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# History of Einstein's General Relativity: Conceptual Development of the Theory

Patrick R. Girard

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Ph.D. Thesis (U.S.A.)

THE CONCEPTUAL DEVELOPMENT OF EINSTEIN'S  
GENERAL THEORY OF RELATIVITY

by

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Date of Examination : 24 November 1981

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THE CONCEPTUAL DEVELOPMENT OF EINSTEIN'S  
GENERAL THEORY OF RELATIVITY

A thesis submitted to the Graduate School of the  
University of Wisconsin-Madison in partial fulfillment of  
the requirements for the degree of Doctor of Philosophy

by

PATRICK RALPH GIRARD

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A thesis submitted in partial fulfillment of the  
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DOCTOR OF PHILOSOPHY  
(History of Science)

at the

UNIVERSITY OF WISCONSIN-MADISON

1981



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History of Einstein's General Relativity :  
Conceptual Development of the Theory

Patrick R. Girard

July 2020



# Avant-propos

The work is a revised latex edition, supplemented by explanatory tables, an index and a few remarks of the Ph.D. thesis of Patrick R. Girard, "The Conceptual Development of Einstein's General Theory of Relativity", The University of Wisconsin-Madison (USA), Ph.D. 1981. Among all the physical theories developed by Albert Einstein, the general theory of relativity (*GTR*) is generally considered as his masterpiece. This theory has yielded the most successful treatment of gravitation known so far, and has, to the present day, brilliantly withstood all the experimental tests to which it has been submitted, while many competing theories have been eliminated. Despite its success, the history of the general theory of relativity has received much less attention than the history of the special theory. Though various aspects of the history of the general theory of relativity have been treated elsewhere, no comprehensive, detailed historical account of the successive stages in the development of the general theory of relativity has, to my knowledge, been published. It is the purpose of this dissertation to fill that gap. The dissertation will not only analyze the mathematical development of the theory in detail, from 1907 to 1917, but will also pay close attention to Einstein's motivations. Though it is generally known that Einstein was motivated by epistemological reasons in the development of *GTR*, this is the first time that this claim is thoroughly documented. The work is intended for students, professors, scientists, historians of science, epistemologists and all those interested in the history of physical theories.

Patrick R. Girard  
Lyon, July 2020



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1. The lines below have been written before the discovery of the original text of Hilbert's conference and reflect the situation which prevailed during most of the twentieth century. See the footnote at the end of the appendix.

# Chapitre 1

## Introduction

Among all the physical theories developed by Albert Einstein, the general theory of relativity (*GTR*) is generally considered as his masterpiece. This theory has yielded the most successful treatment of gravitation known so far, and has, to the present day, brilliantly withstood all the experimental tests to which it has been submitted, while many competing theories have been eliminated.

Despite its success, the history of the general theory of relativity has received much less attention than the history of the special theory. Two basic circumstances are probably responsible for this. First, between the 1930s and the 1950s, the general theory of relativity was relatively neglected by the scientists themselves because experimental methods had not yet reached the sophistication required for the further study of general-relativistic effects. Second, the technicality of the subject matter has hindered historical study of *GTR*.

Though various aspects of the history of the general theory of relativity have been treated elsewhere<sup>1</sup> no comprehensive, detailed historical account of the successive stages in the development of the general theory of relativity has, to my knowledge, been published. It is the purpose of this dissertation to fill that gap. The dissertation will not only analyze the mathematical development of the theory in detail, from 1907 to 1917, but will also pay close attention to Einstein's motivations. Though it is generally known that Einstein was motivated by epistemological reasons in the development of *GTR*, this is the first time that this claim is thoroughly documented. By 1905, Einstein had already successfully applied epistemological considerations to physics : it was his rejection of the concept of absolute time that made the special theory of relativity (*STR*) possible. Einstein's next move was to attempt to eliminate the concept of absolute space (in particular the concept of absolute acceleration) by extending the principle of relativity.

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1. See in particular Jagdish Mehra, *Einstein, Hilbert, and the Theory of Gravitation : Historical Origins of General Relativity Theory* (Dordrecht, Holland/Boston, U.S.A. : D. Reidel, 1974). I shall argue against Mehra's claim that Hilbert discovered the field equations independently of Einstein. For other accounts, see the bibliography

In order to show the continuity of Einstein's concerns in the development of *STR* and *GTR*, I will in the first chapter briefly review the development of the special theory of relativity, and analyze Einstein's dissatisfaction with that theory. I will then deal with : Einstein's first attempts to extend the special theory of relativity via the equivalence principle (1907 – 1912); the first sketch of the general theory of relativity (1913 – 1915); and finally, the general theory of relativity as articulated through 1917, when Einstein introduced the cosmological term in his 1915 field equations.

## Chapitre 2

# The Special Theory of Relativity

### 2.1 Einstein and the development of the special theory of relativity

Among the various accounts of the origin of the theory of relativity, many have attempted to find some decisive influence on Einstein's work either in some experiment or in some previous physical theory. Thereby many of these accounts have underestimated (if not missed) the crucial role epistemological considerations played in the development of Einstein's theory. It is the merit of Tetu Hirosige to have fundamentally reevaluated this role.<sup>1</sup> In his article, Hirosige argues that Mach's refutation of the mechanistic worldview was the most fundamental contribution to the development of Einstein's special theory of relativity. It is not our purpose here to discuss in detail any specific influence on Einstein but rather to analyze the development of the special theory of relativity from Einstein's own point of view. This will show to what extent the special theory of relativity was essentially an epistemological success and will help to understand Einstein's epistemological concerns in the development of the general theory of relativity. After a brief historical survey of the principles of the special theory of relativity, we shall analyze Einstein's approach and his achievement.

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1. Tetu Hirosige, "The Ether Problem, the Mechanistic Worldview, and the Origins of the Theory of Relativity," *Historical Studies in the Physical Sciences*, 7 (1976), 3-82. See also : the articles on the relativity theory in Gerald Holton, *Thematic Origins of Scientific Thought, Kepler to Einstein* (Cambridge : Harvard University Press, 1973) ; Arthur I. Miller, *Albert Einstein's Special Theory of Relativity : Emergence (1905) and Early Interpretation (1905-1911)* (Reading, MA : Addison-Wesley, 1981).

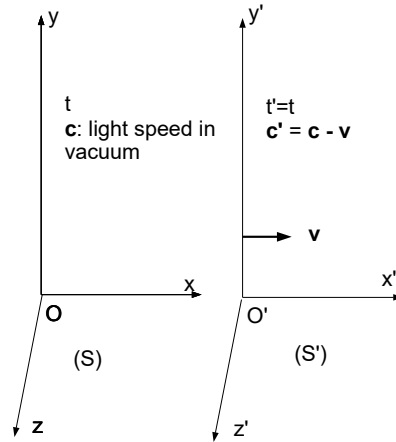


FIGURE 2.1 – Galilean transformation (classical mechanics)

### 2.1.1 Historical Background of the Principles of *STR*

Two basic principles formed the foundation of the special theory of relativity : (1) the principle of relativity, which states that the laws of physics are the same in all inertial systems ; and (2) the principle of the constancy of the velocity of light in vacuum, which states that the velocity of light in an inertial system is equal to a certain velocity  $c$  which is independent of the motion of the source of light.<sup>2</sup> Both of these principles have significant pre-twentieth-century histories.

---

2. Albert Einstein, "Zur Elektrodynamik bewegter Körper," *Annalen der Physik*, **17** (1905) 895.

**Box 1. Classical Mechanics***Absolute time; absolute space**Three postulates :*

a) inertial law;

b)  $\vec{f} = m\vec{\gamma}$ ;

c) action and reaction principle;

$\Rightarrow$  *relativity principle* : the laws of mechanics keep the same form (covariance) in two Galilean reference frames related by a Galilean transform (see Fig. 2.1) :

$$\begin{aligned}t' &= t \\x' &= x - vt \\y' &= y \\z' &= z\end{aligned}$$

reciprocally,

$$\begin{aligned}t &= t' \\x &= x' + vt' \\y &= y' \\z &= z'\end{aligned}$$

*metric (Euclidean) :*

$$\begin{aligned}ds^2 &= dx^2 + dy^2 + dz^2 \\&= ds'^2\end{aligned}$$

 $\Rightarrow$  absolute time interval; $\Rightarrow$  absolute length; $\Rightarrow$  relative light speed in vacuum :  $\mathbf{c}' = \mathbf{c} - \mathbf{v}$ ; $\Rightarrow$  Maxwell's equations (non-covariant form under a Galilean transform).

A mechanical principle of relativity had already been extensively used by Christiaan Huygens and had been firmly integrated in Isaac Newton's theory of mechanics in 1687. Augustin Fresnel's theory of light and later on J. C. Maxwell's electromagnetic theory presented potential challenges to the principle of relativity, since it appeared likely that the optical and electromagnetic laws would be different in reference frames moving with respect to each other. Fresnel showed, however, that a first order (in  $v/c$ ) optical principle of relativity was expected to hold in such reference frames if it was assumed that the light wave was partially dragged along a moving transparent body with the velocity  $(1 - \frac{1}{n^2})v$  where  $v$  is the projection of the velocity of the body on the direction of propagation of the wave and  $n$  the refractive index of the body.<sup>3</sup> This hypothesis, which became later known as Fresnel's principle<sup>4</sup> was confirmed by the experiments of A. H. Fizeau,<sup>5</sup> M. Hoek,<sup>6</sup> G. B. Airy,<sup>7</sup> and particularly by the comprehensive ones of E. Mascart,<sup>8</sup> which won the Grand Prix des sciences mathématiques proposed by the Academy of Sciences in Paris in 1872. In the report of the committee, the general failure, predicted by Fresnel's principle, to detect an influence of the motion of the earth on optical phenomena with terrestrial sources was already interpreted as hinting at the possibility of a general law of nature opposed to the success of such experiments.<sup>9</sup>

In 1892, H. A. Lorentz succeeded in deriving Fresnel's partial drag coefficient for transparent bodies from his electron theory based on Maxwell's equations.<sup>10</sup> Like Fresnel's theory, Lorentz's theory assumed the existence of an ether at rest. In 1895, Lorentz made an attempt to extend the electron theory to electrical as well as optical effects in moving bodies.<sup>11</sup> Introducing a new auxiliary mathe-

---

3. Augustin J. Fresnel, "Sur l'influence du mouvement terrestre dans quelques phénomènes d'optique," *Annales de chimie et de physique*, **9** (1818); *Oeuvres complètes d'Augustin Fresnel* (Paris, 1866); **2**, 627-36; see also Ronald Newburgh, "Fresnel Drag and the Principle of Relativity," *Isis*, **65** (1974), 379-86.

4. "Prix décernés, Année 1872.-Prix extraordinaires. Grand prix des sciences mathématiques. Rapport lu et adopté dans la séance du 14 juillet 1873," *Comptes rendus*, **79** (1874), 1532-33.

5. A. H. L. Fizeau, "Sur les hypothèses relatives à l'éther lumineux, et sur une expérience qui paraît démontrer que le mouvement des corps change la vitesse avec laquelle la lumière se propage dans leur intérieur," *Comptes rendus*, **33** (1851), 349-55.

6. M. Hoek, "Détermination de la vitesse avec laquelle est entraînée une onde lumineuse traversant un milieu en mouvement," *Arch. néerl.*, **3** (1868), 180-85; "Détermination de la vitesse avec laquelle est entraîné un rayon lumineux traversant un milieu en mouvement," *Arch. néerl.*, **4** (1869), 443-50.

7. G. B. Airy, "On a Supposed Alteration in the Amount of Astronomical Aberration of Light, Produced by the Passage of Light Through a Considerable Thickness of Refractive Medium," *Phil. Mag.* **43** (1872), 310-13; "Additional Note to the Paper 'On a Supposed Alteration . . .,'" *Phil. Mag.*, **45** (1873), 306.

8. E. Mascart, "Sur les modifications qu'éprouve la lumière par suite du mouvement de la source lumineuse et du mouvement de l'observateur," *Annales scientifiques de l'École Normale Supérieure*, **1** (1872), 157-214, first part; *ibid.*, **3** (1874), 363-420, second part.

9. "Prix décernés," p. 1533.

10. H. A. Lorentz, "La théorie électromagnétique de Maxwell et son application aux corps mouvants," *Arch. néerl.*, **25** (1892), 363-551.

11. H. A. Lorentz, *Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern* (Leiden, 1895); in H. A. Lorentz, *Collected Papers* (The Hague: Martinus

## 2.1. EINSTEIN AND THE DEVELOPMENT OF THE SPECIAL THEORY OF RELATIVITY 7

mathematical variable  $t' = t - \frac{vx}{c^2}$  which he called the "local time," and making use of the usual Galilean transformation

$$x' = x - vt, y' = y, z' = z$$

(with the usual conventions about the orientation of the axes), Lorentz managed to obtain a first order relativity (in  $v/c$ ) for Maxwell's equations and the Lorentz force. Lorentz's 1895 theory successfully explained the failure of nearly all attempts to detect the motion of the earth relative to the ether, except a few second order experiments—among them the Michelson-Morley experiment (1887)—for which Lorentz introduced ad hoc hypotheses like the Lorentz-Fitzgerald contraction hypothesis. Later on, reacting to criticism by Henri Poincaré and to the growing importance of second order experiments, Lorentz attempted to find a single explanation for the absence of effects of any order. His 1904 theory<sup>12</sup> neared the goal by accumulating the hypotheses, but Lorentz never achieved a strict covariance (i.e., same form of all equations in any inertial system) as is evident from Lorentz's 1915 statement:

Besides the fascinating boldness of its starting point, Einstein's theory has another marked advantage over mine. Whereas I have not been able to obtain for the equations referred to moving axes *exactly* the same form as for those which apply to a stationary system, Einstein has accomplished this by means of a system of new variables slightly different from those which I have introduced.<sup>13</sup>

Lorentz's theories were in fact quite unrelativistic in spirit. His local time variable never had any physical meaning for him up to 1915 as is plain from Lorentz's own remarks :

The chief cause of my failure [to achieve the simplicity Einstein gave to electromagnetism] was my clinging to the idea that the variable  $t$  only can be considered as the true time and that my local time  $t'$  must be regarded as no more than an auxiliary mathematical quantity.<sup>14</sup>

Furthermore, up to 1915 at least, Lorentz believed in the existence of an ether.<sup>15</sup> Poincaré, on the other hand, was ready to reject the ether<sup>16</sup> and must be credited with the explicit formulation of the principle of relativity in 1904.<sup>17</sup>

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Nijhoff, 1937), 5, 1-137.

12. H. A. Lorentz, "Electromagnetic phenomena in a system moving with any velocity smaller than that of light," *Proceedings of the Academy of Sciences of Amsterdam*, 6 (1904); rpt from the English version in H. A. Lorentz, A. Einstein et al., *The Principle of Relativity*, with notes by A. Sommerfeld, trans. W. Perrett and G. B. Jeffery (1923; rpt. New York : Dover, 1952).

13. H. A. Lorentz, *The Theory of Electrons*, 2nd ed. (1915; rpt. New York : Dover, 1952), p. 230, par. 194. Emphasis in original.

14. *Ibid.*, p. 321, par. 72.

15. *Ibid.*, p. 230, par. 194.

16. Henri Poincaré, *La Science et l'Hypothèse* (1902; rpt. Paris : Flammarion, 1968), p. 215.

17. Henri Poincaré, "L'état actuel et l'avenir de la physique mathématique," *Bulletin des Sciences Mathématiques*, 28 (1904), 306. For more on Poincaré's contributions to the special



As to the constancy of the velocity of light, the first important step came in 1676, when Ole R mer found the velocity of light to be finite and calculated its value from astronomical observations. R mer's answer was confirmed in 1727 by James Bradley through the aberration effect. In their calculations, both men already implicitly assumed a constancy (in space and time) of the velocity of light in vacuum. Fresnel's wave theory of light was to make the constancy of the velocity of light with respect to the optical ether (whatever the direction and motion of the light source) a natural assumption. Such an assumption clearly conflicted with the predictions of the emission theory according to which the velocity of light in an inertial system of reference should depend on the motion of the source. Fresnel's optical principle of relativity also ensured that the velocity of light was the same (to the first order in  $v/c$ ) for all inertial systems. The constancy within a given inertial system was confirmed by many experiments, in particular by those of Mascart (1872 – 74)<sup>18</sup>, and became a cornerstone of Lorentz's widely accepted 1895 electron theory. Lorentz progressively extended the first order (in  $v/c$ ) principle of the constancy of the velocity of light to other orders before 1905.

Thus by 1905, both principles of STR were already present to some extent. Yet, it was only in Einstein's hands that these two principles were to be synthesized into an entirely new theoretical edifice.

### 2.1.2 Einstein's approach

Einstein, on various occasions, has explained how he developed the special theory of relativity. The earliest and most detailed account was given by Einstein to his friend Max Wertheimer, who around 1916 questioned him in great detail about the concrete evolution of his thoughts during the development of STR. This account, together with the other available accounts and Einstein's scientific papers, allow the following reconstruction.<sup>19</sup>

#### Einstein's early doubts about the concept of absolute velocity

It seems that already at the age of 16, Einstein had some doubts about the idea of an absolute velocity.<sup>20</sup> Wondering what a light beam would look like to an observer pursuing the beam with the velocity of  $c$  (velocity of light in vacuum), Einstein later remarked :

From the very beginning it appeared to me intuitively clear that, judged from the stand-point of such an observer, everything would have to happen according to the same laws as for an observer who,

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theory of relativity, see G. H. Keswani, "Origin and Concept of Relativity," *British Journal for the Philosophy of Science*, **15** (1965), 286-306 ; **16** (1965), 19-32.

18. Mascart, "Sur les modifications."

19. Max Wertheimer, "Einstein : The Thinking that led to the Theory of Relativity," in Max Wertheimer, *Productive Thinking*, enl. ed. edited by Michael Wertheimer (New York : Harper & Brothers, 1959), chap. 10, pp. 213-33. For a list of Einstein's accounts, see Hirosgie, "Ether Problem" (1976), pp. 52-53.

20. Wertheimer, "Einstein," p. 215.

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relative to the earth, was at rest. For how, otherwise, should the first observer know, i.e., be able to determine, that he is in a state of fast uniform motion? <sup>21</sup>

The passage reveals that Einstein's early notion of the principle of relativity was closely related to his doubts that there could be such a thing as an absolute velocity. Mechanics had already confirmed the principle of relativity for inertial systems. That the principle of relativity should hold exactly for mechanics and be invalid in another domain was for Einstein "*a priori* not very probable" <sup>22</sup>

Two years after he entered the Federal Polytechnic in Zürich, Einstein tried to design an apparatus which would measure the earth's absolute velocity against the ether. Einstein's project failed, however, since there was no opportunity to build the apparatus and because "the skepticism of his teachers was too great, the spirit of enterprise too small." <sup>23</sup> Indeed, Einstein's desire to design this experiment "was always accompanied by some doubt that the thing was really so" <sup>24</sup> (i.e., that one could detect the absolute velocity of the earth).

Einstein's doubts about the concept of absolute velocity were soon to find experimental support in Faraday's electromagnetic induction experiment. In a manuscript entitled (in English translation) *Fundamental Ideas and Methods of Relativity Theory, Presented in Their Development*, discovered by Gerald Holton, and dating according to him from about 1919 or shortly afterwards, Einstein acknowledged that this experiment played a leading role in the construction of *STR* :

According to Faraday, during the relative motion of a magnet with respect to a conducting circuit, an electric current is induced in the latter. It is all the same whether the magnet is moved or the conductor; only the relative motion counts, according to the Maxwell-Lorentz theory. However, the theoretical interpretation of the phenomenon in these two cases is quite different ...

The thought that one is dealing here with two fundamentally different cases was for me unbearable [war mir unerträglich]. The difference between these two cases could be not a real difference, but rather, in my conviction, only a difference in the choice of the reference point. Judged from the magnet there were certainly no electric fields [whereas] judged from the conducting circuit there certainly was. The existence of an electric field was therefore a relative one, depending on the state of motion of the coordinate system being used, and a kind of objective relativity could be granted only to the *electric and magnetic field together*, quite apart from the state of

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21. Einstein, "Autobiographical Notes," in Albert Einstein : Philosopher-Scientist, ed. Paul A. Schilpp, 3rd ed. (La Salle, IL : Open Court, 1969), 1, 53. Trans. Paul A. Schilpp.

22. Albert Einstein, *Relativity, the Special and the General Theory*, authorized trans. by Robert W. Lawson (New York ; Crown, 1961), p. 14. Emphasis in original.

23. Anton Reiser, *Albert Einstein : A Biographical Portrait* (New York : Albert & Charles Boni, 1930), p. 52.

24. Wertheimer, "Einstein," p. 214.

relative motion of the observer or the coordinate system. The phenomenon of the electromagnetic induction forced me to postulate the (special) relativity principle.<sup>25</sup>

The passage reveals that the idea of an absolute velocity was unbearable to Einstein and that he saw in the induction experiment a confirmation of his point of view. Indeed, the experiment led him to postulate the principle of relativity. This train of thought is even more clearly expressed in Einstein's original 1905 paper :

Examples of this sort [such as the Faraday induction experiment], together with the unsuccessful attempts to discover any motion of the earth relative to the "light medium" suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate.<sup>26</sup>

It is likely that Einstein learned about the Faraday induction experiment during his undergraduate years at the Polytechnic, either through the curriculum or through private reading of Kirchhoff, Helmholtz, Föppl, etc..<sup>27</sup> Thus it was probable around that time that Einstein became convinced of the principle of relativity. Yet the principle of relativity is in no way a logical consequence of the Faraday induction experiment. Indeed, most people did not see what Einstein saw. For them there were clearly two different situations while Einstein saw only one. Thus, if Einstein was convinced by the result of the experiment, it was only because he saw in it a necessary consequence of an already intuitively evident principle of relativity. This explain why in his 1905 paper Einstein did

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25. Gerald Holton, "Finding Favor with the Angel of the Lord : Notes toward the Psychobiographical Study of Scientific Genius," in *The Interaction between Science and Philosophy*, ed. Yehuda Elkana (Atlantic Highlands, NJ : Humanities Press, 1974), pp. 369-70 ; trans. G. Holton ; the document is now located in the Einstein Archives at the Princeton Institute for Advanced Study.

26.

"Beispiele ähnlicher Art, sowie die misslungenen Versuche, eine Bewegung der Erde relativ zum 'Lichtmedium' zu konstatieren, führen zu der Vermutung, dass dem Begriffe der absoluten Ruhe nicht nur in der Mechanik, sondern auch in der Elektrodynamik keine Eigenschaften entsprechen, sondern dass vielmehr für alle Koordinatensysteme, für welche die mechanischen Gleichungen gelten, auch die gleichen elektrodynamischen und optischen Gesetze gelten, wie dies für die Grössen erster Ordnung bereits erwiesen ist. Wir wollen diese Vermutung (deren Inhalt im folgenden 'Prinzip der Relativität' genannt werden wird) zur Voraussetzung erheben," Einstein, "Zur Elektrodynamik bewegter Körper" (1905), p. 891 ; trans. from Lorentz et al., *The Principle of Relativity*, pp. 37-38.

27. Gerald Holton, "Influences on Einstein's early work in relativity theory," *The American Scholar*, **37** (1967-68), 59-79 ; rpt in slightly condensed form in Holton, *Thematic Origins of Scientific Thought*, pp. 197-217.

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not much elaborate on the induction experiment, since it was just one illustration among others of an *a priori* principle of relativity which, according to the hypothetico-deductive method he used throughout his life, he needed not to justify in advance.

If Faraday's induction experiment could only confirm but not logically entail Einstein's principle of relativity, then one must look elsewhere in order to explain Einstein's transition from early doubts about the concept of absolute velocity to a strong conviction that such a concept was meaningless and that the principle of relativity was likely to be true. Here, it was undoubtedly Ernst Mach's epistemological influence that was decisive. In his "Autobiographical Notes" Einstein acknowledged this influence :

So to speak, all physicists of the last century saw in classical mechanics a firm and final foundation for all physics, yes, indeed, for all natural science, ...

It was Ernst Mach who, in his History of Mechanics, shook this dogmatic faith; this book exercised a profound influence upon me in this regard while I was a student. I see Mach's greatness in his incorruptible skepticism and independence; in my younger years, however, Mach's epistemological position also influenced me very greatly.<sup>28</sup>

It was his friend Michele Besso who recommended to Einstein, in 1897 or 1898, the reading of Mach's works.<sup>29</sup> At that time, Einstein carefully read Mach's *Mechanics*;<sup>30</sup> he was to read it again together with Mach's *Analyse der Empfindungen*<sup>31</sup> during the regular philosophical discussions he had with his friends Konrad Habicht and Maurice Solovine in Bern, around 1902. Among the other books discussed by this group, the so-called "Olympia Academy", were Henri Poincaré's *La Science et l'Hypothèse* (1902) (Science and Hypothesis) and David Hume's *Treatise of Human Nature*. With respect to the latter book, the discussion focused on Hume's conceptions of substance and causality.<sup>32</sup> Though Poincaré and Hume influenced Einstein, it was Mach who was to be the most central in his thought.

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28. Einstein, "Autobiographical Notes," p. 21; trans. P. A. Schilpp.

29. Besso to Einstein, Oct.-Dec. 1947; dans Albert Einstein and Michele Besso, *Correspondance 1903-1955*, trans., notes, and introduction by Pierre Speziali (Paris : Hermann, 1972), p. 386. For biographical information on Ernst Mach, see John T. Blackmore, *Ernst Mach : His Work, Life, and Influence* (Berkeley : University of California Press, 1972); Erwin Hiebert, "Ernst Mach," *Dictionary of Scientific Biography* (1973), 8, 595-607.

30. Ernst Mach, *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt* (Leipzig, 1883); *The Science of Mechanics : A Critical and Historical Account of Its Development*, English trans. from the 9th German ed. by Thomas J. McCormack, 6th American ed., with new intro. by Karl Menger (LaSalle, IL : Open Court, 1960)

31. Ernst Mach, *Beiträge zur Analyse der Empfindungen* (Jena, 1886); *The Analysis of Sensations and the Relation of the Physical to the Psychological*, trans. from the 1st German ed. by C. M. Williams, rev. and supp. from the 5th German ed. by Sydney Waterlow, paperback.

32. Albert Einstein, *Lettres à Maurice Solovine* (Paris : Gauthier-Villars, 1956), intro. by M. Solovine, p. VIII. See also Carl Seelig, *Albert Einstein : Leben und Werk eines Genius unserer Zeit* (Zürich : Europa Verlag, 1960), pp. 91-93.

In the preface to the first German edition of his *Mechanics*, Mach indicated the purpose of the book "Its aim is to clear up ideas, expose the real significance of the matter, and get rid of metaphysical obscurities"<sup>33</sup> Among the main metaphysical obscurities Mach wanted to eliminate were the concepts of absolute mass, absolute time, absolute space, and absolute motion. Mach argued that since none of these concepts were defined in terms of observable quantities, they were necessarily of a metaphysical nature, and thus, in his view, had to be eliminated from science. With respect to the concept of absolute motion Mach wrote, for example, "A motion may, with respect to another motion, be uniform. But the question whether a motion is *in itself* uniform, is senseless."<sup>34</sup> As to the concept of absolute time, Mach rejected it as being an "idle metaphysical conception"<sup>35</sup>

Mach's desire to eliminate metaphysics from physics and especially his claim that scientific knowledge is restricted to what can be observed –indeed, that "the world consists only of our sensations"<sup>36</sup>– must be viewed within the idealistic tradition as a reaction against the materialistic interpretation of physics that dominated in the late nineteenth century. We shall define materialism as the claim that there is no God and that a non-mind-like matter is the cause of perceptions, and metaphysical idealism as the claim that there is a God and that matter is a mind-like creation of God. An immediate advantage of metaphysical idealism over materialism is that it does not introduce an insurmountable barrier between matter and mind. With the rise of materialistic philosophy shortly after the Scientific Revolution, certain metaphysical idealists like Bishop George Berkeley adopted, in scientific matters, a restricted form of idealism, namely, an epistemological idealism in order to dissociate science from the materialistic philosophy. By epistemological idealism we shall understand the claim that knowledge is restricted to the domain of human consciousness. Thus Berkeley argued that since scientific knowledge can only be derived from human observations, anything which goes beyond, such as a non-mind-like matter, is foreign to science. This allowed Berkeley to show that materialism was essentially a metaphysical position, while at the same time Berkeley's own belief in God (whom he considered to be the source of human perceptions) remained unaffected.

In his works, Berkeley presented at length his epistemological idealism. In 1710, he wrote for example "the *absolute existence of unthinking things* are words without a meaning, or which include a contradiction,"<sup>37</sup> or : "I do not argue against the existence of any one thing we can apprehend, either by sense or reflection. . . . The only thing whose existence we deny is that which *philosophers* call Matter or corporeal substance."<sup>38</sup> and further below :

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33. Mach, *Science of Mechanics*, p. xxii.

34. Ibid., p. 273. Accentué dans l'original.

35. Ibid.

36. Mach, *Analysis of Sensations*, p. 12.

37. George Berkeley, *A Treatise Concerning the Principles of Human Knowledge* (1710), dans *Berkeley : Essay, Principles, Dialogues with Selections from other Writings*, ed. Mary Whiton Calkins (New York : Charles Scribner's Sons, 1929), p. 137, par. 24. Emphasis in original.

38. Ibid., p. 142, par. 35.

How great a friend *material substance* has been to Atheists in all ages were needless to relate. All their monstrous systems have so visible and necessary dependence on it, that when this cornerstone is once removed, the whole fabric cannot choose but fall to the ground.<sup>39</sup>

From the last quote, it is apparent that one –indeed the main one– of Berkeley's purposes was to undermine the materialistic philosophy.

Among the prominent followers of Berkeley's epistemological idealism were David Hume in the eighteenth century and Ernst Mach in the nineteenth century. With respect to the concept of matter, Hume wrote: "The idea of substance . . . is nothing but a collection of simple ideas, that are united by the imagination, and have a particular name assigned to them,"<sup>40</sup> whereas for Mach: "Thing, body, matter, are nothing apart from the combinations of the elements, – the colors, sounds, and so forth–nothing apart from their so-called attributes."<sup>41</sup> Mach, in his early years, had adopted Berkeley's idealism<sup>42</sup> and later on pointed out that "of all approaches to my standpoint, the one by way of idealism seems to me the easiest and most natural."<sup>43</sup> In contradistinction to Berkeley, Mach, however, was not to commit himself publicly as to the cause of the "elements" and simply focused his attention on their relationships. This allowed him to adopt an overall antimetaphysical attitude. Since, however, at the end of the nineteenth century, the materialistic interpretation of physics had become predominant, Mach's antimetaphysical standpoint meant in fact essentially an antimaterialistic standpoint. This is confirmed by the fact that all the absolute concepts Mach rejected were either materialistic by nature or had acquired materialistic overtones by the end of the nineteenth century.

Mach not only uncovered specific metaphysical elements which had crept into physics, but also proposed concrete solutions. Thus he proposed the following operational definition of mass: "The mass-ratio of any two bodies is the negative inverse ration of the mutually induced accelerations of those bodies,"<sup>44</sup> and defined force as the mass times the acceleration. These definitions have become standard in modern textbooks, the latter being the current international definition of force.

Mach's influence was considerable. His *Mechanics* (1883) immediately became influential in scientific circles throughout the world,<sup>45</sup> and by 1897 (or 1898), when Einstein read it, a third German edition of the book had already

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39. Ibid., p. 176, par. 92

40. David Hume, *A Treatise of Human Nature* (1739-40), rpt. from the original edition in 3 vols, and ed., with an analytical index, by L. A. Selby-Bigge (Oxford: Clarendon, 1888, rpt. 1975), p. 16.

41. "Das Ding, der Körper, die Materie ist nichts ausser dem Zusammenhang der Elemente, der Farben, Töne u.s.w., ausser den sogenannten Merkmalen." Mach, *Analysis of Sensations*, pp. 6-7.

42. Ernst Mach, "Die Leitgedanken meiner naturwissenschaftlichen Erkenntnislehre und ihre Aufnahme durch die Zeitgenossen," *Physikalische Zeitschrift*, **11** (1910), 599-606, on 603. See also Mach, *Analysis of Sensations*, p. 362; Blackmore, Ernst Mach, pp. 26-27.

43. Mach, *Analysis of Sensations*, p. 362.

44. Mach, *Science of Mechanics*, p. 303.

45. Blackmore, *Ernst Mach*, p. 117.

been published. Under the influence of Mach and Hume, Einstein was to adopt a strongly idealistic epistemological position. In the meantime, Mach's strong rejection of absolute quantities undoubtedly strengthened Einstein's own doubts about the concept of absolute velocity and supported his belief in a principle of relativity.

### **Vain attempts to reconcile electromagnetism with the principle of relativity**

Einstein's belief in a principle of relativity did not lead him much farther for many years. As he later wrote,

The difficulty that had to be overcome was in the constancy of the velocity of light in vacuum which I had first thought I would give up. Only after groping for years did I notice that the difficulty rests on the arbitrariness of the kinematical fundamental concepts.<sup>46</sup>

Since Maxwell's equations are not invariant under a Galilean transformation, Einstein first thought to modify them by giving up the constancy of the velocity of light (i.e., by making the velocity of light dependent on the motion of the light source). These attempts, however, which probably took place between 1898 and 1903, and for the most part after Einstein's graduation in 1900, led nowhere. At that time, Einstein had acquired some knowledge of Maxwell's electromagnetic theory through his reading of the works of Kirchhoff, Helmholtz, Hertz, etc.<sup>47</sup> Einstein also became familiar with the synthesis of Maxwellian and Continental traditions in electromagnetic theory which H. A. Lorentz had developed in 1892<sup>48</sup> and 1895.<sup>49</sup> Lorentz's electron theory simplified electromagnetism by postulating an ether at rest, and by separating the field from the sources. The theory also showed that it was possible to combine a first-order principle of relativity with a first order principle of the constancy of the velocity of light. Yet this theory, the best available at the time, did not solve Einstein's problem since it already failed to achieve second-order covariance (in  $v/c$ ) of Maxwell's equations and thus violated the principle of relativity by allowing, in principle, the determination of an absolute velocity. The contraction hypothesis which Lorentz introduced to explain the negative result of the Michelson-Morley experiment (1887) was felt by Einstein to be an ad hoc one,<sup>50</sup> since Einstein was already convinced of a strict principle of relativity (valid to any order of approximation). Thus to the extent that Einstein noticed the Michelson-Morley experiment in Lorentz's 1895 book at all, he was not surprised by its result but

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46. Holton, "Finding Favor with the Angel," p. 370; this quotation appeared in a footnote of the document mentioned above.

47. Einstein, "Autobiographical Notes," p. 15.

48. Lorentz, "La théorie électromagnétique" (1892).

49. Lorentz, *Versuch einer Theorie* (1895).

50. Wertheimer, "Einstein," p. 218; Albert Einstein, "Über die Entwicklung unserer Anschauungen über das Wesen und die Konstitution der Strahlung," *Physikalische Zeitschrift*, **10** (1909), 819.

expected it in contradistinction to Lorentz.<sup>51</sup>

### Rejection of the ether

Einstein was soon to abandon Lorentz's ether altogether; this at a time when for most people the existence of the ether seemed to be more certain than ever. Thus in his excellent textbook published in 1902, O. D. Chwolson wrote: "The probability of the hypothesis of the existence of this single agent borders extraordinarily the certitude. We shall call this agent, the ether."<sup>52</sup>

Besides Mach's devastating criticism of mechanistic thinking, Einstein had another reason to reject the ether, which resulted from Planck's successful work of 1900. There, Planck had introduced a quantification of the energy of electric oscillators which was rather incompatible with both the laws of classical mechanics and the laws of electromagnetism.<sup>53</sup> This strengthened Einstein's idea that the electromagnetic laws might not have a strict validity, but might fail on the microscopic level. This conviction, together with the results of Einstein's own investigations on statistical mechanics,<sup>54</sup> led him to consider an emission theory of light.<sup>55</sup> Einstein's quantum investigations were ultimately to crystallize in the first of his three famous 1905 papers<sup>56</sup> With respect to Einstein's relativity problem, an emission theory had the advantage of eliminating the concept of the ether, which conflicted so strongly with the principle of relativity. But at the same time, an emission theory made the principle of the constancy of the velocity of light (whatever the motion of the source) no longer evident. By now, however, Einstein knew of its importance both from his own failure to dispense with it and especially from Lorentz's investigations.<sup>57</sup>

### Epistemological breakthrough

Einstein's failure to achieve a synthesis of the principle of relativity and the principle of the constancy of the velocity of light within an emission theory

51. Wertheimer, "Einstein," p. 217; Robert S. Shankland, "Conversations with Albert Einstein," *American Journal of Physics*, **31** (1963), 55.

52. "Die Wahrscheinlichkeit der Hypothese von der Existenz dieses einen Agens grenzt ausserordentlich nahe an Gewissheit. Dieses Agens wollen wir den Äther nennen," O. D. Chwolson, *Lehrbuch der Physik* (Braunschweig, 1902), **1**, 9. My translations unless otherwise specified. For details on the ether conceptions up to 1900, see *Conceptions of Ether: Studies in the history of ether theories, 1740-1900*, eds. G. N. Cantor and M. J. S. Hodge (Cambridge University Press, 1981).

53. Einstein, "Autobiographical Notes," p. 53.

54. Albert Einstein, "Kinetische Theorie des Wärmegleichgewichtes und des zweiten Hauptsatzes der Thermodynamik," *Annalen der Physik*, **9** (1902), 417-33; "Eine Theorie der Grundlagen der Thermodynamik," *ibid.*, **11** (1903), 170-87; "Zur allgemeinen molekularen Theorie der Wärme," *ibid.*, **14** (1904), 354-62. See also M. J. Klein, "Thermodynamics in Einstein's Thought," *Science*, **157** (4 Aug. 1967), 509-16.

55. Shankland, "Conversations with Albert Einstein" (1963), p. 49.

56. Albert Einstein, "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt," *Annalen der Physik*, **17** (1905), 132-48.

57. Einstein, *Relativity, Special and General Theory*, p. 19. A. Einstein, *La Relativité: La Théorie de la Relativité restreinte et générale; La Relativité et le problème de l'espace* (Paris: Petite Bibliothèque Payot, 1956), p. 28.



led him to a deeper analysis of the problem. Einstein tried to understand what occurred during the measurement of a velocity in general and consequently came to examine the concept of time.<sup>58</sup> His next step was to associate the concept of time with the concept of simultaneity. Poincaré had already associated these two concepts in his book *La Science et l'Hypothèse* (1902), which Einstein had read with his friends Habicht and Solovine in Bern. In that book, Poincaré wrote :

2. There is no absolute time. When we say that two periods are equal, the statement has no meaning, and can only acquire a meaning by a convention.

3. Not only have we no direct intuition of the equality of two periods, but we have not even direct intuition of the simultaneity of two events occurring in two different places.<sup>59</sup>

Once he had become aware of the role of the concept of simultaneity in the definition of time, Einstein's epistemological idealism led him to acquire a definition of this concept. In his book *Relativity, the Special and General Theory* written in 1916 and presenting the main ideas "on the whole, in the sequence and connection in which they actually originated,"<sup>60</sup> Einstein wrote : "The concept does not exist for the physicist until he has the possibility of discovering whether or not it is fulfilled in an actual case. We thus require a definition of simultaneity."<sup>61</sup> This requirement for a definition of the concept of simultaneity, which resulted from the epistemological idealism Einstein had adopted from David Hume and Ernst Mach, already implied the solution of Einstein's problem. In his autobiography, Einstein acknowledged this debt :

all attempts to clarify this paradox [of the apparent lack of relativity in electromagnetism] satisfactorily were condemned to failure as long as the axiom of the absolute character of time, viz., of simultaneity, unrecognizedly was anchored in the unconscious. Clearly to recognize this axiom and its arbitrary character really implies already the solution of the problem. The type of critical reasoning which was required for the discovery of this central point was decisively furthered, in my case, especially by the reading of David Hume's and Ernst Mach's philosophical writings.<sup>62</sup>

Though Hume and Mach had criticized the use of absolute concepts, it was left to Einstein to show how such an antimetaphysical attitude could lead to a scientific revolution. Einstein's definition of simultaneity in terms of light beams not only led to the rejection of the concept of absolute time but also to the rejection of the concept of absolute length, since the measurement of the length

58. Wertheimer, "Einstein," p. 219.

59. Henri Poincaré, *La Science et l'Hypothèse* (Paris : Flammarion, 1968), pp. 111-112. Henri Poincaré, *Science and Hypothesis*, with a preface by J. Larmor, unabridged republication of the first English trans. (1905, rpt. New York : Dover, 1952), p. 90.

60. Einstein, *La Relativité*, préface, pp. 5-6.

61. Ibid., p. 31.

62. Einstein, "Autobiographical Notes," p. 53.

of a moving rod involves two simultaneous measurements (simultaneous determination of the positions of the extremities of the rod). This made it clear to Einstein that new transformation relations between moving inertial reference frames were needed. In order to determine them, Einstein looked for an invariant and first tried to find one by adopting some plausible assumption about matter.<sup>63</sup> After the failure of these attempts, Einstein abandoned such an approach and looked towards a theory of principles such as thermodynamics.<sup>64</sup> The two principles of thermodynamics can be formulated as the impossibility to construct a perpetual mobile of the first and second kind. These two principles can be formulated as follows.

1. It is impossible to construct a machine yielding continuously some work without providing energy from the outside to it.
2. It is impossible to construct a periodical machine furnishing some mechanical work from the cooling of only one heat source.<sup>65</sup>

The second principle excludes the existence of a cruiseship navigating on a warm sea and rejecting behind it blocs of ice (in accordance with the conservation of energy). It occurred to Einstein that the constancy of the velocity of light, which can also be expressed as the experimental impossibility to measure a different velocity of light emitted from a moving source, might serve as an invariant. Furthermore, Einstein's choice of the velocity of light as an invariant rather than say the velocity of sound was guided by experimental evidence as well as Einstein's idea that the velocity of light might be the fastest possible.<sup>66</sup>

From the principle of relativity and the principle of the constancy of the velocity of light, Einstein was able to derive a set of transformation relations (now called the Lorentz transformations) which contained Lorentz's 1895 relations as a limit.<sup>67</sup> Furthermore, Einstein's formalism yielded Lorentz's contraction hypothesis. Lorentz, therefore, had been in the right direction. His contraction hypothesis, however, was no longer an ad hoc assumption in Einstein's theory but a logically derived kinematical effect.

### 2.1.3 Einstein's Achievement

We have seen that both principles of the special theory of relativity existed more or less in 1905. One could indeed argue that Lorentz's and Poincaré's theories contained most elements of *STR* at that time.<sup>68</sup> Even if *all* individual elements of *STR* had existed in 1905, the credit of having built the special

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63. Wertheimer, "Einstein," p. 223.

64. Einstein, "Autobiographical Notes," p. 53.

65. Christian Gerthsen, *Physik* (Berlin : Volk und Wissenschaft, 1948), pp. 54, 190.

66. Wertheimer, "Einstein," p. 224.

67. Ibid.

68. Edmund Whittaker, *A History of the Theories of Aether and Electricity. The Modern Theories 1900-1926* (1953 ; rpt. New York : Humanities Press, 1973), **2**, chap.2 ; G. H. Keswani, "Origin and Concept of Relativity," *British Journal for the Philosophy of Science*, **15** (1965), 286-306 ; **16** (1965), 19-32 ; **16** (1965), 273-94 ; Heinrich Lange, *Geschichte der Grundlagen der Physik* (Freiburg, München : Verlag Karl Alber, 1954), **1**, chap. 10.

theory of relativity would still go to Einstein. In this respect, the case of Einstein's theory is somewhat similar to the case of Newton's theories of mechanics and gravitation.<sup>69</sup> All three principles of Newton's theory of mechanics (principle of inertia, force law, principle of action and reaction) existed more or less separately around 1687. With respect to Newton's gravitational theory, Robert Hooke provided the major insights : idea that a curvilinear motion should be analyzed in an inertial system in terms of just one force instead of two; idea of a central inverse square force; etc.. Yet, the credit of the theories of mechanics and gravitation goes to Newton. Why? Simply for the same reason that a painting is signed by the artist and not by the paint and linen manufacturers. To the same extent that a painting transcends its elements, so does a physical theory since the latter is a highly integrated system and not just a set of loose elements or vague insights. One of the reasons of the uniqueness of Einstein's theory is that although Lorentz and Poincaré realized the necessary character of the principle of relativity and of the principle of the constancy of the velocity of light, it was only Einstein who concretely showed that these two principles were *sufficient* for a consistent theory. Theory making resembles somewhat the reassembling of the pieces of a jigsaw puzzle in which, however (in contradistinction to the child's game), all elements are neither necessary nor perhaps sufficient. This makes it clear that a major creative effort is involved in theory making and shows that the elements of a theory "explain" the latter's origin as little (or as much) as the marble explains the statue. Though Einstein indicated in a letter to Carl Seelig in 1955,<sup>70</sup> that he only knew of Lorentz's 1892 and 1895 works<sup>71</sup> but not of Lorentz's 1904<sup>72</sup> paper nor of its elaboration by Poincaré,<sup>73</sup> the originality of Einstein's theory would remain intact even if Einstein had used all the elements of the above theories.

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69. See for example I. B. Cohen, "Newton, Isaac," *Dictionary of Scientific Biography* (1974), **10**, 42-101.

70. Einstein à Carl Seelig, 19 February 1955. Publié par Carl Seelig dans *Technische Rundschau*, **47** no. 20 (1955), cité en partie par Max Born, "Physics and Relativity," a lecture given at the International Relativity Conference in Berne on 16 July 1955, dans Max Born, *Physics in My Generation* (New York : Springer-Verlag, 1969), p. 104.

71. Specifically : Lorentz, "La théorie électromagnétique" (1892); Lorentz, *Versuch einer Theorie* (1895).

72. Lorentz, "Electromagnetic phenomena" (1904).

73. Henri Poincaré, "Sur la Dynamique de l'électron," *Comptes rendus de l'Académie des Sciences*, **140** (1905), 1504-08, in Henri Poincaré, *Oeuvres* (Paris : Gauthier-Villars et C<sup>ie</sup>, 1954), **9**, 489-493; Henri Poincaré, "Sur la Dynamique de l'Electron," *Rend. Circ. Mat. Palermo*, **21** (1906), 129-76, in *Oeuvres*, **9**, 494-550.

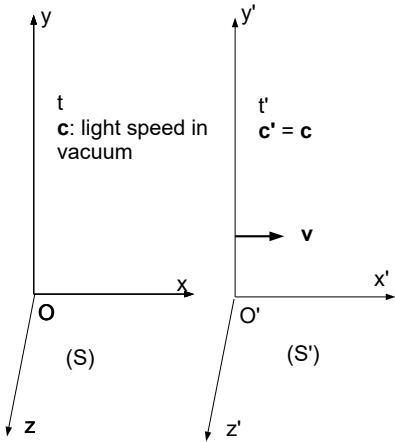


FIGURE 2.2 – Lorentz transformation (special relativity)

**Box 2. Special relativity***relative time; relative space**Two postulates :*

- a) Relativity principle : the physical laws are the same in two Galilean reference systems related by a Lorentz transformation;  
 b) Principle of the constancy of the light velocity in vacuum (independently of the motion of the source)

*Lorentz transformation*(see Fig. 2.2) :

$$\begin{aligned}t' &= \frac{\left(t - \frac{vx}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}} \\x' &= \frac{(x - vt)}{\sqrt{1 - \frac{v^2}{c^2}}} \\y' &= y \\z' &= z\end{aligned}$$

reciprocally,

$$\begin{aligned}t &= \frac{\left(t' + \frac{vx'}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}} \\x &= \frac{(x' + vt')}{\sqrt{1 - \frac{v^2}{c^2}}} \\y &= y' \\z &= z'\end{aligned}$$

*metric (pseudo-euclidean) :*

$$\begin{aligned}ds^2 &= c^2 dt^2 - dx^2 - dy^2 - dz^2 \\&= ds'^2\end{aligned}$$

⇒ relative time interval;

⇒ relative length;

⇒ velocity of light in vacuum :  $\mathbf{c}' = \mathbf{c}$ ;

⇒ Mawell's equations (covariant form under a Lorentz transformation).

## 2.2. EINSTEIN'S EPISTEMOLOGICAL DISSATISFACTION WITH STR21

And yet, we have in fact somewhat exaggerated the similarity of the elements of Einstein's theory with those of Lorentz's and Poincaré's theories. This similarity is only superficial. Whereas for Lorentz and Poincaré the principle of relativity was closely tied with electromagnetism, this was not the case for Einstein. Einstein's transformation relations were of a purely kinematical origin and therefore valid for any theory whereas in Lorentz's theory they resulted from Maxwell's equations, and thus were only valid within that framework.<sup>74</sup> This shows the superiority of Einstein's theory which is a theory about theories rather than just a specific theory. The greatest originality of Einstein's theory lay in its rejection of the concept of absolute time. As we have seen, the epistemological criticism of Mach, Hume, and Poincaré greatly influenced Einstein and was certainly the most important contribution to the development of the special theory of relativity. Yet, Einstein went beyond any of his contemporaries by showing how epistemological concerns could lead to a concrete physical theory. Einstein's theory, in turn, was to deepen the epistemological revolution by making the concept of inertial mass relative to the motion of the observer.<sup>75</sup> Einstein's next step was to attempt to eliminate the concept of absolute acceleration from physics which was to lead to the general theory of relativity.

