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Designing Two-Echelon Distribution Networks under Uncertainty

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Contents

Chapter 1

Introduction

The emergence and rapid expansion of e-commerce are drastically impacting the structure of warehousing and distribution activity. According to the Ecommerce foundation, the European online retailers have shown an increase of 14% in 2017 [112]. Similarly, the United States reaches a growth of 16% according to U.S. Census Bureau [68]. Moreover, the development of e-commerce is boosting the shift to an on-demand economy. This shift is tremendously affecting the distribution schema of several companies that aim to continue improving response time to customers while efficiently offering their products in a multi-channel setting. The delivery service level expectancy has significantly increased in the last decade: it is now expressed in hours rather than days [220]. To this end, several global B-to-C players, and especially companies operating in the retail sector, such as Walmart, Carrefour, Amazon or jd.com, have recently engaged a sustained reengineering of their distribution networks. They have favored a high proximity to customers ship-to location as stores, and relay points among others, in addition to home delivery [83]. This was done without reducing the efficiency of their consolidation policies in warehousing and transportation. When locating their warehouse platforms, companies have followed various optimization rules going from centralization and risk-pooling incentives to sourcing-dependent and financial constraints. Therefore, the warehouses' location and the structure are critically relevant attributes for a distribution network as much as the transportation to meet customer expectations.

With this in mind, having a cost-effective distribution network with a service level improvement mission is a strategic question for companies to increase competitiveness. In the distribution network, finished products flow towards end-customers or aggregated zones from a set of storage/warehouse platforms (WPs). These WPs location should be appropriately chosen in order to cope with the business needs over time. More precisely, such problem consists in deciding the network structure and the transportation scheme. The former determines the number of echelons, facility platform types at each echelon, their number and their location, where an echelon represents one level of the distribution network. Hence, the design of distribution networks involve both strategic location decisions and operational transportation decisions. Clearly, strategic decisions have a direct effect on the operation's costs as well as its ability to serve customers [59, 99].

Distribution network design problems (DDPs) are particularly important. They have attracted the attention of many researchers in the operations research (OR) literature over the last decades. In [139], they have been classified in terms of the number of echelons in the distribution network, echelons in which location decisions are made, and the transportation option involved in each echelon. Consequently, the DDP involves several classical OR problems and innovative ones based on the problem modeling features. Figure 1.1 identifies five modeling features that mostly affect the distribution network. These features are namely the transportation, the customer demand, the number of distribution echelons, the planning horizon and the uncertainty.

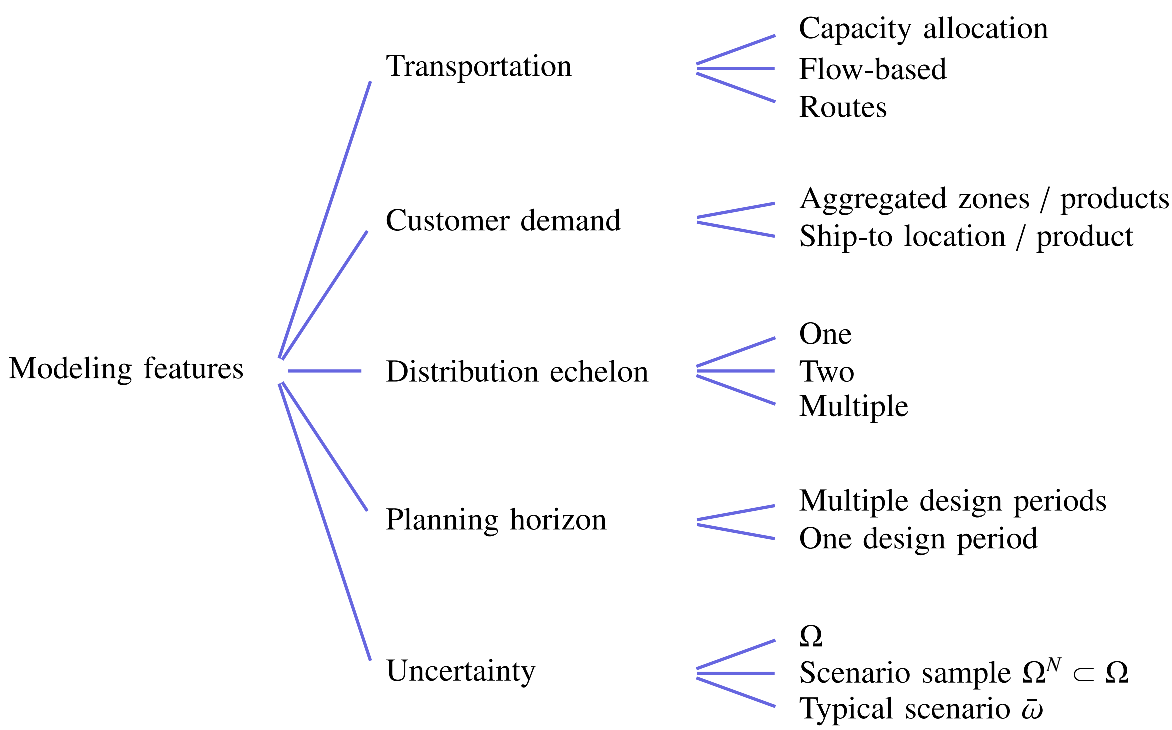


Figure 1.1: *Modeling features options*

Looking at the transportation modeling feature in the upper level of Figure 1.1, we can distinguish between capacity allocation, flow-based arcs and multi-node routes, and each option concerns a well-studied OR model. The capacity-allocation option results in a capacity planning and warehouse location problem in which the aim is about deciding the warehouse location and the amount of capacity assignment [147, 4]. When the transportation is represented as origin-destination flows, a flow-based location-allocation problem is defined where a set of warehouses should be located from a finite set of potential sites and customers are delivered by direct flows from selected warehouses at the minimum cost [54, 15]. The third transportation option concerns multiple node route that visits more than one node. This leads to a location-routing problem (LRP). It integrates vehicle-routing problems (VRPs) that compute a set of minimum-cost routes to meet customer demands [63, 221], with facility location-allocation problems [97, 65]. Therefore, its aim is to find an optimal number of warehouses and their locations, while building routes around them to serve the customers, simultaneously [139]. Moreover, these transportation options influence the modeling of customer demand feature in terms of aggregated zones or products, and of single ship-to location.

On the other hand, most of the DDP models studied so far consider a one-echelon distribution structure where the network includes a set of WPs and customers. Nonetheless, with the growth of e-commerce and continuous increase in cities population [70] contrasted with the rising levels of congestion, such one-echelon networks constrain the companies' ability to provide fast delivery services, and reduce their opportunities to meet today challenges: they are not specifically optimized to provide next day and/or same day deliveries, or to operate efficiently fast fulfillment and shipment services for online orders. In this new context, strategic considerations imply a distribution schema with more than one-echelon that can be dynamically adjusted to the business needs over time, as mentioned in the third level of Figure 1.1. Practitioners are nowadays turning much attention to two-echelon distribution structures. The network topology includes an intermediate echelon of distribution/fulfillment platforms (DPs) located between the initial sites where inventory is held and the customers. According to Tompkins Supply Chain Consortium, more than 25% of retail companies are adapting their distribution networks by adding a new echelon of DPs [220]. For instance, Walmart plans to convert 12 Sam's Club stores into e-commerce fulfillment centers to support the rapid e-commerce growth [122]. In the United Kingdom, Amazon is looking to acquire 42 Homebase stores to expand its network of fulfillment centers and warehouses [157]. Additionally, this two-echelon distribution structure covers recent city logistics models with two tiers of platforms for the case of multiple companies sharing platforms [61, 160]. Postal and parcel delivery also involves two-echelon structure to distribute their products, but it concerns non-substitutable product [236]. From methodological perspective, several authors have recently recalled the need to expand one-echelon networks by considering an intermediate echelon of platforms where merging, consolidation or transshipment operations take place [215, 194]. As far as we know, only few works have investigated this issue for distribution networks. Detailed reviews are presented in [178, 62, 71]. They show that two-echelon distribution structure is still a relatively unexplored area.

Furthermore, strategic design decisions have a long-lasting effect. They are expected to efficiently operate in a long-term period fulfilling future distribution requirements and parameters fluctuations over time, as pointed out by Klibi et al. [130]. Studies have mostly considered a single design period. But, this limits the strategic design decisions capability to be easily adaptable to changes in the business environment over time. As highlighted in Figure 1.1, the planning horizon can be partitioned into a set of design periods characterizing the future opportunities to adapt the design along with the evolution of the business needs. The design decisions should be then planned as a set of sequential decisions to be implemented at different design periods of the horizon (a year, for example).

Finally, there is a significant trend toward reducing the planning horizon in strategic studies. According to Tompkins report in (2011), the length of the re-engineering period defined in strategic network design studies has reduced on average from 4 years to under 2 years due to business uncertainty increasing, and distribution practices becoming more complex [219]. In the literature, models are generally deterministic relying on a single typical scenario for problem parameters. However, incorporating uncertainty as a set of scenarios representing plausible future realizations will provide a better design (see uncertainty feature options in Figure 1.1). The uncertainty can involve the demand level, facility costs and transportation costs, etc. In addition to their uncertainty, the problem parameters vary dynamically over periods following

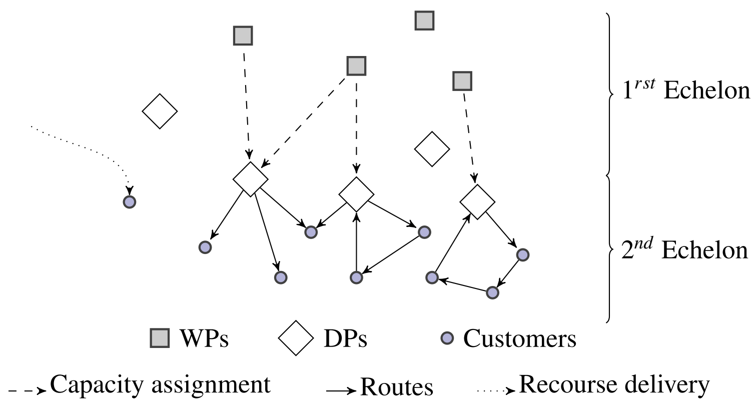


Figure 1.2: *A generic Two-Echelon Distribution Network Design Problem (2E-DDP)*

a trend function. Hence, the traditional deterministic-static representation of the planning horizon is due to be replaced by a more realistic stochastic and multi-period characterization of the planning horizon.

Extensive studies have addressed DDPs models. Nevertheless, the number of papers that investigate the relevant modeling features jointly is very limited. The few works proposed in this regard examine only a subset of these options and are, mostly, dealing with the one-echelon distribution structure omitting the impact of extended two-echelon structure. They also assume that strategic and operational decisions are made simultaneously for the planning horizon.

Our objective in this thesis is to point out the need to take into account the aforementioned issues when designing an effective distribution network that offers more dynamic adjustment to the business requirements over time and copes with the uncertain parameters factors. For this aim, we introduce a comprehensive framework for the stochastic multi-period two-echelon distribution network design problem (2E-DDP) under uncertain customer demand, and time-varying demand and cost. As highlighted above, the 2E-DDP topology includes an advanced echelon of DPs standing between WPs and customers. Figure 1.2 illustrates a typical 2E-DDP partitioned into two capacitated distribution echelons: each echelon involves a specific location-assignment-transportation schema that must be adapted in response to the uncertainty shaping the business horizon.

Our modeling approach in this thesis involves periodically over a set of design periods strategic facility-location decisions and capacity allocation to links between WPs and DPs to efficiently distribute goods to customers' ship-to bases. Then, on a daily basis, the transportation decisions are made on a response to the orders received from customers. This temporal hierarchy gives rise to a hierarchical strategic-operational decision problem and argues the necessity of a stochastic and multi-period characterization of the planning horizon. Besides, our planning horizon allows the design decisions to be adapted periodically at each design period to align the distribution network to its business environment, especially when operating under uncertainty. Therefore, the design of the two-echelon distribution network under a multi-period and stochastic setting leads to a complex multi-stage stochastic decisional problem.

To study this comprehensive modeling approach, several models are proposed and discussed in terms of solvability. The quality of solution is examined through exact based Benders decomposition approach and heuristic.

In the following section, we define the structure of the thesis and subsequently highlight the contributions of each chapter.

Thesis structure

The thesis is made by six chapters including this one. In the following, we present a brief description of the contributions of each chapter.

Chapter 2: Two-echelon Distribution Network Design Problem under Uncertainty: Literature Review

The literature contains a growing body of works focusing on the design of an efficient distribution network. In this chapter, we present a comprehensive review on distribution network design problems (DDPs) and provide a global survey of the current stream of research in this field. We point out several shortcomings related to DDPs. These critical aspects involve two-echelon structure, multi-period setting, uncertainty, and solution approaches. We discuss the DDPs works with respect to these issues and stress their impact on the design of a distribution network. Therefore, this survey emphasizes our interest to this type of problems in the thesis.

Chapter 3: Designing a Two-Echelon Distribution Network under Demand Uncertainty

This chapter first introduces our comprehensive modeling approach for the two-echelon distribution network design problem under uncertain demand, and time-varying demand and cost, formulated as a multi-stage stochastic program. Here, we are interested in 2E-DDPs dealing with a generic context involving the deployment of the distribution network for a retailer. Thus, the problem involves at the strategic level decisions on the number and location of DPs along the set of planning periods as well as decisions on the capacity assignment to calibrate DP throughput capacity. The operational decisions related to transportation are modeled as origin-destination arcs, which correspond to a sufficiently precise aggregate of daily decisions over several products, transportation means, and working periods.

Secondly, considering the curse of dimensionality of the multi-stage stochastic setting and the combinatorial complexity in the 2E-DDP, two approximate modeling approaches are proposed to capture the essence of the problem, while providing judicious accuracy-solvability trade-offs. The two models are: the two-stage stochastic location and capacity-allocation model (LCA) in which DP location decisions and capacity decisions are first-stage decisions, and the two-stage stochastic flow-based location-allocation model (LAF) where capacity decisions are transformed into continuous scenario-dependent origin-destination links within the second-stage.

Finally, we develop a Benders decomposition approach to solve the resulting models. The adequate sample size of scenarios is tuned using the sample average approximation (SAA) approach. A scenario-based evaluation procedure is introduced to assess the quality of the design solutions. The extensive computational experiments validate the proposed models and confirm the efficiency of the solution approaches. They also illustrate the importance of uncertainty in the 2E-DDP. The key findings highlight a significant variability in the design decisions with respect to the demand processes modeling uncertainty. In addition, the analysis of the two alternative models shows the high sensitivity of assignment-capacity decisions to uncertainty comparing to location decisions.

This paper is under revision for the *European Journal of Operational Research*. Preliminary versions of the work have been presented in the following conferences:

Ben Mohamed I., Klibi W., Vanderbeck F. Designing two-echelon distribution network under demand uncertainty. *Fifth meeting of the EURO Working group on Vehicle Routing an Logistics Optimization (VEROLOG)*, Nantes, France, June 6-8, 2016.

Ben Mohamed I., Klibi W., Vanderbeck F. Designing stochastic two-echelon distribution networks. 18^{th} *Edition of the French Society for Operational Research and Decision Aiding Conference (ROADEF)*, Metz, France, February 22-24, 2017.

Ben Mohamed I., Klibi W., Vanderbeck F. Modeling and solution approaches for the stochastic two-echelon distribution network design problem. 21^{st} *Conference of the International Federation of Operational Research Societies (IFORS)*, Quebec, Canada, 17-21 July 17-21, 2017.

Chapter 4: A Benders Decomposition Approach for the Two-Echelon Stochastic Multi-period Capacitated Location-Routing Problem

In this paper, we consider the last-mile delivery in an urban context where transportation decisions involved in the second echelon are characterized through vehicle-routing.

To this end, we define the two-echelon stochastic multi-period capacitated location-routing problem (2E-SM-CLRP). The hierarchical 2E-SM-CLRP aims to decide at each design period on opening, operating and closing of both WPs and DPs, as well as the capacity allocated to links between platforms. In the second level, daily routes are built to visit customers using a vehicle routed from an operating DP. A two-stage stochastic program with integer recourse is introduced that relies on a set of multi-period scenarios generated with a Monte-Carlo approach.

Next, we build on the Benders decomposition approach and on the SAA to solve realistic-size instances for the 2E-SM-CLRP. The operating WPs and DPs as well as the capacity decisions are fixed by solving the Benders master problem. The resulting subproblem is a capacitated vehicle-routing problem with capacitated multi-depot (CVRP-CMD). This latter is formulated as a set partitioning model and then solved by a branch-cut-and-price algorithm. Standard Benders cuts as well as combinatorial Benders cuts are generated from the subproblem solutions in order to converge to the optimal solution of the 2E-SM-CLRP.

Numerical results indicate that our solution method is able to solve optimally a large set of instances, and to get good lower bounds on large-scale instances with up to 50 customers and 25 demand scenarios under a 5-year planning horizon. The impact of the stochastic and multi-period settings is also confirmed in comparison to the static model (i.e., no multi-period setting is considered for design decisions).

We plan in the upcoming weeks to submit this work to *Operations Research* . It has been presented in two conferences:

Ben Mohamed I., Klibi W., Sadykov R., Sen H., Vanderbeck F. Stochastic two-echelon location-routing. 23^{rd} *International Symposium on Mathematical Programming (ISMP)*, Bordeaux, France, July 1-6, 2018.

Ben Mohamed I., Klibi W., Sadykov R., Sen H., Vanderbeck F. Benders approach for the two-echelon stochastic multi-period capacitated location-routing problem. 20^{th} *Edition of the French Society for Operational Research and Decision Aiding Conference (ROADEF)*, Le Havre, France, February 19-21, 2019.

Chapter 5: Rolling Horizon Approach for the Multi-stage Stochastic Two-Echelon Distribution Network Design Problem

In this chapter, we are interested in the multi-stage framework proposed for the two-echelon distribution network design problem under a stochastic multi-period characterization of the planning horizon, and we aim to evaluate its tractability.

Using a scenario tree to handle the set of demand scenarios, we first introduce a compact formulation for the multi-stage stochastic programming model. Then, a rolling horizon approach is developed to solve the problem. The main idea of the algorithm is to use a reduced finite sub-horizon in each iteration, instead of the whole time horizon. Then, the multi-stage model defined over the sub-horizon is solved, fixes the solution for the current period and moves forward the optimization horizon. The fixed decisions are introduced as an initial conditions for the following iterations. A complete solution for the stochastic multi-period 2E-DDP is progressively built by concatenating the decisions related to the first periods of each reduced sub-horizon.

The computational experiments compare our solution method to solving directly the compact formulation. The results show the efficiency of the rolling horizon approach to provide good quality bounds in a reasonable time, and confirm our multi-stage modeling approach proposed in this thesis.

We plan to submit the paper in the upcoming weeks to *Transportation Science*. It has been presented in the following conference:

Ben Mohamed I., Klibi W., Sadykov R., Vanderbeck F. Multi-stage stochastic two-echelon distribution network design. 19^{th} *Edition of the French Society for Operational Research and Decision Aiding Conference (ROADEF)*, Lorient, France, February 21-23, 2018.

Chapter 6: Conclusions and perspectives

This chapter draws conclusions and proposes directions for further research.

Chapter 2

Two-Echelon Distribution Network Design Problem: Literature review

Abstract

Designing an effective and efficient distribution network is a crucial question for companies. It concerns finished products flows from a set of storage/warehouse platforms towards customers, possibly through a set of intermediate distribution platforms. This topic has inspired a growing number of works in the literature. This paper provides a comprehensive review on distribution network design problems (DDPs), and discusses the proposed models. We point out several issues and missing aspects in DDPs such as uncertainty and two-echelon structure. In this work, we survey the DDPs related works with respect to these issues, motivating the inclusion of these missing aspects in DDPs.

Keywords: *Distribution network design problem, two-echelon networks, facility location, routing problem, uncertainty, multi-period.*

2.1 Introduction

In the recent decades, companies are experiencing a fast evolution of the business environment. Given the high competitiveness, companies have to continue improving customer service level expectancy in terms of the delivery lead-time, the delivery period and the customers ship-to location. Designing an efficient and effective distribution network is then a crucial strategic question for companies. In addition, their network must cope with the business needs over the time as pointed out by Klibi et al. [130]. Chopra and Meindl [49] highlight that distribution-related costs make up about 10.5% of the US economy and about 20% of the manufacturing cost. It is therefore not surprising that such topic has attracted many researchers attention in the operations research literature. Thousands of works have been proposed aimed at developing effective optimization models and solution algorithms capable of providing support tools to decision makers.

In the distribution network, finished products flow towards end-customers or ship-to bases from a set of storage/warehouse platforms. When considering distribution network design problems (DDPs), the structure of the network involves the best configuration of the facilities. It must be decided, in addition to the optimization of goods' flow through the network. It consists in determining the number of echelons, facility platform types at each echelon, their number and their location. Thus, an echelon represents one level of the distribution network. Therefore, the design of distribution networks involve strategic decisions which influence the operational decisions [59]. It implies facility location problems at the strategic level and transportation problems at the operational level to supply customers.

The DDP generalizes two hard combinatorial optimization problems that have been the subject of intensive research efforts: facility location problems (FLPs) and transportation problems. In the former, a set of warehouses should be located from a finite set of potential sites and customers are delivered by direct routes from selected warehouses at the minimum cost [97, 65]. On the other hand, vehicle-routing problems (VRPs) are among the most extensively studied classes in transportation problems where the aim is to compute a set of minimum-cost routes to meet customer demands using a fleet of vehicles [63, 221]. These two types of problems have been addressed separately in the literature modeling only some aspects related to the complex network decisions. However, location and transportation are interdependent decisions. Webb [234] and later, Salhi and Rand [192] show that ignoring the interdependence between the two decisions may often lead to sub-optimal solutions. The study of this interdependence has given rise to location-routing problems (LRP) [150] in the context of distribution network design problem. In the LRPs, the aim is to find at a minimum cost which warehouse platforms to open and their number, while building vehicle routes around them to visit the customers, simultaneously [139]. It is worth to mention most production-distribution problems in supply chain management literature generally rely on FLP to formulate the problem. These studies tend towards functional expansions related to procurement, production policies and constraints, and specific manufacturing-linked transportation issues, rather than focusing on the strategic needs of the distribution of finished products stressed here.

LRPs have drawn a consistent attention, and particularly in the last years. Early surveys can be found in [18, 139, 140, 34, 155, 159]. Recent surveys are presented in [71, 178, 62]. It is worth noting that other strategic decisions may be considered in the DDPs such as sourcing, capacity decisions and technology selection. This further complicates the problem. In the following, we refer to all strategic decisions by design decisions. Laporte [139] has proposed a classification in terms of the number of echelons in the distribution network, in which echelons location decisions are made, and the type of routes involved in each echelon. More specifically, the author distinguishes between origin-destination routes or direct routes, and a multiple node routes: a direct route connects a customer and a platform, or two different types of platforms, and a multiple node route visits more than one customer, or more than one platform. Moreover, the author characterizes a location-routing problem by location decisions and multiple node routes at least for one echelon.

In the literature, several models are proposed to formulate and solve DDPs. Nevertheless, the obtained designs may not provide an effective distribution network that copes with the rise of the on-demand economy. Many important real-life issues still need to be incorporated and analyzed to design an efficient distribution network.

First, most approaches proposed for DDPs and LRPs implicitly assume that design and transportation decisions are made simultaneously for the planning horizon. However, in practice, the strategic design decisions and the operational transportation decisions are not made at the same time scales. In fact, the strategic design decisions must be made here-and-now, and then after an implementation period, the distribution network will be used on a daily basis for a planning horizon. Thus, the temporal hierarchy between these decisions should be considered when modeling DDPs.

Next, the models are mostly developed under a single echelon structure. In this case, the network includes a set of warehouse platforms and customers. Given the high growth of e-commerce and the continuing increase in world population, such configuration constrains the companies' capability to provide fast delivery services. It also reduces their opportunities to capture online orders, especially when the customers' bases are located in a large geographical area. In this case, distribution networks should be beyond more than one-echelon to offer more dynamic adjustment to the business needs over time. A typical predisposition schema, claimed now by practitioners, is the two-echelon distribution network. The network topology includes an intermediate echelon of distribution/fulfillment platforms standing between the warehouse platforms (i.e., initial sites) where the inventory is held, and the customers. Savelsbergh and Van Woensel [194] have motivated the need for such multi-echelon structures. These latter offer a tool to reduce urban congestion and nuisance, and increase mobility in cities [216, 61, 45].

Furthermore, strategic design decisions have a long-lasting effect and are time-sensitive. They are expected to efficiently operate in a long-term period handling future business changes and data fluctuations over time. Therefore, the use of a multi-period characterization in the context of distribution network design helps to adjust the network to the evolution of its data. A planning horizon including a set of planning periods should then be considered.

Finally, distribution practices have got more complex over the years, and problem data have higher uncertainty. The adequate design of a distribution network requires the anticipation of its future activity levels. Consequently, relevant models should incorporate the inherited uncertainty of data such as customer demands and costs, as over the last two decades, a large number of publications have emphasized its importance.

Although extensive studies have addressed DDPs, the developed models have raised only a subset of the aforementioned issues. In the recent years, few works have examined these issues. We start our study by reviewing in Section 2.2 works addressing one-echelon distribution network design problems (1E-DDPs), and discuss them with respect to the aforementioned missing aspects. In Section 2.3, we analyze works dealing with two-echelon models, and particularly two-echelon distribution network design problems (2E-DDPs). Section 2.4 discusses uncertainty modeling in the context of DDPs and reviews optimization aspects.

2.2 Deterministic one-echelon distribution network design problems

This section surveys deterministic 1E-DDP works with a particular focus on the temporal hierarchy aspect between decisions and multi-period feature.

Single echelon LRPs are the core of distribution network design problems. The first work is back to Maranzana [150], later a large number of papers on generalized LRP has appeared. Many studies have considered uncapacitated warehouses [222]. Then, since the survey of Nagy and Salhi [159], several authors introduce capacity constraints on warehouses and vehicules, leading to the so-called capacitated LRP (CLRP). However, few works still concern the uncapacitated cases with either uncapacitated vehicles or uncapacitated warehouses. LRP versions mostly formulate the vehicle routing part as a node routing problem (i.e., each pickup or delivery point is considered separately and is identified with a specific location in the network), and a few authors have studied arc routing versions [87, 98].

LRPs, related variants and applications have been largely addressed through exact and heuristic approaches, as illustrated by the recent surveys of Prodhon and Prins [178], Drexl and Schneider [71], Cuda et al. [62] and Schneider and Drexl [196]. Belenguer et al. [25] introduce a two-index vehicle-flow formulation for the CLRP strengthened with several families of valid inequalities. They solve the problem using a branch-and-cut algorithm. Contardo et al. [50] present three new flow formulations from which they derive new valid inequalities for each formulation. For each inequality, the authors either generalize separation methods introduced in [25] or propose new ones. Baldacci et al. [20] describe a new exact method for solving the CLRP based on a set partitioning formulation of the problem. The same formulation in [20] is strengthened by Contardo et al. [51] using new valid inequalities and solved through column-and-cut generation algorithm with subsequent route enumeration. Although approaches by Baldacci et al. [20] and Contardo et al. [51] are the state-of-the algorithms, they are based on an enumeration procedure for the subset of opened platforms. Thus, they cannot be applied for solving instances with more than 10 potential depots. Several heuristics are also proposed to solve LRPs, based on greedy randomized adaptive search procedure (GRASP) [175], variable neighborhood search (VNS) metaheuristics combined with integer-linear programming techniques [172], adaptive large neighborhood search (ALNS) [104], and a GRASP coupled with integer programming methods [52]. Schneider and Löffler [197] present a tree-based search algorithm, which is the best performing heuristics for CLRPs. This approach first explores the space of depot configurations in a tree-like fashion using a customized first improvement strategy. Then, in the routing phase, the multi-depot vehicle-routing problem defined by the depot configuration is solved with a granular tabu search that uses a large composite neighborhood described by 14 operators. These models have considered static and deterministic versions of the LRP ignoring the hierarchical structure of the strategic problem.

Further works involve hierarchical strategic problems as in facility location problems (FLP) [97, 65]. However, most FLP studies approximate the transportation and fulfillment characteristics related to the distribution case by direct routes and ignore the capacity decisions. FLPs consider a single distribution echelon with uncapacitated (UFLP) or capacitated (CFLP) facilities. UFLPs are tackled in many works as in Mirchandani and Francis [156] and Revelle and Eiselt [180] and CFLPs are proposed for example in Sridharan [214]. Owen and Daskin [165] and Klose and Drexl [134] present detailed reviews of the large literature available on these problems. Supply chain management generally relies on FLPs to formulate their problem in addition to other functions of the supply chain such as procurement, production, inventory, and capacity decisions. Static and deterministic models are proposed by Arntzen et al. [14], Martel [151] and Cordeau et al. [55]. A comprehensive review on this area can be found in [228, 180, 154] and in [130].

DDPs are long term planning problems. Several models have been studied under a multi-period setting. We distinguish between static models and multi-period models. Static models involve design decisions only at the beginning of the planning horizon whereas the operational transportation decisions are made over a multi-period setting to better anticipate operational decisions. The static LRP has been tackled in many works, known as a periodic LRP (PLRP) in the literature. This variant has been addressed by Prodhon and Prins [177], Prodhon [176] and Pirkwieser and Raidl [172]. On the other hand, multi-period models allow to review design decisions at the beginning of each period of the planning horizon through opening and/or closing warehouses and adjusting capacity decisions, etc. Laporte and Dejax [141] is the first to address dynamic multi-period uncapacitated LRP (ULRP) with capacitated vehicles. Recently, Albareda-Sambola et al. [11] examine the multi-period LRP with uncapacitated warehouses and vehicles and decoupled time scales for the location and the routing decisions. An approximation of the problem is solved in which routing decisions are approximated by forests rooted at available facilities. Two heuristics are then applied to get solutions for the origin problem.

Multi-period FLPs and production-distribution problems are studied in [77, 207, 66, 42, 69] and [1] among others. Exact and heuristic approaches are developed to solve these multi-period models. Capacity expansion problems are by definition multi-period [123]. Jena et al. [118] introduce a unifying model that generalizes several existing formulations of the multi-period FLP: the problem with facility closing and reopening, the problem with capacity expansion and reduction, and their combination. In [119], the authors propose Lagrangian relaxation based heuristics to solve the same problem with additional features such as relocation and partial facility closing. Surveys on facility location problems with a complete discussion about location problems in a multi-period context are provided by Daskin et al. [67] and Klose and Drexl [134]. A recent review on dynamic FLPs is given in [13].

2.3 Deterministic two-echelon distribution network design problems

As mentioned in the previous section, several models are addressed to handle DDPs with a single distribution echelon. Given the high growth of e-commerce in the recent years, such one-echelon networks limit the companies' capabilities to meet today's challenges. In fact, in the 1E-DDPs, warehouses are not specifically designed to provide next day and/or same day deliveries, or to efficiently operate to fast delivery services. In the new context, more attention is turned to two-echelon distribution structures by adding an intermediate echelon of distribution platforms standing between warehouses and customers. Two-echelon distribution network aims to design a network stucture that offers more flexibility to the future business needs.

LRPs are extended to the two-echelon LRP (2E-LRP). The literature on the 2E-LRPs is still scarce. The first works on the 2E-LRPs are credited to Jacobsen and Madsen [117] and to Madsen [148]. They consider the context of newspapers distribution in which location decisions involve only the intermediate platforms. Further references on the 2E-LRP are much more recent. Gonzalez-Feliu [89] formalizes the multi-echelons LRP providing a unified notation as well as a generic formulation. The 2E-CLRP is the most studied variant of the class of 2E-LRPs. It is formally introduced by Sterle [215] where integer programming models are presented. A tabu search heuristic is proposed in [39]. Later, Nguyen et al. [160, 161] study a particular case of the 2E-CLRP with a single warehouse in the first echelon whose position is known a priori. The authors propose two heuristics: a GRASP metaheuristic enhanced by a learning process [160], and a multi-start iterated local search (MS-ILS) coupled with path-relinking (MS-ILS$\times$PR) [161]. Contardo et al. [53] examine the generic case for the 2E-CLRP where the location decisions concern both echelons. They introduce a branch-and-cut algorithm based on a new two-index vehicle-flow formulation, strengthened by several families of valid inequalities, and also develop an Adaptive Large Neighborhood Search (ALNS) algorithm outperforming previous heuristics. A variable neighborhood search (VNS) for the 2E-CLRP is presented by Schwengerer et al. [200]. Recently, Breuning et al. [41] present a large neighborhood search based heuristic (LNS) for two-echelon routing problems. Their approach finds better solutions on benchmark instances for the 2E-CLRP with a single warehouse. Winkenbach et al. [236] study a particular static-deterministic variant of the 2E-CLRP in which a routing cost approximation formulæare used instead of explicit routing decisions. A literature survey on the two-echelon LRPs can be found in Prodhon and Prins [178], Cattaruzza et al. [45], Drexl and Schneider [71] and in Cuda et al. [62]. These models are static and deterministic versions of the 2E-CLRP. Additionally, they implicitly assume that location and routing decisions are made simultaneously for the planning horizon, without considering the hierarchical structure of the strategic problem.

Further studies have addressed other variants of the two-echelon problems, defined as two-echelon vehicle-routing problems (2E-VRPs) [61, 169]. In this class of problems, the main focus is about the transportation capabilities of the network, without considering location decisions (i.e., platforms are used freely without inducing a setup cost). Generally, 2E-VRP consider a single main warehouse platform in the upper level, and platforms do not have fixed capacities. For a detailed review, the reader is referred to [62].

On the oher hand, some papers use a hierarchical approach to the two-echelon distribution problem, extending the FLP, to the two-echelon FLP (2E-FLP) introduced in [85]. Similarly to FLPs, distribution operations are also represented by direct flows in the 2E-FLPs. Two-echelon hierarchical capacitated FLP (2E-CFLP) are investigated in many works as in [107, 108, 46, 132, 133, 182, 84]. The uncapacitated variant is discussed in [217, 23, 47, 46]. Recently, Ortiz-Astorquiza ett al. [164] study a general class of multi-echelon uncapacitated FLP and propose an exact algorithm based on a Benders reformulation. However, most of these studies consider static and deterministic models. Furthermore, we note that most production-distribution problems have a two-echelon structure, but with a single distribution echelon (see for instance [229, 24, 16, 80]). In this study, we assume that the two echelons are dedicated to distribution operations, i.e., responsible for the delivery of finished goods.

As highlighted in section 2.1, multi-period planning horizon should be considered when designing the distribution network. As far as we know, the only work tackling the multi-period setting in the 2E-LRPs is proposed by Darvish et al. [64]. Nevertheless, several studies consider the multi-period feature in the 2E-FLP variants. Similarly to the 1E-DDPs, we distinguish

Table 2.1: *Network structure, key decisions and solution approaches for deterministic distribution network studies*

Article	Network structure			Planning horizon		Distribution operations	Capacity planning	Specific constraints	Solution approach
	S/P	WP	DP	Static	Multi-period				
Ambrosino and Scutellà [12]	-	$\surd$	$\surd$	$\surd$		R	*Fc*		Lagrangean-based heuristic
Badri et al. [16]	$\surd$	$\surd$			$\surd$	F	*Dv*	$\surd$	Lagrangian-based heuristic
Boccia et al. [40]		$\surd$	$\surd$			R	*Fc*		Commercial solver
Breuning et al. [41]		$\surd$	$\surd$			R	*Fc*		LNS
Canel et al. [43]	-	$\surd$			$\surd$	F	*U*	$\surd$	Branch-and-bound and dynamic programming algorithm
Contardo et al. [53]		$\surd$	$\surd$			R	*Fc*		Branch-and-Cut and ALNS
Correia et al. [56]		$\surd$	$\surd$		$\surd$	F	*Dv*		Commercial solver
Cortinhal et al. [58]	$\surd$	$\surd$			$\surd$	F	*Dv*	$\surd$	Commercial solver
Darvish et al. [64]		-	$\surd$		$\surd$	R	*Fc*		Commercial solver + valid inequalities
Fattahi et al. [80]	$\surd$	$\surd$			$\surd$	F	*Dv*	$\surd$	Simulated annealing with a linear relaxation-based heuristic
Gendron and Semet [84]	-	$\surd$	$\surd$			F	*Dv*		Commercial solver
Georgiadis et al. [86]	-	$\surd$	$\surd$	$\surd$		F	*Dv*	$\surd$	Commercial solver
Hinojosa et al. [110, 109]	$\surd$	$\surd$			$\surd$	F	*Fc*	$\surd$	Lagrangian-based heuristic
Jacobsen and Madsen [117]	-		$\surd$			R	*Fc*		Three constructive heuristics
Nguyen et al. [160]		-	$\surd$			R	*Fc*		GRASP
Nguyen et al. [161]		-	$\surd$			R	*Fc*		MS-ILS
Schwengerer et al. [200]		$\surd$	$\surd$			R	*Fc*		VNS
Winkenbach et al. [236]		$\surd$	$\surd$			RF	*Fc*		Heuristic

S/P: Supplier/Production site, WP: Warehouse platform, DP: Distribution platform.
-: Implicit decision, $\surd$: Explicit decision.
U: Uncapacitated, *Fc*: Fixed capacity a priori, *Dv*: Decision variable.

static from multi-period models. Static models are addressed in [12, 86] where three-echelon production-distribution models compromising a two-echelon distribution schema are studied. In these models, multi-period settings concern the operational decisions, and the strategic decisions are taken once at the beginning of the planning horizon. Multi-period setting is examined by Correia et al. [56] for a two-echelon distribution model in which design decisions are decided over several periods. Other works such as in Hinojosa et al. [110, 109] and Cortinhal et al. [58] tackle a multi-period variants for two-echelon production-distribution problem, but therein, they consider a single distribution echelon. In addition, considering multiple design periods enables making adjustments to location decisions through opening, closing, and/or reopening facilities [64, 58], and adjustments to facility capacities through reduction and expansion [56, 80] in each design period due to demand variability over time. Detailed reviews can be found in [205, 154, 78].

Table 2.1 summarizes the aforementioned deterministic works in terms of the network structure (the echelons involved in the network other than the customer level), and provides the main distinguishing features in terms of the distribution operations (routes (R), flows (F) and route formulæ (RF)), and capacity planning decisions. The specific constraints column identifies works addressing production-distribution problems with additional manufacturing issues, production and inventory constraints. The table also illustrates the solution approaches used to solve the problem.

2.4 DDPs under uncertainty

In section 2.2 and 2.3, we have discussed the proposed deterministic models in the context of DDPs. Nonetheless, when design decisions are deployed, problem parameters such as costs, demand and distances, may fluctuate widely. Thus, the obtained design has no performance guarantee for plausible futures, since the aforementioned models do not handle uncertainties and information inaccuracy about expected plausible future business environments [106]. Hence, integrating uncertainty feature is crucial for more realistic DDPs.

Optimization under uncertainty have followed several modeling approaches. Rosenhead et al. [185] and Sahinidis [191] identify three decision-making environments categories: certainty, risk and uncertainty. This characterization is then used by Snyder [210], Klibi et al. [130] and Govindan et al. [92]. In certainty situations, all parameters are deterministic and known in advance. Risky and uncertainty cases involve randomness. In risk situations, the probability distribution of uncertain parameters is known by the decision maker. These parameters are thus referred as stochastic parameters that can be either continuous or described by discrete scenarios. In this case, problems are known as *stochastic optimization problems*, and their common goal is to optimize the expected value of some objective function. Uncertainty corresponds to the case in which the decision maker has no information about probability distributions of uncertain parameters. Under this setting, *robust optimization models* are developed. They often look to optimize the worst-case performance of the system. Further uncertainty approaches rely on fuzzy numbers to handle random parameters rather than probabilistic approaches. *Fuzzy programming* is thus used to manage the planner's expectations about the level of objective function, the uncertainty range of coefficients, and the satisfaction level of constraints by using membership functions. For further details, the reader may refer to Inuiguchi and Ramık [115], Sahinidis [191] and Govindan et al. [92]. In the DDPs related literature, models under uncertainty are mostly based on a probabilistic approach. Stochastic programming and chance-constraints programming are developed to handle this case. Next, we discuss models for the risky case using probabilistic modeling approach. Then, we briefly describe the robust optimization approach introduced to deal with non-probabilistic cases.

2.4.1 Stochastic programming for DDPs

In the stochastic optimization, uncertainty is modeled by a set of discrete scenarios and each of these scenarios has a probability of occurrence. A stochastic programming approach relates uncertainty and information structure to different stages in time. In this case, we identify decisions that have to be made prior to obtaining information about future parameters' realizations and decisions that are made when information is revealed [105]. Therefore, we recognize two main approaches for stochastic optimization: i) two-stage stochastic programs with recourse (TSSP) and ii) multi-stage stochastic programs (MSSP) [124, 204, 38]. In the following, we will recall the main concept of each approach and overview works using both approaches. Stochastic programming relies on a scenarios approach to build the model. However, this latter raises some difficulties as discussed in Snyder [210]. First, identifying scenarios and assessing their probabilities entail a tremendous effort. Secondly, determining an adequate number of scenarios may result in a large-scale optimization problem.

2.4.1.1 Two-stage stochastic programs

Two-stage stochastic programs with recourse (TSSP) involve two-stage nature of decisions: first- and recourse second-stage decisions [38]. First-stage decisions x are taken in the presence of uncertainty about future realizations of the stochastic parameters vector ξ. In the second-stage, the actual value of ξ is known, and some corrective actions or recourse decisions $y(\omega)$ can be taken depending on each scenario ω. First-stage decisions are, however, chosen by taking their future effects into account. These future effects are measured by a recourse cost function, $Q(x)$, which computes the expected value of taking decision x.

The two-stage stochastic programming approach has been applied in several studies to model stochastic DDPs. In these problems, first-stage decisions generally concern design decisions or long-term decisions such as location and capacity, and are made before knowing the realization of random parameters. When information uncertainty is revealed, operational transportation decisions and routing are made as a recourse second-stage actions to evaluate the obtained design.

Laporte et al. [142] and Albareda-Sambola et al. [10] apply this approach to single echelon ULRP in which depot locations and a priori routes must be specified in the first-stage, and second-stage recourse decisions deal with first-stage failures. Shen [205] proposes a stochastic LRP model based on routing cost estimations. Recently, Klibi et al. [128] formulate a static stochastic location transportation problem (SMLTP) with uncapacitated facilities as a two-stage stochastic program with recourse. The location and mission of depots must be fixed at the beginning of the planning horizon, but transportation decisions are made on a multi-period daily basis as a response to the uncertain customers' demand. They solve the SMLTP by a hierarchical heuristic approach based on sample average approximation (SAA) method. This latter is a sampling-based approach introduced by Shapiro [202]. Hence, to the best of our knowledge, no work has yet addressed stochastic variants or combined stochastic and multi-period settings in the 2E-LRP.

Further works have applied this approach to stochastic variants of FLPs and 2E-FLPs in the context of DDPs with direct flows. A review on stochastic FLPs can be found in Snyder [210]. Klibi et al. [130] and Govindan et al. [92] present a comprehensive survey of studies addressing production-distribution problems under uncertainty. As introduced in the previous sections, we distinguish between static and multi-period stochastic models when a planning horizon is used with stochastic setting. Santoso et al. [193] study a production-distribution variant compromising a single distribution echelon in which stochasticity is assumed for demand and facility capacities. The authors integrate the SAA scheme with an accelerated Benders decomposition algorithm to quickly compute high quality solutions to large-scale stochastic supply chain design problems with a vast number of scenarios. On the other hand, Schütz [199] and Heckmann et al. [101] model a static one-echelon production-distribution problem. In these works, the second-stage includes multiple periods capturing the variability of stochastic parameters over the planning horizon whereas first-stage decisions are determined once at the beginning of planning horizon. Georgiadis et al. [86] also use several combine multiple operational in the second-stage to model a two-echelon distribution configuration where product demand is uncertain and time varying. Other studies consider multi-period stochastic versions of the DDPs, where first-stage decisions are made over multiple periods of the planning horizon. This is investigated in [4, 101, 240] for the supply chain context including a single distribution echelon under demand uncertainty. Zhuge et al. [240] also consider an uncertain and time varying budget. Although few papers apply two-stage stochastic programming approaches in stochastic multi-period 2E-FLPs and production-distribution problems, its application to the two-echelon distribution configuration is still limited.

In Table 2.2, we present works investigating stochastic optimization approaches for DDPs in terms of the number of distribution echelons, the number of location level and optimization aspects. We also identify stochastic models with respect to the planning horizon setting. Therefore, the table points up the few works addressing two-stage stochastic models for the multi-period stochastic 2E-DDPs.

Table 2.2: *Network structure, solution approaches, and modeling approaches for stochastic programming models*

Article	Network			Mathematical modeling	Planning horizon	Solution approach
	Echelons	Distribution echelons	Location levels			
Aghezzaf [4]	2	1	1	TSSP- robust model	MP	Lagrangian relaxation decomposition
Albareda-Sambola et al. [9]	1	1	1	MSSP-and-TSSP	MP	Fix-and-Relax-Coordination algorithm
Fattahi et al. [79]	2	1	2	MSSP	static	Benders decomposition
Georgiadis et al. [86]	3	2	2	TSSP	static	Commercial solver
Heckmann et al. [101]	1	1	1	TSSP	static	Commercial solver
Klibi et al. [128]	1	1	1	TSSP	static	Heuristic + SAA
Klibi and Martel [129]	1	1	1	MSSP framework	MP	-
Nickel et al. [162]	1	1	1	MSSP	MP	Commercial solver
Pimentel et al. [171]	1	1	1	MSSP	MP	Lagrangian-based heuristic
Santoso et al. [193]	2	1	1	TSSP		Benders decomposition + SAA
Schütz et al. [199]	3	1	3	TSSP	static	Lagrangian relaxation + SAA
Zeballos et al. [239]	3	1	3	MSSP	MP	Commercial solver
Zhuge et al. [240]	2	1	1	TSSP	MP	Lagrangian-based heuristic

MP: Multi-period for design decisions.
TSSP: Two-stage stochastic program.
MSSP: Multi-stage stochastic program.

In addition, one should mention that in almost all two-stage stochastic developed models, the second-stage decisions are continuous and positive variables. Therefore, for each feasible first-stage solutions, the value of the recourse problem is a linear program for each scenario. Most stochastic models have developed a mixed integer linear program solved through a commercial solver. Decomposition techniques such as Benders decomposition, called also L-shaped algorithm in stochastic programming [225], are suitable to solve these models. However, when the second-stage problem implies integer variables, this increases the complexity of the problem [143, 224, 44, 8].

2.4.1.2 Multi-stage stochastic programs

Integrating a multi-period setting with uncertainty in long-term problems leads to multi-stage stochastic programming (MSSP) [111]. This is due to the fact that the uncertainty related to stochastic parameters is realized progressively in each period. Thus, MSSPs extend two-stage stochastic programming models by allowing one to adapt decisions in each time stage as more information regarding the uncertainties is revealed. Consequently, the use of multi-stage stochastic programming permits one to better capture the dynamics of decision-making in stochastic multi-period distribution network design problems, and provides more flexibility than does the two-stage model.

Consider a multi-stage stochastic program with T-stages. It includes a sequences of random parameters $\xi_1, \ldots, \xi_{T-1}$ where each $\xi_t, t = 1, \ldots, T$ represents the vector of stochastic parameters such as costs, demand, capacity, etc, at stage t in multi-stage stochastic program for DDPs. A scenario is the realization of these random parameters $\xi_1, \ldots, \xi_{T-1}$. Hence, a scenario tree is built for a discrete representation of the stochastic parameters representing the set of scenarios Ω. Figure 2.1 illustrates a typical scenario tree with eight scenarios for a four-stage stochastic program.

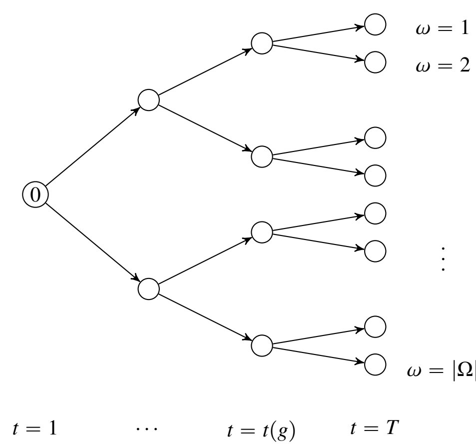


Figure 2.1: *A scenario tree example*

Assume decision vectors $x_1, \ldots, x_T$ corresponding to stages $1, \ldots, T$. In the multi-stage programming, the realizations of the random parameters are revealed in their respective stages. Thus, the values of decision vector x_t in stage t may depend on the information available up to stage t, but not on the future observations which is the basic requirement of non-anticipativity. This distinguishes the multi-stage stochastic programming problems from deterministic multi-period problems, in which all the information is assumed to be available at the beginning of the decision process [204, 38]. The sequence of actions x_t in each stage t, i.e., decisions and realization of random parameters, is then given in figure 2.2, and is called an implementable policy (or a policy).

As pointed out by Dupačová [73], there exist two common approaches for formulating a multi-stage stochastic program. The first one formulates the non-anticipativity settings implicitly, whereas the second approach imposes the non-anticipativity constraints explicitly. For further details on multi-stage stochastic programming, the reader can refer to Kall and Walalce [124], Shapiro et al. [204] and Birge and Louveaux [38].

A illustrated in Table 2.2, most studies have employed two-stage stochastic programs.

$$\begin{array}{ccccccccc} & \xi_1 & & \xi_2 & & & & \xi_T & \\ & \downarrow & & \downarrow & & & & \downarrow & \\ \textit{decision}\ (x_0) & \longrightarrow & \textit{decision}\ (x_1) & \longrightarrow & \textit{decision}\ (x_2) & \longrightarrow & \cdots & \longrightarrow & \textit{decision}\ (x_T) \end{array}$$

Figure 2.2: *Sequencing of decision process in multi-stage stochastic program*

Only few works have applied multi-stage stochastic models to DDPs and are mostly studying production-distribution problems [162, 9, 171, 79]. Ahmed et al. [6] explore this approach to formulate the multi-period capacity expansion problem under uncertain demand and investment cost. Recently, Klibi and Martel [129] and Dunke et al. [72] emphasize the need to such a framework to tackle complex production-distribution problems in the supply chain context. In [129], the authors define a methodological multi-stage framework for the supply chain network design problem, but therein a two-stage stochastic program for the one-echelon location-transportation problem is formulated and solved. Nickel et al. [162] study a multi-stage modeling framework for the supply network design problem with financial decisions and risk management involving a single distribution problem. They first formulate the problem as a multi-stage stochastic program, then propose an alternative formulation based on the paths in the scenario tree. Later, Albareda-Sambola et al. [9] consider the one-echelon stochastic multi-period discrete facility location problem, in which uncertainty affects the costs as well as some of the requirements along the planning horizon. A multi-stage and a two-stage stochastic programming approaches are proposed. Additionally, Pimentel et al. [171] present a mixed-integer multi-stage stochastic programming approach to the stochastic capacity planning and dynamic network design problem which integrates facility location, network design and capacity planning decisions under demand uncertainty. Accordingly, no work addresses the multi-stage stochastic framework for the multi-period stochastic 2E-DDP.

Furthermore, as shown in Table 2.2, most stochastic works use a commercial solver to solve their mathematical formulations. Benders decomposition approach is also applied in several cases to solve two-stage stochastic programs. To solve multi-stage stochastic programs, Albareda-Sambola et al. [9] propose a fix-and-relax coordination algorithm, and Pimentel et al. [171] develop a lagrangian heuristic.

2.4.1.3 Scenario generation

As mentioned, scenarios are used to represent stochastic parameters. This results in a scenario tree and a fan of individual scenarios for multi-stage and two-stage stochastic programs [74]. Under a multi-period setting, not only the parameters can be correlated with each other, but also they can be correlated across the time periods. Therefore, it would be more difficult to generate an appropriate set of scenarios.

This field has attracted the attention of many researchers. Several scenario generation methods and reduction techniques are proposed. We cite for instance, Heitsch and Römisch [102], Dupačová et al. [74, 75], and Høyland and Wallace [113]. Evaluation techniques are also developped to evaluate scenario methods in terms of quality and stability. One can refer to [126] for more details. In DDPs studies, only few ones have proposed an appropriate scenario generation procedure to obtain a set of scenarios [199, 128].

The sample average approximation (SAA) method has been employed to reduce the size of stochastic programs through repeatedly solving the problem with a smaller set of scenarios [202, 204]. This approach is involved in [193, 199, 128].

