97	1098.55	8704.95	1,2,3,4,5,6	647.18
10-10-10	10	<b>757.04</b>	<b>12652.70</b>	1,2,3,4,5, 6,7,8,9,10	53.66	813.63	12636.40	1,2,3,4,5, 6,7,8,9,10	743.98
	15	776.92	12681.30	1,2,3,4,5, 6,7,8,9,10	101.46	<b>752.61</b>	<b>12544.50</b>	1,2,3,4,5, 6,7,8,9,10	901.93
	30	757.04	12652.70	1,2,3,4,5, 6,7,8,9,10	99.59	<b>751.69</b>	<b>12520.50</b>	1,2,3,4,5, 6,7,8,9,10	1085.86
3-15-15	30	3521.44	9502.91	1,2,3	21.01	<b>3417.68</b>	<b>9139.52</b>	1,2,3	234.71
	45	3099.14	8689.43	1,2,3	80.44	<b>3087.26</b>	<b>8578.73</b>	1,2,3	500.24
	90	2939.81	8364.55	1,2,3	145.91	2939.81	8364.55	1,2,3	753.31
6-15-15	30	2385.74	10891.20	1,2,3,4,5,6	28.62	<b>2362.88</b>	<b>10891.20</b>	1,2,3,4,5,6	226.26
	45	2385.74	10891.20	1,2,3,4,5,6	54.82	<b>2362.88</b>	<b>10891.20</b>	1,2,3,4,5,6	396.99
	90	2395.77	10891.20	1,2,3,4,5,6	58.07	<b>2362.88</b>	<b>10891.20</b>	1,2,3,4,5,6	653.52
10-15-15	30	2122.72	15130.30	1,2,3,4,5, 6,7,8,9,10	161.90	<b>2104.68</b>	<b>14990.60</b>	1,2,3,4,5, 6,7,8,9,10	1200.26
	45	2089.62	15177.00	1,2,3,4,5, 6,7,8,9,10	194.25	15153.90	2142.01	1,2,3,4,5, 6,7,8,9,10	1314.55
	90	2100.35	15203.50	1,2,3,4,5, 6,7,8,9,10	179.39	<b>2095.77</b>	<b>13843.90</b>	1,2,4,5,6, 7,8,9,10	1660.11
3-20-20	45	3768.34	10115.00	1,2,3	167.43	<b>3754.40</b>	<b>10066.50</b>	1,2,3	356.51
	60	3747.02	10011.90	1,2,3	177.43	3674.49	10086.30	1,2,3	994.91
	120	<b>3687.69</b>	<b>9792.54</b>	1,2,3	76.28	3689.35	10072.50	1,2,3	1510.81
6-20-20	45	3275.85	12939.60	1,2,3,4,5,6	141.46	3291.17	11892.20	1,2,4,5,6	716.77
	60	3322.85	12914.00	1,2,3,4,5,6	154.20	3371.53	10891.80	2,4,5,6	850.54
	120	3320.60	13006.40	1,2,3,4,5,6	161.74	<b>3291.17</b>	<b>11892.20</b>	1,2,4,5,6	1331.47
10-20-20	45	2727.02	16498.40	1,2,3,4,5, 6,7,8,9,10	386.34	2786.51	15481.50	2,3,4,5,6, 7,8,9,10	2117.74
	60	2727.02	16498.40	1,2,3,4,5, 6,7,8,9,10	457.40	2743.64	15510.90	1,2,4,5, 6,7,8,9,10	2477.12
	120	2749.15	16499.80	1,2,3,4,5, 6,7,8,9,10	437.16	<b>2733.03</b>	<b>16416.00</b>	1,2,3,4,5, 6,7,8,9,10	3267.60
3-25-25	45	4767.00	11735.10	1,2,3	30.49	4770.93	11734.90	1,2,3	111.31
	60	4756.17	11789.10	1,2,3	54.81	<b>4653.72</b>	<b>11288.80</b>	1,2,3	309.58
	120	4675.86	11620.00	1,2,3	58.19	<b>4570.93</b>	<b>11227.70</b>	1,2,3	819.69
6-25-25	45	3881.58	13856.20	1,2,3,4,5,6	224.15	<b>3834.22</b>	<b>13656.30</b>	1,2,3,4,5,6	1323.36
	60	3865.66	13844.70	1,2,3,4,5,6	235.29	<b>3854.10</b>	<b>13710.60</b>	1,2,3,4,5,6	1686.92
	120	3854.80	13710.80	1,2,3,4,5,6	228.92	3854.80	13710.80	1,2,3,4,5,6	3380.92
10-25-25	45	3363.93	17810.10	1,2,3,4,5, 6,7,8,9,10	804.52	<b>3362.46</b>	<b>17647.40</b>	1,2,3,4,5, 6,7,8,9,10	4071.30
	60	3360.95	17755.50	1,2,3,4,5, 6,7,8,9,10	793.12	3423.05	16829.10	1,2,3,5, 6,7,8,9,10	5063.18
	120	3391.20	17792.80	1,2,3,4,5, 6,7,8,9,10	721.73	3392.94	17661.50	1,2,3,4,5, 6,7,8,9,10	6393.38
3-30-30	60	6392.83	14955.90	1,2,3	49.78	<b>6240.27</b>	<b>14611.40</b>	1,2,3	123.57
	90	<b>5608.15</b>	<b>13744.00</b>	1,2,3	100.50	5618.24	13749.40	1,2,3	226.73
	165	5464.48	13386.10	1,2,3	131.74	<b>5372.74</b>	<b>13065.30</b>	1,2,3	1839.07
6-30-30	60	4686.81	15740.40	1,2,3,4,5,6	449.87	<b>4647.83</b>	<b>15588.60</b>	1,2,3,4,5,6	1798.98
	90	<b>4628.17</b>	<b>15659.60</b>	1,2,3,4,5,6	597.96	4640.85	15720.50	1,2,3,4,5,6	2838.03
	165	4672.60	15659.10	1,2,3,4,5,6	603.34	4638.17	15729.70	1,2,3,4,5,6	4647.24
10-30-30	60	4091.23	18437.80	1,2,4,5,6, 7,8,9,10	2256.56	<b>4088.57</b>	<b>18435.00</b>	1,2,4,5,6, 7,8,9,10	5604.37
	90	<b>4028.19</b>	<b>18222.90</b>	1,2,3,4,5, 6,7,8,10	2741.47	4064.15	19487.80	1,2,3,4,5, 6,7,8,9,10	8033.56
	165	4057.87	19415.00	1,2,3,4,5, 6,7,8,9,10	2353.13	4069.25	18427.60	1,2,4,5,6, 7,8,9,10	12915.20

TABLE 7.21 : Results comparison between the single-objective MA of minimizing CO<sub>2</sub> and the bi-objective MA (large instances)

Instance $ H - I - J $	Hub capacity	Single-objective MA-minimum CO <sub>2</sub>				Bi-objective MA-smallest CO <sub>2</sub>			
		$Z_{best}$ (CO <sub>2</sub> )	Cost	Open hub	$T_{total}$ (s)	$Z_{best}$ (CO <sub>2</sub> )	Cost	Open hub	$T_{total}$ (s)
3-35-35	60	6680.11	15459.70	1,2,3	51.20	<b>6241.53</b>	<b>14603.90</b>	1,2,3	212.60
	90	5589.18	13820.00	1,2,3	152.92	5603.89	13765.50	1,2,3	1358.98
	180	5637.21	13627.10	1,2,3	207.26	<b>5608.56</b>	<b>13435.40</b>	1,2,3	3641.68
6-35-35	60	4649.98	16140.20	1,2,3,4,5,6	753.39	4654.08	16111.90	1,2,3,4,5,6	3604.13
	90	4662.20	16158.90	1,2,3,4,5,6	924.47	<b>4653.46</b>	<b>16149.10</b>	1,2,3,4,5,6	4711.90
	180	<b>4645.24</b>	<b>16111.40</b>	1,2,3,4,5,6	1004.74	4655.60	16149.10	1,2,3,4,5,6	7952.50
10-35-35	60	4027.92	19724.80	1,2,3,4,5, 6,7,8,9,10	3915.41	4111.02	18725.50	1,2,3,4,5, 6,7,8,10	11788.30
	90	4053.30	18719.30	1,2,3,4,5, 6,7,8,10	3911.19	4051.28	19779.10	1,2,3,4,5, 6,7,8,9,10	15813.90
	180	4032.45	19724.80	1,2,3,4,5, 6,7,8,9,10	4120.48	4111.79	18740.70	1,2,3,4,5, 6,7,8,10	24082.00
3-40-40	75	6985.67	16412.40	1,2,3	100.51	<b>6853.72</b>	<b>16197.00</b>	1,2,3	325.58
	105	6795.02	16193.70	1,2,3	227.19	<b>6553.53</b>	<b>15637.50</b>	1,2,3	982.89
	210	6770.84	15912.80	1,2,3	464.01	<b>6670.78</b>	<b>15724.40</b>	1,2,3	3048.44
6-40-40	75	5363.58	17558.10	1,2,3,4,5,6	631.12	<b>5316.80</b>	<b>17472.10</b>	1,2,3,4,5,6	4782.40
	105	5370.49	17644.70	1,2,3,4,5,6	1613.69	<b>5340.61</b>	<b>17529.10</b>	1,2,3,4,5,6	6902.36
	210	5383.74	17559.80	1,2,3,4,5,6	1888.00	5353.70	17663.90	1,2,3,4,5,6	11451.80
10-40-40	75	4599.43	21075.10	1,2,3,4,5, 6,7,8,9,10	6401.30	4612.24	20007.70	1,3,4,5,6, 7,8,9,10	18576.50
	105	4582.81	21173.70	1,2,3,4,5, 6,7,8,9,10	6711.77	4588.83	19987.80	1,3,4,5,6, 7,8,9,10	23834.90
	210	4581.85	21022.00	1,2,3,4,5, 6,7,8,9,10	7081.08	4584.12	20178.70	1,3,4,5,6, 7,8,9,10	37141.50
3-45-45	75	6374.80	14728.80	1,2,3	448.65	<b>6204.08</b>	<b>14519.60</b>	1,2,3	626.66
	105	6337.08	14794.30	1,2,3	465.07	<b>6319.74</b>	<b>14600.00</b>	1,2,3	2414.79
	195	6310.66	14265.20	1,2,3	590.70	<b>6176.04</b>	<b>14083.10</b>	1,2,3	4985.50
6-45-45	75	5516.60	16879.70	1,2,3,4,5,6	2472.15	<b>5481.92</b>	<b>16828.20</b>	1,2,3,4,5,6	8130.60
	105	5510.75	16972.70	1,2,3,4,5,6	2818.80	5513.80	15853.30	1,2,4,5,6	5910.32
	195	5530.65	16913.80	1,2,3,4,5,6	2925.16	<b>5511.05</b>	<b>16856.80</b>	1,2,3,4,5,6	7837.90
10-45-45	75	4733.79	20285.50	1,2,3,4,5, 6,7,8,9,10	9782.53	4711.60	20297.90	1,2,3,4,5, 6,7,8,9,10	31991.30
	105	4744.25	20400.30	1,2,3,4,5, 6,7,8,9,10	9153.33	<b>4665.70</b>	<b>19317.10</b>	2,3,4,5,6, 7,8,9,10	38676.10
	195	4787.63	20487.10	1,2,3,4,5, 6,7,8,9,10	11016.85	<b>4669.68</b>	<b>20410.00</b>	1,2,3,4,5, 6,7,8,9,10	50243.80
3-50-50	75	7773.70	17610.60	1,2,3	112.08	<b>7588.11</b>	<b>17157.50</b>	1,2,3	926.05
	120	7223.56	16793.70	1,2,3	332.52	<b>6961.95</b>	<b>16542.90</b>	1,2,3	3074.31
	225	6853.09	16270.40	1,2,3	908.80	<b>6691.41</b>	<b>16097.20</b>	1,2,3	8429.92
6-50-50	75	6039.66	18647.30	1,2,3,4,5,6	1208.74	5953.01	18670.40	1,2,3,4,5,6	293.64
	120	<b>6018.28</b>	<b>18633.50</b>	1,2,3,4,5,6	3564.22	6068.97	18675.60	1,2,3,4,5,6	8532.99
	225	6060.07	18871.80	1,2,3,4,5,6	3846.23	6097.73	17647.40	1,2,3,4,5	5209.28
10-50-50	75	<b>5026.33</b>	<b>22044.80</b>	1,2,3,4,5, 6,7,8,9,10	15082.76	5076.42	22155.30	1,2,3,4,5, 6,7,8,9,10	19480.90
	120	5048.59	22257.20	1,2,3,4,5, 6,7,8,9,10	15516.89	5117.58	20960.60	2,3,4,5,6, 7,8,9,10	29069.90
	225	5022.16	22051.40	1,2,3,4,5, 6,7,8,9,10	14768.26	5136.84	21208.50	1,2,3,4,5, 7,8,9,10	44201.00