## 2.2 Einstein's Epistemological Dissatisfaction with STR

When Einstein had developed the special theory of relativity in 1905, it was clear to him that his theory was not perfect from an epistemological point of view, since the principle of relativity applied only to inertial systems and thus distinguished these systems from all other ones without any good reason. Einstein has himself described his line of thought as follows :

When by the special theory of relativity I had arrived at the equivalence of all so-called inertial systems for the formulation of natural laws (1905), the question whether there was not a further equivalence of coordinate systems followed naturally, to say the least of it. To put in in another way, if only a relative meaning can be attached to the concept of velocity, ought we nevertheless to persevere in treating acceleration as an absolute concept ?

From the purely kinematic point of view there was no doubt about the relativity of all motions whatever ; but physically speaking, the inertial system seemed to occupy a privileged position, which

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74. Einstein à Seelig, 19 February 1955, in Born, *Physics in My Generation*, p. 104. See also T. Kahan, "Sur les origines de la théorie de la relativité restreinte," *Revue d'Histoire des Sciences*, **12** (1959), 159-65.

75. Albert Einstein, "Ist die Trägheit eines Körpers von seinem Energiegehalt abhängig?" *Annalen der Physik*, **18** (1905) 639-41 ; "Das Prinzip von der Erhaltung der Schwerpunktsbewegung und die Trägheit der Energie," *ibid.*, **20** (1906), 627-33 ; "Über die vom Relativitätsprinzip geforderte Trägheit der Energie," *ibid.*, **23** (1907), 371-84.

made the use of coordinate systems moving in other ways appear artificial.

I was of course acquainted with Mach's view, according to which it appeared conceivable that what inertial resistance counteracts is not acceleration as such but acceleration with respect to the masses of the other bodies existing in the world. There was something fascinating about this idea to me, but it provided no workable basis for a new theory.

I first came a step nearer to the solution of the problem when I attempted to deal with the law of gravity within the framework of the special theory of relativity.<sup>76</sup>

We shall analyze Einstein's rejection of the concept of absolute acceleration, after a brief historical review of that concept.

### 2.2.1 Historical Debate about the Concept of Absolute Acceleration

The concepts of absolute motion and in particular of absolute acceleration were introduced by Isaac Newton in 1687 via the concept of absolute space which he defined as follows :

Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces.<sup>77</sup>

He then defined absolute (relative) motion as the translation from one absolute (relative) place to another, and added : "The effects which distinguish absolute from relative motion are, the forces of receding from the axis of circular motion."<sup>78</sup> As experimental evidence Newton presented his famous bucket experiment. The argument runs as follows : imagine a bucket filled with water, which is put into rotation. When the motion starts, the water has not yet a rotational motion and therefore its surface is flat. If, after a while, the water and the vessel rotate at the same speed, then there is no relative motion between the water and the vessel, and the surface of the water is parabolic because of the centrifugal forces. Finally, if the bucket is stopped, the water still rotates for a while and, its surface remains parabolic during that time. In the first and last phases, though there is relative motion between the water and the vessel, centrifugal forces appear only in the latter phase. In the intermediate phase there is no relative motion, yet centrifugal forces are present. Thus, remarked Newton, centrifugal forces in the water are not produced by mere relative motion between the water and the vessel but only by true rotation of the water with respect to absolute space. Hence, he concluded that centrifugal forces furnish a

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<sup>76</sup>. Alber Einstein, "Notes on the Origin of the General Theory of Relativity," in Albert Einstein, *Ideas and Opinions* (1954; rpt. New York : Dell, 1976), p. 279.

<sup>77</sup>. Isaac Newton, *Mathematical Principles of Natural Philosophy*, trans. Motte, revised by F. Cajori (Berkeley : University of California Press, 1962), p. 6.

<sup>78</sup>. *Ibid.*, p.10.

## 2.2. EINSTEIN'S EPISTEMOLOGICAL DISSATISFACTION WITH STR23

basis for distinguishing absolute from relative motion.<sup>79</sup> Newton's bucket experiment did not establish the properties of absolute independence, immutability, immobility, which were rather derived from Newton's own philosophy which considered absolute space as the sensorium of God.<sup>80</sup>

Some people even went so far as to consider space as being God himself. Bishop George Berkeley rejected this identification as well as the belief "that there is something beside God which is eternal, uncreated, infinite, indivisible, immutable" as being "pernicious and absurd notions."<sup>81</sup> Like Gottfried Leibniz and Christiaan Huygens before him,<sup>82</sup> Berkeley tried to overcome Newton's arguments about absolute rotation by attempting to redefine the concept of motion.<sup>83</sup> In 1721, he proposed to replace the concept of absolute space by a relative space, defined in terms of the fixed stars considered at rest.<sup>84</sup> Such a definition, while in accord with established experimental results, made it conceptually possible that the stars, rather than absolute space, might determine the inertial behavior of bodies. Leonhard Euler, a proponent of absolute space, did not like the idea and wrote : "It would be a rather strange proposition and contrary to a lot of other dogmas of metaphysics, to say that the fixed stars govern the bodies in their inertia."<sup>85</sup>

More than a century later, Ernst Mach reintroduced the argument of an influence of the stars on inertial behavior and presented it in a scientific way in his *Mechanics* (1883). Mach was the first to propose a new physical interpretation of Newton's bucket experiment, which he described as follows :

Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick. The one experiment only lies before us, and our business is, to bring it into accord with the other facts known to us, and not with the arbitrary fictions of our imagination.<sup>86</sup>

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79. Ibid., pp. 10-11.

80. Isaac Newton, *Opticks*, based on the 4th ed., London, 1730 (New York : Dover, 1952), Query 28, p. 370 ; Query 31, p. 403. See also H. G. Alexandre ed., *The Leibniz-Clarke Correspondence* (1956 ; rpt. New York : Barnes & Noble, 1976), introduction, pp. XV-XVI, XXXIV ; Max Jammer, *Concepts of Space*, 2nd ed. (Cambridge, MA : Harvard University Press, 1969), chap. 4.

81. Berkeley, *Principles of Human Knowledge*, in *Berkeley : Essay, Principles* (1929), p. 191, par. 117.

82. Jammer, *Concepts of Space* (1969), pp. 119-25.

83. Berkeley, *Principles*, pp. 188-90, pars. 114-15.

84. George Berkeley, *De Motu*, dans David M. Armstrong ed., *Berkeley's Philosophical Writings* (New York : Macmillan, 1965), p. 270, par. 64.

85. Leonhard Euler, "Réflexions sur l'Espace et le Temps," *Histoire de l'Académie Royale des Sciences et Belles Lettres*, Berlin (1748), p. 328.

86. Mach, *La Mécanique*, p. 225. Mach, *Science of Mechanics*, p. 284.



Thus Mach suggested that centrifugal forces might be produced by interactions (the nature of which he did not specify) between masses, rather than by an absolute rotation. We shall refer to this hypothesis as "Mach's hypothesis." Similarly, Mach considered rectilinear inertial motion as relative to distant masses.<sup>87</sup> Mach's hypothesis was to stimulate Einstein in his rejection of the concept of absolute acceleration.

### 2.2.2 Einstein's Rejection of the Concept of Absolute Acceleration

Einstein's remarks quoted above indicate that his initial concern in building *GTR* was to generalize *STR* by eliminating the concept of absolute acceleration (i.e., of absolute space). Einstein expressed the same opinion on various occasions.<sup>88</sup> This opinion is also confirmed by Anton Reiser,<sup>89</sup> Wolfgang Pauli,<sup>90</sup> Arnold Sommerfeld,<sup>91</sup> and Cornelius Lanczos,<sup>92</sup> to mention only a few. We shall see that Einstein's rejection of the concept of absolute acceleration appears explicitly in many of Einstein's papers. In fact, this was a lifelong concern for Einstein. Thus in 1953, Einstein wrote in the preface to Max Jammer's book, *Concepts of Space* :

It required a severe struggle to arrive at the concept of independent and absolute space, indispensable for the development of theory. It has required no less strenuous exertions subsequently to overcome this concept—a process which is probably by no means as yet completed....

The victory over the concept of absolute space or over that of the inertial system became possible only because the concept of the material object was gradually replaced as the fundamental concept of physics by that of the field.<sup>93</sup>

Even as late as a few months before his death Einstein wrote : "The clearest logical characterization of the general theory of relativity can be stated as fol-

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87. Ibid., *Science of Mechanics*, pp. 286-87.

88. (a) Albert Einstein, "Fundamental Ideas and Problems of the Theory of Relativity," lecture delivered to the Nordic Assembly of Naturalists at Gothenburg, 11 July 1923, in *Nobel Lectures, Physics, 1901-1921*, published for the Nobel Foundation (New York : Elsevier, 1967), pp. 482-90, esp. pp. 485-86, 488, 489 ; (b) Einstein, *Relativity, Special and General Theory*, pp. 70-73 ; (c) Einstein, "Autobiographical Notes," pp. 26, 28, 62. (d) Albert Einstein, "Autobiographische Skizze," in Carl Seelig, *Helle Zeit-Dunkle Zeit* (Zürich : Europa Verlag, 1956), p. 13.

89. Reiser, *Albert Einstein* (1930), pp. 109-13, esp. pp. 109-10.

90. Wolfgang Pauli, *Theory of Relativity*, trans. G. Field with supplementary notes by the author (London : New York : Pergamon, 1958), pp. 142-43.

91. Arnold Sommerfeld, "Kurzer Bericht über die allgemeine Relativitätstheorie und ihre Prüfung an der Erfahrung," *Archiv für Elektrotechnik*, Berlin, **9** (1921), 391-99, in Arnold Sommerfeld, *Gesammelte Schriften* (Braunschweig : Friedr. Vieweg & Sohn, 1968), **2**, 270-78 on 273-74.

92. Cornelius Lanczos, *Space through the Ages : The Evolution of Geometrical Ideas from Pythagoras to Hilbert and Einstein* (London, New York : Academic, 1970), p. 94.

93. Albert Einstein, preface of Jammer, *Concepts of Space* (1969), p. XV.

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laws : it is the theory which avoids the introduction of the 'inertial system.'" <sup>94</sup> Einstein's epistemological rejection of the concept of absolute acceleration was to motivate him to extend the special theory of relativity in 1907.

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<sup>94</sup> Albert Einstein and B. Kaufman, "A New Form of the General Relativistic Field Equations," *Annals of Mathematics*, **62** (1955), 128, in Albert Einstein, *Edition of Einstein's Scientific Papers*, Microfilm edition (New York : Readex Microprint, 1960).



## Chapitre 3

# The Extension of the Special Theory of Relativity via the Equivalence Principle (1907 – 1912)

In 1907, Einstein was to undertake the first attempt to generalize the special theory of relativity via the equivalence principle, which postulates a complete physical equivalence between a uniformly accelerated reference system and an inertial system with a uniform gravitational field. In the first part of this chapter, we shall analyze how Einstein came to the idea of the equivalence principle, the significance it had for him, and the uses he made of it up to 1911. In the second part of the chapter, we shall examine Einstein's further use of the equivalence principle in the development of a static theory of gravitation. The static theory of gravitation prepared the transition toward the general theory of relativity by enabling Einstein to arrive at the definitive equation of motion, which in turn conditioned the Riemannian framework of the general theory of relativity.

### 3.1 Early Use of the Equivalence Principle (1907-1911)

#### 3.1.1 The Idea of the Equivalence Principle

##### **From the relativity of electric and magnetic fields to the relativity of the gravitational field**

Einstein himself described how he arrived at the idea of the equivalence principle in a passage of the manuscript discovered by Gerald Holton, which has already been quoted in part in the first chapter. After referring to the Faraday

induction experiment and the relative existence of the electric and magnetic fields in the special theory of relativity, Einstein went on to say :

When, in the year 1907, I was working on a summary essay concerning the special theory of relativity for the *Jahrbuch für Radioaktivität und Elektronik*, I had to try to modify Newton's theory of gravitation in such a way that it would fit into the theory [of relativity]. Attempts in this direction showed the possibility of carrying out this enterprise, but they did not satisfy me because they had to be supported by hypotheses without physical basis. At that point there came to me the happiest thought of my life, in the following form :

Just as is the case with the electric field produced by electromagnetic induction, the gravitational field has similarly only a relative existence. *For if one considers an observer in free fall, e.g. from the roof of a house, there exists for him during his fall no gravitational field* – at least in his immediate vicinity. For if the observer releases any objects they will remain relative to him in a state of rest, or in a state of uniform motion, independent of their particular chemical and physical nature. (In this consideration one must naturally neglect air resistance.) The observer therefore is justified to consider his state as one of "rest."

The extraordinarily curious, empirical law that all bodies in the same gravitational field fall with the same acceleration received through this consideration at once a deep physical meaning. For if there is even a single thing which falls differently in a gravitational field than do the others, the observer would discern by means of it that he is in a gravitational field, and that he is falling into it. But if such a thing does not exist—as experience has confirmed with great precision—the observer lacks any objective ground to consider himself as falling in a gravitational field. Rather, he has the right to consider his state as that of rest, and his surroundings (with respect to gravitation) as fieldfree.

The fact of experience concerning the independence of acceleration in free fall with respect to the material is therefore a mighty argument that the postulate of relativity is to be extended to coordinate systems that move non-uniformly relative to one another. . .

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The quotation shows that though Einstein thought about a relativistic theory

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1. Gerald Holton, "Finding Favor with the Angel of the Lord : Notes toward the Psychobiographical Study of Scientific Genius," in *The Interaction Between Science and Philosophy*, Yehuda Elkana ed., G. Holton trans. (Atlantic Highlands, NJ : Humanities Press, 1974), pp. 370-71. The document is located in the Einstein Archives au Princeton Institute for Advanced Study. Another translation by G. Holton of the above passage is given in Albert Rothenberg, "Einstein's Creative Thinking and the General Theory of Relativity : A Documented Report," *Am. J. Psychiatry*, **136** (1979), 39.

of gravitation while he was writing the review paper<sup>2</sup> on the special theory of relativity, the idea of the relativity of the gravitational field was rather independent of it since it resulted from an extension of the relativity of the electric and magnetic fields to the gravitational field in general. The relevance of the electromagnetic analogy in the genesis of the idea of the relativity of the gravitational field is confirmed by Anton Reiser.<sup>3</sup> Furthermore, in the review paper, Einstein discussed indeed the relativity of the electric and magnetic fields and pointed out that these two fields do not have "an existence by themselves"<sup>4</sup> since, through an appropriate choice of the reference frame, each of them, separately, can be made to vanish in specific cases. Thinking about gravitation, Einstein came naturally to consider the relativity of the gravitational field as well and found it to be in fact greater than that of the electromagnetic field (i.e., the combined electric and magnetic fields) in specific instances. Thus, a uniform gravitational field can be made to disappear altogether in a freely falling elevator, whereas this is not possible in the case of an electromagnetic field because of the relativistic invariants  $[\mathbf{E}^2 - \mathbf{B}^2]$  and  $(\mathbf{E} \cdot \mathbf{B})$ , where  $\mathbf{E}$  and  $\mathbf{B}$  are the electric and magnetic fields respectively.

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2. Albert Einstein, "Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen," *Jahrbuch der Radioaktivität und Elektronik*, **4** (1907), 411-462; **5** (1908), 98-99.

3. Anton Reiser, *Albert Einstein : A Biographical Portrait* (New York : Albert & Charles Boni, 1930), pp. 110-112.

4. Einstein, "Relativitätsprinzip und Folgerungen" (1907), p. 429.

**Box 3. Equivalence principle (1907)**

1) Relativity of the electromagnetic field (see Fig. 3.1) :

$$E'_x = E_x, E'_y = \frac{(E_y - B_z v)}{\sqrt{1 - \frac{v^2}{c^2}}}, E'_z = \frac{(E_z + B_y v)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$B'_x = B_x, B'_y = \frac{(B_y + \frac{E_z v}{c^2})}{\sqrt{1 - \frac{v^2}{c^2}}}, B'_z = \frac{(B_z - \frac{E_y v}{c^2})}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and reciprocally

$$E_x = E'_x, E_y = \frac{(E'_y + B'_z v)}{\sqrt{1 - \frac{v^2}{c^2}}}, E_z = \frac{(E'_z - B'_y v)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$B_x = B'_x, B_y = \frac{(B'_y - \frac{E'_z v}{c^2})}{\sqrt{1 - \frac{v^2}{c^2}}}, B_z = \frac{(B'_z + \frac{E'_y v}{c^2})}{\sqrt{1 - \frac{v^2}{c^2}}}$$

relativistic invariants :

$$\frac{(\vec{E})^2}{c^2} - (\vec{B})^2, \frac{2(\vec{E}) \cdot (\vec{B})}{c}$$

2) Relativity of the gravitational field (see Fig. 3.2)

- ⇒ local annihilation of the gravitational field in a freely falling lift ;
  - ⇒ **Equivalence principle** between a uniform gravitational field and a uniformly accelerated system ;
  - ⇒ hypothesis of a total equivalence (mechanical, electromagnetic, etc.).
- Analysis of an accelerated system and application of the equivalence principle :
- ⇒ inertial mass = gravitational mass ;
  - ⇒ deflection of light rays in a gravitational field ;
  - ⇒ spectral shift of light in a gravitational field.

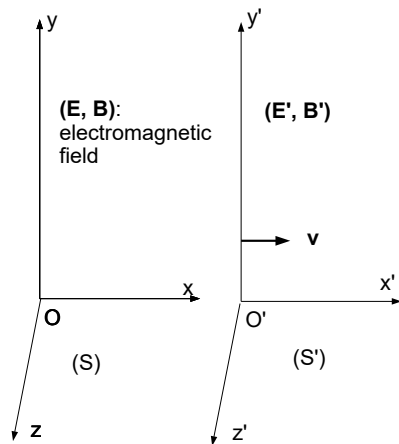


FIGURE 3.1 – Equivalence Principle (1907)

According to Carl Seelig, Einstein had already thought about the situation of a man in a freely falling elevator in his youth.<sup>5</sup> Mach had been interested in that question as well and in his *Mechanics* he dealt at some length with freely falling reference systems: "If [a] table be let fall vertically downwards with the acceleration of free descent  $g$ " Mach wrote, "all pressure on it ceases";<sup>6</sup> furthermore,

We ourselves, when we jump or fall from an elevation, experience a peculiar sensation, which must be due to the discontinuance of the gravitational pressure of the parts of our body on one another—the blood, and so forth. A similar sensation, as if the ground were sinking beneath us, we should have on a smaller planet, to which we were suddenly transported. The sensation of constant ascent, like that felt in an earthquake, would be produced on a larger planet.<sup>7</sup>

The last passage clearly stressed the physical analogy of a constant gravitational field with an accelerated system of reference. Mach also described experiments illustrating the decrease of the apparent weight of a falling body.<sup>8</sup>

5. Carl Seelig, *Albert Einstein: Leben und Werk eines Genies unserer Zeit* (Zürich: Europa Verlag, 1960) p. 118.

6. Ernst Mach, *La Mécanique*, p. 199; *Science of Mechanics*, chap. 2, sec. 4, p. 251.

7. *Ibid.*, *La Mécanique*, p. 200; *Science of Mechanics*, p. 252.

8. *Ibid.*, *Science of Mechanics*, pp. 252-253.



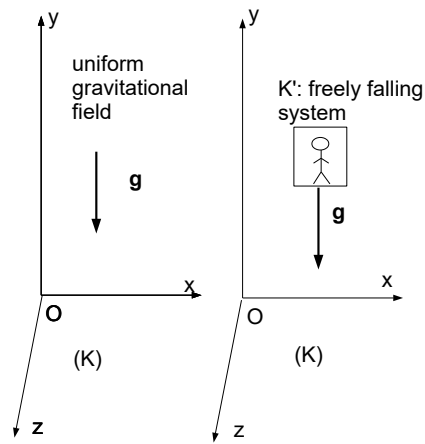


FIGURE 3.2 – Relativity of the gravitational field (the gravitational field vanishes locally in a freely falling lift)

Though it was the relativity of the electric and magnetic fields which was the key element in Einstein's idea of the relativity of the gravitational field, Mach's examples reflect a general interest in such questions.

### **From the relativity of the gravitational field to the equivalence principle**

Though Einstein made no reference to a freely falling reference system in his early papers, the transition from the idea of the relativity of the gravitational field to the equivalence principle was probably as follows. Since Einstein wanted to extend the principle of relativity to accelerated systems, and since a freely falling system behaves like an inertial one for mechanical experiments (if one assumes a strict equality of acceleration of all bodies at a given place of a gravitational field), the idea came to Einstein to consider a freely falling system as a real inertial system at rest, for experiments of any nature (mechanical, electrical, etc.). If this were true, then, the principle of relativity could be extended to such a uniformly accelerated system. Einstein's idea, however, made it necessary that the uniform acceleration field of the system be exactly cancelled by the uniform gravitational field for any arbitrary experiment, which meant that the uniform acceleration field had to be physically equivalent to a uniform gravitational field. This was precisely the formulation Einstein gave to the equivalence principle; he was to use this formulation throughout the development of the general theory of relativity. Einstein, initially, did not use the term "equivalence principle," but introduced the principle as an hypothesis which, in 1912, he called the "equivalence hypothesis."<sup>9</sup>

The main significance of the equivalence principle for Einstein was that it extended the principle of relativity to uniformly accelerated systems and thus offered the prospect of a general relativity. We have seen that for a sufficiently small freely falling reference system this is indeed the case. For a uniform acceleration field of arbitrary origin, this is also true because of its postulated equivalence with an inertial system containing a uniform gravitational field in which the principle of relativity could legitimately be expected to apply. Another closely related reason for Einstein's interest in the equivalence principle was that it eliminated the concept of absolute acceleration<sup>10</sup> by allowing one to view a uniform acceleration field as a real gravitational field created by distant masses.<sup>11</sup> In fact, in this sense, the equivalence principle simply was a particular case of Mach's hypothesis, which assumed any inertial field (i.e., acceleration field) to be a physical field produced by mutual interactions. Whereas Mach had left the nature of these interactions open, Einstein, by assuming a uniform

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9. Albert Einstein, "Lichtgeschwindigkeit und Statik des Gravitationsfeldes," *Annalen der Physik*, **38** (1912) 355-69, on 355.

10. Albert Einstein, "Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes," *Annalen der Physik*, **35** (1911), 898-908, on 899; see also Albert Einstein, "Autobiographical Notes," in Paul Arthur Schlipp ed., *Albert Einstein : Philosopher-Scientist*, 3rd ed. (La Salle, IL : Open Court, 1969), **1**, p. 66.

11. Einstein, "Lichtgeschwindigkeit und Statik des Gravitationsfeldes," p. 356

inertial field to be identical with a uniform gravitational field, had opted for a restricted and specific solution to the problem.

It was Einstein's confidence in the equivalence principle—for the above reasons—that led him to believe in a strict equality of acceleration of all freely falling bodies in a gravitational field (at a given place), at a time when this strict equality had become questionable both from the experimental and theoretical points of view. This equality can also be expressed as the equality of inertial and gravitational masses. That the equality of acceleration in a uniform gravitational field is a necessary consequence of the equivalence principle follows immediately from the fact that, by definition, all bodies have the same acceleration in a uniform inertial field.

Though this equality of acceleration in a gravitational field was known to be generally true, experiments by H. Landolt<sup>12</sup> and Adolf Heydweiller<sup>13</sup> around the turn of the century appeared to cast some doubts on it. Heydweiller, for example, produced chemical reactions in sealed vessels, finding decreases of weight as results of these reactions in certain cases. Thus the weight seemed to depend on the chemical composition of the substance and, if the inertial mass was assumed to be constant, this implied that the gravitational acceleration depended on the chemical structure of the body in contradistinction to previous experiments by Isaac Newton,<sup>14</sup> Friedrich Wilhelm Bessel,<sup>15</sup> and Roland von Eötvös.<sup>16</sup> The main challenge to the hypothesis of equal acceleration, however, was to come from the theoretical side with the advent of *STR*. *STR* had shown that the inertial mass of a body increased with the velocity, but it did not answer the question of whether the gravitational mass also increased with the velocity, and if it did whether it increased proportionally to the inertial mass. All previous experiments involved only low velocities and therefore were irrelevant to this issue. Max Planck, writing in June 1907, was inclined to give up the proportionality of inertial and gravitational masses and to deny that radiation had gravitational mass at all.<sup>17</sup> Poincaré in 1908 stated that there was no way of

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12. H. Landolt, "Untersuchungen über etwaige Änderungen des Gesamtgewichtes chemisch sich umsetzender Körper," *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin*, part 1 (1893), 301-334; esp. p. 303; H. Landolt, "Miscellaneous," *Naturwissenschaftliche Rundschau*, **15** (1900), 66.

13. Adolf Heydweiller, "Ueber Gewichtsänderungen bei chemischer und physikalischer Umsetzung," *Annalen der Physik*, **5** (1901), 394-420.

14. Isaac Newton, *Mathematical Principles of Natural Philosophy*, trans. Motte, revised by F. Cajori (Berkeley : University of California Press, 1962), **2**, 411-412, Book III, Proposition VI, Theorem VI; see also : **1**, 303-304, Book II, Proposition XXIV, Theorem XIX.

15. Friedrich Wilhelm Bessel, "Versuche über die Kraft, mit welcher die Erde Körper von verschiedener Beschaffenheit anzieht," *Annalen der Physik und Chemie*, **25** (1832), 401-417.

16. Roland von Eötvös, "Über die Anziehung der Erde auf verschiedene Substanzen," *Mathematische und Naturwissenschaftliche Berichte aus Ungarn*, **8** (Oct. 1889-Oct. 1890, pub. 1891), 65-68. See also, *ibid.*, pp. 448, 450.

17. Max Planck, "Zur Dynamik bewegter Systeme," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften zu Berlin*, **29** (1907), 542-570, in Max Planck, *Physikalische Abhandlungen und Vorträge* (Braunschweig : Friedr. Vieweg & Sohn, 1958), **2**, 179-180.

deciding that question at the time<sup>18</sup>. Gustav Mie,<sup>19</sup> as late as 1913, rejected the idea of a strict equality of inertial and gravitational masses.

Radioactive substances furnished a new means to test whether energy had weight or not. Since a radioactive substance emits energy, the inertial mass should decrease. If energy has no weight then the weight of the body should remain the same and consequently one would expect the ratio of inertial mass to weight to change in time. Such an experiment was indeed performed in 1910 by L. Southern who compared the ratio of mass to weight of uranium oxide with the ratio of mass to weight of lead oxide.<sup>20</sup> He found that the relative difference was less than 1/200000.

Before that experiment, however, the experiments by Landolt and Heydweiller as well as the new discoveries in electricity and radioactivity had already led the philosophical faculty of the University of Göttingen to propose the Benecke-Prize of 1909 for an "investigation on the proportionality of inertia and gravity." Roland von Eötvös, and the geophysicist Desider Pekár, both of the University of Budapest, and Eugen Fekete from Göttingen, undertook a series of experiments which, among other things, disconfirmed the results of Landolt and Heydweiller and won the first prize. Nothing was published at the time because new experiments promised an even greater precision. It was only after Einstein's general theory of relativity had given a new significance to such experiments that Pekár published in 1919 an account of the 1909 experiments<sup>21</sup> since the latter confirmed the equivalence principle. An abridged version of the prize essay was published in 1922.<sup>22</sup>

When Einstein formulated the equivalence principle in 1907, he was not aware of Eötvös' 1890 experiments<sup>23</sup>, which he mentioned for the first time in 1913,<sup>24</sup> but of course he knew that the question of whether energy has weight or not was an open one. Indeed, Einstein was to deal with that question twice, in 1907 and in 1911. In 1907, Einstein was probably also aware of Planck's inclination to deny weight to radiative energy since he referred to Planck's paper in his own paper.<sup>25</sup> Despite the uncertainty surrounding the equality of inertial and gravitational masses, Einstein believed in a strict equality of both masses

18. Henri Poincaré, "La Dynamique de l'Electron," *Revue Générale des Sciences pures et appliquées*, **19** (1908), 386-402; in Henri Poincaré, *Oeuvres* (Paris, Gauthier-Villars et Cie, 1954), **9**, 577.

19. Albert Einstein, "Zum gegenwärtigen Stande des Gravitationsproblem," *Physikalische Zeitschrift*, **14** (1913), 1249-1266, on 1266.

20. L. Southern, "A Determination of the Ratio Mass to Weight for a Radioactive Substance," *Proc. Roy. Soc. London*, **84** (1910), 325-344.

21. Desider Pekár, "Das Gesetz der Proportionalität von Trägheit und Gravität," *Die Naturwissenschaften*, **7** (1919), 327-331.

22. R. V. Eötvös, D. Pekár, and E. Fekete, "Beiträge zum Gesetze der Proportionalität von Trägheit und Gravität," *Annalen der Physik*, **68** (1922), 11-66.

23. Albert Einstein, "Notes on the Origin of the General Theory of Relativity," dans Albert Einstein, *Ideas and Opinions* (1954; rpt. New York : Dell, 1976), p. 280.

24. Albert Einstein et Marcel Grossmann, "Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation," *Zeitschrift für Mathematik und Physik*, **62** (1913), 225.

25. Einstein, "Relativitätssprinzip und Folgerungen" (1907), p. 414.

because it was a necessary consequence of the equivalence principle. As such, the equality of inertial and gravitational masses became a necessary but not sufficient precondition to the extension of the special theory of relativity.<sup>26</sup> In some of his later accounts,<sup>27</sup> Einstein did not mention the role which the idea of the relativity of the gravitational field had played in the development of the equivalence principle and consequently put more emphasis on the equality of inertial and gravitational masses, despite the fact that the equivalence principle does not logically follow from that equality. In his early years, however, Einstein considered the equivalence principle to be more fundamental : twice, in 1907 and 1911, he derived the equality of inertial and gravitational masses from it.

### 3.1.2 Extension of the Principle of Relativity to Uniformly Accelerated Systems

At the end of 1907, Einstein's "Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen"<sup>28</sup> appeared. The paper was essentially a review article dealing with the consequences of the relativity principle. The first four sections of the paper deal respectively with kinematics, electrodynamics, the mechanics of material points (electrons), and the mechanics and thermodynamics of material systems. The fifth and last section is entitled : "Relativitätsprinzip und Gravitation." It is in this section that Einstein first stated the equivalence principle, attempting through its use to extend the principle of relativity to uniformly accelerated reference systems. (It must be pointed out that although the paper deals with gravitational effects via the equivalence principle, it is not concerned with finding a relativistic extension of Newton's gravitational law. Indeed, neither the latter nor any gravitational force or field law is even mentioned).

Einstein began by remarking that, so far, the principle of relativity had been used only with respect to nonaccelerated reference systems : "Is it conceivable that the principle of relativity also holds for systems which are accelerated relatively to each other ?"<sup>29</sup> Einstein now asked. Since the paper was essentially a review paper, Einstein was aware that this was not the proper place to deal in detail with the new question of an extension of the principle of relativity, but he wanted nevertheless to take a position with respect to it because he felt that anyone who followed the consequences of the principle of relativity would necessarily ask it. Einstein then considered the simplest case of an accelerated system : a uniformly accelerated system. Einstein pointed out that since all objects have the same acceleration in such a system, the situation is therefore physically equivalent to an inertial system with a uniform gravitational

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26. This is confirmed by David Reichstein, *Albert Einstein : sein Lebensbild und seine Weltanschauung*, 3rd enlarged ed. (Prague : Selbstverlag des Verfassers, 1935), p. 230.

27. Einstein : "Notes on the Origin of the General Theory," p. 280 ; Einstein, "Autobiographical Notes," p. 64.

28. Einstein, "Relativitätsprinzip und Folgerungen" (1907).

29. "Ist es denkbar, dass das Prinzip der Relativität auch für Systeme gilt, welche relativ zu einander beschleunigt sind?" *ibid.*, p. 454.

field. Observing that this equivalence was generally true for mechanics (at low velocities), and not seeing any evidence of a restriction to mechanics, Einstein postulated a complete physical equivalence – mechanical, electromagnetic, etc. – between a uniformly accelerated system and an inertial system with a uniform gravitational field, adding that "this assumption extends the principle of relativity to the case of rectilinear uniformly accelerated motion of the reference system."<sup>30</sup> Einstein also remarked that the heuristic value of the postulate resided in the fact that it allowed one to replace a uniform gravitational field by a uniformly accelerated system, which can be treated theoretically to a certain extent.

### Space and time in a uniformly accelerated reference system

Einstein's next step was to analyze the concepts of space and time in a uniformly accelerated system. Einstein did not carefully define the concept of uniform acceleration. In *STR*, acceleration is velocity dependent and therefore the acceleration measured within an accelerated reference frame with nonzero velocity is not the same as the acceleration measured outside with respect to a fixed inertial system. Thus the meaning of the term "uniform acceleration" must be specified, and this Einstein did not do. Einstein rectified this vagueness later on<sup>31</sup> by pointing out that in his 1907 paper the term "uniform acceleration" had to be taken as meaning a constant acceleration with respect to an instantaneous inertial system momentarily at rest with respect to the accelerated one.<sup>32</sup> Despite the uncertainty about the concept of uniform acceleration, Einstein's paper managed to be consistent nevertheless because of the order of approximation adopted.

Einstein's analysis of the concepts of space and time in a uniformly accelerated reference system started with the question of whether or not acceleration influences the shapes of bodies. The question was of importance since acceleration might thereby affect the lengths of measuring rods or the rates of clocks. From the symmetry of the problem, Einstein reasoned that such an influence, if it existed, could only be a constant dilatation in the direction of the acceleration (and in the directions perpendicular to it) and assumed that this dilatation was an even function of the acceleration. In that case, the hypothetical dilatation can be neglected if second and higher powers of the acceleration are neglected. Having adopted this approximation for the rest of his paper, Einstein neglected any specific influence of the acceleration on the shapes of rods or on the rates of clocks. This approximation had the important consequence that a uniformly accelerated system could be considered as physically equivalent to an instantaneous inertial system momentarily at rest with respect to the accelerated system.

The instantaneous inertial system allowed Einstein to introduce what he cal-

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30. "Diese Annahme erweitert das Prinzip der Relativität auf den Fall der gleichförmig beschleunigten Translationsbewegung des Bezugssystem," *ibid.*, p. 454.

31. Albert Einstein, "Berichtigungen," *Jahrbuch der Radioaktivität und Electronik*, **5** (1908), 99.

32. This type of acceleration was later called uniform acceleration in Max Born's sense.

led the "time" of the uniformly accelerated system. Since the latter system, at any given instant, is equivalent to the instantaneous inertial system, a synchronization and thus a time can be defined throughout the accelerated system at each instant. In accord with modern usage, we shall refer to this time as the coordinate time. Einstein defined the coordinate time as the time given by the clock at the origin of the accelerated system which is synchronous (according to the above definition) with the event. Besides this coordinate time, Einstein also introduced what he called the "local time" of the accelerated system, which is simply the time as directly given by local clocks. We shall refer to this time as the proper time. Einstein pointed out that the proper times of two spatially distant events might be the same without the events being synchronous according to the above definition. Thus, for example, if the local clocks of the accelerated system  $\Sigma$  are synchronized with respect to a fixed inertial system  $S$  momentarily at rest with respect to  $\Sigma$ , these clocks will remain synchronized with respect to  $S$  since they undergo the same motions. The local clocks, however, will not in general remain synchronized with respect to the instantaneous inertial systems  $S'$  coinciding with  $\Sigma$  at later instants, because of the relativity of simultaneity in the special theory of relativity. Hence, for identical proper times of spatially distant events, the coordinate times will generally be different and vice versa. The coordinate time is to be used each time a synchronization between spatially distant events is involved; the proper time is adequate when local events are considered.

As to the relation between the proper time and the coordinate time, Einstein established it as follows : Consider the systems  $\Sigma(\xi, \tau, \sigma)$ ,  $S(x, t)$ ,  $S'(x', t')$ , where  $\tau$  is the coordinate time and  $\sigma$  the proper time, and suppose the local clocks synchronized with respect to  $S$  as indicated above (see Fig. 3.3). If two events are synchronous with respect to  $S'$  (and thus with respect to  $\Sigma$ ) then

$$t_1 - \frac{v}{c^2}x_1 = t_2 - \frac{v}{c^2}x_2.$$

Neglecting all terms in  $\tau^2$  or  $v^2$  one has furthermore :

$$x_2 - x_1 = x'_2 - x'_1 = \xi_2 - \xi_1,$$

$$t_1 = \sigma_1 = \tau,$$

$$t_2 = \sigma_2,$$

and

$$v = \gamma t_1 = \gamma \tau,$$

where  $\gamma$  is the acceleration. Hence,

$$\sigma_2 - \sigma_1 = \frac{\gamma \tau}{c^2} (\xi_2 - \xi_1);$$

replacing  $\sigma_2$  and  $\xi_2 - \xi_1$  with  $\sigma$  and  $\xi$  respectively, Einstein finally obtained the relation

$$\sigma = \tau \left( 1 + \frac{\gamma \xi}{c^2} \right),$$

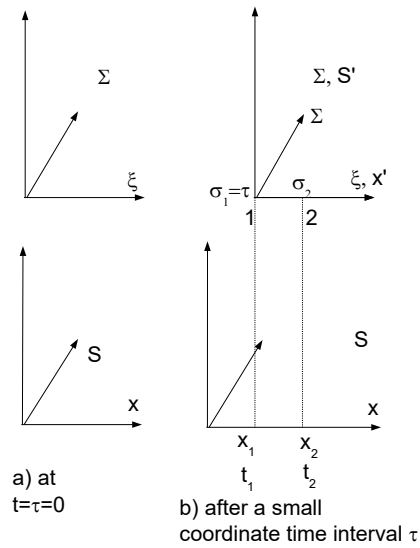


FIGURE 3.3 – Relation between the proper time and the coordinate time; the acceleration is along the  $Ox$  axis



which is valid for small  $\tau$  and  $\xi$ .

At the origin of the uniformly accelerated system ( $\xi = 0$ ), the proper time corresponds to the coordinate time, whereas at the point  $P$  ( $\xi \neq 0$ ) the proper time  $\sigma$  is greater than the coordinate time. Hence, if two local clocks, placed at the origin and at the point  $P$  respectively, are compared anywhere in the accelerated system at a given coordinate time, the clock at  $P$  will appear to run faster than the clock at the origin. By applying the equivalence principle, this becomes true in a uniform gravitational field. In that case, the above formula can be rewritten as

$$\sigma = \tau \left( 1 + \frac{\Phi}{c^2} \right),$$

where  $\Phi = \gamma\varepsilon$  corresponds to the gravitational potential. The greater the gravitational potential, the faster the local clock and, more generally, the local physical phenomenon. Since the gravitational potential at the surface of the sun is smaller than the gravitational potential at the surface of the earth, and assuming the above formula to apply to a non-uniform gravitational field, it follows that the local clocks on the sun should run slower than identical clocks on earth. Einstein concluded that the wavelength of an absorption or emission line in the solar spectrum should be about  $(1 + 2.10^{-6})$  times greater than the wavelength of light emitted by identical atoms on earth but, apparently, he did not know of any experiment which could support his prediction.

### Electromagnetic effects in a uniform gravitational field

Einstein finally proceeded to an analysis of the influence of gravity on electromagnetic processes. His idea was to obtain two modified sets of Maxwell's equations for the uniformly accelerated system, one making use of the proper time, the other using the coordinate time, and then to examine their gravitational consequences via the equivalence principle. Einstein's main results were that the velocity of light in a uniform gravitational field is equal to  $c(1 + \frac{\Phi}{c^2})$  if the coordinate time is used, and that energy  $E$  has gravitational mass  $E/c^2$ , equal to its inertial mass. From the first result, Einstein drew the conclusion that light rays traveling in another direction than the  $\xi$  axis must undergo a deviation in a gravitational field. He found, however, that the effect is so small on earth that he had no hope of verifying it. (At that time, Einstein did not yet have the idea of using a solar eclipse to test the deviation.) With respect to the second result, it is interesting to note that Einstein derived the gravity of energy from the equivalence principle. This shows that Einstein considered the equivalence principle to be more fundamental than any general experimental result about the equality of inertial and gravitational masses he might have been aware of. The reason for Einstein's confidence in the equivalence principle was that Einstein saw in that principle a tool for the extension of the special theory of relativity, as discussed above.

Einstein's derivations of the velocity of light and of the gravitational mass of energy were in fact superfluous. The equality of inertial and gravitational masses follows directly, without any need of further steps, from the postulated

ontological identity of a uniform acceleration field and a uniform gravitational field. Hence inertial mass is one and the same as gravitational mass. The variable velocity of light, on the other hand, is a direct consequence of the definition of the coordinate time. As long as the proper time is used, the velocity of light is everywhere equal to  $c$  in the accelerated system. If the coordinate time  $\tau = \sigma \left(1 - \frac{\Phi}{c^2}\right)$  is used, then the velocity of light becomes equal to

$$c' = \frac{L}{\tau} = \frac{L}{\sigma \left(1 - \frac{\Phi}{c^2}\right)} = c \left(1 + \frac{\Phi}{c^2}\right)$$

(where  $L$  is a length) and thus varies from place to place. Einstein was become aware of these improved derivations in 1911 as we shall see.

Einstein's first attempt to generalize *STR* was not very conclusive and raised more questions than it answered. Einstein apparently had no immediate hope of verifying the equivalence principle. Furthermore, the theory of accelerated systems proved to be rather subtle and difficult. Even in the simple case of uniformly accelerated motion and a first order approximation, there was required a new concept of time—i.e., the coordinate time—which was not directly given by the readings of clocks. Thus, the global time coordinate became a conceptual construction without immediate physical meaning. Later on, Einstein realized that the situation was even worse, and that even the concept of spatial coordinates no longer had its immediate meaning. It took Einstein many years before he could make sense out of that situation. Thus it is not surprising that in 1908 Einstein turned to a more immediate goal : the integration of gravitation within *STR*. Einstein, however, became rapidly convinced that such an attempt was hopeless because it appeared to lead to a relaxation of the strict equality of inertial and gravitational masses, and hence to contradiction of the equivalence principle; he did not publish anything on it at the time. In fact, Einstein probably already realized, at that time, that since the equivalence principle predicts a variable velocity of light, he would have to choose, sooner or later, between the special theory of relativity and a more general theory making use of the equivalence principle. Einstein was to opt for the latter choice, which explains his rather rapid renunciation of the attempt to find a relativistic theory of gravitation within the special theory of relativity. We shall analyze Einstein's brief attempt together with attempts by others to treat gravitation within *STR* in the second part of this chapter.

### 3.1.3 Full Confidence in the Equivalence Principle and in a General Relativity (1911)

When Einstein wrote the 1907 review paper, he was still employed at the Patent Office in Bern, where he was to remain for another two years. In that same year, 1907, Einstein applied as Privatdozent at the University of Bern, where a chair in theoretical physics had been created in 1906. Einstein submitted his revolutionary 1905 paper on *STR* which, however, was rejected as wanting.<sup>33</sup>

33. Seelig, Albert Einstein (1960), p. 150.

Another attempt, the following year, met with success, and Einstein began to lecture in the fall of 1908. Since Einstein's lectures were not brilliant at that time, the prospect of a professorship was rather limited.<sup>34</sup> The situation, however, was to change radically in 1909, when Einstein's scientific contributions began to receive public recognition. In that year, Einstein received an honorary doctorate from the University of Geneva in July,<sup>35</sup> was invited to the Naturforscher conference in Salzburg in September,<sup>36</sup> and became an associate professor in theoretical physics at the university of Zürich in October.<sup>37</sup> From that time on, universities were to compete in their efforts to get Einstein on their faculty. Einstein taught only three semesters at the university of Zürich and then, around Easter 1911, moved to the university of Prague, where he became a full professor in theoretical physics. This was the university where Mach had been active for 28 years (1867 – 1895).<sup>38</sup>

Einstein's appointment at Prague was closely associated with Mach's philosophical influence. The decisive person in the nomination process, the physicist Anton Lampa, a former student of Mach and a strong supporter of Mach's positivistic philosophy, proposed Einstein's nomination, in part, because he thought that Einstein's teaching would embody Mach's line of thought.<sup>39</sup> Mach, who was living in retirement in Vienna at that time, had been in correspondence with Einstein since 1909. Four letters (all from Einstein to Mach) are known. In the first letter, dated August 1909, Einstein wrote :

I thank you very much for the lecture on the conservation of energy which you sent me and which I have already read carefully. Beyond this, I of course know your major works quite well, and among them I admire most the one on mechanics. You have had such an influence on the epistemological conceptions of the younger generation of physicists that even your present opponents such as for example Mr. Planck, one of the physicists, would without doubt have been called "Machians" according to their overall stand, a few decades ago.

Since I do not know how to thank you otherwise, I am sending you a few of my papers. In particular, I suggest that you have a look at the one on the Brownian motion, because we have here a motion which, it is believed, must be interpreted as a "thermal motion . " <sup>40</sup>

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34. Ibid., pp. 154-55.

35. Ibid., 157-60.

36. Ibid., p. 145.

37. Ibid., pp. 162-68.

38. Erwin Hiebert, "E. Mach," *Dictionary of Scientific Biography*, 8 (1973), p. 596.

39. Philippe Frank, *Einstein : His Life and Times*, rev. ed. (New York : Alfred A. Knopf, 1953), p. 78.

40.

"Ich danke Ihnen bestens für den mir übersandten Vortrag über das Gesetz von der Erhaltung der Arbeit, den ich bereits mit Sorgfalt durchgelesen habe. Im übrigen kenne ich natürlich Ihre Hauptwerke recht gut, von denen ich dasjenige über die Mechanik am meisten bewundere. Sie haben auf die erkenntnistheoretischen Auffassungen der jüngeren Physiker-Generation einen solchen Einfluss

From the letter it appears that Mach was interested in the theory of relativity and had sent one of his publications to Einstein, at a time when the latter was still Privatdozent in Bern. In the second communication (postcard dated 17 August 1909), Einstein wrote :

Your friendly letter gave me enormous pleasure, and the paper as well. What you have shared with me personally was already known to me and to all friends of science. I admire your great energy. ... I am very glad that you are pleased with the relativity theory. Unhappily, I do not have any reprints left of the comprehensive paper which I have published in the *Jahrbuch für Radioaktivität und Elektronik*.

Thanking you again for your friendly letter, I remain your student.  
A. E.<sup>41</sup>

The letter expresses Einstein's admiration for Mach and also reveals that, at that time, Mach was pleased with the special theory of relativity. Philipp Frank, who visited Mach in 1910, even got the impression that Mach completely agreed with that theory.<sup>42</sup> It is likely that Mach's initial friendly attitude toward *STR* influenced Lampa's decision to propose Einstein for the nomination.

The move to Prague meant a better financial situation for Einstein, whose salary in Zürich had not been much greater than that at the patent office. It also meant improved working conditions, since Einstein now had access to a fine library. In was in Prague, in June 1911, that Einstein completed the paper "Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes" [On the Influence of Gravitation on the propagation of Light].<sup>43</sup> In the introduction to this paper, Einstein stated that he was returning to the subject of the influence of gravita-

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gehabt, dass sogar Ihre heutigen Gegner, wie z. B. Herr Planck, von einem der Physiker, wie sie vor einigen Jahrzehnten im Ganzen waren, ohne Zweifel für 'Machianer' erklärt würden.

Weil ich nicht weiss, wie ich mich Ihnen sonst dankbar zeigen soll, schicke ich Ihnen einige meiner Abhandlungen. Besonders möchte ich Sie bitten, sich das über die Brown'sche Bewegung kurz anzusehen, weil hier eine Bewegung vorliegt, die man als 'Wärmebewegung' deuten zu müssen glaubt," Friedrich Herneck, "Die Beziehungen zwischen Einstein und Mach, dokumentarisch dargestellt," *Wissenschaftliche Zeitschrift der Friedrich Schiller Universität Jena*, **15** (1966), p. 6.

41.

"Ihr freundlicher Brief hat mich ungemein gefreut und nicht minder die Abhandlung. Was Sie mir Persönliches von sich mitteilen, war mir schon alles bekannt, wie allen Freunden der Wissenschaft. Ich bewundere Ihre grosse Energie. Es scheint, dass ich Ihnen die Abhandlungen zu senden vergessen habe. Aber sie gehen nun zusammen mit der Karte ab. Es freut mich sehr, dass Sie Vergnügen an der Relativitätstheorie haben. Leider habe ich keine Exemplare mehr von der zusammenfassenden Arbeit, die ich im *Jahrbuch für Radioaktivität und Elektronik* darüber publiziert habe.

Indem ich Ihnen nochmals herzlich danke für Ihren freundlichen Brief verbleibe ich Ihr verehrender Schüler. A.E." *ibid.*, p.7

42. *Ibid.*

43. Albert Einstein, "Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes," *Annalen der Physik*, **35** (1911), 898-908.

tion on the propagation of light because his previous 1907 treatment no longer satisfied him, and especially because he now recognized that his prediction of the deviation of light in a gravitational field could be tested experimentally during a solar eclipse. Whereas in 1907 Einstein had had no hope of testing the equivalence principle, the solar eclipse idea suddenly brought that principle within the reach of experimental confirmation. It was probably the eclipse test that underlay Einstein's full conviction of the validity of the equivalence principle and of the necessity of a general theory of relativity. Thus, in a letter to Carl Seelig, Einstein wrote that full conviction on these issues came with the 1911 paper on the deflection of light.