2.4.2 Chance-constrained programming for DDPs

When considering the optimization of a problem under uncertainty, one or multiple constraints need not hold almost surely. Instead, these constraints can be held with some probability or reliability level. In this case, *probabilistic* or *chance-constrained programming* is used as an alternative approach to deal with uncertainties [48]. This method often assumes that the distribution probabilities of uncertain parameters are known for decision makers.

Correia and Saldanha-da Gama [57] introduce a particular case of chance-constrained FLP with stochastic demand under single echelon structure. Using stochastic demand, the decision maker still wants to design an FLP satisfying all the demand realizations. However, the demand amount satisfied from each facility should not exceed its capacity. In this case, one should ensure a certain service level, i.e., ensure that with some pre-specified probability, the overall demand does not exceed the capacity of the operating facilities.

Getting a deterministic equivalent formulations for chance-constrained programs is a challenging task. Lin [146] proposes a deterministic equivalent formulation for the above stated case in which customers' demands are independent and follow poisson or gaussian probability distribution.

Chance-constrained have been considered in few studies for production-distribution problems such as in You and Grossmann [238] and in Vahdani et al. [223]. However, one should mention that it is not always easy to get a deterministic equivalent constraints for the probabilistic constraints. The reader is referred to Sahinidis [191] and Birge and Louveaux [38] for more details about this method.

2.4.3 Robust optimization for DDPs

Robust optimization is an alternative framework to stochastic optimization proposed by Mulvey et al. [158] that allows to handle parameters uncertainties. According to Roy [186], there is different kinds of robustness: it includes model robustness [158, 231], algorithm robustness [211] and solution robustness [185, 158, 237]. In the solution robustness, the solution (obtained decisions) is "nearly" optimal in all scenarios, whereas in model robustness, the solution is "nearly" feasible in all possible realization of uncertain parameters. The definition of "nearly" depends on the modeler's viewpoint. Therefore, many measures of robustness are proposed in the literature: expected value, standard deviation and conditional value at risk, etc, [188, 100]. However, when no probability information is available about the uncertain parameters, these measures are useless. In this case, different robustness measures have been proposed in the literature. These measures distinguish between continuous uncertain parameters and discrete case specified via some scenarios.

Under discrete scenarios without probability information, two common measures are defined for studies in this context: minimax cost and minimax regret. The minimax cost solution looks to minimize the maximum cost over all scenarios. It has been examined by Ahmadi-Javid and Seddighi [5] and Govindan and Fattahi [91] for a production-distribution problem. The second one is a minimax regret in which a regret of a solution is determined. This corresponds to compute the difference (absolute or a percentage) between the cost of a solution and the cost of the optimal solution in a given scenario. Kouvelis et al. [136] explore another approach for getting solution robustness in which some constraints on the maximum regret are added. This approach is defined as p-robustness in Snyder [210] for FLPs. The same approach has been used in [145, 167, 218] for production-distribution variants. Detailed reviews on these methods is provided by Snyder [210] and Govindan et al. [92] for the FLPs and production-distribution problems, respectively.

It is worth noting that several studies consider the risk measures and identify them as robustness measures. This is the case in [121, 189]. However, only Aghezzaf [4], Jin et al. [121] and [189] have examined model robustness measures for production-distribution problems with a single distribution echelon.

For continuous parameters, a pre-defined interval, known as an interval-uncertainty, is generally used to model uncertainty. In this case, robust models have been applied to handle the parameters variability in order to protect the optimal solution from any realization of the uncertainty in a given bounded uncertainty set [31]. This approach is introduced first by Soyster [212], who has looked for a solution that is feasible for all realization of uncertain parameters with their pre-defined uncertainty sets. However, its method leads to over-conservative solutions, the problem cost is much higher than the deterministic problem (i.e., uncertain parameters are fixed to their deterministic values). Then, Ben-Tal a Nemirovski [29, 30] and El-Ghaoui et al. [76] have investigated this issue, and proposed to model the uncertainty sets by ellipsoids. Nonetheless, their robust formulations result in non-linear but convex models, and thereby more difficult to solve. Later, Berstimas and Sim [36] present a less-conservative robust approach in which the conservatism level of robust solutions could be controlled and resulted in a linear optimization model. The literature about robust models for DDPs is limited. Most developed models have examined inventory management problems [2], production-distribution problems [173, 127] and vehicle-routing problems [90].

Furthermore, the aforementioned robust approaches are static. They assume that all decisions are made before the uncertainty is revealed. However, real-life applications, particularly distribution network design problems, have multi-stage nature. Thus, some decisions should be made after the realization of uncertainty. In this context, a *multi-stage robust optimization* models are developed, called affinely adjustable robust counterpart (AARC) [28]. This approach limits the adjustable decisions to be affinely dependent upon the primitive uncertainties. But, it requires further development to be applied to multi-period 2E-DDPs. As far as we know, only few studies have examined this approach for single echelon DDPs [81] and production-distribution problems [201].

Distributionally robust optimization is a recent robust approach introduced to deal with situations when some certain distributional properties of the primitive uncertainties, such as their support and moments are known [195, 88, 235]. In this case, the uncertain parameters are characterized by a probability distribution which is itself subject to uncertainty. A recent review on robust optimization approach is provided in Gabrel et al. [82]. However, one should note that these approaches still need more attention in future works for multi-period problems under uncertainty.

2.5 Conclusion

Distribution network design problems (DDPs) have been extensively studied in the operations research literature. This research field is attracting an increasing attention from practitioners and academics due to their relevant real-life applications, in addition to the challenges induced in their studies.

In this work, we review works addressing DDPs in terms of problem aspects, practical applications, and optimization aspects, as pointed out in the introduction section. Although this research area has been growing steadily, several weaknesses are identified in DDPs literature, and particularly related to the extended two-echelon distribution structures. Some of these issues have been discussed in length all along this survey. However, many aspects and promising research directions are still unexplored. The main research avenues to develop a flexible 2E-DDPs under uncertainty are:

- **Hierarchical methods and use of route length formulæ**: Routing decisions are often used as operations anticipation for the user level (i.e., operational decisions) when designing the strategic distribution network. Instead of explicitly computing vehicle routes, approximation formulæ can be used to speed up the decision process. This area has not attracted many researchers. Therefore, more efforts should be focused on route approximation formulæ, especially for multi-period and stochastic problems where the solution of the routing part may require too much time.

- **Multi-echelon models**: Despite its importance in practical applications, only few works have explored two-echelon structure for DDPs, and are mostly tackling basic problems under static deterministic settings. Further research in this area would include more realistic aspects of distribution problems such time windows for customers and time synchronization for distribution operations at intermediate distribution platforms.

- **Dynamic and stochastic models**: Despite the fact that optimization frameworks under uncertainty are well established in the literature and have followed several modeling approaches, its application to distribution problems is still in its infancy, specifically for two-echelon structures. As far as we know, no work has considered two- and multi-stage stochastic programming for stochastic multi-period 2E-DDPs. Further research directions could use these approaches when designing a distribution network. Additionally, it would be interesting to study robust optimization in multi-period uncertain problems.

- **Solution methods**: Several studies reviewed here use commercial solvers to solve the problem. However, for large-scale problems, commercial solvers' performance is limited. This emphasizes the need for designing specific solution approaches. Decomposition methods such as Benders decomposition or L-shaped method are applied for specific problem structure, in addition to heuristic methods. Further research perspectives would go through developing efficient solution methods for two- and multi-stage stochastic programs and robust models. Another future area is to present hybrid solution algorithms which are based on the combination of exact methods with heuristics.

To conclude, considering the aforementioned issues helps to capture real-life problems. But, this further complicates the design of the distribution network problem. We believe that one should consider the trade-off between the realism and the tractability of the model when designing a flexible and effective network. This survey provides several research directions towards developing a comprehensive methodology for DDPs, and particularly 2E-DDPs.

Chapter 3

Designing Two-Echelon Distribution Network under Demand Uncertainty

Abstract

This paper proposes a comprehensive methodology for the stochastic multi-period two-echelon distribution network design problem (2E-DDP) where product flows towards ship-to-points are directed from an upper layer of primary warehouses to distribution platforms (DPs) before being transported to ship-to-points. A temporal hierarchy characterizes the design level dealing with DPs location decisions and capacity decisions, and the operational level involving transportation decisions as origin-destination flows. These design decisions must be calibrate to minimize the expected distribution cost associated to the two-echelon transportation schema on this network under stochastic demands. We consider a multi-period planning horizon where demand varies dynamically from one planning period to the subsequent one. Thus, the design of the two-echelon distribution network under uncertain customers' demand gives rise to a complex multi-stage decisional problem. Given the strategic structure of the problem, we introduce alternative modeling approaches of the problem based on a two-stage stochastic programming with recourse. The resulting models are solved using a Benders decomposition approach. The size of the scenario set is tuned using the sample average approximation (SAA) approach. Then, a scenario-based evaluation procedure is introduced to post-evaluate the obtained design solutions. Extensive computational experiments based on several types of instances are conducted in order to validate the proposed models and to assess the efficiency of the solution approaches. The evaluation of the quality of the stochastic solution underlines the impact of uncertainty in the two-echelon distribution network design problem (2E-DDP).

Keywords: *Logistics, Two-echelon Distribution Network Design, Location models, Capacity modeling, Uncertainty, Multi-period, Stochastic programming.*

3.1 Introduction

The emergence of e-commerce and the arrival of innovative players, such as Amazon, have unquestionably changed the logistics landscape. More specifically, the major shift to an on-demand economy has tremendously affected the distribution schemas of several companies that aim to continue improving response time to customers while efficiently offering their products in a multi-channel setting. The delivery service level expectancy has significantly increased in the last decade: it is now expressed in hours rather than days. In addition, the ship-to-locations have recently evolved, making use of lockers, relay points, drives, and collection stores as alternatives to home delivery. In such context, several global B-to-C players, and especially companies operating in the retail sector, such as Walmart, Carrefour, Amazon or jd.com, have recently undertaken a sustained reengineering of their distribution networks. They have incorporated extra considerations to be closer to their key customer zones without reducing the efficiency of their consolidation policies in warehousing and transportation. For several global companies, the location of their primary warehouses followed various optimization rules going from centralization and risk-pooling incentives to sourcing-dependent and financial constraints. In practice, companies may have a single centralized warehouse or a reduced set of market-dedicated regional warehouses where they generally keep about a month or a season's worth of inventory depending on the demand dynamics and the production/sourcing cycles. When customers are globally deployed, these storage-locations are not specifically designed to provide next day and/or same day deliveries, especially when the customer bases are located in large geographic and almost urban areas. Such one-echelon networks constrain the companies' ability to provide fast delivery services, and reduce their opportunities to capture online orders.

In this new context, such strategic considerations imply a distribution schema with more than one-echelon that can be dynamically adjusted to the business needs over time. A typical predisposition schema, claimed now by practitioners, is the two-echelon distribution network. The network topology includes an intermediate echelon of distribution/fulfillment platforms located between the initial sites where inventory is held and the ship-to-points. For instance, Walmart plans to convert 12 Sam's Club stores into e-commerce fulfillment centers to support the rapid e-commerce growth [122]. City logistics is probably the most significant example of the shift from a one-echelon to a two-echelon distribution network setting [61, 160]. This is achieved by creating peripheral distribution/consolidation centers dedicated to transferring and consolidating freight from back-level platforms on large trucks into smaller and environmentally friendly vehicles, more suitable for city distribution. Parcel delivery is also a relevant context where two-echelon distribution networks operate [89]. Parcels travel from storing platforms to distribution platforms, and are then loaded onto smaller trucks that ship parcels to relay points, to lockers, or to customer homes.

With this in mind, most of the literature considers the one-echelon network topology to characterize distribution operations where the aim is to find optimal locations, minimize the number of depots, and build routes around the depots to serve customers [178]. At the tactical level, the notion of multi-echelon distribution is well studied from an inventory optimization perspective [93], however, its strategic counterpart is less developed [71, 62]. Several authors recently recalled the need to expand one-echelon networks by considering an intermediate echelon of platforms where storing, merging, consolidation or transshipment operations take place [40].

This is specifically the focus of this paper where we address the two-echelon distribution-network design problem (2E-DDP). This strategic problem aims to decide on the number and location of distribution platforms (DPs), and the capacity allocated to these platforms to efficiently distribute goods to customer ship-to-bases. It also integrates the allocation decisions related to the assignment of ship-to-points to DPs, and DPs to primary warehouses. The overriding challenge of a two-echelon setting is that the location of such DPs does not only depend on the trade-off between ship-to-point demand versus DP capacity and the type of outbound routes that could be designed, but is also influenced by the inbound assignment/replenishment policy and the trade-off between location versus capacity at the preceding echelon. Furthermore, in view of the demand process, the cost variability over time, and demand uncertainty, the question is when and how much distribution capacity to add to the network. We address this related issue by examining the periodic capacity decisions for the distribution network over a multi-period planning horizon. Figure 3.1 illustrates a typical 2E-DDP where the network is partitioned into two capacitated distribution echelons. Each echelon has its own assignment-transportation schema that must be adapted in response to the uncertainty shaping the business horizon.

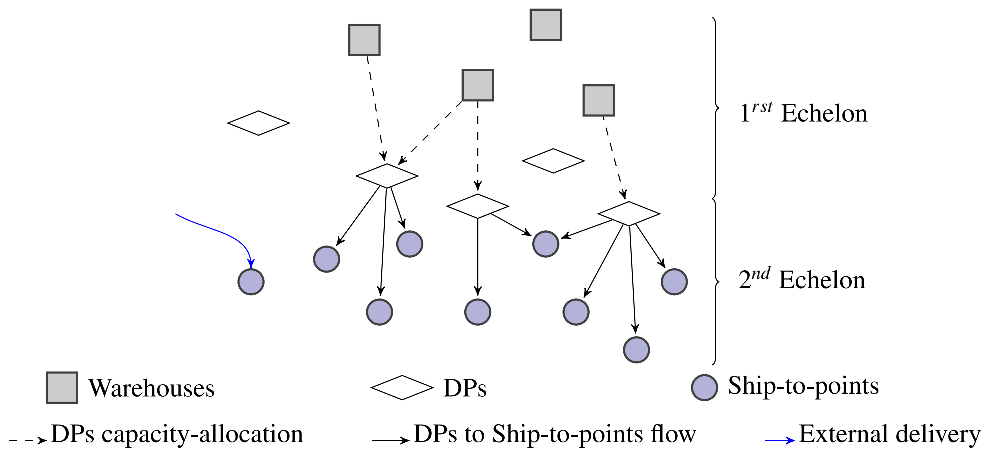


Figure 3.1: *A potential Two-Echelon Distribution Network Design Problem (2E-DDP)*

Some authors consider such a distribution context by explicitly modeling routes to formulate the two-echelon location-routing problem (2E-LRP) as an extension of the well-known location-routing problem (LRP) [139]. The 2E-LRP is formally introduced in Boccia et al. [40] in an urban context and is further studied by Contardo et al. [53]. However, the literature on 2E-LRP is still scarce and considers only a deterministic-static setting [178, 71, 62]. Additionally, 2E-LRP and most LRP modeling approaches implicitly assume that location and routing decisions are made simultaneously for the planning horizon, without considering the hierarchical structure of the strategic problem stressed here. Alternatively, some papers use a hierarchical approach to the two-echelon distribution problem, extending the facility location problem (FLP) [97, 65], to the two-echelon facility location problem (2E-FLP) introduced in [85]. However, most 2E-FLP studies approximate the transportation and fulfillment characteristics related to the distribution case, ignoring the capacity decisions, and studying mostly deterministic versions. Furthermore, we note that most production-distribution problems in the supply chain management literature have a two-echelon structure, but with a single distribution echelon, generally relying on FLP to formulate the problem. These studies tend towards functional expansions related to production policies and constraints, and specific manufacturing-linked transportation issues, rather than focusing on the strategic needs of the distribution businesses stressed above (see for instance [229, 16, 24, 80]). Comprehensive reviews on FLPs can be found in [154, 78].

Moreover, given the strategic nature of decisions in 2E-DDP, the network must be designed to last for several years, fulfilling future requirements. It should also be efficiently adaptable to changes in the business environment over time. According to the Tompkins report in (2011) [219], there is a significant trend toward reducing the planning horizon in strategic studies: the length of the re-engineering period defined in strategic network design studies has reduced on average from 4 years to under 2 years due to business uncertainty increasing and distribution practices becoming more complex. Accordingly, the traditional deterministic-static representation of the planning horizon is replaced by a more realistic stochastic-multi-period characterization of the planning horizon. More specifically, the horizon is modeled with a set of planning periods shaping the evolution/uncertainty of random factors (e.g., demand, costs, etc.), and promoting the structural adaptability of the distribution network. Such decisional framework leads to a multi-stage stochastic programming problem as addressed here. Despite the fact that the multi-stage stochastic programming framework is well established [203, 38], its application to distribution problems is still in its infancy. Recently, some authors [129, 72] raise the need for such a decisional framework to tackle complex supply chain problems. Klibi and Martel [129] propose a multi-stage framework for a supply chain network design problem, but the two-stage stochastic location-transportation model formulated and solved relies on a single distribution echelon. Dunke et al. [72] underline the importance of meeting time-dependent and uncertain ship-to-point demand subject to time-dependent cost parameters with decisions on opening, operating, or closing facilities, and capacity adjustment decisions along the horizon. Further studies, such as Nickel et al. [162], Pimentel et al. [171], and Albareda-Sambola et al. [9], address the multi-stage stochastic setting but only for one-echelon distribution problems. To the best of our knowledge, no study addresses the multi-period and stochastic versions of 2E-LRP and 2E-FLP.

The aim of this paper is thus to first define 2E-DDP under uncertain and time-varying demand, and time-varying DP opening costs, formulated as a multi-stage stochastic program with recourse. Our modeling framework considers that the planning horizon is composed of a set of planning periods shaping the evolution of uncertain ship-to-point demand over time. It also assumes that the number and location of DPs are not fixed a priori and must be decided at the strategic level along the set of planning periods. Furthermore, it considers strategic assignment-transportation decisions to calibrate DP throughput capacity based on transportation capabilities. Our approach looks at a hierarchical strategic-operational decisional framework where the emphasis is on the network design decisions and their impact on the company's distribution performance. The operational decisions related to transportation operations are modeled as origin-destination arcs, which correspond to a sufficiently precise aggregate of daily decisions over several products, transportation means, and working periods, as discussed in [131]. Second, 2E-DDP is an NP-hard stochastic combinatorial optimization problem since it inherits several complexities from the FLP and LRP models, in addition to the curse of dimensionality of its multi-stage stochastic setting. This justifies the development of approximate modeling approaches that allow handling realistic instances and guarantee the generation of good-quality design solutions. Consequently, we propose solvable two-stage stochastic versions of the 2E-DDP. These models differ in the modeling of distribution operations, and we discuss their solvability with respect to the capabilities of current solvers. Our solution methodology builds on a Benders decomposition [32] and on the sample average approximation (SAA) method [203] to formulate the deterministic equivalent model with an adequate sample size of scenarios. The results obtained from the extensive experiments show the trade-off between solvability and the quality of the designs produced by both models. They also illustrate the importance of tackling the stochastic problem by evaluating the quality of the stochastic solutions. Several managerial insights are also derived on the behavior of design decisions under the stochastic-multi-period characterization of the planning horizon.

The remainder of this paper is organized as follows. Section 3.2 summarizes the related works on distribution network design models under deterministic and stochastic settings, and the exact solution methods applied to such models. Section 3.3 defines the 2E-DDP, presents the characterization of the uncertainty by scenarios, and provides a comprehensive multi-stage stochastic formulation. Section 3.4 discusses the solvability of our model and introduces two-stage stochastic program approximations of the multi-stage model that make it tractable. Section 3.5 presents the solution approaches proposed to solve the problem using Benders decomposition and the SAA technique. It also introduces an evaluation procedure to assess the quality of the stochastic solutions, and evaluate the performance of the designs obtained. Section 3.6 reports our computational results and discusses the quality of the solutions obtained in conjunction with the solvability effort of the associated models. Section 3.7 provides conclusions and outlines future research avenues.

3.2 Related works

2E-DDP is closely related to several classes of problems in the operations research (OR) literature that we classify according to their modeling options. Table 3.1 presents the main studies related to 2E-DDP and classifies the related works in terms of network structure (the echelons involved in the network other than the ship-to-point level), and provides the main distinguishing features in terms of the distribution operations (routes (R) versus flows (F)) and capacity planning decisions under single/multi-period (SP vs MP) and deterministic (D)/stochastic (S) settings. Table 3.1 also highlights the mathematical modeling and the solution approaches used to tackle the problem.

As mentioned in Table 3.1, existing studies include different numbers and types of echelons. The echelons differ in terms of location decisions as an implicit or explicit decision. Boccia et al. [40] and Sterle [215] formulate a static two-echelon distribution problem as a 2E-LRP where location decisions involve warehouses and distribution centers, and distribution operations are modeled by routes. They introduce a two-index and three-index vehicle-flow formulation, and a set-partitioning formulation. Nguyen et al. [160, 161] study a special case of 2E-LRP in the urban context including one single main warehouse at the first echelon and several potential distribution centers. Correia et al. [56] focus on a multi-period two-echelon network with central and regional distribution centers where distribution decisions are approximated by flows. Supply chain networks have provided suitable applications of FLPs in the last decades. Ambrosino and Scutellà [12] and Georgiadis et al. [86] address designing a supply chain network comprising a two-echelon distribution schema in addition to inbound flows (i.e., from suppliers/plants). Further studies use the two-echelon structure in supply chain network design problems [58, 80, 240], in closed loop supply chains [239], and in production-distribution problems [43, 229], but only considering a single distribution echelon. These studies are oriented toward functional expansions, such as production policies and constraints, and specific manufacturing-linked transportation issues, rather than focusing on the strategic needs of the distribution businesses. In this study, we assume that the two echelons are dedicated to distribution operations, i.e., responsible for the delivery of finished goods (see for instance [56, 86]).

The complexities of distribution systems in real world applications lead to integrating operational decisions at the strategic level. More specifically, capacity decisions as strategic/design planning decisions and transportation and distribution policies as operational planning decisions are included in 2E-DDPs, as well as classic location-allocation decisions. Moreover, considering a time horizon of multiple periods is of great importance to decision-makers, since facility location decisions are long-term problems. Ambrosino and Scutellà [12], Georgiadis et al. [86] and Heckmann et al. [101] limit the multi-period settings to operational decisions, and the strategic decisions are taken once at the beginning of the planning horizon, whereas Correia et al. [56] and Darvish et al. [64] consider multi-period settings for strategic decisions. In [128], a single design period is used and coupled with multiple operational periods. Albareda-Sambola et al. [11] tackle a multi-period LRP where two interconnected time scales for design and operational decisions are considered.

In addition, considering multiple design periods enables making adjustments to location decisions through opening (O), closing (C), and/or reopening (Re) facilities [162, 64, 171], and adjustments to facility capacities through reduction and expansion [56, 4, 240] in each design period due to demand variability over time (see Table 3.1). Pimentel et al. [171] focus on the one-echelon stochastic capacity planning and dynamic network design problem in which warehouses can be opened, closed, and reopened more than once during a planning horizon. This is more suited to new warehouses that are being rented instead of built, since lower fixed setup costs are incurred. Heckmann et al. [101] introduce a single echelon risk-aware FLP in which capacity expansions with different levels are possible in each facility. Capacity expansion is executed at the second-stage decision when uncertainty is revealed. Jena et al. [118] introduce the one-echelon dynamic facility location problem with generalized modular capacities that generalizes several existing formulations for the multi-period FLP: the problem with facility closing and reopening, the problem with capacity expansion and reduction, and their combination.

One-echelon distribution problems under uncertainty are studied in the literature as in [193, 101] and [199] where stochasticity is assumed for demand and facility capacities. Additionally, incorporating a multi-period planning horizon with uncertainty is investigated in Aghezzaf [4], and Zhuge et al. [240] in the supply chain context with one distribution echelon to meet the variability of demand uncertainty. Zhuge et al. [240] also consider an uncertain and time varying budget. Georgiadis et al. [86] combine multiple operational periods with the stochastic setting in a two-echelon distribution configuration where product demand is uncertain and time varying.

In 2E-DDP, distribution operations may take the form of routes (R) or direct flows (F) generalizing the classic problems in the literature: 2E-LRPs [62] and 2E-FLPs [154, 78], respectively. Table 3.1 shows that the majority of papers using routes are studied in a deterministic-static setting, and to the best of our knowledge, no work has yet addressed the multi-period and stochastic settings simultaneously. However, some papers consider the multi-period and stochastic settings in 2E-FLPs and supply chain network design, but its application to the two-echelon distribution configuration is still limited.

Furthermore, Table 3.1 shows that most studies in the deterministic-static and multi-period setting are formulated with a mixed-integer linear program (MILP) (for instance, [53, 56]). Nevertheless, under uncertainty, stochastic programming approaches are more appropriate as in [86, 101, 240], where the problem is modeled as a mixed-integer two-stage stochastic program with recourse (TSSP).

Integrating a multi-period setting with uncertainty in long-term problems leads to multi-stage stochastic programming (MSSP) [111]. This approach is applied in Nickel et al. [162], Albareda-Sambola et al. [9], and Pimentel et al. [171] for the one-echelon distribution network, and in Zeballos et al. [239] for the closed-loop supply chain context. Ahmed et al. [6] formulate the multi-period capacity expansion problem as a MSSP where both the demand and investment costs are uncertain.

The multi-stage modeling approach adds complexity to the problems. Theoretical developments and approximations are proposed in the literature: Guan et al. [95] present cutting planes for multi-stage stochastic integer programming enhanced by inequalities that are valid for individual scenarios. When a set of scenarios is assumed for modeling uncertainty in a multi-stage setting, it is possible to build a scenario tree. However, in practice, accurate approximations of a complex stochastic process with a modest-sized scenario tree represent a very difficult problem. Thus, several scenario generation methods and reduction techniques are proposed. We refer the reader to Heitsch and Römisch [102], Dupačová et al. [74, 75], and Høyland and Wallace [113]. Römisch and Schultz in [183] explore the scenario tree and propose a path-based alternative modeling framework. Later, Huang and Ahmed [114] used the same framework to improve the modeling of uncertainties in a MSSP context.

Accordingly, no work addresses the multi-stage stochastic framework for the multi-period stochastic 2E-DDP. Furthermore, although some works propose tackling mixed-integer multi-stage stochastic programs using exact and heuristic methods based on decomposition techniques and/or scenario sampling methods [198, 38], further progress is required to solve realistic two-echelon distribution design problems.

Table 3.1: *Network structure, key decisions and solution approaches in multi-period planning horizon distribution network studies*

Article	Network structure			SP/MP	D/S	Distribution operations	Capacity Planning	Location status per time period	Mathematical modeling	Solution approach
	S/P	W	DC							
Jacobsen and Madsen [117]	-		√	SP	D	R	*Fc*	1|O		Three constructive heuristics
Boccia et al. [40]		√	√	SP	D	R	*Fc*	1|O	Three MILPs	Commercial solver
Nguyen et al. [160]		-	√	SP	D	R	*Fc*	1|O	ILP	GRASP
Nguyen et al. [161]		-	√	SP	D	R	*Fc*	1|O	MILP	MS-ILS
Contardo et al. [53]		√	√	SP	D	R	*Fc*	1|O	MILP	Branch-and-Cut and ALNS
Darvish et al. [64]		-	√	MP	D	R	*Fc*	T|O	MILP	Commercial solver + valid inequalities
Ambrosino and Scutellà [12]	-	√	√	MP	D	R	*Fc*	1|O	MILP	Lagrangean-based heuristic
Correia et al. [56]		√	√	MP	D	F	*Dv*	T|O	MILP	Commercial solver
Georgiadis et al. [86]	-	√	√	MP	S	F	*Dv*	1|O	TSSP	Commercial solver
Heckmann et al. [101]		√		MP	S	F	*Dv*	1|O	TSSP	Commercial solver
Aghezzaf [4]	-	√		MP	S	F	*Dv*	T|O	Robust model	Lagrangian relaxation decomposition
Zhuge et al. [240]	-		√	MP	S	F	*Dv*	T|O	TSSP	Lagrangian-based heuristic
Albareda-Sambola et al. [9]		√		MP	S	F	*U*	T|O/C	MSSP-and-TSSP	Fix-and-Relax-Coordination algorithm
Nickel et al. [162]			√	MP	S	F	*Fc*	T|O	MSSP	Commercial solver
Pimentel et al. [171]		√		MP	S	F	*Dv*	T|O/C/Re	MSSP	Lagrangian-based heuristic
Zeballos et al. [239]	√	√	√	MP	S	F	*Fc*	T|O/C	MSSP	Commercial solver
Our work		-	√	MP	S	F	*Dv*	T|O/C/Re	MSSP-and-TSSP	Benders decomposition+SAA

S/P: Supplier/Plant, W: Warehouse, DC: Distribution center.
-: Implicit decision, √ : Explicit decision.
U: Uncapacitated, *Fc*: Fixed capacity a priori, *Dv*: Decision variable.

O: Opening new locations, C: Closing existing locations, Re: Reopening closed locations.

The aforementioned studies mostly use commercial solvers to solve their mathematical programs as in [56, 86, 40]. Exact solution methods are proposed to solve the two-echelon distribution configuration, such as the branch-and-cut algorithm in [53]. Lagrangian-based heuristics in [12] are the most popular heuristics used in this context. Metaheuristics are also proposed to solve the deterministic and static 2E-DDP with routes such as the Greedy Randomized Adaptive Search Procedure (GRASP) in [160], the Multi-Start Iterated Local Search (MS-ILS) in [161], and the Adaptive Large Neighborhood Search (ALNS) [53]. Sampling methods [202] such as sample average approximation (SAA) are also used in stochastic models to limit the large number of scenarios. This is applied in supply chain network design problems with a one distribution echelon in [193, 230, 199, 128]. Santoso et al. [193] integrate the SAA scheme with an accelerated Benders decomposition algorithm to quickly compute high quality solutions to large-scale stochastic supply chain design problems with a vast number of scenarios.

3.3 The two-echelon multi-period distribution network design problem

3.3.1 Problem definition

We consider the business context of a retail company that sources a range of products from a number of supply sites (e.g., suppliers, manufacturing plants), and stores them at primary warehouses. Without loss of generality, these products are aggregated in a single product family in our modeling approach because they are relatively uniform and share the same handling and storage technology [152]. This is done by taking average cost and demand information related to the entire product family. Under a make-to-stock policy, the company operates a set of primary warehouses, formerly designed to centralize inventories and ensure distribution to demand zones periodically. However, the locations of the company warehouses are not necessarily designed to provide next day and/or same day delivery. To do so, the company needs to deploy an advanced set of distribution resources to serve ship-to-points with an adequate service level. Strategic facility-location decisions concern a new intermediate echelon, dimensioning capacitated DPs used to fulfill orders and deliver finished goods to ship-to-points. This 2E-DDP is illustrated in Figure 3.1: strategic decisions concern the location of DPs at the intermediate level and the capacity of the links between the warehouses and the ship-to-locations.

Ship-to-point orders vary in quantity of product demanded on a daily basis. Once a given set of DPs is deployed, the company periodically determines the quantities of goods to be allocated to each DP: this translates into a number of full-load trucks required from warehouses to deliver products to a DP. Then, on a daily basis, the goods are delivered to ship-to-locations through common or contract carriers for each single ship-to-point. Thus, while the allocation decisions are modeled as periodic origin-destination quantities from warehouses to DPs, the transportation decisions are modeled as daily transportation links from DPs to ship-to-points. Our focus is on strategic capacity allocation decisions made by distribution platforms and their transportation capabilities. The operational shipping decisions are modeled to capture the running cost of strategic decisions. To ensure feasibility, a recourse delivery option is allowed if the ship-to-points cannot be satisfied from the set of deployed DPs on a given day. This recourse comes at a higher shipment cost.

Our model considers a long-term planning horizon $\mathcal{T}$ that covers a set of successive design planning periods $\mathcal{T} = \{1, \ldots, T\}$. Such periods must be defined in accordance with the operational dynamics. For instance, one can assume that a period corresponds to a year, which is typical in the context of DPs to lease (in practice, it could be up to 2 years in the case of building or renovating DPs). Each planning period covers a set of operational periods, represented generally in a discrete way by "typical" business days. Figure 3.2 illustrates the relationship between the decision planning periods and operational days, and the hierarchical structure of the decision problem. It also shows that strategic design decisions (location and capacity decisions) could be adapted periodically at each design period t to align the distribution network to its business environment, especially when operating under uncertainty. Worth noting is that the design decisions must be made prior to their deployment period with partial information on the future business environment. After an implementation period, they will be available for use as shown by the positioning of the arrows in Figure 3.2. This assumes information asymmetry between the design level and the operational level, mainly due to the fact that the decisions are not made at the same time.

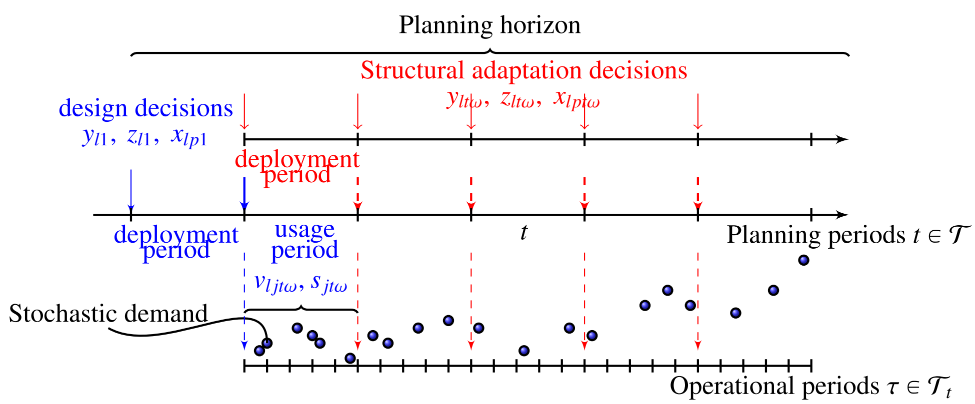


Figure 3.2: *Decision time hierarchy for the planning horizon under uncertainty*

Therefore, our model has a multi-stage decision structure. At the beginning of the planning horizon, the here and now design decisions are made, and are thus considered as the first-stage DP design decisions. Next, at the beginning of each subsequent period ($t > 1$), a new opportunity to adapt the distribution network structure to its future environment is offered, based on the information available at that time. Decisions made at the beginning of the period t depend on the design decisions up to that period, as illustrated in Figure 3.2 and described mathematically in the stochastic 2E-DDP model below.

3.3.2 Scenario building and tree representation

Uncertain daily demand of ship-to-points is represented by a random variable, which is estimated by a given probability distribution. Let d_j be the random variable for ship-to-point j

demand that follows a distribution probability F_j with a mean value μ_{j0} estimated from historical data until $t = 0$. Under a multi-period planning horizon setting, the random demand process is time-varying. More specifically, a multi-period plausible future allows capturing factor transitions (inflation-deflation, population density, market stores, etc.) that perturb the a priori estimation of demand behavior and could thus impact the design decisions. This means that a trend function is associated with the random variable and the associated distribution probability F_{jt} with a mean value μ_{jt} to shape demand realization. The demand process is stationary if the trend function is fixed at zero, and non-stationary otherwise. Worth noting is that the uncertainty concerns the order quantity, and can thus shift the demand level from one location to another in the network.

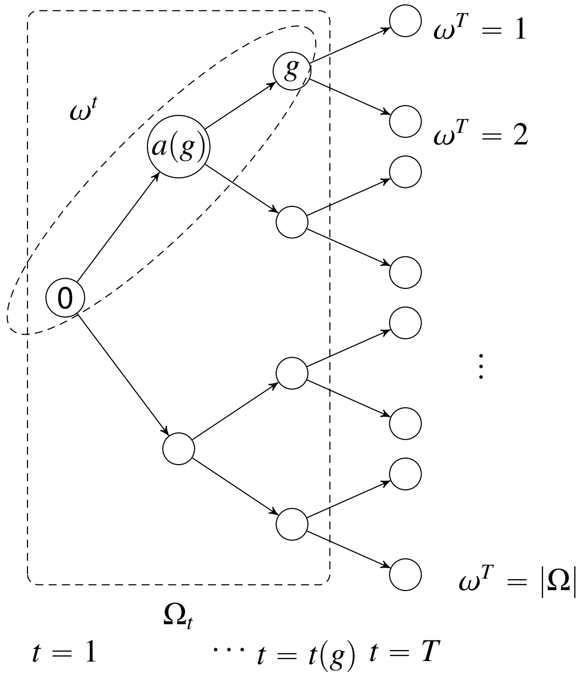
The uncertainty is characterized by a set Ω of plausible future scenarios where a scenario ω encompasses the demand realization for each period t and for all the ship-to-points during a typical business day. Then, at the beginning of each period t, the information available is updated according to the additional data revealed up to t. Let Ω_t be the subset of distinct scenarios of Ω that share the same realization up to stage t. Hence, $\Omega_t = \{\omega^t : \omega \in \Omega\}$ and $\Omega_T = \Omega$. Thus, $d_{jt\omega}$ will denote the demand of ship-to-point j at period t under scenario $\omega \in \Omega_t$ following the distribution probability F_{jt} with the parameters μ_{jt}. Given the entire planning horizon, a scenario tree $\mathfrak{T}$ should be built to characterize the realization of demand for each planning period. When using such stochastic process, scenario instances can be generated with Monte Carlo methods. At each stage t, a discrete number of nodes represent points in time where realizations of the uncertain parameters take place and decisions have to be made. Each node g of the scenario tree, except the root, is connected to a unique node at stage $t-1$, called the ancestor node $a(g)$, and to nodes at stage $t+1$, called the successors. We denote with $\pi_{a(g),g}$ the conditional probability of the random process in node g given its history up to the ancestor node $a(g)$. The path from the root node to a terminal (leaf) node corresponds to a scenario ω, and represents a joint realization of the problem parameters over all periods $1, \ldots, T$. Partial paths from the root node to intermediate nodes correspond to the restricted scenarios up to stage t denoted ω^t.

Figure 3.3a illustrates a typical example of a multi-stage scenario tree. The scenario probability is obtained by multiplying the conditional probabilities through the path. The non-anticipativity principle [181] is implemented by requiring that the decisions related to scenarios that are identical up to a given stage are the same and can therefore be represented by a single variable. In the following, we use branches of the scenario tree to define recourse variables where each branch represents a restricted scenario $\omega \in \Omega_t$. Accordingly, using restricted scenarios, one can avoid to write the non-anticipativity constraints explicitly.

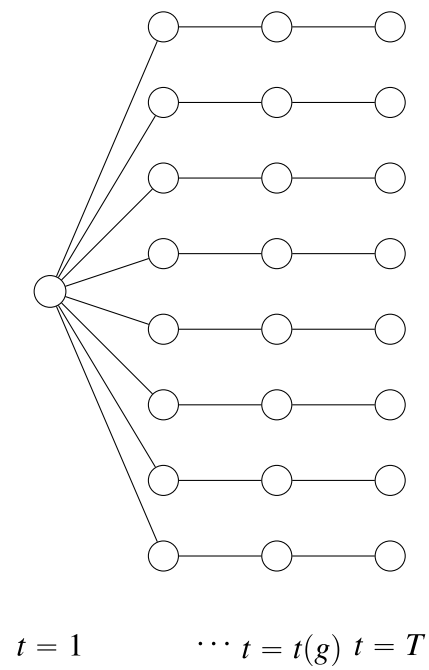
3.3.3 The multi-stage stochastic formulation

The stochastic 2E-DDP can be modeled as a multi-stage stochastic program with a set of scenarios. As mentioned above, Ω_t represents the set of distinct restricted scenarios up to stage t, for all $t = 1, \ldots, T$: $\Omega_t = \{\omega^t : \omega \in \Omega\}$. We also define:

prec(ω) the set of direct ancestors of scenario $\omega \in \Omega_t$, for all $t = 1, \ldots, T-1$: *prec*$(\omega) =$



(a) *Multi-stage scenario tree*



(b) *Fan of individual scenarios*

Figure 3.3: *Scenario tree representation*

$\{\omega' \in \Omega_{t-1} : \omega_\tau(\omega) = \omega_\tau(\omega'), \forall \tau < t\}$.

$p(\omega)$ the probability of each scenario $\omega \in \Omega_t$. Note that $\sum_{\omega \in \Omega_t} p(\omega) = 1$ for all t.

Additionally, we consider the following notations:

- Sets

  $\mathcal{P}$ set of primary warehouses.

  $\mathcal{L}$ set of distribution platforms (DPs).

  $\mathcal{J}$ set of ship-to-points.

- Parameters

  C_p is the maximum throughput capacity of primary warehouse $p \in \mathcal{P}$ (expressed in flow unit for a given period).

  C_l is the maximum capacity of the DP $l \in \mathcal{L}$.

  C_{lp} is the maximum capacity of transportation used for flows from warehouse $p \in \mathcal{P}$ to DP $l \in \mathcal{L}$.

  c_{ljt} is the transportation cost per product unit from a DP $l \in \mathcal{L}$ to ship-to-point $j \in \mathcal{J}$ at period $t \in \mathcal{T}$.

c_{lpt} is the unit transportation cost per flow unit from warehouse $p \in \mathcal{P}$ to DP $l \in \mathcal{L}$ at period $t \in \mathcal{T}$.

f^s_{lt} is the cost of opening a DP $l \in \mathcal{L}$ at period $t \in \mathcal{T}$.

f^u_{lt} is the cost of operating a DP $l \in \mathcal{L}$ at period $t \in \mathcal{T}$.

c_{jt} is the shipment cost when recourse delivery is employed to cover a proportion of the demand of a ship-to-point $j \in \mathcal{J}$ at period $t \in \mathcal{T}$.

Since transportation decisions are made within typical business day, the operational transportation costs c_{ljt} and c^r_{jt} are annualized to cover their daily aspect in the objective function.

The decision variables are:

$z_{lt\omega}$ = 1 if DP $l \in \mathcal{L}$ is opened at period t under scenario $\omega \in \Omega_t,\ t = 1, \ldots, T$, 0 otherwise.

$y_{lt\omega}$ = 1 if DP $l \in \mathcal{L}$ is operating at period t under scenario $\omega \in \Omega_t,\ t = 1, \ldots, T$, 0 otherwise.

$x_{lpt\omega}$ = Inbound allocation from warehouse $p \in \mathcal{P}$ to DP $l \in \mathcal{L}$ expressed in number of truckload units contracted to deliver from the warehouse under scenario $\omega \in \Omega_t,\ t = 1, \ldots, T$.

$v_{ljt\omega}$ = fraction of demand delivered from DP $l \in \mathcal{L}$ to ship-to-point $j \in \mathcal{J}$ under scenario $\omega \in \Omega_t,\ t = 1, \ldots, T$.

$s_{jt\omega}$ = fraction of demand of ship-to-point $j \in \mathcal{J}$ satisfied from a recourse delivery under scenario $\omega \in \Omega_t,\ t = 1, \ldots, T$ (i.e., external shipment, not from DPs).

The deterministic equivalent formulation for the multi-stage stochastic problem takes the form:

$$\text{(MS-M)} \min \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega_t} p(\omega)\Bigg(\sum_{l\in\mathcal{L}} \Big[(f^u_{lt}\, y_{lt\omega} + f^s_{lt}\, z_{lt\omega}) + \sum_{p\in\mathcal{P}} c_{lpt}\, C_{lp} x_{lpt\omega} \Big] + \sum_{j\in\mathcal{J}} d_{jt\omega} \Big[\sum_{l\in\mathcal{L}} c_{ljt}\, v_{ljt\omega} + c_{jt}\, s_{jt\omega} \Big] \Bigg) \quad (3.1)$$

$$\text{S. t.} \quad \sum_{l\in\mathcal{L}} C_{lp} x_{lpt\omega} \leqslant C_p \qquad \forall p \in \mathcal{P}, t \in \mathcal{T}, \omega \in \Omega_t \quad (3.2)$$

$$\sum_{p\in\mathcal{P}} C_{lp} x_{lpt\omega} \leqslant C_l y_{lt\omega} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T}, \omega \in \Omega_t \quad (3.3)$$

$$y_{lt\omega} - y_{ltprec(\omega)} \leqslant z_{lt\omega} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T}, \omega \in \Omega_t \quad (3.4)$$

$$\sum_{j\in\mathcal{J}} d_{jt\omega}\, v_{ljt\omega} \leqslant \sum_{p\in\mathcal{P}} C_{lp} x_{lpt\omega} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T}, \omega \in \Omega_t \quad (3.5)$$

$$\sum_{l\in\mathcal{L}} v_{jlt\omega} + s_{jt\omega} = 1 \qquad \forall j \in \mathcal{J}, t \in \mathcal{T}, \omega \in \Omega_t \quad (3.6)$$

$$x_{lpt\omega} \in \mathbb{N} \qquad \forall l \in \mathcal{L}, p \in \mathcal{P}, t \in \mathcal{T}, \omega \in \Omega_t \quad (3.7)$$

$$y_{lt\omega}, z_{lt\omega} \in \{0, 1\} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T}, \omega \in \Omega_t \quad (3.8)$$

$$v_{ljt\omega} \geqslant 0 \qquad \forall l \in \mathcal{L}, j \in \mathcal{J}, t \in \mathcal{T}, \omega \in \Omega_t \quad (3.9)$$

$$s_{jt\omega} \geqslant 0 \qquad \forall j \in \mathcal{J}, t \in \mathcal{T}, \omega \in \Omega_t \quad (3.10)$$

The objective function (3.1) seeks to minimize the total expected cost for the design and operational costs throughout the planning horizon. The first and second terms refer to the design costs that include the operating and opening costs for DPs, and the inbound allocation cost to DPs from warehouses. The third term represents the transportation cost in the second echelon (from DPs to ship-to-points) as well as the external delivery costs. Constraints (3.2)-(3.3) express capacity limit on warehouses and DPs, respectively, over stages. In addition, constraints (3.3) force that a delivery is possible to a DP only if it is operating. Constraints (3.4) define the setup of the DPs over stages. These constraints manage the status of the DPs operating from one stage to the next and set their opening. Constraints (3.5) aim to cover the demand of ship-to-points from the operating DPs without exceeding their inbound allocation from warehouses. Constraints (3.6) ensure that a ship-to-point is either satisfied through the opened set of DPs or through a recourse delivery. All the other constraints (3.7)-(3.10) define the decision variables of the problem. Observe that $|\Omega_1| = 1$ and therefore decisions (x, y, z) at the first stage represent here and now decisions concerning the design. Their counterpart at later periods represents recourse on the design in this multi-stage process. On the other hand, allocation decisions (v, s) allow evaluating the operational cost of the distribution system given design (x, y, z) that is fixed in each period.

3.4 Two-stage stochastic program approximations

The above multi-stage stochastic formulation for the stochastic 2E-DDP presented highlights the hierarchical structure of the problem and the scenario-based relation between the different decisions. For realistic size problems, directly tackling such multi-stage stochastic programs using exact and heuristic methods is beyond the scope of current technologies [198, 38]. We build here on the reduction [125, 209] and relaxation [204] approaches recently applied to transform the multi-stage stochastic program to a two-stage stochastic program, and transform the multi-stage stochastic program (3.1)-(3.10) into a two-stage stochastic program that is sufficiently accurate to capture the essence of the problem while being solvable in practice.

Accordingly, one modeling approach consists in transferring from the MS-M model all the design decisions of the T periods (location and capacity-allocation) to the first-stage in order to be set at the beginning of the horizon. In this case, only first-stage design decisions ($t = 1$) are made here and now, but subsequent design decisions ($t > 1$) (see Figure 3.2) are essentially used as an evaluation mechanism. These latter design decisions for periods ($t > 1$) are deferrable in time according to their deployment period. Therefore, the obtained model offers an approach to set the design decisions for ($t = 1$) with an optimistic evaluation at the beginning of the horizon of subsequent design decisions without losing its hedging capabilities. This approach gives rise to the two-stage stochastic location capacity-allocation model. The obtained model is challenging to solve due to the combinatorial difficulty of the resulting mixed-integer program and the high number of scenarios. Another modeling approach consists in transferring from the MS-M model only the T periods' design decisions related to the location decisions to the first-stage, and relaxing the capacity-allocation decisions to the second-stage for all periods. These latter capacity-allocation decisions now become part of the recourse problem and thus scenario-dependent (see Figure 3.2). This approach gives rise to the two-stage stochastic flow-based location-allocation model.

The discussion above impacts the scenario building approach: for stages $t \geqslant 2$, the scenario tree construction algorithm can reduce the number of nodes to a fan of individual scenarios that prescribes the random parameter value for the full time horizon with a probability $p(\omega), \omega \in \Omega_t$. This is illustrated in Figure 3.3b where scenarios are independent of the number of periods, as shown in [74]. Thus, the scenario representation fan of the planning horizon fits well with our two-stage stochastic programming and clearly simplifies the generation of scenarios. With this in mind, the remainder of the section provides the formulation of these two design models for the 2E-DDP.

3.4.1 Location and capacity-allocation model (LCA)

In this two-stage model, the design decisions for ($t > 1$) can be taken at the beginning of the planning horizon and do not depend on the history up to period t. In this case, the first-stage decisions consist in deciding the DPs to open and to operate as well as the capacities assigned to DPs from warehouses (i.e., the first echelon of the network) during T periods. In complement, the second-stage decisions look forward to the distribution operations in the second echelon of the network (i.e., from DPs to ship-to-points) and the recourse deliveries. This formulation is referred to as the location and capacity-allocation model (LCA).

The decision variables are defined below:

x_{lpt} = the number of full truckloads assigned from warehouse $p \in \mathcal{P}$ to DP $l \in \mathcal{L}$ at period $t \in \mathcal{T}$.

y_{lt} = 1 if DP $l \in \mathcal{L}$ is operating at period $t \in \mathcal{T}$.

z_{lt} = 1 if DP $l \in \mathcal{L}$ is opened at period $t \in \mathcal{T}$.

$v_{lj\omega t}$ = fraction of demand delivered from DP $l \in \mathcal{L}$ to ship-to-point $j \in \mathcal{J}$ under demand scenario $\omega \in \Omega_t$.

$s_{j\omega t}$ = fraction of demand of ship-to-point $j \in \mathcal{J}$ satisfied through external delivery under scenario $\omega \in \Omega_t$ of period t (i.e., that ship-to-point is not delivered to from DPs).

We denote the set of feasible combinations of first-stage decisions y_{lt} with y, z_{lt} with z and x_{lpt} with x. Then, the LCA is formulated as a mixed-integer two-stage stochastic linear program:

$$\text{(LCA)} \quad \min_{x,y,z,v,s} \mathbb{E}_{\Omega} h(x,y,z,\omega) = \min \sum_{t\in\mathcal{T}} \sum_{l\in\mathcal{L}} (f^u_{lt}\, y_{lt} + f^s_{lt}\, z_{lt}) + \sum_{t\in\mathcal{T}} \sum_{l\in\mathcal{L}} \sum_{p\in\mathcal{P}} c_{lpt}\, C_{lp} x_{lpt} + \sum_{t\in\mathcal{T}} \mathbb{E}_{\Omega}[Q_t(x,\omega)] \quad (3.11)$$

$$\text{S. t.} \quad \sum_{l\in\mathcal{L}} C_{lp} x_{lpt} \leqslant C_p \qquad \forall p \in \mathcal{P}, t \in \mathcal{T} \quad (3.12)$$

$$\sum_{p\in\mathcal{P}} C_{lp} x_{lpt} \leqslant C_l\, y_{lt} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T} \quad (3.13)$$

$$y_{lt} - y_{lt-1} \leqslant z_{lt} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T} \quad (3.14)$$

$$x_{lpt} \in \mathbb{N} \qquad \forall l \in \mathcal{L}, p \in \mathcal{P}, t \in \mathcal{T} \quad (3.15)$$

$$y_{lt}, z_{lt} \in \{0,1\} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T} \quad (3.16)$$

where $Q_t(x,\omega)$ is the solution of the second-stage problem:

$$Q_t(x,\omega) = \min_{v,s} \sum_{j\in\mathcal{J}} d_{j\omega t}[\sum_{l\in\mathcal{L}} c_{ljt}\, v_{lj\omega t} + c_{j\omega t}\, s_{j\omega t}] \quad (3.17)$$

$$\text{S. t.} \quad \sum_{j\in\mathcal{J}} d_{j\omega t}\, v_{lj\omega t} \leqslant \sum_{p\in\mathcal{P}} C_{lp} x_{lpt} \qquad \forall l \in \mathcal{L} \quad (3.18)$$

$$\sum_{l\in\mathcal{L}} v_{jl\omega t} + s_{j\omega t} = 1 \qquad \forall j \in \mathcal{J} \quad (3.19)$$

$$v_{lj\omega t} \geqslant 0 \qquad \forall l \in \mathcal{L}, j \in \mathcal{J} \quad (3.20)$$

$$s_{j\omega t} \geqslant 0 \qquad \forall j \in \mathcal{J} \quad (3.21)$$

The objective function (3.11) is the sum of the first-stage costs and the expected second-stage costs. The first-stage costs represent the opening DP cost, the operating DP cost, as well as the capacity cost induced by the number of truckloads associated with DPs from warehouses. The objective function of the second stage (3.17) consists in minimizing the cost of the total flow delivered from DPs to ship-to-points and the cost of the recourse when the ship-to-point is satisfied partially or totally through an extra delivery. Constraints (3.12) ensure that the quantity delivered from a warehouse do not exceed its capacity. Constraints (3.13) guarantee the capacity restriction at an operating DP. Constraints (3.14) define the location setup. To use a DP, opening decisions must be activated in the same period, unless set as active in a preceding period. Constraints (3.15)-(3.16) describe the feasible set for the first-stage variables. Constraints (3.18) aim to cover the ship-to-point demand from the opened DPs. Constraints (3.19) ensure that a ship-to-point is either satisfied totally or partially through the designed network or from the extra delivery option. Constraints (3.20)-(3.21) are the non-negativity constraints for the second-stage variables.

