



The Effectiveness of an Enrichment Program Using Dynamic Geometry Software in Developing Mathematically Gifted Students' Geometric Creativity in High Schools

Mohamed El-Sayed Ahmed El-Demerdash

► To cite this version:

Mohamed El-Sayed Ahmed El-Demerdash. The Effectiveness of an Enrichment Program Using Dynamic Geometry Software in Developing Mathematically Gifted Students' Geometric Creativity in High Schools. Humanities and Social Sciences. Pädagogischen Hochschule Schwäbisch Gmünd, 2010. English. <NNT: >. <tel-02065030>

HAL Id: tel-02065030

<https://hal.science/tel-02065030v1>

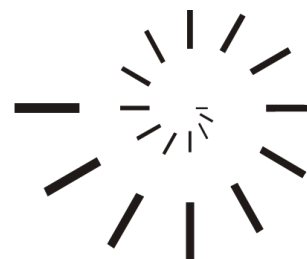
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THE EFFECTIVENESS OF AN ENRICHMENT PROGRAM USING DYNAMIC
GEOMETRY SOFTWARE IN DEVELOPING MATHEMATICALLY GIFTED
STUDENTS' GEOMETRIC CREATIVITY IN HIGH SCHOOLS

DISSERTATION

Submitted to the University of Education Schwäbisch Gmünd
for the degree of

DOCTOR OF PHILOSOPHY

in

Mathematics Education

Presented by

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Von der Pädagogischen Hochschule Schwäbisch Gmünd
zur Erlangung des Grades eines

Doktors der Philosophie (Dr. phil.)

angenommene Dissertation von

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Fach: Mathematik

Abgabetermin der Dissertation: 08.01.2010

Abstract

It goes without saying that students can learn and develop their creative potential if appropriate programs are used that successfully teach them the creative skills and operations necessary. However, mathematics in general and geometry in particular, by their own nature, have a lot of possibilities that can be used in developing creativity. The purpose of the present study was threefold: First, to identify the principles of preparing a suggested enrichment program in Euclidean geometry using dynamic geometry software to develop the mathematically gifted students' geometric creativity in high schools; second, to develop an enrichment program based on the identified principles; third, to investigate the effectiveness of this program by testing it with high school students.

By reviewing the prior literature and studies related to the subject of the study, the principles of developing the suggested enrichment program were identified. Then, a suggested enrichment program based on these identified principles using the Interactive Geometry Software, Cinderella was developed. Moreover, within the present study an instrument, a geometric creativity test (GCT), was developed and a grading method for assessing the mathematically gifted students geometric creativity was established.

The material was tested with a group of 7 mathematically gifted students in the Landesgymnasium für Hochbegabte (LGH), a public high school for highly gifted and talented students in Schwäbisch Gmünd – Baden-Württemberg, Germany. The students came from 11th (5), 10th (1) and 9th (1) grade, two of them were male, five female.

In the study, the researcher used a one-group pretest – intervention – posttest pre-experimental design. In this context the GCT was administered to students as a pretest at the beginning of the study; then the suggested enrichment program was introduced to them in 12 weekly 90-minutes sessions during the first semester of the academic year 2008/09. The students retook the GCT as a posttest at the end of the study.

The results indicated there were statistically significant differences between the mean ranks of the subjects' scores on the pre-post measurements of the geometric creativity test and its subscales in favor of the post measurement. The results also revealed that the suggested enrichment program was significantly effective in developing geometric creativity as a whole ability and its four sub-components – fluency, flexibility, originality, and elaboration.

The results of the study suggest that the prepared enrichment program using DGS has a positive impact on the mathematically gifted students' geometric creativity. The positive impact can be traced back to the content of the suggested enrichment program and its open-ended and divergent-production geometric situations and problems. Also, the positive impact can be attributed to the use of DGS along the program sessions that provide the subjects with not only many opportunities to explore, experiment, and make new mathematical conjectures, but also to solve problems and pose related problems.

However, the study requires replication and improvements before any firm conclusions can be made. One of the biggest improvements would be to have more subjects so that the results become more generalized and meaningful. Moreover, pertaining to the experimental design, further studies are needed to investigate the effectiveness of the suggested enrichment program using both quasi-experimental and true-experimental designs.

Other avenues for research may focus on students' affective and emotional domains (e.g. self confidence, attitudes, and achievement motivation, among others), and might as well include an analysis of gender-related individual differences.

Acknowledgments and Thanks

Praise is to Allah the Lord of the Worlds, and peace and blessings of Allah on His Messenger, his family, companions and those who follow them until the last Day.

I praise Allah subhanahu wa taala for His favor to me in completing this work, praying Him to accept this as a sincere service for His sake, and for the benefit of mankind.

The financial support of the Egyptian government represented in the ministry of high education and the cultural office in Berlin toward this study is hereby acknowledged. Without such support it would have been extremely difficult to carry out this study.

I would like to thank my supervisor, Professor Dr. Ulrich Kortenkamp for his encouragement, direction, and advice provided throughout my work in this dissertation, especially his insights into the use of Cinderella in preparing the suggested enrichment program.

Special thanks are also due to the co-examiner of the dissertation, Professor Dr. Astrid Beckmann, for sincerely encouraging me through the process of the study, in validating the Geometric Creativity Test (GCT) and the developed Enrichment Program, and for having the time to evaluate the dissertation before holding her new position as a president of the University of Education Schwäbisch Gmünd.

I also appreciate Dipl. Päd. Axel M. Blessing in his support along my work in the dissertation, especially his support in developing the html indices of the suggested enrichment program and being there whenever I need him.

A big thank you is due to my special colleague M.A. Enas Mohamed Farahat who supported me along my work through her accurate editing, proofreading, encouragements, and deep, insightful suggestions. May Allah (swt) guide her to the Straight Path!

I also thank Dr. Uwe Heim-Dreger who provided me with all statistical knowledge and

expertise in qualifying the Geometric Creativity Test (GCT) and in testing the study hypotheses.

Special thanks are also due to all jurors and experts who validated the geometric creativity test and the enrichment program for their generous spirit and the sincere support they gave me.

I wish to acknowledge all the help and advice given by my colleagues at the University of Education Schwäbisch Gmünd, especially Christian Dohrmann, Simon Zell, and Dr. Silke Ladel whose beneficial discussions enhanced this work.

Special thanks to the students of the University of Education Schwäbisch Gmünd students, especially Markus Böckler, Jonathan Härtel, and Samuel Preuß, who helped me in writing and editing the German text in both the program applets and the students handouts.

Thanks also go to Dr. Albert Organian and Dr. Olga Lomonosova, the mathematics teachers at Landesgymnasium für Hochbegabte (LGH) school who supported me during the experimental study at the Landesgymnasium für Hochbegabte and helped me to identify the study subjects.

I also wish to express my indebtedness, gratitude, and love to my parents, my brothers, my wife and my special daughters, for standing by and believing in me when I was not ready to believe in myself.

It is Allah Who bestows success, and guides to the Straight Path.

Mohamed El-Demerdash

Schwäbisch Gmünd – Karlsruhe, Germany

January 2010 – Muharram 1431

Contents

1	Problem of the Study	1
1.1	Background of the Problem	1
1.2	Problem of the Study	4
1.3	Terms of the Study	5
1.3.1	Dynamic Geometry Software (DGS)	5
1.3.2	Mathematically Gifted Students	5
1.3.3	Geometric Creativity	5
1.4	Aims of the Study	6
1.5	Significance of the Study	6
1.6	Limitations of the Study	7
1.7	Variables of the Study	8
1.8	Methodology of the Study	8
1.9	Instrument of the Study	8
1.10	Experimental Design of the Study	8
1.11	Hypotheses of the Study	9
1.12	Procedures of the Study	10
1.13	Structure of the Thesis	11
2	Review of Literature	15
2.1	The Use of Dynamic Geometry Software	15
2.1.1	Definitions of Dynamic Geometry Software	16
2.1.2	Key Features of Dynamic Geometry Software	17

1) Drag Mode	18
2) Generating Locus/ Loci of Points	18
3) Developing Macro-constructions	18
2.1.3 Types of Dynamic Geometry Software	19
1) Examples of 2-dimensional Packages	19
2) Examples of 3-dimensional Packages	22
2.1.4 Purposes and Functions of Dynamic Geometry Software	23
1) DGS as a Construction Tool	23
2) Measurement	24
3) DGS as a Visualization Tool	24
4) Exploration and Discovery	24
5) DGS as a Modeling Tool	25
6) Experimentation	25
7) DGS as a Problem Solving and Problem Posing Tool	26
8) Transformation	26
9) Proof	27
2.1.5 The Role of DGS in Geometry Education	27
1) The Role of DGS in Developing Geometric Thinking	28
2) The Role of DGS in Developing the Academic Achievement	31
3) The Role of DGS in Developing the Affective Domain	32
2.2 The Mathematically Gifted Education	33
2.2.1 The Nature of Giftedness	33
1) Lewis Terman View of Giftedness	34
2) Joseph Renzulli's Three-Ring Conception of Giftedness	34
2) François Gagné's Differentiated Model of Giftedness	36
2.2.2 The Nature of Mathematical Giftedness	39
2.2.3 Characteristics of the Mathematically Gifted Students	42
1) Cognitive Characteristics	43
2) Affective Characteristics	45
3) Creative Characteristics	46

2.2.4	Educational Differentiation for Gifted Students	46
1)	Acceleration	47
2)	Enrichment	50
2.2.5	Teaching and Learning Models and Strategies for Gifted Students	52
1)	Enrichment Triad Model	52
2)	Multiple Talent Model	53
3)	Autonomous Learner Model	54
4)	Creative Problem Solving (CPS) Process	55
2.2.6	General Principles of Developing Enrichment Programs	56
1)	Program Aims	57
2)	Program Content	57
3)	Program Instruction	57
4)	Program Assessment	58
2.3	Geometric Creativity Development	58
2.3.1	The Nature of Creativity	59
1)	Fluency	59
2)	Flexibility	60
3)	Originality/Novelty	60
4)	Elaboration	60
2.3.2	The Nature of Geometric Creativity	61
2.4	The Significance of Using Dynamic Geometry Software for Geometric Creativity	64
3	Related Studies	67
3.1	Studies Related to the Use of Dynamic Geometry Software	67
3.1.1	Studies Concerned with Developing New Interactive Systems or Software	68
3.1.2	Studies Concerned with Developing Students' Learning Outcomes	70
3.1.3	Studies Concerning Interaction with the Software	77
3.2	Studies Related to the Mathematically Gifted Education	80

3.2.1	Studies Concerned with Identifying Mathematically Gifted Students' Characteristics	80
3.2.2	Studies Concerned with Providing Tailored Educational Differentiations	88
3.3	Studies Related to Geometric Creativity Development	94
3.3.1	Studies Concerning the Development of Creative Potential in Geometry	94
3.3.2	Studies Concerning the Assessment of Creative Potential in Geometry	98
3.4	Commentary on Related Studies	99
3.5	The Principles of Developing the Suggested Enrichment Program	106
4	Method and Procedures	109
4.1	The Suggested Enrichment Program	109
4.1.1	Why Cinderella?	110
4.1.2	Principles of Developing the Suggested Enrichment Program	111
4.1.3	Aims of the Suggested Enrichment Program	111
4.1.4	Content of the Suggested Enrichment Program	113
	1) Student's Handouts	114
	2) Teacher's Guide	114
	3) CD-ROM	116
4.1.5	Appropriateness of the Suggested Enrichment Program	116
4.1.6	Time-range for the Program Activities	117
4.1.7	The Student's Role	118
4.1.8	The Teacher's Role	120
4.2	The Geometric Creativity Test	121
4.2.1	Specification of the Aim of the Test	121
4.2.2	Specification of the Components that the Test Measures	122
4.2.3	Creation of a Preliminary Form of the Test	122
4.2.4	Setting-up of a Grading Method for the Test	126

4.2.5	Content Validity Check	127
4.2.6	The Piloting of the Test	128
	1) The Reliability Coefficient	129
	2) Item-internal Consistency Reliability	129
	3) The Experimental Validity	132
	4) The Suitable Time-range	132
4.3	Procedures of the Experimental Study	133
4.3.1	Subjects of the Study	133
4.3.2	Experimental Design of the Study	134
4.3.3	Before the Intervention	134
4.3.4	Administering the Pretest	134
4.3.5	Administering the Suggested Enrichment Program	135
4.3.6	Administering the Posttest	137
4.4	Researcher's Notes During the Experimental Study	137
5	Findings and Interpretation	141
5.1	Analysis of Findings	142
5.1.1	Results of the Pretest	142
5.1.2	Results of the Posttest	143
5.1.3	Results of Testing Hypothesis One	144
5.1.4	Results of Testing Hypothesis Two	146
5.1.5	Results of Testing Hypothesis Three	147
5.1.6	Results of Testing Hypothesis Four	149
5.1.7	Results of Testing Hypothesis Five	149
5.1.8	Results of Testing Hypothesis Six	152
5.1.9	Results of Testing Hypothesis Seven	152
5.1.10	Results of Testing Hypothesis Eight	153
5.1.11	Results of Testing Hypothesis Nine	153
5.1.12	Results of Testing Hypothesis Ten	154
5.2	Discussion and Implications	155

5.3	Conclusions	159
6	Summary, Conclusions and Recommendations	161
6.1	Background of the Problem	161
6.2	Problem of the Study	162
6.3	Aims of the Study	163
6.4	Significance of the Study	163
6.5	Principles of Developing the Enrichment program	164
6.6	The Suggested Enrichment Program	165
6.7	Instrument of the Study	166
6.8	The Experimental Study	166
6.9	Results of the Study	167
6.10	Conclusions and Implications	168
6.11	Recommendations	169
6.12	Suggestions for Further Research	171
	Bibliography	173
	Appendices	185
A	Student's Handouts (English Version)	187
	Handout 1. The Suggested Enrichment Program Outcomes	191
	Handout 2. Launching and Exploring Cinderella	193
	Handout 3. Basic Commands and Modes of Cinderella	199
	Handout 4. Dragging and Measuring Facilities of Cinderella	209
	Handout 5. Automatic Proving Facilities of Cinderella	211
	Handout 6. Developing Macro-constructions	214
	Handout 7. Animating and Tracing Loci Facilities of Cinderella	219
	Handout 8. Midpoints of the Sides of a Quadrilateral	222
	Handout 9. Angular Bisectors of a Parallelogram	223
	Handout 10. Constructing a Parallelogram	224
	Handout 11. Constructing a Rhombus	227

Handout 12. Constructing a Rectangle	230
Handout 13. Constructing a Square	233
Handout 14. Posing Geometric Problems	236
Handout 15. Finding Geometric Relationships	237
B Student's Handouts (German Version)	239
Handout 1. Lernziele des Enrichment Programms	243
Handout 2. Starten und Erkunden von Cinderella	245
Handout 3. Grundlegende Befehle und Konstruktionen von Cinderella	252
Handout 4. Zugmodus und Messwerkzeuge von Cinderella	263
Handout 5. Der Automatische Beweiser von Cinderella	265
Handout 6. Entwicklung von Makros	268
Handout 7. Animation und Definition von Ortskurven mit Cinderella	274
Handout 8. Mittelpunkte der Seiten eines Vierecks	277
Handout 9. Winkelhalbierenden eines Parallelogramms	278
Handout 10. Konstruieren eines Parallelogramms	279
Handout 11. Konstruieren eines Rhombus	282
Handout 12. Konstruieren eines Rechtecks	285
Handout 13. Konstruieren eines Quadrats	288
Handout 14. Geometrische Aufgabenstellungen	291
Handout 15. Finden Geometrischer Beziehungen	292
C Teacher's Guide	293
To the Teacher	297
The Content of the Teacher's Guide	302
Aims of the Suggested Enrichment Program	304
Cinderella Getting Started	306
The Enrichment Activities of the Program	308
Enrichment Activity 1. Dragging and Measuring Facilities of Cinderella	309
Enrichment Activity 2. Automatic Proving Facilities of Cinderella	320
Enrichment Activity 3. Developing Macro-constructions	333

Enrichment Activity 4. Animating and Tracing Loci Facilities of Cinderella . .	342
Enrichment Activity 5. Midpoints of the Sides of a Quadrilateral	362
Enrichment Activity 6. Angular Bisectors of a Parallelogram	378
Enrichment Activity 7. Constructing a Parallelogram	386
Enrichment Activity 8. Constructing a Rhombus	395
Enrichment Activity 9. Constructing a Rectangle	395
Enrichment Activity 10. Constructing a Square	395
Enrichment Activity 11. Posing Geometric Problems	396
Enrichment Activity 12. Finding Geometric Relationships	410
Resources Used in Developing the Suggested Enrichment Program	421
D Experts Who Validated the Enrichment Program	423
A List of the Experts Who Validated the Suggested Enrichment Program . . .	425
E A Letter to Experts for Judging on the Program Appropriateness	427
A Letter to the Experts for Judging on the Appropriateness of the Program .	429
F A Letter to Judges for Validating the Geometric Creativity Test	431
A Letter to the Judges for Validating the Geometric Creativity Test	433
G Judges Who Validated the Geometric Creativity Test	435
A List of the Judges Who Validated the Geometric Creativity Test	437
H The Geometric Creativity Test (GCT – English Version)	439
The Geometric Creativity Test	443
Index of the Mathematical Symbols Used in the Test	445
Item 1	447
Item 2	449
Item 3	451
Item 4	453
Item 5	455
Item 6	457

Item 7	459
Item 8	461
Item 9	463
Item 10	465
Item 11	467
Item 12	469
 I The Geometric Creativity Test (GCT – German Version)	471
Test zur Geometrischen Kreativität	475
Verzeichnis der im Test verwendeten mathematischen Symbole	477
Aufgabe 1	479
Aufgabe 2	481
Aufgabe 3	483
Aufgabe 4	485
Aufgabe 5	487
Aufgabe 6	489
Aufgabe 7	491
Aufgabe 8	493
Aufgabe 9	495
Aufgabe 10	497
Aufgabe 11	499
Aufgabe 12	501

List of Tables

4.1	<i>Time-range for the Program Activities</i>	118
4.2	<i>Specifications of the Geometric Creativity Test</i>	123
4.3	<i>Item 1 and Assessing Different Geometric Creativity Components</i>	124
4.4	<i>Test Items and Geometric Creativity Components</i>	125
4.5	<i>Originality/Novelty Scores for the Geometric Creativity Test</i>	126
4.6	<i>Statistics Attributes of the Pilot Study of the Geometric Creativity Test</i>	131
4.7	<i>Statistical Attributes of the Test Time in the Pilot Testing</i>	132
4.8	<i>The Schedule of the Suggested Enrichment Program</i>	135
5.1	<i>Pretest Results</i>	143
5.2	<i>Posttest Results</i>	143
5.3	<i>Geometric Fluency Test Results</i>	145
5.4	<i>Geometric Flexibility Test Results</i>	146
5.5	<i>Geometric Originality/Novelty Test Results</i>	148
5.6	<i>Geometric Elaboration Test Results</i>	149
5.7	<i>Overall Geometric Creativity Test Results</i>	150
5.8	<i>The Effect Size of the Enrichment Program on Geometric Fluency</i>	152
5.9	<i>The Effect Size of the Enrichment Program on Geometric Flexibility</i>	152
5.10	<i>The Effect Size of the Enrichment Program on Geometric Originality</i>	153
5.11	<i>The Effect Size of the Enrichment Program on Geometric Elaboration</i>	154
5.12	<i>The Effect Size of the Enrichment Program on Overall Geometric Creativity</i>	154

List of Figures

2.1	<i>Graphical Representation of Renzullis Three-Ring Conception of Giftedness . .</i>	35
2.2	<i>Graphical Representation of Gagné’s Differentiated Model of Giftedness and Talent</i>	37
5.1	<i>Mean Scores of the Subjects on the Pretest and Posttest</i>	144
5.2	<i>Fluency Scores of the Subjects on the Pretest and Posttest</i>	145
5.3	<i>Flexibility Scores of the Subjects on the Pretest and Posttest</i>	147
5.4	<i>Originality/Novelty Scores of Subjects on the Pretest and Posttest</i>	148
5.5	<i>Elaboration Scores of the Subjects on the Pretest and Posttest</i>	150
5.6	<i>Overall Geometric Creativity Scores of the Subjects on the Pretest and Posttest</i>	151

Chapter 1

Problem of the Study

1.1 Background of the Problem

Nowadays, many countries all over the world seek to give due care to the gifted and talented, believing in the importance of their role in setting and carrying out the goals of scientific and technical developments. Since education is considered the most important means of human development, these countries sponsor the gifted and talented students through special educational programs and curricula to promote their creative potentials and prepare them as scientists capable of thinking properly and contributing to the scientific and technological developments of the computer and information age.

The development of creativity and innovation in the field of mathematics for gifted and talented students has become a major topic discussed in many international conferences and meetings. It focuses on the role of the teacher in promoting mathematical creativity and calls for offering qualified programs and curricula for gifted and talented students in the field of mathematics. Consequently, many educational programs have been developed to provide the gifted with various experiences in order to promote their creative potential (Cho *et al.*, 2004; Eraky, 2004; Mohamed, 2003; Velikova, 2004, among others). In this concern, two main approaches emerge. Some researchers see that creativity can be learned and developed directly using specific programs that teach creativity skills and

its operations regardless of the subject matters. On the other hand, other researchers assert that teaching creativity should be associated with the subject matters and it should be a part of the corresponding lesson plans that teachers prepare (Jerwan, 2002, p. 38).

With respect to mathematics as a subject matter, there has been a growing interest in using mathematical content to develop creativity. As a result, new terminologies about creativity emerge, such as mathematical creativity which refers to creativity in the field of mathematics. Also, recent studies used geometric content to develop the creative potential among students (e.g., Al-Baz, 1999; El-Rayashy and Al-Baz, 2000; Mohamed, 2003; Weth, 1998b).

In the context of growing concern with developing creativity in the field of mathematics, a new trend appears that calls for using technology as a way to help promote creativity. For addressing this call, many studies were conducted adopting the integrated learning environment with technology in order to develop creativity (e.g., Kakihana, Fukuda, and Watanabe, 2008; Wurnig, 2008).

On the other hand, examining the status quo of the mathematically gifted education, supported by both research and experience, indicates that there are not any series of mathematics textbooks that are written specially for the mathematically gifted students and most of the textbooks are written for the general population of students. Namely, they are not always appropriate for the gifted students because their needs dictate a curriculum that is deeper, broader and faster than what is delivered to the other students (Johnson, 2000, p. 4). That is, these students have needs that differ in nature from those of other students and therefore they require some curricula and instruction differentiation.

Furthermore, many programs for the mathematically gifted students, as literature indicates (Rotigel and Fello, 2004, p. 47), are inadequate and poorly designed; leaving the classroom teachers behind, struggling in an attempt to effectively meet the educational needs of the mathematically gifted. Even though rich and varied learning programs that offer the mathematically gifted possibilities for investigation and exploration, can give

them further possibilities to reach new levels in mathematics.

Studying geometry is very important as a requirement for many scientific, technical and occupational areas as well as in studying mathematics itself. The reason why it is important is that through teaching and learning geometry a wide range of creativity and divergent thinking such as conjecturing, experimenting, comparing, and applying can be developed. However, geometry is often neglected as a result of the extension of other disciplines of mathematics such as algebra and calculus in most high schools. Some possible reasons for this negligence would be the lack of resources such as concrete materials, computer software, and the lack of knowledge and expertise about how to use computers and other materials for instructional purposes (Olkun, Sinoplu, and Derzakulu, 2005, p. 1).

As a result of the radical development in the field of computer technology in both of its levels, hardware and software, and the widespread of computers uses and their powerful computing tools in the schooling system, many educational systems and applications have emerged, which did not exist before, (e.g., Intelligent Tutoring System ITS, and Intelligent Learning Environment (ILE)) (Eraky, 2004, p. 21). Moreover, with respect to mathematics education in general and geometry education in particular, a sort of dynamic applications known as “Dynamic Geometry Software” (DGS) has emerged over the last two decades as a new system in the teaching and learning of geometry and as an attractive alternative to ruler and compass. This alternative can be used to construct with points, lines, circles, conics and possibly more types of objects. It enables users to drag free elements later while automatically maintaining the construction, i.e., the relation between the objects (Kortenkamp, 1999, pp. 163-164).

The appearance of DGS with its constructing and dragging facilities (among others) is considered a paradigm shift in the teaching and learning of geometry since it has established new norms and standards and deepened our understanding of tool-situated learning (Talmon and Yerushalmy, 2003, p. 6). It supports students’ understanding of properties of geometrical objects and enables them to become geometric experimenters and make their own discoveries. Now, dynamic geometry software has become a stan-

dard tool for students, teachers, and mathematicians in schools and universities. Furthermore, the teaching and learning of geometry utilizing dynamic geometry software have been explicitly articulated in Principals and Standards for School Mathematics of the National Council of Teachers of Mathematics (NCTM) (National Council of Teachers of Mathematics (NCTM), 2000).

Leading from this, the researcher attempts this study seeking to address those previously-mentioned calls and the increasing need for qualified programs and curricula in the field of teaching and learning mathematics to the mathematically gifted students by developing an enrichment program in Euclidean geometry using Dynamic Geometry Software (DGS) and investigates its effectiveness in promoting the mathematically gifted students' geometric creativity. The suggested enrichment program could be considered a step forward toward designing and using precisely tested programs for the mathematically gifted students by using the latest powerful computing tools and software to develop the mathematically gifted students' geometric creativity in high schools.

1.2 Problem of the Study

The present study investigates the following basic question: "How far is a suggested enrichment program using dynamic geometry software effective in developing the mathematically gifted students' geometric creativity in high schools?"

This question is branched out into the following sub-questions:

1. What are the principles of developing a suggested enrichment program using dynamic geometry software for developing the mathematically gifted students' geometric creativity in high schools?
2. What is the suggested enrichment program using dynamic geometry software for developing the mathematically gifted students' geometric creativity in high schools?
3. How far is the suggested enrichment program using dynamic geometry software

effective in developing the mathematically gifted students' geometric creativity in high schools?

1.3 Terms of the Study

In the present study, the researcher deals with some terms; namely, “dynamic geometry software”, “mathematically gifted students”, and “geometric creativity” To elaborate on these terms, the researcher made use of current literature in the field (e.g., Al-Baz, 1999; Cho *et al.*, 2004; Choi and Do, 2008; El-Rayashy and Al-Baz, 2000; Eraky, 2004; Kortenkamp, 1999; Mohamed, 2003; Sträßer, 2002; Velikova, Bilchev, and Georieva, 2004)

1.3.1 Dynamic Geometry Software (DGS)

DGS, in this study, is a type of software which provides an intelligent environment for mathematics education in general and for geometry education in particular, and in which the mathematically gifted students can engage in constructing, exploring, conjecturing, testing, and confirming geometric notations through the suggested enrichment program.

1.3.2 Mathematically Gifted Students

Mathematically gifted students in this study are those who are determined by highly qualified and professional persons, and those students have high mathematical abilities and the ability to set a high performance in solving mathematical problems.

1.3.3 Geometric Creativity

Geometric creativity is to be known, in the present study, as a directed intellectual activity for generating new geometric ideas over the known or familiar ones of the mathematically gifted students in the high schools in a non-routine geometric situation,

and the new ideas reflect the abilities of fluency, flexibility, originality/novelty, and elaboration. The term geometric creativity is objectively defined, in this study, as the student's score on the geometric creativity test prepared by the present researcher.

1.4 Aims of the Study

The present study aims at:

1. Identifying the principles of the suggested enrichment program using dynamic geometry software for developing the mathematically gifted students' geometric creativity in high schools.
2. Developing the suggested enrichment program using dynamic geometry software for developing the mathematically gifted students' geometric creativity in high schools.
3. Measuring the effectiveness of the enrichment program using dynamic geometry software in developing the mathematically gifted students' geometric creativity in the high schools.

1.5 Significance of the Study

The present study can contribute to the following domains:

1. Helping the teachers and program designers of the mathematically gifted students to acknowledge the ways DGS contribute to developing the mathematically gifted students' geometric creativity.
2. Providing some enrichment topics in mathematics using dynamic geometry software for the mathematically gifted students in high schools.
3. Enabling gifted students to use the dynamic geometry software to mathematically communicate with others through the World Wide Web, the electronic mail, and web pages.

4. Developing a validated and reliable instrument for assessing the geometric creativity of the mathematically gifted students in high schools.
5. Meeting the increasing need for a large number of studies in the field of teaching mathematics to the mathematically gifted students. The suggested program is considered a step forward toward designing and using precisely tested programs for such students.

1.6 Limitations of the Study

The present study is bound by the following limitations:

Purpose: Developing the four adopted components of the geometric creative potential (fluency, flexibility, originality/novelty, and elaboration) among the mathematically gifted students in high schools.

Instructional Limitations: Using the Interactive Geometry Software, Cinderella as dynamic geometry software along the suggested enrichment program.

One instrument is used; the geometric creativity test, which is designed by the researcher.

Population Subjects: The study is limited to a sample of 7 (5 eleventh grader, 1 tenth grader, and 1 ninth grader - 2 male, 5 female) mathematically gifted students at Landesgymnasium für Hochbegabte (LGH), a public high school for gifted and talented students in Schwäbisch Gmünd – Baden-Württemberg, Germany.

Content Subjects: the content is limited to the content of the suggested enrichment program prepared by the researcher.

Time Limit: The suggested enrichment program only covers 12 weekly 90-minute sessions during the first semester of the academic year 2008/09.

1.7 Variables of the Study

The variables, in this study, are classified as follows:

1. **Independent Variable:** This is the variable whose effect is to be measured in this study. This variable is represented in the suggested enrichment program using DGS and its use (in teaching) with the mathematically gifted.
2. **Dependent Variable:** This is the variable whose effect on the independent variable is to be measured before and after the experimental manipulation. This variable is represented in the geometric creativity of the mathematically gifted as measured by the geometric creativity test prepared by the present researcher.

1.8 Methodology of the Study

The researcher in this study made use of both the pre-experimental and descriptive method. The pre-experimental method is used in investigating the effectiveness of the suggested enrichment program using the dynamic geometry software in developing the mathematically gifted students' geometric creativity in high schools. On the other hand, the descriptive method is used as well to collect and analyze data, and describe the suggested enrichment program and the instrument of the study.

1.9 Instrument of the Study

The researcher prepared a geometric creativity test to assess the mathematically gifted students' geometric creativity before and after the intervention.

1.10 Experimental Design of the Study

The researcher made use of the pre-experimental one-group, pretest – intervention – posttest design to investigate the effectiveness of the suggested enrichment program

in developing geometric creativity among the mathematically gifted students in high schools.

1.11 Hypotheses of the Study

To answer the third question of the present study, which is: “How far is the suggested enrichment program using dynamic geometry effective in developing the mathematically gifted students’ geometric creativity in high schools?” the following ten hypotheses¹ were formulated:

1. There is a statistically significant difference at the level 0.05 between the mean ranks of subjects’ scores on the pre-post measurements of the geometric fluency component, as measured by the designed geometric creativity test, in favor of the post measurement.
2. There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects’ scores on the pre-post measurements of the geometric flexibility component, as measured by the designed geometric creativity test, in favor of the post measurement.
3. There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects’ scores on the pre-post measurements of the geometric originality/novelty component, as measured by the designed geometric creativity test, in favor of the post measurement.
4. There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects’ scores on the pre-post measurements of the geometric elaboration component, as measured by the designed geometric creativity test, in favor of the post measurement.
5. There is a statistically significant difference at the level 0.05 between the mean

¹The hypotheses of the study were formulated in a predictive directive and alternative way based on the indicators of the prior studies and literature related to the subject of the study. For more details see: Chapter 2. Review of Literature and Chapter 3. Related Studies.

ranks of the subjects' scores on the pre-post measurements of geometric creativity as a whole ability, as measured by the designed geometric creativity test, in favor of the post measurement.

6. The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric fluency component in high schools.
7. The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric flexibility component in high schools.
8. The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric originality/novelty component in high schools.
9. The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric elaboration component in high schools.
10. The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' overall geometric creative potential in high schools.

1.12 Procedures of the Study

To answer the study questions and examine the validity of its hypotheses, the researcher went through the following procedures:

1. Specifying the principles of preparing the suggested enrichment program using dynamic geometry software for developing the mathematically gifted students' geometric creativity in high schools.
2. Preparing the enrichment program in its initial form in the light of the principles specified before.

3. Presenting the program, in its initial form, to a group of experts and education-
alists who are experienced in teaching and learning mathematics to decide on its
appropriateness to the mathematically gifted students in high schools and suggest
any changes to modify it.
4. Preparing an instrument, a geometric creativity test, to assess the geometric cre-
ativity of the mathematically gifted students in high schools.
5. Presenting the geometric creativity test, in its initial form, to a group of judges
specialized in teaching and learning mathematics to decide on its validity and
suggest any changes to modify it.
6. Conducting the experimental study.
7. Collecting data and treating them statistically using the appropriate methods.
8. Interpreting and discussing results.
9. Presenting the suitable suggestions and recommendations to make full use of the
study results.

1.13 Structure of the Thesis

Chapter 2. Review of Literature: reviews prior literature related to the subject
of the present study in three domains:

- The use of dynamic geometry software in the teaching and learning of math-
ematics
- The mathematically gifted education
- Geometric creativity development

Based on the given data and results of prior literature, in this chapter, the re-
searcher attempts to determine the significance of using dynamic geometry soft-
ware in developing the mathematically gifted students' geometric creativity in
high schools.

Chapter 3. Related Studies: reviews the prior studies related to the subject of the present study in three main domains:

- Studies related to the use of dynamic geometry software in the teaching and learning of mathematics.
- Studies related to the mathematically gifted education.
- Studies related to geometric creativity development.

Then, the researcher presents his commentary on related studies, and attempts to figure out the principles of preparing the suggested enrichment program. By the end of this chapter, the first question of the study which is: *“What are the principles of developing a suggested enrichment program using dynamic geometry software for developing the mathematically gifted students’ geometric creativity in high schools?”* is answered.

Chapter 4. Method and Procedures: presents in detail the development of both the suggested enrichment program and the geometric creativity test as well as the procedures of the experimental study. In this chapter, the second question of the study, which is: *“What is the suggested enrichment program using dynamic geometry software for developing the mathematically gifted students’ geometric creativity in high schools?”* is answered.

Chapter 5. Findings and Interpretation: presents the statistical analysis results of testing the study hypotheses as well as interpretation and implications of these results in the light of the literature review and purposes of the study. By the end of this chapter, the third question of the study which is: *“How far is the suggested enrichment program using dynamic geometry effective in developing the mathematically gifted students’ geometric creativity in high schools?”* is answered.

Chapter 6. Summary, Conclusions and Recommendations: includes a summary of the study. It also includes recommendations pertinent to the findings of the program and concludes with the suggestion for further research related to the use of dynamic geometry software in the teaching and learning of mathematics,

the mathematically gifted education, and geometric creativity development.

Chapter 2

Review of Literature

In this chapter, the researcher reviews prior literature related to the subject of the present study and surveys it under three main domains: The use of dynamic geometry software in the teaching and learning of mathematics, the mathematically gifted education, and the geometric creativity development. By the end of the chapter, based on the given data and the results of prior literature, the researcher attempts to determine the significance of using dynamic geometry software in developing the mathematically gifted students' geometric creativity in high schools.

2.1 The Use of Dynamic Geometry Software in the Teaching and Learning of Mathematics

DGS is the short form of Dynamic Geometry Software. It is also known as Dynamic Geometry Environment (DGE) and Dynamic Geometry Systems (DGS).

It is a sort of computer software that provides a range of tools for constructing geometric objects from a range of “primitive” objects such as points, straight objects (line segments, lines, rays, and vectors) and circles (or arcs) to “classical” constructions (e.g., midpoint, parallel lines, perpendicular lines). It also provides tools for “transformational” geometry (e.g., reflection, translation, rotation, similarity). Once the geometric

object is drawn using the software, measurements can be taken from objects (e.g., length, angle, area) (Oldknow, 1997, p. 4). The dynamic feature comes from the ability to drag free or semi-free objects, such as points, around the computer screen with the mouse (Jones, 2005, p. 27). The dragging facilities deform the constructed object; however, some aspects of the geometric object remain the same. An important task is to observe the invariant aspects of the geometric object under the dragging facilities.

Furthermore, some dynamic geometry software embraces other facilities (e.g., animating and tracing loci facilities, developing macro-constructions facilities, and automatic proving facilities). More details about these facilities will be explained in the next sections.

Over the last two decades, dynamic geometry software has become one of the most widely used pieces of mathematics software in schools, colleges, and universities all over the world¹ (Jones, 2005, p. 28). Unlike other mathematics software, the dynamic geometry software is not question-and-answer based tutorial. It is context-free learning software suitable for implementation in an experimental discovery-based learning environment as well as for demonstration purposes. It is common in middle school, high school, and college geometry classrooms but is occasionally used in other subject areas or with younger students (Hull and Brovey, 2004).

In this part of the dissertation the researcher surveys in detail some aspects of the DGS such as its definitions, types, functions and purposes, and its role in geometry education.

2.1.1 Definitions of Dynamic Geometry Software

There are many definitions for the dynamic geometry software based on one or more of its main characteristics, features, purposes or functions. One such definition, defined by Wikipedia encyclopedia, emphasizing the drag mode of the dynamic geometry environment is: “Dynamic geometry environment is a computer program which allows one to

¹There are versions of the software in Arabic, Chinese, Dutch, English, French, German, Hungarian, Italian, Japanese, Korean, Polish, Portuguese, and Spanish among others.

create and then manipulate geometric constructions”. Namely, one starts construction by putting a few points and using them to define new objects (lines, circles, other points, etc). After some construction is done, one can move the points he/she starts with and see how the construction changes.

The term dynamic geometry software is carried further by [Cho *et al.* \(2004\)](#), who emphasized the functions of the software as an experimental discovery-based learning environment and stressed the role of learners in this environment. In this respect, [Cho *et al.*](#) define the dynamic geometry software as “an experimental environment in which the learners hypothesize, justify, and confirm geometric and algebraic properties through mouse exploration”. Accordingly, the use of such software changes the role of mathematics students from passive spectators into conceptual participants involved in a scientific process for exploring theorems through: hypothesizing, justifying, and confirming.

[Sträßer \(2002\)](#) also defines dynamic geometry software as “a certain type of software which is predominantly used for construction and analysis of tasks and problems in elementary geometry”.

Reviewing these samples of definitions and many others, the present researcher adopts a definition that combines some attempts to define the term in one comprehensive definition that fits the purposes of the present study. Dynamic geometry software, in this study, is a type of software which provides an intelligent environment for mathematics education in general and for geometry education in particular, and in which the mathematically gifted students can engage in constructing, exploring, conjecturing, testing, and confirming geometric notations through the suggested enrichment program.

2.1.2 Key Features of Dynamic Geometry Software

Even though there are many dynamic geometry software packages, which differ in their conceptual design, most of them still share the following three key features ([Sträßer, 2002](#), p. 65): (1) Drag mode, (2) Generating a locus/loci of points, and (3) Developing macro-constructions.

1) Drag Mode

The dragging facilities, which are embedded in all dynamic geometry software, are the fundamental improvements versus the technical devices: the straightedge and the compass (Kortenkamp, 1999, p. 16). Dragging in DGS simply means the possibility to move free object or semi-free object (such as a point, a straight line, or even an edge of a circle) of a geometric figure by dragging the mouse so that the rest of the figure follows the moves and changes/responses accordingly and the figures is updated automatically to show the new position of the figure objects, which depend on the moved object.

The use of these facilities has great educational implications in the teaching and learning of geometry in particular and in mathematics in general since it produces a dynamic behavior for each element in the construction that allows users to move certain elements of the construction and to observe how others respond to the altered conditions (Talmon and Yerushalmy, 2003, p. 1). In other words, dragging facilities allow free elements of the construction to be moved, while the rest of the construction follows the moves and changes/responses accordingly. But some aspects of the construction remain the same. The software allows a focus on the important geometric ideas of invariance. Therefore, dragging within DGS is considered a tool for investigating relationships in geometry figures at both perceptual and formal levels (Jones, 2005, p. 27).

2) Generating Locus/ Loci of Points

Many of the dynamic geometry software packages include the facilities of generating and tracing locus/loci of points – drawing the traces of elements when other elements are moved on a path, such as a line or a circle – as well as performing automatic animations.

3) Developing Macro-constructions

Dynamic geometry software usually has the ability to group or capsule a sequence of construction commands into a new command or tool. For example, constructing

the perpendicular bisector using two circles and their intersections can be encapsulated to a macro, a new command or tool, that issues the necessary steps automatically (Kortenkamp and Richter-Gebert, 1998, p. 2)

2.1.3 Types of Dynamic Geometry Software

There are two main types of DGS packages: 2-dimensional packages and 3-dimensional packages.

1) Examples of 2-dimensional Packages

Cabri-Geometry² (also known as Cabri-Géomètre) is a dynamic geometry application offering an interactive environment for doing Euclidean, transformational, and analytical geometries. Cabri-Geometry was initially developed at IMAG, as a joint research of the department of Information Technology and Applied Mathematics Université Joseph Fourier and CNRS (the National Centre for Scientific Research), France, and it started in 1985 in order to make it easier to teach and learn geometry. Cabri-Geometry is considered the first dynamic geometry software package developed for the teaching and learning of geometry. It was presented by Jean-Marie Laborde and Franck Bellemain for the first time on ICME-6 (1988) Budapest (Hungary). In 1989, Cabri-Geometry was available on French educational market and supported by the ministry of education, then later on in many countries as MacOS and DOS version. Since 1992, Cabri-Geometry has been integrated and become compatible with TI-92 graphic calculators.

Through Cabri-Java, Cabri-Geometry has the possibility to create and export its geometric constructions into web pages on Java-enabled Web browsers.

Cinderella³ is interactive geometry software. Cinderella is designed to cover a wide range of different geometric disciplines. It provides native supports for doing Euclidean geometry, hyperbolic geometry, elliptic geometry, and projective geometry on the com-

²Laborde, J. M. and Bellemain, F. (1988). Cabri-Geometry. Online at: <http://www.cabri.com/>

³Richter-Gebert, J. and Kortenkamp, U. (2006). The Interactive Geometry Software Cinderella, Version 2.0. Online at: <http://cinderella.de>

puter. It is a joint project that has been conducted by Jürgen Richter-Gebert and Ulrich Kortenkamp since 1996 and it is a development of an earlier project carried out by Jürgen Richter-Gebert and Henry Crapo between 1993 and 1995. The latest version of the software, Cinderella.2.0, consists of three main environments: Cinderella – *an interactive dynamic geometry environment* for doing geometry, *CindyLab* – an experimental physics simulation environment for doing interactive physics experiments, and *CindyScript* – a high-level programming environment for fast flexible and freely programmable interactive scenarios. The three environments can be used almost independent from each other; however, Cinderella has its full power/potential when the three environments are integrated (Richter-Gebert and Kortenkamp, 2006). Cinderella has the possibility to easily publish and export geometric constructions into web pages on Java-enabled Web browsers.

Compass and Ruler⁴ (also known as C.a.R.) is dynamic geometry software simulating compass and ruler on the computer. C.a.R. is the short form of Compass and Ruler as construction tools or Construct and Rule as user's roles. René Grothmann is the designer of this software. It is a free and open source available under the GPL license. It can be used as a tool for constructing and exploring Euclidean and non Euclidean geometries. C.a.R. is a java-based program running on many platforms. C.a.R. has the possibility to export the geometric constructions and assignments into web pages on Java-enabled browsers.

Euklid DynaGeo⁵ is a Windows program for doing geometry on the computer. It was originally developed in the mid 90's as a German program, and later on in 1997 was available in English. Roland Mechling is the main designer of the program. Euklid DynaGeo has many features starting from the possibility to create geometric objects (e.g., points, segments, circles, polygons) going through the built-in constructions (e.g., perpendicular, bisector, perpendicular bisector) ending by the built-in transformations (e.g., reflection, translation, rotation, dilation). Euklid DynaGeo also has the possibility

⁴Grothmann, R. (2004). C.a.R. – Compass and Ruler. Online at: http://zirkel.sourceforge.net/doc_en/index.html

⁵Mechling, R. (1994). Euklid DynaGeo. Online at: <http://www.dynageo.com/>

to export its geometric construction into web pages using its own viewer, which is called Euklid DynaGeo X viewer. It is not a java applet but rather it is an Active X control.

GeoGebra⁶ is dynamic software that connects geometry, algebra, and calculus. Markus Hohenwarter was the main designer of the software in 2001/2. GeoGebra is written in Java and thus available for multiple platforms. It allows users to do constructions with points, vectors, segments, lines, conic sections as well as functions and change them dynamically. Equations and coordinates can be entered directly. Thus, it can deal with variables for numbers, vectors and points and find derivatives and integrals of functions. It is possible to publish and export the constructions made by GeoGebra into web pages on Java-enabled browsers.

Geometer⁷ is dynamic geometry software that runs on Macintosh or Windows for doing geometry. Tom Davis is the designer of the program. Geometer has two amazing features. The first is the *proof mechanism* feature that enables teachers to prepare step-by-step proofs that can be used to lead students gradually through the proof. The second is the *test diagram* feature that reports and shows many geometric relationships about the configuration that include reporting points on a line, concurrent lines, triangles of equal areas, equal angles, supplementary angles, equal length segments, and ratios.

The Geometer's Sketchpad⁸ is interactive dynamic geometry software for exploring Euclidean geometry, algebra, calculus and other areas of mathematics. The Geometer's Sketchpad is considered one of the first two software packages that could be used for dynamic geometry which first appeared in 1989, and Cabri-Geometry, dating back to 1988 (Kortenkamp, 1999, p. 17). The Geometer's Sketchpad began as an outgrowth of the Visual Geometry Project (VGP) at Swarthmore College, headed by Eugene Klotz and Doris Schattschneider in 1985 (Scher, 2000, p. 42). Nicholas Jackiw was the main designer and programmer of the Sketchpad. After several years of development in the

⁶Hohenwarter, M. (2001). GeoGebra. Online at: <http://www.geogebra.org/cms/>

⁷Davis, T. (2002). Geometer. Online at: <http://www.geometer.org/geometer/index.html>

⁸Jackiw, N. (1989). The Geometer's Sketchpad. Online at: <http://www.keypress.com/x5521.xml>

academic setting, The Sketchpad underwent a year of field-test in schools across the United States, and in 1991, the first commercial Macintosh version was released. In 1993, the Sketchpad version for Microsoft Windows was produced. In 1995, a third version extended the scope of the program from Euclidean and transformational geometry to include analytic geometry. More significant changes came in 2001, when the fourth version of Sketchpad extended dynamic geometry to the teaching and learning of algebra and calculus as well.

The Geometer's Sketchpad needs additional software called JavaSketchpad (JSP) in order to create and publish sketches from the Geometer's Sketchpad on the Internet.

In addition to the dynamic geometry software packages mentioned and surveyed above, there are many others for doing and exploring geometry on the computer. For a list of other dynamic geometry packages see the link: http://en.wikipedia.org/wiki/Dynamic_geometry_software. What the present researcher would like to emphasize here is that even though there is a wide range of different types of dynamic geometry software, there is no significant difference between them if one needs only basic compass and straightedge constructions.

2) Examples of 3-dimensional Packages

Cabri 3D⁹ is an interactive software package for doing geometry in three dimensions, in which a 3D environment containing objects such as points, lines, planes, and polyhedra is represented on a 2D screen. Jean-Marie Laborde presented it for the first time at CabriWorld in Roma in September 2004. Cabri 3D can run on Mac and Windows platforms. The last version, Cabri 3D v2, includes both numeric and geometric tools and unique 3D visualization tools. The key features of Cabri 3D can be summed up in the following points:

- Create solid geometric construction with just a few clicks of the mouse.
- Integrate numeric data using measurements and calculation tools.

⁹Laborde, J. M. (2004). Cabri 3D. Online at: <http://www.cabri.com/>

- Manipulate and animate constructions and reshape objects using only the mouse.
- Print out patterns from virtual constructions and transform them into real objects.

In addition to Cabri 3D as an example of the 3-dimensional packages for constructing, manipulating and exploring space geometry, there are many others such as: Archimedes Geo3D¹⁰, Geometria¹¹, Geomview¹², just to mention some. The reader might use their URLs inserted in footnote to check their own features.

2.1.4 Purposes and Functions of Dynamic Geometry Software

Dynamic geometry software has a variety of purposes and functions. A big body of literature and studies pertaining to DGS focus on its uses as a mediation tool for performing geometric constructions and transformations that support the inquiry approach in teaching and learning mathematics and encourage students to use problem solving and posing processes such as modeling, conjecturing, experimenting, and generalization (Christou *et al.*, 2005). There is also emphasis on its characteristics in providing a window onto the students' mathematical meaning that enables deeper discussion and directed mediation (Budge, 1999, p. 86). In more detail, the researcher surveys these purposes and functions in the following framework:

1) DGS as a Construction Tool

Dynamic geometry software provides an accurate constructor for creating geometric configurations and has the ability to automatically adjust and preserve the variant and invariant properties of constructed geometric configurations under dragging in a visual, efficient, and dynamic manner.

¹⁰Archimedes Geo3D. Online at: <http://www.raumgeometrie.de/drupal/en>

¹¹Geometria. Online at: <http://geocentral.net/geometria/>

¹²Geomview. Online at: <http://www.geomview.org/>

2) Measurement

Most of the dynamic geometry software packages have facilities of measurements that can be used to measure the distance between two points, the measurement of angles, and the areas of polygons as well as automatically updating these measurements under dragging of free elements.

3) DGS as a Visualization Tool

Dynamic geometry software, as a visualization tool in mathematics classrooms, can help students to visualize mathematical and geometric situations. In this concern, research indicated that uses of the dynamic geometry environment allow mathematical notations to become visible to students (Budge, 1999, pp. 86-94). Christou *et al.* (2005) in their research observed that DGS helps students to construct accurate images of the geometric situations and also helps them to visually explore the geometric situations and reflect on them. In addition, the extracted images from the constructed images enable students to explore at a perceptual level and make conjectures about the geometric situations. (Patsiomitou, 2008, p. 386) also asserted “creating visual mathematical representation in the software can make it easier for students to grasp the relevant concepts by allowing the continuous manipulation of mathematical objects in real time”.

4) Exploration and Discovery

Dynamic geometry is considered as an ideal tool in mathematics classrooms for exploration and discovery through utilizing its special facilities of constructing, dragging, measuring, and calculating. Constructing and dragging are two important features embedded in all dynamic geometry software that provide students with not only one case of geometric configuration but also many cases of this configuration that can enable students to explore and make mathematical conjectures. Measuring and calculating features can then enable students to check the correctness of their conjectures during exploration and discovery processes. Furthermore, using these facilities of DGS during

the exploration and discovery processes allow students to consider not only special or general cases of geometric configuration but also extreme cases, cases that textbook authors never imagine (Christou *et al.*, 2005, pp. 136-137). Accordingly, using DGS can help teachers to implement an inquiry approach for teaching and learning mathematics. In this respect, Contreras (2003) also recommended “All students should be engaged in discovering mathematical ideas on their terms and DGS can facilitate such discoveries within geometric tasks”.

5) DGS as a Modeling Tool

Dynamic geometry software can efficiently be used for mathematical modeling and simulation in the field of teaching and learning of geometry (Schumann, 2000). Mathematical modeling and simulation with DGS is an important example of the usage of computer tools in geometry since it opens up a wide range of creative activities for both teachers and students (Schumann, 2003).

The different embedded facilities in dynamic geometry software, which allow the user to perform calculations on measurements and use such results to plot the coordinates of a point on specific axes, open the gate for mathematical modeling using such software in mathematics education (Oldknow, 2003, p. 16). Furthermore, an interesting form of modeling becomes available in DGS by virtue of its possibility to import a picture and construct axes and graphs on it, which allows the users to model captured images either geometrically or algebraically or both (Oldknow, 2003, pp. 16-19).

6) Experimentation

The use of dynamic geometry software in mathematics classrooms opens new horizons for experimentation and could turn mathematics into a laboratory science (Olive, 2000, p. 17), in which students’ main duties will be observing, recording, manipulating, predicting, conjecturing and testing, and developing theory to explain the phenomena concerned (Man and Leung, 2005, p. 2).

For using DGS in mathematics experimentation, Friedrich (1999) articulated a working scheme (experiment, guess, test, and proof) supported with an example that designs an experiment of the locus of the heights intersection points of a triangle. For further research about the use of DGS in mathematics experimentation, refer to (Haja, 2005), (Kortenkamp, 2004), and (Man and Leung, 2005).

7) DGS as a Problem Solving and Problem Posing Tool

Dynamic geometry software is considered as a tool that mediates students strategies in solving and posing problems (Christou *et al.*, 2005). Christou *et al.* further contend “In the DGS environment the problem solving processes involve the generation of new problems, supporting the relationship between problem-solving strategies and the tendency to pose extension problems”.

Contreras (2003) also states that the constructing, dragging, measuring, and calculating facilities of the dynamic geometry software can enable students to make and investigate mathematical conjectures, solve problems, and pose related problems. Moreover, Budge (1999) concludes that the use of the dynamic geometry environment generally encourages investigation and problem solving in geometry.

8) Transformation

Most dynamic geometry software allow several kinds of geometric transformations (reflections, translations, rotations, and dilations, among others) that can perform on figures or their parts in the front of students’ eyes (Schattschneider and King, 1997). Man and Leung (2005) provided two illustrative examples on how to use dynamic geometry software in teaching transformational geometry in an undergraduate mathematics course, in order to deepen the student teachers’ understanding of the concepts behind. Olive (2000) points out, in implications of using dynamic geometry technology, the importance of dynamic geometry in investigating transformational concepts in different levels and grades and constructing understanding of mental transformational

images.

9) Proof

Even though DGS cannot actually produce proofs in deductive logical steps, the interactive, dynamically visual evidence it provides produces a strong conviction that can motivate the desire for proof (Schattschneider and King, 1997). In the same concern, Kortenkamp (2004) articulates that the experimental techniques of computer software (including DGS) could supplement, enrich, and enhance the proving process. He also displays how these experimental techniques can be used to motivate proofs, eliminate proofs, make proofs understandable, and find proofs. Furthermore literature (Patsiomitou, 2008, p. 386) asserts that “the software tools can assist cognitive functions with a view to analyzing the data and composing the solution and make it easier for students to reach decisions and teachers can thus improve their students’ knowledge by eliciting mental schemas from them, which students can be guided to reach conclusions which form a step-by-step visual proof”.

In terms of van Hiele geometric thinking levels and the logical proving process, Jones (2002) points out that such use of DGS does help students to connect between the informal explorations and logical deductive arguments, which in turn help them in writing proof argumentation.

2.1.5 The Role of DGS in Geometry Education

The role of DGS in the teaching and learning of geometry is surveyed in terms of three main domains: (1) The role of DGS in developing geometric thinking, (2) The role of DGS in developing the academic achievement, and (3) The role of DGS in developing the affective domain.

1) The Role of DGS in Developing Geometric Thinking

There is no doubt that a central aim of geometry education is to raise the students' geometric thinking levels. In this context, Two Dutch mathematics educators, Pierre van Hiele and Dina van Hiele-Geldof, developed a pedagogical theory describing levels of thinking in Euclidean geometry that students pass through as they develop their understanding of new geometric topics. The van Hiele theory originated in the doctoral dissertations of Pierre van Hiele and his wife Dina van Hiele-Geldof in 1957.

The van Hiele Theory of Geometric Thinking¹³

The van Hieles originally numbered the levels from 0 to 4. Since their work has been translated into English the levels are often numbered from 1 to 5. Additionally, multiple terms are used to describe some levels. For clarity and for a reason to be mentioned later, the present researcher used the 1-to-5 numbering scheme and multiple terms to describe levels as needed.

Level 1 (Visualization or Recognition)

Students recognize geometric shapes by their general appearance, often through comparing them by prototype. At this level, students are not able to explicitly identify the properties of geometric shapes and make decisions based upon perception rather than reasoning.

Level 2 (Analysis or Descriptive)

Students begin to analyze and name the properties of geometric shapes, but they do not see the relationships between these properties. At this level, students are not able to identify which properties are necessary and which are sufficient to indicate or describe the geometric object.

Level 3 (Abstraction, Ordering, Informal Deduction or Argumentation)

Students can see the relationships between the properties of geometric shape and be-

¹³ The van Hiele theory has been investigated and validated by numerous studies (e.g., Clements and Battista, 1991; Johnson, 2002; Mason, 1998; Usiskin, 1982) and the researcher made use of them to elaborate on this theory.

tween geometric shapes in order. At this level, students can create meaningful definitions and make brief deductive arguments but the role and significance of formal deduction, however, is not understood.

Level 4 (Deduction, formal Deduction or formal logic)

Students can construct formal deductive proofs and understand the role of axioms, definitions and theorems in deduction. At this level, students can specify the necessary and the sufficient conditions, which indicate or describe the geometric object.

Level 5 (Rigor)

Students can compare different mathematical deductive systems based upon different axioms. At this level, students can understand the use of indirect proof and proof-by-contradiction and can understand non-Euclidean geometry.

Literature also proposes the existence of “Level 0” which is called “pre-recognition”. At this level, students notice only a subset of the visual characteristics of a geometric shape. For this point of view the present researcher prefers to use the 1-to-5 schemes because this scheme allows adding the pre-recognition, which is called level 0.

Other Aspects of the Theory

According to the van Hiele theory, the learner must go through the levels in order, and cannot achieve one level without mastering all the prior levels. Movement from one level to the next is more dependent on instruction than on age or maturation. Instruction can accelerate movement both within a level and to a higher level. The instruction leading to complete understanding should have five phases of learning entitled:

- Information
- Guided orientation
- Explication
- Free orientation
- Integration

Language plays an important role in instruction. Namely, each level of thought has its own linguistic symbols and its own network of relationships connecting those symbols. Thereby, for effective classroom communication, teacher and student should talk and discuss geometric concepts at the same level of thought. Moreover, the teacher should ensure that students are interpreting concepts correctly.

The Role of DGS in Developing Geometric Thinking Levels

Regarding the role of DGS in developing geometric thinking levels, literature indicates that dynamic environments help students analyze dynamic shapes to discover their characteristic properties and build mental models for thinking about shapes. Hence, using such software encourages students to move to higher levels of geometric thinking and involves them as conceptual participants, not passive spectators, in the process of doing geometry (Battista, 2002, p. 339).

Furthermore, findings of Patsiomitou's study reveal that the use of DGS promote students' van Hiele levels of geometric thinking through the combination of linking visual active representation and reflective visual reaction in the prepared activities with the questions asked during the software procedure.

In the same concern, Gawlick (2004) introduces some dynamic construction activities to illustrate how different van Hiele geometric thinking levels correspond to the use of the three-main features (tools) embedded in any DGS; namely drag mode, macro-constructions, and the locus of points. Moreover, Gawlick recommends "we utilize the dynamic tools for advancing to higher levels . . . the drag mode is the key to advance from level 1 to level 2. Macro and loci support passing on from level 2 to level 3. Families of loci can be used to progress from level 3 to level 4". Battista (2002) also introduces some activities using the Shape Makers micro-world to stimulate the development of higher levels of geometric thinking.

On the other hand, Johnson's experimental study (2002) results indicate that using DGS, The Geometer's Sketchpad, does not improve the van Hiele levels among high school geometry students. Johnson also concludes that a longer period of time using

DGS is needed for students to make significant gains in the van Hiele levels. So, based on the mixed results regarding the role of DGS in developing van Hiele levels of geometric thinking the present researcher recommends further research in this concern.

2) The Role of DGS in Developing the Academic Achievement

With respect to the role of dynamic geometry in developing students' academic achievement, related studies¹⁴ have shown mixed results regarding this issue. Studies (e.g., Almeqdadi, 2000; Chomskis and Hinkle, 2006; Hull and Brovey, 2004) show that there are positive effects on students' academic achievement performance due to the use of DGS, whereas others (e.g., Abu Bakar *et al.*, 2009; Johnson, 2002) reveal that there is no effect or negative effects of the use of DGS on students' academic achievement performance.

These mixed reviews of the effectiveness of the use of DGS on enhancing the academic achievement performance would be traced back to two main research limitations. The first limitation is the time duration needed for the students to be able to learn and explore using DGS. Namely, the time duration might be one of the factors that negatively affect the use of DGS. The second limitation is represented in the students' comfort and familiarity with using computers in general and DGS in particular. That is, it should be firstly ensured that students are familiar and comfortable with the use of computer; and secondly, students should be also familiar with whatever DGS used before conducting the intervention.

These two limitations, the time duration and the familiarity and comfort of students with the use of computer and software and their impact on the students achievement, are supported in the studies of Johnson (2002) and Abu Bakar *et al.* (2009). That is, The results of Johnson's study (2002) indicate that a long period of time using DGS is needed for students to make significant gains (Johnson, 2002, p. 111) and the results of Abu Bakar *et al.* (2009) study also indicate that "once the subjects are familiar and have the opportunity to explore the software, then the subjects will not be overly anxious

¹⁴For more details: See Chapter 3. Related Studies

into wanting to concentrate on too many new things at the same time. The subjects then have to concentrate on the content that is supposed to be learned” (Abu Bakar *et al.*, 2009, p. 25).

3) The Role of DGS in Developing the Affective Domain

Numerous studies have investigated the role of dynamic geometry in developing students’ affective domain. These studies have investigated the issue from two different perspectives. Some studies (e.g., Chomskis and Hinkle, 2006) emphasize investigating how the use of DGS would influence students’ attitudes towards geometry or mathematics while others (e.g., Abu Bakar *et al.*, 2009; Hull and Brovey, 2004) investigate the attitudes of students towards the use of DGS itself as a teaching and learning approach.

Abu Bakar *et al.* (2009) investigate the effectiveness of the use of DGS on the students’ attitudes towards learning in a dynamic geometry environment in terms of four dimensions: enthusiasm, enjoyment, anxiety, and avoidance. The findings reveal that there are positive attitudinal changes in the four detected dimensions of students’ attitudes, which are traced back to the use of DGS.

In another study Hull and Brovey (2004) conclude that the use of DGS provides students with a special type of exploration, which is more convincing than a formal proof. Also, within the same study, even though students’ responses on the attitude questionnaire show that students gain more enjoyment from lessons in the computer lab than traditional lessons that are more teacher-centered, results did not indicate that the use of DGS positively or negatively impacted the students scores.

Another study Chomskis and Hinkle (2006) found that students were very engaged in learning during the use of DGS and enjoyed the lessons. Additionally, the findings show that the attitudes of the class changed after experiencing the use of dynamic software and students wanted to be involved in learning using dynamic software.

2.2 The Mathematically Gifted Education

In this part of chapter 2, the present researcher briefly outlines important concepts and theories related to the field of the mathematically gifted education. A brief outline is provided about the nature of giftedness and the difference between gifts and talents. Then, some definitions and indicators of mathematical giftedness are given. This is followed by a description of the characteristics of the mathematically gifted and talented students concerning three of their behavioral domains: cognitive, affective, and creative. Some educational provisions for the mathematically gifted education are presented. Some teaching and learning models and strategies that are designed to cater for the gifted students' potentials and needs are described. This part ends by briefly outlining the general principles of developing enrichment programs for the mathematically gifted students and classifying them under four categories in terms of program aims, content, instruction, and assessment.

2.2.1 The Nature of Giftedness

Understanding the nature of “giftedness” or “gifted” is a problematic and complex issue like many other psychological and educational hypothetical constructs (e.g., learning, motivation, creativity) in the field of behavioral sciences. There is no single accepted perspective or model of giftedness. Rather, there is a wide range of perspectives and models that have been suggested, dealing with the term giftedness and looking into how it is defined, measured, and developed. Amongst these perspectives and models of giftedness are Lewis Terman view of giftedness, Renzulli's three-ring conception of giftedness, and Gagné's differentiated model of giftedness and talent. In this section, the notion of giftedness is surveyed within the framework of three main perspectives showing how the meaning of giftedness evolved from being a high performance at intelligent tests in Terman's view to a predictor of individual's creative achievement or production resulting from the interaction of three psychological clusters of individuals traits in Renzulli's theory of giftedness to a natural ability (or potential) that must be nurtured

according to Gagné's model.

1) Lewis Terman View of Giftedness

In the early of 1920s, Lewis Terman at Stanford University, in his longitudinal study of gifted children, defined gifted children as children with IQs exceeding 140. The results of his study showed that the subjects had superior intelligence, health, social adjustment, and a moral attitude (El-Tantawy, 2001, p. 12). Moreover, the results indicated that depending on IQ alone could not predict success in the children's future, which led to the conception that giftedness should not only depend on IQ, but should also include a predictor of individuals' creative-productive achievement or gifted behavior and that was the fundamental base of Joseph Renzulli's view of giftedness coming in the next section.

2) Joseph Renzulli's Three-Ring Conception of Giftedness

Joseph Renzulli's three-ring conception of giftedness is a theory that attempts to portray the main dimensions of human gifted behavior. In his theory of giftedness, Renzulli posits two types of giftedness: schoolhouse (academic) giftedness and creative-productive giftedness. Schoolhouse giftedness is the form of giftedness most often emphasized in school. In other words, it is test-taking or lesson-learning giftedness. On the other hand, those who display creative-productive giftedness are excellent producers of knowledge; in contrast those high in schoolhouse giftedness are superior consumers of knowledge. Therefore, Renzulli, in his theory, focuses on the development of the creative-productive giftedness (Kaufman and Sternberg, 2007, p. 389). In this theory, Renzulli posits that the creative-productive gifted individual possesses three interactive clusters of psychological traits: well-above-average ability, task commitment, and creativity and their relationships with general and specific areas of human performance (see Figure 2.1)¹⁵ (Renzulli, 2005, pp. 246-279).

¹⁵The graphical representation of "Renzulli's Three-Ring Conception of Giftedness" in Figure 2.1 is taken from: Giger, M. (2007). Renzulli's three-ring conception of giftedness. Retrieved October 21, 2009, from: http://www.gigers.com/matthias/gifted/three_rings.html



Figure 2.1: *Graphical Representation of Renzulli's Three-Ring Conception of Giftedness*

Well-Above-Average Ability

Renzulli defines the well-above-average ability in two ways: general abilities and specific abilities. **General abilities** consist of traits that can be applied across all domains or broad domains. Examples of these abilities are verbal and numerical reasoning, spatial relation, memory, and word fluency.