As concerns the general theory of relativity, it is even less possible to speak of a definite birthdate. The first decisive idea came in 1911 (equivalence principle). The title of the corresponding publication is "On the Influence of Gravitation on the Propagation of Light" ("Annalen der Physik," 1911). From then on, there was the conviction concerning the general theory of relativity.<sup>44</sup>

Though Einstein appears to have been fully convinced of the validity of the equivalence principle in 1911, he was aware that his colleagues would probably not share his conviction so readily. In the first section of the paper, Einstein tried to justify the equivalence principle the best he could but was aware of the difficulty of the task. One argument Einstein gave in favor of the equivalence principle was that the latter provided a "very satisfactory interpretation"<sup>45</sup> of the experimental equality of acceleration of freely falling bodies in a uniform gravitational field. Einstein did not mention any specific experiment, but considered the above equality to be well established. Einstein admitted, however, that the equality of acceleration of freely falling bodies can only justify, at most, a mechanical equivalence principle, whereas the heuristic power of the equivalence principle resided precisely in the overall equivalence. Another argument Einstein presented was that the equivalence principle eliminated the concept of absolute acceleration, since it allowed one to view a uniform acceleration field as a uniform gravitational field. Thus he wrote :

With this conception [of exact physical equivalence], one cannot speak of the *absolute acceleration* of the reference system any more than one can speak of the *absolute velocity* of a system in the usual relativity theory.<sup>46</sup>

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44. "Bei der Allgemeinen Relativitätstheorie kann man erst recht nicht von einem Geburtstag sprechen. Die erste entscheidende Idee kam in 1911 (Äquivalenzprinzip). Der Titel der betreffenden Publication ist 'Einfluss der Schwerkraft auf die Ausbreitung des Lichtes' ('Annalen der Physik,' 1911). Von da an war die Überzeugung der Allgemeinen Relativitätstheorie vorhanden," Seelig, *Einstein* (1960), p. 114.

45. Einstein, "Einfluss der Schwerkraft," p. 899.

46.

"Mann kann bei dieser Auffassung ebensowenig von der *absoluten Beschleunigung*

Einstein added in a footnote that of course it is impossible to transform away an arbitrary gravitational field via a single state of motion of the reference system, just as it is impossible to transform to rest all points of a medium in arbitrary motion through one relativistic transformation in special relativity.

As further support for the equivalence principle, Einstein, in the second section of the paper, derived the gravitational mass of energy from relativistic considerations and the equivalence principle. Here again, as with the 1907 derivation, one might wonder about the utility of the argument, since, as Einstein himself pointed out, the equivalence principle yields the gravitational mass of energy at once : if energy has a given inertial mass in a uniformly accelerated system, it automatically has an identical gravitational mass in a uniform gravitational field. As to the question why, after all, energy should have a gravitational mass, Einstein pointed out that if the gravitational mass of a body was independent of its energy content, the acceleration of the body in the same gravitational field would be different for different energy contents. Furthermore, gravitational mass would still be conserved in the prerelativistic sense instead of satisfying a broader principle of conservation of energy as is the case for inertial mass. Such an asymmetry seemed highly unlikely to Einstein.<sup>47</sup> This, taken together with his interest in the equivalence principle itself, probably explain why Einstein assumed a strict equality of inertial and gravitational masses, which he then presented as support for the equivalence principle.

In the third section of the 1911 paper, Einstein gave a simplified derivation of the velocity of light in a uniform gravitational field. Whereas, in 1907, Einstein had obtained the velocity of light from modified Maxwell's equations, he was now to derive it directly from the relation between the coordinate and local times. Consider a uniformly accelerated system  $K'$ , with the acceleration  $\gamma$  along the  $z$  axis, and an inertial system  $K_0$  (see Fig. 3.4). At the initial instant, the system  $K'$  is assumed to be rest with respect to  $K_0$ ; furthermore, the measurement instruments located at the points  $S_1$  and  $S_2$  of  $K'$  are supposed to be small with respect to the distance  $h$  between these two points. If, at the time origin,  $S_2$  sends radiation of frequency  $\nu_2$  (as measured locally at  $S_2$ ) in the direction of  $S_1$ ,  $S_1$  will in first approximation, receive radiation of frequency

$$\nu_1 = \nu_2 \left( 1 + \frac{\gamma h}{c^2} \right)$$

(as measured locally) because of the Doppler effect resulting from the velocity  $\gamma \frac{h}{c}$  (with respect to  $K_0$ ) acquired by  $S_1$  during the time of propagation of the radiation. By applying the equivalence principle, the same is true for a uniform gravitational field, and the above relation becomes

$$\nu_1 = \nu_2 \left( 1 + \frac{\Phi}{c^2} \right),$$

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*gung* des Bezugssystems sprechen, wie man nach der gewöhnlichen Relativitätstheorie von der *absoluten Geschwindigkeit* eines Systems reden kann," *ibid.*, p. 899. Emphasis in original.

47. *Ibid.*, p. 901.

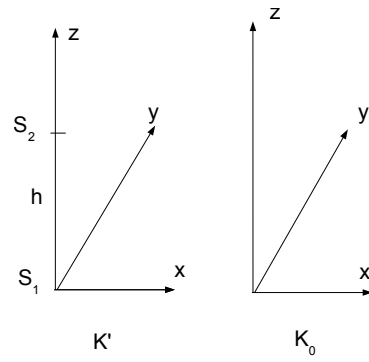


FIGURE 3.4 – Uniformly accelerated system  $K'$  (the acceleration is along the  $Oz$  axis)

where  $\Phi$  is the gravitational potential of  $S_2$  relative to  $S_1$ . If  $S_2$  is a point on the sun, it follows that a red-shift should be observed on earth (since  $\Phi < 0$ ). Einstein mentioned that such a red-shift had indeed been observed by L. E. Jewell,<sup>48</sup> and in particular by Ch. Fabry and H. Boisson,<sup>49</sup> but indicated that these authors had attributed the red-shift to a pressure effect.

From the result that the number of oscillation emitted and received per unit proper time is different, Einstein concluded that a new time (i.e., the coordinate), defined in such a way that the number of oscillations between  $S_2$  and  $S_1$ , is independent of the absolute value of the time, had to be introduced in the reference system. Otherwise, the physical laws would not be invariant with respect to time translation. Specifically, Einstein stated that the clock at  $S_2$  must be modified in such a way that it runs  $(1 + \frac{\Phi}{c^2})$  times slower than the local clock at  $S_1$ , when both clocks are compared to the same place. In that case the frequency emitted at  $S_2$  and measured there becomes equal to  $\nu_2 (1 + \frac{\Phi}{c^2})$  and is therefore equal to the frequency received at  $S_1$ . Similarly, the velocity of light at  $S_2$  now becomes equal to

$$c = c_0 \left( 1 + \frac{\Phi}{c^2} \right)$$

where  $c_0$  is the velocity of light at  $S_1$  (Einstein, here again, assumed the velocity of light, if measured locally by a proper clock, to have the same value everywhere in the reference system). It must be pointed out that in all the above considerations, Einstein as he did in 1907, assumed the acceleration or the gravitational field to have no intrinsic effect on the clocks and rods, i.e., he considered the rods to be rigid and the clocks to be perfect.

From the variation of the velocity of light *in vacuo* with the gravitational potential, it follows that, as in the case of a medium with variable index of refraction, in which light-rays are deflected towards regions of higher refractive index, light-rays in a gravitational field are bent toward regions of lower gravitational potential. Using Huygens' principle, Einstein found that light rays grazing the sun would be deflected through an angle  $\theta = 0,83''$  seconds of arc, and he proposed to use stars as light sources during a total solar eclipse (see Fig. 3.5). It is interesting to note that J. Soldner in 1801, considering light as made of particles subject to Newton' laws, found a deviation of  $0,84''$  seconds of arc near the sun.<sup>50</sup> Einstein urged astronomers to take up the question of the ve-

48. L. E. Jewell, "The coincidence of solar and metallic lines, a study of the appearance of lines in the spectra of the electric arc and the sun," *The Astrophysical Journal*, **3** (1896), 90-113; abstract by E. Bouty, same title, *Journal de physique Théorique et Appliquée*, **6** (1897), 84. L. E. Jewell, J. F. Mohler, and W. J. Humphreys, "Note on the pressure of the reversing layer of the solar atmosphere," *The Astrophysical Journal*, **3** (1896), 138; abstract by E. Bouty, same title, *Journal de Physique Théorique et Appliquée*, **6** (1897), 84-85.

49. Ch. Fabry et H. Boisson, "Comparaison des raies du spectre de l'arc électrique et du Soleil. Pression de la couche renversante de l'atmosphère solaire." *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences* (Paris), **148** (1909), 688-690.

50. J. Soldner, "Über die Ablenkung eines Lichtstrahls von seiner geradlinigen Bewegung durch die Attraktion eines Weltkörpers, an welchem er nahe vorbeigeht," *Berliner Astronomisches Jahrb.*, **1804**, p. 161; rpt. in parts and with comments by P. Lenard, same title, *Annalen der Physik*, **65** (1921), 593-604.



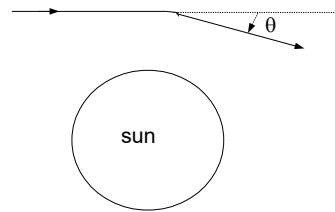


FIGURE 3.5 – Deflection of light rays near the sun

rification of a deflection of light rays in a gravitational field—"even if the above considerations should appear insufficiently founded or even adventurous"<sup>51</sup>—since, independently of any theory the above verification had its own intrinsic interest according to Einstein. Einstein was thus fully aware that his theory, and in particular the equivalence principle, might appear as rather speculative to his contemporaries.

The first person to become actively involved in the experimental verification of the effects derived from the equivalence principle was Erwin Finlay Freundlich.<sup>52</sup> Freundlich had studied mathematics, physics, and astronomy at the University of Göttingen, and had, in 1910, obtained a Ph.D. in mathematics from that university under the direction of Felix Klein.<sup>53</sup> On 1 July 1910, he was appointed as assistant at the Royal Observatory in Berlin. Freundlich had first come in contact with Einstein through Leo Wenzel Pollak, Demonstrator at the German University in Prague, during a visit the latter made to the Berlin Observatory in August 1911.<sup>54</sup> Einstein, shortly afterwards, asked Pollak to transmit to Freundlich the proofs of his paper "Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes" [On the Influence of Gravitation on the Propagation of Light]<sup>55</sup>—which was published on 1 September 1911—and requested Freundlich to verify the deviation of light rays in a gravitational field.<sup>56</sup> Freundlich then wrote to observatories throughout the world, asking them to send photographic plates of total solar eclipses. C. D. Perrine of Cordoba (Argentina) even promised to take special photographs of the solar eclipse of October 1912, but was prevented from doing so because of poor weather.<sup>57</sup> With the photographic plates he received, however, Freundlich could not verify the deflection of starlight because of insufficiently focused star images and also because the sun was not centered on the plates, making it impossible, in particular, to detect twice the deflection for stars on both sides of the sun. In 1913, in a short paper describing the light deflection effect, Freundlich communicated the negative results obtained so far

51. "auch wenn die im vorigen gegebenen Überlegungen ungenügend fundiert oder gar abenteuerlich erscheinen sollten," Einstein, "Einfluss der Schwerkraft" (1911), p. 908; this quotation, curiously, has been left out from the English translation in A. Einstein, H. A. Lorentz et al., *The Principle of Relativity*, W. Perrett and G. B. Jeffery trans. (1923; rpt. New York: Dover, 1952), p. 108.

52. Albert Einstein, preface to Erwin Freundlich, *Die Grundlagen der Einsteinschen Gravitationstheorie*, 2nd enl. ed. (Berlin: J. Springer, 1917).

53. For biographical information and the relationship of Freundlich with Einstein, see Eric Forbes, "Freundlich, E. F.," *Dictionary of Scientific Biography*, 5 (1972), 181-84, and Lewis Robert Pyenson, "The Goettingen Reception of Einstein's General Theory of Relativity" (Ph. D. dissertation, John Hopkins University, 1974), pp. 313-38. For a complete list of Freundlich's publications, see H. von Klüber, "Erwin Finlay-Freundlich," *Astronomische Nachrichten*, 288 (May 1964-Dec. 1965), 281-86.

54. Pyenson, "Goettingen Reception", diss., p. 315.

55. Albert Einstein, *Annalen der Physik*, 35 (1911), 898-908.

56. Erwin Freundlich, "Über einen Versuch, die von A. Einstein vermutete Ablenkung des Lichtes in Gravitationsfeldern zu prüfen," *Astronomische Nachrichten*, 193, no. 4628 (1913), cols. 369-72, on col. 369.

57. *Ibid.*, col. 371. See also John Earman and Clark Glymour, "Relativity and Eclipses: The British Eclipse Expeditions of 1919 and Their Predecessors," *Historical Studies in the Physical Sciences*, 11, part 1, (1980), 49-85, on 61-62.

and appealed to astronomers and observatories to take photographs of future solar eclipses, in particular, of the coming August 1914 eclipse.<sup>58</sup> Prior to any experimental test, Einstein was to use the equivalence principle to develop his static theory of gravitation.

## 3.2 Further use of the equivalence principle : the static theory of gravitation

Einstein's static theory of gravitation was to break new ground in gravitational theories insofar as it was based on the equivalence principle and did not try to integrate gravitation within *SRT*. We shall examine Einstein's static theory of gravitation after briefly reviewing the main prerelativistic and (special) relativistic attempts (including an unpublished one by Einstein) to improve upon Newton's gravitational work at the beginning of the twentieth century.<sup>59</sup>

### 3.2.1 Historical Background

#### Prerelativistic theories of gravitation

When Newton presented his gravitational law in 1687,<sup>60</sup> he was perfectly aware that he did not "explain" gravitation. Indeed, the concept of an instantaneous action at a distance seemed absurd to most people, Newton included. Thus efforts immediately began to interpret the successful gravitational law in terms of mechanical models viewing gravitation either as a deformation effect (static theory) or as a kinetic effect (dynamical theory).<sup>61</sup> With the advent of electromagnetic theories in the second half of the 19th century, it became natural to attempt to reduce gravitation to the electromagnetic formalism. By now, it was also known that there existed experimental discrepancies with respect to Newton's law, the most important being the unexplained residual precession of the perihelion of Mercury; this was discovered by Urbain Le Verrier and amounted to about 40" per century. Among the other discrepancies were anomalies in the motion of Venus and Mars and in the motion of Encke's comet.<sup>62</sup> All this

58. Freundlich, "Ablenkung des Lichtes" (1913).

59. For review articles see (a) P. Drude, "Ueber Fernwirkungen," *Annalen der Physik*, **62** (1897), I-XLIX; (b) J. Zenneck, "Gravitation," *Encyklopädie der Mathematischen Wissenschaften*, V, part 1, 2, (1901), pp. 25-67; S. Oppenheim, "Kritik des Newtonschen Gravitationsgesetzes," *Encyklopädie der Mathematischen Wissenschaften*, VI, part 2, 22 (article completed in 1920; Leipzig, 1922-1934), pp. 80-158. See also Pyenson, "Goettingen Reception," diss.; J. D. North, *The Measure of the Universe : A History of Modern Cosmology* (Oxford : Clarendon, 1965).

60. Isaac Newton, *Mathematical Principles of Natural Philosophy*, trans. Motte, rev. by F. Cajori (Berkeley : University of California Press, 1962), Book III, Propositions I-VII, pp. 406-15.

61. See for example Drude, "Fernwirkungen"; Walther Ritz, "Die Gravitation," *Scientia*, **5** (1909), 241-55.

62. S. Newcomb, *The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy, Supplement to the American Ephemeris and Nautical Almanac for 1897* (Washington, 1895).

clearly suggested that Newton's gravitational theory might perhaps be replaced by a better one.

In 1900, H. A. Lorentz attempted to solve the problem of Mercury's perihelion by integrating gravitation within his electron theory.<sup>63</sup> His theory made use of a hypothesis which had already been used by O. F. Mossotti and Friedrich Zöllner,<sup>64</sup> to the effect that the attraction between charges of opposite signs is slightly greater than the repulsion between charges of the same sign. This difference in magnitude of electric forces leads to a net attraction between neutral bodies at rest, the gravitational field  $\vec{E}_g$  being the resultant of the two kinds of electric fields. To each kind of the electric fields there corresponds in the dynamical case a magnetic field, the resultant of which produces a gravitational field  $\vec{B}_g$ . Thus the gravitational field is described by two vectors,  $\vec{E}_g$  and  $\vec{B}_g$  satisfying field equations analogous to Maxwell's equations. The gravitational force law is given by  $\vec{F} = m [\vec{E}_g + \vec{v} \wedge \vec{B}_g]$  and therefore depends on the velocity of the particle. One of the difficulties of such a theory, already mentioned by Maxwell,<sup>65</sup> is that the energy density  $u = -\frac{1}{2} [(\vec{E}_g)^2 + (\vec{B}_g)^2]$  is negative. This implies that the energy of a region containing a gravitational field is lower than when the region is empty. Thus when a gravitational wave penetrates an empty region, energy is supposed to move out in the opposite direction. Lorentz's theory did not achieve what it was designed to achieve, namely, the resolution of the problem of the precession at Mercury's perihelion. Furthermore, despite the electromagnetic formalism, it was not yet relativistic since the absolute velocity of the solar system appeared in Lorentz's gravitational force law.<sup>66</sup>

### The force laws of Poincaré and Minkowski

The first attempt to develop a relativistic theory of gravitation was made by Henri Poincaré. His approach, which makes use of group-theoretical arguments, appears particularly original even today. A summary of Poincaré's theory was read to the Academy of Sciences in Paris on 5 June 1905<sup>67</sup>—before Einstein submitted his 1905 paper on *STR*—and the whole theory was completed in July 1905, before the publication of Einstein's paper in September; Poincaré's paper, however, was published only in 1906.<sup>68</sup> The historical importance of Poincaré's

63. H. A. Lorentz, "Considérations sur la pesanteur," *Verh. Kon. Akad. Wetensch. Amsterdam*, **8** (1900), 603, in H. A. Lorentz, *Collected Papers* (The Hague : Martinus Nijhoff, 1937), **5**, 198-215.

64. O. F. Mossotti, *Sur les forces qui régissent la constitution intérieure des corps* (Turin, 1836); F. Zöllner, *Erklärung der universellen Gravitation* (Leipzig, 1882).

65. J. C. Maxwell, "A Dynamical Theory of the Electromagnetic Field," *Roy. Soc. Trans.*, **155** (1865), 492, in J. C. Maxwell, *Scientific Papers* (Cambridge, 1890), **1**, 570-71.

66. H. A. Lorentz, "Considérations sur la pesanteur," pp. 212-15; see also F. Kottler, "Gravitation und Relativitätstheorie," *Encyklopädie der Mathematischen Wissenschaften*, VI, part 2, 22a (1922), p. 170.

67. Henri Poincaré, "Sur la Dynamique de l'Electron," *Comptes rendus de l'Académie des Sciences*, **140** (1905), 1504-08, in Henri Poincaré, *Oeuvres* (Paris, 1954), **9**, 489-93.

68. Henri Poincaré, "Sur la Dynamique de l'Electron," *Rend. Circ. Mat. Palermo*, **21** (1906), 129-76, in *Oeuvres* (1954) **9**, 494-550.

theory lies in the fact that between 1905 and 1911 no major relativistic gravitational force law was developed which was not already contained in the formulas given by Poincaré. Poincaré's purpose in the paper was to discuss what he named the Lorentz transformation<sup>69</sup> from a group-theoretical point of view in order to improve the mathematical basis of the Lorentz 1904 theory.<sup>70</sup> At the same time, Poincaré also wanted to analyze the consequences, in particular with respect to gravitation, of Lorentz's hypothesis that all forces, whatever their origin, transform like the electromagnetic forces in a change of inertial systems. Thus Poincaré set out to find a Lorentz invariant gravitational force law yielding Newton's force law as an approximation for low velocities of the bodies.

Poincaré analyzed the problem from the most general point of view and assumed that the force  $\vec{F}$  exerted by a body  $A$  (situated at the point  $\vec{r}_0 + \vec{r}$  at the time  $t_0 + t$ ) on a body  $B$  (located at the point  $\vec{r}_0$  at the time  $t_0$ ) at the time  $t_0$  depended on the position and velocity  $\vec{v}_B$ , of  $B$  at  $t_0$  and on the position and velocity  $\vec{v}_A$  of  $A$  at the time  $t_0 + t$  ( $t < 0$ ).<sup>71</sup> Thus the force was a priori a function of  $t$ ,  $\vec{r}$ ,  $\vec{v}_A$ , and  $\vec{v}_B$ . Furthermore, Poincaré assumed a finite propagation of the gravitational interaction and a general equation of propagation of the type

$$\Phi(t, r, \vec{v}_A, \vec{v}_B) = 0.$$

Poincaré's next step was to find the invariants of the Lorentz group which are :

$$r^2 - t^2, \frac{t - \vec{r} \cdot \vec{v}_A}{\sqrt{1 - v_A^2}}, \frac{t - \vec{r} \cdot \vec{v}_B}{\sqrt{1 - v_B^2}}, \frac{1 - \vec{v}_A \cdot \vec{v}_B}{\sqrt{(1 - v_A^2)(1 - v_B^2)}}$$

(where  $c = 1$ ), and remarked that the four quantities :

$$q = (-r; \vec{r}),$$

$$f = (k_0 T; k_0 \vec{F}),$$

$$u_B = (k_0; k_0 \vec{v}_B),$$

$$u_A = (k_1; k_1 \vec{v}_A),$$

$$\left[ \text{with } k_0 = \frac{1}{\sqrt{1 - v_B^2}}, k_1 = \frac{1}{\sqrt{1 - v_A^2}} \text{ and } T = \vec{F} \cdot \vec{v}_B \right]$$

transform in the same way under a Lorentz transformation. Thereby Poincaré introduced what Arnold Sommerfeld later called four-vectors.<sup>72</sup> Since the equation of propagation is a priori a function of the invariants of the Lorentz group,

69. Ibid., p. 490.

70. H. A. Lorentz, "Electromagnetic phenomena in a system moving with any velocity smaller than that of light," *Proceedings of the Academy of Sciences of Amsterdam*, **6** (1904), 809; in H. A. Lorentz, A. Einstein, et al., *The Principle of Relativity* (1923; rpt. New York: Dover, 1952), pp. 9-32.

71. Poincaré used component notation instead of the vector notation used here

72. A. Sommerfeld, "Zur Relativitätstheorie. I. Vierdimensionale Vektoralgebra," *Annalen der Physik*, **32** (1910), 749-76, in Arnold Sommerfeld, *Gesammelte Schriften*, F. Sauter ed. (Braunschweig: Friedr. Vieweg & Sohn, 1968), **2**, 190.

Poincaré chose for  $\Phi$  the function  $r^2 - t^2 = 0$ , and retained the solution  $t = -r$ . This equation of propagation implies that the velocity of propagation is the velocity of light. For the four-force, Poincaré adopted the linear combination

$$f = \alpha q + \beta u_B + \gamma u_A$$

where the coefficients  $\alpha, \beta, \gamma$  are unknown functions of the invariants. By making various approximations with respect to the velocities and acceleration ( $\frac{v^2}{c^2} \ll 1, c^2 \gg \text{acceleration} \times \text{distance}$ ), Poincaré finally obtained for the relativistic gravitational force

$$f = \frac{q}{B^3} - \frac{A}{B^3} u_A,$$

where  $A = -r_1 + \vec{r} \cdot (\vec{v}_A - \vec{v}_B)$  and  $B = -r_1; \vec{r}_0 + \vec{r}_1$  is the position of the attracting body at the time  $t_0$ . It follows that the ordinary force  $\vec{F}$  is composed of two forces : one, which is parallel to the vector connecting the two points; the other, which is parallel to the velocity of the attracting body. As mentioned above, the respective positions and velocities of the bodies  $A$  and  $B$  are the positions and velocities when the gravitational wave (Poincaré speaks of an "onde gravifique"<sup>73</sup>) leaves the body  $A$  and reaches the body  $B$ . Poincaré remarked that the above solution is not unique and showed that many others are possible. Furthermore, he also established that the force  $\vec{F}$  can be rewritten in such a way as to reveal an analogy with the electromagnetic force. As to the question whether these formulas were in accord with astronomical observations, Poincaré expressed hope that the divergence (of the order of  $\frac{v^2}{c^2}$ ) with respect to Newton's law would not be too great although he left the final decision to a more thorough discussion.<sup>74</sup>

Three years later, in 1908, Poincaré was to come back to gravitation. At that time, there were still doubts about Einstein's theory. Indeed, Einstein in his 1907 paper admitted that Walter Kaufmann's experiments were in much better agreement with the electron theories of Max Abraham and A. H. Bucherer than with his own theory.<sup>75</sup> Yet, despite the negative experimental results, Einstein remained convinced of the validity of his theory because of the greater generality of its axioms. Planck<sup>76</sup> and Poincaré<sup>77</sup> also were inclined to doubt the experiments because of the harmony provided by the relativity principle. Thus Einstein's theory was looked at favorably before it was really confirmed. This made the need for a relativistic theory of gravitation apparent by 1908. In that year, Poincaré examined several gravitational theories, based respectively on : (a) Abraham's hypothesis of undeformable electrons and Newton's gravitational force law ; (b) Lorentz's hypothesis of deformable electrons and Newton's

73. Poincaré, "Sur la Dynamique de l'Electron," p. 548.

74. Ibid., p. 550.

75. Einstein, "Relativitätssprinzip und Folgerungen" (1907), p. 439.

76. Max Planck, "Das Prinzip der Relativität und die Grundgleichungen der Mechanik," *Be-richte der Deutschen Physikalischen Gesellschaft*, 4 (1906), 136, dans Planck, *Abhandlungen* (Braunschweig, 1958), 2, p. 115.

77. Henri Poincaré, "La Dynamique de l'Electron," *Revue générale des Sciences pures et appliquées*, 19, (1908), 386-402, dans Poincaré, *Oeuvres*, 9, (1954), 572.

gravitational force law; and (c) Lorentz's hypothesis of deformable electrons and a relativistic gravitational force law.<sup>78</sup> His conclusion was that none of these theories could account for the precession of the Mercury perihelion. Perhaps as an alternative, Poincaré discussed a relativistic kinetic theory of gravitation on the model of Lesage's theory. The negative verdict was even more definitive in that case, as Poincaré found that the temperature of the earth would increase by  $10^{13}$  degrees every second in that theory!<sup>79</sup>

In 1908, Hermann Minkowski too dealt with the question of a relativistic gravitational force law.<sup>80</sup> Instead of deriving the latter from general considerations as Poincaré did, Minkowski simply proceeded by analogy with the relativistic electromagnetic force between two electrons replacing the product of the charges by the negative product of the masses. Minkowski's force law was a particular case of Poincaré's force law, as Sommerfeld showed in 1910.<sup>81</sup> Neither Poincaré's nor Minkowski's gravitational theories, which were based on force law, were able to account for the precession of Mercury's perihelion.

### Einstein's relativistic field approach

At the end of 1907, Einstein made an attempt to develop a relativistic theory of gravitation. According to the manuscript discovered by Gerald Holton quoted earlier, Einstein made a step in that direction while he was writing the review paper. Furthermore, in a letter to his friend Conrad Habicht, dating from Christmas 1907, Einstein wrote :

During the months of October and November, I was very busy with a partly review and partly new paper ["Relativitätsprinzip und Folgerungen"] on the principle of relativity.... Now I am occupied with a likewise relativistic consideration on the gravitational law with which I hope to explain the yet unexplained secular variations of the perihelion motion of Mercury. So far it does not seem to succeed.<sup>82</sup>

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78. Ibid., *Oeuvres*, **9**, 580-81.

79. Ibid., p. 586.

80. H. Minkowski, "Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern," *Nachrichten der K. Gesellschaft der Wissenschaften zu Göttingen, mathematisch-physikalische Klasse*, 1908, pp. 53-111, dans H. Minkowski, *Gesammelte Abhandlungen von Hermann Minkowski*, ed. David Hilbert with the collaboration of Andreas Speiser and Hermann Weyl (1911; rpt. New York : Chelsea, 1967), 2 vol. in one, **2**, 401-04.

81. A. Sommerfeld, "Zur Relativitätstheorie. II. Vierdimensionale Vektoranalysis," *Annalen der Physik*, **33**, (1910), 649-89, in Sommerfeld, *Gesammelte Schriften* (Braunschweig, 1968), **2**, 252-57.

82. "In den Monaten Oktober und November war ich sehr stark beschäftigt mit einer teils referierenden, teils Neues behandelnden Arbeit über das Relativitätsprinzip. ... Jetzt bin ich mit einer ebenfalls relativitätstheoretischen Betrachtung über das Gravitationsgesetz beschäftigt, mit der ich die noch unerklärten säkularen Änderungen der Perihellänge des Merkur zu erklären hoffe. Bis jetzt scheint es aber nicht zu gelingen," A. Einstein, lettre à C. Habicht, Noël 1907, in C. Seelig, *Albert Einstein* (Zürich : Europa Verlag, 1960), pp. 127-28.

Since, according to Einstein's various accounts,<sup>83</sup> it was in 1908 that Einstein became convinced that gravitation could not be integrated within *STR*, Einstein's attempts to find a relativistic theory of gravitation within *STR* probably took place between the end of 1907 and the end of 1908.

Instead of trying to find a relativistic generalization of Newton's gravitational law, as Poincaré and Minkowski did, Einstein approached the problem from a field theoretical point of view and looked for relativistic extensions of Poisson's field equation and of the equation of motion of a particle in a gravitational field. That Newton's theory was unsatisfactory within *STR* resulted from the latter's rejection of the concept of absolute time (i.e., absolute simultaneity) and thus of the concept of instantaneous interaction as well, since an instantaneous interaction in one reference frame is no longer instantaneous in another reference frame. Consequently, Newton's gravitational force law, which assumes an instantaneous interaction between masses, is no longer adequate within *STR*. Einstein obtained a relativistic field equation by adding a term  $\frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2}$  to Poisson's equation so as to obtain the equation

$$\square \Phi = 4\pi G\rho,$$

where  $\square$  is the operator

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2},$$

$\Phi$  the gravitational potential,  $\rho$  the rest density of matter, and  $G$  the gravitational constant.<sup>84</sup> As to the relativistic generalization of the classical equation of motion,

$$\vec{F} = -m \overrightarrow{\text{grad}} \Phi,$$

we do not know how Einstein proceeded. We only know that Einstein found that the gravitational acceleration of the particle varied with its internal energy and, in particular, with its horizontal velocity.

According to Einstein it was the result that the acceleration varied with the velocity which, in 1908, led him to abandon his attempt to integrate gravitation within *STR*.<sup>85</sup> In fact, Einstein's specific failure did not necessarily entail the conclusion that *STR* was an inadequate framework for a gravitational theory, since Gustav Nordström was to develop, within *STR*, a theory of gravitation that satisfied the equality of inertial and gravitational masses to a certain extent. Why then was the founder of *STR* so quickly convinced, in contradistinction to most other people, that *STR* was an inadequate framework for gravitation (and this despite the circumstance that, at the time, Einstein did not know of Eötvös' experiments on the equality of acceleration of freely falling bodies)? The reason

83. Einstein, "Autobiographical Notes," p. 67; see also : Einstein, "Notes on the Origin of the General Theory," pp. 280-81.

84. Einstein, "Notes on the Origin of the General Theory," p. 280; Einstein, "Autobiographical Notes," p. 62.

85. Einstein, "Autobiographical Notes," p. 65; Einstein, "Notes on the Origin of the General Theory," p. 280.



is that Einstein had wanted to generalize the principle of relativity from the very beginning. Thus he was naturally inclined to place great emphasis on the tool which offered the prospect of achieving that goal, namely the equivalence principle. This association of the equivalence principle with the generalization of the principle of relativity appears explicitly in Einstein's 1907 review paper as we have seen, as well as in most of Einstein's later accounts. Thus Einstein wrote for example :

If this principle held good for any events whatever (the 'principle of equivalence'), this was an indication that the principle of relativity needed to be extended to coordinate systems in non-uniform motion with respect to each other, if we were to reach a natural theory of the gravitational fields. Such reflections kept me busy from 1908 to 1911, and I attempted to draw special conclusions from them, of which I do not propose to speak here. For the moment the one important thing was the discovery that a reasonable theory of gravitation could only be hoped for from an extension of the principle of relativity.

What was needed, therefore, was to frame a theory whose equations kept their form in the case of nonlinear transformations of the coordinates. Whether this was to apply to arbitrary (continuous) transformations of coordinates or only to certain ones, I could not for the moment say.<sup>86</sup>

Since the equivalence principle predicted consequences reaching clearly beyond *STR*, such as a variable velocity of light, Einstein was aware that he had to choose between *STR* and the prospect of a more general relativistic theory incorporating the equivalence principle. Because Einstein wanted to generalize the principle of relativity anyway, he opted for the latter choice. That Einstein was indeed concerned with generalization of the principle of relativity during the years 1908 – 1911 appears from a letter of 29 September 1909 to Arnold Sommerfeld, in which Einstein wrote :

The treatment of the uniformly rotating rigid body seems to me to be of great importance on account of an extension of the relativity principle to uniformly rotating systems along analogous lines of thought to those that I tried to carry out for uniformly accelerated translation in the last section of my paper published in the *Zeitschrift für Radioaktivität*.<sup>87</sup>

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<sup>86</sup>. Einstein, "Notes on the Origin of the General Theory," pp. 280-81; see also Albert Einstein, "Autobiographische Skizze," in Carl Seelig, *Helle Zeit-Dunkle Zeit* (Zürich : Europa-Verlag, 1956), p. 13.

<sup>87</sup>. "Die Behandlung des gleichförmig rotierenden starren Körpers scheint mir von grosser Wichtigkeit wegen einer Ausdehnung des Relativitätsprinzips auf gleichförmig rotierende Systeme nach analogen Gedankengänge, wie ich sie im letzten meiner in der *Zeitschr. f. Radioaktivität*. publizierten Abhandlung für gleichförmig beschleunigte Translation durchzuführen versucht habe," John Stachel, "Einstein and the Rigidly Rotating Disc," dans Alan Held, ed., *General Relativity and Gravitation : One Hundred Years after the Birth of Albert Einstein*, 2 vols. (New York : Plenum, 1980), 1, 2.

Einstein's hope of developing a general theory of relativity via the equivalence principle explains why he did not insist on finding a theory of gravitation within *STR*. Notwithstanding Einstein's early, private renunciation, several attempts were made later on to find a satisfactory (special) relativistic theory of gravitation, the main ones being those by Max Abraham, Gunnar Nordström, and Gustav Mie.

### Abraham's theories

In December 1911, Max Abraham initiated a new approach to gravitation by making use of Einstein's idea of a variable velocity of light. The historical significance of Abraham's theory was that it prompted Einstein to develop his static theory of gravitation two months later. Although Einstein's line of thought was completely independent of that of Abraham, Einstein probably profited from the mathematical level and the clarity of Abraham's papers, and in particular from Abraham's clear presentation of the concept of the energy-momentum tensor of a field.

Max Abraham<sup>88</sup> was born in 1875 in Danzig, studied in Berlin under Max Planck, and after graduation became his assistant. In 1900, Abraham went to Göttingen, where he taught as Privatdozent until 1909. It was during the Göttingen period that Abraham developed his theory of the rigid electron which seemed at first (in contradistinction to Einstein's theory) to be confirmed by Bucherer's experiments; at this time Abraham also wrote the two volume textbook *Theorie der Elektrizität*, which became the standard work in electrodynamics in Germany. The first volume was an adaptation of Föppl's *Einführung in die Maxwell'sche Theorie der Elektrizität*; the second volume dealt with electromagnetic radiation. It was Abraham who gave to classical electrodynamics its modern vectorial form. Abraham had been an early proponent of the use of vectors in physics and indeed wrote the first comprehensive review article on vectors in 1901.<sup>89</sup> Yet, despite his outstanding achievements, Abraham was to remain Privatdozent for nine years because of his somewhat polemical nature. Abraham just could not resist criticizing his colleagues for scientific or personal reasons. This personal trait had won him many enemies and explains why such a competent physicist could not find an adequate position in Germany. In 1909, Abraham finally accepted a professorship in theoretical mechanics in Milan, where he remained until the war obliged him to leave Italy in 1915. Abraham himself delicately acknowledged his polemical inclination. To the question "How do you stand with your colleagues in Milan?" Abraham answered, "Superbly, I do not yet have a complete command of the language."<sup>90</sup>

88. M. Born and M. v. Laue, "Max Abraham," *Physikalische Zeitschrift*, **24** (1923), 49-53, in Max Born, *Ausgewählte Abhandlungen* (Göttingen : Vandenhoeck & Ruprecht, 1963), **2**, 599-603 ; S. Goldberg, "Abraham, Max," *Dictionary of Scientific Biography* (1970), **1**, 23-25.

89. Max Abraham, "Geometrische Grundbegriffe," *Encyklopädie der Mathematischen Wissenschaften*, **4**, part 3, article completed in 1901, pp. 3-47.

90. "Vortrefflich, ich beherrsche die Sprache noch nicht so ganz," Born and v. Laue, "Max Abraham," in Born, *Abhandlungen*, **2**, 603.

It was during the Milan period that Abraham developed his theories of gravitation and entered into a debate with Einstein. Abraham had never liked Einstein's special theory of relativity because it directly challenged the electromagnetic worldview and in particular the absolute ether. According to Max Born and Max von Laue, Abraham "loved his absolute ether, its field equations, his rigid electron, just as a youth loves his first flame, whose memory no later experience can extinguish."<sup>91</sup> Yet, despite his strong antipathy for Einstein's theory, Abraham came to accept it for a while and even made valuable contributions to the development of Minkowski's electrodynamics, in particular with respect to the energy-momentum tensor and its symmetry.<sup>92</sup> Furthermore, when Abraham presented his first theory of gravitation, he thought at first that it satisfied the principle of relativity.<sup>93</sup> Abraham's theory was based on the following field equation

$$\square\Phi = 4\pi G\rho,$$

(with  $\square = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$ ) where  $G$  is the gravitational constant,  $\rho$  the rest-density of matter, and  $\Phi$  the gravitational potential, which Abraham assumed to be an invariant with respect to four-rotations. The equation of motion per unit mass was

$$F = -\text{Grad}\Phi,$$

where  $\text{Grad}$  represents the four-operator  $\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \frac{\partial}{\partial(ict)}\right)$ . Assuming a variable velocity of light, Abraham was able to derive Einstein's relation between the gravitational potential and the velocity of light without using the equivalence principle. Abraham proceeded as follows. Differentiating the identity

$$\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2 + \left(\frac{dict}{d\tau}\right)^2 = -c^2$$

with respect to the proper time  $\tau$ , and using the equation of motion, Abraham obtained the relation :

$$d\Phi = cdc \tag{3.1}$$

and, after integration,

$$\frac{c^2 - c_0^2}{2} = \Phi - \Phi_0.$$

Neglecting terms in  $\left(\frac{\Phi}{c^2}\right)^2$ , Abraham finally obtained Einstein's formula

$$c = c_0 \left(1 + \frac{\Phi - \Phi_0}{c^2}\right).$$

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91. Ibid., p. 602.

92. Max Abraham, "Sull'Elettrodinamica Di Minkowski," *Rendiconti del circolo matematico di Palermo*, **30** (1910), 33-46; "Zur Frage der Symmetrie des elektromagnetischen Spannungstensor," *Annalen der Physik*, **44** (1914), 537-44.

93. Max Abraham, "Zur Theorie der Gravitation," *Physikalische Zeitschrift*, **13** (1912), 1-5, on p. 1.

Hence Abraham's theory yielded the deviation of light rays in a gravitational field predicted by Einstein, but without using the equivalence principle. In the same paper, Abraham also analyzed the conservation of momentum and energy in terms of a symmetric field energy-momentum tensor which, in particular, yielded a positive energy density and thus solved the problem inherent to the older vector theories of gravitation. Initially, Abraham succeeded in deriving the conservation of energy only for a constant velocity of light, but removed this restriction shortly afterwards.<sup>94</sup>

In three other papers,<sup>95</sup> Abraham examined various consequences of his theory. In particular, Abraham concluded from relation (3.1) that the theory satisfied Lorentz covariance only for infinitesimal space-time domains, since  $\Phi$  is assumed to be an invariant.<sup>96</sup> Einstein was to criticize this claim, showing that as soon as the constancy of the velocity of light is given up, Lorentz covariance no longer holds even for infinitesimal space-time domains. Abraham also found that the gravitational force between two moving particles is independent of the velocity of the attracted particle.<sup>97</sup> This could be expected from the very beginning, since the velocity of the attracted particle does not enter explicitly in the equation of motion. Abraham did not elaborate, but later, under the influence of Einstein, he realized that such force was unsatisfactory, because it led to a gravitational acceleration which depended on the velocity of the attracted body. Subsequently, Abraham was to develop a second rather elaborate theory, which again assumed a variable velocity of light and was specifically designed to incorporate the equality of gravitational acceleration.<sup>98</sup> The theory, however, satisfied neither the principle of relativity nor Einstein's equivalence principle, two principles Abraham was willing to reject.<sup>99</sup> In Einstein's view, Abraham's theory appeared logically sound but rather artificial. In a letter to Sommerfeld dated 29 October 1912, Einstein wrote

Abraham's new theory is indeed, so far as I can see, logically correct, but only a monstrosity [Missgeburt] of embarrassment. The existing relativity theory is certainly not as false as Abraham believes.<sup>100</sup>

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94. Max Abraham, "Die Erhaltung der Energie und der Materie im Schwerkraftfelde," *Physikalische Zeitschrift*, **13** (1912), 311-14.

95. Max Abraham, "Das Elementargesetz der Gravitation," *Physikalische Zeitschrift*, **13** (1912), 4-5; "Der freie Fall," *ibid.*, 310-11; "Erhaltung der Energie" (1912), 311-14.

96. Abraham, "Erhaltung der Energie" (1912), p. 312.

97. Abraham, "Das Elementargesetz der Gravitation," p. 5.

98. Max Abraham, "Das Gravitationsfeld," *Physikalische Zeitschrift*, **13** (1912; reception date : 12 juillet 1912), 793-97; "Eine neue Gravitationstheorie," Conference held on le 19 octobre 1912 at the Società italiana per il progresso delle scienze, *Archiv der Mathematik und Physik*, Leipzig, 3rd series, **20** (1912), 193-209, trans. by author.

99. Abraham, "Das Gravitationsfeld" (1912), p. 794; "Eine neue Gravitationstheorie" (1912), p. 209.

100. "Abrahams neue Theorie ist zwar, soweit ich sehe, logisch richtig, aber nur eine Missgeburt der Verlegenheit. So falsch, wie Abraham meint, ist die bisherige Relativitätstheorie sicherlich nicht," Einstein to Sommerfeld, 29 Octobre 1912, in Albert Einstein und Arnold Sommerfeld, *Briefwechsel*, Armin Hermann ed. (Basel : Schwabe, 1968), p. 26.

### Nordström's and Mie's theories

Whereas Abraham's and Einstein's theories gave up the constancy of the velocity of light, Nordström's and Mie's theories assumed the validity of that postulate and of the relativity principle. The historical importance of Nordström's second theory was that Einstein considered it to be the main rival theory to the Einstein-Grossmann theory. Mie's theory, on the other hand, provides an example of a theory which rejected a strict equality of inertial and gravitational masses from the outset. Furthermore, Mie's derivation of the theory from relativistic invariants was quite original and was to influence David Hilbert's approach to gravitation. Though Nordström's and Mie's theories were developed only after the publication of Einstein's 1912 static theory of gravitation, we shall briefly discuss them here in order to show the uniqueness of Einstein's approach.

Gunnar Nordström<sup>101</sup> (1881–1923) was born in Helsinki, studied in that city as well as in Göttingen (1906–1907), and obtained his doctorate in Helsinki in 1909. From 1910 to 1918 Nordström taught as assistant professor at the University of Helsinki. In face of the difficulties which Abraham's and Einstein's theories raised with respect to the relativity principle, Nordström proposed to maintain the two postulates of *STR*. His first theory, which was completed in October 1912, was based, like Abraham's theory, on the field equation

$$\square\Phi = 4\pi G\rho,$$

and the equation of motion for the unit mass,

$$F = -Grad\Phi,$$

where  $\rho$  is the rest density,  $F$  the four-force, and  $Grad$  the four-gradient.<sup>102</sup> Since the velocity of light is assumed to be constant, the four-velocity must be perpendicular to the four-acceleration. In order to satisfy this condition Nordström adopted a variable rest-mass and found that the mass varied according to the formula

$$m = m_0 e^{\frac{\Phi}{c^2}}.$$

In an addendum Nordström pointed out that Einstein had written to him, indicating that he had already examined such a theory, but had rejected it because he found the gravitational acceleration of a rotating body to differ from that of a non-rotating one. Nordström, however, remarked that the difference was too small to be experimentally detectable. He admitted that his theory violated Einstein's equivalence principle, but saw no reason therein to abandon his theory. Indeed, Nordström thought that the equivalence principle presented

101. For biographical information see Hjalmar Tallqvist, "Gunnar Nordström," *Finska Vetenskaps-Societeten*, Helsingfors, Minnesteckningar och Föredrag, **3** (1924), 14 pp.; J. C. Poggendorf, *Biographisch-literarisches Handwörterbuch für Mathematik, Astronomie, Physik ...*, **5** (1926), 911-12; *Ibid.*, **6** (1938), 1875.

102. Gunnar Nordström, "Relativitätsprinzip und Gravitation," *Physikalische Zeitschrift*, **13** (1912), 1126-29, on 1126.

serious difficulties : "though Einstein's hypothesis is extraordinarily ingenious, it presents on the other hand great difficulties."<sup>103</sup> Nordström did not specify these difficulties. He was to confirm Einstein's objection in another paper, where he also found that in a static gravitational field, the acceleration of a particle decreased with its velocity, independently of the direction of the velocity.<sup>104</sup>

In a second theory,<sup>105</sup> which he completed in Zürich in July 1913, Nordström tried to implement the equality of inertial and gravitational masses to a greater extent. To this effect, following a proposition by von Laue and Einstein, he defined the rest-density of matter as proportional to the trace of the energy-momentum tensor of matter. He also introduced a variable "gravitational factor"  $g(\Phi)$  in the field equation and in the equation of motion, which thus became, respectively,

$$\square\Phi = g(\Phi)\rho,$$

$$F = -g(\Phi)\rho\text{Grad}\Phi,$$

where  $F$  is the four-force density. Nordström determined the function  $g(\Phi)$  in such a way that for a material system at rest in a static gravitational field, the equality of the gravitational acceleration was satisfied and obtained the relation

$$g(\Phi) = \frac{c^2}{\Phi}.$$

Hence, the basic equations of Nordström's theory became

$$\Phi\square\Phi = c^2\rho,$$

$$F = -c^2\rho\frac{\text{Grad}\Phi}{\Phi}.$$

Nordström established that the equality of acceleration also held for rotating bodies, but not for a body thrown horizontally, for which he found the acceleration to be smaller than that of a body at rest.<sup>106</sup> Thus the universal equality of the gravitational acceleration predicted by the equivalence principle was still not incorporated in Nordström's theory.

Gustav Mie's theory,<sup>107</sup> on the other hand rejected the idea of a strict equality of the gravitational acceleration from the outset. Mie, who was born in 1868 and was a full professor of physics at the University of Greifswald from 1905 to 1916, initially wanted to develop a comprehensive, relativistic, electromagnetic field theory of matter, in which electrons were viewed as field singularities. His idea was to formulate a relativistic invariant which he called "Hamilton's

103. "Obwohl die Einsteinsche Hypothese ausserordentlich geistreich ist, bietet sie doch anderseits grosse Schwierigkeiten," *ibid.*, p. 1129.

104. Gunnar Nordström, "Träge und schwere Masse in der Relativitätsmechanik," *Annalen der Physik*, **40** (1913), 856-78, on 878.

105. Gunnar Nordström, "Zur Theorie der Gravitation vom Standpunkt des Relativitätsprinzips," *Annalen der Physik*, **42** (1913), 533-54.

106. *Ibid.*, p. 554.

107. Gustav Mie, "Grundlagen einer Theorie der Materie. I," *Annalen der Physik*, **37** (1912), 511-534; same title, "II," *ibid.*, **39** (1912), 1-40; same title, "III," *ibid.*, **40** (1913), 1-66.

function"<sup>108</sup> in terms of the electric and magnetic fields and the four-current, and to derive relativistic field equations through differentiation. Initially, Mie had hoped to explain gravitational effects within this electromagnetic world-view but in the end found that this was not possible.<sup>109</sup> Thus he saw himself obliged to add further variables. He also assumed that the gravitational mass was proportional not to the energy of the body, but to Hamilton's function. This made the ratio of inertial and gravitational masses dependent on the temperature. Mie pointed out, however, that the difference was too small to be detectable experimentally.<sup>110</sup>

### 3.2.2 Einstein's Static Theory of Gravitation

In Prague, in February 1912, Einstein was to develop a static theory of gravitation that was fundamentally different from all other contemporary gravitational theories, in that it was based on the equivalence principle rather than on a postulated field law or equation of motion. Another important difference was that Einstein, adopting a variable velocity of light, knew that the Lorentz invariance would no longer hold in general even for infinitesimal space-time domains.<sup>111</sup> Thus, far from trying to integrate gravitation within *STR*, Einstein was consciously leaving the restricted framework of *STR* and was venturing into unknown territory. Einstein expected the allowed transformations to include more general ones than the Lorentz transformations,<sup>112</sup> but at that time, this was more a hope than a certitude. To avoid losing ground, Einstein therefore proceeded cautiously, restricting the discussion to the case of a static gravitational field. The main external motivation for Einstein was probably the fact that, by that time, Max Abraham had already used Einstein's idea of a variable velocity of light to develop his own theory of gravitation. Einstein's criticism of Abraham's theory was to lead to a public debate which obliged both men to clarify their own ideas. The main historical significance of Einstein's static theory of gravitation was that its equation of motion was to lead Einstein to the definitive equation of motion of the general theory of relativity. The latter equation of motion, in turn, was to determine the Riemannian framework of the general theory of relativity.