In the LCA model presented above, the first-stage decisions are projected out in the recourse problem through the capacity assignment variables x_{lpt}, as expressed in constraints (3.18). However, the inclusion of assignment-transportation x_{lpt} as integer first-stage decision variables may complicate its resolution, particularly for large size instances.

3.4.2 Flow-based location-allocation model (LAF)

As mentioned, the first echelon (warehouses-DPs) distribution operations are represented by a throughput capacity based on the transportation capabilities in the LCA model. Such modeling option necessitates the inclusion of capacity assignment-transportation as the integer first-stage decision variables x_{lpt}, and may complicate its resolution. A common alternative modeling approach is to consider continuous flows as capacity-allocation variables, which are part of the second-stage and thus scenario-dependent variables. In such case, the first echelon distribution operations are represented by a set of origin-destination links, $x_{lpt\omega}$, and refer to the proportion of truckloads assigned from warehouse p to DP l under scenario ω at period t. With such modeling approach, a two-stage stochastic program is obtained where location decisions (opening and operating DPs) are first-stage decisions, and the flow-based transportation decisions in both echelons are second-stage decisions, denoted as the flow-based location-allocation model (LAF). Given the LCA program, constraints (3.12), (3.13), and (3.15) should be adjusted to be part of the recourse problem, and thus replaced by (3.24), (3.25), and (3.28), respectively. Constraint (3.18) is also substituted by (3.26). The LAF can then be written as:

$$\text{(LAF)} \min_{y,z,x,v,s} \mathbb{E}_\Omega h(y,z,\omega) = \min \sum_{t\in\mathcal{T}} \sum_{l\in\mathcal{L}} (f^u_{lt}\, y_{lt} + f^s_{lt}\, z_{lt}) + \sum_{t\in\mathcal{T}} \mathbb{E}_\Omega[Q_t(y,\omega)] \tag{3.22}$$

$$\text{S. t.} \qquad (3.14) \text{ and } (3.16)$$

where $Q_t(y,\omega)$ is the solution of the second-stage problem of the LAF formulation:

$$Q_t(y,\omega) = \min_{x,v,s} \sum_{l\in\mathcal{L}} \sum_{p\in\mathcal{P}} c_{lpt}\, C_{lp} x_{lpt\omega} + \sum_{j\in\mathcal{J}} d_{j\omega t} [\sum_{l\in\mathcal{L}} c_{ljt}\, v_{lj\omega t} + c_{j\omega t}\, s_{j\omega t}] \tag{3.23}$$

$$\text{S. t.} \quad \sum_{l\in\mathcal{L}} C_{lp} x_{lpt\omega} \leqslant C_p \qquad \forall p \in \mathcal{P} \tag{3.24}$$

$$\sum_{p\in\mathcal{P}} C_{lp} x_{lpt\omega} \leqslant C_l\, y_{lt} \qquad \forall l \in \mathcal{L} \tag{3.25}$$

$$\sum_{j\in\mathcal{J}} d_{j\omega t}\, v_{lj\omega t} - \sum_{p\in\mathcal{P}} C_{lp} x_{lpt\omega} \leqslant 0 \qquad \forall l \in \mathcal{L} \tag{3.26}$$

$$\sum_{l\in\mathcal{L}} v_{jl\omega t} + s_{j\omega t} = 1 \qquad \forall j \in \mathcal{J} \tag{3.27}$$

$$x_{lpt\omega} \geqslant 0 \qquad \forall l \in \mathcal{L}, p \in \mathcal{P} \tag{3.28}$$

$$v_{lj\omega t} \geqslant 0 \qquad \forall l \in \mathcal{L}, j \in \mathcal{J} \tag{3.29}$$

$$s_{j\omega t} \geqslant 0 \qquad \forall j \in \mathcal{J} \tag{3.30}$$

In the LAF model, the recourse problem is linked to the problem by constraints (3.25), where first-stage decisions are projected out through the operating DP variables (y_{lt}). Given the LCA and LAF models, the next section proposes a solution methodology to solve these multi-period stochastic versions of the 2E-DDP. Hereafter, we denote for each model $o \in$ {LCA, LAF}, the design vector $X(o)$ with $X(\text{LCA}) = \{x,y,z\}$ and $X(\text{LAF}) = \{y,z\}$, respectively.

3.5 Solution methodology

For real-scale instances of the aforementioned models, one would have to manage the inherent combinatorial complexity and the extremely large set of demand scenarios of the 2E-DDP under uncertainty. Therefore, our solution methodology combines the scenarios sampling approach and the decomposition schema of the stochastic models. The sample average approximation (SAA) is used to handle the large set of scenarios and determine the most adequate sample size [204]. This approach has been applied to network design problems in [193, 199] and to stochastic multi-period location transportation problem in [128]. For more details, interested readers are referred to Appendix A.1. In this section, we first propose a reformulation of both models based on the Benders decomposition scheme, adapted to the two-stage and multi-period setting of the stochastic programs. Then, a scenario-based evaluation procedure is introduced to post-evaluate the alternative design solution produced by the LCA and the LAF models, and thus discuss their design structure and the quality of the stochastic solutions obtained.

3.5.1 The Benders decomposition

Given the combinatorial complexity of the stochastic 2E-DDP, cutting plane algorithms such as Benders decomposition could be suitable to enhance its solvability. The Benders decomposition is a well-known partitioning method applicable to mixed-integer programs [32, 33]. In the Benders decomposition, the original problem is separated into a master problem and a number of sub-problems, which are typically easier to solve than the original problem. By using linear programming duality, all sub-problem variables are projected out, and the master problem contains the remaining variables and an artificial variable representing the lower bound on the cost of each sub-problem. The resulting model is solved by a cutting plane algorithm. In each iteration, the values of the master problem variables are first determined, and the sub-problems are solved with these variables fixed. An optimal solution of the master problem provides a lower bound, and this solution is transmitted to the sub-problems to construct new ones. If the sub-problems are feasible and bounded, an optimality cut is added to the master problem, otherwise a feasibility cut is added. The solution of the feasible sub-problems provides the valid upper bound. By adding the new Benders cuts, the master problem is resolved, and the Benders decomposition algorithm is repeated continuously until the difference between the lower bound and the upper bound is small enough or zero.

In this paper, we introduce a Benders decomposition algorithm to solve the two SAA programs related to the LCA and LAF models based on a set of sampled scenarios Ω^N. In this subsection, we present the Benders reformulation developed for the (LCA(Ω^N)) program and keeping the reformulation for the (LAF(Ω^N)) in Appendix A.2. In the (LCA(Ω^N)) program, integer variables are first-stage decisions, while continuous variables belong to the second-stage. If the first-stage decisions are fixed, the resulting sub-problem can be decomposed into $|T| \times |\Omega^N|$ sub-problems, one for each period $t \in \mathcal{T}$ and each scenario $\omega \in \Omega^N$. This is inspired by the original idea of applying Benders decomposition to stochastic integer programs, also known as the L-shaped method [225, 38]. Given $X(\mathrm{LCA}(\Omega^N)) = \{x, y, z\}$, the fixed vector related to the (LCA(Ω^N)), the expected total cost of the second-stage decisions, denoted with $\phi(x,y,z)$ can be calculated as $\phi(x,y,z) = \frac{1}{N}\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega_t^N}\phi_{t\omega}(x,y,z)$, where $\phi_{t\omega}(x,y,z)$ is the total second-stage cost for each period t and each scenario ω. This cost is obtained through solving the following primal sub-problem (PS$_{\omega t}$):

$$
\begin{aligned}
(\mathrm{PS}_{\omega t}) \quad \phi_{t\omega}(x,y,z) = Q_t(x,\omega) = \min \sum_{j\in\mathcal{J}} d_{j\omega t}[\sum_{l\in\mathcal{L}} c_{ljt}\, v_{lj\omega t} + c_{j\omega t}\, s_{j\omega t}] \qquad (3.31)\\
\text{S. t.} \qquad (3.18) - (3.21)
\end{aligned}
$$

The (PS$_{\omega t}$) is always feasible because the demand can be satisfied from an extra delivery option, i.e., $s_{j\omega t}$, which is uncapacitated. We denote with α_l and β_j the dual variables associated with constraints (3.18)and (3.19), respectively. Accordingly, the dual of (PS$_{\omega t}$) for each t and ω, called the dual sub-problem (DS$_{\omega t}$), can be formulated as:

$$
(\mathrm{DS}_{\omega t}) \quad \phi_{t\omega}(x,y,z) = \max \sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} C_{lp}\, x_{lpt}\, \alpha_l + \sum_{j\in\mathcal{J}} \beta_j \tag{3.32}
$$
$$
\text{S. t.} \qquad d_{j\omega t}\, \alpha_l + \beta_j \leqslant d_{j\omega t}\, c_{ljt} \qquad \forall l \in \mathcal{L}, j \in \mathcal{J} \tag{3.33}
$$
$$
\beta_j \leqslant d_{j\omega t}\, c_{j\omega t} \qquad \forall j \in \mathcal{J} \tag{3.34}
$$
$$
\alpha_l \leqslant 0 \qquad \forall l \in \mathcal{L} \tag{3.35}
$$
$$
\beta_j \in \mathbb{R} \qquad \forall j \in \mathcal{J} \tag{3.36}
$$

We define as $\Delta_{t\omega}$ the polyhedron under the constraints (3.33) and (3.34) of (DS$_{\omega t}$). Let $P_{\Delta_{t\omega}}$ be the set of extreme points of $\Delta = \cup_{t\omega}\Delta_{t\omega}$. We introduce an additional variable $u_{t\omega}$ representing the total expected second-stage decision cost per t and ω. Thus, the Benders master problem is written as:

$$
(\mathrm{BMP}) \min \sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}} (f_{lt}^u\, y_{lt} + f_{lt}^s\, z_{lt}) + \sum_{t\in\mathcal{T}}\sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}} c_{lpt} C_{lp} x_{lpt} + \frac{1}{N}\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega_t^N} u_{t\omega} \tag{3.37}
$$
$$
\text{S. t.} \qquad (3.12) - (3.16)
$$
$$
u_{t\omega} - \sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} C_{lp}\, x_{lpt}\, \alpha_l \geqslant \sum_{j\in\mathcal{J}} \beta_j \qquad \forall t \in \mathcal{T}, \omega \in \Omega_t^N,\ (\alpha_l,\beta_j) \in P_{\Delta_{t\omega}} \tag{3.38}
$$
$$
u_{tw} \geqslant 0 \qquad \forall t \in \mathcal{T}, \omega \in \Omega_t^N \tag{3.39}
$$

Constraints (3.38) represent the Benders optimality cuts.

3.5.2 Evaluation procedure

When using a sampling approach, the assessment of the design solutions produced by the two models presented in Section 3.4 is restricted to the set of scenarios considered. To better appreciate the performance of the produced design solutions, we develop a complementary evaluation procedure. This relies on the fact that the higher the evaluation sample size, the more precise the assessment of the design solution. It also builds on the ability of this post-optimization phase to introduce additional performance measures to appreciate the robustness of the solution, which were not part of the optimized model. In practice, a design solution evaluation procedure would be as close as possible to the company's real operational problem. Hereafter, we consider the second-stage formulation of the LCA and LAF models as an evaluation model that practically refers to operational-level decisions. In addition, we base the evaluation on a much larger sample of Monte Carlo scenarios, $N^e \gg N$ ($N^e = |\Omega^{N^e}|$), than those used to generate the candidate designs. Thus, for a given design vector $\widehat{X}_t(o)$ at period t obtained from the SAA program $o \in \{\text{LCA}(\Omega^N), \text{LAF}(\Omega^N)\}$, we compute the cost value for each scenario $\omega \in \Omega^{N^e}$, $Q_t(\widehat{X}_t(o), \omega)$, using the respective sub-model. More specifically, in the (LCA(Ω^N)) program, the design vector is $\widehat{X}_t(\text{LCA}(\Omega^N))=\{\hat{x}_t, \hat{y}_t, \hat{z}_t\}$, and the $Q_t(\widehat{X}_t(\text{LCA}(\Omega^N)), \omega)$ is evaluated by (3.17)-(3.21). On the other hand, for (LAF(Ω^N)), the design vector is $\widehat{X}_t(\text{LAF}(\Omega^N))=\{\hat{y}_t, \hat{z}_t\}$, and the $Q_t(\widehat{X}_t(\text{LAF}(\Omega^N)), \omega)$ is evaluated by (3.23)-(3.30). To note is that the evaluation model used in the procedure is separable per scenario since the design decisions are fixed, which allows considering a much larger scenarios sample.

First, a measure of the expected value $\mathcal{V}_t^e(o)$ for each design period t is computed using the evaluation sample Ω^{N^e}. Second, a downside risk measure is computed to assess the variability of each design at period t. More specifically, the variability measure is the upper semi-deviation from the mean, $MSD_t(o)$, for each model $o \in \{\text{LCA}(\Omega^N), \text{LAF}(\Omega^N)\}$, and is formulated as:

$$MSD_t(o) = \frac{1}{N^e} \sum_{\omega=1,\dots,N^e} \max\left(0; Q_t(\widehat{X}_t(o), \omega) - \mathbb{E}_{\Omega^{N^e}}[Q_t(\widehat{X}_t(o), \omega)]\right)$$

This measure was introduced by Shapiro et al. in [204], and helps assess the penalization of an excess of a realization ω over its mean. The evaluation procedure is detailed in the algorithms 3.1 and 3.2 for LCA(Ω^N)and LAF(Ω^N), respectively.

Algorithm 3.1 Evaluation procedure LCA(Ω^N) model

1: **for all** $t = 1, \dots, T$ **do**
2: **for all** $\omega = 1, \dots, N^e$ **do**
3: Evaluate $Q_t(\widehat{X}_t(LCA(\Omega^N)), \omega)$ using the program (3.17)-(3.21)
4: **end for**
5: **return** $\mathcal{V}_t^e(LCA(\Omega^N)) = \sum_{l\in\mathcal{L}}(f_{lt}^u\, \hat{y}_{lt} + f_{lt}^s\, \hat{z}_{lt}) + \sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} c_{lpt}\, C_{lp}\hat{x}_{lpt} + \mathbb{E}_{\Omega^{N^e}}[Q_t(\widehat{X}_t(LCA(\Omega^N)), \omega)]$
6: Evaluate $MSD_t(LCA(\Omega^N))$
7: **end for**

Algorithm 3.2 Evaluation procedure for LAF(Ω^N) model

for all $t = 1, \dots, T$ **do**
  for all $\omega = 1, \dots, N^e$ **do**
    Evaluate $Q_t(\widehat{X}_t(LAF(\Omega^N), \omega)$ using the program (3.23)-(3.30)
  end for
  return $\mathcal{V}_t^e(LAF(\Omega^N)) = \sum_{l \in \mathcal{L}} (f_{lt}^u \, \hat{y}_{lt} + f_{lt}^s \, \hat{z}_{lt}) + \mathbb{E}_{\Omega^{N^e}}[Q_t(\widehat{X}_t(LAF(\Omega^N)), \omega)]$
  Evaluate $MSD_t(LAF(\Omega^N))$
end for

3.6 Computational results

In this section, we describe the experimental study carried out and the related results. First, we present the data instances used in the experiments. Second, we provide the calibration of the design models using the SAA method. Then, we discuss the results in terms of the solvability of the stochastic models and the value of the stochastic solutions. Third, we summarize the obtained results and evaluate the different designs produced by the two models in terms of design structure and design value. All experiments were run using a cluster Haswell Intel Xeon E5-2680 v3 2.50 GHz of two processors with 12 Cores each and 128 Go of memory. We used CPLEX 12.7 to solve the linear programs.

3.6.1 Test data

In our experiments, we generated several 2E-DDP instances based on the following factors: the problem size, the network characteristics, and the demand processes. The tested size problems are shown in Table 3.2. The network incorporates several possible configurations depending on the number of different DP locations (# DPs) and the number of different capacity configurations per DP location (# capacity configurations). Thus, multiplying these two parameters gives the number of potential DPs, $|\mathcal{L}|$. In the case of several capacity configurations, the second configuration has a higher capacity. Ship-to-points are realistically scattered in the geographic area covered (25000 km^2). The number of warehouses ($|\mathcal{P}|$) is also given. A 5-year planning horizon is considered in this context, which is partitioned into 5 design periods in the tests (i.e., $|\mathcal{T}| = T = 5$).

We consider simple and compound demand processes. The simple process refers to a normally distributed demand level per ship-to-point j and per period t. The compound process refers to a Bernoulli-normal distribution, where the Bernoulli process shapes the demand occurrence for a given ship-to-point j at period t with a probability p_{jt}, $j \in \mathcal{J}$, $t \in \mathcal{T}$. In both processes, the normal distribution refers to the demand quantity with a mean value μ_{jt} and a standard deviation σ_{jt}, $j \in \mathcal{J}$, $t \in \mathcal{T}$. In addition, we consider a network including large-size (L) and medium/small-size (S) ship-to-points reflected by the historical mean value μ_{j0}^L and μ_{j0}^S for a given ship-to-point j, respectively. We set large-size ship-to-points that represent 20% of the network with an associated demand occurrence rate of $p_{jt}^L = 0.95$ (in contrast to a rate of $p_{jt}^S = 0.8$ for small-size ship-to-points). Moreover, we assume that each ship-to-point mean

Table 3.2: *Test problems size*

Problem instance (P)	$\lvert\mathcal{J}\rvert$	$\lvert\mathcal{P}\rvert$	# DPs	# capacity configurations	$\lvert\mathcal{L}\rvert$
P1	50	2	4	1	4
P2	50	3	8	1	8
P3	50	4	8	2	16
P4	75	3	8	1	8
P5	75	4	8	2	16
P6	100	3	8	1	8
P7	100	4	8	2	16

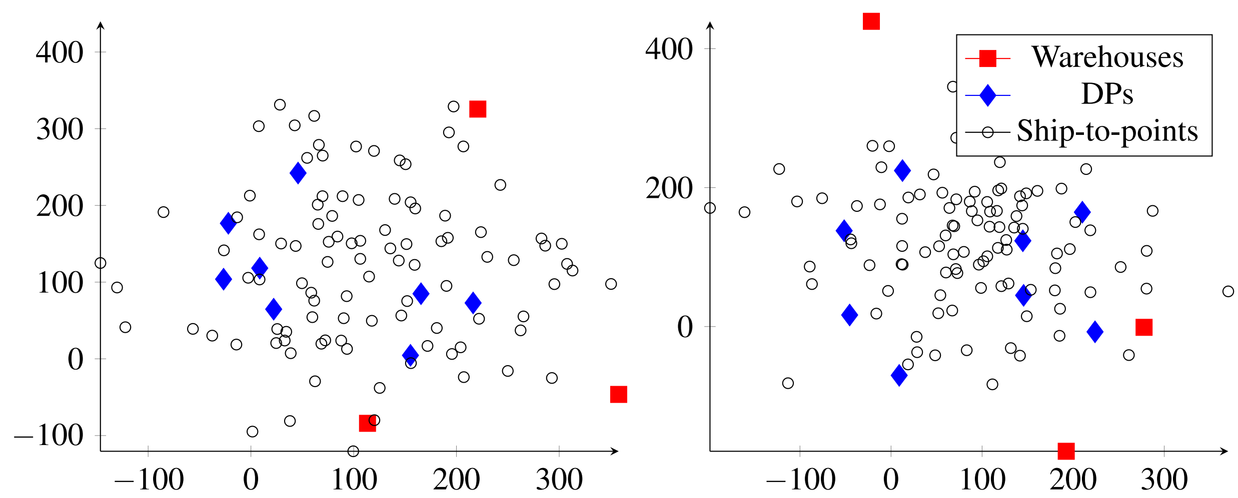
demand μ_{jt} depends on a time varying trend, based on a factor δ_j and on the historical mean μ_{j0}. The coefficient of variation ($\frac{\sigma_{jt}}{\mu_{jt}}$) is a fixed parameter for each ship-to-point over periods. Two alternative time-varying trends are tested here with respect to the two demand processes to obtain the three problem instances shown in Table 3.3. The normal distribution with a regular trend (NRT) refers to a ship-to-point mean value that is related to the historical mean at $t = 0$, and following a regular inflating factor δ_j over the periods of the planning horizon. The same regular trend is applied with the compound Bernoulli-normal distribution and is denoted with CRT. Finally, the normal distribution with a dynamic trend (NDT) refers to the case where each ship-to-point mean demand varies dynamically over periods according to a perturbation factor δ_{jt} linked to the preceding period. The values and ranges for all the parameters with regard to the demand process are given in Table 3.3.

Table 3.3: *Demand processes*

Normal distribution	NRT	$\mu_{jt} = \mu_{j0}(1 + \delta_j \times t)$	$\delta_j \in [0, 0.1]$
	NDT	$\mu_{jt} = \mu_{jt-1}(1 + \delta_{jt})$	$\delta_{jt} \in \{0.15, -0.2, 0\}$
Compound Bernoulli-Normal distribution	CRT	$\mu_{jt} = \mu_{j0}(1 + \delta_j \times t)$	$\delta_j \in [0, 0.1]$
$\mu^L_{j0} \in [300; 400]$, $\mu^S_{j0} \in [150; 220]$, $\frac{\sigma_{jt}}{\mu_{jt}} = 0.25$, $p^L_{jt} = 0.95$ and $p^S_{jt} = 0.8$			

Warehouse and DP capacities are uniformly generated with respect to the demand level of the problem instance in the unit intervals $[25k, 32k]$ and $[7k, 11k]$, respectively. The truckload capacities between warehouses and DPs are estimated in the interval $[1700, 2500]$. High and low levels of fixed costs are defined and denoted with LL and HL, respectively. The fixed costs for each DP are generated per period t, respectively in the ranges $[100k, 150k]$ and $[3500, 6500]$, proportionally to its maximum capacity. An inflation factor is also considered to reflect the increase of the cost of capital on a periodic basis with $r = 0.005$. All the locations of the network (warehouses, DPs, ship-to-points) are normally scattered in a given space as shown in the instance of Figure 3.4. We separate in the instances two possible configurations according to the disposition of the ship-to-points: dispersed (Dis) or concentric (Con). The transportation costs between the network nodes correspond to the Euclidean distances, multiplied by a unit load cost per distance unit and the inflation factor r. The unit load cost per distance unit is different in each echelon to reflect the different loading factors. The external delivery cost c_{jt}

is calibrated to be higher than internal distribution costs with $c_{jt} \in [2200; 3000]$). To apply the SAA technique, we used in our experiments $M = 6$ samples, a reference sample $N' = 2000$, and running $N \in \{100, 200, 500\}$ generated scenarios. We refer to $s \in \{s1, s2, \ldots, s6\}$ as the sample used, and denote with $\bar{s}$ the average value over the 6 samples. We considered an evaluation sample size $N^e = 2000$.



(a) *Dispersed ship-to-points instance (Dis)* **(b)** *Concentric ship-to-points instance (Con)*

Figure 3.4: *Ship-to-points scattering in the space for P6*

Combining all the elements above yielded 84 problem instances. Each instance type is a combination of $(a, b, c, d), a \in \{Dis, Con\}, b \in \{P1, P2, P3, P4, P5, P6, P7\}$, $c \in \{LL, HL\}$, $d \in \{NRT, NDT, CRT\}$. Each instance problem is denoted by the ship-to-points configuration a, the problem size b, the cost level c, the demand process d, the sample size N, and its references s. It has the format a-b-c-dNs (for example, Dis-P1-LL-NRT-500s1).

3.6.2 Results

3.6.2.1 Models' solvability analysis

The first solution quality-seeking step for stochastic models is the calibration of the number N of scenarios to include in the optimization phase. Such calibration is carried out using the SAA algorithm A.1 (see Appendix A.1), and the quality of the obtained solutions is evaluated using the statistical optimality gap. Table 3.4 summarizes the average optimality gap values for the sample size $N \in \{100, 200, 500\}$ for instances P1 to P4. The optimality gap is expressed as a percentage of the objective function value of the best design found for a given problem instance.

We can see in this table that (LCA(Ω^N)) provides satisfactory results, generally less than 1% for most instances starting from $N = 500$, for all the demand processes investigated. Our experiments show that the optimality gap value improves as the sample size N grows and converges to 0%. Moreover, we note that the design solutions produced with alternative samples

Table 3.4: *Average statistical optimality gap values*

		$gap^{N,2000}$ (%)				
		LCA(Ω^N)			**LAF(Ω^N)**	
Sample size N		100	200	500	200	500
Instance	P1-NRT-LL-N$\bar{s}$	0.26	0.26	0.21	0.01	-0.01
	P1-NDT-LL-N$\bar{s}$	0.03	0.01	-0.01	**0**	-0.02
	P1-CRT-LL-N$\bar{s}$	0.80	0.31	0.07	-0.01	-0.02
	P2-NRT-LL-N$\bar{s}$	0.61	0.23	0.08	0.06	**0**
	P2-NDT-LL-N$\bar{s}$	0.28	0.12	-0.01	**0**	-0.02
	P2-CRT-LL-N$\bar{s}$	0.60	0.24	0.06	-0.01	-0.02
	P4-NRT-LL-N$\bar{s}$	0.22	0.09	0.05	0.02	0.02
	P4-NDT-LL-N$\bar{s}$	0.15	0.01	0.01	**0**	0.01
	P4-CRT-LL-N$\bar{s}$	0.97	0.42	0.28	-0.04	-0.04

($M = 6$) of 500 scenarios provide the same location decisions. In the same way, regarding the (LAF(Ω^N)) model, the results show a very low gap, almost less than 0.1%, which underlines the sufficiency of this sample size. Accordingly, the sample size of $N = 500$ scenarios is considered satisfactory in terms of solvability and solution quality, and retained for the rest of the experiments with both models (LCA(Ω_N)) and (LAF(Ω_N)).

With this in mind, we next propose exploring the deterministic counterparts of both models and evaluate the difference between the deterministic and the stochastic solutions using the indicators proposed in [149]. These indicators are detailed in **??**. Table 3.5 provides an evaluation of the estimated value of stochastic solution ($\widehat{VSS}$), the estimated loss using the skeleton solution ($\widehat{LUSS}$), and the estimated value of loss of upgrading the deterministic solution ($\widehat{LUDS}$) as a percentage of the expected value of the recourse problem ($\widehat{RP}$) for problem sizes P2, P4, and P6 with both models.

Table 3.5 highlights a large value for the $\widehat{VSS}$ compared to $\widehat{RP}$ in the (LCA(Ω^N)) program, and a lower value in the (LAF(Ω^N)) program: it can reach 96% in the (LCA(Ω^N)) with problem size (P6), and 8% in (LAF(Ω^N)) with (P4), under the NRT demand process. The large $\widehat{VSS}$ values obtained in some cases with (LCA(Ω^N)) are partly due to the high variability of the objective function value for these instances. This variability is due to the information assumed when the capacity-allocation decisions of all the periods are anticipated at the first-stage. Conversely, this effect is attenuated in the case of (LAF(Ω^N)) because the capacity-allocation decisions are scenario-dependent and thus part of the recourse problem.

Regarding the skeleton solution from the deterministic model, we obtain $\widehat{ESSV}$ equal to or higher than $\widehat{RP}$ in the (LCA(Ω^N)) model. This leads to a $\widehat{LUSS}$ greater than zero, but remains less than the $\widehat{VSS}$ (i.e., $0 < \widehat{LUSS} \leqslant \widehat{VSS}$). The positive $\widehat{LUSS}$ obtained in our evaluation of the (LCA(Ω^N)) model means that the deterministic solution tends to open non-optimal DPs, and sub-optimally allocate DP capacity from warehouses. The same statement is observed with (LAF(Ω^N)) where the variability reaches 8% under the NRT demand process. In addition, for many problem instances with the (LAF(Ω^N)) program, the perfect skeleton

Table 3.5: *Evaluation of the stochastic solution*

	P2-		P4-		P6-	
Indicators	LCA(Ω^N)	LAF(Ω^N)	LCA(Ω^N)	LAF(Ω^N)	LCA(Ω^N)	LAF(Ω^N)
			LL-NRT-500$\bar{s}$			
$\widehat{VSS}\,(\%)$	22.82	8.13	45.47	0.6	95.98	0
$\widehat{LUSS}\,(\%)$	13.67	8.13	0	0.6	26.3	0
$\widehat{LUDS}\,(\%)$	4.13	0.51	0	0.16	1.2	0
			LL-NDT-500$\bar{s}$			
$\widehat{VSS}\,(\%)$	3.85	0	93.63	0	101.42	0
$\widehat{LUSS}\,(\%)$	2.33	0	28.6	0	0.87	0
$\widehat{LUDS}\,(\%)$	0.95	0	3.38	0	1.0	0
			LL-CRT-500$\bar{s}$			
$\widehat{VSS}\,(\%)$	89.49	0	57.08	0	222.01	0.04
$\widehat{LUSS}\,(\%)$	11.26	0	11.7	0	0.09	0.04
$\widehat{LUDS}\,(\%)$	0.59	0	1.62	0	0.4	0.04

Table 3.6: *The average computational time (CPU)*

P	**LCA(Ω^N)**						**LAF(Ω^N)**				
	Variables			Constraints	CPU		Variables		Constraints	CPU	
	Binary	Integer	Continuous		DEF	BD	Binary	Continuous		DEF	BD
P1	40	80	625000	135050	23m7s	2m54s	40	645000	150020	9m49s	2m29s
P2	80	120	1125000	145095	9h10m	47m49s	80	1185000	172540	2h11m	10m23s
P3	160	320	2125000	145180	35h20m	13h3m	160	2285000	285080	21h30m	1h17s
P4	80	120	1687500	187595	10h38m	1h5m43s	80	1747500	235040	4h	14m41s
P5	160	320	3187500	187180	-	15h15m	160	3347500	347580	-	2h53m
P6	80	120	2250000	270095	39h42m	2h15m	80	2310000	297540	8h26m	20m45s
P7	160	320	4250000	270180	-	40h40m	160	4410000	410080	-	5h

-: Not solved within time limit 48 hours

solution is captured. The last measure evaluates the upgradability of the deterministic solution to the stochastic solution. The table indicates that (LCA(Ω^N)) presents a small and non-zero $\widehat{LUDS}$ value (about 4% and 1% for P2 and P6, respectively, in (LCA(Ω^N)) and less than 0.5% for P2 in (LAF(Ω^N)). This confirms the non-upgradability of the deterministic solution for problem sizes P2 and P6. The results using the NDT and the CRT demand processes also validate the high value of the stochastic solution compared to the deterministic counterpart for (LCA(Ω^N)). In this case, the $\widehat{VSS}$ value increases as the demand variability grows, and reaches 200% of the $\widehat{RP}$ for the CRT demand process. On the other hand, the (LAF(Ω^N)) model shows a low variability of the stochastic solutions with the NDT and CRT demand processes. These primary results show the worthiness of investigating the stochastic formulations of the 2E-DDP. One may also conclude that (LCA(Ω^N)) is much more sensitive to demand uncertainty due to the anticipation of the capacity-allocation decisions at the first stage.

Finally, solvability is a crucial issue that needs to be taken into consideration given the complexity of stochastic programming models. Table 3.6 provides the average computational time (CPU) for the two proposed models. It compares the running time of the deterministic equivalent formulation (DEF) when solving with a commercial solver (Cplex) and when applying the Benders decomposition (BD). The results show the efficiency of the BD approach in considerably reducing the running time. The BD running time is 4 to 18 times less than using Cplex for DEF for (LCA($C\Omega^N$)) where the higher difference (18 times faster) is observed for P6. In the case of (LAF(Ω^N)), the efficiency of BD is also significant, namely, 25 times faster for P6. Moreover, the table indicates that the CPU(s) grows as the problem size increases, clearly a further complexity with P3, P5, and P7 where two capacity configurations are considered. We also observe that (DEF) cannot be solved to optimality for problem instance (P5) and (P7) within the time limit of 48 hours. Nevertheless, the (P7) problem instance is solved to optimality with the BD approach within 40 hours and 5 hours for the (LCA(Ω^N)) and the (LAF(Ω^N)) model, respectively. Further, Table 3.6 confirms the huge discrepancy between the solved models in terms of complexity and consequently running times. The (LAF(Ω^N)) program seems easier to tackle, where the CPU(s) is 3 to 12 times less than in (LCA(Ω^N)). The high computational time observed in (LCA(Ω^N)) is mainly due to the complexity of the integer capacity variables, and thus the combinatorial nature in the problem, which makes it intractable for larger instances. These results also confirm the importance of the development of the Benders decomposition approach to solve a large scale 2E-DDP.

3.6.2.2 Design solutions analysis

In this subsection, we provide an analysis of the design solutions produced by the two models proposed to deal with the stochastic 2E-DDP. Three facets are relevant: 1) the global performance of the design solutions in terms of the expected value and the expected mean semi-deviation (MSD) value, 2) the sensitivity of DP location decisions to uncertainty under various problem attributes, and 3) the behavior of the capacity-allocation decision in a multi-period and uncertain setting. These analyses are based on the numerical results of the 84 problem instances described above. We recall that the evaluation is based on the procedures given in 3.1 and 3.2 for the LCA and LAF models, respectively, with a large evaluation sample of $N^e = 2000$ scenarios.

Table 3.7: *Mean value and MSD deviations for (LCA(Ω^N)) under (P6,P7)-LL-500 attributes*

		P6-LL-500						**P7-LL-500**					
		NDT		**NRT**		**CRT**		**NDT**		**NRT**		**CRT**	
		% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD
Dis-	s1	0.14	**0**	**0**	1.06	0.48	15.37	0.13	3.93	0.02	**0**	0.31	38.2
	s2	0.19	28.50	0.11	12.97	0.05	**0**	0.11	11.5	**0**	6.88	0.02	**0**
	s3	**0**	5.15	**0**	1.06	**0**	6.09	0.09	17.52	0.07	13.36	**0**	8.04
	s4	0.11	15.65	0.02	3.21	0.1	11.99	0.15	15.45	0.42	79.87	0.02	2.73
	s5	0.15	33.6	**0**	1.06	0.2	1.3	**0**	**0**	0.06	12.27	0.01	0.48
	s6	0.21	49.19	0.04	**0**	0.05	0.77	0.04	4.01	0.16	24.72	0.51	42.26
Con-	s1	0.06	3.87	0.31	69.10	0.05	6.09	**0**	94.33	0.49	107.16	**0**	6.89
	s2	**0**	6.10	**0**	19.82	0.11	7.99	**0**	94.33	0.45	3.84	0.52	36.24
	s3	0.15	27.67	0.03	9.43	**0**	**0**	1.87	**0**	0.48	**0**	0.16	8.71
	s4	0.20	47.90	**0**	19.82	0.10	13.38	**0**	94.33	0.19	4.73	0.04	1.33
	s5	**0**	6.10	0.21	**0**	0.40	30.02	**0**	94.33	**0**	19.17	0.12	14.27
	s6	0.05	**0**	**0**	19.82	0.11	7.99	0.32	87.98	0.15	6.56	0.22	**0**

To start, Table 3.7 describes relative deviations to the best value recorded on the expected value, and the MSD for the (LCA(Ω^N)) model using large-size problems (P6 and P7) contrasted with the NRT, NDT, and CRT demand processes. When looking at the solutions' performance in terms of expected value, we observe that the designs produced by the alternative samples present a small deviation, often less than 0.5%, which clearly indicates the stability of the design structure over the samples. Verter and Dincer in [227] show that location decisions tend to be highly driven by the network topology, as is the case in our results. The highest deviations observed are 0.48% for P6-CRT and 1.87% for P7-NDT, deriving from the lesser sensitivity of the capacity-allocation decisions to the optimization sample. Furthermore, when inspecting the MSD measure, we first observe a higher deviation between the design solutions, which is due to the relatively small mean semi-deviation values (about 8k). Also, we underline that for a given problem instance, the design solution presenting the lowest MSD is always different from that providing the highest expected value. This offers distribution network designers a better insight with regard to Pareto optimality. As illustrated in Table 3.7, these observations are still valid under all the demand processes and network typologies. For problem sizes P2 to P5, the results led to similar conclusions and are detailed in Tables B.1 and B.2 in Appendix B. In the case of the (LAF(Ω^N)) model, the mean value and MSD deviation results indicate a pronounced similarity in the design structure, since all the samples lead to the same design value. This is clearly due to modeling the capacity decisions as scenario-dependent in the second stage in (LAF(Ω^N), which bases the design evaluation only on the location decisions. Therefore, one can conclude that the DP location is well stabilized, but the capacity-allocation decisions are sensitive to the demand scenarios. These results are congruent with the insights derived from the statistical gaps, and confirm the good quality solutions produced by both design models.

Next, we look closely at the design decisions produced by both models presented in Tables 3.8 and 3.9 for medium- and large-size problems (P4 to P7) with (LCA(Ω^N)) and (LAF(Ω^N)) respectively. We note that the represented design corresponds to the best solution in terms of expected value based on Table 7. For each demand process, these tables provide DP opening decisions and their capacity configuration, where value 1 corresponds to an opened DP with a low capacity configuration, 2 refers to a high capacity configuration, and blanks refer to DPs kept closed. The third row corresponds to the list of potential DPs and the first column lists the instance labels in terms of problem size and cost structure. The first part of the tables is dedicated to instances with dispersed ship-to-points, and the second part depicts instances with concentric ship-to-points. First, Tables 3.8 and 3.9 show that the opened DP number is quite stable in the instances, but the location of DPs and the capacity level are correlated with the demand process, the ship-to-point dispersion, and the DP fixed cost. In almost all the instances, DP locations 6 and/or 7 are opened, because they benefit from a centralization effect due to their positioning in the grid (see Figure 4) and the importance of inbound and outbound transportation costs. Table 3.8 also reveals the impact of the opening DP costs on the strategic location decisions where in several cases the design structure varies between high and low DP opening costs. For instance, we observe in Table 3.8 with P5-LL-Dis-NRT that the network design opens four DPs at capacity level 1, whereas with P5-HL-Dis-NRT, only three DPs are opened, but two at capacity level 2. Additionally, the results point out the impact of ship-to-point dispersion (i.e., Dis vs Con) on the DP location decisions, mainly under LL attributes.

For instance, Table 3.8 illustrates the difference between P7-HL-Dis-NDT and P7-HL-Con-NDT where DPs 7 and 8 are opened instead of DPs 3 and 4, respectively. In the same way, Table 3.9 reports different design solutions for all P7 instances and several P5 instances, which are the problems considering two capacity levels, and thus offer more distribution capabilities to deal with demand uncertainty.

Moreover, a key finding is the sensitivity of the network design in terms of the opened DPs and their location with respect to demand uncertainty. First, essential to mention is that all the opening decisions depicted in these figures are fixed from the first design period and no further opening are made at periods two to five. This behavior is explained by the fact that in our context, the DP opening costs follow an increasing trend function along the planning horizon, and it is thus more efficient to anticipate future DP openings, if any, at design period one. This anticipation effect is only possible with the explicit modeling of a multi-period design setting, as is the case in this work, in contrast to a static modeling approach to design decisions. In addition, these results reveal that the number of opened DPs under the three demand processes is in general the same for a given problem size. However, for some instances, the number of DPs under NRT is higher than the two other demand processes. For example, considering instance P4-LL-Dis, four DPs are opened under NRT, whereas only three DPs are opened with NDT and CRT. This behavior is observed with the solutions produced by both the LCA and LAF models, and can be seen as the flexibility hedging of both models to avoid opening additional DPs when uncertainty increases. We also note a high variability in the location of opened DPs when comparing solutions from the different demand processes. To emphasize this result, one can closely observe the instance P4-LL-Dis, where we obtain 66.7% identical DPs for NDT vs CRT, 50% for NRT vs NDT, and 75% for NRT vs CRT, when the LCA model is solved. Similarly, when the LAF model is solved, we obtain 50% of identical DPs for both NRT vs NDT, and NRT vs CRT, and 100% for CRT vs NDT. The worst case is observed for instance P6-HL-Con with only 33.3% identical DPs for NDT vs CRT, and 66.7% for NRT vs NDT, and for NRT vs CRT when the LCA model is solved. These results confirm that the stochastic multi-period demand process is adequately captured by both the two-stage reformulations of the multi-stage problem.

In complement to the above analysis, Table 3.10 provides similarity statistics on DPs opening and their capacity level for several pairs of instances when LCA is solved. In general, we observe a very high similarity in the DPs' location, over 70% identical positions, but a much lower similarity in the capacity configuration (about 40%). Looking closely at instance P7-LL-Con, we see that CRT and NDT give two identical DPs location out of three, but these identical DPs are not opened with the same capacity configuration level. Therefore, the results confirm the high variability in solutions in terms of location when capacity levels are considered, and the sensitivity of the two-echelon capacitated distribution network to uncertainty.

Furthermore, we investigate the design solutions to discuss the behavior of the capacity-allocation decisions in a multi-period and uncertain setting. In Figure 3.5, we examine in depth the capacity decisions modeled in the LCA model and contrast it to the evolution of each demand process along the planning horizon. This figure provides the results of the instance P6-LL-Dis for the three demand processes, which for all cases produced a design solution with three opened DPs. Each solid line corresponds to the capacity level allocated to an opened DP

Table 3.8: *Best location decisions for (LCA(Ω^N))*

	Dispersed ship-to-points (Dis)																							
	NRT-500								NDT-500								CRT-500							
DPs	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**
P4-LL	1			1	1		1		1					1	1		1			1			1	
P4-HL	1			1		1	1		1					1	1		1					1	1	
P5-LL	1			1	1		1		1					1	1		1					1	2	
P5-HL	1					2	2		1					1	1		1					1	2	
P6-LL		1				1	1			1				1	1			1				1	1	
P6-HL		1				1	1			1				1	1			1				1	1	
P7-LL		1				1	2			1				1	1			1				1	1	
P7-HL		1				1	1				2	2						1				1	1	
	Concentric ship-to-points (Con)																							
P4-LL	1		1		1	1			1				1		1		1				1		1	
P4-HL	1				1		1		1				1		1		1				1		1	
P5-LL	1				2		2		1				1		1		1				1		1	
P5-HL	1				2		2						1	1	1		1				1		1	
P6-LL	1						1	1	1						1	1	1						1	1
P6-HL	1						1	1						1	1	1	1		1					1
P7-LL	1						2	1							2	2	1						1	1
P7-HL	1						2	1							2	2	1		1					1

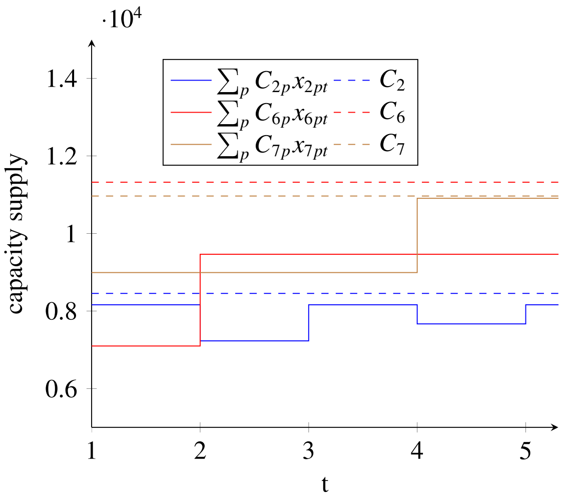
Table 3.9: *Best location decisions for (LAF(Ω^N))*

	Dispersed ship-to-points (Dis)																							
	NRT-500								NDT-500								CRT-500							
DPs	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**
P4-LL	1			1	1		1		1					1	1		1					1	1	
P4-HL	1					1	1		1					1	1		1					1	1	
P5-LL	2					1	1		1					1	1		1					1	1	
P5-HL	2					1	1		1					1	1		1					1	1	
P6-LL		1				1	1			1				1	1			1				1	1	
P6-HL		1				1	1			1				1	1			1				1	1	
P7-LL		1				1	1			1				1	1			1				1	1	
P7-HL		1				1	1			1				1	1			1				1	1	
	Concentric ship-to-points (Con)																							
P4-LL	1				1		1				1		1	1					1		1	1		
P4-HL	1				1		1				1		1	1					1		1	1		
P5-LL	1				1		1				1		1	1					1		1	1		
P5-HL	1				1		1				1		1	1					1		1	1		
P6-LL	1						1	1	1		1					1					1		1	1
P6-HL					1		1	1						1	1	1						1	1	1
P7-LL	1						1	1							2	2					1		1	1
P7-HL					1		1	1						1	1	1						1	1	1

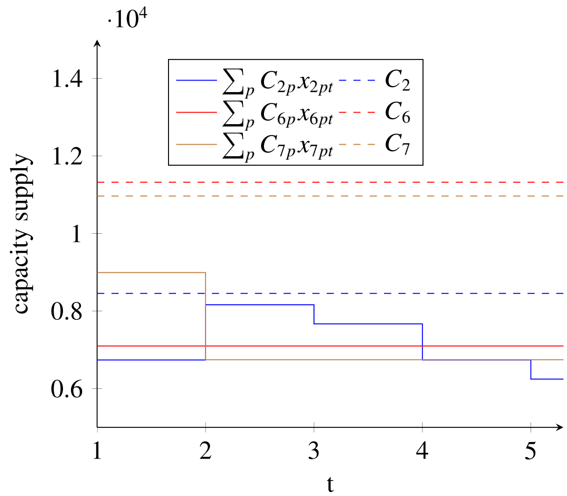
Table 3.10: *The impact of capacity configuration on the location decisions*

	P2-LL vs P3-LL		P4-LL vs P5-LL		P6-LL vs P7-LL	
	%loc	%cap	%loc	%cap	%loc	%cap
NRT	75%	79.17%	75%	62.5%	75%	75%
CRT	70.83%	45.83%	100%	100%	83.33%	58.33%
NDT	70.83%	41.67%	79.17%	64.58%	97.22%	100%

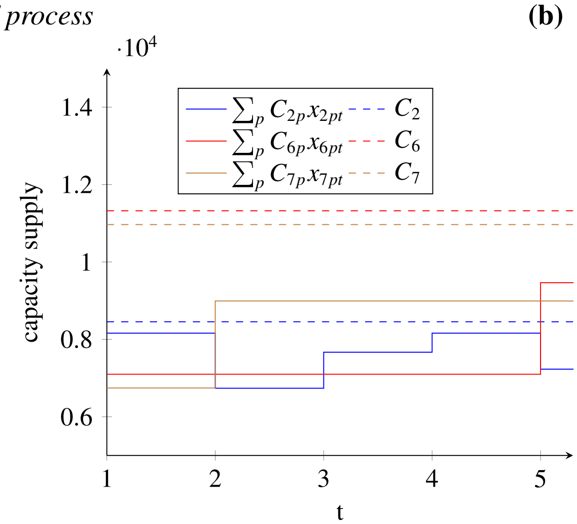
(i.e., $\sum_p C_{lp}x_{lpt}$) for each design period t, and each dashed line its related predetermined capacity C_l (each color distinguishes a separate opened DP). Figure 3.5 clearly illustrates the impact of the multi-period modeling approach where the capacity-allocation decisions for each opened DP are clearly adapted periodically. Even if for the three demand processes of this instance the LCA model produces the same DP location decisions, the capacity-allocation decisions behave differently under each demand process to follow the time-varying demand process. This means that the two-stage LCA mimics the dynamic capacity model with the inclusion of multi-period capacity-allocation decisions.



(a) *NRT process*



(b) *NDT process*

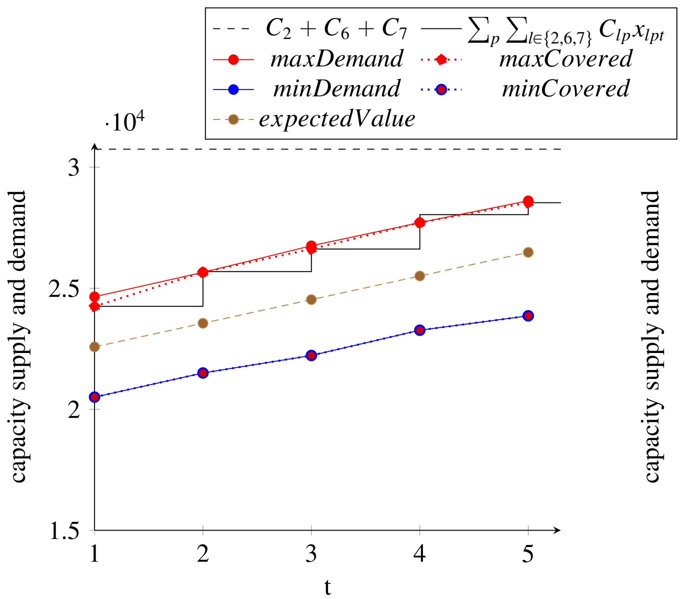


(c) *CRT process*

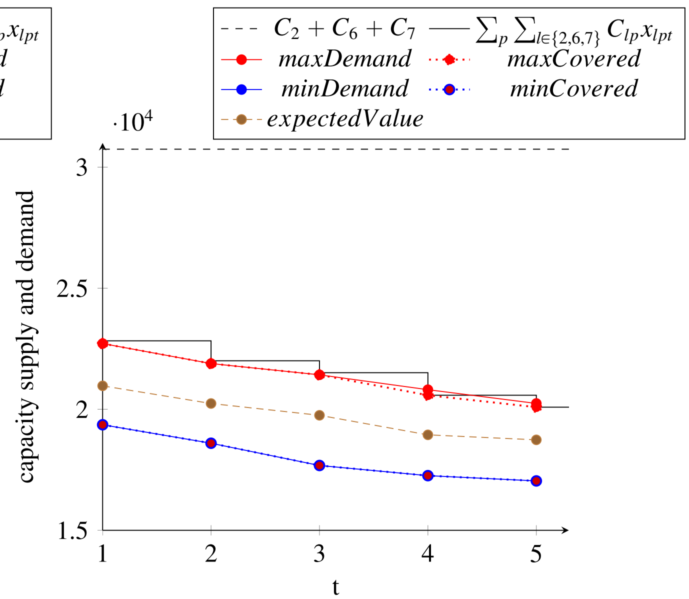
Figure 3.5: *Capacity-allocation decisions versus the a priori capacity C_l for Dis-P6-LL-500 in (LCA(Ω^N))*

Figure 3.6 completes the analysis, with the same instance under the three demand processes, as it aggregates the capacity-allocation $\sum_l \sum_p C_{lp}x_{lpt}$ over all the opened DPs at each period t (solid line in black) and the capacity limit $\sum_l C_l$ (dashed line). The figure contrasts the global network capacity with the average demand scenario, the maximum demand scenario, and the minimum demand scenario. It also illustrates the relation between these typical scenarios and the effective demand covered by the opened DPs with a dotted line (i.e., $\sum_{j=1..|\mathcal{J}|} d_{j\omega t} \sum_{l=1..|\mathcal{L}|} v_{lj\omega' t}$, at each t, as formulated in constraint 3.13). The main insight here is that the capacity-allocation decisions follow the periodic demand under the regular, and most importantly, the dynamic demand setting. This insight is accentuated by the observation that the capacity-allocation decisions follow in all cases the maximum demand scenario. This means that this first stage decision takes into account the worst case demand (highest demand scenario) and provides the necessary capacity level to hedge against it. Clearly, this latter point means that the capacity available at DPs at each period precedes the minimum and the expected demand scenarios. Therefore, one can conclude that the modeling framework employed in the LCA model provides a capacity hedging approach under uncertain demand with a stationary or non-stationary process. For further illustration, the example of the instance P6-HL-Con is reported in Figure B.1 and in Figure B.2 in B.