Specific abilities consist of the capacity to acquire knowledge, skill, or the ability to perform an activity of specialized kind of human experience; for example, arts, leadership, administration, mathematics. Through these abilities, individuals can express themselves in the life.

Task Commitment

With regard to task commitment as the second cluster of individual's traits in the ring, it can be described as motivation turned into actions (e.g., perseverance, endurance, hard work, dedicated practice, self-confidence, confidence in one's ability to execute important work, and a special fascination with a special subject). According to this view, high achievement is not possible without task commitment (Giger, 2007).

Creativity

Creativity is the third cluster of traits that is required for the development of creative-productive gifted behavior. The creativity traits are represented in fluency, flexibility,

and originality of thought, openness to experience, sensitivity to stimulations, and a willingness to take risks.

According to Renzulli, each cluster plays an important role in the development of gifted behaviors. The existence of one of these cluster traits is not sufficient for emerging gifted behavior; only if trait clusters from all three rings **work together** can high achievement or gifted behavior be witnessed (Giger, 2007).

The main criticism to Renzulli's theory of giftedness is that it strongly emphasize creative-productive achievement or performance as an indicator of giftedness and fails to explain the notion of giftedness in the case of the gifted underachiever – the individual who may have well above average ability but who, for whatever reason, has not yet been able to translate this into above average performance (Gagné, 1985, p. 105). This criticism leads to the differentiated model of Gagné that recognizes the existence, and the dilemma, of the gifted underachiever and avoids this problem by defining giftedness as outstanding potential rather than outstanding performance (Gross *et al.*, 2005, Core Module 1, p. 4).

2) François Gagné's Differentiated Model of Giftedness and Talent (DMGT)

François Gagné's differentiated model of giftedness and talent posits a clear distinction between giftedness and talent. In this model, the term giftedness refers to the possession and use of untrained natural abilities, which are called aptitudes or gifts, in at least one ability domain, to an extent that places an individual at least among the top of 10% of his age-peers. On the other hand, the term talent refers to the superior mastery of well-trained systematically developed abilities and knowledge in at least one field of human activity or performance to an extent that places an individual's achievement within at least the top 10% of his age-peers, who are, or have been, active in that field or fields (Gagné, 1985, 2004, pp. 103-112).

Gifts

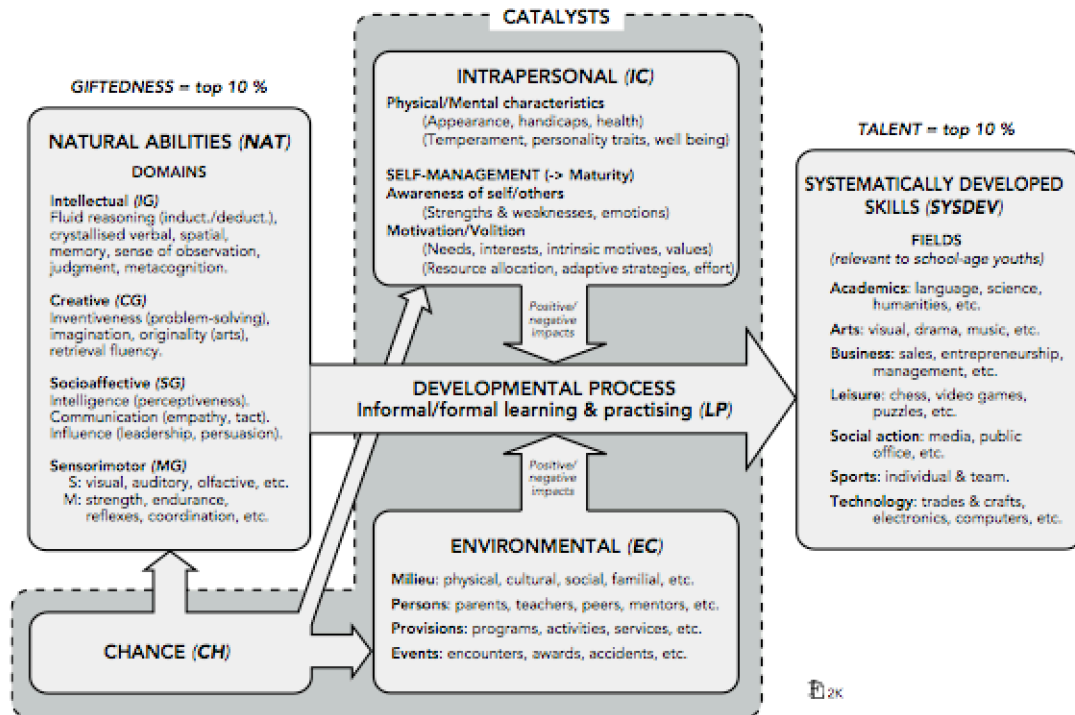


Figure 2.2: Graphical Representation of Gagné's Differentiated Model of Giftedness and Talent

Gagné defines four sets of aptitude domains (see Figure 2.2)¹⁶ that have a clear genetic basis. These aptitude domains are (Gagné, 2004):

- Intellectual abilities that are needed to learn to read, speak a foreign language, understand mathematical concepts, etc.
- Creative abilities that are needed to generate many different types of responses toward the existed problems and produce original work in science and arts.
- Socioaffective abilities that children use daily in interactions with classmates, teachers, and parents.

Talents

According to the model, the natural abilities or aptitudes are considered the raw compo-

¹⁶The graphical representation of “Gagné’s differentiated model of giftedness and talent” presented in Figure 2.2 is taken from: Gross, M., MacLeod, B., Bailey, S., Chaffey, G., Merrick, C., and Targett, R. (2005). *Gifted and talented education: Professional development package for teachers*. Gifted Education Research, Resource and Information Center, The University of New South Wales, Australia. (Core Module 1, p. 5)

nents of talents and these talents progressively emerge from the transformation of these aptitudes into well-trained and systematically developed skills pertaining to a particular field of human activity. Therefore, the process of talent development manifests itself when the individual engages in learning, training, and practicing.

Developmental Process

The model presents four sets of inner and outer catalysts that facilitate or hinder the developmental process of the talent. They include intrapersonal catalysts, environmental catalysts, chance, and learning/practice (Kaufman and Sternberg, 2007, p. 392).

Intrapersonal Catalysts

The intrapersonal catalysts include five main characteristics of the person. They include:

- Physical characteristics (e.g., appearance, handicaps, health).
- Motivation (e.g., needs, interests, values, intrinsic motives).
- Volition (e.g., resource allocation, adaptive strategies, effort, will power, persistence).
- Self-management or awareness of self (concentration, work habits, initiative, scheduling).
- Mental characteristics or personality (temperament, self-esteem, adaptability, well-being).

Environmental Catalysts

The model contains four distinct environmental factors that influence the development of talents. They include:

- Milieu or social environment (society, culture, family, etc.).
- Persons (parents, teachers, peers, mentors, etc.).
- Provisions (programs, activities, services, etc.).

- Events (the death of a parent, winning a prize or award, suffering a major accident or illness, etc.).

Chance

Associating it with the four environmental catalysts mentioned before, Gagné considers chance as a fifth significant factor in the developmental process of talents, which include the socioeconomic environment in which children are born and grow up, the quality of parenting they receive or the transmission of hereditary characteristics.

Learning and Practice

Learning and practice are assumed to take four different forms in this model. They include maturation, informal learning, formal noninstitutional learning, and formal institutional learning.

In conclusion, the surveyed perspectives and models of the notion of giftedness show how its meaning has been developed from a unidimensional view, emphasizing only the individual's intellectual traits as represented by a high intelligent quotient measurement ($IQ > 140$) according to Lewis Terman view of giftedness, to a multidimensional view encompassing other individual's psychological and personal traits while interacting with the surrounding environmental factors added to the individual's intellectual traits as they are described by Renzulli and Gagné.

2.2.2 The Nature of Mathematical Giftedness

In the literature, there are many definitions of “mathematical giftedness”; but according to Bicknell (2009) it seems there is no commonly accepted single definition of mathematical giftedness. However, many attempts have been undertaken to define mathematical giftedness based on one or more of its characteristics or qualities.

One such definition, which emphasizes having high performance on mathematics standardized tests as a quality of mathematical giftedness and which has been widely used in the United States in many school districts, defines the mathematically gifted and

talented children as “the 3 to 5 percent of top-scoring students on some standardized tests” (Sheffield, 2000, p. 416). This definition, as Sheffield criticizes, does not take into consideration a wide range of mathematical abilities that are fundamental in recognizing and nurturing young mathematicians (Sheffield, 2000, p. 416).

In Mohamed’s study (2003), two qualities of mathematical giftedness are adopted so as to identify the study subjects – the mathematically gifted students. One quality entails having high scores on intelligent test measurements. Another quality pertains to having high performance in mathematics achievement tests. Namely, in Mohamed’s study (2003), the mathematically gifted students are considered “those who have achievement scores exceeding 90% in their previous mathematical test and IQs more than 120”.

There are two other qualities of mathematical giftedness; namely, the broader nature of mathematical abilities and nurturing different aspects of mathematical thinking. These two qualities are the bases of a definition concluded from the literature in (Bicknell, 2009, p. 11). The definition considers the mathematically gifted students as “those who have special mathematical abilities or those who engage in qualitatively different mathematical thinking”.

Another definition of the mathematically gifted students is adopted in Choi and Do’s (2008). It focuses on having both a creative-productive performance and high interests in mathematical exploration processes. It also marks out the mathematically gifted students as “those who have talent or potential to get creative achievement in mathematics and a strong desire for mathematical exploration” (Choi and Do, 2008, p. 33).

To sum up, the previously reviewed definitions of mathematical giftedness suggest a collection of qualities or characteristics that can be used as indicators of mathematical giftedness, as shown below. The mathematically gifted students are those who have:

- High performance on mathematics standardized tests or other regular academic achievement tests in the field of mathematics.
- High IQs on intelligent test measurements.

- Special abilities or potentials in the field of mathematics.
- High interests in mathematical exploration processes.
- Ability or potential to get creative-production achievement in the field of mathematics.
- Successful involvement in different mathematical thinking activities.

Additionally, Rotigel and Fello (2004), by reviewing the related literature in the field of mathematically gifted education, provide another collection of characteristics of mathematical giftedness. They characterize the mathematically gifted students as individuals who:

- Are often able to pick out answers of mathematical problems with unusual speed and accuracy.
- Can see relationships among topics, concepts, and ideas without formal instruction intervention.
- Have intuitive understanding of mathematical functions and processes that enables them to skip over steps to find an answer.
- Often want to know more about the “how” and “why” of mathematics ideas than the computational “how-to” processes.
- Prefer to learn all they can about a particular mathematics idea before leaving it for a new concept.
- Get frustrated when the regular classroom schedule demands moving to another activity.

Adapting Renzulli’s notion of giftedness, Velikova, Bilchev, and Georieva (2004, p. 84), in their comparison of schoolhouse (academic) giftedness and creative-productive giftedness in mathematics, provide another collection of characteristics of creative-productive giftedness in mathematics. It includes that the creative-productive individuals are those who:

- Think originally but not quickly.
- Slowly build step-by-step reasoning and result is harmonious mathematical theory.
- Are not able to solve easy problems with limited time but are able to solve very hard problems in unlimited time.
- Have solid mathematical interests.
- Have the ability for overall evaluation of the problem, investigate many facts in the mathematical area of interest, and create original problems.
- Have the capacity to work on interesting problems within a long period of time.

Additionally, in the same study of Velikova, Bilchev, and Georieva, they provide a diagnostic technological approach to identify the creative-productive gifted students in the field of mathematics in terms of three parameters of mathematical giftedness, as follows (Velikova, Bilchev, and Georieva, 2004, pp. 85-86):

1. Measuring above average general abilities by IQ. IQ has to be more than 80%.
2. Measuring creativity by CQ test (Mensa). CQ has to be more than 70%.
3. Measuring above average mathematical abilities by combining both assessments of teachers and experts. The sum of assessments has to be more than 50%.

Surveying these collections of definitions and indicators of mathematical giftedness, the present researcher was able to put forward a definition for the mathematically gifted students that fits the purposes of the study. The mathematically gifted students, in this study, are those who are determined by highly qualified and professional persons, and those students have high mathematical abilities and the ability to set a high performance in solving mathematical problems.

2.2.3 Characteristics of the Mathematically Gifted Students

In this section, the present researcher outlines some of the mathematically gifted and talented students' characteristics, articulated in the prior studies and literature, con-

cerning three of their behavioral domains: cognitive, affective, and creative.

1) Cognitive Characteristics

Depth of Understanding

The mathematically gifted and talented students differ from the regular students in their depth of understanding (Johnson, 2000, p. 2) of mathematical concepts and mastery of mathematical skills as key indicators of achievement in mathematics. Their depth of understanding of a range of mathematical concepts and ideas enable them to know how and when to use these mathematical skills and knowledge. That is, they differ in their ability to apply their mathematical knowledge and skills to unfamiliar situations, and their preparedness for further study.

Mathematical Reasoning and Generalization Abilities

Sriraman's study (2003), in its comparison of gifted and non-gifted students' behaviors regarding generalization and reflection, concludes that the mathematically gifted students showed outstanding abilities in forming abstraction and generalization; specifically, in identifying similarities in the structure and problem solutions, in using analogical reasoning, and in verbalizing common principles (Sriraman, 2003, p.158). Moreover, in the same study, Sriraman illustrated that the mathematically gifted students showed competence in conjecturing, examining examples and non-examples, and abstracting structural similarities in the problems and solutions.

Problem-solving Abilities

With respect to problem-solving abilities, the mathematically gifted students differ from the general group of regular students in diverse abilities of problem solving, which include problem finding and posing abilities, and problem solving strategies – how they approach the solution of the problem. In this concern, Lee and Hwang (2005) experimentally found that the mathematically gifted students do not only differ from the regular students in problem solving abilities but also differ in problem finding and posing abilities (Lee and Hwang, 2005, p. 3).

Research findings further indicate that the mathematically gifted tend to use diversified and flexible strategies while solving non-routine mathematics problems (e.g., making representation, drawing figures, working backward, introducing auxiliary elements, and attempting mistakes) (Huang, 2003).

Additionally, results of Sriraman's study (2003) concerning comparison of the gifted and non-gifted problem-solving behaviors indicate that the mathematically gifted students differ from the non-gifted in the four phases of problem solving: orientation, organization, execution, and verification (Sriraman, 2003, p. 157).

One more characteristic of the mathematically gifted concerning the phases of problem-solving processes, as yielded in the findings of Huang's study (2003), is that the mathematically gifted were able to provide correct solutions to non-routine mathematical problems without going through the verification phase of problem solving.

Geometric Thinking Abilities and Proofs

Due to van Hiele levels of geometric thinking for the mathematically gifted, Literature (Mason, 1998, p. 5) indicates that the mathematically gifted students appear to skip levels, maybe because they develop logical reasoning skills in different ways than those of regular students.

Experimental research emphasizes this point of view about the mathematically gifted students. In a study conducted by Mason (1995) to investigate geometric thinking according to van Hiele model of 120 gifted students in the sixth through eighth grades. Results showed that some of the gifted students, 38.5% of the subjects, skipped levels in van Hiele's model, because of their capability of handling inclusion on relationships if they have suitable definition of the elements. In the same study of Mason (1995), it is further concluded that the gifted students demonstrate higher overall van Hiele levels of geometric thinking than regular students.

Moreover, in the same concern of geometric thinking and mathematical proof, research (Choi and Do, 2008, p. 44) showed that the mathematically gifted students have the ability to provide extraordinary proofs and intuition to understand mathematical struc-

ture and proof.

2) Affective Characteristics

Concerning the affective domain, in general, many clear evidences have been articulated in prior studies related to the domain of gifted education. These evidences indicate that the mathematically gifted students hold affective characteristics such as concentration and task focus in problem-solving processes as well as positive attitudes towards mathematics (confidence and preference) in comparison to their peers of regular students (Choi and Do, 2008, p. 40).

Hui-Chin's study (2004) also investigated another set of affective characteristics of the mathematically gifted students. These characteristics include: interests in problem solving, emotions to mathematics tests, reasons to like mathematics, emotions to face challenges, confidence in problem solving, and response to the difficulty level of problem.

In another study, Huang (2003) aimed at analyzing the affective characteristics of the mathematically gifted while solving non-routine mathematical problems. Results indicated that the subjects of gifted students exhibited positive affective characteristics (e.g., patience and perseverance) and some other creative characteristics (e.g., personal mathematics curiosity, excitement, and confidence).

Attitudes Towards Mathematics

In terms of attitudes towards mathematics, in general, research revealed that the mathematically gifted students hold positive attitudes towards mathematics (confidence and preference) compared to their peers of regular students (Choi and Do, 2008, p. 40). Hui-Chin's study (2004) further asserted the positive attitudes towards mathematics held by the mathematically gifted students such as “free from fear of mathematics”, “enjoyment of mathematics”, and “motivation of mathematics”.

Social Adaptation

Choi and Do point out that the mathematically gifted students have much stability in social adaptation compared to their peers of regular students (Choi and Do, 2008, p. 40).

3) Creative Characteristics

Regarding the creative characteristics of the mathematically gifted students, results of experimental research indicated that the mathematically gifted students were statistically different from their peer group of regular students in the originality and abstractness components of creativity while their fluency, elaboration, and creativity index were not statistically different (Choi and Do, 2008, p. 45).

Hui-Chin's study (2004) results further asserted that the creativity of mathematically gifted students was better than that of regular students. That might be traced back to many aspects that include: openness to prior experience, tolerance of anxiety, interests in problem solving, affective pleasure in challenge, and positive confidence in problem solving.

2.2.4 Educational Differentiation for Gifted Students

Due to the fact that gifted and talented students have their own abilities, qualities, and interests; it is essential to differentiate educational practices in classrooms to provide tailored learning opportunities to cater their strengths and needs as well as help them achieve further development.

Educational differentiation means offering a variety of educational provisions for students who differ in abilities, knowledge, and skills that capitalize on their strengths and needs. In this differentiation, there should be different approaches to what students learn (content), how students learn (process), and how students demonstrate what they have learned (product) (NSW, Department of Education and Training, 2004, p. 11). In addition to these three differentiating elements, content, process, and product, the student's learning environment should be also optimized for the differentiation.

These elements of differentiation are further elaborated in (Ministry of Education, 2000, pp. 36-37), as follows:

- **Content:** What is taught or learned – The concepts, skills, information, ideas, and facts within the curriculum.
- **Process:** How the content is taught or learned – how new materials are presented, what teaching strategies and methods are used to present this content, which activities students are involved in.
- **Product:** How students' learning is evidenced.
- **Learning environment:** learning environment for the gifted and talented students should be:
 - Student-centered.
 - Open to new people, materials, and things.
 - Complex and filled with materials.
 - Flexible in all aspects of management especially scheduling.

The most two common educational differentiation provisions, associated with the education of gifted and talented students, which also provide horizontal and vertical movements from the regular school curriculum, are **acceleration** and **enrichment**. In the coming sections, both differentiation provisions, acceleration and enrichment, are presented in terms of their definitions, forms, advantages, and disadvantages.

1) Acceleration

Acceleration can be defined as an educational differentiation of a school regular curriculum based on giving students schoolwork that matches their abilities, without regard to age (King, 1996). In the light of the pre-mentioned definition, the (Institute of Research and Policy on Acceleration (2009) provide two broad categories of acceleration: content-based acceleration and grade-based acceleration. These two categories have

different forms and ways of providing accelerative options (Institute of Research and Policy on Acceleration, 2009, pp. 12-14).

Content-based Acceleration

These strategies provide students with advanced content, skills, or understandings without regard to age. Forms of content-based acceleration include (Institute of Research and Policy on Acceleration, 2009, pp. 12-14); (DeLacy, 1996):

- Single-subject acceleration, which involves offering the student an advanced curriculum in a single subject.
- Curriculum compacting, which involves skipping material that the student has already mastered.
- Concurrent or dual enrollment, enabling a student to attend more than one school at a time.
- Correspondence courses.
- Credit by examination or prior experience.
- Mentoring: individual instruction at an advanced level in a single subject offered by an expert in that subject.
- Extracurricular programs
- Advanced placement and talent search program.

Grade-based Acceleration

These strategies reduce the number of years students spend in the k-12 system. Forms of grade-based acceleration include (Institute of Research and Policy on Acceleration, 2009, pp. 12-14); (DeLacy, 1996):

- Early admission to school (Types are: early admission to kindergarten and early admission to the first grade).
- Ungraded classrooms where students of varying ages are grouped together and

the curriculum is based on individual mastery rates rather than the age of the student.

- Whole-grade acceleration (grade or year skipping).
- Self-paced instruction.
- Telescoping curriculum, which involves completing a program that usually requires a fixed number of years to finish in less than the usual time.
- Combined classes.
- Early entrance to middle school, high school, or college.
- Early graduation.
- Acceleration in college

There are several **benefits** and **advantages** of providing accelerative opportunities for the gifted students as illustrated in pertinent literature (e.g., King, 1996; Pyryt, 1998):

- Acceleration reduces the amount of time gifted students should spend in content that they already know.
- Acceleration is a way of keeping gifted students motivated and interested all the time and preventing behavioral problems caused by boredom, frustration, or anger.
- Acceleration intervention provides flexible curricular options. That is, it is possible for gifted students to combine the content of two years in one year and that will save time to cover additional areas of the subjects.
- Reducing the time needed for study and providing the flexible curricular options of acceleration have another advantage in reducing the educational costs for both the school system and the student.
- Acceleration provision is an effective way of fulfilling the academic needs of gifted students.

On the other hand, there are some **dangers** and **disadvantages** of acceleration revealed in pertinent literature reviews, which indicate that (Neihart, 2007, p. 330); (Ministry of Education, 2000, p. 38)):

- Although, acceleration has socioaffective benefits for gifted students who are selected on the basis of their demonstrated academic, social, and emotional maturity, it might be harmful to unselected students.
- In content-based acceleration, students may miss out some learning aspects related to new tasks or content and that will create gaps in their learning.
- In grade-based acceleration, moving into a higher grade with little or no adjustments made to teaching methods or materials, it may not adequately address individual strengths and interests.
- Some gifted and talented students may feel different or isolated if acceleration means removal from their well-established social, emotional, cultural peer group.
- Some teachers may feel uncomfortable or threatened by student abilities that outstrip their own.

2) Enrichment

Enrichment is an educational differentiation provision based on providing students with a variety of learning situations, materials, and activities that provide depth and breadth of learning experiences beyond those offered in a school's regular curriculum (Clark and Zimmerman, 1994, p. 9).

In the light of this definition, it seems there are two forms or types of enrichment: vertical enrichment and horizontal enrichment. **Vertical enrichment** means providing students with learning activities so as to deepen the regular curriculum according to students' abilities. **Horizontal enrichment** includes providing students with new learning activities that broaden and extend their knowledge and skills related to the regular school curriculum.

Using Enrichment as a provision to cater gifted students needs and interests has several **advantages** (Clark and Zimmerman, 1994, p. 9); (Ministry of Education, 2000, p. 39) such as:

- Carrying out enrichment programs is easy for school and school districts. That is, enrichment can provide a means for differentiated education for gifted students in schools and school districts that are not large enough to offer specialized accelerated classes.
- Enrichment is viewed as a democratic means of differentiated education because it does not call for the division of students for competitive groups.
- Enrichment may cater varied grouping of students such as students with similar abilities, similar interests, and same-age peers.

On the other hand, there are some **drawbacks** or **disadvantages** of enrichment provision, such as (Ministry of Education, 2000, p. 39):

- The difficulty of defining enrichment and distinguishing it from being extension or busy work.
- Considering the common claim about enrichment, which reveals that enrichment is good for all students, it should be examined whether enrichment is the appropriate solution to cater for the needs and interests of gifted students.

In the field of mathematics education, according to Bicknell (2009), enrichment means “broadening students’ mathematical experiences by examining mathematics outside of the prescribed curriculum and also making connections with other curriculum areas” (Bicknell, 2009, p. 35).

The present study uses the enrichment provision to provide the mathematically gifted students with some geometric activities and situations in Euclidean geometry using dynamic geometry software so as to broaden and deepen their mathematical knowledge and skills with emphasis on the investigation and the encouragement of creative thinking in the field of geometry (see Chapter 4 – Section 4.1 The Suggested Enrichment

Program).

2.2.5 Teaching and Learning Models and Strategies for Gifted Students

Numerous teaching and learning models and strategies have been developed for gifted and talented students in order to cater for their potentials and needs. The various models and strategies call for developing gifted students' gifts and talents to the maximum extent possible by enhancing academic performance, developing creative abilities, improving problem-solving abilities, promoting self-directed learning skills, and developing self-confidence. In the coming sections, the researcher describes some of these models and strategies that are designed for gifted students and used in their teaching and learning.

1) Enrichment Triad Model

The Enrichment Triad Model (ETM) is developed by Joseph Renzulli based on his notion of creative-productive giftedness as a result of three interactive clusters of psychological traits: well-above-average ability, task commitment, and creativity (see Chapter 2 – Section 2.2.1 2)).

The Enrichment Triad Model is designed to encourage gifted students' creative productivity by exposing them to various topics, areas of interest, and fields of study. It is also designed to further train them to apply advanced content and methodological training to real areas of interest. The model advocates three types of enrichments (Renzulli and Reis, 1998):

Type I: General Exploratory Activities

This type of enrichment is designed to expose gifted students with a wide range of educational experiences, knowledge, and skills as possible through: excursions, clubs, interest centers, visiting speakers and brainstorming sessions.

Type II: Group Training Activities

These activities are designed to promote thinking and feeling processes. Students are engaged in designing, experimenting, recording, comparing, analyzing, synthesizing, and evaluating. Training activities include the development of creative and critical thinking, skills of learning how to learn using advanced level reference materials of written, oral, and visual communication.

Type III: Individual and Small Group Investigations of Real Problems

In Type III enrichment, students apply the knowledge and skills they have already developed while working through Type I and Type II activities. The activities in type III enrichment aim at:

- Providing opportunities for applying interests, knowledge, and creative ideas.
- Acquiring advanced understanding level of the knowledge and methodology that are used within particular disciplines or fields.
- Developing authentic products.
- Developing self-directed learning skills.
- Developing task commitment, self-confidence, and feelings of creative accomplishment.

2) Multiple Talent Model

The Multiple Talent Model (MTM) is developed by Calvin Taylor based on the aspects of Guilford's Structure of Intellect (SOI) model. The model assumes that nearly all students are talented or gifted in at least one way and that if programs were developed to cater for a wide range of talent and gift areas, greater numbers of students would benefit. The model considers multiple abilities in which students could display talents and gifts. Amongst these abilities are: academic, creativity, decision-making, planning, forecasting, communication, interpretation, human relationships, and discerning opportunities (Western Australian Department of Education, 2009).

Taylor's model encourages the following teaching and learning considerations (Western Australian Department of Education, 2009):

- Providing open-ended activities that will allow the development of talents.
- Focusing on talent development rather than only the acquisition of knowledge.
- Providing various and different approaches to learning.
- Emphasizing contact work within the classroom.
- Developing students' self-confidence and self-esteem.

3) Autonomous Learner Model

The Autonomous Learner Model (ALM) is developed by George Betts. The model is mainly designed to cater for both the emotional and cognitive needs of gifted and talented students. The major goal of the model is to facilitate the growth of students as independent, self-directed learners, with the development of skills, concepts and positive attitudes within the cognitive, emotional, and social domains. Accordingly the model changes the role of both the student and the teacher. The student becomes an autonomous learner, who (1) solves problems or develops new ideas through a combination of divergent and convergent thinking and (2) functions with minimal external guidance in selected fields of endeavor. The teacher, on the other hand, becomes a facilitator of the learning process and guides students through five dimensions of the model (Idaho State Department of Education, 2005, pp. 79-80).

The model consists of five major dimensions (Idaho State Department of Education, 2005, pp. 79-80); (Western Australian Department of Education, 2009):

Orientation

It represents the emotional dimension of the model and contains the areas: understanding the notion of giftedness, self or personal development, group building activities, understanding program opportunities and responsibilities.

Individual Development

It includes inter-personal or intra-personal understanding, learning skills, use of technology, university or career awareness, organizational and productivity skills.

Enrichment Activities

The student takes more responsibility for his/her learning by selecting from the different activities that include exploration, investigation, cultural activities, service activities, and adventure trips.

Seminars

Seminars include small group presentation in futuristic, controversial, and problematic topics of general interest or require advanced knowledge.

In-Depth Study

In-depth study includes individual projects, group projects, mentorship, presentations, and self-assessment.

4) Creative Problem Solving (CPS) Process

The Creative Problem Solving (CPS) process or strategy is developed by Alex Osborn to encourage the gifted and talented students to generate several solutions to problems by following a six-step strategy. Through the six-step strategy, the model provides a structured procedure for identifying challenges, generating ideas, implementing innovative solutions, and evaluating the solutions (Idaho State Department of Education, 2005, p 61); (Western Australian Department of Education, 2009).

The creative problem-solving strategy of Osborn flows through six sequentially steps (Idaho State Department of Education, 2005, p 61); (Western Australian Department of Education, 2009):

Objective Finding

The goal, challenge, or future direction is identified.

Fact Finding

This step is concerned with gathering all possible data and information about the problem or challenge, and observing the problem as objectively as possible.

Problem Finding

In this step, students generate a list of questions that capture the central core of the problem.

Idea Finding

This is the brainstorming step in which students generate as many ideas and solutions as possible regarding the assigned problem or challenge that might be a solution of that problem or challenge.

Solution Finding

In this step, students evaluate their brainstorming ideas or responses and select one or more of the best responses that would be most appropriate to address the problem or the challenge.

Acceptance Finding

Students incorporate the best idea or ideas for action plan.

2.2.6 General Principles of Developing Enrichment Programs for the Mathematically Gifted Students

There are many sets of general principles of developing enrichment programs for the mathematically gifted students, which have been articulated in the literature and prior studies related to the domain of gifted and talented education(e.g., [Johnson, 2000](#); [Lee and Hwang, 2005](#); [Sheffield, 2000](#)). Reviewing these sets of principles, the present researcher was able to determine general principles of developing enrichment programs for the mathematically gifted students and classify them under four categories in terms of program aims, content, instruction, and assessment.

1) Program Aims

Program aims should:

- Concern with not only the intellectual and cognitive domains but also the emotional and social ones of the mathematically gifted students.
- Emphasize interdisciplinary approaches so as to connect different mathematics disciplines to each other and further connect mathematics to other scientific disciplines.

2) Program Content

Program content should:

- Include various educational experiences based on the characteristics of the mathematically gifted.
- Provide a wide range of concepts and knowledge to illustrate ideas.
- Be an extension of the proper curriculum, which is introduced in a regular mathematics classroom.
- Challenge students to formulate concepts, apply content to real problems, and develop authentic products.

3) Program Instruction

Program instruction should:

- Emphasize the enhancement of problem solving, creative thinking, and higher-order thinking.
- Provide opportunities for students to explore mathematical ideas in a creative fashion.

- Inspire students to think like mathematicians by encouraging them to take time to explore the depth and complexities of problems, patterns, and connections through the program activities.
- Use deductive and inductive learning techniques.
- Use inquiry-based and discovery learning approaches that emphasize open-ended problems with multiple solutions or multiple paths to solutions.
- Provide useful concrete experiences such as manipulatives and “hands on” activities.
- Use powerful computing tools such as computer software and graphical calculators.

4) Program Assessment

Program assessment should:

- Allow for differences in understanding, accomplishment, and creativity by giving students opportunities to express what they have learned.
- Require students to explain their reasoning orally and in writing.
- Use a lot of higher-level questions in justification and discussion of problems by asking “why” and “what if” questions.

2.3 Geometric Creativity Development

In this part of the dissertation, a brief outline is provided for the meaning of creativity in general, followed by a description of the meaning of creativity in the field of geometry and its adopted components that can be used in its assessment. Then, the objective definition of geometric creativity is presented.

2.3.1 The Nature of Creativity

In the psychology literature there are literally thousands of commentaries offered or studies conducted on the nature of creativity, its distribution within the population, and its origins and manifestations in human experience (Silver, 1997, p. 75).

However, there is a considerable confusion about the nature of creativity. There are at least four trends in which the term is defined. One trend of definitions is devoted to the cognitive operations of thinking that intertwine in the creativity process. The second trend emphasizes the new products, which are generated during the creativity process and perceived to be creative. The third one describes creativity through the behavior of individuals who are generally considered to have been creative (Pehkonen, 1997, p. 63). The fourth one considers the individuals intellectual abilities and capabilities involved in the creativity process (e.g., fluency, flexibility, originality, sensitivity to problems, reorganization, and elaboration). Consequently, the definition of creativity and its assessment techniques have been varied according to the adopted trend.

A research trend was adopted by Guilford and his associates to identify the factors responsible for the emergence of the individual's creative thinking. Based on the individual's intellectual abilities and capabilities and a clear distinction between convergent and divergent thinking, some factors were identified that can be used to describe how creativity can be assessed. These factors include sensitivity to problems, fluency, flexibility, originality, reorganization, and elaboration, among others (there is a good deal of research and studies concerning creativity which is attributed to both Guilford and Torrance, and the reader is referred to their original works). Then again, based on Guilford and his associates' works in the field of creativity, creativity can be defined as a combination of four intellectual factors or components of abilities, as follows:

1) Fluency

It is shown by the production of many ideas in a short time. There are four sub-factors of the fluency component:

- Verbal Fluency
- Ideational Fluency
- Associational Fluency
- Expressive Fluency

2) Flexibility

It is the ability to overcome fixation or rigidity in thinking and to break from mental sets (Haylock, 1997, p. 69). Flexibility can be shown in two types:

- Adaptive Flexibility
- Spontaneous Flexibility

3) Originality/Novelty

It is the ability to come up with new and unique ideas. i.e., it is the statistical infrequency of the responses in relation to peer group. Guilford claimed that people generating infrequent responses are expected to be more creative. Originality can also be manifested in two types:

- Adaptive Originality
- Spontaneous Originality

4) Elaboration

Elaboration is the ability to come up with new ideas or improve upon existing ideas by adding depth or detail. Elaboration can appear in nine sub-factors:

- **Substitution.** Can I put something in place of something else?
- **Combination.** Can I combine two ideas into one?
- **Adapt.** How could I make something similar work?

- **Modify.** How can I alter the use of this idea?
- **Magnify.** How can I expand the use of this idea?
- **Put to other use.** What other applications is this idea useful for?
- **Eliminate.** What parts of the idea can I get rid of?
- **Rearrange.** How can I rearrange the components?
- **Reserve.** What are some opposite ideas?

2.3.2 The Nature of Geometric Creativity

Given the lack of an accepted definition for creativity in general, it is not surprising that there is not a clear-cut, single definition of geometric creativity. Many researchers try to define the term of geometric creativity. However, their definitions' focus is on identifying the kinds of thinking in the field of geometry that qualify for the description "creative".

El-Rayashy and Al-Baz (2000) proposed a definition in which geometric creativity is described as the ability to produce the greatest number of ideas and different solutions toward a geometric problem. In their definition, they considered geometric creativity as a product that is characterized by fluency, flexibility, originality/novelty of ideas, and sensitivity to problems. They further provided a list of abilities that characterize the creative behavior in the field of geometry. That is, the student will be creative in the field of geometry if he/she has the following abilities (El-Rayashy and Al-Baz, 2000, p. 84)):

- Identifying the aspects of the geometric problem.
- Recognizing new relationships among the aspects of the geometric problem.
- Reformulating the geometric problem.
- Reorganizing his/her experience toward the given geometric problem to discover new related relationships.

- Generating many different and varied proofs related to the geometric problem.
- Generating many different and varied solutions related to the geometric problem.
- Getting many original, non-routine solutions of the geometric problem.
- Criticizing and evaluating the non-routine solutions of the geometric problem.

The concept of geometric creativity, according to [El-Rayashy and Al-Baz \(2000\)](#), is obviously set within the framework of problem-solving skills in the field of geometry. The authors further suggest that geometric creativity in this framework is characterized by identification, formulation, and reformation of the geometric problem; and flexibility and originality of solutions.

Another definition of geometric creativity is proposed in terms of the creativity components (fluency, flexibility, and originality) of the generated ideas, solutions or responses toward a geometric problem or situation. In this respect, geometric creativity is defined as the student's ability to produce the greatest possible number of relevant responses toward a geometric problem or situation. These responses should reflect different abilities of creativity: fluency, flexibility and originality/novelty that are defined as follows ([Mohamed, 2003](#), p. 7):

1. **Fluency**, which is the student's ability to think of in a large number of ideas toward a geometric problem or situation. It is measured by the number of ideas that are related to the geometric problem or situation in direct or indirect ways and that represents "ideas' fluency" or "availability".
2. **Flexibility**, which means the diversity and variety of ideas that are generated for the geometric problem or situation by the student. Thus, it refers to how easy the student can change his/her viewpoint of a certain mentality and is measured by the number of different categories of the generated ideas.
3. **Originality/novelty**, which refers to the new and unusual generated ideas for a geometric problem or situation. In other words, it is the statistical infrequency of ideas or relevant responses in relation to the peer group.

To sum up, both reviewed definitions of El-Rayashy and Al-Baz (2000) and Mohamed (2003) emphasize the quantitative and qualitative aspects of geometric creativity. The quantitative aspect is represented in producing a large number of responses related to a geometric problem whereas the qualitative aspect is represented in both the cognitive process in generating responses (e.g., breaking the mental set and overcoming fixation of responses) and novelty of responses.

Reviewing these samples of definitions and many others, the present researcher proposes a definition that fits the purposes of the present study. So, geometric creativity, in this study, is defined as a directed intellectual activity of the mathematically gifted students in high schools for generating new geometric ideas over the known ones in a non-routine geometric problem or situation; and the new ideas reflect the abilities of fluency, flexibility, originality, and elaboration.

Consequently, geometric creativity can be defined as a combination of the following four components, which reflect the researcher's notion of geometric creativity:

1. **Fluency**, which means the student's ability to pose or come up with many geometric ideas or configurations related to a geometric problem or situation in a short time.
2. **Flexibility**, which refers to the student's ability to vary the approach or suggest a variety of different methods toward a geometric problem or situation.
3. **Originality**, which means the student's ability to try novel or unique approaches toward a geometric problem or situation.
4. **Elaboration**, which is the student's ability to redefine a single geometric problem or situation to create others, which is not the geometric problem, situation itself, or even its solutions but rather the careful thinking upon the particular aspects that govern the geometric problem or situation, changing one or more of these aspects by substituting, combining, adapting, altering, expanding, eliminating, rearranging, or reversing and then speculating on how this single change would have a ripple effect on other aspects of the problem or the situation at hand.

In short, the term of **Geometric Creativity** is objectively defined in this study as the student's score on the geometric creativity test prepared by the present researcher.

2.4 The Significance of Using Dynamic Geometry Software in Developing the Mathematically Gifted Students' Geometric Creativity in High Schools

Based on the given data and results of prior literature in the field of teaching and learning of mathematics in relation to the subject of the present study, the researcher was able to determine the significance of using dynamic geometry software in developing mathematically gifted students' geometric creativity in high schools as follows:

- DGS may help the mathematically gifted students to overcome the static features of diagrams and mathematical shapes in the textbook; thereby the use of DGS is considered a sort of curriculum differentiation that is required for the mathematically gifted students.
- DGS may encourage the mathematically gifted students to invent their own mathematics by making it easy to create simple geometric figures and make conjectures about their properties. Learning geometry could then become an open-ended exploration of relationships in geometric figures rather than a rerun of proofs and theorems.
- DGS is a good tool for providing an appropriate environment for exploring and experimenting with various properties. So, it may help the mathematically gifted students in designing experiments to test the validity of geometric statements. Namely, it may help the mathematically gifted students to experiment with moving shapes on the screen, which was impossible to take place for the generations whose tools consisted of pen and paper. Moreover, the constructing, dragging, and measuring facilities which are embedded in dynamic geometry software can enable the mathematically gifted students not only to explore, experiment and make

2.4. The Significance of Using Dynamic Geometry Software for Geometric Creativity

mathematical conjectures but also to solve problems and pose related problems.

- Using DGS can create a healthy atmosphere in mathematics classrooms that provides the mathematically gifted students with a good chance of simulation, which is very close to the real-life situation.
- Using such software may make instruction for the mathematically gifted students more student-centered. It may also encourage cooperative learning and increasingly stimulate teacher-student interaction.
- It is possible for the mathematically gifted students to conjecture certain geometric properties through dragging and manipulation, then through “theorem proving facilities”, which are embedded in some DGS (e.g., The Interactive Geometry Software, Cinderella); they can automatically check the correctness of their conjectures and immediately get feedback about these conjectures.
- Using DGS may help the mathematically gifted students to develop their ability to correctly parse/analyze the geometric statements as well as to figure out the conditional statements (necessary and sufficient conditional statements) that govern the geometric statements.
- DGS environment may provide the mathematically gifted students with opportunities to discover variant/ invariant properties of geometric shapes by mouse dragging.
- Uses of dynamic geometry environment may allow the mathematically gifted students to visually and dynamically express their mathematical thoughts while they are working in geometric situations.
- DGS environment may improve the mathematically gifted students’ abilities to construct geometric objects and solve a variety of geometric problems in novel and unusual methods. Furthermore, it may provide them with powerful means for extending and posing new related problems by implementing new approaches and strategies.

- Using such software may encourage the mathematically gifted students to pass on to higher levels of geometric thinking and engage them in the process of doing geometry.
- Carefully designed activities using DGS can provide the mathematically gifted students with the basis on which to build deductive proofs.
- DGS, with carefully designed activities in the context of geometry, can be used to train the mathematically gifted students on the scientific method processes through: hypothesizing, justifying, and confirming.

Chapter 3

Related Studies

In this chapter, the researcher reviews the prior studies related to the subject of the present study and surveys them under three main domains: Studies related to the use of dynamic geometry software in the teaching and learning of mathematics, studies related to the mathematically gifted education, and studies related to geometric creativity development. Then, the researcher presents his commentary on related studies. By the end of the chapter, based on the given data of prior literature and the results of the prior studies, the researcher tries to specify the principles of developing the suggested enrichment program using the dynamic geometry software for developing the mathematically gifted students' geometric creativity in the high schools.

3.1 Studies Related to the Use of Dynamic Geometry Software in the Teaching and Learning of Mathematics

Numerous prior studies were conducted related to the use of dynamic geometry software in the teaching and learning of mathematics. They tackled different issues and concerns according to different motives underlying them. However, they can be categorized under three main topics: Studies concerned with developing new interactive systems and software, studies concerned with developing students' learning outcomes, and studies

concerning interacting with the software.

3.1.1 Studies Concerned with Developing New Interactive Systems or Software

Under this category, some studies criticize the infrastructures of the existing software and go behind that to develop new interactive systems and software. For instance, the study attempted by [Kortenkamp \(1999\)](#) emphasizes the necessity of mathematical foundations of dynamic geometry software and the importance of a correct mathematical treatment of geometry on computer, and it comes up with new interactive geometry software, Cinderella that addresses this issue. On the other hand, [Mavrikis \(2001\)](#) seeks designing more intelligent and educational dynamic geometry environment by emphasizing the importance of the integration between fields of artificial intelligence in computer science and education in the teaching and learning of mathematics for achieving more significant results. His study also comes up with a new feedback mechanism system attached to DGE called DANTE. More details about both studies come within the following sections.

Foundations of dynamic geometry is the title of the study attempted by [Kortenkamp \(1999\)](#), which is an interdisciplinary study about the foundations of dynamic geometry with respect to mathematics, computer science, and education. Accordingly, the study was divided into three main parts: mathematics, computer science, and education.

In the first part, the mathematical part, the study laid the theoretical foundation for dynamic geometry based on both projective geometry and Cayley-Klein geometries, and solved the problem of continuity in dynamic geometry by assigning suitable Riemann surface to the construction input elements.

The second part of the study addressed the various computational problems and their solutions that arose by implementing the mathematical theory in developing the interactive geometry software, Cinderella.

The third part of the study covered many pedagogical issues of implementing the in-

teractive dynamic software, Cinderella (e.g., explaining why even for the level of school geometry such a complex mathematics foundation for the software is necessary, the importance of a correct mathematical treatment of geometry on computer, promoting creativity with the help of the computer in mathematics classrooms, interactive exercise as Internet-ware of the interactive dynamic geometry, Cinderella, powered by its automatic proving facilities, among others).

The other study, under this category, is attempted by [Mavrikis \(2001\)](#) and seeks for more intelligent and educational dynamic geometry environment. More specifically, the study aims at:

1. Conducting an experiment in order to observe how students interact when working with DGE; in other words, observe what kind of help they need and when they need it, and what information the system can use to decide about their state.
2. Using these observations to design and implement a feedback mechanism that can be attached to DGE and help students more effectively.
3. Designing and implementing the system so as to provide authoring and feedback capabilities to the teachers themselves.

To achieve that, an experiment was conducted to observe the students' behaviors while they are working with DGE focusing on when and what kind of help they need and how they manifest emotions like boredom, irritation and unease. The subjects were fifteen students of a Greek private institute, called (NEON), who had just finished the first grade of Greek high school (lycee).

Then, based on these observations and literature about feedback and help-seeking, a feedback mechanism called DANTE (Dynamic Authoring aNd Tutoring Environment) was designed and implemented so that it provides feedback based on students' interactions. The mechanism monitors students while they are working on a dynamic geometry environment and provides feedback to them according to the activity's goals, the object manipulation and the mouse activity.

Afterwards, another experiment was conducted to assess the prototype's effects of the

designed system. The subjects of this experiment were five students of Stewart's college aged 11-12. This experiment illustrated the positive effects of the mechanism feedback and based on students' actions during the experimentation a slight change was applied to the interface, which brought a significant improvement to the feedback mechanism system.

One major implication of this study is that the integration between artificial intelligence and education in the current teaching and learning practices can yield significant results.

3.1.2 Studies Concerned with Developing Students' Learning Outcomes

The second category of the related studies delved deeply into how far DGS can be used so as to promote students' learning outcomes (including: mathematical performance, academic achievement, attitudes, mastery level of mathematical concepts and skills, and van Hiele levels of geometric thinking, among others). For instance, studies of [Hull and Brovey \(2004\)](#), [Chomskis and Hinkle \(2006\)](#), and [Abu Bakar *et al.* \(2009\)](#) aimed at investigating the role of DGS in developing students' mathematical performance, academic achievement, and attitudes. Studies of [Budge \(1999\)](#) and [Almeqdadi \(2000\)](#) were more targeted at investigating the role of DGS in promoting students' understandings and knowledge of geometric concepts. Studies of [Johnson \(2002\)](#) and [Patsiomitou \(2008\)](#) focused on investigating the role of DGS in developing van Hiele levels of geometric thinking. More details are discussed about these studies and their concerns in the following sections.

With regard to the use of dynamic geometry software in developing students' mathematical performance, academic achievement and attitudes, [Hull and Brovey \(2004\)](#) conducted an experimental study on three geometry classes in a public high school in an attempt to investigate the effects of using DGS – The Geometer's Sketchpad – on students' achievement and attitudes toward geometry.

Subjects were 68 students, thirty-four male and thirty-four female, from ninth-grade geometry classes. The researchers designed some activities using the Geometer's Sketchpad to help students to discover some geometric properties related to circles. Students were taught twice a week a 3-week unit on circles.

A student-attitude questionnaire was administered at the beginning and at the end of the study. And at the end of the study a paper-and-pencil unit test was also administered.

Results of the study indicated that the students taught using the Geometer's Sketchpad performed slightly better than the students taught last year using the usual method. In terms of gender, male students performed at a higher level than female students on the unit test. Regarding the students' attitudes, results did not indicate whether the use of DGS – The Geometer's Sketchpad – positively or negatively impacted the students' scores on the attitude questionnaire.

Chomskis and Hinkle (2006) conducted another study, concerning students' achievement and attitudes, to examine three research questions:

1. What is the effect of using the Geometer's Sketchpad on students' mathematics achievement?
2. How does the use of Geometer's Sketchpad for geometry instruction influence students' attitudes toward mathematics?
3. What are the experiences of the classroom teacher as a user of the Geometer's Sketchpad in teaching geometry?

Subjects were 22 students, 14 male and 8 female, 16 of them were tenth graders and 6 of them were eleventh graders. The Geometer's Sketchpad was used to teach a unit on circles entailing inscribed angle, tangents to circle, secant to a circle, segment relationships, and equation of a circle. Instruction was delivered during 90-minutes class periods and the classes extended over 6 days.

At the beginning of the circle unit, the researcher administered a geometry achievement

pre-test and a pre-attitudinal survey. Prior to presenting the theorems, the Geometer's Sketchpad was used to guide student discovery of the theorems. After the theorems were discovered, they were presented in writing. Then, students were provided examples involving the theorems. Students also completed practice problems, which required the use of their newly learned theorems. During the intervention, two teachers observed the class and provided feedback on the integration of the Geometer's Sketchpad into the mathematics lessons and on the involvement of the students. At the end of the circle unit, data were gathered from classroom observations, a teacher journal, mathematics achievement tests, student-attitude surveys and student interviews.

Findings from mathematics tests indicated that students' achievement of knowledge and skills regarding the theorems and postulates for the study of circles unit increased. Students' attitudes toward mathematics improved as indicated by data from the attitude survey, peer/teacher observation, and students' interviews. In addition, the teacher's journal provided data to support the continued use of the Geometer's Sketchpad for geometry instruction.

Abu Bakar *et al.* (2009) also attempted an experimental study to investigate the effectiveness of integrating the Geometer's Sketchpad compared to the traditional teaching strategy on the mathematical performance and the attitudes of secondary school mathematics students on the topic of "Quadratic Functions".

Subjects were 92 students from grade 4 of the national secondary school in Malaysia. The subjects were randomly assigned into two groups: the Geometer's Sketchpad (experimental) group and traditional (control) group. The total number of students in the experimental group was 45 students whereas that of the control group was 47 students.

The experimental group was taught by using the Geometer's Sketchpad, while the control group was taught using the traditional teaching strategy. The study used four phases for the experimental group, namely: 1) Introduction to the Geometer's Sketchpad; 2) Introduction to Quadratic Functions 3) Integrated teaching and learning using the Geometer's Sketchpad; and 4) Assessment using a set of Quadratic Equations test

as a posttest. On the other hand, the control group experienced only Introduction to Quadratic Functions (phase 2 of the experimental group) followed by a session containing further exercises. Then, the control group took the same posttest as the experimental group. The time duration of the intervention was six hours. The attitudes of both groups were measured after the intervention according to four dimensions: level of enthusiasm, level of enjoyment, level of anxiety, and level of avoidance.

Results of the study indicated that there was no significant difference between the mean performance scores of the control group compared to the scores of the experimental group.

Regarding the students' attitudes, findings of the study confirmed that students in the experimental group were more enthused and enjoyed their lessons more than the students of the control group. On the other hand, findings showed that the control group experienced less anxiety than the experimental group.

With regard to the studies concerned with investigating the role of DGS in promoting students' understandings and knowledge of geometric concepts, [Budge \(1999\)](#) attempted a qualitative study to trace students' thoughts and understandings in their investigations of Fagnano's theorem using the Geometer's Sketchpad.

Subjects were 4 students in the second and third years of university studying mathematics, and all were competent computer users. The inquiry approach was used to present Fagnano's theorem to the students. Students were asked to find the inscribed triangle whose perimeter is as small as possible in a given acute-angled triangle using the Geometer's Sketchpad. Before students were given this problem, they participated in hands-on activities workshops and were asked to make a conjecture and then to attempt a proof.

Results of the study revealed that using the Geometer's Sketchpad allowed mathematical notations to become visible to students. The Geometer's Sketchpad also encouraged investigation and solving problems in geometry. In addition, the Geometer's Sketchpad provided a window onto the students' mathematical meaning-making, enabling deeper

discussion and directed mediation. The study recommended using dynamic software environment in teaching and learning mathematics, on the grounds that it plays a vital role in addressing particular shortcomings in the present situation of the teaching and learning of mathematics.

Also, [Almeqdadi \(2000\)](#) conducted an experimental study to investigate the effect of using the Geometer's Sketchpad on students' understandings of some geometric concepts.

Subjects were 52 male-students from ninth grade in the Model School of Yarmouk University, Irbid, Jordan. There were 26 male-students in the experimental group and 26 male-students in the control group. The same teacher was teaching both groups. The experimental group was taught the geometric part of the curriculum by using the textbook and the Geometer's Sketchpad, while the control group was taught the same part using only the textbook. The students in the experimental group used the Geometer's Sketchpad once a week.

A test, designed by the researcher, was administered at the beginning and at the end of the study to measure students' understanding of some geometric concepts focusing on the relationships between the area and the perimeter of polygons (e.g., rectangle and triangle).

Results of the study indicated that there was a significant difference between the means of the students' scores on the post-test in favor of the experimental group. The results also indicated that there were more gains in the scores of the post-test than the pre-test in case of the experimental group. According to the results of the study, the researcher generally suggested more emphasis on the use of computer and its applications in mathematics education and particularly recommended further studies to evaluate dynamic geometry software features and capabilities in the teaching and learning of mathematics.

With regard to the use of DGS in developing van Hiele's levels of geometric thinking, Patsiomitou (2008) conducted a qualitative study aimed at exploring:

1. Ways in which students in small groups develop problem representations, reasoning and problem-solving, making decisions, and receiving feedback about their ideas and strategies in a DGS-supported environment.
2. Ways in which students develop rigorous proofs through “building linking visual active” representations.
3. Ways to develop students’ van Hiele levels of geometric thinking using DGS.

The study subjects were 28 students from a public secondary school in Athens, Greece. There were 14 students in the experimental group and 14 students in the control group. The two groups were equivalent with respect to their van Hiele levels of geometric thinking based on their results on a van Hiele levels test. The mathematical problem with which the students engaged with – either in the Geometer’s Sketchpad dynamic geometry environment or in the static environment – was the treasure problem (Lost treasure of the pirates). Students in the experimental group had the opportunity to explore the assigned problem using static or dynamic means, while students in the control group had to examine the problem using static means only.

The researcher followed five steps for the inquiry process as follows:

1. Two pairs of the experimental group explored the problem within a dynamic geometry environment – The Geometer’s Sketchpad.
2. The discussions were videotaped and examined simultaneously with the researcher’s notes.
3. The control group explored the problem using static means.
4. The researcher formulated the problem taking into account the retroactions by the study subjects and linking all the software functions and actions using the interaction techniques, supported and facilitated by the Geometer’s Sketchpad environment to better allow students to discover solution paths and to reason by rigorous proofs.
5. Both the control and experimental group explored the problem after reformulation.

The findings of the study revealed that students were led to:

1. Develop strategies to solve problems using the software.
2. Develop mental schemes through constructions of the utilization schemes.
3. Improve their reflective visual reaction through building and designing the activities as linking visual active representations.
4. Develop their van Hiele levels of geometric thinking through the combination of linking visual active representations in the activities with the questions asked during the software procedures.

The other study – in this survey – which is also concerned with the role of DGS in developing the van Hiele levels of geometric thinking – was attempted by [Johnson \(2002\)](#). It aimed at investigating whether the Geometer's Sketchpad can be used in high school geometry classes to improve students' achievement or van Hiele levels.

Subjects were 105 high-school students enrolled in an advanced geometry course. There were 60 students in the experimental group and 45 students in the control group. Students in both groups were pre-tested using a non-proof geometry achievement test and a van Hiele level test.

The experimental group students spent at least one class period per week doing geometric exploration using the Geometer's Sketchpad for the first twelve weeks of the class. On the other hand, the control group students were not exposed to computer exploration for the first twelve weeks of the class. During the course, both groups of students were tested twice more on both proof and non-proof geometry content and post-tested for van Hiele levels.

Results indicated that the Geometer's Sketchpad did not improve the achievement and the van Hiele levels. Results also provides some evidence that technology implementation might actually hinder the development of proof-writing ability among some students. The results further indicated that a long period of time is needed for students to make significant gains in van Hiele levels.

3.1.3 Studies Concerning Interaction with the Software

A number of studies considered how students interact with the software. Amongst these studies are: The study of Connor *et al.* (2004) that aimed at investigating how pre-service teachers interact with DGS while exploring the validity of geometric statements. Another study is that of Haja (2005) who tried to investigate how pre-service teachers explore problems in dynamic geometry environments – Cabri II. Moreover, the study of Christou *et al.* (2005) explored how students interact with DGS as a mediation tool for problem solving, problem finding, and problem posing. More details about these studies will be given within the following sections.

The study of Connor *et al.* (2004) is attempted to investigate how students use DGS to explore the validity of geometric statements using data gathered from interviews with 6 pre-service secondary school teachers.

Subjects were 6 junior or senior pre-service secondary school teachers enrolled in the second quarter of a required geometry sequence. Each interviewee had taken an introduction to the proofs course as a pre-requisite course for the geometry sequence, or had taken other proof-based courses. The interviewees were each asked to explore the validity of five geometrical statements, presented one at a time, using the Geometer's Sketchpad. The first three statements the interviewees were asked to explore were:

1. If the in-center and the circum-center of a triangle coincide, then the triangle is an equilateral triangle.
2. If ABCD is a cyclic quadrilateral, then $m(\angle BAC) + m(\angle BDC) = 180^\circ$
3. If a cyclic quadrilateral is a parallelogram, then it is a rectangle.

The interviewees were video and audio taped and video out-put of the interviewee's work was also recorded on a videotape.

The interviewees' justifications were classified. The sophistication of use of the Geometer's Sketchpad was also noted; this was measured, in part, by counting the variety of tools used during a demonstration.

Observations of the study indicated that the interviewees used definitions in a correct and economical manner in the dynamic geometry environment and correctly parsed the interior conditional statement.

The study recommended some issues of concern such as the difficulties the interviewees had in generating justification by using DGS, and the difficulties the interviewees had in mathematical statements. The study also recommended further research concerning the issue of how to incorporate the use of dynamic geometry software into developing strategies for exploring the correctness of assertion and, in particular, designing experiments to test the validity of a statement.

Haja (2005) attempted a study to investigate the problem-solving competencies of pre-service secondary school teachers as they explored problems in a dynamic geometry environment, Cabri II.

Subjects were 4 secondary school pre-service teachers (3 male, 1 female) in University of London. None of the subjects had used Cabri before taking part in the experiment but they had studied geometry in high school/O level.

The researcher developed a “Knowledge-in-Action” model to illustrate the problem solving processes for pre-service teachers. The model was based on applying content knowledge of pre-service teachers to understand the given problem, construct the dynamic figures, make conjectures, verify the conjectures, and solve similar problems. Accordingly, three main competencies were defined as follows:

1. Construct figures in DGE.
2. Make conjectures in DGE.
3. Verify the made conjectures in DGE.

The researcher hypothesized that if the pre-service teachers were able to solve the given problem using their knowledge by constructing, conjecturing and verifying the properties in DGE then their competency would be testified.

The experiment was conducted on a one-to-one basis on different days. Two open-ended

problems, Varignon's quadrilateral and medial triangle, were constructively designed for the experiment.

Observations on the problem solving processes of the four pre-service teachers while they attempted to solve Varignon's quadrilateral and medial triangle problems in the Cabri dynamic geometry environment showed that:

1. Pre-service teachers' content knowledge is adequate to understand the given problem.
2. Pre-service teachers are competent enough to apply the content knowledge to construct the dynamic configurations.
3. Pre-service teachers are competent enough to apply the content knowledge to make conjectures.
4. Pre-service teachers are competent enough to apply the content knowledge to verify conjectures.
5. Pre-service teachers are able to use Cabri to justify their solution.

In an attempt to investigate how DGS mediates students' strategies in solving and posing problems, Christou *et al.* (2005) conducted a qualitative study to examine two research questions:

1. In which ways does DGS mediate students' problem solving processes in geometry problems?
2. In which ways does DGS provide opportunities for students to pose and solve their own geometry problems?

Subjects were 6 voluntary pre-service teacher students with prior experience in dynamic geometry. Two non-routine geometry problems were presented to students to show how constructing, dragging, and measuring facilities of dynamic geometry software can enable students to explore and make mathematical conjectures, solve problems, and pose related problems. Students were interviewed while they were working on the two problems. There were two sessions. During the first session, interviewees were asked to

solve the problems. During the second session, the researchers discussed the interviewees' solutions and their possible ways of extending, posing and solving new problems. The two sessions were video recorded. Interpretative techniques were used to analyze data.

Results revealed that DGS acts as a mediation tool and plays a significant role in engendering problem solving and posing through its dragging and measurement facilities. That is, the dragging facility is an important tool for problem solving and posing and the measurement facility is an important tool for checking the correctness of students' conjectures. Results further showed that DGS supports the relationship between the problem solving strategy and the tendency to pose extension problems.

3.2 Studies Related to the Mathematically Gifted Education

Studies regarding the mathematically gifted education can be categorized according to their varying concerns into two main categories. One category concerns with identifying the mathematical gifted students' special characteristics in comparison with their regular peers. The other category concerns with providing the mathematically gifted students with tailored educational differentiations, which include developing enrichment programs, providing extracurricular work, and suggesting alternative instructional strategies.

3.2.1 Studies Concerned with Identifying Mathematically Gifted Students' Characteristics

The main educational motive for conducting the studies under this category is twofold. The first is to look into the characteristics differences between the mathematically gifted students and their regular peer group of students, which include students' abilities, attitudes, creativity, emotions, interests, etc. The second is to provide some indicators

that can be used to identify the mathematically gifted students based on their unique characteristics.

As a brief outline of this category, the study of [Sriraman \(2003\)](#) clinically explored the difference between the gifted and non-gifted behaviors in their problem-solving experiences and how they formulate generalization and abstraction. [Lee and Hwang's study \(2005\)](#) was more targeted to experimentally identifying the differences between gifted and regular students in different abilities of problem solving, which include not only the problem solving but also the problem finding and problem posing in their responses to problem solving ability tests. [Huang's study \(2003\)](#) aimed at analyzing the gifted children's processes, strategies, and emotional characteristics, while solving non-routine mathematical problems. [Hui-Chin's study \(2004\)](#) compared the creative and emotional characteristics of the primary school gifted pupils in mathematics with their regular peers during mathematics problem solving processes. [Choi and Do's study \(2008\)](#) also experimentally compared the intellectual, emotional, social, and creative characteristics of gifted and non-gifted students using a mathematics aptitude test, a problem-solving test, and Figural Torrance Test of Creative Thinking. [Velikova, Bilchev, and Georieva \(2004\)](#) developed a diagnostic technology approach of three characteristics indicators for identifying the creative-productive gifted students in mathematics. This approach is based on both theoretical and experimental research related to psychology, pedagogy, and methods of teaching and learning mathematics. More details about these studies are given within the following sections.

[Sriraman \(2003\)](#) attempted a clinical-qualitative study to examine three research questions:

1. What are the problem-solving behaviors in which high school students engage?
2. What are the differences in problem-solving behaviors of gifted and non-gifted students?
3. How do gifted students abstract and generalize mathematical concepts?

Subjects were 9 freshmen, 4 male and 5 female; 4 of them were gifted students and 5

of them were non-gifted students, in a ninth-grade accelerated algebra class in a rural high school.

Journal writing was an integral part of the accelerated algebra class. The researcher assigned 5 combinatorial problems for the study as a journal assignment, starting with the problem of lowest complexity. These 5 problems were assigned over a time period of 3 months. The researcher asked the students to record everything they tried, including scratch work. Students were given three cues from the researcher: 1) Restate the problem in your own words. In other words, what is the problem asking about? 2) How would you begin solving the problem? 3) Solve the problem and write a summary of what worked and what did not work. Students were given 7-10 days to solve each problem.

The researcher collected the journals weekly in order to read the solutions developed by the students. The researcher then recorded in his journal possible questions to ask students in the interview.

Data were collected in the second semester of the school year through students' journal writings, clinical interviews, and the researcher's journal writings.

Results of the study, pertaining to students' problem-solving behaviors, indicated that there were major differences between gifted and non-gifted students laying in the four phases of problem solving: orientation, organization, execution, and verification.

In the **orientation** phase, the 4 gifted students showed consistent comprehension of the problem situation, identification of (and understanding) of the assumptions of the problem situation and distinction between the interrogative and declarative statements. The 5 non-gifted students, on the other hand, showed miscomprehension of the problem situation, poor understanding of assumptions underlying the given situation, and no distinction between interrogative and declarative statements.

In the **organization** phase, the 4 gifted students showed control of the variability of the problem solving situation and global planning, whereas the 5 non-gifted students showed no control of the variability of the problem situation.

The 4 gifted students, in the **execution** phase, showed performance of correct local action, continuous monitoring progress and consistency of plans, whereas the 5 non-gifted students showed performance of unusual local actions, and no careful monitoring progress and consistency plans.

The 4 gifted students, in the **verification** phase, showed checking results of local actions, verifying consistency of results with implemented plans, and use of the particular cases for better understanding why a phenomenon occurred. On the other hand, the 5 non-gifted students showed inconsistencies in results of local actions, results with implemented plans, and use of examples and non-examples to reach conclusions.

Results of the study, according to students' behaviors in generalization, reflection, and how gifted students formulate abstraction and generalization, indicated that the 4 gifted students were successful in forming generalization. In details, the 4 gifted students were successful in identifying similarities in the structure and solution of the problems, using analogical reasoning, refining methods where appropriate, extending the domain of validity, and verbalizing common principles. On the contrary, the 5 non-gifted students showed superficial identification of similarities in the structure of the problems, inconsistencies in verbalization of similarities in the solutions of the problems, and verbalization barriers.

In reflection, the 4 gifted students showed competence in conjecturing and examining plausible examples and non-examples, relating to previous experience, decision making during and after execution and verification, thinking about similarities in the problems and solutions, and abstracting structural similarities in the problems and solutions over an extended time period. On the other hand, the 5 non-gifted students showed conjecturing but no examining of the plausibility of a conjecture, sometimes little or no conjecturing, and poor decision making during execution and verification.

Overall, the study validates the hypothesis that there exists a relationship between mathematical giftedness, problem-solving ability, and the ability to generalize.

Lee and Hwang (2005) conducted a comparative study to:

1. Examine and analyze how differently gifted and regular students from seventh grade respond to the Mathematical Problem Solving Ability Test (MPSAT).
2. Determine differences between mathematically gifted and regular students in the (MPSAT).

Subjects were 318 seventh grade students, especially, 168 regular students from B middle school in Daejeon and 150 gifted students from the Gifted Center of a university in Suwon.

The MPSAT was administered to both groups of students. The MPSAT is used to investigate the three components of mathematical problem solving: finding, solving and posing.