# IV

## Two-phase methods for the single-objective HLRP





## Two-phase model and memetic algorithm

As discussed in Chapter 5, the global model of the single-objective HLRP can find optimal solutions for small instances. However it has difficulty solving large problems and consumes a large computing time. The proposed MA is able to solve small problems optimally and find feasible solutions for the medium and large problem instances with limited gaps compared to the lower bounds of CPLEX within a reasonable computing time. The sensitivity analysis of the MA (Section 5.5, Chapter 5) also shows its robust stability. Since the difficulty of solving the HLRP, the CPLEX solver can only find solutions with a large gap for the large instances. Considering that it is necessary to take further investigation on the performance of the MA for solving large problems, we propose a two-phase method by means of decomposing the global HLRP into two simpler problems, the HLP and the VRP, and solve them separately to obtain the solutions of the HLRP. The results are then investigated by being compared to those obtained by the global method. Notice that, the two-phase method is inspired based on the previously proposed global method for the single-objective HLRP by doing simple decomposition of the global mathematical model and the MA. The main purpose of proposing the two-phase model and the MA lies in providing another way to solve the global problem and evaluate the performance of the proposed MA.

Section 8.1 presents the mathematical formulations of the HLP and the VRP for separate collection and delivery process which are based on the formulations in Chapter 3. In Section 8.2, the Memetic Algorithm (MA) proposed in Chapter 4 is adapted to solve the HLRP by two steps : in the first phase, the MA aims to solve the HLP while in the second phase, the local search operators are implemented to search for the collection and delivery routes.

### 8.1 Two-phase model for minimizing cost

The main idea of our two-phase approach is to decompose complex HLRP into two simpler relevant problems (HLP and VRP) and solving them separately. The hub location problems and the vehicle routing problems have been abundantly researched. Regarding the HLPs, we refer the reader to the relevant reviews : [Campbell \[1994\]](#) gave the first survey of the discrete HLP, [Alumur and Kara \[2008\]](#) provided a comprehensive review of the HLP covering the period from 1987 to 2007, [Campbell and O’Kelly \[2012\]](#) presented the origins and motivations of 25 years of HLP research, and [Farahani et al. \[2013\]](#) highlighted the aspects of HLP published after 2007 and current trends. A state-of-the art of the HLPs also can be found in Chapter 2. Regarding the VPRs, we refer the reader to the reviews of [Laporte \[1992\]](#), [Toth and Vigo \[2002\]](#), [Laporte \[2007\]](#), [Golden et al. \[2008\]](#) and [Koç and Laporte \[2017\]](#).

The model of the HLP in the first phase adapts the hub location constraints of the single-objective

model of the HLRP in Chapter 3 (Formula (3.2)-(3.7)) which is mainly derived from the model in Ernst and Krishnamoorthy [1999]. It takes into consideration the fixed hub location cost, the freight handling cost, the inter-hub transfer cost and the allocation cost between the hubs and non-hub nodes. In the second phase, it assumes that the hub location and allocation scheme are pre-determined and taken from the solution in the first phase. The classic Vehicle Routing Problem (VRP) is tackled, scheduling the distinct collection and delivery routes for every open hub (collection VRP and delivery VRP). The two models of the collection VRP and the delivery VRP are based on the collection routing and delivery routing constraints which are inspired by Toth and Vigo [2014] (Formula (3.8)-(3.18), Formula (3.19)-(3.29), Chapter 3), accordingly. The second phase aims at minimizing the collection and delivery routing costs for each open hub.

Figure 8.1 presents the schematic illustration of the two-phase approach. Given the set of potential hubs  $H$ , the supplier set  $I$  and client set  $J$ , we assume that variable  $Y_{kl}^i$  is the fraction of flow shipped from supplier  $i$  via hubs  $k$  to  $l$ ,  $l \in H$ ,  $k \neq l$  and  $i \in I$ .  $z_{ik}$  represent the 0-1 allocation variable of a node  $i$  to a hub  $k$ ;  $z_{kk}=1$  indicates that hub  $k$  is opened.  $x_{ij}$  are the 0-1 routing variables and  $f_{ij}$  represent the vehicle load variables. The HLRP is decomposed to solve a CSAHLP at first, obtaining the set of open hubs which we defined as  $H'$ , the supplier set  $I_k$  in which supplier  $i$ ,  $i \in I_k$ , is allocated to hub  $k$ ,  $k \in H'$ , and the client set  $J_k$  in which client  $j$ ,  $j \in J_k$ , is allocated to hub  $k$ ,  $k \in H'$ . When it comes to the second phase, for each open hub  $k \in H'$ ,  $z_{kk}=1$ ; for the suppliers allocated to hub  $k$ ,  $z_{ik}=1$ ,  $i \in I_k$ ; for the clients allocated to hub  $k$ ,  $z_{jk}=1$ ,  $j \in J_k$ . The collection routes and delivery routes for each open hub  $k$  are then scheduled in separate process by means of solving the collection VRP and delivery VRP.

After solving the models in two phases, the open hubs, the inter-hub flow transfer and allocation scheme are generated by the first phase. Based on the hub location and allocation scheme, the collection and delivery routes, as well as the number of vehicles used, are scheduled by the second phase. These decisions from the HLP and the two VRPs constitute the solution of the HLRP. Accordingly, we obtain the fixed cost of locating hubs, the inter-hub transfer cost and the freight handling cost from the first phase (we do not retain the allocation cost); we obtain the collection and delivery routing cost and the cost of using vehicles in the second phase. At last, the sum of these costs constitutes the total cost of the HLRP (Figure 8.1).

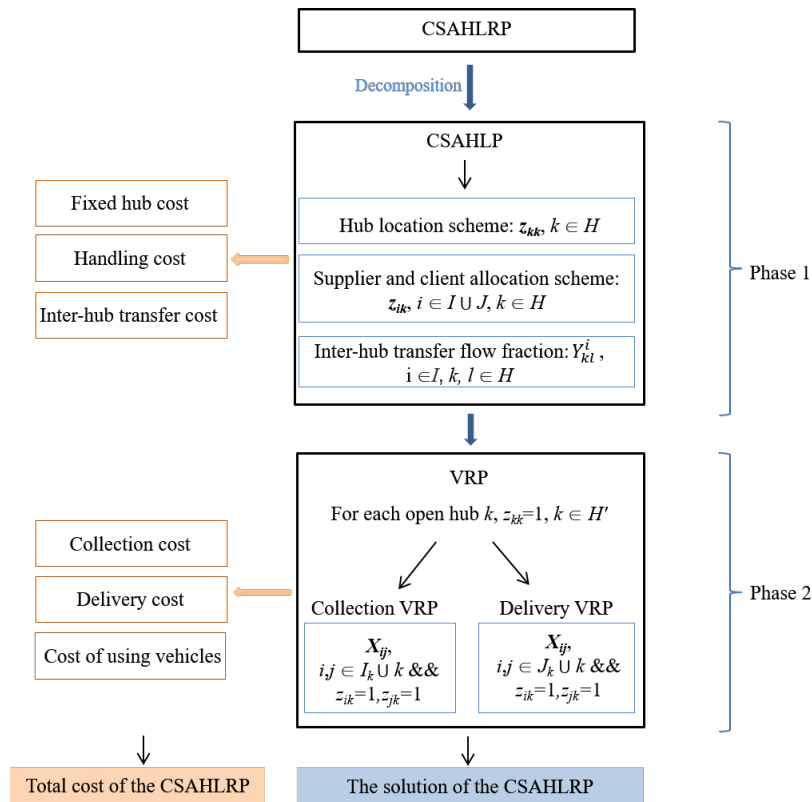


FIGURE 8.1 : Schematic illustration of building the two-phase models

### 8.1.1 First phase of the model : CSAHLP

As previously stated, the first phase deals with the Capacitated Single Allocation Hub Location Problem (CSAHLP), tackling the location of hub facilities and the assignment of customers to the hubs. It assumes that the demand of suppliers and clients has to be served by only one open hub with a capacity and a fixed opening cost. The freight can be transferred between at most two hubs before being delivered from the suppliers to the clients. The hubs and the suppliers/clients are fully connected. Direct transportation is organized between the hubs and suppliers/clients, and no local tours are considered. The model of the CSAHLP is based on the hub location constraints of the single-objective model of the HLRP (Constraints (3.2) to (3.7), Chapter 3) which is derived from the model of Ernst and Krishnamoorthy [1999]. This problem is defined on a complete graph  $G = (N, A)$ , in which  $N$  is the set of  $n$  nodes and  $A$  is the set of arcs. The sets and parameters are explained as follows (similar to the model of the single-objective HLRP) :

#### Sets

- $H$  – Set of hub nodes,  $H = \{k \mid k = 1, 2, \dots, h\}$ ;
- $I$  – Set of supplier nodes,  $I = \{i \mid i = h + 1, h + 2, \dots, h + m\}$ ;
- $J$  – Set of client nodes,  $J = \{j \mid j = h + m + 1, h + m + 2, \dots, h + m + n\}$ ;
- $N$  – Set of all nodes,  $N = H \cup I \cup J$ ;
- $A_1$  – Set of arcs in collection routing,  $A_1 = \{(i, j) : i, j \in I \cup H\}$ ;
- $A_2$  – Set of arcs in delivery routing,  $A_2 = \{(i, j) : i, j \in J \cup H\}$ ;
- $A$  – Set of all arcs,  $A = \{(i, j) : i, j \in N\}$

#### Parameters

- $F_k$  – Fixed cost of operating hub  $k$ ;
- $\Gamma_k$  – Capacity of hub  $k$ ;
- $c_k$  – Handling cost to operate one unit product in hub  $k$ ,  $k \in H$ ;
- $C$  – Fixed cost of a vehicle;
- $Q$  – Capacity of a vehicle;
- $q_{ij}$  – Flow quantity from supplier  $i \in I$  to client  $j \in J$ ;
- $d_{ij}$  – Distance between two nodes  $i$  and  $j$ , arc  $(i, j) \in A$ ;
- $\alpha$  – Unit cost parameter for the inter-hub transport;
- $\beta$  – Unit cost parameter for the collection tour;
- $\gamma$  – Unit cost parameter for the delivery tour;
- $O_i$  – Total quantity of flow originating at supplier  $i$ ,  $O_i = \sum_{j \in J} q_{ij}$ ;
- $D_j$  – Total quantity of flow for client  $j$ ,  $D_j = \sum_{i \in I} q_{ij}$ .

The proposed model in this phase is named **CSAHLP-COST**. The objective function (8.1) of the model constitutes of the fixed open hub cost, the inter-hub transfer cost, the allocation cost of collection and delivery, and the freight handling cost in hubs. Constraints (8.2) to (8.7) are hub location constraints mainly derived from the model in Ernst and Krishnamoorthy [1999] (the same as Constraints (3.2) to (3.7) of the global model of the HLRP). Constraints (8.8) and (8.9) define the domain of the decision variable  $z_{ik}$  and  $Y_{kl}^i$ . The main differences of the model from that of Ernst and Krishnamoorthy [1999] is that we consider two sets of customers (suppliers and clients) and limit the total collection and delivery load on hubs in two separate constraints (Constraints (8.4) and (8.5)). We also include the freight handling cost which Ernst and Krishnamoorthy [1999] did not consider. Besides, the valid inequalities (8.10) are added to ensure the minimum number of open hubs.