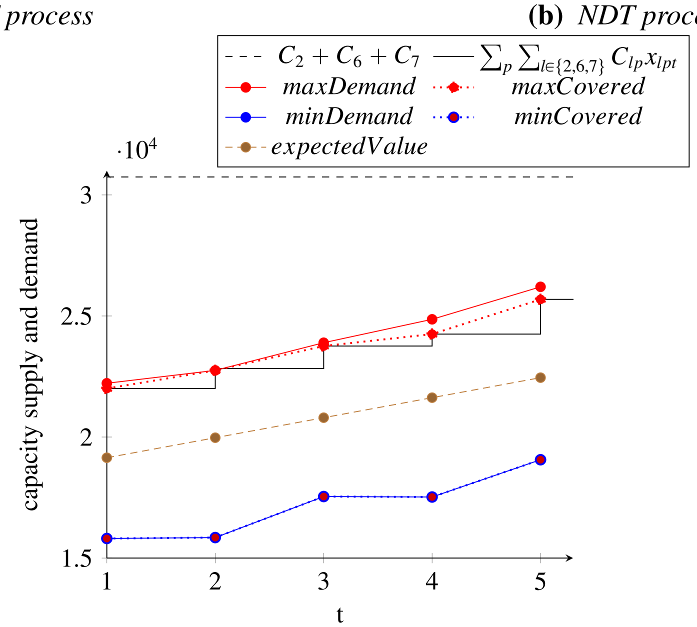
Similarly, we plot in Figure 3.7 demand vs capacity for the LAF model, which shows that capacity follows demand for each typical scenario. This means that the scenario-dependent flows are adjusted to each scenario, and that the demand covered by the designed network is superposed with the demand scenario. This indicates that the LAF model converges to a solution driven by the expected value criterion, but does not anticipate any capacity hedging. It seems that LAF solutions are prone to greater efficiency in terms of capacity-allocation, but this comes at the price of no flexibility. Finally, to further underline is that in the 2E-DDP, the demand is covered with respect to the inbound allocation to DPs from warehouses, and not the DPs' predefined capacity. This is in accordance with the flow balance constraints at DPs, and emphasizes the impact of the two-echelon context considered in this work.



(a) *NRT process*

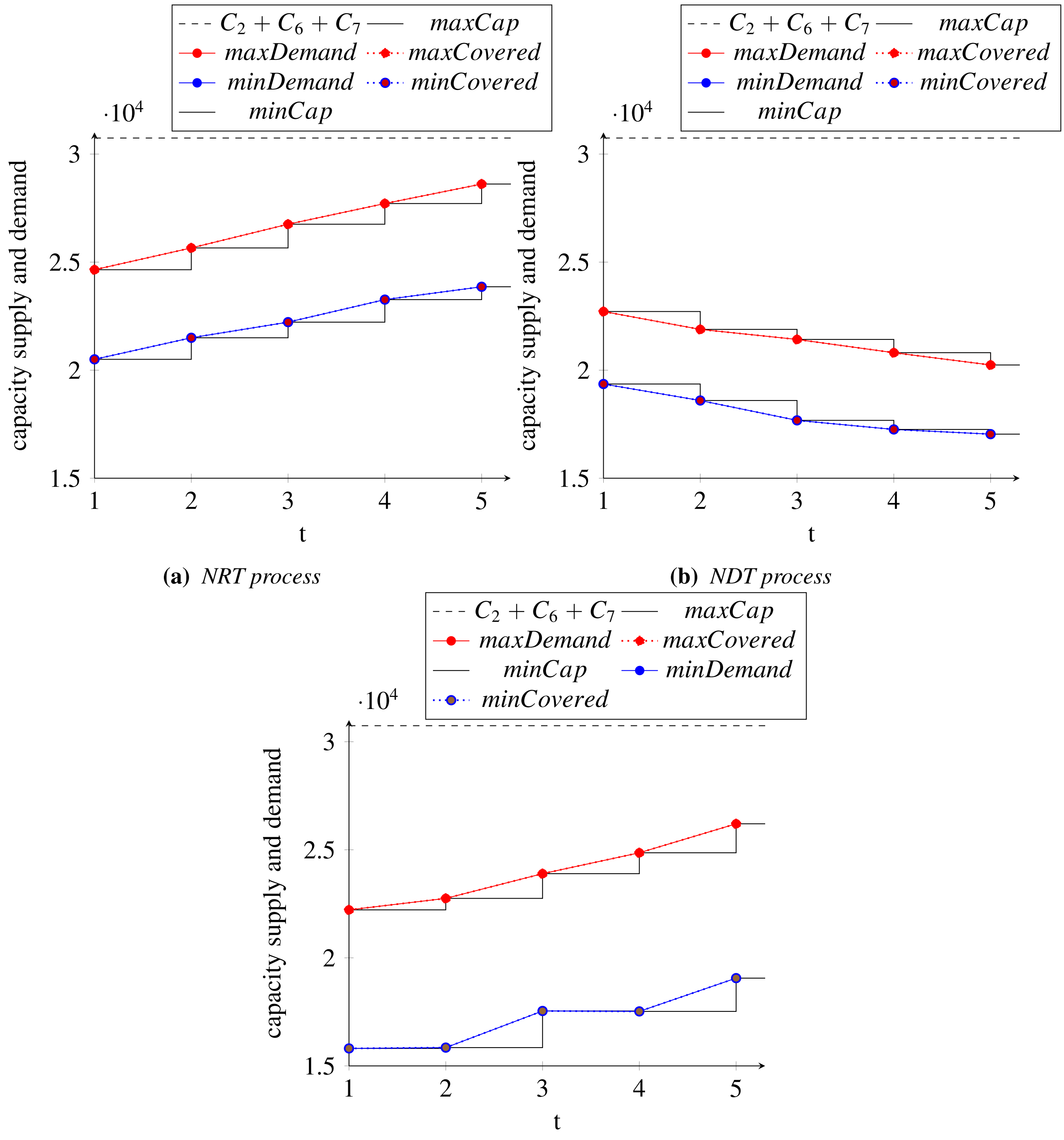


(b) *NDT process*



(c) *CRT process*

Figure 3.6: *Capacity decisions versus demand for Dis-P6-LL-500 in (LCA(Ω^N))*



(a) *NRT process*

(b) *NDT process*

(c) *CRT process*

Figure 3.7: *Capacity decisions versus demand for Dis-P6-LL-500 in (LAF(Ω^N))*

3.7 Conclusion

In this paper, we introduce a comprehensive methodology for the stochastic multi-period two-echelon distribution network design problem (2E-DDP) where products are directed from primary warehouses to distribution platforms (DPs) before being transported to ship-to-points from DPs. The problem is characterized by a temporal hierarchy between the design level dealing with DP location decisions and capacity decisions, and the operational level involving transportation decisions as origin-destination flows. A stochastic multi-period characterization of the planning horizon is considered, shaping the evolution of the uncertain ship-to-point demand and DP opening costs. This problem is initially formulated as a multi-stage stochastic program, and we propose alternative two-stage multi-period modeling approaches to capture the essence of the problem, while providing judicious accuracy-solvability trade-offs. Consequently, the two models proposed are: the two-stage stochastic location and capacity-allocation model (LCA) in which DP location decisions and capacity decisions are first-stage decisions, and the two-stage stochastic flow-based location-allocation model (LAF) where capacity decisions are transformed into continuous scenario-dependent origin-destination links within the second-stage. To solve these models, we develop a Benders decomposition approach integrated with the sample average approximation (SAA) method. We also examine the value of the stochastic solution and compare it to the deterministic solution. The extensive computational experiments based on 84 realistic problem instances validate the modeling approaches and the efficiency of the solution approaches.

These computational experiments lead to some important managerial insights regarding the impact of uncertainty on 2E-DDP. The findings highlight a significant variability in the design decisions when three demand processes are involved. Moreover, the results point out that considering several capacity configurations (i.e., low and high configuration on the same position) changes the design of the network. In some cases, the number of opened DPs is even reduced. Further, when inspecting the badness/goodness of the deterministic solutions, the results confirm the positive impact of uncertainty. It emphasizes that the LCA model is very sensitive to uncertainty. The value of the stochastic solution (VSS) increases as uncertainty grows in the network. On the other hand, the LAF is affected by uncertainty, but less significantly. This leads to conclude that assignment-capacity decisions are more sensitive to uncertainty than location decisions.

Although the Benders decomposition provides good solutions, its performance is limited for large-scale instances (more than 40 hours). It might be worthwhile improving the considered algorithm to reduce the run time and develop efficient heuristics to solve larger problem instances. We believe that our framework provides ample opportunities for additional research. Future works could consider stochastic multi-period two-echelon distribution network design problems with more complex features where the location decisions of the two echelons (warehouses and DPs) are questioned, and where operational decisions are modeled by multi-drop routes. Additionally, it would be interesting to add risk measures to the objective function, such as mean semi-deviation and conditional value at risk. Another interesting research direction consists in tackling the multi-stage stochastic program, and further proposing performance measures on the quality of the obtained solutions compared to two-stage stochastic models.

However, this remains a challenging task, given the curse of dimensionality of multi-stage stochastic problems.

Chapter 4

A Benders Approach for the Two-Echelon Stochastic Multi-period Capacitated Location-Routing Problem

Abstract

In the two-echelon stochastic multi-period capacitated location-routing problem (2E-SM-CLRP), one has to decide at each period on the number and the location of warehouse platforms as well as intermediate distribution platforms; while fixing the capacity of the links between them. The system must be dimensioned to enable an efficient distribution of goods to customers under a stochastic and time-varying demand. In the second echelon, the goal is to construct vehicle routes that visit customers from operating distribution platforms. The objective is to minimize the total expected cost. For this hierarchical decision problem, the model is a two-stage stochastic program with integer recourse. The first-stage includes location and capacity decisions to be fixed at each period over the planning horizon, while routing decisions in the second echelon are determined in the recourse problem. We develop a Benders decomposition approach to solve the 2E-SM-CLRP. In our approach, the location and capacity decisions are taken by solving the Benders master problem. When these first-stage decisions are fixed, the resulting subproblem is a capacitated vehicle-routing problem with capacitated multi-depot (CVRP-CMD) that is solved by a branch-cut-and-price algorithm. Computational experiments show that several instances of realistic size can be solved optimally, and that relevant managerial insights are derived on the behavior of the design decisions under the stochastic multi-period characterization of the planning horizon.

Keywords: Two-echelon Capacitated Location-Routing, uncertainty, multi-period, Benders decomposition

4.1 Introduction

Distribution problems have drawn many researchers attention over the last decades as their applications are of great interest for companies. These latter are always looking to improve the efficiency of their distribution network in terms of the facilities location and transportation schemes. Vehicle-routing problems (VRPs) are among the most extensively studied classes of distribution problems in the operational research literature where the aim is to compute a set of minimum-cost routes to meet customer demands using a fleet of vehicles [63, 221]. Facility location problems (FLPs) have also been the subject of intensive research efforts. In the FLPs, a set of warehouses should be located from a finite set of potential sites and customers are delivered by direct routes from selected warehouses at the minimum cost [97, 65]. However, it is now commonly believed that the integration of the two decision levels into a location-routing problem (LRP), often leads to better network design solutions as introduced in [192] and recently discussed in [206] and [131]. In LRPs, the aim is to find an optimal number of warehouses and their locations, while building routes around them to serve the customers. An extensive literature review including a description of different applications for location-routing and a classification scheme is given in [159]. However, as pointed out in [71], most of the LRP models studied so far considered a one-echelon distribution setting.

Nonetheless, the high growth of e-commerce in recent years, and the emergence of omnichannel sales approach, have drastically changed the distribution landscape. They have significantly increased the delivery service level expectation and favored a high proximity to customers ship-to location as home, stores and relay points among others [83]. Such challenges are especially experienced in urban areas due to the increase in cities population, which is contrasted with the rising levels of congestion and the regulations to limit pollutant emissions. Therefore, practitioners oversee limits in the capabilities of the one-echelon distribution network to meet today's challenges. They have nowadays turned much more attention to two-echelon distribution structures. For instance, most of retailers have used to operate a single centralized warehouse per region/market that is optimized for risk pooling and for sourcing and delivery efficiency. However, in the past, the location of such warehouse was not necessarily optimized to provide next-day/same-day deliveries, or to operate efficiently fast fulfillment and shipment services for online orders. To deal with, several retailers such as Walmart, JD.com or Amazon have reconfigured their distribution networks by adding an advanced echelon of distribution/fulfillment platforms, mostly in urban areas. According to [122], Walmart plans to convert 12 Sam's Club stores into e-commerce fulfillment centers to support the rapid e-commerce growth. Two-echelon structure is nowadays promoted in several City logistics applications [61, 160]. This is done by creating peripheral distribution/consolidation centers, where freights coming from warehouses on large trucks are loaded into smaller and environmentally friendly vehicles, more suitable for city center distribution. Parcel delivery also represents a relevant application for the two-echelon capacitated location-routing [89]. Parcels travel from primary platforms to secondary distribution platforms, and they are then sorted and loaded onto smaller trucks that ship parcels to relay points, and to customers' homes. This is studied in [236] considering a route length approximation formulas instead of explicit routing decisions under a static-deterministic setting.

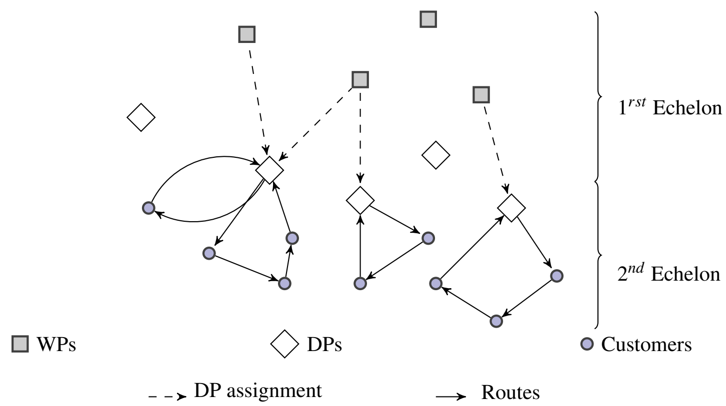


Figure 4.1: *A potential two-echelon capacitated location-routing problem (2E-CLRP)*

In this work, a two-echelon distribution problem is studied with the aim to design a network structure that offers more flexibility to the future business needs of a given company. More specifically, this strategic problem aims to decide on the number and location of warehousing/storage platforms (WPs) and distribution/fulfillment platforms (DPs), and on the capacity allocated from first echelon to second echelon platforms. It also determines transportation decisions between platforms. As consolidation is generally more pronounced at the primary echelon, we consider direct assignment with full truckloads transportation option departing from WPs to DPs. Then, since DPs are generally devoted to more fragmented services, the transportation activity from the second echelon is shaped by multi-drop routes. This problem description gives rise to the two-echelon capacitated location-routing problem (2E-CLRP). Figure 4.1 illustrates a typical 2E-CLRP partitioned into two capacitated distribution echelons: each echelon involves a specific location-assignment-transportation schema that must cope with the future demand.

Contrary to the VRP and the LRP, the literature on the 2E-CLRPs is still scarce. It has been formally introduced by [215]. Later, it is studied by [53] where they introduce a branch-and-cut algorithm based on a new two-index vehicle-flow formulation, strengthened by several families of valid inequalities. They also develop an Adaptive Large Neighborhood Search (ALNS) algorithm that outperforms the other heuristics proposed for the 2E-CLRP. [160, 161] have examined a particular case of the 2E-CLRP with a single warehouse in the first echelon with a known position and proposed two heuristics to solve it. A literature survey on the two-echelon distribution problems can be found in [178, 71] and in [62]. These models considered static and deterministic versions of the 2E-CLRP.

Moreover, given the strategic nature of the 2E-CLRP, it must be designed to last for several years, fulfilling future distribution requirements. To this end, the horizon must be partitioned into a set of periods shaping the uncertainty and time variability of demand and cost. The location and capacity decisions should be planned as a set of sequential decisions to be implemented at different design periods of the horizon (a year, for example) and promoting the structural adaptability of the network. This is more critical nowadays as distribution practices have got more complex over the years, and have higher uncertainty in terms of demand level, demand location, and cost evolution. Accordingly, the traditional deterministic-static representation of the planning horizon is due to be replaced by a more realistic stochastic and multi-period characterization of the planning horizon. Hereafter, we refer to the stochastic multi-period version by the 2E-SM-CLRP. [128] examine a stochastic variant of the one-echelon LRP, but therein only the transportation level is decided under a multi-period setting, whereas in our study the multi-period feature involves the design decisions (i.e., location and capacity decisions). [27] investigate a stochastic multi-period version of the two-echelon location-allocation problem, where routes are substituted by multi-period inter-facility flow decisions. Hence, as far as we know, stochastic-multi-period setting has not been tackled yet in the 2E-CLRP. We further note that existing 2E-CLRP and most LRP modeling approaches implicitly assume that location and routing decisions are made simultaneously for the planning horizon, without considering the hierarchical structure of the strategic problem that we stress here. To end with, Laporte [139] has shown that the LRP is NP-hard and thus, the 2E-CLRP inherits the same NP-hardness property from LRP. When considering uncertainty, the 2E-CLRP is an NP-hard stochastic combinatorial optimization problem. However, the few exact methods and metaheuristics proposed in the literature are designed for deterministic-static setting [62]. This justifies the development of advanced exact solution approaches to solve realistic-size instances of the problem. Decomposition methods such as L-shaped methods [225, 38] and Benders decomposition [32] are crucial to solve the two-stage stochastic programs.

With this in mind, the contribution of this paper is threefold. First, we provide a more precise definition of the 2E-SM-CLRP under uncertain demand, and time-varying demand and cost. The problem is casted as a two-level organizational decision process: Location and capacity decisions are made on a yearly basis, whereas routing decisions are made on a daily basis in a response to the customer orders received. This temporal hierarchy gives rise to a hierarchical decision problem and argues the necessity of a stochastic and multi-period modeling approach. More precisely, the decision process aims to decide at each design period on opening, operating and closing of WPs and DPs, as well as the capacity allocated to links between platforms. In the second level, the goal is to periodically build routes that visit customers using a vehicle routed from an operating DP (as illustrated in Figure 4.1). The objective is to minimize the total expected cost based on strategic and operational cost components. Second, we introduce a formulation of the 2E-SM-CLRP as a two-stage stochastic program with recourse. The scenario-based approach relies on a set of multi-period scenarios generated with a Monte-Carlo approach. The location and capacity decisions are taken here-and-now for the set of design periods considered. Second-stage decisions consist in building daily routes in the second echelon once customer orders are revealed, replicated for high number of scenarios. The two-stage stochastic formulation reduces the combinatorial complexity of the multi-stage process. However, its solvability is still challenging due to the presence of multiple design periods in the first stage, in addition to the integer recourse problem [44, 143, 8]. As a third contribution, we propose an exact approach to solve the 2E-SM-CLRP. Our approach builds on the Benders decomposition approach [32] and on the sample average approximation (SAA) [204] to solve realistic-size instances. The proposed Benders approach first fixes the operating WPs and DPs as well as the capacities allocated to links between platforms by solving the Benders master problem. Then, the resulting subproblem is a capacitated vehicle-routing problem with capacitated multi-depot (CVRP-CMD), which is harder variant than the uncapacitated case.

The CVRP-CMD is formulated as a set partitioning model strengthened by a lower bound on the number of vehicles obtained from the solution of a bin packing problem in preprocessing. Then, it is decomposed by period and by scenario. These latter are solved in parallel using the state-of-art branch-cut-and-price algorithm from Sadykov et al. [190]. Subproblem solutions are used to generate standard Benders cuts as well as combinatorial Benders cuts in order to converge to the optimal solution of the 2E-SM-CLRP. Extensive computational experiments emphasize the performance of our algorithm to solve optimally a large set of instances, and to get good lower bounds on large-scale instances with up to 50 customers and 25 demand scenarios under a 5-year planning horizon. Finally, from the results are also derived insights on the impact of the stochastic and multi-period settings on the 2E-CLRP.

The reminder of this paper is organized as follows. Section 4.2 briefly surveys the related works on the 2E-CLRP under stochastic and multi-period settings. Section 4.3 describes the mathematical formulation. Section 4.4 presents our exact approach to solve the problem. The computational results are presented and analyzed in Section 4.5. Section 4.6 concludes the study and outlines future research avenues.

4.2 Related works

As mentioned in the introduction, the literature on the 2E-CLRP is limited. Hereafter, we review the contributions in link with problems related to the 2E-CLRP.

The two-echelon vehicle-routing problem (2E-VRP) is a particular case of the 2E-CLRP, in which the location of all WPs and DPs is known in advance. Platforms do not have fixed capacities and can be used freely without inducing a setup cost. Generally, a single main WP is considered in the upper level of the 2E-VRP. Several exact methods [168, 169, 22] and heuristics [60, 104, 41, 233] have been developed in the literature. To the best of our knowledge, the best performing heuristics are the large neighbourhood search (LNS) heuristic introduced by [41] and the metaheuristic proposed by [233] for the 2E-VRP with environmental considerations. A well performing exact algorithm is presented in [22]. As for the stochastic variant, [232] is the first to propose a genetic algorithm for 2E-VRP with stochastic demand. However, as far as we know, no previous study has considered both stochastic and multi-period features in 2E-VRP. For further details, interested readers are referred to the recent survey by [62].

The capacitated location-routing problem (CLRP) is also a related class of problems for the 2E-CLRP, where the location of main warehouses is known in advance and the cost between WPs and DPs are neglected. This problem combines the multi-depot vehicle-routing problem and the facility location problem. Several studies focus on the LRP and its variants. They are mostly in deterministic setting, and propose exact methods [20, 25, 50, 51] and heuristics [52, 103, 172, 175] to solve the problem. However, a few authors have studied stochastic cases as in [142], [205], [10] and [128]. [25] introduce a two-index vehicle-flow formulation strengthened with several families of valid inequalities for the CLRP that they solve through branch-and-cut algorithm. [50] present three new flow formulations from which they derive new valid inequalities. The authors also propose new improved separation routines for the inequalities introduced in [25]. [20] introduce a set partitioning formulation for the problem, strengthened with new families of valid inequalities. [51] improve the set partitioning formulation by new valid inequalities and solve the problem using an exact algorithm based on cut-and-column generation. [175] develop a greedy randomized adaptive search procedure (GRASP). [172] also introduce the first hybrid metaheuristic for the CLRP combining the variable neighborhood search (VNS) metaheuristics and integer-linear programming techniques. [104] present an ALNS for the 2E-CVRP and also test it on the CLRP instances. Finally, [52] introduce a hybrid metaheuristic combining a GRASP with integer programming methods based on column generation. Their method provides very good results compared to previous methods.

Furthermore, the multi-period feature was also considered in some LRP models. [141] examine a multi-period uncapacitated LRP (ULRP) with capacitated vehicles. [11] consider the multi-period ULRP with decoupled time scales, in which the routing decisions and the location decisions follow two different time scales. On the other hand, stochastic setting is tackled in the ULRP by [142] and by [10]. In the considered stochastic model, depot locations and a priori routes must be specified in the first-stage, and second-stage recourse decisions deal with first-stage failures. [205] proposes a stochastic LRP model based on routing cost estimations. [128] examine the stochastic multi-period location transportation problem (SMLTP) where distribution centers are uncapacitated. The location and mission of depots must be fixed at the beginning of the planning horizon, but transportation decisions are made on a multi-period daily basis as a response to the uncertain customers' demand. They formulate the SMLTP as a two-stage stochastic program with recourse, and solve it by a hierarchical heuristic approach based on SAA method. This latter is a sampling-based approach introduced by [202] and has been successfully applied in [193] and [199].

This review confirms the literature shortcomings to address the stochastic and multi-period 2E-CLRP. It also relates a lack of exact methods to deal with the stochastic version of this novel problem.

4.3 Mathematical formulation

Our 2E-SM-CLRP considers a long-term planning horizon $\mathcal{T}$ that covers a set of successive design planning periods $\mathcal{T} = \{1, \dots, T\}$. Such periods are defined in accordance with the evolution of the uncertain customers' demand over time (typically a year). Each planning period encompasses a set of operational periods represented generally in a discrete way by "typical" business days. Under uncertainty, the routing decisions depend on the actual realization of the demand along each period t. Thus, each realization defines a demand scenario ω representing a "typical" day of delivery. All potential scenarios characterize the set of demand scenarios Ω_t modeling the uncertainty behavior of customers' demand at that period t. Each scenario has a probability of occurrence $p(\omega)$. The set of all scenarios is $\Omega = \cup_t \Omega_t$.

The 2E-SM-CLRP is defined on a graph with three disjoint sets of nodes, potential locations for warehouse platforms (WPs), $\mathcal{P} = \{p\}$, potential locations for distribution platforms (DPs), $\mathcal{L} = \{l\}$ and the customers $\mathcal{J} = \{j\}$. WPs and of DPs can be opened, maintained operating or closed along the sequence of the planning periods, and to each decision, a time-varying fixed cost is associated (i.e. fw_{pt},fw^s_{pt},fw^c_{pt} for WP and $f_{lt}, f^s_{lt}, f^c_{lt}$ for DP). Each WP (resp DP) has a limited capacity denoted C_p (resp C_l). Additionally, each customer j has an uncertain demand $d^{t\omega}_j$ at period t under scenario ω. Therefore, two echelons are defined.

At the first echelon, we consider an undirected bipartite graph $\mathcal{G}^1 = (\mathcal{V}^1, \mathcal{E}^1)$, with the vertex set $\mathcal{V}^1 = \mathcal{P} \cup \mathcal{L}$, and the edge set $\mathcal{E}^1 = \{(p, l) : p \in \mathcal{P}, l \in \mathcal{L}\}$ represents the links between the WPs and DPs. These links help to calibrate the DPs throughput. To each link we can assign one or several full truckloads, each of which has a capacity Q^1_{lp} and a fixed cost h_{lpt}. Moreover, multi-sourcing strategy is allowed in the first echelon where each DP can be supplied from more than one operating WP with respect to its capacity C_l and the WPs' capacity C_p. At the second echelon, an undirected graph $\mathcal{G}^2 = (\mathcal{V}^2, \mathcal{E}^2)$ is defined where $\mathcal{V}^2 = \mathcal{L} \cup \mathcal{J}$, and $\mathcal{E}^2 = \{(i, j) : i, j \in \mathcal{V}^2,\ j \notin \mathcal{L}\}$. It is worth to mention that no lateral transshipment between DPs is performed, i.e., there are no direct edges between DPs. A routing cost c^t_{ij} is associated with each edge $(i, j) \in \mathcal{E}^2$ at period t in the second echelon.

We consider an unlimited set $\mathcal{K}$ of identical vehicles with capacity Q^2 used to visit customers in the second echelon, where $Q^2 < Q^1_{lp}$. If used, a fixed cost is paid for each vehicle to which we assign a route in the second echelon. We assume that this cost is already incorporated into the routing cost in the following way. For each t and for each edge $(i, j) \in \mathcal{E}^2$ adjacent to a DP, value c^t_{ij} is increased by the half of the fixed vehicle cost.

The proposed model decides on the opening, closing and operating periods for each WP and DP, and well as the number of full truckloads assigned to each link $(p, l) \in \mathcal{E}^1$ defining thus the capacity allocated to DPs. In the second-stage, the goal is to build vehicle routes so that each customer is visited exactly once in each period and each scenario. The quantity delivered to customers from each operating DP under each scenario is less or equal to the capacity assigned to that DP from WPs. Our aim is to minimize the total expected cost by minimizing the sum of the expected transportation cost and the design cost (location and capacity).

The 2E-SM-CLRP is formulated as follows. At the first-stage, let y_{pt}, y^+_{pt} and y^-_{pt} be binary variables equal to 1 if WP $p \in \mathcal{P}$ is selected for operating, opening and closing in period t. Similarly, we define z_{lt}, z^+_{lt} and z^-_{lt} for each DP $l \in \mathcal{L}$. Let x_{lpt} be an integer variable denoting the number of full truckloads assigned from WP $p \in \mathcal{P}$ to DP $l \in \mathcal{L}$ in period $t \in \mathcal{T}$.

Given a fixed first-stage design solution, the second-stage problem is modeled using a set partitioning formulation [221]. Let us denote by $\mathcal{R}^{t\omega}_l$ the set of all routes starting and ending at an operating DP l satisfying capacity constraints for period t under scenario $\omega \in \Omega_t$, and let $\mathcal{R} = \cup_{t\in\mathcal{T}} \cup_{\omega\in\Omega_t} \cup_{l\in\mathcal{L}} \mathcal{R}^{t\omega}_l$. Note that a route $r \in \mathcal{R}$ is not necessarily elementary, i.e. it can visit a client more than once. Let ψ^r_{ij} denotes the number of times edge (i, j) participates in route $r \in \mathcal{R}$, ξ^r_j denotes the number of times customer j is visited in route $r \in \mathcal{R}$. Then $\sum_{j\in\mathcal{J}} d^{t\omega}_j \xi^r_j \leqslant Q^2$ for every $r \in \mathcal{R}^{t\omega}_l$. Cost c^t_r of route $r \in \mathcal{R}^{t\omega}_l$ is calculated as $c^t_r = \sum_{(i,j)\in\mathcal{E}^2} \psi^r_{ij} c^t_{ij}$.

Let $\lambda^{t\omega}_{lr}$ be a binary variable indicating whether a route $r \in \mathcal{R}^{t\omega}_l$ is selected in the optimal solution. The two-stage stochastic integer program with recourse is then written as:

$$\min \sum_{t\in\mathcal{T}}\sum_{p\in\mathcal{P}}(fw_{pt}\,y_{pt}+fw^{s}_{pt}\,y^{+}_{pt}+fw^{c}_{pt}\,y^{-}_{pt})+\sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}}(f_{lt}\,z_{lt}+f^{s}_{lt}\,z^{+}_{lt}+f^{c}_{lt}\,z^{-}_{lt})+\sum_{t\in\mathcal{T}}\sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}}h_{lpt}x_{lpt}+\sum_{t\in\mathcal{T}}\mathbb{E}_{\Omega_t}[\phi_{t\omega}(x)] \tag{4.1}$$

$$\text{S. t.} \quad \sum_{l\in\mathcal{L}} Q^{1}_{lp}x_{lpt} \leqslant C_p\,y_{pt} \qquad \forall p,t \tag{4.2}$$

$$\sum_{p\in\mathcal{P}} Q^{1}_{lp}x_{lpt} \leqslant C_l\,z_{lt} \qquad \forall l,t \tag{4.3}$$

$$y_{pt}-y_{pt-1} \leqslant y^{+}_{pt} \qquad \forall p\in\mathcal{P}, t\in\mathcal{T} \tag{4.4}$$

$$z_{lt}-z_{lt-1} \leqslant z^{+}_{lt} \qquad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{4.5}$$

$$y_{pt-1}-y_{pt} \leqslant y^{-}_{pt} \qquad \forall p\in\mathcal{P}, t\in\mathcal{T} \tag{4.6}$$

$$z_{lt-1}-z_{lt} \leqslant z^{-}_{lt} \qquad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{4.7}$$

$$\sum_{t} y^{+}_{pt} \leqslant 1 \qquad \forall p\in\mathcal{P} \tag{4.8}$$

$$\sum_{t} y^{-}_{pt} \leqslant 1 \qquad \forall p\in\mathcal{P} \tag{4.9}$$

$$\sum_{t} z^{+}_{lt} \leqslant 1 \qquad \forall l\in\mathcal{L} \tag{4.10}$$

$$\sum_{t} z^{-}_{lt} \leqslant 1 \qquad \forall l\in\mathcal{L} \tag{4.11}$$

$$x_{lpt}\in\mathbb{N} \qquad \forall l\in\mathcal{L}, p\in\mathcal{P}, t\in\mathcal{T} \tag{4.12}$$

$$y^{+}_{pt}, y_{pt}, y^{-}_{pt}\in\{0,1\} \qquad \forall p\in\mathcal{P}, t\in\mathcal{T} \tag{4.13}$$

$$z^{+}_{lt}, z_{lt}, z^{-}_{lt}\in\{0,1\} \qquad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{4.14}$$

where $\phi^{t\omega}(x)$ is the solution of the recourse problem:

$$(\text{SPF}_{t\omega}) \quad \phi_{t\omega}(x) = \min \sum_{l\in\mathcal{L}}\sum_{r\in\mathcal{R}^{t\omega}_{l}} c^{t}_{r}\,\lambda^{t\omega}_{lr} \tag{4.15}$$

$$\text{S. t.} \quad \sum_{l\in\mathcal{L}}\sum_{r\in\mathcal{R}^{t\omega)}_{l}} \xi^{r}_{j}\lambda^{t\omega}_{lr} = 1 \qquad \forall j\in\mathcal{J} \tag{4.16}$$

$$\sum_{r\in\mathcal{R}^{t\omega}_{l}}\left(\sum_{j\in\mathcal{J}} d^{t\omega}_{j}\xi^{r}_{j}\right)\lambda^{t\omega}_{lr} \leqslant \sum_{p\in\mathcal{P}} Q^{1}_{lp}x_{lpt} \qquad \forall l\in\mathcal{L} \tag{4.17}$$

$$\sum_{l\in\mathcal{L}}\sum_{r\in\mathcal{R}^{t\omega}_{l}} \lambda^{t\omega}_{lr} \geqslant \Gamma^{t\omega} \tag{4.18}$$

$$\lambda^{t\omega}_{lr}\in\{0,1\} \qquad \forall l\in\mathcal{L}, r\in\mathcal{R}^{t\omega}_{l} \tag{4.19}$$

The objective function (4.1) minimizes the sum of the first-stage costs and the expected second-stage costs. The first-stage costs represent the operating, opening and closing WPs and DPs cost, and the capacity cost induced by the number of truckloads associated to DPs from WPs. The objective function (4.15) of the second-stage consists in minimizing the routing cost and the fixed cost for using vehicles. Constraints (4.2) and (4.3) guarantee the capacity restriction at operating WP and DP, respectively. Constraints (4.4) and (4.5) determine the platform opening. These constraints manage the status of the WPs and the DPs operating from one period to the next and set their opening. Constraints (4.6) and (4.7) build on the same way to manage the location closing status for WPs and DPs, respectively. Constraints (4.8)–(4.11) specify that each WP and DP is opened or closed at most once during the planning horizon. Constraints (4.12)–(4.14) describe the feasible set for the first-stage variables. Constraints (4.16) ensure that each customer is served exactly once. Constraints (4.17) are the depot capacity inequalities. They guarantee that the demand satisfied from an operating DP will not exceed its throughput capacity. Inequalities (4.18) are added to strengthen the linear relaxation of (SPF) by imposing lower bound $\Gamma^{t\omega}$ on the number of vehicles required to serve customers' demand in period t in scenario ω. This lower bound is obtained through the solution of a bin packing problem (BPP) in which the aim is to pack a given set of items having different weights into a minimum number of equal-sized bins [153]. Here, the set of items represents the set of customers $\mathcal{J}$ where each item has a weight $d_j^{t\omega}$, i.e. the customer demand, and the bin capacity is the vehicle capacity Q^2. More precisely, $\Gamma^{t\omega}$ is the value of the linear relaxation of (BPP) obtained for period t and scenario ω.

4.4 Benders approach

In this section, we develop a Benders decomposition approach to solve the 2E-SM-CLRP where the integer recourse is handled through two steps iteratively. The algorithm separates the problem into a Benders master problem (MP) and a number of Benders subproblems, which are easier to solve than the original problem. By using linear programming duality, all subproblem variables are projected out and the relaxed MP contains only the remaining master variables and artificial variables representing the lower bounds on the cost of each subproblem. In the first-stage, location (WPs and DPs) and capacity assignment decisions are taken by solving the Benders master problem. When these first-stage decisions are fixed in the original problem, the resulting subproblem is a capacitated vehicle-routing problem with capacitated multi-depot (CVRP-CMD) that can be decomposed by period and by scenario. However, solving these subproblems as an integer program does not produce dual values to generate standard Benders cuts. In order to overcome this difficulty, we iteratively tackle the second-stage integer program to get valid and useful Benders cuts. The main steps of our solution approach are summarized in Figure 4.2. As a preprocessing step, we solve the linear relaxation of a bin packing problem (BPP) for each period and each scenario through column generation. Rounded up values of the obtained bounds are then determined and introduced to the Benders subproblems as a lower bound on the number of vehicles required for each period and each scenario. Further details on the BPP are given in Appendix C. Then, at each iteration of the Benders approach, a relaxed integer MP, including only a small subset of Benders cuts, is optimally solved to obtain a valid dual bound and the first-stage solution. Using such fixed first-stage decisions (operating DPs and capacity assignment), we first relax the integrality restrictions in the Benders subproblems (i.e., CVRP-CMD). For each period and scenario, we solve the LP of the set partitioning formulation (SPF) (4.15)–(4.19) using the column generation approach to generate a Benders optimality cuts from the dual solutions. These cuts are then added to the MP. If there are no new Benders optimality cuts, the integer subproblems are then solved using the branch-cut-and-price algorithm from [190]. This algorithm is demonstrated to be the best performing exact approach for many classical variants of the vehicle routing problem. The expected cost from integer feasible solutions of subproblems yields a primal bound for the 2E-SM-CLRP. If the gap is still large, combinatorial Benders cuts are added to the MP to eliminate the current master solution. This process is repeated until an optimal solution is found or the relative gap is smaller than a given threshold ϵ.

In the following, we first describe the Benders master problem (MP). Then, we present the Benders subproblems and the Benders cuts generated. Finally, we give a complete description of the Benders algorithm used to solve the 2E-SM-CLRP.

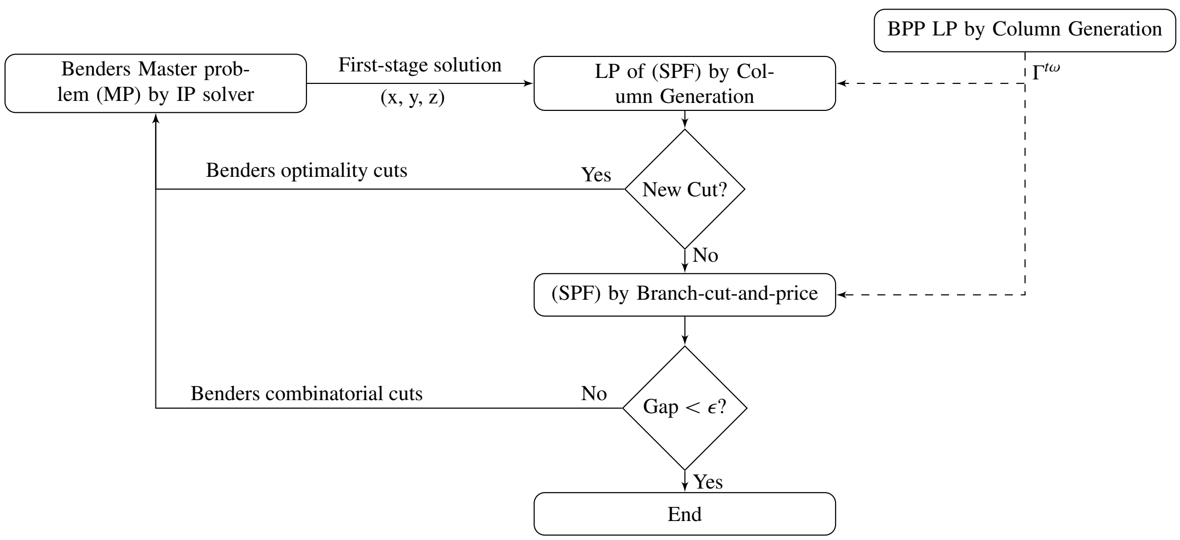


Figure 4.2: *Main steps of our solution approach*

4.4.1 Benders master problem

The Benders master problem (MP) includes first-stage decisions: location decisions for WPs and DPs and capacity assignment decisions. Introducing an additional variable $\theta_{t\omega}$ representing the total second-stage decisions cost for period t and scenario ω, we then formulate the MP as:

$$\text{(MP)} \min \sum_{t\in\mathcal{T}}\sum_{p\in\mathcal{P}}(fw_{pt}\, y_{pt} + fw^s_{pt}\, y^+_{pt} + fw^c_{pt}\, y^-_{pt}) + \sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}}(f_{lt}\, z_{lt} + f^s_{lt}\, z^+_{lt} + f^c_{lt}\, z^-_{lt}) + \sum_{t\in\mathcal{T}}\sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}} h_{lpt} x_{lpt} + \sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega} p(\omega)\theta_{t\omega} \tag{4.20}$$

$$\text{S. t.} \qquad (4.2)-(4.14)$$

$$\theta_{t\omega} \geqslant 0 \quad \forall t \in \mathcal{T}, \omega \in \Omega \tag{4.21}$$

It is worth to note that MP contains only the integer design variables and $|\mathcal{T}| \times |\Omega_t|$ additional continuous variables. Benders optimality cuts and combinatorial cuts are added to the model iteratively after solving subproblems.

Let k represent the current iteration number. In each iteration, the optimal solution of the master problem provides a dual bound (v^k_{MP}) for the 2E-SM-CLRP. We define $v^k_{design} = \sum_{t\in\mathcal{T}}\sum_{p\in\mathcal{P}}(fw_{pt}y_{pt}+fw^s_{pt}y^+_{pt}+fw^c_{pt}y^-_{pt})+\sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}}(f_{lt}z_{lt}+f^s_{lt}z^+_{lt}+f^c_{lt}z^-_{lt})+\sum_{t\in\mathcal{T}}\sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}}h_{lpt}x_{lpt}$ as the cost value of the design variables.

4.4.2 Benders subproblems

For fixed operating DPs and capacity assignment, the recourse problem can be decomposed into $|T| \times |\Omega|$ subproblems, which are CVRP-CMD problems, one for each period $t \in \mathcal{T}$ and each scenario $\omega \in \Omega_t$.

Let $\bar{x}^k$ denote the vector of fixed variables in iteration k. Solution values $\phi_{t\omega}(\bar{x}^k)$ for the subproblems can be used to compute the expected total cost $\phi(\bar{x}^k)$ of the second-stage decisions: $\phi(\bar{x}^k) = \sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega_t} p(\omega)\phi_{t\omega}(\bar{x}^k)$.

4.4.2.1 Generating Benders optimality cuts

As pointed out at the beginning of Section (4.4), we start by solving the linear relaxation (SPLP$_{t\omega}$) of the set partitioning formulation (SPF$_{t\omega}$) for all periods $t \in \mathcal{T}$ and for all scenarios $\omega \in \Omega_t$. Due to the exponential size of $\mathcal{R}^{t\omega}_l$, every (SPLP$_{t\omega}$) is solved using column generation. It is an iterative approach in which in every iteration a subset of variables λ is considered, and a restricted set-partitioning linear program (RSPLP$_{t\omega}$) is solved. Let ($\tau^{t\omega}$, $\rho^{t\omega}$, $\iota^{t\omega}$) be an optimal dual solution of (RSPLP$_{t\omega}$), corresponding to constraints (4.16), (4.17), and (4.18). To determine whether this dual solution is optimal for (SPLP$_{t\omega}$), the pricing problem should be solved. In it, we search for a route $r \in \mathcal{R}^{t\omega}_l$, $l \in \mathcal{L}$, with a negative reduced cost. If such routes are found, the corresponding variables λ are added to (RSPLP$_{t\omega}$), and we pass to the next iteration. The pricing problem is decomposed into $|\mathcal{L}|$ problems, one for each DP. Reduced cost $\hat{c}^{t\omega}_{lr}$ of a route $r \in \mathcal{R}^{t\omega}_l$ is computed as

$$\hat{c}^{t\omega}_{lr} = c^t_r - \sum_{j\in\mathcal{J}} \tau^{t\omega}_j \xi^r_j + \left(\sum_{j\in\mathcal{J}} d^{t\omega}_j \xi^r_j\right) \rho^{t\omega}_l - \iota^{t\omega}. \tag{4.22}$$

By replacing ξ by ψ and removing the constant part in (4.22), the pricing problem for (RSPLP$_{t\omega}$) and DP $l \in \mathcal{L}$ can be formulated as

$$\min_{r\in\mathcal{R}^{t\omega}_l} \hat{c}^{t\omega}_{lr} = \min_{r\in\mathcal{R}^{t\omega}_l} \sum_{(i,j)\in\mathcal{E}^2} (c^t_{ij} - \tau^{t\omega}_j)\psi^r_{ij}. \tag{4.23}$$

Each pricing problem is a Resource Constrained Shortest Path (RCSP) problem. The aim here is to find a minimum cost path linking a source vertex to a sink vertex that satisfy the capacity constraint. This problem can be efficiently handled using dynamic programming labeling algorithms [116, 179]. In this study, we use the bucket graph based labeling algorithm from [190].

Note that set $\mathcal{R}^{t\omega}_l$ of routes can be restricted to only elementary ones (passing by each customer at most once) without eliminating any feasible solution of the 2E-SM-CLRP. Relaxation

($\text{SPLP}_{t\omega}$) is tighter in this case. However, considering only elementary routes in the pricing can make it hard to be solved. Instead, we use *ng*-route relaxation [21], known to have a good trade-off between formulation strength and pricing difficulty. An *ng*-route can only revisit a customer i if it first passes by a customer j such that i is not in a pre-defined neighbourhood of j. In many instances, reasonably small neighborhoods (for example, of size 8) already provide tight bounds that are close to those that would be obtained by pricing elementary routes [174].

To insure the feasibility of each ($\text{SPLP}_{t\omega}$), we add slack variables to constraints (4.16)–(4.18). We set the coefficients for these variables in the objective function large enough, so that the second-stage decisions are feasible for every period and every scenario in the final solution obtained for the 2E-SM-CLRP.

Dual solution $(\bar{\tau}^{t\omega}, \bar{\rho}^{t\omega}, \bar{\iota}^{t\omega})$ is optimal for ($\text{SPLP}_{t\omega}$) if no route with negative reduced cost if found after solving the pricing problem. Thus this solution is feasible for the dual of ($\text{SPLP}_{t\omega}$), and by linear programming duality theorem, the following inequality is valid for the (MP):

$$\theta_{t\omega} + \sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}} Q^1_{lp} \bar{\rho}^{t\omega}_l x_{lpt} \geqslant \sum_{j \in \mathcal{J}} \bar{\tau}^{t\omega}_j + \Gamma^{t\omega} \bar{\iota}^{t\omega} \tag{4.24}$$

In iteration k of our Benders algorithm, we solve ($\text{SPLP}_{t\omega}$) for all periods t and all scenarios ω. All inequalities (4.24) violated by $\bar{x}^k$ are then added to (MP). Such constraints are called Benders optimality cuts in the literature.

4.4.2.2 Generating combinatorial Benders cuts

If none of inequalities (4.24) is violated, formulations ($\text{SPF}_{t\omega}$) for each period t and each scenario $\omega \in \Omega_t$ are then solved using the state-of-the-art branch-cut-and-price algorithm from [190].

The branch-cut-and-price algorithm is based on a combination of column generation, cut generation and branching. For each node of the branch-and-bound, the lower bound on the optimal cost is computed by solving the problem ($\text{SPLP}_{t\omega}$) enhanced with Rounded Capacity Cuts (RCC) [144] and limited memory Rank-1 Cuts (R1C) [120, 166] using column generation as described above in Section 4.4.2.1. Column generation convergence is improved with the automatic dual pricing stabilization technique proposed in [170]. After each convergence, the algorithm performs bucket arc elimination procedure based on reduced costs [190] Then, a bi-directional enumeration procedure [19] is called to try to generate all improving elementary routes with reduced cost smaller than the current primal-dual gap. If the pricing problem corresponding to a DP $l \in \mathcal{L}$ is successfully enumerated, the current ($\text{RSPLP}_{t\omega}$) is updated by excluding non-elementary columns corresponding to routes in $\mathcal{R}^{t\omega}_l$. This pricing problem is then solved by inspection. When pricing problems for all DPs $l \in \mathcal{L}$ are enumerated and the total number of routes is small, the node is finished by a MIP solver.

On the other hand, if there exists at least one non-enumerated pricing problem that becomes too time consuming to solve and the tailing off condition for cut generation is reached, branching is performed. Three different branching strategies are used, they can be expressed as constraints over the following aggregated variables:

- branching on the number of vehicles used from a DP l: $\sum_{r\in\mathcal{R}_l^{t\omega}}\sum_{j\in\mathcal{J}}\frac{1}{2}\psi^r_{lj}\lambda^{t\omega}_{lr},\ \forall l\in\mathcal{L}$,
- branching on the assignment of a customer $j\in\mathcal{J}$ to an operating DP l: $\sum_{r\in\mathcal{R}_l^{t\omega}}\xi^j\lambda^{t\omega}_{lr},\ \forall j\in\mathcal{J},\ l\in\mathcal{L}$,
- branching on the edges of the original graph:$\sum_{l\in\mathcal{L}}\sum_{r\in\mathcal{R}_l^{t\omega}}\psi^r_{ij}\lambda^{t\omega}_{lr},\ (i,j)\in\mathcal{E}^2$.

Branching variables are selected according to a sophisticated hierarchical strong branching procedure, inspired from [166].

Remember that $\phi_{t\omega}(\bar{x}^k)$ is the solution value of (SPF$_{t\omega}$). The branch-cut-and-price algorithm finds values $\underline{\theta}^k_{t\omega}$ and $\hat{\theta}^k_{t\omega}$, which are lower and upper bounds on value $\phi_{t\omega}(\bar{x}^k)$. If (SPF$_{t\omega}$) is solved to optimality, both values coincide. After solving all problems (SPF$_{t\omega}$), primal bound on the solution value of the 2E-SM-CLRP can be computed as

$$v^k_{design}+\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega_t}p(\omega)\hat{\theta}^k_{t\omega}. \tag{4.25}$$

If the optimality gap, i.e. difference between dual bound v^k_{MP} and primal bound (4.25) is sufficiently small, the algorithm is stopped. Otherwise, combinatorial Benders cuts are generated, which we will now describe.

The intuition behind combinatorial cuts is the following. Given period $t\in\mathcal{T}$, if for every $l\in\mathcal{L}$ the capacity of DP l induced by the first-stage decisions is not larger than $\sum_{p\in\mathcal{P}}Q^1_{lp}\bar{x}^k_{lpt}$, then for any scenario $\omega\in\Omega_t$ the value $\theta_{t\omega}$ of the second-stage decisions cannot be smaller than $\underline{\theta}^k_{t\omega}$. So, mathematically we can write

$$\forall t\in\mathcal{T}:\quad \begin{cases}\theta_{t\omega}\geqslant\underline{\theta}^k_{t\omega},\forall\omega\in\Omega_t, & \text{if } \sum_{p\in\mathcal{P}}Q^1_{lp}x_{lpt}\leqslant\sum_{p\in\mathcal{P}}Q^1_{lp}\bar{x}^k_{lpt},\forall l\in\mathcal{L},\\ \theta_{t\omega}\geqslant 0,\forall\omega\in\Omega_t, & \text{otherwise.}\end{cases} \tag{4.26}$$

In order to linearize conditions (4.26), additional variables are introduced. We define a^k_t as a non-negative variable representing the maximum increase of capacity of a DP in period t in comparison with DP capacities in iteration k:

$$a^k_t=\max\left(0,\ \max_{l\in\mathcal{L}}\left(\sum_{p\in\mathcal{P}}Q^1_{lp}x_{lpt}-\sum_{p\in\mathcal{P}}Q^1_{lp}\bar{x}^k_{lpt}\right)\right) \tag{4.27}$$

Let also q^k_t be a binary variable equal to 1 if and only if the value of a^k_t is strictly positive. In addition, we define binary variables b^k_{lt} and b^k_{0t} involved in the linearisation of expression (4.27). Let M' be a value which is larger than any possible value variables a can take, i.e. $M'\geqslant C_l$, $l\in\mathcal{L}$. Combinatorial Benders cuts are then formulated as:

$$a_t^k \leqslant M' q_t^k \qquad \forall t \in \mathcal{T} \tag{4.28}$$
$$q_t^k \leqslant a_t^k \qquad \forall t \tag{4.29}$$
$$\theta_{t\omega} \geqslant \underline{\theta}_{t\omega}^k (1 - q_t^k) \qquad \forall t \in \mathcal{T}, \omega \in \Omega_t \tag{4.30}$$
$$\sum_{p \in \mathcal{P}} Q_{lp}^1 x_{lpt} - \sum_{p \in \mathcal{P}} Q_{lp}^1 \bar{x}_{lpt}^k \leqslant a_t^k \qquad \forall t \in \mathcal{T}, l \in \mathcal{L} \tag{4.31}$$
$$a_t^k \leqslant \sum_{p \in \mathcal{P}} Q_{lp}^1 x_{lpt} - \sum_{p \in \mathcal{P}} Q_{lp}^1 \bar{x}_{lpt}^k + M'(1 - b_{lt}^k) \qquad \forall t \in \mathcal{T}, l \in \mathcal{L} \tag{4.32}$$
$$a_t^k \leqslant M'(1 - b_{0t}^k) \qquad \forall t \in \mathcal{T} \tag{4.33}$$
$$b_{0t}^k + \sum_l b_{lt}^k = 1 \qquad \forall t \in \mathcal{T} \tag{4.34}$$
$$a_t^k \in \mathbb{R}_+ \qquad \forall t \in \mathcal{T} \tag{4.35}$$
$$b_{lt}^k, b_{0t}^k, q_t^k \in \{0, 1\} \qquad \forall t \in \mathcal{T}, l \in \mathcal{L} \tag{4.36}$$

Constraints (4.28) and (4.29) link variables a and g. Constraints (4.30) impose lower bounds variables θ according to the first condition in (4.26). As for constraints (4.31)–(4.34), they express a linearization the definition (4.27) of variables a.