Significant differences were found between gifted students and regular students on the scores of MPSAT. More particularly, gifted students had more mathematical problem solving abilities than did regular students as measured by the MPSAT.

Results also indicated that not only the problem solving ability but both also both problem finding and problem posing abilities could be the characteristics of giftedness.

Another study was conducted by Huang (2003) to analyze mathematics problem-solving processes of gifted children. The study aimed at:

1. Analyzing processes, strategies, and affective characteristics of gifted children in primary school using Schoenfeld's mathematics problem-solving model.
2. Providing some suggestions and recommendations for gifted and non-gifted teachers.

Subjects were 6 third-grade gifted children in a primary school in Kaohsiung.

There were four non-routine mathematics problems introduced to the subjects. Think-aloud protocols were used to analyze the subjects' responses while solving the four problems.

Findings of the study indicated that:

1. Some subjects followed the Schoenfeld's model of problem solving, which includes exploration, design, and verification. Only one of the subjects appeared to provide correct solutions skipping the verification stage of the model.
2. Subjects exhibited diversified and flexible strategies while solving the non-routine mathematics problems (e.g., making representation, drawing figures, working backward, introducing auxiliary elements, and attempting mistakes).
3. Subjects exhibited positive affective characteristics (e.g., patience and perseverance) and further exhibited some creative characteristics (e.g., personal mathematics curiosity, excitement, and confidence) while solving the problems.

The study provided some suggestions and recommendations for the gifted and non-gifted teachers that included:

1. The non-routine mathematics problems would be used to organize students' metacognitive ability in problem-solving processes, which include exploration, design, and verification.
2. Generating many solving strategies should be encouraged in mathematics classroom group discussions.
3. The problem solving characteristics of the gifted students would be used as materials or resources for learning in mixed mathematics classrooms, which include gifted and non-gifted students.

Hui-Chin (2004) conducted a further study to investigate the creative and emotional characteristics of mathematically gifted students during mathematics problem-solving processes.

Subjects were 207 fifth and sixth grade mathematically gifted and regular primary school pupils. There were 115 mathematically gifted pupils elected from the Pre-Semifinal of 2003 Asia Pacific Mathematical Olympiad for Primary School in Taiwan, and 92 regular pupils from a primary school in Taipei County in Taiwan.

A paper-pencil test was administered to both groups of pupils. Then, 8 pupils were

selected from the two groups, considering 2 high-score and 2 low-score pupils in each group to analyze their creative and emotional characteristics during problem-solving processes.

Findings of the study can be summed up in the following:

1. The creativity of the gifted group of pupils was better than that of the regular group of pupils and their creativity leads to some positive attitudes (e.g., free from fear of mathematics, enjoyment of mathematics, and motivation in mathematics).
2. The group of gifted pupils was different from the group of regular students in their emotional and affective characteristics during the problem-solving processes. These characteristics include interest in problem solving, affect to mathematics tests, reasons to like mathematical problems, affect to face challenges, confidence in problem solving, and response to the difficulty level of problem.
3. The group of gifted pupils exhibited many emotional, effective, and creative characteristics. They include: openness to prior experience of problem-solving, tolerance of anxiety, interests in and passions for problem solving, affective pleasure in challenge, positive confidence in problem solving, relaxation to mathematical tests, persistence in problem solving, and positive mathematical attitudes.

Also, [Choi and Do \(2008\)](#) were concerned with analyzing the intellectual, affective, and creative characteristics of mathematically gifted students in comparison to their peer groups of scientifically gifted and regular students. The study was guided by four research questions:

1. What intellectual characteristics do mathematically gifted students have?
2. Are mathematically gifted students different from scientifically gifted and ordinary students in their mathematics disposition?
3. What are the characteristics of mathematically gifted students' social coping?
4. Are mathematically gifted students different from ordinary students in general creativity?

Subjects of the study were eighth graders – mathematically gifted students – from 4 middle schools in Seoul.

To analyze the intellectual characteristics of the mathematically gifted students, a problem-solving test was administered to 17 mathematically gifted students and 57 scientifically gifted students.

For analyzing the affective characteristics of the mathematically gifted students, a self-questionnaire on mathematical disposition was administered to 109 mathematically gifted students, 64 scientifically gifted students, and 142 ordinary ones.

As to look into the general creativity of the mathematically gifted students, a Figural Torrance Test of Creative Thinking was administered to 28 mathematically gifted students and 28 ordinary students.

Results of the study regarding the intellectual characteristics of the mathematically gifted students showed that mathematically gifted students have the ability to provide extraordinary proofs, which they have not dealt with in their previous learning, and some of them have the intuition to understand the nature of mathematical structure and proof.

In terms of mathematical disposition, mathematically gifted students showed positive characteristics when they were compared with scientifically gifted or ordinary students and they were also stable in social adaptation.

In exploring the creative characteristics of the mathematically gifted students, results showed that the mathematically gifted students were statistically different in the originality and abstractness components of creativity while their fluency, elaboration, and creativity index were not statistically different.

For providing some indicators that can be used in identifying creative-productive gifted students in mathematics, Velikova, Bilchev, and Georieva (2004) developed a diagnostic technological approach for identifying creative-productive gifted students in mathematics based on both theoretical and experimental research concerning psychology, pedagogy, methods of teaching mathematics, and statistics.

The study approach for identifying creative-productive gifted students in mathematics was based on two steps:

First step: identifying creative-productive gifted students considering the following criteria:

1. Measuring above average general abilities by IQ. IQ has to be more than 80%.
2. Measuring creativity by CQ test (Mensa). CQ has to be more than 70%.
3. Measuring above average mathematical abilities by combining both assessments of teachers and experts. The sum of assessments has to be more than 50%.

Second step: considering the social relationship between the students and their parents by defining longings of both of them.

The identifying approach was applied in finding creative-productive students among 69 students in (6-12) classes.

The analysis of the results shows that the research instruments have a high level of reliability in identifying the creative-productive students in mathematics.

3.2.2 Studies Concerned with Providing Tailored Educational Differentiations

The surveyed studies under this category seek to differentiate the educational contexts and practices of the mathematically gifted, either by developing enrichment programs, providing extracurricular work or suggesting alternative instructional strategies, in order to provide the mathematically gifted students with tailored educational opportunities so as to not only cater their mathematical needs and interests but to also further develop their mathematical powers and learning outcomes.

Concerning the development of enrichment programs, Lee, Hwang, and Lee's study (2004) aimed at developing an enrichment program for the gifted students in high schools, focusing on conic section according to the Enrichment Triad Model of Joseph Renzulli (see Chapter 2 – Section 2.2.5-1). Eraky's study (2004) was also targeted

at developing an enrichment program for the purpose of enhancing the gifted students' problem-solving abilities and attitudes towards autonomous learning in middle schools.

With respect to providing extracurricular work, Velikova (2004) suggested a new Model of Joint and Independent Creative Work (MJICW) between a leading teacher (or a team) and the creative-productive gifted students.

As for proposing other alternative strategies, Diezmann and Watters's study (2004) provided some alternative instructional strategies that could be used to laterally differentiate the regular mathematics curriculum for the gifted students.

More details about these studies are provided in the coming sections.

Lee, Hwang, and Lee (2004) performed a project that developed teaching and learning contents of an enrichment program for the mathematically gifted high school students focusing on the conic section. The program was based on the characteristics of the mathematically gifted, the nature of enrichment programs and the principles of developing programs for the gifted.

Class activities in the contents were selected and organized based on topic-centered, activity oriented, open, interdisciplinary and student-selected approaches. They were also designed according to the Triad Enrichment Model developed by Joseph Renzulli (see Chapter 2 – Section 2.2.5-1)).

Sixteen class activities were composed as project performing procedures in four steps:

First step: It calls interest and active participation of students into the subject through the various activities

Second step: This is to provide the students with sufficient knowledge concerning the conic section through activities.

Third step: This enables students to express the conic curve as a polar equation based on the knowledge and function acquired from the performance, and to study on cryptosystem and crypto experiment on the elliptic curve as well as the algorithms. And it in-

structs them to compose a subject report based on the properties of elliptic curve.

Fourth step: This is the presentation and evaluation/reflection step. In this step, students are encouraged to present and discuss on their subject report and evaluate it according to the objective followed by the overall review and modification on the project procedures and products (subject report or idea suggestion) based on the evaluation.

Another study conducted by Eraky (2004) pertained to developing an enrichment program for the purpose of promoting the mathematically gifted students' problem-solving ability and the attitudes towards autonomous learning in middle schools.

The study aimed at developing an enrichment program in mathematics based on the open-ended mathematical problem solving approach as well as using the computer technology represented in dynamic geometry software in developing the mathematically gifted students' both ability of problem solving and attitude towards mathematics self-learning in middle schools.

The subjects of the study included two classes of mathematically gifted students in George Center School at Halle, Sachsenanhalt, Germany. The Subjects consisted of two experimental groups. The first experimental group included 17 students at 7th grade. The second experimental group included 14 students at 9th grade.

The proposed enrichment program comprised of two sections: the first one contained open-ended mathematics problems characterized by complexity and challenge the mathematically gifted students.

The second section also included open-ended mathematics problems; however, it was based on using the interactive dynamic geometry software.

The results of the study revealed that the proposed enrichment program was effective in developing the mathematically gifted students problem solving ability as well as improving the students attitudes toward mathematics self-learning.

The proposed enrichment program contributed to the development of the verbal and

symbolic mathematical communication abilities of the students. It also improved the students' attitudes towards mathematics self-learning through critical reading, understanding, planning for the solution, exploration, implementation, verification, generalization, and verbally and symbolically communicating with others.

In addition, the different facilities of the interactive dynamic geometry software helped the students to generate new ideas and approaches in solving the given problem that led to improving their problem solving abilities.

As for providing an educational differentiation in the form of extracurricular work, Velikova's study (2004) suggested a new Model of Joint and Independent Creative Work (MJICW) between a leading teacher (or a team) and the creative-productive gifted students.

In this model (MJICW), a suggested combination of activities was designed, which was mainly directed at developing of the productive potential of the students, enhancing the development of their creative products, stimulating their creativity in specific areas of mathematics, and providing conditions for the realization of mathematical creativity.

The model (MJICW) included three inter-related actions:

1. **Independent research and creative work of the teacher** himself for planning and organizing the enrichment activities.
2. **Joint work between the teacher and the students** to prepare students for the coming independent creative work.
3. **Independent creative work of the gifted students** including developing, formulating, and presenting new products in front of an appropriate audience.

The model (MJICW) was fundamentally based on the Enrichment Triad Model of Joseph Renzulli (see Chapter 2 – Section 2.2.5-1)) and it, accordingly, had three types of enrichment activities:

Activities of Type I: They include general preparation and exploratory activities,

which aim at developing a strong interest in a specific area of mathematics.

Activities of Type II: They include preparation for creative work and group training activities, which aim at developing the creative abilities, acquiring knowledge to a degree of applicability in the field of the students interests, acquiring the methods of creative work, and promoting the motivation level required for the creative work.

Activities of Type III: They include the creative work, which aim at providing conditions for the realization of mathematical creativity and presenting the creative products in front of an appropriate audience.

The program content comprised many mathematical topics including geometry, algebra, vectors, inequalities, and others.

The model was experimented with students from ninth to twelfth grades to decide on its significance. The experimental results indicated that the use of the model (MJICW), as a set of stimuli, provided conditions for developing the productive potential of students and enhancing the development of their creative products.

With respect to developing alternative instructional strategies for the mathematically gifted, [Diezmann and Watters \(2004\)](#) proposed some lateral instructional strategies to differentiate the regular mathematics curriculum in order to support mathematically gifted elementary students in the regular classroom.

These strategies aimed at developing rich mathematical learning opportunities within regular classrooms. That is, these strategies provided ways to enrich learning experiences **laterally** rather than **sequentially** so as to enhance gifted students' reasoning, accommodate their capacity for learning, and foster their interests in accordance with best practice in gifted education.

The study provided four lateral instructional strategies to challenge the mathematically gifted in regular mathematics classrooms. These strategies include:

1. **Problematising mathematical tasks** – used to increase the level of challenge in regular classroom tasks. *Problematisation* includes inserting obstacles to the solu-

tion, removing some problem information, or requiring students to use particular representations or develop generalizations.

2. **Implementing mathematical investigations** – used to encourage students to apply and create mathematical knowledge by posing and solving novel and open-ended problems.
3. **Extending manipulative use** – used to promote students’ visual-spatial or kinaesthetic representations and accordingly support their higher-level thinking.
4. **Modifying educational games** – used to provide rich mathematical and social learning opportunities for the gifted.

Findings indicated that using lateral instructional strategies expands gifted students’ mathematical knowledge through challenging experiences that are connected to the regular curriculum. The study, further, revealed that the lateral strategies have five particular advantages.

1. Lateral strategies are not add-ons or extensions but take the existing curriculum and problematize, adapt and enrich its experiences for gifted students.
2. These approaches can be used in both collaborative and independent learning.
3. Lateral strategies provide underachieving gifted students with opportunities to move back and forth between regular activities and more challenging activities according to their capability, confidence and motivation.
4. Lateral approaches provide teachers with opportunities to develop their confidence and competence in working with gifted students.
5. Lateral strategies are thought-revealing activities that provide opportunities for the identification and development of mathematically gifted students.

3.3 Studies Related to Geometric Creativity Development

In this domain, only a limited number of related studies is found and surveyed only six studies due to the novelty of the educational research on the topic of the development of creativity in the field of geometry. Four of them (Al-Baz, 1999; El-Rayashy and Al-Baz, 2000; Mohamed, 2003; Nakin, 2003) were concerned with proposing strategies, approaches, or programs for the development of the creative potential in the field of geometry. The other two studies (Lee and Shim, 2005; Yang, 2007) were particularly concerned with the assessment of that potential. Accordingly, the related studies in this domain are classified under two categories: Studies concerning the development of creative potential in geometry and studies concerning the assessment of creative potential in geometry.

3.3.1 Studies Concerning the Development of Creative Potential in Geometry

Under this category, an experimental study conducted by (Al-Baz, 1999) aimed at:

1. Developing a proposed strategy of teaching and learning solid geometry for developing general creativity, geometric creativity, and creative perceptions.
2. Determining which is more effective in developing creativity, geometric creativity, and creative perceptions: the strategy proposed by the researcher or the strategy of De Bono.
3. Determining which strategy is more effective – the proposed one or De Bono's – in retaining creativity, geometric creativity, and creative perceptions for a long time.

Subjects were students from the third grade of secondary stage in the scientific section who studied solid geometry. Two experimental groups of students were assigned for the study. One group studied the content of solid geometry using the proposed strategy and the other studied the same content of solid geometry using De Bono's strategy.

The proposed strategy for teaching and learning the content of solid geometry in the secondary stage was developed. The proposed strategy involves many instructional adaptations such as autonomous learning, problem-solving strategy, inquiry-based learning, and brainstorming. Moreover, a geometric creativity test was developed to assess the subjects' geometric creativity before and after the educational treatment.

The developed geometric creativity test and Williams scale for assessing creativity and creative perceptions were administered to the students before and after the intervention to assess students' general creativity, geometric creativity, and creative perceptions.

The results indicated the effectiveness and superiority of the proposed strategy to that of De Bono's in developing creativity, geometric creativity, and creative perceptions and retaining them for a long time among secondary school students.

Another experimental study concerning the development of geometric creativity was conducted by [El-Rayashy and Al-Baz \(2000\)](#). The study attempted to answer the following research questions:

1. What is the proposed strategy based on the group mastery learning approach for developing geometric creativity and reducing problem-solving anxiety among preparatory stage students?
2. What is the effect of teaching geometry using the group mastery learning approach on developing geometric problem solving skills among preparatory stage students?
3. What is the effect of teaching geometry using the group mastery learning approach on developing the geometric creativity components among preparatory stage students?
4. What is the effect of teaching geometry using the group mastery learning approach on reducing problem solving anxiety among preparatory stage students?
5. What is the relation between geometric problem solving skills, geometric creativity, and problem solving anxiety among preparatory stage students?

Subjects were 96 students from eighth grade at a public school in Zagazig governorate, Egypt. There were 48 students in the experimental group and 48 students in the control group.

The experimental group studied the geometrical part of the curriculum, which is represented in two units (Unit 1: Areas and Unit 2: The relation between the lengths of sides of the right-angled triangle) using the group mastery learning approach. On the other hand, the control group studied the same part using the regular way of teaching and learning.

Geometric creativity test, problem solving anxiety scale, and geometric problem solving skills test, designed by the two researchers, were administered at the beginning and at the end of the experimentation to measure students' geometric creativity, students' levels of problem solving anxiety, and students' geometric problem solving skills.

The study revealed many results that can be summarized in the following points:

1. Teaching geometry using the group mastery learning approach is effective in developing geometric problem solving skills among the students of the experimental group.
2. Teaching geometry using the group mastery learning approach is effective in developing the geometric creativity components and each component of geometric creativity (fluency, flexibility, originality, and sensitivity to problems) individually among the students of the experimental group.
3. Teaching geometry using the group mastery learning approach is effective in reducing problem solving anxiety among the students of the experimental group.
4. There is a negative significant correlation between students' performance of solving geometric problems and their level of anxiety of solving problems.
5. There is a negative significant correlation between students' scores of geometric creativity and their anxiety levels of solving geometric problems.
6. There is a positive significant correlation between students' scores of geometric

creativity and their geometric problem solving skills.

The third experimental study was attempted by [Mohamed \(2003\)](#) and aimed at investigating the effect of using an enrichment program in Euclidean geometry for developing creative thinking in mathematics among mathematically gifted girls in middle schools.

For achieving this, the researcher prepared an enrichment program in two Euclidean geometry units (Unit 1: Congruency, and Unit 2: Inequalities of triangles), an achievement test for the selected units, and a creative thinking test in mathematics.

Subjects were 38 mathematically gifted girls in the eighth grade.

Results of the study showed that there were statistically significant differences between the mean scores of the students in the pre and post measurements of the achievement test in congruency and inequalities of triangles, the creative thinking test in mathematics in its fluency, flexibility, and originality components in favor of the post measurement.

Results also showed that there was a positive significant correlation between the achievement in geometry and the creative thinking in mathematics.

The fourth study under this category was attempted by [Nakin \(2003\)](#). The study aimed at determining to what extent creativity and divergent thinking could enhance the acquisition of geometric concepts using the problem-based approach and students' self-discovery.

Subjects of the study were 12 students at Northcrest High School, Mthatha, South Africa. Students were guided to discover the meaning of geometric concepts through their self-discovery. Furthermore, students were asked to use metaphors and analogies; write poems, essays, and posters; and tessellate colored polygons in various patterns to produce works of art to express their understandings of geometric concepts.

Results of the study revealed the following recommendations:

1. Students should re-invent geometry and develop their own strategies in problem

solving.

2. Activities of creativity and divergent thinking should be valued in the teaching and learning of geometry since they enhance the acquisition of geometric concepts. Workgroup should be used while students are tackling such activities.

3.3.2 Studies Concerning the Assessment of Creative Potential in Geometry

As for assessing the creative potential in the field of geometry, Lee and Shim (2005) attempted to establish a scoring system for assessing creativity in a mathematical activity program.

The developed scoring system assumed creativity as a construct that consists of three components; **fluency** (based on the number relevant responses), **flexibility** (based on the number of different categories of the relevant responses), and **originality** (based on the degree of rareness of responses).

Using some geometric resources such as paper folding, pattern blocks, pentomino and geoboard, the study provides some examples of assessment items that could be used to assess mathematical creativity and some examples of expected responses of using these activities. Then, the study proposed a general scoring method and criteria for checking mathematical creativity.

In the same concern of assessment, Yang (2007) conducted a study to investigate the validity issues of the Computerized Assessment of Geometric Creativity (CAGC).

Subjects were two different samples of students in fourth and sixth grades.

The researcher used the structure equation model to evaluate the proposed model. In this model, geometric creativity, with creativity, is considered as an independent variable, geometric creativity as a mediator variable, and the mathematical ability as a dependent variable.

The students' responses on the Computerized Assessment of Geometric Creativity were

analyzed.

Results of the study can be summarized in the following points:

1. The internal consistency of the CAGC was 0.73. The correlation coefficient between CAGC and related variables (mathematical ability, spatial sense, and creativity) were around 0.38 to 0.48.
2. The proposed model assumes that creativity affects geometric creativity and geometric creativity as a mediator also affects mathematical ability. In this concern, results show that students' creativity significantly predicts their geometric creativity and students' geometric creativity also significantly predicts their mathematical performances.
3. There were significant differences between grades on the CAGC performances. The accuracy in discriminating gifted fourth and sixth graders by CAGC was 89.8% and 77.9%.

3.4 Commentary on Related Studies

With regard to the first domain of related studies – studies related to the use of dynamic geometry software in the teaching and learning of mathematics – the results of the reviewed studies are consistent with each other indicating the effectiveness of using dynamic geometry software in the teaching and learning mathematics as well as emphasizing that the use of DGS has positive effects on different learning outcomes of students (e.g., mathematical performance, academic achievement, mastery levels of mathematical concepts and skills, mathematical thoughts and understandings, and van Hiele levels of geometric thinking, among others).

However, prior studies in the domain of using dynamic geometry software in the teaching and learning mathematics also emphasized important limitations of this use. One of these limitations entails that the use of DGS cannot provide a self-contained environment and that the software itself does not necessarily mean that students will learn geometry.

Another limitation, suggested by research, is that it can take quite a long period of time for the benefits of using DGS to emerge but that this investment is beneficial in developing students' understandings and knowledge of geometry. Moreover, the sorts of tasks that students tackle, the form of teacher input and the general classroom atmosphere are all important factors (Jones, 2005, p. 27).

Overall, results of the prior studies in this domain indicate that successful access to geometric theory using DGS does not take place without carefully designed tasks, professional teacher input, and opportunities for students to conjecture, to make mistakes, to reflect, to interpret relationships among objects, and to offer tentative mathematical explorations (Jones, 2005, p. 29).

However, reviewing the prior studies related to this domain was a very beneficial procedure in the present study since it was utilized in elaborating and dealing with many issues and concerns at both levels of the study: the theoretical level and the experimental one. That is, reviewing the prior studies – equally with the given data of prior literature – had a significant role and helped a good deal in developing the following issues and concerns underlying the present study:

1. Formulating the study hypotheses in a predictive way as they are presented in Chapter 1 – Section 1.11.
2. Determining the role of dynamic geometry software in geometry education, which is presented in Chapter 2 – Section 2.1.5.
3. Determining the significance of using dynamic geometry software in developing mathematically gifted students' geometric creativity in high schools, which is presented in Chapter 2 – Section 2.4.
4. Choosing the suitable dynamic geometry software that can be effectively used in developing and designing the suggested enrichment program so as to develop the mathematically gifted students' geometric creativity.
5. Developing suitable teaching and learning strategies for the enrichment activities using dynamic geometry software to develop the mathematically gifted students'

geometric creativity, which are presented within The Suggested Enrichment Program – The Teacher’s Guide (see Appendix [C](#)).

Nevertheless, the present study is consistent with the related studies of this domain in emphasizing the importance of using DGS in mathematics classrooms for better students’ learning outcomes. There are three main research parameters in terms of the study learning inputs, learning outputs, and subjects that are clearly distinct in the present study from the other related ones.

More specifically, with regard to the research parameter concerning the study learning inputs, the present study makes use of the dynamic geometry software in developing and designing a whole enrichment program (see Chapter 4 – Section [4.1](#) The Suggested Enrichment Program). In contrast of that, the related studies used the software to introduce some regular classroom geometric content – for instance: circles unit ([Chomskis and Hinkle, 2006](#); [Hull and Brovey, 2004](#)), to investigate some open-ended problems – for instance: Varignon’s quadrilateral, medial triangle, and Fagnano’s theorem ([Budge, 1999](#); [Christou *et al.*, 2005](#); [Haja, 2005](#)), to explore the validity of some geometric statements – for instance: If a cyclic quadrilateral is a parallelogram, then it is a rectangle ([Connor, Moss, and Grover, 2004](#)), or to look into some mathematical concepts and skills – for instance: the relationship between area and perimeter of polygons, and quadratic functions ([Abu Bakar *et al.*, 2009](#); [Almeqdadi, 2000](#)).

With regard to the second research parameter concerning the study learning outputs, the present study aimed at using DGS to develop creative thinking abilities in the field of geometry, whereas the other related studies investigated the effectiveness of using the software in promoting other learning outputs, for instance: mathematical performance, academic achievement, and attitudes ([Abu Bakar *et al.*, 2009](#); [Chomskis and Hinkle, 2006](#); [Hull and Brovey, 2004](#)), mathematical understandings and knowledge of geometric concepts ([Almeqdadi, 2000](#); [Budge, 1999](#)), or van Hiele levels of geometric thinking ([Johnson, 2002](#); [Patsiomitou, 2008](#)) as mentioned before in the review.

As for the third research parameter concerning the study subjects, the present study mainly targeted the mathematically gifted and talented students during using the dy-

dynamic geometry software, whilst other related studies dealt with regular students in middle schools or high schools (Abu Bakar *et al.*, 2009; Almeqdadi, 2000; Chomskis and Hinkle, 2006; Hull and Brovey, 2004; Johnson, 2002; Patsiomitou, 2008) or pre-service teacher students (Budge, 1999; Christou *et al.*, 2005; Connor *et al.*, 2004; Haja, 2005).

With regard to the second domain of related studies – studies related to mathematically gifted education – the reviewed studies highlighted the importance of providing the mathematically gifted students with tailored educational differentiations. Moreover, they asserted on the positive effects of using these differentiations on developing the gifted students' mathematical knowledge, problem-solving abilities, attitudes and motivations towards mathematics, different types of thinking, and achievement (Diezmann and Watters, 2004; Eraky, 2004; Lee, Hwang, and Lee, 2004; Velikova, 2004).

The reviewed studies also emphasized the importance of experimenting and evaluating the different programs and educational differentiations provided in schools to the mathematically gifted students in order to ascertain their appropriateness in catering the mathematically gifted students' needs and interests.

With regard to the category of studies concerned with identifying the characteristics of the mathematically gifted students, they involve multiple research techniques and methods including: observation, clinical interview, self-questionnaire, journal writings, Figural Torrance Test of Creative Thinking, etc. However, they are consistent with each other indicating that the mathematically gifted students had high abilities for achievement, knowledge construction, and solving problems compared with regular students.

Reviewing this domain of related studies was beneficial for the present study in elaborating and developing many sections in the dissertation (e.g., Nature of Mathematical Giftedness, Characteristics of the Mathematically Gifted Students, General Principles of Developing Enrichment Programs for The Mathematically Gifted students, The principles of Developing the Suggested Enrichment Program)

Furthermore, this related studies review was fruitful for the present researcher in deepening his realization of the study's problem and the importance of providing well qualified and precisely tested programs for the mathematically gifted students. It was also beneficial in strengthening the present researcher insights in the field of mathematically gifted education, which were required for developing the suggested enrichment program and its enrichment activities.

There are some similarities and variations between the present study and the other reviewed studies in this domain with regard to studies' dependent variables, independent variables, and subjects. More specifically, with regard to the dependent variables, the present study uses the same educational differentiation technique – the enrichment program – similar to some of the related studies (Eraky, 2004; Lee, Hwang, and Lee, 2004; Velikova, 2004) but the developed enrichment program in the present study is different from other researchers' attempts in terms of its principles, aims, content, and process (see Chapter 4 – Section 4.1 The Suggested Enrichment Program).

As for the independent variables and students' learning outcomes, the present study clearly seeks the development of students' creativity in the field of geometry, while others sought other learning outcomes, for instance Eraky's study (2004) aimed at the development of students' problem-solving ability and their attitudes towards autonomous learning. Further, Velikova's study (2004) targeted at developing students' productive potential and enhancing their creativity.

As for subjects, the present study was similar to a range of prior studies in selecting mathematically gifted subjects from high schools (e.g., Lee, Hwang, and Lee, 2004; Sriraman, 2003; Velikova, 2004). However, it was different from the others that use subjects from primary and middle schools (e.g., Choi and Do, 2008; Eraky, 2004; Huang, 2003; Hui-Chin, 2004; Lee and Hwang, 2005).

With regard to the third domain of related studies – studies related to geometric creativity development – the studies, under the category of the development of creative potential in geometry, clearly pointed out the effectiveness of using different teaching and learning methods, approaches, and strategies in developing students'

creative potential in geometry. Instances of these methods include: the group mastery learning approach in [El-Rayashy and Al-Baz \(2000\)](#), the proposed strategy that involves different teaching styles (e.g., autonomous-learning approach, problem-solving strategy, inquiry-based approach, and brainstorming) in [Al-Baz \(1999\)](#) the problem-based approach, and students' self-discovery in [Nakin \(2003\)](#).

Moreover, they emphasized the teacher's role in developing creativity and creative thinking in geometry classrooms. They also revealed the importance of presenting special programs to develop creative thinking among students with special needs including gifted and underachieving students. Finally, they called for evaluating the programs introduced in the universities in terms of their qualities in developing creative thinking among the university graduates.

Combining some results of the surveyed studies in this domain ([El-Rayashy and Al-Baz, 2000](#); [Mohamed, 2003](#)) can lead to an important educational conclusion in the field of teaching and learning geometry. This conclusion indicates that there is a positive significant correlation between students' achievement in geometry, their geometric problem-solving skills, and their creative potential in geometry.

Reviewing this domain of related studies was very beneficial for the present study in elaborating and developing many sections in the dissertation concerning different issues and concerns. Amongst those sections are:

1. The nature of creativity (see Chapter 2 – Section [2.3.1](#)).
2. The nature of geometric creativity (see Chapter 2 – Section [2.3.2](#)).
3. Specification of the components that the developed Geometric Creativity Test (GCT) measures, which is presented in Chapter 4 – Section [4.2.2](#).
4. Developing a grading system for the GCT, which is presented in Chapter 4 – Section [4.2.4](#).
5. Formulating the items of the GCT (see The Geometric Creativity Test (GCT – English Version – Appendix [H](#))).

In addition to the previously mentioned gains, reviewing the prior studies in this domain provides the present researcher with a good deal of expertise in the methods and ways that can be used to promote creativity in geometry classrooms, which was needed in designing and developing the enrichment activities and developing suitable teaching and learning strategies for presenting them to mathematically gifted students (see Chapter 4 – Section 4.1 The Suggested Enrichment Program, The Student’s Handouts – Appendix A, and The Teacher’s Guide – Appendix C).

Comparing the present study to the other reviewed ones in this domain, some similarity can be discerned between the present study and the studies of (Al-Baz, 1999; El-Rayashy and Al-Baz, 2000; Mohamed, 2003; Nakin, 2003). This similarity is manifest in some teaching and learning techniques that they used and led to the development of the creative potential such as (open-ended problem-solving strategy, self and guided discovery approach, brainstorming activities, among others). On the other hand, it is different from these studies in its contribution to enriching students’ learning environment with an interactive mediation represented in the use of the interactive dynamic geometry software, Cinderella.

Furthermore, in terms of the subjects used, the present study involved high-school mathematically gifted subjects. However, the study of Mohamed (2003) involved middle-school mathematically gifted students, and the studies of (Al-Baz, 1999; El-Rayashy and Al-Baz, 2000; Nakin, 2003) had subjects consisting of regular students either in high schools or middle schools.

Moreover, the study differentiates itself from the related studies of (Lee and Shim, 2005; Yang, 2007) that were concerned with the assessment of the creative potential in geometry. This difference lies in developing a specially designed instrument – The Geometric Creativity Test (GCT) – and deciding on its appropriateness, validity and readability for assessing the creative potential in geometry among the mathematically gifted students (see Chapter 4 – Section 4.2 The Geometric Creativity Test)

In conclusion, the review of related studies and the previously provided commentary on its three domains reveal the importance of the present attempt. As far as the present

researcher traced, developing an enrichment program using the interactive geometry software – as a mediation tool – and deciding on its effectiveness in developing the mathematically gifted students’ geometric creativity in high schools is a research area that was not covered by prior research in the field of mathematics education.

3.5 The Principles of Developing the Suggested Enrichment Program

Reviewing prior literature and studies related to the topic of the study, the present researcher was able to determine the following principles underlying the development of the suggested enrichment program (using dynamic geometry software for the purpose of promoting geometric creativity among the mathematically gifted students in high schools):

- The suggested enrichment program should provide the mathematically gifted students with opportunities to explore some mathematical ideas using the DGS in a creative fashion.
- Activities within the suggested enrichment program should provide the mathematically gifted students with opportunities to reinvent the mathematical ideas through both exploration and refining of earlier ideas.
- The enrichment activities should be designed and presented in a constructivist way that encourages the mathematically gifted students to make new connections to their prior experiences and construct their own understanding.
- Teaching the instructional activities, within the suggested enrichment program, should follow van Hiele phases of learning geometric concepts: Information, guided orientation, explanation, free orientation, and integration.

Information: Through discussion, the teacher uses DGS to identify what students already know about the topic. Through this discussion, the students become oriented to the topic.

Guided Orientation: Using DGS facilities of dragging, constructing, measuring, calculating and dynamic manipulation, students can explore specific geometric concepts.

Explication: Using IDGS, students can describe what they have learned about a topic in their own construction.

Free Orientation: Using DGS, students can apply the relationships while trying to solve problems and investigate more open-ended tasks through theorem proving facilities, which are embedded in the DGS.

Integration: Using DGS, students can develop a new network of objects and relations.

- The suggested enrichment program activities should correspond to the students' skills; since they should experience success in order to stay motivated.
- The suggested enrichment activities should challenge students' thinking, enhance students' achievement, and develop students' geometric creativity.
- The instructional activities, within the suggested enrichment program, should be designed to be effective in revealing geometric creativity and in distinguishing among the mathematically gifted students in terms of geometric creativity and their responses.
- The suggested enrichment program activities should address standards for school mathematics recommended by the National Council of Teachers of Mathematics (NCTM) as one of the most popular standard in the field of teaching and learning mathematics.

Thus, in this part of the thesis, the researcher has answered the first question of the study, which states: “*What are the principles of developing a suggested enrichment program using dynamic geometry software for developing the mathematically gifted students' geometric creativity in high schools?*”

Chapter 4

Method and Procedures

In this chapter, the researcher presents in detail the suggested enrichment program, the designed geometric creativity test, and the procedures of the experimental study. For the purpose of designing the suggested enrichment program, the researcher reviews literature related to the use of DGS in teaching and learning mathematics and developing geometric creativity as well as current perspectives and methods of teaching and learning for the mathematically gifted education. The geometric creativity test designed by the researcher is also explained in this chapter. Reviewing literature on the development of geometric creativity, the researcher designed the geometric creativity test for the purpose of assessing the geometric creativity of the mathematically gifted students before and after administering the suggested enrichment program. Details about the suggested enrichment program, the geometric creativity test, and the procedures of the experimental study are further discussed in the coming sections.

4.1 The Suggested Enrichment Program

The purpose of the study is to introduce a suggested enrichment program in Euclidean geometry to enhance the geometric creativity of the mathematically gifted students in high schools using the interactive geometry software, Cinderella as DGS (Richter-Gebert and Kortenkamp, 2006). But, why has Cinderella been used?

4.1.1 Why Cinderella?

Even though there are many computer software applications for doing mathematics in general and for doing geometry in particular, the present researcher chose Cinderella to use in developing the suggested enrichment program because Cinderella may provide the mathematically gifted students with a suitable dynamic environment in which they can engage in exploring, conjecturing, testing, and confirming geometric relationships. Moreover, Cinderella has special features (see [Kortenkamp, 1999](#); [Materlik, 2003](#); [Richter-Gebert and Kortenkamp, 1999, 2006](#)) that could be useful in teaching and learning geometry among the mathematically gifted students such as:

1. Cinderella provides an environment, with a range of geometric objects and tools, in which the mathematically gifted students can not only simply construct geometric constructions with extreme accuracy and ease but also interactively explore the dynamic behavior of the constructed figures through the dynamic tools, which are embedded in Cinderella.
2. Cinderella special facilities of constructing, dragging, measuring, calculating, animating, and tracing loci provide many opportunities to the mathematically gifted students to dynamically visualize a variety of geometric situations and problems.
3. Cinderella has built-in automatic proving facilities. This feature enables the mathematically gifted students to automatically check the correctness of their mathematical ideas and conjectures.
4. Cinderella has the possibility to directly publish interactive figures onto the Web. This enables the mathematically gifted students not only to immediately publish their work on interactive web pages but also to mathematically communicate with others through the World Wide Web, the electronic mail, and the web pages.
5. Cinderella has powerful modes for geometric transformations (e.g., reflection, translation, rotation, and similarity). These modes of geometric transformations could be very useful in simplifying constructions for the mathematically gifted students.

6. Cinderella has the possibility to design interactive exercises as web pages that mathematically gifted students can solve on their own.
7. Cinderella has a correct mathematical treatment of geometry on computer, which is considered an additional advantage for software rationality and stability, while doing mathematics on computer.

4.1.2 Principles of Developing the Suggested Enrichment Program

By the end of chapter 3, the present researcher specifies the principles¹ of developing the suggested enrichment program, which are derived from the following resources:

- The characteristics of the mathematically gifted students and the nature of mathematical giftedness.
- The nature of creativity in general and geometric creativity in particular.
- The general principles of developing enrichment programs for the mathematically gifted students.
- The contemporary trends in planning and organizing enrichment programs for the mathematically gifted students.
- The principles and standards of school mathematics.
- The characteristics of dynamic geometry software.

4.1.3 Aims of the Suggested Enrichment Program

The overall aim of the suggested enrichment program is to cultivate and develop keen creative abilities in geometry among the mathematically gifted students in high schools using Cinderella application as a mediation environment. This overall aim can be interpreted in the light of the definition of geometric creativity, and the main characteristics of Cinderella. It branches out into the following specific aims:

¹For more details about the principles of developing the suggested enrichment program, see Chapter 3. Related Studies – Section 3.5.

Through the suggested enrichment program, the mathematically gifted students will hopefully be able to use Cinderella to:

1. Construct dynamic figures.
2. Come up with many construction methods to construct dynamic configurations for an assigned figure.
3. Come up with many various and different construction methods to construct dynamic configurations for an assigned figure.
4. Come up with novel and unusual methods to construct dynamic configurations to an assigned figure.
5. Produce many relevant responses (ideas, solutions, proofs, conjectures, and new formulated problems) toward a geometric problem or situation.
6. Produce many various and different categories of relevant responses (ideas, solutions, proofs, conjectures, and new formulated problems) toward a geometric problem or situation.
7. Generate many unusual (“way-out”), unique, clever responses or products toward a geometric problem or situation.
8. Make new conjectures and relationships by recognizing their experience toward the aspects of the given problem or situation.
9. Investigate the made conjectures by different methods in different situations.
10. Generate many different and varied proofs using the formal logical and deductive reasoning toward a geometric problem or situation.
11. Generate many follow-up problems by redefining (modifying, adapting, expanding, or altering) a given geometric problem or situation.
12. Apply different learning aspects of geometry (concepts, generalizations, and skills) in solving a geometric problem or situation.

4.1.4 Content of the Suggested Enrichment Program

The suggested enrichment program consists of three interrelated portions: student's handouts, a teacher's guide, and a CD-ROM.

They (student's handouts, the teacher's guide, and the CD-ROM) cover 12 enrichment activities. These enrichment activities are open-ended and divergent-production geometric situations and problems that require many various and different responses.

These enrichment activities are designed in four categories to develop the geometric creativity components based upon specific teaching and learning strategies using different facilities of the interactive geometry software Cinderella, which are:

1. **Problem solving activities**, where the student is given a geometric problem with a specific question and then invited not only to find many various and different solutions but also to pose many follow-up problems related to the original problem (activities 1, 5, and 6).
2. **Problem posing activities**, where the student is given a geometric situation and asked to make up as many various and different questions, relationships, or conjectures as he/she can, that can be answered in direct or indirect ways using the given information (activities 11 and 12).
3. **Construction activities**, where the student is asked to come up with as many various and different methods as he/she can to construct a geometric figure (e.g., Parallelogram) using the constructing facility of Cinderella application (activities 7, 8, 9, and 10).
4. **Redefinition activities**, where the student is given a geometric problem or situation and invited to pose as many problems as possible by redefining – substituting, adapting, altering, expanding, eliminating, rearranging or reversing – the aspects that govern the given problem (activities 2 and 4).

These are not presented as hard-and-fast categories, but as a framework for designing activities that might help the mathematically gifted students to develop their geometric

creativity.

1) Student's Handouts

The first main portion of the suggested enrichment program is represented in the student's handouts². There are 15 handouts prepared to guide the student throughout the program. For each activity, a student's handout is prepared to assist students to go through the activity and promote discussions between the teacher and the students and discussions among the students as well.

2) Teacher's Guide

The second main portion of the suggested enrichment program is the teacher's guide³. It is designed to make the teacher's work and progress in the course easier and more effective. The guide does not restrict the teacher's work, but is flexible enough for any creative additions. The teacher's guide includes the following:

Introduction

An introduction for the teacher to the suggested enrichment program is provided in the beginning of the teacher's guide. It aims at introducing the suggested enrichment program, its purposes, its content, and its teaching and learning strategies, as well as a brief explanation of the concept of geometric creativity.

Aims of the Suggested Enrichment Program

The overall aim and the specific aims of the suggested enrichment program are objectively formulated to guide both the teacher and the student toward achieving them through the course. They precisely describe what could be expected from the mathematically gifted students by the end of the course (see Handout 1).

Cinderella Getting Started

²See Appendix A: Suggested Enrichment Program: Student's Handouts.

³See Appendix C: Suggested Enrichment Program: Teacher's Guide.

In this part the researcher introduces some teaching hints to the teacher to help him/her manage the introductory session, which is about Cinderella getting started through the designed handouts (see Handout 2 and Handout 3).

Moreover, the teacher's guide covers 12 enrichment activities; each of them includes the following elements:

The title of the activity The researcher writes the title for each activity in the teacher's guide in a way that consists with the title written in the student's handouts.

The activity problem In each activity, the problem statement of the activity is presented.

Activity content analysis The researcher presents a mathematical content analysis for each activity that includes three main categories of contents: concepts, generalizations, and skills to inform the teacher of the learning aspects that might be covered in the activity.

Objectives Instructional objectives are objectively formulated for each activity to guide both the teacher and the students during teaching and learning processes. They precisely describe what could be expected from the mathematically gifted students by the end of the activity.

Materials A list of proposed materials is suggested for each activity, which includes: computers with Cinderella application installed on them, LCD projector, and student's handout of the activity.

Vocabulary A list of new mathematical vocabulary is provided for each activity.

Prerequisites A list of Cinderella prerequisite skills is given for each activity.

Teaching and learning strategies Specific teaching and learning strategies using different facilities of Cinderella application are presented for each activity, including warm up that aim at developing the mathematically gifted students' geometric creativity.

3) CD-ROM

The third main portion of the suggested enrichment program is the accompanying CD-ROM that contains all dynamic configurations prepared for the program in two formats cdy and html. In addition, it contains html index, in both languages English and German, for all activities of the program and their dynamic configurations that can be used to easily access any activity and any configuration within it.

4.1.5 Appropriateness of the Suggested Enrichment Program

For judging the appropriateness of the suggested enrichment program, the researcher presented it, in its initial form, to a group of experts⁴, who are experienced in teaching and learning mathematics. These experts were asked to decide on the appropriateness of the suggested enrichment program (the student's handouts, the teacher's guide, and the CD-ROM) and suggest all changes necessary to modify it within the framework of the following criteria⁵:

- To what extent the suggested enrichment program is appropriate to the level of the mathematically gifted students in high schools
- To what extent the enrichment activities included in the suggested enrichment program are appropriate to develop the mathematically gifted students' geometric creativity in high schools.
- To what extent the use of Cinderella application is appropriate to the learning actions⁶ adopted in the suggested enrichment program.
- To what extent the teacher's guide is appropriate to guide the teaching and learning processes during the program.
- To what extent the directions set in the teacher's guide are clear.

⁴A list of the experts who validated the suggested enrichment program is presented in Appendix [D](#).

⁵A letter to the experts used for judging the appropriateness of the suggested enrichment program is presented in Appendix [E](#).

⁶Learning Actions are: Constructing, observing, conjecturing, investigating, proving, posing, and elaborating. For more details, see Section [4.1.7](#) The Student's Role.

- Further comments for development.

The experts stated that the suggested enrichment program is appropriate to the level of the mathematically gifted students in high schools and confirmed that the enrichment activities are also appropriate to develop the students' geometric creativity. The experts also remarked that the use of Cinderella as proposed in the suggested enrichment program is appropriate to access and manipulate the program's mathematical content. As for the teacher's guide, the experts asserted that the teacher's guide is appropriate to guide the teacher during the course of the program and the directions set in the teacher's guide are clear. Finally and more importantly, it is recommended that it would be much appropriate to administer the suggested enrichment program to students in grades 9-12 as the students in German high schools study geometry in grades 7-9, so that they can have enough experience and background in geometry that allows them to work with the suggested enrichment program.

4.1.6 Time-range for the Program Activities

After finishing the preparation of the suggested enrichment program and deciding on its appropriateness, the researcher attempted a pilot experimentation for the program that aimed at determining the suitable time-range for each activity and ensuring the experimental appropriateness of the instructional treatment using Cinderella. In this respect, the student's handouts were translated into German⁷ and one of the pre-service student teachers⁸ was trained to be able to administer the program in German. Afterwards, the suggested enrichment program was administered to a sample of 11 pre-service student teachers of mathematics, 7 male and 4 female, in the University of Education Schwäbisch Gmünd at the beginning of the summer semester of the academic year 2008. In the light of the pilot experimentation of the suggested enrichment program, some modifications were made and the time-range for each activity of the program was determined as shown in Table 4.1.

⁷See Appendix B: Suggested Enrichment Program: Student's Handouts German Version.

⁸Markus Böckler

Table 4.1: *Time-range for the Program Activities*

The Activity	No. of Sessions	Time
Introductory Session: Cinderella Getting Started	1	90 min.
Activity 1: Dragging and Measuring Facilities of Cinderella	1	90 min.
Activity 2: Automatic Proving Facilities of Cinderella	1	90 min.
Activity 3: Developing Macro-constructions	1	90 min.
Activity 4: Animating and Tracing Loci Facilities of Cinderella	1	90 min.
Activity 5: Midpoints of the Sides of a Quadrilateral	1	90 min.
Activity 6: Angular Bisectors of a Parallelogram	1	90 min.
Activity 7: Constructing a Parallelogram	1	90 min.
Activity 8: Constructing a Rhombus	1	90 min.
Activity 9: Constructing a Rectangle	1	90 min.
Activity 10: Constructing a Square		
Activity 11: Posing Geometric Problems	1	90 min.
Activity 12: Finding Geometric Relationships	1	90 min.

4.1.7 The Student's Role

The student in the suggested enrichment program has several roles, which are varying according to the learning actions the student will move through during the enrichment activities of the program. There are 7 main learning actions the student will move through most of them during the enrichment activities and therefore there are 7 different roles of the student corresponding to them.

Here are the learning actions and the student's roles that correspond to each of them.

Constructing: The first learning action in most of the enrichment activities is to construct dynamic configurations using the constructing facilities of Cinderella. In construction activities, the student's role is to use the constructing facilities of Cinderella to come up with various and different dynamic configurations for the assigned figure. In other enrichment activities, the student's role is to come up

with only one dynamic configuration for the assigned problem or situation.

Observing: Observing action includes using the dragging facilities of Cinderella to alter the constructed configuration and visually observe the ripple effect. The student's role is to drag free points or semi free points and visually observe what is invariant under dragging. Knowing what is invariant when a configuration is dragged is not always obvious. However, very often the student is guided through the observation process using the problem statement to see what is invariant.

Conjecturing: Here the observation should be reported in the form of mathematical theorems; so the student's role is to think up mathematical conjectures based upon his/her own observation and formulate them in the form of mathematical theorems.

Investigating: This action is concerned with looking into the correctness of the formulated conjectures using different facilities of Cinderella by different methods in different situations. The student's role is to use different facilities of Cinderella to design endeavors to visually and dynamically examine the correctness of his/her conjectures.

Proving: Deductive reasoning and producing formal proofs are vital learning actions to decide on the correctness of the formulated conjecture as well as logically convince students of the validity of the conjectures. The student's role is to build various and different mathematical proofs for the formulated conjectures using the formal logical and deductive reasoning.

Elaborating: The elaboration action includes changing the original situation by re-defining – substituting, combining, adapting, altering, expanding, eliminating, rearranging, or reversing – one or more of the situation's aspects then speculating on how this single change would have a ripple effect on other aspects of the situation at hand. The student's role is to think carefully to pick one or more of the situation's aspects and redefine it/them and then again use different facilities of Cinderella to design an endeavor to explore the ripple effect on the situation,

and then formulate new conjectures or situations.

Posing: The posing action in the suggested enrichment program includes using the information that is available in the problem or the situation with different facilities of Cinderella to make up and find new problems that could be answered or deduced, in direct or indirect ways, from the given information in the situation. The student's role is to use the given information with different facilities of Cinderella to find and pose new problems. In this action, the student's role is extended to be not only problem solver but also problem finder and problem poser.

Consequently, the roles of the student during the suggested enrichment program could be described as: dynamic-configuration constructor, observer, conjecture maker, endeavor designer, investigator, explorer, elaborator, problem solver, problem finder, and problem poser. All of these roles are supposed to be played in a collaborative context in which the student actively interacts with the teacher and other students using the proposed materials.

4.1.8 The Teacher's Role

The role of teacher is important to facilitate the learning actions that the students move through during the enrichment activities of the program. So it is expected that the teacher manage the enrichment session by:

1. Presenting the activity problem to the students.
2. Encouraging the students to use Cinderella to construct a dynamic configuration for the problem or the situation.
3. Helping the students through the observation process to see what is invariant in the problem configuration under dragging.
4. Encouraging the students to express their geometric ideas in their own words and in different forms.
5. Assisting the students to use the different facilities of Cinderella to investigate the

correctness of their ideas in different methods.

6. Encouraging the students to produce mathematical proofs for their investigated conjectures using the formal logical steps. Here also the teacher might help by giving the students some hints in the form of questions to guide/direct their attention toward the proving process.
7. Directing the students' attention towards the problem aspects that could be re-defined in order to find or pose new related problems.

Thus, in this part of the thesis, the researcher has answered the second question of the study, which states: “*What is the suggested enrichment program using dynamic geometry software for developing the mathematically gifted students' geometric creativity in high schools?*”

4.2 The Geometric Creativity Test

Since the purpose of the present study is to develop the geometric creativity of the mathematically gifted students using the suggested enrichment program, so the present researcher had to design a tool, a geometric creativity test, to assess the geometric creativity of the mathematically gifted students before and after administering the suggested enrichment program. In designing the geometric creativity test, the present researcher passed through the following steps:

4.2.1 Specification of the Aim of the Test

The aim of the geometric creativity test is to assess the geometric creativity of the mathematically gifted students in terms of its components before and after administering the suggested enrichment program.

4.2.2 Specification of the Components that the Test Measures

By reviewing literature and prior studies⁹ related to the subject of creativity, mathematical creativity, and geometric creativity, the present researcher was able to determine the geometric creativity components¹⁰ to be measured by the test measures as follows:

1. **Fluency:** the student's ability to pose or come up with many geometric ideas or configurations related to a geometric problem or situation in a short time.
2. **Flexibility:** the student's ability to vary the approach or suggest a variety of different methods toward a geometric problem or situation.
3. **Originality/Novelty:** the student's ability to try novel or unusual approaches toward a geometric problem or situation.
4. **Elaboration:** the student's ability to redefine a single geometric problem or situation to create others, which are not the geometric problem, the situation itself, or even its solutions but rather the careful thinking upon the particular aspects that govern the geometric problem or situation changing, one or more of these aspects by substituting, combining, adapting, altering, expanding, eliminating, rearranging, or reversing and then speculating on how this single change would have a ripple effect on other aspects of the problem or the situation at hand.

The combination of these four components defines the notion of geometric creativity as tackled in this study.

4.2.3 Creation of a Preliminary Form of the Test

This step includes identifying test specifications, item types, writing items, and writing directions of the test.

For the test specification, Table 4.2 shows the geometric creativity components of the test, the items that were designed to measure each component, the number of items

⁹(see Al-Baz, 1999; El-Rayashy and Al-Baz, 2000; Haylock, 1997; Lee and Shim, 2005; Mann, 2005; Mohamed, 2003; Nakin, 2003; Park, 2004; Weth, 1998b)

¹⁰For more details about creativity components see Chapter 2 – Section 2.3.

corresponding to each component, and the percentage of each component.

Table 4.2: *Specifications of the Geometric Creativity Test*

Components of Geometric Creativity	Items	No. of Items	Percentage
Fluency	1, 2, 3, 4	4 items	33%
Flexibility	5,6,9	3 items	25%
Originality/Novelty	7,8,12	3 items	25%
Elaboration	10,11	2 items	17%
Overall Geometric Creativity Test	1 to 12	12 items	100%

Concerning the item types in the test, the GCT includes open-ended, and non-routine geometric situations and problems that require producing many various and different responses. In designing these situations and problems the researcher took into consideration some criteria for a task to be effective in revealing geometric creativity and in distinguishing among students in a particular population in terms of their creativity of their responses (Haylock, 1997, p. 72):

1. The students' responses should show a wide range of geometric and mathematical ideas.
2. A large number of appropriate responses are possible for these students.
3. The students' responses should show a consistent interpretation of the instruction in the task.
4. There should be several clear responses that can be obtained by most students.
5. There should be a number of appropriate responses that are obtained by relatively few students.
6. These original responses should have a degree of face validity for indicating creative ability in geometry and they should not be geometrically trivial.

Regarding the writing items of the test, they were written in verbal and nonverbal (symbols and figures) ways and a vision of the expected responses for each item of the

test was put into account.

The GCT, in its preliminary form, consists of 12 items that are distributed among the four components of geometric creativity: fluency, flexibility, originality, and elaboration. Items 1, 2, 3, and 4 were designed to assess students' geometric fluency. Items 5, 6, and 9 were designed to allow students to come up with not only many ideas but also many categories of ideas in order to assess their geometric flexibility. Items 7, 8, and 12 were designed to allow students to show unusual and unique ways of solutions to find out how many original/novel geometric ideas they have. Items 10 and 11 were designed to assess students' geometric creativity in elaborating a geometric problem or situation.

Before moving to the next section, it is important to explain how each item, though designed to measure a certain component of creativity, is used to measure some other components at the same time. Item 1 is used to explain how each item can be used to measure different components of creativity. Item 1 – originally designed for testing fluency – requires a student to write down as many geometric concepts and terminologies as possible that starts with the letter p. The researcher assumed these responses: parallel, parallelogram, perpendicular, polyeder, pyramid, point, and point of symmetry as shown in Table 4.3.

Table 4.3: *Item 1 and Assessing Different Geometric Creativity Components*

Student's Responses	Fluency	Flexibility	Originality
Parallel	1	C1	0
Parallelogram	1	C1	0
Perpendicular	1	C1	4
Polyeder	1	C2	1
Pyramid	1	C2	2
Point	1	C1	0
Point of symmetry	1	C3	3
Score	7	3	10

In this example, as the student came up with 7 relevant responses, his/her fluency score

on this item were 7 points. Moreover, as the responses could be classified into three different categories according to different domains of geometry: Euclidean geometry, space geometry, and transformational geometry, which reflects the student's ability to vary his/her approach and break from mental sets to come up with not only different responses but also varied ones, therefore his/her flexibility score on this item was 3 points. Similarly, the student's originality score could be assessed on this item, as the statistical infrequency of responses in relation to peer group responses. Each response took zero, one, two, three or four points as an originality score based on its frequency. More details about scoring originality component will be given in the following section.

Table 4.4: *Test Items and Geometric Creativity Components*

	Fluency	Flexibility	Originality	Elaboration
Item 1	×	*	*	
Item 2	×	*	*	
Item 3	×	*	*	
Item 4	×	*	*	
Item 5	*	×	*	*
Item 6	*	×	*	*
Item 7	*	*	×	
Item 8	*	*	×	
Item 9	*	×	*	
Item 10	*	*	*	×
Item 11	*	*	*	×
Item 12	*	*	×	

For completeness, Table 4.4 shows which other components could be assessed by the test items in addition to the ones they were designed for. With the notice that in Table 4.4, × indicates that the item was intentionally designed to assess this component and * indicates that the item would be used to assess this component and it was not intentionally designed to assess it.

As for the writing directions of the test, simple directions were written for the students, including some instructions that stimulate students' creative thinking. Instructions to inform students of the time allowed for the test and how to answer the test items were also included. The directions also indicate that the answer to each item is not restricted.

4.2.4 Setting-up of a Grading Method for the Test

Reviewing literature and prior studies¹¹ related to the subject of creativity in general and mathematical and geometric creativity in particular, the researcher identified a grading method for the test. Through this method, each student should have 4 scores for fluency, flexibility, originality, and elaboration for each item of the test as well as the overall score of geometric creativity, as follow:

Fluency: The number of relevant responses. Each relevant response is given one point.

Flexibility: The number of different categories of relevant responses: answers, methods, or questions. Each flexibility category is given one point.

Originality/Novelty: It is the statistical infrequency of responses in relation to peer group. The more statistical infrequency the response has, the more originality it manifests. Each response is given zero, one, two, three or four points according to Table 4.5:

Table 4.5: *Originality/Novelty Scores for the Geometric Creativity Test*

The Number of Students Who Registered the Response	1 Student	2 Students	3 Students	4 Students	5 Students
Originality/Novelty Scores	4	3	2	1	0

Elaboration: It is graded by the number of follow-up questions or problems that are posed by redefining – substituting, combining, adapting, altering, expanding, eliminat-

¹¹(see Al-Baz, 1999; El-Rayashy and Al-Baz, 2000; Lee and Shim, 2005; Mann, 2005; Mohamed, 2003; Park, 2004)

ing, rearranging, or reversing – one or more aspects of the given geometric problem or situation. Each correct response is given one point.

Overall Geometric Creativity: It is the sum of fluency, flexibility, originality, and elaboration scores that represents the creative thinking ability in geometry.

4.2.5 Content Validity Check

For validating the GCT, the researcher presented it, in its preliminary form, to a group of judges¹² specialized in teaching and learning mathematics in China, Egypt, and Germany. These judges reviewed the items, in their initial form, for clarity, readability, and appropriateness to measure what it is designed to measure and the level of mathematically gifted students in the high schools¹³.

Most changes suggested by the judges had to do with rhetorical and sequencing considerations. For one thing, upon the judges' request for the readability of the test items, the researcher used different fonts and font styles within the test items so that students could easily distinguish between the items statement and the items directions as well as quickly recognize the items tasks.

The judges also found that the question example given in item 3 is too complicated and it should be split into two questions. The question example was “Is it a plane figure such as a rectangle or a solid figure such as a sphere?” Thus, it was changed to: “Is it a plane figure such as a rectangle? Is it a solid figure such as a sphere?” For the same item, the judges recommended adding one more question, which is not Yes/No question. So, the researcher added one more question, which is “Does it have vertices?”, “How many?” For item 9, the judges suggested changing the given example, which was “ $\triangle AEF$ and $\triangle BDC$ is a pair of equivalent triangles” as it would restrict the students' thinking, causing them to only think about equivalent figures in terms of triangles. Accordingly, the researcher changed it to: “Triangle BCE and parallelogram

¹²A list of the judges who validated the geometric creativity test is presented in Appendix [G](#)

¹³A letter to the judges used for validating the items of the geometric creativity test is presented in Appendix [F](#).

ABDE is a pair of equivalent figures”.

Finally, and more importantly, in items 5, 6, 7, 8, 9, 10, and 11 the judges were afraid that the mathematical symbols used in these items might not be recognized by the students in German schools as they use another system of symbols. For example: German students would not recognize $\overrightarrow{AM}$ as a ray; rather, it would be recognized as a vector. So the researcher was directed to use the same symbols used in German schools as shown in the German version¹⁴. The test, in its final form, is presented in Appendix H.

In conclusion, the judges were confident that the students being able to answer the test items show a certain degree of geometrical creativity, and vice-versa. They also asserted that the test items were appropriate to assess the assigned geometric components, which they are designed for.

4.2.6 The Piloting of the Test

The researcher attempted a test piloting aiming at calculating:

1. The reliability coefficient for the test.
2. Item-internal consistency reliability for the test items.
3. Experimental validity for the test.
4. The suitable time-range for the test.

In this respect, the GCT was translated into German and administered to a sample of (30) students, 15 male and 15 female, in the university of education Schwäbisch Gmünd at the end of the summer semester of the academic year 2008. Students' responses on the test were analyzed to calculate the scores of the geometric creativity components for each student.

¹⁴The German version of the geometric creativity test is presented in Appendix I.

1) The Reliability Coefficient

The reliability coefficient (Cronbach's α) for all test items as they measure geometric creativity was calculated using SPSS16. It was 0.83, a high reliability coefficient. Consequently, the GCT prepared by the researcher was proven reliable to measure the geometric creativity ability as a whole.

2) Item-internal Consistency Reliability

As for the item-internal consistency reliability, (Cronbach's α) is calculated for each of the geometric creativity component scores (i.e., fluency, flexibility, originality, and elaboration) as subscales of the test, as follows:

For **fluency** as a component of geometric creativity and a subscale of the geometric creativity test, the reliability coefficient Cronbach's α was calculated for the fluency scores of the 12 items of the test as 0.62. To improve the reliability coefficient of the fluency component as a subscale of the test, SPSS suggested that if items 9 and 11 were deleted it might result in a better reliability coefficient for fluency. Indeed, deleting items 9 and 11 from the statistical analysis of the test items gave a reliability coefficient that equals 0.72, which is a good reliability coefficient. Consequently, measuring the fluency component of geometric creativity using items 1, 2, 3, 4, 5, 6, 7, 8, 10, and 12 of the prepared test is reliable.

Regarding **flexibility** as a component of geometric creativity and a subscale of the test, the reliability coefficient was calculated for the flexibility scores of the 12 items of the test as 0.55. Again, to improve the reliability coefficient of the flexibility component as a subscale of the test, SPSS also suggested that if item 9 and 11 were deleted it might result in a better reliability coefficient for the flexibility, and accordingly deleting items 9 and 11 gave a reliability coefficient that equals 0.64, which is an accepted reliability coefficient for flexibility as a subscale of the test. Consequently, measuring the flexibility component of geometric creativity using items 1, 2, 3, 4, 5, 6, 7, 8, 10, and 12 of the prepared test can be considered reliable.

As for **originality**, as a component of geometric creativity and a subscale of the test, the reliability coefficient was calculated for the originality scores of the 12 items of the test as 0.59. Here SPSS suggested that if item 11 was deleted it might result in a better reliability coefficient for the originality. The deletion of item 11 only improved the coefficient slightly to be 0.60, but again this is an accepted reliability coefficient for originality as a subscale of the test. Consequently, it is considered that measuring the originality component of geometric creativity using items 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 12 of the prepared test is reliable.

Regarding **elaboration** component of the test, the reliability coefficient was calculated for the elaboration scores of items 5, 6, 10, and 11 that include the elaboration component according to Table 3. It was found to be only 0.41. To improve the reliability coefficient of the elaboration component as a subscale of the test, SPSS also suggested that if item 11 was deleted it might result in a better reliability coefficient for the elaboration component. Deleting item 11 from the statistical analysis of the test items gives a reliability coefficient (Cronbach's α) for the elaboration component that equals 0.51, which is a low reliability coefficient for elaboration as a subscale of the test. Consequently measuring the elaboration component of geometric creativity using items 5, 6, and 10 of the prepared test is not reliable.

Since the above-mentioned results suggest that items¹⁵ 9 and 11 should not be used for the total creativity measure, then the researcher had to recalculate the reliability coefficients again. After deleting items 9 and 11 from the statistical analysis of the test, the statistical attributes (mean, standard deviation, and Cronbach's α) of the overall geometric creativity test and its subscales (fluency, flexibility, originality, and elaboration) were recalculated as shown in Table 4.6. The table shows that the subjects of the pilot study of the test had a mean of 120.50 ($SD = 42.08$) and the reliability coefficient (Cronbach's α) for the geometric creativity test as a whole scale is 0.85 (high reliability coefficient) which means that the prepared geometric creativity test after

¹⁵An explanation for why items 9 and 11 do not go with the other test items is that item 9 contains concepts about the area of triangle and parallelogram, which are heavily stressed during different levels of the subjects' learning in primary school, high school, and even in their preparation in the university. As for item 11 it was difficult for the subjects since 13 out of 30 gave wrong or no response to this item.

deleting the two items is still reliable to measure the geometric creativity ability as a whole.

Table 4.6: *Statistics Attributes of the Pilot Study of the Geometric Creativity Test*

Components of Geometric Creativity	M	SD	Cronbach's α
Fluency	39.67	11.76	0.72
Flexibility	23.77	5.94	0.64
Originality/Novelty	44.53	21.83	0.60
Elaboration	12.53	4.92	0.51
Overall Geometric Creativity Test	120.50	42.08	0.85

Regarding the geometric creativity component, **fluency**, the subjects had a mean of 39.67 ($SD = 11.76$) and the reliability coefficient (Cronbach's α) was 0.72 (good reliability coefficient), which means that after deleting the two items, the prepared test was suitable for measuring the fluency component of geometric creativity.

As for the second geometric creativity component, **flexibility**, the subjects had a mean of 23.77 ($SD = 5.94$) and the reliability coefficient (Cronbach's α) was 0.64 (accepted reliability coefficient), which means that after deleting the two items, the prepared test was suitable for measuring the flexibility component of geometric creativity.

Regarding the originality component, the subjects had a mean of 44.53 ($SD = 21.83$) and the reliability coefficient (Cronbach's α) was 0.60 (accepted reliability coefficient), which means that after deleting the two items, the prepared test was suitable for measuring the originality component of geometric creativity.

Concerning the **elaboration** component, the subjects had a mean of 12.53 ($SD = 4.92$) and the reliability coefficient (Cronbach's α) was 0.51 (low reliability coefficient). One interpretation for low consistency of the elaboration component would be because the elaboration component of geometric creativity has many subscales (aspects) to measure, which had a negative effect on the consistency of the component items. Even though the reliability coefficient for the elaboration component was low, the researcher believes

that the elaboration construction is an important component of geometric creativity and during the intervention; the researcher measured it using the prepared test. These findings evoke the need for further studies with bigger sample size of students to get higher reliability coefficients for the prepared test and its subscales as well.