#### Development of the theory

In the introduction to his first paper on the static theory, Einstein reaffirmed his commitment to the equivalence principle, despite the fact that it limited the validity of the constancy of the velocity of light, and thus of *STR*, to domains of constant gravitational potential. Einstein wrote :

108. Mie, "Grundlagen, I," p. 523.

109. Mie, "Grundlagen, III," p. 5.

110. Ibid., p. 64.

111. Albert Einstein, "Lichtgeschwindigkeit und Statik des Gravitationsfeldes," *Annalen der Physik*, **38** (1912), 355-68, on 368-69.

112. Ibid., p. 369.

In my opinion at least, the hypothesis that the "acceleration field" is a special case of a gravitational field has such a great probability, in particular with respect to the consequences already derived in the first paper about the gravitational mass of the energy content, that a detailed development of its consequences seems to be indicated.<sup>113</sup>

The formulation of the passage clearly reveals a Machian line of thought, in that Einstein considered an acceleration field to be a real physical field, and more specifically a gravitational field. To those who might have wondered about the source of such a gravitational field, Einstein indicated in a note on the next page that "the masses which produce this field [uniform acceleration field] must be thought of as located at infinity."<sup>114</sup> From space-time considerations in a uniformly accelerated system and the equivalence principle, Einstein was to derive the two key elements of his theory : the field equation, giving the field in terms of the distribution of matter ; and the equation of motion of a material particle in a given field. As field variable, Einstein adopted the velocity of light, which he assumed to be constant in time since the theory was restricted to the static case.

Einstein first derived the transformation relations up to the second order (with respect to the coordinate time  $t$ ) between a uniformly accelerated system  $K(x, y, z, t)$  (in Born's sense) and a fixed inertial system  $\Sigma(\xi, \eta, \zeta, \tau)$  ; (see Fig. 3.6).

Here again Einstein assumed that the acceleration has no influence on the rods of the system, and found the following relations :

$$\begin{aligned}\xi &= x + \frac{ac}{2}t^2, \\ \eta &= y, \\ \zeta &= z, \\ \tau &= ct,\end{aligned}$$

where  $c = c_0 + ax$  is the velocity of light in  $K$ , and  $a$  is a constant ; the time  $\tau$  is measured in such a way that the velocity of light in  $\Sigma$  is equal to 1.

Having obtained the velocity of light in the accelerated system, Einstein looked for a differential equation (analogous to Poisson's equation) which the function  $c = c_0 + ax$  would satisfy and found immediately the equation

$$\Delta c = \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} = 0.$$

By applying the equivalence principle, this equation became the gravitational field equation of a static gravitational field for empty space. In the presence of matter, Einstein generalized the equation to the equation

$$\Delta c = kcp$$

113. Einstein, "Lichtgeschwindigkeit und Statik des Gravitationsfeldes" (1912), p. 355.

114. Ibid., p. 356.



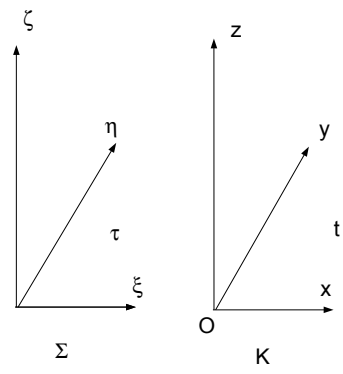


FIGURE 3.6 – Uniformly accelerated system  $K$  (the acceleration is along the  $Ox$  axis)

where  $k$  is a universal constant and  $\rho$  the density of matter, which Einstein defined independently of  $c$  by the convention that the mass of  $1\text{cm}^3$  of water is 1, whatever the gravitational potential.<sup>115</sup>

To determine the equation of motion of a material particle in a given gravitational field, Einstein first established the equation of motion of a free particle in the accelerated system by applying the transformation equations to a rectilinear inertial motion in  $\Sigma$ . Having obtained the analytic expression of the motion in  $K$ , Einstein then inferred from it the differential equation

$$\frac{d}{dt} \left( \frac{1}{c^2} \frac{d\vec{r}'}{dt} \right) = -\frac{1}{c} \overrightarrow{\text{grad}} c.$$

Via the equivalence principle this equation became the equation of motion for a particle in a static gravitational field. Multiplying the equation of motion by  $\frac{c}{\sqrt{1-\frac{v^2}{c^2}}}$ , which he showed to be a constant of the motion, Einstein rewrote the equation

$$\frac{d}{dt} \left( \gamma \frac{m}{c} \vec{v}' \right) = -\gamma m \overrightarrow{\text{grad}} c,$$

where  $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ .

Einstein interpreted the quantity  $\gamma \frac{m}{c} \vec{v}'$  as the momentum of the particle, and the right side of the equation as the force exerted by the gravitational field on the particle. As to the energy of the particle, Einstein defined it as the quantity  $\gamma mc$ .<sup>116</sup>

### Consequences

In March 1912, Einstein examined various electromagnetic and thermal consequences<sup>117</sup> of his static theory of gravitation and derived further consequences in a paper entitled "Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktionswirkung analog ist?" [Is there a gravitational action analogous to the electrodynamic induction?],<sup>118</sup> which shows explicitly that Einstein was concerned with Mach's views on inertia in Prague. Since the electromagnetic and thermal consequences had no particular influence, we shall only examine the latter consequences, which constituted the basis of what Einstein later called the relativity of inertia. Einstein's paper on the gravitational induction must have been written before May 1912, since Einstein stated that the equations of motion of the static theory of gravitation had not yet been published.

115. Ibid., 360.

116. The dimensions of Einstein's quantities differ from the usual ones because of the choice  $c = 1$  dans  $\Sigma$ .

117. Albert Einstein, "Zur Theorie des statischen Gravitationsfeldes," *Annalen der Physik*, **38** (1912), 443-58.

118. Albert Einstein, "Gibt es eine Gravitationswirkung die der elektrodynamischen Induktionswirkung analog ist?" *Vierteljahrsschrift für gerichtliche Medizin und öffentliches Sanitätswesen*, 3rd ser., **44** (1912), 37-40.

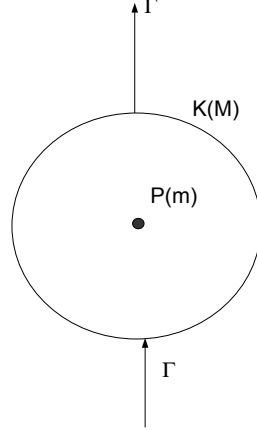


FIGURE 3.7 – Accelerated massive hollow sphere

In the paper, Einstein asked whether a particle  $P$  at rest inside a massive hollow shell  $K$  is submitted to an induced force when the shell is accelerated (see Fig. 3.7). To answer the question, Einstein first established that the mass  $M$  of the shell influences the mass  $m$  of a particle located at its center. Qualitatively the effect can be understood as follows : From the definition of the momentum of a particle,  $\gamma \frac{m}{c} \vec{v}$ , it follows that its inertial rest-mass is given by  $m/c$ , or  $m \frac{c_0}{c}$  in standard dimensions, if  $c_0$  is a given velocity. Since the shell modifies the value of  $c$  it thereby influences the mass of the particle. From the equation of motion and the usual definition of the gravitational potential,

$$\frac{d^2 \vec{r}}{dt^2} = -\overrightarrow{\text{grad}} \Phi,$$

Einstein, for low velocities, obtained the relation

$$\overrightarrow{\text{grad}} \Phi = c \overrightarrow{\text{grad}} c,$$

which after integration yields, in first approximation

$$\begin{aligned} \frac{c_0}{c} &= \left( 1 - \frac{\Phi_0 - \Phi}{c_0^2} \right) \\ &= 1 - \frac{kM}{Rc_0^2}, \end{aligned}$$

where  $R$  is the radius of the shell, and where  $c_0$  is the velocity of light at infinity. Thus the new inertial mass of the particle in the presence of the shell is

$$m' = m + k \frac{mM}{Rc_0^2},$$

where  $m$  represents the inertial mass of the particle when it is alone at infinity. Concerning this effect Einstein remarked :

The result is in itself of great interest. It shows that the presence of the inertial shell  $K$  increases the inertial mass of the enclosed material point  $P$ . This suggests the idea that the whole inertia of a material point might be an effect of the presence of all other masses, based on a kind of interaction with the latter.

He added in a note, "This is very precisely the point of view which E. Mach, in his penetrating investigations, has advanced on the subject."<sup>119</sup> In the same note Einstein referred the reader to the second chapter of Mach's *Mechanics*. This was the first time Einstein mentioned Mach's name in one of his papers. We have seen that Einstein had been in correspondence with Mach since 1909. Einstein also visited Mach in Vienna, probably in 1911 or the beginning of 1912.<sup>120</sup> From then on, Mach's name was to appear frequently in Einstein's papers.

After having shown also that the mass of the shell is increased by the presence of the particle, Einstein finally came to the question of whether an induced force acts on the particle when the shell is accelerated. If so, the particle would exert an opposite force on the shell because of the equality of action and reaction. Thus Einstein assumed that the total external force  $\vec{F}$  which must be applied to the shell in order to communicate to it the acceleration  $\vec{\Gamma}$  and the (eventual) acceleration  $\vec{\gamma}$  to the particle is of the form

$$\vec{F} = A\vec{\Gamma} + \alpha\vec{\gamma},$$

where  $A$  and  $\alpha$  are coefficients to be determined. Similarly, if a total external force  $\vec{f}$  is applied to the particle and communicates to it the acceleration  $\vec{\gamma}$  and the (eventual) acceleration  $\vec{\Gamma}$  to the shell then one has

$$\vec{f} = a\vec{\gamma} + \alpha\vec{\Gamma},$$

where the coefficient  $\alpha$ , is the same as the one above because of the equality of action and reaction. To determine the three coefficients  $A, a$  and  $\alpha$ , Einstein examined three particular cases :

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119. Ibid., p. 39.

120. Herneck, "Beziehungen zwischen Einstein und Mach," p. 8; see also I. B. Cohen, Interview of Einstein with I. B. Cohen, *Scientific American*, **193**, no. 1 (July 1955), 68-73, on 72-73; P. Franck, Einstein (1953), pp. 104-105.

1. The shell and the particle have the same acceleration  $\gamma$ , leading to the equation

$$\vec{F} + \vec{f} = (A + a + 2\alpha) \vec{\gamma} = \left( M + m - \frac{kMm}{Rc^2} \right) \vec{\gamma},$$

where the enclosed expression on the right is the inertial mass of the whole system (i.e., its rest energy divided by  $c^2$ ), and where  $c$  is the velocity of light at the location of the system.

2. The particle only is accelerated and the shell is maintained at rest; thus

$$\vec{f} = a \vec{\gamma} = \left( m + \frac{kMm}{Rc^2} \right) \vec{\gamma},$$

where the enclosed expression is the inertial mass of the particle in the presence of the shell.

3. The shell only is accelerated and the particle is maintained at rest; thus

$$\vec{F} = A \vec{\Gamma} = \left( M + \frac{kMm}{Rc^2} \right) \vec{\Gamma},$$

where the enclosed expression is the mass of the shell in the presence of the particle.

From these questions it follows that

$$\alpha = -\frac{3}{2} \frac{kMm}{Rc^2}.$$

Thus, if only the shell is accelerated, the force which must be exerted on the particle to keep it at rest is equal to  $\vec{f} = \alpha \vec{\Gamma}$ . Einstein concluded that the induced force exerted by the shell on the particle is equal to  $-\alpha \vec{\Gamma}$  and is thus in the same direction as the acceleration of the shell (since  $\alpha$  is negative), in contradistinction to the similar effect between electric charges of the same sign. Hence it appeared that Einstein's theory vindicated to a certain extent Mach's idea that inertial mass and inertial forces are relative to other bodies. We shall see that Einstein was to attribute great significance to such effects.

### Problems and modification of the theory

In March 1912, shortly after he had presented his static theory of gravitation, Einstein expressed doubts about the validity of the whole theory because the equation of motion, taken together with the field equation, violated conservation of momentum.<sup>121</sup> Specifically, Einstein found that for matter at rest, which he supposed to be fixed on a rigid massless structure, the integral over the whole

121. Einstein, "Zur Theorie des statischen Gravitationsfeldes," p. 453.

space (with  $c$  being constant at infinity),

$$\begin{aligned}\int \vec{F} d\tau &= - \int \rho \overrightarrow{\text{grad}} c d\tau \\ &= -\frac{1}{k} \int \frac{\Delta c}{c} \overrightarrow{\text{grad}} c d\tau,\end{aligned}$$

where  $\vec{F}$  is the force per unit volume acting on the matter, did not vanish in general because of the last integral. To obtain a vanishing integral, Einstein first tried to attribute a gravitational mass to the stresses of the rigid structure, but he found that this violated the equality of inertial and gravitational masses. Einstein's next step was to examine the basic assumptions of his theory, i.e., the equation of motion and the field equation. Since the force expression is a direct consequence of the equation of motion, a modification of the first required a modification of the second. Einstein, however, showed that it would be rather difficult to modify the equation of motion unless one wanted to give up the core of the whole theory, i.e., the determination of the gravitational potential by the velocity of light  $c$ .

As to modification of the field equation, Einstein first transformed the above integral and inferred that the field equation

$$c \Delta c - \frac{1}{2} \left( \overrightarrow{\text{grad}} c \right)^2 = kc^2 \rho$$

would make the integral vanish. Einstein, however, stated that he was reluctant to take that step, because it seemed to restrict the use of the equivalence principle to infinitesimal domains.<sup>122</sup> Einstein did not give a specific explanation, but he seems to have concluded from the new theory that the linear relation

$$c = c_0 + ax,$$

which resulted from the equivalence principle, could be maintained only locally, because the coefficient  $a$  now became a function of the coordinates. Hence, the applicable domain of the equivalence principle became restricted as well. Einstein pointed out that the equation of motion of a material particle remained unaffected, since its derivation made only use of the transformation equations for an infinitesimal domain. He also managed to interpret the supplementary term  $\frac{1}{2k} \frac{(\overrightarrow{\text{grad}} c)^2}{c}$  as an energy density of the gravitational field, but he was not to make any further use of the new field equation. By now, then, it appeared that with his static theory of gravitation Einstein had given up the constancy of the velocity of light, the Lorentz invariance (even for infinitesimal domains), and the macroscopic equivalence principle. Such apparent renunciations or limitations could become tempting targets for criticism, as the debate between Einstein and Abraham was to show.

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122. Ibid., p. 456.

### Debate with Max Abraham

Between June and August 1912, a public debate took place between Einstein and Abraham which was initiated by Einstein's remark that "his [Abraham's] conception of time and space is already unacceptable from a purely formal mathematical point of view."<sup>123</sup> Einstein supported his assertion by showing that as soon as the constancy of the velocity of light is given up, there can be no Lorentz invariance even for infinitesimal space-time domains, in contradistinction to what Abraham had initially thought. Einstein reasoned as follows: If there is a Lorentz transformation for an infinitesimal domain, then one has the relations

$$dx' = \gamma(dx - vdt),$$

and

$$dt' = \gamma\left(-\frac{v}{c^2}dx + dt\right),$$

with  $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ . Since  $dx'$  and  $dt'$  must be total differentials, it follows that one must have the relations

$$\frac{\partial}{\partial t}(\gamma) = -\frac{\partial}{\partial x}(\gamma v),$$

and

$$\frac{\partial}{\partial t}\left(-\gamma\frac{v}{c^2}\right) = \frac{\partial}{\partial x}(\gamma).$$

If in the unprimed system the gravitational field is a static one, then  $c$  is a function of  $x$  but not of  $t$ . If the primed system is, for example, a system moving with a uniform velocity, then  $v$  must (for constant  $x$ ) be independent of  $t$ . Thus the left sides must vanish and hence the right sides as well. The latter vanishing, however, is impossible in the case of an arbitrary function  $c(x)$ , which concludes the demonstration.

Abraham answered Einstein's criticism by rejecting the principle of relativity and *STR* altogether. He argued that if Einstein had convinced himself that the constancy of the velocity of light had to be given up in order to arrive at a consistent theory of gravitation, and that if the founder of *STR* had now even shown that the principle of relativity is inconsistent with such a theory, then it was clear that Einstein had himself given "the coup de grâce" (den Gnadenstoss) to the theory of relativity.<sup>124</sup> In fact, Abraham was rather pleased by such a development and did not hide his satisfaction:

One, who, like the present author, has had to warn repeatedly against the siren song of that theory [STR], can with legitimate satisfaction welcome the fact that its very originator has now convinced himself of its untenability.<sup>125</sup>

<sup>123</sup> Einstein, "Lichtgeschwindigkeit und Statik des Gravitationsfeldes," p. 355.

<sup>124</sup> Max Abraham, "Relativität und Gravitation. Erwiderung auf eine Bemerkung des Hrn. A. Einstein," *Annalen der Physik*, **38** (1912), 1056-58, on 1056.

<sup>125</sup> "Wer, wie der Verfasser, wiederholt vor den Sirenenklängen dieser Theorie hat warnen müssen, der darf es mit Genugtuung begrüßen, dass ihr Urheber selbst sich nunmehr von ihrer Unhaltbarkeit überzeugt hat," *ibid.*, p. 1056.

Modestly, Abraham added that if he allowed himself to polemicize against such a deserving work as Einstein's last paper, it was only because of the criticism Einstein had "bestowed" on his—Abraham's—theory of gravitation. Abraham not only rejected the principle of relativity of *STR*, but even proposed to distinguish among all reference frames the one in which the gravitational field is a static one. He proposed to call the latter reference system an "absolute" one, and saw in it evidence for the existence of the ether.<sup>126</sup> With respect to the equality of inertial and gravitational masses, Abraham became convinced through Einstein of its importance and was prepared to consider it as a basic postulate of a new gravitational theory. Yet, in contradistinction to Einstein, Abraham saw no need to associate this with the "questionable 'equivalence hypothesis.'" <sup>127</sup> Indeed, at the end of 1912, Abraham was to develop a second theory of gravitation, incorporating the equality of inertial and gravitational masses, but not the principle of relativity.<sup>128</sup>

Abraham's charge that Einstein had himself given the coup de grâce to the special theory of relativity obliged Einstein to present his own position with respect to the theory. In his response, in July 1912, Einstein reaffirmed his commitment to the principle of relativity which he now formulated as follows :

Let  $\Sigma$  be an isolated system with respect to all other physical systems [in the sense of the usual language of physics), and let  $\Sigma$  be referred to a coordinate system  $K$  such that the laws which the spatio-temporal variations of  $\Sigma$  satisfy are as simple as possible; then there are infinitely many coordinate systems with respect to which these laws are the same, namely all those coordinate systems which, relatively to  $K$ , are in uniform rectilinear motion.<sup>129</sup>

Einstein mentioned that one could also define a principle of relativity with respect to the whole universe, but that the latter was beyond experimental confirmation. Adopting the first formulation—which he also restated in shortened form as "The relative velocity of the reference system  $K$  with respect to the remaining system  $U$  does not enter the physical laws"<sup>130</sup>—Einstein remarked that powerful arguments would have to be presented before this relativity principle could be doubted. He stressed repeatedly that he saw no reason for such a doubt and made it clear how important he considered that principle to be by rejecting any theory which did not incorporate it :

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126. Ibid., p. 1058

127. Max Abraham, "Nochmals Relativität und Gravitation. Bemerkung zu A. Einsteins Erwiderung," *Annalen der Physik*, **39** (1912), 444-48, on 446.

128. Abraham, "Das Gravitationsfeld," pp. 793-97.

129. "Es sei  $\Sigma$  ein von allen übrigen physikalischen Systemen (im Sinne der geläufigen Sprache der Physik) isoliertes System, und es sei  $\Sigma$  auf ein solches Koordinatensystem  $K$  bezogen, dass die Gesetze, welchen die räumlich-zeitlichen Änderungen von  $\Sigma$  gehorchen, möglichst einfache werden; dann gibt es unendlich viele Koordinatensysteme, in bezug auf welche jene Gesetze die gleichen sind, nämlich alle diejenigen Koordinatensysteme, die sich relativ zu  $K$  in gleichförmiger Translationsbewegung befinden," Albert Einstein, "Relativität und Gravitation. Erwiderung auf eine Bemerkung von M. Abraham." *Annalen der Physik*, **38** (1912; reception date : 4 juillet 1912), 1060.

130. Ibid., p. 1061.



The considerations outlined above imply, in my opinion, that any theory is to be rejected, which distinguishes a reference system from another one in uniform translational motion with respect to the first.<sup>131</sup>

Consequently, Einstein rejected Abraham's attempt to distinguish among all others the reference frame in which the gravitational field is a static one.

With respect to the second principle of *STR*, i.e., the principle of the constancy of the velocity of light, which says that there is a reference system  $K$  in which the velocity of light in vacuum is equal to the universal value  $c$  whatever the motion of the emitting body with respect to  $K$ , Einstein remarked that he thought its validity to be limited to space-time domains of constant gravitational potential. Einstein believed that something of *STR* had clearly to be given up, since it appeared impossible to integrate gravitation within *STR* without violating the equality of inertial and gravitational masses. He did not, however, interpret this situation as meaning a failure of the relativity principle, but saw in it only the need to limit the validity of the principle of the constancy of the light velocity to space-time domains of constant gravitational potential. Thus he claimed that *STR* was still valid within that restricted domain.

The fact that Einstein's own static theory of gravitation did not satisfy a local Lorentz invariance could not really be brought up against the relativity principle, since Einstein was well aware of the restricted and provisional character of that theory. Einstein knew that a more general and relativistic scheme satisfying the equality of inertial and gravitational masses would have to be developed. He wrote : "the task in the immediate future must be to build a relativistic theoretical scheme, in which the equivalence between inertial and gravitational mass finds its expression ." <sup>132</sup> Einstein stated that with his static theory of gravitation he had intended to make a contribution toward that goal. Since the theory was based on the equivalence principle it provided for the gravity of energy. Einstein admitted that he had only been able to use a local equivalence principle and was unable to give a satisfactory reason for this. Yet, despite this limitation, he was not ready to follow Abraham and give up the equivalence principle altogether.

Besides the gravity of energy, Einstein had another reason to hold to the equivalence principle, namely the prospect of a general relativity. He wrote :

On the other hand, the equivalence principle opens up the interesting perspective that the equations of a relativity theory encompassing gravitation might also be invariant with respect to accelerated (and rotating) systems. The path towards that goal seems, however,

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131. "Die im vorigen angedeuteten Überlegungen bringen es nach meiner Ansicht mit sich, dass jede Theorie abzulehnen ist, welche *ein* Bezugssystem gegenüber den relativ zu ihm in gleichförmiger Translation befindlichen Bezugssystemen auszeichnet," *ibid.*, p. 1061. Emphasis in original.

132. "Aufgabe der nächsten Zukunft muss es sein, ein relativitätstheoretisches Schema zu schaffen in welchem die Äquivalenz zwischen träger und schwerer Masse ihren Ausdruck findet," *ibid.*, p. 1063.

to be a rather difficult one".<sup>133</sup>

Among the difficulties which hindered his progress toward that goal in July 1912, Einstein mentioned the foreseeable loss of the immediate physical meaning of the space-time coordinates and the fact that he did not yet know the form of the general space-time transformations. Einstein had already encountered a loss of physical meaning with respect to the time coordinate, but now the space coordinates were affected as well. In view of these difficulties, Einstein publicly asked his colleagues for help : "I would like to ask all colleagues to try their hands at this important problem."<sup>134</sup> Einstein's call for collaboration was to be answered by his mathematician friend Marcel Grossmann, who introduced Einstein to the absolute differential calculus, which was to make Einstein's dream of general relativity reality.

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133. "Andererseits eröffnet uns dies Äquivalenzprinzip die interessante Perspektive, dass die Gleichungen einer auch die Gravitation umfassenden Relativitätstheorie auch bezüglich Beschleunigungs- (und Drehungs-) Transformationen invariant sein dürften. Allerdings scheint der Weg zu diesem Ziele ein recht schwieriger zu sein," *ibid.*, p. 1063.

134. "Ich möchte alle Fachgenossen bitten, sich an diesem wichtigen Problem zu versuchen," *ibid.*, p. 1064.



## Chapitre 4

# First Sketch of the General Theory of Relativity (1913 – 1915)

At the end of the summer semester of 1912 Einstein moved back from Prague to Zurich, where he became a professor Ordinarius at the Zurich Eidgenössischen Technischen Hochschule (ETH). In Prague, the mathematician Georg Pick, one of Einstein's closest colleagues there, had suggested to Einstein the use of the differential calculus of Ricci and Levi-Civita to solve his mathematical difficulties in generalizing *STR*.<sup>1</sup> In Zurich, his friend Marcel Grossmann, whose notes Einstein had used as undergraduate and who was now professor of mathematics at the ETH, responded immediately with enthusiasm to Einstein's request for help on that subject. In 1913, Einstein and Grossmann presented the first sketch of a general theory of relativity. A generally covariant equation of motion was obtained, but not generally covariant field equations. Shortly afterwards, Einstein made a renewed attack on the problem of generally covariant field equations, which was characterized by alternating hope and disappointment. Einstein first tried to rationalize his failure, and then for a time thought he could indeed achieve a satisfactory covariance for his 1913 field equations; finally, in 1915, he realized that this was not the case and abandoned the field equations of 1913.

After a few historical remarks on tensor calculus, we shall examine the 1913 Einstein-Grossmann theory, in particular with respect to the development of the equation of motion, the field equations, and the main consequences of the theory. We shall then analyze Einstein's further elaboration of the theory and briefly survey its reception.

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1. Philipp Frank, *Einstein, His Life and Times* (New York : Alfred A. Knopf, 1953), p. 82.

## 4.1 Historical Remarks on the Tensor Calculus

Since tensors, like any other physical quantities are representations of groups, the concept of a tensor is defined only relatively to a specific group.<sup>2</sup> Historically, the tensor concept entered physics along two distinct lines. Along one line, the physicists Josiah Willard Gibbs and Woldemar Voigt introduced the tensor concept as an extension of the vector calculus, which was itself an outgrowth of Hamilton's quaternion calculus. These tensors were initially defined with respect to the rotation group in 3-space and were later generalized to tensors with respect to the rotation group in 4-space. Following standard notation we shall refer to these groups as  $O(3)$  and  $O(4)$  respectively (the letter  $O$  standing for orthogonal transformation). Along the other line, the mathematician Elwin Bruno Christoffel, in 1868, laid the mathematical foundation of a tensor calculus with respect to a more general group, namely, the group of transformations associated with a quadratic differential form. Christoffel's considerations were used in 1900 by Ricci and Levi-Civita to develop what they called the "absolute differential calculus." We shall analyze successively these two lines of development.

### 4.1.1 Tensors with Respect to the Groups $O(3)$ and $O(4)$

Since vectors and hence tensors with respect to the groups  $O(3)$  and  $O(4)$  were historically rooted in William Rowan Hamilton's quaternion calculus, it will be appropriate to say a few words about quaternions.<sup>3</sup> The influence of Hamilton's quaternion calculus is reflected in current terminology : the terms scalar,<sup>4</sup> vector,<sup>5</sup> and tensor<sup>6</sup> were all introduced by Hamilton; the first two terms have kept the meaning Hamilton gave them, whereas the meaning of the last term was generalized by Voigt to its present meaning. Hamilton's discovery of quaternions resulted from his desire to generalize the complex numbers. Introducing ordered sets of four real (or complex) numbers  $q = (d, a, b, c)$  which he called quaternions (biquaternions) and wrote  $q = d + ai + bj + ck$ , Hamilton analyzed the properties of  $1, i, j, k$ , and discovered in 1843<sup>7</sup> what is now called the quaternion group, which is constituted by 8 elements :  $\pm 1, \pm i, \pm j, \pm k$ , with

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2. See Paul Kustaanheimo, *On the equivalence of some calculi of transformable quantities*, *Societas Scientiarum Fennica : Commentationes Physico-Mathematicae*, **17**, no. 9 (1955), p. 4; Eric Lord, *Tensors, Relativity and Cosmology* (New Delhi : Tata McGraw-Hill, 1976), chaps. 1, 2.

3. On the role quaternions played in the development of vector calculus see Michael J. Crowe, *A History of Vector Analysis : The Evolution of the Idea of a Vectoriel System* (Notre Dame, London : University of Notre Dame Press, 1967).

4. William Rowan Hamilton, "On Quaternions, or on a New System of Imaginaries in Algebra," *Philosophical Magazine*, **29** (1846), 26-27; mentioned by Crowe, *History of Vector Analysis*, pp. 31-32.

5. Ibid.

6. William Rowan Hamilton, *Lectures on Quaternions* (Dublin, 1853), p. 57, par. 63.

7. For an account of the discovery, see Thomas L. Hankins, *Sir William Rowan Hamilton* (Baltimore and London : Johns Hopkins University Press, 1980), chap. 7; see also Crowe (1967), chap. 2.

the multiplication rule :

$$\begin{aligned} i^2 &= j^2 = k^2 = -1 \\ ij &= k, jk = i, ki = j \\ ji &= -k, kj = -i, ik = -j. \end{aligned}$$

Reciprocally, Hamilton's multiplication rule of the quaternion algebra can be obtained from the multiplication table of the quaternion group (by identifying  $a(-i)$  with  $-a(i)$ , etc.). Hamilton called  $d$  the "scalar part" of the quaternion  $q$ ,  $(a, b, c)$  the "vector part," and the quantity  $\sqrt{d^2 + a^2 + b^2 + c^2}$  the "tensor of  $q$ ."

Despite Hamilton's conviction of the usefulness of quaternions for physical applications—which was shared in particular by Peter Guthrie Tait and James Clerk Maxwell (the latter made use of quaternions in his *Treatise on Electricity and Magnetism*)—most physicists saw no need for such an abstract mathematical tool in their science. The influence of Maxwell, Tait, and others, however, together with the growing importance of electromagnetism, stimulated the development of a quaternion calculus for the practical man, namely the vector calculus. One of the key figures in the growth of the vector calculus was Josiah Willard Gibbs. Gibbs separated the scalar part from the vector part of the quaternion and introduced two kinds of products : the dot product and the vector product, instead of one quaternion product. Gibbs also made use of Hamilton's linear functions of quaternions and introduced operators he called dyadics which are equivalent to modern tensors with respect to the  $O(3)$  group. In Gibbs' notation,<sup>8</sup> the equation

$$\vec{r}' = \Phi \cdot \vec{r},$$

where  $\Phi$  is the dyadic operating on  $\vec{r}$ , corresponded to the following linear transformation :

$$\begin{aligned} x' &= a_{11}x + a_{12}y + a_{13}z, \\ y' &= a_{21}x + a_{22}y + a_{23}z, \\ z' &= a_{31}x + a_{32}y + a_{33}z, \end{aligned}$$

where the coefficients  $a_{ij}$  are numbers. The set  $a_{ij}$  is called today a tensor of second rank. In the nineteenth century, such linear functions were used in the theories of elasticity, hydrodynamics, and electricity. Gibbs<sup>9</sup> introduced the term "right tensor" for a dyadic corresponding to a pure strain.

It was precisely the problem of describing elongations which, around the turn of the century, led the theoretical physicist and crystallographer Woldemar

8. Josiah Willard Gibbs and E. B. Wilson, *Vector Analysis* (New Haven : Yale University Press, 1901), p. 265.

9. Josiah Willard Gibbs, *Elements of Vector Analysis* (New Haven, 1881-84, not published), p. 57 ; mentioned by Max Abraham, "Geometrische Grundbegriffe," *Encyklopädie der Mathematischen Wissenschaften, IV*, part 3 (article completed in 1901), p. 28.

Voigt to look for a new mathematical tool which would allow one to describe stresses and strains of crystals. Since an elongation along one axis is characterized by a number and a direction without orientation, a vector is inappropriate to describe such a state since  $\vec{a}$  and  $-\vec{a}$  are essentially two different vectors. Thus Voigt, who was not aware of Gibbs's privately printed work, introduced what he called tensors : a tensor was characterized by a number and a direction without orientation.<sup>10</sup> With respect to the choice of the term "tensor," Voigt adopted Hamilton's term because he considered the new meaning to be an extension of the old one;<sup>11</sup> whereas for Hamilton a tensor was a magnitude, for Voigt it meant a magnitude plus a direction. Since any pure deformation can be characterized by three tensors in three perpendicular directions, Voigt introduced the term "tensor triple"<sup>12</sup> and defined the latter in terms of six independent components in an arbitrary coordinate system.<sup>13</sup> Voigt<sup>14</sup> was soon to write these components in the standard notation :  $T_{xx}, T_{yy}, T_{zz}, T_{yz}, T_{zx}, T_{xy}$  and also introduced tensors of higher ranks.<sup>15</sup> Voigt's "tensor triple" corresponds to a symmetric tensor. Gibbs's dyads and Voigt's tensors constituted representations of the group of rotations  $O(3)$ .

The next step in this line of development was Poincaré's introduction of what Sommerfeld<sup>16</sup> later called quadrivectors, which by construction constituted a representation of the group of linear orthogonal transformations leaving  $x^2 + y^2 + z^2 - t^2$  invariant. (This group corresponds to  $O(3, 1)$  and was denoted by Poincaré the Lorentz group.<sup>17</sup>) It must be pointed out that Voigt,<sup>18</sup> in 1887, had already made use of the Lorentz transformation.

Hermann Minkowski, following Poincaré and Einstein, took the relativistic invariance seriously and derived the fundamental electrodynamic equations for moving bodies from a four-dimensional Lorentz-invariant formalism. The four-formalism, as with Poincaré, associated the spatial coordinates and the time coordinate within a new four-quantity which Minkowski<sup>19</sup> wrote  $(x, y, z, it)$  or  $(x_1, x_2, x_3, x_4)$ . Through the introduction of the complex number  $i$ , the Lorentz

10. Woldemar Voigt, *Die fundamentalen Eigenschaften der Krystalle in Elementarer Darstellung* (Leipzig, 1898), p. 20.

11. Ibid., préface, p. VI.

12. Ibid., p. 22.

13. Ibid., p. 23.

14. W. Voigt, "Etwas über Tensoranalysis," *Nachr. Ges. Gött.*, math.-physikalische Klasse (1904), pp. 495-513, on p. 499.

15. W. Voigt, "Ueber die Parameter der Krystallphysik und über gerichtete Grössen höherer Ordnung," *Gött. Nachr.*, Heft 4 (1900), pp. 355-79, sur p. 358; Voigt, "Tensoranalysis" p. 499.

16. Arnold Sommerfeld, "Zur Relativitätstheorie, I. Vierdimensionale Vektoralgebra," *Annalen der Physik*, **32** (1910), 749-76, in Arnold Sommerfeld, *Gesammelte Schriften*, ed. F. Sauter (Braunschweig : Friedr. Vieweg & Sohn, 1968), **2**, 190.

17. Henri Poincaré, "Sur la Dynamique de l'Electron," *Rend. Circ. Mat. Palermo*, **21** (1906), 129-76, in Henri Poincaré, *Oeuvres* (Paris, 1954), **9**, 547.

18. W. Voigt, "Über das Doppler'sche Princip," *Nachr. Ges. Gött.*, (1887), pp. 44-51, on p. 45. The paper was reprinted *Physikalische Zeitschrift*, **16** (1915), 381-86.

19. Hermann Minkowski, "Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern," *Nachr. Ges. Gött.*, math.-physikalische Klasse, (1908), pp. 53-111, in Hermann Minkowski, *Gesammelte Abhandlungen*, ed. D. Hilbert (Leipzig, 1911; rpt. New York : Chelsea, 1967), 2 vols. in one, **2**, 354.

invariant  $x^2 + y^2 + z^2 - t^2$  could be written  $x_1^2 + x_2^2 + x_3^2 + x_4^2$ . Consequently, a Lorentz transformation could be viewed as a rotation in four-space. It seems likely that Minkowski came to the idea of using the norm  $x_1^2 + x_2^2 + x_3^2 + x_4^2$  through the quaternion calculus in which this quantity plays a major role. Minkowski's awareness of quaternions is established by the fact that in a footnote of the same paper in which he introduced the four-formalism, he suggested that one could have thought of using the quaternion calculus rather than the matrix calculus adopted in the paper.<sup>20</sup> In fact, Minkowski's four-vector  $(x, y, z, it)$  was just a special case of Hamilton's complex quaternions (which Hamilton called biquaternions).<sup>21</sup>

The historical importance of Minkowski's work in physics was that it led to the development of the four-dimensional tensor calculus. Initially, Einstein had been rather critical of Minkowski's work : according to Max Born, Einstein, around 1909, saw in Minkowski's work little more than "superfluous mathematical accessories" ("überflüssiges mathematisches Beiwerk").<sup>22</sup> Einstein later acknowledged, however, that without the four-dimensional tensor calculus, *GTR* would not have been possible.<sup>23</sup> The four-dimensional tensor calculus was to provide the mathematical background against which Einstein could appreciate the potential usefulness of the generally covariant tensor calculus. Indeed, within this context the latter was to appear as a natural extension of the tensor calculus with respect to  $O(4)$ .

#### 4.1.2 Tensors with Respect to the Groups Associated with a Quadratic Differential Form

The theoretical basis of the absolute differential calculus of Ricci and Levi-Civita was developed by Elwin Bruno Christoffel in his 1869 paper entitled "Ueber die Transformation der homogenen Differentialausdrücke zweiten Grades."<sup>24</sup> There Christoffel introduced the concept of what Ricci later called the covariant derivative<sup>25</sup> and also gave an analytic expression of what was later called the Riemann-Christoffel curvature tensor. Prior to Christoffel, Bernhard Riemann had introduced the latter concept in general terms in his Habilitationsschrift of 1854, which was published only after his death.<sup>26</sup> Christoffel briefly mentioned

20. Minkowski, *Abhandlungen*, **2**, 375.

21. William Rowan Hamilton, *Elements of Quaternions*, ed. C. J. Joly, 3rd ed. (London, 2nd ed., 1899 ; rpt. New York : Chelsea, 1969), **1**, 133.

22. Max Born, "Erinnerungen an Hermann Minkowski zur 50. Wiederkehr seines Todestages," *Naturwiss.*, **46** (1959), 501-05, in Max Born, *Ausgewählte Abhandlungen* (Göttingen : Vandenhoeck & Ruprecht, 1963), **2**, 688.

23. Einstein to Besso, 6 Jan. 1948, *ibid.*, p. 391.

24. E. B. Christoffel, "Ueber die Transformation der homogenen Differentialausdrücke zweiten Grades," *Crelle's Journal*, **70** (1869), 46-70, in E. B. Christoffel, *Gesammelte mathematische Abhandlungen*, ed. L. Maurer (Leipzig-Berlin, 1910), **I**, 352-77.

25. G. Ricci, "Sulla derivazione covariante ad una forma quadratica differenziale." *Rendiconti Accad. Lincei* (4), **3**, part I (1887), 15-18.

26. B. Riemann, "Ueber die Hypothesen, welche der Geometrie zu Grunde liegen," Habilitationsschrift, 1854, *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen*, vol. 13, in B. Riemann, *Gesammelte Mathematische Werke und Wissenschaftlicher*



Riemann's lecture at the very end of his paper, but apparently developed his considerations quite independently of it.

Christoffel's problem was as follows. If the independent variables  $x^i$  of a differential expression<sup>27</sup>  $ds^2 = \omega_{ik} dx^i dx^k$ , where  $\omega_{ik}$  are arbitrary functions of  $x^i$ , are replaced by another set of independent variables  $x'^i$ , one obtains a new differential expression  $ds'^2 = \omega'_{ik} dx'^i dx'^k$  such that  $ds^2 = ds'^2$ . If on the other hand, one starts from the differential expressions  $ds^2, ds'^2$  with given  $\omega_{ik}, \omega'_{ik}$ , the question arises whether there are transformations  $x^\mu = x'^\mu (x'^\sigma)$  such that  $ds^2 = ds'^2$  and, if there are, what conditions these transformations have to satisfy. From algebraic invariant theory it follows that the transformations  $x^\mu = x'^\mu (x'^\sigma)$  have to satisfy the condition  $E' = r^2 E$  where  $E'$  and  $E$  are the determinants of  $\omega'_{ik}$  and  $\omega_{ik}$  respectively, and where  $r$  is the Jacobian of the transformation. The above condition would be sufficient if  $\omega_{ik}, \omega'_{ik}$  were constant. In the present case, however, further conditions have to be satisfied so that the  $dx^\mu$  may be total differentials. Through differentiation operations, Christoffel found that the necessary and sufficient conditions (except for initial conditions) imposed on the transformations are given by

$$\frac{\partial^2 x^r}{\partial x'^\alpha \partial x'^\beta} + \Gamma_{ik}^r \frac{\partial x^i}{\partial x'^\alpha} \frac{\partial x^k}{\partial x'^\beta} = \Gamma_{\alpha\beta}^{\prime\lambda} \frac{\partial x^r}{\partial x'^\lambda} \quad (4.1)$$

where  $\Gamma_{il}^r = \sum_k \frac{E_{rk}}{E} \Gamma_{k,il}$ ,  $E_{rk}$  being the cofactor of  $\omega_{rk}$  in  $E$ , and

$$\Gamma_{k,il} = \frac{1}{2} \left[ \frac{\partial \omega_{ik}}{\partial x^l} + \frac{\partial \omega_{lk}}{\partial x^i} - \frac{\partial \omega_{il}}{\partial x^k} \right].$$

The coefficients  $\Gamma_{ik}^r, \Gamma_{k,il}$  which Christoffel wrote respectively  $\binom{ik}{r}, \left[ \begin{smallmatrix} il \\ k \end{smallmatrix} \right]$  are now known as Christoffel symbols.

Condition (4.1) allowed Christoffel to introduce the concept of the covariant derivative as follows. From the postulate of the invariance of  $ds^2 = \omega_{\mu\nu} dx^\mu dx^\nu = ds'^2$ , Christoffel pointed out that the transformation law of the  $\omega_{\mu\nu}$  is given by

$$\omega'_{\mu\nu} = \omega_{\alpha\beta} u_\mu^\alpha u_\nu^\beta,$$

where  $u_\mu^\alpha = \frac{\partial x^\alpha}{\partial x'^\mu}$ . This is the relation Einstein was to adopt for the definition of a tensor  $\omega_{ik}$ . From this definition, which entails the invariance of  $ds^2$ , and similar definitions for the other tensors, it follows that tensors constitute a representation of the group of transformations with respect to which  $ds^2$  is an invariant. In the general theory of relativity, this group is the group of arbitrary transformations of four-dimensional space-time. We shall refer to this group as the *GTR* group. Christoffel now considered a general form

$$G_\mu = \sum_{i_1 i_2 \dots i_\mu} (i_1 i_2 \dots i_\mu) dx^{i_1} dx^{i_2} \dots dx^{i_\mu},$$

*Nachlass*, ed. Heinrich Weber, 2nd ed. (Leipzig, 1892), pp. 272-87.

27. From here on, we adopt the summation convention Einstein introduced in 1916; each time an index appears twice, once in the upper position and once in the lower position, a summation over this index is assumed.

where the coefficients (which are tensors of the type  $T_{i_1 i_2 \dots i_\mu}$ ) are functions derived from  $\omega_{\mu\nu}$ . The transformation law for the coefficients of  $G_\mu$  is

$$(\alpha_1 \alpha_2 \dots \alpha_\mu)' = \sum_{i_1 i_2 \dots i_\mu} (i_1 i_2 \dots i_\mu) u_{\alpha_1}^{i_1} u_{\alpha_2}^{i_2} \dots u_{\alpha_\mu}^{i_\mu}.$$

By taking the derivative  $\frac{\partial(\alpha_1 \alpha_2 \dots \alpha_\mu)}{\partial x'^{\alpha}}$ , and replacing the second derivatives  $\frac{\partial^2 x^\lambda}{\partial x'^{\alpha} \partial x'^{\alpha s}}$  in terms of equation (4.1), Christoffel, after rearrangement of the terms on both sides of the equation, obtained the relation<sup>28</sup>

$$(\alpha \alpha_1 \dots \alpha_\mu)' = \sum_{i_1 \dots i_\mu} (ii_1 \dots i_\mu) u_\alpha^i u_{\alpha_1}^{i_1} \dots u_{\alpha_\mu}^{i_\mu} \quad (4.2)$$

where

$$(ii_1 i_2 \dots i_\mu) = \frac{\partial(i_1 i_2 \dots i_\mu)}{\partial x^i} - \sum_\lambda [\Gamma_{ii_1}^\lambda (\lambda i_2 \dots i_\mu) + \Gamma_{ii_2}^\lambda (i_1 \lambda \dots i_\mu) + \dots] \quad (4.3)$$

and similarly for  $(ii_1 \dots i_\mu)'$ . Thus from a "system of transformation relations of order  $\mu$ " Christoffel obtained a system of transformation relations of order  $\mu + 1$ . Relation (4.3) is the definition of the covariant derivative of a covariant tensor  $(i_1 i_2 \dots i_\mu)$  and equation (4.2) shows that the covariant derivative transforms like a tensor.

In the same paper, Christoffel analyzed the integrability conditions of the system of equations (4.1). He found that the latter system is integrable if the following relation is satisfied

$$(\alpha \delta \beta \gamma)' = \sum_{ghik} (gkhi) u_\alpha^g u_\beta^h u_\gamma^i u_\delta^k,$$

where

$$(gkhi) = \frac{1}{2} \left( \frac{\partial^2 \omega_{gi}}{\partial x^h \partial x^k} + \frac{\partial^2 \omega_{hk}}{\partial x^g \partial x^i} - \frac{\partial^2 \omega_{gh}}{\partial x^i \partial x^k} - \frac{\partial^2 \omega_{ik}}{\partial x^g \partial x^h} \right) + \sum_{\alpha\beta} \frac{E_{\alpha\beta}}{E} (\Gamma_{\alpha,gi} \Gamma_{\beta,hk} - \Gamma_{\alpha,gh} \Gamma_{\beta,ik}).$$

The expression  $(gkhi)$  is the Riemann-Christoffel tensor<sup>29</sup> (or curvature tensor) which is the cornerstone of Einstein's final field equations.

28. Christoffel "Differentialausdrücke (1869), p. 363.

29. The term is used by Einstein and A. D. Fokker, "Die Nordströmsche Gravitationstheorie vom Standpunkt des absoluten Differentialkalküls," *Annalen der Physik*, **44** (1914), 321-28, on 328.