## Phase 1 : CSAHLP-COST

$$\begin{aligned} \min \sum_{k \in H} F_k z_{kk} + \sum_{i \in I} \sum_{k \in H} \sum_{l \in H} \alpha d_{kl} O_i Y_{kl}^i + \sum_{i \in I} \sum_{k \in H} \beta d_{ik} z_{ik} + \sum_{j \in J} \sum_{k \in H} \gamma d_{jk} z_{jk} \\ + \sum_{i \in I} \sum_{k \in H} c_k O_i z_{ik} + \sum_{i \in I} \sum_{(k,l) \in A_3} c_l O_i Y_{kl}^i \end{aligned} \quad (8.1)$$

Subject to

$$z_{ik} \leq z_{kk} \quad \forall i \in N, \forall k \in H \quad (8.2)$$

$$\sum_{k \in H} z_{ik} = 1 \quad \forall i \in I \cup J \quad (8.3)$$

$$\sum_{i \in I} O_i z_{ik} \leq \Gamma_k z_{kk} \quad \forall k \in H \quad (8.4)$$

$$\sum_{j \in J} D_j z_{jl} \leq \Gamma_l z_{ll} \quad \forall l \in H \quad (8.5)$$

$$\sum_{l \in H} Y_{kl}^i = z_{ik} \quad \forall i \in I, \forall k \in H \quad (8.6)$$

$$\sum_{l \in H} Y_{lk}^i O_i = \sum_{j \in J} q_{ij} z_{jk} \quad \forall i \in I, \forall k \in H \quad (8.7)$$

$$z_{ik} \in \{0, 1\} \quad \forall i \in N, \forall k \in H \quad (8.8)$$

$$0 \leq Y_{kl}^i \leq 1 \quad \forall i \in I, \forall k, l \in H \quad (8.9)$$

$$\sum_{k \in H} \sum_{i \in I} x_{ki} \geq \lceil \frac{\sum_{i \in I} O_i}{Q} \rceil \quad (8.10)$$

## 8.1.2 Second phase of the model : CVRP

Following the hub location and allocation scheme for each open hub from phase 1, the second phase determines the collection and delivery routes and minimize the routing costs for every open hub by means of solving distinct collection VRP and delivery VRP. There is thus for each open hub one collection VRP model and one delivery VRP model. The model of the two VRPs are based on the collection routing and delivery routing constraints of the global MILP model of the HLRP which are inspired by [Kara et al. \[2007\]](#) and [Toth and Vigo \[2014\]](#).

Since the collection VRP and delivery VRP for each open hub are solved separately and the models are similar, only the model of the collection VRP is presented. It is assumed that  $H'$  is the set of open hubs,  $I_k$  is the set of suppliers allocated to hub  $k$ ,  $k \in H'$ , which means that  $z_{ik}$  ( $i \in I_k, k \in H'$ ) is always equal to 1. For a given open hub  $k$ ,  $k \in H'$ , and the set of suppliers  $I_k$ , we describe the collection VRP model :

## Phase 2 : Collection VRP-COST

$$\min \sum_{i,j \in I_k \cup k} \beta d_{ij} x_{ij} + \sum_{i \in I_k} C x_{ki} \quad (8.11)$$

Subject to

$$\sum_{j \in I_k \cup k} x_{ij} = 1 \quad \forall i \in I_k \quad (8.12)$$

$$\sum_{i \in I_k \cup k} x_{ij} - \sum_{i \in I_k \cup k} x_{ji} = 0 \quad \forall j \in I_k \cup k \quad (8.13)$$

$$\sum_{j \in I_k \cup k} f_{ij} - \sum_{j \in I_k \cup k} f_{ji} = O_i \quad \forall i \in I_k \quad (8.14)$$

$$\sum_{i \in I_k} f_{ik} = \sum_{i \in I_k} O_i \quad (8.15)$$

$$f_{ij} \leq (Q - O_j)x_{ij} \quad \forall i \in I_k \cup k, \forall j \in I_k \quad (8.16)$$

$$O_i x_{ij} \leq f_{ij} \quad \forall i \in I_k, \forall j \in I_k \cup k \quad (8.17)$$

$$\sum_{i \in I_k} f_{ki} = 0 \quad (8.18)$$

$$f_{ik} \leq Qx_{ik} \quad \forall i \in I_k \quad (8.19)$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in I_k \cup k \quad (8.20)$$

$$f_{ij} \geq 0 \quad \forall i, j \in I_k \cup k \quad (8.21)$$

$$x_{ij} + x_{ji} \leq 1 \quad \forall i, j \in I_k \quad (8.22)$$

$$\sum_{i \in I_k} x_{ki} \geq \lceil \frac{\sum_{i \in I_k} O_i}{Q} \rceil \quad (8.23)$$

The objective function (8.11) minimizes the collection routing cost for the hub  $k$  and the cost of using vehicles. Constraint (8.12) to (8.19) are inspired by the VRP constraints proposed in Kara et al. [2007] and Toth and Vigo [2014] and are the same as the collection routing constraints of the global model of the HLRP (Constraints (3.8) and (3.9), (3.13) to (3.18), Chapter 3). Constraints (8.20) and (8.21) define the domain of the collection flow variable  $f_{ij}$  and the 0-1 routing variable  $x_{ij}$ . Valid inequality (8.22) eliminate sub-tours. Valid inequalities (8.23) provide a lower bound of the total number of vehicles required.

Since for each open hub, two VRP models are built and experimented separately, the total routing cost of one open hub is the sum of the collection cost and delivery cost obtained from the two models. The models are experimented on all the open hubs from phase 1. At last, the collection and delivery routes for all the open hubs are generated and the total cost of collection and delivery are calculated.

## 8.2 Two-phase MA for minimizing cost

In addition to the proposition of the two-phase model for solving the single-objective HLRP, in this section, a two-phase MA is presented, solving the HLP and VRP continuously to obtain the solutions of the HLRP. As previously discussed in Chapter 2 and Chapter 4, many researches have applied the MA to solve the HLPs and the VRPs due to its characteristics of being able to hybridized with other methods flexibly. For more details, one can refer to the reviews of the HLP by Alumur and Kara [2008] and of the VPR by Potvin [2007]. In this section, we propose the two-phase MA based on the single-objective MA of the HRLP in Chapter 4.

Figure 8.2 presents the general framework of the two-phase MA. In the first phase (Figure 8.2.(a)), a CSAHLP is solved to locate hubs and allocate the suppliers and clients to the open hubs. The objective of the problem is to minimize the total cost of the HLP (the sum of the fixed hub cost, inter-hub transfer cost, handling cost and allocation cost). Adopting the same deigning method of the global MA proposed in Chapter 4, the two-phase MA firstly generates an initial population with heuristic and random individuals. Each individual is evaluated by a fitness function based on the objective function (8.1) of the CSAHLP presented

in Section 8.1. The genetic operators (selection, crossover, mutation) (Fortin and Parizeau [2013]) and the local searches (Manzour-al Ajdad et al. [2012]) from the global MA are adapted to generate offsprings. The process iterates until the stopping criteria are satisfied.

The second phase deals with the collection VRP and delivery VRP for each open hub. We limit the two VRPs to a local search derived from that in Chapter 4 which is inspired by Manzour-al Ajdad et al. [2012]. The procedure of the local search on the collection routing is presented in Figure 8.2.(b) which is also applied on the delivery routing. After the chromosome representing the solution of the HLP is generated in the first phase, with aims of minimizing the total collection cost (Equation (8.11)) for each open hub from the first phase, the local search operators are applied by means of changing the sequence of the suppliers which are allocated to the open hub on the chromosome. At the same time, for the clients assigned to the hub, the local searches are also applied by similar procedure. The following sections describe the main phases of the two-phase MA in detail.

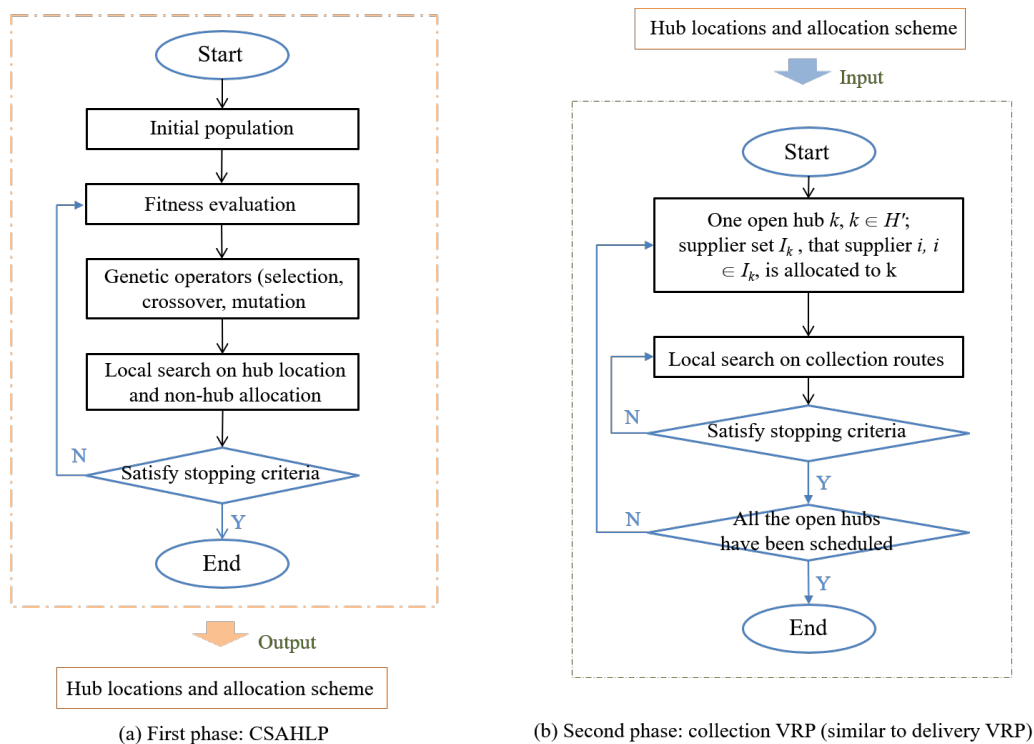


FIGURE 8.2 : Schematic illustration of the two-phase MA

### 8.2.1 First phase of the MA : CSAHLP

This section introduces the process of the MA solving the CSAHLP. Since all the procedures are derived from the global MA of the HLRP, the main idea for each operation of the MA is presented and more details can be found in Chapter 4.

#### (1) Chromosome and fitness function

The individuals in the MA are represented by chromosomes shown in Figure 8.3. Chromosome  $P(x)$  includes a hub location vector  $H(x) = \{h_1, h_2, \dots, h_n\}$  and a non-hub nodes vector  $A(x) = \{a_1, a_2, \dots, a_n\}$  (Prins et al. [2006b]). As one supplier/client must be served by only one hub, vector  $A(x)$  does not contain duplicates. For every position  $i$  ( $i = \{1, 2, \dots, n\}$ ), if  $a_i = j$  and  $h_i = k$ , the non-hub node  $j$  is assigned to hub  $k$ . Consider a problem scale : 3 potential hubs numbered 1, 2 and 3; five suppliers numbered from 4 to 8 (grey boxes in Figure 8.3); 5 clients numbered from 9 to 13 (white boxes in Figure 8.3). Figure 8.3 shows a chromosome  $P(x)$  that hub 1 and 3 are selected open in vector  $H(x)$ . According to the relevant positions,

suppliers 4, 5, clients 9, 12, 13 are allocated to hub 1 ; suppliers 6, 7, 8 and clients 10, 11 are allocated to hub 3.