3) The Experimental Validity

The experimental validity of the test as an estimation of the test validity was also calculated by taking the square root of the test reliability coefficient (Angoff, 1988, p. 20). It was calculated before deleting items 9 and 11 as 0.913 and after deleting items 9 and 11 it was 0.922, which shows that the geometric creativity test has a high experimental validity.

4) The Suitable Time-range

The time each subject took to finish the test was calculated. Table 4.7 shows the statistical attributes of the time taken by the subjects in the pilot test. The subjects had a mean 94 ($SD = 17.16$), median = 90, mode = 85.

Table 4.7: *Statistical Attributes of the Test Time in the Pilot Testing*

Mean	Median	Mode	SD	Minimum	Maximum
94	90	85	17.16	60	145

To determine the suitable time-range for the test, the researcher calculated the time each student tested took then calculated the mean of the time the first student took (60 minutes) and the last one took (145 minutes), so the suitable time of the test was calculated as approximately 100 minutes, as given by $\frac{60 + 145}{2} = 102.5$.

4.3 Procedures of the Experimental Study

After developing the suggested enrichment program, deciding on its appropriateness to develop the mathematically gifted students' geometric creativity, developing the geometric creativity test, and deciding on its validity and reliability to assess geometric creative thinking and its components, the researcher was able to attempt the experimental study to investigate the effectiveness of the suggested enrichment program in developing the mathematically gifted students' geometric creativity.

4.3.1 Subjects of the Study

The setting for the experimental study was at Landesgymnasium für Hochbegabte (LGH), a public high school for gifted and talented students in Schwäbisch Gmünd, Baden-Württemberg, Germany. The enrolment of the school is approximately 185 students. The students' population in the academic year 2008/09 was comprised of: 179 German, 1 Turkish, 2 Russian, and 3 Chinese.

All students in LGH should have at least 130 as a score on IQ test as a prerequisite to be accepted in this school. Up to grade 10, students study the same programs. For grade 11 and 12, students are allowed to choose between science, languages and humanities, or arts sections. All students in LGH must study mathematics up to twelfth grade.

In the academic year 2007/08, approximately 89% of the students attended universities and the rest attended colleges and approximately 63% of the students attended technical universities.

The subjects of the study were 7 mathematically gifted students. The subjects consisted of 5 eleventh grader, 1 tenth grader, and 1 ninth grader (2 male, 5 female). The subjects voluntarily participated in the study and they were identified by their mathematics teachers¹⁶ based on their scores in the last mathematics evaluation (at least 14 out of 15), their high interest in mathematics in comparison to their peer group, and their

¹⁶ Dr. Olga Lomonosova – Mathematics Teacher in LGH
Dr. Albert Oganian – Mathematics Teacher in LGH

high performance in solving mathematical problems.

None of the subjects had used Cinderella before taking part in this experiment.

4.3.2 Experimental Design of the Study

As for the experimental design of the study, the pre-experimental one-group, pretest – intervention – posttest design was used to investigate the effectiveness of the prepared enrichment program in developing the mathematically gifted students' geometric creativity.

4.3.3 Before the Intervention

Before starting the experimental study, the researcher prepared the computers in the LGH computer lab by installing Cinderella and its license on the students' PCs. In this concern, a Cinderella license was issued to LGH and was put with Cinderella application install folder on LGH server. Moreover, a copy of the prepared CD ROM, which contained the student's handouts and the cdy files with the English and German html indices that were used along the intervention, were also put on the LGH server.

Finally, and more importantly, the researcher met with the subjects of the study explaining to them the aims, the nature of the study, what is required from them during the study (Starting from the pretest, going through administering the enrichment program, and ending with the posttest) and what is expected from them by the end of the study.

4.3.4 Administering the Pretest

The geometric creativity test was administered to the subjects as a pretest at the beginning of the experimental study to assess their geometric creativity before administering the suggested enrichment program. It took 100 minutes, which is the suitable time-range calculated for the designed geometric creativity test.

4.3.5 Administering the Suggested Enrichment Program

The suggested enrichment program was administered to the study subjects during the first semester of the academic year 2008/2009. To administer the program, it took approximately 18 hours distributed on 12 weekly 90-minute sessions. The intervention took place in the computer lab of LGH where each student of the subjects had a PC with Cinderella installed on it. The schedule of the intervention is shown in Table 4.8.

Table 4.8: *The Schedule of the Suggested Enrichment Program*

Sessions	The Activity
1st session	Introductory Session: Cinderella Getting Started
2nd Session	Activity 1: Dragging and Measuring Facilities of Cinderella
3rd Session	Activity 2: Automatic Proving Facilities of Cinderella
4th Session	Activity 3: Developing Macro-constructions
5th Session	Activity 4: Animating and Tracing Loci Facilities of Cinderella
6th Session	Activity 5: Midpoints of the Sides of a Quadrilateral
7th Session	Activity 6: Angular Bisectors of a Parallelogram
8th Session	Activity 7: Constructing a Parallelogram
9th Session	Activity 8: Constructing a Rhombus
10th Session	Activity 9: Constructing a Rectangle
	Activity 10: Constructing a Square
11th Session	Activity 11: Posing Geometric Problems
12th Session	Activity 12: Finding Geometric Relationships

In the first session, the outcomes of the suggested enrichment program were presented to the students to read them before receiving instruction. Then, students got started with Cinderella and explored its dynamic environment. In the same session, students were also taught the basic commands and modes of Cinderella. In addition to the learning contents prepared for the first session, students were also asked to do some Cinderella interactive exercises using the Internet – on MathsNet website¹⁷, which introduces a

¹⁷MathsNet Website: <http://www.mathsnet.net/>

special course in interactive geometric constructions at three levels: foundation, intermediate and advanced. Doing these interactive exercises helped students to become more accustomed to the use of different construction modes of Cinderella.

Starting from the second session, the researcher began to introduce the enrichment activities to the students. In these sessions, one purpose was to teach the students how to use different facilities of Cinderella and another was to enhance their geometric creativity thinking in terms of the creativity components (fluency, flexibility, originality, and elaboration). That is, in the second session for instance the students were instructed to use the dragging and measuring facilities of Cinderella to conjecture and investigate solutions for the activity problem and were also trained to develop their geometric creativity in the context of the given problem by encouraging them to pose many follow-up problems related to the activity problem. Furthermore, in the third session the automatic proving facilities of Cinderella were presented to the students and they were also asked to redefine the activity problem using different strategies. Developing Macro-constructions using Cinderella was the focus of the fourth session. In the fifth session, the animating and tracing loci facilities of Cinderella were introduced to the students, and then they were trained to elaborate the given situation by altering its conditions.

By the fifth session, the main facilities of Cinderella (measuring and dragging facilities, automatic proving facilities, macro-construction facilities, animating and tracing loci facilities) had been already introduced to the students and the instructions given to the students with regard to using Cinderella became less. Therefore, in the sixth and seventh sessions the students started to use the different facilities of Cinderella they had learned so far to construct dynamic configurations for the activity problems, make intelligent conjectures and relationships in the context of the given problems, investigate their conjectures by different methods in different situations, and elaborate the given problems to generate many other follow-up problems.

Sessions eight, nine, and ten were pertinent to introducing the constructions activities to the students. Students came up with many various and different methods to construct a parallelogram, a rhombus, a rectangle and a square using Cinderella.

The problem posing activities were presented in the eleventh and twelfth sessions. In these sessions, the students were trained to use the different facilities of Cinderella to formulate many various and different conjectures and relationships related to the given problems in different domains of geometry (e.g., areas, congruency, and similarity, among others) and investigate them by different ways.

As for the teaching and learning strategies used during the intervention, they were varied according to the type of the enrichment activity (problem solving activity, problem posing activity, construction activity, or redefining activity) and were executed as they were described in the teacher's guide¹⁸.

4.3.6 Administering the Posttest

The geometric creativity test was administered to the study subjects as a posttest at the end of the experimental study to assess their geometric creativity after administering the suggested enrichment program. The test administered took 100 minutes, which is the suitable time-range calculated for the designed geometric creativity test.

4.4 Researcher's Notes During the Experimental Study

During the experimental study, the researcher was concerned with recording some notes and reflections in terms of the students' involvement during the intervention, the students' creative behavior, and the students' use of Cinderella. For doing so, the researcher kept a regular record of his notes and reflections during the program's sessions in a journal. In this journal, some entries were recorded after each session to address the following topics:

1. The involvement of the students during the intervention.
2. The creative behavior of the students.
3. The ability of the students to use Cinderella.

¹⁸Teacher's guide is presented in Appendix [C](#).

4. Other pertinent notes.

Notes and reflections in the journal indicated many responses that can be summed up within the framework of its four topics as follows:

With regard to the first topic, which is related to the involvement of the students during the intervention, responses in the reflective journal indicated that:

1. The students were on task and engaged in the activities during the intervention.
2. The students were engaged in taking notes and asking relevant questions toward the sessions' problems or situations.
3. The students could handle the given situations in which they were involved and were able to plan and organize.

With regard to the second topic, the creative behavior of the students during the intervention, responses in the reflective journal indicated that:

1. The students could produce many relevant responses within the given situations.
2. Using the suggested enrichment program helped the students to express their geometric ideas in different forms verbal and figurative ways.
3. Using Cinderella encouraged the students to pose and find many conjectures and relationships related to different domains of geometry.
4. As the students moved through the sessions of the intervention, their geometric creativity in terms of its four components was getting better and better.
5. The students could overcome their mind fixation and produce many different categories of relevant responses toward the given situations and problems.
6. The suggested enrichment program with the help of using the dynamic geometry software encouraged the students to change different aspects of the problem and investigate the ripple effect of this changing.
7. The students could generate new responses and products within the given situations and problems.

As for the third topic, regarding the ability of the students to use Cinderella the journal revealed that:

1. The students were able to use Cinderella to construct dynamic configurations for the given situations and problems.
2. The students were able to use Cinderella to make and investigate conjectures and relationships within the given situations and problems.
3. The students were able to use Cinderella to modify, adapt, or alter the given problems to pose many follow-up problems.
4. The students were attaining the level expected of them in using Cinderella within the program's sessions.

Finally, with regard to the fourth topic covering other pertinent notes, many responses supported by the students' comments were recorded in the reflective journal, which revealed that the intervention was interesting and beneficial to the students. As for the enjoyable and interesting aspect of the intervention, many of the students' comments that indicate that the intervention was interesting and enjoyable for them were recorded along the program's sessions such as: "I had a lot of fun using Cinderella", "I enjoyed the program", and "It was very nice participating in the project".

In addition to the above-mentioned responses, which reported the students' comments about the intervention, three more responses that indicated the fruitfulness of the intervention to the students were also reported.

The First, two students of the study subjects willingly volunteered to hold a presentation in their mathematics class and used Cinderella to present and solve the presentation problem. The students said that it was very nice and helpful using Cinderella in the presentation and their teacher appreciated using Cinderella in the presentation as well. As a result they got 15 out of 15 in this class.

The second interesting response in this concern that might show the practical effectiveness of the intervention is that in a mathematics competition called "Bundeswettbewerb

Mathematik (BWM)”, one student of the study subjects used Cinderella to solve the geometry problem introduced to the school team in the competition.

The Third response showed that one of the study subjects willingly carried out her research project based on the intervention topics describing Cinderella as dynamic geometry software, elaborate in its tools and facilities, and giving some activities that she learned during the intervention.

Chapter 5

Findings and Interpretation

In this chapter, the researcher presents the results of the geometric creativity pretest and posttest of the geometric creativity test, the statistical analysis results of testing the study hypotheses, as well as interpretation and implications of these results in the light of the literature review and the purposes of the study.

For testing the study hypotheses, the designed geometric creativity test was administered to the study subjects at the beginning and end of the intervention (administering the suggested enrichment program). Both tests, the pretest and the posttest, were identical in format and content. The test^I consisted of 12 questions, which were distributed along the four adopted components of geometric creativity (fluency, flexibility, originality, and elaboration).

In testing the study hypotheses from 1 till 5, which are concerned with investigating the statistical differences between the pre and post measurements of the study subjects and their significance, the non-parametric statistical styles were used in comparing the pretest and posttest scores of the study subjects due to the small number of the subjects. So, both the “Sign Test” and the “Wilcoxon Signed Ranks Test” were used in the statistical analysis of these hypotheses to compare the differences between the mean ranks of the scores of the study subjects on the geometric creativity pretest and

¹The Geometric Creativity Test (GCT), which is used in the study as a pretest and posttest, is presented in Appendix II

on the posttest and to check the significance level of these differences at a significant level 0.05. All the data were statistically analyzed using SPSS16.

With regard to testing the study hypotheses from 5 till 10, which are concerned with deciding on the effectiveness of the suggested enrichment program, the Cohen's effect size indicator d^2 (Cohen, 1988, p. 20-22) was used to qualify the differences between the pre and post measurements of the study subjects in order to decide on the effectiveness of the suggested enrichment program in developing their geometric creativity.

In the coming sections, in detail, the researcher presents the results of the pretest, posttest, and the hypotheses statistical analysis results.

5.1 Analysis of Findings

5.1.1 Results of the Pretest

After administering the geometric creativity test as a pretest to the study subjects at the beginning of the intervention, the subjects' responses on the pretest were analyzed in the light of the grading method suggested by the present researcher³ to calculate the geometric creativity scores for each subject before the intervention.

Results of the pretest are shown in Table 5.1. Results of the pretest indicated that the subjects had a mean of 34.00($SD = 11.23$) on the fluency component, a mean of 20.29($SD = 7.93$) on the flexibility component, 86.14($SD = 37.34$) on the originality component, 10.57($SD = 4.16$) on the elaboration component, and 151.00($SD = 58.38$) on the overall geometric creativity test.

²Cohen's effect size indicator d is defined as the difference between two means divided by the pooled standard deviation of the two means and it is given by the formula:

$$d = \frac{M_1 + M_2}{\sqrt{\frac{SD_1^2 + SD_2^2}{2}}}$$

where d = Effect size index,

M_1, M_2 = The means of the subjects' scores on the pretest and posttest respectively, and

SD_1, SD_2 = The standard deviation of the subjects' scores on the pretest and posttest respectively.

³For more details about the grading system of the geometric creativity test, see Chapter 4 – Section 4.2.4.

Table 5.1: *Pretest Results*

Components of Geometric Creativity	M	SD
Fluency	34.00	11.23
Flexibility	20.29	7.93
Originality/Novelty	86.14	37.34
Elaboration	10.57	4.16
Overall Geometric Creativity Test	151.00	58.38

5.1.2 Results of the Posttest

At the end of the intervention, the geometric creativity test was administered to the study subjects as a posttest. The subjects' responses on the posttest were also analyzed to calculate their geometric creativity scores after the intervention.

In the posttest, as shown in Table 5.2, the subjects had a mean of 63.57($SD = 15.80$) on the fluency component, a mean of 33.00($SD = 5.45$) on the flexibility component, 172.29($SD = 59.43$) on the originality component, 23.86($SD = 9.42$) on the elaboration component, and 292.71($SD = 87.38$) on the overall geometric creativity test.

Table 5.2: *Posttest Results*

Components of Geometric Creativity	M	SD
Fluency	63.57	15.80
Flexibility	33.00	5.45
Originality/Novelty	172.29	59.43
Elaboration	23.86	9.42
Overall Geometric Creativity Test	292.71	87.38

The mean scores of the study subjects in both the pretest and posttest, as shown in Table 5.1 and Table 5.2, and as also seen in the chart of Figure 5.1, showed that the means of the subjects' scores of the four components of geometric creativity (fluency, flexibility, originality, and elaboration) and the overall score of geometric creativity on the posttest

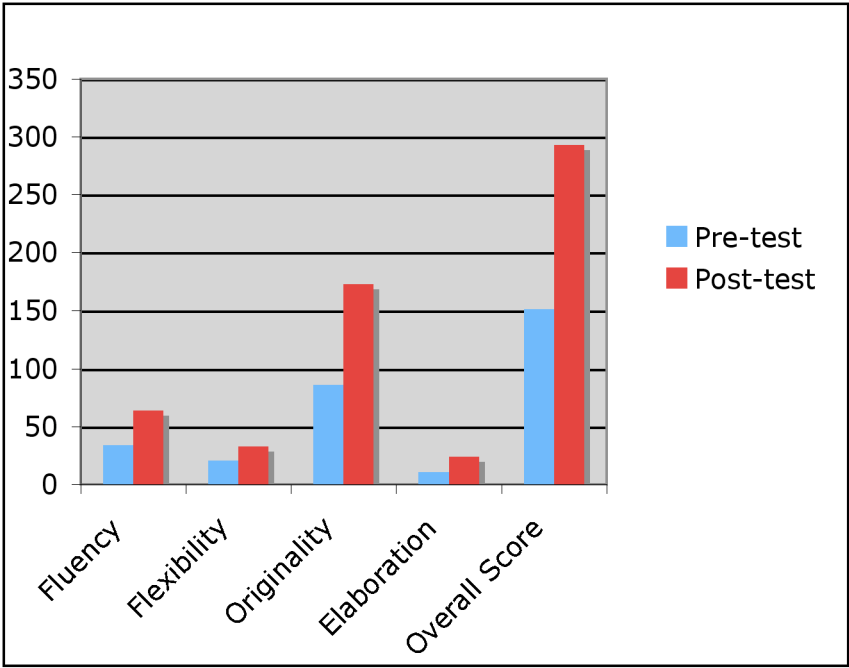


Figure 5.1: Mean Scores of the Subjects on the Pretest and Posttest

are higher than their corresponding means on the pretest, which can be considered positive indicators for the existence of statistical differences and the effectiveness of the suggested enrichment program.

5.1.3 Results of Testing Hypothesis One

The first hypothesis is: “There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects’ scores on the pre-post measurements of the geometric fluency component, as measured by the designed geometric creativity test, in favor of the post measurement.”

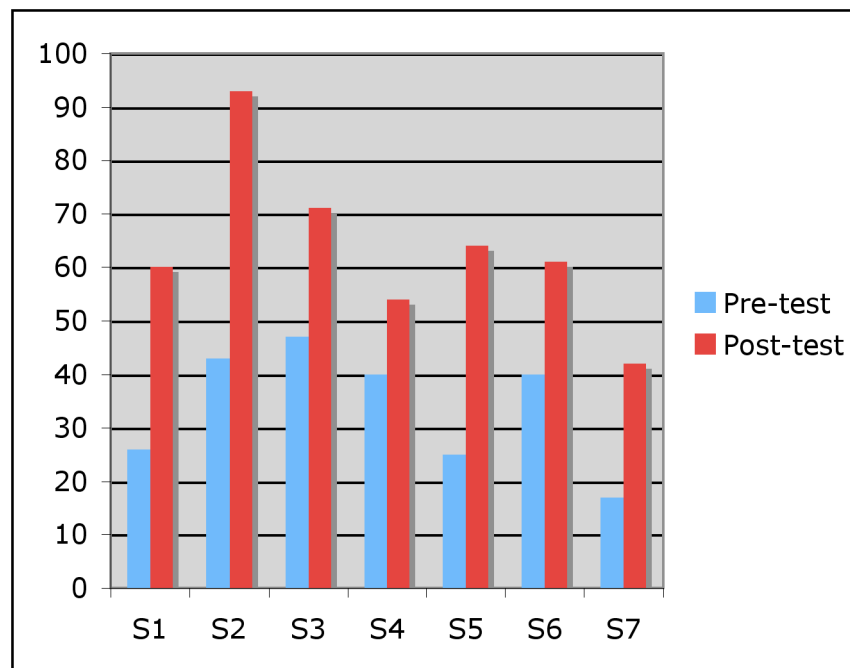
For testing the first hypothesis, the “Wilcoxon Signed Ranks Test” was used to identify the difference between the mean ranks of the subjects’ scores on the pre-post measurements of the geometric fluency component. Moreover, both of the “Wilcoxon Signed Ranks Test” and the “Sign Test” were used to check the significance level of this difference.

Table 5.3 shows the statistical analysis results of the pre-post measurements of geometric

Table 5.3: *Geometric Fluency Test Results*

Component of Geometric Creativity	Pretest		Posttest		Mean	Sign.
	M	SD	M	SD	Diff.	Level
Geometric Fluency	34.00	11.23	63.57	15.80	2.366	0.05

fluency, which indicates that regarding the geometric creativity component, *fluency*, the subjects had a mean of 34.00($SD = 11.23$) on the pretest and a mean of 63.57($SD = 15.80$) on the posttest with a mean difference of 2.366; this difference is statistically significant at the level 0.05. That is, there is a statistically significant difference between the mean ranks of the subjects' scores on the pre-post measurements of the geometric fluency component, as measured by the designed geometric creativity test, in favor of the post measurement; consequently the first hypothesis is verified.

Figure 5.2: *Fluency Scores of the Subjects on the Pretest and Posttest*

Furthermore, as seen in the chart of Figure 5.2, which presents the fluency scores of the subjects on the pretest and posttest, all the subjects' scores on the posttest were higher than theirs on the pretest, which indicates, together with the statistical analysis shown in Table 5.3, that the subjects gained significant improvement in their geometric

fluency as they were exposed to the intervention.

5.1.4 Results of Testing Hypothesis Two

The second hypothesis is: “There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects’ scores on the pre-post measurements of the geometric flexibility component, as measured by the designed geometric creativity test, in favor of the post measurement.”

Again, for testing the second hypothesis, the “Wilcoxon Signed Ranks Test” was used to identify the difference between the mean ranks of the subjects’ scores on the pre-post measurements of the geometric flexibility component. Both of the “Wilcoxon Signed Ranks Test” and the “Sign Test” were also used to check the significance level of the difference. Table 5.4 shows the results of the pre-post measurements of the geometric flexibility component.

Table 5.4: *Geometric Flexibility Test Results*

Component of Geometric Creativity	Pretest		Posttest		Mean	Sign.
	M	SD	M	SD	Diff.	Level
Geometric Flexibility	20.29	7.93	33.00	5.45	2.371	0.05

Regarding the geometric creativity component, *flexibility*, the subjects had a mean 20.29($SD = 7.93$) on the pretest and a mean of 33.00($SD = 5.45$) on the posttest with a mean difference of 2.371; this difference is statistically significant at the level 0.05. That is, there is a statistically significant difference between the mean ranks of subjects’ scores on the pre-post measurements of the geometric flexibility component, as measured by the designed geometric creativity test, in favor of the post measurement. The second hypothesis, therefore, is verified.

The chart in Figure 5.3, which represents the flexibility scores of the subjects on the pretest and posttest, shows that all the subjects’ scores on the posttest were higher than theirs on the pretest, which indicates, together with the statistical analysis shown in

Table 5.4, that the subjects gained significant improvement in their geometric flexibility as they were exposed to the intervention.

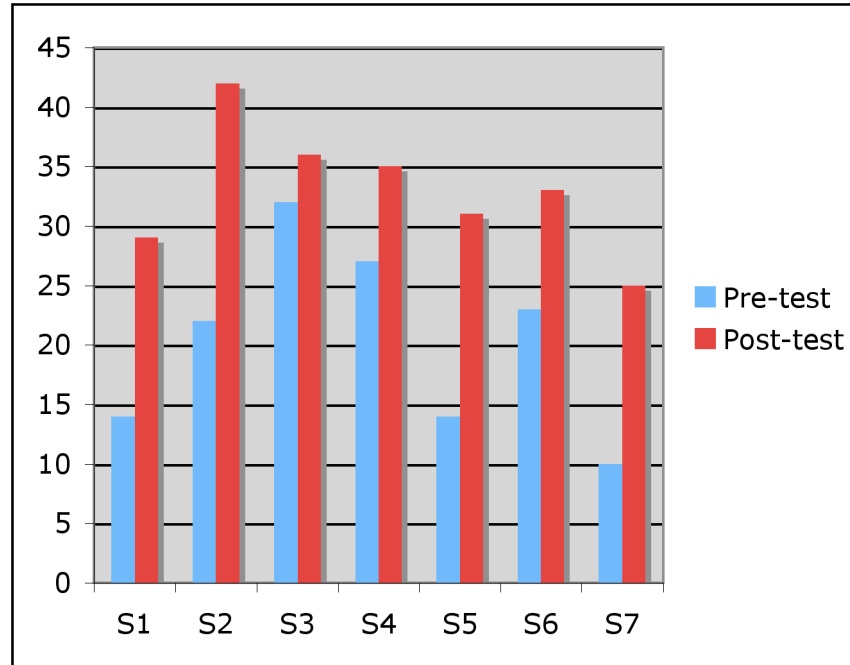


Figure 5.3: Flexibility Scores of the Subjects on the Pretest and Posttest

5.1.5 Results of Testing Hypothesis Three

The third hypothesis is: “There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects’ scores on the pre-post measurements of the geometric originality/novelty component, as measured by the designed geometric creativity test, in favor of the post measurement”.

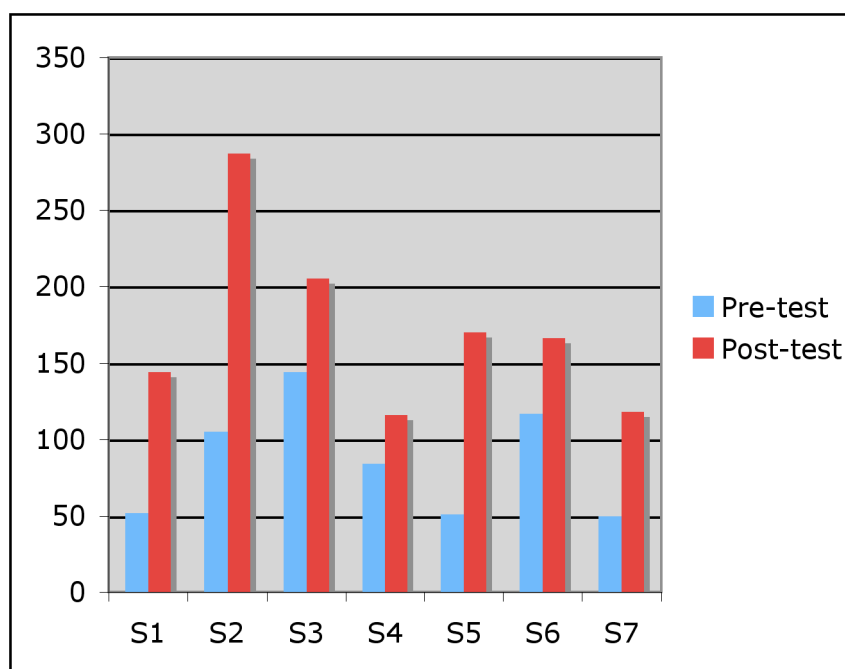
The third hypothesis concerned with testing the difference with regard to the *originality/novelty* component of geometric creativity. In the originality/novelty component of geometric creativity, as it is shown in Table 13, the subjects had a mean of 86.14($SD = 37.34$) on the pre-test and a mean of 172.29($SD = 59.43$) on the post-test. A mean difference of 2.366 was found (the difference was identified by using the “Wilcoxon Signed Ranks Test”). This difference was significant at level 0.05 (the significance level was checked by using both of the “Wilcoxon Signed Ranks Test” and

the “Sign Test”). Consequently, there is a statistically significant difference at the level 0.05 between the mean ranks of the subject scores on the pre-post measurements of the geometric originality/novelty component, as measured by the designed geometric creativity test, in favor of the post measurement. Therefore, the third hypothesis is verified.

Table 5.5: *Geometric Originality/Novelty Test Results*

Component of Geometric Creativity	Pretest		Posttest		Mean	Sign.
	M	SD	M	SD	Diff.	Level
Geometric Originality/Novelty	86.14	37.34	172.29	59.43	2.366	0.05

In addition, as seen in the chart of Figure 5.4, which presents the originality/novelty scores of the subjects on the pretest and posttest, all the subjects’ scores on the posttest were higher than theirs on the pretest, which indicates together with the statistical analysis shown in Table 5.5, that the subjects gained significant improvement in their geometric originality/novelty as they were exposed to the intervention.

Figure 5.4: *Originality/Novelty Scores of Subjects on the Pretest and Posttest*

5.1.6 Results of Testing Hypothesis Four

The fourth hypothesis is: “There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects’ scores on the pre-post measurements of the geometric elaboration component, as measured by the designed geometric creativity test, in favor of the post measurement”.

Table 5.6: *Geometric Elaboration Test Results*

Component of Geometric Creativity	Pretest		Posttest		Mean	Sign.
	M	SD	M	SD	Diff.	Level
Geometric Elaboration	10.57	4.16	23.86	9.424	2.366	0.05

Elaboration was the next component for testing. As shown in Table 5.6, the subjects had a mean of 10.57 ($SD = 4.16$) on the pretest and a mean of 23.86 ($SD = 9.424$) on the posttest. A mean difference of 2.366 was found, which is statistically significant at the level 0.05. Consequently, there is a statistically significant difference at the level 0.05 between the mean ranks of the subjects’ scores on the pre-post measurements of the geometric elaboration component, as measured by the designed geometric creativity test, in favor of the post measurement. The fourth hypothesis, therefore, is verified.

Observing the subjects’ scores on the elaboration component as shown in the chart of Figure 5.5 and supported with the statistical analysis shown in Table 5.6, It was found out that the subjects’ scores on the posttest were higher than theirs on the pretest, which means that the study subjects gained significant improvement after receiving instruction using the attempted suggested enrichment program.

5.1.7 Results of Testing Hypothesis Five

The fifth hypothesis is: “There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects’ scores on the pre-post measurements of geometric creativity as a whole ability, as measured by the designed geometric creativity test, in favor of the post measurement”.

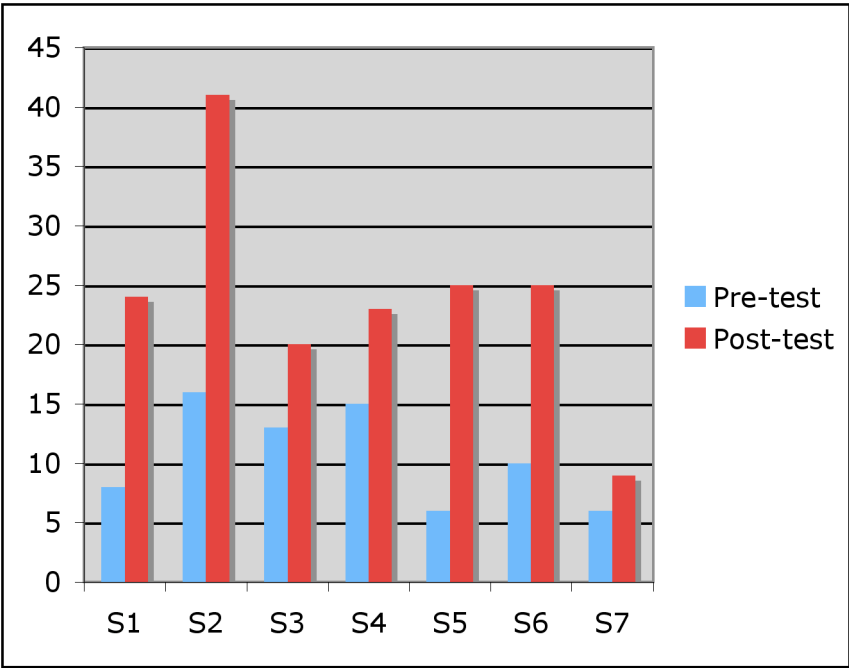


Figure 5.5: *Elaboration Scores of the Subjects on the Pretest and Posttest*

The fifth hypothesis concerned with testing the statistical difference regarding the geometric creative potential as a whole ability as measured by the overall score of the geometric creativity test. Table 5.7 shows the statistical analysis of hypothesis 5. For geometric creativity as a whole ability the subjects had a mean of 151.00($SD = 58.38$) on the geometric creativity pretest and a mean of 292.71($SD = 87.384$) on the geometric creativity posttest with a mean difference of 2.366, which is statistically significant at the level 0.05. That is, there is a statistically significant difference at the level 0.05 between the mean ranks of the subjects' scores on the pre-post measurements of geometric creativity as a whole ability, as measured by the designed geometric creativity test, in favor of the post measurement. Consequently, the fifth hypothesis is totally verified.

Table 5.7: *Overall Geometric Creativity Test Results*

Component of Geometric Creativity	Pretest		Posttest		Mean	Sign.
	M	SD	M	SD	Diff.	Level
Overall Geometric Creativity Test	151.00	58.38	292.71	87.384	2.366	0.05

Furthermore, observing the subjects' scores on the pretest and posttest as seen in the chart of Figure 5.6, it was found out that the subjects' scores of the overall geometric creativity on the posttest were always higher than their corresponding scores of the overall geometric creativity on the pretest. This means together with the statistical analysis shown in Table 5.7, that the suggested enrichment program was successfully effective in achieving a significant difference in the study subjects' geometric creativity potential as a whole ability.

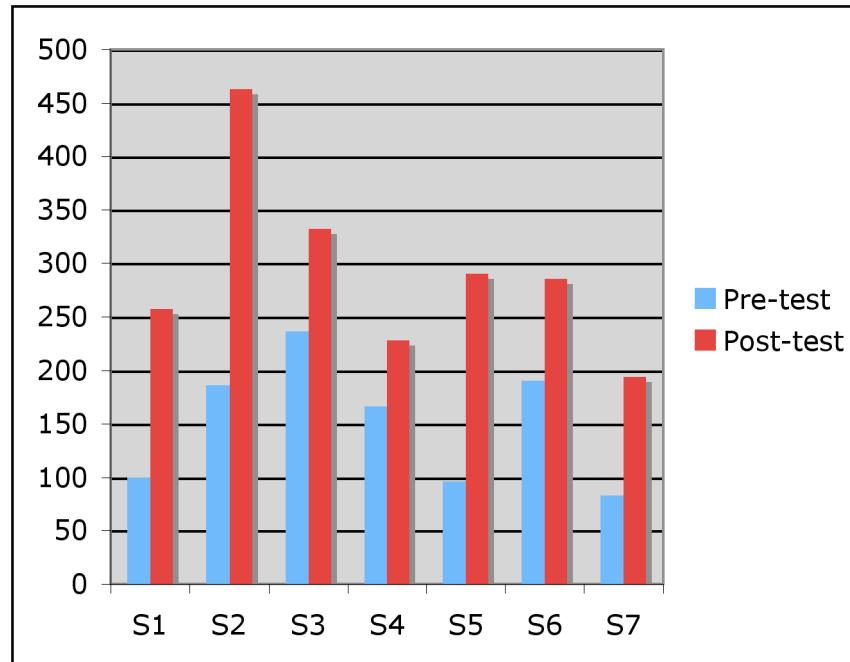


Figure 5.6: Overall Geometric Creativity Scores of the Subjects on the Pretest and Posttest

In the previous sections, results of testing hypotheses from 1 till 5 revealed that there were statistically significant differences between the subjects' performances on the pre and post measurements of the four components of geometric creativity (fluency, flexibility, originality/novelty, and elaboration) as well as the geometric creative potential, as a whole ability, in favor of the post measurement. In the coming sections, in testing hypotheses 6 till 10, the researcher investigates whether these statistically significant differences are large enough to be of practical concern.

5.1.8 Results of Testing Hypothesis Six

The sixth hypothesis is: “The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students’ geometric fluency component in high schools”.

Table 5.8: *The Effect Size of the Enrichment Program on Geometric Fluency*

Component of Geometric Creativity	Pretest		Posttest		Cohen’s	Effect
	M	SD	M	SD	d	Size
Geometric Fluency	34.00	11.23	63.57	15.80	2.16	Large

For testing the sixth hypothesis, the Cohen’s effect size indicator d was calculated in order to decide on the effectiveness of the suggested enrichment program in developing the subjects’ *fluency* component. In Table 5.8, the Cohen’s effect size indicator d was calculated ($d = 2.16$), which is a large effect size (since it is more than 0.8) and that indicates the effectiveness of the suggested enrichment program in developing the subjects’ fluency component. Consequently, the sixth hypothesis was verified. That is, the suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students’ geometric fluency component in high schools.

5.1.9 Results of Testing Hypothesis Seven

The seventh hypothesis is: “The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students’ geometric flexibility component in high schools”.

Table 5.9: *The Effect Size of the Enrichment Program on Geometric Flexibility*

Component of Geometric Creativity	Pretest		Posttest		Cohen’s	Effect
	M	SD	M	SD	d	Size
Geometric Flexibility	20.29	7.93	33.00	5.45	1.87	Large

For the *flexibility* component, Table 5.9 shows that the calculated Cohen's effect size indicator d was 1.87, which is a large effect size since it is more than 0.8 and that indicates that the suggested enrichment program was effective in developing the subjects' geometric flexibility component. Therefore, the seventh hypothesis is verified, a result which implies that the suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric flexibility component in high schools.

5.1.10 Results of Testing Hypothesis Eight

The eighth hypothesis is: "The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric originality/novelty component in high schools".

Table 5.10: *The Effect Size of the Enrichment Program on Geometric Originality*

Component of Geometric Creativity	Pretest		Posttest		Cohen's d	Effect Size
	M	SD	M	SD		
Geometric Originality/Novelty	86.14	37.34	172.29	59.43	1.74	Large

As for the *originality/novelty* component, Table 5.10 shows that the calculated Cohen's effect size indicator d was 1.74, which is a large effect size since it is more than 0.8 and that indicates that the suggested enrichment program was effective in developing the subjects' geometric originality/novelty component. Accordingly, the eighth hypothesis is verified, which means that the suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric originality/novelty component in high schools.

5.1.11 Results of Testing Hypothesis Nine

The ninth hypothesis is: "The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted

students' geometric elaboration component".

Table 5.11: *The Effect Size of the Enrichment Program on Geometric Elaboration*

Component of Geometric Creativity	Pretest		Posttest		Cohen's	Effect
	M	SD	M	SD	d	Size
Geometric Elaboration	10.57	4.16	23.86	9.424	1.82	Large

Again, the Cohen's effect size indicator d was calculated to decide on the effectiveness of the suggested enrichment program in developing the subjects' elaboration. As seen in Table 5.11, the calculated effect size was 1.82 (more than 0.8). Therefore the suggested enrichment program was effective in developing the study subjects' elaboration component of geometric creativity among the study subjects. Consequently, the ninth hypothesis is verified. Namely, the suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric elaboration component in high schools.

5.1.12 Results of Testing Hypothesis Ten

The tenth hypothesis is: "The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' overall geometric creative potential in high schools".

Table 5.12: *The Effect Size of the Enrichment Program on Overall Geometric Creativity*

Component of Geometric Creativity	Pretest		Posttest		Cohen's	Effect
	M	SD	M	SD	d	Size
Overall Geometric Creativity Test	151.00	58.38	292.71	87.384	1.91	Large

For testing this hypothesis, the Cohen's effect size indicator d was also calculated to decide on the effectiveness of the suggested enrichment program in developing the subjects' overall geometric creativity potential. Table 5.12 shows that the calculated effect size was 1.91 (more than 0.8). Thus, the suggested enrichment program was effective in developing the overall geometric creativity potential among the study subjects. Con-

sequently, the tenth hypothesis is verified, which again confirms that the suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' overall geometric creative potential in high schools.

5.2 Discussion and Implications

The results of the present study revealed the verifications of the study's hypotheses. That is the results indicated there were statistically significant differences between the mean ranks of the subjects' scores on the pre-post measurements of the geometric creativity test and its subscales in favor of the post measurement. Moreover, by calculating the effect size of these differences, it was found that the suggested enrichment program was significantly effective in developing the mathematically gifted students' geometric creativity as a whole ability and its four sub-components (fluency, flexibility, originality, and elaboration).

The results of the present study can be traced back to many factors and reasons. In the coming sections, discussion and interpretations are drawn out for these results in the light of some boundaries, which include:

- The nature of geometric creativity.
- The content of the suggested enrichment program.
- The teaching and learning techniques used in the intervention.
- The use of the dynamic geometry software during the intervention.
- The characteristics of the subjects.

The results of the present study concerning the development of the geometric creativity, as a whole ability, can be interpreted in the light of the content of the suggested enrichment program. This may be attributed to the idea that it includes a lot of open-ended and divergent-production geometric situations and problems that require many new,

creative, and different responses, which in turn contribute to the development of the subjects' geometric creative ability.

More specifically, the results concerning the development of the fluency component can be attributed to the different learning resources that are provided to the subjects throughout the suggested enrichment program. Equally, promoting discussion and discourse during the intervention was apt to motivate the students to produce a great deal of fluency.

As for the results concerning the development of the flexibility component, they can be ascribed to the use of many non-routine geometric situations and problems within the suggested enrichment program that helped the subjects to overcome their mind fixation and to produce many categories of ideas in different domains of geometry (e.g., perpendicularity, parallelism, collinearity, measurements, congruency, similarity, circles, triangles, quadrilaterals, and areas).

With regard to the development of both the originality/novelty and elaboration components, the results can be interpreted in terms of the teaching and learning techniques, which are used during the intervention. For instance: Activities such as brainstorming, what if, and changing properties, did in turn help the subjects to produce many new responses and to elaborate new geometric problems and situations.

Moreover, in the light of the content of the suggested enrichment program, two more factors that contributed to the development of the subjects' geometric creativity can be also mentioned here. First, the suggested enrichment program included a lot of geometric constructions and interactive exercises that challenged the subjects' abilities; for instance, how to trisect a given line segment and how to construct a square by using specific tools. Second, the subjects did engage during the intervention in seeking different ways to logically prove the validity of their conjectures, which in turn deepened and widened the learning aspects included in the suggested enrichment program.

The results of the present study can also be traced back to the use of the interactive geometry software, Cinderella, and its different facilities (constructing, dragging,

measuring, and calculating, among others) during the intervention, which helped the subjects to express themselves and their mathematical ideas visually and dynamically. The use of Cinderella along the program sessions provided the subjects with many opportunities not only in solving problems but also in posing, finding, and formulating new problems.

Finally, the results of the present study can be attributed to the characteristics of the study subjects, as they are mathematically gifted students. For instance: their special abilities to pose and find problems, their curiosity, their tendency to solve the problems that challenge their abilities, and their high abilities on divergent and convergent thinking, all lent weight and depth to the program sessions and to the eventual learning outcomes.

The results of the present study are consistent with the studies' results of (Al-Baz, 1999; El-Rayashy and Al-Baz, 2000; Mohamed, 2003; Nakin, 2003) since they all revealed the possibility of developing the creative potential in the field of geometry, if the appropriate programs that successfully teach creativity skills and operations were used. In addition, the results of the present study also confirm the results of prior studies (e.g. Kakihana, Fukuda, and Watanabe, 2008; Wurnig, 2008) that adopt the use of integrated learning environment with technology represented in the use of dynamic geometry software to promote students' creativity and help them to approach situations numerically, graphically, and symbolically. The results of the present study are also consistent with the results of Eraky (2004) in that they stress the potential contribution of the instruction differentiation using dynamic geometry software contributes in achieving better learning outcomes among the mathematically gifted students. However, the present study does differ from the previous studies in many aspects (e.g., the subjects, the use of interactive geometry software Cinderella, and the nature of the attempted enrichment program). The present study is the only one among them that makes use of the latest powerful computing tools and software to enhance geometric creativity among the mathematically gifted students in high schools.

However, as any research has drawbacks, the present study has its own limitations. One

problem lies in the size of the study subjects (7 subjects), which means that one of the biggest improvements would be to have more subjects so that the results may become more generalized and meaningful. Also, for one thing pertaining to the experimental design, further studies are needed to investigate the effectiveness of the suggested enrichment program using both quasi-experimental and true experimental designs. A further problem may also emerge in the duration of the attempted program (12 sessions), the matter which incites the researcher of this study to recommend further rather longitudinal studies in the future (see Chapter 6 – Section [6.12](#) Suggestions for Further Research). The problems encountered in this study, however, may open up new avenues for further research on using dynamic geometry software, the development of geometric creativity, and the mathematically gifted students education.

Although this study is limited in terms of the time and sampling boundaries, it provides important implications for classroom practice:

1. It confirms that the enrichment of the classroom-learning environment with technology is optimized to promote creativity in general and geometric creativity in particular.
2. Sometimes, it is very important to just give students situations or activities without specific tasks to perform and asking them to pose and find as many various and different problems as possible that could be deduced or solved in direct or indirect ways from the given information in these situations. The use of these activities would be beneficial on two levels; one to allow students to become more independent and productive in creating and generating their ideas and the other to help teachers to find out how their students think and which level of creativity they have already reached.
3. It highlights the importance of using the integrated environment with technology within mathematics classrooms for approaching mathematical situations in different ways and promoting creativity. It also emphasize the role of dynamic geometry software not only in enhancing the abilities of problem solving, problem posing, and problem finding, but also in constructing concepts and other learning

aspects.

4. It recommends that teachers should take into consideration the vast array of creative thinking and learning strategies of their students allowing them to engage in enrichment activities, whereby they have the support, flexibility, and autonomy to express and explore their own ideas and thoughts.
5. It suggests that learning and developing creativity, its operations and its skills, should be taken into consideration and associated with mathematics curricula while they are designed. Furthermore, it should be a part of daily lesson plans that mathematics teachers prepare for their classrooms.
6. Finally, it also recommends that teachers should think about the difference between teaching problem solving and teaching creative problem solving as well as help their students to recognize this difference in their classrooms.

5.3 Conclusions

The results of the present study have proven that the prepared enrichment program using the interactive geometry software Cinderella is an effective means of enhancing the mathematically gifted students' geometric creativity and its four subcomponents (fluency, flexibility, originality/novelty, and elaboration) in high school. More specifically the designed enrichment activities within the prepared program empowered with using different facilities of the dynamic geometry software enhances students' divergent production ability in geometry and more likely affects students' performance and deepens their learning aspects in geometry.

The use of dynamic geometry software in teaching and learning mathematics for the mathematically gifted students can be used alternatively as an effective differentiation approach to adjust the usual one-way learning environment to facilitate the integration of both learning structure and learning process. Its uses add many dimensions to the learning environment (e.g., creating the meaning of the mathematical concepts, pro-

moting mathematical communication and understanding, enhancing the development of mathematical insight through exploration and experimentation, and promoting students' thinking).

The findings of the present study also suggest that the use of dynamic geometry software as a differentiation approach of teaching and learning mathematics for the mathematically gifted students provides them with an innovative medium that sustains their interest and knowledge by engaging them in a dynamic learning environment that promotes their mathematical performance. These findings come consistent with previous studies that adopt the use of technology as a differentiation approach of teaching and learning mathematics for the mathematically gifted students for the purpose of reaching better outcomes [Eraky \(2004\)](#).

The findings of the present study indicate the possibility of teaching and learning creativity skills and its operations using geometric content if this content is well prepared and enriched for this purpose. These findings also echo on the results of the previous studies ([Al-Baz, 1999](#); [El-Rayashy and Al-Baz, 2000](#); [Mohamed, 2003](#)) and supports the claim that learning and developing creativity should be associated with the mathematical content and it should be part of corresponding lesson plans teachers prepare ([Jerwan, 2002](#), p. 38).

Finally, the findings of the present study emphasize the importance of using the integrated learning environment with technology represented in the use of dynamic geometry software to promote students' creativity and in providing many opportunities to explore complex problems and ideas as well as offering opportunities for the mathematically gifted students to advance at their own pace ([Rotigel and Fello, 2004](#), p. 51).

Chapter 6

Summary, Conclusions and Recommendations

This chapter includes a summary of the study questions, aims, significance, the identified principles of the suggested enrichment program, the suggested enrichment program, instrument of the study, the experimental study, and the results. It also presents conclusions, implications, and recommendations pertinent to the findings of the program and ends with the suggestions for further research related to the topic of the present study.

6.1 Background of the Problem

The development of creativity and innovation among the gifted students has become a major topic discussed in many international conferences. Consequently, many educational programs were developed to provide the gifted with various experiences in order to promote their creative potential (e.g., [Cho *et al.*, 2004](#); [Eraky, 2004](#); [Mohamed, 2003](#); [Velikova, 2004](#)). In this concern, two main approaches appear. Some researchers see that creativity can be learned and developed directly using specific programs that teach creativity skills and its operations regardless of the subject matters, while others assert

that teaching creativity should be associated with the subject matters and it should be a part of the corresponding lesson plans that teachers prepare (Jerwan, 2002, p. 38).

With respect to mathematics as a subject matter, there has been a growing interest in using mathematical content to develop creativity. As a result, new terminologies about creativity emerge, such as mathematical creativity which refers to creativity in the field of mathematics. Also, recent studies used geometric content to develop the creative potential among students (e.g., Al-Baz, 1999; El-Rayashy and Al-Baz, 2000; Mohamed, 2003; Weth, 1998b).

In the context of growing concern with developing creativity in the field of mathematics, a new trend appears that calls for using technology as a way to help promote creativity. For addressing this call, many studies were conducted adopting the integrated learning environment with technology in order to develop creativity (e.g., Kakihana, Fukuda, and Watanabe, 2008; Wurnig, 2008).

On these grounds, there is a need to address more qualified programs and curricula in the field of mathematics for mathematically gifted students. It was in response to this need that the present researcher developed an enrichment program in Euclidean geometry using DGS for the purpose of investigating its effectiveness in promoting the mathematically gifted students' geometric creativity. The suggested enrichment program could be considered a forward step towards designing and using precisely tested programs for mathematically gifted students.

6.2 Problem of the Study

Again, to address the increasing need for qualified programs and curricula in the field of mathematics for mathematically gifted students, the researcher undertook this study, attempting to answer the following question: "How far is a suggested enrichment program using dynamic geometry software effective in developing mathematically gifted students' geometric creativity in high schools?"

This question is branched out into the following sub-questions:

1. What are the principles of developing a suggested enrichment program using dynamic geometry software for developing mathematically gifted students' geometric creativity in high schools?
2. What is the suggested enrichment program using dynamic geometry software for developing mathematically gifted students' geometric creativity in high schools?
3. How far is the enrichment program using dynamic geometry software effective in developing mathematically gifted students' geometric creativity in high schools?

6.3 Aims of the Study

The present study aims at:

1. Identifying the principles of preparing an enrichment program in Euclidean geometry using dynamic geometry software to develop mathematically gifted students' geometric creativity in high schools.
2. Developing the enrichment program based on the identified principles.
3. Investigating the effectiveness of the enrichment program by administering it with high school students.

6.4 Significance of the Study

The study's significance lies within the following parameters:

1. Helping the teachers and program designers of mathematically gifted students to acknowledge the ways DGS contribute to developing the mathematically gifted students' geometric creativity.
2. Providing some enrichment topics in mathematics using dynamic geometry software for mathematically gifted students in high schools.

3. Enabling gifted students to use the dynamic geometry software to communicate mathematically with others through the World Wide Web, the electronic mail, and web pages.
4. Developing a validated and reliable instrument for assessing the mathematically gifted students' ability of geometric creativity.
5. Meeting the increasing need for a large number of studies in the field of teaching mathematics to the mathematically gifted students. The suggested program is considered a step forward toward designing and using precisely tested programs for such students.

6.5 Principles of Developing the Enrichment program

The principles underlying the process of developing the enrichment program were identified as follows:

1. The activities should provide students with opportunities to explore some mathematical ideas using the DGS in a creative fashion.
2. The activities should provide students with opportunities to reinvent mathematical ideas through both exploration and refining of earlier ideas.
3. The enrichment activities should be designed and presented in a constructivist way that encourages mathematically gifted students to make new connections to their prior experiences and construct their own understanding.
4. Teaching the instructional activities should follow van Hiele phases of learning geometric concepts: Information, guided orientation, explication, free orientation, and integration.
5. The activities should correspond to students' skills, since students should experience success in order to stay motivated.
6. The enrichment activities should challenge students' thinking, enhance students'

achievements, and develop students' geometric creativity.

7. The instructional activities should be designed to be effective in revealing geometric creativity and showing the differences between students in terms of their geometric creativity and their responses.
8. The suggested enrichment program activities should address standards for school mathematics recommended by the National Council of Teachers of Mathematics (NCTM) as one of the most popular standard in the field of teaching and learning mathematics.

6.6 The Suggested Enrichment Program

Based on these principles and using the interactive geometry software Cinderella [Richter-Gebert and Kortenkamp \(2006\)](#) as DGS, an enrichment program was developed. The program includes three interrelated portions: student's handouts, a teacher's guide, and a CD-ROM, which cover 12 enrichment activities. All of them are open-ended and divergent-production geometric situations and problems that require many various and different responses.

There are 15 handouts prepared to guide the student throughout the suggested enrichment program. The student's handouts were prepared in both English and German. The teacher's guide was designed to make the teacher's work and progress along the program sessions easier and more effective. The guide does not restrict the teacher's work, but is flexible enough for any creative additions. The accompanying CD-ROM contains all dynamic configurations prepared for the program as construction files and also as web pages, together with an index that provides easy access to all activities and examples.

6.7 Instrument of the Study

Since the purpose of the study was to develop the geometric creativity of the mathematically gifted students using the suggested enrichment program, the researcher had to design an instrument, a geometric creativity test, to assess the geometric creativity of the mathematically gifted students before and after administering the enrichment program. In designing the geometric creativity test, a 6-step process was used:

1. Specification of the aim of the test.
2. Specification of the components that the test measures.
3. Creation of a preliminary form of the test.
4. Setting-up a grading method for the test.
5. Content validity check.
6. Test-piloting.

6.8 The Experimental Study

The materials were tested with a group of 7 mathematically gifted students in the Landesgymnasium für Hochbegabte (LGH), a public high school for highly gifted and talented students in Schwäbisch Gmünd, Baden-Württemberg, Germany. The students came from 11th (5), 10th (1) and 9th (1) grades, two of them were male and five female.

In the study, a one-group pretest – intervention – posttest pre-experimental design was used to investigate the effectiveness of the suggested enrichment program. In this context the GCT was administered to students as a pretest at the beginning of the study, then the suggested enrichment program was introduced to them in 12 weekly 90-minutes sessions during the first semester of the academic year 2008/09. The students retook the GCT as a posttest at the end of the study.

6.9 Results of the Study

The results of testing the study hypotheses can be eventually summed up as follows:

1. There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects' scores on the pre-post measurements of the geometric fluency component, measured by the designed geometric creativity test, in favor of the post measurement.
2. There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects' scores on the pre-post measurements of the geometric flexibility component, measured by the designed geometric creativity test, in favor of the post measurement.
3. There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects' scores on the pre-post measurements of the geometric originality/novelty component, measured by the designed geometric creativity test, in favor of the post measurement.
4. There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects' scores on the pre-post measurements of the geometric elaboration component, measured by the designed geometric creativity test, in favor of the post measurement.
5. There is a statistically significant difference at the level 0.05 between the mean ranks of the subjects' scores on the pre-post measurements of geometric creativity as a whole ability, measured by the designed geometric creativity test, in favor of the post measurement.
6. The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric fluency component in high schools.
7. The suggested enrichment program using dynamic geometry software has a suit-

able level of effectiveness in developing the mathematically gifted students' geometric flexibility component in high schools.

8. The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric originality/novelty component in high schools.
9. The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' geometric elaboration component in high schools.
10. The suggested enrichment program using dynamic geometry software has a suitable level of effectiveness in developing the mathematically gifted students' overall geometric creative potential in high schools.

6.10 Conclusions and Implications

The results of this study suggest that the prepared enrichment program using the interactive geometry software Cinderella has a positive impact on developing the mathematically gifted students' geometric creativity as a whole ability and its four sub-components (fluency, flexibility, originality/novelty, and elaboration). The positive impact can be traced back to the content of the suggested enrichment program and its open-ended and divergent-production geometric situations and problems. Also, the positive impact can be attributed to the use of DGS along the program sessions that provide the subjects with many opportunities not only to explore, experiment and make mathematical conjectures, but also to solve problems, and pose related problems.

However, the study requires replication and improvements before any firm conclusions can be made. One of the biggest improvements would be to have more subjects that the results become more generalized and meaningful. Also, for one thing pertaining the experimental design, further studies are needed to investigate the effectiveness of the suggested enrichment program using both quasi-experimental and true experimental

designs. See [Suggestions for Further Research](#) in the coming sections.

It is important here to ensure that the classroom-learning environment is optimized to promote creativity in general and geometric creativity in particular. In this context, using different facilities of the interactive geometry software Cinderella along the program sessions enriched the learning environment by offering the students accurate tools for constructing and measuring, encouraging the students to postulate new conjectures toward the given problem or situation and immediately investigate the validity of these conjectures and motivating the students to look for logical proofs for their formulated conjectures. In addition, using dynamic geometry software, as a mediation environment was a great advantage to the students that enables them to approach the given problems and situations in various mathematical representations; numerically, graphically, and symbolically. It also enables students to mathematically communicate and express themselves more easily.

Accordingly, the consistency of the present study results with other prior studies of [Kakihana, Fukuda, and Watanabe \(2008\)](#) and [Wurnig \(2008\)](#) that adopt the use of technology to promote students' creativity would provide an important implication, which confirms that the integrated learning environment with technology, which is represented here by using DGS, is optimized to promote creativity.

6.11 Recommendations

In light of the findings, the following recommendations are provided:

1. School mathematics curricula should be reshaped in a way so that they include experiences (e.g., activities, situations, and problems) that would contribute to the development of students' creative thinking.
2. The school mathematics textbooks should include open-ended and divergent-production questions similar to the ones addressed in the geometric creativity test.

3. The mathematically gifted students should be given the opportunity to think like mathematicians do and re-invent mathematics by going through the process which leads them to the product and not merely learn the product (theorems and axioms).
4. The mathematically gifted students should be trained to be creative through using designed activities of creativity to enhance their creativity and divergent thinking. Group work should be used during such activities.
5. Providing special schools or classes for the mathematically gifted students at different educational levels and stages.
6. Developing other enrichment programs in the field of geometry and other mathematics disciplines based on the use of the integrated learning environment with technology, for instance using Computer Algebra Systems (CAS) and Dynamic Geometry Software (DGS), for the purpose of developing mathematical creativity and innovation.
7. Developing creativity tests in the field of mathematics that can be used beside regular norm-reference tests to discover and identify the mathematically gifted students.
8. Supplying tools for measuring creative thinking abilities in different disciplines of mathematics in different educational levels and stages.
9. Supplying tools of measurements to discover the mathematically gifted students in different educational levels and stages.
10. Using questions that develop creative thinking abilities such as “what if” questions, and “Brainstorming” questions.
11. Using divergent-production questions that require deleting, changing, or adding parts in the problem and noticing the ripple effect.
12. Using questions that measure creative thinking abilities; for instance fluency of ideas and figures, adaptive and spontaneous flexibility, originality and elaboration.

13. During evaluation, attention should be paid to measuring creative abilities and training the teachers and the students on these tests.
14. Students should be asked to perform various activities that develop their creative abilities (fluency, flexibility, originality/novelty, and elaboration).
15. Using open-ended and divergent production questions in mathematics classroom will promote creativity.
16. Developing the creative thinking abilities of the teacher-students of mathematics to be reflected on their students in the future.
17. Training the teacher-students of mathematics on teaching and learning methods of mathematically gifted students, how to meet their needs, and methods of developing creative thinking through mathematics education courses and field training.
18. Training the teacher-students of mathematics on the ways and techniques that can be used to enrich the mathematical content for mathematically gifted students and the techniques of developing creative thinking in the field of mathematics.
19. Preparing training courses for in-service teachers and supervisors at different educational levels to raise their awareness of the uses of dynamic geometry software.

6.12 Suggestions for Further Research

Efforts to supplement findings of the study are needed to develop feasible but differential instructional programs in other disciplines of mathematics (solid geometry, analytical geometry, algebra, and calculus, among others) for the mathematically gifted students in order to help them achieve better mathematical performance as well as mathematical creativity.

Further research is needed to examine how the three factors; students' learning environment, teacher, and methods interact with each other. Given that the student-learning environment is the most important factor, it is important for the learning processes to

succeed to determine the specific components in the learning environment that promote creative thinking in the classroom.

A great deal of research remains to be done on many aspects of the uses of dynamic geometry software as well as its impact on developing other types of thinking (e.g., critical thinking, deductive and inductive thinking, reflective thinking, analytical thinking, and intuitive thinking) in different disciplines of mathematics and in different stages in the educational process.

Other avenues of research might focus on students' affective and emotional domains (e.g., self confidence, attitudes, and achievement motivation), and might as well include an analysis of gender-related individual differences.

The framework of the present study would go beyond its limitations by developing enrichment programs in different mathematics disciplines and investigate their effectiveness in developing the creative thinking abilities for gifted and non-gifted students. Also in terms of the study limitations, the duration of the attempted program, which covered 12 sessions, indicates the need for further rather longitudinal studies in the future.

Further studies are suggested for investigating the effect of using the integrated learning environment with technology; such as using Computer Algebra Systems (CAS) and Dynamic Geometry Software (DGS) in mathematics classrooms in order to promote and develop creativity in the field of mathematics.

In terms of students' performance and its relationship with the development of creative thinking in geometry, further research is needed to determine the correlation between students' performance in geometry and the development of creative thinking.

Last and not least, the findings of the piloting of the geometric creativity test, which are related to test reliability as well as the factor analysis of test subscales, evoke the need for further study to qualify the test in the light of these issues with a bigger sample of students.

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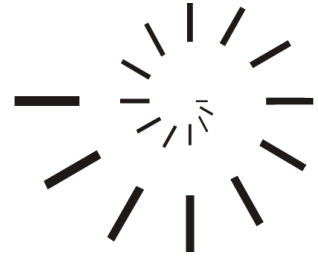
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Appendices

Appendix A
Student's Handouts
(English Version)



The Suggested Enrichment Program

Student's Handouts

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Handout 1

The Suggested Enrichment Program Outcomes

Dear student,

The following are the suggested enrichment program outcomes. Please, read them before receiving instruction:

Throughout the suggested enrichment program you will hopefully be able to use Cinderella to:

- Outcome 1 Construct dynamic figures.
- Outcome 2 Come up with many construction methods to construct dynamic configurations for a certain figure.
- Outcome 3 Come up with many different and varied construction methods to construct dynamic configurations for a certain figure.
- Outcome 4 Come up with novel and unusual methods to construct dynamic configurations for a certain figure.
- Outcome 5 Produce many relevant responses (ideas, solutions, proofs, conjectures, and new formulated problems) toward a geometric problem or situation.
- Outcome 6 Produce many different and varied relevant responses (ideas, solutions, proofs, conjectures, and new formulated problems) toward a geometric problem or situation.
- Outcome 7 Generate many unusual (“way out”), unique, clever responses or products toward a geometric problem or situation.
- Outcome 8 Make new conjectures and relationships by recognizing your experience toward the aspects of the given problem or situation.

- Outcome 9 Investigate the made conjectures and relationships in different situations and methods.
- Outcome 10 Generate many different and varied proofs toward a geometric problem or situation.
- Outcome 11 Generate many follow-up problems by redefining (modifying, adapting, expanding, or altering) a given geometric problem or situation.
- Outcome 12 Apply different learning aspects of geometry (concepts, generalizations, and skills) in solving a geometric problem or situation.

Handout 2

Launching and Exploring Cinderella

In this session you will learn how to:

- Launch Cinderella.
- Explore Cinderella window.
- Explore Cinderella menus.

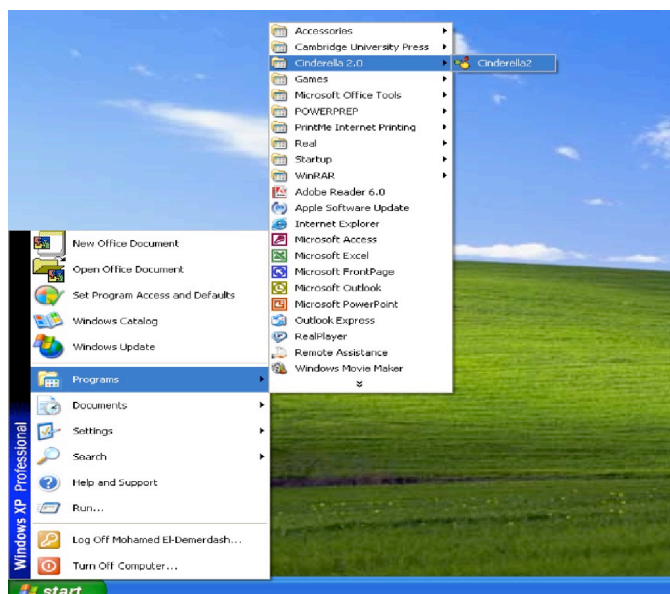
Student Activity: Launch Cinderella

To start Cinderella, It depends on where Cinderella installing folder is; however you may either:

- Double-click **Cinderella.2 shortcut** on the desktop, If it exists.



- Click **Start** button on the taskbar. Click **Programs**. Find and click **Cinderella.2**.



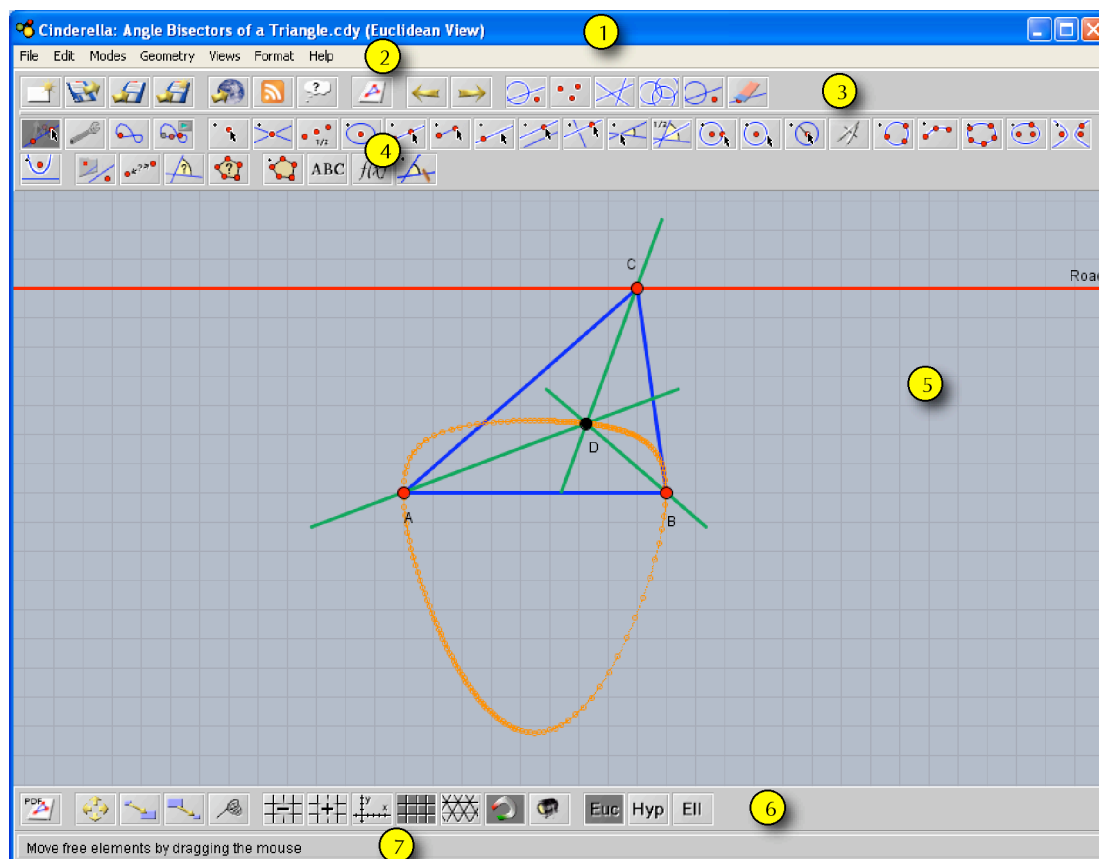
- Double-click a file created by Cinderella. Files that are created by Cinderella usually have the extension “.cdy” and have icons similar to the icon shown below.