**Box 4. Tensors of first rank**

Pseudo-euclidean space (curvilinear coordinates) or Riemannian space.  
Transformation of the coordinate system (arbitrary continuous function)

$$(x^0, x^1, x^2, x^3) \rightarrow (x'^0, x'^1, x'^2, x'^3)$$

$$x^i = f^i(x'^0, x'^1, x'^2, x'^3)$$

$$dx^i = \frac{\partial x^i}{\partial x'^k} dx'^k$$

(Einstein's summation convention on  $k$ );

invariant :

$$ds^2 = g_{ik} dx^i dx^k = g'_{ik} dx'^i dx'^k.$$

1) Contravariant quadrivector :  $A^i$

$$A^i = \frac{\partial x^i}{\partial x'^k} A'^k$$

and reciprocally

$$A'^i = \frac{\partial x'^i}{\partial x^k} A^k$$

2) Covariant quadrivector :  $A_i$

$$A_i = \frac{\partial x'^k}{\partial x^i} A'_k$$

reciprocally

$$A'_i = \frac{\partial x^k}{\partial x'^i} A_k$$

example : (with  $\varphi$  : scalar)

$$A_i = \frac{\partial \varphi}{\partial x^i} = \frac{\partial \varphi}{\partial x'^k} \frac{\partial x'^k}{\partial x^i} = \frac{\partial x'^k}{\partial x^i} A'_i$$

$\Rightarrow$  invariant

$$A^i B_i = \frac{\partial x^i}{\partial x'^k} \frac{\partial x'^m}{\partial x^i} A'^k B'_m = \delta_k^m A'^k B'_m = A'^k B'_k$$

(with  $\frac{\partial x^i}{\partial x^k} = \delta_k^i = \frac{\partial x^i}{\partial x'^m} \frac{\partial x'^m}{\partial x^k}$ )

**Box 5. Tensors of second rank**

Pseudo-euclidean space (curvilinear coordinates) or Riemannian space.  
Transformation of the system of coordinates (arbitrary continuous function)

$$(x^0, x^1, x^2, x^3) \rightarrow (x'^0, x'^1, x'^2, x'^3)$$

$$x^i = f^i(x'^0, x'^1, x'^2, x'^3)$$

$$dx^i = \frac{\partial x^i}{\partial x'^k} dx'^k$$

(with Einstein's summation convention on  $k$ );

invariant :

$$ds^2 = g_{ik} dx^i dx^k = g'_{ik} dx'^i dx'^k.$$

1a) Contravariant tensors of second rank :  $A^{ik}$  ( $i, k \in 0, 1, 2, 3$ )

$$A^{ik} = \frac{\partial x^i}{\partial x'^l} \frac{\partial x^k}{\partial x'^m} A'^{lm}$$

and reciprocally

$$A'^{ik} = \frac{\partial x'^i}{\partial x^l} \frac{\partial x'^k}{\partial x^m} A^{lm}$$

b) Covariant tensors of second rank :  $A_{ik}$

$$A_{ik} = \frac{\partial x'^l}{\partial x^i} \frac{\partial x'^m}{\partial x^k} A'_{lm}$$

reciprocally

$$A'_{ik} = \frac{\partial x^l}{\partial x'^i} \frac{\partial x^m}{\partial x'^k} A_{lm}$$

$\Rightarrow$  invariant

$$A^{ik} B_{ik} = \frac{\partial x^i}{\partial x'^l} \frac{\partial x^k}{\partial x'^m} \frac{\partial x'^p}{\partial x^i} \frac{\partial x'^q}{\partial x^k} A'^{lm} B'_{pq} = \delta_l^p \delta_m^q A'^{lm} B'_{pq} = A'^{pq} B'_{pq}$$

(with  $\frac{\partial x'^p}{\partial x'^n} = \delta_l^p = \frac{\partial x'^p}{\partial x^i} \frac{\partial x^i}{\partial x'^n}$ )

c) Mixt tensor

$$A^i_k = \frac{\partial x^i}{\partial x'^l} \frac{\partial x'^m}{\partial x^k} A^l_m$$

reciprocally

$$A^i_k = \frac{\partial x'^i}{\partial x^l} \frac{\partial x^m}{\partial x'^k} A^l_m$$

2) Other relations

$$A^i = g^{ik} A_k, A_i = g_{ik} A^k$$

$$g_{ik} g^{kl} = \delta_i^l$$

Christoffel's notion of a covariant derivative was used by Gregorio Ricci-Curbastro to develop what he called the "absolute differential calculus." Together with Tullio Levi-Civita, he published a comprehensive memoir<sup>30</sup> in which they developed various applications (analytical, geometrical, mechanical, and physical) of the tensor calculus. The considerations of Christoffel and those of Ricci and Levi-Civita must be seen within the context of the researches done in the mathematical field of invariant theory associated with a quadratic differential form which was an active field around the turn of the century. Thus Marcel Grossmann was naturally aware of it. In 1908, J. Edmund Wright gave a survey of the field mentioning in particular the method used by Christoffel, Ricci, and Levi-Civita and those used by Lie and Maschke<sup>31</sup>.

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30. G. Ricci et T. Levi-Civita, "Méthodes de calcul différentiel absolu et leurs applications," *Mathematische Annalen*, **54** (1901), 125-201; for a translation and comments, see Robert Hermann, Ricci and Levi-Civita's tensor analysis paper : translation, comments and additional material (Brookline, MA : Math Sci Press, 1975).

31. Joseph Edmund Wright, *Invariants of Quadratic Differential Forms* (1908 ; rpt. New York : Hafner, 1972).

**Box 6. Covariant derivative**

Pseudo-euclidean space (curvilinear coordinates) or Riemannian space

$$ds^2 = g_{ik} dx^i dx^k.$$

1) Covariant differential of a contravariant vector

$$\begin{aligned} DA^i &= \left( \frac{\partial A^i}{\partial x^l} + \Gamma_{kl}^i A^k \right) dx^l \\ &= A^i_{;l} dx^l \end{aligned}$$

hence, the covariant derivative

$$A^i_{;l} = \frac{\partial A^i}{\partial x^l} + \Gamma_{kl}^i A^k$$

2) Covariant differential of a covariant vector

$$\begin{aligned} DA_i &= \left( \frac{\partial A_i}{\partial x^l} - \Gamma_{kl}^i A_k \right) dx^l \\ &= A_{i;l} dx^l \end{aligned}$$

hence, the covariant derivative

$$A_{i;l} = \frac{\partial A_i}{\partial x^l} - \Gamma_{il}^k A_k$$

with Christoffel's symbols :

$$\Gamma_{kl}^i = \frac{1}{2} g^{im} \left( \frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right)$$

$$\begin{aligned} \Gamma_{i,kl} &= g_{im} \Gamma_{kl}^m \\ &= \frac{1}{2} \left( \frac{\partial g_{ik}}{\partial x^l} + \frac{\partial g_{il}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^i} \right) \end{aligned}$$

The first physical application of the tensor calculus of Ricci and Levi-Civita was made by Friedrich Kottler, who in 1912 developed generally covariant electromagnetic field equations<sup>32</sup> as a natural generalization of Minkowski's conceptions. The work was published when Einstein and Grossmann were already working on their joint paper of 1913.<sup>33</sup>

## 4.2 The Einstein-Grossmann Tensor Theory (1913)

The efforts by Einstein and Grossmann to develop a general theory of relativity were to result in a paper entitled "Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation" [Sketch of a generalized relativity theory and a theory of gravitation],<sup>34</sup> which we shall refer to as the Entwurf paper. The paper contained two parts : a physical part written by Einstein and a mathematical part written by Grossmann. It was in this paper that Einstein was to mention Eötvös experiments on the equality of inertial and gravitational masses for the first time; he was to refer to these experiments frequently thereafter, as support for the equivalence principle. We shall analyze successively the development of the equation of motion, of the field equations, and of the main results of the theory.

### 4.2.1 The Generally Covariant Equation of Motion

A key step toward the generally covariant equation of motion was Einstein's discovery that the equation of motion of his static theory of gravitation could be derived from Hamilton's principle in a particularly simple way. The discovery must have taken place between March and May 1912, i.e., during the time of submission and publication of Einstein's paper "Zur Theorie des statischen Gravitationsfeldes"<sup>35</sup> since the finding appeared in an addendum which was published together with that paper. In the addendum, Einstein showed that the equation of motion of a free particle in a static gravitational field can be expressed in the following simple form

$$\delta \left( \int \sqrt{c^2 dt^2 - dx^2 - dy^2 - dz^2} \right) = 0$$

where  $c$  is a function of the spatial coordinates. Einstein commented :

It is apparent—as Planck has already shown for the ordinary relativity theory—, that the equations of analytical mechanics have a

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32. Friedrich Kottler, "Über die Raumzeitlinien der Minkowski'schen Welt," *Sitzungsb. Ak. Wiss. Wien*, Mathematisch-Naturwissenschaftliche Klasse, Section IIa, part 2, **121** (1212), 1659-1759.

33. Albert Einstein and Marcel Grossmann, "Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation," *Zeitschrift für Mathematik und Physik*, **62** (1913), 225-61, on 245.

34. Ibid.

35. Albert Einstein, "Zur Theorie des statischen Gravitationsfeldes," *Annalen der Physik*, **38** (1912), 443-58, on 458.

significance which goes far beyond the Newtonian mechanics. The Hamiltonian equation [above] gives a hint of how the equations of motion of a material point are constituted in a dynamical gravitational field.<sup>36</sup>

Since Planck,<sup>37</sup> in 1906, had shown that the above equation (with  $c$  being constant) represents the equation of motion of a free particle in *STR*, the similarity of these equations was striking. Thus the equation of motion of the static theory of gravitation appeared as a natural extension of the equation of motion of a free particle in *STR*.

From the Entwurf paper, as we shall see, it appears that Einstein's next step was to assume that an equation of the type  $\delta \int ds = 0$ , where  $ds$  is a certain invariant to be found, represented the equation of motion of a particle in a dynamical gravitational field. Einstein could easily get some idea of the form of  $ds$  by taking the  $ds$  of the static theory of gravitation and by making a Lorentz transformation, or some other general transformation that would transform the static gravitational field into a dynamical one. He thus realized that more functions would enter the  $ds^2$  than just  $c^2$ . It is probably in this sense that Einstein's final remark in the addendum must be interpreted. Yet, by July 1912, we have seen that Einstein still did not know the form of the most general transformation of coordinates and that he asked his colleagues for help on that question. Thus, at that time, Einstein probably was still not certain that the invariant he was looking for was the quadratic differential form,  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ . But at least Einstein knew in what direction to look and what kind of questions to ask.

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36. "Auch hier zeigt sich - wie dies für die gewöhnliche Relativitätstheorie von Planck dargetan wurde - , dass den Gleichungen der analytischen Mechanik eine über die Newtonsche Mechanik weit hinausreichende Bedeutung zukommt. Die zuletzt hingeschriebene Hamiltonsche Gleichung lässt ahnen, wie die Bewegungsgleichungen des materiellen Punktes im dynamischen Gravitationsfelde gebaut sind," *ibid.*, p. 458.

37. Max Planck, "Das Prinzip der Relativität und die Grundgleichungen der Mechanik," *Berichte der Deutschen Physikalischen Gesellschaft*, **4** (1906), 136-41, in Max Planck, *Physikalische Abhandlungen und Vorträge* (Braunschweig : Friedr. Vieweg & Sohn, 1958), **2**, 119.



**Box 7. Equation of motion in general relativity**

Pseudo-euclidean space (curvilinear coordinates) or Riemannian space.

1) Inertial motion in special relativity

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

$$-mc\delta \int ds = 0$$

$$\Rightarrow \frac{du^i}{ds} = 0$$

(with  $u^i = \frac{dx^i}{ds}$  the four-velocity).

2) Geodesic in general relativity

$$ds^2 = g_{ik} dx^i dx^k.$$

$$-mc\delta \int ds = 0$$

$$\Rightarrow \frac{Du^i}{ds} = 0$$

(with  $u^i = \frac{dx^i}{ds}$  the four-velocity)

$$\Rightarrow \frac{du^i}{ds} + \Gamma_{kl}^i u^k u^l = 0$$

or

$$\frac{d^2 x^i}{ds^2} + \Gamma_{kl}^i \frac{dx^k}{ds} \frac{dx^l}{ds} = 0$$

Marcel Grossmann was to provide answers to Einstein's questions. Grossmann<sup>38</sup> was born in 1878 in Budapest, studied with Einstein at the Zürich ETH, and in 1907 became professor of descriptive geometry at the ETH. Grossmann was a close friend of Einstein and had been helpful to the latter not only through his lecture notes at the ETH but especially through his father's recommendation of Einstein to the director of the patent office in Bern. Einstein judged that without the recommendation he would probably not have obtained that vital job.<sup>39</sup> Grossmann had written his first papers on non-Euclidean geometry and could at once inform Einstein of the major role quadratic differential forms played in mathematics both from an algebraic and geometric points of view. By introducing Einstein to the tensor calculus developed by Christoffel, Ricci, and Levi-Civita, Grossmann undoubtedly saved Einstein a lot of time searching the literature. The two men were to publish two papers together before the collaboration was interrupted by the war. Though above we have referred to the Entwurf theory as the Einstein-Grossmann theory, Einstein must nevertheless be considered as the discoverer of the theory since it was he who directed the search. Einstein's colleagues generally referred to the theory as Einstein's theory, which we shall often do as well. In a letter to Sommerfeld, Einstein described what he saw as Grossmann's contribution : "Grossmann will never claim to be codiscoverer. He only helped me to orient myself in the mathematical literature, but contributed nothing materially to the results."<sup>40</sup> The last sentence perhaps underestimates somewhat the contribution of Grossmann's mathematical expertise.

Thus, at the end of 1912, guided by Grossmann, Einstein knew that the invariant for the equation of motion was of the form  $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$  and that the  $g_{\mu\nu}$  could be interpreted as the metric of a Riemannian space. At that time, Vladimir Varičák had already presented a non-Euclidean, Lobachevskian interpretation of the special theory of relativity.<sup>41</sup> Einstein was probably aware of Varičák's ideas since he wrote a reply to a paper by Varičák on another topic at about the same time.<sup>42</sup> Besides the equation of motion, Einstein had another reason to be interested in a non-Euclidean space, namely, the result that for a

38. For biographical information see J. J. Burckhardt, "Grossmann, Marcel," *Dictionary of Scientific Biography*, **5** (1972), 554-55; Louis Kollros, "Prof. Dr. Marcel Grossmann," *Verhandlungen der Schweizerischen Naturforschenden Gesellschaft*, **118** (1937), 325-29; Walter Saxer, "Marcel Grossmann" *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*, **81** (1936), 322-26.

39. Einstein à Besso, 6 mars 1952, *Correspondance* (1972), pp. 464-65; see also Seelig, *Einstein* (1960), pp. 86-87.

40. "Grossmann wird niemals darauf Anspruch machen, als Mitentdecker zu gelten. Er half mir nur bei der Orientierung über die mathematische Litteratur, trug aber materiell nichts zu den Ergebnissen bei," Einstein à Sommerfeld, 15 juillet 1915, in Albert Einstein and Arnold Sommerfeld, *Briefwechsel*, ed. Armin Hermann (Basel : Schwabe, 1968), p. 30.

41. Vladimir Varičák, "Anwendung der Lobatschefschijschen Geometrie in der Relativtheorie," *Physikalische Zeitschrift*, **11** (1910), 93-96; "Die Relativtheorie und die Lobatschefschijsche Geometrie," *ibid.*, pp. 287-93; see also Vladimir Varičák, "Über die nichteuclidische Interpretation der Relativtheorie," *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Leipzig, **21** (1912), 103-27.

42. Albert Einstein, "Zum Ehrenfestschen Paradoxon," *Physikalische Zeitschrift*, **12** (1911), 509-10; reply to Vladimir Varičák, same title, *ibid.*, p. 169-70.

rotating body the Euclidean geometry was no longer valid.<sup>43</sup> Since Einstein wanted to include rotations among the allowed transformations, non-Euclidean spaces had to be considered.

The importance of the Hamiltonian formulation of the equation of motion of Einstein's static theory in the transition toward the generally covariant equation of motion is confirmed by the fact that Einstein devoted the first section of the Entwurf paper to that subject. The section, entitled "Equations of motion of the material point in a static gravitational field," reproduced the arguments presented in the addendum, but in the reverse order. Now, Einstein presented the equation of motion of a particle in a static gravitational field as a natural extension of the equation of motion of a free particle in *STR*:  $\delta \int ds = 0$  where  $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$ , the difference being that  $c$  became a function of the spatial coordinates in the static theory instead of being a constant. In the second section of the Entwurf paper, Einstein introduced the equation of motion of a particle in an arbitrary gravitational field as an extension of the equation of motion in a static gravitational field. Considering an arbitrary transformation  $x^\mu \rightarrow x'^\mu(x^\mu)$ , Einstein observed that the equation of motion of a particle in a static gravitational field would be transformed into the equation  $\delta \int ds' = 0$  with  $ds'^2 = g'_{\mu\nu} dx'^\mu dx'^\nu$  where the coefficients  $g'_{\mu\nu}$  which are symmetric in  $\mu, \nu$  are functions of the coordinates. Since, physically, the above transformation corresponds to the passage from a static field to a dynamical one, Einstein associated the  $g'_{\mu\nu}$  with a dynamical gravitational field. Einstein's next step was to assume that any gravitational field could be characterized by ten functions  $g_{\mu\nu}$ . Thereby, the static theory and *STR* were reduced to particular cases with

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & c^2 \end{bmatrix},$$

with  $\mu, \nu = 1, 2, 3, 4$  and where  $c$  is a function of  $x, y, z$  in the first case and a constant in the second. Furthermore, Einstein now assumed the equation of motion  $\delta \int ds = 0$ , where  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ , to hold for any gravitational field. The latter equation of motion is the same in any reference system only if  $ds$  is an invariant. In *STR* the postulate of the invariance of  $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$  is equivalent to the restricted principle of relativity. By postulating the invariance  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ , which implies that  $g_{\mu\nu}$  is a tensor, Einstein made his equation of motion generally covariant. Furthermore, the tensor calculus now enabled Einstein to formulate generally covariant equations (i.e., equations which keep the same form under the *GTR* group of transformations) simply by writing down tensor equations.

Though the postulate of the invariance of  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$  might have appeared as a rather straightforward step, the latter had serious consequences, of which Einstein was fully aware, with respect to the meaning of the space-time

43. Albert Einstein, "Lichtgeschwindigkeit und Statik des Gravitationsfeldes," *Annalen der Physik*, **38** (1912), 355-69, on 356.

coordinates. In *STR*,  $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$  is computed by measuring  $dt$  and  $dl = \sqrt{dx^2 + dy^2 + dz^2}$  directly with instruments. With the new metric  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ , where the  $g_{\mu\nu}$  which represent the gravitational field are considered to be given, the proper length of an infinitesimal rod is no longer determined only by  $dx^1, dx^2, dx^3$ , but by the potentials  $g_{11}, g_{22}, g_{33}, g_{12}, g_{13}, g_{23}$  as well. Similarly, the proper time is not given by  $dx^4$  but by  $ds^2 = g_{44} (dx^4)^2$ . Thus the coordinates  $x^1, x^2, x^3, x^4$  have no longer any immediate physical meaning.<sup>44</sup> With respect to the coordinate  $x^4$ , this was already the case in Einstein's static theory of gravitation as we have seen.

From what Einstein called the Hamilton function (and now usually called the Lagrangian),  $H = -m \frac{ds}{dt}$ , Einstein then derived the expressions for the momentum and energy of a particle in the gravitational field as well as the expression for the force exerted by the gravitational field on the particle. Generalizing these expressions to the case of a continuous distribution of matter and introducing the energy momentum tensor of the material distribution :  $T^{\mu\nu} = \rho_0 \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}$ , where  $\rho_0$  the rest density of matter, Einstein finally found for the densities of momentum, energy, and force, respectively :

$$\frac{J_x}{V} = -\sqrt{-g} g_{1\mu} T^{\mu 4}, \text{ etc.}$$

$$-\frac{E}{V} = -\sqrt{-g} g_{4\mu} T^{\mu 4},$$

and

$$\frac{K_x}{V} = -\frac{1}{2} \sqrt{-g} \frac{\partial g_{\mu\nu}}{\partial x^1} T^{\mu\nu} \text{ etc.}$$

With respect to the energy-momentum conservation equations for matter, Einstein expected them to be of the form  $(\sqrt{-g} g_{\sigma\mu} T^{\mu\nu})_{;\nu} = 0$  or, explicitly

$$\sum_{\mu\nu} \frac{\partial}{\partial x^\nu} (\sqrt{-g} g_{\sigma\mu} T^{\mu\nu}) - \frac{1}{2} \sum_{\mu\nu} \sqrt{-g} \frac{\partial g_{\mu\nu}}{\partial x^\sigma} T^{\mu\nu} = 0,$$

where  $\sigma = 1, 2, 3, 4$ , and the semicolon stands for the covariant derivative. This equation resulted from a generalization of the corresponding conservation equations  $T^{\mu\nu}_{;\mu} = 0$  (the comma standing for the ordinary derivative) of *STR*. As to the physical significance of the terms of the above equations, Einstein considered the first sum to represent the derivatives of the energy-momentum distribution of matter and the second sum to express the action of the gravitational field on matter.<sup>45</sup> In September 1913, Einstein was to recast the above equation in the following form<sup>46</sup>

$$\frac{\partial}{\partial x^\nu} T'^{\nu\sigma} = \frac{1}{2} \left( \frac{\partial g_{\mu\nu}}{\partial x^\sigma} g^{\mu\tau} \right) T'^{\nu\tau},$$

44. Einstein/Grossmann, "Entwurf" (1913), pp. 230-31.

45. Ibid., p. 233.

46. Albert Einstein, "Zum gegenwärtigen Stande des Gravitationsproblems," *Physikalische Zeitschrift*, **14** (1913), 1249-66, on 1257.

where  $T_{\sigma}^{\nu} = \sqrt{-g}g_{\sigma\mu}T^{\mu\nu}$ .

As the right side represents the four-force of the gravitational field on matter, Einstein came to view the expression

$$\frac{1}{2} \left( \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}} g^{\mu\tau} \right)$$

as the "natural expression of the components of the gravitational field."<sup>47</sup> This interpretation was to hinder Einstein's successful use of the Riemann-Christoffel curvature tensor later on because it led him to express that tensor in terms of the above components, which greatly complicates the formulas.

### 4.2.2 The Linearly Covariant Field Equations

Once Einstein had developed the equation of motion, his next goal was to develop field equations determining the gravitational field  $g_{\mu\nu}$  in terms of a given energy-momentum tensor  $T^{\mu\nu}$  of matter. Here, however, the path was not as uniquely determined as for the equation of motion. We shall first present Einstein's assumptions before analyzing the development of the field equations.

Einstein first assumed that the field equations have the form  $\Gamma^{\mu\nu} = \chi T^{\mu\nu}$ , where  $\chi$  is a constant and  $\Gamma^{\mu\nu}$  a contravariant tensor of rank 2 obtained through differentiation of the fundamental tensor  $g_{\mu\nu}$ . By analogy with Poisson's equation  $\Delta\Phi = 4\pi k\rho$ , Einstein required the differential equation to be of second order. Since the Riemann-Christoffel curvature tensor, which was ultimately to lead to the final field equations, is precisely a differential tensor of second order, Einstein and Grossmann tried to make use of the latter. Yet, they thought that they had several reasons against its use. First, they thought that the curvature tensor did not reduce to  $\Delta\Phi$  for an infinitely weak static gravitational field.<sup>48</sup> Secondly, they believed that the conservation laws were not satisfied.<sup>49</sup> Finally, according to a student who visited Einstein in May 1917, Einstein was reluctant to use the curvature tensor because he could not discern in it an immediate physical meaning.<sup>50</sup> This relative lack of motivation resulted perhaps in a lack of perseverance which may have prevented them from discovering their errors. These errors were to cost Einstein three years of extremely hard work until he finally returned to the curvature tensor and made a successful use of it.

In the meantime, the alternatives Einstein and Grossmann saw were either that the differential equations were of a higher order or that the expression  $\Gamma^{\mu\nu}$  was a tensor only with respect to a restricted group of transformations.<sup>51</sup> Einstein rejected the first option because he considered it to be premature.

47. Albert Einstein, "Zur allgemeinen Relativitätstheorie," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften, Berlin*, part 2 (1915), pp. 777, 778-86, on p. 782.

48. Einstein/Grossmann, "Entwurf," pp. 233, 257; see also Einstein to Sommerfeld, 28 Nov. 1915, *Briefwechsel* (1968), p. 33; Einstein to Besso, 10 Dec. 1915, *Correspondance* (1972), p. 60.

49. Einstein à Besso, 10 Déc. 1915, *Correspondance* (1972), p. 60.

50. Seelig, *Einstein* (1960), p. 260.

51. Einstein/Grossmann, "Entwurf," pp. 233-34, 257.

Consequently, he saw himself obliged to opt for a restricted covariance of the field equations.<sup>52</sup> Despite the fact that he relinquished general covariance, Einstein still had some hope of achieving a general relativity. These two concepts were distinct in Einstein's mind : by general relativity, Einstein, as he was later to explain, understood a covariance of physical laws with respect to transformations corresponding to physical three dimensional relative motions of the reference system,<sup>53</sup> whereas by general covariance he meant a covariance with respect to arbitrary four-dimensional transformations. Though a general relativity was highly plausible to Einstein for epistemological reasons, he had no good justification for the requirement of general covariance except that it contained the general relativity. In the Entwurf paper, Einstein acknowledged this lack of support for general covariance : "It must be pointed out by the way that we do not have any criterion ['Anhaltspunkte'] whatever for a general covariance of the gravitational field equations."<sup>54</sup>

Having opted for a restricted covariance of the field equations, Einstein did not exactly know what covariance group he should adopt. Since *STR* postulates a covariance with respect to linear orthogonal transformations, Einstein required that the group of linear transformations be included in the covariance group.<sup>55</sup> To find a  $\Gamma^{\mu\nu}$  tensor with respect to linear transformations, Einstein first tried to use the relation  $div(grad\Phi) = \Delta\Phi$  as guiding principle. Both operations, the gradient and divergence can be extended to general tensors. Thus the gradient of a tensor  $T^{\mu\nu}$  corresponds to  $T^{\mu\nu}_{;\sigma}$  and the divergence to  $T^{\mu\nu}_{;\mu}$ . Einstein, initially, tried to apply these operations to the fundamental tensor  $g_{\mu\nu}$  but since  $g_{\mu\nu;\sigma} = 0$  vanishes identically, no tensor could be obtained. Einstein then attempted a similar approach but with tensors relative to the group of linear transformations. Proceeding this time with ordinary derivatives instead of covariant ones, he obtained the field equation

$$\Gamma^{\mu\nu} = \frac{\partial}{\partial x^\alpha} \left[ g^{\alpha\beta} \frac{\partial g^{\mu\nu}}{\partial x^\beta} \right] = \chi T^{\mu\nu},$$

and found that it yielded Newton's gravitational theory as first approximation.

Hence, it seemed that the above relation could be considered as a candidate for the field equation. Einstein remarked, however, that other tensors with respect to the linear group, such as

$$\frac{\partial g_{\alpha\beta}}{\partial x^\mu} \frac{\partial g^{\alpha\beta}}{\partial x^\nu}$$

might enter the equation since such tensors would vanish within the above approximation. In order to determine such terms and hence the field equations, Einstein made use of the momentum-energy conservation laws for matter plus

52. Ibid., p. 234.

53. Albert Einstein, "Die Grundlage der allgemeinen Relativitätstheorie," *Annalen der Physik*, **49** (1916), 769-822, on 772 and 776.

54. "Es ist übrigens hervorzuheben, dass wir keinerlei Anhaltspunkte für eine allgemeine Kovarianz der Gravitationsgleichungen haben," Einstein, "Entwurf," p. 234.

55. Ibid., p. 234.

the field. Einstein's idea was the following. Since he knew the expressions for the momentum and energy densities transferred from the gravitational field to matter, by writing out the conservation equations he would be able to determine the field equations. To illustrate his method, Einstein gave an electrostatic example. If  $\rho$  is the electric charge density, the momentum density transferred per unit time to the associated matter is

$$\frac{dp_\nu}{dt} = -\frac{\partial\Phi}{\partial x^\nu}\rho,$$

where  $\Phi$  is the electrostatic potential. A solution to the problem of finding a field equation satisfying the conservation of momentum is the equation  $\Delta\Phi = -\rho$ , since the relation

$$\frac{dp_\nu}{dt} = -\frac{\partial\Phi}{\partial x^\nu}\rho = \frac{\partial\Phi}{\partial x^\nu}\Delta\Phi \quad (\nu = 1, 2, 3)$$

can be rewritten in the form

$$\frac{\partial}{\partial x^\mu}(T^{\sigma\mu} + t^{\sigma\mu}) = 0 \quad (4.4)$$

by making use of the identity

$$\frac{\partial\Phi}{\partial x^\nu}\Delta\Phi = \sum_\mu \frac{\partial}{\partial x^\mu} \left( \frac{\partial\Phi}{\partial x^\nu} \frac{\partial\Phi}{\partial x^\mu} \right) - \frac{\partial}{\partial x^\nu} \left[ \frac{1}{2} \sum_\mu \left( \frac{\partial\Phi}{\partial x^\mu} \right)^2 \right]. \quad (4.5)$$

Equations (4.4), which contain the energy-momentum tensors  $T^{\sigma\mu}$  and  $t^{\sigma\mu}$ , pertaining to matter and the field respectively, are the conservation equations for momentum and energy. Furthermore, if any of the terms of equation (4.5) is known then the other terms can be found.

Einstein proceeded similarly with the problem of finding the field equations  $\Gamma^{\mu\nu} = \chi T^{\mu\nu}$ . He knew that the momentum and energy densities transferred from the gravitational field to matter were given by

$$\frac{1}{2}\sqrt{-g}\frac{\partial g_{\mu\nu}}{\partial x^\sigma}T^{\mu\nu},$$

or, equivalently,  $\frac{1}{2\chi}\sqrt{-g}\frac{\partial g_{\mu\nu}}{\partial x^\sigma}\Gamma^{\mu\nu}$ . Furthermore, from his previous findings, he assumed that  $\Gamma^{\mu\nu}$  contained the term

$$\frac{\partial}{\partial x^\alpha} \left( g^{\alpha\beta} \frac{\partial g^{\mu\nu}}{\partial x^\beta} \right),$$

along with other terms disappearing in first approximation. By rewriting the known term of  $\Gamma^{\mu\nu}$  in terms of differential quotients, Einstein and Grossmann were able to arrive at the identity

$$\frac{\partial}{\partial x^\nu} (\sqrt{-g}g_{\sigma\mu}\chi t^{\mu\nu}) = \frac{1}{2}\sqrt{-g}\frac{\partial g_{\mu\nu}}{\partial x^\sigma} (-\Delta^{\mu\nu}(g) + \chi t^{\mu\nu}). \quad (4.6)$$

where

$$-2\chi t^{\mu\nu} = \left( g^{\alpha\mu} g^{\beta\nu} \frac{\partial g_{\tau\rho}}{\partial x^\alpha} \frac{\partial g^{\tau\rho}}{\partial x^\beta} - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta} \frac{\partial g_{\tau\rho}}{\partial x^\alpha} \frac{\partial g^{\tau\rho}}{\partial x^\beta} \right)$$

(Einstein called  $t^{\mu\nu}$  the "contravariant stress-energy tensor of the gravitational field,"<sup>56</sup> and

$$\Delta^{\mu\nu}(g) = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\alpha} \left( g^{\alpha\beta} \sqrt{-g} \frac{\partial g^{\mu\nu}}{\partial x^\beta} \right) - g^{\alpha\beta} g_{\tau\rho} \frac{\partial g^{\mu\tau}}{\partial x^\alpha} \frac{\partial g^{\nu\rho}}{\partial x^\beta}.$$

By taking for  $\Gamma^{\mu\nu}$  the expression  $\Gamma^{\mu\nu} = \Delta^{\mu\nu}(g) - \chi t^{\mu\nu}$ , Einstein finally obtained the following gravitational field equations

$$\Delta^{\mu\nu}(g) = \chi (T^{\mu\nu} + t^{\mu\nu}).$$

By adding the equation of energy-momentum conservation of matter

$$\frac{\partial}{\partial x^\nu} (\sqrt{-g} g_{\sigma\mu} T^{\mu\nu}) - \frac{1}{2} \sqrt{-g} \frac{\partial g_{\mu\nu}}{\partial x^\sigma} T^{\mu\nu} = 0$$

to equation (4.6), Einstein obtained :

$$\frac{\partial}{\partial x^\nu} [\sqrt{-g} g_{\sigma\mu} (T^{\mu\nu} + t^{\mu\nu})] = 0 \quad (\sigma = 1, 2, 3, 4)$$

which showed that the laws of conservation of momentum-energy for matter and the field together are indeed satisfied. By this point, Einstein had developed field equations consistent with the conservation laws and could move on to the consequences of the theory.

### 4.2.3 Consequences of the Theory

In the Entwurf paper Einstein did not analyze any solution of the field equations, but simply illustrated his theory by giving the electromagnetic equations in a generally covariant form,<sup>57</sup> which Friedrich Kottler had already done before. Among the consequences of his theory that Einstein was to present in the next months, were the Newtonian gravitational field approximation, including the deviation of light rays and the red-shift, and the relativity of inertia.

#### The Newtonian gravitational field approximation

At the 85 Versammlung deutscher Naturforscher und Ärzte in Vienna (21–28 Sept. 1913), Einstein,<sup>58</sup> on 23 September 1913, derived the Newtonian gravitational field from his field equations as follows. IN STR, the  $g_{\mu\nu}$  have the values  $\eta_{\mu\nu}$  where

$$\eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

<sup>56</sup>. Ibid., p. 238

<sup>57</sup>. Ibid., pp. 240-42.

<sup>58</sup>. Einstein, "Zum Gegenwärtigen Stande des Gravitationsproblems" (1913).



If a weak gravitational field,  $g_{\mu\nu} = \eta_{\mu\nu} + g_{\mu\nu}^*$ , with  $|g_{\mu\nu}^*| \ll |\eta_{\mu\nu}|$ , is considered, the field equations become, to first approximation

$$\square g_{\mu\nu}^* = \chi T_{\mu\nu},$$

where  $T_{\mu\nu}$  is the energy-momentum tensor of the distribution of masses. To obtain the Newtonian gravitational theory, Einstein introduced several assumptions :

1. the velocity of the field-producing masses is neglected (i.e., the field is considered to be a static one) ;
2. in the equation of motion only first order terms relative to the velocity and acceleration of the material point need be taken into account ;
3. the  $g_{\mu\nu}^*$  vanish at infinity.

From these assumptions it follows that only the component  $T_{44} = \rho_0 c^2$ , ( $\rho_0$  being the rest density) is different from zero, which led Einstein to the following equations :

$$\begin{aligned} \Delta g_{\mu\nu}^* &= 0 \text{ (except for } \mu = \nu = 4), \\ \Delta g_{44}^* &= \chi c^2 \rho_0. \end{aligned}$$

The solution is straightforward :

$$\begin{aligned} g_{\mu\nu}^* &= 0 \text{ (except for } \mu = \nu = 4), \\ g_{44}^* &= \frac{\chi c^2}{4\pi} \int_{3 \text{ vol}} \frac{\rho_0 dV}{r} \end{aligned}$$

where  $r$  is the distance between  $dV$  and the field point. Since the equation of motion is given  $\delta \int H dt = 0$  where  $H = -m \frac{ds}{dt}$  and  $ds = dt \sqrt{g_{44} - v^2}$ , or equivalently,

$$\ddot{x} = -\frac{1}{2} \frac{\partial g_{44}^*}{\partial x},$$

Einstein concluded that this is the Newtonian equation of motion if the constant  $\chi$  is taken equal to  $\chi = \frac{8\pi K}{c^2}$  where  $K$  is the usual gravitational constant.

Within the same approximation, Einstein found for the line element

$$ds = \sqrt{-dx^2 - dy^2 - dz^2 + g_{44} dt^2},$$

with  $g_{44} = c^2 \left(1 - \frac{\chi}{4\pi} \int \frac{\rho_0 dV}{r}\right)$ . From the line element, Einstein concluded that the coordinate length ( $dt = 0$ ) is equal to the natural length but that the rate of clocks, which depends on the factor  $\frac{ds}{dt} = \sqrt{g_{44}}$  (with  $dx = dy = dz = 0$ ), is influenced by the gravitational potential. If masses are present in the neighborhood of a clock, it runs slower. Similarly, Einstein found that the velocity of light, which is

$$\left( \sqrt{\frac{dx^2 + dy^2 + dz^2}{dt^2}} \right)_{ds=0} = \sqrt{g_{44}},$$

depends on the gravitational potential but is independent of the direction of propagation. From this he concluded that light rays are curved in a gravitational field.

### The relativity of inertia

We have seen that Einstein had already examined the question of the variation of inertia in 1912 within the framework of his static theory of gravitation. In the first section of his Entwurf paper, in which he gave a Hamiltonian derivation of the equation of motion of the static theory, Einstein came back to the topic. Starting from the Lagrangian

$$H = -\frac{ds}{dt}m = -m\sqrt{c^2 - v^2},$$

Einstein derived the momentum and energy of a material particle :

$$p_x = \frac{\partial H}{\partial \dot{x}} = \frac{m\dot{x}}{\sqrt{c^2 - v^2}}, \text{ etc.},$$

$$E = \frac{\partial H}{\partial \vec{v}} \cdot \vec{v} - H = \frac{mv^2}{\sqrt{c^2 - v^2}} + m\sqrt{c^2 - v^2}.$$

Since for low velocities these formulas reduce to

$$p_x = \frac{m}{c}\dot{x}, \text{ etc.},$$

$$E - mc = \frac{1}{2} \frac{mv^2}{c},$$

it follows that both the momentum and the kinetic energy are proportional to the factor  $m/c$ , which corresponds to the inertial mass of the particle ( $m$  being the rest mass, which Einstein assumed to be independent of the gravitational potential). If masses are brought into the neighborhood of a material point,  $c$  diminishes and hence the inertia of the material point is increased. To this Einstein remarked : "This agrees with Mach's bold idea that inertia is due to an interaction of the material point considered, with all the others."<sup>59</sup>

To the question of whether the new theory yielded similar results, Einstein was to answer positively during a talk he gave at the annual meeting of the Schweizer Naturforschende Gesellschaft in Frauenfeld on 9 September 1913.<sup>60</sup> The address was entitled "Physikalische Grundlagen einer Gravitationstheorie" [Physical foundations of a gravitational theory], and presented the main results of the gravitational theory developed in the Entwurf paper. In the conclusion of the talk Einstein pointed out that the theory eliminated an "epistemological defect" stressed by Ernst Mach, namely, the concept of absolute acceleration,

59. "Es passt dies zu Machs kühnem Gedanken, dass die Trägheit in einer Wechselwirkung des betrachteten Massenpunktes mit allen übrigen ihren Ursprung habe," *ibid.*, p. 228.

60. Albert Einstein, "Physikalische Grundlagen einer Gravitationstheorie," *Naturforschende Gesellschaft, Zürich, Vierteljahrsschrift*, **58** (1913), 284-90, on p. 290.

by making inertia relative to the presence of other bodies.<sup>61</sup> More specifically, Einstein remarked that the rejection of the concept of absolute acceleration leads one to require what he called the "relativity of inertia,"<sup>62</sup> namely, that the inertia of a body  $A$  increase in the presence of other bodies  $B$  and that the increase vanish when the masses  $B$  and  $A$  undergo the same acceleration. Einstein stated, without proof, that his theory satisfied the relativity of inertia, and ended the talk by saying that "this circumstance constitutes one of the most important supports of the theory here outlined."<sup>63</sup>

Einstein was to come back at length to the relativity of inertia at the 85. Versammlung deutscher Naturforscher und Ärzte in Vienna on 23 September 1913. There, he was to support the claim made two weeks earlier that the relativity of inertia followed from his new theory. His approach was here again to start from the Lagrangian

$$H = -m \frac{ds}{dt} = -m \sqrt{g_{44} - v^2}$$

of a particle in a Newtonian gravitational field with

$$g_{44} = c^2 \left[ 1 - \frac{\chi}{4\pi} \int \frac{\rho_0 dv}{r} \right],$$

and to derive the momentum and energy of a slowly moving particle. From the latter expressions Einstein found that the inertial mass of a particle is given by

$$\frac{m}{c} \left[ 1 + \frac{\chi}{8\pi} \int \frac{\rho_0 dv}{r} \right]$$

and thus increases when masses are present in its neighborhood. Einstein described the result as being of "high theoretical interest"<sup>64</sup> since, according to him, the above increase of inertia made it plausible that the whole inertia of a material point was determined by other masses. Einstein then restated his conviction that the concept of absolute acceleration is an absurd notion since one can only speak of motion of a body relatively to other bodies; he referred the listener to Mach's *Mechanics* and added that, although he expected the relativity of inertia "a priori" he did not think that it was a "logical necessity"<sup>65</sup>

61. "Durch die skizzierte Theorie wird ein erkenntnistheoretischer Mangel beseitigt, der nicht nur der ursprünglichen Relativitätstheorie, sondern auch der Galilei'schen Mechanik anhaftet und insbesondere von E. Mach betont worden ist. Es ist einleuchtend, dass dem Begriff der Beschleunigung eines materiellen Punktes ebensowenig eine absolute Bedeutung zugeschrieben werden kann wie demjenigen der Geschwindigkeit. Beschleunigung kann nur definiert werden als Relativbeschleunigung eines Punktes gegenüber andern Körpern. Dieser Umstand lässt es als sinnlos erscheinen, einem Körper einen Widerstand gegen eine Beschleunigung schlechthin zuzuschreiben (Trägheitswiderstand der Körper im Sinne der klassischen Mechanik); es wird vielmehr gefordert werden müssen, dass das Auftreten eines Trägheitswiderstandes an die Relativbeschleunigung des betrachteten Körpers gegenüber andern Körpern geknüpft sei," *ibid.*, p. 290.

62. *Ibid.*

63. "Dieser Umstand bildet eine der wichtigsten Stützen der skizzierten Theorie," *ibid.*

64. Einstein, "Zum gegenwärtigen Stande des Gravitationsproblems," p. 1260.

65. *Ibid.*, p. 1261.

(i.e., scientific necessity), but stated that a theory incorporating the relativity of inertia was to be preferred to a theory introducing privileged inertial systems.

Einstein also showed that his theory not only yielded an increase of inertia of a particle  $A$  in the presence of other masses  $B$ ,  $C$ , but also the vanishing of the effect when both the particle  $A$  and the masses  $B$ ,  $C$  undergo the same acceleration. More specifically, Einstein showed that the acceleration of the masses  $B$ ,  $C$  induces on  $A$  a force which is in the same direction as their common acceleration. He also found that the plane of oscillation of a pendulum inside a rotating shell precesses in the same direction as the rotation.<sup>66</sup> In Einstein's view this effect also was to be expected from the relativity of inertia, and he found it very satisfying that the theory also agreed on this point, though he realized that the effect was too small to be observable either by terrestrial or astronomical means.

The importance which Einstein attached to the fulfillment of the relativity of inertia is clearly revealed in the conclusion of the address, where Einstein invoked the relativity of inertia as a decisive advantage of his theory over Nordström's rival theory.<sup>67</sup> Einstein was to repeat the argument at a conference held in Zürich on 9 February 1914.<sup>68</sup> Experimentally, Einstein expected the eclipse of August 1914 to decide between the two theories, since it provided a test for the deflection of light rays.

## 4.3 Further Elaboration of the Theory

### 4.3.1 The Problem of the Covariance of the Field Equation

Despite Einstein's initial desire to develop a generally covariant theory<sup>69</sup>, we have seen that Einstein and Grossmann had been unable to derive generally covariant field equations, in particular because they thought that the Riemann-Christoffel tensor, which was the most likely tool for such equations, did not yield the Newtonian approximation.<sup>70</sup> They knew that the field equations which they had developed were covariant with respect to linear transformations—they had built in at least that much—but they did not know how far the actual covariance group of the field equations might go beyond the linear group.<sup>71</sup> Einstein considered the question of the existence of a larger covariance group as "the most important"<sup>72</sup> one concerning the considerations given in the paper.

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66. *Ibid.*, pp. 1261-62.

67. *Ibid.*, p. 1262.

68. Albert Einstein, "Zur Theorie der Gravitation," *Naturforschende Gesellschaft, Zürich, Vierteljahrsschrift*, **59** (1914), 4-6, on 5-6.

69. For a detailed analysis from a contemporary philosophical point of view, see John Earman and Clark Glymour, "Lost in the Tensors: Einstein's Struggles with Covariance Principles 1912-1916," *Studies in History and Philosophy of Science*, **9** (1978), 251-78.

70. Einstein to Sommerfeld, 28 Nov. 1915, in Einstein/Sommerfeld, *Briefwechsel* (1968), p. 33; Einstein to Besso, 10 Dec. 1915, in Einstein/Besso, *Correspondance* (1972), p. 60.

71. Einstein, "Entwurf," p. 240.

72. *Ibid.*

From the fact that he had been able to formulate a generally covariant energy-momentum conservation equation for material systems, Einstein was inclined to postulate the general covariance of all physical equations except for those of gravitation. At first, he related the particular character of the latter to the fact that only the gravitational field equations are allowed to contain second derivatives of the fundamental tensor.<sup>73</sup> Searching for a deeper reason, Einstein came up shortly afterwards with what he thought to be a proof of the impossibility of generally covariant field equations. The proof was based on the requirement that the field be uniquely determined by the distribution of matter ( $T_{\mu\nu}$ ). Einstein probably developed the supposed proof in late August or early September 1913; he referred to it in a talk he gave on 9 September 1913,<sup>74</sup> and stated at the end of the month that he had developed that proof "in recent days."<sup>75</sup> The argument is given in full in the remarks Einstein added to the Entwurf paper and is as follows. Consider a space-time domain  $L$  such that the components of the energy-tensor  $T^{\mu\nu}$  vanish within  $L$  and have arbitrary values outside of  $L$ ; these components  $T^{\mu\nu}$  determine the  $g_{\mu\nu}$  everywhere. If a new coordinate system  $x'^{\mu}$  is introduced instead of the old one  $x^{\mu}$  such that  $x'^{\mu} = x^{\mu}$  outside of  $L$  and  $x'^{\mu} \neq x^{\mu}$  (for at least a part of  $L$  and for at least one index) then it follows that  $g'^{\mu\nu} \neq g^{\mu\nu}$  (at least for a part of  $L$ ). Since  $T'^{\mu\nu} = T^{\mu\nu} = 0$  (inside of  $L$ ) and  $T'^{\mu\nu} = T^{\mu\nu}$  (outside of  $L$  because  $x'^{\mu} = x^{\mu}$ ), Einstein concluded that if general covariance is postulated, a single distribution of matter could give rise to two different sets of  $g_{\mu\nu}$ . Hence the gravitational field is not uniquely determined and, according to Einstein, causality is violated.<sup>76</sup>

In his eagerness to explain his failure to develop generally covariant field equations, Einstein had simply misused the tensor calculus.<sup>77</sup> In a transformation of coordinates,  $g_{\mu\nu}$  is transformed into  $g'_{\mu\nu}$ , but it is still the same field (by definition of a tensor). Einstein, however, as well as the opponents to the theory, were to refer to this argument repeatedly<sup>78</sup> during the next year. In October 1914, Einstein gave a modified version of the argument.<sup>79</sup> He now correctly indicated that  $g_{\mu\nu}(x_k)$  and  $g'_{\mu\nu}(x'_k)$  represent the same gravitational field, but added that a new gravitational field  $g'_{\mu\nu}(x_k)$  can be obtained by replacing the  $x'_k$  by the  $x_k$  in  $g'_{\mu\nu}(x'_k)$ . If  $g_{\mu\nu}(x_k)$  is a solution of the field equations, so is  $g'_{\mu\nu}(x'_k)$ , as well as  $g'_{\mu\nu}(x_k)$ . Thus, Einstein concluded that two different

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73. Ibid.

74. Einstein, "Physikalische Grundlagen einer Gravitationstheorie" (1913), p. 289.

75. Einstein, "Zum gegenwärtigen Stande des Gravitationsproblems" (1913), p. 1257.

76. Einstein, "Entwurf," p. 260.

77. Earman/Glymour, "Lost in the Tensors"; Banesh Hoffmann, "Einstein and Tensors", *Tensor*, **26** (1972), 157-62.

78. Einstein, "Physikalische Grundlagen einer Gravitationstheorie" (1913), p. 289; "Zum gegenwärtigen Stande des Gravitationsproblems" (1913), p. 1257; "Prinzipielles zur verallgemeinerten Relativitätstheorie und Gravitationstheorie," *Physikalische Zeitschrift*, **15** (1914), 176-80, on 178; Albert Einstein and Marcel Grossmann, "Kovarianzeigenschaften der Feldgleichungen der auf die verallgemeinerte Relativitätstheorie gegründeten Gravitationstheorie," *Zeitschrift für Mathematik und Physik*, **63** (1914), 215-25, on 217-18.

79. Albert Einstein, "Die formale Grundlage der allgemeinen Relativitätstheorie," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften*, Berlin, part 2 (1914), pp. 1030-85, on p. 1067.

gravitational fields,  $g_{\mu\nu}(x_k)$  and  $g'_{\mu\nu}(x_k)$ , relative to the same coordinate system, with the same boundary conditions, are solutions of the same differential equations; hence, in his view the principle of causality was again violated. This argument is also wrong, as Einstein later realized : in a letter to Besso dated 3 January 1916, Einstein indicated that what was wrong with this new argument was that it is meaningless to conceive two solutions in the same manifold.<sup>80</sup>

In September 1913, Einstein presented another argument against general covariance. Assuming that the energy-momentum conservation laws for the energy-momentum distribution of matter  $T^\nu_\sigma$  together with that of the field  $t^\nu_\sigma$  are likely to have the form

$$\frac{\partial}{\partial x^\nu} (T^\nu_\sigma + t^\nu_\sigma) = 0,$$

and assuming  $t^\nu_\sigma$  to be a tensor, Einstein remarked that such equations are probably covariant only with respect to linear transformations of the coordinates.<sup>81</sup> Since the field equations are to be consistent with the conservation laws, Einstein concluded that the covariance of the field equations must probably be restricted to linear transformations. In the fall of 1913 and early 1914, Einstein, in his papers, presented this conclusion as a certitude.<sup>82</sup> Similarly, in a New Year's letter to Mach, probably written around New Year 1913/14, Einstein wrote :

To begin with, the events are described in terms of four entirely arbitrary space-time variables. The latter must then, if the conservation laws of momentum and energy are to be satisfied, be specialized in such a way that only (strictly [ganz]) linear transformations lead from one legitimate reference system to another.<sup>83</sup>

Later, Einstein was to realize that what was wrong with the above argument is that  $t^\nu_\sigma$  of the field is not necessarily a tensor.

Though Einstein's first arguments against general covariance did not exclude the possibility of a general relativity (i.e., a covariance with respect to transformations between arbitrary three-dimensional physical motions of the reference

80. Einstein à Besso, 3 janvier 1916, *Correspondance* (1972), p. 63.

81. Einstein, "Zum gegenwärtigen Stande des Gravitationsproblems" (1913), p. 1258.

82. Einstein, "Physikalische Grundlagen einer Gravitationstheorie" (1913), p. 289 ; "Zum gegenwärtigen Stande des Gravitationsproblems" (1913), p. 1258 ; "Entwurf" (1913), addendum, p. 260 ; "Prinzipielles" (1914), p. 178 ; see also a draft of a letter of Besso to Einstein, 20 March 1914, *Correspondance* (Paris, 1972), p. 55.