The fitness function  $F_{eva}(x)$  to evaluate each individual is defined as

$$F'_{eva}(x) = ObjValue(x) + Penalty(x) \tag{8.24}$$

$$Penalty(x) = \sigma \sum_{k \in H} \max\{0, \sum_{i \in I} z_{ik} O_i - \Gamma_k\} + \sigma \sum_{k \in H} \max\{0, \sum_{j \in J} z_{jk} D_j - \Gamma_k\} \tag{8.25}$$

where  $ObjValue(x)$  denotes the objective value of solution  $x$  calculated by Equation (8.1).  $\sigma$  is the penalty parameter with a large value,  $\sum_{i \in I} z_{ik} O_i$  and  $\sum_{j \in J} z_{jk} D_j$  are the total demand of suppliers and clients allocated to an open hub  $k$ ,  $\Gamma_k$  is the capacity of hub  $k$ . Thus if the collection or delivery quantities for one open hub violate the hub capacity, the violation demand is multiplied by  $\sigma$ . The total sum of the violation demand multiplying  $\sigma$  for all the open hubs constitutes the penalty cost  $Penalty(x)$  for solution  $x$ .

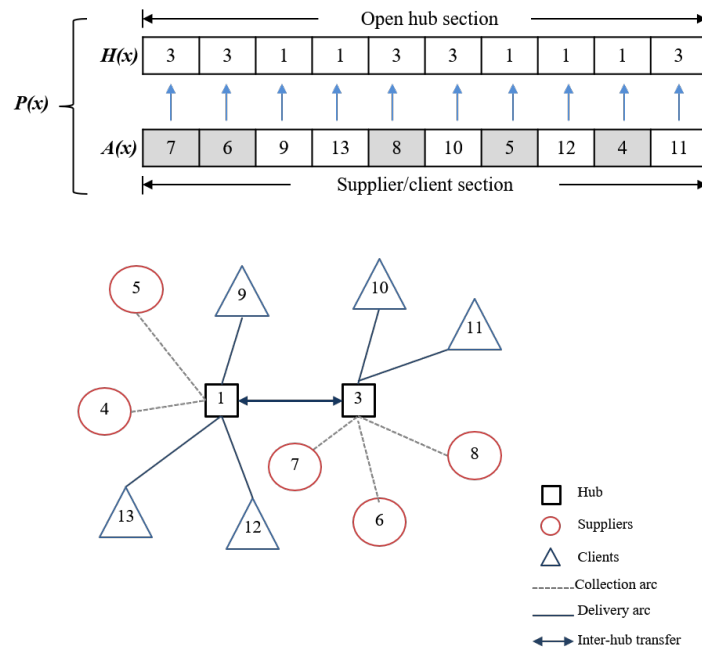


FIGURE 8.3 : Solution representation of the HLP

(2) Initial population

Similar to the single-objective MA of the HLRP, the initial population of the HLP includes both heuristic and random solutions. The method to generate heuristic solutions is inspired from the Extended Clarke and Wright Algorithm (ECWA) proposed by [Karaoglan and Altiparmak \[2015\]](#). At first, the minimum number of hubs  $MinH$  needed are calculated by the formulation of  $MinH \leftarrow \left\lceil \frac{D}{\Gamma} \right\rceil$ , where  $D$  is the total demand and  $\Gamma$  is hub capacity. Then hubs are selected open randomly with the number of at least  $MinH$ . For each supplier node, the first and second nearest open hubs are searched and the saving (distance differences) is calculated. The supplier node with the largest saving is allocated to the nearest hub and the capacity of the hub is updated. After that, the supplier with the second largest saving is allocated. The process continues until all the supplier nodes are assigned to a hub. When some suppliers cannot be allocated to their nearest hub, their saving is recalculated. Furthermore, random new hubs may be opened if there are still suppliers left because the remaining capacity of the open hubs is not enough to serve them. Depots without suppliers are closed. Notice that the process of allocating the suppliers and clients are independent.



The same procedure is implemented to allocate clients to the hubs. More details of the algorithm can be found in **Algorithm 3** in Chapter 4. Due to the limited number of open hub combinations, the heuristic procedure may fail to reach the required population size. The remaining solutions are randomly produced.

(3) Genetic operators

The genetic operators (selection, crossover and mutation) of the global MA are adapted to the chromosomes of the HLP to generate offsprings (Chapter 4). On current generation of the MA, solutions are selected by a unique fitness binary tournament selection (Fortin and Parizeau [2013]) which defines a unique fitness set storing the fitnesses of the individuals from current generation without repeating values. Each time, two fitnesses in the unique fitness set are randomly compared and the best fitness is selected. One of the individuals in current generation sharing the same best fitness is randomly selected to survive. The selection process continues until the number of the selected individuals meets the population size.

In the next step, the selected individuals are arranged randomly in pairs and a one-point crossover with a probability is applied on the parent pairs. Figure 8.4 illustrates the crossover procedure on chromosomes. Two crossover points  $P_L$  and  $P_R$  are defined for the location part and the non-hub nodes part, respectively. On the hub location vector  $H(x)$ , a new offspring combines the code of Parent 1 before crossover point  $P_L$  and the code of Parent 2 after  $P_L$ ; on the non-hub nodes vector  $A(x)$ , the new offspring takes the code of Parent 1 before  $P_R$  as the first part. The second part sequentially copies the code of  $A(x)$  of Parent 2 except for the nodes that have been copied from Parent 1.

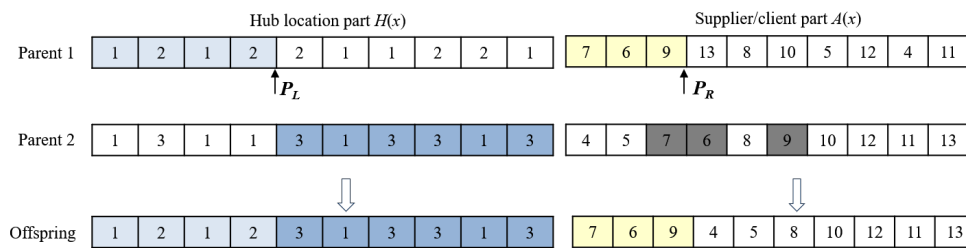


FIGURE 8.4 : The crossover operator of the two-phase MA

Followed by the crossover, a mutation operator mutates the chromosome with a probability. As shown in Figure 8.5. In the location section  $H(x)$ , random positions are replaced by another hub. In the non-hub nodes section  $A(x)$ , the random locations of two points are exchanged.

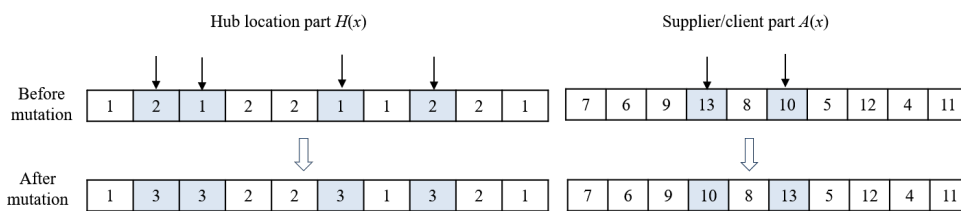


FIGURE 8.5 : The mutation operator of the two-phase MA

(4) Local search

The same local searches of the global MA of the HLRP are applied on both the non-hub node part  $A(x)$  and the hub location part  $H(x)$  of the chromosome. On set  $A(x)$ , two random non-hub nodes exchange their positions while other nodes remain unchanged. Next, one random non-hub node is selected to be inserted into a random position of  $A(x)$ . On set  $H(x)$ , four local search neighbourhood are used : (1) when not all the potential hubs are open, random previously closed hub is opened to replace the position of a current open hub ; (2) the position of one open hub is replaced by another open hub randomly ; (3) several positions are randomly chosen and a new potential hub is opened to the hubs on the selected positions ; (4) two random hubs exchange their positions.

### 8.2.2 Second phase of the MA : CVRP

In the second phase, the local searches based on that of the global MA (Chapter 4) are applied to explore the distinct collection and delivery routes. After the first phase of the MA, a chromosome representing the final solution of the HLP is generated. The hub location set is denoted as  $H'$ . For each open hub  $k, k \in H'$ , there are two sets of customers that are allocated to hub  $k$  : supplier set  $I_k$  and client set  $J_k$ . The initial collection route scheme of hub  $k$  is obtained based on the permutation order of the suppliers  $i, i \in I_k$ , in the chromosome. The vehicle capacity constraint is checked by each decoding procedure. Once the vehicle capacity is violated by the insertion of a new supplier to current route, this supplier is assigned to a new collection route. The initial route scheme of delivery also follows these steps. For example, assume that one problem scales is with 3 hubs, 20 suppliers and 20 clients. After Phase 1, hub 3 is one of the hubs selected to open. Suppliers 4 to 8 and client 9 to 13 are allocated to hub 3. Figure (8.6.(a)) shows the part of the chromosome presenting hub 3 and the suppliers and clients assigned to it. Firstly, the suppliers are arranged together according to the sequence on the chromosome, as well as the clients (8.6.(b)). The initial collection and delivery routes are built separately according to the supplier and the client sequences of the chromosome without violating vehicle capacity (8.6.(c)). The value of collection cost is calculated by Equation (8.11). The value of delivery cost is calculated by a similar equation.

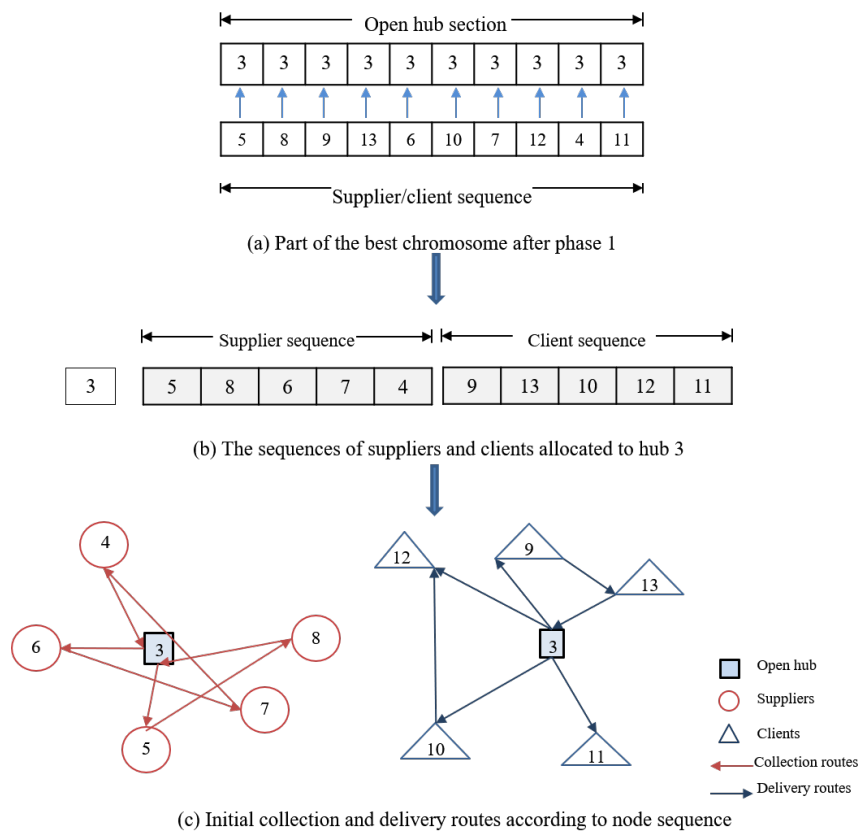


FIGURE 8.6 : Initial collection and delivery routes after Phase 1

Next step, on the level of collection, two local searches are applied on the supplier sequence (Figure (8.7)) : swapping the positions of two supplier nodes (Figure (8.7.(b))); inserting one supplier into a random position (Figure (8.7.(c))). Each time the offspring is generated, the value of collection cost is computed. If the solution is improved (reducing collection cost), the offspring replaces its parent and the local search operator is applied on the offspring. The process iterated until the stopping criteria are satisfied. The same procedure is also implemented to explore delivery routes and compute the delivery routing cost.

At last, the sum of the fixed cost of opening hubs, the inter-hub transfer cost and the freight handling cost from phase 1, along with the collection cost and deliver cost from phase 2 constitutes the total cost of the HLRP.