After a few moments, Cinderella will launch and be ready to work.

Exploring Cinderella Window

After you launch Cinderella, you will see a window, called “**Unnamed in (Euclidean View)**”. It looks similar to the figure shown below. The main window of Cinderella consists of seven major elements. Here are the main elements of the window in order from the top to the bottom.



1. Title Bar

Here you will find the name of the program – Cinderella – and the current construction file name and its corresponding view as well. If you haven't named the construction file yet, you will see something like **“Unnamed in (Euclidean View)”**.

2. Menu Bar

It contains the menus with the commands that control Cinderella. It comprises of seven menus, which are: **File, Edit, Modes, Geometry, Views, Format, and Help**. Click on a menu to pull it down. Select a command by clicking on it.

3. Standard Toolbar

It contains buttons for basic/general commands such as **opening, saving, printing, exporting, undoing, redoing, and selecting**.

4. Geometric Modes Toolbar

This toolbar contains tools for **all** geometric construction modes ranging from adding point to generating loci.

5. Construction Area

This is the area where you can create and modify your geometric constructions such as points, straight lines, and circles. Here, you can also make your geometric explorations using Move mode.

6. Specific-view Actions Toolbar

This toolbar contains buttons for **specific-view actions** such as **zooming in and out, translating, drawing coordinates axes, drawing rectangular or triangular**

grids, and showing hidden elements.

7. Message Bar

In this bar Cinderella tells you in which mode you are and what actions you should do.

Exploring Cinderella Menus

Cinderella menu bar comprises of 7 pull-down menus, which are: **File**, **Edit**, **Modes**, **Geometry**, **Views**, **Format**, and **Help** respectively. It should be mentioned here that **all** functions and facilities of Cinderella could be accessed through the main menus, whereas the toolbars only include the most important tools.

1. File Menu

The **File** menu contains a list of file-related commands, such as **New**, which creates a new Cinderella file, **Open** which opens or loads a Cinderella saved file, **Save** which saves the current Cinderella file, **Close Window** which closes the current Cinderella file, etc. Notice the combination next to some of the commands in the file menu. For instance, if you want to save a Cinderella file use the combination **Ctrl+S** (Control key and S key).

New	Ctrl+N
Open	Ctrl+O
Recent Constructions	▶
Save	Ctrl+S
Save as	Ctrl+Shift+S
Close Window	Ctrl+W
Edit Scripts	Ctrl+9
Open Exercise Editor	Ctrl+8
Export to HTML	Ctrl+E
Export Graphics to	▶
Print	Ctrl+P
Choose Toolbars	
Exit	Ctrl+Q

2. Edit Menu

This menu contains tools for **undoing**, **redoing**, and **copying** and **pasting** parts of construction. In addition, under this menu you can create **Macro-constructions** using the tool, **Create Tool from Selection**. It also contains several tools for selection that enables you either to **select all**, **select points**, **select lines**, **select conics**, or **deselect all**. Here you also get access to the **inspector**, a tool that can be used to edit the geometric elements within your construction.

Undo	Ctrl+Z
Redo	Ctrl+Y
Copy	Ctrl+C
Paste	Ctrl+V
Create Tool from Selection	Ctrl+Shift+N
Delete Elements	Ctrl+Delete
Select all	Ctrl+A
Select points	Ctrl+Shift+P
Select lines	Ctrl+Shift+L
Select conics	Ctrl+Shift+C
Deselect all	Ctrl+Shift+A
Fetch	
Information	Ctrl+I

3. Modes Menu

The **Modes** menu contains a collection of construction tools for **all** geometric constructions ranging from adding point to generating loci. Most of these tools can be accessed via geometric modes toolbar.

✓ Move	Ctrl+M
Redefine Point	
Point	▶
Line	▶
Circle	▶
Conic	▶
Graphics	▶
Measure	▶
Polygon	▶
Transformation	▶
Special	▶
Simulation	▶
API	▶
	Free Line
	Segment
	by two points Ctrl+L
	Horizontal Line
	Vertical Line
	Through Point
	Parallel
	Orthogonal
	Fixed Angle
	Angular Bisector
	Polar of Point

4. Geometry Menu

One of the main features of Cinderella is that it supports different kinds of geometries. Here you can choose the kind of geometry you wish to work in (the default is

Euclidean Geometry, but you can also work in **Hyperbolic Geometry** or **Elliptic**

✓ Euclidean Geometry
Hyperbolic Geometry
Elliptic Geometry

Geometry). It should be mentioned here that switching to another kind of geometry does affect the behavior of the geometric elements you will construct.

5. Views Menu

This menu enables you to open your constructions in different views (the default is **Euclidean View**, but you can also open your construction in **Hyperbolic View**, **Spherical View**, **Polar Euclidean View**, and **Polar Spherical View**). It also enables you to open the **Construction Text**

Euclidean View	Ctrl+1
Hyperbolic View	Ctrl+2
Spherical View	Ctrl+3
Polar Euclidean View	Ctrl+Shift+1
Polar Spherical View	Ctrl+Shift+3
Construction Text	Ctrl+4
Information Window	Ctrl+5

(Cinderella special window that shows a detailed description for each element within your construction) as well as the **Information Window** that shows and reports non-trivial facts about your construction.

6. Format Menu

The **Format** menu enables you to choose the default format in which units and equations should be displayed.

Angle	▶	
Length	▶	1
Line	▶	✓ cm
Point	▶	mm
Circle	▶	inch

7. Help Menu

The Help menu contains **Help on mode** that provides help for the current mode, **Cinderella complete Manual**, and references to online help, updates and licensing issues.

Help on mode	Ctrl+F1
Cinderella Manual	Ctrl+Shift+F1
About Cinderella	
Cinderella Homepage	
Springer Homepage	
Enter License...	

Handout 3

Basic Commands and Modes of Cinderella

In this session you will learn about:

- Cinderella basic commands.
- Cinderella basic modes.

Cinderella Basic Commands

Open New Cinderella Window

To get a new construction window in Cinderella, do one of the following:

- From the **File** menu select **New**.
- Press the keyboard shortcut **Ctrl+N**.
- Click on the **New** button  on the standard toolbar.

The New command opens a new, blank Cinderella window in Euclidean View for creating and modifying geometric constructions. In this new window, everything is reset to the default state.

Open/Load Cinderella Saved File

To open/load a Cinderella saved file, do one of the following:

- From **File** menu select **Open**.
- Press the keyboard shortcut **Ctrl+O**.
- Click on the **Open** button  on the standard toolbar.
- Double-click the Cinderella file icon; it could be similar to this icon .

The open command opens a dialog box for opening an existing Cinderella file.

Save Construction

To save the current construction, do one of the following:

- From **File** menu, select **Save**.
- Press the keyboard shortcut **Ctrl+S**.
- Click on the **Save** button  on the standard toolbar.

The save command opens a dialog box that allow you to save the current Cinderella construction to a name that you specify.

Save Construction with New Name

To save the current construction with a new name, do one of the following:

- From **File** menu select **Save as**.
- Press the keyboard shortcut **Ctrl+Shift+S**.
- Click on the **Save as** button  on the standard toolbar.

The save as command opens a dialog box that allows you to save the current Cinderella construction to another name that you specify.

Create an Interactive Web Page

To create an interactive web page of the current construction, do one of the following:

- From **File** menu select **Export to HTML**.
- Press the keyboard shortcut **Ctrl+E**.
- Click on the **Create an Interactive Web Page** button  on the standard toolbar.

This action will open a dialog box that allows you to save the current Cinderella file as a web page.

Two **warnings** should be taken into consideration, while you are creating interactive web pages:

1. The construction data has to be saved as Cinderella file with the extension “**cdy**” before you export it to HTML.
2. The construction data has to be saved into the same directory your webpage will be resided in.

Undo 

Undo command enables you to take one or more actions backward. You can undo as many backward actions as you want; however each click on the undo button takes you one action backward. The undoable actions are:

- Construction steps.
- Movements.
- Appearance changes.
- Deletion of elements.

Redo 

Redo command enables you to take one or more actions forward. Here you can also redo as many forward actions as you want.

Delete 

Delete command is used to delete currently selected elements and other elements that depend on them.

Selection Commands

Under the **Edit** menu, there are several tools for selection that enable you either to **select all elements**, **all points**, **all lines**, or **all conics and circles**. **All** of these tools are also accessible via corresponding buttons on the standard toolbar.

Select All Elements 

It is used to select all geometric elements in the construction.

Select All Points 

It is used to select all points in the construction.

Select All Lines 

It is used to select all lines in the construction.

Select All Conics 

It is used to select all conics and circle in the construction.

Deselect Everything 

It is used to clear the current selection in the construction.

Cinderella Basic Modes

Move Mode 

The move mode is one of the most important tools that provide Cinderella with the dynamic behavior – the ability to move free elements and the whole constructions moves

accordingly without destroying the configuration which is a very beneficial characteristic in exploring geometric relationships.

Generally, move mode has twofold functions that enable you to select and move elements around the construction area.

There are two types of points: **free/movable points** and **fixed points**.

Free points: these are not dependent on the elements of the construction and can be moved around the construction area freely and the rest of the construction moves/changes accordingly.

Fixed points: are points that are dependent on the position of other free elements of the construction and cannot be moved.

In the construction area, you can visually distinguish between the free and fixed points by their appearance: fixed points appear darker.

Moving Points

To move a free/movable point, move the mouse pointer over the point you want to move. Press the left mouse button. Hold it pressed and drag the mouse. The point follows the mouse pointer.

Selecting Elements

The move mode can also be used to select one or more geometric elements in the construction area. To select an element, just click over the element you want to select. By holding the shift key you can select several elements at the same time.

You can easily recognize the selected elements, since they are highlighted in all views. You may select elements for the following reasons:

- To individualize the appearance of geometric elements using the inspector which is available under the item menu **“Edit/Information”**.

- To delete the selected elements using “**Delete**” action.
- To simultaneously move the selected elements.

Add a Point



Add a point mode construct a single new point by using a single press-drag-release sequence mouse input. Constructing a point can be described as follows:

- **Pressing** the left mouse button generates the point.
- **Dragging** the mouse while the left mouse button is pressed changes the position and definition of the point.
- **Releasing** the mouse button fixes the definition of the point.

The definition of the point depends on the position on which the mouse was released. In other words, if at the moment of release the mouse is ...

- ...over no element at all, then a free point is added and can be freely moved in “**Move**” mode.
- ...over an existing element (line, circle, conic), then a semi free point is added on the existing element and can be moved along the existing element in “**Move**” mode.
- ...over the intersection of two elements (line, circle, conic) ,then the intersection of these elements is added as a fixed point that can not be moved in “**Move**” mode.
- ...over an existing point, then no point will be added.

Add a Line



Add a line constructs two points and a straight line joining them by using a single press-drag-release sequence mouse input. Constructing two points and a line joining them can be described as follows:

- Pressing the left mouse button generates the first point. The definition of this point depends on the position of the mouse when the button is pressed as explained earlier in “**Add a Point**” mode.
- Dragging the mouse while the left mouse button is pressed generates the line and the second point. The definition of the second also depends on the position of the mouse.
- Releasing the mouse button freezes the second point and ends up with the straight line that joins the two points.

Line Through Point

Line through point mode constructs a line through a point with a certain slope so that when the point is moved, the slope of the line remains constant. However, in move mode it is also possible to select the line and change its slope. Constructing line through point can be described as follows:

- **Pressing** the left mouse button generates the point.
- **Dragging** the mouse while the left mouse button is pressed generates the line.
- **Releasing** the mouse button freezes and finishes the construction.

Add a Parallel

Add a parallel mode constructs a line through a point parallel to another line by using a single press-drag-release sequence mouse input. Constructing the parallel can be described as follows:

- **Move** the mouse pointer over the line you want to construct a parallel to it. Then **press** the left mouse button. This action generates the parallel line and a point, which the parallel line passes through it.

- **Drag** the mouse while the left mouse button is pressed. This action moves the parallel line and the generated point to the desired position.
- **Release** the mouse button. This action freezes the definition of the parallel line and the new point, and finishes the construction.

Add a Perpendicular



Add a perpendicular constructs a line through a point perpendicular to another line by a single press-drag-release sequence using the mouse. Constructing the perpendicular can be described as follows:

- **Move** the mouse pointer over the line you want to construct a perpendicular to it. Then **press** the left mouse button. This action generates the perpendicular line and a point, which the perpendicular line passes through it.
- **Drag** the mouse while the left mouse button is pressed. This action moves the perpendicular line and the generated point to the desired position.
- **Release** the mouse button. This action freezes the definition of the perpendicular line and the new point, and finishes the construction.

Add a Circle



Add a circle mode constructs a circle given by its center and a point on its circumference by using a single press-drag-release sequence mouse input. Constructing a circle in this mode can be described as follows:

- **Pressing** the left mouse button generates the center of the circle.
- **Dragging** the mouse while the left mouse button is pressed generates the circle and the circumference point.
- **Releasing** the mouse button freezes the definition of the circumference point and finishes the construction.

Midpoint



Midpoint mode constructs two points and the point that is the midway between them using a single press-drag-release sequence with the mouse. Constructing the midpoint can be described as follows:

- **Pressing** the left mouse button generates the first point.
- **Dragging** the mouse generates the second point and the midpoint as well.
- **Releasing** the mouse freezes the definition and finishes the construction.

Segment



Segment mode constructs two points and a line segment joining them by using a single press-drag-release sequence mouse input. Constructing two points and a line segment joining them can be described as follows:

- **Pressing** the left mouse button generates the first point.
- **Dragging** the mouse while the left mouse button is pressed generates the second point and the line segment joining the two points. The definition of the second also depends on the position of the mouse.
- **Releasing** the mouse freezes the second point and ends up with the line segment line that joins the two points.

Student Activity

Use Cinderella to come up with dynamic configurations for each of the following figures:

- Triangle
- Equilateral triangle
- Isosceles triangle

- Right-angled triangle
- Quadrilateral
- Trapezium
- Parallelogram
- Rectangle
- Rhombus
- Square
- Circle
- Tangent to a Circle

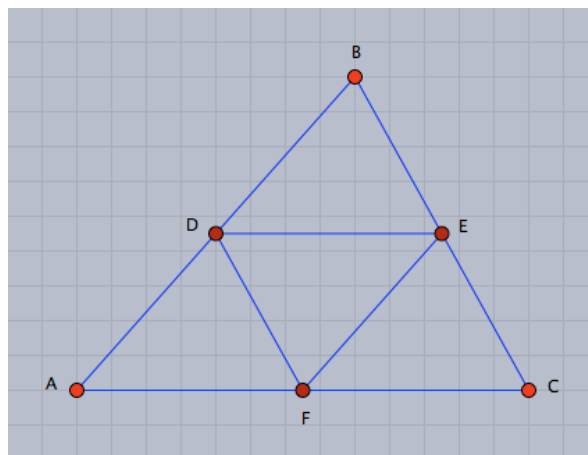
Handout 4

Dragging and Measuring Facilities of Cinderella

Cinderella has a wide range of facilities; among these facilities are the dragging and measuring facilities. In this activity, you will experience the dragging and measuring facilities of Cinderella to make and investigate new geometric conjectures.

Student Activity

1. Launch Cinderella or open new Cinderella file.
2. Use Cinderella to construct any triangle. (Use “**Add Point**” tool  to construct three points A, B , and C as three vertices of the triangle, then switch to “**Draw a Segment**” tool  to join up the three points using a press-drag-release sequence with the mouse to end up with triangle ABC)
3. Construct the midpoints of the sides of the triangle. (Use “**Midpoint**” tool  with the mouse press-drag-release sequences to bisect the three sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ at D, E , and F respectively)
4. Join the midpoints of the sides to get the four triangles: $\triangle ADF$, $\triangle DBE$, $\triangle EDF$, and $\triangle CEF$. (Use “**Draw a Segment**” tool  to join the midpoints D, E , and F). Your construction should look like the figure shown below before you continue.



5. Switch to **“Move”** mode by pressing the button  in the toolbar. Use Cinderella's dragging facilities to drag free points (A , B , or C), visually observe the four triangles and try to make a conjecture about the area of the four triangles. In other words, what can you say about the area of the four triangles?
6. Using measuring facilities of Cinderella, the area of each triangle can be measured. For doing so, switch to the **“Define a Polygon”** mode by pressing the button  in the toolbar, define the four triangles as polygons: $Poly0$, $Poly1$, $Poly2$, and $Poly3$, then switch to the **“Measure Area of a Polygon”** mode by pressing the button  in the toolbar and just click inside each triangle (polygon) to get its area.
7. Use dragging to alter the triangle ABC and visually observe the area of the four triangles. What can you conclude about the area of the four triangles obtained by joining the midpoints of the sides of a triangle? Write down your conjecture in the form of mathematical theorems.
8. Produce a mathematical proof for the investigated conjecture.
9. Pose follow-up problems related to this geometric situation.

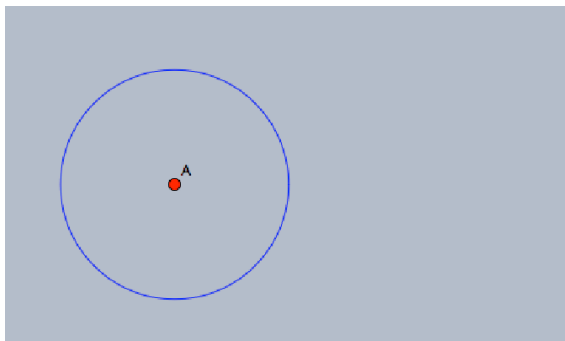
Handout 5

Automatic Proving Facilities of Cinderella

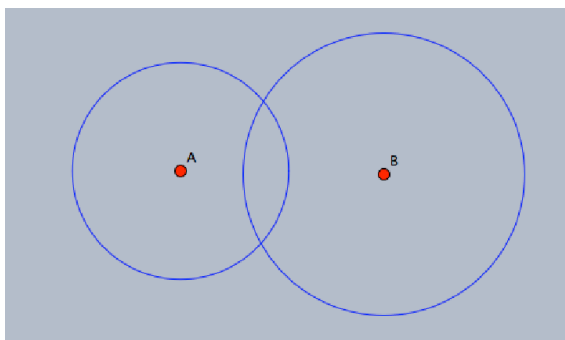
Cinderella has built-in automatic proving facilities that could be used to immediately check the correctness of your geometric conjectures. In this activity, you will see how to use these facilities to check the collinearity of the three points.

Student Activity

1. Launch Cinderella or open new Cinderella window.
2. Switch to “**Circle by Radius**” mode using “**Modes**” menu or by pressing the button  in the toolbar.
3. Move the mouse pointer over the construction area. Use a press-drag-release sequence with the mouse to add a circle with centre A .

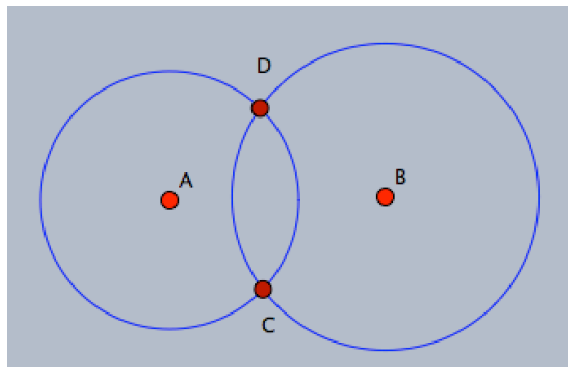


4. Add a second circle using the same mode such that the two circles intersect.

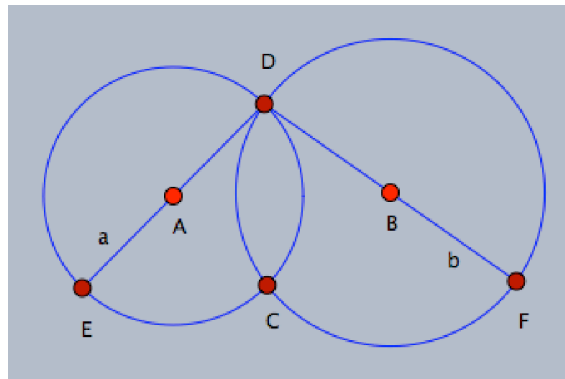


5. Generate the intersection points between the two circles, for doing so, switch to

“Intersection” mode by pressing the button  in the toolbar, then select the two circles one at a time to generate their intersection points C and D . Your construction should look like the following figure before you continue.



6. Switch to **“line by two points”** mode by choosing the menu item **“Modes/line/by two points”**, by pressing the keyboard shortcut **“Ctrl+L”**, or by pressing the button  in the toolbar. Use a press-drag-release sequence with the mouse to add a line connecting the two points A and D .
7. Add another line connecting the two points D and B .
8. Switch to **“Intersection”** mode by pressing the button  in the toolbar. Select Circle A and Line a to get the intersection point E .
9. Using the same mode, select Circle B and Line b to get the intersection point F .
10. Open **“Cinderella’s Inspector”** by choosing the menu item **“Edit/Information”**.
11. Select all lines by pressing the button .
12. Clip the two lines by choosing the clipping button  on the inspector. After clipping the two lines to their endpoints your construction should look like the following figure.
13. Use Cinderella’s dragging facilities to drag free elements in your construction and visually observe the behavior of the three points F , C , and E ? What can you say about the three points F , C , and E in terms of collinearity? i.e. Are they collinear? Anyway make a conjecture about the collinearity of the three points



and Cinderella will check it for you.

14. To check the correctness of the made conjecture about the collinearity of the three points, open Cinderella's information window by choosing the menu item **"View/Information Window"** or pressing the keyboard shortcut **"Ctrl+5"**. A console window pops up in which an automatic report about the configuration will appear. Now, switch to the **"Line, by two points"** mode by pressing the button  to construct the line that connects the two points F and E , and then move the mouse pointer toward the point F press the left mouse button, hold it then drag the mouse over point E then release the mouse button. Notice the message that is given in the Cinderella Console Window, which indicates that point C lies on the connecting line of F and E .
15. Produce as many mathematical proofs as possible to prove the investigated conjecture.
16. Use different facilities of Cinderella to make up as many new conjectures as possible by elaborating – extending, modifying, or adapting – the conditions of the given situation.

Handout 6

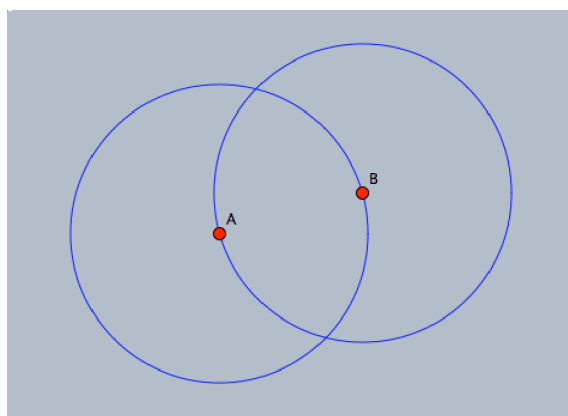
Developing Macro-constructions

Cinderella has the ability to capsule a sequence of construction commands into a new command/tool, which is called Macro-constructions. In this activity, you will experience how to use Macro-construction facilities to develop some geometric tools such as: perpendicular bisector and circum-center point of a triangle.

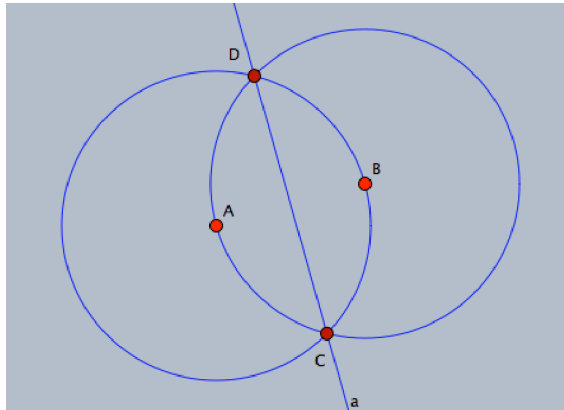
Student Activity 1

Developing a Macro Tool for Perpendicular Bisector

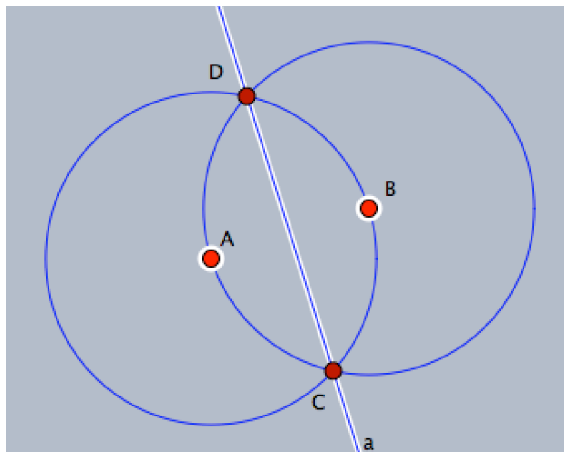
1. Launch Cinderella or open new Cinderella window.
2. Switch to **“Circle by Two Points”** mode using **“Modes”** menu or by pressing the button  in the toolbar. This mode allows you to construct a circle with two points one at the center and the other at the circumference of the circle.
3. Use a press-drag-release sequence with the mouse twice to construct two points A and B and two circles that use A and B as centers and B and A as point on the circle.



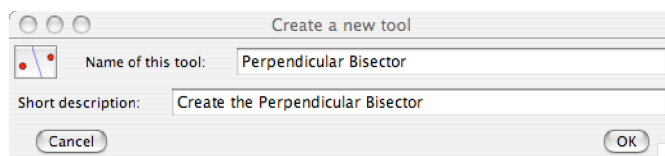
4. Generate the intersection points between the two circles at C and D , using the **“Intersection”** mode . Then, switch to **“Add a Line”** mode  to construct the perpendicular bisector of A and B .



5. To define a macro you have to select the initial elements – here the two points A and B – and the final elements – here the line a . So, Switch to **“Move”** mode, and then use Shift key to select the two points A and B and the connecting line a .



6. Now, the three selected elements, the two points A , B and the line a , which represent the initial and final elements, can be used to create a macro tool for the perpendicular bisector. For doing so, choose menu item **“Edit/Create Tool from Selection”** or press the keyboard shortcut **“Ctrl+Shift+N”**. A new window of two fields pops up that requires a name and a description for the new tool. Enter the name and the description for the new tool – Perpendicular Bisector and Create the Perpendicular Bisector are suggested to be a name and description for the new tool – then, click OK button to confirm the creation of the tool. The icon is created automatically and will be available in the toolbar.

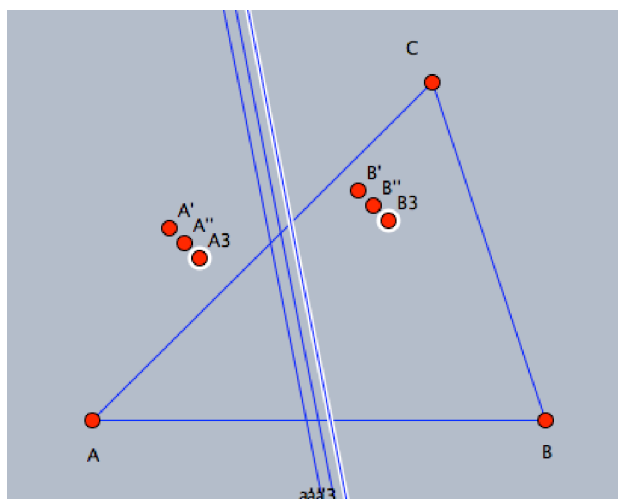


Student Activity 2

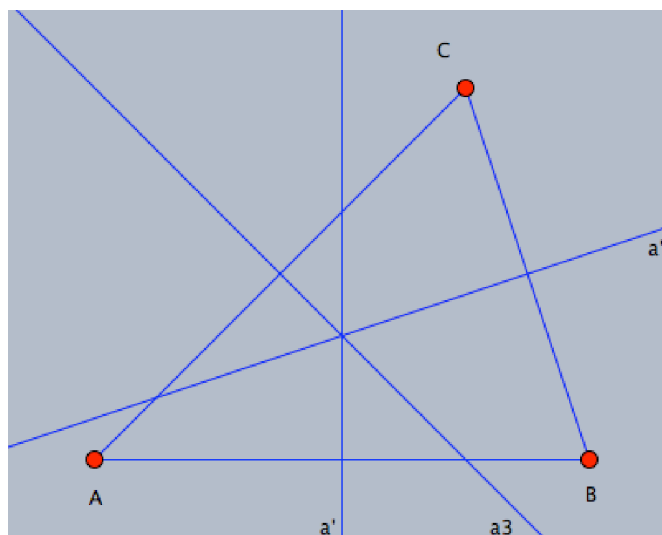
Developing a Macro Tool for Circum-center Point of a Triangle

In this activity, you will be guided to use the perpendicular bisector tool to construct the circum-center point of a triangle and develop a new macro tool for the circum-center point of a triangle as well.

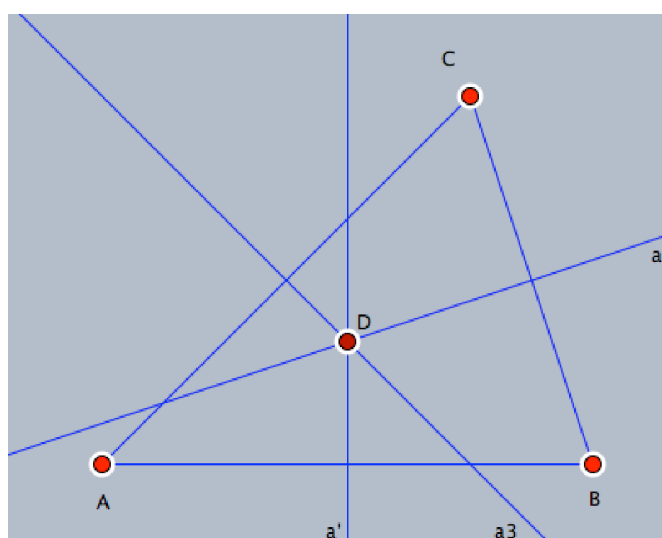
1. In the last Cinderella window that contains the developed perpendicular bisector tool. Clean the construction area by selecting all elements using the item menu **“Edit/Select all”** or by clicking the button  or by pressing the keyboard shortcut **“Ctrl+A”**. Then, delete elements using the menu item **“Edit/Delete Elements”** or by clicking the button  or pressing the keyboard shortcut **“Ctrl+Delete”**.
2. Construct a triangle ABC .
3. To construct the perpendicular bisectors for the triangle sides, click the developed perpendicular bisector tool button  three consecutive times this action will instantly paste the points A' and B' ; A'' and B'' ; A_3 and B_3 and the perpendicular bisector lines a' , a'' and a_3 .



4. Switch to “**Redefine Point**” by using the menu item “**Modes/Redefine Point**” or by clicking the button  in the toolbar mode and then move A' to A , B' to B , A'' to B , B'' to C , A_3 to A , and B_3 to C . That will end up with a configuration of a triangle and its side perpendicular bisectors.



5. Generate the intersection point of the perpendicular bisectors at D  to get the circum-center point of triangle ABC .
6. Switch to “**Move**” mode, and use Shift key to select the points A , B , and C as initial elements and D as final element for the macro.



¹Remark: The perpendicular bisectors of the sides of a triangle are concurrent. The automatic proving facilities of Cinderella can be used to show that, while you are generating the intersection point D.

7. After you select the initial and final elements of the macro, use “**Create Tool from Selection**” command from edit menu to develop a new macro tool for the circum-center point of a triangle, and give it a name and a description. A new icon for the circum-center point of a triangle will be created in the toolbar.

Assignment Project

Use Cinderella to develop a macro tool for each of the following:

- In-center point of a triangle.
- Orthocenter point of a triangle.
- Center of mass or centroid point of a triangle.
- Euler line of a triangle.

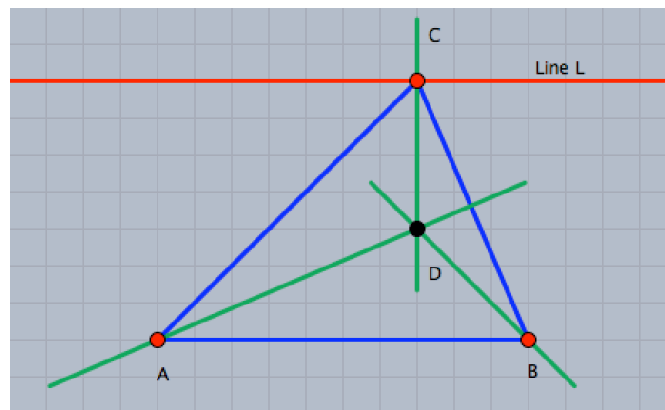
Handout 7

Animating and Tracing Loci Facilities of Cinderella

In this activity, you will experience the use of animating and tracing loci facilities of Cinderella to discover and generate geometric loci.

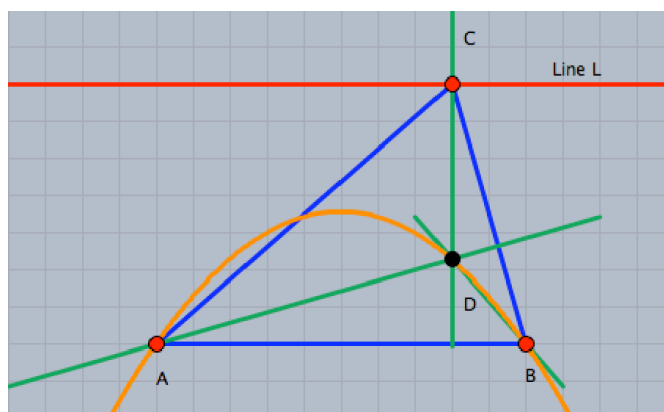
Student Activity

1. Construct a triangle ABC in a way that it is possible to move the point C along a straight line L , which is parallel to the base $\overline{AB}$.
2. Construct the three heights of the triangle ABC and generate their intersection point at D ². Your construction should look like the following figure.

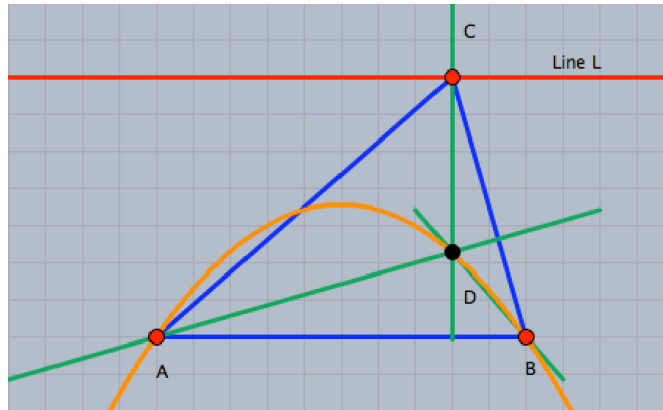


3. Move the point C along the straight line L , which is parallel to $\overline{AB}$, and observe the point D (the heights' intersection point). Can you expect the locus of the point D , while the point C moves along the straight line L ? In other words, which curve does the point D trace, while the point C moves along the straight line L ? Make a conjecture?
4. The locus of the point D can be automatically generated using tracing locus facilities of Cinderella. For doing so, switch to “**Create a Locus**” mode by choosing the menu item “**Modes/Special/Loci**” or by clicking the button 

²Remark: The heights of a triangle are concurrent. The automatic proving facilities of Cinderella can be used to show that, while you are generating the intersection point D .



For example: the situation can be elaborated to discover and generate the locus of the medians intersection point of a triangle when a triangle vertex moves along a straight line. In this case, as shown in the figure below, the locus of the medians intersection point is a straight line, which is parallel to the road straight line.



Handout 8

Midpoints of the Sides of a Quadrilateral

Student Activity

Use Cinderella to construct any quadrilateral. Construct the midpoints of each side. Connect the midpoints of the adjacent sides.

Make a conjecture about the obtained figure. i.e., What is the figure formed by joining the midpoints of the adjacent sides of a quadrilateral?

Use Cinderella objects and tools to investigate the conjecture.

State your conjecture in the form of mathematical theorems.

Can you figure out a mathematical proof for this conjecture?

Can you pose some follow-up problems related to the original problem?

Handout 9

Angular Bisectors of a Parallelogram

Student Activity

Use Cinderella to construct a parallelogram. Construct the angular bisectors of the interior angles of the parallelogram. Generate the four intersection points of the angular bisectors.

Make a conjecture about the obtained figure. i.e., What is the figure formed by joining the four intersection points the angular bisectors of the interior angles of parallelogram?

Use Cinderella objects and tools to investigate the conjecture.

State your conjecture in the form of mathematical theorems.

Can you figure out a mathematical proof for this conjecture?

Can you pose some follow-up problems related to the original problem?

Handout 10

Constructing a Parallelogram

Student Activity

1. Write down as many generalizations (theorems, definitions, properties, and corollaries) as you can that are related to the parallelogram.

For example: A parallelogram is a quadrilateral, in which each two opposite sides are parallel.

(We shall want you to discuss your responses with the rest of the group, so please write them on OHP transparency).

2. Think about which of these generalizations can be used to construct a parallelogram using Cinderella. Come up with as many different and various methods as you can to construct a parallelogram using Cinderella. In each construction method, you should:

- List the construction algorithm.
- Explain the theoretical background of the construction algorithm.
- Justify the construction algorithm order.

3. How can the different methods of construction be classified?

Classification criteria:

Handout 11

Constructing a Rhombus

Student Activity

1. Write down as many generalizations (theorems, definitions, properties, and corollaries) as you can that are related to the rhombus.

For example: A rhombus is a parallelogram, in which two adjacent sides are equal in length.

(We shall want you to discuss your responses with the rest of the group, so please write them on OHP transparency).

2. Think about which of these generalizations can be used to construct a rhombus using Cinderella. Come up with as many different and various methods as you can to construct a rhombus using Cinderella. In each construction method, you should:

- List the construction algorithm.
- Explain the theoretical background of the construction algorithm.
- Justify the construction algorithm order.

3. How can the different methods of construction be classified?

Classification criteria:

Handout 12

Constructing a Rectangle

Student Activity

1. Write down as many generalizations (theorems, definitions, properties, and corollaries) as you can that are related to the rectangle.

For example: A rectangle is a parallelogram with a right angle.

(We shall want you to discuss your responses with the rest of the group, so please write them on OHP transparency).

2. Think about which of these generalizations can be used to construct a rectangle using Cinderella. Come up with as many different and various methods as you can to construct a rectangle using Cinderella. In each construction method, you should:

- List the construction algorithm.
- Explain the theoretical background of the construction algorithm.
- Justify the construction algorithm order.

3. How can the different methods of construction be classified?

Classification criteria:

Handout 13

Constructing a Square

Student Activity

1. Write down as many generalizations (theorems, definitions, properties, and corollaries) as you can that are related to the square.

For example: A square is a parallelogram with a right angle and two adjacent sides equal in length.

(We shall want you to discuss your responses with the rest of the group, so please write them on OHP transparency).

2. Think about which of these generalizations can be used to construct a square using Cinderella. Come up with as many different and various methods as you can to construct a square using Cinderella. In each construction method, you should:
 - List the construction algorithm.
 - Explain the theoretical background of the construction algorithm.
 - Justify the construction algorithm order.

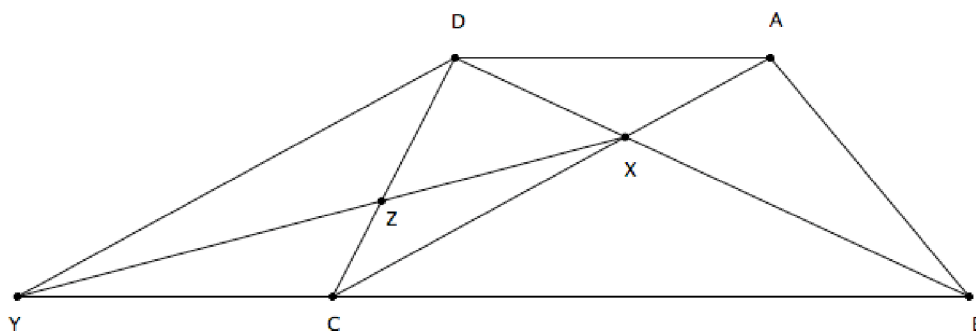
3. How can the different methods of construction be classified?

Classification criteria:

Handout 14

Posing Geometric Problems

Student Activity



In the figure above, $ABCD$ is a trapezium whose parallel sides are $\overline{AD}$ and $\overline{BC}$. X is the point where the diagonals $\overline{AC}$ and $\overline{BD}$ intersect. $\overrightarrow{DY}$ is drawn so that $\overrightarrow{DY} \parallel \overrightarrow{AC}$ and cuts $\overline{BC}$ at Y .

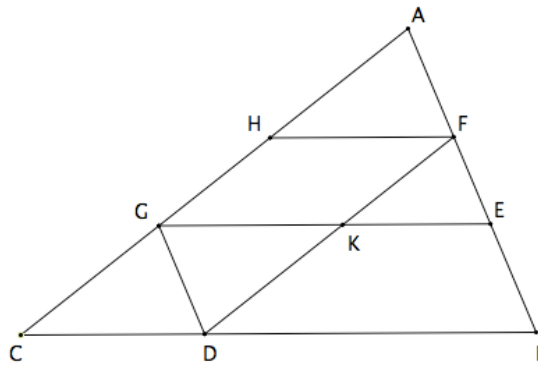
Use different facilities of Cinderella – constructing, dragging, measuring, and calculating facilities – to **pose** as many various and different **geometric problems** as possible, which could be answered either in direct or indirect ways based on the given information.

Handout 15

Finding Geometric Relationships

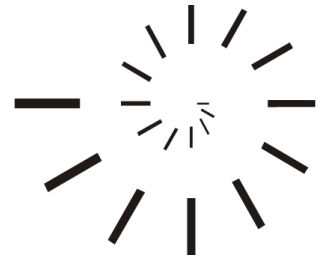
Student Activity

In the following figure, $AF = BE$, $\overline{FH} \parallel \overline{EG} \parallel \overline{BC}$ and $\overline{GD} \parallel \overline{AB}$.



Use different facilities of Cinderella – constructing, dragging, measuring, and calculating facilities – to **find** as many various and different **geometric relationships** as possible, which could be deduced in direct or indirect ways based on the given information.

Appendix B
Student's Handouts
(German Version)



Das Vorgeschlagene Enrichment Programm

Handouts für Studierende

Erstellt von

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Handout 1

Lernziele des Enrichment Programms

Sehr geehrte Studentin,

sehr geehrter Student,

nachfolgend finden Sie die einzelnen Lernziele aufgelistet, die Sie in Cinderella mit Hilfe des Enrichment Programms erreichen können. Bitte lesen Sie diese durch, bevor Sie die Anleitung erhalten.

- Lernziel 1 Dynamische Figuren konstruieren.
- Lernziel 2 Viele Konstruktionsmethoden entwerfen/entwickeln, um eine bestimmte Figur zu konstruieren.
- Lernziel 3 Viele verschiedene und unterschiedliche Konstruktionsmethoden entwerfen/entwickeln, um eine bestimmte Figur zu konstruieren.
- Lernziel 4 Neuartige und ungewöhnliche Konstruktionsmethoden entwerfen/entwickeln, um eine bestimmte Figur zu konstruieren.
- Lernziel 5 Viele sachbezogene Antworten (Ideen, Lösungen, Beweise, Hypothesen, neu formulierte Aufgabestellungen) zu einer geometrischen Aufgabe oder Situation entwickeln.
- Lernziel 6 Viele verschiedene und unterschiedliche sachbezogene Antworten (Ideen, Lösungen, Beweise, Hypothesen, neu formulierte Aufgabestellungen) zu einer geometrischen Aufgabe oder Situation entwickeln.
- Lernziel 7 Viele ungewöhnliche, einzigartige, raffinierte Antworten oder Ergebnisse zu einer geometrischen Aufgabe oder Situation entwickeln.
- Lernziel 8 Neue Hypothesen und Beziehungen zu einer gegebenen Aufgabe oder Situation aufstellen, indem Sie auf Ihre Erfahrungen zurückgreifen.

- Lernziel 9 Die aufgestellten Hypothesen und Beziehungen in verschiedenen Situationen und mittels unterschiedlicher Methoden untersuchen.
- Lernziel 10 Viele verschiedene und unterschiedliche Beweise zu einer geometrischen Aufgabe oder Situation finden.
- Lernziel 11 Viele weiterführende Aufgabenstellungen durch die Neudefinition (Modifikation, Überarbeitung, Erweiterung, Abwandlung) einer geometrischen Aufgabe oder Situation finden.
- Lernziel 12 Unterschiedliche Lernaspekte der Geometrie (Konzepte, Verallgemeinerungen und Kompetenzen) bei der Lösung einer geometrischen Aufgabe oder Situation anwenden.

Handout 2

Starten und Erkunden von Cinderella

In dieser Sitzung werden Sie

- lernen, Cinderella zu starten,
- das Cinderella Fenster kennen lernen,
- die Cinderella Menüs kennen lernen.

Aufgabe: Cinderella starten

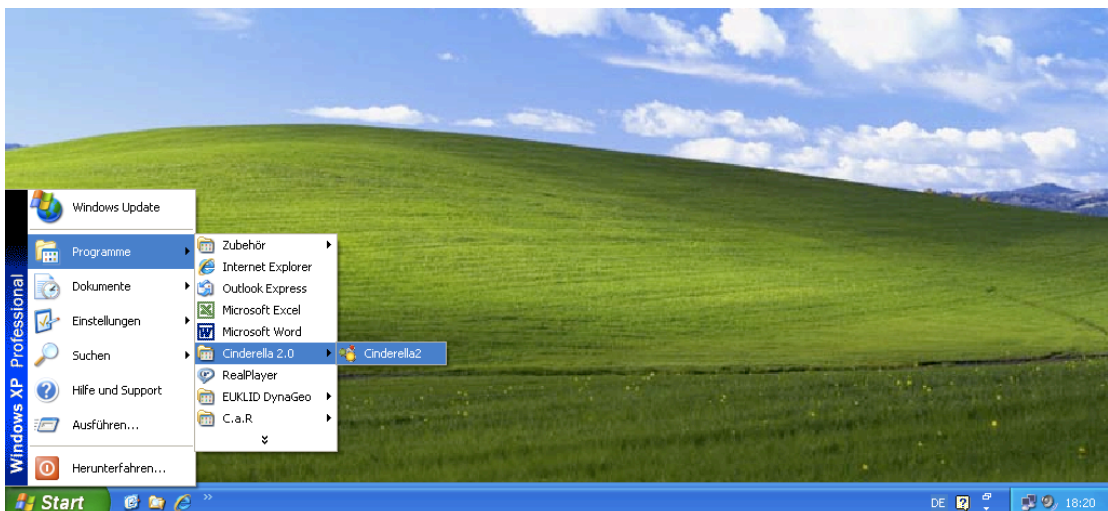
Um Cinderella zu starten, hängt es davon ab, wo der Cinderella-Ordner gespeichert ist.

Es gibt drei Möglichkeiten:

- Wenn auf Ihrem Desktop eine Verknüpfung Cinderella.2 vorhanden ist, doppelklicken Sie auf das folgende Symbol.



- Klicken Sie auf die Schaltfläche **Start** auf der Taskleiste. Klicken Sie auf **Programme**. Klicken Sie auf **Cinderella. 2**.



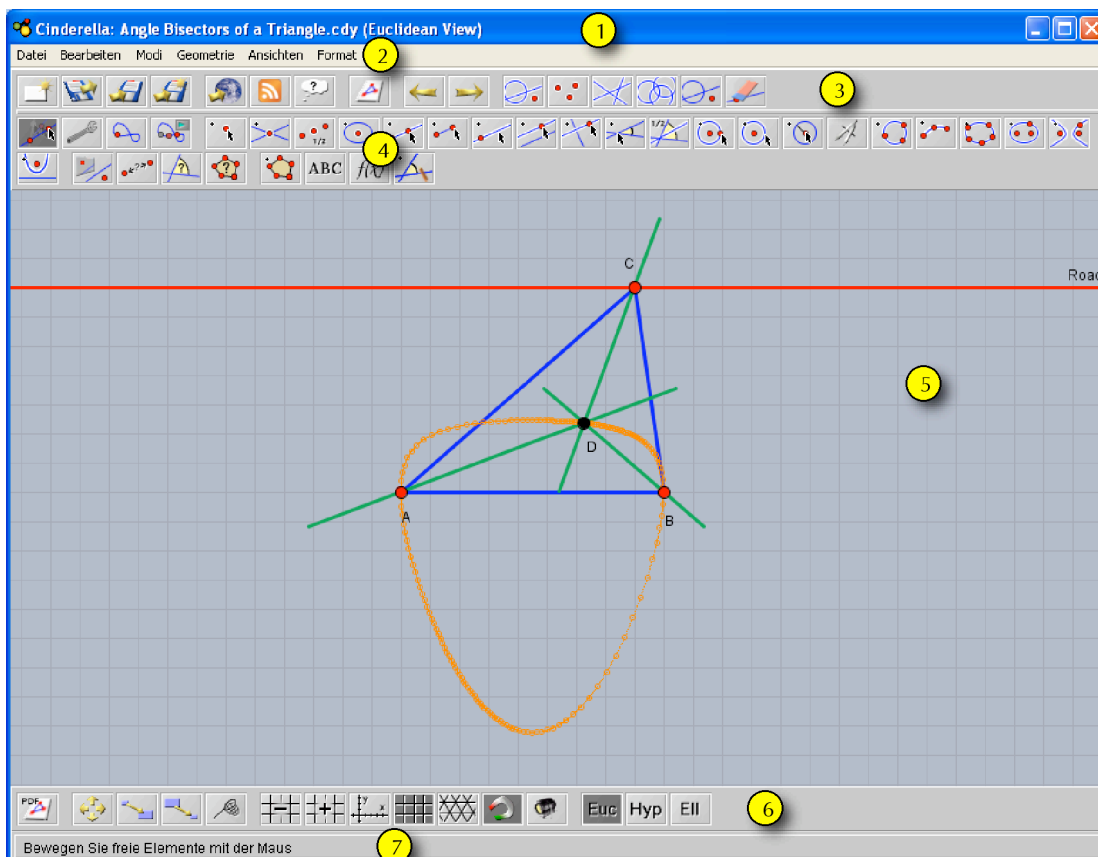
- Doppelklicken Sie auf eine von Cinderella erstellte Datei. Diese Dateien haben in der Regel die Endung „.cdy“ und haben ein ähnliches Symbol wie das folgende.



Nach einer Weile wird Cinderella gestartet.

Das Cinderella-Fenster

Nach dem Start von Cinderella sehen Sie das Fenster „**Unnamed (Euclidean View)**“. Es sieht ähnlich aus wie die Abbildung unten. Das Hauptfenster von Cinderella besteht aus sieben Hauptelementen. Hier sind die wichtigsten Elemente (von oben nach unten gesehen):



1. Die Titelleiste

Hier finden Sie den Namen des Programms – **Cinderella**, den Namen der aktuellen Konstruktions-Datei und der entsprechenden Ansicht. Wenn die Konstruktions-Datei noch keinen Namen hat, steht dort so etwas wie „**Unnamed (Euclidean View)**“.

2. Die Menüleiste

Sie enthält die Menüs mit den Befehlen, mit denen Cinderella bedient wird. Die sieben Menüs sind: **Datei**, **Bearbeiten**, **Modi**, **Geometrie**, **Ansichten**, **Format** und **Hilfe**. Klicken Sie auf ein Menü, um es zu öffnen. Wählen Sie einen Befehl aus, indem Sie darauf klicken.

3. Die Standard-Werkzeugleiste für allgemeine Aktionen

Sie enthält Schaltflächen für allgemeine Befehle: **Neu**, **Öffnen**, **Speichern**, **Drucken**, **HTML erzeugen**, **Rückgängig**, **Wiederholen** und **Markierungswerkzeuge**.

4. Die Werkzeugleiste mit geometrischen Modi

Diese Werkzeugleiste enthält Schaltflächen für alle geometrischen Konstruktionsmodi, von Punkt hinzufügen bis hin zu Ortskurve definieren.

5. Der Zeichenbereich

Dies ist der Bereich, in dem Sie geometrische Konstruktionen wie Punkte, Geraden und Kreise durchführen können. Hier können Sie auch geometrische Erkundungen durchführen, indem Sie den Zugmodus „**Elemente bewegen**“ benutzen.

6. Die Werkzeugleiste mit speziellen Aktionen für die aktuelle Ansicht

Diese Werkzeugleiste enthält Schaltflächen für Aktionen für spezifische Ansichten, wie „Vergrößern und Verkleinern“, „Zeichenblatt verschieben“, „Koordinatenachsen einzeichnen“, „Kästchen einzeichnen“, „Dreiecksgitter einzeichnen“, „versteckte Elemente anzeigen“ und die Auswahlmöglichkeit zwischen den drei Ansichtsmodi: **euklidische**, **hyperbolische** und **elliptische** Geometrie.

7. Die Meldungszeile

In dieser Leiste teilt Ihnen Cinderella mit, in welchem Modus Sie sich gerade befinden und welche Aktionen Sie gerade durchführen sollten.

Die Cinderella-Menüs

Die Cinderella-Menüleiste besteht aus 7 Hauptmenüs: **Datei**, **Bearbeiten**, **Modi**, **Geometrie**, **Ansichten**, **Format** und **Hilfe**. Es sollte hier erwähnt werden, dass Sie auf alle Funktionen und Werkzeuge von Cinderella über die Hauptmenüs zugreifen können, während die Werkzeugleisten nur die wesentlichsten Befehle beinhalten.

1. Datei

Das Menü **Datei** enthält eine Liste von datei-bezogenen Befehlen: Der Befehl **Neue Konstruktion** legt eine neue (leere) Datei an, der Befehl **Öffnen** öffnet oder lädt eine zuvor gespeicherte Datei, der Befehl **Speichern** speichert die aktuelle Datei, der Befehl **Fenster schließen** schließt die aktuelle Datei usw. Beachten Sie die Tastenkombinationen neben einigen der Befehle im Menü **Datei**. Zum Beispiel

Neue Konstruktion	Ctrl+N
Öffnen	Ctrl+O
Benutzte Konstruktionen	▶
Speichern	Ctrl+S
Speichern als	Ctrl+Shift+S
Fenster schließen	Ctrl+W
Scripte erstellen	Ctrl+9
Übungsaufgabe erstellen	Ctrl+8
HTML-Export	Ctrl+E
Graphik exportieren nach	▶
Drucken	Ctrl+P
Toolbars auswählen	
Beenden	Ctrl+Q

können Sie, wenn Sie eine Cinderella-Datei speichern wollen, die Tastenkombination **[Strg]+[S]** verwenden: Drücken und halten Sie die Taste Strg und drücken Sie die Taste S, während Sie Strg gedrückt halten.

2. Bearbeiten

Dieses Menü enthält Werkzeuge für das **Rückgängigmachen, Wiederholen** und für das **Kopieren und Einfügen** von Teilen der Konstruktion. Darüber hinaus können Sie unter diesem Menü mit dem Befehl **Werkzeug erzeugen** Makro-Konstruktionen erstellen. Es enthält auch mehrere Auswahl-Werkzeuge, mit denen Sie alles, nur Punkte, nur Geraden oder nur Kegelschnitte auswählen oder die Auswahl aufheben können. Hier erhalten Sie auch Zugang zum Inspektor, einem Werkzeug, das zur Bearbeitung der geometrischen Elemente innerhalb ihrer Konstruktion verwendet werden kann.

Rückgängig	Ctrl+Z
Wiederholen	Ctrl+Y
Kopieren	Ctrl+C
Einfügen	Ctrl+V
Werkzeug erzeugen	Ctrl+Shift+N
Elemente löschen	Ctrl+Delete
Alles auswählen	Ctrl+A
Punkte auswählen	Ctrl+Shift+P
Geraden auswählen	Ctrl+Shift+L
Kegelschnitte auswählen	Ctrl+Shift+C
Nichts auswählen	Ctrl+Shift+A
Punkte Fangen	
Informationen einblenden	Ctrl+I

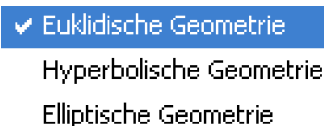
3. Modi

Das Menü **Modi** enthält eine Sammlung von Konstruktions-Werkzeugen für alle geometrischen Konstruktionen, vom Hinzufügen eines Punktes bis zum Generieren von Ortskurven. Die meisten dieser Werkzeuge sind in der Werkzeugleiste der geometrischen Modi zu finden.

✓ Bewegen	Ctrl+M
Punkt umdefinieren	
Punkt	
Gerade	
Kreis	
Kegelschnitt	
Grafik	
Messen	
Polygon	
Transformation	
Spezial	
Simulation	
API	
	Freie Gerade
	Strecke
	durch zwei Punkte Ctrl+L
	Horizontale Gerade
	Vertikale Gerade
	durch einen Punkt
	Parallele
	Senkrechte
	fester Winkel
	Winkelhalbierende
	Punkt-Polare

4. Geometrie

Eines der wichtigsten Merkmale von Cinderella ist, dass es verschiedene Arten von Geometrien unterstützt. Hier können Sie wählen, mit welcher Art der Geometrie



Sie arbeiten möchten (die Standardeinstellung ist die **euklidische Geometrie**, aber Sie können auch mit der **hyperbolischen Geometrie** oder der **elliptischen Geometrie** arbeiten). Es sollte hier erwähnt werden, dass die Umstellung auf eine andere Art von Geometrie Auswirkungen auf das Verhalten der geometrischen Elemente hat, die Sie konstruieren.

5. Ansichten

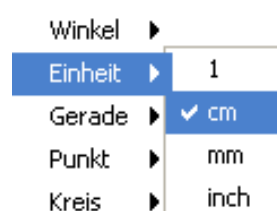
Dieses Menü ermöglicht es Ihnen, Ihre Konstruktionen in verschiedenen Ansichten zu öffnen (die Standardeinstellung ist die **euklidische Zeichenoberfläche**, aber Sie können Ihre Konstruk-

Euklidische Zeichenoberfläche	Ctrl+1
Hyperbolische Zeichenoberfläche	Ctrl+2
Sphärische Zeichenoberfläche	Ctrl+3
Polare Euklidische Zeichenoberfläche	Ctrl+Shift+1
Polare Sphärische Zeichenoberfläche	Ctrl+Shift+3
Konstruktionsbeschreibung	Ctrl+4
Informationsfenster	Ctrl+5

tion auch in **hyperbolischer Zeichenoberfläche**, **sphärischer Zeichenoberfläche**, **polar-euklidischer Zeichenoberfläche** und **polar-sphärischer Zeichenoberfläche** öffnen). Darüber hinaus können Sie auch die **Konstruktionsbeschreibung** öffnen (ein spezielles Cinderella-Fenster, das eine detaillierte Beschreibung für jedes Element innerhalb Ihrer Konstruktion anzeigt) sowie das **Informationsfenster**, das Fakten über Ihre Konstruktion anzeigt und darüber informiert.

6. Format

Im Menü **Format** können Sie das Standard-Format wählen, in dem Einheiten und Gleichungen angezeigt werden sollen.



7. Hilfe

Das Menü Hilfe enthält die Werkzeuge „**Hilfe zum Modus**“, „**Online Dokumentation**“ und Verweise auf die Online-Hilfe, sowie Updates und Lizenzfragen.

Hilfe zum Modus	Ctrl+F1
Online Dokumentation	Ctrl+Shift+F1
Über Cinderella	
Cinderella Homepage	
Springer Homepage	
Lizenz eingeben...	

Handout 3

Grundlegende Befehle und Konstruktionen von Cinderella

In dieser Sitzung erfahren Sie mehr über:

- Grundlegende Befehle von Cinderella.
- Grundlegende Konstruktionen (Modi) von Cinderella

Grundlegende Befehle von Cinderella

Öffnen von neuen Konstruktions-Fenstern

Um ein neues Konstruktions-Fenster in Cinderella zu öffnen, führen Sie einen der folgenden Schritte aus:

- Aus dem Menü **Datei** wählen Sie **Neue Konstruktion**.
- Drücken Sie die Tastenkombination **[Strg]+[N]**.
- Klicken Sie auf die Schaltfläche  **Neue Konstruktion** in der Standard-Werkzeugleiste.

Der Befehl **Neue Konstruktion** öffnet ein neues, leeres Cinderella Fenster in Euklidischer Ansicht für das Erstellen und Ändern von geometrischen Konstruktionen. Alle Programmeinstellungen werden in den Ursprungszustand versetzt.

Öffnen / Laden von gespeicherten Dateien in Cinderella

Zum Öffnen / Laden einer gespeicherten Datei in Cinderella führen Sie einen der folgenden Schritte aus:

- Aus dem Menü **Datei** wählen Sie **Öffnen**.
- Drücken Sie die Tastenkombination **[Strg]+[O]**.

- Klicken Sie auf die Schaltfläche  **Konstruktion laden** in der Standard-Werkzeugleiste.

Nach Ausführung eines der oben genannten Schritte öffnet sich ein Explorer-Fenster zum Auswählen der Cinderella-Datei.

Konstruktion Speichern

Zum Speichern der aktuellen Konstruktion führen Sie einen der folgenden Schritte aus:

- Aus dem Menü **Datei** wählen Sie **Speichern**.
- Drücken Sie die Tastenkombination **[Strg]+[S]**.
- Klicken Sie auf die Schaltfläche  **Konstruktion Speichern** auf der Standard-Werkzeugleiste.

Nach Ausführung eines der oben genannten Schritte öffnet sich ein Explorer-Fenster, in dem Sie die aktuelle Cinderella-Konstruktion unter einem selbst bestimmten Namen speichern können.

Konstruktion unter neuem Namen speichern

Zum Speichern der aktuellen Konstruktion unter einem neuen Namen führen Sie einen der folgenden Schritte aus:

- Aus dem Menü **Datei** wählen Sie **Speichern als**.
- Drücken Sie die Tastenkombination **[Strg]+[Umschalt]+[S]**.
- Klicken Sie auf die Schaltfläche  **Konstruktion unter neuem Namen speichern** auf der Standard-Werkzeugleiste.

Nach Ausführung eines der oben genannten Schritte öffnet sich ein Explorer-Fenster, in dem Sie die aktuelle Cinderella-Konstruktion unter einem neuen Namen speichern können.

HTML erzeugen

Um eine interaktive Web-Seite der aktuellen Konstruktion zu erstellen, führen Sie einen der folgenden Schritte aus:

- Aus dem Men **Datei** wählen Sie **HTML-Export**.
- Drücken Sie die Tastenkombination **[Strg]+[E]**.
- Klicken Sie auf die Schaltfläche  **Interaktive WWW-Seite erzeugen** auf der Standard-Werkzeugleiste.

Nach Ausführung eines der oben genannten Schritte öffnet sich ein Explorer-Fenster, in dem Sie die aktuelle Cinderella-Konstruktion als Webseite speichern können.

Zwei **Schritte** müssen bei der Erstellung von interaktiven Web-Seiten beachtet werden:

1. Die Konstruktion muss zunächst als Cinderella-Datei (Endung „**cdy**“ gespeichert werden, bevor Sie als HTML-Datei exportiert werden kann.
2. Die HTML-Datei muss in dem gleichen Verzeichnis gespeichert werden, in dem zuvor die Konstruktions-Datei gespeichert wurde.

Rückgängig

Mit dem Befehl **Rückgängig** können Sie eine oder mehrere Aktionen zurücksetzen. Die Anzahl der rückgängig zu machenden Schritte ist dabei unbegrenzt. Mit jedem Klick auf die **Rückgängig** Schaltfläche gelangen Sie jeweils einen Konstruktionsschritt zurück.

Folgende Aktionen können rückgängig gemacht werden:

- Konstruktionsschritte
- Bewegungen
- Änderungen im Aussehen
- Zoomen, Verschieben und Drehen von Ansichten

- Löschen von Elementen

Wiederholen



Mit dem Befehl **Wiederholen** können Sie eine oder mehrere Aktionen erneut durchführen. Die Anzahl der Wiederholungsschritte ist unbegrenzt.

Löschen



Der Befehl **Löschen** dient zum Löschen aktuell ausgewählter Elemente und anderer Elemente die von ihnen abhängen.

Markierungswerkzeuge

Unter dem Menü „**Bearbeiten**“ gibt es verschiedene Möglichkeiten, um Elemente, Punkte, Geraden oder Kegelschnitte und Kreise auszuwählen bzw. zu markieren. Alle diese Aktionen sind auch über entsprechende Schaltflächen in der Standard Werkzeugleiste abrufbar.

Alles markieren



Es wird verwendet, um alle geometrischen Elemente in der Konstruktion auszuwählen.

Punkte markieren



Es wird verwendet, um alle Punkte in der Konstruktion auszuwählen.

Geraden markieren



Es wird verwendet, um alle Geraden in der Konstruktion auszuwählen.

Kegelschnitte markieren



Es wird verwendet, um alle Kegelschnitte und Kreise in der Konstruktion auszuwählen.

Markierung aufheben



Es wird verwendet, um die aktuelle Auswahl in der Konstruktion aufzuheben.