83. "Das Geschehen wird zunächst auf vier ganz willkürliche raum-zeitliche Variable bezogen. Diese müssen dann, wenn den Erhaltungssätzen des Impulses und der Energie Genüge geleistet werden soll, derart spezialisiert werden, dass nur (ganz) lineare Substitutionen von einem berechtigten Bezugssystem zu einem andern führen," Einstein to Mach, New Year's letter, in Friedrich Herneck, "Die Beziehungen zwischen Einstein und Mach dokumentarisch dargestellt," *Wissenschaftliche Zeitschrift der Friedrich-Schiller-Universität Jena*, **15** (1966), 1-14, on 8 ; Herneck (p. 8) gives New Year 1912/13 as the probable date of the letter ; see also Friedrich Herneck, "Zum Briefwechsel Albert Einsteins mit Ernst Mach," *Forschungen und Fortschritte*, **37** (1963), 239-43, on 241-42. To me, the date 1913/14 seems more probable because Einstein speaks of his certitude of a specialization to linear transformations ; Einstein had this certitude around 1913/14. See also John T. Blackmore, *Ernst Mach* (Berkeley : University of California Press, 1972), p. 255.

systems), this last argument seemed definitely to preclude that possibility since the restriction to linear transformations would have excluded acceleration transformations. In fact, according to Einstein, some of his colleagues were already disposed to reject the theory because of the failure to come up with generally covariant field equations : "and this is the reason why the colleagues believe it necessary to strangle our theory."<sup>84</sup> Einstein, however, was not disposed to give up, and in January 1914 interpreted the rather difficult situation as follows. He argued that in principle there still ought to exist generally covariant gravitational field equations corresponding to his own field equations, if the theory was to have a physical content, but thought that for various reasons (unique determination of the  $g_{\mu\nu}$ , and the conservation laws) the covariance of the field equations became restricted to linear transformations.<sup>85</sup> In his New Year letter to Mach, Einstein expressed a similar idea : "With the aid of the energy conservation law, the reference system is so to speak adapted to the existing world and loses its nebulous, a priori existence."<sup>86</sup> Thus, Einstein thought that physical reasons in particular restricted the covariance group a posteriori. Although Einstein, throughout 1914 and most of 1915, was to remain convinced that the conservation laws restrict the covariance group, he was soon to remove the restriction to linear transformations, which despite his somewhat wishful thinking about the existence of generally covariant field equations would have meant a serious blow to his program of general relativity.

### 4.3.2 New Hope for an Extended Covariance the Field Equations

By March 1914, before he was to leave Zurich for Berlin at the end of the month, Einstein thought he could extend the covariance group of the field equations. In a letter to Besso, dating from the beginning of March 1914, Einstein indicated that, after "horrible" efforts, he had finally arrived at the following straightforward results. From the gravitational field equations

$$\left( \sqrt{-g} g^{\alpha\beta} g_{\sigma\mu} \frac{\partial g^{\mu\nu}}{\partial x^\beta} \right)_{,\alpha} = k (T_\sigma^\nu + t_\sigma^\nu),$$

and the conservation laws,

$$(T_\sigma^\nu + t_\sigma^\nu)_{,\nu} = 0,$$

Einstein derived the following condition :

$$\left( \sqrt{-g} g^{\alpha\beta} g_{\sigma\mu} \frac{\partial g^{\mu\nu}}{\partial x^\beta} \right)_{,\alpha,\nu} = 0 \quad (4.7)$$

84. "und dieser Umstand ist es, aus dem die Fachkollegen unserer Theorie den verhängnisvollen Strick drehen zu sollen glauben," Einstein, "Prinzipielles" (1914), p. 177.

85. Ibid., p. 178.

86. "Das Bezugssystem ist der bestehenden Welt mit Hilfe des Energiesatzes sozusagen angemessen und verliert seine nebulöse apriorische Existenz," Herneck, "Beziehungen zwischen Einstein und Mach" (1966), p. 8.

which he now interpreted as a condition restricting the choice of the reference system. Einstein then stated that he had proved that the field equations are valid for any reference system satisfying the above condition and concluded :

From this it follows that there are acceleration transformations of the most various kind, which transform the equations into themselves (for example also rotations), so that the equivalence hypothesis holds in its original form, indeed to a greater extent indeed than was expected.<sup>87</sup>

Thus, by now, Einstein believed that acceleration transformations were included in the covariance group of the field equations.

Einstein was to present his new finding in the second and last paper he wrote together with Grossmann (the collaboration being interrupted by Einstein's move to Berlin and the war).<sup>88</sup> In the introduction to the paper, Einstein stated two reasons why the question of the existence of an extended covariance of the field equations was important to him : first, the answer would decide to what extent the basic idea of the special theory of relativity could be generalized and thus would have "great significance for the doctrine [Lehre] of space and time"<sup>89</sup> ; and second, the answer would allow one to evaluate the physical content of the theory. With respect to the second point, Einstein explained that the equivalence principle is particularly convincing only if an acceleration field can be interpreted as a "real" gravitational field, i.e., if acceleration transformations, which are nonlinear, are allowed among the legitimate transformations. Consequently, Einstein was anxious to see at least acceleration transformations included in the covariance group of the field equations. In fact, he thought that this was now the case. As in the letter to Besso, Einstein stated that acceleration transformations of "various kinds"<sup>90</sup> were now included in the covariance group, although he did not specify which ones. In a footnote,<sup>91</sup> Einstein withdrew the argument he had given earlier about a restriction to linear transformations because of the conservation laws and correctly stated that the energy-momentum distribution of the field  $t_{\sigma}^{\nu}$  need not be a generally covariant tensor.

In the body of the paper, Einstein and Grossmann established the covariance of the field equations with respect to transformations between coordinate systems satisfying condition (4.7). They called such coordinate systems adapted ("angepasst") coordinate systems and any transformation between them a legiti-

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87. "Hieraus geht hervor, dass es Beschleunigungstransformationen mannigfaltigster Art gibt, welche die Gleichungen in sich selbst transformieren (z.B. auch Rotation), sodass die Aequivalenzhypothese in ihrer ursprünglichsten Form gewahrt ist, sogar in ungeahnt weitgehendem Masse," Einstein to Besso, March 1914, in *Correspondance* (1972), p. 53, see also comments, *ibid.*, p. 55.

88. Einstein et Grossmann, "Kovarianzeigenschaften" (1914), 215-25. From the notation used in the paper and for other logical reasons, it appears that this paper was written before Einstein's "Formale Grundlage" (1914).

89. Einstein/Grossmann, "Kovarianzeigenschaften," p. 215.

90. *Ibid.*, p. 216.

91. *Ibid.*, p. 218.



mate ("berechtigt") transformation.<sup>92</sup> In order to establish the covariance of the field equations with respect to legitimate transformations, Einstein and Grossmann made use of a Hamiltonian formulation of the gravitational field equations and established the covariance of the action integral. In the conclusion Einstein and Grossmann admitted that the above considerations still did not make the meaning of the adapted coordinate systems and the legitimate transformations entirely clear but stated that the theory became more convincing through this extended covariance. In fact, the extended covariance seems to have been largely illusory since, in 1915, Einstein was to reject the field equations in part because the gravitational field in a uniformly rotating system did not satisfy the field equations.

So far Einstein and Grossmann had shown that the field equations could be derived from a certain Lagrangian and that the latter was covariant with respect to legitimate transformations. Consequently, the field equations possessed the same covariance. On 29 October 1914, Einstein thought that he could derive the Lagrangian itself in a natural way and thus provide a better derivation of his 1915 field equations.<sup>93</sup> After an elaborate demonstration, Einstein thought that he had finally achieved that goal. Thus, it appeared that after two years of hard efforts Einstein had at last reached a satisfactory theory. A year later, however, Einstein was to abandon the theory altogether when he found unsatisfactory consequences of the field equations and realized that his derivation of the Lagrangian had been defective.

## 4.4 Einstein's Philosophical Views and the Reception of the Theory

Though we have dealt so far essentially with the mathematical development of Einstein's theory, Einstein's efforts must be seen within his overall epistemological position. The latter also explains to a certain extent the rather negative reception of the theory. We shall examine these two issues successively.

### 4.4.1 Einstein's Epistemological Idealism

Around 1913, Einstein was still predominantly an epistemological idealist. There is various evidence for this. Thus, in 1912, Einstein signed a manifesto calling for the formation of an association with the goal of furthering positivistic (i.e., epistemological idealistic) theories. The manifesto, which was signed among others, by Föppl, Freud, Hilbert, Klein, Mach, Popper, von Seeliger, and Petzoldt, reads in part as follows :

There has long been felt the need of a philosophy which should grow in a natural manner out of the facts and problems of natural science. The mechanical view of nature no longer satisfies this need...

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<sup>92</sup>. Ibid., p. 221.

<sup>93</sup>. Einstein, "Formale Grundlage" (1914).

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...there has grown up from the soil of natural science itself a strictly empirical and positivistic point of view quite indifferent to metaphysical speculation and to so-called critical, transcendental doctrines. Its principles are however not yet accepted in their essential meanings and systematic relations throughout considerable scientific circles. ...

In the theory of relativity it [physics] touches the most searching question thus far of epistemology : Is absolute or is only relative knowledge attainable? Indeed : Is absolute knowledge conceivable? It comes here directly upon the question of man's place in the world, the question of the connection of thought with the brain. What is thought? What are concepts? What are laws? .....

Those who take interest in these progressive inquiries will find it to their advantage to have a scientific association which shall declare itself opposed to all metaphysical undertakings, and have for its first principle the strictest and most comprehensive ascertainment of facts in all fields of research and in the development of organization and technique. All theories and requirements are to rest exclusively on this ground of facts and find here their ultimate criterion.<sup>94</sup>

The manifesto clearly reveals a strongly antimetaphysical attitude which was the source of Einstein's rejection of absolute quantities such as the concept of absolute space.

Further evidence for Einstein's epistemological idealism around 1913 is provided by the Einstein-Mach correspondence. Einstein had sent the Entwurf paper to Mach and on 25 June 1913 wrote to the latter that the solar eclipse of 1914 was to provide a test for the equivalence principle by showing whether or not light rays are curved near the sun. In the case of a positive answer, Einstein remarked,

Then your inspired investigations into the foundations of mechanics - - despite Planck's unjust criticisms- -will receive a splendid confirmation. For it is a necessary consequence that inertia has its origin in a kind of mutual interaction of bodies, fully in the sense of your critique of Newton's bucket experiment.<sup>95</sup>

Einstein supported his assertion by referring Mach to the Entwurf paper and by mentioning that from the theory it followed furthermore that : (a) an accelerated shell induces a force on a particle inside ; (b) a rotating shell (with respect to the fixed stars) induces a Coriolis-field inside the shell. In conclusion,

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94. "Notes and News," *The Journal of Philosophy, Psychology, and Scientific Methods*, 9 (1912), 419-20.

95. "Wenn ja, so erfahren Ihre genialen Untersuchungen über die Grundlagen der Mechanik-Plancks ungerechtfertigter Kritik zum Trotz- -eine glänzende Bestätigung. Denn es ergibt sich mit Notwendigkeit, dass die Trägheit in einer Art Wechselwirkung der Körper ihren Ursprung hat, ganz im Sinne Ihrer Überlegungen zum Newtonschen Eimerversuch," Herneck, "Beziehungen zwischen Einstein und Mach" (1966), p. 9 ; trans. from Blackmore, *Ernst Mach* (1972), p. 254.

Einstein wrote, "It is a great joy for me to be able to communicate this to you since Planck's criticism has always appeared to me as most unjustified."<sup>96</sup> The letter clearly expresses Einstein's joy to see that his theory supported Mach's ideas in that it entailed the relativity of inertia, at least to a certain extent. It also shows that Einstein sided with Mach in the Mach-Planck debate, to which we shall return.

In another undated New Year Letter to Mach, which, for the reasons given above, was probably written around 1913/14, Einstein wrote :

I am very glad about the friendly interest you are showing for the new theory. The mathematical difficulties which one encounters in the development of these ideas are unhappily also very great for me. I am extremely pleased that in the development of the theory the depth and importance of your studies on the foundations of classical mechanics becomes manifest. Even now, I don't understand how Planck, for whom I have otherwise a unique esteem, could bring so little understanding to your efforts. His stand on my new theory [of general relativity] is also one of refusal by the way.

I can't blame him for that. Indeed, that epistemological argument is, so far, the only thing which I can bring forward in favor of my theory. For me it is absurd to attribute physical properties to "space." The totality of masses creates a  $G_{\mu\nu}$ -field [ $g_{\mu\nu}$  field] (gravitational field) which on its part governs the evolution of all processes including the propagation of light rays and the behavior of rods and clocks.<sup>97</sup>

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96. "Es ist mir eine grosse Freude, Ihnen dies mitteilen zu können, zumal jene Kritik Plancks mir schon immer höchst ungerechtfertigt erschienen war," Herneck, "Beziehungen zwischen Einstein und Mach" (1966), p. 9.

97.

"Ich freue mich sehr über das freundliche Interesse, das Sie der neuen Theorie entgegenbringen. Die mathematischen Schwierigkeiten, auf die man bei Verfolgung dieser Gedanken stösst, sind leider auch für mich sehr grosse. Es freut mich ausserordentlich, dass bei der Entwicklung der Theorie die Tiefe und Wichtigkeit Ihrer Untersuchungen über das Fundament der klassischen Mechanik offenkundig wird. Ich kann heute noch nicht begreifen, wie Planck, den ich sonst wie kaum einen zweiten hochschätzen gelernt habe, Ihren Bestrebungen so wenig Verständnis entgegenbringen konnte. Er steht übrigens auch meiner neuen Theorie ablehnend gegenüber.

Ich kann ihm das nicht verargen. Denn bis jetzt ist jenes erkenntnistheoretische Argument das Einzige, was ich zugunsten meiner neuen Theorie vorbringen kann. Für mich ist es absurd, dem 'Raum' physikalische Eigenschaften zuzuschreiben. Die Gesamtheit der Massen erzeugt ein  $G_{\mu\nu}$ -Feld (Gravitationsfeld), das seinerseits den Ablauf aller Vorgänge, auch die Ausbreitung der Lichtstrahlen und das Verhalten der Massstäbe und Uhren regiert," *ibid.*, p. 8.

If the dating of the letter is correct, then it seems that Mach was still expressing a friendly attitude toward the Entwurf theory in his correspondence with Einstein, even after Mach had declined, in July 1913, to adopt STR for himself in the preface of his book : Ernst Mach, *Die Prinzipien der physikalischen Optik, Historisch und erkenntnispsychologisch entwickelt* (Leipzig : J. A. Barth, 1921). The content of the preface became known to Einstein only after the publication of the book in 1921.

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The letter reveals that Mach was interested in the Entwurf theory, and more important, that Einstein considered the epistemological rejection of the concept of absolute space to be the main argument he could advance in favor of his theory. For Einstein it was "absurd" to attribute physical properties to space itself. We shall see that this epistemological belief was to lead Einstein, in 1917, to introduce the cosmological term into his field equations. It was the same philosophical requirement that underlay Einstein's conception of the relativity of inertia, according to which a body ought not to have inertia on its own but only relatively to other bodies. The rejection of absolute properties also facilitated Einstein's giving up of the principle of the constancy of the velocity of light. He wrote: "It seems to me unbelievable that the evolution of any process (for example, that of the propagation of light) might be construed as being independent of all other events in the world."<sup>98</sup> Einstein was to repeat his rejection of absolute properties of space (i.e., absolute acceleration) over and over again in his scientific publications.<sup>99</sup>

Einstein's New Year's letter to Mach also indicates that Planck was against Einstein's Entwurf theory. Planck's opposition must be viewed within the context of the Mach-Planck debate. The heart of the debate concerned the usefulness of metaphysics in physics. Initially, Planck had been an enthusiastic follower of Mach, but later on he changed his mind because, in contradistinction to Mach who rejected any metaphysics (whether materialistic or idealistic) in science, Planck came to the view that, ultimately, physics cannot dispense with the metaphysical idea of an objective reality existing independently of the observer.<sup>100</sup> In 1908, Planck was to defend this point of view in opposition to the Machian line of thought, going so far as to imply that Mach, who was 70 years old and half paralyzed, was a "false prophet."<sup>101</sup> Mach<sup>102</sup> answered with dignity and reserve and did not further respond to Planck's reply.<sup>103</sup> In the latter reply,

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98. "Es erscheint mir unglaublich, dass der Ablauf irgendeines Vorganges (z.B. der der Lichtausbreitung im Vakuum) als unabhängig von allem übrigen Geschehen in der Welt aufgefasst werden könne," Einstein, "Prinzipielles" (1914), p. 176.

99. During the period 1913-15, such statements can be found in Einstein, "Physikalische Grundlagen einer Gravitationstheorie" (1913), p. 290; "Zum gegenwärtigen Stande des Gravitationsproblems" (1913), p. 1255; "Prinzipielles" (1914), p. 176; "Zur Theorie der Gravitation" (1914), p. v; "Formale Grundlage" (1914), pp. 1031-32.

100. "Zählte ich mich doch in meiner Kieler Zeit (1885-1889) zu den entschiedenem Anhängern der Machschen Philosophie, die, wie ich gerne anerkenne, eine starke Wirkung auf mein physikalisches Denken ausgeübt hat. Aber ich habe mich später von ihr abgewendet, hauptsächlich, weil ich zur der Ansicht gelangte, dass die Machsche Naturphilosophie ihr glänzendes Versprechen, das ihr wohl die meisten Anhänger zugeführt hat: Die Eliminierung aller metaphysischen Elemente aus der physikalischen Erkenntnislehre, keineswegs einzulösen vermag," Max Planck, "Zur Machschen Theorie der physikalischen Erkenntnis," *Physikalische Zeitschrift*, **11** (1910), 1186-90, on 1187.

101. M. Planck, "Die Einheit des physikalischen Weltbildes," *Physikalische Zeitschrift*, **10** (1909), 62-75; Vortrag gehalten am 9 Dezember 1908 in der naturwissenschaftlichen Fakultät des Studentenkorps an der Universität Leiden, in Max Planck, *Physikalische Abhandlungen und Vorträge* (Braunschweig, 1958), **3**, 29.

102. Ernst Mach, "Die Leigedanken meiner naturwissenschaftlichen Erkenntnislehre und ihre Aufnahme durch die Zeitgenossen," *Physikalische Zeitschrift*, **11** (1910), 599-606.

103. Planck, "Zur Machschen Theorie der physikalischen Erkenntnis," (1910).

Planck, in particular, attacked Mach's idea of a relativity of all rotational motions as being "physically totally useless."<sup>104</sup> He argued that such a view would go against the result that an angular velocity of an infinitely distant body (with a rotation axis at a finite distance) cannot be finite (Planck apparently assumed a finite velocity of the body) and would question the great progress achieved by the Copernican world view. Since Einstein's theory tried very precisely to implement Mach's idea of a general relativity, one can easily see why Planck, initially, was strongly against it.

The difference between Mach's and Planck's philosophical positions was that existing between epistemological idealism and metaphysical idealism. Whereas Mach had restricted himself (at least in public) to an epistemological idealism, Planck had moved on to a metaphysical idealism which explains his concept of reality. Planck had grown up in a religious family and had distinguished professors of theology among his ancestors (grandfather and great-grandfather).<sup>105</sup> In letters to the religion historian Alfred Bertholet, Planck acknowledged that since his childhood he had maintained a strong belief in an almighty and benevolent God.<sup>106</sup> It appears that since the world for Planck was a creation of God and since God has an objective existence beyond the perception of any human being it was natural for Planck to believe that the world too had objective features which existed independently of the perceptions of any human being. For Planck, indeed, such objective features and more specifically, the constant pattern behind the diversity of phenomena constituted his concept of reality.<sup>107</sup> In fact, in his later years, Planck came to associate this constant world pattern with God himself and moved toward a pantheistic world view in the sense of Spinoza or Goethe.<sup>108</sup> Thus Planck wrote: "Hence, nothing prevents us, and our desire for a unified worldview requires it, to identify the two universally effective and yet mysterious forces, the world order of science and the God of religion."<sup>109</sup>

104. Ibid., p. 1189.

105. Hans Hartmann, *Max Planck : als Mensch und Denker* (Berlin : Verlag Karl Siegmund, 1938), p. 13-14.

106. Alfred Bertholet, "Erinnerungen an Max Planck," *Physikalische Blätter*, **4** (1948), 161-80 on 162.

107. "Das konstante einheitliche Weltbild ist aber gerade, wie ich zu zeigen versucht habe, das feste Ziel, dem sich die wirkliche Naturwissenschaft in allen ihren Wandlungen fortwährend annähert, ... Dieses Konstante, von jeder menschlichen, überhaupt jeder intellektuellen Individualität Unabhängige ist nun eben das, was wir das Reale nennen," Planck "Die Einheit des physikalischen Weltbildes," in Planck, *Abhandlungen*, **3**, 27.

108. Friedrich Herneck, "Ein Brief Max Plancks über sein Verhältnis zum Gottesglauben," *Forschungen und Fortschritte*, **32** (1958), 364-66, on 366; "Bemerkungen zur Religiosität Max Planck," *Physikalische Blätter*, **16** (1960), 382-84, on 384; Lise Meitner, "Max Planck als Mensch," *Die Naturwissenschaften*, **45** (1958), 406-08, on 408.

109. "Nichts hindert uns also, und unser nach einer einheitlichen Weltanschauung verlangender Erkenntnistrieb fordert es, die beiden überall wirksamen und doch geheimnisvollen Mächte, die Weltordnung der Naturwissenschaft und der Gott der Religion, miteinander zu identifizieren. Danach ist die Gottheit, die der religiöse Mensch mit seinen anschaulichen Symbolen sich nahezubringen sucht, wesensgleich mit der naturgesetzlichen Macht, von der dem forschenden Menschen die Sinnesempfindungen bis zu einem gewissen Grade Kunde geben," Max Planck, "Religion und Naturwissenschaft," Vortrag gehalten im Baltikum, Mai 1937, in Max Planck, *Vorträge und Erinnerungen*, 5th enlarged ed. of *Wege zur Physikalischen Erkenntnis* (Stuttgart : S. Hirzel Verlag, 1949), p. 331.

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In parallel with Planck's philosophical evolution from epistemological idealism to metaphysical idealism, Einstein was to move toward a metaphysical idealism in his later years. Though Einstein was not religious in the sense of practising religious rituals, Einstein throughout his life maintained a strong belief that the world was the work of a higher mind.<sup>110</sup> Already in his childhood, Einstein associated the harmony of the world with God and thus developed religious views which were in affinity with Spinoza's pantheism.<sup>111</sup> This is confirmed by Einstein's later statement: "I believe in the God of Spinoza, who reveals himself in the harmony of the world," and further, "The spinozistic conception has always been close to me and I have always admired this man and his teaching."<sup>112</sup> As pantheism identifies God and nature, it follows that if God is considered to be mind-like, then nature is mind-like too. That Einstein considered God to be mind-like follows explicitly from his statement dating from about 1927: "My religiosity consists in a humble admiration of the infinitely superior spirit that reveals itself in the little that we, with our weak and transitory understanding, can comprehend of reality."<sup>113</sup> Consequently, Einstein's philosophical position was that of a metaphysical idealism in which reality is not only mind-like but closely related to God himself.

As for Planck, Einstein's metaphysical idealism was the source of his belief in the independent existence of the world with respect to the human mind. Since God was closely associated with the universe, there existed basic objective principles which it was the task of the scientist to find. For Einstein, finding these principles was similar to guessing God's thoughts about the universe. This characterization is confirmed by Max Born who, referring to Einstein, wrote: "He believed in the power of reason to guess the laws according to which God has built the world."<sup>114</sup> Similarly, Arnold Sommerfeld, another close friend of Einstein, wrote: "Many a time, when a new theory appeared to him arbitrary or forced, he remarked: 'God doesn't do anything like that.' I have often felt and occasionally also stated that Einstein stands in particularly intimate relation to the God of Spinoza."<sup>115</sup> One might perhaps think that such statements apply only to Einstein in his later years. In fact, there is evidence that Einstein practiced that kind of guessing throughout his career. Thus, in 1905, in a letter

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110. Carl Seelig, *Albert Einstein* (Zürich: Europa Verlag, 1960), p. 111.

111. Anton Reiser, *Albert Einstein: A Biographical Portrait* (New York: Albert & Charles Boni, 1930), pp. 28-30; see also Einstein, "Autobiographical Notes," pp. 3-5.

112. "Ich glaube an Spinozas Gott, der sich in der Harmonie des Seienden offenbart"; "Die spinozistische Auffassung ist mir immer nahe gewesen und ich habe diesen Mann und seine Lehre stets bewundert," quoted in Seelig, *Albert Einstein* (1960), p. 258.

113. "Meine Religiosität besteht in einer demütigen Bewunderung des unendlich Überlegenen Geistes, der sich in dem Wenigen offenbart, was wir mit unserer schwachen und hilfälligen Vernunft von der Wirklichkeit zu erkennen vermögen," trans. and German original in Helen Dukas and Banesh Hoffmann, *Albert Einstein, the Human Side* (Princeton, NJ: Princeton University Press, 1979), p. 66.

114. Max Born, "Physics and Relativity," a lecture given at the International Relativity Conference in Berne on July 16, 1955, in Max Born, *Physics in My Generation*, 2nd revised ed. (New York: Springer, 1969), p. 114.

115. Arnold Sommerfeld, "To Albert Einstein's Seventieth Birthday," in Schilpp, *Albert Einstein*, 1, 103.

to Habicht Einstein wrote : "The consideration is amusing and fascinating ; but whether the Lord laughs at it and has had me, I cannot know." <sup>116</sup>

Einstein's belief in the guessing of the first principles of a theory explains why he did not think that there was a systematic way to find the first principles of a physical theory. Thus, on 2 July 1914, in his inaugural address to the Berlin Academy, after describing the method of the theoretician as consisting of (a) a search for the first principles, and (b) the development of the consequences of the principles, Einstein contrasted the first activity to the second as follows :

The first of the mentioned tasks, namely, the one consisting in the formulation of the principles, which are to form the basis for deduction, is of an entirely different kind. Here, there is no learnable, systematic method which can be used to attain the goal. Rather, in order to obtain those general principles, the scientist must so to speak listen in on nature and detect certain general features, suitable for a sharp formulation, in larger complexes of experimental facts. <sup>117</sup>

The passage shows that the discovery of the principles was for Einstein essentially an intuitive, unsystematic, personal process consisting in a sympathetic listening to nature. Four years later, Einstein was to stress the same point : "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built by pure deduction. There is no logical path to these laws ; only intuition, resting on sympathetic understanding of experience, can reach them." <sup>118</sup> In his 1914 inaugural address to the Berlin Academy, Einstein supported his views on theoretical creativity by referring to the contemporary example of thermal radiation. Though the experimental data was there and the precise law of thermal radiation was known, the principles of a new "mechanics" were still missing despite all efforts and were to remain so for another decade. On the other hand, Einstein presented the general theory of relativity as a case in which a clearly formulated principle, i.e., the general principle of relativity existed without the experimental data being there to check its consequences. Einstein's main justification of the general principle of relativity was that it eliminated what he perceived to be a deficiency of *STR*, namely the fact that it privileged uniform motions over accelerated ones. <sup>119</sup>

Einstein was to come back to the epistemological considerations which had

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116. "Die Überlegung ist lustig und bestechend ; aber ob der Hergott nicht darüber lacht und mich an der Nase herumgeführt hat, das kann ich nicht wissen," Einstein à Conrad Habicht, no date, in Seelig, *Albert Einstein* (1960), p. 126.

117. "Die erste der genannten Aufgaben, nämlich jene, die Prinzipie aufzustellen, welche der Deduktion als Basis dienen sollen, ist von ganz anderer Art. Hier gibt es keine erlernbare, systematisch anwendbare Methode, die zum Ziele führt. Der Forscher muss vielmehr der Natur jene allgemeinen Prinzipie gleichsam ablauschen, indem er an grösseren Komplexen von Erfahrungstatsachen gewisse allgemeine Züge erschaut, die sich scharf formulieren lassen," Albert Einstein, "Antrittsrede," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften, Berlin*, par 2 (1914), pp. 739-42, on p. 740.

118. Albert Einstein, "Principles of Research," address delivered at a celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin, in Albert Einstein, *Ideas and Opinions* (1954; rpt., New York : Dell, 1976), p. 221.

119. Einstein, "Antrittsrede" (1914), pp. 741-42.

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led him to develop the general theory of relativity in his first scientific paper to the Berlin Academy.<sup>120</sup> By now, as a prominent member of the Academy, Einstein felt free to discuss such epistemological issues at length at the very beginning of his papers. Einstein justified a general theory of relativity as follows : since all systems of reference are kinematically equivalent, it seems unjustified to privilege inertial systems over other ones ; therefore, a generalization of *STR* is required. Newton's argument about centrifugal forces, however, seems to deny such a possibility since it shows that kinematically equivalent systems are not necessarily dynamically equivalent, to which Einstein replied :

This argument however—as Mach in particular has shown—is not valid. We do not have to attribute the existence of centrifugal forces necessarily to a motion of  $K'$  [uniformly rotating system] ; we can rather attribute them as well to the average relative rotation of the surrounding distant ponderable masses with respect to  $K'$ , and treat thereby  $K'$  as being 'at rest'.<sup>121</sup>

Einstein remarked that the fact that Newton's laws of mechanics and gravitation do not allow such an interpretation of centrifugal forces in terms of Mach's hypothesis might well be due to deficiencies of Newton's theory. Einstein saw an important argument in favor of rotational relativity in the fact that there is no way of distinguishing a centrifugal field from a gravitational field, since centrifugal and gravitational forces are both proportional to the same mass constant. Thus a centrifugal field in a rotating system can be viewed via Mach's hypothesis as a physical (gravitational) field in the same system, considered to be at rest. Einstein remarked that there is a parallel between this case and the situation in *STR*, where the magnetic force ( $q\vec{v} \wedge \vec{B}$ ) acting on an electric charge moving in a magnetic field can be viewed as an electric force  $q\vec{E}$  in the rest frame of the particle<sup>122</sup> (the magnetic and electric forces corresponding respectively to the centrifugal and gravitational forces). The above considerations make it plain that Einstein's main motivation in developing his theory was to eliminate the concept of absolute space. At the end of the paper Einstein referred again to the relativity of inertia as being completely in accord with the "spirit" (*Geist*) of his theory, according to which space cannot have physical properties of its own.<sup>123</sup>

To sum up : Einstein around 1913 was still essentially an epistemological idealist wanting to eliminate absolute properties from physics because of their metaphysical nature. The success of *STR* had indeed proven the fruitfulness of such an epistemological position. Yet, Einstein's epistemological idealism was

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120. Einstein, "Formale Grundlage" (1914).

121. "Dies Argument ist aber- -wie insbesondere E. Mach ausgeführt hat- -nicht stichhaltig. Die Existenz jener Zentrifugalkräfte brauchen wir nämlich nicht notwendig auf eine Bewegung von  $K'$  [réceptien en rotation uniforme] zurückzuführen ; wir können sie vielmehr ebensogut zurückführen auf die durchschnittliche Rotationsbewegung der ponderablen fernen Massen der Umgebung in bezug auf  $K'$ , wobei wir  $K'$  als 'ruhend' behandeln," *ibid.*, p. 1031.

122. *Ibid.*, p. 1032.

123. *Ibid.*, p. 1085.



only a tool within a broader world view, based on his belief in God, which was to evolve more and more explicitly toward a metaphysical idealism.<sup>124</sup>

#### 4.4.2 Reception of the Theory

Though the reception of Einstein's theory is somewhat beyond the scope of this dissertation and has been treated to some extent elsewhere,<sup>125</sup> we shall nevertheless say a few words about it in order to show that Einstein's Entwurf theory was, on the whole, rather negatively received by his colleagues, in particular by Gustav Mie, Max Planck, and Max Abraham.

The negative reception started at the 85th *Versammlung deutscher Naturforscher und Ärzte* in Vienna in September 1913,<sup>126</sup> where Einstein had been invited to give a summary of his new theory. At a time when there were still doubts about the validity of the restricted principle of relativity, Einstein's difficult task was to present a theory which some people might have considered as even more incomprehensible, and which was based on the epistemological idea of a general relativity. This explains why Einstein decided to proceed carefully. He would first start with a set of reasonable requirements and then present his theory as having these features, as well as extra ones, such as embodying general relativity—at least to a certain extent—and the relativity of inertia. Thus, at the beginning of his talk Einstein mentioned four plausible hypotheses which one could postulate (but which one needed not necessarily require all together) for a gravitational theory :

1. the conservation laws of momentum and energy ;
2. the equality of inertial and gravitational mass for closed systems ;
3. the validity of the restricted relativity theory ;
4. independence of the physical laws from the absolute value of the gravitational potential.

Einstein was aware of the fact that, except for the first hypothesis, the remaining ones were not yet universally accepted : "I am fully aware of the circumstance that the postulates 2-4 resemble more a scientific creed than a secured basis."<sup>127</sup>

As examples of gravitational theories, Einstein was to discuss Nördström's second scalar theory and the Entwurf theory, which he considered to be the "most natural" generalizations of Newton's theory.<sup>128</sup> In his Entwurf paper, Einstein had already examined the possibility of a scalar theory of gravitation

124. Compare with Gerald Holton, "Mach, Einstein, and the Search for Reality," *Deadalus*, Spring 1968, pp. 636-73 ; in Gerald Holton, *Thematic Origins of Scientific Thought* (Cambridge, MA : Harvard University Press, 1973), pp. 219-59.

125. Lewis Robert Pyenson, "The Goettingen Reception of Einstein's General Theory of Relativity" (Ph.D. dissertation, Johns Hopkins University, 1974).

126. Einstein, "Zum gegenwärtigen Stande des Gravitationsproblems" (1913).

127. "Ich bin mir des Umstandes wohl bewusst dass die Postulate 2-4 mehr einem wissenschaftlichen Glaubensbekenntnis als einem gesicherten Fundamente ähnlich sind," *ibid.*, p. 1251.

128. *Ibid.*

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but had mainly rejected it because of his conviction that the relativity principle was valid with respect to a much larger group than the group of linear orthogonal transformations.<sup>129</sup> At the Vienna conference, Einstein could not use this argument since he did not include the requirement of a general relativity among the four plausible hypotheses. Einstein was to examine in detail Nordström's yet unpublished second theory.<sup>130</sup> Although that theory did not yield a deflection of light rays in a gravitational field because of the assumed constancy of the velocity of light, Einstein showed that the theory satisfied all four plausible requirements. The only objection Einstein expressed was that although the theory predicted an influence on the inertia of a particle by other bodies, the inertia did not seem to be "caused" by the latter since the inertia increased with the removal of the remaining bodies.<sup>131</sup> In fact, in the absence of any experimental decision about the deflection of light rays in a gravitational field, this was the only argument Einstein could advance in favor of his theory. In 1914, Einstein and A. D. Fokker showed that Nordström's theory could even be formulated in a generally covariant form.<sup>132</sup> Hence, Nordström's much simpler scalar theory appeared as a strong rival to the Entwurf theory. As late as 1917, Max von Laue was to write a comprehensive survey article in defense of Nordström's gravitational theory.<sup>133</sup>

Though Einstein considered Nordström's second theory as the main rival theory to his own and had dealt exclusively with that theory and the Entwurf theory during his Vienna address, other physicists such as Gustav Mie, and Max Abraham did not share this view and argued in favor of their own theories of gravitation. This led them to a critical analysis of Einstein's theory in which they were joined by Max Planck. The criticism of all these physicists shows that most of Einstein's basic assumptions were regarded as rather questionable at the time. Mie's criticism concerned mainly the assumption of a strict equality of inertial and gravitational mass and Einstein's attempt to generalize the principle of relativity. With respect to the equality of inertial and gravitational mass, Mie made a distinction between a strict equality and an equality consistent with experiment. He rejected the idea of a strict equality and did not think that one could base a theory on it: "I have indeed abandoned the principle of the identity of the gravitational and inertial mass and believe that one cannot build a theory on it."<sup>134</sup> Mie thought that the ratio of inertial and gravitational

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129. "Ich muss freilich zugeben, dass für mich das wirksamste Argument dafür, dass eine derartige Theorie [théorie scalaire] zu verwerfen sei, auf der Überzeugung beruht, dass die Relativität nicht nur orthogonalen linearen Substitutionen gegenüber besteht, sondern einer viel weiteren Substitutionsgruppe gegenüber," Einstein/Grossmann, "Entwurf" (1913), p. 244.

130. Gunnar Nordström, "Zur Theorie der Gravitation vom Standpunkt des Relativitätsprinzips," *Annalen der Physik*, **42** (1913), 533-54.

131. Einstein, "Zum gegenwärtigen Stande des Gravitationsproblems" (1913), p. 1254.

132. Albert Einstein et A. D. Fokker, "Die Nordströmsche Gravitationstheorie vom Standpunkt des absoluten Differentialkalküls," *Annalen der Physik*, **44** (1914), 321-28, on 328.

133. M. v. Laue, "Die Nordströmsche Gravitationstheorie (Bericht)," *Jahrbuch der Radioaktivität und Elektronik*, **14** (1917), 263-313.

134. Ich habe allerdings das Prinzip von der Identität der schweren und der trägen Masse fallen gelassen und glaube auch, dass man darauf keine Theorie gründen kann," Einstein, "Zum gegenwärtigen Stande des Gravitationsproblems" (1913), Discussion, p. 1266.

mass depended on factors such as the velocity of the body and its temperature and believed that he had good arguments for his case.<sup>135</sup> Mie's arguments are complicated and were not answered directly by Einstein. The interesting point is that Mie's arguments illustrate the diversity of responses to the experimental equality of inertial and gravitational mass. Far from inferring a strict theoretical equality<sup>136</sup> as Einstein did, Mie thought that such a strict equality was in fact theoretically impossible in the general case but accepted of course a restricted equality in accord with experiment. Thus, Mie argued that his theory, though it did not incorporate a strict identity of the two types of masses, was nevertheless consistent with experiment.

Mie's second criticism concerned Einstein's general principle of relativity, though he was aware that Einstein's theory did not fully incorporate it. At the Vienna conference, at which Einstein presented his theory in september 1915, Mie presented his objection as follows :

I have understood from Mr. Einstein's presentation that he wants to further develop a Machian idea, according to which it would ... be impossible to detect absolute accelerations. Against such a conception of the generalized principle of relativity one must, as a physicist, raise serious objections.<sup>137</sup>

As illustration of his objection, Mie discussed the case of a railroad train isolated from the outside. According to the classical point of view the jolts felt inside the train by the passengers are attributed to inertial effects due to the irregular motion of the train. If the general principle of relativity is accepted then the train can be considered at rest and the jolts are attributed to gravitational effects due to the irregular motion of masses surrounding the train. Mie admitted that mathematically such a point of view could be very convenient, but rejected it as physically implausible.<sup>138</sup> Later on, Mie drew support for his position from Einstein's own supposed proof of the impossibility of completely covariant field equations.<sup>139</sup>

Like Mie, Max Planck believed that the idea of a general relativity was physically unsound and also invoked Einstein's supposed proof against the idea. Planck was to state his opposition publicly on the very occasion of Einstein's

135. Gustav Mie, "Bemerkungen zu der Einsteinschen Gravitationstheorie, I, II," *Physikalische Zeitschrift*, **15** (1914), 115-22 and 169-76, on 118.

136. Since an experimental equality cannot entail a mathematical equality, Mie's position is logically sound. A similar position was adopted by Gustav Robert Kirchhoff with respect to his radiation law which he justified theoretically rather than experimentally; see for example Daniel M. Siegel, "Balfour Steward and Gustav Robert Kirchhoff : Two Independent Approaches to 'Kirchhoff's Radiation law,'" *Isis*, **67** (1976), 565-600.

137. "Ich habe eben in seinem Vortrag Herrn Einstein so verstanden, als ob er eine Machsche Idee weiter verfolgen wollte, wonach es auch nicht möglich sein dürfte, die Beschleunigungen absolut nachzuweisen. Gegen eine solche Auffassung des verallgemeinerten Relativitätsprinzips muss man aber als Physiker sehr schwerwiegende Bedenken erheben," Einstein, "Zum gegenwärtigen Stande des Gravitationsproblems" (1913), p. 1264.

138. Ibid. For Einstein's later answer to the example of the train, see A. Einstein, "Dialog über Einwände gegen die Relativitätstheorie," *Die Naturwissenschaften*, **6** (1918), 697-702, on 700 f.

139. Mie, "Bemerkungen II" (1914), p. 176.

inaugural address to the Berlin Academy on 2 July 1914. To Einstein's statement that *STR* is unsatisfactory because it privileges uniform motion, Planck replied that one could as well be of the opposite opinion and see in the preference of *STR* for uniform motion a valuable characteristic of the theory.<sup>140</sup> He pointed out that Newton's gravitational law is not found unsatisfactory because the power 2 appears in it as a privileged number; rather, physicists relate that number to the 3-dimensionality of space (i.e., to the spherical symmetry). Similarly, Planck wondered whether the preference for uniform motion was not related to the privileged position of the straight line over all other spatial lines. Though Planck's objections were very carefully phrased on that solemn occasion to avoid offending Einstein, the fact that Planck nevertheless brought up his objections on that day indicates how important the matter was to him.

Among the critics of Einstein's theory, the most expert was undoubtedly Max Abraham. We have seen that, in contradistinction to Mie and Nordström, Abraham in his second theory rejected the restricted principle of relativity and the constancy of the velocity of light, but accepted the idea of a strict equality of inertial and gravitational mass.<sup>141</sup> On the other hand, Abraham shared the doubts about Einstein's equivalence principle. In his comprehensive review article on gravitational theories,<sup>142</sup> completed in December 1914, Abraham in particular noted that Einstein saw in the implementation of the relativity of inertia a decisive advantage of his tensor theory over scalar theories. Abraham argued, however, that the relativity of inertia was not quantitatively secured in the Entwurf theory unless one postulated the existence of enormous invisible masses.<sup>143</sup> To Abraham, the introduction of such hypothetical masses was as objectionable as the introduction of a hypothetical ether. Hence, Abraham argued that there was no reason to opt for a tensor theory and added wittily that scalar theories ought to be preferred in the name of Mach's "economy of thought."<sup>144</sup>

In summary, the main reasons for the overall negative reception of the Entwurf theory were: (a) the existence of simpler scalar theories, (b) the more or less general rejection of the idea of a general principle of relativity, and (c) the imperfection of the Entwurf theory. By the end of 1914, Einstein thought that he had succeeded in reducing the imperfection of the theory by supposedly enlarging the covariance group of the field equations and by giving what he believed to be a natural derivation of the field equations. This was to give him an increased confidence in the validity of his theory until he suddenly abandoned the field equations altogether in favor of new ones at the end of 1915.

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140. Max Planck, reply to Einstein's inaugural address, *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften, Berlin*, part 2, 1914, pp. 742-44, on p. 743.

141. Max Abraham, "Neuere Gravitationstheorien," *Jahrbuch der Radioaktivität und Elektronik*, **11** (1914), 470-520.

142. Max Abraham, "Neuere Gravitationstheorien," *Jahrbuch der Radioaktivität und Elektronik*, **11** (1914), 470-520.

143. *Ibid.*, p. 520.

144. *Ibid.*, p. 520.



## Chapitre 5

# The General Theory of Relativity (1915 – 1917)

In November 1915, Einstein finally was to achieve a generally covariant theory, after three years of hard work. The resulting field equations are the standard field equations in use today. In that sense, one can speak of Einstein's final field equations. Einstein, however, was not entirely satisfied with them and in 1917 proposed generalized equations containing a supplementary term—the cosmological term. Einstein was to retain these new equations for many years, and only after the discovery of the expansion of the universe at the end of the 1920s and new theoretical developments in the 1930s did he drop the supplementary term. The episode of the modified field equations was not in vain, however, since it led to the development of relativistic cosmologies. Another reason to analyze the development of Einstein's 1917 theory is that it sheds light on the basic motivations that animated Einstein during those years.

### 5.1 The Generally Covariant Theory (1915-1916)

#### 5.1.1 The Return to the Riemann-Christoffel Tensor

Having developed at the end of 1914 what he thought to be a natural derivation of the gravitational field equations of the Entwurf theory, Einstein was to remain convinced of the validity of that theory during most of 1915. Einstein's satisfaction appears in a letter he wrote to a student in May 1915 : "To have actually attained this goal [general relativity], constitutes the greatest satisfaction of my life, even though no colleague has recognized so far the depth and necessity of this path."<sup>1</sup> In the same letter, Einstein also indicated that one of

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1. "Dies Ziel [Allgemeine Relativität] nun wirklich erreicht zu haben, ist die höchste Befriedigung meines Lebens, wenn auch kein Fachgenosse die Tiefe und Notwendigkeit dieses Weges bis jetzt erkannt hat," Einstein to Carl Seelig, 31 May 1915, in Carl Seelig, *Albert Einstein* (Zürich : Europa Verlag, 1960), p. 240.

the two important experimental tests, namely, the spectral shift in a gravitational field (the other test being the deflection of light rays in a gravitational field), had already been "brilliantly confirmed." Though this statement is somewhat in contrast with the more cautious presentation he gave to the Berlin Academy on 25 March 1915,<sup>2</sup> Einstein had indeed some indication of success at that time. In addition to this positive news, Einstein also had the pleasure of seeing his theory well understood at a talk he gave in Göttingen during the summer of 1915. Thus, in a letter to Arnold Sommerfeld dated 15 July 1915, Einstein wrote: "In Göttingen I had the great joy of seeing everything understood, and in detail. I am very enthusiastic about Hilbert. A great man! I am very curious to know your opinion."<sup>3</sup> In the same letter, Einstein, after some initial reluctance, expressed his willingness to accept Sommerfeld's proposal to include some of Einstein's papers on general relativity in a new edition of the book *Das Relativitätsprinzip* (originally published in 1913), of which Sommerfeld was the editor. Einstein's initial reluctance had been due to the fact that he considered none of his papers to give a complete exposition of the theory; Einstein furthermore expressed his intention to write an introductory book to the relativity theory with a presentation aiming at a general relativity from the beginning.<sup>4</sup> All this indicates that Einstein still had full confidence in his theory during the summer of 1915.

Einstein's confidence was to persist until October 1915, when he suddenly realized that the theory was untenable. In a postcard to David Hilbert, dated 7 November 1915, Einstein indicated that he had been aware for "about four weeks" that his 1914 derivation of the field equations was "delusive."<sup>5</sup> Furthermore, in a letter to Sommerfeld dated 28 November 1915, Einstein explained in detail the reasons which led him to abandon his theory:

1. I proved that the gravitational field in a uniformly rotating system does not satisfy the field equations.
2. The motion of the perihelion of Mercury came out to be 18" instead of 45" per century.
3. The covariance consideration of my paper of last year did not yield the Hamiltonian function  $H$ . It allows, if properly generalized, an arbitrary  $H$ . From this it followed that the covariance with respect to "adapted"

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2. A. Einstein, "Über den Grundgedanken der allgemeinen Relativitätstheorie und Anwendung dieser Theorie in der Astronomie," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften, Berlin*, part 1 (1915), p. 315; this is just a summary. The topic is also discussed in a letter from Einstein to Besso, 12 February 1915, in Albert Einstein and Michele Besso, *Correspondance 1903-1955* (Paris: Hermann, 1972), p. 57.

3. "In Göttingen hatte ich die grosse Freude, alles bis ins Einzelne verstanden zu sehen. Von Hilbert bin ich ganz begeistert. Ein bedeutender Mann! Ich bin auf Ihre Meinung sehr neugierig," Einstein to Sommerfeld, 15 July 1915, in Albert Einstein and Arnold Sommerfeld, *Briefwechsel*, ed. Armin Hermann (Basel: Schwabe, 1968), p. 30.

4. *Ibid.*

5. Einstein Papers, Princeton University Press, microfilm reel I.B.1, no 13. Quoted by J. Earman, C. Glymour, "Einstein and Hilbert: Two Months in the History of General Relativity," *Archive for the History of Exact Sciences*, **19** (1978), 291-308, on 294.

coordinate systems was a flop.<sup>6</sup>

Einstein mentioned the same three reasons in a letter to H. A. Lorentz in January 1916, but inverted items (1) and (2).<sup>7</sup> Since, with Sommerfeld, Einstein felt quite at ease to discuss his ideas, it is possible that the ordering of the reasons that Einstein gave to Sommerfeld represents historical sequence. If this is the case, the sequence of events would appear to have been the following. Since in his 1914 paper to the Berlin Academy Einstein had strongly argued in favor of a covariance group containing in particular rotation transformations,<sup>8</sup> it was natural for him, though perhaps not immediate from a mathematical point of view, to check whether his field equations had indeed such a covariance. Finding that this was not the case, Einstein probably decided to test the theory on the precession of the perihelion of Mercury. Meeting once more a failure, Einstein must have come to a critical examination of what he had thought to be a "natural" derivation of the field equations, and found it to be defective.