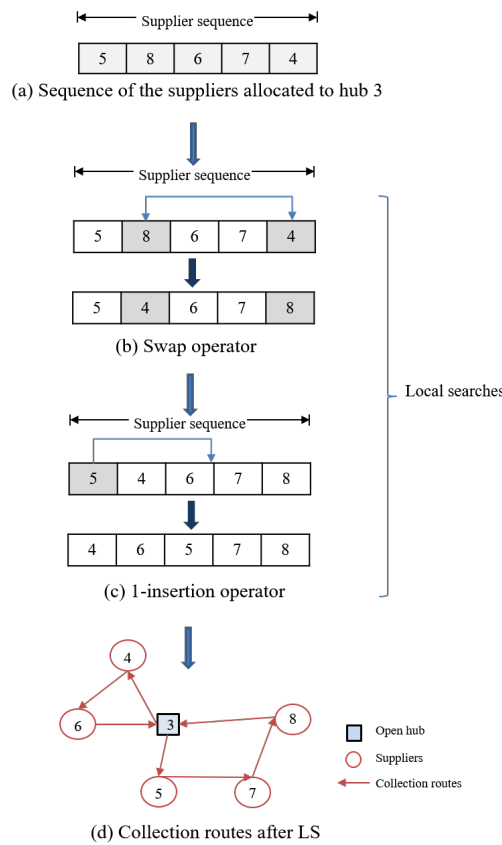


FIGURE 8.7 : Local searches on collection routing in phase 2

### 8.3 Conclusion

This chapter proposes another approach to solve the HLRP by splitting the complex problem into two sub-problems : one sub-problem of HLP; one sub-problem of collection VRP and delivery VRP. It is obvious that the two sub-problems are simpler due to the reduction of the decision variables and constraints. We limit the research on the two-phase method by directly decomposing and adapting the global model and the MA for the HLRP proposed in chapters 3 and 4. In the following chapter, the computational experiments of the two-phase method are conducted and the results are analysed. Furthermore, the performance of the global model and MA of the HLRP is also investigated by means of comparing their results with those obtained by the two-phase method.

## Computational experiments for the two-phase method

In this chapter, the computational experiments on both the two-phase model and the MA are conducted with the objective of minimizing cost. In order to compare the solutions of the two phase approaches with that of the global single-objective method from Chapter 5, the computational experiments are carried on the same platforms (the computer and the versions of CPLEX/C++) and based on the same instances as the single-objective HLRP.

Section 9.1 describes the data sets and parameters used for the experiments. Section 9.2 shows the results of the two-phase model with CPLEX solver. Section 9.3 presents the results obtained by the two-phase MA and compares them with the CPLEX results of the two-phase model, as well as the results of the global single-objective MA.

### 9.1 Data and parameters

We have generated the data set for solving the global problem on the basis of the AP data set (Ernst and Krishnamoorthy [1999]) (Chapter 5). The data set contains 27 instances with three hub types for each. Our computational experiments on the two-phase method are also conducted based on the generated data set. We select 18 instances including 6 instances for each small, medium and large problem scale (Table 9.1). The names of the instances  $|H|-|I|-|J|-|\Gamma|$  stands for the number of potential hubs, the number of supplier and client nodes, the capacity of hubs, correspondingly. The candidate hub number set  $|H| \in \{3, 6, 10\}$ . The sets of supplier and client nodes  $|I|=|J| \in \{10, 15, 20, 30, 40, 50\}$ .  $\Gamma$  stands for the hub type with integer capacity corresponding to 1/2 of the total demand of each instance problem. The fixed cost and unit transportation cost of vehicles are from the logistics data of the French Comité National Routier CNR<sup>1</sup> data base. The unit cost of handling freight in hubs was communicated by a French logistic company (see Section 5.1, Chapter 5).

The models of the HLP, the collection VRP and the delivery VRP proposed are coded in Visual studio C++ 2012 and solved with CPLEX 12.6.1.. The two-phase MA is implemented in Visual studio C++ 2012 on PCs with 3.07 GHz and 8 GB RAM memory.

The general notations of the tables in the following sections are explained below :

- $UB$  : best objective value of CPLEX in three hours for each experiment ;

<sup>1</sup><http://www.cnr.fr/en>

- $LB$  : lower bound found by CPLEX in three hours ;
- $Gap$  (%) : deviation in % between the best objective found by CPLEX and the lower bound found by CPLEX for each experiment,  $\%Gap = \frac{UB - LB}{UB} \times 100\%$  ;
- $Z_{best}$  : best objective value found by the MA in 10 runs for each instance ;
- CPU time (s) : total CPU time of CPLEX in seconds to obtain the best objective ;
- $GapMA$  (%) : deviation in % between the best objective found by MA and the lower bound found by CPLEX for each instance,  $\%GapMA = \frac{Z_{best} - LB}{Z_{best}} \times 100\%$  ;
- $T_{total}$  (s) : total computing time of the MA (10 runs) ;
- Open hub : hub location scheme of the best solution ;
- Route numbers : total number of collection and delivery routes of the best solution which also indicate the number of vehicles used for collection and delivery.

TABLE 9.1 : Data structures for computational experiments

Size	Notation $ H - I - J $	Hub capacity $ \Gamma $
Small	3/6/10-10-10	15
	3/6/10-15-15	45
Medium	3/6/10-20-20	60
	3/6/10-30-30	90
Large	3/6/10-40-40	105
	3/6/10-50-50	120

## 9.2 CPLEX results of the two-phase model

In this section, the models of the HLP (phase 1), the collection VRPs and delivery VRPs (phase 2) are solved by CPLEX. In order to be comparable to the results of the global model, the maximum running time of the CPLEX is restricted to 3 hours and the limitation of the tree memory is 1500MB for each model. The CPLEX parameter “MIPEmphasis” is set to 2 and “Probe” is set to 3.

### CPLEX results of the HLP (phase 1)

The model of the HLP in the first phase minimizes the total cost of the fixed hub cost, the inter-hub transfer cost, the freight handling cost and the allocation cost of collections and deliveries. Tables 9.2 gives the details of the CPLEX results regarding the hub location scheme, the upper bound, the gap, the values of different cost components and the CPU times. It reveals that the CPLEX solver is able to solve all the instances to optimality with up to 10 hubs and 100 non-hub nodes in a very fast computing time. The average CPU time for solving all the instances is 2.76 seconds. Observe the values of the different cost components for each instance, the allocation cost and the hub location cost account for more than 80% of the total cost. Finally, regarding the instance problems with the same number of non-hub nodes, the solutions are improved, reaching lower costs, when there are more choices of potential hubs.

TABLE 9.2 : CPLEX results of the first phase (HLP)

Instance $ H - I - J $	Hub capacity	Open hub	UB (Cost)	Gap (%)	Cost of open hub	Inter-hub cost	Allocation cost		Handling cost	CPU time (s)
							Collection	Delivery		
3-10-10	15	1,3	3854.04	0.00	2000	172.91	747.96	860.55	72.62	0.23
6-10-10	15	3,4	3833.58	0.00	2000	123.53	771.42	869.71	68.92	0.27
10-10-10	15	4,8	3778.09	0.00	2000	109.87	958.21	640.22	69.79	0.94
3-15-15	45	1,3	5889.24	0.00	2000	454.01	1643.95	1597.67	193.61	0.22
6-15-15	45	4,5	5444.12	0.00	2000	253.27	1288.23	1708.96	193.66	0.33
10-15-15	45	7,10	5196.12	0.00	2000	285.30	1180.92	1537.57	192.33	0.42
3-20-20	60	2,3	6562.01	0.00	2000	429.41	2336.72	1517.22	278.66	0.31
6-20-20	60	2,5	6539.69	0.00	2000	526.80	2244.55	1489.68	278.66	26.23
10-20-20	60	2,8	6442.93	0.00	2000	466.35	2239.55	1458.37	278.66	3.11
3-30-30	90	1,2	9507.39	0.00	2000	995.98	2868.94	3246.42	396.05	0.63
6-30-30	90	4,6	8611.68	0.00	2000	634.84	2590.81	2986.88	399.15	0.64
10-30-30	90	4,6	8611.68	0.00	2000	634.84	2590.81	2986.88	399.15	2.16
3-40-40	105	1,2,3	12343.66	0.00	3000	1453.00	3429.96	3928.97	513.74	0.84
6-40-40	105	4,5	11108.51	0.00	2000	511.41	3783.69	4315.57	497.83	0.89
10-40-40	105	2,10	10577.83	0.00	2000	749.02	3467.83	3849.30	511.68	5.77
3-50-50	120	1,2,3	13616.69	0.00	3000	1367.47	4474.37	4183.00	591.85	0.45
6-50-50	120	2,3,5	13236.22	0.00	3000	1074.99	4337.94	4208.07	615.22	1.16
10-50-50	120	2,3,10	13086.26	0.00	3000	1148.19	4290.22	4049.66	598.19	5.13

**CPLEX results of the collection VRP and the delivery VRP (phase 2)**

The second phase of the proposed model deals with several collection VRPs and delivery VRPs. Based on the hub location and allocation scheme from the first phase, the distinct collection VRP and delivery VRP are solved for each open hub, minimizing the routing cost and the cost of using vehicles. Tables 9.3 and 9.4 present the results of the separate collection VRPs and delivery VRPs. Table 9.5 shows the general results for each problem instance based on Tables 9.3 and 9.4. It records the total routing cost, the total number of routes numbers, and the total computing time of the collection VRP and the delivery VRP. The column of  $\sum UB$  and  $\sum LB$  represent the total value of the upper bounds and lower bounds of all the collection and delivery VRPs (see Tables 9.3 and 9.4). The column of  $Gap'$  records the deviation of the  $\sum UB$  and  $\sum LB$  where  $Gap' = (\sum UB - \sum LB) / \sum UB \times 100\%$ . The optimal solutions for all the small and medium size instances can be found by CPLEX with a short computing time. High quality solutions of all the large instances can be found with a small gap which, however, consumes a big computing time.

**Results comparison with the global model of the HLRP**

As mentioned in Chapter 8, the total network cost includes : the hub location cost, the inter-hub cost and the handling cost during the first phase ; the collection cost, the delivery cost and the cost of using vehicles during the second phase. Table 9.6 compares the CPLEX results by the global model in Chapter 5 and the two-phase model in terms of the total cost, the gap, the location scheme, the route numbers and the computing times. Here the column of  $Gap_1$  records the gaps of the HLP model while the column of  $Gap_2$  presents the general gaps of the VRP models taking the values of the  $Gap'$  in Table 9.5. The column of Total time sums the CPU time of the first and second phase. Since it is difficult for the CPLEX solver to solve large instances for the global model which yields a large gap, only the results of the small and medium instances are compared in Table 9.6. The results of large problem instances of the two-phase method can be found in Table 9.9.