4.4.3 Overall algorithm

The complete description of the Benders approach is given in Algorithm 4.1.

We will now prove that our Benders approach converges to an optimum solution of the 2E-SM-CLRP under certain conditions.

Proposition 4.1. *Algorithm 4.1 finds an optimum solution to the 2E-SM-CLRP after a finite number of iterations if $\epsilon = 0$ and second-stage integer problems (SPF$_{t\omega}$) are solved to optimality at every iteration.*

Proof. The validity of Benders optimality cuts follows from the strong duality of linear programming. The validity of combinatorial Benders cuts follows from the fact that they follow a linearization of conditions (4.26).

The overall number of cuts is finite, as the number of different solutions $\bar{x}$ is finite. Therefore, there exists finite iteration k such that solution $\bar{x}^k$ is the same as solution $\bar{x}^{k'}$ at some previous iteration $k' < k$. The lower bound in iteration k is not smaller than $v_{design}^k + \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega_t} p(\omega) \bar{\theta}^k$. As all problems (SPF$_{t\omega}$) were solved to optimality in iteration k', upper bound is not larger than $v_{design}^{k'} + \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega_t} p(\omega) \underline{\theta}^{k'}$.

As $\bar{x}^{k'} = \bar{x}^k$, we have $v_{design}^k = v_{design}^{k'}$. Also from the construction of combinatorial cuts, we have $\bar{\theta}^k \geqslant \underline{\theta}^{k'}$ for all periods t and all scenarios ω. Thus, the lower and upper bounds match in iteration k, and feasible solution obtained in iteration k' is optimal for the 2E-SM-CLRP. □

Algorithm 4.1 Benders approach for the 2E-SM-CLRP

ϵ is set to the maximum optimality gap
$ub \leftarrow \infty$, $lb \leftarrow -\infty$, $k \leftarrow 0$
for all $t \in \mathcal{T}$, $\omega \in \Omega_t$ **do**
    Solve the linear relaxation of the bin packing problem (BPP) to obtain $\Gamma^{t\omega}$
end for
while $(ub - lb)/ub > \epsilon$ **do**
    Solve the (MP) to obtain solution $(\bar{x}^k, \bar{\theta}^k)$ of value v_{MP}^k
    $lb \leftarrow \max\{lb, v_{MP}^k\}$
    $newCut \leftarrow false$
    for all $t \in \mathcal{T}$, $\omega \in \Omega_t$ **do**
        Solve (SPLP$_{t\omega}$) by column generation to obtain dual solution $(\bar{\tau}^{t\omega}, \bar{\rho}^{t\omega}, \bar{\iota}^{t\omega})$ of value $\phi_{t\omega}^{LP}(\bar{x}^k)$
        if $\bar{\theta}_{t\omega}^k < \phi_{t\omega}^{LP}(\bar{x}^k)$ **then**
            Add the Benders optimality cut (4.24) to the (MP)
            $newCut \leftarrow true$
        end if
    end for
    if $newCut = false$ **then**
        for all $t \in \mathcal{T}$, $\omega \in \Omega_t$ **do**
            Solve the (SPF$_{t\omega}$) by branch-cut-and-price to obtain lower bound $\underline{\theta}_{t\omega}^k$ and upper bound $\hat{\theta}_{t\omega}^k$
            if $\bar{\theta}_{t\omega}^k < \underline{\theta}_{t\omega}^k$ **then**
                Add combinatorial Benders cuts (4.28)–(4.36) to the (MP)
            end if
        end for
        $\phi(\bar{x}^k) \leftarrow \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega_t} p(\omega)\hat{\theta}_{t\omega}(\bar{x}^k)$
        $ub \leftarrow \min\{ub, v_{design}^k + \phi(\bar{x}^k)\}$
        if $\underline{\theta}_{t\omega}^k < \hat{\theta}_{t\omega}^k$ for some t, ω and no violated cuts (4.28)–(4.36) were added to the (MP) **then**
            stop
        end if
    end if
    $k \leftarrow k + 1$
end while

The algorithm may stop before reaching predefined gap ϵ if the overall time limit is reached or if not all second-stage problems (SPF$_{t\omega}$) are solved to optimality. In this case, we are not guarantees to obtain a feasible solution. However, the results of our experiments below show that we were always able to obtain it in such cases.

Table 4.1: *Test problems size*

$\lvert\mathcal{P}\rvert$	4								
$\lvert\mathcal{L}\rvert$	8			12			16		
$\lvert\mathcal{J}\rvert$	15	20	50	15	20	50	15	20	50

4.5 Computational results

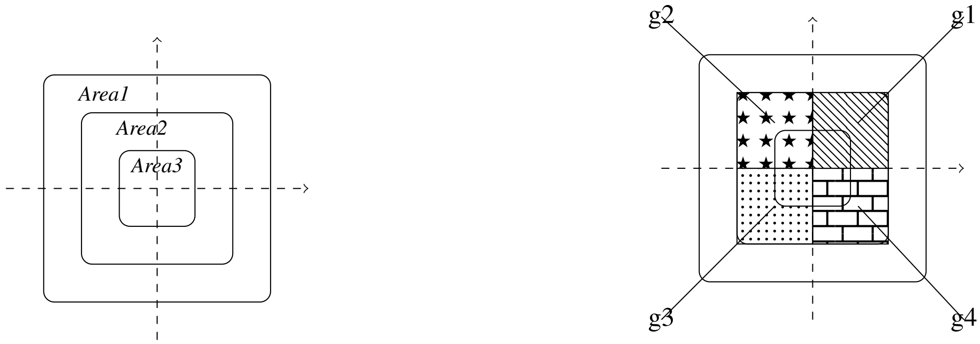
In this section, we describe our experimental results. First, we present the data instances used in the experiments. Then, we report and discuss the obtained results.

Our approach is implemented in the C++ programming language and compiled with GCC 5.3.0. BaPCod package [226] is used to handle the branch-cut-and-price framework. We use *CPLEX* 12.8.0 as the linear programming solver in column generation and as the integer programming solver for the set partitioning problem with enumerated columns as well as for the Benders master problem (MP). All tests are run on a cluster of 2 dodeca-core Haswell Intel Xeon E5-2680 v3 server running at 2.50 GHz with 128Go RAM. The OpenMP API for parallel computing [163] is considered to solve the $|T| \times |\Omega_t|$ CVRP-CMD subproblems using 24 cores in parallel.

4.5.1 Test data

To test our approach, several 2E-SM-CLRP instances have been randomly generated based on the following attributes: the problem size, the network characteristics, the demand process, the cost structure as well as the capacity dimension. Problems of nine different sizes are tested as shown in Table 4.1. In each case, it is either the number of DPs ($|\mathcal{L}|$) or the number of customers ($|\mathcal{J}|$) that varies. The number of potential WPs ($|\mathcal{P}|$) is also provided. We mention that the sets of potential DPs given in instances with $|\mathcal{L}| = 8$ DPs and $|\mathcal{L}| = 12$ DPs are subsets of the large set with $|\mathcal{L}| = 16$ DPs. A 5-year planning horizon is considered, which is partitioned into 5 design periods (i.e., $|\mathcal{T}| = T = 5$).

Platforms (i.e., WPs and DPs) and customers are realistically scattered in a geographic area within concentric square of increasing size. We assume that the covered geographic territory is composed of three urban areas *Area1*, *Area2* and *Area3*, where *Area3* represents the central area. Figure 4.3a illustrates the partition of the three urban areas made in a way to scatter realistically the customers locations and the 2-echelon platforms. Coordinates of customers and platforms potential locations are randomly generated in the defined urban area using the following criteria. Customers are randomly located within *Area2* and *Area3*. α is the ratio of customers randomly located in *Area3* and $(1 - \alpha)$ is the ratio of customers within *Area2*. DPs are located within *Area2* and *Area3*. 20% of the total number of DPs is randomly located in the central *Area3*, and 80% of DPs are in *Area2*. WPs are randomly located within *Area1*. Depending on the ratio α, two instance types are defined: I1 refers to concentric customers case where $\alpha = 0.8$ of customers are located within *Area3* and 20% in *Area2*. I2 corresponds to dispersed case where $\alpha = 0.6$.



(a) *The three urban area* **(b)** *Customers dispersion zones*

Figure 4.3: *Representation of two-echelon urban area*

Euclidean distances between nodes are computed, and two unit costs are defined to compute platform costs and transportation costs. Higher unit cost is attributed to DPs within *Area3* compared to the other areas. Two transportation costs configurations are tested: low transportation cost (LT) where the transportation cost represents 40% of the total network cost, and a high transportation cost (HT) where the trade-off is 60%. These ratios are determined based on some preliminary tests. At the second echelon, vehicle capacity is fixed to $Q^2 = 75$, and its fixed cost is $f = 600$ under the (LT) attribute and $f = 1300$ under (HT) attribute. The fixed WPs and DPs opening costs are generated, respectively in the ranges $fw_{pt} \in [14000, 25000]$ and $f_{lt} \in [7000, 10000]$ per location and period. An inflation factor is considered to reflect the increase of the cost of capital on a periodic basis with $r = 0.005$. The operating cost for both WPs and DPs is $fw^s_{pt} = 0.12 fw_{pt}$, and the closing cost fw^c_{pt} is about $0.2 fw_{pt}$. Additionally, we define two capacity configurations: a tight level (TC) where C_p and C_l are uniformly generated in the intervals $[600, 900]$ and $[220, 400]$ respectively; and a large level (LC) in which WPs and DPs capacities are uniformly generated in the intervals $[850, 1400]$ and $[550, 800]$, respectively. The truckload capacities Q^1_{lp} between WPs and DPs are generated in the interval $[150, 250]$.

Furthermore, we assume here, without loss of generality, that the demand scenario $d^{t\omega}_j$ of a customer j in period t follows the normal distribution with mean value μ_{jt} and standard deviation σ_{jt}. The customers are dispersed over four zones *g1, g2, g3* and *g4* within *Area2* and *Area3* as mentioned in Figure 4.3b. For each zone g and each period t, we define an inflation-deflation factor δ_{gt}. Each customer mean demand μ_{jt} depends on the time-varying trend δ_{gt} and on its mean value at the previous period. The coefficient of variation $(\frac{\sigma_{jt}}{\mu_{jt}})$ is a fixed parameter for each customer over periods. Two time-varying trends are tested providing two demand processes. The normal distribution with an increasing trend (NIT) refers to a customer mean value that follows an increasing factor $\delta_{gt} \in [0, 0.4]$ such that $\sum_g \delta_{gt} = 0.4$. The second one is the normal distribution with a variable trend (NVT) where each customer mean demand varies according to an increasing trend $\delta_{gt} \in [0, 0.9]$ for periods $t = 1..3$ and then according to a decreasing trend for $t = 4, 5$. The values and ranges for all the parameters regarding to the demand process are given in Table 4.2. Combining all the elements above yields several problem instances. Each instance is denoted by a problem size T-$|\mathcal{P}|/|\mathcal{L}|/|\mathcal{J}|$-, a scenario sample size N and a combination of customer dispersion (I1, I2), transportation

Table 4.2: *Demand processes*

Demand process	Mean value	Trend
NIT	$\mu_{jt} = \mu_{jt-1}(1+\delta_{z_jt})$	$\delta_{z_jt} \in [0, 0.4]$ such that $\sum_{z_jt} \delta_{z_jt} = 0.4$
NVT	$\mu_{jt} = \mu_{jt-1}(1+\delta_{z_jt})$	$\delta_{z_jt} \in [0, 0.9]$ such that $\sum_{z_jt} \delta_{z_jt} = 0.9$
	$\mu_{jt} = \mu_{jt-1}(1-\delta_{z_jt})$	$\delta_{z_jt} \in [0, 0.4]$
$\mu_{j0} \in [5, 25]$, $\frac{\sigma_{jt}}{\mu_{jt}} = 0.25$		

attribute (LT, HT), capacity configuration (TC, LC) and the demand process (NIT, NVT).

4.5.2 Parameters setting

The Benders decomposition algorithm terminates when one of the following criteria is met: (i) the optimality gap between the upper and lower bounds is below a threshold value ϵ, i.e., $(ub - lb)/ub < \epsilon$, or (ii) the maximum time limit $time_{max}$ is reached. We set the parameter values as $\epsilon = 0.0005$, and $time_{max} = 72$ *hours*. A time limit of 50 minutes is considered for each CVRP-CMD.

Next, an important parameter to calibrate for stochastic models is the number N of scenarios to include in the optimization phase. Under a scenario-based optimization approach, generating the adequate set of scenarios Ω could be complex due to the high enumeration issue induced by continuous normal distribution [204]. Assessing their probabilities also entails a tremendous effort. The combination of the Monte Carlo sampling methods [202] and the sample average approximation technique (SAA) [204] helps in finding a good trade-off in terms of the scenarios' probability estimation and the sufficient number of scenarios to consider in the model. This approach has been applied to network design problems in [193] and to stochastic multi-period location transportation problem in [128]. The SAA consists in generating at each time period t, before the optimization procedure, an independent sample of N equiprobable scenarios $\Omega_t^N \subset \Omega_t$ from the initial probability distribution, which removes the need to explicitly compute the scenario demand probabilities $p(\omega)$. The quality of the solution obtained with this approach improves as the scenario sample size N increases. However, one would choose N taking into account the trade-off between the quality of the obtained design and the computational effort needed to solve the problem. Thus, to determine the best value of N, solving the problem with M independent samples of demand repeatedly can be more efficient. This leads to a maximum of M different design decisions (i.e. location and capacity). It is worth to note that some samples may provide identical design decisions. The average value from the M expected cost based on N scenarios leads to a statistical lower bound. Then, we evaluate the different obtained designs based on the expected daily routing cost. We fix the first-stage according to each of these different designs and solve the resulting problem for $N' = |\Omega_t^{N'}| \gg N$ independent scenarios to get an upper bound on the optimal solution of the problem. Finally, a statistical optimality gap is computed for each obtained design from these lower and upper bounds. For more details, interested readers are referred to [193] and [199]. We notice that an external recourse option is added here, at a high cost, in order to guarantee the feasibility for all scenarios.

Table 4.3: *Average statistical optimality gap values for (I1,LT,TC,NIT) instances*

$\text{Gap}_{N,120}$ (in %)				
Problem size	Sample size (N)			
	5	10	15	25
5-4/8/15-N-	2.11	0.85	0.39	-0.14
5-4/8/20-N-	2.55	1.07	1.14	0.34

To apply the SAA technique, we solved $M = 10$ demand samples and used sample sizes of $N = 5, 10, 15$ and 25 scenarios for each time period t. The best feasible solution of each SAA sample is then stored as a candidate solution for valuation in the reference sample. The size of the reference sample per period t is set to $N' = |\Omega_t^{N'}| = 120$ scenarios. The average gap values for problem sizes 5-4/8/15-N- and 5-4/8/20-N- under (I1,LT,TC,NIT) instance configuration using the different values of N are summarized in Table 4.3.

Table 4.3 shows that the optimality gap improves as the sample size N increases and converges to 0%. Samples of $N = 15$ and 25 scenarios provide satisfactory results, generally less than 1% for both instances. Moreover, we note that the decisions produced with alternative samples ($M = 10$) present a high similarity in terms of the opened DPs and the inbound allocation. However, the solution time increases considerably with the sample size. Accordingly, the sample size of $N = 15$ is retained as the best trade-off to use in the experiments for instances with 50 customers and $N = 25$ is selected for instances with 15 and 20 customers. Recall that when N scenarios are used in the SAA model, $5 \times N$ instances are then sampled from the probability distribution as the planning horizon includes 5 periods.

4.5.3 Results

In this section, we evaluate the performance of the Benders approach and provide an analysis of the design solutions produced to deal with the 2E-SM-CLRP. Further, we examine the sensitivity of WP and DP location decisions to uncertainty under various problem attributes, and the behavior of the capacity decision in a multi-period and uncertain setting.

In order to evaluate the performance of our approach, we solve the deterministic equivalent formulation (DEF) of the problem using a commercial solver (Cplex). To this end, in the DEF, we reformulate the CVRP-CMD as a three-index vehicle-flow formulation as introduced in [215]. Two instances of size 5-4/8/15-1- and 5-4/8/20-1- under attributes (I1, LT, LC, NIT) are tested. The results show that Cplex is not able to solve to optimality these two instances with one scenario and is stopped after 13 hours of run due to a lack of memory with a gap of 13% for 5-4/8/15-1- instance and a gap of 25% for 5-4/8/20-1- instance.

Next, the 2E-SM-CLRP is solved using the Benders approach on the nine size-instances of Table 4.1, for each sample size. The results are reported in Table 4.4. The two first columns give the instance attribute and size associated with the sample size N. The third column *#Opt* refers to the number of optimal solutions found under each scenario sample size N. The next three columns *#CombCuts*, *#OptCuts* and *#Iter* provide the average number of generated combinato-

Table 4.4: *Average results per problem-instance*

Instance	N	#Opt	#CombCuts	#OptCuts	#Iter	Gap(%)	Parallel computing time			Sequential computing time
							MP	CVRP-CMD	total	
5-4/8/15-	5	24/24	2	6.3	8.5	0.00	4s	50s	2m47s	3m48s
	10	24/24	1.9	6.3	8.1	0.01	5s	50s	4m6s	6m55s
	15	24/24	1.9	6.3	8.2	0.00	6s	1m4s	5m51s	8m39s
	25	24/24	2	6.7	8.7	0.00	9s	1m27s	9m21s	13m49s
5-4/12/15-	5	15/15	1.9	6.8	8.7	0.00	8s	47s	2m32	8m20s
	10	15/15	2	7.4	9.4	0.00	12s	59s	4m22s	11m32s
	15	15/15	2.1	7.3	9.3	0.00	15s	1m8s	6m5s	18m2s
	25	15/15	2.1	7.5	9.6	0.00	25s	1m28s	9m44s	24m7s
5-4/16/15-	5	12/12	2.25	7.3	9.6	0.00	12s	2m35s	4m24s	11m39s
	10	12/12	2.3	7	9.3	0.00	20s	1m	4m34s	11m29s
	15	12/12	2.3	7.25	9.5	0.00	24s	1m11s	6m24s	17m29s
	25	12/12	1.9	7	8.9	0.01	27s	1m32s	9m50s	38m21s
5-4/8/20-	5	24/24	2.1	9	11.1	0.00	6s	1m19s	3m39s	36m7s
	10	24/24	2.1	8.7	10.7	0.00	7s	2m40s	7m14s	1h2m
	15	24/24	1.9	8.8	10.7	0.01	10s	3m23s	10m15s	1h46m
	25	24/24	2.1	9	11.1	0.00	16s	5m48s	17m14s	1h59m
5-4/12/20-	5	15/15	2.1	9.3	11.5	0.00	12s	2m28s	4m55s	39m9s
	10	15/15	1.9	8.5	10.5	0.01	16s	1m51s	6m37s	1h25m
	15	15/15	2	8.7	10.7	0.00	22s	5m26s	12m29s	2h11m
	25	15/15	1.9	7.9	9.9	0.00	30s	14m21s	26m3s	1h34m
5-4/16/20-	5	12/12	1.9	8.8	10.75	0.00	16s	1m47s	4m20s	29m2s
	10	12/12	2.2	9.7	11.8	0.01	37s	2m59s	8m7s	1h10m
	15	12/12	2.5	9.9	12.4	0.00	1m3s	11m47s	18m33s	2h49m
	25	12/12	2.25	9.67	11.91	0.01	1m34s	18m9s	30m56s	5h25m
5-4/8/50-	15	9/15	7.4	20.4	27.8	0.03	39m13s	16h54s	17h54m	
5-4/12/50-	15	8/15	7.1	26.5	33.6	0.03	2h50m	13h3m	16h14m	
5-4/16/50-	15	9/12	4.8	19.7	24.7	0.02	35m48s	7h34m	8h52m	

rial Benders cuts (4.28)–(4.36), the average number of generated Benders optimality cuts (4.24) and the required number of iterations to converge, respectively. The column *Gap(%)* indicates the average optimality gap. The three next columns under the heading *Parallel computing time* give the average CPU time spent for solving the Benders master problem (MP), the $T \times N$ CVRP-CMD subproblems and the total time needed to obtain an optimal solution of the problem using parallel computing. This average is computed over the instances that could be solved within the time limit (72 *hours*). For comparison purpose, the last column presents the total running time using sequential computing for the solved problems. It is worth noting that the total time includes the MP time, the CVRP-CMD time and the BPP time.

Table 4.4 shows that the developed approach is able to solve most instances in reasonable time within the threshold value $\epsilon = 0.05\%$. It solves all instances of 15 and 20 customers, in less than 30 minutes average time with parallel computing. It is worth to mention that for instances with 15 customers, the solution of the BPP takes the most computing time (two to seven minutes) compared to the MP+CVRP-CMD time. This is essentially due to the sequential computing of the BPP for each period and each scenario. When inspecting the sequential computing time, it is clear that the parallel computing helps to reduce drastically the running time, mainly in moderate-size instances with 20 customers. As illustrated in Table 4.4, the sequential solving approach could take from 4 to 10 times more computing time for instances with 20 customers, reaching an average running time of 5 hours and 25 minutes in instances with 16 DPs and 20 customers (which are solved in about 30 minutes in average with parallelization). Worth noting is that most computing time (about 70%) is spent by the branch-cut-and-price to get an integer solution to the $T \times N$ CVRP-CMD subproblems (see details in Table D.1 in Appendix D). This justifies the use of a parallel computing approach to solve the $T \times N$ CVRP-CMD subproblems.

Furthermore, when inspecting large-scale instances with 50 customers, our algorithm is able to solve to optimality 26 out of 42 instances in average time of 14 hours 20 minutes. The average optimality gap in the remaining 16 instances is generally below 0.5% which underlines the efficiency of the developed Benders approach. These instances are not solvable with the sequential approach within the maximum allocated time, as solving the $T \times N$ CVRP-CMD subproblems is time consuming. Moreover, Table 4.4 emphasizes the inherent complexity of the stochastic setting where computing time grows as the size of the scenarios sample N increases. Additionally, the Benders master problem (MP) takes only few seconds to be solved in most of the instances with 15 and 20 customers and grows up to two hours in the instances (5-4/12/50-.) using 15 scenarios. The MP resolution is also highly correlated to the number of potential DPs in the considered instance. The increase in computing time here is mainly due to the large number of cuts that are added to the MP iteratively.

Next, we evaluate the performance of the solution approach on the set of problem instances, given the combination of attributes: customer dispersion, transportation cost, capacity configuration and demand process, as defined in Section 5.1. Table 4.5 shows the results obtained for instances with 8 DPs and 15, 20, 50 customers, respectively, in terms of the best upper bound (ub), the best lower bound (lb), the optimality gap (%) and the computation time. For consistency purpose, for each instance, three random instantiation of the input parameters is made as indicated in the second column. For problem instances with 12 DPs and 16, the results are provided in Tables D.2 and D.3, respectively, in Appendix D.

Table 4.5: *Detailed results for 5-4/8/15-25-, 5-4/8/20-25- and 5-4/8/50-15-*

		5-4/8/15-25-				5-4/8/20-25-				5-4/8/50-15-			
		ub	lb	Gap(%)	Time	ub	lb	Gap(%)	Time	ub	lb	Gap(%)	Time
(I2, LT, TC, NIT)	1	74486.0	74486.0	0.0	10m36s	82611.2	82611.2	0.0	12m52	205012.0	204953.0	0.03	3h49m
	2	72344.1	72344.1	0.0	8m55s	96468.5	96468.5	0.0	13m19s	188933.0	188856.0	0.04	53h50m
	3	84334.7	84334.7	0.0	8m58s	89290.4	89290.4	0.0	33m53s	200191.0	200116.0	0.04	18h9m
average		77054.9	77054.9	0.0	9m30s	89456.7	89456.7	0.0	20m2s	194562.0	194486.0	0.04	36h
(I1, LT, TC, NIT)	1	74387.2	74387.2	0.0	9m44s	91276.5	91276.5	0.0	14m43s	203537.8	203367.7	0.08	-
	2	73465.3	73465.3	0.0	8m48s	84255.3	84255.3	0.0	13m40s	187485.0	187396.0	0.05	56h16m
	3	82051.9	82051.9	0.0	9m30s	89212.5	89212.5	0.0	28m4s	199104.0	199045.0	0.03	6h53m
average		76634.8	76634.8	0.0	9m21	88248.1	88248.1	0.0	18m49s	196708.9	196602.9	0.05	31h34m
(I1, HT, TC, NIT)	1	104470.0	104470.0	0.0	10m21s	128554.0	128554.0	0.0	15m54s	286317.3	286003.7	0.11	-
	2	103992.0	103992.0	0.0	9m24s	123425.0	123425.0	0.0	13m52s	268163.6	267670.7	0.18	-
	3	113685.0	113685.0	0.0	9m56s	126692.0	126692.0	0.0	21m18s	283578.6	282502.6	0.38	-
average		107382.3	107382.3	0.0	9m54s	126223.7	126223.7	0.0	17m2s	279353.2	278725.7	0.22	-
(I1, LT, LC, NIT)	1	84275.7	84275.7	0.0	8m54s	96880.2	96880.2	0.0	12m41s	190453.0	190453.0	0.0	35m10s
	2	82808.3	82808.3	0.0	8m30s	100817.0	100817.0	0.0	13m41s	188179.0	188179.0	0.0	34m5s
	3	86280.8	86280.8	0.0	9m34s	93665.0	93665.0	0.0	20m36s	196985.0	196898.0	0.04	1h16m
average		84454.9	84454.9	0.0	9m	97120.7	97120.7	0.0	15m40s	191872.3	191843.3	0.01	48m25s
(I1, LT, TC, NVT)	1	75856.1	75856.1	0.0	9m4s	107509.0	107465.0	0.04	13m22s	189925.8	189346.1	0.31	-
	2	57568.5	57568.5	0.0	8m46s	96637.7	96637.7	0.0	16m56s	218641.0	218553.0	0.04	19h43m
	3	82664.3	82664.3	0.0	9m13s	98871.2	98871.2	0.0	13m41s	208253.5	208028.5	0.11	-
average		72029.6	72029.6	0.0	9m1s	101006.0	100991.3	0.01	14m40s	205606.8	205309.2	0.15	19h43m

We observe that instances under large capacity attribute (LC) are easier to solve than the other attributes, and are all solved to optimality. Nevertheless, the tight capacity attribute (TC) makes data set more difficult to solve. Only few instances with 50 customers are solved within the time limit and the other ones present a reduced gap, below 1%. This is consistent with former results on capacitated LRPs and VRPs. Furthermore, the instances with network configuration I2 are solved more efficiently compared to I1-based instances. This is mainly due to the fact that in I2, where customers are more dispersed in the urban area (Figure 3b),the routing subproblems are easier to solve. Indeed, two instances out of three with 50 customers are optimally solved under I1, whereas the three instances are solved to optimality under I2. Cost attribute also impacts the complexity of the problem. In fact, for 5-4/8/50-15-(I1,.,TC,NIT), two instances out of three are optimally solved under LT attribute. However, no optimality is obtained under the HT attribute. This is mainly due to the increase of transportation costs under HT, which makes location-routing cost trade-offs more contrasting. A difference in solvability is also observed regarding the demand process attribute. Instances under NVT process are more difficult to solve compared to those under NIT, which is due to the augmented variability of the demand process in the former. As seen in Table 4.5 for instance, one instance sized 5-4/8/50-15-(I1,LT,TC,.) is optimally solved under NVT process compared to two under NIT.

Table 4.6: *Location decisions and their operating periods for 5-4/./20-25-(.,.,.,.)-3*

	(I1,LT,TC,NIT)			(I1,HT,TC,NIT)			(I1,LT,TC,NVT)			(I1,LT,LC,NIT)			Presence (%)
$\lvert\mathcal{L}\rvert$	8	12	16	8	12	16	8	12	16	8	12	16	
$\underline{l1}$	0	0	0	0	0	0	0	0	0	0	0	0	0
$\underline{l2}$	0	0	0	0	0	0	0	0	0	0	0	0	0
l3	0	0	0	0	0	0	1	1	0	0	0	0	14
l4	1	0	1	1	0	0	0	0	1	0	0	0	35
l5	0	2	0	0	2	0	0	0	0	1	0	0	29
l6	1	0	0	1	0	0	1	0	0	0	0	0	29
l7	0	0	0	0	0	0	$2^{\star}$	1	0	0	1	0	21
l8	0	0	0	0	0	0	0	0	0	0	0	0	0
$\underline{l9}$		0	0		0	0		0	0		0	0	0
l10		0	0		0	0		0	0		0	0	0
l11		0	0		0	0		0	0		0	0	7
l12		1	0		1	0		0	0		0	0	14
$\underline{l13}$			0			0			0			0	0
l14			0			1			0			1	14
l15			1			1			1			0	21
l16			0			0			0			0	0
Closed at t=							5						

Next, we closely look at the design decisions produced by our model under various problem-instances. The results are presented in Tables 4.6 and 4.7 for the different problem sizes with 20 and 50 customers, respectively. The results for instances with 15 customers in the network are given in Table D.4 in Appendix D. These tables provide the DP opening decisions and their operating periods: value 0 refers to DPs kept closed and a value in the range $[1,5]$ corresponds to the DP opening period t. A particular mark (star (*) or diamond ($^{\diamond}$)) next to a value indicates when that DP has been closed and indicates then the closing period. If no mark has been mentioned, the DP has remained operating until the end of the planning horizon. The first row corresponds to the combination of attributes and the second row provides the size of potential DPs. The first column corresponds to the list of potential DPs. The underlined DPs are those

Table 4.7: *Location decisions and their operating periods for 5-4/./50-15-(.,.,.,.)-3*

	(I1,LT,TC,NIT)			(I1,HT,TC,NIT)			(I1,LT,TC,NVT)			(I1,LT,LC,NIT)			Presence (%)
$\lvert\mathcal{L}\rvert$	8	12	16	8	12	16	8	12	16	8	12	16	
l1	0	0	0	0	0	0	0	0	0	0	0	0	0
l2	0	0	0	0	0	0	1	0	0	0	0	0	7
l3	4	0	0	2	1	0	0	1	0	0	0	0	42
l4	1	1	1	1	0	1	1	2⋆	1	0	1	1	85
l5	1	1	0	1	2	0	1⋆	1	0	1	0	0	64
l6	2	0	0	4	0	0	1⋄	0	0	0	0	0	28
l7	1	1	2	1	1	1	3	1	2	0	1	1	92
l8	0	0	0	0	0	0	0	0	0	1	0	0	14
l9		0	0		0	0		0	0		0	0	0
l10		4	0		0	0		0	0		0	0	7
l11		0	1		4	1		0	3		0	0	28
l12		1	0		1	0		1	1		0	0	35
l13			0			0			0			0	0
l14			0			0			0			0	0
l15			4			4			1⋆			0	21
l16			1			1			0			0	14
Closed at t=							⋆5,⋄4	⋆4	⋆5				

located in the central urban area *Area3* of the network. Tables 4.6 and 4.7 reveal that the opened DP number increases as the customer size grows, since this latter size impacts the total demand level of the network. Under tight capacity attribute (TC), two to three DPs are opened for instances with 20 customers. This number reaches five to six opened DPs for instances with 50 customers. Nevertheless, under large capacity (LC), the number of opened DPs is smaller than under TC as DPs can, in the former case, accommodate more inbound flows from WPs. Regarding WPs, the opened number is quite stable in the instances: three WPs at most are opened with instances covering 50 customers.

Additionally, the location of both WPs and DPs and the throughput capacity level are correlated with the demand process, the customers dispersion, and costs. The tables also highlight the sensitivity of the strategic location decisions where in several cases the design structure varies between high and low transportation cost. For instance, we observe in Table 4.6 with 5-4/8/20-25-(I1,LT,TC,NIT) that the network opened DPs 4 and 6, whereas with 5-4/8/20-25-(I1,HT,TC,NIT), DPs 3 and 7 are opened. We noticed that only in few instances the centralized DPs (i.e., positioned in *Area3* and are underlined in tables) were opened, which is due to their higher fixed costs. In addition, the customer dispersion (i.e., I1 vs I2) impacts the DP location decisions, mainly under (.,LT,TC,NIT) attributes, as illustrated in Table 4.8. For example, for the instance size 5-4/12/50-15-, DPs 4, 5, 7, 8 and 12 are opened under I2 attribute instead of 4, 5, 7, 10 and 12 under I1. Even though the DP 12 is identical under the two attributes I1 and I2, this latter is not opened at the same period: $t = 1$ for I1 and $t = 4$ for I2.

Moreover, the presence of additional potential DPs has an impact on the location decisions. It offers additional DPs' position for the network. In general, we observe a very low similarity in the DPs' location between instances with 8, 12 and 16 DPs, respectively. Looking closely at the instance with 20 customers in Table 4.6, no identical DPs' location is observed under (I1,HT,TC,NIT) configuration. However, under (I1,LT,TC,NIT), the instances 5-4/8/20-25- and 5-4/16/20-25- have DP 4 in common. Additionally, the change in full truckloads capacities

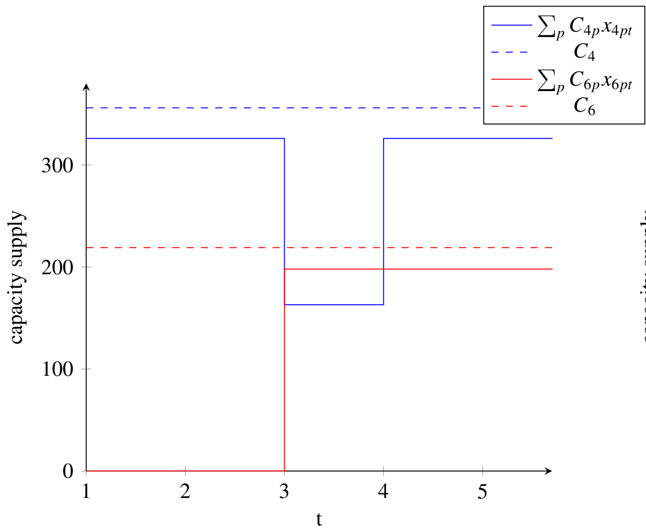
Table 4.8: *Location decisions and their operating periods for the customer dispersion attribute (I1 vs I2)*

	5-4/./20-25-				5-4/./50-15-			
	(I1,LT,TC,NIT)		(I2,LT,TC,NIT)		(I1,LT,TC,NIT)		(I2,LT,TC,NIT)	
$\lvert\mathcal{L}\rvert$	8	12	8	12	8	12	8	12
l1	0	0	0	0	0	0	0	0
l2	0	0	0	0	0	0	0	0
l3	0	0	0	0	4	0	3	0
l4	1	0	1	0	1	1	1	1
l5	0	2	0	1	1	1	1	1
l6	1	0	1	0	2	0	1	0
l7	0	0	0	0	1	1	1	1
l8	0	0	0	0	0	0	0	1
l9		0		0		0		0
l10		0		0		4		0
l11		0		1		0		0
l12		1		0		1		4
Closed at t=								

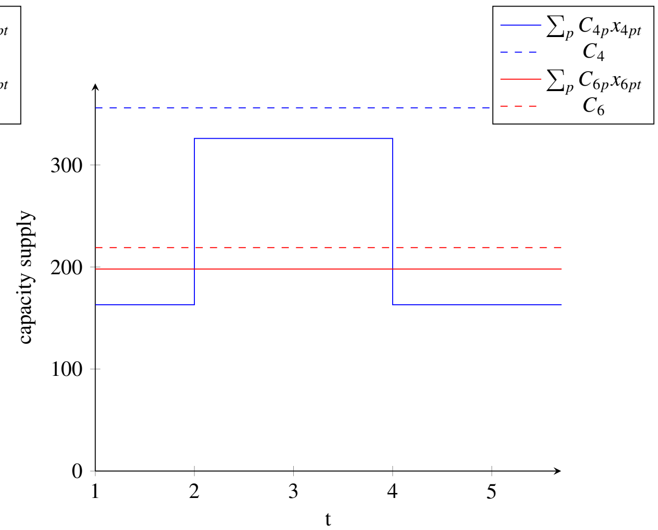
impacts the location decisions as illustrated in instances 5-4/./50-15 under (I1,LT,LC,NIT).

Furthermore, a key finding is the sensitivity of the problem to demand uncertainty in terms of location decisions and capacity decisions. The results affirm the multi-period design setting. In fact, we notice in almost all cases that the opening decisions follow the trend function and are planned over the different design periods. For instance, under the (I1,LT,TC,NIT) configuration, the problem size 5-4/12/20-25- fixes DP 12 from the first design period, and further at period $t = 2$ it opens DP 5. The same instance under the NVT demand process fixes all the opening locations from the first design period. The problem instance 5-4/8/20-25-(I1,LT,TC,NVT) also points out the multi-period design flexibility that we propose in this work. We can see that DP 7 is opened at $t = 2$ to meet the increasing trend of the demand at periods one to three. This latter is then closed at $t = 5$ as the demand decreases at periods four and five. In several instance cases, the tables do not show the multi-period opening/closing of location decisions. Instead, the problem adjust its capacity level to meet the demand variability which we report in Figures 4.4 and 4.5. In addition, we note a high variability in the location of opened DPs when comparing solutions from the two demand processes. To emphasize this result, one can closely observe the instance 5-4/12/20-25-(I1,LT,TC,.) where we have no identical DPs for NIT vs NVT. As for 5-4/16/15-25-(I1,LT,TC,.), we obtain 100% identical DPs for NIT vs NVT, but they differ in opening and closing periods (see Table D.4). Therefore, the obtained results confirm that the stochastic multi-period demand process is adequately captured by the two-stage stochastic formulation we present in this work.

In complement to the above analysis, we investigate the evolution of the capacity decisions in the multi-period and uncertain setting, since in the two-echelon structure, the demand is covered with respect to the inbound allocation to DPs from WPs, and not the DPs' predefined capacity. In Figures 4.4 and 4.5, we examine in depth the capacity decisions modeled in the 2E-SM-CLRP and contrast it to the evolution of each demand process along the planning horizon. These figures provide the results of instances 5-4/8/15-25-(I1,LT,TC,.) and 5-4/8/20-25-(I1,LT,TC,.) for both the NIT and NVT demand processes, which produces each a design

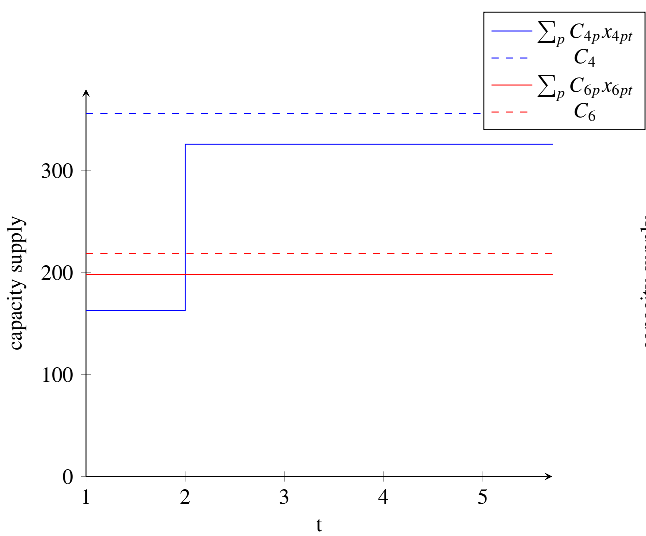


(a) *NIT process*

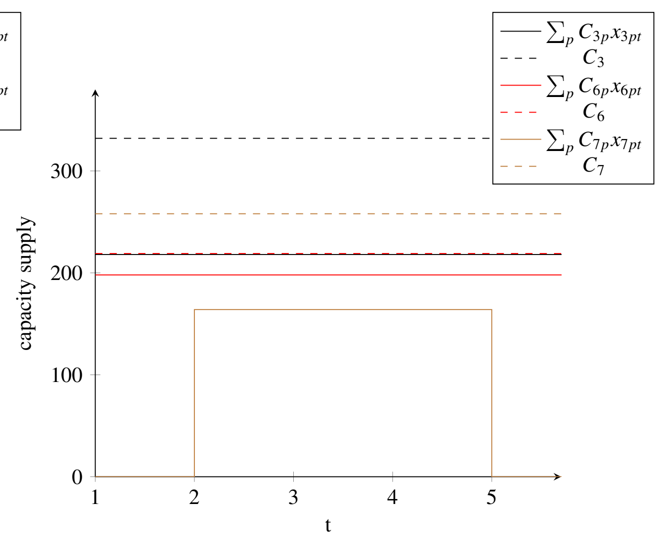


(b) *NVT process*

Figure 4.4: *Capacity-allocation decisions from multi-period modeling approach versus the a priori capacity C_l for 5-4/8/15-25-(I1,LT,TC,.)*



(a) *NIT process*



(b) *NVT process*

Figure 4.5: *Capacity-allocation decisions from multi-period modeling approach versus the a priori capacity C_l for 5-4/8/20-25-(I1,LT,TC,.)*

solution with two or three DPs. Each solid line corresponds to the capacity available at each opened DP (i.e., $\sum_p C_{lp} x_{lpt}$) in period t, and each dashed line its related predetermined capacity C_l (each color distinguishes a separate opened DP). We use the same color if the same DP is opened by the two processes. Figure 4.4 and 4.5 clearly illustrate the impact of the multi-period modeling approach where the capacity decisions for each opened DP are clearly adapted periodically. Even if the two demand processes of the instance 5-4/8/15-25-(I1,LT,TC,.) produce the same DP location decisions, the capacity decisions behave differently under each demand process to follow the time-varying demand process. This means that the two-stage model for the 2E-SM-CLRP mimics the dynamic capacity model with the inclusion of multi-period capacity decisions.

4.5.4 Multi-period design setting vs static setting

In this subsection, we further explore the multi-period design setting and contrast it to the static modeling approach, in which all design decisions should be fixed from the first design period and no further design adaptability is possible over the planning horizon. Table 4.9 provides the best upper and lower bounds from the static design modeling approach, the optimality gap(%) as well as an evaluation of the cost loss in respect to the multi-period approach (i.e., $\frac{\left(ub^{static} - ub^{multi-period}\right)}{ub^{multi-period}} \times 100$).

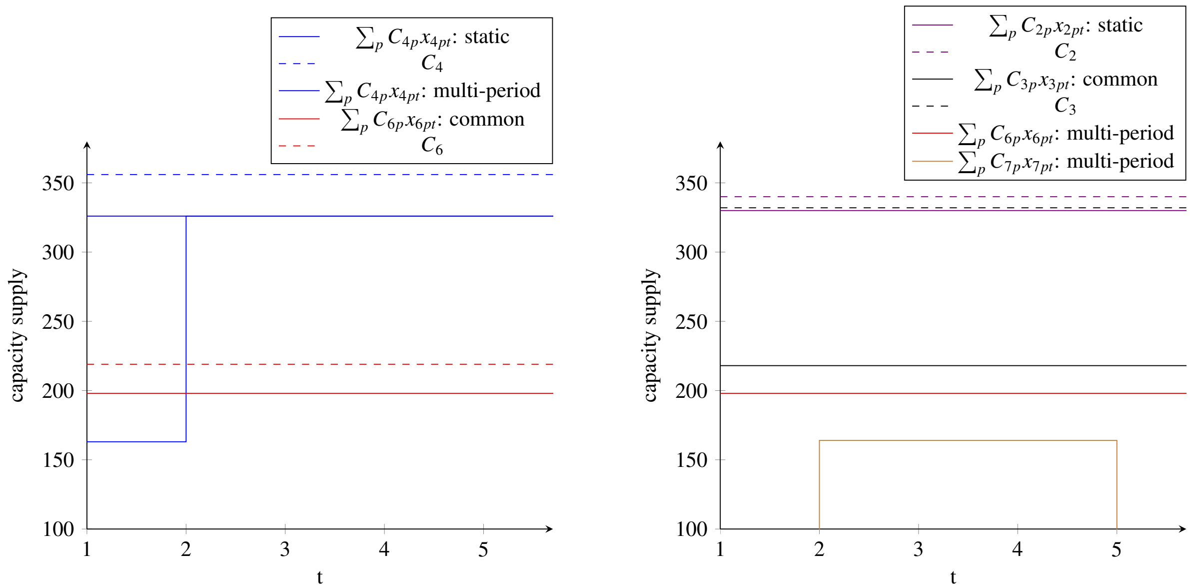
In Table 4.9, we can see that in almost all cases the static design setting provides higher expected cost. The cost loss increases with the problem size and it reaches more than 5%. The largest losses are observed under the NVT process as it presents more variability. This is in accordance with the static setting as the model anticipates the DP openings and the required capacity level at design period one and does not allow further changes at the subsequent periods resulting in an over-estimation of the capacity allocated. In other instances as in 5-4/12/20-25-(I1,LT,TC,NIT)-1, we observe that both models converge to the same optimal value. This is explained by the fact that particular instance fixes its design decisions from the first period under the multi-period design setting. This behavior is generally detected with small instances of 15 or 20 customers, and are mostly under NIT process. In addition, the static approach can solve optimally instances that cannot be solved with the multi-period setting, as it is the case of 5-4/8/50-15- under (I1,HT,TC,NIT) attributes. It is worth noting that a high variability of the solutions in terms of location and capacity is observed between both modeling approaches. For further illustration, see the results reported in Table D.5 in Appendix D.

Figure 4.6 completes the analysis by reporting the obtained capacity decisions for instance 5-4/8/20-25-(I1,LT,TC,.) under both demand processes using the static setting. Each color identifies an opened DP. For comparison purpose, we also mention in Figure 4.6 the capacity obtained using the multi-period approach. We note that we use the same color for an opened DP, if it is common in both approaches. From Figure 4.6a, under the NIT process, we notice that both static and multi-period modeling settings converge to the same location openings, but therein the static setting fixes all its capacity at the first period for DP 4 contrary to the multi-period approach. Under the NVT process, both approaches have only DP 3 in common fixing the same level of capacity from the beginning of the planning horizon. As mentioned, the

Table 4.9: *Comparison between static and multi-period modeling approach*

			Static modeling approach			Cost loss (%)
			ub	lb	Gap(%)	
5-4/8/20-25-	(I1,LT,TC,NIT)	1	91585.8	91585.8	0	0.34
		2	84255.3	84255.3	0	0.0
		3	90082.1	90082.1	0	0.97
	(I1,HT,TC,NIT)	1	129205	129205	0	0.51
		2	123425	123425	0	0.0
		3	128426	128383	0.03	1.37
	(I1,LT,TC,NVT)	1	110947	110947	0	3.2
		2	98895.8	98895.8	0	2.34
		3	99970.8	99970.8	0	1.11
5-4/8/50-15-	(I1,LL,TC,NIT)	1	208249.7	208181.9	0.03	2.31
		2	192102.2	192066.5	0.02	2.46
		3	206563.9	205694.3	0.42	3.75
	(I1,HT,TC,NIT)	1	292578.7	292541.7	0.01	2.19
		2	276247.5	276170.9	0.03	3.01
		3	292412.5	292281.1	0.04	3.12
	(I1,LT,TC,NVT)	1	194168.8	194130.8	0.02	2.23
		2	231215.2	231184.8	0.01	5.75
		3	216878.5	216770.3	0.05	4.14
5-4/12/20-25-	(I1,LT,TC,NIT)	1	79842.3	79842.3	0	0.00
		2	88448.6	88425	0.03	0.04
		3	84441.5	84441.5	0	1.96
	(I1,HT,TC,NIT)	1	117079	117079	0	0.00
		2	126107	126078	0.02	0.00
		3	121762	121762	0	1.56
	(I1,LT,TC,NVT)	1	98547.9	98547.9	0	1.76
		2	94633.7	94633.7	0	2.33
		3	92953.5	92953.5	0	0.00
5-4/12/50-15-	(I1,LT,TC,NIT)	1	192350.3	192350.3	0	1.87
		2	186311.6	186311.6	0	2.35
		3	194780.5	194651.3	0.07	3.12
	(I1,HT,TC,NIT)	1	281270.5	281136	0.04	2.24
		2	270558	270492.9	0.02	2.85
		3	281297.2	281042.4	0.1	3.27
	(I1,LT,TC,NVT)	1	186052.5	185785.7	0.14	2.40
		2	224365.8	224352.7	0.005	4.58
		3	203203.1	203143.9	0.03	4.55

static approach over-estimates its capacity level to hedge against the variability of the demand. Looking closely at the aforementioned example, the system allocates in total about 2620 units of capacity with the static setting versus 2457 for the multi-period case under the NIT process (resp, 2740 vs 2572 under NVT). This confirms our statement about the inherent approximation of the static setting of the problem. Therefore, one can conclude about the effectiveness of the multi-period modeling approach, as it offers the flexibility to adapt its hedging capabilities over time.



(a) *NIT process* **(b)** *NVT process*

Figure 4.6: *Capacity-allocation decisions from static modeling approach versus the a priori capacity C_l for 5-4/8/20-25-(I1,LT,TC,.)*

4.6 Conclusion

In this paper, we have addressed the two-echelon stochastic multi-period capacitated location-routing problem (2E-SM-CLRP). The problem is characterized as a hierarchical decision process involving a design level taking network location and capacity decisions, and an operational level dealing with transportation decisions in the second echelon. A stochastic multi-period characterization of the planning horizon is considered, shaping the evolution of the uncertain customer demand and costs. This temporal hierarchy is formulated as a two-stage stochastic integer program with recourse. To solve the 2E-SM-CLRP, we have presented an exact Benders decomposition algorithm. In the first-stage, location (WPs and DPs) and capacity assignment decisions are taken by solving the Benders master problem. When these first-stage decisions are fixed, the resulting subproblem is a capacitated vehicle-routing problem with capacitated multi-depot (CVRP-CMD) which is further decomposed by period and scenario. Each CVRP-CMD is then solved using the state-of-the-art branch-cut-and-price algorithm. Combinatorial Benders cuts are also proposed to cut off the current solution in order to converge to the optimal solution of the 2E-SM-CLRP. To the best of our knowledge, this is the first exact method that has been proposed for this class of problems. The method is able to optimally solve several realistic instances containing up to 50 customers and 25 demand scenarios under a 5-year planning horizon, and provides good lower bounds for the instances that cannot be solved to optimality. Extensive computational experiments provide relevant managerial insights regarding the impact of uncertainty on the 2E-SM-CLRP, in addition to the effectiveness of multi-period modeling setting.

Although the Benders decomposition provides good solutions, its performance is limited for large-scale instances. It might be worthwhile improving the considered algorithm to reduce the running time. In some instances with 50 customers, the CVRP-CMD cannot be solved in an hour, whereas all multi-depot CVRP instances in the literature are solved within the time limit. This points out the complexity of the CVRP-CMD involved in the 2E-SM-CLRP, with respect to the uncapacitated variant. Thus, it would be interesting to propose new cuts adapted to the CVRP-CMD to strengthen its solution algorithm. Another interesting research direction would be to develop an efficient heuristic solution method adapted to the 2E-SM-CLRP to solve larger problem instances. Future works would include more realistic features such as synchronization constraints at intermediate distribution platforms, and route length limit constraints. Moreover, since routing decisions are often used as operations anticipation for the operational level, another research perspective of this work would examine route approximation formulæinstead of explicitly computing vehicle routes. This may speed up the decision process. Last, it would be interesting to add risk measures to the objective function, such as mean semi-deviation and conditional value at risk.