Geometrische Werkzeuge von Cinderella (Modi)

Elemente bewegen (Zugmodus)



Der Zugmodus gehört zu den wichtigsten Eigenschaften, die Cinderella ausmachen – die Möglichkeit, freie Elemente zu bewegen, wobei sich die gesamte Konstruktion entsprechend mitbewegt ohne die Konfiguration zu verändern. Es stellt eine wichtige Besonderheit dar, mit der sich geometrische Relation entdecken lassen können.

Generell ermöglicht dieser Modus zwei Funktionen: die Auswahl und das Bewegen von Elementen auf dem Konstruktionsbereich.

Es gibt zwei Punktarten: **freie/bewegliche Punkte** und **fixierte Punkte**.

Freie Punkte: Sie sind nicht von anderen Elementen der Konstruktion abhängig und können frei auf dem Konstruktionsbereich bewegt werden. Alle von ihnen abhängige Elemente bewegen sich entsprechend mit.

Fixierte Punkte: Wenn Punkte von den Positionen anderer freier Punkte abhängig sind, lassen sie sich nicht bewegen.

Im Konstruktionsbereich lassen sich die beiden Punktarten an ihrer Erscheinung unterscheiden: Fixierte Punkte sind dunkler dargestellt.

Punkte bewegen

Um einen freien/beweglichen Punkt zu bewegen, fahren sie mit der Maus über den entsprechenden Punkt, drücken die linke Maustaste und bewegen die Maus während sie sie gedrückt lassen. Der Punkt folgt dem Mauszeiger.

Elemente auswählen

Eine zweite Funktion des Zugmodus ist die Auswahl eines oder mehrerer Elemente auf dem Konstruktionsbereich. Um ein Element auszuwählen klicken sie einfach auf das gewünschte Element. Mehrere Elemente lassen sich mit gedrückter Shift-Taste auswählen.

Die ausgewählten Elemente werden in allen Ansichten markiert dargestellt, was sie leicht erkennbar macht. Für folgende Zwecke müssen sie Elemente auswählen:

- Um ein geometrisches Element mit dem Fenster Elementeigenschaften, das sie über das Menü „**Bearbeiten/Informationen einblenden**“ erreichen, individuell zu gestalten.
- Zum Löschen von ausgewählten Elementen mit dem Werkzeug „**Löschen**“.
- Um mehrere ausgewählte Elemente gleichzeitig zu bewegen.

Punkt hinzufügen



Dieser Modus konstruiert einen neuen Punkt mit einer Klicken-Ziehen-Loslassen-Aktion mit der Maus, die sich wie folgt beschrieben lässt:

- **Klicken** der linken Maustaste erzeugt den Punkt.
- **Ziehen** der Maus, während die linke Maustaste gedrückt ist, ändert die Position und die Definition des Punktes.
- **Loslassen** der Maustaste definiert den Punkt.

Ein Punkt ist danach definiert, an welcher Stelle sich der Mauszeiger befindet, wenn die Maustaste losgelassen wird. Wenn die Maus auf ...

- ... keinem Element war, dann wird ein freier Basispunkt erzeugt, der im Bewege-Modus frei bewegbar ist.
- ... einem Element (Linie, Kreis, Kegelschnitt) war, dann ist ein Objektpunkt entstanden, der entlang des Element, zu dem es konstruiert worden ist, im Bewege-Modus beweglich ist.
- ... einer Kreuzung von zwei Elementen (Linie, Kreis, Kegelschnitt), dann ist der neue Punkt als fixierter Schnittpunkt entstanden der sich nicht direkt bewegen lässt.
- ... einem Punkt war, dann wird kein neuer Punkt gezeichnet.

Zwei Punkte mit Verbindungsgrade



Dieser Modus konstruiert zwei Punkte und eine Gerade die durch beide Punkte verläuft. Man erzeugt eine Gerade indem eine Klicken-Ziehen-Loslassen-Aktion durchgeführt wird:

- **Klicken** der linken Maustaste erzeugt den ersten Punkt. Der Punkt ist danach definiert, wo sich die Maus auf dem Konstruktionsbereich befindet. (Beschrieben beim Modus „**Punkt hinzufügen**“)
- **Ziehen** der Maus, während die linke Maustaste gedrückt ist, generiert die Gerade und den zweiten Punkt. Der zweite Punkt wird ebenfalls nach der Position der Maus definiert.
- **Loslassen** der Maustaste zeichnet den zweiten Punkt und man hat eine Gerade, die beide Punkte verbindet, als Ergebnis.

Gerade durch einen Punkt



Dieser Modus konstruiert eine Gerade durch einen Punkt, so dass die Steigung der Geraden konstant bleibt, wenn der Punkt bewegt wird. Im Bewege-Modus kann allerdings auch die Steigung der Geraden verändert werden. Eine Gerade durch einen Punkt wird folgendermaßen konstruiert:

- **Klicken** der linken Maustaste erzeugt den Punkt.
- **Ziehen** der Maus, während die linke Maustaste gedrückt ist, generiert die Gerade.
- **Loslassen** der Maustaste beendet die Konstruktion.

Parallele definieren



Dieser Modus konstruiert eine parallele Gerade durch einen Punkt zu einer gegebenen Geraden. Mit folgender Klicken-Ziehen-Loslassen-Aktion wird eine Parallele erzeugt:

- **Bewegen** die Maus über die Gerade, zu der Sie die Parallele konstruieren möchten. Klicken Sie dann auf die linke Maustaste. Diese Aktion erzeugt eine parallele Gerade und einen Punkt, durch den die Parallele verläuft.
- **Ziehen** der Maus, während die linke Maustaste gedrückt ist. Die Parallele und der erzeugte Punkt bewegen sich zu der gewünschten Position.
- **Loslassen** der Maustaste zeichnet den zweiten Punkt und die Parallele, was die Konstruktion beendet.

Senkrechte definieren



Dieser Modus konstruiert eine senkrechte Gerade durch einen Punkt zu einer gegebenen Geraden. Mit folgender Klicken-Ziehen-Loslassen-Aktion wird eine Senkrechte erzeugt:

- **Bewegen** Sie die Maus über die Gerade, zu der sie die Senkrechte konstruieren möchten. **Klicken** Sie dann auf die linke Maustaste. Diese Aktion erzeugt eine senkrechte Gerade und einen Punkt, durch den die Senkrechte verläuft.
- **Ziehen** der Maus, während die linke Maustaste gedrückt ist. Die Senkrechte und der erzeugte Punkt bewegen sich mit zu der gewünschten Position.
- **Loslassen** der Maustaste zeichnet den zweiten Punkt und die Senkrechte, was die Konstruktion beendet.

Zwei Punkte und ein Kreis



Dieser Modus konstruiert einen Kreis über einen Mittelpunkt und einem Punkt auf dem Kreisumfang. Mit folgender Klicken-Ziehen-Loslassen-Aktion wird ein Kreis erzeugt:

- **Klicken** der linken Maustaste erzeugt den Mittelpunkt.
- **Ziehen** der Maus, während die linke Maustaste gedrückt ist, generiert den Kreis und den Punkt auf dem Umfang.
- **Loslassen** der Maustaste platziert den Punkt auf dem Umfang und beendet die Konstruktion.

Mittelpunkt zweier Punkte



Dieser Modus konstruiert zwei Punkte und den Punkt, der die Strecke zwischen den beiden in zwei gleich große Abschnitte teilt. Mit folgender Klicken-Ziehen-Loslassen-Aktion wird ein Kreis erzeugt:

- **Klicken** der linken Maustaste erzeugt den ersten Punkt.
- **Ziehen** der Maus, während die linke Maustaste gedrückt ist, generiert den zweiten und den Mittelpunkt.

- **Loslassen** der Maustaste platziert den zweiten Punkt und beendet die Konstruktion.

Strecke zwischen zwei Punkten



Dieser Modus konstruiert zwei Punkt und eine Strecke, die die beiden Punkte verbindet. Mit folgender Klicken-Ziehen-Loslassen-Aktion wird eine Strecke erzeugt:

- **Klicken** der linken Maustaste erzeugt den ersten Punkt.
- **Ziehen** der Maus, während die linke Maustaste gedrückt ist, generiert den zweiten und die Strecke zwischen den beiden Punkten. Der zweite Punkt ist nach der Mausposition definiert.
- **Loslassen** der Maustaste platziert den zweiten Punkt und man hat die Strecke zwischen den beiden Punkten als Ergebnis.

Aufgaben

Verwenden sie Cinderella, um dynamische Konfigurationen der folgenden Figuren zu erzeugen:

- Dreieck
- Gleichseitiges Dreieck
- Gleichschenkliges Dreieck
- Rechtwinkliges Dreieck
- Viereck
- Trapez
- Parallelogramm
- Rechteck
- Raute/Rhombus

- Quadrat
- Kreis
- Tangente eines Kreises

Handout 4

Zugmodus und Messwerkzeuge von Cinderella

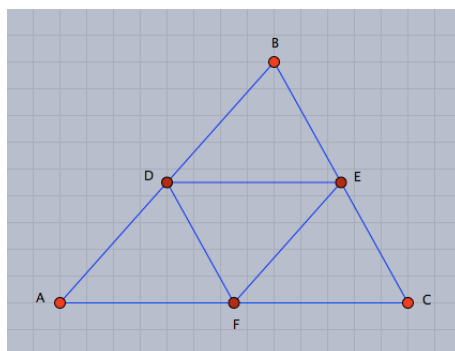
Cinderella beinhaltet eine breite Palette von Werkzeugen. Zwei dieser Werkzeuge sind der Zugmodus und die Messwerkzeuge. In dieser Aufgabe erleben wir den Zugmodus und die Messwerkzeuge von Cinderella, um neue geometrische Hypothesen aufzustellen und Untersuchungen damit durchzuführen.

Aufgaben

1. Starten Sie Cinderella oder öffnen Sie eine neue Cinderella Konstruktion.
2. Verwenden Sie Cinderella, um ein Dreieck zu konstruieren. (Klicken Sie auf das Symbol „**Punkt hinzufügen**“  in der Werkzeugleiste, um die Scheitelpunkte A , B , und C des Dreiecks zu konstruieren. Verwenden Sie dann das Werkzeug „**Strecke zwischen zwei Punkten**“  um die drei Punkte durch Klicken-Ziehen-Loslassen mit der Maus zu einem Dreieck ABC zu verbinden)
3. Konstruieren Sie die Mittelpunkte der Dreiecksseiten. (Klicken Sie auf „**Mittelpunkt zweier Punkte**“ . Durch Klicken-Ziehen-Loslassen werden die drei Strecken $\overline{AB}$, $\overline{BC}$, und $\overline{AC}$ geteilt. Es entstehen die Mittelpunkte D , E , und F).
4. Verbinden Sie die Mittelpunkte, um die vier Dreiecke $\triangle ADF$, $\triangle DBE$, $\triangle EDF$, und $\triangle CEF$ zu konstruieren. (Verwenden Sie das Werkzeug „**Strecke zwischen zwei Punkten**“  um die Mittelpunkte D , E , und F zu verbinden).

Bevor Sie nun weiter arbeiten, sollten Sie sicher gehen, dass Ihre Konstruktion folgendermaßen aussieht:

5. Wechseln Sie in den Modus „**Elemente bewegen**“ . Verwenden Sie den Zugmodus, um die freien Punkte (A , B oder C) zu bewegen. Schauen Sie die vier Dreiecke genauer an und stellen Sie Hypothesen über ihren Flächeninhalt auf. Oder anders gesagt: Was können Sie über die Flächeninhalte der vier Dreiecke sagen?



6. Durch Verwenden der Messwerkzeuge in Cinderella ist es möglich, die Fläche jedes einzelnen Dreiecks zu messen. Um dies zu tun, müssen wir zuerst die vier Dreiecke als Polygone definieren. Hierzu klicken Sie auf die Schaltfläche „**Polygon definieren**“ in der Werkzeugleiste  und definieren die vier Polygone: *Poly0*, *Poly1*, *Poly2*, und *Poly3* indem Sie nacheinander die Eckpunkte der jeweiligen Flächen anklicken. Danach wechseln Sie in den Modus „**Fläche messen**“  um die Fläche zu messen. Klicken Sie hierzu auf die innere Fläche jedes einzelnen Dreiecks (Polygons).
7. Verändern Sie das Dreieck $\triangle ABC$ durch Ziehen der Scheitelpunkte. Beobachten Sie dabei den Flächeninhalt der vier Dreiecke. Was können Sie über die Fläche der vier Dreiecke sagen, welche sie durch das Verbinden der Mittelpunkte der Dreiecksseiten erhalten haben? Schreiben Sie Ihre Hypothese in Form eines mathematischen Satzes.
8. Erstellen Sie einen mathematischen Beweis für ihre Hypothese.
9. Stellen Sie einige weiterführende Aufgaben im Zusammenhang mit der ursprünglichen Aufgabe auf.

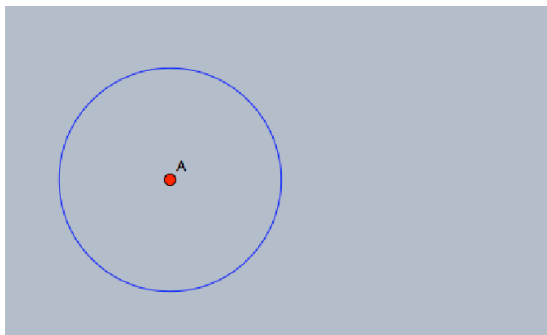
Handout 5

Der Automatische Beweiser von Cinderella

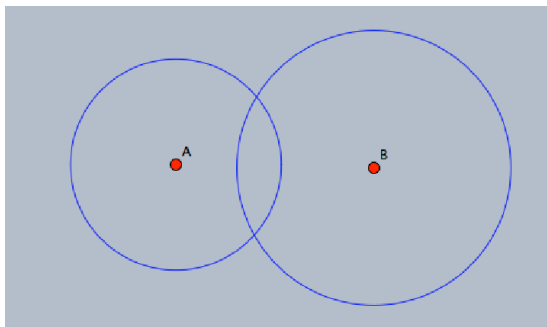
Cinderella besitzt einen Automatischen Beweiser, der die Richtigkeit von geometrischen Verknüpfungen direkt überprüfen kann. Sie werden in dieser Aufgabe den Beweiser einsetzen um zu prüfen, ob sich drei Punkte auf einer Linie befinden.

Aufgaben

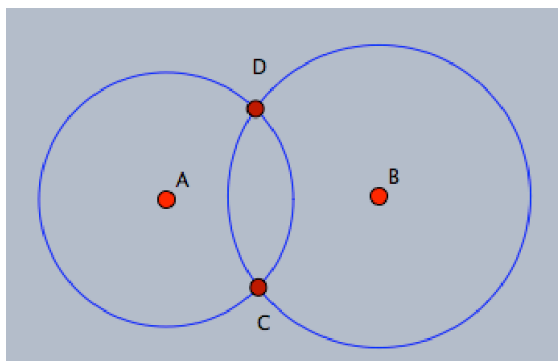
1. Starten Sie Cinderella oder öffnen Sie eine neue Konstruktion.
2. Wechseln Sie in den Modus „**Kreis um einen Punkt**“ über das Menü „**Mo-di/Kreis/Mittelpunkt + Radius**“ oder über das Symbol  in der Werkzeugleiste.
3. Bewegen Sie die Maus über den Konstruktionsbereich. Durch Klicken-Ziehen-Loslassen mit der Maus erhalten sie einen Kreis mit dem Mittelpunkt *A*.



4. Erzeugen Sie auf die gleiche Weise einen zweiten Kreis, so dass sich die beiden Kreise schneiden.

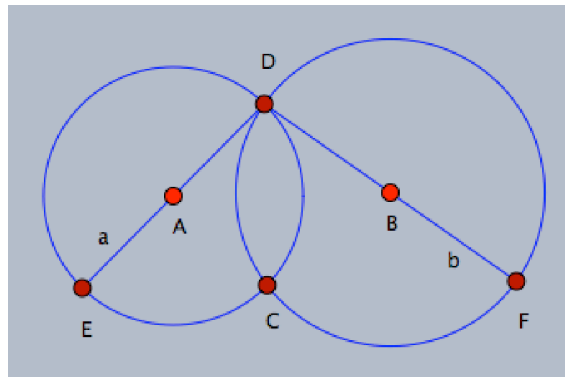


5. Um die Schnittpunkte zu erzeugen, wechseln sie in den Modus „**Schnittpunkte**“, den Sie über das Symbol  in der Werkzeugleiste aktivieren. Wählen Sie danach die beiden Kreise nacheinander aus, indem Sie auf sie klicken und erstellen Sie so die Schnittpunkte C und D . Prüfen Sie bitte bevor sie weiter fortfahren, ob Ihre Konstruktion wie die folgende Abbildung aussieht.



6. Wechseln Sie in den Modus „**Gerade durch zwei Punkte**“ indem Sie im Menü „**Modus/Gerade/Durch zwei Punkte**“ wählen, durch die Tastenkombination „**[Strg]+[L]**“ oder über das Symbol  in der Werkzeugleiste. Sie verbinden die beiden Punkte A und D durch Klicken-Ziehen-Loslassen der beiden Punkte.
7. Erzeugen Sie eine weitere Gerade durch die Punkte D und B .
8. Wechseln Sie in den Modus „**Schnittpunkte**“ über das Symbol  in der Werkzeugleiste. Markieren Sie nacheinander den Kreis A und die Gerade, um den Schnittpunkt E zu erhalten.
9. Bleiben Sie im gleichen Modus und wählen Sie den Kreis B und die Gerade b nacheinander aus, um den Punkt F zu erhalten.
10. Öffnen Sie die „**Elementeigenschaften**“ über das Menü „**Bearbeiten /Informationen einblenden**“.
11. Markieren Sie alle Geraden, indem Sie den Schalter  in der Werkzeugleiste betätigen.
12. Schneiden Sie die Geraden a und b , wenn Sie im Fenster der Elementeigenschaften den Schalter  betätigen. Nachdem die Geraden an den Endpunkten abge-

schnitten sind, sollte Ihre Konstruktion wie folgt aussehen.



13. Benutzen Sie den Bewege-Modus in Cinderella um die freien Elemente zu bewegen und beobachten Sie das Verhalten der drei Punkte F , C und E . Welche Aussagen lassen sich über die Kollinearität der drei Punkte machen? Liegen sie auf einer Gerade? Stellen Sie eine Hypothese auf und lassen Sie sie von Cinderella überprüfen.
14. Öffnen Sie zur Überprüfung der Hypothese über die Kollinearität das Informationsfenster, indem Sie im Menü den Pfad „**Ansichten/Informationsfenster**“ wählen oder indem Sie den Tastatur-Kurzbefehl „**[Strg]+[5]**“ ausführen. Es öffnet sich ein Konsolenfenster, worin ein automatisierter Bericht über die Anordnung der Konstruktion erscheinen wird. Wechseln Sie nun in den Modus „**Gerade durch zwei Punkte**“ über den Schalter , um eine Gerade, die die Punkte E und F verbindet, zu zeichnen, indem Sie auf F klicken und halten, die Maus auf E bewegen und dort die Maustaste loslassen. Beachten Sie die Mitteilung im Konsolenfenster, in der angegeben wird, dass der Punkt C auf der Verbindungsgeraden von E und F liegt.
15. Erstellen Sie so viele mathematische Beweise wie möglich, um die untersuchte Hypothese zu beweisen.
16. Nutzen Sie verschiedene Anordnungen, um möglichst viele neue Vermutungen über die Konstruktion zu finden. Verändern Sie die Situation, indem Sie Bedingungen weglassen, hinzufügen, erweitern, modifizieren oder anpassen.

Handout 6

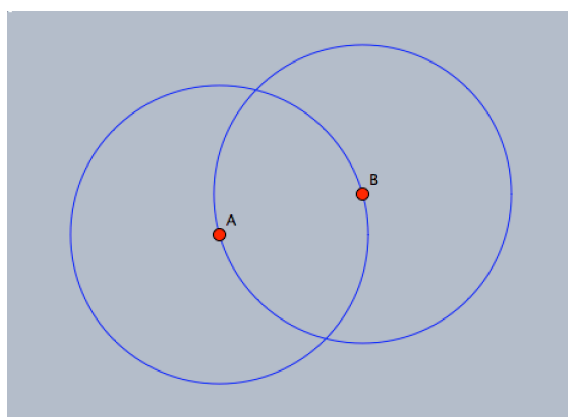
Entwicklung von Makros

Cinderella kann eine Sequenz von Konstruktionsbefehlen aufzeichnen und sie als neues Werkzeug speichern. Dies bezeichnet man als Makro-Konstruktion. Mit diesen Aufgaben erfahren Sie, wie Sie mit Makros geometrische Werkzeuge, wie Mittelsenkrechten oder Umkreismittelpunkte von Dreiecken, erzeugen können.

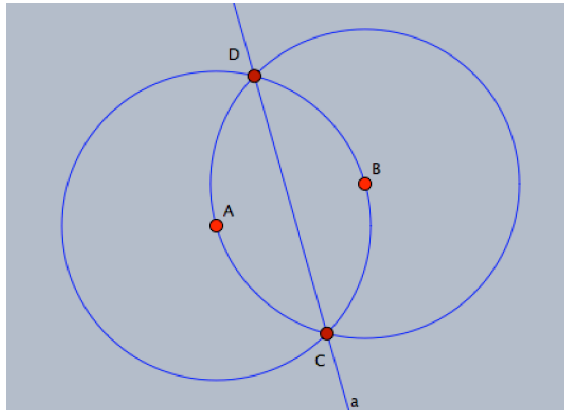
Aufgabe 1

Entwickeln eines Makros für Mittelsenkrechten

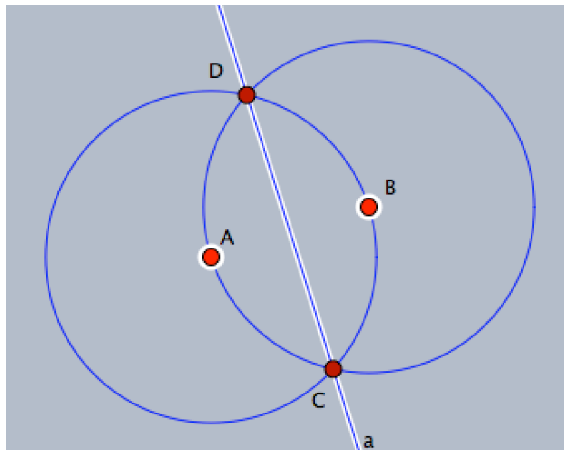
1. Starten Sie Cinderella oder öffnen Sie eine neue Konstruktion.
2. Wechseln Sie in den Modus „**Zwei Punkte und ein Kreis**“ über das Menü „**Modi/Kreis/Zwei Punkte**“ oder über Auswahl des Schalters  in der Werkzeugleiste. Mit diesem Modus erstellt man einen Kreis über zwei Punkte: einen Mittelpunkt und einen auf dem Umfang des Kreises.
3. Erstellen Sie durch zweimaliges Klicken-Ziehen-Loslassen mit der Maus zwei Punkte A und B und zwei Kreise, die A und B als Mittelpunkt und B und A als Punkt auf dem Kreis besitzen.



4. Erzeugen Sie die Schnittpunkte C und D der beiden Kreise, indem Sie den Modus „**Schnittpunkte**“ über den Schalter  betätigen. Wechseln Sie danach in den Modus „**Geraden**“  um die Mittelsenkrechte von A und B zu konstruieren.

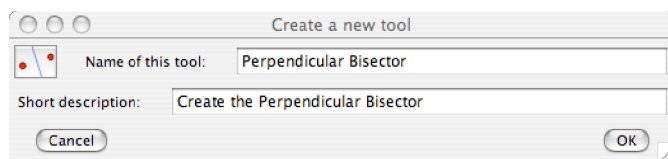


5. Um ein Makro zu definieren müssen Sie die Ausgangselemente – hier die beiden Punkte A und B – und die gewünschten abhängigen Elemente – hier die Gerade a – markieren. Wechseln Sie also in den Bewege-Modus und wählen Sie die Elemente gleichzeitig aus, indem Sie sie nacheinander mit gedrückter Shift-Taste auswählen.



6. Die drei ausgewählten Elemente, die beiden Punkte A und B und die Gerade a , können nun für die Erstellung eines Makros für Mittelsenkrechten verwendet werden. Wählen Sie dafür im Menü die Auswahl „**Bearbeiten/Eigenes Werkzeug erzeugen...**“ oder drücken Sie den Tastatur-Kurzbefehl „**Strg+Shift+N**“. Es öffnet sich ein neues Fenster, wo ein Name und eine Beschreibung für das neue Werkzeug verlangt wird. Geben Sie den Namen und die Beschreibung ein – „**Mittelsenkrechte**“ und „**Erstellen einer Mittelsenkrechten**“ sind Vorschläge für Namen und Beschreibung – und klicken Sie dann OK, um die Erstellung des Werkzeuges zu bestätigen. Das Symbol wird automatisch erstellt und ist auf der

Werkzeuggeste zu finden.

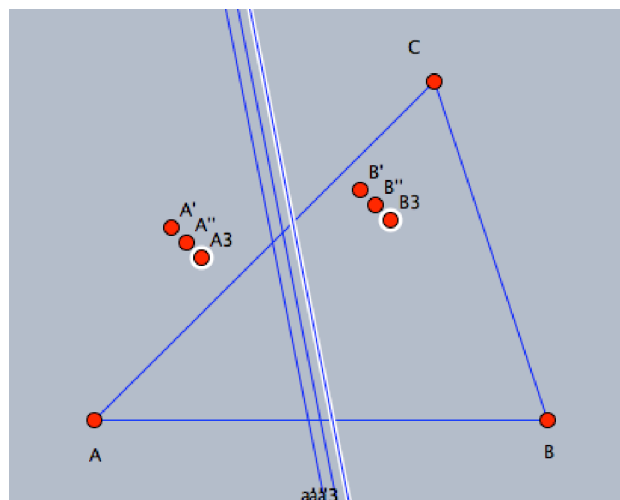


Aufgabe 2

Entwickeln eines Makros für den Umkreismittelpunkt eines Dreiecks

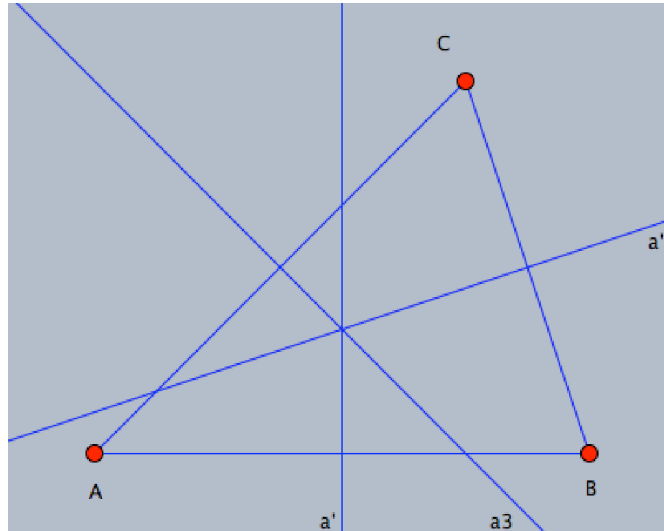
Sie werden in dieser Aufgabe dazu angeleitet, das Werkzeug für Mittelsenkrechten zum Konstruieren des Umkreismittelpunktes eines Dreiecks zu verwenden und ein neues Makro für den Umkreismittelpunkt eines Dreiecks zu erstellen.

1. Arbeiten Sie in dem letzten Cinderella-Fenster, welches das Werkzeug für Mittelsenkrechten enthält. Löschen Sie alle Elemente des Konstruktionsfensters, indem Sie alle Elemente über das Menü „**Bearbeiten/Alles Auswählen**“, über den Schalter  oder über den Tastatur-Kurzbefehl **Strg+A** auswählen. Löschen Sie dann alle Elemente mit dem Menüeintrag „**Bearbeiten/Elemente löschen**“, durch Klicken auf den Schalter  oder über den Kurzbefehl **Strg+Entf**.
2. Konstruieren Sie ein Dreieck ABC .
3. Um die Mittelsenkrechten der Dreiecksseiten zu erzeugen, klicken Sie drei Mal auf das entwickelte Mittelsenkrechten-Werkzeug . Dadurch werden direkt die Punkte A' und B' ; A'' und B'' ; A_3 und B_3 und die Mittelsenkrechten a' , a'' und a_3 erzeugt.



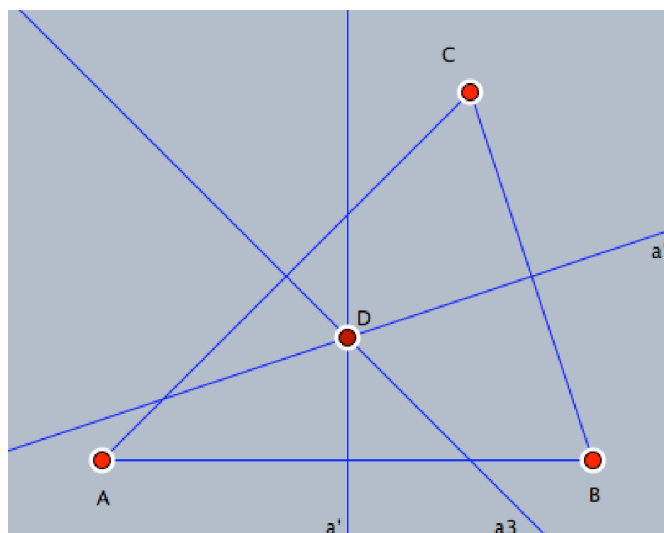
4. Wechseln Sie in den Modus „**Ablösen und Ankleben von Punkten**“ über das Menü „**Modi/Punkt umdefinieren**“ oder klicken Sie auf den Schalter  in der Werkzeugleiste und bewegen Sie dann den Punkt A' zu A , B zu B' , A'' zu

B , B'' zu C , $A3$ zu A , und $B3$ zu C . Damit haben sie die Mittelsenkrechten des Dreiecks dargestellt.



5. Erstellen Sie den Schnittpunkt der Mittelsenkrechten D^3 , um den Umkreismittelpunkt des Dreiecks ABC zu erhalten.

Wechseln Sie in den Bewege-Modus und Markieren Sie mit gedrückter Shift-Taste die Punkte A , B , C als Ausgangselemente und D als gewünschtes abhängiges Element für das Makro.



6. Nachdem Sie die Elemente ausgewählt haben, wählen Sie „**Bearbeiten/Eigenes**

³Bemerkung: Die Mittelsenkrechten der Seiten eines Dreiecks schneiden sich in einem Punkt. Der automatische Beweiser von Cinderella kann dies anzeigen, während der Schnittpunkt erzeugt wird.

Werkzeug erzeugen“ aus dem Menü, um ein neues Werkzeug für den Umkreismittelpunkt zu erzeugen und geben Sie einen Namen und eine Beschreibung an. Ein neues Symbol für den Umkreismittelpunkt eines Dreiecks wird in der Werkzeugleiste angezeigt.

Projektaufgabe

Verwenden Sie Cinderella, um Makros für die folgenden Konstruktionen zu erstellen:

- Inkreismittelpunkt eines Dreiecks.
- Höhenschnittpunkt eines Dreiecks.
- Schwerpunkt eines Dreiecks.
- Eulersche Gerade eines Dreiecks.

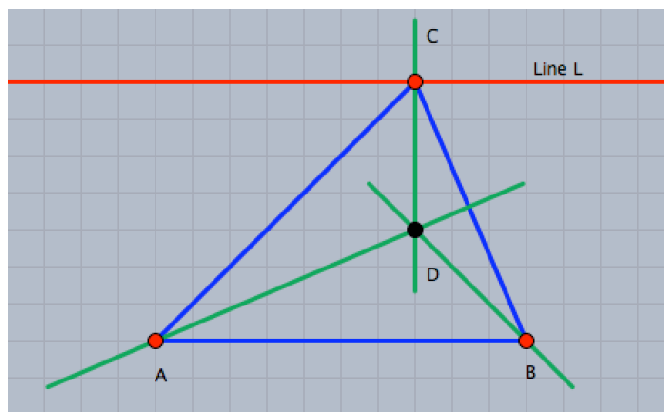
Handout 7

Animation und Definition von Ortskurven mit Cinderella

In dieser Aufgabe werden Sie die Verwendung der Animation-Funktion und der Zeichnung von Ortskurven mit Cinderella erleben, um geometrische Ortskurven zu entdecken und zu generieren.

Aufgaben

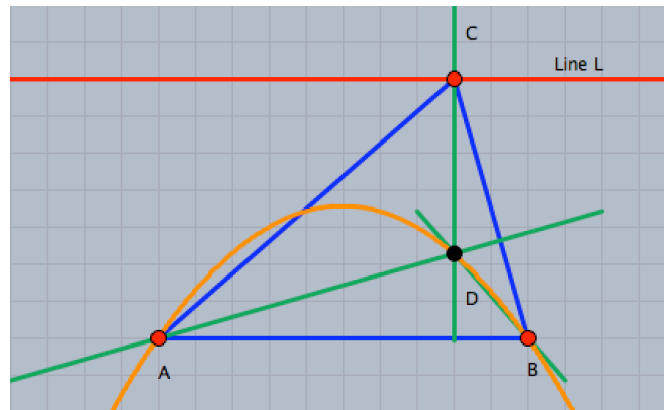
1. Konstruieren Sie ein Dreieck ABC in einer Weise, so dass es möglich ist, den Punkt C entlang einer geraden Linie L , die parallel zur Basis $\overline{AB}$ ist, zu verschieben.
2. Konstruieren Sie die drei Höhen des Dreiecks ABC und generieren Sie ihren Schnittpunkt in D^4 . Ihre Konstruktion sollte wie die folgende Abbildung aussehen.



3. Bewegen Sie den Punkt C entlang der Geraden L , die parallel zu $\overline{AB}$ ist, und beobachten Sie den Punkt D (den Höhen-Schnittpunkt). Können Sie die Ortskurve durch den Punkt D errahnen, während der Punkt C sich entlang der Geraden L bewegt? Mit anderen Worten: Welche Kurve entsteht durch den Punkt D während sich der Punkt C entlang der Geraden L bewegt? Stellen Sie eine Hypothese auf!
4. Die Ortskurve des Punktes D kann automatisch mit einem Werkzeugen von Cinde-

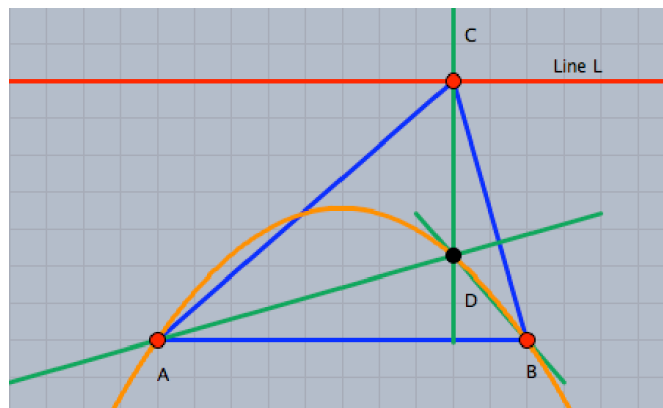
⁴Bemerkung: Die drei Höhen des Dreiecks schneiden sich in einem Punkt. Der automatische Beweiser von Cinderella kann dies anzeigen, während der Schnittpunkt erzeugt wird.

rella generiert werden. Schalten Sie zu diesem Zweck auf den Modus „**Ortskurven definieren**“, indem Sie die Menüelemente „**Modi/Spezial/Ortskurve**“ klicken oder betätigen Sie die Schaltfläche  in der Symbolleiste. Um Ortskurven zu generieren, sollten drei Objekte definiert werden: Das sich bewegende Element (der Punkt C), die Straße (die Gerade L) und der zu verfolgende Punkt (der Punkt D bzw. der Höhen-Schnittpunkt). Wählen Sie den Punkt C , dann die Gerade L und schließlich den Punkt D aus. Innerhalb einer Sekunde wird die Ortskurve automatisch generiert werden. Nach dem Erzeugen der Ortskurve sollte Ihre Konstruktion wie in der Abbildung unten aussehen.



5. Schalten Sie in den Zugmodus und beobachten Sie die Entstehung der Ortskurve, wenn Sie den Punkt C entlang der Geraden L verschieben.
6. Darüber hinaus hat Cinderella eine leistungsfähige Animations-Einrichtung, die den Punkt C automatisch entlang der Geraden für Sie bewegen kann. Zur Erzeugung einer Animation, bei der der Punkt C entlang der Geraden L bewegt wird, wechseln Sie zum Modus „**Animation**“, indem Sie die Menüelemente „**Modi/Spezial/Animation**“ betätigen oder auf die Schaltfläche  in der Symbolleiste klicken. Dann sollten zwei Elemente definiert werden: Das sich bewegende Element, das ist der Punkt C , und die Straße, das ist die Gerade L . Wählen Sie den Punkt C und dann die Gerade L . Nachdem Sie die Animation definiert haben, erscheint ein Eingabefeld mit drei Tasten und einem Geschwindigkeitregler in der linken unteren Ecke des Fensters.

- Zum Beispiel: die Situation kann erweitert werden, um die Ortskurve „**der Schwerpunkte eines Dreiecks**“ zu entdecken und zu generieren, wenn ein Eckpunkt eines Dreiecks sich entlang einer geraden Linie bewegt. In diesem Fall, wie in der Abbildung unten, ist die Ortskurve des Schwerpunkts eine gerade Linie, die parallel zur Geraden verläuft.*



Handout 8

Mittelpunkte der Seiten eines Vierecks

Aufgaben

Verwenden Sie Cinderella, um ein Viereck zu konstruieren! Konstruieren Sie den Mittelpunkt jeder Seite. Verbinden Sie diese Mittelpunkte mit den Mittelpunkten der benachbarten Seiten.

Stellen Sie eine Hypothese über die geformte Figur auf, d. h., welche Figur entsteht, wenn wir die Mittelpunkte der benachbarten Seiten miteinander verbinden?

Verwenden Sie die verschiedenen Werkzeuge von Cinderella (z.B. Konstruktionswerkzeuge, Zugmodus, Messwerkzeuge, Berechnungen), um die Hypothese zu untersuchen.

Schreiben Sie Ihre Hypothese in Form eines mathematischen Satzes.

Können Sie einen mathematischen Beweis für diese Hypothese finden?

Können Sie einige weiterführende Aufgaben im Zusammenhang mit der ursprünglichen Aufgabe aufstellen?

Handout 9

Winkelhalbierenden eines Parallelogramms

Aufgaben

Verwenden Sie Cinderella, um ein Parallelogramm zu konstruieren. Konstruieren Sie die inneren Winkelhalbierenden des Parallelogramms. Erzeugen Sie die vier Schnittpunkte der Winkelhalbierenden.

Stellen Sie dabei eine Hypothese über die geformte Figur auf, d. h., welche Figur entsteht, wenn wir die vier Schnittpunkte der Winkelhalbierenden eines Parallelogramms miteinander verbinden?

Verwenden Sie die verschiedenen Werkzeuge von Cinderella (z.B. Konstruktionswerkzeuge, Zugmodus, Messwerkzeuge, Berechnungen), um die Hypothese zu untersuchen.

Schreiben Sie Ihre Hypothese in Form eines mathematischen Satzes.

Können sie einen mathematischen Beweis für diese Hypothese finden?

Können Sie einige weiterführende Aufgaben im Zusammenhang mit der ursprünglichen Aufgabe aufstellen?

Handout 10

Konstruieren eines Parallelogramms

Aufgaben

1. Schreiben Sie so viele Verallgemeinerungen (Sätze, Definitionen, Eigenschaften und Folgerungen) auf, wie Sie können, die im Zusammenhang mit dem Parallelogramm stehen.

Zum Beispiel: Ein Parallelogramm ist ein Viereck, dessen gegenüberliegenden Seiten parallel sind.

(Bitte diskutieren sie Ihre Antworten mit dem Rest der Gruppe. Sammeln sie Ihre Ergebnisse auf der Folie).

2. Denken Sie darüber nach, welche dieser Verallgemeinerungen verwendet werden können, um in Cinderella ein Parallelogramm zu konstruieren. Versuchen Sie so viele unterschiedliche und verschiedene Methoden wie möglich zu finden, um ein Parallelogramm mit Cinderella zu konstruieren. Bei jeder Konstruktions-Methode sollten Sie:

- Die Konstruktionsbeschreibung machen. Siehe oben!
- Den theoretischen Hintergrund der Konstruktionsbeschreibung erläutern.
- Die einzelnen Konstruktionsschritte begründen.

3. Wie können die verschiedenen Konstruktions-Methoden klassifiziert werden?

Kriterien der Klassifikation:

Handout 11

Konstruieren eines Rhombus

Aufgaben

1. Schreiben Sie so viele Verallgemeinerungen (Sätze, Definitionen, Eigenschaften und Folgerungen) auf, wie Sie können, die im Zusammenhang mit dem Rhombus stehen.

Zum Beispiel: Ein Rhombus ist ein Parallelogramm, dessen zwei benachbarten Seiten gleich lang sind.

(Bitte diskutieren sie Ihre Antworten mit dem Rest der Gruppe. Sammeln sie Ihre Ergebnisse auf der Folie).

2. Denken Sie darüber nach, welche dieser Verallgemeinerungen verwendet werden können, um in Cinderella einen Rhombus zu konstruieren. Versuchen Sie so viele unterschiedliche und verschiedene Methoden wie möglich zu finden, um einen Rhombus mit Cinderella zu konstruieren. Bei jeder Konstruktions-Methode sollten Sie:

- Die Konstruktionsbeschreibung machen. Siehe oben!
- Den theoretischen Hintergrund der Konstruktionsbeschreibung erläutern.
- Die einzelnen Konstruktionsschritte begründen.

3. Wie können die verschiedenen Konstruktions-Methoden klassifiziert werden?

Kriterien der Klassifikation:

Handout 12

Konstruieren eines Rechtecks

Aufgaben

1. Schreiben Sie so viele Verallgemeinerungen (Sätze, Definitionen, Eigenschaften und Folgerungen) auf, wie Sie können, die im Zusammenhang mit dem Rechteck stehen.

Zum Beispiel: Ein Rechteck ist ein Parallelogramm mit einem rechten Winkel.

(Bitte diskutieren sie Ihre Antworten mit dem Rest der Gruppe. Sammeln sie Ihre Ergebnisse auf der Folie).

2. Denken Sie darüber nach, welche dieser Verallgemeinerungen verwendet werden können, um in Cinderella ein Rechteck zu konstruieren. Versuchen Sie so viele unterschiedliche und verschiedene Methoden wie möglich zu finden, um ein Rechteck mit Cinderella zu konstruieren. Bei jeder Konstruktions-Methode sollten Sie:

- Die Konstruktionsbeschreibung machen. Siehe oben!
- Den theoretischen Hintergrund der Konstruktionsbeschreibung erläutern.
- Die einzelnen Konstruktionsschritte begründen.

3. Wie können die verschiedenen Konstruktions-Methoden klassifiziert werden?

Kriterien der Klassifikation:

Handout 13

Konstruieren eines Quadrats

Aufgaben

1. Schreiben Sie so viele Verallgemeinerungen (Sätze, Definitionen, Eigenschaften und Folgerungen) auf, wie Sie können, die im Zusammenhang mit dem Quadrat stehen.

Zum Beispiel: Ein Quadrat ist ein Parallelogramm mit einem rechten Winkel und zwei benachbarten Seiten, die gleich lang sind.

(Bitte diskutieren sie Ihre Antworten mit dem Rest der Gruppe. Sammeln sie Ihre Ergebnisse auf der Folie).

2. Denken Sie darüber nach, welche dieser Verallgemeinerungen verwendet werden können, um in Cinderella ein Quadrat zu konstruieren. Versuchen Sie so viele unterschiedliche und verschiedene Methoden wie möglich zu finden, um ein Quadrat mit Cinderella zu konstruieren. Bei jeder Konstruktions-Methode sollten Sie:

- Die Konstruktionsbeschreibung machen. Siehe oben!
- Den theoretischen Hintergrund der Konstruktionsbeschreibung erläutern.
- Die einzelnen Konstruktionsschritte begründen.

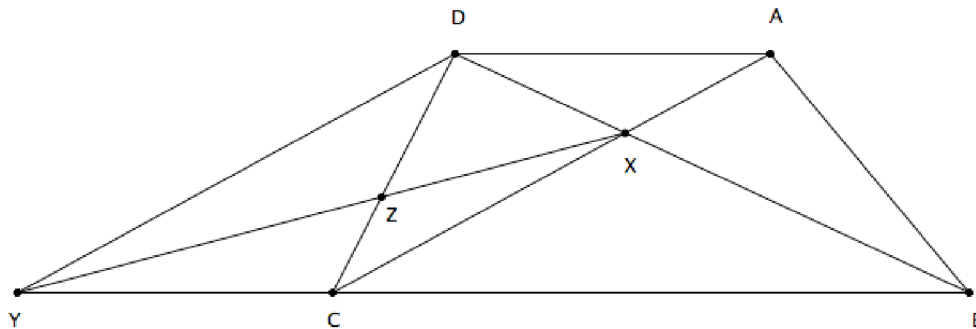
3. Wie können die verschiedenen Konstruktions-Methoden klassifiziert werden?

Kriterien der Klassifikation:

Handout 14

Geometrische Aufgabenstellungen

Aufgaben



Die obige Figur $ABCD$ ist ein Trapez, dessen Seiten $\overline{AD}$ und $\overline{BC}$ parallel sind. X ist der Schnittpunkt der Diagonalen. Es gilt: der Strahl $\overrightarrow{DY}$ ist parallel zum $\overrightarrow{AC}$ und der Strahl $\overrightarrow{DY}$ schneidet den Strahl $\overrightarrow{BC}$ in Y .

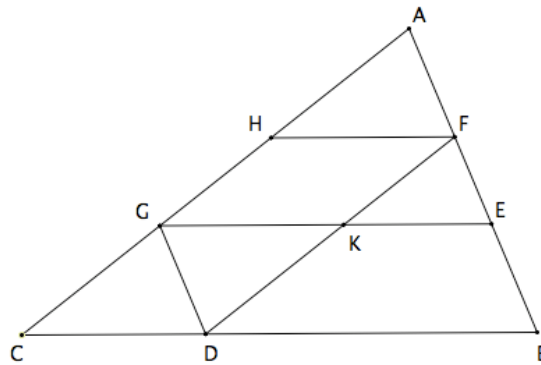
Verwenden Sie die verschiedenen Werkzeuge von Cinderella (z.B. Konstruktionswerkzeuge, Zugmodus, Messwerkzeuge, Berechnungen), um so viele verschiedene und unterschiedliche **geometrische Aufgabestellungen** wie möglich zu finden, die direkt oder indirekt aus den gegebenen Informationen abgeleitet werden können.

Handout 15

Finden „Geometrischer Beziehungen“

Aufgaben

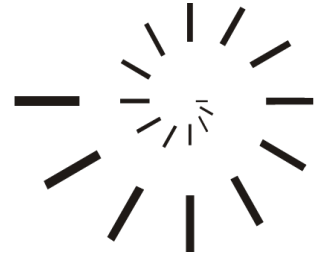
In der folgenden Figur gilt: $AF = BE$, $\overline{FH} \parallel \overline{EG} \parallel \overline{BC}$ und $\overline{GD} \parallel \overline{AB}$.



Verwenden Sie die verschiedenen Werkzeuge von Cinderella (z.B. Konstruktionswerkzeuge, Zugmodus, Messwerkzeuge, Berechnungen), um so viele verschiedene und unterschiedliche **geometrische Beziehungen** wie möglich **zu finden**, die direkt oder indirekt aus den vorgegebenen Informationen abgeleitet werden können.

Appendix C

Teacher's Guide



The Suggested Enrichment Program

Teacher's Guide

Prepared by

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To the Teacher

Improving students' geometric thinking is one of the major aims of mathematics education. That is, geometric thinking is very important in many scientific, technical, and occupational fields as well as in studying mathematics. However, important as geometric thinking is, geometry is often neglected in school mathematics especially in most high schools. Some possible reasons for this negligence would be the lack of resources, such as concrete materials, computer applications, and the lack of knowledge and expertise about how to use computers and other materials for instructional purposes (Olkun, Sinoplu, and Derzakulu, 2005). In this concern, this guide is an attempt to provide some knowledge and expertise about the use of computers and interactive dynamic geometry software in the teaching and learning of some enrichment activities in Euclidean geometry.

This guide is a part of a larger experimental study that aims at developing an enrichment program, using dynamic geometry software and deciding on its effectiveness in enhancing the mathematically gifted students' geometric creativity in high school.

In this study, the principles of developing the suggested enrichment program were first identified. Then, based on these principles and using Cinderella as dynamic interactive geometry software, the enrichment program was developed. The program consists of three interrelated portions: Student's Handouts, a CD-ROM, and this guide.

The **Student's Handouts** comprise the first portion of the suggested enrichment program, which contains 15 handouts prepared to guide the student throughout the program. For each activity, a student's handout is prepared to assist the students to go through the activity and promote discussion between the teacher and the students and discussion among the students as well.

The second main portion of the suggested enrichment program is the accompanying **CD-ROM** that contains all dynamic configurations prepared for the program in two formats, cdy and html. In addition, it contains two html indices, in both languages English and German, for all activities in the program and their dynamic configurations

that can be used to easily access any activity and any configuration within it.

This **Teacher's Guide** is the third portion of the enrichment program. It is designed to make the teacher's work and progress in the course easier and more effective. The guide does not restrict the teacher's work, but it is flexible enough for any creative addition.

It should be mentioned for the teacher that the present study uses the enrichment provision to provide the mathematically gifted students with some geometric activities and situations in Euclidean geometry using dynamic geometry software so as to broaden and deepen their mathematical knowledge and skills with emphasis on investigation and encouragement of creative thinking in the field of geometry.

The activities, within the suggested enrichment program, are designed using the interactive geometry software, Cinderella, as a mediation tool for developing the geometric creativity thinking components among the mathematically gifted students through active student participation, self directed-exploration, and reinvention of geometric relations instead of teaching a specific geometric content.

In the suggested enrichment program, the interactive geometry software, Cinderella, is introduced in three levels:

- Cinderella basic commands (e.g., open new Cinderella window, open/load Cinderella saved file, and save construction).
- Cinderella basic tools (e.g., add point, add line including line segment, parallel, perpendicular, and add circle with different modes).
- Cinderella different facilities (dragging facilities, measuring and calculating facilities, automatic proving facilities, developing macro-constructions facilities, and generating loci facilities)

With the hope that the mathematically gifted students become more accustomed to the use of Cinderella as a mediation tool throughout the suggested enrichment program so as to develop their geometric creativity and later on during their study of mathematics.

That is, one purpose of the suggested enrichment program is to teach the students how to use different facilities of Cinderella and another is to enhance their geometric creativity thinking in terms of the creativity components.

Geometric creativity can be defined as a combination of the following four components, which reflect the notion of the geometric creativity:

- **Fluency**, which means the student's ability to pose or come up with many geometric ideas or configurations related to a geometric problem or situation in a short time.
- **Flexibility**, which refers to the student's ability to vary the approach or suggest a variety of different methods toward a geometric problem or situation.
- **Originality**, which means the student's ability to try novel or unique approaches toward a geometric problem or situation.
- **Elaboration**, which is the student's ability to redefine a single geometric problem or situation to create others. It is not the geometric problem, situation itself, or even its solutions but rather the careful thinking upon the particular aspects that govern the geometric problem or situation. The elaboration ability also involves changing one or more of these aspects by substituting, combining, adapting, altering, expanding, eliminating, rearranging, or reversing and then speculating on how this single change would have a ripple effect on other aspects of the problem or the situation at hand.

In the development of instructional activities, the researcher adopted some criteria developed by (Haylock, 1997, p. 72) for the activity to be effective in developing geometric creativity and in distinguishing among the mathematically gifted students in terms of their geometric creativity and their responses:

1. The students' responses should show a wide range of geometric and mathematical ideas.
2. A large number of appropriate responses are possible for these students.

3. The students' responses should show a consistent interpretation of the instruction in the activity.
4. There should be several clear responses that can be obtained by most students.
5. There should be a number of appropriate responses that are obtained by relatively few students.
6. These original responses should have a degree of face validity for indicating creative ability in geometry and they should not be geometrically trivial.

Four categories of open-ended, divergent-production activities are developed within the suggested enrichment program, meeting the criteria mentioned before, to develop the geometric creativity components based upon specific teaching and learning strategies using different facilities of the interactive geometry software, Cinderella. The categories of activities include:

1. **Problem solving activities**, where the student is given a geometric problem with a specific question and then invited not only to find many various and different solutions but also to pose many follow-up problems related to the original problem (e.g., activities 1, 5, and 6).
2. **Problem posing activities**, where the student is given a geometric situation and asked to make up as many various and different questions, relationships, or conjectures as he/she can, that can be answered in direct or indirect ways using the given information (e.g., activities 11 and 12).
3. **Construction activities**, where the student is asked to come up with as many various and different methods as he/she can to construct a geometric figure (e.g. Parallelogram) using the constructing facilities of Cinderella application (e.g., activities 7, 8, 9, and 10).
4. **Redefinition activities**, where the student is given a geometric problem or situation and invited to pose as many problems as possible by redefining – substituting, adapting, altering, expanding, eliminating, rearranging or reversing – the aspects

that govern the given problem (e.g., activities 2 and 4).

These are not presented as hard-and-fast categories, but as a framework for designing activities that might help the mathematically gifted students to develop their geometric creativity.

However, the researcher figured out the suggested enrichment program, with the following objectives in mind:

- To develop suitable geometric activities that can introduce Cinderella basic tools and features to the mathematically gifted and enhance their geometric creativity thinking.
- To provide the mathematically gifted students with enrichment geometric activities that would promote effective outcomes related to geometric creativity components.
- To challenge the mathematically gifted to express their geometric ideas in different forms.
- To encourage the mathematically gifted to develop keen geometric creative abilities.
- To address several standards for school mathematics recommended by the National Council of Teachers of Mathematics (NCTM) such as: geometry standard, mathematics and problem solving standard, mathematics as communication standard, mathematics as reasoning standard, mathematical connections standard, mathematical representation standard, computation and estimation standard, and measurement standard.
- To involve the mathematically gifted students in both inductive and deductive reasoning.

The Content of the Teacher's Guide

This guide includes the following sections:

Aims of the Suggested Enrichment Program

In this section, the overall aim and the specific aims of the suggested enrichment program are objectively formulated. They precisely describe what could be expected from the mathematically gifted students by the end of the course.

Cinderella Getting Started

In this section, some teaching hints are introduced to the teacher to help him/her manage the introductory session, which is about Cinderella Getting Started, through the designed handouts (see Handout 2 and Handout 3).

The Enrichment Activities of the Program

This section is concerned with introducing the enrichment activities of the program. The 12 enrichment activities are covered considering the following elements for each activity:

The Title of the Activity

The title for each activity in the teacher's guide is written in a way that consists with the title written in the students handouts.

The Activity Problem

In each activity, the problem statement of the activity is presented.

Activity Content Analysis

A mathematical content analysis was provided for each activity that includes three main categories of content: concepts, generalizations, and skills to inform the teacher of the learning aspects that might be covered in the activity.

Objectives

Instructional objectives are objectively formulated for each activity to guide both the teacher and the students during the teaching and learning processes. They precisely describe what could be expected of the mathematically gifted students by the end of the activity.

Materials

A list of proposed materials is suggested for each activity, which includes: computers with Cinderella application installed on them, LCD projector, and student's handout of the activity.

Vocabulary

A list of new mathematical vocabulary is provided for each activity.

Prerequisites

A list of Cinderella prerequisite skills is given for each activity.

Teaching and Learning Strategies

Specific teaching and learning strategies using different facilities of Cinderella application are provided for each activity, which aim at developing the mathematically gifted students' geometric creativity.

Aims of the Suggested Enrichment Program

The overall aim of the suggested enrichment program is to cultivate and develop keen creative abilities in the field of geometry among the mathematically gifted students in high schools using Cinderella application as a mediation environment. This overall aim can be rendered in the light of the adopted definition of geometric creativity and the main characteristics of Cinderella to the following specific aims:

Throughout the suggested enrichment program the mathematically gifted students will hopefully be able to use Cinderella to:

1. Construct dynamic figures.
2. Come up with many construction methods to construct dynamic configurations for a certain figure.
3. Come up with many different and varied construction methods to construct dynamic configurations for a certain figure.
4. Come up with novel and unusual methods to construct dynamic configurations to a certain figure.
5. Produce many relevant responses (ideas, solutions, proofs, conjectures, and new formulated problems) toward a geometric problem or situation.
6. Produce many different and varied categories of relevant responses (ideas, solutions, proofs, conjectures, and new formulated problems) toward a geometric problem situation.
7. Generate many unusual ("way-out"), unique, clever responses or products toward a geometric problem or situation.
8. Make new conjectures and relationships by recognizing their experience toward the aspects of the given problem or situation.
9. Investigate the made conjectures in different situations or methods.

10. Generate many different and varied proofs using the formal logical and deductive reasoning toward a geometric problem or situation.
11. Generate many follow-up problems by redefining (modifying, adapting, expanding, or altering) a given geometric problem or situation.
12. Apply different learning aspects of geometry (concepts, generalizations, and skills) in solving a geometric problem or situation.

The teacher should inform the students by these outcomes before they receive the program instruction (see Student's Handout 1).

Cinderella Getting Started

The Student's Handouts 2 and 3 are designed to help the mathematically gifted students to get started with Cinderella; including launching Cinderella and exploring both Cinderella window and Cinderella menus. They also help the students become accustomed to both Cinderella basic commands (e.g., Open New Construction Window, Open/Load Saved Construction, Save as, etc.) and Cinderella basic constructions or modes (e.g., Add Point, Add Line Modes including the line segment and parallel and perpendicular, Add Circle Modes, etc.)

Launch Cinderella

For launching Cinderella, detailed instructions are given, in Student's Handout 2, that enable the students to launch Cinderella using three different ways – Cinderella.2 shortcut, Start/Programs/ Cinderella.2 sequence, and double click a file created by Cinderella (see Student's Handout 2). Have the students try the three mentioned ways to launch Cinderella.

Exploring Cinderella Window

The main elements of Cinderella window are also provided in Handout 2. These elements include: Title Bar, Menu Bar, Standard toolbar, Geometric Modes Toolbar, Construction Area, Specific-view Actions Toolbar, and Message Bar (see Student's Handout 2).

Exploring Cinderella Menus

Handout 2 also gives explanations for the 7 pull-down menus of Cinderella accompanied with screen shots, which include: File, Edit, Geometry, Views, Format, and Help menus respectively (see Student's Handout 2). For more details, see online Cinderella documentation of Cinderella menus available at: <http://doc.cinderella.de/tiki-index>.

[php?page=The+Main+Menu](#).

Basic Commands of Cinderella

Using Student's Handout 3, the students can be taught the basic commands of Cinderella, which include: Open New Cinderella Window, Open/Load Cinderella Saved File, Save Construction, Save Construction with New Name, Create an interactive Web Page, Undo, Redo, and Selection Commands (see Student's Handout 3). For more details, we refer the teacher to online Cinderella documentation of Cinderella menus available at: <http://doc.cinderella.de/tiki-index.php?page=General+Operations>.

Basic Constructions or Modes of Cinderella

Using also Student's Handout 3, the students can also be taught the basic constructions or modes of Cinderella, which include: Move Mode, Add a Point, Add a Line, Line through Point, Add a Parallel, Add a Perpendicular, Add a Circle, Midpoint, Segment (see Student's Handout 3). For more details, see online Cinderella documentation of Cinderella menus available at: <http://doc.cinderella.de/tiki-index.php?page=Move+Mode> and <http://doc.cinderella.de/tiki-index.php?page=Interactive+Modes>. By the end of the getting started session, the students should be encouraged to use Cinderella to come up with dynamic configurations for some geometric objects.

MathsNet Resources

As supplementary and interesting resources in addition to the learning contents prepared in Student's Handout 3 for the constructions or modes of Cinderella, the teacher can use online Cinderella interactive exercises – on MathsNet website⁵, which introduces a special course in interactive geometric constructions at three levels: foundation, intermediate and advanced.

⁵MathsNet Website: <http://www.mathsnet.net/>

The Enrichment Activities of the Program

The suggested enrichment program is composed of 12 enrichment activities that are designed to enable the mathematically gifted students to skillfully use the interactive geometry software, Cinderella, as a mediation tool to enhance their geometric creativity. In this section of the guide, The researcher provides – in detail – how the teacher can introduce these enrichment activities to the mathematically gifted students using the interactive geometry software, Cinderella, in order to develop their creative potential in the field of geometry.

The titles of the enrichment activities are:

1. Dragging and Measuring Facilities of Cinderella
2. Automatic Proving Facilities of Cinderella
3. Developing Macro-constructions
4. Animating and Tracing Loci Facilities of Cinderella
5. Midpoints of the Sides of a Quadrilateral
6. Angular Bisector of a Parallelogram
7. Constructing a Parallelogram
8. Constructing a Rhombus
9. Constructing a Rectangle
10. Constructing a Square
11. Posing Geometric Problems
12. Finding Geometric Relationships

Enrichment Activity 1

Dragging and Measuring Facilities of Cinderella

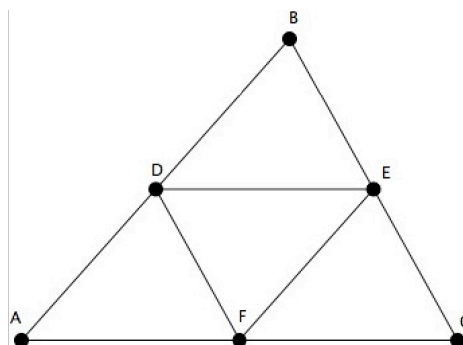
In this activity, the mathematically gifted students experience the dragging and measuring facilities of Cinderella to make and investigate new geometric conjectures.

The Activity Problem⁶

What is the relationship between the areas of the four triangles obtained by joining the midpoints of the sides of a triangle?

In this activity, detailed instructions are given to the students, as shown in Handout 4, to construct a dynamic configuration for a triangle $\triangle ABC$ and bisect its three sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ at D , E , and F respectively.

Then join between the three midpoints D , E , and F as shown in the opposite figure. After that, the students are directed to use the dragging facilities of Cinderella through its “Move” mode to make a conjecture about the area of the four triangles $\triangle ADF$, $\triangle DBE$, $\triangle EDF$, and $\triangle CEF$ by



the question: “What can you say about the area of the four triangles?” Then, the students are instructed how to use Cinderella measuring facilities to check the correctness of their conjecture about the area of the four triangles. Afterwards, the students are asked to use Cinderella to pose follow-up problems related to this geometric situation (see Student’s Handout 4).

⁶Resources used in designing this activity:

Olkun, S., Sinoplu, N. B. and Derzakulu, D. (2005). Geometric explorations with dynamic geometry applications based on van Hiele levels. Retrieved February 20, 2006, from: <http://www.cimt.plymouth.ac.uk/journal/default.htm>

Haja, S. (2005). Investigating the problem-solving competency of pre-service teachers in dynamic geometry environment. In H. L. Chick and J. L. Vincent, editors, *Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education*, volume 3, pages 81–87. Melbourne, Australia.

Activity Content Analysis

Concepts

- Area of a triangle
- Congruency of two triangles
- Similarity of two triangles

Generalizations

- The line segment joining between midpoints of two sides of a triangle is parallel to the third side and its length is equal to half of it.
- Two triangles are congruent if the length of each side of the first triangle is equal to the length of the corresponding side in the second triangle.
- A parallel to a side of a triangle that intersects the other two sides cuts off a triangle similar to the original triangle.

Skills

- Skill of constructing dynamic figures using Cinderella application.
- Skill of making and investigating geometric conjectures using dragging and measuring facilities of Cinderella.

Objectives

The activity is designed to help the mathematically gifted students to:

- Construct a dynamic configuration for a triangle and the four triangles obtained by joining the midpoints of the sides of the triangle using Cinderella application.
- Use dragging facilities of Cinderella through its “**Move**” mode to make a geometric conjecture about the area of the four triangles obtained by joining the

midpoints of the sides of a triangle.

- Use measuring facilities of Cinderella to investigate the conjecture made about the area of the four triangles obtained by joining the midpoints of the sides of a triangle.
- Produce mathematical proofs for the investigated conjecture about the area of the four triangles obtained by joining the midpoints of the sides of a triangle.
- Pose follow-up problems related to the original problem.

Materials

■ Computers	■ Handout 4
■ Cinderella application	■ CD-ROM of the program
■ LCD projector	

Vocabulary

■ Area of a triangle	■ Right-angled triangle
■ Type of a triangle	■ Obtuse-angled triangle
■ Equilateral triangle	■ Two equivalent triangles
■ Isosceles triangle	■ Two congruent triangles
■ Scalene triangle	■ Two similar triangles
■ Acute-angled triangle	■ Polygon

Prerequisites

The students should know how to use the following tools before they start working in this activity:



Launch Cinderella or open new Cinderella window.



Use “**Move**” mode to move free elements by dragging the mouse.



Construct a point.



Construct a line segment joining two points.

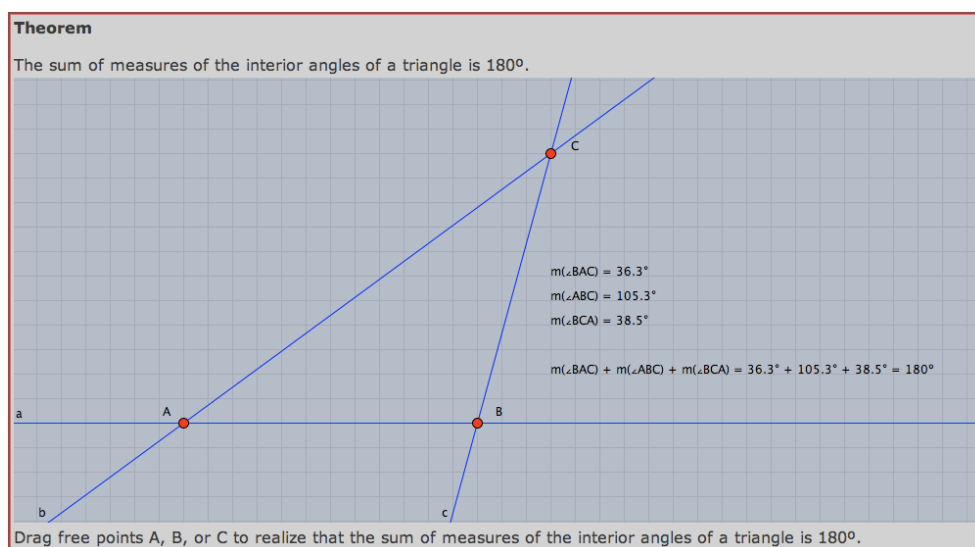


Construct a midpoint of a given line segment.