According to the results, the two-phase model solves all the small and medium instances to optimality with a quite short computing time, while the global model of the HLRP can only solve small instances to optimality. It finds feasible solutions for medium instances with a computing time of 3 hours (Table 9.6). Regarding the solutions of the small instances obtained by the HLRP model, the results are better than which obtained by the two-phase model even if the solutions are optimal in both phases. It is obvious that

TABLE 9.3 : CPLEX results of the collection VRP

Instance $ H - I - J $	Hub capacity	Open hub	$UB$ (Cost)	$LB$ (Cost)	$Gap$ (%)	CPU time (s)	Route numbers
3-10-10	15	1	585.45	585.45	0.00	0.03	1
		3	631.25	631.25	0.00	0.06	1
6-10-10	15	3	512.98	512.98	0.00	0.08	1
		4	637.25	637.25	0.00	0.03	1
10-10-10	15	4	451.40	451.40	0.00	0.05	1
		8	832.03	832.03	0.00	0.05	1
3-15-15	45	1	1577.93	1577.93	0.00	0.06	2
		3	1010.66	1010.66	0.00	10.00	4
6-15-15	45	4	576.67	576.67	0.00	0.22	3
		5	1699.09	1699.09	0.00	0.52	3
10-15-15	45	7	809.10	809.10	0.00	0.23	3
		10	1264.55	1264.55	0.00	0.06	3
3-20-20	60	2	2251.14	2251.14	0.00	1.05	4
		3	1394.19	1394.19	0.00	0.20	4
6-20-20	60	2	2251.14	2251.14	0.00	1.13	4
		5	1341.03	1341.03	0.00	0.13	4
10-20-20	60	2	2251.14	2251.14	0.00	1.00	4
		8	1314.97	1314.97	0.00	0.14	4
3-30-30	90	1	2731.86	2731.86	0.00	0.50	5
		2	1951.16	1951.14	0.00	621.83	7
6-30-30	90	4	1712.68	1712.67	0.00	44.20	6
		6	2685.78	2685.78	0.00	0.33	5
10-30-30	90	4	1712.68	1712.67	0.00	43.81	6
		6	2685.78	2685.78	0.00	0.31	5
3-40-40	105	1	1425.93	1425.93	0.00	0.91	4
		2	2547.75	2532.77	0.59	10806.55	8
		3	992.39	992.39	0.00	0.16	3
6-40-40	105	4	2909.31	2871.27	1.31	10806.97	7
		5	2614.78	2496.60	4.52	2109.70	8
10-40-40	105	2	3028.62	3028.59	0.00	674.17	8
		10	2139.47	2079.75	2.79	7155.92	7
3-50-50	120	1	1443.67	1443.66	0.00	3.15	4
		2	2789.64	2721.88	2.43	4738.75	9
		3	1630.02	1630.02	0.00	1.11	4
6-50-50	120	2	2244.04	2244.02	0.00	33.67	6
		3	1092.73	1092.73	0.00	0.05	3
		5	2229.07	2229.04	0.00	883.20	7
10-50-50	120	2	2921.79	2860.40	2.10	3562.27	9
		3	1092.73	1092.73	0.00	0.09	3
		10	1507.09	1507.08	0.00	36.19	4

TABLE 9.4 : CPLEX results of the delivery VRP

Instance $ H - I - J $	Hub capacity	Open hub	$UB$ (Cost)	$LB$ (Cost)	$Gap$ (%)	CPU time (s)	Route numbers
3-10-10	15	1	507.49	507.49	0.00	0.08	1
		3	608.53	608.53	0.00	0.02	1
6-10-10	15	3	772.01	772.01	0.00	0.03	1
		4	497.11	497.11	0.00	0.05	1
10-10-10	15	4	425.00	425.00	0.00	0.02	1
		8	755.56	755.56	0.00	0.03	1
3-15-15	45	1	1172.78	1172.78	0.00	0.05	3
		3	1229.36	1229.36	0.00	0.36	3
6-15-15	45	4	704.73	704.73	0.00	0.05	2
		5	1724.79	1724.79	0.00	3.41	4
10-15-15	45	7	1113.64	1113.64	0.00	0.08	3
		10	1287.45	1287.45	0.00	0.16	3
3-20-20	60	2	1321.66	1321.66	0.00	0.41	4
		3	1508.43	1508.42	0.00	2.92	5
6-20-20	60	2	1321.66	1321.66	0.00	0.31	4
		5	1457.22	1457.22	0.00	2.20	5
10-20-20	60	2	1321.66	1321.66	0.00	0.38	4
		8	1430.89	1430.89	0.00	1.66	5
3-30-30	90	1	2577.86	2577.86	0.00	0.48	5
		2	2364.96	2364.94	0.00	225.69	7
6-30-30	90	4	1969.22	1969.22	0.00	5.15	6
		6	2652.07	2652.07	0.00	8.14	6
10-30-30	90	4	1969.22	1969.22	0.00	5.10	6
		6	2652.07	2652.07	0.00	8.16	6
3-40-40	105	1	1033.47	1033.47	0.00	0.05	3
		2	3067.06	3013.17	1.76	3507.99	8
		3	1466.10	1466.10	0.00	0.34	4
6-40-40	105	4	3149.65	3149.61	0.00	675.08	7
		5	2674.08	2621.85	1.95	3113.94	7
10-40-40	105	2	2775.43	2775.40	0.00	542.49	7
		10	2497.90	2448.25	1.99	2204.20	8
3-50-50	120	1	1414.71	1414.69	0.00	4026.40	5
		2	2692.72	2617.58	2.79	5592.06	9
		3	1225.44	1225.44	0.00	0.31	3
6-50-50	120	2	1799.47	1799.45	0.00	52.65	6
		3	1356.18	1356.18	0.00	0.14	3
		5	2277.85	2228.99	2.14	2385.32	7
10-50-50	120	2	2458.83	2458.81	0.00	122.85	8
		3	1225.44	1225.44	0.00	0.28	3
		10	1523.21	1523.21	0.00	22.81	5



TABLE 9.5 : General CPLEX results of the second phase (VRP)

Instance	Hub capacity	Open hub	$\sum UB$ (cost)	$\sum LB$ (cost)	$Gap'$ (%)	Total route numbers	Total time (s)
3-10-10	15	1,3	2332.72	2332.72	0.00	4	0.19
6-10-10	15	3,4	2419.35	2419.35	0.00	4	0.19
10-10-10	15	4,8	2463.99	2463.99	0.00	4	0.15
3-15-15	45	1,3	4990.73	4990.73	0.00	12	10.47
6-15-15	45	4,5	4705.28	4705.28	0.00	12	4.20
10-15-15	45	7,10	4474.74	4474.74	0.00	12	0.53
3-20-20	60	2,3	6475.42	6475.42	0.00	17	4.58
6-20-20	60	2,5	6371.05	6371.05	0.00	17	3.77
10-20-20	60	2,8	6318.66	6318.66	0.00	17	3.18
3-30-30	90	1,2	9625.84	9625.84	0.00	24	848.50
6-30-30	90	4,6	9019.75	9019.75	0.00	23	57.82
10-30-30	90	4,6	9019.75	9019.75	0.00	23	57.38
3-40-40	105	1,2,3	10532.70	10463.83	0.65	30	14316.00
6-40-40	105	4,5	11347.82	11139.33	1.84	29	16705.69
10-40-40	105	2,10	10441.42	10331.99	1.05	30	10576.78
3-50-50	120	1,2,3	11196.20	11053.27	1.28	34	14361.78
6-50-50	120	2,3,5	10999.34	10950.41	0.44	32	3355.03
10-50-50	120	2,3,10	10729.09	10667.67	0.57	32	3744.49

the two-phase model leads to sub-optimal results. Since the HLRP model is hard to solve by means of CPLEX solver, the results subjects to a big gap for the medium instances while the two-phase model is able to yield optimal solutions in a short computing time.

TABLE 9.6 : Results comparison between the global model of the HLRP and the two-phase model

Instance $ H - I - J $	Hub capacity	Global model of the HLRP (CPLEX)					Two-phase model (CPLEX)					
		Total cost	Gap (%)	Open hub	Route numbers	CPU Time (s)	Total cost	$Gap_1$ (%) (HLP)	$Gap_2$ (%) (VRP)	Open hub	Route numbers	Total Time (s)
3-10-10	15	<b>4269.15</b>	0.00	1,2	4	217.22	4578.26	0.00	0.00	1,3	4	<b>0.42</b>
6-10-10	15	<b>4269.15</b>	2.07	1,2	4	10800.00	4611.80	0.00	0.00	3,4	4	<b>0.46</b>
10-10-10	15	<b>4363.80</b>	4.45	8,10	4	10800.20	4643.65	0.00	0.00	4,8	4	<b>1.09</b>
3-15-15	45	<b>7584.04</b>	3.15	1,3	12	10536.69	7638.35	0.00	0.00	1,3	12	<b>10.69</b>
6-15-15	45	<b>7107.65</b>	3.56	3,5	12	10800.15	7152.21	0.00	0.00	4,5	12	<b>4.53</b>
10-15-15	45	7013.07	9.72	7,10	12	10800.11	<b>6952.37</b>	0.00	0.00	7,10	12	<b>0.95</b>
3-20-20	60	<b>8825.23</b>	1.75	2,3	16	10800.09	9183.49	0.00	0.00	2,3	17	<b>4.89</b>
6-20-20	60	9257.22	11.60	4,5	16	10800.23	<b>9176.50</b>	0.00	0.00	2,5	17	<b>30.00</b>
10-20-20	60	9520.10	19.98	5,9	17	10800.30	<b>9063.67</b>	0.00	0.00	2,8	17	<b>6.29</b>
3-30-30	90	<b>12945.40</b>	9.71	1,2	24	10800.20	13017.87	0.00	0.00	1,2	24	<b>849.13</b>
6-30-30	90	13838.80	24.40	2,6	27	10800.90	<b>12053.74</b>	0.00	0.00	4,6	23	<b>58.46</b>
10-30-30	90	15289.90	32.63	4,5,8	27	10800.70	<b>12053.74</b>	0.00	0.00	4,6	23	<b>59.54</b>

### 9.3 MA results of the two-phase method

Similar to the parameter settings of the global MA in Chapter 5, the two-phase MA runs 10 times and keeps the best solution. In the first phase of the CSAHLP, the population of the generation is 200. The probability of crossover and mutation are set to 0.8 and 0.7, respectively. The local search in the first phase is called after every 10 iterations of the genetic part. In the second phase, the local search stops when it has been called on all the supplier and client nodes. Two stopping criteria were set for each run of the MA : the iterations were stopped whenever no improvement was obtained every 100 iterations or the total iteration reaches 200.

The first phase of the MA solves the hub location problems and the results are compared with that obtained by CPLEX in Table 9.7. The MA finds the optimal solutions for the small instances and is able to find feasible solutions for the medium and large instances with a small gap. In all cases, the numbers of open hubs are identical for CPLEX and the MA and only four out of the eighteen instances open different hubs.

The second phase of the MA deals with several collection VRPs and delivery VRPs by calling the local searches. The general results of the VPRs are shown in Table 9.8 and are compared with the results of the two-phase model. In most cases, the model of the VPRs is able to find better solutions than the MA but with a longer computing time in solving large instances. For some instances, the MA can find the solutions that are better than the optimal solutions of the model of the VPRs. The reason is that scheduling of the collection and delivery routes is based on the location and allocation in the first phase. Opening different hubs or allocating different suppliers or clients to the same hub will greatly influence the solutions of the second phase.

Table 9.9 shows the final results of the two-phase model and the MA. Since most of the solutions by CPLEX are optimal in both phases of the CSAHLP and CVRP, CPLEX performs better than the MA in most cases. On the other hand, the MA is able to find feasible solutions in a shorter computing time than CPLEX for solving the medium and large instances. However, for all the instances, the results of the global MA are better than the two-phase MA with a slightly longer computing time (Table 9.10).