Chapter 5

Rolling Horizon Approach for the Multi-stage Stochastic Two-Echelon Distribution Network Design Problem

Abstract

The delivery of products from the warehouse platforms (WPs) to customers is managed through an advanced echelon of distribution/fulfillment platforms (DPs) where transshipment and consolidation activities are performed. This type of distribution systems gives the two-echelon distribution network design problem (2E-DDP). In this paper, we provide a comprehensive methodology for the stochastic multi-period 2E-DDP, in which we stress the temporal hierarchy between the design level dealing with DP location and capacity decisions, and the operational level involving transportation decisions as origin-destination flows. The system must be dimensioned to enable an efficient distribution of goods to customers under a stochastic and multi-period planning horizon. Thus, the design of the two-echelon distribution network under uncertain customer demand gives rise to a complex multi-stage decisional problem. We develop a rolling horizon approach to solve the stochastic multi-period 2E-DDP. Our solution approach solves a sequence of stochastic models with a reduced planning horizon over the scenario tree. Preliminary experiments using small and medium sized data instances validate our modeling approach and show the effectiveness of our approach.

Keywords: *Two-echelon distribution network design problem, uncertainty, multi-period, multi-stage, rolling horizon.*

5.1 Introduction

E-commerce has experienced a sustained progress over the last decades. Its growth is influencing the way the logistics and supply chain management is played. Moreover, the development of e-commerce is boosting the expansion of an on-demand economy. This shift is tremendously affecting the distribution schema, more specifically the location and transportation networks. It thus results in more uncertainty and more complexity for the distribution network than logistics' practitioners are used to, especially with the increase of the customers' delivery service level. In addition, the rapid growth of e-commerce has led to the development of alternatives to home delivery such as lockers, relay points, and collection stores. In such context, several retail players, such as Amazon that continues an inexorable march toward distribution and order-fulfillment dominance, are examining promising opportunities for all stakeholders modal sector in designing their distribution networks [213]. In fact, having logistics space closer to customer zones plays an important role in improving the efficiency and reducing distribution costs and time to deliver. Consequently, the distribution network should be transformed to manage the new challenges impacting the distribution activity. In practice, the majority of companies and studies in the literature considers a single echelon distribution network where centralized warehouse platforms are deployed and transportation schema are built around [178, 78]. However, such configuration limits the efficiency of the distribution network.

Given the business requirements, distribution networks should be beyond more than a single echelon to be in line with new business conditions and meet strategic objectives. Therefore, considering an intermediate echelon of distribution/fulfillment platforms (DPs) standing between the upper echelon of warehouse platforms (WPs) and the customers' ship-to bases would be a good compromise. This type of configuration allows considerable savings compared to direct deliveries from one main warehouse. In particular, exploiting consolidation synergies allows for a reduction of the number of freight vehicles going into cities. Hence, the need for such a two-echelon structure becomes crucial in today's business environment to offer a good performance in distribution activities. For instance, Amazon is looking to acquire 42 Homebase stores around the UK to expand its network of fulfillment centers and warehouses [157]. Many real-life applications involve two-echelon distribution network. They are employed in city logistics applications for the case of multi-stakeholders sharing network structures [61, 160, 161]. In this case, peripheral distribution centers are created where no storage is allowed, and freight transfer and consolidation coming from back-level platforms on large trucks is carried out into smaller and environmentally friendly vehicles, more suitable for city distribution. Dedicated two-echelon distribution networks are also deployed for the case of non-substitutable products as in postal and parcel delivery [89], where parcels transit over one or multiple intermediate distribution centers, before being delivered to relay points, to lockers or to customers' homes.

With this in mind, we define the two-echelon distribution network design problem (2E-DDP). The problem involves facility location, capacity allocation and transportation decisions. More precisely, this strategic problem aims to decide on the number and the location of DPs, as well as the capacity allocated from first echelon to second echelon platforms. It also determines transportation decisions between platforms. Since first echelon network is generally devoted for consolidation, direct assignment with full truckloads transportation option departing from WPs to DPs is thus considered. In the second echelon, goods are transported as origin-destination flows from deployed DPs to customers. The objective is to minimize the total cost. Under this type of configuration, DPs location decisions strongly depend on the inbound allocation from upper WPs in addition to the trade-off between locations and capacities in the first echelon, as well as on the transportation decisions in the second echelon designed according to the trade-off

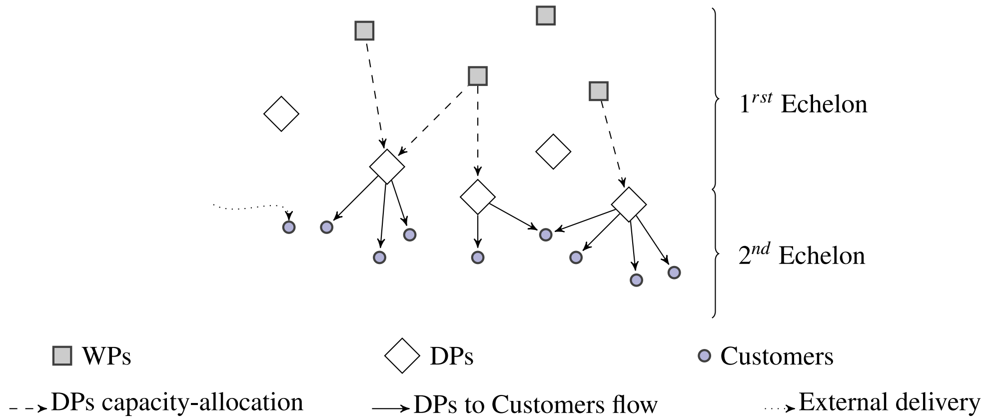


Figure 5.1: *A potential Two-Echelon Distribution Network Design Problem (2E-DDP)*

between customers demand versus DPs capacity. In Figure 5.1, we illustrate a typical example of 2E-DDP where the network includes two capacitated distribution echelons. Each echelon has its own assignment-transportation schema that must be adjusted in response to the evolution of the business horizon.

Existing studies related to our problem have been discussed in the literature. Sterle et al. [215] and Contardo et al. [53] formulate a deterministic-static two-echelon distribution network as a two-echelon location-routing problem (2E-LRP). In this case, distribution operations are modeled by explicit routes and location decisions may involve both WPs and DPs. In [53], the authors propose a branch-and-cut algorithm and an Adaptive Large Neighbourhood Search algorithm (ALNS). Nguyen et al. [160, 161] study a special case of 2E-LRP including one main warehouse and several potential distributon centers. To the best of our knowledge, only deterministic-static setting of the 2E-LRPs is addressed [71, 178, 62]. Moreover, the majority of studies suppose that location and routing decisions are made simultaneously for the planning horizon, without considering the hierarchical structure of the strategic problem stressed here. Further works focus on a hierarchical approach to two-echelon distribution problem. They extend the facility location problems (FLP) [97, 65] to the two-echelon facility location problem (2E-FLP) as introduced in [85]. Nonetheless, most 2E-FLP studies represent distribution operations by direct flows and ignore the capacity decisions, studying mostly deterministic versions. Additionally, most production-distribution problems and supply chain network design problem have a two-echelon structure, but with a single distribution echelon. The main focus of these studies is towards functional expansions such as production policies and constraints, and specific manufacturing-linked transportation issues, rather than the strategic needs of the distribution business (see for instance [80, 229]). Comprehensive reviews on FLPs can be found in [154, 78].

Nevertheless, location decisions have to last several years to meet future requirements. Such decisions should be planned as a set of sequential actions to be implemented at different periods of a given planning horizon [11]. They should also be tuned to the evolution of problem data over time. Thus, the 2E-DDP must be designed to last for several years, and robust enough to cope with the random environmental factors affecting the normal operations of a company. Moreover, the business uncertainty continues to increase and distribution practices are getting more complex. The uncertainty may come from strategic trends as well as operational variations, and both affect the design level [199, 125, 209]. Hence, a stochastic multi-period characterization of the planning horizon is considered in this case to capture the truly dynamic behaviour of most real-world applications, specifically in the strategic problem addressed here. The planning horizon is modeled by a set of planning periods (typically years) shaping the evolution of uncertain parameters (e.g., demand, costs, etc.), and promoting the structural adaptability of the distribution network. Such decisional framework leads to a multi-stage stochastic decisional problem as introduced in [129, 27].

With the development of stochastic programming [124, 38], scenarios with associated probabilities are increasingly used to represent uncertainties. Two-stage stochastic programming approach is used in [86, 240] for production-distribution problems under uncertain and multi-period settings. Georgiadis et al. [86] address designing a supply chain network comprising a two-echelon distribution schema in addition to inbound flows (i.e., from suppliers/plants) under uncertain and time-varying demand. But, therein periods involve only operational decisions, i.e., transportation. Zhuge et al. [240] consider a multi-period supply chain network design problem with a single distribution echelon to meet the variability of uncertain and time-varying demand and budget. In these variants, design decisions are determined before the uncertainty is realized for the entire multi-period planning horizon, and only a limited number of recourse can be take afterwards. However, such two-stage modeling approach cannot capture well the dynamic decision process in design problems.

Multi-stage models extend two-stage stochastic models by allowing revised decisions in each time stage as more information regarding the uncertainties is revealed. Consequently, multi-stage models are more appropriate for stochastic multi-period distribution network problems and offer more flexibility. A scenario tree may be built to handle the set of scenarios representing the uncertainty, and the optimization problem consists of designing the network that hedges against the scenario tree. The multi-stage stochastic approach is applied in Ahmed et al. [6] where they formulate the multi-period capacity expansion problem under uncertain demand and investment cost as a multi-stage stochastic program. Recently, Klibi and Martel [129] and Dunke et al. [72] evoke the need to such a framework to tackle complex supply chain problems. Klibi and Martel [129] define a methodological multi-stage framework for the supply chain network design problem, but therein a two-stage stochastic program for the one-echelon location-transportation problem is formulated and solved. Nickel et al. [162] study a multi-stage modeling framework for the supply network design problem with financial decisions and risk management involving a single distribution problem. They first formulate the problem as a multi-stage stochastic program, then propose an alternative formulation based on the paths in the scenario tree. Pimentel et al. [171] present a mixed-integer multi-stage stochastic programming approach to the stochastic capacity planning and dynamic network design problem which integrates facility location, network design and capacity planning decisions under demand uncertainty. They have also developed a lagrangian heuristic to get good approximate solutions when a large number of scenarios is considered. Later, Albareda-Sambola et al. [9] consider the one-echelon stochastic multi-period discrete facility location problem, in which uncertainty affects the costs as well as some of the requirements along the planning horizon. A multi-stage and a two-stage stochastic programming approaches are proposed, and solved through a Fix-and-Relax coordination scheme. In [3], the authors introduce a a two-stage and a multi-stage stochastic formulation for the production-distribution problem under demand uncertainty. In the first-stage, production setup and customer visit decisions are determined. The subsequent stages involve production, inventory, and delivery flow decisions, which are made when the demand becomes known. A Benders decomposition is developed to solve both formulations that they improve through the use of a single branching tree for the master problem, lower-bound lifting inequalities, scenario group cuts, and Pareto-optimal cuts.

Theoretical developments and approximations are proposed in the literature [38]. Solution methods include nested Benders decomposition [37, 187], progressive hedging [181] and lagrangian decomposition [184]. However, when integer variables are involved in the subsequent stages ($\geqslant 2$), the complexity of the problem increase significantly, and methods would require further development to provide integer solution to the problem. Sampling methods are also developed [202] for multi-stage stochastic programs. It is worthwhile to mention, although in principle, scenario trees can be used to handle uncertainty, in practice, accurate approximations of a complex stochastic process with a modest-sized scenario tree represent a difficult problem. Thus, several scenario generation methods and reduction techniques are proposed. We refer the reader to Heitsch and Römisch [102], Dupačová et al. [74, 75], and Høyland and Wallace [113]. Therefore, considering the aforementioned studies, mostly have developed a multi-stage stochastic program for single echelon distribution network design problems. To the best of our knowledge, no work has addressed the multi-stage stochastic framework for the multi-period stochastic 2E-DDP. Our paper contributes to fill this gap in the literature.

The main contribution of this work is first to address a comprehensive methodology for the two-echelon distribution network design problem over stochastic and multi-period settings. Our modeling framework assumes that the number and location of DPs are not fixed a priori and must be decided at the strategic level along the set of planning periods. It also considers strategic assignment-transportation decisions to calibrate DP throughput capacity based on transportation capabilities. Our approach emphasizes the temporal hierarchy between the design level dealing with DPs location decisions and the capacity decisions, and the operational level involving transportation decisions. The transportation decisions are modeled as origin-destination flows, which correspond to a sufficiently precise aggregate of daily decisions over several products, transportation means and working periods, as discussed in [131]. A stochastic multi-period characterization of the planning horizon is considered shaping the evolution of the uncertain and time-varying customers' demand. Second, a mixed-integer multi-stage stochastic program is introduced. Our formulation points out the temporal hierarchy between the strategic design decisions and the operational transportation decisions. Third, network design problems and facility location problems are NP-hard even in their deterministic versions [7]. However, decision-making under uncertainty is further complicated, and leads to large-scale optimization models [191]. To solve the multi-stage problem, we propose a rolling horizon approach. In this approach, a feasible solution is computed by solving a sequence of stochastic programming models having a reduced time horizon. Computational experiments validate our modeling approach and show the good quality bounds obtained for the original problem in reasonable computing time.

The reminder of this paper is organized as follows. In Section 5.2, we further detail the 2E-DDP under stochastic and multi-period settings. In Section 5.3, the problem is formulated as a mixed-integer multi-stage model. The rolling horizon approach is described in Section 5.4. Section 5.5 evaluates the solution procedure for different problem instances. Finally, Section 5.6 concludes the work and points out future research directions.

5.2 Problem description

In the two-echelon stochastic multi-period distribution network problem, strategic facility-location decisions concern the intermediate echelon of capacitated DPs used to distribute goods to customers. When a set of DPs is deployed, a number of full-load trucks is periodically determined to deliver products from warehouse platforms to each DP. Then, on a daily basis, the goods are delivered from each DP to the set of customers location. Thus, the allocation decisions are represented as periodic origin-destination full truckloads from warehouses to DPs, whereas the transportation decisions are represented as daily transportation links from DPs to customers. Our focus is on strategic capacity allocation decisions made by distribution platforms and their transportation capabilities. The operational decisions are modeled to evaluate their impact on the strategic decisions. To ensure total demand satisfaction on a given day, an external recourse delivery option is allowed at a higher shipment cost. A typical 2E-DDP is illustrated in Figure 5.1.

We consider a long-term planning horizon $\mathcal{T}$ that covers a set of successive design planning periods $\mathcal{T} = \{1, \dots, T\}$ defined in accordance with the operational dynamics. In practice, a period may correspond to year in the case of DPs to lease, and may be up to 2 years in the case of building or renovating DPs. Each planning period $t \in \mathcal{T}$ encompasses a set of operational periods $\tau \in \mathcal{T}_t$, represented generally in a discrete way by "typical" business days. Figure 5.2 illustrates the relationship between the decision planning periods and operational days. The figure underlines the hierarchical structure of the decision problem. It also shows that strategic design decisions (location and capacity decisions) could be adapted periodically at each design period t in response to the uncertainty shaping the business environment. In fact, design decisions are made prior to their deployment period with a partial information on the future business environment, and then after an implementation period, they will be available for use as shown by the positioning of the arrows in Figure 5.2. This emphasizes the information asymmetry between the design level and the operational level, mainly due to their different time scale (i.e., strategic vs operational period).

To capture the effect of operational uncertainty about customer demand on design decisions, our stochastic multi-period 2E-DDP is casted as a multi-stage stochastic program [38]. At time $t = 1$, design decisions are made under uncertainty and represent first-stage decisions. Then, at the beginning of each working period $\tau \in \mathcal{T}_{t=1}$, the value of the uncertain parameters becomes known providing a set of plausible future scenarios, and operational transportation decisions are determined. Based on the information available at the beginning of each subsequent time period ($t > 1$), design decisions are revised to offer a new opportunity to adapt the network to

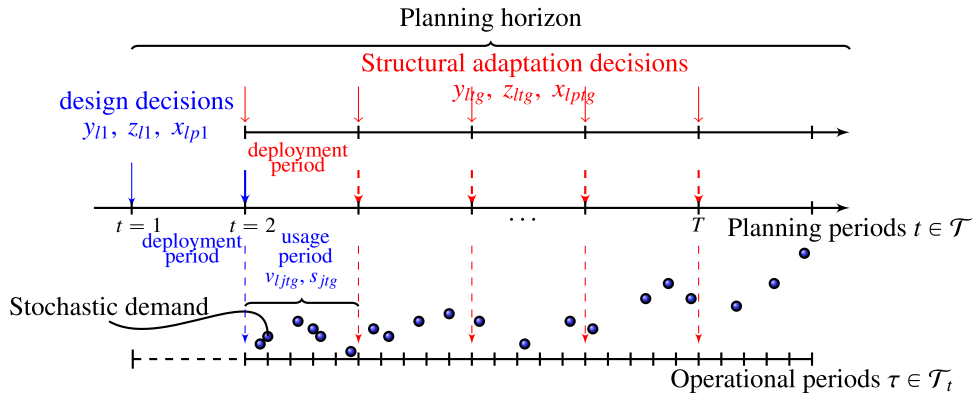


Figure 5.2: *Decision time hierarchy for the planning horizon under uncertainty*

its future environment. Therefore, design decisions made at the beginning of a time period t are first-stage decisions for that period, but they also depend on the previous design decisions up to that period, as illustrated in Figure 5.2. This underlines the dynamics of multi-stage decision structure (see Figure 5.3a for a generic multi-stage stochastic tree). Moreover, our model for a given period of the stochastic multi-period 2E-DDP under uncertain and time-varying demand, is itself a two-stage stochastic program. It is worth noting that, in practice, decisions are made under a rolling horizon basis: only first-stage decisions are implemented. Subsequent decisions ($t > 1$) are referred as structural adaptation decisions, and essentially used as an evaluation mechanism.

5.3 Problem formulation

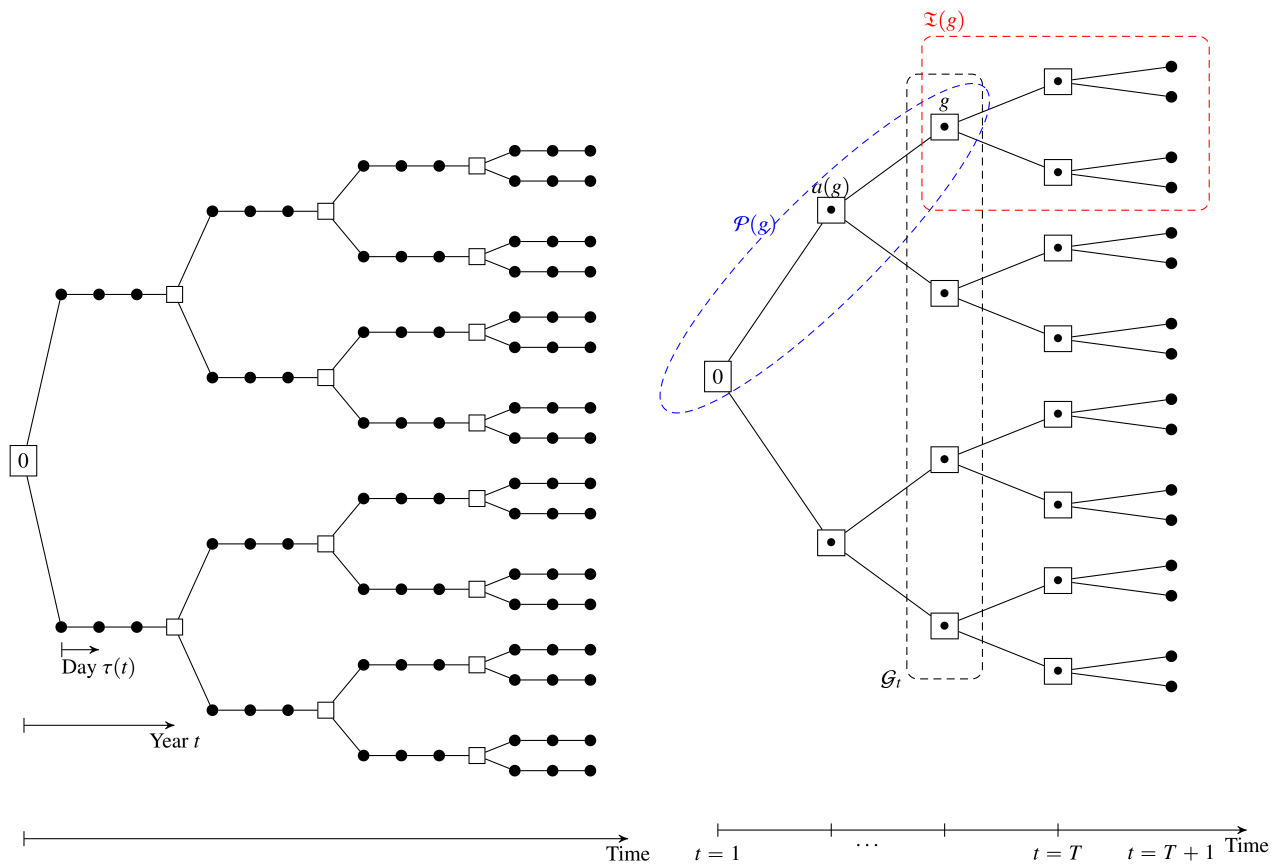
In this section, we present our mathematical model for the two-echelon stochastic multi-period distribution network design problem.

Let d_j be the random variable for customer j demand. Under a multi-period planning horizon setting, the random demand process is time-varying. Thus, the operational uncertain demand follows a stationary process, and is estimated from a discrete time stochastic process with a finite probability space. Accordingly, we can model the demand uncertainty as a scenario tree $\mathfrak{T}$.

We start by introducing the necessary terminology for the multi-stage stochastic program and the related scenario tree. In the scenario tree, a stage corresponds to a decision time point when new information is received. A period denotes the time interval between two consecutive time-discretization points in which the uncertainty is realized. Thus, the first time point in the tree defines a stage.

Nodes in the tree $g \in \mathfrak{T}$ correspond to time points when decisions are made. They represent distinguishable states of the world of information available up to a given stage. We define two node types, strategic nodes (illustrated in the figure with $\square$) for design decisions and operational nodes (illustrated with $\bullet$) for operational transportation decisions. $\mathcal{N}^d$ and $\mathcal{N}^o$ denote the set of design and operational transportation nodes, respectively. Leaf nodes start a last operational period illustrating the usage of last design decisions.

With this in mind, our framework for designing two-echelon distribution network problem under operational demand uncertainty provides a generic multi-stage stochastic. Figure 5.3a illustrates an example of the generic multi-stage program and its tree with both strategic and operational periods [209]. We see in the figure how the transportation decisions made at working times (i.e., operational days $\tau(t)$) are associated with a particular design stage (i.e. year t), and the design decisions (location and capacity decisions) are made subject to operational uncertainty.



(a) *Generic multi-stage stochastic program with operational uncertainty*

(b) *Our multi-stage stochastic program with operational uncertainty*

Figure 5.3: *Multi-stage stochastic trees with both strategic and operational time periods*

In our study, operational periods are represented by a typical business day. Under a stationary process, it is sufficient to replicate either the number of operational periods or the number of scenarios. Hence, the typical business day is sufficiently accurate. Recall that transportation decisions are made with certainty depending on the design decisions made before the realization of the demand for a given stage t. For the sake of simplification, we relax the notation $\tau(t)$, and defer transportation decisions at the beginning of stage $t + 1$. This helps to reduce the size of the tree. Figure 5.3b illustrates our multi-stage stochastic program with operational demand uncertainty over the tree as well as notations for the problem.

Each node g of the scenario tree, except the root ($g = 0$), has a unique parent $a(g)$ (i.e., ancestor node), and each non-terminal node g is the root of a sub-tree $\mathfrak{T}(g)$. Thus, $\mathfrak{T}(0)$ denotes the entire tree. Nodes at any given level of the tree are all related to the same time stage t and are referred as the set $\mathcal{G}_t$. The time stage corresponding to a node g is denoted $t(g)$. The path from root node to a node g is $P(g)$. A scenario $\omega \in \Omega$ corresponds to the path $P(g)$ till a terminal (leaf) node of the tree and represents a joint realization of problem parameters over all stages. All leaf nodes refer to the last operational period $T + 1$. Thus, $d_{jt\ g}$ will denote the demand of customer j at node g for stage $t = 2, \ldots, T + 1$ following a stochastic time space. Let $p(.)$ be the probability associated to a given state g and let π_{g_i,g_j} represent a transition between two consecutive nodes g_i and g_j. The probability of the realization of any node $p(g)$ is then given by multiplying the set of (g_i, g_j) transitions through the path from the root node up to g. Note that $p(g) = 1$ for the root node $g = 0$.

Using the aforementioned uncertainty information structure, we can now state a formulation for the problem. As pointed out by Dupačová [73], there are two approaches to impose the non-anticipativity constraints in the multi-stage program: the split variable for an explicit modeling and the compact formulation for an implicit modeling of the non-anticipativity setting.

Hereafter, we will provide a compact formulation. Consider the following notations:

- Sets

  $\mathcal{T} = \{1, \ldots, T\}$ set of time design stages.

  $\mathcal{T}^\star = \{2, \ldots, T + 1\}$ set of operational time stages.

  $\mathcal{P}$ set of warehouse platforms (WPs).

  $\mathcal{L}$ set of distribution platforms (DPs).

  $\mathcal{J}$ set of customers.

- Parameters

  C_p is the maximum throughput capacity of WP $p \in \mathcal{P}$ (expressed in flow unit for a given period).

  C_l is the maximum capacity of the DP $l \in \mathcal{L}$.

  C_{lp} is the maximum capacity of transportation used for flows from warehouse $p \in \mathcal{P}$ to DP $l \in \mathcal{L}$.

  c_{ljt} is the transportation cost per product unit from a DP $l \in \mathcal{L}$ to customer $j \in \mathcal{J}$ at period $t \in \mathcal{T}^\star$.

  c_{lpt} is the unit transportation cost per flow unit from warehouse $p \in \mathcal{P}$ to DP $l \in \mathcal{L}$ at period $t \in \mathcal{T}$.

  f^s_{lt} is the cost of opening a DP $l \in \mathcal{L}$ at period $t \in \mathcal{T}$.

  f^u_{lt} is the cost of operating a DP $l \in \mathcal{L}$ at period $t \in \mathcal{T}$.

c^r_{jt} is the shipment cost when recourse delivery is employed to cover a proportion of the demand of a customer $j \in \mathcal{J}$ at period $t \in \mathcal{T}^\star$.

It is worth to mention that operational transportation costs c_{ljt} and c^r_{jt} are annualized to cover the daily aspect of transportation decisions in the objective function made within typical business day.

Then, the decision variables are:

y_{ltg} = 1 if DP $l \in \mathcal{L}$ is operating at node $g \in \mathcal{G}_t$ in stage $t \in \mathcal{T}$, 0 otherwise.

z_{ltg} = 1 if DP $l \in \mathcal{L}$ is opened at node $g \in \mathcal{G}_t$ in stage $t \in \mathcal{T}$, 0 otherwise.

x_{lptg} = Inbound allocation from warehouse $p \in \mathcal{P}$ to DP $l \in \mathcal{L}$ expressed in number of full truckloads to deliver from the warehouse at node $g \in \mathcal{G}_t$ in stage $t \in \mathcal{T}$.

v_{ljtg} = fraction of demand delivered from DP $l \in \mathcal{L}$ to customer $j \in \mathcal{J}$ at node $g \in \mathcal{G}_t$ in stage $t \in \mathcal{T}^\star$.

s_{jtg} = fraction of demand of customer $j \in \mathcal{J}$ satisfied from an external shipment at node $g \in \mathcal{G}_t$ in stage $t \in \mathcal{T}^\star$ (i.e. recourse delivery, not from DPs).

The compact formulation for the multi-stage stochastic problem takes the form:

$$\min \sum_{t=1..T} \sum_{g \in \mathcal{G}_t} p(g) \sum_{l \in \mathcal{L}} \left[(f^u_{lt}\, y_{ltg} + f^s_{lt}\, z_{ltg}) + \sum_{p \in \mathcal{P}} c_{lpt}\, C_{lp} x_{lptg} \right] + \sum_{t=2..T+1} \sum_{g \in \mathcal{G}_t} p(g) \sum_{j \in \mathcal{J}} d_{jtg} \left[\sum_{l \in \mathcal{L}} c_{ljt}\, v_{ljtg} + c_{jt}\, s_{jtg} \right] \tag{5.1}$$

$$\sum_{l \in \mathcal{L}} C_{lp} x_{lptg} \leqslant C_p \qquad \forall p \in \mathcal{P},\ g \in \mathcal{G}_t,\ t = 1..T \tag{5.2}$$

$$\sum_{p \in \mathcal{P}} C_{lp} x_{lptg} \leqslant C_l y_{ltg} \qquad \forall l \in \mathcal{L},\ g \in \mathcal{G}_t,\ t = 1..T \tag{5.3}$$

$$y_{ltg} - y_{lt-1a(g)} \leqslant z_{ltg} \qquad \forall l \in \mathcal{L},\ g \in \mathcal{G}_t,\ t = 1..T \tag{5.4}$$

$$\sum_{j \in \mathcal{J}} d_{jtg}\, v_{ljtg} \leqslant \sum_{p \in \mathcal{P}} C_{lp} x_{lpt-1a(g)} \qquad \forall l \in \mathcal{L},\ g \in \mathcal{G}_t,\ t = 2..T+1 \tag{5.5}$$

$$\sum_{l \in \mathcal{L}} v_{jltg} + s_{jtg} = 1 \qquad \forall j \in \mathcal{J},\ g \in \mathcal{G}_t,\ t = 2..T+1 \tag{5.6}$$

$$x_{lptg} \in \mathbb{N} \qquad \forall l \in \mathcal{L},\ p \in \mathcal{P},\ g \in \mathcal{G}_t,\ t = 1..T \tag{5.7}$$

$$y_{ltg}, z_{ltg} \in \{0, 1\} \qquad \forall l \in \mathcal{L},\ g \in \mathcal{G}_t,\ t = 1..T \tag{5.8}$$

$$v_{ljtg}, s_{jtg} \geqslant 0 \qquad \forall l \in \mathcal{L},\ j \in \mathcal{J},\ g \in \mathcal{G}_t,\ t = 2..T+1 \tag{5.9}$$

The objective function (5.1) minimizes the total expected cost for the design and operational costs over the planning horizon. The first and second terms refer to the design costs that include the operating and opening costs for DPs, and the inbound allocation cost to DPs from WPs. The third term corresponds to the transportation cost in the second echelon (from DPs to customers) as well as the external delivery costs. Constraints (5.2)-(5.3) ensure capacity restrictions on WPs and DPs, respectively, over stages. In addition, constraints (5.3) guarantee that a delivery is possible to a DP only if it is operating. Constraints (5.4) represent the DPs setup over stages. These constraints manage the status of the DPs operating from one stage to the next and set their opening. Constraints (5.5) cover the customers demand from the operating DPs without exceeding their inbound allocation from WPs. Constraints (5.6) force demand total satisfaction for each customer. Constraints (5.7)-(5.9) define the decision variables of the problem. Worth noting is that $\mathcal{G}_1 = \{g = 0\}$ refers to the root node. Therefore, its associated decisions (x, y, z) are first stage decisions and represent here and now decisions concerning the design. Their counterpart at later stages represents recourse on the design in this multi-stage process. Operational allocation decisions (v, s) allow to evaluate the quality of the strategic design decisions (x, y, z) in each stage.

5.4 Rolling horizon approach

With the scenario tree specified, the stochastic multi-period 2E-DDP is a large-scale deterministic equivalent mixed-integer program (MIP). This formulation can be handled by standard mixed-integer solvers. However, such scheme will computationally be very expensive. Therefore, the use of alternative decomposition techniques [94] is often required to produce good results in less time. In the literature, rolling horizons have successfully been applied to several problems under uncertainty in order to deal with large MIPs. The approach drastically reduces the complexity of the model by solving a sequence of stochastic programming models over a reduced time horizon. We refer to Kusy and Ziemba [138], Kouwenberg [137], Guigues and Sagastizábal [96], Silvente et al. [208] and Bertazzi and Maggioni [35] for applications of this approach to different problems. A recent review about the rolling horizon is provided by Kopanos and Pistikopoulos [135]. The comparison of the total cost of the Rolling horizon approach with the optimal total cost is often missing in the literature, as the optimal solution cannot be computed.

To solve the multi-stage program for the stochastic multi-period 2E-DDP, we apply the rolling horizon approach, a common business practice in dynamic stochastic environment [17]. In this approach, instead of considering the whole time horizon, we build a sequence of models with reduced finite time horizon (i.e., sub-horizon), which are easier to solve. Each sub-horizon, referred to as a rolling sub-horizon $T^{RH} = r + H$, consists of two time periods: the roll stages r that will be fixed in iteration k, and the look-ahead stages H to foresee future realizations. In this study, we assume that we have a roll of length $r = 1$. Thus, decisions made at r refer to first-stage decisions.

With this in mind, the rolling horizon heuristic solves a model over a rolling sub-horizon of size $T^{RH} = H + 1$ in iteration k, and fixes its first-stage solution in roll r^k. Before solving the problem for the sub-horizon in iteration $k + 1$, the rolling sub-horizon is shifted forward in the optimization horizon to reveal new information so that the whole or the first part of the look-ahead H^k stages becomes r^{k+1}. The model starting from the next time period is then solved and again we store its first-stage solution. The process is repeated until we reach the end of the original time horizon. A complete solution over the horizon is progressively built by concatenating the solutions related to the first-stage decisions of a each reduced sub-horizon. In this manner, the rolling horizon approach helps reducing the size of the problem and thus improves the computational time required to get a solution for the original problem.

Given the multi-stage tree representing the uncertainty, the rolling horizon heuristic should go through all the tree nodes $\mathcal{G}_t$ related to each stage t. Each rolling sub-horizon is represented as restriction over T^{RH} stages from the scenario tree providing a sub-tree including T^{RH} stages ($r = 1$). Thus, in each iteration, the obtained MIP over the sub-tree $\mathfrak{T}_{T^{RH}}(g)$ of root node g is solved. Figure 5.4 illustrates an example of the rolling horizon approach with $T^{RH} = 2$ ($H = 1$, $r = 1$). In our illustrative example in Figure 5.4, the sub-tree embeds two stages ($T^{RH} = 2$), then a two-stage stochastic program is solved in each iteration. After solving the resulting MIP, an optimal solution is obtained for all decision nodes m in the sub-tree $\mathfrak{T}_{T^{RH}}(g)$. Therefore, we fix the solution of the root representing the first-stage design decisions (DPs and capacity). In our example, we refer to a fixed decision by ■, as mentioned in Figure 5.4b. In the next iterations, these fixed DPs location decisions are introduced as an initial conditions $\hat{y}(a(g))$ coming from the parent $a(g)$ of the current root node g. Concatenating all the fixed decisions leads a feasible solution for the original multi-stage program. In view of this, the process sequence in the example is as follows: it starts from Figure 5.4a, visits the nodes as in Figures 5.4b, 5.4c, 5.4d and 5.4e when moving forward in time, and finishes with a full solution as in Figure 5.4f.

In the following, we denote by v_{RH} the total cost obtained from the rolling horizon algorithm for the original problem. Using the fixed solution, v_{RH} is computed as defined in the objective function (5.1). The complete description of the rolling horizon approach applied to the stochastic multi-period 2E-DDP is given in Algorithm 5.1.

Algorithm 5.1 Rolling horizon approach

```
Planning horizon $\mathcal{T} \cup \mathcal{T}^\star = \{1, \ldots, T+1\}$, roll $r = 1$, look-ahead $H$, rolling sub-horizon $T^{RH} = H + 1$,
$(\hat{x}, \hat{y}, \hat{z}, \hat{v}, \hat{s}) = 0$ initial network state
for all $h = 1, \ldots, T + 1 - T^{RH}$ do
    for all $g \in \mathcal{G}_h$ do
        Solve the multi-stage stochastic program (5.1)-(5.9) over sub-tree $\mathfrak{T}_{T^{RH}}(g)$ defined on stages $h, \ldots, h + T^{RH}$ with initial conditions $\hat{y}_{a(g)}$
        Let $\{x^*_m, y^*_m, z^*_m, v^*_m, s^*_m\}$ be the optimal solution for decision nodes in the sub-tree $m \in \mathfrak{T}_{T^{RH}}(g)$.
        Fix the solution of the root node $g$ in $\mathfrak{T}_{T^{RH}}(g)$: $\hat{x}_g = x^*_g$, $\hat{y}_g = y^*_g$, $\hat{z}_g = z^*_g$, $\hat{v}_g = v^*_g$, $\hat{s}_g = s^*_g$.
    end for
end for
return feasible solution $(\hat{x}_g, \hat{y}_g, \hat{z}_g, \hat{v}_g, \hat{s}_g)$ for all $g \in \mathfrak{T}$ and total expected cost $v_{RH}$:
```

$$v_{RH} = \sum_{t=1..T} \sum_{g \in \mathcal{G}_t} p(g) \sum_{l \in \mathcal{L}} \left[(f^u_{lt} \hat{y}_{ltg} + f^s_{lt} \hat{z}_{ltg}) + \sum_{p \in \mathcal{P}} c_{lpt} C_{lp} \hat{x}_{lptg} \right] + \sum_{t=2..T+1} \sum_{g \in \mathcal{G}_t} p(g) \sum_{j \in \mathcal{J}} d_{jtg} \left[\sum_{l \in \mathcal{L}} c_{ljt} \hat{v}_{ljtg} + c_{jt} \hat{s}_{jtg} \right]$$

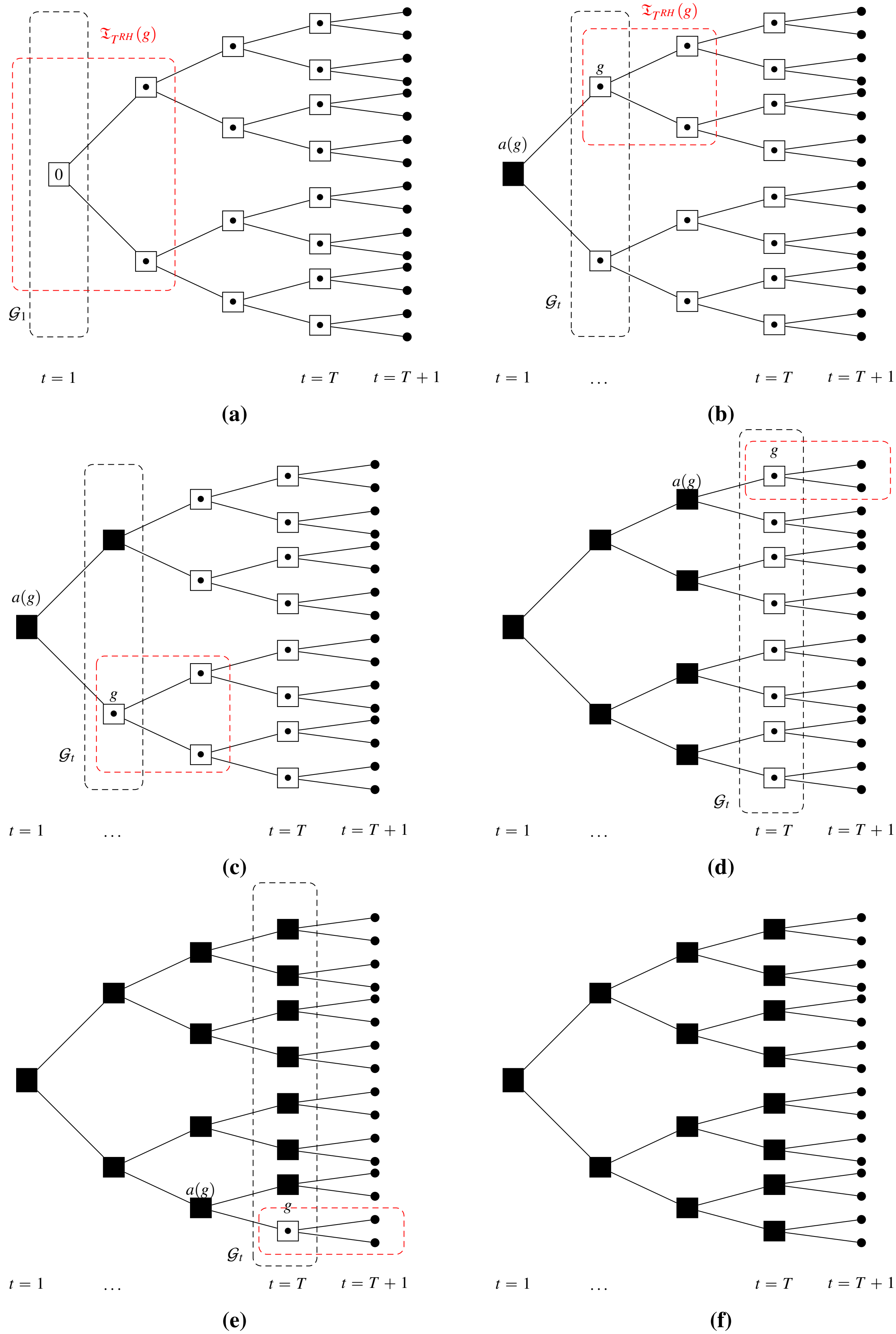


Figure 5.4: *An illustration of the rolling horizon algorithm with* $T^{RH}=1$

5.5 Computational results

In order to evaluate the solvability of our stochastic multi-period 2E-DDP, we run a set of computational tests. In this section, we first describe the data instances used in the expriments. Then, we report and discuss the obtained results.

Our approach is implemented in the C++ programming language and compiled with GCC 5.3.0. We use Cplex 12.8.0 as the mixed-integer programming solver for the the resulting programs in the rolling horizon approach. All experiments are run on a cluster of 2 dodeca-core Haswell Intel Xeon E5-2680 v3 server running at 2.50 GHz with 128Go RAM.

5.5.1 Test data

Several stochastic multi-period 2E-DDP instances have been randomly generated. In all instances, a 6-year planning horizon is considered, providing five design stages ($|\mathcal{T}| = T = 5$). The tested size problems are shown in Table 5.1. The network incorporates several possible configurations depending on the number of potential DPs location (# DPs) and the number of different capacity configurations per DP location (# capacity configurations). Thus, multiplying these two parameters gives the number of potential DPs, $|\mathcal{L}|$. When several capacity configurations are used, the second configuration has a higher capacity. Customers are realistically scattered in the geographic area covered. The number of WPs ($|\mathcal{P}|$) is also given.

Table 5.1: *Test problems size*

$\lvert\mathcal{J}\rvert$	$\lvert\mathcal{P}\rvert$	# DPs	# capacity configurations	$\lvert\mathcal{L}\rvert$
25	3	8	1	8
25	3	8	2	16
50	3	8	1	8
50	3	8	2	16

Customer demand is assumed to be uncertain, and can be seen as continuous random variables. In stochastic programming, scenario trees are often used to approximately model continuous distributions of the uncertain parameters. Demands are generated according to an auto-regressive process: $d_{jtg} = d_{jt-1a(g)} + \epsilon, \forall t \in \mathcal{T}^\star$ where the demand at the first period (d_{j10}) follows a log-normal probability distribution with parameters $m = 4.5$ and $s = 0.25$ and ϵ is a white noise generated according to the same log-normal to which we subtract the mean value of the distribution. To build the scenario tree, we proceed as follows. From each node, starting from the root, we associate to each node, starting from the root, a number of successor nodes C representing the number of stochastically independent events. Then, at each successor, the customer demands d_{jtg} is randomly generated from the auto-regressive process. We test cases with two, three, five and six independent events, respectively leading to different scenario sizes. Equiprobability between different events is considered. Table 5.2 present the problem number of events in each stage $|C|$, the number of design nodes $|\mathcal{N}^d|$, the number of operational transportation nodes $|\mathcal{N}^o|$ and the number of scenarios $|\Omega|$.

Table 5.2: *Test problems size*

$\lvert\mathcal{C}\rvert$	$\lvert\mathcal{N}^d\rvert$	$\lvert\mathcal{N}^o\rvert$	$\lvert\Omega\rvert$
2	31	62	32
3	121	263	243
4	341	1364	1024
5	781	3905	3125
6	1555	9330	7776

WP and DP capacities are uniformly generated with respect to the demand level of the problem instance in the unit intervals $[3500, 6000]$ and $[1200, 2200]$, respectively. The truck-load capacities between WPs and DPs are estimated in the interval $[700, 1000]$. The fixed DPs costs are generated, respectively in the ranges $f^s_{lt} \in [18000, 35000]$ and $f^u_{lt} \in [1300, 2500]$, proportionally to its maximum capacity. An inflation factor is also considered to reflect the increase of the cost of capital on a periodic basis with $r = 0.005$. The transportation costs between the network nodes correspond to the Euclidean distances, multiplied by a unit load cost per distance unit and the inflation factor r. The unit load cost per distance unit is different in each echelon to reflect the different loading factors. The external delivery cost c_{jt} is calibrated to be higher than internal distribution costs with $c_{jt} \in [2000; 4000]$. As for the rolling horizon approach, we test three sub-horizon lengths $T^{RH} \in \{2, 3, 4\}$ with a look-ahead size $H = 1, 2$ and 3, respectively.

Each instance is denoted by a problem size T-$|\mathcal{P}|/|\mathcal{L}|/|\mathcal{J}|$- and a number of scenarios $|\Omega|$. The next section presents an illustrative instance to compare the solution produced with our solution approach and solving the compact formulation directly using cplex. Then, in section 5.5.3, main computational results are reported and discussed.

5.5.2 Illustrative instance

In order to illustrate the solutions produced with our solution approach, we present the results for one instance of size 5-3/16/25 and three independent events at each stage. The problem instance includes three WPs (i.e., $|\mathcal{P}| = 3$), $|\mathcal{L}| = 16$ potential DPs and $|\mathcal{J}| = 25$ customers and $T = 5$ design stages. A scenario tree of $|\Omega| = 243$ scenarios are thus used.

Figure 5.5 depicts the optimal location decisions produced with the rolling horizon approach using a sub-horizon of size $T^{RH} = H + 1 = 2$ (i.e., a roll $r = 1$ and a look-ahead $H = 1$). For clarity purposes, we only draw few scenarios in the last level of the tree. Using a sub-horizon $T^{RH} = 2$, our approach proposes to open three DPs (2, 4 and 6 with capacity level 1) at the first stage ($t = 1$). These DPs are available at the first stage ($t = 1$) for all scenarios. Then, as more uncertainty is revealed, our multi-stage approach allows to adapt these decisions to the different scenarios under consideration. At stage $t = 4$, DP 4 with the capacity level 2 is opened according to the uncertainty realized. Afterwards, an additional DP 5 with the capacity level 1 is opened depending on scenarios revealed. For instance, looking at the 27 downmost scenarios at stage $t = 5$ in the scenario tree, we have 81 scenarios at the end of the time horizon. Different location decisions are observed in stage 5 and 4 yielding to three combinations for their respective descendant leaves over the five design stages where the vector in each stage t provides opened DPs position in that stage and the value 1 or 2 next to the DP number refer to the capacity level. The obtained combinations are: $\left(\begin{pmatrix}2^1\\4^1\\6^1\end{pmatrix} \quad () \quad () \quad () \quad ()\right)$, $\left(\begin{pmatrix}2^1\\4^1\\6^1\end{pmatrix} \quad () \quad () \quad \begin{pmatrix}4^2\end{pmatrix} \quad ()\right)$, and $\left(\begin{pmatrix}2^1\\4^1\\6^1\end{pmatrix} \quad () \quad () \quad () \quad \begin{pmatrix}5^1\end{pmatrix}\right)$ Therefore, under $T^{RH} = 2$, our approach suggests three different combinations of location decisions over all the considered scenarios. Further experiments with $T^{RH} = 3$ and $T^{RH} = 4$ (i.e., look-ahead period $H = 2$ and 3 respectively) provides different location decisions. They propose to open DPs 2^1, 6^1 and 7^1 with $T^{RH} = 3$, and DPs 2^1, 5^1 and 7^1 with $T^{RH} = 4$ at the first-stage, and no further opening decisions are proposed in latter stages. In fact using larger sub-horizons, more information are introduced in the optimization phase to offer a higher flexibility in the location decisions which justifies the difference observed in the obtained design.

Table 5.3: *Results for the illustrative case 5-3/16/25-243*

	$T^{RH}=2$		$T^{RH}=3$		$T^{RH}=4$		Compact formulation	
	v_{RH}	CPU(s)	v_{RH}	CPU(s)	v_{RH}	CPU(s)	Best bound	Gap(%)
5-3/16/25-243	184033	7	176391	100	176110	1122	176449	7.24*

*: 24 hours of run

Moreover, Table 5.3 provides the total expected cost v_{RH} obtained using $T^{RH} = 2$, 3 and 4 as well as the CPU time in seconds spent to solve each case. The results show that the total expected cost decreases as the sub-horizon length T^{RH} grows. This is in accordance with our approach as more information are involved with $T^{RH} = 3$ and 4. However, the computational increases to reach 1100 seconds (i.e., 18 minutes) with $T^{RH} = 4$ compared to 7 seconds with $T^{RH} = 2$. With $T^{RH} = 4$, a four-stage stochastic program is solved in each iteration, and thus more difficult to solve. When solving the compact formulation directly with Cplex, the optimality gap is 7.24% after 24 hours of run. In addition, the returned bound is greater than the bounds obtained with $T^{RH} = 3$ and 4. This points out the good quality of solutions from the rolling horizon approach obtained in reasonable time.

5.5.3 Preliminary results

In this section, we evaluate the performance of our approach and provide an analysis of the produced solutions based on some preliminary experiments.

To start, Tables 5.4 and 5.5 describe the average computational results for instances presented in Tables 5.1 and 5.2. We report the results from the rolling horizon approach with $T^{RH} = 1$, 2 and 3 in terms of the total expected cost v_{RH}, the computing time in seconds (i.e., CPU(s)) and the gap between v_{RH} and the optimal value $v^*_{Compact}$ obtained with the compact formulation solved directly with Cplex ($\frac{(v_{RH}-v^*_{Compact})}{v_{RH}}$). We also provide the computing time spent

Table 5.4: *Average results for 5-3/8/.-.*

$\lvert\mathcal{J}\rvert$	$\lvert\Omega\rvert$	$T^{RH}=2$			$T^{RH}=3$			$T^{RH}=4$			CPU(s) from Compact
		v_{RH}	CPU(s)	Gap(%)	v_{RH}	CPU(s)	Gap(%)	v_{RH}	CPU(s)	Gap(%)	
25	32	184927	1	1.74	181768	3.5	0.00	181768	13.5	0.00	1296
	243	182476.5	4	2.19	178566.5	22	0.00	178566.5	302	0.00	88591.5
	1024	180831.5	11		176444	116		176443	8388		-
	3125	182011.5	30		177237.5	505		177231	75480		-
	7776	181441	69		176649.5	2281		-	-		-
50	32	246706.5	1	0.00	246706.5	3.5	0.00	246706.5	7.5	0.00	118.5
	243	252194.5	5	0.23	252040.5	18	0.17	252040.5	57.5	0.17	23423.5
	1024	251984	17.5	0.17	251944	72	0.16	251944	347	0.16	135175.5
	3125	252906.5	51.5		252881.5	236.5		252093.5	5754		-
	7776	252522	124.5		252486	689.5		251669.5	69634		-

-: time limit of 48 hours

Table 5.5: *Average results for 5-3/16/.-.*

$\lvert\mathcal{J}\rvert$	$\lvert\Omega\rvert$	$T^{RH}=2$			$T^{RH}=3$			$T^{RH}=4$			CPU(s) from Compact
		v_{RH}	CPU(s)	Gap(%)	v_{RH}	CPU(s)	Gap(%)	v_{RH}	CPU(s)	Gap(%)	
25	32	193986.5	2	6.24	183780.5	19	0.65	182591	134	0.00	12236
	243	190629	7.5		179294	76		179153	1867.5		-
	1024	187736	23		176974.5	244.5		176914	34727		-
	3125	189373.5	63		178399	1131.5		-	-		-
	7776	189168	159		177942.5	5580.5		-	-		-
50	32	245760.5	2.5	2.01	242552	6.5	0.68	240910	16	0.00	1269.5
	243	245081	11		244893	37		244884	203		-
	1024	244812	40.5		244762.5	162		244755	3456		-
	3125	245869	106		245856.5	546.5		245850	109496		-
	7776	245416.5	260		245366	1887.5		-	-		-

-: time limit of 48 hours

at solving the compact formulation.