Einstein's next thought was as follows :

After any confidence in the results and method of the former theory had thus vanished, I clearly recognized that only a generally covariant theory, i.e., one making use of Riemann's covariants, could allow for a satisfactory solution.<sup>9</sup>

Having completely lost confidence in his previous field equations, Einstein decided to find new ones. This time, however, he was to secure a satisfactory covariance group from the very beginning by making use of the Riemann-Christoffel tensor. The relevance of that tensor for finding field equations involving the  $g_{\mu\nu}$  and their first and second derivatives stemmed from the mathematical result that any tensor derived from the  $g_{\mu\nu}$  and their derivatives can be obtained from the Riemann-Christoffel tensor. We have seen that Einstein and Marcel Grossmann were fully aware of this point from the very beginning and had already considered using the curvature tensor in the 1913 Entwurf paper. Yet, they thought that they had several reasons against its use.

During the years 1913 – 15, Einstein was to gradually overcome the objections against the use of the Riemann-Christoffel tensor. The failure to obtain

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6. "1) Ich bewies, dass das Gravitationsfeld auf einem gleichförmig rotierenden System den Feldgleichungen nicht genügt.

2) Die Bewegung des Merkur-Perihels ergab sich zu 18" statt 45" pro Jahrhundert.

3) Die Kovarianzbetrachtung in meiner Arbeit vom letzten Jahre liefert die Hamilton-Funktion H nicht. Sie lässt, wenn sie sachgemäss verallgemeinert wird, ein beliebiges H zu. Daraus ergab sich, dass die Kovarianz bezüglich 'angepasster' Koordinatensysteme ein Schlag ins Wasser war," Einstein to Sommerfeld, 28 November 1915, *Briefwechsel* (1968), pp. 32-33.

7. Einstein to Lorentz, 1 January 1916, Einstein Papers, Princeton University, microfilm reel I.B. 1, no. 16; mentioned by Earman/Glymour, "Einstein and Hilbert," p. 295.

8. A. Einstein, "Die formale Grundlage der allgemeinen Relativitätstheorie," *Sitzungsberichte der Preussischen Akademie der Wissenschaften, Berlin*, part 2 (1914), pp. 1030-85, on pp. 1031-32, 1068.

9. "Nachdem so jedes Vertrauen in Resultate und Methode der früheren Theorie gewichen war, sah ich klar, dass nur einen Anschluss an die allgemeine Kovariantentheorie, d.h. an Riemann's Kovariante, eine befriedigende Lösung gefunden werden konnte," Einstein to Sommerfeld, 28 November 1915, *Briefwechsel* (1968), p. 33.



Newton's theory as approximation was overcome by February 1914, when Einstein and A. D. Fokker gave a generally covariant formulation of Nordström's theory making use of the Riemann tensor. In a footnote, Einstein and Fokker stated without elaboration that the objection concerning the Newtonian approximation had been found to be invalid.<sup>10</sup> Since Nordström's scalar theory of gravitation, which yielded Newton's gravitational theory as approximation, was derived in the paper via the curvature scalar, the above objection had indeed been refuted in that case. Einstein then probably generalized the refutation to the case of a tensor theory. One might perhaps wonder why Einstein, having used successfully the Riemann-Christoffel tensor, did not immediately try to build gravitational field equations directly on the curvature tensor. The reason is that, at that stage, Einstein was convinced of the validity of his previous field equations and was only looking, at most, for an eventual connection of the latter with the curvature tensor. Finding such a connection was certainly not evident if, indeed, there is any at all.

What apparently hindered, in particular, Einstein's more extensive use of the curvature tensor at that time was that he wrote it out in terms of the  $g_{\mu\nu}$  instead of the Christoffel symbols because, as we have seen, he considered the quantities

$$\frac{1}{2} \left( g^{\tau\mu} \frac{\partial g_{\mu\nu}}{\partial x^\alpha} \right)$$

to be the "natural" components of the gravitational field.<sup>11</sup> In November 1914, Einstein still defended this view<sup>12</sup> despite the fact that, at that time, he wrote conservation equations and, even more important, the equation of motion in terms of the Christoffel symbols. This was the first time Einstein gave the equation of motion in the standard form

$$\frac{d^2 x^\tau}{ds^2} + \Gamma_{\mu\nu}^\tau \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = 0$$

( $s$  being the proper time), whereas previously, Einstein had used less simple expressions containing the  $g_{\mu\nu}$  explicitly. Einstein, at that point, realized that the Christoffel symbols might be viewed as the components of the gravitational field<sup>13</sup> but thought to have counterarguments. It was only a year later that Einstein was to reverse his position, a reversal which was to make Einstein's use of the Riemann-Christoffel tensor successful this time because of the simplifications it introduced into the formulas.

10. Albert Einstein et A. D. Fokker, "Die Nordströmsche Gravitationstheorie vom Standpunkt des absoluten Differentialkalküls," *Annalen der Physik*, **44** (1914), 321-28, on 328.

11. A. Einstein, "Zur allgemeinen Relativitätstheorie," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften, Berlin*, part 2 (1915), pp. 777, 778-86, Nachtrag, pp. 789, 799-801, on 782.

12. A. Einstein, "Formale Grundlage" (1914), pp. 1058, 1060-61.

13. *Ibid.*, p. 1060.

### 5.1.2 The Generally Covariant Field Equations

Though Einstein had removed the objections against the use of the Riemann-Christoffel tensor by November 1915, Einstein was not to attain generally covariant field equations immediately but obtained them only after intense efforts, which took place during November 1915. The intensity of the efforts is reflected in the frequency of Einstein's communications to the Berlin Academy (one per regular weekly session), and was probably due in part because of Einstein's excitement to feel close to the long desired goal and perhaps also because of some fear that following indications he gave to David Hilbert, the latter might anticipate him in some way. Three phases can be distinguished in Einstein's struggle, which we shall examine successively. In the first phase, to be discussed in section (a) below, Einstein restricted the covariance group to transformations of Jacobian 1; in the second and third phases, discussed in sections (b) and (c), Einstein achieved a general covariance, first with the aid of an ad hoc hypothesis and finally without.

#### Covariant field equations with respect to transformations of Jacobian 1

The results of the first phase of Einstein's struggle for new gravitational field equations were published in a paper entitled "Zur allgemeinen Relativitätstheorie"<sup>14</sup> [On the General Theory of Relativity], which was presented at the general meeting of the Prussian Academy of sciences on 4 November 1915. Einstein opened the paper with the following remarks: "Over the past years, I have endeavored to develop a general theory of relativity on the assumption of a relativity including nonuniform notions. I thought in fact that I had found the only gravitational law compatible with ... the general postulate of relativity."<sup>15</sup> He then explained why he abandoned his former field equations, stating that he had found his previous derivation of the field equations<sup>16</sup> to be delusive, and that, consequently, he completely lost confidence in the field equations. Looking for a natural way to restrict the theoretical possibilities, he then came back to the requirement of general covariance of the field equations which he had abandoned reluctantly, "with a heavy heart,"<sup>17</sup> three years before. In fact, Einstein's reason for abandoning his previous field equations is not convincing. The failure of a single particular justification of the field equations does not imply, that these field equations were false. Curiously, Einstein did not mention in the paper that the former field equations did not yield the correct value of the precession of the perihelion of Mercury. The reason for the omission was probably that Einstein did not know for sure at that point whether his new field equations would give

14. Einstein, "Zur allgemeinen Relativitätstheorie" (1915).

15. "In den letzten Jahren war ich bemüht, auf die Voraussetzung der Relativität auch nicht gleichförmiger Bewegungen eine allgemeine Relativitätstheorie zu gründen. Ich glaubte in der Tat, das einzige Gravitationsgesetz gefunden zu haben, das dem sinnemäss gefassten, allgemeinen Relativitätspostulate entspricht." Ibid., p. 785.

16. Einstein, "Formale Grundlage" (1914), pp. 1066-77.

17. Einstein, "Zur allgemeinen Relativitätstheorie" (1915), p. 778.

a better value of the perihelion precession than the previous ones.

The basic postulate of the paper was the postulate of the covariance of all equations with respect to transformations of Jacobian 1.<sup>18</sup> Though Einstein did not elaborate on the choice of this postulate, it appears that Einstein was motivated by his desire to see rotations and acceleration transformations included in the covariance group in order to avoid any further disappointment with respect to the covariance of the theory; we have seen that the absence of rotational covariance was the first reason Einstein mentioned to Sommerfeld for abandoning the field equations. Indeed, in the conclusion of the paper, Einstein was to verify explicitly that rotations and acceleration transformations were included in the covariance group. It was this extensive relativity of motion which probably justified in Einstein's eyes the title of his paper "On the general theory of relativity" despite the fact that the theory was not yet generally covariant.

Besides the extensive relativity, the covariance postulate had also the advantage of introducing various simplifications in the formulas. Since

$$d\tau' = \frac{\partial (x'^1 \dots x'^4)}{\partial (x^1 \dots x^4)} d\tau,$$

it follows that  $d\tau' = d\tau$  and also  $\sqrt{-g'} = \sqrt{-g}$  (with  $g = |g_{\mu\nu}|$ ) because of the invariance of  $\sqrt{-g}d\tau$ . The invariance of  $\sqrt{-g}$ , on the other hand, entails that the contracted Christoffel symbol  $\Gamma_{s\tau}^s = \frac{\partial \ln \sqrt{-g}}{\partial x^\tau}$  is a tensor (with respect to the covariance group here considered), which leads to a simplified contracted Riemann tensor. Einstein introduced the latter as follows. Looking for a tensor of second rank derived from the  $g_{\mu\nu}$  and their first and second derivatives, Einstein stated that "mathematics teaches" that all such tensors can be derived from the Riemann-Christoffel tensor. Since the latter is a tensor of fourth rank, Einstein made the only possible relevant contraction (because of the various symmetries) and obtained the tensor

$$G_{im} = R_{im} + S_{im},$$

with :

$$R_{im} = \frac{\partial \Gamma_{im}^l}{\partial x^l} + \Gamma_{il}^\rho \Gamma_{\rho m}^l,$$

$$S_{im} = -\frac{\partial \Gamma_{il}^l}{\partial x^m} - \Gamma_{im}^\rho \Gamma_{\rho l}^l,$$

and where

$$\Gamma_{kl}^m = \frac{1}{2} g^{mi} \left( \frac{\partial g_{ik}}{\partial x^l} + \frac{\partial g_{li}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^i} \right)$$

are the standard Christoffel symbols.<sup>19</sup> From the result that  $\Gamma_{il}^l$  is a tensor, it follows that  $S_{im}$  is a tensor because it is the covariant derivative of  $\Gamma_{il}^l$ . From this, Einstein reached the conclusion that  $R_{im}$  is a tensor as well, because of the tensor character of  $S_{im}$  and  $G_{im}$ .

18. Ibid., p. 779.

19. Einstein's  $\Gamma_{kl}^m$  have the opposite sign.

After explaining in detail his shift from his earlier conviction that

$$\frac{1}{2}g^{\tau\mu}\frac{\partial g_{\mu\nu}}{\partial x^\alpha}$$

represented the "natural expression of the components of the gravitational field"<sup>20</sup> to his new conviction that the Christoffel symbols ought to play this role because of their symmetry and their presence in the equation of motion, Einstein introduced, without justification, the following field equations

$$R_{\mu\nu} = -\chi T_{\mu\nu},$$

or explicitly

$$-\frac{\partial\Gamma_{\mu\nu}^\alpha}{\partial x^\alpha} + \Gamma_{\mu\beta}^\alpha\Gamma_{\nu\alpha}^\beta = -\chi T_{\mu\nu}. \quad (5.1)$$

Einstein's choice of the field equations seems to have been conditioned by his initial intention to assume that  $\sqrt{-g} = 1$  (which is a natural choice since  $STR$  is supposed to be valid locally) because in that case  $S_{im} = 0$ . Later on, however, he found that he could not assume  $\sqrt{-g} = 1$  and then decided to introduce the field equations without justification.<sup>21</sup> Einstein was to remove the objection against the use of the condition  $\sqrt{-g} = 1$  in his next paper, and was to adopt that condition permanently thereon. From the field equations, Einstein derived energy-momentum conservation equations of the form

$$\frac{\partial}{\partial x^\lambda} (T_\sigma^\lambda + t_\sigma^\lambda) = 0$$

where

$$t_\sigma^\lambda = \frac{1}{2}\delta_\sigma^\lambda g^{\mu\nu}\Gamma_{\mu\beta}^\alpha\Gamma_{\nu\alpha}^\beta - g^{\mu\nu}\Gamma_{\mu\sigma}^\alpha\Gamma_{\nu\alpha}^\lambda.$$

Einstein called  $t_\sigma^\lambda$  the "energy tensor" of the gravitational field but was aware that it was only a tensor with respect to linear transformations. He obtained two further relations by multiplying the field equations (5.1) respectively by  $g^{\mu\nu}$  (and summing over  $\mu, \nu$ ) and  $g^{\nu\lambda}$  (and summing over  $\nu$ ). Generalizing slightly the resulting second equation

$$\frac{\partial}{\partial x^\mu} \left( \frac{\partial^2 g^{\alpha\beta}}{\partial x^\alpha \partial x^\beta} - g^{\sigma\tau}\Gamma_{\sigma\beta}^\alpha\Gamma_{\tau\alpha}^\beta \right) = 0$$

to the requirement

$$\frac{\partial^2 g^{\alpha\beta}}{\partial x^\alpha \partial x^\beta} - g^{\sigma\tau}\Gamma_{\sigma\beta}^\alpha\Gamma_{\tau\alpha}^\beta = 0, \quad (5.2)$$

and applying it to the first relation, Einstein obtained for the latter

$$\frac{\partial}{\partial x^\alpha} \left( g^{\alpha\beta} \frac{\partial \ln \sqrt{-g}}{\partial x^\beta} \right) = -\chi T_\sigma^\sigma.$$

From this equation, Einstein concluded : "it is impossible to choose the coordinate system in such a way, that  $\sqrt{-g}$  becomes equal to 1; for the scalar of the energy tensor cannot be made equal to zero."<sup>22</sup> This remark makes it likely

20. Einstein, "Zur allgemeinen Relativitätstheorie" (1915), p. 782.

21. Ibid., p. 778.

22. Ibid., p. 785.

that Einstein had initially considered adopting the condition  $\sqrt{-g} = 1$  when he developed the field equations.

As to the consequences of his theory, Einstein showed that it yielded Newton's law in first approximation as follows. From the relations (5.2), Einstein obtained in first approximation

$$\frac{\partial^2 g^{\alpha\beta}}{\partial x^\alpha \partial x^\beta} = 0.$$

Adopting the solution  $\frac{\partial g^{\alpha\beta}}{\partial x^\beta} = 0$ , he found for the field equations (in first approximation)

$$\frac{1}{2} \frac{\partial^2 g_{\mu\nu}}{\partial (x^\alpha)^2} = \chi T_{\mu\nu} \quad (x^4 = ict),$$

which yield Poisson's equation for  $\mu, \nu = 4$ . Einstein's enthusiasm for the theory appears in his statement : "The magic of this theory will hardly fail to touch anybody who has really understood it ; the theory represents a real triumph of the method of the general differential calculus founded by Gauss, Riemann, Christoffel, Ricci and Levi-Civiter [sic]." <sup>23</sup>

#### Generally covariant field equations with the assumption $T_\mu^\mu = 0$

Einstein was to remove the objection against using coordinate systems with  $\sqrt{-g} = 1$  in an addendum to his paper "Zur allgemeinen Relativitätstheorie," which he presented to the Berlin Academy of Sciences on 11 November 1915. <sup>24</sup> The removal of that obstacle was to allow Einstein to develop generally covariant field equations but was achieved only through the introduction of a daring hypothesis, namely the assumption that the contracted energy-momentum tensor  $T_\mu^\mu$  vanishes for matter. In the introduction to the addendum, Einstein acknowledged the boldness of that hypothesis but was willing to take that step in order to endow the theory with an "even firmer logical structure." <sup>25</sup>

Einstein tried to justify the assumption  $T_\mu^\mu = 0$  for matter as follows. Whereas for the electromagnetic field  $T_\mu^\mu = 0$ , for matter, the scalar of the energy tensor does not generally vanish. Einstein then remarked : "It must now be remembered, that according to our knowledge, 'matter' is not to be conceived as something primitively given or physically simple. Indeed, there are not a few who hope to reduce matter to purely electromagnetic processes." <sup>26</sup> Adopting in part such an electromagnetic world view, Einstein then reasoned as follows. If one considers ordinary matter as being made of electromagnetic matter (for

23. "Dem Zauber dieser Theorie wird sich kaum jemand entziehen können, der sie wirklich erfasst hat ; sie bedeutet einen wahren Triumph der durch GAUSS, RIEMANN, CHRISTOFFEL, RICCI und LEVI-CIVITER [sic] begründeten Methode des allgemeinen Differentialkalküls," *ibid.*, p. 779.

24. *Ibid.*, Nachtrag, pp. 799-801.

25. *Ibid.*, p. 799.

26. "Es ist nun daran zu erinnern, dass nach unseren Erkenntnissen die 'Materie' nicht als ein primitiv Gegebenes, physikalisch Einfaches aufzufassen ist. Es gibt sogar nicht wenige, die hoffen, die Materie auf rein elektromagnetische Vorgänge reduzieren zu können," *Ibid.*, p. 799.

which  $T_\mu^\mu = 0$  in an electromagnetic world view) and of gravitational fields (for which  $t_\mu^\mu \neq 0$ ), then one could have for the whole matter  $T_\mu^\mu + t_\mu^\mu \neq 0$ . Since in the gravitational field equations  $T^{\mu\nu}$  represents matter without the gravitational field, the assumption  $T_\mu^\mu = 0$ , appears plausible within such a world view.

As to the field equations, Einstein proposed the generally covariant field equations

$$G_{\mu\nu} = -\chi T_{\mu\nu},$$

which, through specialization of the coordinate system systems such that  $\sqrt{-g} = 1$ , yielded the previous ones, namely  $R_{\mu\nu} = -\chi T_{\mu\nu}$ . Hence, Einstein's assumption  $T_\mu^\mu = 0$  led indeed to a more integrated theory because: (a) it eliminated the arbitrary restriction of the covariance group to transformations of Jacobian 1; (b) it eliminated the arbitrariness in the previous choice of the field equations; (c) coordinate systems such that  $\sqrt{-g} = 1$  were now allowed. The assumption  $T_\mu^\mu = 0$  itself, however, was rather shaky and left Einstein uncomfortable. He was to remove it two weeks later.

### Generally covariant field equations without the assumption $T_\mu^\mu = 0$

At the 25 November 1915 session Einstein communicated a paper entitled "Die Feldgleichungen der Gravitation"<sup>27</sup> [The field equations of gravitation], in which he succeeded in removing the ad hoc assumption  $T_\mu^\mu = 0$  while maintaining the general covariance of the field equations. With this paper, Einstein's general theory of relativity, in its classic form, was finally completed. In the short paper of four pages, Einstein, after briefly retracing the development of his generally covariant field equations, pointed out that he had recently found that one could do without the hypothesis  $T_\mu^\mu = 0$  by introducing the energy-momentum tensor in a slightly different way in the field equations. For the latter, Einstein adopted the equations

$$G_{im} = -\chi \left( T_{im} - \frac{1}{2} g_{im} T \right),$$

where  $T = T_\mu^\mu$  is the scalar of the energy-momentum tensor of matter and  $G_{im} = R_{im} + S_{im}$ , with

$$R_{im} = \frac{\partial \Gamma_{im}^l}{\partial x^l} + \Gamma_{il}^\rho \Gamma_{m\rho}^l,$$

and

$$S_{im} = -\frac{\partial \Gamma_{il}^l}{\partial x^m} - \Gamma_{im}^\rho \Gamma_{\rho l}^l.$$

By choosing a coordinate system such that  $\sqrt{-g} = 1$  these field equations reduce to

$$R_{im} = -\chi \left( T_{im} - \frac{1}{2} g_{im} T \right). \quad (5.3)$$

<sup>27</sup> A. Einstein, "Die Feldgleichungen der Gravitation," Sitzungsberichte der K. Preussischen Akademie der Wissenschaften, Berlin, part 2 (1915), pp. 843, 844-47.

Einstein stated that the reasons which led him to introduce the supplementary term  $\frac{1}{2}\chi g_{im}T$  in the field equations resulted from considerations completely analogous to those given in his first November paper.<sup>28</sup> Relying basically on his former presentation, Einstein just gave the following indications. Multiplying the field equation (5.3) by  $g^{im}$  and summing over  $i$  and  $m$ , Einstein obtained the equation

$$\frac{\partial^2 g^{\alpha\beta}}{\partial x^\alpha \partial x^\beta} - \chi(T + t) = 0 \quad (5.4)$$

where  $t = t_\sigma^\sigma$ . Einstein stressed the fact that the energy-momentum tensor of the gravitational field enters this equation in the same way as the energy tensor of matter, which he pointed out, was not the case in his former treatment. Finally, referring to the derivation of the equation

$$\frac{\partial}{\partial x^\mu} \left( \frac{\partial^2 g^{\alpha\beta}}{\partial x^\alpha \partial x^\beta} - g^{\sigma\tau} \Gamma_{\sigma\beta}^\alpha \Gamma_{\tau\alpha}^\beta \right) = 0 \quad (5.5)$$

of that same paper, Einstein found it to be replaced by the equation

$$\frac{\partial}{\partial x^\mu} \left( \frac{\partial^2 g^{\alpha\beta}}{\partial x^\alpha \partial x^\beta} - \chi(T + t) \right) = 0.$$

Einstein concluded that as this equation is automatically satisfied because of (5.4), it follows that no restriction is imposed on the energy-momentum tensor of matter except for the conservation laws.

From Einstein's indications, it appears that he came to introduce the term  $\frac{1}{2}\chi g_{im}T$  in the field equations as follows. Since he wanted to eliminate the assumption  $T_\mu^\mu = 0$ , he naturally came back to the source of the problem, namely, the considerations in section 3 of his paper "Zur allgemeinen Relativitätstheorie." There, as we have seen above, Einstein had multiplied the field equations<sup>29</sup>

$$\frac{\partial \Gamma_{\mu\nu}^\alpha}{\partial x^\alpha} + \Gamma_{\mu\beta}^\alpha \Gamma_{\nu\alpha}^\beta = -\chi T_{\mu\nu}$$

by  $g^{\mu\nu}$  (with a contraction over  $\mu$  and  $\nu$ ) and  $g^{\mu\nu}$  (with contraction over  $\nu$ ) and had respectively obtained the equations :<sup>30</sup>

$$\begin{aligned} \frac{\partial^2 g^{\alpha\beta}}{\partial x^\alpha \partial x^\beta} (-g^{\sigma\tau} \Gamma_{\sigma\beta}^\alpha \Gamma_{\tau\alpha}^\beta) + \frac{\partial}{\partial x^\alpha} \left( g^{\alpha\beta} \frac{\partial \ln \sqrt{-g}}{\partial x^\beta} \right) &= -\chi T_\sigma^\sigma, \\ \frac{\partial}{\partial x^\alpha} (g^{\nu\lambda} \Gamma_{\mu\nu}^\alpha) - \frac{1}{2} \delta_\mu^\lambda \Gamma_{\mu\beta}^\alpha \Gamma_{\nu\alpha}^\beta &= -\chi \left( T_\mu^\lambda + \frac{1}{\chi} t_\mu^\lambda \right). \end{aligned}$$

By adopting the restriction  $\sqrt{-g} = 1$  and using the definition of  $t_\sigma^\lambda$  given in the same section, Einstein probably rewrote these equations respectively in the form :

$$\frac{\partial^2 g^{\alpha\beta}}{\partial x^\alpha \partial x^\beta} + \frac{\partial}{\partial x^\alpha} \left( g^{\alpha\beta} \frac{\partial \ln \sqrt{-g}}{\partial x^\beta} \right) = -\chi T + t,$$

28. Einstein, "Zur allgemeinen Relativitätstheorie" (1915).

29. In order to stay close to Einstein's original notation, we shall temporarily use in this paragraph Einstein's  $\Gamma_{\mu\nu}^\alpha$  which have the opposite sign of the standard Christoffel symbols.

30. In Einstein's paper, the factor  $1/\chi$  on the right side of the second equation is missing.

$$\frac{\partial}{\partial x^\alpha} (g^{\nu\lambda} \Gamma_{\mu\nu}^\alpha) = -\chi \left( T_\mu^\lambda + \frac{1}{\chi} t_\mu^\lambda \right) + \frac{1}{2} \delta_\mu^\lambda t.$$

By redefining  $t_\mu^\lambda$  as  $\chi t_\mu^\lambda$  these equations become respectively :

$$\frac{\partial^2 g^{\alpha\beta}}{\partial x^\alpha \partial x^\beta} = \chi (-T + t), \quad (5.6)$$

$$\frac{\partial}{\partial x^\alpha} (g^{\nu\lambda} \Gamma_{\mu\nu}^\alpha) = -\chi (T_\mu^\lambda + t_\mu^\lambda) + \frac{1}{2} \delta_\mu^\lambda \chi t. \quad (5.7)$$

Both equations reveal a suspicious asymmetry in  $t$  and  $T$  which Einstein probably tried to eliminate. A straightforward attempt is to add  $\frac{1}{2} \delta_\mu^\lambda \chi t$  on the right side of (5.7). In that case, through the use of

$$\frac{\partial}{\partial x^\lambda} (T_\sigma^\lambda + t_\sigma^\lambda) = 0,$$

the latter equation yields, after various rearrangements,

$$\frac{\partial}{\partial x^\mu} \left[ \frac{\partial^2 g^{\alpha\beta}}{\partial x^\alpha \partial x^\beta} - \chi (t + T) \right] = 0, \quad (5.8)$$

which replaces equation (5.5). The analogy of the expression in parentheses of (5.8) with equation (5.6) is striking. By adding  $2\chi T$  on the right side of (5.6), equation (5.8) would be automatically satisfied and, consequently, the condition which led Einstein to introduce the assumption  $T_\mu^\mu = 0$  would disappear.

Finally, it would have been easy for Einstein to infer the modifications of the field equations which the supplementary term called for since (5.6) was obtained by multiplying the field equation by  $g^{\mu\nu}$  (with contractions over  $\mu$  and  $\nu$ ). In order to get  $2\chi T = \frac{1}{2} g^{im} (g_{im} \chi T)$ , a term  $\frac{1}{2} g_{im} \chi T$  must be added on the right side of the field equation  $R_{im} = -\chi T_{im}$ . Once the modification of the field equation was arrived at, Einstein's remaining task was to make sure that the supplementary term did not have undesirable consequences for the theory. Hence, the transition toward the final field equations appears to have been a quite natural one, which was conditioned essentially by Einstein's previous work. That Einstein, indeed, seems to have followed the path described here is supported by the remarks Einstein made in the comprehensive review paper he was to write a few months later<sup>31</sup>

About the same time that Einstein presented his 25 November field equations, David Hilbert,<sup>32</sup> in a lecture presented similar equations which he derived from the curvature invariant (i. e., the twice contracted Riemann-Christoffel tensor). Hilbert's approach was not independent of Einstein since it appears that Hilbert got the idea of using the curvature invariant from Einstein, at the latest through the proofs of Einstein's 4 November 1915 paper which Einstein

31. A. Einstein, "Die Grundlagen der allgemeinen Relativitätstheorie," *Annalen der Physik*, **49** (1916), 769-822, on 806-07.

32. David Hilbert, "Die Grundlagen der Physik, I," *Nachrichten von der Königl. Gesellschaft der Wissenschaften zu Göttingen*, mathematisch-physikalische Klasse (1915), pp. 395-407.



had sent to Hilbert at the beginning of November.<sup>33</sup> For a mathematician expert in invariant theory and, furthermore, familiar with Gustav Mie's physical use of invariants, it was then straightforward to derive field equations from the curvature invariant. In any case, Hilbert never claimed any priority concerning the field equations. Though the field equations must be considered as Einstein's, Hilbert's formulation constituted an elegant mathematical derivation of the field equations. Another contribution of Hilbert was showing that the field equations yielded four identities—the so-called Bianchi identities.<sup>34</sup>

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33. For a detailed analysis, see the appendix.

34. Voir Jagdish Mehra, *Einstein, Hilbert, and the Theory of Gravitation* (Dordrecht, Holland/Boston, USA : D. Reidel, 1974), pp. 49-50.

**Box 8. Einstein's field equations**

Riemannian space (with curvature, without torsion) :

$$ds^2 = g_{ik} dx^i dx^k.$$

Riemann tensor :

$$R_{iklm}^i = \frac{\partial \Gamma_{km}^i}{\partial x^l} - \frac{\partial \Gamma_{kl}^i}{\partial x^m} + \Gamma_{nl}^i \Gamma_{km}^n - \Gamma_{nm}^i \Gamma_{kl}^n$$

Ricci tensor :

$$\begin{aligned} R_{ik} &= R_{ilk}^l \\ &= \frac{\partial \Gamma_{ik}^l}{\partial x^l} - \frac{\partial \Gamma_{il}^l}{\partial x^k} + \Gamma_{ik}^l \Gamma_{lm}^m - \Gamma_{il}^m \Gamma_{km}^l \end{aligned}$$

Curvature :

$$R = g^{ik} R_{ik}$$

Einstein's field equations :

$$R_{ik} = \frac{8\pi G}{c^4} \left( T_{ik} - \frac{1}{2} g_{ik} T \right)$$

(with  $T_{ik}$  energy-momentum tensor of matter and  $T = T_i^i$ ,  $G$  : gravitational constant)

## 5.2 Consequences of the Theory and Experimental Verification

Among the main consequences of his theory that Einstein developed during 1915–16 were first and second order effects. Einstein discussed these effects on 18 November 1915<sup>35</sup> and on March 1916 in a comprehensive review paper.<sup>36</sup> Since the November derivations only involved the field equations for vacuum, they remained valid with respect to the final field equations.

### 5.2.1 First Order Effects : Newton's Law of Gravitation, Deflection of Light Rays, Behavior of Clocks and Rods in a Gravitational Field

Einstein defined the first order approximation as corresponding to a metric  $g_{\mu\nu}$ , differing from the *STR* metric

$$\eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(with  $x^4 = ct$ ) only by first order terms<sup>37</sup>

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}, |\gamma_{\mu\nu}| \ll |\eta_{\mu\nu}|.$$

He further restricted the  $g_{\mu\nu}$  as follows :

1. all  $g_{\mu\nu}$  are independent of the time (static case) ;
2. the solution is spatially symmetric around the origin of the coordinates ;
3.  $g_{\rho 4} = g_{4\rho} = 0$  for  $\rho = 1, 2, 3$  (these coefficients are involved, for example in rotating frames of reference<sup>38</sup>) ;
4. the  $g_{\mu\nu}$  tend toward the  $\eta_{\mu\nu}$  at spatial infinity (boundary conditions) .

Einstein also considered the sun to be at the origin of the coordinate system and the latter to be such that  $g = |g_{\mu\nu}| = -1$ .

With these assumptions, the field equations

$$-\frac{\partial \Gamma_{\mu\nu}^{\alpha}}{\partial x^{\alpha}} + \Gamma_{\mu\beta}^{\alpha} \Gamma_{\nu\alpha}^{\beta} = 0,$$

35. A. Einstein, "Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften*, Berlin, part 2 (1915), pp. 831-39.

36. A. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie" (1916).

37. Einstein, "Perihelbewegung des Merkur" (1915), p. 833.

38. Max Abraham, "Neuere Gravitationstheorien," *Jahrbuch der Radioaktivität und Elektronik*, **11** (1914), 470-520, on 512.

reduce in first approximation to

$$\frac{\partial \Gamma_{\mu\nu}^{\alpha}}{\partial x^{\alpha}} = 0.$$

Einstein proposed the solution

$$g_{\rho\sigma} = -\delta_{\rho\sigma} - \alpha \frac{x^{\rho} x^{\sigma}}{r^3} \quad (\rho, \sigma = 1, 2, 3),$$

$$g_{44} = 1 - \frac{\alpha}{r}$$

where  $\alpha$  is a constant of integration. From the  $g_{\mu\nu}$ , Einstein then computed the Christoffel symbols :

$$\Gamma_{\rho\sigma}^{\tau} = -\alpha \left( \delta_{\rho\sigma} \frac{x^{\tau}}{r^2} - \frac{3}{2} \frac{x^{\rho} x^{\sigma} x^{\tau}}{r^5} \right),$$

$$\Gamma_{44}^{\sigma} = \Gamma_{4\sigma}^4 = -\frac{\alpha}{2} \frac{x^{\sigma}}{r^3}.$$

With respect to the equation of motion,

$$\frac{d^2 x^{\nu}}{ds^2} = \Gamma_{\sigma\tau}^{\nu} \frac{dx^{\sigma}}{ds} \frac{dx^{\tau}}{ds},$$

Einstein assumed the moving body to have a small velocity as compared to that of the velocity of light and thus obtained

$$\frac{d^2 x^{\nu}}{ds^2} = \Gamma_{44}^{\nu} = -\frac{\alpha}{2} \frac{x^{\nu}}{r^3} \quad (\nu = 1, 2, 3),$$

$$\frac{d^2 x^4}{ds^2} = 0.$$

Assuming  $s \simeq x^4$ , the first three equations correspond to the Newtonian equation  $a^i = KM \frac{x^i}{r^3}$ , if one sets  $\alpha = \frac{2KM}{c^2}$ ; since  $\Gamma_{44}^{\nu} \simeq \frac{1}{2} \frac{\partial g_{44}}{\partial x^{\nu}}$ , it follows that only  $g_{44}$  is needed in order to obtain Newton's law of gravitation.

The fact that already in first approximation,  $g_{11}, g_{22}, g_{33}$  are different from zero, leads to other effects such as the deflection of light rays and the effects of the gravitational field on clocks and rods. For the deflection of light rays, Einstein found twice the value he had obtained previously in the Entwurf theory. Thus, for a light ray grazing the sun, he found for the deflection  $1,7''$  (instead of the previous  $0,85''$ ).<sup>39</sup> Einstein derived the effect as follows. From the equation of propagation of light rays  $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = 0$ , which is a generalization of the STR equation for light rays  $ds^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu} = 0$ , Einstein computed for a given direction (i. e. for given ratios  $dx^1 : dx^2 : dx^3$ ) the components  $dx^1/dx^4, dx^2/dx^4, dx^3/dx^4$ , and thus obtained the velocity of light. Since the velocity of light depends on the  $g_{\mu\nu}$ , the situation is similar to a medium with

39. Einstein, "Perihelbewegung des Merkur," p. 834.

a variable refractive index; in both cases, the resulting deflection of light rays is obtained by applying Huygen's principle.

Concerning the effects of the gravitational field on clocks and rods, Einstein established them as follows.<sup>40</sup> Considering a clock at rest at the origin of coordinates [ $dx^1 = dx^2 = dx^3 = 0$ ,  $ds^2 = g_{44} (dx^4)^2$ ], and adopting a unit proper time interval  $ds = 1$ , Einstein obtained, in first approximation, for the coordinate time,

$$dx^4 = \frac{1}{\sqrt{g_{44}}} = 1 + \frac{K}{c^2} \int \frac{\rho d\tau}{r},$$

where  $\rho$  is the density and  $d\tau$  a three-dimensional volume element. Hence,  $dx^4$  increases if masses are brought into the neighborhood of the clock, which led Einstein to conclude that the clock slows down (i.e., for a given proper time interval, the conceptual coordinate times differ; consequently, for the same coordinate time interval, the proper time intervals indicated by the clocks will differ). A consequence of this is that the solar spectral lines must appear to be shifted toward the red when observed from the earth. As to the effect of the gravitational field on rods, Einstein proceeded similarly. If a unit rod ( $ds^2 = -1$ ) is placed radially from the sun along the  $x^1$  axis, its coordinate length  $dx^1$  is given by

$$ds^2 = -1 = g_{11} (dx^1)^2 \quad (dx^2 = dx^3 = dx^4 = 0),$$

or

$$dx^1 = 1 - \frac{\alpha}{2r},$$

since

$$g_{11} = - \left( 1 + \frac{\alpha}{r} \right).$$

From this, it follows that the rod appears contracted in the radial direction. If the rod is placed tangentially at  $x^1 = r$  (with  $x^2 = x^3 = 0$ ,  $dx^1 = dx^3 = dx^4 = 0$ ), then  $ds^2 = -1 = g_{22} (dx^2)^2$ , and since  $g_{22} = -1$ , there is no influence on the rod in that direction.

### 5.2.2 Second Order Effect : Residual Precession of the Perihelion of Mercury

In his 18 November paper,<sup>41</sup> Einstein derived the residual precession of the perihelion of Mercury (i.e., the precession which could not be explained on the basis of Newton's theory) from the second order solution of the field equations and the second order equations of motion. For the latter equations, Einstein found after various approximations

$$\frac{d^2 x^\nu}{ds^2} = -\frac{\alpha}{2} \frac{x^\nu}{r^3} \left[ 1 + \frac{\alpha}{r} + 2u^2 - 3 \left( \frac{dr}{ds} \right)^2 \right] \quad (5.9)$$

40. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie" (1915), pp. 818-20.

41. Einstein, "Perihelbewegung des Merkur" (1915).

for  $\nu = 1, 2, 3$ , where

$$u^2 = \frac{dr^2 + r^2 d\Phi^2}{ds^2}$$

and  $r$  is the distance of the planet from the sun; for the fourth component (which needs only be given up to the first order), he obtained

$$\frac{dx^4}{ds} = 1 + \frac{\alpha}{r}.$$

After various transformations of variables, Einstein integrated equation (5.9) and found that during a period of revolution, the perihelion of a planet would advance in the direction of the revolution of the planet by

$$\epsilon = 24\pi^3 \frac{a^2}{T^2 c^2 (1 - e^2)},$$

where  $a$  is the semimajor axis,  $e$  the eccentricity, and  $T$  the period of revolution. In the case of Mercury, Einstein calculated the advance to be  $43''$  per century. Comparing it to the astronomical value of the residual precession,  $45'' \pm 5''$ , Einstein concluded with satisfaction that the agreement was complete. In the case of the Earth and Mars, Einstein found that the computed values, respectively  $4''$  and  $1''$ , differed from the values, respectively  $11''$  and  $9''$ , inferred from astronomical data. Einstein, however, had reasons to doubt the latter values on the basis of uncertainty margins provided by Newcomb (communicated to Einstein by Erwin Freundlich) and argued that the perihelion advance appeared to be really established only in the case of Mercury.

Einstein's approximate solution of the field equations was confirmed in early 1916 by the astronomer Karl Schwarzschild who, in two papers,<sup>42</sup> gave rigorous solutions of the field equations for a material point and for an incompressible fluid sphere acting as sources. In the first paper, Schwarzschild, adopting Einstein's general assumptions (about the metric, etc.), derived the metric of a spherically symmetric field created by a material point, now known as the Schwarzschild metric

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \frac{dR^2}{1 - \frac{\alpha}{R}} - R^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

where  $R = (r^3 + \alpha^3)^{\frac{1}{3}}$  and where  $\alpha$  is a constant depending on the mass of the material point. The paper not only confirmed Einstein's approximate solution of the field equations but also established its necessary character, whereas Einstein had only established its sufficiency. In the second paper, Schwarzschild examined, in particular, the consequences of what is now known as the "Schwarzschild-radius." If an incompressible fluid star of a given mass has a radius smaller than

42. K. Schwarzschild, "Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften, Berlin*, part 1 (1916), pp. 42, 189-96; "Über das Gravitationsfeld einer Kugel inkompressibler Flüssigkeit nach der Einsteinschen Theorie," *ibid.*, pp. 313, 424-34.

the Schwarzschild radius, it collapses and becomes a black hole, a solution which Schwarzschild, at the time, considered to be physically meaningless because it involved an infinite pressure at the center. For the sun, Schwarzschild found the value of the radius to be equal to  $3km$ .<sup>43</sup>

### 5.2.3 Experimental verification

We have already mentioned Erwin Freundlich's early efforts to verify the predictions of the equivalence principle. In fact, according to Einstein, Freundlich was the only active supporter of *GTR*; in a letter to Sommerfeld dated 2 February 1916, Einstein wrote : "Freundl [ich] was the only colleague who has actively supported me hitherto in my endeavors in the domain of general relativity."<sup>44</sup> Einstein was to refer to Freundlich's papers several times<sup>45</sup> and also wrote a preface to Freundlich's book, *Die Grundlagen der Einsteinschen Gravitationstheorie*, published in 1916.<sup>46</sup>

Freundlich organized an expedition to Russia for the solar eclipse of August 1914, but neither he nor Einstein could obtain funding either from the Berlin Observatory or from the Berlin Academy. Funding finally came through private sources, from the chemist Emil Fischer and the industrialist Krupp.<sup>47</sup> As to the outcome of the eclipse test, Einstein, apparently, awaited the result with a certain equanimity. In 1912, to the question of a student asking whether it was not tormenting ("quälend") to him to have to wait years for the next solar eclipse, Einstein answered "Well, you know, if someone like myself has had to throw so much of the stuff he has brooded over in the paper basket, then he is no longer so bent on knowing whether he will be right or not in the end."<sup>48</sup> Einstein's reply suggests that his published work constituted only the tip of the iceberg and that a great deal of his efforts, in the end, found their way into the paper basket. Having his eye on longterm goals, Einstein did not feel threatened by temporary setbacks and was determined to pursue the search. On the other hand, once Einstein had a theory showing a sufficient inner consistency, his confidence in the theory could survive isolated experimental setbacks. Thus, in a letter to Michele Besso, written probably at the beginning of March 1914, Einstein wrote : "Now I am fully satisfied and no longer doubt the correctness of the whole system, whether the observation of the solar eclipse succeeds or

43. Schwarzschild, "Gravitationsfeld einer Kugel," p. 434.

44. "Freundl [ich] war der einzige Fachgenosse, der mich bis jetzt in meinen Bestrebungen auf dem Gebiete der allgemeinen Relativität wirksam unterstützte," Einstein à Sommerfeld, 2 Feb. 1916, *Briefwechsel* (1968), p. 39.

45. Einstein, "Über den Grundgedanken und Anwendung" (1915), p. 315 ; "Perihelbewegung des Merkur" (1915), pp. 831, 839 ; "Die Grundlage der allgemeinen Relativitätstheorie" (1916), p. 820.

46. Erwin Freundlich, *Die Grundlagen der Einsteinschen Gravitationstheorie*, 2nd enl. and improved ed (Berlin : J. Springer, 1917).

47. Lewis R. Pyenson, "The Goettingen Reception of Einstein's General Theory of Relativity" (Ph. D. dissertation, Johns Hopkins University, 1974), pp. 324-25.

48. "Ach wissen Sie, wer wie ich soviel vom dem, an dem er herumgegrübelt hat, in den Papierkorb schmeissen musste, ist nicht mehr so darauf versessen, zu wissen, ob man nun recht behalten wird oder nicht, " in Seelig, *Albert Einstein* (1960), p. 169.

not. The good sense of the matter is too evident."<sup>49</sup> Freundlich's solar eclipse expedition to Russia, however, collapsed when the war broke out, Freundlich himself becoming a prisoner for a short time. Had the expedition been successful, exactly twice the value of the predicted deviation would have been observed, because Einstein's field equations were still unsatisfactory at that time.

In 1914, Freundlich also attracted attention to the prediction of a shift of spectral lines in a gravitational field.<sup>50</sup> Freundlich, initially, had the impression that the effect existed but, in another paper,<sup>51</sup> indicated that evidence by Schwarzschild<sup>52</sup> seemed to go in the opposite direction. In his paper, Freundlich nevertheless found restricted support for a spectral shift for certain stars, to which Einstein referred twice.<sup>53</sup>

As to the precession of the perihelion of Mercury, Freundlich in 1915 endeavored to establish its anomalous nature by showing that it could not be explained on the basis of Newton's theory.<sup>54</sup> The question was of importance to Einstein, who referred to Freundlich's paper as a "noteworthy paper" ("beachtenswerten Aufsatz"<sup>55</sup>) because only the absence of a good explanation could justify a search for a better one.<sup>56</sup> To establish his case, Freundlich attacked a recent attempt to explain the anomaly - made by the distinguished astronomer Hugo von Seeliger - which had found widespread acceptance.<sup>57</sup> In 1906, von Seeliger, who between 1902 and 1916 was president of the German Astronomical Society,<sup>58</sup> had attempted to explain the residual precession of the Mercury perihelion by making certain assumptions on the density of the particles producing

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49. "Nun bin ich vollkommen befriedigt und zweifle nicht mehr an der Richtigkeit des ganzen Systems, mag die Beobachtung der Sonnenfinsternis gelingen oder nicht. Die Vernunft der Sache ist zu evident," Einstein to Besso, March 1914, *Correspondance* (1972), p. 53.

50. Erwin Freundlich, "Über die Verschiebung der Sonnenlinien nach dem roten Ende auf Grund der Hypothesen von Einstein und Nordström," *Physikalische Zeitschrift*, (1914), **15**, 369-71; "Über die Verschiebung der Sonnenlinien nach dem roten Ende des Spektrums auf Grund der Äquivalenzhypothese von Einstein," *Astronomische Nachrichten*, **198**, no. 4742 (1914), cols. 265-70.

51. Erwin Freundlich, "Über die Gravitationsverschiebung der Spektrallinien bei Fixsternen," *Physikalische Zeitschrift*, **16** (1915), 115-17.

52. K. Schwarzschild, "Über die Verschiebungen der Bande bei 3883 Å im Sonnenspektrum," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften*, Berlin, part 2 (1914), pp. 1201-13.

53. Einstein, "Über den Grundgedanken und Anwendung" (1915), p. 315; "Grundlage der allgemeinen Relativitätstheorie" (1916), p. 820.

54. Erwin Freundlich, "Über die Erklärung der Anomalien im Planetensystem durch die Gravitationswirkung interplanetarer Massen," *Astronomische Nachrichten*, **201**, no. 4803 (1915), cols. 49-56.

55. Einstein, "Perihelbewegung des Merkur" (1915), p. 831.

56. For an analysis of the various explanation attempts, see Warren Z. Watson, "An Historical Analysis of the Theoretical Solutions to the Problem of the Advance of the Perihelion of Mercury" (Ph.D. dissertation, University of Wisconsin, 1969)

57. Freundlich, "Anomalien im Planetensystem" (1915), col. 50.

58. F. Schmiedler, "Seeliger, Hugo von," *Dictionary of Scientific Biography*, **12** (1975), 282-83, on 282.



the zodiacal light.<sup>59</sup> Von Seeliger defended his theory<sup>60</sup> against Freundlich's criticism and, subsequently, criticized Freundlich for a calculational error the latter made in another paper, the result of all this being that Freundlich's relations with the astronomical establishment became rather strained. Einstein defended Freundlich<sup>61</sup> and later helped him to find an adequate position.

Concerning the subsequent verifications of *GTR*, these lie beyond the scope of this dissertation. In brief, in the period from 1915 up to the early 1950s, only the prediction of the anomalous precession of the perihelion of Mercury appeared to be securely established. Though the deflection of light rays seemed to be verified in 1919,<sup>62</sup> other eclipse tests up to 1952 gave a variety of results which did not reproduce the initial success. The verification of the shift of spectral lines, which had been controversial from the very beginning, was to remain so for many decades.<sup>63</sup> Beginning in the 1950s, scientific experimentation finally reached the level of sophistication needed for *GTR* and by now (1981) all three classical tests have been confirmed. New tests such as signal retardation (based on the variation of  $c$  in a gravitational field) have been confirmed and others such as the detection of gravitational waves, the Lense-Thirring effect and the geodetic precession are being undertaken.<sup>64</sup>

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59. H. V. Seeliger, "Das Zodiakallicht und die empirischen Glieder in der Bewegung der inneren Planeten," *Sitzungsberichte der K. B. Akademie der Wissenschaften, München, mathematisch-physikalische Klasse*, **36** (1906), 595-622.

60. H. v. Seeliger, "Über die Anomalien in der Bewegung der inneren Planeten," *Astronomische Nachrichten*, **201** (1915), cols. 273-80.

61. Einstein to Sommerfeld, 2 Feb. 1916, *Briefwechsel* (1968), pp. 38-39.

62. D. F. Moyer, "Revolution in Science : the 1919 Eclipse Test of General Relativity," in *On the Path of Albert Einstein*, eds. Arnold Perlmutter and Linda F. Scott (New York : Plenum, 1979), pp. 55-101.

63. John Earman and Clark Glymour, "The Gravitational Red Shift as a Test of General Relativity : History and Analysis," *Studies in History and Philosophie of Science*, **11** (1980), 175-214. See also E. Forbes, "A History of the Solar Red Shift Problem," *Annals of Science*, **17** (1961), 129-164.

64. See for examples : (a) Irwin I. Shapiro, "Experimental Challenges Posed by the General Theory of Relativity," in *Some Strangeness in the Proportion*, ed. Harry Woolf (Reading, MA : Addison-Wesley, 1980), pp. 115-36 ; (b) Ramanath Cowsik, "Relativity Experiments in Space," in *Gravitation, Quanta and the Universe*, eds. A. R. Prasanna, J. V. Narlikar, C. V. Vishveshawara (New York : John Wiley & Sons, 1980), pp. 18-40 ; (c) C. M. Will, "The confrontation between gravitation theory and experiment," in *General Relativity : An Einstein Centenary Survey*, eds. S. W. Hawking, W. Israel (Cambridge : Cambridge University Press, 1979), chap. 2, pp. 24-89. (d) D. H. Douglass and V. B. Braginsky, "Gravitational-radiation experiments," *ibid.*, chap. 3, pp. 90-137 ; (e) C. W. F. Everitt, "Gravitation, Relativity, and Precise Experimentation," in *Proceedings of the First Marcel Grossmann Meeting on General Relativity*, ed. Remo Ruffini (Amsterdam : North-Holland, 1977), pp. 545-615 ; (f) for references on tests up to 1964, see Marie-Antoinette Tonnelat, *Les vérifications expérimentales de la Relativité Générale* (Paris : Masson et Cie, 1964), pp. 279-98.