TABLE 9.7 : Results comparison between the model and the MA of the first phase (HLP)

Instance $ H - I - J $	Hub capacity	The model of HLP (CPLEX)				Phase 1 of the MA (HLP)			
		$UB$ (Cost)	$Gap$ (%)	Open hub	CPU time (s)	$Z_{best}$ (Cost)	$GapMA$ (%)	Open hub	CPU time (s)
3-10-10	15	<b>3854.04</b>	<b>0.00</b>	1,3	<b>0.23</b>	<b>3854.04</b>	<b>0.00</b>	1,3	1.16
6-10-10	15	<b>3833.58</b>	<b>0.00</b>	3,4	<b>0.27</b>	<b>3833.58</b>	<b>0.00</b>	3,4	1.57
10-10-10	15	<b>3778.09</b>	<b>0.00</b>	4,8	<b>0.94</b>	3810.67	0.85	1,10	2.05
3-15-15	45	<b>5889.24</b>	<b>0.00</b>	1,3	<b>0.22</b>	<b>5889.24</b>	<b>0.00</b>	1,3	1.68
6-15-15	45	<b>5444.12</b>	<b>0.00</b>	4,5	<b>0.33</b>	<b>5444.12</b>	<b>0.00</b>	4,5	2.25
10-15-15	45	<b>5196.12</b>	<b>0.00</b>	7,10	<b>0.42</b>	5227.25	0.60	9,10	4.39
3-20-20	60	<b>6562.01</b>	<b>0.00</b>	2,3	<b>0.31</b>	6584.72	0.34	2,3	2.63
6-20-20	60	<b>6539.69</b>	<b>0.00</b>	2,5	26.23	6550.50	0.17	3,4	<b>4.58</b>
10-20-20	60	<b>6442.93</b>	<b>0.00</b>	2,8	<b>3.11</b>	6509.57	1.02	2,8	14.01
3-30-30	90	<b>9507.39</b>	<b>0.00</b>	1,2	<b>0.63</b>	9558.45	0.53	1,2	4.10
6-30-30	90	<b>8611.68</b>	<b>0.00</b>	4,6	<b>0.64</b>	8645.20	0.39	4,6	22.58
10-30-30	90	<b>8611.68</b>	<b>0.00</b>	4,6	<b>2.16</b>	8662.85	0.59	2,6	36.25
3-40-40	105	<b>12343.66</b>	<b>0.00</b>	1,2,3	<b>0.84</b>	12446.11	0.82	1,2,3	8.17
6-40-40	105	<b>11108.51</b>	<b>0.00</b>	4,5	<b>0.89</b>	11147.99	0.35	4,5	35.45
10-40-40	105	<b>10577.83</b>	<b>0.00</b>	2,10	<b>5.77</b>	10596.33	0.17	2,10	175.46
3-50-50	120	<b>13616.69</b>	<b>0.00</b>	1,2,3	<b>0.45</b>	13639.32	0.17	1,2,3	11.55
6-50-50	120	<b>13236.22</b>	<b>0.00</b>	2,3,5	<b>1.16</b>	13249.07	0.10	2,3,5	219.81
10-50-50	120	<b>13086.26</b>	<b>0.00</b>	2,3,10	<b>5.13</b>	13105.70	0.15	2,3,10	335.96

In order to assess the performance of the two-phase MA, we computed the coefficient of relative standard deviation (RSD) for 10 runs,  $RSD = \frac{SD}{\bar{Z}} \times 100\%$ .  $SD$  is the standard deviation between the average cost value  $\bar{Z}$  in 10 runs and the best value  $Z_i$  found by the MA for the  $i$ th run,  $SD = \sqrt{\frac{\sum_{i=1}^{10} (\bar{Z} - Z_i)^2}{10}}$ . Figure 9.1 gives an insight of the  $RSD$  value for different problem scales. Most of the  $RSD$  values are below 2% and the curves show relatively smooth evolution, which proves the stability of the two-phase MA for decision-makings.

TABLE 9.8 : Results comparison between the model and the MA of the second phase (VRP)

Instance $ H - I - J $	Hub capacity	Two-phase model-VRP						Two-phase MA-VRP			
		Open hub	$\sum UB$ (cost)	$\sum LB$ (cost)	$Gap'$ (%)	Routes numbers	Total time (s)	Open hub	Routing cost	Routes numbers	Total time (s)
3-10-10	15	1,3	<b>2332.72</b>	2332.72	0.00	4	<b>0.19</b>	1,3	<b>2332.72</b>	4	10.03
6-10-10	15	3,4	<b>2419.35</b>	2419.35	0.00	4	<b>0.19</b>	3,4	<b>2419.35</b>	4	12.51
10-10-10	15	4,8	2463.99	2463.99	0.00	4	<b>0.15</b>	1,10	<b>2354.41</b>	4	25.67
3-15-15	45	1,3	<b>4990.73</b>	4990.73	0.00	12	<b>10.47</b>	1,3	5005.85	12	13.90
6-15-15	45	4,5	<b>4705.28</b>	4705.28	0.00	12	<b>4.20</b>	4,5	4749.66	12	21.36
10-15-15	45	7,10	<b>4474.74</b>	4474.74	0.00	12	<b>0.53</b>	9,10	4608.48	12	62.64
3-20-20	60	2,3	6475.42	6475.42	0.00	17	<b>4.58</b>	2,3	<b>6466.41</b>	16	27.99
6-20-20	60	2,5	<b>6371.05</b>	6371.05	0.00	17	<b>3.77</b>	3,4	6400.58	17	48.65
10-20-20	60	2,8	6318.66	6318.66	0.00	17	<b>3.18</b>	7,8	<b>6285.88</b>	18	124.12
3-30-30	90	1,2	<b>9625.84</b>	9625.84	0.00	24	848.50	1,2	9770.02	24	<b>44.90</b>
6-30-30	90	4,6	<b>9019.75</b>	9019.75	0.00	23	<b>57.82</b>	4,6	9385.50	25	171.43
10-30-30	90	4,6	<b>9019.75</b>	9019.75	0.00	23	<b>57.38</b>	2,6	9456.10	24	532.87
3-40-40	105	1,2,3	<b>10532.70</b>	10463.83	0.65	30	14316.00	1,2,3	11215.94	30	<b>84.12</b>
6-40-40	105	4,5	<b>11347.82</b>	11139.33	1.84	29	16705.69	4,5	11865.65	30	<b>377.53</b>
10-40-40	105	2,10	<b>10441.42</b>	10331.99	1.05	30	10576.78	2,10	11214.13	30	<b>1851.97</b>
3-50-50	120	1,2,3	<b>11196.20</b>	11053.27	1.28	34	14361.78	1,2,3	12250.97	32	<b>97.21</b>
6-50-50	120	2,3,5	<b>10999.34</b>	10950.41	0.44	32	3355.03	2,3,5	11945.54	32	<b>2014.74</b>
10-50-50	120	2,3,10	<b>10729.09</b>	10667.67	0.57	32	3744.49	2,3,10	11596.89	34	5254.44

TABLE 9.9 : Results comparison between the two-phase model and the two-phase MA

Instance $ H - I - J $	Hub capacity	Two-phase model (MA)				Two-phase model (CPLEX)					
		Total cost	Open hub	Routes number	CPU time (s)	Total cost	$Gap_1$ (%) (HLP)	$Gap_2$ (%) (VRP)	Open hub	Routes number	Total time (s)
3-10-10	15	<b>4578.26</b>	1,3	4	11.10	<b>4578.26</b>	0.00	0.00	1,3	4	<b>0.42</b>
6-10-10	15	<b>4611.80</b>	3,4	4	13.37	<b>4611.80</b>	0.00	0.00	3,4	4	<b>0.46</b>
10-10-10	15	<b>4515.46</b>	1,10	4	26.54	4643.65	0.00	0.00	4,8	4	<b>1.09</b>
3-15-15	45	7653.46	1,3	12	16.08	<b>7638.35</b>	0.00	0.00	1,3	12	<b>10.69</b>
6-15-15	45	7196.60	4,5	12	23.34	<b>7152.21</b>	0.00	0.00	4,5	12	<b>4.53</b>
10-15-15	45	7156.23	9,10	12	64.67	<b>6952.37</b>	0.00	0.00	7,10	12	<b>0.95</b>
3-20-20	60	9201.86	2,3	16	31.40	<b>9183.49</b>	0.00	0.00	2,3	17	<b>4.89</b>
6-20-20	60	<b>9142.46</b>	3,4	17	52.06	9176.50	0.00	0.00	2,5	17	<b>30.00</b>
10-20-20	60	9137.27	2,8	18	127.85	<b>9063.67</b>	0.00	0.00	2,8	17	<b>6.29</b>
3-30-30	90	13274.77	1,2	24	<b>46.88</b>	<b>13017.87</b>	0.00	0.00	1,2	24	849.13
6-30-30	90	12479.14	4,6	25	179.56	<b>12053.74</b>	0.00	0.00	4,6	23	<b>58.46</b>
10-30-30	90	12464.06	2,6	24	542.20	<b>12053.74</b>	0.00	0.00	4,6	23	<b>59.54</b>
3-40-40	105	16308.54	1,2,3	30	<b>98.36</b>	<b>15517.44</b>	0.00	0.39	1,2,3	30	14316.00
6-40-40	105	14965.79	4,5	30	<b>394.27</b>	<b>14408.83</b>	0.00	1.95	4,5	29	16705.69
10-40-40	105	14492.03	2,10	30	<b>1868.14</b>	<b>13747.41</b>	0.00	1.20	2,10	30	10576.78
3-50-50	120	17322.89	1,2,3	32	<b>136.67</b>	<b>16210.13</b>	0.00	0.87	1,2,3	34	14361.78
6-50-50	120	16693.08	2,3,5	32	<b>2037.77</b>	<b>15689.55</b>	0.00	0.36	2,3,5	32	3355.03
10-50-50	120	16406.31	2,3,10	34	5277.73	<b>15483.41</b>	0.00	0.35	2,3,10	32	<b>3744.49</b>

TABLE 9.10 : Results comparison between the global MA and the two-phase MA

Instance $ H - I - J $	Hub capacity	HLRP model (MA)				Two-phase model (MA)			
		Total cost	Open hub	Routes number	CPU time (s)	Total cost	Open hub	Routes number	CPU time (s)
3-10-10	15	<b>4269.15</b>	1,2	4	12.60	4578.26	1,3	4	<b>11.10</b>
6-10-10	15	<b>4269.15</b>	1,2	4	59.04	4611.80	3,4	4	<b>13.37</b>
10-10-10	15	<b>4258.45</b>	2,10	4	82.25	4515.46	1,10	4	<b>26.54</b>
3-15-15	45	<b>7638.35</b>	1,3	12	34.19	7653.46	1,3	12	<b>16.08</b>
6-15-15	45	<b>7107.65</b>	3,5	12	40.88	7196.60	4,5	12	<b>23.34</b>
10-15-15	45	<b>6861.88</b>	7,10	12	131.45	7156.23	9,10	12	<b>64.67</b>
3-20-20	60	<b>9048.58</b>	2,3	17	82.64	9201.86	2,3	16	<b>31.40</b>
6-20-20	60	<b>9022.21</b>	3,4	17	116.44	9142.46	3,4	17	<b>52.06</b>
10-20-20	60	<b>8725.87</b>	7,8	17	244.04	9137.27	2,8	18	<b>127.85</b>
3-30-30	90	<b>12994.00</b>	1,2	24	164.00	13274.77	1,2	24	<b>46.88</b>
6-30-30	90	<b>11973.20</b>	2,6	24	411.62	12479.14	4,6	25	<b>179.56</b>
10-30-30	90	<b>11987.10</b>	4,6	22	847.83	12464.06	2,6	24	<b>542.20</b>
3-40-40	105	<b>15233.90</b>	2,3	29	295.92	16308.54	1,2,3	30	<b>98.36</b>
6-40-40	105	<b>14317.30</b>	4,5	30	507.60	14965.79	4,5	30	<b>394.27</b>
10-40-40	105	<b>13838.00</b>	2,10	29	2591.37	14492.03	2,10	30	<b>1868.14</b>
3-50-50	120	<b>16324.10</b>	1,2	32	723.49	17322.89	1,2,3	32	<b>136.67</b>
6-50-50	120	<b>15564.30</b>	2,5	32	2558.65	16693.08	2,3,5	32	<b>2037.77</b>
10-50-50	120	<b>15471.72</b>	8,10	32	5645.03	16406.31	2,3,10	34	<b>5277.73</b>

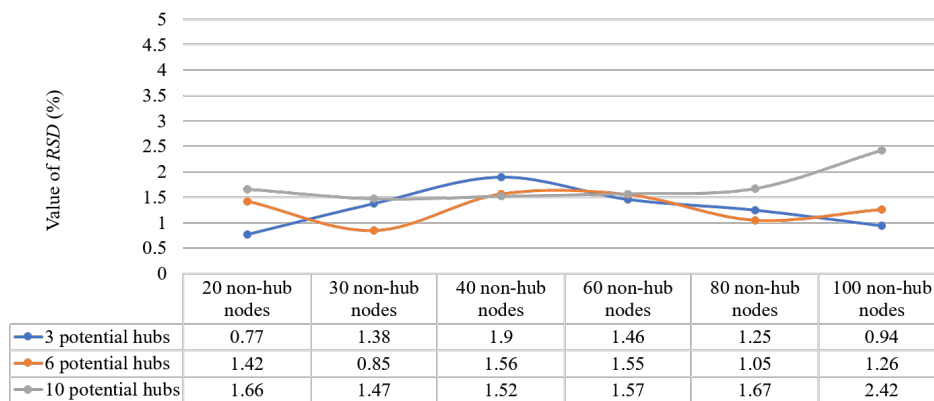


FIGURE 9.1 : RSD indicator with different numbers of instance nodes (the two-phase MA)

## 9.4 Conclusion

In this chapter we investigated experimental results of a proposed two-phase method for solving the HLRP by solving an HLP and two VRPs in sequence for each selected hub. We solved the corresponding MILP models with CPLEX as well as with our adapted memetic algorithm in order to assess the performance of this two phase method. To that purpose, the same data set and parameters were adopted as we used for the single-objective HLRP in Chapter 5. Eighteen instances ranging from 20 to 100 nodes have been used for these experiments.