When inspecting the results, we observe that the total expected cost v_{RH} decrease as the rolling sub-horizon length T^{RH} grows. In fact, with a larger T^{RH}, more information about uncertainty are considered at each iteration of the rolling horizon approach. This yields tighter bounds for the original problem. Additionally, we compare the obtained expected values v_{RH} to the optimal value $v^*_{Compact}$ with the compact formulation. We see that the gap decreases as the rolling sub-horizon T^{RH} is extended. In several cases, the optimality is attained with $T^{RH} = 4$. Looking at instance 5-3/16/25- with 32 scenarios for example, the gap goes down from 6.24% with $T^{RH} = 2$ to 0.65% with $T^{RH} = 3$, and it reaches the optimality with $T^{RH} = 4$. Therefore, larger sub-horizon T^{RH} helps to get tighter bounds as more uncertainty is involved.

Nevertheless, the computing time augments drastically with T^{RH} and the problem size. We notice that the CPU reaches 30 hours in average with large instance up 5-3/16/50- and 3125 scenarios for $T^{RH} = 4$. The time limit of 48 hours is also attained in some instances sized 5-3/16/25- and 5-3/16/50- with 7776 scenarios using $T^{RH} = 4$. However, our approach makes only few minutes to solve the same size with $T^{RH} = 2$ and 3. Moreover, the tables indicate that the CPU(s) grows as the number of scenarios increases, clearly a further complexity. Our rolling horizon approach takes few minutes to solve small trees with 32 and 243 scenarios and needs several hours to solve large trees with up to 3125 and 7776 scenarios. Consequently, one should consider T^{RH} providing a best compromise between the running time and the quality of the solution. Thus, $T^{RH} = 3$ is considered satisfactory and is retained for the rest of experiments.

From the tables, we observe in some cases for this randomly generated set that instances with 5-3/./50- are easier to solve compared to instances with 5-3/./25-. This is due to the reduced combinatorial complexity in instances with 50 customers. In fact, with 25 customers, a maximum of four DPs are opened whereas with 50 customers, five to six DPs are opened among eight potential DPs. Hence, in the latter, the combinatorial number is drastically reduced.

Further, the obtained results in Tables 5.4 and 5.5 highlight the efficiency of the developed approach in considerably reducing the running time. The efficiency of the rolling horizon is significant: it returns good bounds (a gap of 2% to 6%) of the original problem in reasonable time and may reach the optimality in some cases. The approach is ten to thousands times faster than the compact formulation as in 5-3/8/50 with 1024 scenarios. We also observe that only small instances with relatively small scenario trees are optimally solved with the compact formulation. The model becomes intractable for larger instances, whereas our approach can solve these instances in few seconds/minutes. Therefore, Tables 5.4 and 5.5 confirms the huge discrepancy in our stochastic problem in terms of complexity and consequently running times. The results also emphasize the importance of the development of such rolling horizon approach to solve a large scale stochastic multi-period 2E-DDP.

Table 5.6: *Design decisions for 5-3/8/.-. with* $T^{RH} = 3$

$\lvert\mathcal{J}\rvert$	$\lvert\Omega\rvert$	Combination for the design vector
25	32	$\left(\begin{pmatrix}4^1\\6^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$
	243	$\left(\begin{pmatrix}4^1\\6^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$
	1024	$\left(\begin{pmatrix}4^1\\6^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$
	3125	$\left(\begin{pmatrix}4^1\\6^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$
	7776	$\left(\begin{pmatrix}4^1\\6^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$ $\left(\begin{pmatrix}4^1\\6^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ (2^1)\right)$
50	32	$\left(\begin{pmatrix}3^1\\4^1\\5^1\\6^1\\8^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$
	243	$\left(\begin{pmatrix}3^1\\4^1\\5^1\\6^1\\8^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$
	1024	$\left(\begin{pmatrix}3^1\\4^1\\5^1\\6^1\\8^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$
	3125	$\left(\begin{pmatrix}3^1\\4^1\\5^1\\6^1\\8^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$
	7776	$\left(\begin{pmatrix}3^1\\4^1\\5^1\\6^1\\8^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$

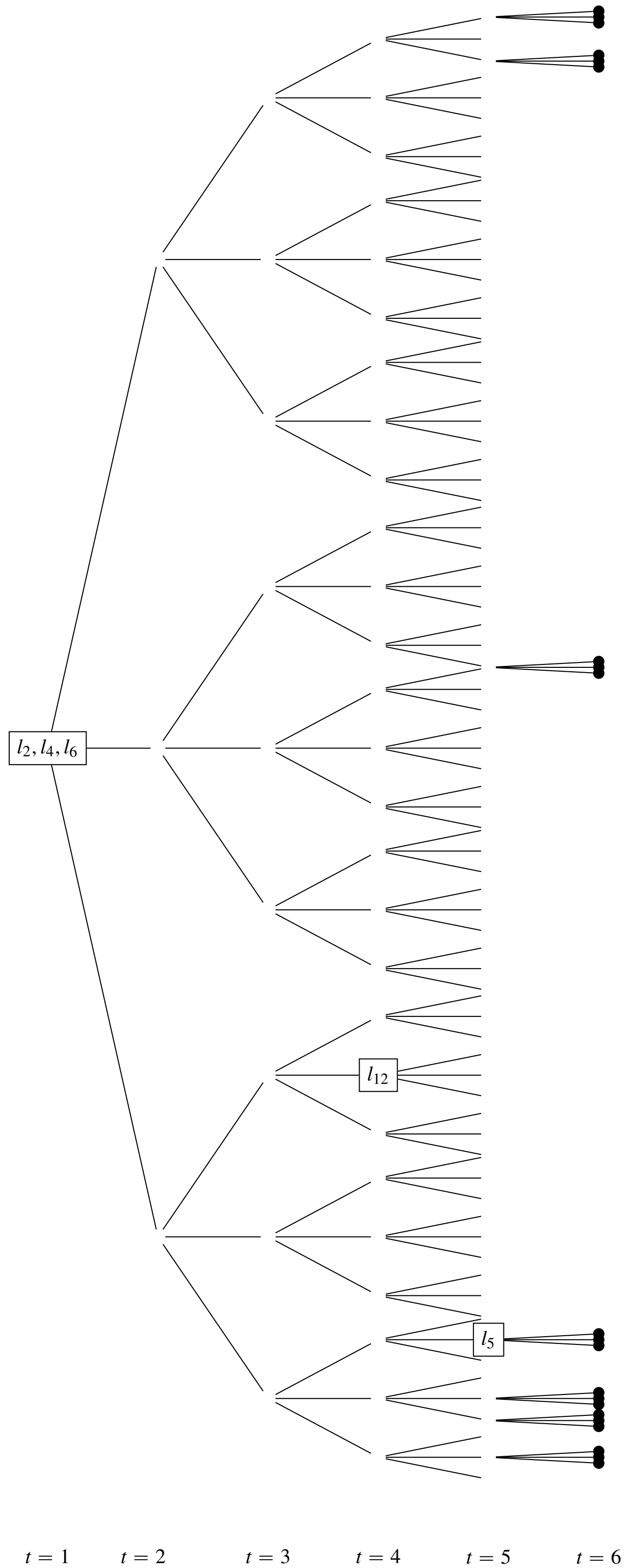


Figure 5.5: *The obtained design for 5-3/16/25-243 with $H = 1$*

Table 5.7: *Design decisions for 5-3/16/.-. with* $T^{RH} = 3$

$\lvert\mathcal{J}\rvert$	$\lvert\Omega\rvert$	Combination for the design vector			
25	32	$\left(\begin{pmatrix}2^1\\6^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$			
	243	$\left(\begin{pmatrix}2^1\\6^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$			
	1024	$\left(\begin{pmatrix}2^1\\5^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$	$\left(\begin{pmatrix}2^1\\5^1\\7^1\end{pmatrix}\ ()\ ()\ (4^1)\ ()\right)$	$\left(\begin{pmatrix}2^1\\5^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ (6^2)\right)$	$\left(\begin{pmatrix}2^1\\5^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ (4^1)\right)$
	3125	$\left(\begin{pmatrix}2^1\\5^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$	$\left(\begin{pmatrix}2^1\\5^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ (4^1)\right)$		
	7776	$\left(\begin{pmatrix}2^1\\5^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$	$\left(\begin{pmatrix}2^1\\5^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ (4^1)\right)$	$\left(\begin{pmatrix}2^1\\5^1\\7^1\end{pmatrix}\ ()\ ()\ ()\ (6^2)\right)$	
50	32	$\left(\begin{pmatrix}3^{1,2}\\4^1\\5^1\\8^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$			
	243	$\left(\begin{pmatrix}3^{1,2}\\4^1\\5^1\\8^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$			
	1024	$\left(\begin{pmatrix}3^{1,2}\\4^1\\5^1\\8^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$			
	3125	$\left(\begin{pmatrix}3^{1,2}\\4^1\\5^1\\8^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$			
	7776	$\left(\begin{pmatrix}3^{1,2}\\4^1\\5^1\\8^1\end{pmatrix}\ ()\ ()\ ()\ ()\right)$			

Next, we closely look at the obtained design decisions presented in Tables 5.6 and 5.7 for the problem sizes with 25 and 50 customers and the different scenario trees' size produced by the rolling horizon for $T^{RH} = 3$. In these tables, we illustrate the DP opening decisions and their capacity configuration, where value 1 or 2 next to a DP position corresponds to the capacity configuration of the opened platform. The first and second columns give the number of customers $|\mathcal{J}|$ and the number of scenarios $|\Omega|$, respectively. Then, the different combinations of design vectors over the planning horizon according to the revealed uncertainty are provided. First, we remark that the number of opening DPs grows with the number of customers: three DPs are opened at the first-stage with 25 customers versus five for instances with 50 customers. The results also depict the impact of considering two capacity levels. Different designs are observed between solutions with 8 and 16 DPs, as illustrated in Tables 5.6 and 5.7 respectively. In several instances 5-3/16/50-., the same DP position is opened with both capacity levels 1 and 2. Based on this, the consideration of different capacity configurations offers more distribution capabilities to deal with demand uncertainty.

Moreover, a key finding is the sensitivity of the network design in terms of the opened DPs and their location with respect to demand uncertainty. We can see that in many instances from this preliminary data set, the design decisions are fixed from the first-stage. However, in others, design decisions are adjusted over the stages according the realization of uncertainty offering thus a higher flexibility in the location decisions. The latter leads to different combinations of location decisions along the time horizon. This is in accordance with the multi-stage modeling approach in which design decisions are revised with respect to the uncertainty realization. In practice, only first-stage are implemented, and subsequent periods ($t > 1$) decisions will not be implemented and help essentially to the evaluation. This then confirms the rolling horizon basis.

5.6 Conclusion

In this paper, we introduce a comprehensive methodology for the stochastic multi-period two-echelon distribution network design problem (2E-DDP). In the 2E-DDP, products are transferred from warehouse platforms (WPs) to distribution platforms (DPs) before being transported to customers. The problem is characterized by a temporal hierarchy between the design level dealing with DP location decisions and DP throughput capacity based on transportation capabilities, and the operational level involving transportation decisions as origin-destination flows. A stochastic multi-period characterization of the planning horizon is considered, shaping the evolution of the uncertain and time-varying customers' demand and promoting the structural adaptability of the distribution network over time. Our emphasis here is on the network design decisions and their impact on the company's distribution performance.

The stochastic multi-period 2E-DDP is a multi-stage stochastic decisional framework as introduced in [27]. The first-period design decisions are made here-and-now. Subsequent stages are considered as structural adaptation decisions depending on the history up to that period, offering thus a new opportunity to adapt the network to its future environment based on the realization of the random demand at that time. A scenario tree is assumed to handle the set of scenarios representing demand uncertainty. Thus, a compact formulation of the multi-stage stochastic 2E-DDP is presented stressing the hierarchical structure of the problem.

To solve the multi-stage problem, we develop a rolling horizon approach based on the solution of a sequence of models defined on a reduced time horizon over the scenario tree. Our solution approach is motivated by the fact that in practice design decisions are made on a rolling horizon basis: only first-stage decisions are implemented, and subsequent stages are essentially used for an anticipation mechanism. The preliminary computational experiments based on a set of randomly generated instances validate our modeling approach proposed in this paper and show the efficiency of the solution approach. It provides good quality bounds for the multi-stage stochastic program in a reasonable time. To confirm our modeling approach, extensive experiments based on more data sets should be conducted. One should generate more instances using other scenario generation methods and demand processes to ensure that all plausible future realizations are covered. Additionally, one have to vary problem parameters in order to examine the sensitivity of design decisions and the performance of rolling horizon algorithm to different problem attributes.

Although the preliminary results show that our modeling approach seems to be promising to deal with multi-period 2E-DDPs under uncertainty, the performance of its solution methodologies is still limited for real-life and large-scale instances. From this perspective, alternative two-stage multi-period modeling approaches are proposed to capture the essence of the problem, while providing judicious accuracy-solvability trade-offs. These approaches are examined in depth in [27] yielding to two models based on two-stage stochastic modeling setting for the stochatic multi-period 2E-DDP. Further complex features have been addressed in [26] for the stochastic multi-period 2E-DDP. The location decisions of the two echelons (WPs and DPs) are questioned, and multi-drop routes visiting customers are considered, leading thus to a stochastic multi-period two-echelon capacitated location-routing problem. We solve these two-stage stochastic models using a Benders decomposition technique, and the size of the scenario set in these models is tuned using the sample average approximation (SAA) method [204]. These research applications point out the effectiveness of the stochastic and multi-period settings in 2E-DDPs. They also confirm our methodology to get judicious accuracy-solvability trade-offs.

We believe that our comprehensive framework for the 2E-DDP helps opening new good perspectives for the development of additional comprehensive models. Future works could develop efficient heuristic and exact solution methods capable to handle huge stochastic programs. Exploring hybrid solution methods that combines the rolling horizon approach with a decomposition method such as Benders decomposition algorithm may improve the computing time. In this case, the Benders decomposition technique may be used to solve the resulting model in each iteration of the rolling horizon approach. Additionally, it would be interesting to add performance measures on the quality of obtained solutions between the multi-stage and two-stage stochastic modeling approaches. Another interesting research direction would also consider the optimization of the objective function in a risk averse framework.

Chapter 6

Conclusions and perspectives

Distribution network design problems have drawn the attention of many researchers in the operations research literature over the last decades. This is due to their relevant real-life applications (ex: retail sector, city logistics, parcel and postal delivery) where the network structure and its locations impact the distribution activity performance, in addition to the computational challenges induced in their studies. These problems are looking to improve the efficiency of their distribution network in terms of the strategic location decisions and operational transportation schemes.

In the literature, distribution network design problems generally rely on a single echelon distribution structure. In addition, they implicitly assume that strategic and operational decisions are made simultaneously for the planning horizon. However, such structure may limit the network capabilities to meet today challenges and to offer a good delivery service level in terms of the delivery lead-time and the delivery destination.

A first contribution of this thesis has reviewed the operations research literature on this topic. The survey highlights some weaknesses that should be taken into account to design an efficient distribution network. These shortcomings are essentially related to ignoring the two-echelon distribution structure and the temporal hierarchy between the strategic and the operational level. In addition, omitting an advanced anticipation of future requirements and the integration of data uncertainty deeply affect the distribution activities' performance. We have considered these critical issues in the survey and explored them in the thesis.

To this end, we have proposed a comprehensive framework for the design of an efficient two-echelon distribution network under multi-period and stochastic settings where products are directed from warehouse platforms (WPs) to distribution platforms (DPs) before being transported to customers from DPs. The network should cope with the business changes over time and be efficiently adaptable to the uncertainty shaping the business horizon. For this purpose, our contributions to this topic are presented through three research papers as highlighted hereafter.

The first work introduces the comprehensive modeling approach for the stochastic multi-period two-echelon distribution network design problem. The problem is characterized by a temporal hierarchy between the design level dealing with DPs location decisions and capacity decisions, and the operational level involving transportation decisions as origin-destination flows. A stochastic multi-period characterization of the planning horizon is considered, shaping the evolution of the uncertain customer demand and time-varying demand and costs. The problem is initially formulated as multi-stage stochastic program. Then, given the strategic structure of the problem, approximate multi-period modeling approaches are proposed based on two-stage stochastic programming with recourse to capture the essence of the problem, while providing judicious accuracy-solvability trade-offs. The two models differ in the modeling of transportation decisions: the location and capacity-allocation model (LCA) in which DP location decisions and capacity decisions are first-stage decisions, and the flow-based location-allocation model (LAF) where capacity decisions are transformed into continuous scenario-dependent origin-destination links within the second-stage decisions. Next, we develop a solution method that integrates the Benders decomposition approach with the sample average approximation technique to produce efficient and robust designs. An extensive computational study validates the modeling approaches and the efficiency of the solution approach. The results also reveal some important managerial insights regarding the impact of uncertainty on the two-echelon distribution network design problem. The findings highlight a significant variability in the design decisions when several demand processes are involved. Moreover, the inspection of the stochastic solutions in respect to the deterministic solutions for both two-stage models confirm the positive impact of uncertainty. Additionally, the analysis of the two alternative models shows the high sensitivity of assignment-capacity decisions to uncertainty comparing to location decisions.

The second work is also carried out to validate our methodological framework. First, the two-echelon stochastic multi-period capacitated location-routing problem is defined. Given the temporal hierarchy between design and routing decisions, the problem is characterized as a hierarchical decision problem. Second, we have formulated the problem as two-stage stochastic program with recourse, in which the uncertainty is handled through a set of scenarios. The location and capacity decisions for both WPs and DPs are taken here-and-now for the set of design periods considered. Second-stage decisions consist in building daily routes that visit customers in the second echelon. As a third contribution to this study, we have introduced an exact Benders decomposition algorithm to solve the problem. In the first-stage, location and capacity assignment decisions are taken by solving the Benders master problem. When these first-stage decisions are fixed, the resulting subproblem is a capacitated vehicle-routing problem with capacitated multi-depot (CVRP-CMD) which is solved using the state-of-the-art branch-cut-and-price algorithm. Standard and combinatorial Benders cuts are also generated in order to converge to the optimal solution. The results emphasize the performance of our algorithm to optimally solve a large set of instances containing up to 50 customers and 25 demand scenarios under a 5-year planning horizon, and to provide good lower bounds for the instances that cannot be solved to optimality. Further analysis on the produced design illustrate how the stochastic and multi-period settings are adequately captured by the two-stage stochastic formulation. Moreover, the findings point out the effectiveness of the stochastic and multi-period modeling approach in comparison to the static model (i.e., no multi-period setting is considered for design decisions).

Lastly, we have been interested in the multi-stage modeling framework introduced in this thesis. It offers more flexibility to adapt the network its future environment as more information regarding the uncertainties is revealed in each time stage. A scenario tree is built to handle the set of scenarios representing demand uncertainty. We have presented a compact formulation for the multi-stage stochastic two-echelon distribution network design problem. Then, we have developed a heuristic method that can provide design solutions for the multi-stage program. Our solution method is a rolling horizon approach that solves a sequence of stochastic models defined on a reduced time horizon (sub-horizon), fixes the solution for the current stage and moves forward the optimization horizon. A complete solution for the original problem is progressively built by concatenating the decisions related to the first stage of a each reduced sub-horizon. In the preliminary experiments we have conducted, our algorithm confirms the good quality bounds obtained for the problem in reasonable times and validates our modeling approach.

In this thesis work, models and solution methods are investigated to approve our framework in designing an effective two-echelon distribution networks under uncertainty. They also show that our modeling approach is promising for dealing with multi-period two-echelon distribution network design problems. At the time of concluding this thesis, several perspectives are identified to improve this thesis work and to extend it to include additional comprehensive models.

From a methodological point of view, future works would be interested in the development of efficient solution methods that can handle the multi-stage modeling approach. The objective in Chapter 5 was not about developing the best performing solution method. However, we sought for an approach that allows us to compute bounds on the multi-stage problem and derive conclusive results. We may enhance the proposed algorithm in several ways. First, one would be interested in developing hybrid solution method that combines the rolling horizon approach with a decomposition method such as Benders decomposition algorithm. In this case, the Benders decomposition will be used to solve the multi-stage approximation at each iteration of rolling horizon algorithm, instead of commercial solvers such as Cplex. This hybrid solution algorithm will help to handle large-scale instances. The second enhancement could involve a backward scheme in the rolling horizon algorithm in which we move backwards in time. In our algorithm, we use a forward scheme over time. Combining forward and backward steps may improve the quality of obtained bounds. Moreover, it would be interesting to discuss the relevance of using a multi-stage stochastic models in contrast with the deterministic problem. Additionally, exploring the quality of bounds from two-stage stochastic problems and a multi-stage models could be of great interest.

As another research perspective of our work, we could focus on the development of further exact and heuristic solution methods to handle large-scale instances for the two- and multi-stage stochastic models proposed in this thesis. To the best of our knowledge, the Benders decomposition algorithm presented in Chapter 4 is the first exact method proposed to solve the two-echelon stochastic multi-period capacitated location-routing problem classes. Even though it provides good solutions, its performance is limited for large-scale instances. It might be worthwhile improving the considered algorithm to reduce the running time. One may introduce an initial good solution and an upper bound on the problem that are computed from some heuristic. Moreover, it has been observed that the CVRP-CMD cannot be solved in an hour for some instances with 50 customers. This points out the complexity of the CVRP-CMD instances in comparison with the uncapacitated multi-depot CVRP instances in the literature. These latter are all solved within the time limit. Thus, it would be interesting to propose new cuts adapted to the CVRP-CMD to strengthen its solution algorithm. Additional works could explore other methods such as local search based heuristics and hybrid methods developed for deterministic models to efficiently solve stochastic two-echelon distribution network design problems.

Furthermore, in this thesis, we have limited uncertainty to customer demand and used a scenario set sampled from Monte-Carlo procedure to model uncertainty. However, future requirements are getting higher uncertainty. Thus, a natural extension of our work would incorporate more uncertainty, specifically the strategic uncertainty that can affect fixed costs and capacities, for instance. Another extension of the work is to test other scenario generation and sampling methods to ensure that all plausible future realizations are covered by the generated scenario samples. Besides, the consideration of uncertain problem data with correlations may lead to better sampled scenarios.

An additional promising research direction is to discuss the proposed stochastic optimization models from a point of view of risk averse optimization. In stochastic optimization, the total objective cost is optimized on average over all the scenarios. But, for a particular realization of the data process, the costs could be much higher than its expected value. To this end, risk averse approaches aim at finding a compromise between minimizing the average cost and trying to control the upper limit of the costs for some possible realizations of the data set. Thus, it would be interesting to propose a modeling framework for such cases.

From a practical perspective, we could include more real-life constraints in the framework such as synchronization constraints at intermediate distribution platforms, and route length limit constraints. In the two-echelon distribution network design problem, the emphasis is on the design decisions and transportation decisions are used as operations anticipation for the operational level. Consequently, it would be interesting to propose route approximation formulæ to speed up the decision process. Moreover, the incorporation of further decisions such as transportation modes, and inventory may have an impact on the design decisions. This will lead to a more complex problem that should be modeled adequately.

Appendix A

Solution methodology

A.1 Sample Average Approximation (SAA)

Under a scenario-based optimization approach, building scenarios $\omega \in \Omega$ and assessing their probabilities $p(\omega)$ could entail a tremendous effort. Moreover, generating the adequate sample of scenarios $|\Omega|$, $\Omega = \cup_{t \in T} \Omega_t$, could be complex due to the large number of scenarios required under a high degree of uncertainty and the enumeration issue induced by continuous probability distributions [204]. The combination of the Monte Carlo sampling methods [202] and the SAA technique [204] helps in finding a good trade-off in terms of the scenarios' probability estimation and the sufficient number of scenarios to consider in the design model. The approach is to generate, outside the optimization procedure, an independent sample of N equiprobable scenarios $\{\omega^1, \ldots, \omega^N\} = \Omega_t^N \subset \Omega_t$ from the initial probability distribution, which also eliminates the need to explicitly compute the scenario probabilities $p(\omega)$. Then, the SAA program is built and the adequacy of the sample size validated by optimization. Hereafter, for instance, the SAA program related to the LCA model is provided, denoted with (LCA(Ω^N)):

$$
(\text{LCA}(\Omega^N)) \quad \mathcal{V}^N = \min \sum_{t \in \mathcal{T}} \sum_{l \in \mathcal{L}} (f_{lt}^u\, y_{lt} + f_{lt}^s\, z_{lt}) + \sum_{t \in \mathcal{T}} \sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}} c_{lpt}\, C_{lp} x_{lpt} + \frac{1}{N} \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega_t^N} \sum_{j \in \mathcal{J}} d_{j\omega t} [\sum_{l \in \mathcal{L}} c_{ljt}\, v_{lj\omega t} + c_{j\omega t}\, s_{j\omega t}] \tag{A.1}
$$

$$
\text{S. t.} \qquad (3.12) - (3.16)
$$

$$
\sum_j d_{j\omega t}\, v_{lj\omega t} \leqslant \sum_p C_{lp} x_{lpt} \qquad \forall l \in \mathcal{L}, t \in \mathcal{T}, \omega \in \Omega_t^N \tag{A.2}
$$

$$
\sum_l v_{jl\omega t} + s_{j\omega t} = 1 \qquad \forall j \in \mathcal{J}, t \in \mathcal{T}, \omega \in \Omega_t^N \tag{A.3}
$$

$$
v_{lj\omega t}, s_{j\omega t} \geqslant 0 \qquad \forall l \in \mathcal{L}, j \in \mathcal{J}, \omega \in \Omega_t^N, t \in \mathcal{T} \tag{A.4}
$$

It is well established that the optimal value $\mathcal{V}^N(o)$ of the optimal design vector $\widehat{X}^N(o)$ of each SAA program $o \in \{\text{LCA}(\Omega^N), \text{LAF}(\Omega^N)\}$ converges to optimality as the sample size increases [204]. This suggests that the quality of the SAA models improves as the size of the sample employed grows. However, one would in practice choose N, taking into account the trade-off between the quality of the obtained design for each SAA program o and the computational effort needed to solve it ([131]). In such case, solving the SAA program with M

independent samples repeatedly can be more insightful than increasing the sample size N. This leads to calculating a statistical optimality gap for each obtained design vector $\widehat{X}^N(o)$ by estimating the lower and upper bounds. The SAA-based procedure employed to calibrate the samples-size for both the LCA and LAF models is given in A.1.

Algorithm A.1 The SAA method to give model o

1. Generate M independent samples Ω_m^N, $m = 1, \ldots, M$ of N scenarios and solve the SAA program for each sample.
   Let $\mathcal{V}_m^N(o)$ and $\widehat{X}_m^N(o)$ be the corresponding optimal value and the optimal design solution, respectively.
2. Compute the statistical lower bound:

$$\bar{\mathcal{V}}_M^N(o) = \frac{1}{M} \sum_{m=1..M} \mathcal{V}_m^N(o)$$

3. Choose a feasible solution $\widehat{X}_m^N(o)$
4. Estimate the upper bound using a reference sample $N' \gg N$, $\mathcal{V}^{N'}(\widehat{X}_m^N(o))$
5. Compute an estimate of the statistical optimality gap of solution $\widehat{X}_m^N(o)$:

$$gap_M^{N,N'}(\widehat{X}_m^N(o)) = \mathcal{V}^{N'}(\widehat{X}_m^N(o)) - \bar{\mathcal{V}}_M^N(o)$$

   An estimate of the average gap for the sample of size N is given by:

$$gap^{N,N'}(o) = \frac{1}{M} \sum_{m=1..M} gap_M^{N,N'}(\widehat{X}_m^N(o))$$

6. **if** *acceptable*(gap) **then**
7.   **return** $gap^{N,N'}$
8. **else**
9.   Repeat step 1-5 using larger N and/or M.
10. **end if**

Worth noting is that the upper bound $\mathcal{V}^{N'}(\widehat{X}_m^N(o))$ proposed in step 4 for each model o is computed as follows: in the case of the (LCA(Ω^N)) model,

$$\mathcal{V}^{N'}(\widehat{X}_m^N(\text{LCA}(\Omega^N))) = \sum_{t\in\mathcal{T}} \sum_{l\in\mathcal{L}} (f_{lt}^u\, \hat{y}_{lt} + f_{lt}^s\, \hat{z}_{lt}) + \sum_{t\in\mathcal{T}} \sum_{l\in\mathcal{L}} \sum_{p\in\mathcal{P}} c_{lpt}\, C_{lp} \hat{x}_{lpt} + \frac{1}{N'} \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega_t^{N'}} Q_t(\hat{x}_m^N, \omega)$$

and in the case of (LAF(Ω^N)),

$$\mathcal{V}^{N'}(\widehat{X}_m^N(\text{LAF}(\Omega^N))) = \sum_{t\in\mathcal{T}} \sum_{l\in\mathcal{L}} (f_{lt}^u\, \hat{y}_{lt} + f_{lt}^s\, \hat{z}_{lt}) + \frac{1}{N'} \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega_t^{N'}} Q_t(\hat{y}_m^N, \omega)$$

A.2 The Benders reformulation for (LAF(Ω^N))

The primal sub-problem for each ω and t is:

$$(\mathrm{PS}_{\omega t}) \quad \phi_{t\omega}(y,z) = \min \sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}} c_{lpt}\, C_{lp} x_{lpt\omega} + \sum_j d_{j\omega t}[\sum_l c_{ljt}\, v_{lj\omega t} + c_{j\omega t}\, s_{j\omega t}] \tag{A.5}$$

$$\text{S. t.} \qquad \sum_{l\in\mathcal{L}} C_{lp} x_{lpt\omega} \leqslant C_p \qquad \forall p \in \mathcal{P} \tag{A.6}$$

$$\sum_{p\in\mathcal{P}} C_{lp} x_{lpt\omega} \leqslant C_l\, y_{lt} \qquad \forall l \in \mathcal{L} \tag{A.7}$$

$$\sum_j d_{j\omega t}\, v_{lj\omega t} - \sum_p C_{lp} x_{lpt\omega} \leqslant 0 \qquad \forall l \tag{A.8}$$

$$\sum_l v_{lj\omega t} + s_{j\omega t} = 1 \qquad \forall j \tag{A.9}$$

$$x_{lpt\omega} \geqslant 0 \qquad \forall l, p \tag{A.10}$$

$$v_{lj\omega t} \geqslant 0 \qquad \forall l, j \tag{A.11}$$

$$s_{j\omega t} \geqslant 0 \qquad \forall j \tag{A.12}$$

Its dual is :

$$(\mathrm{DS}_{\omega t}) \quad \phi_{t\omega}(y,z) = \max \sum_p C_p\theta_p + \sum_l C_l\, y_{lt}\, \gamma_l + \sum_j \beta_j \tag{A.13}$$

$$\text{S. t.} \qquad C_{lp}\theta_p + C_{lp}\gamma_l - C_{lp}\alpha_l \leqslant c_{lpt} C_{lp} \qquad \forall l, p \tag{A.14}$$

$$d_{j\omega t}\, \alpha_l + \sum_j \beta_j \leqslant d_{j\omega t}\, c_{ljt} \qquad \forall l, j \tag{A.15}$$

$$\beta_j \leqslant d_{j\omega t}\, c_{j\omega t} \qquad \forall j \tag{A.16}$$

$$\theta_p \leqslant 0 \qquad \forall p \tag{A.17}$$

$$\gamma_l, \alpha_l \leqslant 0 \qquad \forall l \tag{A.18}$$

$$\beta_j \in \mathbb{R} \qquad \forall j \tag{A.19}$$

Then, the master problem is as follows:

$$(\mathrm{MastP}) \min \sum_{lt} (f^u_{lt}\, y_{lt} + f^s_{lt}\, z_{lt}) + \frac{1}{N} \sum_{t\omega} u_{t\omega} \tag{A.20}$$

S. t. (3.14) and (3.16)

$$u_{t\omega} - \sum_l C_l\, y_{lt}\, \gamma_l \geqslant \sum_j \beta_j + \sum_p C_p\, \theta_p \qquad \forall t, \omega, (\theta_p, \gamma_l, \alpha_l, \beta_j) \in P_{\Delta_{t\omega}} \tag{A.21}$$

$$u_{tw} \geqslant 0 \qquad \forall t, \omega \tag{A.22}$$

A.3 The deterministic model (EV)

$$\text{(EV)}\min \sum_{l\in\mathcal{L}}\sum_{t\in\mathcal{T}}(f^{u}_{lt}\, y_{lt} + f^{s}_{lt}\, z_{lt}) + \sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}} c_{lpt}\, C_{lp}x_{lpt} + \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}}\mu_{jt}[\sum_{l\in\mathcal{L}} c_{ljt}\, v_{ljt} + c_{jt}\, s_{jt}] \tag{A.23}$$

$$\text{S. t.}\quad \sum_{l\in\mathcal{L}} C_{lp}x_{lpt} \leqslant C_p \qquad \forall p\in\mathcal{P}, t\in\mathcal{T} \tag{A.24}$$

$$\sum_{p\in\mathcal{P}} C_{lp}x_{lpt} \leqslant C_l\, y_{lt} \qquad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{A.25}$$

$$y_{lt} - y_{lt-1} \leqslant z_{lt} \qquad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{A.26}$$

$$\sum_{j}\mu_{jt}\, v_{ljt} \leqslant \sum_{p} C_{lp}x_{lpt} \qquad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{A.27}$$

$$\sum_{l} v_{jlt} + s_{jt} = 1 \qquad \forall j\in\mathcal{J}, t\in\mathcal{T} \tag{A.28}$$

$$x_{lpt}\in\mathbb{N} \qquad \forall l\in\mathcal{L}, p\in\mathcal{P}, t\in\mathcal{T} \tag{A.29}$$

$$y_{lt}, z_{lt}\in\{0,1\} \qquad \forall l\in\mathcal{L}, t\in\mathcal{T} \tag{A.30}$$

$$v_{ljt}\geqslant 0 \qquad \forall l\in\mathcal{L}, j\in\mathcal{J}, t\in\mathcal{T} \tag{A.31}$$

$$s_{jt}\geqslant 0 \qquad \forall j\in\mathcal{J}, t\in\mathcal{T} \tag{A.32}$$

A.4 Quality of the stochastic solution

To characterize stochastic solutions, several indicators can be computed [38, 149]. The first is the statistical optimality gap estimate of the best solution identified by the SAA method for different sample sizes N as described in subsection A.1. Furthermore, the value of the stochastic solution (VSS), the loss using the skeleton solution (LUSS), and the loss of upgrading the deterministic solution (LUDS) proposed in [149] are also assessed as indicators of the quality of the stochastic solutions produced. Let $X^{*}(o)$ be the optimal solution obtained from the model $o\in\{\text{LCA, LAF}\}$, the here and now solution, under all $\omega\in\Omega$, and let RP be the optimal value of the associated objective function. More specifically, RP is computed by (A.33). Under a sampling approach, $\Omega^{N}\subset\Omega$, $\widehat{X}(o)$ represents the near-optimal solution obtained from the SAA program $o\in\{\text{LCA}(\Omega^{N}), \text{LAF}(\Omega^{N})\}$. A good estimate of RP is given by (A.34).

$$RP = \mathbb{E}_{\Omega}[h(X^{*}(o),\Omega)] \tag{A.33}$$

$$\widehat{RP} = \mathcal{V}^{N} = \mathbb{E}_{\Omega^{N}}[h(\widehat{X}(o),\Omega^{N})] \tag{A.34}$$

A common approach in the literature is to consider the expected value problem, where all the random variables are replaced by their expected value $\bar{\omega} = \mathbb{E}(\omega)$, solving the deterministic model (EV) as given in (A.35). Let $\bar{X}(o)$ be the optimal solution to (A.35), referred to as the expected value solution. The evaluation of its expected value (EEV) is computed as given in (A.36). Thus, the estimate of the value of the stochastic solution VSS is defined by formula (A.37), and measures the expected gain from solving a stochastic model rather than its deterministic counterpart.

$$\begin{aligned} EV &= \min h(X(o), \bar{\omega}) && \text{(A.35)} \\ \widehat{EEV} &= \mathbb{E}_{\Omega^N}[h(\bar{X}(o), \Omega^N)] && \text{(A.36)} \\ \widehat{VSS} &= \widehat{EEV} - \widehat{RP} && \text{(A.37)} \end{aligned}$$

The expected value solution $\bar{X}(o)$ may behave very badly in a stochastic environment. We here propose investigating how the expected value solution relates to its stochastic counterpart. For this purpose, we compute the loss using the skeleton solution (LUSS) and the loss of upgrading the deterministic solution (LUDS), which provide deeper information than the VSS on the structure of the problem [149]. To calculate the LUSS, we fix at zero all first-stage variables, which are at zero in the expected value solution $\bar{X}(o)$, and solve the SAA stochastic program $o \in \{\text{LCA}(\Omega^N), \text{LAF}(\Omega^N)\}$ with these additional constraints. Considering the program (LCA(Ω^N)), the additional constraints are:

$$\begin{aligned} x_{lpt} &= 0 \qquad \forall l, p, t \in \mathcal{K}(\bar{x}, 0) && \text{(A.38)} \\ y_{lt} &= 0 \qquad \forall l, t \in \mathcal{K}(\bar{y}, 0) && \text{(A.39)} \\ z_{lt} &= 0 \qquad \forall l, t \in \mathcal{K}(\bar{z}, 0) && \text{(A.40)} \end{aligned}$$

where $\mathcal{K}(\bar{x}, 0)$, $\mathcal{K}(\bar{y}, 0)$ and $\mathcal{K}(\bar{z}, 0)$ are sets of indices for which the components of the expected value solution $\bar{X} = \{\bar{x}, \bar{y}, \bar{z}\}$ are at zero. In the (LAF(Ω^N)), only constraints (A.39)-(A.40) are added. The solution of this program gives the near-optimal solution $\check{X}(o)$. Thus, an estimate of the expected skeleton solution value (ESSV) is computed with (A.41). Comparing it to the $\widehat{RP}$ estimate leads to an estimate of the LUSS as computed in (A.42). This test allows investigating why a deterministic solution may behave badly. In the case of $\widehat{LUSS} = 0$, this corresponds to the perfect skeleton solution in which the stochastic solution takes the deterministic solution values. Otherwise, in case of $0 < \widehat{LUSS} \leqslant \widehat{VSS}$, it means that alternative decision variables are chosen for the expected solution, thus providing alternative solution values.

$$\begin{aligned} \widehat{ESSV} &= \mathbb{E}_{\Omega^N}[h(\check{X}(o), \Omega^N)] && \text{(A.41)} \\ \widehat{LUSS} &= \widehat{ESSV} - \widehat{RP} && \text{(A.42)} \end{aligned}$$

The last indicator considers the expected value solution $\bar{X}(o)$ as a starting point (input) to the stochastic program $o \in \{\text{LCA}(\Omega^N), \text{LAF}(\Omega^N)\}$, and compares, in terms of objective functions, the same program without such input. This helps test whether the expected value solution $\bar{X}(o)$ is upgradeable to become good (if not optimal) in the stochastic setting. To do so, one needs to solve the program o with additional constraints that ensure the expected value solution $\bar{X}(o)$ as a starting solution, thus obtaining the solution $\widetilde{X}(o)$. More specifically, for the (LCA(Ω^N)) program, additional constraints are:

$$x_{lpt} \geqslant \bar{x}_{lpt} \qquad \forall l, p, t \tag{A.43}$$
$$y_{lt} \geqslant \bar{y}_{lt} \qquad \forall l, t \tag{A.44}$$
$$z_{lt} \geqslant \bar{z}_{lt} \qquad \forall l, t \tag{A.45}$$

However, the (LAF(Ω^N)) program considers only constraints (A.44)-(A.45). The expected input value is then computed with with (A.41), and this value is used to provide an estimate of the LUDS with formula (A.41). Worth noting is that $\widehat{LUDS} = 0$ corresponds to perfect upgradability of the deterministic solution, whereas $0 < \widehat{LUDS} \leqslant \widehat{VSS}$ leads to no upgradability.

$$\widehat{EIV} = \mathbb{E}_{\Omega^N}[h(\widetilde{X}(o), \Omega^N)] \tag{A.46}$$
$$\widehat{LUDS} = \widehat{EIV} - \widehat{RP} \tag{A.47}$$

Appendix B

Further results For 2E-DDP

Table B.1: *Mean value and MSD deviations for $(LCA(\Omega^N))$ for problem sizes P4 and P5*

		P4-LL-500						P5-LL-500					
		NDT		NRT		CRT		NDT		NRT		CRT	
		% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD
Dis-	s1	**0**	**0**	**0**	**0**	0.14	42.15	0.34	67.77	0.03	**0**	0.02	15.18
	s2	**0**	**0**	**0**	**0**	**0**	**0**	0.01	1.27	**0**	9.16	0.02	8.30
	s3	**0**	**0**	**0**	**0**	0.14	42.15	0.05	31.11	**0**	9.16	**0**	**0**
	s4	**0**	**0**	**0**	**0**	1.20	78.42	0.07	30.91	0.03	0.05	0.02	15.18
	s5	**0**	**0**	**0**	**0**	**0**	**0**	**0**	12.45	**0**	9.16	0.02	15.18
	s6	**0**	**0**	**0**	**0**	0.14	42.15	**0**	**0**	**0**	9.05	0.02	15.18
Con-	s1	**0**	**0**	**0**	1.29	**0**	1.42	0.04	10.07	0.08	**0**	0.01	12.36
	s2	**0**	**0**	**0**	1.29	0.09	**0**	**0**	**0**	0.08	**0**	0.06	9.40
	s3	**0**	**0**	**0**	1.29	**0**	1.42	0.04	10.07	**0**	42.32	**0**	**0**
	s4	**0**	**0**	0.11	**0**	**0**	1.42	0.04	10.07	**0**	42.32	0.01	12.36
	s5	**0**	**0**	**0**	1.29	0.21	14.22	0.04	10.07	0.08	**0**	**0**	**0**
	s6	**0**	**0**	**0**	1.29	0.12	15.63	0.04	10.07	0.08	**0**	0.01	12.36

Table B.2: *Mean value and MSD deviations for (LCA(Ω^N)) for problem sizes P2 and P3*

		P2-LL-500						P3-LL-500					
		NDT		NRT		CRT		NDT		NRT		CRT	
		% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD	% $\mathcal{V}^e$	% MSD
Dis-	s1	**0**	4.71	0.07	3.74	0.28	**0**	**0**	1.09	0.15	7.62	0.48	29.32
	s2	0.05	**0**	0.02	1.38	0.03	3.39	**0**	1.09	0.03	**0**	**0**	1.47
	s3	0.13	14.77	**0**	**0**	**0**	4.56	**0**	1.09	0.01	10.19	0.01	**0**
	s4	0.01	3.61	0.02	1.38	0.04	6.70	0.04	**0**	0.01	12.06	0.01	**0**
	s5	0.04	9.18	**0**	**0**	0.12	2.31	**0**	1.09	**0**	2.77	0.01	**0**
	s6	0.06	7.01	0.09	3.26	0.09	2.56	**0**	1.09	0.22	20.56	0.01	**0**
Con-	s1	0.02	2.84	**0**	**0**	0.03	3.56	**0**	**0**	**0**	12.11	**0**	**0**
	s2	**0**	**0**	**0**	**0**	0.06	5.28	**0**	**0**	**0**	12.11	0.16	12.68
	s3	**0**	0.07	0.06	20.49	0.06	5.28	**0**	**0**	0.08	**0**	0.21	7.83
	s4	0.03	4.08	**0**	**0**	0.01	1.55	**0**	**0**	**0**	12.11	**0**	**0**
	s5	0.01	1.91	**0**	**0**	**0**	**0**	**0**	**0**	0.08	**0**	**0**	**0**
	s6	**0**	0.67	**0**	**0**	0.04	6.02	**0**	**0**	0.08	**0**	**0**	**0**

Table B.3: *Best location decisions for (LCA(Ω^N))*

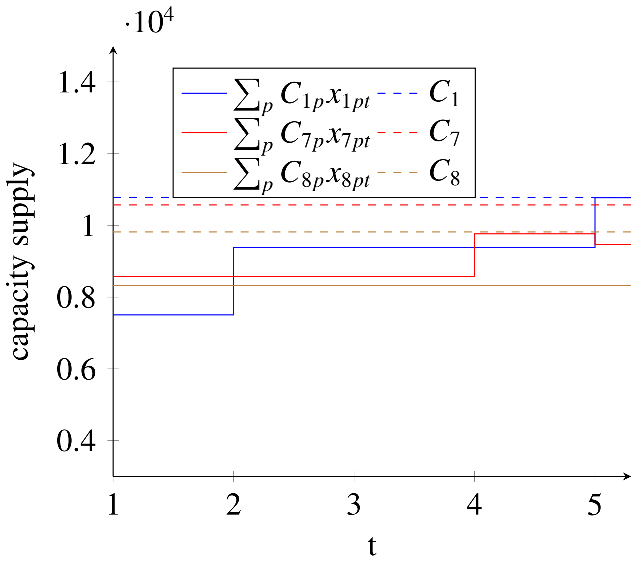
	Dispersed ship-to-points (Dis)																							
	NRT-500								**NDT-500**								**CRT-500**							
DPs	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**
P1-LL	1		1						1		1						1		1					
P1-HL	1		1						1		1						1		1					
P2-LL		1				1		1		1				1		1		1				1		1
P2-HL		1				1		1		1				1		1		1				1		1
P3-LL		2			1	1				2	1					1		2	2					1
P3-HL	1				1	1				2	1					1		2	2					1
	Concentric ship-to-points (Con)																							
P1-LL	1			1					1			1					1			1				
P1-HL	1			1					1			1					1			1				
P2-LL	1		1			1		1	1		1			1		1	1		1			1		1
P2-HL	1		1			1		1	1		1					1	1		1					1
P3-LL	2		1			1		1	2		1			2			2		1					1
P3-HL	2		2					1	1		1			2			1		1					1

1 and 2 refer to the Capacity configuration

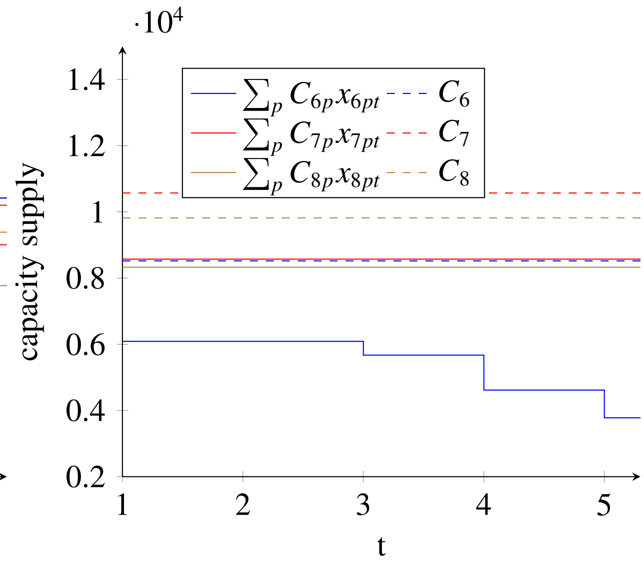
Table B.4: *Best location decisions for (LAF(Ω^N))*

	Dispersed ship-to-points (Dis)																							
	NRT-500								**NDT-500**								**CRT-500**							
DPs	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**	**l1**	**l2**	**l3**	**l4**	**l5**	**l6**	**l7**	**l8**
P1-LL	1		1						1		1						1		1					
P1-HL	1		1						1		1						1		1					
P2-LL		1				1		1		1				1		1		1				1		1
P2-HL		1				1		1		1				1		1		1				1		1
P3-LL		1	1					1		1	1					1		1	1					1
P3-HL		1	1					1						2		2		1	1					1
	Concentric ship-to-points (Con)																							
P1-LL	1			1					1			1					1			1				
P1-HL	1			1					1			1					1			1				
P2-LL	1		1					1	1		1			1			1		1					1
P2-HL	1		1					1	1		1			1			1		1					1
P3-LL	2		1					1	1		1			1			1		1					1
P3-HL	2		1					1	1		1			1			1		1					1

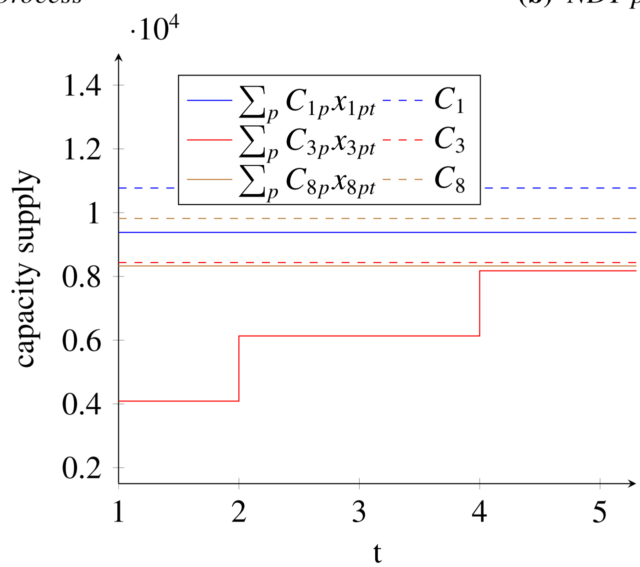
1 and 2 refer to the Capacity configuration



(a) *NRT process*

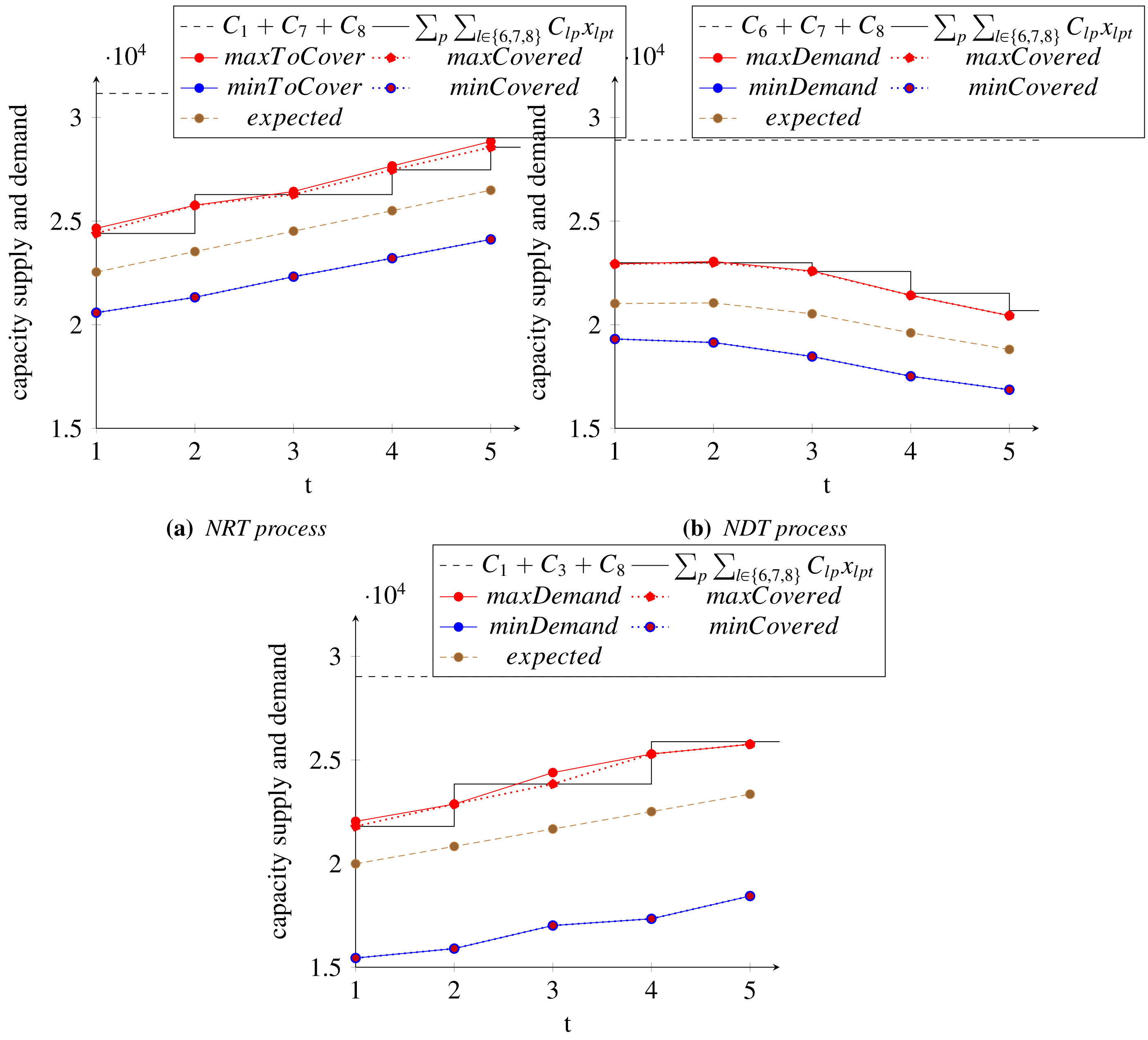


(b) *NDT process*



(c) *CRT process*

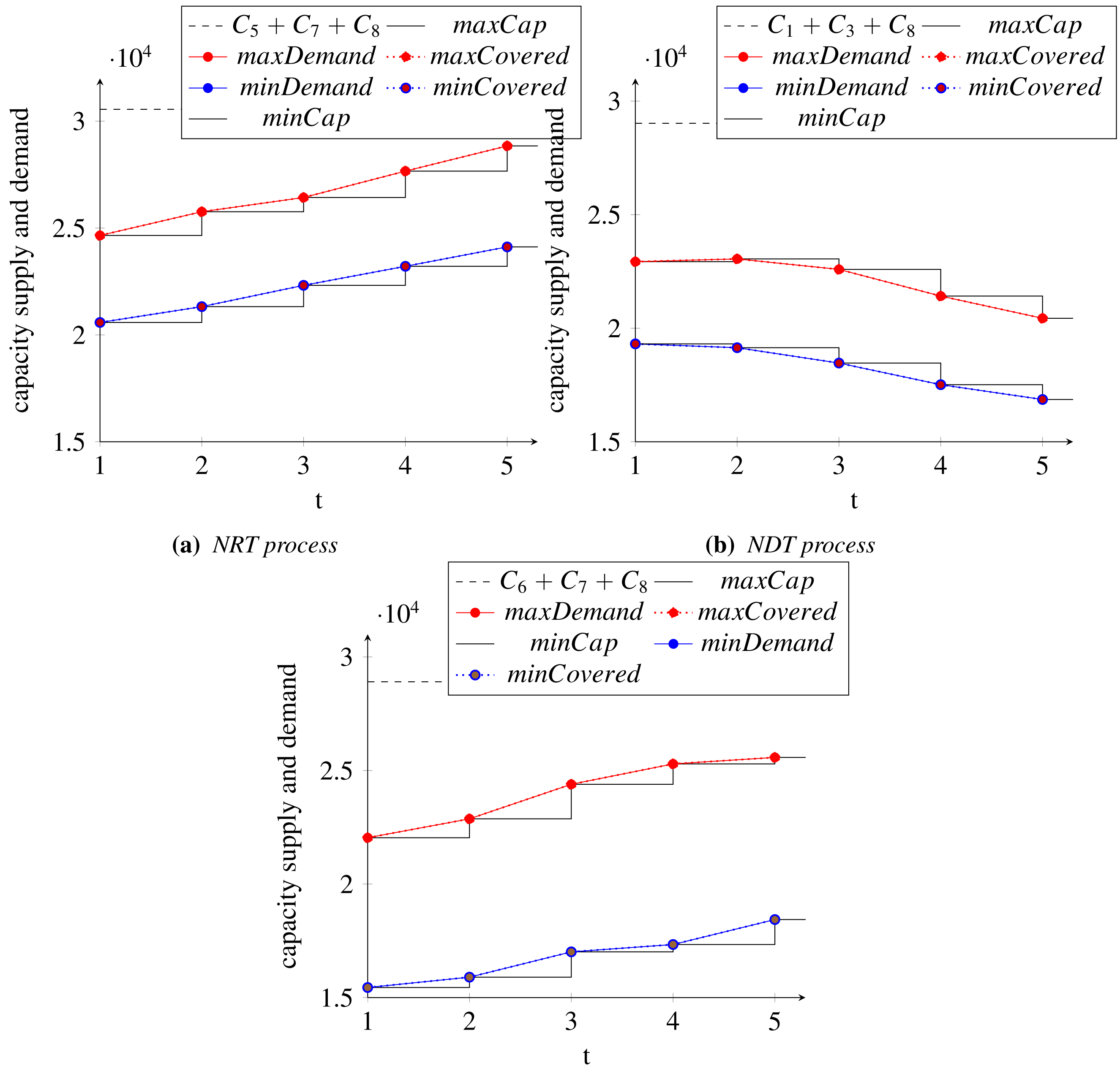
Figure B.1: *Capacity allocation decisions versus the a priori capacity for Con-P6-HL-500 in (LCA(Ω^N))*



(a) *NRT process* **(b)** *NDT process*

(c) *CRT process*

Figure B.2: *Capacity decisions versus demand for Con-P6-HL-500 in (LCA(Ω^N))*



(a) *NRT process*

(b) *NDT process*

(c) *CRT process*

Figure B.3: *Capacity decisions versus demand for Con-P6-HL-500 in (LAF(Ω^N))*

Appendix C

Bin packing problem

In this section, we give the description of the bin packing problem formulated for each period and each scenario, denoted as (BPP$_{t\omega}$), and present its mathematical formulation. Then, we briefly describe the column generation algorithm used to solve the linear relaxation of the (BPP$_{t\omega}$).