## 5.3 Extension of the Theory : The Cosmological Constant (1917)

Though Einstein's 1915 theory was (and still is) quite successful, Einstein was not entirely satisfied with his theory and, in 1917, modified the field equations. We shall examine successively Einstein's dissatisfaction and the modification of the field equations.

### 5.3.1 Einstein's Epistemological Dissatisfaction

#### Epistemological background

We have seen that Einstein was guided by epistemological considerations during the development of both *STR* and *GTR*. In the case of *STR*, it was the rejection of the concept of absolute time that finally led to the theory, whereas in the case of *GTR*, it was the rejection of the concept of absolute space which motivated Einstein. Einstein's interest in and defense of epistemological considerations as guides to theoretical physics is probably nowhere better stated than in his eulogy of Ernst Mach,<sup>65</sup> who died on 19 February 1916. There Einstein described Mach as a man "who had a most important influence on the epistemological orientation of the scientists of our time."<sup>66</sup>

Rejecting the implicit criticism of some colleagues who argued that there were more valuable things to do than epistemological researches, Einstein pointed out that anybody who is not just interested in science because of superficial reasons, is naturally led to ask epistemological questions concerning the goal of science, the truth content of its results, and the relative importance to be attributed to various developments.<sup>67</sup> Einstein added that the "truth" (Wahrheit) in these questions had to be reformulated again and again by "strong characters" (kräftigen Naturen) if it was not to be lost altogether.<sup>68</sup> Einstein made it clear that far from being a "vain game" (müßige Spielerei), he considered historical analyses of familiar concepts, which often lead to an appreciation of the limitations of these concepts, to be of great importance for the progress of science. Einstein remarked that such analyses often appear to the specialized scientist as "superfluous, pompous, sometimes even ridiculous";<sup>69</sup> yet, when scientific progress makes a conceptual shift necessary, he added :

Then, those who did not handle their own concepts properly, vigorously protest and complain about a revolutionary threat to the most sacred possessions. In this outcry, they are then joined by those philosophers who think they cannot dispense that particular concept because they had placed it in their jewelry box of the "Absolute,"

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65. A. Einstein, "Ernst Mach," *Physikalische Zeitschrift*, **17** (1916), 101-04.

66. *Ibid.*, p. 101.

67. *Ibid.*

68. *Ibid.*, pp. 101-02

69. "Überflüssig, gepreizt, zuweilen gar lächerlich," *ibid.*, p. 102.

or the "a priori," or simply because they had proclaimed its immutability as a matter of principle.<sup>70</sup>

Einstein added that he was of course referring in particular to the concepts of space and time and other mechanical concepts that were modified by *STR*. Einstein acknowledged the decisive influence which the epistemologists had on that development : "Nobody can take it from the epistemologists that they have here smoothed the paths of that development ; concerning myself at least, I know that I have been very stimulated directly and indirectly, in particular by Hume and Mach."<sup>71</sup> Einstein referred the reader to Sections 6 and 7 of the second chapter of Mach's *Mechanik*, where he stated one would find thoughts "masterly presented which by no means have yet become the common property of the physicists";<sup>72</sup> Einstein quoted at length a "few pearls" (einige Rosinen) dealing with the concepts of absolute time, absolute space and Mach's "very interesting" criticism of Newton's bucket experiment. Einstein's conclusion was that he considered Mach to be a forerunner of *STR* and *GTR* :

The quoted lines show that Mach clearly recognized the weak points of classical mechanics and was not very far from requiring a general theory of relativity, and this already about half a century ago! It is not improbable that Mach would have discovered the theory of relativity...

The considerations on Newton's bucket experiment show how close to his mind was the requirement of relativity in the generalized sense (relativity of acceleration).<sup>73</sup>

According to Einstein, what prevented Mach from developing *STR* and *GTR* was, in the first case, that Mach did not feel the necessity for a new definition of simultaneity for spatially separated events, because the constancy of the velocity of light had not yet received widespread attention ; in the case of *GTR*,

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70. "Dann erheben diejenigen, welche den eigenen Begriffen gegenüber nicht reinlich verfahren sind, energischen Protest und klagen über revolutionäre Bedrohung der heiligsten Güter. In dies Geschrei mischen sich dann die Stimmen derjenigen Philosophen, welche jenen Begriff nicht entbehren zu können glauben, weil sie ihn in ihr Schatzkästlein des "absoluten" des "a priori" oder kurz derart eingereiht hatten, dass sie dessen prinzipielle Unabänderlichkeit proklamiert hatten," *ibid.*, p. 102.

71. "Niemand kann es den Erkenntnistheoretikern nehmen, dass sie der Entwicklung hier die Wege geebnet haben ; von mir selbst weiss ich mindestens, dass ich insbesondere durch Hume und Mach direkt und indirekt sehr gefördert worden bin," *ibid.*, p. 102.

72. "Meisterhaft dargelegt, die noch keineswegs Gemeingut der Physiker geworden sind," *ibid.*, p. 102.

73.

"Die zitierten Zeilen zeigen, dass Mach die schwachen Seiten der klassischen Mechanik klar erkannt hat und nicht weit davon entfernt war, eine allgemeine Relativitätstheorie zu fordern, und dies schon vor fast einem halben Jahrhundert ! Es ist nicht unwahrscheinlich, dass Mach auf die Relativitätstheorie gekommen wäre, ...

Die Betrachtungen über Newtons Eimerversuch zeigen, wie nahe seinem Geiste die Forderung der Relativität im allgemeineren Sinne (Relativität der Beschleunigungen) lag," *ibid.*, p. 103.

### 5.3. EXTENSION OF THE THEORY : THE COSMOLOGICAL CONSTANT (1917)139

Einstein attributed the lost opportunity to a lack of awareness of the idea that the equality of inertial and gravitational masses calls for an extension of the relativity principle, because of the impossibility of distinguishing an inertial field from a gravitational field.<sup>74</sup>

About the same time that Einstein wrote the eulogy, he also completed his comprehensive review paper on *GTR*<sup>75</sup> of 1916. The review paper, which became the standard version of *GTR*, confirms the importance of epistemological considerations in the development of *GTR*. There, Einstein again presented the elimination of the concept of absolute space (absolute acceleration) as the main argument for the extension of the *STR* relativity principle, and adopted Mach's hypothesis as solution to the "inherent epistemological defect"<sup>76</sup> of classical mechanics and *STR*, namely the failure to provide a causal explanation for the origin of inertial effects.

To illustrate his point, Einstein returned to a favorite example.<sup>77</sup> Let  $S_1$  and  $S_2$  be two initially identical fluid masses at rest ( see Fig. 5.1) ; if  $S_2$  is rotating, two observers placed on  $S_1$  and  $S_2$  respectively will observe the same magnitude of the relative speed of rotation, yet the first observer will find  $S_1$  to be a sphere whereas the second observer will find  $S_2$  to be an ellipsoid. Asking for the cause of the behavior, Einstein remarked that if it is to be epistemologically sound the cause must refer to an observable experimental fact. Consequently, Einstein dismissed Newton's explanation in terms of absolute space as providing only a "fictitious cause"<sup>78</sup> and presented his own conclusions as follows

We have to take it that the general laws of motion, which in particular determine the shapes of  $S_1$  and  $S_2$ , must be such that the mechanical behaviour of  $S_1$  and  $S_2$  is partly conditioned, in quite essential respects, by distant masses which we have not included in the system under consideration. These distant masses and their motions relative to  $S_1$  and  $S_2$  must then be regarded as the seat of the causes (which must be susceptible to observation) of the different behaviour of our bodies  $S_1$  and  $S_2$ .<sup>79</sup>

Thus, Einstein adopted Mach's hypothesis - - that is, the assumption that the relative motion of the fluid masses with respect to distant masses is the cause of the inertial forces.

Since the goal of Mach's hypothesis is to eliminate the concept of absolute space, Einstein remarked that a general relativity must be postulated at the same time if privileged systems are not to be introduced once again. Einstein formulated the principle of general relativity as follows : "The laws of physics must be of such a nature that they apply to systems of reference in any kind

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74. Ibid.

75. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie" (1916), pp. 769-822.

76. Ibid., p. 771.

77. A. Einstein, "Zum Relativitätsproblem," *Scientia* (Bologna), **15** (1914), 337-48, on 344-46.

78. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie" (1916), p. 771.

79. Ibid., p. 772 ; trans. by W. Perrett and G.B. Jeffery, in Lorentz, Einstein et al., *The Principle of Relativity* (New York : Dover, 1952), p. 113.

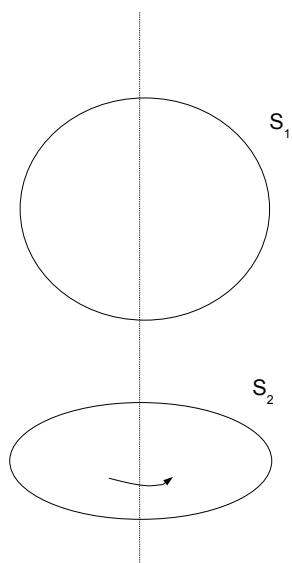


FIGURE 5.1 – Relative rotation of two fluid masses

of motion."<sup>80</sup> A few pages later, Einstein also defined the principle of general covariance: "The general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are covariant with respect to any substitutions whatever (generally covariant)."<sup>81</sup> Specifying the difference between what he meant by general relativity and general covariance, Einstein stated that the first referred to all relative motions of three-dimensional coordinate systems whereas the second referred to any four-transformation. By requiring general covariance, general relativity was automatically fulfilled. Besides the above "weighty epistemological argument,"<sup>82</sup> Einstein presented also the equality of inertial and gravitational masses as another evidence for the need to generalize *STR*; again he stated that the impossibility to distinguish an inertial field from a gravitational one, makes the concept of absolute acceleration meaningless.<sup>83</sup>

Einstein apparently thought that his theory by being generally covariant eliminated the concepts of absolute space and absolute time. Thus on 18 November 1915, he remarked that the theory "bereft" (beraubt) time and space of the "last trace of objective reality";<sup>84</sup> he made a similar remark in the 1916 review paper.<sup>85</sup> Einstein's joy of having achieved, in particular general covariance appears in a postcard, dated 10 December 1915, to Besso: "The boldest dreams have now been fulfilled. General covariance. Motion of the perihelion of Mercury, wonderfully precise."<sup>86</sup> As to the relativity of inertia, Einstein was soon to check whether his theory yielded that effect.

### The problem of the relativity of inertia

That Einstein was concerned with the relativity of inertia in early 1916, is clear from a communication he made to the Berlin Academy on 23 March 1916. There, Einstein discussed, among other things, the effect that the earth's rotation has on a Foucault pendulum according to the new theory.<sup>87</sup> Similarly, in a postcard to Besso, Einstein explained how to find the Coriolis and centrifugal fields created by a rotating ring.<sup>88</sup>

80. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie" (1916), p. 772; trans. in *Principle of Relativity* (1952), p. 113.

81. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie" (1916), p. 772; trans. in *Principle of Relativity* (1952), p. 113.

82. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie" (1916), p. 772.

83. *Ibid.*, pp. 772-73.

84. "Durch welche Zeit und Raum der letzten Spur objektiver Realität beraubt werden," in Einstein, "Perihelbewegung des Merkur" (1915), p. 831.

85. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie" (1916), p. 776.

86. "Die kühnsten Träume sind nun in Erfüllung gegangen. Allgemeine Kovarianz. Perihelbewegung des Merkur wunderbar genau," Einstein to Besso, 10 Dec. 1915, *Correspondance* (1972), p. 60.

87. A. Einstein, "Über einige anschauliche Überlegungen aus dem Gebiete der Relativitätstheorie," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften, Berlin*, part 1 (1916), p. 423.

88. Einstein to Besso, 31 July 1916, *Correspondance* (1972), p. 77; see also Einstein to Besso, 31 Dec. 1916, *ibid.*, p. 86.

The historical significance of the relativity of inertia during that period was that it led Einstein to cosmological considerations, which in turn led him to modify the field equations; in a letter to Besso dated 14 May 1916, Einstein wrote: "With respect to gravitation, I am now looking for the boundary conditions at infinity; it is after all interesting to consider to what extent a finite world can exist, namely, a world of finite extension, in which all inertia is indeed relative."<sup>89</sup> The problem Einstein faced was as follows. The gravitational field equations being differential equations, boundary conditions must be given in order to determine a solution. So far, in the problem of planetary motion, Einstein had simply assumed the  $g_{\mu\nu}$  to have the *STR* metric as limit at spatial infinity. For an extensive distribution of matter, however, such boundary conditions were unsatisfactory from the point of view of the relativity of inertia since the inertia of a body (which depends on the  $g_{\mu\nu}$ ) would not vanish at spatial infinity. Einstein, however, was not to find suitable boundary conditions. The alternative he proposed was a closed universe; in another letter to Besso, Einstein remarked that the main point here was to ensure that the  $g_{\mu\nu}$ , were causally determined by the matter of the universe in order to secure the relativity of inertia.<sup>90</sup>

### 5.3.2 Modification of the Field Equation

On 8 February 1917, Einstein argued in favor of a spatially closed universe and a modification of the field equations in a paper entitled "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie."<sup>91</sup> This paper opened the era of relativistic cosmologies. The paper deals with two main themes: (a) the problem of boundary conditions in *GTR*; (b) the solution adopted to that problem, namely a spatially closed universe—which, in Einstein's view, made a modification of the field equations necessary.

In order to illustrate the problem of boundary conditions and the modification of the field equations, Einstein first presented the case of the Newtonian cosmology as follows. Starting from the well known fact that Poisson's differential equation  $\Delta\Phi = 4\pi K\rho$  determines a solution only if boundary conditions are given, Einstein indicated that for the latter the condition  $\Phi \rightarrow \text{constant}$  at spatial infinity is usually adopted. Einstein pointed out, however, that it is "a priori not at all evident"<sup>92</sup> that such boundary conditions can be used for cosmological purposes. Indeed, Einstein's own conclusion was that such conditions do not work. Adopting, however, for the moment the above boundary condition, Einstein analyzed its Newtonian cosmological consequences. If  $\Phi \rightarrow \text{constant}$  at

89. "In der Gravitation suche ich nun nach den Grenzbedingungen im Unendlichen; es ist doch interessant, sich zu überlegen, inwiefern es eine endliche Welt gibt, d.h. eine Welt von natürlich gemessener endlicher Ausdehnung, in der wirklich alle Trägheit relativ ist," Einstein à Besso, 14 May 1916, *Correspondance* (1972) p. 69.

90. Einstein to Besso, Dec. 1916, *Correspondance* (1972), p. 96.

91. A. Einstein, "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften, Berlin*, part 1 (1917), pp. 142–52.

92. *Ibid.*, p. 142.

spatial infinity, this implies that  $\rho$  must tend toward zero more rapidly than  $1/r^2$ . Hence, the world resembles a finite island in an infinite empty universe. Since such a world would lose energy through radiation and escape of stars, one could attempt to make at least the diffusion of stars impossible by postulating a very high gravitational potential at spatial infinity. Einstein, however, rejected this solution as being in contradiction with the low velocities of stars observed at the time. Einstein took the observational result of the low velocities of stars (which was found in the late 1920s to be erroneous) very seriously and referred to it four times as being a hard fact.<sup>93</sup> Besides this objection, he also indicated that the finite island model must already be rejected for statistical reasons since a finite ratio of the gravitational potentials entails a finite ratio of the densities (in the case of statistical equilibrium). If  $\rho$  is zero at infinity, it must vanish at the center of the island as well. Hence, there is no solution within the Newtonian system.

Placing himself outside the Newtonian system, Einstein then presented another non-relativistic attempt in order to prepare the reader for the introduction, at the end of the paper, of a supplementary term in the *GTR* field equations. Einstein's idea was to adopt the modified Poisson's equation  $\Delta\Phi - \lambda\Phi = 4\pi K\rho$ , which makes an infinite universe of uniform density (with a constant potential  $\Phi = -\frac{4\pi K}{\lambda}\rho_0$  at spatial infinity) possible.

Concerning the boundary conditions within *GTR*, Einstein described the "somewhat rough and winding road"<sup>94</sup> of how he tried and failed to find suitable boundary conditions. Einstein stated that the guideline he followed in that search was the relativity of inertia :

The opinion which I entertained until recently, as to the limiting conditions to be laid down at spatial infinity, took its stand on the following considerations. In a consistent theory of relativity there can be no inertia *relative to "space"*, but only an inertia of masses *relative to one another*. If therefore, I have a mass at a sufficient distance from all other masses in the universe, its inertia must fall to zero.<sup>95</sup>

Mathematically, Einstein tried to incorporate the relativity of inertia as follows. Assuming an isotropic metric of the form

$$ds^2 = -A(dx_1^2 + dx_2^2 + dx_3^2) + Bdx_4^2,$$

where  $A$  and  $B$  are functions of the coordinates, together with the simplifying restriction  $\sqrt{-g} = 1 = \sqrt{A^3B}$ , Einstein obtained respectively for the energy

93. Ibid., pp. 143, 146 148, 152.

94. "Etwas indirekten und holperigen Wege," *ibid.*, p. 144.

95. "Meine bis vor kurzem gehegte Meinung über die im räumlich Unendlichen zu setzenden Grenzbedingungen fusste auf folgenden Überlegungen. In einer konsequenten Relativitätstheorie kann es keine Trägheit *gegenüber dem 'Raume'* geben, sondern nur eine Trägheit der Massen gegeneinander. Wenn ich daher eine Masse von allen anderen Massen der Welt räumlich genügend entferne, so muss ihre Trägheit zu Null herabsinken," *ibid.*, p. 145, emphasis in original; trans. from *The Principle of Relativity* (1952), p. 180.



(in the case of rest) and the momentum (in first approximation and for small velocities) of a particle :

$$E = m\sqrt{B};$$

$$P_1 = m \frac{A}{\sqrt{B}} \frac{dx_1}{dx_4}, P_2 = m \frac{A}{\sqrt{B}} \frac{dx_2}{dx_4}, P_3 = m \frac{A}{\sqrt{B}} \frac{dx_3}{dx_4}.$$

From these expressions it follows that the inertia of the particle is given by

$$m \frac{A}{\sqrt{B}},$$

where  $m$  is a constant independent of the position. If the inertia is to decrease at zero at spatial infinity as suggested by the relativity of inertia, then one must have the boundary conditions  $A \rightarrow 0, B \rightarrow \infty$  (since  $\sqrt{A^3 B} = 1$ ). The condition  $B \rightarrow \infty$  implies, however, that the (potential) energy  $m\sqrt{B}$  becomes infinite at infinity which, according to Einstein was in contradiction with the observed low star velocities.<sup>96</sup> In the end, it was this experimental fact that ruined Einstein's attempt.

Having admitted failure, Einstein saw two other options : (a) to adopt the *STR* metric as boundary condition at spatial infinity ; (b) to renounce generally valid boundary conditions and to assume specific boundary conditions for each case. Einstein rejected possibility (a) because it privileged a reference system, which was contrary to the spirit of the relativity principle, and also because it violated the relativity of inertia. A supplementary objection against (a) came from statistical considerations similar to those of the Newtonian case. As to the possibility (b), Einstein did not like it because it meant renunciation of a general solution, and he saw this only as a last resort.

Having rejected all these approaches, Einstein reasoned that if the universe were spatially closed, then there would be no need for boundary conditions at all. Einstein found that the metric corresponding to the closed world model did not satisfy the 1915 field equations but a slightly modified set of equations

$$R_{\mu\nu} - \lambda g_{\mu\nu} = -\varkappa \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right).$$

These generally covariant equations were compatible with both the available experimental data and the conservation equations of energy-momentum. In the conclusion of the paper, Einstein made the interesting remark that even without the coefficient  $\lambda$  (which is now called the cosmological constant), there would be a positive curvature of space and that  $\lambda$  was only needed in order to ensure a quasi-static distribution of matter as demanded by the "fact" of the low velocities of stars.<sup>97</sup>

Aside from the question of boundary conditions, Einstein had another reason for modifying the 1915 field equations. Since these questions allow the solution  $g_{\mu\nu} = \text{constant}$  for  $T_{\mu\nu} = 0$ , this metric can exist without any matter. Hence,

96. Einstein, "Kosmologische Betrachtungen" (1917), p. 146.

97. Ibid., p. 152.

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there existed inertial properties (determined by  $g_{\mu\nu}$ ) not related to matter, which was unacceptable to Einstein because it was in violation of the relativity of inertia. Thus, in a letter to De Sitter dated 24 March 1917, Einstein wrote :

In my opinion it would be unsatisfactory, if a conceivable world existed without matter. The  $g_{\mu\nu}$  field must rather be *determined by matter, and vanish in the absence the latter*. This is the essence, of what I understand under the requirement of the relativity of inertia.<sup>98</sup>

Initially, Einstein<sup>99</sup> had hoped that his new field equations would not allow the existence of an empty universe (i.e., would not have a solution  $g_{\mu\nu}$  for  $T_{\mu\nu} = 0$ ) ; but De Sitter<sup>100</sup> showed, Einstein to be wrong on that point. Later on, following Hubble's discovery of the expansion of the universe and the theoretical work by A. Friedman,<sup>101</sup> Einstein proposed to drop the cosmological term in the name of "logical economy ."<sup>102</sup>

In 1918, in a paper entitled "Prinzipielles zur allgemeinen Relativitätstheorie" Einstein was to raise the relativity of inertia to the rank of an independent principle, to which he gave the name "Mach's principle," on an equal footing with the general covariance principle and the equivalence principle. Einstein defined Mach's principle as follows :

The  $G$ -field [ $g_{\mu\nu}$  tensor] is completely determined by the masses of the bodies. Since mass and energy are identical according to the results of the special theory of relativity and since the energy is formally described by the symmetric energy tensor ( $T_{\mu\nu}$ ), this means that the  $G$ -field is conditioned and determined by the energy tensor of matter.<sup>103</sup>

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98. "Es wäre nach meiner Meinung unbefriedigend, wenn es eine denkbare Welt ohne Materie gäbe. Das  $g_{\mu\nu}$ -Feld soll vielmehr durch die Materie bedingt sein, ohne dieselbe nicht bestehen können. Das ist der Kern dessen, was ich unter der Forderung von der Relativität der Trägheit verstehe," W. De Sitter, "On the relativity of inertia. Remarks concerning EINSTEIN's latest hypothesis," *Proc. K. Akad. Wet., Amsterdam, Section Sciences*, **19** (1917), 1217-25, on 1225 ; see also C. Kahn and F. Kahn, "Letters from Einstein to De Sitter on the nature of the Universe," *Nature*, **257** (9 Oct. 1975), 451-54.

99. A. Einstein, "Prinzipielles zur allgemeinen Relativitätstheorie," *Annalen der Physik*, **55** (1918), 241-44, on 243.

100. W. De Sitter, "On Einstein's Theory of Gravitation, and Its Astronomical Consequences," *Monthly Notices of the Royal Astronomical Society*, **78** (Nov. 1917), 3-28, on 7.

101. A. Friedman, "über die Krümmung des Raumes," *Zeitschrift für Physik*, **10** (1922), 377-86 ; "Über die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes," *ibid.*, **21** (1924), 326-32 ; see also A. Einstein, "Zum kosmologischen Problem der allgemeinen Relativitätstheorie," *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften, Berlin*, (1931), pp. 235-37.

102. Albert Einstein, *The Meaning of Relativity*, 5th ed. (1956 ; rpt. Princeton, NJ : Princeton University Press, 1974), p. 127.

103. "*Machsches Prinzip* : Das  $G$ -Feld ist restlos durch die Massen der Körper bestimmt. Da Masse und Energie nach den Ergebnissen der speziellen Relativitätstheorie das gleiche sind und die Energie formal durch den symmetrischen Energie-tensor ( $T_{\mu\nu}$ ) beschrieben wird, so besagt dies, dass das  $G$ -Feld durch den Energietensor der Materie bedingt und bestimmt sei," Einstein, "Prinzipielles zur allgemeinen Relativitätstheorie" (1918), pp. 241-42.

Einstein pointed out that the above three principles were not at all independent of each other and indicated in a footnote that hitherto he had not distinguished the principle of general covariance from Mach's principle; this, in his view, had had a confusing effect. Einstein justified the term "Mach's principle," by pointing out that the principle was a generalization of Mach's requirement that inertia be reduced to interactions among bodies.<sup>104</sup> Einstein admitted that his allegiance to Mach's principle was not widely shared by his colleagues, but averred that he himself considered it to be "absolutely necessary."<sup>105</sup> For several years, Einstein was to give a central place to Mach's principle in his publications.<sup>106</sup> Later on he proposed to drop Mach's principle as an independent postulate because he thought that in a unified field theory there would be no distinction between matter and field, and, therefore, the question of an influence of the first on the second would become meaningless.<sup>107</sup> From the number of papers still published on various "Mach's principles," it appears that the principle in some form or another survived Einstein's later rejection of it and remains an active topic of debate.

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104. Ibid., p. 241.

105. Ibid., p. 242.

106. See for example Einstein's Nobel Prize lecture, "Fundamental ideas and problems of the theory of relativity," lecture delivered to the Nordic Assembly of Naturalists at Gothenburg, 11 July 1923. In *Nobel Lectures, Physics, 1901-1921*, published for the Nobel Foundation (Amsterdam : Elsevier, 1967), pp. 482-90, esp. p. 489.

107. Gerald Holton, "Mach, Einstein, and the Search for Reality," in Gerald Holton, *Thematic Origins of Scientific Thought* (Cambridge, MA : Harvard University Press, 1973), p. 227 ; see also Einstein, *The Meaning of Relativity*, p. 140.

## Chapitre 6

# Conclusion

The purpose of the dissertation was to trace the development and the motivations which led Einstein to the general theory of relativity. We have seen that Einstein was essentially motivated by his epistemological rejection of the concept of absolute space (absolute acceleration). This led him to require an extension of the special theory of relativity which, in turn, led him in part to the equivalence principle. During the years 1907 – 1912, Einstein developed various consequences of the equivalence principle and built a static theory of gravitation on it. The equation of motion of that theory led Einstein to the definitive equation of motion, which conditioned the Riemannian and tensorial framework of the 1913 Einstein-Grossmann theory. After various errors and enormous efforts, Einstein, in 1915, succeeded, despite the widespread skepticism of the physics community, to develop generally covariant field equations, which proved to be extraordinarily successful. In 1917, Einstein generalized these questions because of his desire to implement the concept of the relativity of inertia. Though the issue of whether the general theory of relativity has eliminated the concept of absolute space is still a matter of debate, it appears that Einstein's epistemological idealism (rejecting absolute time and absolute space) has twice been very fruitful since together with Einstein's theoretical skills and his physical insights it gave to the twentieth century two of its major theories : the special and the general theories of relativity.



## Annexe A

# EINSTEIN'S PRIORITY IN THE MATTER OF THE FIELD EQUATIONS <sup>1</sup>

Though the majority of physicists correctly attribute the priority of the field equations to Einstein, a priority issue between Einstein and Hilbert is sometimes raised. Thus, I shall briefly analyze that question for the sake of historical accuracy. Shortly before Einstein's communication to the Berlin Academy on 25 November 1915, David Hilbert gave a lecture to the Mathematische Gesellschaft in Göttingen in which he developed a sweeping theory yielding in particular a set of generally covariant gravitational field equations. The talk appears to have been given on 20 or 23 November. (The printed version of the lecture indicates "Presented at the session of 20 November 1915" <sup>2</sup> whereas Hilbert on a postcard to Einstein dated 14 November gives 23 November as the date of the lecture. <sup>3</sup> Since, in the lecture Hilbert apparently did not mention Einstein's contributions but did so extensively in the printed version, the latter was clearly a revised one. Hence, having only this document available we do not know for sure which field equations exactly were given during the lecture but can only assume them to have been the same as the ones of the printed version, which appears likely.

This assumption having usually been made, the apparent similarity of Hilbert's field equations with Einstein's final ones has led to various claims of priority. Thus, Hermann Weyl, in his 1918 book, stated that Hilbert formula-

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1. The lines below have been written before the discovery of the original text of Hilbert's conference and reflect the situation which prevailed during most of the twentieth century. See the footnote at the end of the appendix.

2. David Hilbert, "Die Grundlagen der Physik, I," *Nachrichten von der K. Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-physikalische Klasse* (1915), pp. 395-407, on p. 395.

3. John Earman and Clark Glymour, "Einstein and Hilbert : Two Months in the History of General Relativity," *Archive for History of Exact Sciences*, **19** (1978), 291-308, on 301; the authors have no explanation for the discrepancy.

ted the gravitational field equations "about the same time as Einstein, though only in the framework of Mie's theory";<sup>4</sup> Wolfgang Pauli, in 1921, went further and stated "at the same time as Einstein, and independently, Hilbert formulated the generally covariant field equations."<sup>5</sup> Similarly, Pascual Jordan in his book *Schwerkraft und Weltall* speaks of the Einstein-Hilbert equations.<sup>6</sup> More recently, the claim of independent discovery has been rejected by Eugene Guth<sup>7</sup> who referred to it as a "myth" but was taken over again by Jagdish Mehra who wrote : "During the third period, 1915 – 16, the field equations of gravitation were formulated by Einstein, were independently derived by Hilbert ...,"<sup>8</sup> and further below, added : "There can be no doubt that Hilbert's derivation was entirely independent."<sup>9</sup> The whole issue of the origin of the field equations was analyzed in detail by John Earman and Clark Glymour,<sup>10</sup> who surprisingly left the claim of independent discovery undecided and even suggested the possibility of an influence of Hilbert on Einstein with respect to Einstein's renunciation of the Einstein-Grossmann theory and the discovery of the final field equations. I shall side with Hilbert himself and Guth and show : (a) that the priority of the field equations belongs to Einstein; (b) that the suggestion of an influence of Hilbert on Einstein is essentially unfounded. We shall examine successively the claims of simultaneous, independent discovery and the suggestions of an influence of Hilbert on Einstein.

The expression "simultaneous discovery" implying that the discovery relates to the same object at about the same time, it must first be pointed out that Hilbert's theory as a whole was basically different from Einstein's. Whereas Einstein's theory, in 1915, dealt only with gravitation, Hilbert's theory, by attempting to synthesize Gustav Mie's and Einstein's ideas, aimed, as the title of the paper "Die Grundlagen der Physik" indicates, at deriving all of physics from two axioms, namely :

Axiom I (Mie's axiom of the world function) : *The law of physical evolution is determined by a world function  $H$ , which contains the following terms :*

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4. "Etwa gleichzeitig mit Einstein, wenn auch nur im Rahmen der Mieschen Theorie," Hermann Weyl, *Raum, Zeit, Materie : Vorlesungen über allgemeine Relativitätstheorie* (Berlin : Verlag von Julius Springer, 1918), p. 230, footnote 5, chap. 4. See also H. Weyl, "Zu David Hilberts siebzigsten Geburtstag," *Die Naturwissenschaften*, **20** (1932), 57-58, on 58 ; "50 Jahre Relativitätstheorie," *ibid.*, **38** (1951), 73-83, on 80.

5. Wolfgang Pauli, *Theory of Relativity*, trans. by G. Field with supplementary notes by the author (London, New York : Pergamon, 1958), p. 145, footnote 277. The same assertion is also made in Max Born, "Hilbert und die Physik," *Die Naturwissenschaften*, **10** (1922), 88-93, in Max Born, *Ausgewählte Abhandlungen* (Göttingen : Vandenhoeck & Ruprecht, 1963), **2**, 594.

6. Pascual Jordan, *Schwerkraft und Weltall*, 2nd ed. (Braunschweig : Vieweg und Sohn, 1955), pp. 65, 68.

7. Eugene Guth, "Contribution to the History of Einstein's Geometry as a Branch of Physics," dans *Relativity*, eds. Moshe Carmeli, Stuart I. Fickler, Louis Witten (New York : Plenum, 1970), pp. 161-207, on pp. 183-84.

8. Jagdish Mehra, *Einstein, Hilbert and the Theory of Gravitation* (Dordrecht, Holland/Boston, U.S.A. : D. Reidel, 1974), p. 2.

9. *Ibid.*, p. 25.

10. Earman/Glymour, "Einstein and Hilbert," pp. 291-308.

$$(1) g_{\mu\nu}, g_{\mu\nu l} = \frac{\partial g_{\mu\nu}}{\partial w_l}, g_{\mu\nu lk} = \frac{\partial^2 g_{\mu\nu}}{\partial w_l \partial w_k}$$

$$(2) q_s, q_{sl} = \frac{\partial q_s}{\partial w_l} \quad (l, k = 1, 2, 3, 4);$$

the variation of the integral

$$\int H \sqrt{g} dw \quad (g = |g_{\mu\nu}|, dw = dw_1 dw_2 dw_3 dw_4)$$

must vanish for each of the 14 potentials  $g_{\mu\nu}, q_s$ .

Axiom II (Axiom of the general invariance) :

The world function  $H$  is an invariant with respect to arbitrary transformations of the world parameters  $w_s$ .<sup>12</sup>

The great hopes Hilbert expressed for his theory in the conclusion of the first paper<sup>13</sup> were not to be fulfilled and the theory as a whole was a failure, in contradistinction to Einstein's more modest theory.

Though the theories of Hilbert and Einstein were different, Hilbert's gravitational field equations appear very similar to Einstein's field equations. By applying a variational principle to the invariant  $H = K + L$ , where  $K$  is the Riemannian curvature and  $L$  an, at first, unspecified invariant, Hilbert obtained the field equations<sup>14</sup>

$$\sqrt{g} \left( K_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \right) = - \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}} \quad (\text{A.1})$$

where  $K_{\mu\nu}$  is the contracted Riemann-Christoffel tensor. These equations must now be compared with Einstein's final field equations

$$G_{\mu\nu} = -\chi \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right). \quad (\text{A.2})$$

Since,  $G_{\mu\nu} = K_{\mu\nu}$ , the two equations are the same if

$$-\frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}} + \frac{1}{2} K g_{\mu\nu}$$

can be identified with  $-\chi T_{\mu\nu} + \frac{1}{2} g_{\mu\nu} T$ . By multiplying (A.1) and (A.2) with  $g^{\mu\nu}$  and summing over  $\mu, \nu$  one obtains respectively :

$$(K - 2K) = -\frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}} g^{\mu\nu} = -K,$$

$$G = -\chi (T - 2T) = \chi T.$$

From  $G = K$ , it follows that the identification is complete if  $\chi T$  is identical to

$$\frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}} g^{\mu\nu},$$

12. Hilbert, "Die Grundlagen der Physik, I" (1915), p. 396.

13. Ibid., p. 407.

14. Ibid., p. 404.



and  $\chi T_{\mu\nu}$  to

$$\frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}},$$

the second condition containing the first. This, however, is not the case. Needing an invariant independent of the  $g_{\mu\nu}$  for  $L$ , Hilbert adopted, rather arbitrarily, the two simplest invariants (out of four) given by G. Mie.<sup>15</sup> Furthermore, whereas Mie's invariants were only invariants with respect to Lorentz transformations, Hilbert assumed them to satisfy a general invariance (with respect to the *GTR* group). In any case, Hilbert's invariant  $L$  contained only the electromagnetic potentials and their derivatives whereas Einstein's final  $T_{\mu\nu}$  represented matter macroscopically without reference to an electrodynamical world-view. Hence, as correctly pointed out by Earman and Glymour,<sup>16</sup> Hilbert's field equations are not the same as Einstein's field equations. On the other hand, one might perhaps argue that since

$$\frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}}$$

is equivalent to an energy tensor, the above difference is only a minor one and that, forgetting for a moment this point, Hilbert's and Einstein's field equations are basically identical.

Let us now momentarily adopt this position and examine whether at least Hilbert discovered his field equations independently of Einstein. Since Hilbert's field equations were derived from the curvature invariant via a variation principle, the crucial point to examine is how Hilbert arrived at that invariant. I shall argue that Hilbert arrived at the curvature invariant through Einstein. The argumentation goes as follows. In 1913, Einstein and Grossmann had already considered using the Riemann-Christoffel tensor but, as we have seen above, had serious reasons against its use. Throughout most of 1915, Einstein thought he had a satisfactory theory and was not looking for a new one. That theory was apparently well received in Göttingen during the talk Einstein gave in the summer. At that point, Einstein had no need of the Riemann-Christoffel tensor and perhaps had not yet completely overcome all the objections against its use. Hilbert, unless guided by private talks with Einstein, had even less reasons to ponder using it since he was faced with an apparently successful theory which made no use of it. How could Hilbert have guessed, a priori, before November 1915, that there was something physically meaningful in the curvature invariant? H. A. Lorentz, who, on 30 January 1915, presented a derivation of Einstein's 1914 field equations from a variational principle certainly did not have that thought.<sup>17</sup> Even if Hilbert did, what sense would he have made out of the resulting field equations since they would have been different from the Einstein-Grossmann equations.

15. Ibid., p. 407.

16. Earman/Glymour, "Einstein and Hilbert," p. 303.

17. H. A. Lorentz, "On Hamilton's principle in Einstein's theory of gravitation," original paper in *Versl. Kon. Wetensch. Amsterdam*, **23** (1915), 1073; English trans. : *Proc. Roy. Acad. Amsterdam, Section of Sciences*, **19**, part 1 (1915), 751-65, in H. A. Lorentz, *Collected Papers* (1937), **5**, 229-45.

The situation, however, was to change dramatically when Einstein presented his 4 November 1915 paper in which he derived the field equations from the Riemann-Christoffel tensor. On a postcard to Hilbert dated 7 November, Einstein indicated that he was sending by the same post the proofs of his 4 November paper to Hilbert and concluded with the remarks : "I am curious whether you will be well disposed towards this solution."<sup>18</sup> It is very likely that, unless guided by other previous exchange of information with Einstein, Hilbert got the idea of using the curvature invariant from that paper. Since Hilbert was an expert in invariant theory who, furthermore, was familiar with G. Mie's use of invariants in physics, it was immediate for him to see that the essence of Einstein's approach rested on the use of the curvature invariant which is the only scalar invariant which can be derived linearly from the Riemann-Christoffel tensor. Once the invariant under a certain group of transformations is known, field equations can be derived at once through a variational principle, which is precisely what Hilbert did.

Further extensive correspondance with Einstein must have strenghtened Hilbert's conviction that he was on the right track. On 12 November, the day following the presentation of his addendum, Einstein thanked Hilbert for his "kind letter" (which probably answered Einstein's 4 November letter) and wrote to him that he had obtained generally covariant field equations.<sup>20</sup> By 14 November, Hilbert's investigations were advanced to the point that he invited Einstein to attend a lecture on 25 November. Too tired by his own efforts to attend and, also, because of illness, Einstein declined the offer on 15 November, and asked Hilbert to send the proofs of his lecture.<sup>21</sup> On 18 November, Einstein wrote to Hilbert that, as far as he could see Hilbert's equations corresponded to his own 11 November equations and also indicated that he had derived the correct value for the secular precession of the perihelion of Mercury, a result for which Hilbert congratulated Einstein the next day.<sup>22</sup>

This friendly exchange of information between Hilbert and Einstein came probably abruptly to an end when Einstein learned that Hilbert had given the planned lecture presenting the gravitational field equations without mentioning Einstein's contribution. Einstein was probably outraged and in any case deeply offended because of the trust he had placed in Hilbert. Einstein's resentment filters through a letter dated 20 November 1915 he wrote to Hilbert :

I want to take this opportunity to say something to you which is important to me.

There has been a certain spell of coolness between us, the cause of which I do not want to analyze. I have, to be sure, struggled against my resentment, and with complete success. I think of you once again with untroubled friendliness, and I ask you to try to think of me in

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18. Einstein Papers, Princeton University, microfil reel I. B. 1, no. 13 ; quoted and trans. by Earman/Glymour, "Einstein and Hilbert," p. 297.

20. Einstein Papers, Princeton University Press, microfilm reel I.B. 1, no. 13 ; mentioned by Earman/Glymour, "Einstein and Hilbert," pp. 299, 301.

21. *Ibid.*, p. 301.

22. *Ibid.*, pp. 301-302.

the same way. It is really a shame when two such real fellows, whose work has taken them above this shabby world, give one another no pleasure.<sup>23</sup>

Not having mentioned Einstein in his lecture was a serious error on the part of Hilbert because the latter, undoubtedly, got the idea of using the curvature invariant from Einstein and very likely through the proofs of Einstein's first November paper which Einstein had generously sent to Hilbert. Since Einstein sent the proofs on 7 November and since the paper was published on 11 November, Hilbert must have gained at least four days. Without these supplementary days it is probable that Hilbert would not have been able to present his field equations before Einstein. It is not clear why Hilbert made such a mistake. E. Guth mentions Hilbert's "legendary"<sup>24</sup> absent-mindedness in that context.

In any case, Hilbert tried to correct the error the best he could. In the printed version of his lecture, he referred to all of Einstein's November papers on gravitation including the one presenting the final field equations. Furthermore, he gave Einstein credit for the idea of general covariance which found expression in Hilbert's second axiom and remarked, with respect to the gravitational field equations arrived at in the paper, that they seemed to be in accord with the "bold theory of general relativity developed by Einstein in his later papers."<sup>25</sup> Hilbert never claimed any priority over the field equations and freely and frequently admitted in his lectures that the "great physical idea was Einstein's."<sup>26</sup> According to Philipp Frank, Hilbert once said :

Every boy in the streets of Göttingen understands more about four-dimensional geometry than Einstein. Yet, despite that, Einstein did the work and not the mathematicians.<sup>27</sup>

According to Max Born, what Hilbert considered as his main contribution was to have shown that among the  $n$  differential equations (involving four space-time coordinates), obtained via the variation principle, there were always 4 which were a consequence of the  $n - 4$ .<sup>28</sup> It was probably Hilbert's printed version of the lecture and (or) perhaps other signs of good will which prompted Einstein to accept to forget the incident of the lecture by December 1915, as

23. "Bei dieser Gelegenheit drängt es mich dazu, Ihnen noch etwas zu sagen, was mir wichtiger ist.

Es ist zwischen uns eine gewisse Verstimmung gewesen, deren Ursache ich nicht analysieren will. Gegen das damit verbundene Gefühl der Bitterkeit habe ich gekämpft, und zwar mit vollständigem Erfolge. Ich gedenke Ihrer wieder in ungetrübter Freundlichkeit, und bitte Sie, dasselbe bis [sic] mir zu versuchen. Es ist objectiv [?] schade, wenn sich zwei wirkliche Kerle, die sich aus dieser schätzbaren Welt etwas herausgearbeitet haben, nicht gegenseitig zur Freude erreichen," Einstein Papers, Princeton University, microfilm reel I.B. 1, no. 13; original and trans. in Earman/Glymour, "Einstein and Hilbert," p. 306.

24. Guth, "Contribution to the History," p. 184.

25. Hilbert, "Die Grundlagen der Physik, I," p. 405.

26. Born, "Hilbert," pp. 594-95; see also Constance Reid, *Hilbert* (Berlin, New York : Springer-Verlag, 1970), pp. 141-42.

27. Philipp Frank, *Einstein, His Life and Times* (New York : Alfred A. Knopf, 1953), p. 206.

28. Born, "Hilbert," pp. 595-96; see also Hilbert, "Die Grundlagen der Physik, I," p. 397.

we have seen. The conclusion of all this is that the gravitational field equations should be (and generally are) called Einstein's equations. To Hilbert goes the credit of having presented another formulation of Einstein's physical insights .

Finally I shall briefly analyze the suggestions made by Earman and Glymour that Einstein might have received some help from Hilbert : (a) in the rejection of the Einstein-Grossmann theory, and (b) in the formulation of his final field equations. In the conclusion of their paper "Einstein and Hilbert...", the authors summarize their suggestions as follows :

The evidence of the correspondance raises the possibility that HILBERT was in part responsible for heightening EINSTEIN's discontent with the EINSTEIN-GROSSMANN theory, and it suggests, but does not prove, that HILBERT's results determined EINSTEIN's transition from (14) [ $G_{im} = \chi T_{im}$ ] to (18) [ $G_{im} = \chi (T_{im} - \frac{1}{2} g_{im} T)$ ].<sup>30</sup>

The "evidence" given for (a) is the postcard dated 7 November 1917 which Einstein sent to Hilbert. According to Earman and Glymour this postcard "raises the intriguing possibility that HILBERT was the sower of the seed [of doubt against the Einstein-Grossmann theory]." <sup>31</sup> On the postcard, Einstein indicated that Sommerfeld had told him in a letter that Hilbert had raised some objections against Einstein's 1914 comprehensive paper. <sup>32</sup> Besides the fact that the objections in question are not known, there is no reason to believe that Hilbert's objections were "the seed of doubt." Rather, as we have seen above, it appears from Einstein's letter to Sommerfeld dated 28 November 1915 (as well as the letter to Lorentz dated 1 January 1916) that the discovery of the faultiness of the derivation might in fact have come after Einstein's discovery that the theory did not satisfy a rotational relativity and did not yield the correct precession of the perihelion of Mercury. At that stage, the theory was probably already dead in Einstein's mind. In any case, as already mentioned, the faultiness of one, a posteriori, derivation did not entail the invalidity of the field equations. It is likely that Hilbert's arguments just confirmed what Einstein already suspected, namely that the derivation could not be sound given the two other failures. As to the suggestion (b), the authors rely on the assumption that Hilbert communicated the field equations of the printed version to Einstein prior to 25 November 1915 but do not specify how Einstein could have been influenced. We have seen that Einstein believed the equations communicated by Hilbert to be equivalent to his 11 November equations. Einstein, perhaps, did not even analyze Hilbert's theory carefully because he disliked Hilbert's overall approach. On a postcard to Paul Ehrenfest dated 24 May 1916, Einstein wrote : "I do not like Hilbert's formulation. It is unnecessarily specialized in what concerns matter, unnecessarily complicated, not honest (= Gaussian) in its development (attempt to play the superman by veiling the methods)." <sup>33</sup> In any case, there is no evidence that

30. Earman/Glymour, "Einstein and Hilbert," p. 307.

31. Ibid., p. 296.

32. Einstein, "Formale Grundlage" (1914).

33. "Hilberts Darstellung gefällt mir nicht. Sie ist unnötig speziell, was die Materie anbelangt, unnötig kompliziert, nicht ehrlich (=gaussisch) im Aufbau (Vorspiegelung des Über-

Hilbert's approach influenced Einstein. On the other hand we have seen that Einstein's transition from the field equations  $G_{\mu\nu} = -\chi T_{\mu\nu}$  to the final ones  $G_{\mu\nu} = -\chi (T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$  was entirely natural. Thus, the above suggestion can safely be dismissed as being insufficiently documented. Hilbert, except for his initial faux pas, cannot be blamed for the subsequent priority claims in his favor. As long as the Einstein-Hilbert correspondance was inaccessible such claims could invoke some plausibility which, in my view, is no longer possible.<sup>34</sup>

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menschen durch Verschleierung der Methoden)," Einstein à Ehrenfest, 24 May 1916, in Carl Seelig, *Albert Einstein* (1960), p. 276.

34. A recent paper has disclosed the original text of Hilbert's conference : Leo Corry et al., "Belated Decision in the Hilbert-Einstein Priority Dispute", *Science*, **278**, 1270-1273 (1997). The paper shows that Hilbert's equations of his conference are not covariant and are not the same as those of Einstein, thus putting a definitive end to the debate in favor of Einstein.

## Annexe B

# Principal Figures

Abraham, Max (1875 – 1922)  
Berkeley, George Bishop (1685 – 1753)  
Born, Max (1882 – 1970)  
Christoffel, Elwin Bruno (1829 – 1900)  
Einstein, Albert (1878 – 1955)  
Eötvös, Roland Baron von (1848 – 1919)  
Freundlich, Erwin (1885 – 1964)  
Friedman, Aleksandr (1888 – 1925)  
Grossmann, Marcel (1878 – 1936)  
Herglotz, Gustav (1881 – 1953)  
Hilbert, David (1862 – 1943)  
Hume, David (1711 – 1776)  
Klein, Felix (1849 – 1925)  
Kottler, Friedrich (1886 – 1965)  
Laue, Max von (1879 – 1960)  
Levi-Civita, Tulio (1873 – 1941)  
Lipschitz, Rudolf (1832 – 1903)  
Lorentz, Hendrik Antoon (1853 – 1928)  
Mach, Ernst (1838 – 1916)  
Mie, Gustav (1868 – 1957)  
Minkowski, Hermann (1864 – 1909)  
Nordström, Gunnar (1881 – 1923)  
Petzoldt, Joseph (1862 – 1929)  
Planck, Max (1858 – 1947)  
Poincaré, Henri (1854 – 1912)  
Ricci-Curbastro, Gregorio (1853 – 1925)  
Riemann, Bernhard (1826 – 1866)  
Schwarzschild, Karl (1873 – 1916)  
Seeliger, Hugo von (1849 – 1924)  
Sitter, Willem de (1872 – 1934)  
Sommerfeld, Arnold (1868 – 1951)

Varičák, Vladimír (1865 – 1942)  
Voigt, Woldemar (1850 – 1919)

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