Teaching and Learning Strategies

Warm up

It could be a suitable warm up to this activity to explain the meaning of Cinderella dragging facilities of the “**Move**” mode by presenting some pre-made constructions to the students.

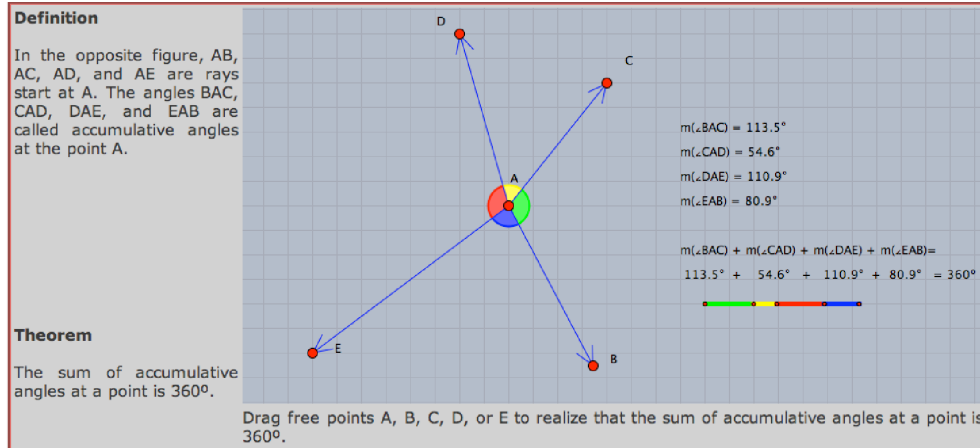


`/files/move_mode/the_sum_of_measures_of_the_interior_angles_of_a_triangle`

There are two pre-made constructions prepared by the researcher and placed on the `/files/move_mode/` folder on the accompanying CD-ROM⁷. The first construction (shown above) shows that the sum of measures of the interior angles of the triangle is 180° .

⁷The accompanying CD-ROM contains all dynamic configurations prepared for the program in two formats cdy and html. In addition, it contains two html indices, in both languages English and German, for all activities of the program and their dynamic configurations that can be used to easily access any activity and any configuration within it. The name and the path of the two files – the cdy file and the html file – are provided under the figure.

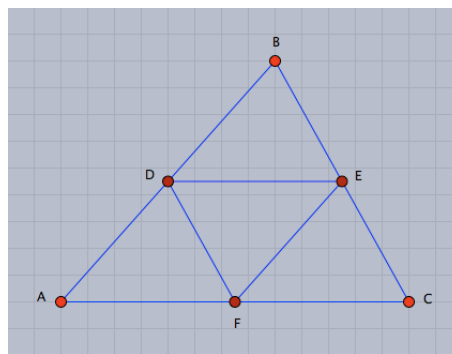
The second construction (shown below) demonstrates that the sum of the measures of the accumulative angles at a point is 360° .



/files/move_mode/the_sum_of_measures_of_the_accumulative_angles_at_a_point

Both constructions illustrate the dragging facilities of Cinderella in a dynamic and interactive manner. When a free element is dragged, it can be visually and dynamically observed that even though under dragging mode the measures of angles change, the sums of their measures are still the same.

Construction



/files/move_mode/triangle_midpoints_activity/problem_statement

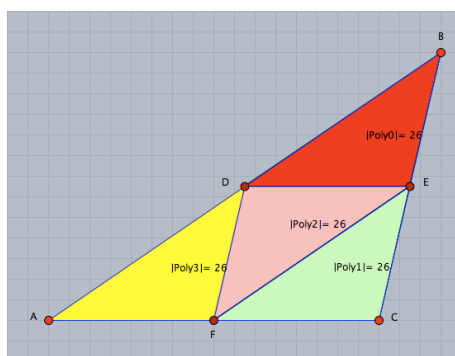
Encourage the students to follow the activity handout steps and use Cinderella to construct a triangle, midpoint of each side, and then join the midpoints of the sides of the triangle to end up with a dynamic configuration for the problem as shown in the figure above. In this figure, the vertices of the triangle A , B , and C are completely arbitrary

and can be moved freely, while D , E , and F are fixed as midpoints that cannot be moved.

Making Conjecture

Have the students use Cinderella dragging facilities of “**Move**” mode to drag free points (A , B , or C) and make a primary conjecture about the area of the four triangles obtained by joining the midpoints of the sides of a triangle.

Investigating Conjecture



`/files/move_mode/triangle_midpoints_activity/problem_investigation`

Have the students use Cinderella measuring facilities to measure the area of the four triangles obtained by joining the midpoints of the sides of a triangle. For doing so, the students should switch to the “**Define a Polygon Mode**” first to define the four triangles as four Polygons: $Poly0$, $Poly1$, $Poly2$, and $Poly3$, then switch to the “**Measure Area of a Polygon Mode**” and just click inside each triangle (Polygon) to get the measurement of its area. Have the students use Cinderella dragging facilities through Cinderella “**Move**” mode to alter the triangle ABC several times, and visually observe the area of the four triangles in each time. When the vertices of the triangle ABC are dragged, the area of the four triangles remains equal, as shown in the figure [above](#), and this shows that the area of the four triangles obtained by joining the midpoints of the sides of a triangle is equal.

Producing Mathematical Proofs of the Conjecture

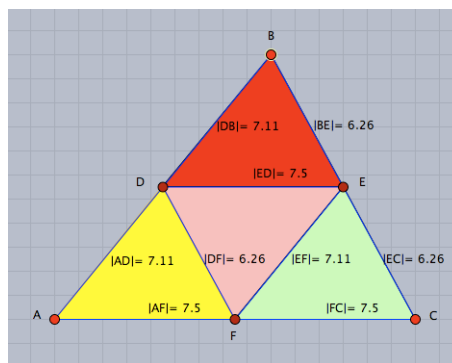
The next step in the activity is to encourage the students to produce as many mathematical proofs as possible for the investigated conjecture. So, have the students think of how to prove the investigated conjecture using the formal logical steps. It may be helpful to give them some hints in the form of questions to guide/direct their attention toward the proving process, such as:

- What is the relationship between the line segment joining the midpoints of two sides of a triangle and the third side?
- What is the relationship between the length of the line segment joining midpoints of two sides of a triangle and the length of the third side?
- What is the relationship between the heights of the triangles $\triangle ADF$, $\triangle DBE$, $\triangle EDF$, and $\triangle CEF$?

Posing Follow-up Problems

In addition to the conjecture previously investigated, many follow-up problems related to the original problem can be posed if the following conjectures are considered:

Conjecture 1

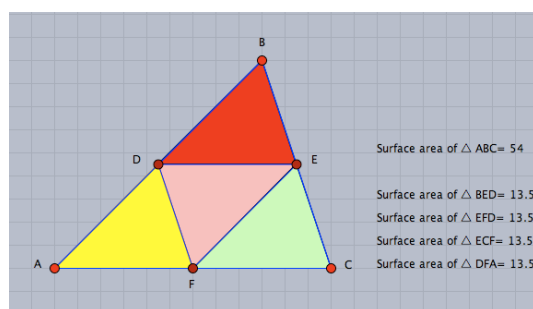


/files/move_mode/triangle_midpoints_activity/proposed_problem_1

The configuration above shows that the sides of each of the four obtained triangles

are equal in length to their corresponding sides in the other three triangles. Since the perimeter of the triangle is the sum of lengths of its sides, so the four triangles have the same perimeter. This leads to suggesting a new conjecture, which is: *“The perimeter of the four triangles obtained by joining the midpoints of a triangle is equal”*. The dynamic configuration of the conjecture can be used to visually convince the students of the validity of the conjecture through the dragging and measuring facilities of Cinderella. When any vertex of the original triangle is dragged, the sides of each triangle of the four triangles obtained remain equal in length to their corresponding sides in the other three triangles.

Conjecture 2



[/files/move_mode/triangle_midpoints_activity/proposed_problem.2](/files/move_mode/triangle_midpoints_activity/proposed_problem.2)

Using the measuring facilities of Cinderella, It is also possible to get a new conjecture about the area of the original triangle in comparison to the area of each of the four triangles. Have the students use **“Measure Area of a Polygon”** mode to measure the area of the original triangle. Have the students use Cinderella dragging facilities through **“Move”** mode to drag free points (A , B , or C) to alter the original triangle ABC and visually observe the area of the original triangle in comparison to the area of each of the four triangles obtained to make a new conjecture. The students can easily end up with a new conjecture, which is: *“The area of the original triangle equals four times the area of each of the four obtained triangles”*. The conjecture can be written in other words as: *“The area of each triangle of the four obtained triangles equals one quarter the area of the original triangle”*.

i.e.,

$$\text{Surface Area of } \triangle ABC = 4 \text{ Surface Area of } \triangle BED,$$

$$\text{Surface Area of } \triangle ABC = 4 \text{ Surface Area of } \triangle EFD,$$

$$\text{Surface Area of } \triangle ABC = 4 \text{ Surface Area of } \triangle ECF,$$

$$\text{Surface Area of } \triangle ABC = 4 \text{ Surface Area of } \triangle DFA.$$

or

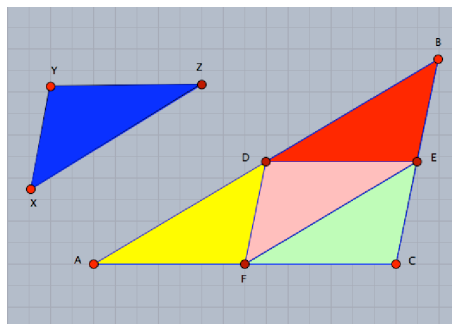
$$\text{Surface Area of } \triangle BED = \frac{1}{4} \text{ Surface Area of } \triangle ABC,$$

$$\text{Surface Area of } \triangle EFD = \frac{1}{4} \text{ Surface Area of } \triangle ABC,$$

$$\text{Surface Area of } \triangle ECF = \frac{1}{4} \text{ Surface Area of } \triangle ABC,$$

$$\text{Surface Area of } \triangle DFA = \frac{1}{4} \text{ Surface Area of } \triangle ABC.$$

Conjecture 3



/files/move_mode/triangle_midpoints_activity/proposed_problem_3

The configuration above shows that the sides of each of the four triangles obtained are equal in length to their corresponding sides in the other three triangles. This is sufficient evidence to say that the four triangles obtained are congruent, based on the theorem which states that “Two triangles are congruent if the lengths of the sides of the first triangle are equal to their corresponding lengths of the sides of the second triangle”. This also leads to suggesting a new conjecture, which is: “*The four triangles obtained by joining the midpoints of a triangle are congruent*”. Symbolically, the conjecture can be written as: $\triangle BED \equiv \triangle EFD \equiv \triangle ECF \equiv \triangle DFA$

Conjecture 4

Based on the theorem which states that “A parallel to a side of a triangle that intersects the other two sides cuts off a triangle similar to the original triangle”, this conjecture can be easily deduced: *“The four triangles obtained by joining the midpoints of a triangle are similar to each other, and each one of them is similar to the original triangle”*. In other words, based on the theorem mentioned above the conjecture can be symbolically figured out as follow:

$\therefore$ The four triangles obtained are congruent.

$\therefore$ They are similar to each other.

i.e., $\triangle BED \sim \triangle EFD \sim \triangle ECF \sim \triangle DFA$

In $\triangle ABC$

$\therefore \overline{DE} \parallel \overline{AC}$

$\therefore \triangle DBE \sim \triangle ABC$

Thus the four obtained triangles are similar to each other, and each one of them is similar to the original triangle.

i.e., $\triangle BED \sim \triangle EFD \sim \triangle ECF \sim \triangle DFA \sim \triangle ABC$

Conjecture 5

“The four obtained triangles are of the same type as the original triangle either according to the lengths of sides or the measures of angles”. For example, if the original triangle is an equilateral triangle, then the four triangles obtained by joining the midpoints of the original triangle are equilateral triangles. Equally, if the original triangle is an acute-angled triangle then the four triangles are acute-angled, etc. According to this conjecture many related problems can be posed, such as:

What type would the four triangles obtained by joining the midpoints of an equilateral triangle be?

What type would the four triangles obtained by joining the midpoints of an isosceles triangle be?

What type would the four triangles obtained by joining the midpoints of a scalene triangle be?

What type would the four triangles obtained by joining the midpoints of an acute-angled triangle be?

What type would the four triangles obtained by joining the midpoints of a right-angled triangle be?

What type would the four triangles obtained by joining the midpoints of an obtuse-angled triangle be?

The students can easily test the validity of this conjecture by either two ways. The first way is starting a new construction with a specific type of triangle. The second way is altering the triangle in the original construction to a specific type and visually observing the type of the four resulted triangles according to the lengths of sides or the measures of angles.

Enrichment Activity 2

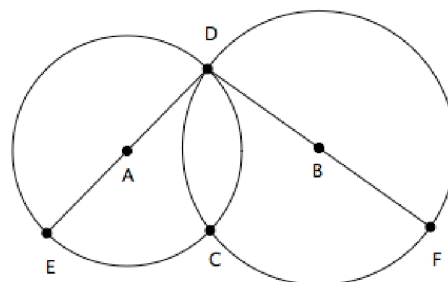
Automatic Proving Facilities of Cinderella

In this activity, the mathematically gifted students experience the automatic proving facilities of Cinderella to check the correctness of a geometric conjecture. Moreover, they are provided with opportunities to develop their ability to elaborate on a given geometric situation by extending, modifying or adapting its conditions to make up as many new conjectures as possible.

The Activity Problem⁸

In this activity, detailed instructions are given to the students to construct a dynamic configuration for two circles A and B with two intersection points C and D and two diameters $\overline{DE}$ and $\overline{DF}$ as shown in the opposite figure.

After that, the students are directed to make a conjecture about the collinearity of the three points F , C , and E by the questions: “What can you say about the three points F , C , and E in terms of collinearity? – i.e., are they collinear?”



Then, the students are instructed how to use Cinderella proving facilities to check the correctness of the made conjecture. Afterwards, the students are asked to use Cinderella to elaborate on the given situation by extending, modifying, or adapting its conditions to make up as many new conjectures as possible (see Student's Handout 5).

⁸Adapted from: Keyton, M. (1997). A search for an all-encompassing problem in elementary geometry. *Eightysomething!*, 12–13.

Activity Content Analysis

Concepts

- Collinearity
- Parallelism
- Quadrilateral
- Perpendicularity
- Congruency of two triangles
- Similarity of two triangles
- Area

Generalizations

- The inscribed angle in a semi-circle is a right angle.
- The line of centers of two intersecting circles is perpendicular to the common chord and bisects it.
- Two triangles are congruent if the length of each side of the first triangle is equal to the length of the corresponding side in the second triangle.
- The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length equals half of it.
- Congruent triangles are similar.
- A parallel to a side of a triangle that intersects the other two sides cuts off a triangle similar to the original triangle.

Skills

- Skill of constructing dynamic configurations using Cinderella application.
- Skill of checking the correctness of geometric conjectures using the automatic proving facilities of Cinderella application.

- Skill of elaborating on a given geometric situation by extending, modifying, or adapting its conditions to make up new geometric conjectures or situations using different facilities of Cinderella application.

Objectives

The activity is designed to help the mathematically gifted students to:

- Construct a dynamic configuration for the given situation using Cinderella application.
- Make a geometric conjecture about the collinearity of three points in the given situation.
- Use the automatic proving facilities of Cinderella to check the correctness of the made conjecture.
- Produce as many mathematical proofs as possible to prove the collinearity of the three points.
- Make up new conjectures by elaborating the given situation using Cinderella application.

Materials

▪ Computers	▪ Handout 5
▪ Cinderella application	▪ CD-ROM of the program
▪ LCD projector	▪ Overhead projector

Vocabulary

▪ Collinearity	▪ Parallelism
▪ Two intersecting circles	▪ Area
▪ The line of centers of two circles	▪ Kite
▪ Common chord	▪ Quadrilateral
▪ Perpendicular bisector	▪ Rectangle
▪ Two congruent triangles	▪ Parallelogram
▪ Two similar triangles	

Prerequisites

The students should know how to use the following tools before they start working in this activity:



Launch Cinderella or open new Cinderella window.



Use “**Move**” mode to move free elements by dragging the mouse.



Construct a point.



Construct a line connecting two points.



Generate the intersection of two geometric objects.



Measure the length of line segments.



Measure angles.



Define polygons.



Measure areas.

Teaching and Learning Strategies

Warm up

Two brainstorming questions could be used as a starting point for this situation. The questions are:

1. Write down as many geometric concepts and terminologies as possible that are related to the topic of circles.

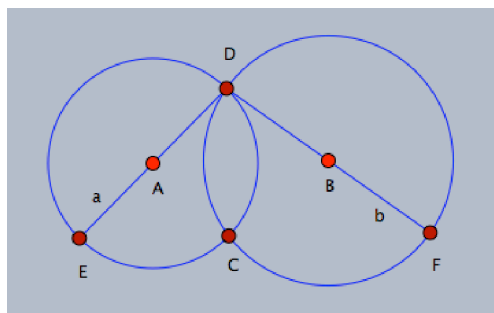
For example: inscribed angle.

2. Write down as many generalizations (theorems, definitions, properties or corollaries) as possible that are related to the topic of circles.

For example: The inscribed angle in a semi-circle is a right angle.

Present the two questions to the students, one at a time using the overhead projector, and encourage them to respond. After the students respond to each question, the teacher may spend a few minutes to revise and clarify some of the students' terminologies and generalizations that are needed in the current situation to ensure that the students interpret concepts correctly for effective classroom communication.

Construction



/files/automatic.proving.facilities/the_problem_statement

Have the students follow the directions of the activity handout (steps 1 to 13) and use Cinderella to construct two circles A and B with two intersection points C and D and two diameters $\overline{DE}$ and $\overline{DF}$ to end up with a dynamic configuration similar to the figure shown above. In this figure, the two points A and B , which represent the centers of the circles, are completely arbitrary and can be moved freely, while E is a semi-free point that can be moved along the radius of circle A , F is also a semi-free point that can be moved along the radius of circle B , and both C and D are semi-free points that can be only moved along an arc path that has end-points at the two intersection points of the two circles.

Making Conjecture

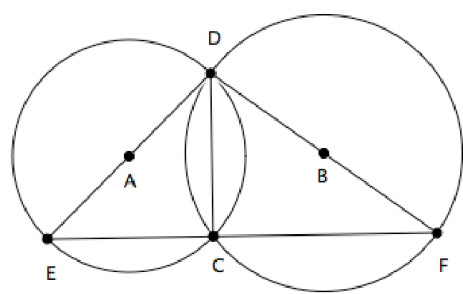
Have the students follow the directions of the activity handout (step 13) to drag free elements (free points (A and B), semi free points(C and D), or even the edge of either circle) to deform the situation and observe the behavior of the three points F , C , and E in terms of the collinearity. i.e., Whether the three points F , C , and E are collinear or not.

Investigating Conjecture

Have the students follow the directions of the activity handout (step 14) for using automatic proving facilities of Cinderella to automatically check/investigate the correctness of the made conjecture about the collinearity of the three points F , C , and E . To end up with a conclusion about the collinearity of the three points, present the pre-made construction using the overhead projector to the students to visually and dynamically convince them of the collinearity of the three points and promote more discussion about why the three points are collinear, which can be used to motivate the students to come up with mathematical proofs for the collinearity of the three points.

Producing Mathematical Proofs of the Conjecture

After the students use Cinderella automatic proving facilities to check the collinearity of the three points and have become visually and dynamically convinced, they should produce mathematical proofs that logically convince them. So here the students should be encouraged to pro-



duce as many ways as they can to prove the collinearity of the three points. In fact, “proving collinearity” is a difficult task for most high school students, but in this situation it can be reduced to prove that the two adjacent angles at C are supplementary and that is enough to prove the collinearity of the three points F , C , and E . In the

given situation, since each of the two angles $\angle FCD$ and $\angle ECD$ is an inscribed angle in a semi-circle, then each of them is a right angle and thus a pair of supplementary angles.

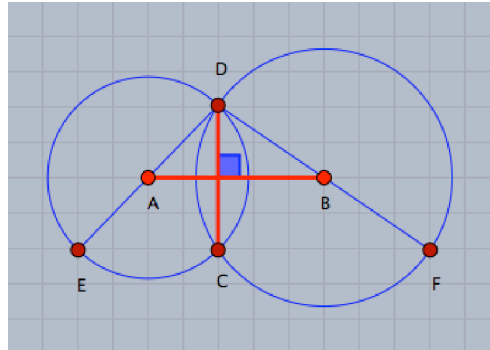
Another way to prove the collinearity of the three points F , C , and E include proving that $\overline{AB}$ is parallel to $\overline{EF}$ and that $\overline{AB}$ is perpendicular to $\overline{DC}$ at G , letting $\overline{DC}$ intersects $\overline{EF}$ at a point X , showing that G is the midpoint of $\overline{DC}$ and that G is the mid point of $\overline{DX}$ (using either one that parallels divide a straight line into segments of equal lengths, then they divide any other straight line into segments of equal lengths; or the line drawn from the midpoint of a side of a triangle parallel to another side bisects the third side) implies that $C = X$.

Making up New Related Conjectures

In addition to the conjecture previously investigated, many other discoveries or conjectures can be made by elaborating the conditions of the original situation. These discoveries or conjectures unite several different basic areas in elementary geometry such as: perpendicularity, parallelism, collinearity, measurements, congruency, similarity, circles, triangles, quadrilaterals, and areas. It is worth mentioning here that presenting a situation like this one enables students to see new mathematical connections among the different areas embedded in the situation. Furthermore, it provides students with opportunities to construct their own understanding by linking their previous and current geometric knowledge during their discoveries.

So, in this section, the teacher should guide the students to use Cinderella to make up and discover new conjectures by elaborating on the conditions of the given situation. Presented here are some of the conjectures the students may discover in which the basic areas of elementary geometry are connected. A cdy file and an html file are prepared for each conjecture and placed on the /files/automatic_proving_facilities/ folder on the accompanying CD-ROM that can be used to confirm the students' new discoveries.

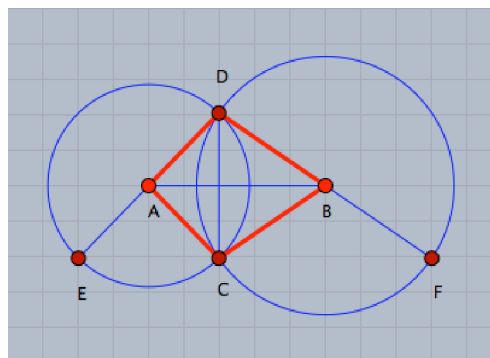
Conjecture 1



/files/automatic-proving-facilities/conjecture_1

By constructing the two line segments $\overline{AB}$ and $\overline{DC}$, a new conjecture can be made up, which is: *“The line segment $\overline{AB}$ is the perpendicular bisector of the line segment $\overline{DC}$ ”,* which can be easily proven using concurrency of triangles. This conjecture can be generalized to any two intersecting circles. That is, it can be said that: *The line of centers of two intersecting circles is perpendicular to the common chord and bisects it.*

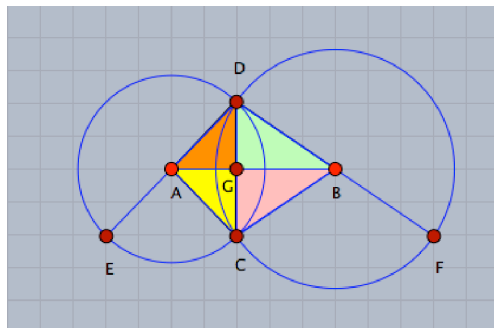
Conjecture 2



/files/automatic-proving-facilities/conjecture_2

As a result of conjecture 1, and by constructing the two line segments $\overline{BC}$ and $\overline{CA}$, a new conjecture related to quadrilaterals can be made; so it can be said that: *“The quadrilateral ACBD is a kite”.*

Conjecture 3

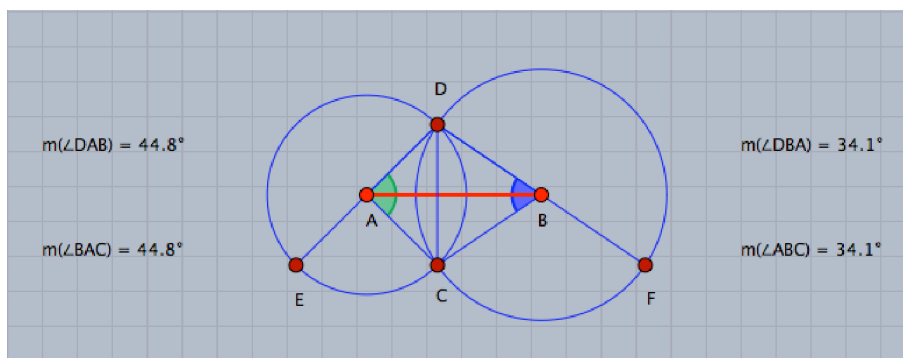


/files/automatic_proving_facilities/conjecture_3

Regarding to congruency, by using the (S, S, S) theorem of concurrency of two triangles, it can be said that: “*There are three pairs of congruent triangles*”, which are:

1. $\triangle DBG \equiv \triangle CBG$
2. $\triangle DAG \equiv \triangle CAG$
3. $\triangle BDA \equiv \triangle BCA$

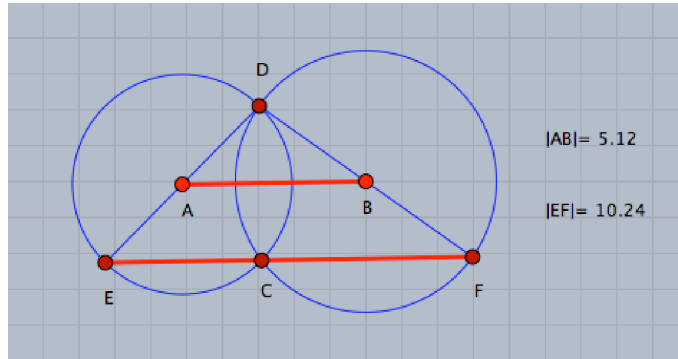
Conjecture 4



/files/automatic_proving_facilities/conjecture_4

Concerning the measurement of angles, the line segment $\overline{AB}$ bisects the two angles $\angle DBC$ and $\angle DAC$; i.e., $m(\angle DBA) = m(\angle ABC)$ and $m(\angle DAB) = m(\angle BAC)$ a result which is closely related to the congruency of triangles in conjecture 3.

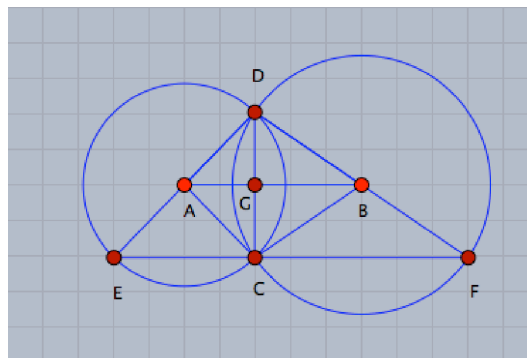
Conjecture 5



/files/automatic_proving_facilities/conjecture_5

By constructing the line segments $\overline{AB}$ and $\overline{EF}$ and considering the theorem which states that “The line segment joining the mid-points of two sides of a triangle is parallel to the third side and its length equals half of it”, a conjecture of two results can be made up: “The line segments $\overline{AB}$ is parallel to the line segment $\overline{EF}$ and its length equals half of it”. i.e., $\overline{AB} \parallel \overline{EF}$ and $AB = \frac{1}{2} EF$.

Conjecture 6

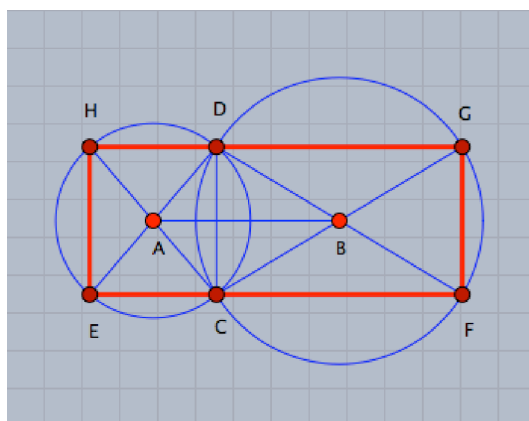


/files/automatic_proving_facilities/conjecture_6

As for similarity of triangles, by constructing the line segments $\overline{AB}$, $\overline{EF}$ and $\overline{DC}$, as in the figure shown above, and considering the conjectures related to congruency of triangles and parallelism, the situation can be further extended for conjecturing 8 pairs of similar triangles:

1. $\triangle DBG \sim \triangle CBG$	5. $\triangle DAG \sim \triangle DEC$
2. $\triangle DAG \sim \triangle CAG$	6. $\triangle DBG \sim \triangle DFC$
3. $\triangle BDA \sim \triangle BCA$	7. $\triangle DEC \sim \triangle CBG$
4. $\triangle DAB \sim \triangle DEF$	8. $\triangle DFC \sim \triangle CAG$

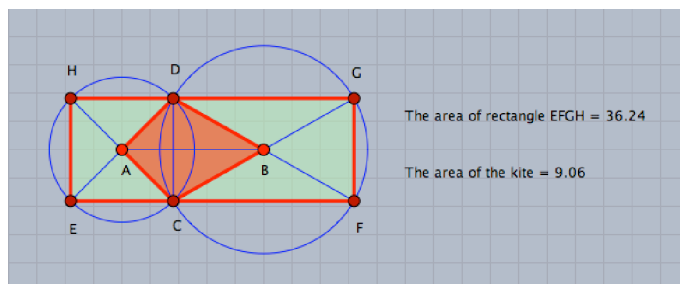
Conjecture 7



/files/automatic-proving-facilities/conjecture_7

The situation can be further elaborated by constructing the diameters $\overline{CH}$ of circle A and $\overline{CG}$ of circle B , and constructing the quadrilateral $EFGH$ to end up with a configuration similar to the figure shown above for a new conjecture, which is: “*Quadrilateral $EFGH$ is a rectangle*”.

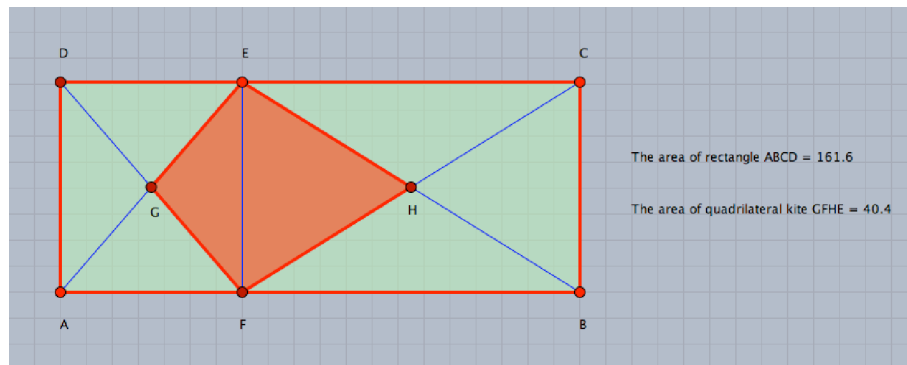
Conjecture 8



/files/automatic-proving-facilities/conjecture_8

Here, a new conjecture can be discovered to identify the relationship between the area of the rectangle $EFGH$ and that of the kite $ACBD$, which is: “The area of rectangle $EFGH$ is four times the area of the kite $ACBD$ ”.

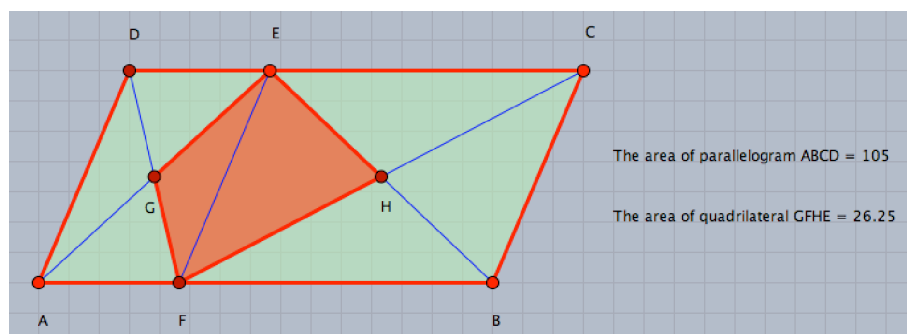
Conjecture 9



/files/automatic_proving_facilities/conjecture_9

The situation can be elaborated by generalizing the rectangle in conjecture 8 to any rectangle. Given that $ABCD$ is a rectangle, in which E lies between D and C and F lies between A and B such that $\overline{EF}$ is parallel to $\overline{CB}$, $\overline{AE}$ intersects $\overline{DF}$ at G , and $\overline{BE}$ intersects $\overline{CF}$ at H , as in the figure shown above, then the area of rectangle $ABCD$ is four times the area of the quadrilateral kite $GFHE$.

Conjecture 10



/files/automatic_proving_facilities/conjecture_10

Furthermore, the situation can be also elaborated by generalizing the rectangle to any parallelogram and the inner quadrilateral kite to any quadrilateral. Given that $ABCD$

is a parallelogram, in which E lies between D and C , F lies between A and B , such that $\overline{DA} \parallel \overline{EF} \parallel \overline{CB}$, $\overline{DF}$ intersects $\overline{AE}$ at G , $\overline{BE}$ intersects $\overline{CF}$ at H , as in the figure shown above, then the area of the parallelogram $ABCD$ is equal to four times the area of quadrilateral $GFHE$.

Enrichment Activity 3

Developing Macro-constructions

In this activity, the mathematically gifted students experience the macro-construction facilities of Cinderella in developing two macro-construction tools for the perpendicular bisector of a given line segment and the circum-center point of a triangle. Moreover, they are afforded the opportunity to develop their own macro-constructions for other geometric situations; for instance, in-center point of a triangle, and orthocenter point of a triangle.

Student Activity

In this activity, detailed instructions are given to the students to develop two macro-construction tools for the perpendicular bisector of a given line segment and the circum-center point of a triangle. Then, the students are asked to develop a macro-construction tool for each of the following:

- In-center point of a triangle
- Orthocenter point of a triangle
- Center of mass or centroid point of a triangle
- Euler line of a triangle

(see Student's Handout 6)

Activity Content Analysis

Concepts

- In-center point of a triangle
- Orthocenter point of a triangle
- Center of mass or centroid point of a triangle

- Euler line of a triangle
- Euler ratio
- Euler circle or nine-points circle

Generalizations

- The heights of a triangle are concurrent.
- The medians of a triangle are concurrent.
- The perpendicular bisectors of triangle sides are concurrent.
- The angle bisectors of interior angles of a triangle are concurrent.

Skills

- Skill of constructing dynamic configurations using Cinderella application.
- Skill of developing a macro-construction tool for perpendicular bisector of a given line segment using Cinderella application.
- Skill of developing a macro-construction tool for circum-center point of a triangle using Cinderella application.
- Skill of using Cinderella application to develop a macro-construction tool for each of the following:
 - In-center point of a triangle
 - Orthocenter point of a triangle
 - Center of mass or centroid point of a triangle
 - Euler line of a triangle

Objectives

The activity is designed to help the mathematically gifted students to:

- Experience the development of two macro-construction tools for the perpendicular bisector of a line segment and circum-center point of a triangle using Cinderella application.
- Develop their own macro-construction tools for:
 - In-center point of a triangle
 - Orthocenter point of a triangle
 - Center of mass or centroid point of a triangle
 - Euler line of a triangle

Materials

■ Computers	■ Handout 6
■ Cinderella application	■ CD-ROM of the program
■ LCD projector	

Vocabulary

■ Perpendicular bisector	■ Circum-center point of a triangle
■ Triangle height	■ In-center point of a triangle
■ Triangle median	■ Orthocenter point of a triangle
■ Angle bisector	■ Euler line of a triangle
■ Euler ratio of a triangle	■ Euler circle or nine-points circle
■ Right bisector	■ Mediator
■ Centroid point of a triangle	■ Center of mass of a triangle

Prerequisites

The students should know how to use the following tools before they start working in this activity:



Launch Cinderella or open new Cinderella window.



Use “**Move**” mode to move free elements by dragging the mouse.



Construct a point.



Construct a line connecting two points.



Generate the intersection of two geometric objects.



Construct a midpoint of a given line segment.



Construct the angular bisectors of two lines.



Construct a perpendicular to a line through a given point.



Construct a circle by a given center and a point on its circumference.



Construct a circle that passes through three given points.

Teaching and Learning Strategies

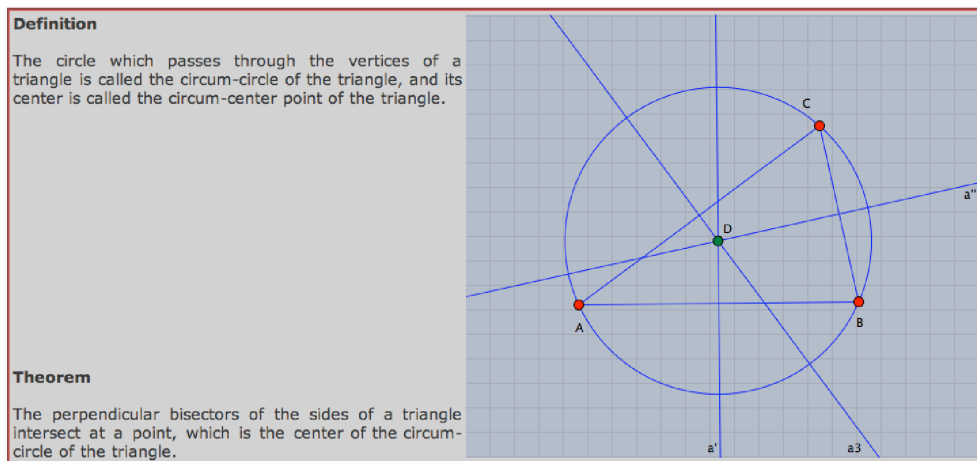
Have the students follow the instructions given in the first part of the activity's handout to develop a macro-construction tool for the perpendicular bisector of a line segment using Cinderella. Next, have them use the developed perpendicular bisector tool and also follow the given instructions to develop a macro-construction tool for the circum-center point of a triangle.

It is important to ensure that the students have the correct interpretation for both concepts: the perpendicular bisector of a line segment and circum-center point of a triangle before they start to develop their macro-construction tools. Here are the definitions for both concepts.

The Perpendicular Bisector

The perpendicular bisector is the straight line through the midpoint of a given line segment and is perpendicular to it. It is also known as right bisector or mediator.

The Circum-center Point of a Triangle



/files/developing_macro-constructions/circum-center_point_of_a_triangle

The circum-center point of a triangle is the point in which the three perpendicular bisectors of a triangle's sides meet, and it is known as the center of the circum-circle of the triangle (The circle which passes through the vertices of a triangle).

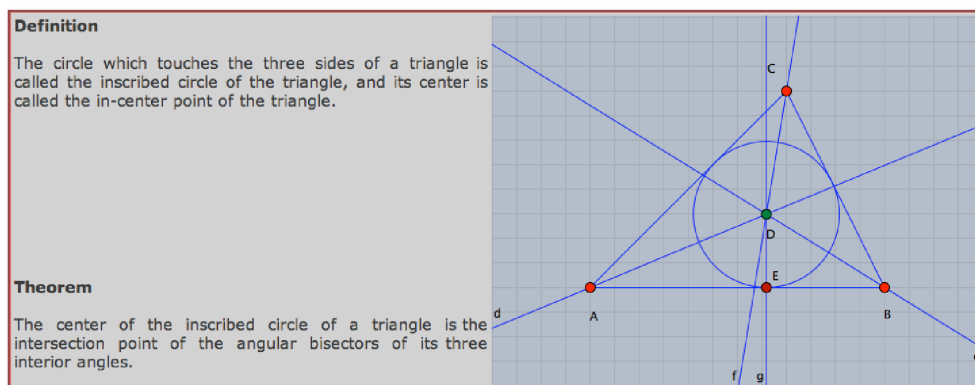
The second aim of this activity is to encourage the students to use the same technique to further develop macro-construction tools for other geometric situations. For addressing this, the teacher should ask the students to suggest other special geometric cases or situations that can be considered for developing other macro-construction tools. The teacher might say, for instance: “what if we consider the intersection point of the triangle's medians instead of its three perpendicular bisectors”, “what if we consider the intersection point of the triangles heights instead of its three perpendicular bisectors”, etc.

So, some possible suggestions would be to develop macro-construction tools for in-center point of a triangle, orthocenter point of a triangle, center of mass or centroid point of a triangle, and Euler line of a triangle. For each of these concepts, a macro-construction

tool is developed and placed on the /files/developing_macro-constructions/ folder on the accompanying CD-ROM.

Again, the teacher should ensure that the students have correct interpretations of these concepts before they start developing their macro-construction tools. For that purpose, the teacher can use the prepared applets, which are also placed on the same /files/developing_macro-constructions/ folder as a teacher presentation tool to convey the correct interpretation of the concepts to the students. Here are the definitions of the above-mentioned concepts:

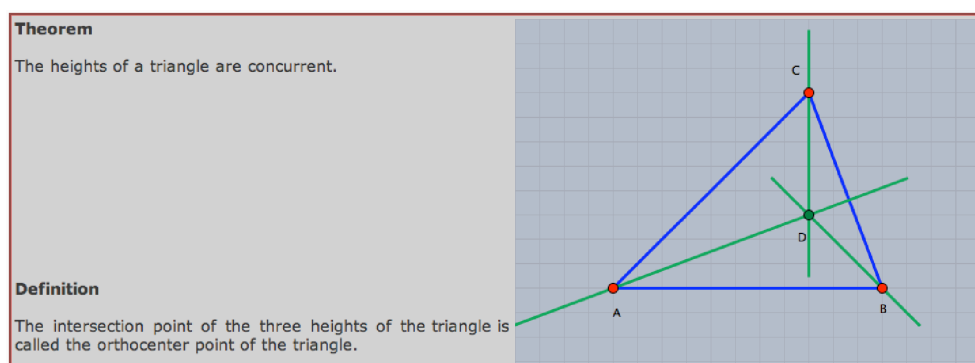
In-center Point of a Triangle



/files/developing_macro-constructions/in-center_point_of_a_triangle

In-center point of a triangle is the point in which the three angular bisectors of the triangle's angles meet, which is the center of the inscribed circle of the triangle (The circle which touches the three sides of a triangle).

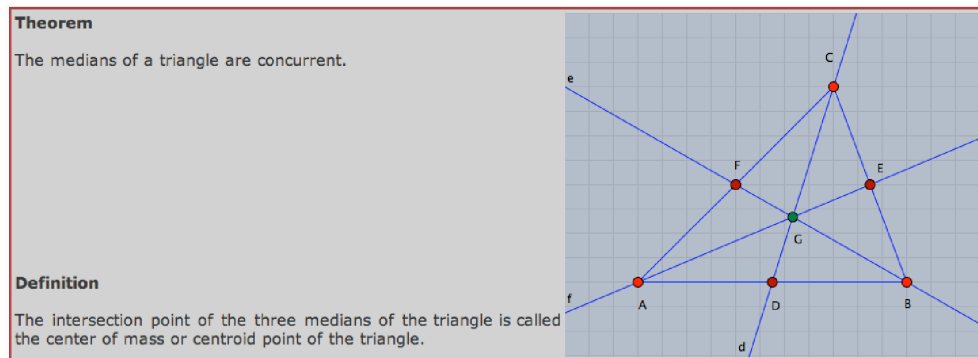
Orthocenter Point of a Triangle



/files/developing_macro-constructions/orthocenter_point_of_a_triangle

Orthocenter point of a triangle is the point in which the three heights of the triangle meet. (Triangle's height is a line through the vertex of the triangle perpendicular to the opposite side).

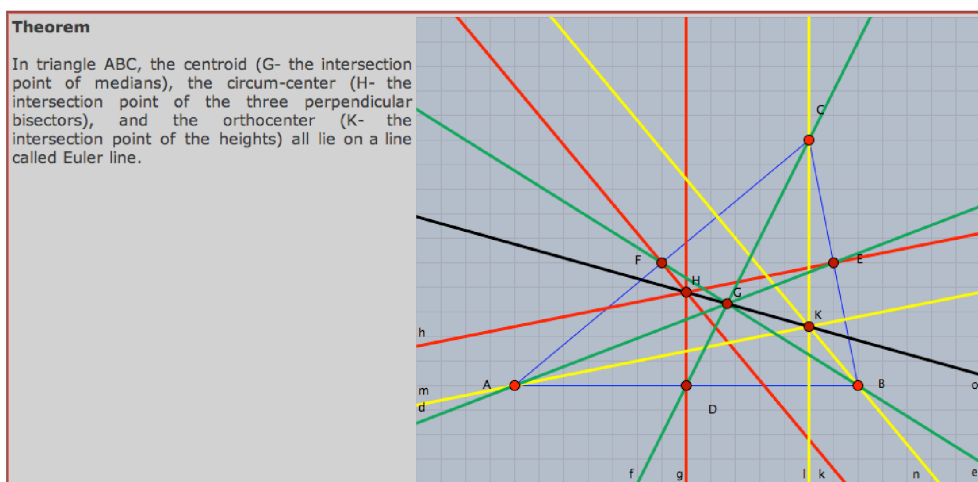
Center of Mass or Centroid Point of a Triangle



/files/developing_macro-constructions/centroid_point_of_a_triangle

Centroid point of a triangle is the point in which the three medians of a triangle meet (Triangle's median is a line joining the vertex of a triangle to the midpoint of the opposite side). It is also known as the center of mass of a triangle.

Euler Line of a Triangle



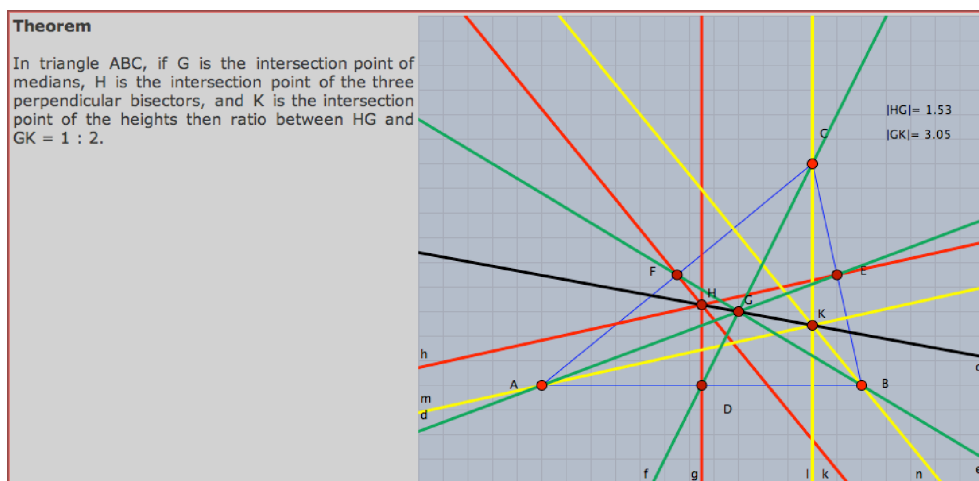
/files/developing_macro-constructions/euler_line

In any triangle, the centroid (the intersection point of medians), the circum-center (the intersection point of the three perpendicular bisectors), and the orthocenter (the intersection point of the heights) all lie on a line, which is called Euler⁹ line.

⁹Euler, Leonard (170783)

After the students develop the macro tool for the Euler line of a triangle, it is a good chance to encourage them to discover two more concepts (Euler Ratio and Euler Circle) related to the Euler line.

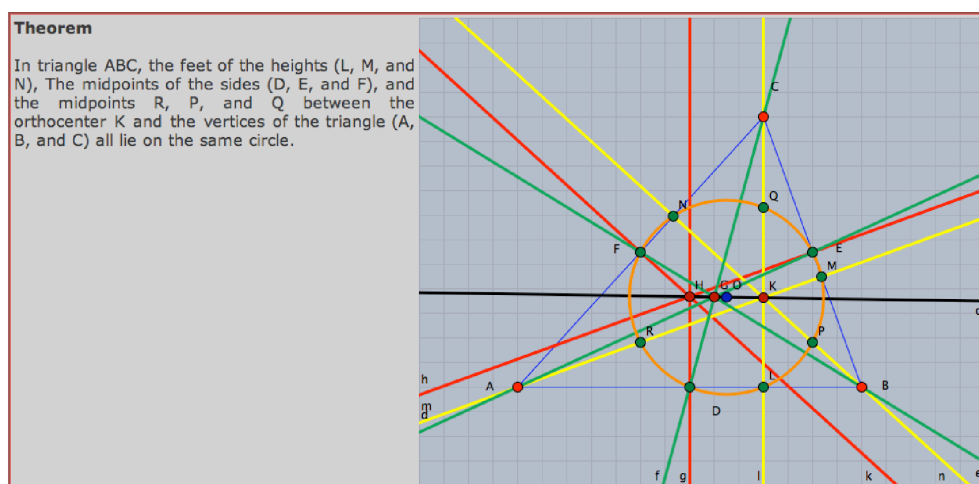
Euler Ratio



/files/developing_macro-constructions/euler_ratio

In any triangle, the ratio between the length of the line segment joining the centroid and circum-center points and that joining the centroid and the orthocenter points is 1 : 2 .

Euler Circle or Nine Points Circle



/files/developing_macro-constructions/euler_circle

In any triangle, the feet of the heights, the midpoints of the sides, and the midpoints between the orthocenter and the vertices of the triangle, all lie on the same circle called the Euler circle or nine points circle.

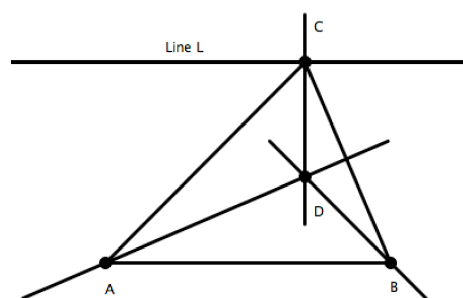
Enrichment Activity 4

Animating and Tracing Loci Facilities of Cinderella

In this activity, the mathematically gifted students experience the use of animating and tracing loci facilities of Cinderella to discover and generate geometric loci as well as elaborate on the activity situation by altering its conditions to discover and generate other geometric loci.

Student Activity¹⁰

In this activity, the students are asked to construct a dynamic configuration for a triangle ABC in a way that makes it possible to move the point C along a straight line L , which is parallel to the base $\overline{AB}$. They are also asked to construct the three heights of the triangle ABC and generate their intersection at the point D . Then, they are directed to make a conjecture about the locus of the heights' intersection point D , while the vertex C moves along the straight line L . After that, the students are instructed to use Cinderella's animating and tracing loci facilities to discover the locus of the heights' intersection point. Afterwards, the students are asked to use Cinderella to elaborate on the given situation by altering its conditions to discover and generate other geometric loci related to triangle (see Student's Handout 7).



¹⁰Resources used in designing this activity:

Friedrich, H. (1999). DGS in schools. Retrieved April 17, 2008, from: <http://math-www.uni-paderborn.de/~hugo/artikel/pdf>

Weth, T. (1998). Kreative Zugänge zum Kurvenbegriff. *Mathematikunterricht*, 44(4-5), 38–60.

Weth, T. Kegelschnitte und höhere Kurven als Ortslinien in Dreiecken. Retrieved September 28, 2007, from: http://www.didmath.ewf.uni-erlangen.de/kegel_weth/index.html

Activity Content Analysis

Concepts

- Geometric locus
- Height of a triangle
- Median of a triangle
- Angular bisector
- Perpendicular Bisector

Generalizations

- The heights of a triangle are concurrent.
- The medians of a triangle are concurrent.
- The perpendicular bisectors of triangle sides are concurrent.
- The angle bisectors of interior angles of a triangle are concurrent.

Skills

- Skill of constructing dynamic configurations using Cinderella application.
- Skill of using animating and tracing loci facilities of Cinderella to discover and generate geometric loci.
- Skill of elaborating on the given geometric situation by altering its conditions to discover and generate geometric loci using Cinderella.

Objectives

The activity is designed to help the mathematically gifted students to:

- Construct a dynamic configuration for the given situation using Cinderella application.

- Make a geometric conjecture about the locus of the heights' intersection point of a triangle, while a triangle vertex moves along a straight line, which is parallel to the opposite base of the movable triangle vertex.
- Use the animating and tracing loci facilities of Cinderella to discover and generate the geometric locus of the heights' intersection point of a triangle while a triangle vertex moves along a straight line, which is parallel to the opposite base of the movable triangle vertex.
- Elaborate on the given situation by altering its conditions to discover and generate other geometric loci related to triangle.

Materials

- Computers
- Cinderella application
- LCD projector
- Handout 7
- CD-ROM of the program

Vocabulary

- Geometric locus
- Triangle height
- Triangle median
- Angle bisector
- Perpendicular bisector
- Conic
- Ellipse
- Parabola
- Hyperbola
- Curve of high degree polynomial

Prerequisites

The students should know how to use the following tools before they start working in this activity:



Launch Cinderella or open new Cinderella window.



Use “**Move**” mode to move free elements by dragging the mouse.



Construct a point.



Construct a line connecting two points.



Construct parallel lines.



Construct perpendicular lines.



Generate the intersection of two geometric objects.



Construct a midpoint of a given line segment.



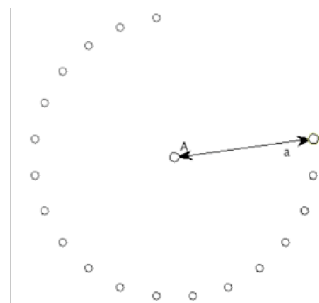
Construct the angular bisectors of two lines.

Teaching and Learning Strategies

Warm up

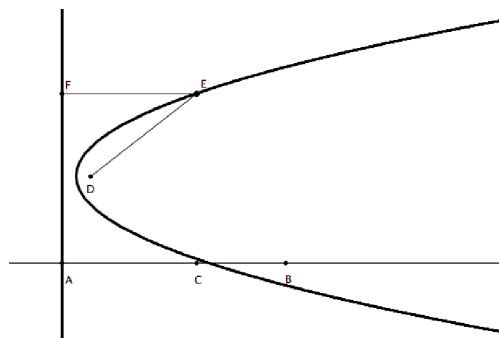
Before introducing the animating and tracing loci facilities of Cinderella to the students, the meaning of the geometric locus has to be explained to them. For doing so, the teacher should first present a definition for a geometric locus (locus is a set of points on a curve that share a property or satisfy particular conditions). Then, present some examples for geometric loci that help the students to visualize the meaning of geometric loci. Here are two examples for geometric loci that might be helpful:

Circle is defined as the locus of a point C that moves in the plane such that it always has a certain distance AC (the radius) to a fixed point A (the center).



Parabola is defined as the locus of a point E that moves in the plane such that it is equidistant from a point D (the focus) and a line AF (the directrix).

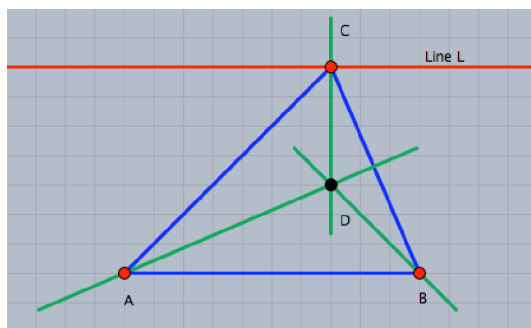
Regarding the parabola, A Cinderella file named “**Parabola**” is placed on the /files/animating_and_tracing_loci_facilities/ folder on the accompanying CD-ROM that can be



/files/animating-and-tracing-loci-facilities/parabola

used to visualize the meaning of geometric locus in the case of defining the parabola. Have the students open the “**Parabola**” file, move the free elements, and use animation buttons to experience and visualize the meaning of the locus in the case of parabola.

Construction

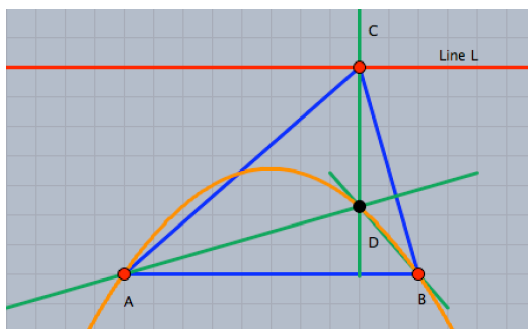


/files/animating-and-tracing-loci-facilities/the-problem-statement

Have the students follow the instructions of the activity handout (steps 1 and 2) for using Cinderella to construct a triangle ABC so that it is possible to move the vertex C along a straight line L , which is parallel to the base $\overline{AB}$, and then construct the three heights of the triangle and generate their intersection point at D . The students should come up with a dynamic configuration similar to the figure shown above, in which the triangle vertices A , B and C are completely arbitrary and can be moved freely, while the heights' intersection point D is a fixed point that is dependent on the other free points that represent the triangle vertices.

Investigating Conjecture

1. **The mover** is a free element that its movement generates the locus.
2. **The road** is an element incident to the mover. The mover will be moved along the road.
3. **The tracer** is the element that its trace is calculated and presented as a geometric locus.



The three elements in this activity are: The point C , the straight line L , and the point D , respectively. So, have the students select the point C , the straight line L , and the point D in this order. That will automatically generate the locus of the heights

intersection point as shown in the above figure.

Have the students switch to “**Move**” mode to experience the locus when you move the point C along the line L .

For more investigation, we can use the animation facilities of Cinderella to automatically move the point C along the straight line L instead of dragging it with the mouse. Have the students follow the directions of the activity handout (step 6) for using Cinderella to generate an animation that automatically moves the point C along the straight line. For generating an animation, two elements should be defined in order:

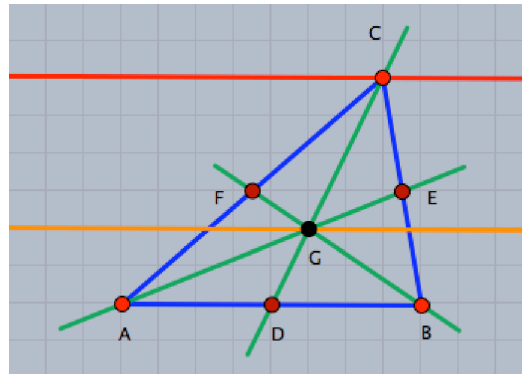
1. **The mover** is a free element that its movement generates the animation.
2. **The road** is an element incident to the mover. During the animation, the mover will be moved along the road.

The two elements in this activity are: The point C and the straight line L , respectively. It is also possible to generate the same animation by selecting the locus. In this case, the animation will automatically select the mover and the road of the locus. So, have the students either select the point C then the straight line L or the locus curve to generate the animation. An animation control panel  pops up in the lower left corner of the window. It has three buttons like a CD player to start, stop, and pause the animation. It also has a speed slide to control the animation speed. Have the students use the animation buttons with the speed slide to experience the locus of the point D when the point C automatically moves along the straight line L .

Other Geometric Loci Related to Triangle

This situation provides many opportunities to discover and generate other geometric loci related to triangle if it is elaborated by altering its conditions. One possibility to alter the situation is to consider other lines of triangle instead of the heights such as medians, angular bisectors, and perpendicular bisectors.

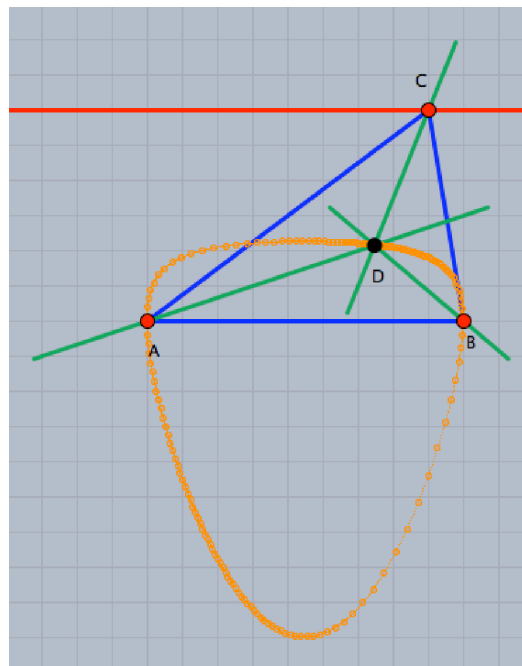
The Locus of the Intersection Point of Medians



/files/animating_and_tracing_loci_facilities/medians_of_a_triangle

As explained in the example, which is given in the activity handout, the situation can be elaborated on to discover and generate the locus of the medians' intersection point of a triangle when a triangle's vertex moves along a straight line. In this case, as shown in the figure above, the locus of the medians' intersection point is a straight line, which is parallel to the road straight line and the triangle base.

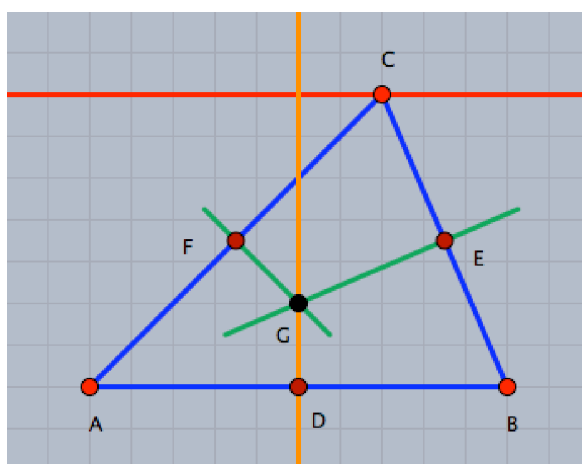
The Locus of the Intersection Point of Angular Bisectors



/files/animating_and_tracing_loci_facilities/angular_bisectors_of_a_triangle

Since the angular bisectors of the interior angles of a triangle are concurrent, the given situation can be altered to discover and generate the locus of the angular bisectors' intersection point instead of the heights' intersection point. This situation produces an algebraic curve of higher degree as a locus of the intersection point of the angular bisectors.

The Locus of the Intersection Point of Perpendicular Bisectors



/files/animating-and-tracing-loci-facilities/perpendicular-bisectors-of-a-triangle

Similarly, since the perpendicular bisectors of the sides of a triangle are congruent, so substituting the heights' intersection point by the perpendicular bisectors' intersection point in the given situation produces a new situation to discover and generate the locus of the perpendicular bisectors point of the sides of a triangle. The situation produces a straight-line locus, which is perpendicular to the road line.

Furthermore, the situation can be elaborated on to discover and generate other loci by considering the intersection point of two different lines in the triangle such as the intersection point of a height and a median, the intersection point of a median and a perpendicular bisector, etc.

In this context, an assignment project could be presented to the students to design new situations for discovering and generating other loci using the intersection point of two different lines in the triangle. The table below can be used in organizing the situations

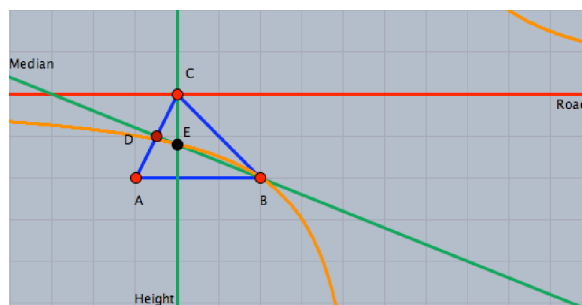
with a notice that the line in the header of the table is the line which moves through the movable vertex (the vertex C).

For this assignment project, the teacher would present the table to the students and ask them to design a new situation for each pair of lines in the table cells so that they can discover and generate the locus of the intersection point of the two lines, then describe the traced curve by the intersection point.

	Height (H)	Median (M)	Angular Bisector (A. B.)
Height (H)	Hs. inter. point	$M + H$	A. B. + H
Median (M)	$H + M$	Ms. inter. point	A. B. + M
Angular Bisector (A. B.)	$H + A. B.$	$M + A. B.$	A. Bs. inter. point
Perpendicular Bisector (P. B.)	Height + P. B.	$M + P. B.$	A. B. + P. B.

In the following sections, the 9 situations are presented and the locus for each situation is also generated and described. For each situation, A Cinderella file is designed and placed on the /files/animating_and_tracing_loci_facilities/using_movable_vertex/ folder on the accompanying CD-ROM that can be used in discussing the assignment project.

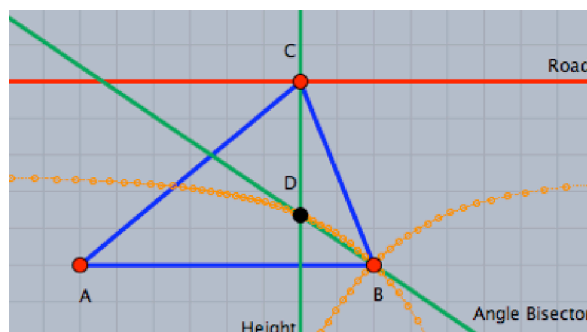
The Locus of the Intersection Point of a Height and a Median



/files/animating_and_tracing_loci_facilities/using_movable_vertex/
height_and_median

The situation that includes the intersection point of a height and a median produces a hyperbola locus.

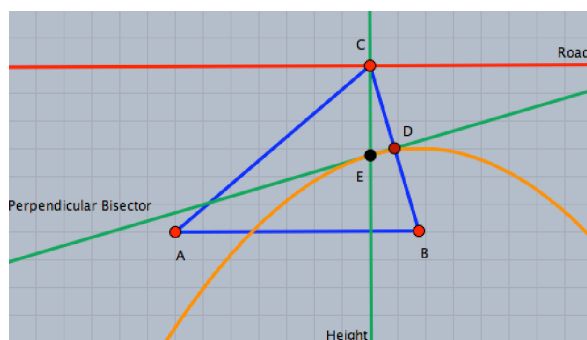
The Locus of the Intersection Point of a Height and an Angular Bisector



/files/animating_and_tracing_loci_facilities/using_movable_vertex
/height_and_angle_bisector

In the case of the intersection between a height and an angular bisector, the locus is an algebraic curve of higher degree.