According to the results, the CPLEX solver is able to obtain optimal solutions of the HLP for all the instances with up to 100 nodes in a quite short computing time. It also solves all the small and medium problem instances of the VRP's to optimality while finding high quality solutions for large instances with a small gap. In comparison, solving the global HLRP model performs better than the two-phase model in solving small instances even if the solutions of the two-phase method are optimal. This was to be expected. On the other hand, solving the two-phase model yields better solutions than the global model in solving medium instances because the solutions of the global model are subject to a big gap.

The proposed global MA is capable to be adapted to solve the HLP, finding the optimal solutions for the small instances and high quality feasible solutions for the medium and large instances with a small gap

(the average gap for all the instances is 0.35%). However, since we implemented simple local searches for solving the routing schemes, the final solutions of the two-phase MA are inferior to those of the global MA for solving the HLRP. Sensitivity analyses of the two-phase MA shows its stability and robust in a decision-making process. Regarding computing times, the two-phase MA is slightly faster than the global MA in obtaining solutions for all the test instances. The two-phase model is faster than the two-phase MA in solving small and medium instances while it needs clearly higher running times to solve the large instances.

These results validate the interest of using this two-phase method for solving large instances of the HLRP in a reasonable computing time. In this chapter, we limited the research to straightforward adaptations of our MILP model and memetic algorithm to investigate the two phase method. The validation obtained would encourage to test efficient specialized algorithms for the HLP and the VRP to solve the two-phase method.



## **General conclusions and prospects**



## General conclusions and prospects

In this thesis, we addressed the Capacitated Single Allocation Hub Location-Routing Problem (CSAHLRP) when collection and delivery routes to and from the hubs are organized separately for logistical reasons. After a comprehensive state of art of the HLRP and related problems, we first addressed the single objective problem aimed at the hub network design and optimization for minimum total cost. We proposed and experimented a MILP formulation as well as a memetic algorithm (MA) to solve the problem. Then we extended the formulation and MA solution technique to address the bi-objective green HLRP in order to exhibit efficient solutions for the joint minimization of costs and CO<sub>2</sub> emissions from transport. Finally, we proposed a two-step procedure in order to solve the single objective problems by combining the solutions of the corresponding Hub Location Problem (HLP) and the associated Vehicle Routing Problems (VRP). Solutions of all these models and algorithms were experimented and compared on the basis of a set of realistic instances of different sizes and characteristics which we have developed. While the MILP models could only solve small to medium size problems, our Memetic Algorithms proved to be able to find high quality solution for all instances in a reasonable computing time.

Four major contributions were made in this thesis. The first contribution was to provide a state-of-the-art of the HLRP where we compared all variations of this class of problems addressed so far. Reviews of related problems, the Hub Location Problem (HLP) and the Location and Routing Problem (LRP), were also presented supplemented by recent works. The models and solution methods for the HLP have been summarized and analyzed in several review papers. The latest published survey paper covers the research on this problem conducted before the year 2012 (Farahani et al. [2013]). We discussed the main criteria for classifying the HLPs and presented some fundamental mathematical models. Further, the articles on the HLPs published after the year 2012 were reviewed to bridge the gap until now. Regarding the LRP, we focused on the standard Capacitated Location-routing Problem (CLRP). The problem definition, mathematical models and solution methods to solve this problem were summarized. Furthermore, reviews of the CLRP from the year of 2002 to 2017 were discussed. We also highlighted the main differences between the HLRP, the HLP and the LRP. The published models and solution techniques for the HLRP were reviewed. Finally, we discussed environmental aspects and modelling techniques for evaluating the CO<sub>2</sub> emissions from transport and some corresponding works on the VRP. The main goal of surveying the literature was to identify research directions that would be promising for addressing the HLRP in terms of problem modelling and solution methods.

The second contribution was the proposition of a single-objective mathematical model of the Capacitated Single Allocation Hub Location-Routing Problem (CSAHLRP) and a memetic metaheuristic to solve large-size problems. Our HLRP model combines a classical HLP formulation with recent flow formulations that have been proposed for the vehicle routing and location-routing problems. Our proposed MA combines



a heuristic for the generation of the initial population, a specifically designed genetic algorithm and several local searches on the location and routing parts of the problem. To generate the solutions of least total cost, extensive comparative experiments were conducted with the CPLEX solver and with the MA on the basis of the set of instances which we have generated. Even with specially tuned parameters, CPLEX could solve the model to optimality only for small instances and found feasible solutions for larger instances with a gap, whose size grows with that of the instances. The MA was able to find feasible solutions to all types of test instances in a reasonable computing time and with limited gaps compared to the lower bounds of CPLEX.

Our third contribution was to address the green HLRP and develop a bi-objective MILP model and memetic algorithm to solve the CSAHLRP in order to jointly minimize total cost and the CO<sub>2</sub> emissions from transport. In order to do so, we combined our single-objective MA with a fast elitist Non-dominated Sorting Genetic Algorithm (NSGAI). This enabled us to sort the solutions of the initial generation according to different non-dominance levels. Each time one offspring was generated by the GA operators (selection, crossover and mutation) and local searches, an efficient non-domination level update (ENLU) method was employed to add a new individual and delete a previously selected one, on the basis of a crowded comparison operator. The bi-objective MA captures the trade-off between minimizing total cost and CO<sub>2</sub> emissions and exhibits approximations of the Pareto front in a reasonable computing time. Computational experiments were conducted and the results were favorably compared with those obtained by solving our bi-objective MILP model with an epsilon constraint method.

Our fourth contribution was to propose a two-phase methods to solve the single objective HLRP, with the goal of comparing the efficiency of this approach with that of our previously developed global approach. In the first phase, the HLRP was reduced to a classical Capacitated Single Allocation Hub Location Problem (CSAHL). This problem dealt with the location of hubs and direct allocation of suppliers and clients to the open hubs, as well as the determination of the inter-hub flows. In the second phase, based on the solution of the first phase, two classical Vehicle Routing Problem (VRP) remained to be solved for each selected hub, for the determination of the collection and delivery routes each open hub. Corresponding MILP models were proposed on the basis of our initial global model and our global single-objective MA was adapted to sequentially solve the HLP in the first phase and then the VRP with separate collection and delivery processes in the second phase. Experimental results showed that the two-phase method could solve the problem to optimality with a short computing time while the solutions were inferior to the optimal solutions obtained by the global model for small instances, but this could provide an alternative way to determine good approximate solutions to very large problems. This work also showed the flexibility and applicability of our proposed MA, that could be adapted to solve other related problems.

Pursuing the research conducted for this thesis, several further research directions could be considered. Regarding the exact solution method, our MILP modeling for the single objective HLRP was solved directly using the CPLEX solver. One could study different formulations and more efficient solution techniques such as adding efficient valid inequalities or developing a branch and cut algorithm.

Another straightforward future research direction would be to study other cases of the HLRP, such as the HLRP with simultaneous pickup and delivery (HLRPSPD). A major focus of our research has been on the HLRP for freight transport where the collections and deliveries are separated, which corresponds to realistic cases for some classes of logistical problems of goods transportation. In other cases, pick up and deliveries can be handled in the same tours. This is for example the case for postal services where mail or small parcels are simultaneously delivered and collected to and from the same entities and where all nodes may be hub candidates. In another area, beverage companies tend to deliver beverages to clients and collect the empty bottles at the same time (Karaoglan et al. [2012]). Our models and solution techniques could be adapted to handle such situations of pick-up and delivery.

Thirdly, recall that we adapted our single objective procedure in a straightforward manner to the two phases procedure, separating the HLP part and the VRP part. The procedure might be enhanced by using more efficient or exact algorithms to solve the HLP problem of phase one and the VRP problems of phase two.

Another future research direction would be to pursue the study of the green HLRP by improving the

efficiency of our algorithms or adopting another bio-objective method.

Finally, it would be challenging to apply our model and solution techniques to the solution of a real industrial case.



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# Thèse de Doctorat

Xiao YANG

Le problème de Localisation de Hubs et Routage  
dans le contexte de logistique verte

Green hub location-routing problem  
for LTL transport

## Résumé

Le problème de localisation de hubs et tournées combinées (Hub Location-Routing Problem, HLRP), concerne la conception d'un réseau de transport performant entre de nombreuses origines (fournisseurs) et destinations (clients). Ce système est basé sur la localisation de plates formes (hubs) permettant de concentrer les flux et l'organisation de tournées pour la collecte des marchandises des fournisseurs et la distribution vers les clients. Nous étudions le cas spécifique du HLRP à capacités et allocations uniques (CSAHLRP) et de processus de tournées de collecte et distribution séparés. Nous proposons un modèle de programmation linéaire mixte (MILP) et un Algorithme Mémétique (MA) pour ce problème en vue de la minimisation du coût total du réseau de transport. De plus, nous étendons le modèle MILP pour le cas bi-objectif afin de minimiser à la fois le coût total et les émissions de CO<sub>2</sub> du transport. Notre algorithme Mémétique (MA) est adapté et combiné à un algorithme génétique de tri non-dominé élitiste rapide (NSGAI) afin de déterminer des approximations du front de Pareto. Enfin, nous proposons une procédure en deux phases pour résoudre le HLRP mono objectif, comportant la résolution du problème de localisation des hubs (HLP) suivi pour chaque hub de la résolution de deux problèmes de tournées relatifs à la collecte et la livraison. Notre modèle MILP mono objectif est décomposé et notre MA est adapté pour résoudre le problème suivant ces deux étapes. Un ensemble d'instances de différentes tailles et caractéristiques a été développée afin de conduire des expérimentations et de valider nos approches de résolution de ces différents problèmes.

## Mots clés

Problème de localisation de hubs et tournées, logistique verte, transport de fret, modèle bi-objectif, algorithme mémétique.

## Abstract

We study the Hub Location-Routing Problem (HLRP) aiming at the design of an efficient freight transportation network for LTL (less-than-truck) transport between many origins (suppliers) and destinations (clients). Such a network relies on the location of consolidation hubs, the organization of routings for the collection/distribution of freight from suppliers to hubs and from hubs to clients, as well as direct shipment of consolidated freight between hubs. We focus on the Capacitated Single Allocation Hub Location-Routing Problem (CSAHLRP) in the case of distinct collection and delivery processes. We propose mixed integer linear programming (MILP) model and a Memetic Algorithm (MA) to solve the problem for minimizing the total cost of the network. Then we extend the model into a bi-objective model for minimizing both the total cost and CO<sub>2</sub> emissions of transport. A modified memetic algorithm (MA) combined with a fast elitist non-dominated sorting genetic algorithm (NSGAI) is developed to capture the trade-off between minimizing total cost and CO<sub>2</sub> emissions and exhibit approximations of the Pareto front. At last, a two step procedure is proposed to solve the single-objective HLRP based on a hub location problem (HLP) and two distinct vehicle routing problems for suppliers and clients allocated to each hub by the first step. Our single objective MILP model is decomposed accordingly and our MA is adapted to solve the HLRP following these two steps. A data base of instances of different sizes and characteristics has been developed in order to conduct extensive experiments for solving all these problems using the different solution techniques and validate our approaches.

## Key Words

Hub Location-routing Problem, MILP, green logistics, LTL transport, bi-objective model, memetic algorithm.