Consider a large set of bins (i.e. vehicles) with capacity q and a set of $|\mathcal{J}|$ items (i.e. customers) with weights $d_{j\omega t}$ to pack into bins. The objective is to find the minimum number of bins required to pack the set of items so that the capacity of the bins is not exceeded. Let $\mathcal{B}$ be the family of all the subsets of items which fit into one bin, i.e., the solutions to a subproblem. We define the parameter x_j^B that takes 1 if item $j \in \mathcal{J}$ is in set $B \in \mathcal{B}$. Let λ^B be the binary variable taking value 1 if the corresponding subset of items B is selected to fill one bin. The set covering reformulation is:

$$\min \sum_{B \in \mathcal{B}} \lambda^B \tag{C.1}$$

$$\text{S. t.} \qquad \sum_{B \in \mathcal{B}} x_j^B \lambda^B \geqslant 1 \qquad j = 1, ..., \mathcal{J} \tag{C.2}$$

$$\lambda_k \in \{0, 1\} \tag{C.3}$$

The linear relaxation of (C.1)-(C.3) is solved by column generation to provide a lower bound Γ. This lower bound is obtained by iteratively solving:

- the restricted master problem (RMP) which is the linear relaxation of (C.1)-(C.3) with a restricted number of variables;
- and the pricing problem which determines whether there exists a variable λ^B to be added to (RMP) in order to improve its current solution; this refers to solve a knapsack problem to get the set $B \in \mathcal{B}$, satisfying capacity constraints, and yielding to the minimum reduced cost column for (RMP).

Let π_j be the dual variable associated to constraints (C.2), the pricing problem for the (BPP$_{t\omega}$) is written as:

$$\max_{j=1..\mathcal{J}} \pi_j z_j \tag{C.4}$$

$$\text{S. t.} \quad \sum_{j=1..\mathcal{J}} d_{j\omega t} z_j \leqslant q \tag{C.5}$$

$$z_j \in \{0,1\} \qquad j \in \mathcal{J} \tag{C.6}$$

A column generated by solving the knapsack problem (C.4)-(C.6) will terminate column generation procedure in case its reduced cost $1 - \sum_j \pi_j z_j$ turns negative.

We apply the MIP solver *CPLEX* to the pricing formulation (C.4)-(C.6). Then, the linear relaxation of (C.1)-(C.3) is terminated by LP solver *CPLEX*. This leads to the lower bound Γ.

Appendix D

Further results for the 2E-SM-CLRP

Table D.1 provides detailed results of the sequential optimization approach.

Table D.1: *Average results under attributes (.,.,.,.) with sequential approach*

Instance	N	#Opt	#CombCuts	#OptCuts	#Iter	Gap(%)	Sequential computing time		
							MP	CVRP-CMD	total
5-4/8/15-	5	24/24	2	6.4	8.4	0.0	30s	2m	3m48s
	10	24/24	1.8	6.3	8.1	0.0	40s	3m33s	6m55s
	15	24/24	2	6.2	8.2	0.0	1m	3m40s	8m39s
	25	24/24	2	6.7	8.7	0.0	1m52s	5m29s	13m49s
5-4/12/15-	5	15/15	2	6.8	8.8	0.0	2m47s	4m6s	8m20s
	10	15/15	1.8	7.4	9.2	0.0	2m50s	6m	11m32s
	15	15/15	2	7.2	9.2	0.0	3m	11m	18m2s
	25	15/15	2	7.6	9.6	0.0	2m50s	13m38s	24m8s
5-4/16/15-	5	12/12	2.2	7.4	9.7	0.0	2m16s	7m	11m39s
	10	12/12	2.2	7.1	9.3	0.0	2m34s	6m41s	11m29s
	15	12/12	2.3	7.2	9.5	0.01	2m28s	10m7s	17m29s
	25	12/12	2	7	9	0.01	8m30s	22m39s	38m21s
5-4/8/20-	5	24/24	2.1	8.9	11	0.0	2m	33m	36m
	10	24/24	2	8.7	10.7	0.0	2m30s	55m54s	1h2m
	15	24/24	2	8.8	10.8	0.0	2m27s	1h38m	1h46m
	25	24/24	2.1	9	11.1	0.0	4m20s	1h45m	2h
5-4/12/20-	5	15/15	2	9.4	11.4	0.01	1m50s	35m32s	39m9s
	10	15/15	2.2	8.6	10.8	0.0	2m56s	1h19m	1h25m
	15	15/15	2.6	8.6	11.2	0.01	7m7s	1h57m	2h11m
	25	15/15	2	8.2	10.2	0.0	4m27s	1h19m	1h34m
5-4/16/20-	5	12/12	2	8.9	10.9	0.0	4m38s	22m32s	29m2s
	10	12/12	2.1	9.8	11.9	0.0	4m2s	1h2m	1h10m
	15	12/12	2.5	9.8	12.3	0.0	16m30s	2h26m	2h49m
	25	12/12	2.4	9.9	12.3	0.01	1h16m	3h58m	5h25m

Table D.2: *Detailed results for 5-4/12/15-25-, 5-4/12/20-25- and 5-4/12/50-15-*

		5-4/12/15-25-				5-4/12/20-25-				5-4/12/50-15-			
		ub	lb	Gap(%)	Time	ub	lb	Gap(%)	Time	ub	lb	Gap(%)	Time
(I2, LT, TC, NIT)	1	71146.4	71146.4	0.00	8m52s	76597.5	76597.5	0.00	1h50m	192216.0	192136.0	0.04	11h9m
	2	75246.1	75246.1	0.00	9m36s	90515.9	90489.4	0.03	13m5s	183250.0	183165.0	0.05	57h16m
	3	82505.5	82505.5	0.00	10m12s	84316.8	84316.8	0.00	1h2m	190410.9	190228.3	0.10	-
average		76299.3	76299.3	0.00	9m33s	83810.1	83801.2	0.01	1h2m	188625.6	188509.8	0.06	34h12m
(I1, LT, TC, NIT)	1	69862.2	69862.2	0.00	9m51s	79842.3	79842.3	0.00	17m30s	188827.0	188539.1	0.15	-
	2	75536.5	75536.5	0.00	9m39s	88416.3	88416.3	0.00	15m41s	182033.0	181944.0	0.05	5h36m
	3	76998.7	76998.7	0.00	9m48s	82822.0	82822.0	0.00	15m33s	188881.3	188686.4	0.10	-
average		74132.5	74132.5	0.00	9m46s	83693.5	83693.5	0.00	16m14s	186580.4	186389.8	0.10	5h36m
(I1, HT, TC, NIT)	1	98256.0	98256.0	0.00	9m35s	117079.0	117079.0	0.00	19m21s	275113.3	272759.1	0.86	-
	2	105654.0	105654.0	0.00	10m28s	126107.0	126098.0	0.01	14m36s	263067.0	262956.0	0.04	17h43m
	3	109252.0	109251.0	0.00	10m28s	119896.0	119896.0	0.00	23m55s	272392.7	269664.1	1.00	-
average		104387.3	104387.0	0.00	10m44s	121027.3	121024.3	0.00	19m17s	270191.0	268459.7	0.63	17h43m
(I1, LT, LC, NIT)	1	85410.0	85410.0	0.00	10m1s	92902.8	92902.8	0.00	13m31s	190269.0	190259.0	0.01	36m44s
	2	85285.8	85285.8	0.00	9m19s	93485.5	93485.5	0.00	13m23s	180619.0	180619.0	0.00	34m56s
	3	83886.4	83886.4	0.00	9m42s	89401.7	89401.7	0.00	13m15s	190580.0	190551.0	0.02	36m55s
average		84860.7	84860.7	0.00	9m40s	91930.0	91930.0	0.00	13m23s	187156.0	187143.0	0.01	36m12s
(I1, LT, TC, NVT)	1	71174.0	71174.0	0.00	9m31s	96847.2	96847.2	0.00	13m32s	181685.2	181146.7	0.30	-
	2	56085.6	56085.6	0.00	9m4s	92481.1	92481.1	0.00	20m45s	214547.0	214475.0	0.03	36h21m
	3	77253.9	77253.9	0.00	9m47s	92953.2	92953.2	0.00	23m41s	194353.6	194160.2	0.10	-
average		68171.2	68171.2	0.00	9m27s	94093.8	94093.8	0.00	19m19s	196861.9	196594.0	0.14	36h21m

Table D.3: *Detailed results for 5-4/16/15-25-, 5-4/16/20-25- and 5-4/16/50-15-*

		5-4/16/15-25-				5-4/16/20-25-				5-4/16/50-15-			
		ub	lb	Gap(%)	Time	ub	lb	Gap(%)	Time	ub	lb	Gap(%)	Time
(I1, LT, TC, NIT)	1	68308.6	68308.6	0.00	10m31s	77457.4	77457.4	0.00	14m40s	176344.0	176273.0	0.04	4h10m
	2	68268.8	68268.8	0.00	9m31s	83721.4	83721.4	0.00	45m22s	174587.0	174570.0	0.01	6h30m
	3	69865.8	69865.8	0.00	9m43s	84059.2	84059.2	0.00	31m36s	182674.0	182632.0	0.02	19h38m
average		68814.4	68814.4	0.00	9m55s	81746.0	81746.0	0.00	30m32s	177868.3	177825.0	0.02	10h6m
(I1, HT, TC, NIT)	1	97063.5	97063.5	0.00	10m47s	115242.0	115242.0	0.00	14m35s	261513.0	261384.0	0.05	6h28m
	2	97640.0	97640.0	0.00	11m3s	122079.0	122025.0	0.04	1h55m	254798.0	254725.0	0.03	29h1m
	3	102520.0	102491.0	0.03	9m57s	120723.0	120723.0	0.00	48m17s	267690.8	265847.9	0.69	-
average		99074.5	99064.8	0.01	10m35s	119348.0	119330.0	0.01	59m22s	261333.9	260652.3	0.26	17h44m
(I1, LT, LC, NIT)	1	84768.3	84768.3	0.00	9m16s	92933.6	92933.6	0.00	14m26s	187444.0	187444.0	0.00	43m30s
	2	68358.8	68358.8	0.00	9m18s	79351.7	79351.7	0.00	15m57s	151987.0	151987.0	0.00	39m28s
	3	69606.2	69606.2	0.00	10m12s	75279.5	75279.5	0.00	22m14s	158576.0	158522.0	0.03	52m5s
average		74244.4	74244.4	0.00	9m35s	82521.6	82521.6	0.00	17m32s	166002.3	165984.3	0.01	45m1s
(I1, LT, TC, NVT)	1	69772.4	69748.9	0.03	9m10s	93171.1	93171.1	0.00	13m58s	169987.0	169949.0	0.02	11h46m
	2	52241.9	52241.9	0.00	9m18s	86603.9	86560.8	0.01	18m1s	206768.8	206072.7	0.34	-
	3	69025.6	69025.6	0.00	9m14s	85880.8	85880.8	0.00	16m55s	192274.7	191670.8	0.31	-
average		63680.0	63672.1	0.01	9m14s	90003.9	90003.9	0.00	16m18s	189676.8	189230.8	0.22	11h46m

Table D.4: *Location decisions and their operating periods for 5-4/./15-25-(.,.,.,.)-3*

	(I1,LT,TC,NIT)			(I1,HT,TC,NIT)			(I1,LT,TC,NVT)			(I1,LT,LC,NIT)			Presence (%)
$\vert\mathcal{L}\vert$	8	12	16	8	12	16	8	12	16	8	12	16	
l1	0	0	0	0	0	0	0	0	0	0	0	0	0
l2	0	0	0	0	0	0	0	0	0	0	0	0	7
l3	0	0	0	0	0	0	0	0	0	0	0	0	0
l4	1	0	1	1	0	1	1	0	1	0	0	0	50
l5	0	0	0	0	0	0	0	0	0	1	0	0	7
l6	3	0	0	3	0	0	1	0	0	0	0	0	36
l7	0	4	0	0	4	0	0	2	0	0	1	0	36
l8	0	0	0	0	0	0	0	0	0	0	0	0	0
l9		0	0		0	0		0	0		0	0	0
l10		0	0		0	0		0	0		0	0	0
l11		0	0		0	0		0	0		0	0	0
l12		1	0		1	0		1	0		0	0	14
l13			0			0			0			0	0
l14			0			0			0			1	7
l15			0			0			0			0	0
l16			3			3			2*			0	21
Closet at t=									4				

Table D.5: *Location decisions under static modeling approach for 5-4/./20-25-(.,.,.,.)-3*

	(I1,LL,TC,NIT)		(I1,HL,TC,NIT)		(I1,LL,TC,NVT)	
$\vert\mathcal{L}\vert$	8	12	8	12	8	12
l1	0	0	0	0	0	0
l2	0	0	0	0	1	0
l3	0	0	0	0	1	1
l4	1	0	1	0	0	0
l5	0	0	0	0	0	0
l6	1	0	1	0	0	0
l7	0	1	0	1	0	1
l8	0	0	0	0	0	0
l9		0		0		0
l10		0		0		0
l11		0		0		0
l12		1		1		0

Appendix E

Résumé en français

Design de réseaux de distribution à deux échelons sous incertitude

E.1 Introduction

L'émergence et l'expansion rapide du e-commerce ont un impact considérable sur la structure des activités d'entreposage et de distribution. Selon la fondation Ecommerce, les détaillants européens en ligne ont connu une augmentation de 14% en 2017 [112]. De même, les États-Unis enregistrent une croissance de 16% selon le bureau U.S. Census [68]. De plus, le développement du e-commerce stimule le passage à une économie à la demande. Ce changement affecte considérablement le schéma de distribution de plusieurs entreprises qui cherchent à continuer l'amélioration du temps de réponse aux clients tout en offrant efficacement leurs produits dans un environnement multi-canal. D'où, nous observons une augmentation de l'attente au niveau de service de livraison au cours de la dernière décennie: cette dernière est maintenant exprimée en heures plutôt qu'en jours [220]. A cette fin, plusieurs acteurs mondiaux du B-to-C, et notamment des entreprises opérant dans le secteur du commerce de détail comme Walmart, Carrefour, Amazon ou jd.com, ont récemment engagé une réingénierie soutenue de leurs réseaux de distribution. Ils ont favorisé une grande proximité pour les points de livraisons aux clients, en utilisant des magasins, des points de relais, entre autres, en plus de la livraison à domicile [83]. Cela a été fait sans réduire l'efficacité de leurs politiques de consolidation en matière d'entreposage et de transport. Lors de la localisation de leurs plateformes d'entrepôts, les entreprises ont suivi diverses règles d'optimisation allant de la centralisation et de la mutualisation des risques aux contraintes financières et dépendantes de l'approvisionnement. Par conséquent, la localisation et la structure des entrepôts sont des attributs essentiels pour un réseau de distribution, tout comme le transport, afin de répondre aux attentes des clients.

Dans cet esprit, disposer d'un réseau de distribution rentable avec une mission d'amélioration du niveau de service est une question stratégique pour les entreprises afin d'accroître leur compétitivité. Dans le réseau de distribution, les produits finis sont acheminés vers les clients finaux ou les zones agrégées à partir d'un ensemble de plateformes de stockage/d'entrepôt (warehouse platforms (WPs)). La localisation de ces WPs devrait être choisie de façon appropriée afin de répondre aux besoins opérationnels au fil du temps. Plus précisément, un tel problème consiste à décider de la structure du réseau et du schéma de transport. La première détermine le nombre d'échelons, les types de plateformes à chaque échelon, leur nombre et leurs localisations, où un échelon représente un niveau du réseau de distribution. Par conséquent, le design des réseaux de distribution implique à la fois des décisions stratégiques de localisation et des décisions opérationnelles de transport. De toute évidence, les décisions stratégiques ont un effet direct sur les coûts opérationels, ainsi que sur sa capacité à servir les clients [59, 99].

Les problèmes de design des réseaux de distribution (distribution network design problem (DDPs)) sont particulièrement importants. Ils ont attiré l'attention de nombreux chercheurs dans la littérature de la recherche opérationnelle (RO) au cours des dernières décennies. Dans [139], ils ont été classés en fonction du nombre d'échelons dans le réseau de distribution, des échelons dans lesquels les décisions de localisation sont prises et de l'option de transport utilisée dans chaque échelon. Par conséquent, le DDP implique plusieurs problèmes classiques de la RO, ainsi que des problèmes innovants basés sur les caractéristiques de modélisation des problèmes. Figure E.1 identifie cinq caractéristiques de modélisation qui affectent principalement le réseau de distribution. Ces caractéristiques sont notamment le transport, la demande de la clientèle, le nombre d'échelons de distribution, l'horizon de planification et l'incertitude.

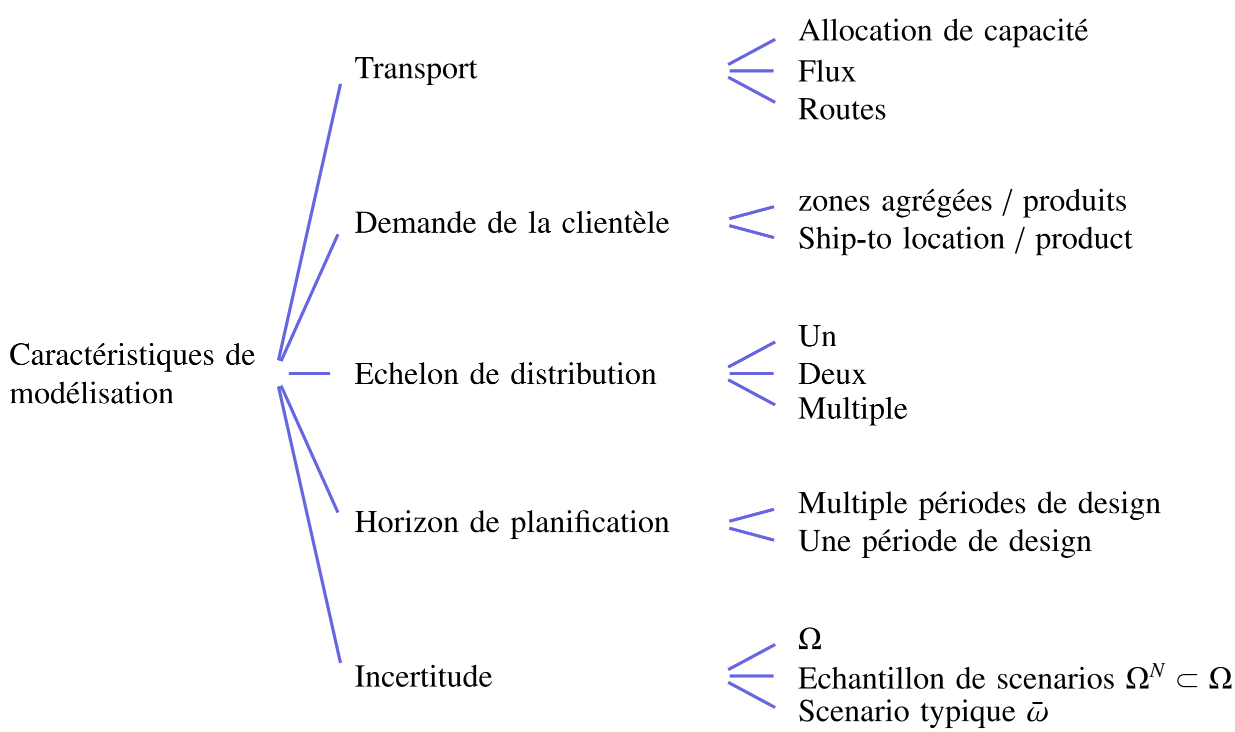


Figure E.1: *Les caractéristiques de modélisation*

En regardant la fonction de modélisation de transport située au niveau supérieur de la Figure E.1, nous pouvons distinguer entre l'allocation de capacité, les flux et les routes à plusieurs nœuds, où chaque option concerne un modèle bien étudié dans la RO. L'option d'affectation de capacité entraîne un problème de planification de la capacité et de localisation d'entrepôts dans lequel il faut décider la localisation des entrepôts et le niveau de capacité à allouer [147, 4]. Lorsque le transport est représenté par des flux origine-destination, un problème de localisation-affectation basé sur les flux est défini où un ensemble d'entrepôts devrait être choisi à partir d'un ensemble fini de sites potentiels et les clients sont livrés par flux directs à partir d'entrepôts sélectionnés au coût minimum [54, 15]. La troisième option de transport concerne les routes à plusieurs nœuds qui visitent plus d'un client par route. Il en résulte un problème de localisation-routage (Location-routing problem (LRP)). Il intègre des problèmes des tournées de véhicules (Vehicle-routing problem (VRP)) qui calculent un ensemble de routes à coût minimum pour répondre aux demandes des clients [63, 221], avec des problèmes de localisation des entrepôts [97, 65]. Son objectif est donc de trouver un nombre optimal d'entrepôts et leur localisation, tout en construisant des tournées autour d'eux pour servir les clients, simultanément [139]. De plus, ces options de transport influencent la modélisation des caractéristiques de la demande des clients en termes de zones ou de produits agrégés et de lieu de livraison unique.

D'autre part, la plupart des modèles DDP étudiés jusqu'à présent considèrent une structure de distribution à un seul échelon où le réseau comprend un ensemble de WPs et de clients. Néanmoins, avec la croissance du e-commerce et l'augmentation continue de la population des villes [70] contrastée avec des niveaux croissants de congestion, ces réseaux à un seul échelon limitent la capacité des entreprises à fournir des services de livraison rapide et réduisent leurs possibilités de relever les défis actuels : ils ne sont pas spécifiquement optimisés pour fournir des livraisons le lendemain et/ou le jour même, ou pour opérer efficacement et rapidement des services de traitement des commandes en ligne. Dans ce nouveau contexte, les considérations stratégiques impliquent un schéma de distribution à plus d'un échelon pouvant être ajusté de manière dynamique aux besoins de l'entreprise au fil du temps, comme mentionné au troisième niveau de la Figure E.1. Les praticiens accordent de plus en plus d'attention aux structures de distribution à deux échelons. La topologie du réseau comprend un échelon intermédiaire de plateformes de distribution / fulfillment (distribution platforms (DP)) situé entre les sites initiaux où les stocks sont tenus et les clients. Selon le Tompkins Supply Chain Consortium, plus de 25% des entreprises de détail adaptent leurs réseaux de distribution en ajoutant un nouvel échelon de DPs [220]. Par exemple, Walmart prévoit de convertir 12 magasins du Sam's Club en centres de fulfillment pour soutenir la croissance rapide du e-commerce [122]. Au Royaume-Uni, Amazon cherche à acquérir 42 magasins de Homebase pour étendre son réseau de centres de fulfillement et d'entrepôts [157]. En outre, cette structure de distribution à deux échelons couvre des modèles récents de logistique urbaine avec deux types de plateformes pour le cas de plusieurs entreprises partageant les plateformes [61, 160]. La distribution de courriers et de colis implique également une structure à deux échelons pour distribuer leurs produits, mais elle concerne des produits non substituables [236]. D'un point de vue méthodologique, plusieurs chercheurs ont récemment rappelé la nécessité d'étendre les réseaux à un seul échelon en considérant un échelon intermédiaire de plateformes où se déroulent des opérations de fusion, de consolidation ou de transbordement [215, 194]. Pour autant que nous le sachions, peu de papiers ont examiné ce type de réseaux de distribution. Quelques revues sont présentées dans [178, 62, 71]. Elles montrent que la structure de distribution à deux échelons est relativement inexplorée.

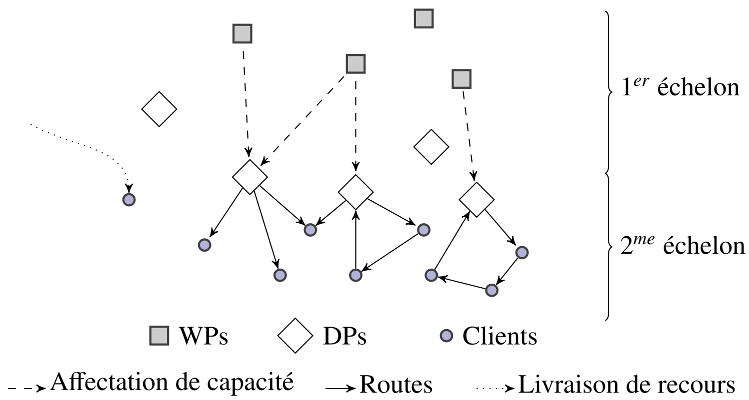


Figure E.2: *A generic Two-Echelon Distribution Network Design Problem (2E-DDP)*

De plus, les décisions stratégiques de design ont un effet durable dans le temps. Elles sont censées fonctionner efficacement sur une longue période et répondre aux futurs besoins de distribution et aux fluctuations des paramètres, comme le soulignent Klibi et al. [130]. La plupart des études ont considéré une seule période de design. Cependant, cela limite la capacité des décisions stratégiques de design à s'adapter facilement aux changements de l'environnement au fil du temps. Comme le montre la Figure E.1, l'horizon de planification peut être divisé en un ensemble de périodes de design caractérisant les possibilités futures d'adapter le design en fonction de l'évolution des besoins de l'entreprise. Les décisions de design doivent ensuite être planifiées sous la forme d'un ensemble de décisions séquentielles à mettre en œuvre à différentes périodes de design de l'horizon (un an, par exemple).

Enfin, on observe une tendance importante à réduire l'horizon de planification dans les études stratégiques. Selon le rapport de Tompkins (2011), la durée de la période de réingénierie définie dans les études de design de réseaux est passée en moyenne de 4 ans à moins de 2 ans en raison de l'incertitude croissante des entreprises et de la complexité des opérations de distribution [219]. Dans la littérature, les modèles sont généralement déterministes et s'appuient sur un seul scénario typique pour les paramètres du problème. Cependant, l'intégration de l'incertitude sous forme d'un ensemble de scénarios représentant des réalisations futures plausibles offrira un meilleur design (voir les options de l'incertitude dans la Figure E.1). L'incertitude peut concerner le niveau de la demande, les coûts des plateformes et les coûts de transport, etc. En plus de leur incertitude, les paramètres du problème varient dynamiquement au cours des périodes suivant une fonction de tendance. Par conséquent, la représentation traditionnelle déterministe-statique de l'horizon de planification doit être remplacée par une caractérisation plus réaliste, stochastique et multi-périodes, de l'horizon de planification.

Des études approfondies ont examiné les modèles de DDPs. Néanmoins, le nombre d'articles qui étudient conjointement les caractéristiques de modélisation pertinentes est très limité. Les quelques travaux proposés à cet égard n'examinent qu'un sous-ensemble de ces options et traitent, pour la plupart, une structure de distribution à un seul échelon, en omettant l'impact de la structure étendue à deux échelons. Ils supposent également que les décisions stratégiques et opérationnelles sont prises simultanément pour l'horizon de planification.

Notre objectif dans cette thèse est de souligner la nécessité de prendre en compte les enjeux susmentionnés lors du design d'un réseau de distribution efficace qui offre un ajustement plus dynamique des besoins de l'entreprise au fil du temps et qui fait face aux aléas des paramètres incertains. Pour ce but, nous introduisons un cadre global pour le problème stochastique et multi-période de design de réseau de distribution à deux échelons (Two-echelon distribution network design problem (2E-DDP)) avec une demande incertaine de la clientèle, et des demande et des coût variables dans le temps. Comme indiqué ci-dessus, la topologie 2E-DDP comprend un échelon avancé de DPs qui se situe entre les WPs et les clients. La figure E.2 illustre un 2E-DDP typique comprenant deux échelons de distribution avec capacité: chaque échelon implique un schéma spécifique de localisation-affectation-transport qui doit être adapté en fonction de l'incertitude qui impacte l'horizon.

Notre approche de modélisation dans cette thèse implique périodiquement, sur un ensemble de périodes de design, des décisions stratégiques concernant la localisation des plateformes et l'affectation de la capacité aux liens entre les WPs et les DPs afin de distribuer efficacement les produits aux clients. Ensuite, chaque jour, les décisions de transport sont prises en réponse aux commandes reçues des clients. Cette hiérarchie temporelle donne lieu à un problème de hiérarchie décisionnelle stratégique-opérationnelle et fait valoir la nécessité d'une caractérisation stochastique et multi-périodique de l'horizon de planification. De plus, notre horizon de planification permet d'adapter périodiquement les décisions de design à chaque période de design afin d'aligner le réseau de distribution à son environnement, notamment en cas d'incertitude. Par conséquent, le design du réseau de distribution à deux échelons dans un contexte multi-période et stochastique donne lieu à un problème décisionnel stochastique à plusieurs étapes.

Pour étudier cette approche de modélisation globale, plusieurs modèles sont proposés et discutés en termes de solvabilité. La qualité de la solution est examinée à l'aide d'une approche exacte basée sur la décomposition de Benders et une heuristique.

E.2 Design du réseau de distribution à deux échelons avec demande incertaine

Ce chapitre présente d'abord notre approche de modélisation globale pour le problème de design de réseau de distribution à deux échelons soumis à une demande incertaine, ainsi qu'une demande et des coûts variables dans le temps, formulée sous la forme d'un programme stochastique à plusieurs étapes. Ici, nous sommes intéressés aux 2E-DDPs qui traitent un contexte générique impliquant le déploiement du réseau de distribution pour un détaillant. Ainsi, le problème implique au niveau stratégique, sur un ensemble de périodes de design, des décisions sur le nombre et la localisation des DPs, ainsi que des décisions sur l'affectation des capacités pour calibrer la capacité de débit des DPs. Les décisions opérationnelles relatives au transport sont modélisées par des arcs origine-destination, qui correspondent à un agrégat suffisamment précis de décisions quotidiennes sur plusieurs produits, les moyens de transport et les périodes de travail.

Ensuite, compte tenu du caractère dimensionnel du problème stochastique à plusieurs étapes et de la complexité combinatoire du 2E-DDP, deux approches de modélisation alternatives sont proposées pour capturer l'essentiel du problème, tout en offrant des compromis judicieux en termes de précision et de solvabilité. Les deux modèles sont: le modèle stochastique à deux étapes de localisation et d'allocation de capacité (location and capacity-allocation model (LCA)), dans lequel les décisions de localisation des DPs et de capacité sont des décisions de première étape, et le modèle stochastique à deux étapes de localisation-allocation basé sur les flux (flow-based location-allocation model (LAF)), dans lequel les décisions de capacité sont transformées en liens origine-destination dépendant de scénarios déterminées dans la deuxième étape. Ces dernières sont de type continu.

Enfin, nous développons une approche de décomposition de Benders pour résoudre les modèles résultants. La taille adéquate de l'échantillon des scénarios est ajustée à l'aide de la technique d'approximation moyenne d'échantillon (SAA). Une procédure d'évaluation basée sur des scénarios est introduite pour évaluer la qualité des solutions de design. Les expérimentations numériques approfondies valident les modèles proposés et confirment l'efficacité des approches de solution. Ils illustrent également l'importance de l'incertitude dans le 2E-DDP. Les principales constatations mettent en évidence une variabilité importante dans les décisions de design en rapport avec l'incertitude de la modélisation des processus de la demande. En outre, l'analyse des deux modèles alternatifs montre la grande sensibilité des décisions relatives à l'affectation de capacité à l'incertitude par rapport aux décisions de localisation.

E.3 Approche de décomposition de Benders pour le problème multi-période stochastique de localisation-routage à deux échelons avec capacité

Dans ce chapitre, nous nous intéressons à la livraison du dernier kilomètre dans un contexte urbain où les décisions de transport dans le deuxième échelon sont caractérisées par des tournées de véhicules.

A cette fin, nous définissons le problème multi-période stochastique de localisation-routage à deux échelons avec capacité (Two-echelon stochastic multi-period capacitated location-routing problem (2E-SM-CLRP)). Le 2E-SM-CLRP, une variante hiérarchique, cherche à décider à chaque période de design l'ouverture, le maintien opérationnel et la fermeture des WPs et des DPs, ainsi que de la capacité allouée aux liaisons entre les deux types de plateformes. Au deuxième échelon, les tournées quotidiennes sont construites pour visiter les clients à l'aide d'un véhicule acheminé à partir d'un DP opérationnel. Un programme stochastique à deux étapes avec recours en nombre entier est introduit, qui repose sur un ensemble de scénarios multi-périodes générés avec une approche de Monte-Carlo.

Puis, nous nous basons sur l'approche de décomposition de Benders et sur le SAA pour résoudre des instances de taille réaliste pour le 2E-SM-CLRP. Les WPs et DPs opérationnels ainsi que les décisions de capacité sont fixés en résolvant le problème maître de Benders. Le sous-problème qui en résulte est un problème de tournées de véhicules multi-dépôts avec contrainte de capacité (capacitated vehicle-routing problem with capacitated multi-depot (CVRP-CMD)).

Ce dernier est formulé sous la forme d'un modèle de partitionnement, puis résolu par un algorithme de Branch-cut-and-price. Les coupes de Benders standards ainsi que les coupes de Benders combinatoires sont générées à partir des solutions de sous-problèmes afin de converger vers la solution optimale du 2E-SM-CLRP.

Les résultats numériques indiquent que notre méthode de solution est capable de résoudre de manière optimale un grand nombre d'instances, et d'obtenir de bonnes bornes inférieures sur des instances à grande échelle comprenant jusqu'à 50 clients et 25 scénarios de demande sur un horizon de planification de 5 ans. L'impact des attributs stochastiques et multi-périodes est également confirmé par rapport au modèle statique (c.-à-d. aucune modélisation multi-période n'est considérée pour les décisions de design: elles sont fixées au début de l'horizon et ne sont plus modifiées).

E.4 Approche d'horizon glissant pour le problème stochastique multi-étapes du design de réseau de distribution à deux échelons

Ce chapitre adresse le cadre multi-étapes proposé pour le problème de design de réseau de distribution à deux échelons sous une caractérisation stochastique multi-période de l'horizon de planification.

En utilisant un arbre de scénarios pour gérer l'ensemble des scénarios de demande, nous introduisons d'abord une formulation compacte pour le modèle de programmation stochastique à plusieurs étapes. Ensuite, une approche à horizon glissant est développée pour résoudre le problème. L'idée principale de l'algorithme est d'utiliser un sous-horizon réduit et fini à chaque itération, au lieu de l'horizon complet. Puis, un nouveau modèle multi-étapes est défini sur le sous-horizon et résolu. Il fixe la solution pour la période en cours et fait avancer l'horizon d'optimisation. Les décisions fixées sont introduites en tant que conditions initiales pour les itérations suivantes. Une solution complète pour le 2E-DDP stochastique multi-période est progressivement construite en concaténant les décisions relatives aux premières périodes de chaque sous-horizon réduit.

Les expériences numériques comparent notre méthode de résolution à la résolution directe de la formulation compacte par un solveur. Les résultats montrent l'efficacité de l'approche à horizon glissant pour fournir des bornes de bonne qualité dans un temps de calcul raisonnable, et confirment notre approche de modélisation à plusieurs étapes proposée dans cette thèse.

E.5 Conclusions & perspectives

Les problèmes de design des réseaux de distribution ont attiré l'attention de nombreux chercheurs dans la littérature de recherche opérationnelle au cours des dernières décennies. Ceci est dû à leurs applications réelles pertinentes (ex : secteur du commerce de détail, logistique urbaine, distribution de courriers et colis) où la structure du réseau et sa localisation ont un impact sur la performance de l'activité de distribution, en plus des défis informatiques induits dans leurs études. Ces problèmes cherchent à améliorer l'efficacité de leur réseau de distribution en termes de décisions stratégiques de localisation et de schémas de transport opérationnels.

Dans la littérature, les problèmes de design des réseaux de distribution reposent généralement sur une structure de distribution à un seul échelon. En outre, ils supposent implicitement que les décisions stratégiques et opérationnelles sont prises simultanément pour l'horizon de planification. Cependant, une telle structure peut limiter les capacités du réseau pour faire face aux défis actuels et pour offrir un bon niveau de service de livraison en termes de délai de livraison et de destination.

Une première contribution de cette thèse a passé en revue la littérature de recherche opérationnelle sur ce sujet. L'étude met en évidence certaines lacunes qu'il convient de les prendre en compte pour concevoir un réseau de distribution efficace. Ces lacunes sont essentiellement liées à l'ignorance de la structure de distribution à deux échelons et de la hiérarchie temporelle entre le niveau stratégique et le niveau opérationnel. En outre, le fait d'omettre une anticipation avancée des besoins futurs et l'intégration de l'incertitude des données affecte profondément la performance des activités de distribution. Nous avons examiné ces questions cruciales dans l'étude et les avons explorées dans la thèse.

À cette fin, nous avons proposé un cadre complet pour le design d'un réseau de distribution efficace à deux échelons, dans un contexte multi-période et stochastique, dans lequel les produits sont acheminés depuis des plateformes de stockage vers des plateformes de distribution (DPs) avant d'être acheminés vers les clients des DPs. Le réseau doit faire face aux variations opérationnelles au fil du temps et être efficacement adaptable à l'incertitude impactant l'horizon. À cet égard, nos contributions à ce sujet sont présentées à travers trois papiers de recherche, comme nous l'avons souligné dans les sections E.2, E.3 and E.4.

Dans cette thèse, des modèles et des méthodes de résolution sont étudiés pour approuver notre cadre pour le design de réseaux de distribution efficaces à deux échelons sous incertitude. Ils montrent également que notre approche de modélisation est prometteuse pour traiter les problèmes multi-périodes de design de réseau de distribution à deux échelons. De ce travail, plusieurs perspectives ont été identifiées pour améliorer ce travail de thèse et l'étendre à d'autres modèles compréhensifs.

D'un point de vue méthodologique, les travaux futurs seraient intéressés par le développement de méthodes de résolution efficaces pouvant gérer l'approche de modélisation à plusieurs étapes. L'objectif du chapitre 5 ne visait pas à développer la méthode la plus performante. Cependant, nous avons cherché une approche qui nous permet de calculer des bornes du problème à plusieurs étapes et d'obtenir des résultats concluants. Nous pouvons améliorer l'algorithme proposé de plusieurs manières. Tout d'abord, il serait intéressant de développer une méthode de solution hybride combinant l'approche à horizon glissant et une méthode de décomposition telle que la décomposition de Benders. Dans ce cas, la décomposition de Benders sera utilisée pour résoudre l'approximation à plusieurs étapes résultant à chaque itération de l'algorithme de l'horizon glissant, au lieu d'utiliser des solveurs commerciaux tels que Cplex. Cet algorithme de solution hybride aidera à gérer les instances à grande échelle. La deuxième amélioration pourrait impliquer un schéma en remontant dans l'algorithme de l'horizon glissant dans lequel nous nous déplaçons en remontant dans le temps. Dans notre algorithme, nous utilisons un schéma descendant dans le temps. La combinaison des schémas descendant et remontant peut améliorer la qualité des bornes obtenues. De plus, il serait intéressant de discuter la pertinence d'utiliser des modèles stochastiques à plusieurs étapes en contraste avec le problème déterministe. Aditionellement, explorer la qualité des bornes à partir de problèmes stochastiques à deux étapes et de modèles multi-étapes pourrait être d'un grand intérêt.

Comme autre perspective de recherche de notre travail, nous pourrions nous concentrer sur le développement de méthodes de solutions exactes et heuristiques supplémentaires pour gérer des instances à grande échelle pour les modèles stochastiques à deux et plusieurs étapes proposées dans cette thèse. A notre connaissance, l'algorithme de décomposition de Benders présenté dans le chapitre 4 est la première méthode exacte proposée pour résoudre les classes de problèmes stochastique multi-périodes de localisation-routage à deux échelons. Même s'il offre de bonnes solutions, ses performances sont limitées pour les instances à grande échelle. Il pourrait être intéressant d'améliorer l'algorithme considéré pour réduire le temps de calcul. On peut introduire une bonne solution initiale et une borne supérieure sur le problème, calculées à partir d'une heuristique. D'autre part, il a été observé que le CVRP-CMD ne peut pas être résolu en une heure dans certains cas avec 50 clients. Cela met en évidence la complexité des instances CVRP-CMD par rapport aux instances CVRP multi-dépôts sans contrainte de capacité traitées dans la littérature. Ces dernières sont toutes résolues dans le temps limite. Il serait donc intéressant de proposer de nouvelles coupes adaptées au CVRP-CMD pour renforcer leur algorithme de résolution. D'autres travaux pourraient explorer d'autres méthodes telles que les heuristiques basées sur la recherche de voisinage et les méthodes hybrides développées pour les modèles déterministes, afin de résoudre efficacement les problèmes stochastiques de design de réseau de distribution à deux échelons.

De plus, dans cette thèse, nous avons limité l'incertitude à la demande des clients et utilisé un jeu de scénarios échantillonné à l'aide de la procédure de Monte-Carlo pour modéliser l'incertitude. Cependant, les besoins futurs deviennent de plus en plus incertaines. Ainsi, une extension naturelle de nos travaux incorporerait plus d'incertitude, notamment l'incertitude stratégique pouvant affecter les coûts fixes et les capacités, par exemple. Une autre extension du travail consiste à tester d'autres méthodes de génération de scénario et d'échantillonnage pour s'assurer que toutes les réalisations futures plausibles sont couvertes par les échantillons de scénario générés. En outre, la prise en compte de données incertaines avec des corrélations peut conduire à des meilleurs échantillons de scénarios.

Une autre direction de recherche prometteuse consiste à examiner les modèles d'optimisation stochastique proposés du point de vue de l'optimisation du risque. En optimisation stochastique, le coût total dans la fonction objective est optimisé en moyenne sur tous les scénarios. Mais, pour une réalisation particulière du processus de données, les coûts pourraient être beaucoup plus élevés que sa valeur moyenne. À cette fin, les approches d'aversion au risque visent à trouver un compromis entre la minimisation du coût moyen et la maîtrise de la limite supérieure des coûts pour certaines réalisations de l'ensemble des données. Il serait donc intéressant de proposer un cadre de modélisation pour de tels cas.

D'un point de vue pratique, nous pourrions inclure davantage de contraintes réelles dans notre cadre, telles que des contraintes de synchronisation au niveau des plateformes de distribution intermédiaires et des contraintes sur la longueur des routes. Dans le problème de design du réseau de distribution à deux échelons, l'accent est plutôt sur les décisions de design et les décisions de transport sont utilisées comme anticipation des opérations pour le niveau opérationnel. En conséquence, il serait intéressant de proposer des formules d'approximation des routes pour accélérer le processus de décision. De plus, l'intégration d'autres décisions, comme les modes de transport et la gestion du stock, peut avoir une incidence sur les décisions de design. Cela conduira à un problème plus complexe qui devrait être modélisé adéquatement.

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Designing Two-Echelon Distribution Networks under Uncertainty

Keywords: two-echelon distribution network design problems, stochastic optimization, Benders decomposition

Abstract:

With the high growth of e-commerce and the continuous increase in cities population contrasted with the rising levels of congestion, distribution schemes need to deploy additional echelons to offer more dynamic adjustment to the requirement of the business over time and to cope with all the random factors. In this context, a two-echelon distribution network is nowadays investigated by the practitioners.

In this thesis, we first present a global survey on distribution network design problems and point out many critical modeling features, namely the two-echelon structure, the multi-period setting, the uncertainty and solution approaches. The aim, here, is to propose a comprehensive framework for the design of an efficient two-echelon distribution network under multi-period and stochastic settings in which products are directed from warehouse platforms (WPs) to distribution platforms (DPs) before being transported to customers. A temporal hierarchy characterizes the design level dealing with facility-location and capacity decisions over a set of design periods, while the operational level involves transportation decisions on a daily basis.

Then, we introduce the comprehensive framework for the two-echelon distribution network design problem under uncertain demand, and time-varying demand and cost, formulated as a multi-stage stochastic program. This work looks at a generic case for the deployment of a retailer's distribution network. Thus, the problem involves, at the strategic level, decisions on the number and location of DPs along the set of design periods as well as decisions on the capacity assignment to calibrate DP throughput capacity. The operational decisions related to transportation are modeled as origin-destination arcs. Subsequently, we propose alternative modeling approaches based on two-stage stochastic programming with recourse, and solve the resulting models using a Benders decomposition approach integrated with a sample average approximation (SAA) technique.

Next, we are interested in the last-mile delivery in an urban context where transportation decisions involved in the second echelon are addressed through multi-drop routes. A two-echelon stochastic multi-period capacitated location-routing problem (2E-SM-CLRP) is defined in which facility-location decisions concern both WPs and DPs. We model the problem using a two-stage stochastic program with integer recourse. To solve the 2E-SM-CLRP, we develop a Benders decomposition algorithm. The location and capacity decisions are fixed from the solution of the Benders master problem. The resulting subproblem is a capacitated vehicle-routing problem with capacitated multi-depot (CVRP-CMD) and is solved using a branch-cut-and-price algorithm.

Finally, we focus on the multi-stage framework proposed for the stochastic multi-period two-echelon distribution network design problem and evaluate its tractability. A scenario tree is built to handle the set of scenarios representing demand uncertainty. We present a compact formulation and develop a rolling horizon heuristic to produce design solutions for the multi-stage model. It provides good quality bounds in a reasonable computational times.

Design de réseaux de distribution à deux échelons sous incertitude

Mots clefs : Design de réseaux de disribution à deux échelons, optimisation dans l'incertain, décomposition de Benders

Résumé :

Avec la forte croissance du e-commerce et l'augmentation continue de la population des villes impliquant des niveaux de congestion plus élevés, les réseaux de distribution doivent déployer des échelons supplémentaires pour offrir un ajustement dynamique aux besoins des entreprises au cours du temps et faire face aux aléas affectant l'activité de distribution. Dans ce contexte, les praticiens s'intéressent aux réseaux de distribution à deux échelons.

Dans cette thèse, nous commençons par présenter une revue complète des problèmes de design des réseaux de distribution et souligner des caractéristiques essentielles de modélisation. Ces aspects impliquent la structure à deux échelons, l'aspect multi-période, l'incertitude et les méthodes de résolution. Notre objectif est donc, d'élaborer un cadre complet pour le design d'un réseau de distribution efficace à deux échelons, sous incertitude et multi-périodicité, dans lequel les produits sont acheminés depuis les plateformes de stockage (WP) vers les plateformes de distribution (DP) avant d'être transportés vers les clients. Ce cadre est caractérisé par une hiérarchie temporelle entre le niveau de design impliquant des décisions relatives à la localisation des plateformes et à la capacité allouée aux DPs sur une échelle de temps annuelle, et le niveau opérationnel concernant des décisions journalières de transport.

Dans une première étude, nous introduisons le cadre complet pour le problème de design de réseaux de distribution à deux échelons avec une demande incertaine, une demande et un coût variables dans le temps. Le problème est formulé comme un programme stochastique à plusieurs étapes. Il implique au niveau stratégique des décisions de localisation des DPs ainsi que des décisions d'affectation des capacités aux DPs sur plusieurs périodes de design, et au niveau opérationnel des décisions de transport sous forme d'arcs origine-destination. Ensuite, nous proposons deux modèles alternatifs basés sur la programmation stochastique à deux étapes avec recours, et les résolvons par une approche de décomposition de Benders intégrée à une technique d'approximation moyenne d'échantillon (SAA).

Par la suite, nous nous intéressons à la livraison du dernier kilomètre dans un contexte urbain où les décisions de transport dans le deuxième échelon sont caractérisées par des tournées de véhicules. Un problème multi-période stochastique de localisation-routage à deux échelons avec capacité (2E-SM-CLRP) est défini, dans lequel les décisions de localisation concernent les WPs et les DPs. Le modèle est un programme stochastique à deux étapes avec recours en nombre entier. Nous développons un algorithme de décomposition de Benders. Les décisions de localisation et de capacité sont déterminées par la solution du problème maître de Benders. Le sous-problème résultant est un problème multi-dépôt de tournées de véhicule avec des dépôts et véhicules capacitaires qui est résolu par un algorithme de branch-cut-and-price.

Enfin, nous étudions le cadre à plusieurs étapes proposé pour le problème stochastique multi-période de design de réseaux de distribution à deux échelons et évaluons sa tractabilité. Pour ceci, nous développons une heuristique à horizon glissant qui permet d'obtenir des bornes de bonne qualité et des solutions de design pour le modèle à plusieurs étapes.