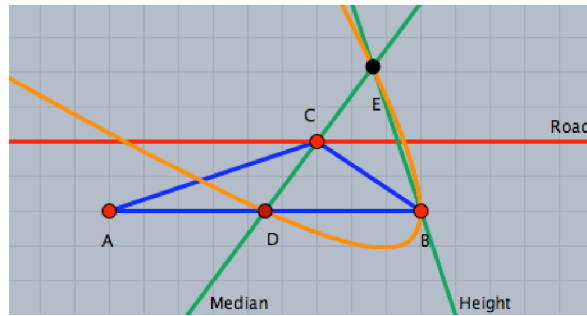
The Locus of the Intersection Point of a Height and a Perpendicular Bisector



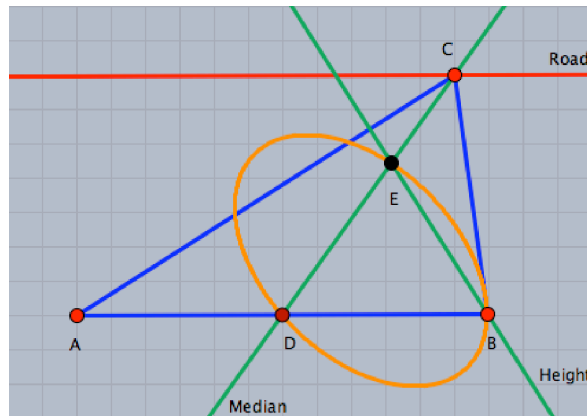
/files/animating_and_tracing_loci_facilities/using_movable_vertex /
height_and_perpendicular_bisector

As shown in the figure above, the situation produces a parabola locus of the intersection point of a height and a perpendicular bisector.

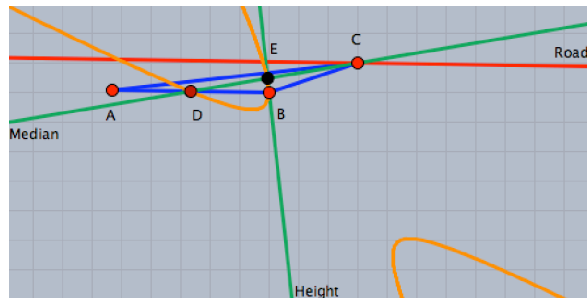
The Locus of the Intersection Point of a Median and a Height



(a) Parabola



(b) Ellipse

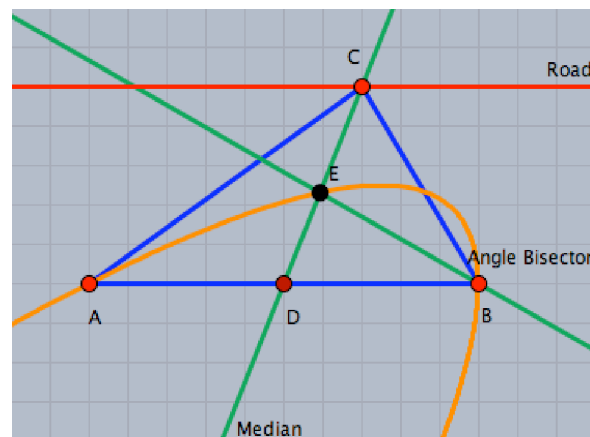


(c) Hyperbola

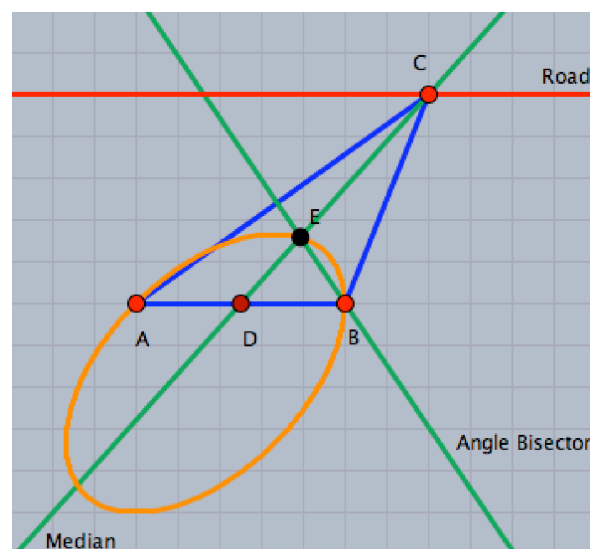
/files/animating_and_tracing_loci_facilities/using_movable_vertex/
median_and_height

In this case, the locus depends on the height of the triangle and it could be parabola, ellipse, or hyperbola. The figures above show the locus in three different cases of the triangle ABC .

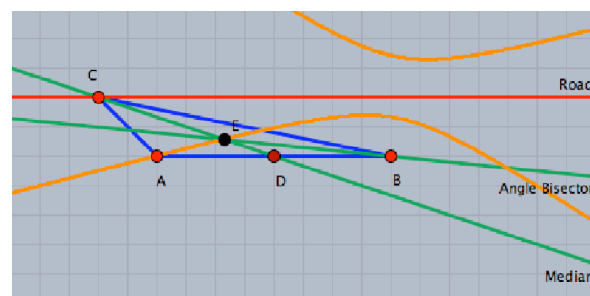
The Locus of the Intersection Point of a Median and an Angular Bisector



(a) Parabola



(b) Ellipse

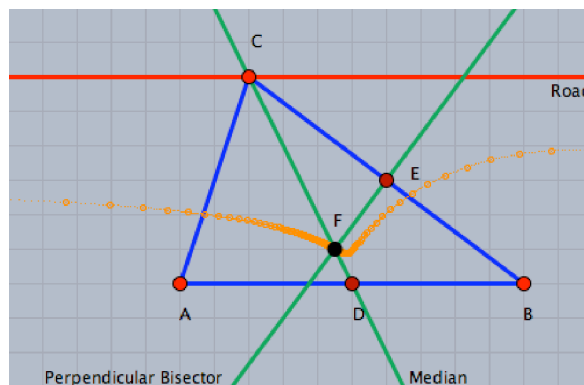


(c) Hyperbola

/files/animating_and_tracing_loci_facilities/using_movable_vertex
/median_and_angle_bisector

The locus of the intersection point of a median and an angular bisector of a triangle could also be parabola, ellipse, or hyperbola as shown above.

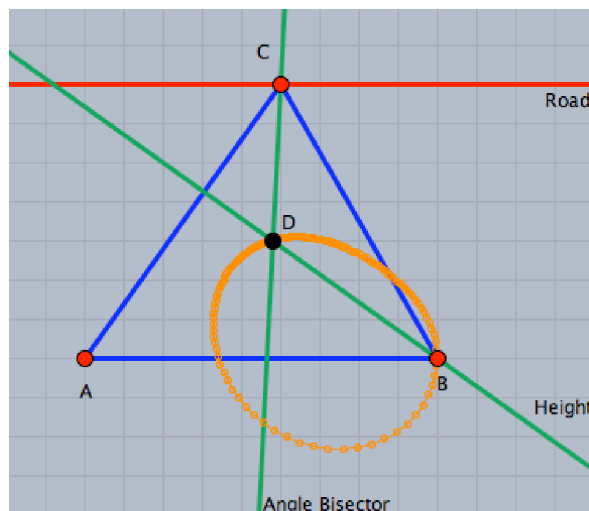
The Locus of the Intersection Point of a Median and a Perpendicular Bisector



/files/animating_and_tracing_loci_facilities/using_movable_vertex
/median_and_perpendicular_bisector

The locus of the intersection point of a median and a perpendicular bisector of a side of the situation provides an algebraic curve of higher degree.

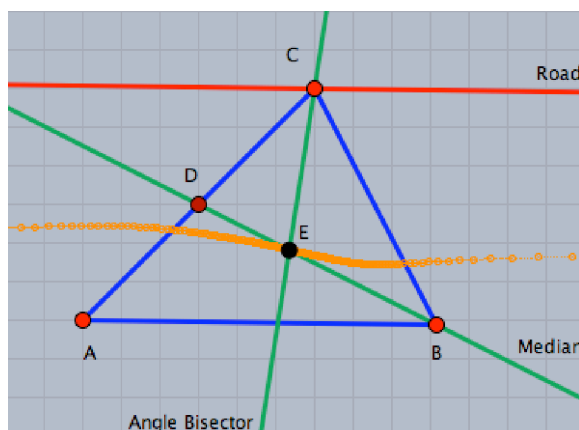
The Locus of the Intersection Point of an Angular Bisector and a Height



/files/animating_and_tracing_loci_facilities/using_movable_vertex
/angle_bisector_and_height

Here Cinderella calculates an incorrect locus, considering that the locus has changed while C moves along the road line. Here the locus is an algebraic curve higher than the fourth order.

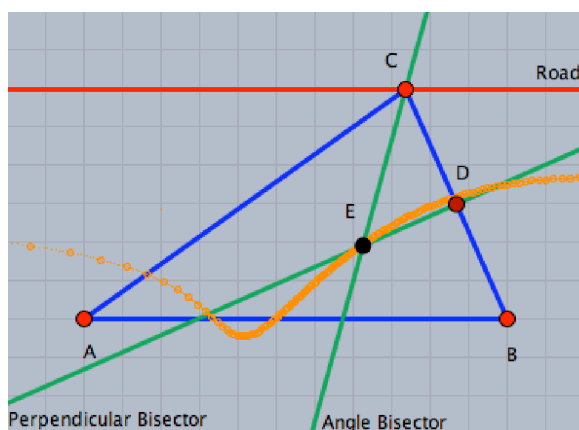
The Locus of the Intersection Point of an Angular Bisector and a Median



/files/animating_and_tracing_loci_facilities/using_movable_vertex
/angle_bisector_and_median

Here, the situation provides an algebraic curve higher than the third degree.

The Locus of the Intersection Point of an Angular Bisector and a Perpendicular Bisector



/files/animating_and_tracing_loci_facilities/using_movable_vertex
/angle_bisector_and_perpendicular_bisector

This situation also provides an algebraic curve of higher degree.

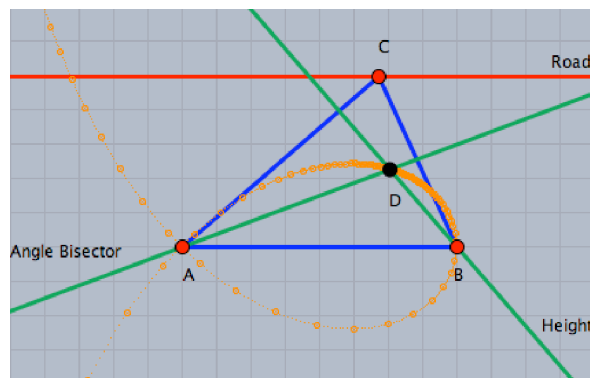
The situation can be further elaborated on to discover and generate other interesting loci related to the triangle, if the two intersecting lines of the triangle are constructed so that they do not pass through the movable vertex. The following table should help a lot in organizing the situations.

	Height (H)	Perpendicular Bisector (P. B.)	Median (M))
Angular Bisector (A. B.)	$H + A. B.$	$P. B. + A. B.$	$M + A. B.$
Height (H)		$P. B. + H$	$M + H$
Perpendicular Bisector (P. B.)			$M + P. B.$

A complementary part of the assignment project could be also presented to the students for designing situations that include the intersection point of two lines of the triangle that do not pass through the movable vertex guided by the table above.

Here, six more situations are presented and their loci are also generated and described. For each situation, A Cinderella file is designed for each situation and placed on the /files/animating_and_tracing_loci_facilities/without_movable_vertex/ folder on the accompanying CD-ROM that can be used in discussing these situations.

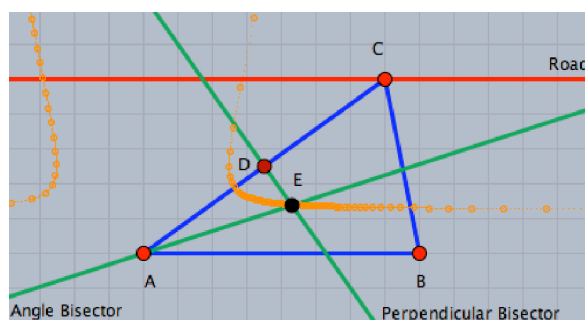
The Locus of the Intersection Point of a Height and an Angular Bisector



/files/animating_and_tracing_loci_facilities/without_movable_vertex
/height_and_angle_bisector

This situation produces a strophoid curve as a locus of the intersection point of a height and an angular bisector that do not pass through the movable vertex.

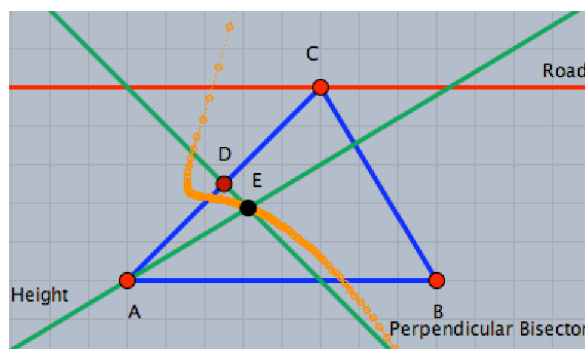
The Locus of the Intersection Point of a Perpendicular Bisector and an Angular Bisector



/files/animating_and_tracing_loci_facilities/without_movable_vertex/
perpendicular_bisector_and_angle_bisector

The situation provides an algebraic curve of higher degree locus for the intersection point of a perpendicular bisector and an angular bisector of a triangle.

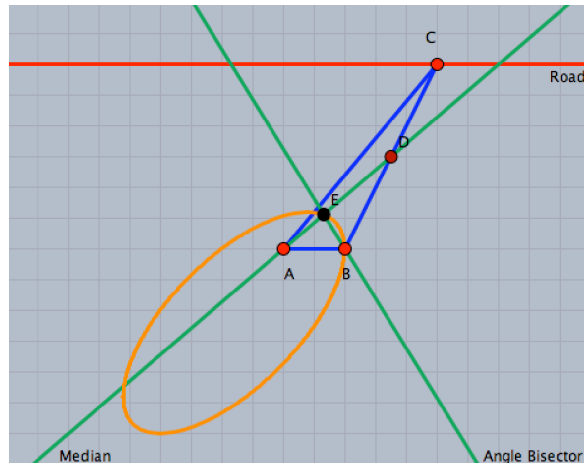
The Locus of the Intersection Point of a Perpendicular Bisector and a Height



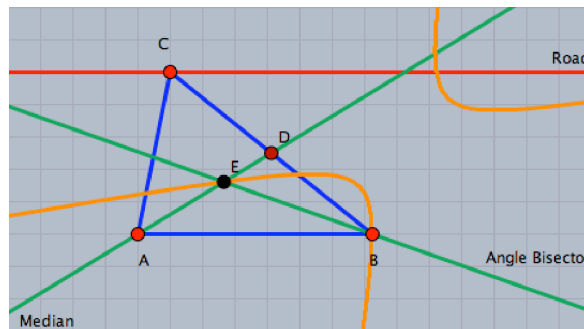
/files/animating_and_tracing_loci_facilities/without_movable_vertex/
perpendicular_bisector_and_height

The situation provides a locus as an algebraic curve of higher degree.

The Locus of the Intersection Point of a Median and an Angular Bisector



(a) Ellipse

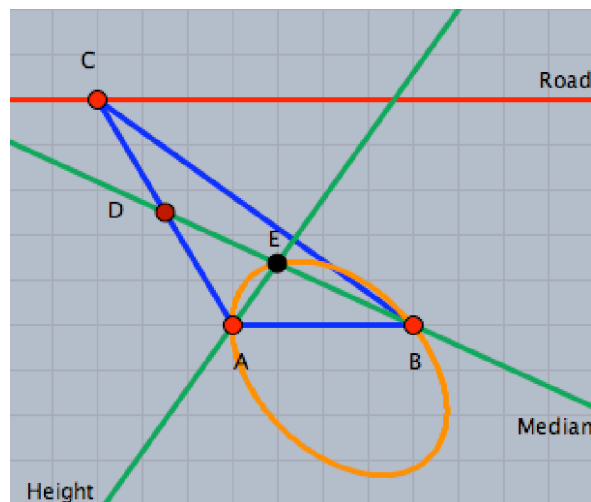


(b) Hyperbola

/files/animating_and_tracing_loci_facilities/using_movable_vertex
/median_and_angle_bisector

The locus of the intersection point of a median and an angular bisector of a triangle could be ellipse, or hyperbola as shown above.

The Locus of the Intersection Point of a Median and a Height



(a) Ellipse

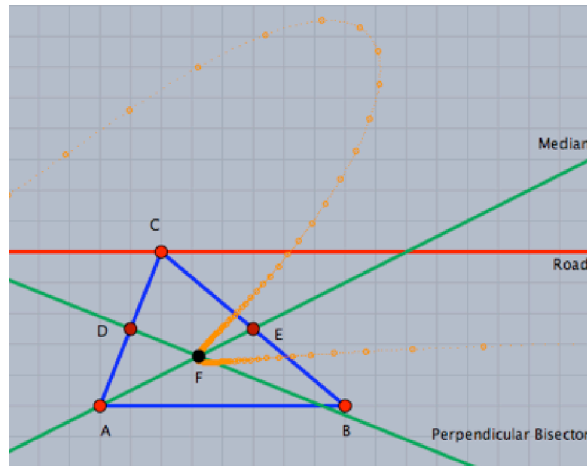


(b) Hyperbola

/files/animating_and_tracing_loci_facilities/without_movable_vertex
/median_and_height

The situation provides an ellipse, or a hyperbola locus.

The Locus of the Intersection Point of a Median and a Perpendicular Bisector



/files/animating_and_tracing_loci_facilities/without_movable_vertex
/median_and_perpendicular_bisector

The situation provides an algebraic curve of higher degree locus.

Enrichment Activity 5

Midpoints of the Sides of a Quadrilateral

This activity is designed to help the mathematically gifted students make and investigate a geometric conjecture about the figure obtained by joining the midpoints of the adjacent sides of a quadrilateral. It also helps them figure out mathematical proofs for this conjecture and generate new problems by altering the problem situation (considering a special case, a general case, or even an extreme case of the problem situation) or reformulating the given problem in different ways.

The Activity Problem¹¹

What is the figure obtained by joining the midpoints of the adjacent sides of a quadrilateral? (see Student's Handout 8)

Activity Content Analysis

Generalizations

- A parallelogram is a quadrilateral, in which each two opposite sides are equal in length.
- A parallelogram is a quadrilateral, in which each two opposite sides are parallel.

¹¹Resources used in designing this activity:

Christou, C., Mousoulides, N., Pittalis, M., and Pitta-Pantazi, D. (2005). Problem solving and problem posing in a dynamic geometry environment. *The Montana Mathematics Enthusiast*, **2**(2), 125–143.

Contreras, J. (2003). Using dynamic geometry software as a springboard for making conjectures, solving problems and posing problems. Retrieved February 20, 2007, from: <http://www.usm.edu/pt3/pa/jc01.htm>

Haja, S. (2005). Investigating the problem-solving competency of pre-service teachers in dynamic geometry environment. In H. L. Chick and J. L. Vincent, editors, *Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education*, volume 3, pages 81–87. Melbourne, Australia.

Loy, J. (2003). Varignons theorem. Retrieved July 3, 2008, from: <http://www.jimloy.com/cindy/varignon.htm>

Loy, J. (2003). Wittenbauers theorem. Retrieved July 3, 2008, from: <http://www.jimloy.com/cindy/witten.htm>

- A parallelogram is a quadrilateral, in which each two opposite angles are equal in measure.
- A parallelogram is a quadrilateral, in which the sum of each two consecutive angles is supplementary.
- The slopes of two parallel lines are equal.
- The position vectors of two parallel lines are equal.
- The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is equal to half of it.

Skills

- Skill of constructing dynamic figures using Cinderella application.
- Skill of making geometric conjectures using different facilities of Cinderella application.
- Skill of investigating geometric conjecture using different facilities of Cinderella in different ways.
- Skill of elaborating on the given geometric situation using different facilities of Cinderella application.

Objectives

The activity is designed to help the mathematically gifted students to:

- Construct a dynamic configuration for a quadrilateral and the figure obtained by joining the midpoints of the quadrilateral using Cinderella application.
- Make a geometric conjecture about the figure obtained by joining the midpoints of the adjacent sides of the quadrilateral.
- Investigate the conjecture made about the figure obtained by joining the midpoints of the adjacent sides of the quadrilateral in different ways.

- Produce as many mathematical proofs as possible to prove the investigated conjecture of the figure obtained by joining the midpoints of this quadrilateral.
- Pose follow-up problems related to the original problem.

Materials

- Computers
- Cinderella application
- LCD projector
- Handout 8
- CD-ROM of the program

Vocabulary

- Quadrilateral
- Adjacent sides
- Opposite sides
- Convex quadrilateral
- Concave quadrilateral
- Parallelogram
- Rectangle
- Rhombus
- Square
- Trapezium
- Kite
- Position vector
- Slope of a straight line

Prerequisites

The students should know how to use the following tools before they start working in this activity:



Launch Cinderella or open new Cinderella window.



Use “**Move**” mode to move free elements by dragging the mouse.



Construct a point.



Construct a line segment joining two points.



Construct a midpoint of a given line segment.



Generate the intersection of two geometric objects.



Measure the length of line segments.



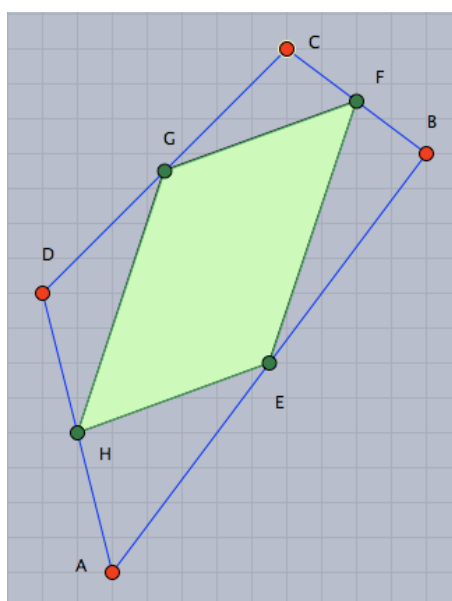
Measure angles.



Define polygons.

Teaching and Learning Strategies

Have the students use Cinderella to construct a quadrilateral, midpoint of each side, and connect the midpoints of the adjacent sides of the quadrilateral to end up with a dynamic configuration for the problem as shown below, so that four vertices of the quadrilateral $ABCD$ – A , B , C , and D – are completely arbitrary and can be altered freely, while E , F , G , and H are fixed as midpoints that can't be altered. A Cinderella file named “the_problem_statement” is placed on the `/files/quadrilateral_midpoints_activity/` folder on the accompanying CD-ROM.

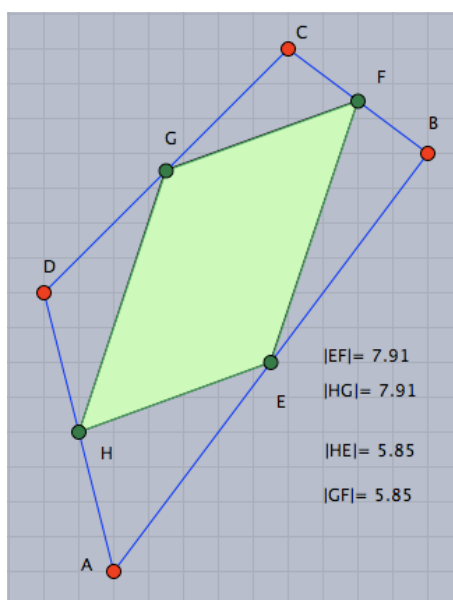


`/files/quadrilateral_midpoints_activity/the_problem_statement`

Making Conjecture

Have the students figure out a geometric conjecture about the obtained figure $EFGH$. When the vertices of the quadrilateral $ABCD$ are dragged using Cinderella's "Move" mode, the figure obtained by joining the midpoints of its adjacent sides seems to be a parallelogram no matter what the type of the original quadrilateral is.

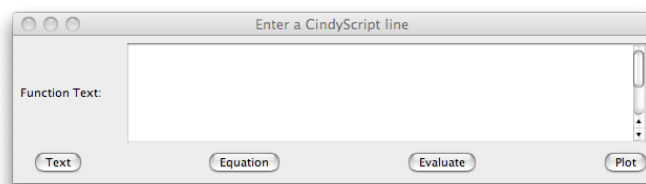
Investigating Conjecture



/files/quadrilateral_midpoints_activity/the_problem_investigation_1

To investigate the made conjecture, have the students use Cinderella's measuring facilities to find the lengths of each two opposite sides of the inner quadrilateral $EFGH$. Then, have the students use Cinderella's dragging facilities through Cinderella's "Move" mode to alter the quadrilateral $ABCD$, and visually observe the obtained figure $EFGH$ and the lengths of its opposite sides. When the vertices of the quadrilateral $ABCD$ are dragged, the lengths of each two opposite sides of the inner quadrilateral $EFGH$ are still equal, as shown above, and this is enough to show that the obtained figure $EFGH$ is a parallelogram based on one of the parallelogram's properties, which is: "A parallelogram is a quadrilateral, in which each two opposite sides are equal in length".

Another way to examine whether the obtained figure is a parallelogram or not is to calculate the slopes of each two opposite sides using CindyScript language by entering little CindyScript lines. For doing so, have the students switch to the “**Define a Function**” mode by clicking the button  and click somewhere on the construction area; then the following window will pop up:



“Define a Function” Dialog Box

Then, have the students enter the following text to calculate the slopes of each two opposite sides of the obtained figure:

```
"The slope of EF =" +e.slope
```

```
"The slope of HG =" +g.slope
```

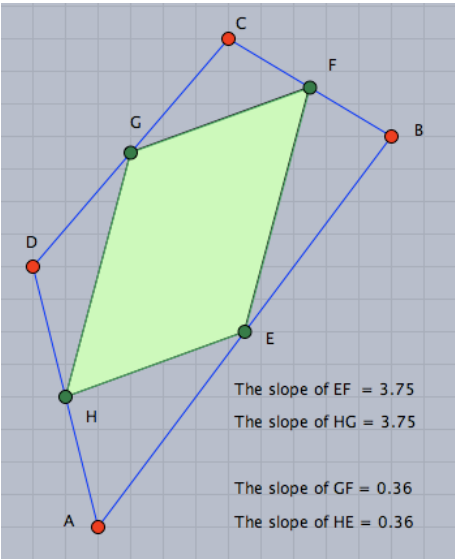
```
"The slope of GF =" +f.slope
```

```
"The slope of HE =" +h.slope
```

Afterwards, have them click on the “**Text**” button on the dialog box. This will calculate the slopes of each two opposite sides of the obtained figure.

Again, have the students use Cinderella’s “**Move**” mode to alter the quadrilateral $ABCD$ and observe the obtained figure $EFGH$ and the slopes of its opposite sides. When the vertices of the quadrilateral $ABCD$ are dragged, the slopes of each two opposite sides of the inner quadrilateral $EFGH$ remain equal, as shown in the figure . Since the slopes of each two opposite sides of the inner quadrilateral $EFGH$ are equal then they are parallel. This confirms that the obtained figure $EFGH$ is a parallelogram based on the parallelogram’s definition which states that “A parallelogram is a quadrilateral, in which each two opposite sides are parallel”.

Cinderella has a special window to describe the construction elements, which is called “**Construction Text**” window. We can get it by using the menu item “**Views/Construction Text**”. In this window, each geometric element of the construction has



/files/quadrilateral_midpoints_activity/the_problem_investigation_3

an information row that describes this element. The figure below shows the construction text window of the problem construction.

Cinderella: Midpoints of the Sides of a Quadrilat...			
	Who?	What?	Where?
● A	A	Point(19.36 -15.8)	(19.36 -15.8)
● B	B	Point(28.64 -2.96)	(28.64 -2.96)
● C	C	Point(24 1)	(24 1)
● D	D	Point(16.36 -6.2)	(16.36 -6.2)
— a	a	Segment(A;B)	-
— b	b	Segment(B;C)	-
— c	c	Segment(C;D)	-
— d	d	Segment(D;A)	-
● E	E	Mid(A;B)	(24 -9.38)
● F	F	Mid(B;C)	(26.32 -0.98)
● G	G	Mid(C;D)	(20.18 -2.6)
● H	H	Mid(D;A)	(17.86 -11)
■ Poly0	Poly0	Polygon(E;F;G;H)	-
K	K	Mid(E;F)	(25.16 -5.18)
dist0	dist0	Distance(E;F)	8.71
L	L	Mid(G;H)	(19.02 -6.8)
M	M	Mid(E;H)	(20.93 -10.19)
dist1	dist1	Distance(E;H)	6.35
N	N	Mid(F;G)	(23.25 -1.79)
e	e	Segment(H;E)	-
f	f	Segment(E;F)	-

Move free elements by dragging the mouse

“Construction Text” Window

Using the highlighted information rows of the four midpoints, the students can investigate the conjecture by considering the Cartesian coordinates of the four midpoints E , F , G , and H . These coordinates can be used to find either the lengths of each two opposite sides of the obtained figure, using the mathematical formula: The distance between

two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, or the slopes of each two opposite sides using the mathematical formula: The slope of a line joining between two points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

So, as another way to investigate the conjecture, have the students use the above formulas to find either the lengths of each two opposite sides or the slopes of each opposite sides and examine whether the obtained figure is a parallelogram or not.

The Cartesian coordinates can also be used to show that the obtained figure is a parallelogram in another way by finding the position vectors of the sides of the obtained figure and using them to examine whether the obtained figure is a parallelogram or not.

In an arbitrary case of the above configuration of the problem, the coordinates of the four midpoints E , F , G , and H are: $E = (24, -9.38)$, $F = (26.32, -0.98)$, $G = (20.18, -2.6)$ and $H = (17.86, -11)$. The position vectors of the sides of the obtained figure can be calculated as follows:

$$\begin{aligned} EF &= OF - OE \\ &= (26.32, -0.98) - (24, -9.38) \\ &= (2.32, 8.4) \end{aligned}$$

Similarly,

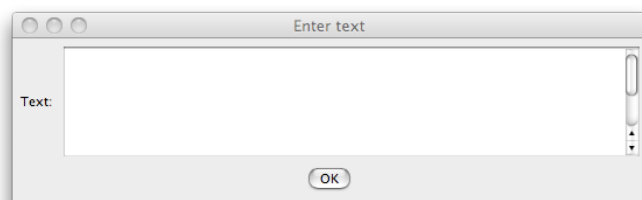
$$\begin{aligned} HG &= OG - OH \\ &= (20.18, -2.6) - (17.86, -11) \\ &= (2.32, 8.4) \end{aligned}$$

$$\begin{aligned} HE &= OE - OH \\ &= (24, -9.38) - (17.86, -11) \\ &= (6.14, 1.62) \end{aligned}$$

$$\begin{aligned} GF &= OF - OG \\ &= (26.32, -0.98) - (20.18, -2.6) \\ &= (6.14, 1.62) \end{aligned}$$

Noticing the above coordinates of the position vectors of the sides of the obtained figure; it can be easily deduced that the position vectors of each two opposite sides of the obtained figure are the same. Since the position vectors of each two opposite sides of the obtained figure are equal then each two opposite sides of the obtained figure are parallel. Consequently, the obtained figure $EFGH$ is a parallelogram based on the parallelogram's definition which states that "A parallelogram is a quadrilateral, in which each two opposite sides are parallel".

Furthermore, Cinderella's calculation facilities afford another opportunity to calculate interactive position vectors of each two opposite sides of the obtained figure using either CindyScript language in "**Define a Function**" mode or referencing text in "**Add Text**" mode. Now, to calculate the interactive position vectors of the sides of the obtained figure have the students switch to "Add Text" mode by clicking the button  and click somewhere on the construction area; then the following window will pop up:



"Add Text" Window

Then, have the students enter the following referencing text in the "**Add Text**" window:

The position vector of $GF = F - G = @\{F-G\}$

The position vector of $HE = E - H = @\{E-H\}$

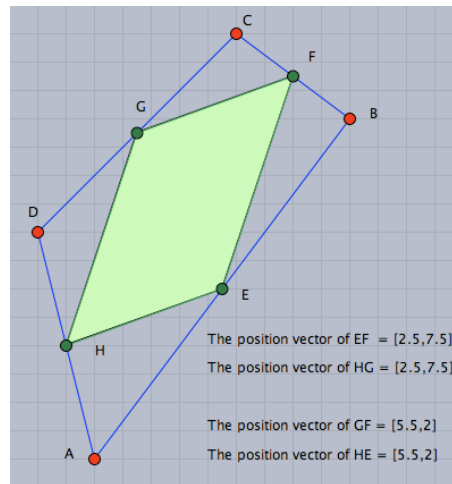
The position vector of $EF = F - E = @\{F-E\}$

The position vector of $HG = G - H = @\{G-H\}$

After that, have them click on the OK button on "**Add text**" window. This will calculate the interactive position vectors of the sides of the obtained figure, as shown in the figure .

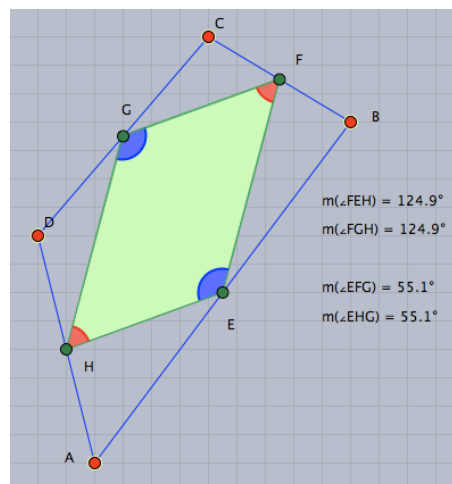
Again, have the students alter the original quadrilateral $ABCD$ by dragging one of its

vertices and observe the position vectors of the sides of the obtained figure.



/files/quadrilateral_midpoints_activity/the_problem_investigation_4

There is another possibility to show that the obtained figure is a parallelogram using both dragging and measuring facilities of Cinderella. Have the students measure each two opposite angles in the obtained figure, then alter the quadrilateral $ABCD$, and visually observe the obtained figure and the measures of each two opposite angles. When the vertices of the quadrilateral $ABCD$ are dragged, the measures of each two opposite angles of the obtained figure remain equal, as shown in the figure [below](#), which is also enough evidence to reveal that the obtained figure is a parallelogram based on another property of the parallelogram, which is: “A parallelogram is a quadrilateral, in which each two opposite angles are equal in measure”



/files/quadrilateral_midpoints_activity/the_problem_investigation_2

There is one more possibility to show that the obtained figure is a parallelogram by considering the sum of each two consecutive angles in the obtained figure. In this respect, Cinderella's measuring and dragging facilities are used to investigate the sum of each two consecutive angles in the obtained figure, based on the property of the parallelogram which states that "A parallelogram is a quadrilateral, in which the sum of each two consecutive angles is supplementary".

However, the teacher should encourage the students to use constructing, dragging, measuring, and calculating facilities of Cinderella to investigate the conjecture in different ways.

Here, it is worth mentioning that providing the students with opportunities to use Cinderella to investigate the conjecture in different ways and different situations affects them in many positive ways. For one thing, it trains the students to overcome mind fixation in thinking and break their mental sets while they are designing their endeavors using Cinderella to examine the conjecture and accordingly helps the students to develop their flexibility. Moreover, It encourages the students to make new mathematical connections between Euclidean geometric concepts included in the given situation and another set of concepts belonging to analytical geometry (e.g., slope of the straight line, position vector, and Cartesian coordinates) that are used in the investigation.

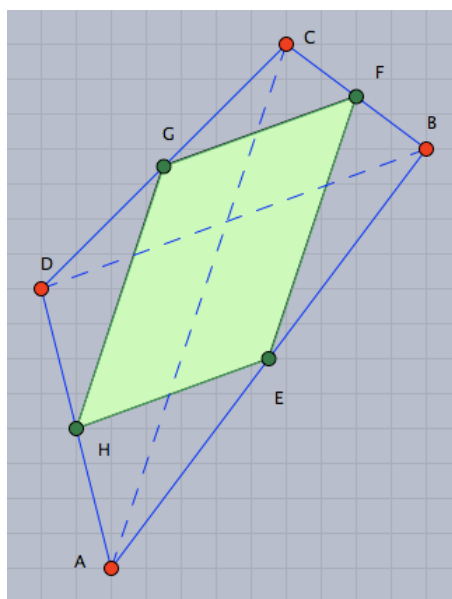
However, after the students investigate the made conjecture in different ways, have them figure out the investigated conjecture in the form of mathematical theorems. For example, it can take this form: *"The quadrilateral obtained by joining the midpoints of the sides of any quadrilateral is a parallelogram"*.

Producing Mathematical Proofs of the Conjecture

After the students use Cinderella to visually and dynamically investigate the validity of their conjecture, the next step then is to logically convince them of the validity of the conjecture by producing a logical mathematical proof. So, have the students think of how to prove the investigated conjecture using the formal logical steps. Encourage them

to come up with as many methods as they can to prove the investigated conjecture. It may be helpful to mention some hints while they are thinking of the proof to help them come up with many proof methods, such as:

- Construct the diagonal AC .
- In $\triangle ABC$, think about the relationship between EF and AC .
- In $\triangle ADC$, think about the relationship between HG and AC .
- Construct the diagonal BD .
- In $\triangle BCD$, think about the relationship between FG and BD .
- In $\triangle ABD$, think about the relationship between EH and BD .
- Consider the lengths of each two opposite sides in the obtained quadrilateral $EFGH$.
- Consider the measures of each two opposite angles in the obtained quadrilateral $EFGH$.
- Consider the measures of each two consecutive angles in the obtained quadrilateral $EFGH$.



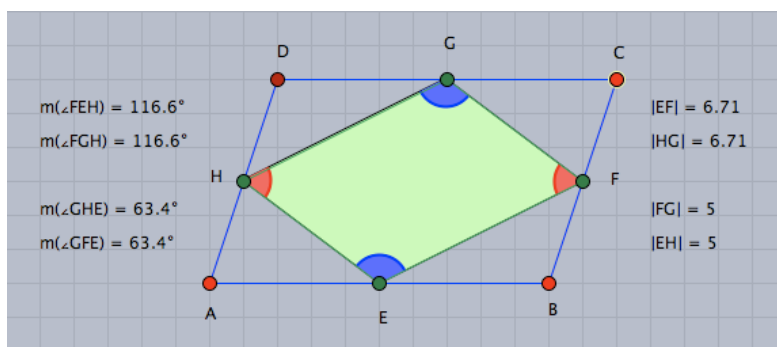
Constructing the Two Diagonals AC , and BD

After providing the mathematical proof of the investigated conjecture, it can become a mathematical theorem for the students.

Posing Follow-up Problems

Many follow-up problems related to the given problem can be posed if special or even extreme cases are considered. Namely, special cases of the problem can be considered if the quadrilateral is altered to a trapezium, a parallelogram, a rhombus, a rectangle, or a square and the question remains what is the figure obtained in each case? Extreme cases of the problem can be considered if the convex quadrilateral is altered to a concave quadrilateral or a quadrilateral that its lines intersect each other. The use of the interactive geometry software, Cinderella, provides the students with opportunities to alter the configuration to any of its special or extreme cases and immediately through its dragging and measuring facilities enables them to easily conclude a conjecture.

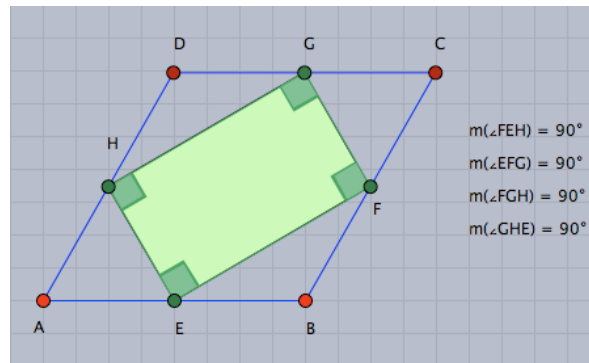
Start by altering the quadrilateral to a parallelogram. Have the students either use Cinderella's **"Move"** mode with the free points in the problem configuration (A , B , C , or D) they have constructed until it becomes a parallelogram or construct a new configuration starting with a parallelogram instead of the quadrilateral and construct the midpoint of each side. Then, have them connect the midpoints of the adjacent sides of the parallelogram to end up with a configuration, as shown in the figure [below](#). The configuration shows a new conjecture, which is: *"the figure obtained by joining the midpoints of the sides of a parallelogram is another parallelogram"*.



/files/quadrilateral_midpoints_activity/proposed_problem_1

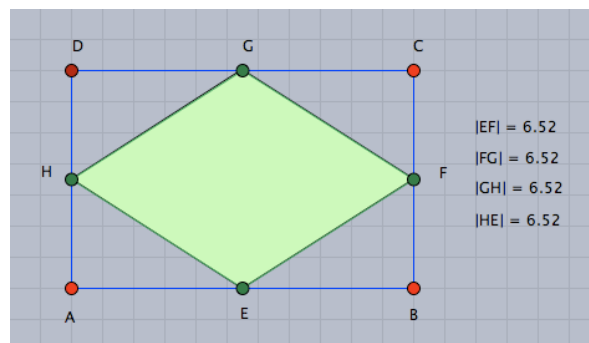
The second special case of the problem is to start with a rhombus instead of the quadri-

lateral. The configuration [below](#) shows that “the figure obtained by joining the midpoints of the sides of a rhombus is a rectangle”.



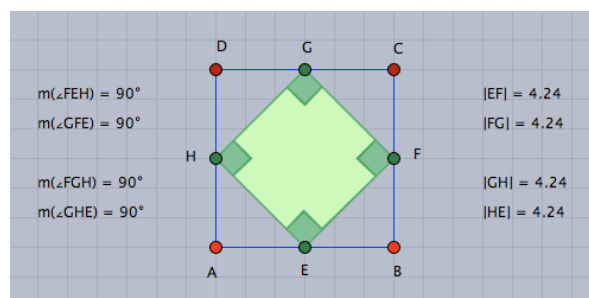
/files/quadrilateral_midpoints_activity/proposed_problem_2

The third special case of the problem is to start with a rectangle instead of the quadrilateral in the original problem. In this case, as shown [below](#), “the figure obtained by joining the midpoints of the sides of a rectangle is a rhombus”.



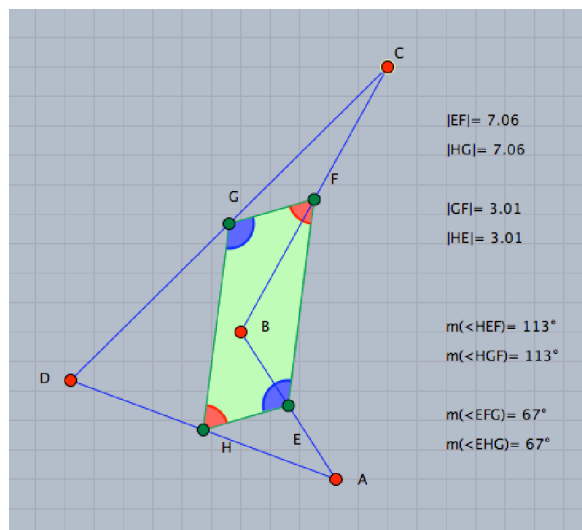
/files/quadrilateral_midpoints_activity/proposed_problem_3

The fourth special case of the problem is to start with a square. In this case, as shown [below](#), “the figure obtained by joining the midpoints of the sides of a square is another square”.

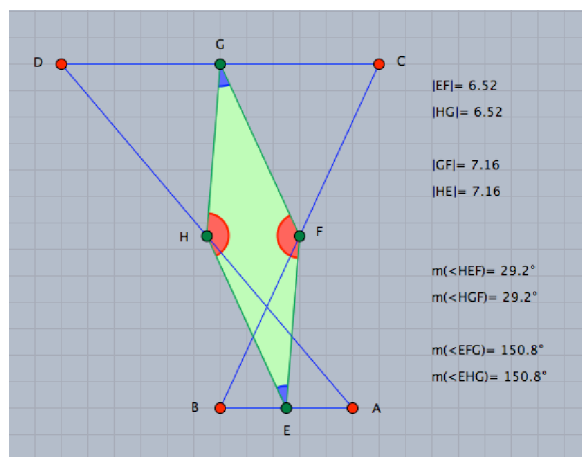


/files/quadrilateral_midpoints_activity/proposed_problem_4

General or extreme cases of the problem can also be considered if the convex quadrilateral, in the original problem, is altered to a concave quadrilateral or even to a quadrilateral that its sides intersect each other. The configurations [below](#) show that the figure obtained by joining the midpoints of a concave quadrilateral or a quadrilateral that its sides intersect each other is a parallelogram.



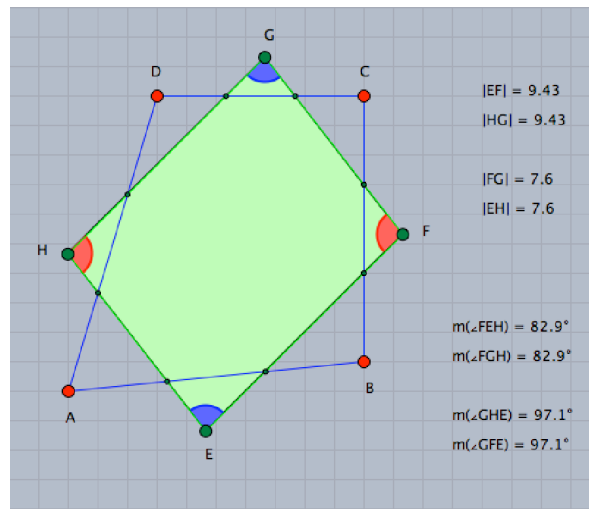
Joining Midpoints of Sides of a Concave Quadrilateral



Joining Midpoints of Sides of a Quadrilateral That its Sides Intersect Each Other

Another set of problems can be posed by altering the midpoints' condition in the original problem to be trisecting points. That is, in the case of a quadrilateral, if the trisecting points of each side are found, what would the figure obtained by connecting and extending adjacent points on either side of a vertex be? In this case, as shown [below](#), the

obtained figure is a parallelogram.



/files/quadrilateral_midpoints_activity/proposed_problem_5

Similarly, other problems can be posed in the case of a parallelogram, a rhombus, a rectangle, and a square with the same consideration of the trisection points (continue with the developed applets on the /files/quadrilateral_midpoints_activity/ folder on the accompanying CD-ROM for more suggested problems).

Enrichment Activity 6

Angular Bisectors of a Parallelogram

This activity is designed to help the mathematically gifted students make and investigate a geometric conjecture about the figure formed by the angular bisectors of the interior angles of a parallelogram, provide a mathematical proof for this conjecture, and generate new problems by altering the problem situation.

The Activity Problem¹²

What is the figure formed by the angular bisectors of the interior angles of a parallelogram? (see Student's Handout 9)

Activity Content Analysis

Skills

- Skill of constructing dynamic figures using Cinderella application.
- Skill of making geometric conjectures using different facilities of Cinderella application.
- Skill of investigating geometric conjectures using different facilities of Cinderella application.
- Skill of elaborating the given geometric situation using different facilities of Cinderella application.

¹²Resources used in designing this activity:

Christou, C., Mousoulides, N., Pittalis, M., and Pitta-Pantazi, D. (2005). Problem solving and problem posing in a dynamic geometry environment. *The Montana Mathematics Enthusiast*, 2(2), 125–143.

Contreras, J. (2003). Using dynamic geometry software as a springboard for making conjectures, solving problems and posing problems. Retrieved February 20, 2007, from: <http://www.usm.edu/pt3/pa/jc01.html>

Olivero, F. (2002). *Proving process within a dynamic geometry environment*. Ph.D. thesis, Graduate School of Education, University of Bristol, UK.

Objectives

The activity is designed to help the mathematically gifted students to:

- Construct a dynamic configuration for a parallelogram and the figure formed by the angular bisectors of the interior angles of this parallelogram using Cinderella application.
- Make geometric conjecture about the figure formed by the angular bisectors of the interior angles of a parallelogram.
- Investigate the conjecture made about the figure formed by the angular bisectors of the interior angles of a parallelogram.
- Produce a mathematical proof of the investigated conjecture of the figure formed by the angular bisectors of the interior angles of a parallelogram.
- Pose follow-up problems related to the original problem.

Materials

▪ Computers	▪ Handout 9
▪ Cinderella application	▪ CD-ROM of the program
▪ LCD projector	

Vocabulary

▪ Parallelogram	▪ Rectangle
▪ Rhombus	▪ Square
▪ Trapezium	▪ Isosceles trapezium
▪ Kite	▪ Interior angle
▪ Angular bisector	

Prerequisites

The students should know how to use the following tools before they start working in this activity:



Launch Cinderella or open new Cinderella window.



Use “**Move**” mode to move free elements by dragging the mouse.



Construct a point.



Construct a line connecting two points.



Construct parallel lines.



Generate the intersection of two geometric objects.



Construct the angular bisectors of two lines.

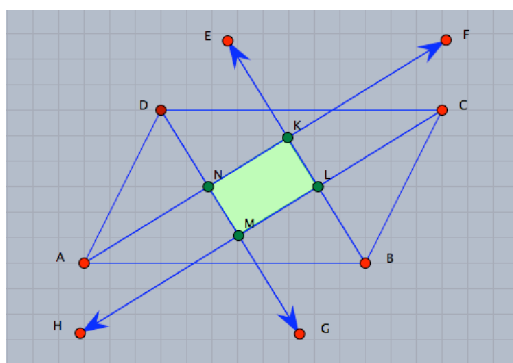


Define polygons.

Teaching and Learning Strategies

Construction

Encourage the students to use Cinderella to construct a parallelogram and the angular bisectors of its interior angles end up with a dynamic configuration for the problem as shown in the figure below, in which, A , B , and C are completely arbitrary and can be moved freely, while D that controls the parallelogram is a fixed point and L , M , N , and K are also fixed.

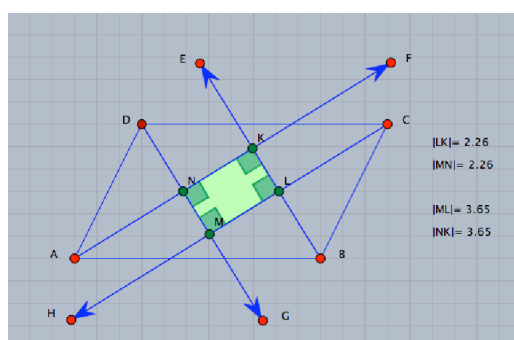


/files/parallelogram_angular_bisectors_activity/the_problem_statement

Have the students figure out a mathematical conjecture about the formed figure $KLMN$. Notice that when the free vertices of the parallelogram – A , B , or C – are dragged, the figure formed by the bisectors of the interior angles of the parallelogram seems to be a rectangle.

Investigating Conjecture

At this stage, it is important to have the mathematically gifted students use on-screen measurements for sides lengths and angles measurements to precisely make and investigate a conjecture about the formed figure. So, have the students use Cinderella's measuring facilities to find the lengths of each two opposite sides and measures of the angles of the inner quadrilateral $KLMN$. Then, have them alter the quadrilateral $ABCD$, and visually observe the formed figure $KLMN$, the lengths of each two opposite sides, and the measures of its interior angles. When the vertices of the parallelogram $ABCD$ are dragged, the lengths of each two opposite sides of the inner quadrilateral $KLMN$ remain the same and its interior angles are always right, as shown in the figure below, and this is enough to reveal that the formed figure $KLMN$ is a rectangle.



/files/parallelogram_angular_bisectors_activity/the_problem_investigation

After the students investigate the conjecture, have them figure out the investigated conjecture in the form of mathematical theorems. For example, it could take this form: “The figure formed by the angular bisectors of the interior angles of a parallelogram is a rectangle”.

Producing Mathematical Proofs of the Conjecture

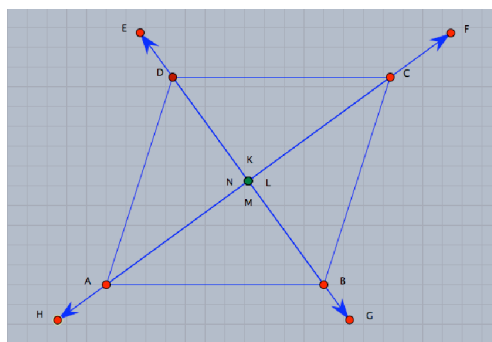
The next step in this activity is to produce a mathematical proof of the conjecture. So, have the students think of how to prove the investigated conjecture using the formal logical steps.

Posing Follow-up Problems

The problem can be extended to generate many other follow-up problems if a special case or a general case of the problem is considered or if the conditions of the problem are reformulated in different ways.

Special cases of the problem can be generated if the parallelogram in the original problem is altered to be a rhombus, a rectangle, or a square and the question remains: “What is the formed figure in each case?”.

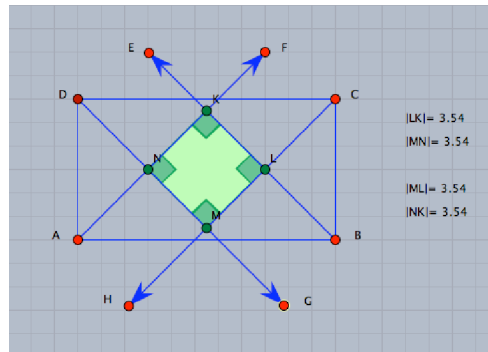
So, for the first special case, have the students move the free points in the original configuration (A , B , or C) they have constructed until it becomes a rhombus and observe the interior figure. In this case, the interior figure disappears. It is not a rectangle but becomes a point (the intersection point of the two diagonals of the rhombus) as shown in the figure below.



The Figure Formed by the Bisector of a Rhombus

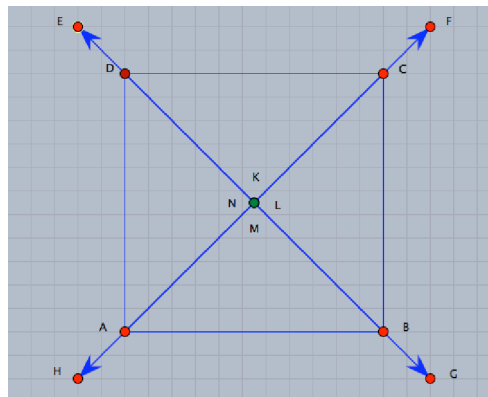
The second special case of the problem is to alter the parallelogram into a rectangle. Again, have the students move the free points (A , B , or C) in the original configuration until it becomes a rectangle and observe the figure formed by the bisectors of the interior

angles. It turns out to be a square as shown in the figure [below](#).



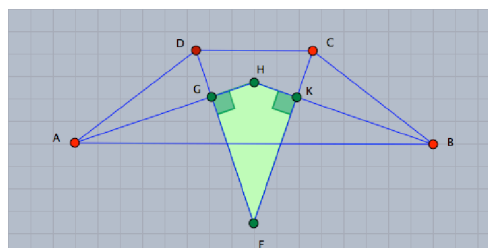
/files/parallelogram_angular_bisectors_activity/proposed_problem_1

The third special case of the problem is to alter the parallelogram into a square. In this case, the interior figure disappears again as shown in the figure [below](#).



The Figure Formed by the Bisector of a Square

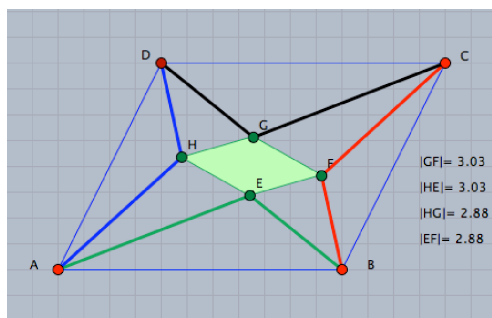
Now, let us consider a general case of the problem by starting with isosceles trapezoid instead of the parallelogram. So, have the students construct an isosceles trapezoid and bisect its interior angles and encourage them to make and investigate a conjecture about the figure formed by the bisectors of the interior angles. In this case, the figure formed by the bisectors is a kite with two right angles, as shown in the figure [below](#).



/files/parallelogram_angular_bisectors_activity/proposed_problem_2

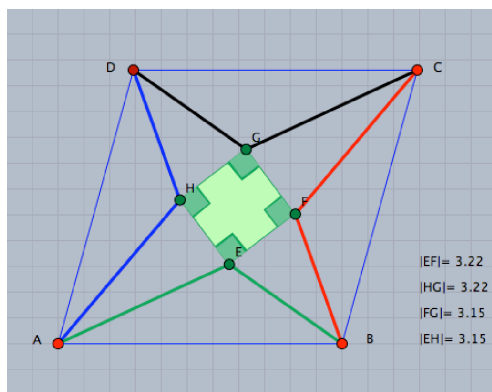
Furthermore, a new set of interesting problems related to the original problem can be posed by reformulating the given conditions in the original problem in different ways; by considering other geometric objects instead of the angular bisectors; for example, the angular trisectors so that the problem question would turn to be: “What would the figure formed by joining the four intersection points of the adjacent angular trisectors of the interior angles of a parallelogram, a rhombus, a rectangle, or a square be?”

In the case of a parallelogram, as shown below, the figure formed by joining the four intersection points of the adjacent angular trisectors of the interior angles is another parallelogram.



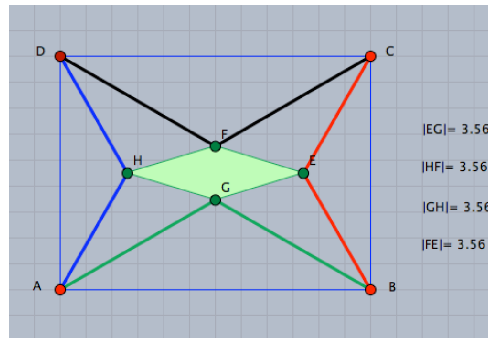
/files/parallelogram_angular_bisectors_activity/proposed_problem.3

As for the rhombus, the formed figure is investigated as a rectangle.



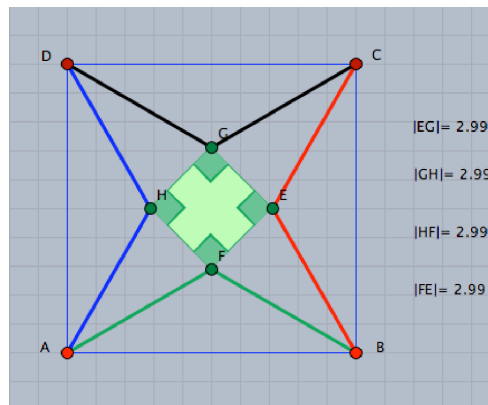
/files/parallelogram_angular_bisectors_activity/proposed_problem.4

The formed figure in the case of a rectangle is a rhombus, as shown below.



/files/parallelogram_angular_bisectors_activity/proposed_problem.5

As for the square, the formed figure is investigated as another square, as shown be-
low.



/files/parallelogram_angular_bisectors_activity/proposed_problem.6

For more details about the problems related to the use of the angular trisectors (see the developed applets on the /files/parallelogram_angular_bisectors_ activity/ folder on the accompanying CD-ROM).

The same technique of reformulating the problems conditions can be used again to suggest two other sets of problems if other geometric objects; for instance, medians or heights are considered instead of the angular bisectors. So, the given problem may become: what is the figure formed by the medians of parallelogram? Or: what is the figure formed by the heights of a parallelogram? (Continue with the developed applets on the /files/parallelogram_angular_ bisectors_activity/ folder on the accompanying CD-ROM).

Enrichment Activity 7

Constructing a Parallelogram

This activity is designed to help the mathematically gifted students come up with many various and different methods to construct a parallelogram.

Activity Problem¹³

In this activity, as a warm up, an interactive exercise to construct a parallelogram is provided. That is followed by giving a brainstorming activity to write down as many generalizations as possible that are related to the parallelogram. Then, the students are asked to think of which of these generalizations can be used in constructing a parallelogram using Cinderella. They are also encouraged to come up with many different and various methods of construction. The activity ends with asking the students to think of classification criteria to classify the construction methods (see Student's Handout 10).

Activity Content Analysis

Concepts

- Parallelogram
- Base of a parallelogram
- Center of a parallelogram
- Diagonal of a parallelogram

¹³Resources used in designing this activity:

Eraky, E. M. (2004). *The effectiveness of a computer-based mathematics enrichment program in developing the problem solving ability of mathematical ly gifted students at the preparatory stage and their attitudes towards autonomous learning*. Ph.D. thesis, Mansoura University, Egypt.

MathsNet (1998). Interactive Geometry with Cinderella. Retrieved July 3, 2008, from: <http://www.mathsnet.net/dynamic/cindy/index.html>

Olivero, F. (2002). *Proving process within a dynamic geometry environment*. Ph.D. thesis, Graduate School of Education, University of Bristol, UK.

Pereira, P. (2002). *Dynamic geometry*. Paper presented at the Technology and Education Conference sponsored by the Norwegian University of Science and Technology (NTNU), Trondheim, Norway, September 17.

Generalizations

- A parallelogram is a quadrilateral, in which each two opposite sides are parallel.
- A parallelogram is a quadrilateral, in which each two opposite sides are equal in length.
- A parallelogram is a quadrilateral, in which diagonals bisect each other.
- A parallelogram is a quadrilateral, in which each two opposite angles are equal in measure.
- A parallelogram is a quadrilateral, in which each two consecutive angles are supplementary.
- If two sides of a quadrilateral are parallel and equal in length, then the quadrilateral is a parallelogram.

Skills

- Skill of constructing many various and different dynamic configurations for the parallelogram using Cinderella application.

Objectives

This activity is designed to help the mathematically gifted students to:

- Produce many geometric generalizations related to the parallelogram.
- Dynamically discover parallelogram properties.
- Construct many various and different dynamic configurations for the parallelogram using Cinderella application.
- Try novel and unusual methods to construct dynamic configurations for the parallelogram using Cinderella application.
- List the construction algorithm for each method.

- Explain the theoretical background of each method.
- Logically justify the order of each method algorithm.
- Figure out criteria to classify the different methods of constructions.

Materials

- Computers
- Cinderella application
- LCD projector
- Handout 10
- CD-ROM of the program

Vocabulary

- Parallelogram
- Diagonal of a parallelogram
- Opposite angles
- Supplementary angles
- Center of parallelogram
- Quadrilateral
- Consecutive angle
- Parallelism
- Bisecting a line segment
- Base of parallelogram

Prerequisites

The students should know how to use the following tools before they start working in this activity:



Launch Cinderella or open new Cinderella window.



Use “**Move**” mode to move free elements by dragging the mouse.



Construct a point.



Construct a line connecting two points.



Construct a midpoint of a given line segment.



Construct parallel lines.



Generate the intersection of two geometric objects.



Construct an angular bisector.



Construct circles in different modes: (Draw Two Points and Circle, Draw Circle Around Point, Draw Circle with Fixed Radius, and Use a Compass).



Use the mirror tool and the other different modes of geometric transformation; such as translation, rotation, ...



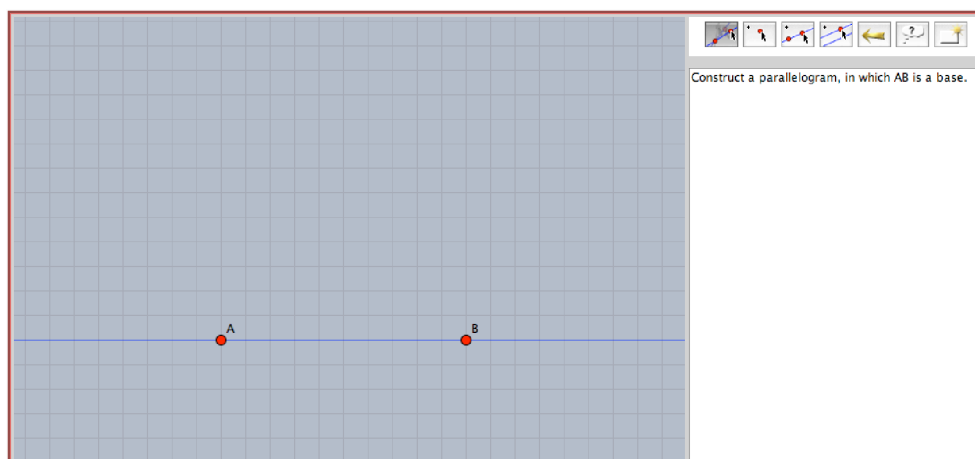
Define polygons.

Teaching and Learning Strategies

Warm up

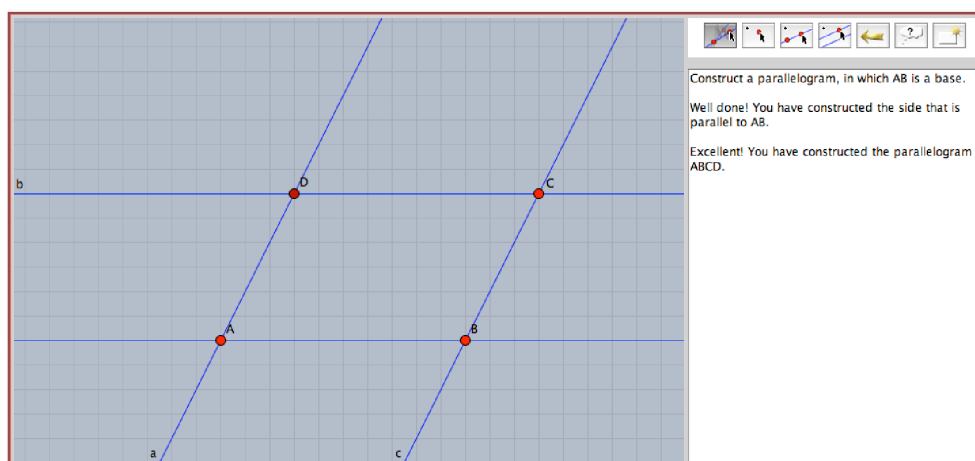
A Cinderella interactive exercise for constructing a parallelogram with a given base is designed to prepare the students for work in this activity. In the interactive exercise, the students are given a straight line $\overleftrightarrow{AB}$ and a toolbox. The toolbox includes the tools for “Add a Point” , “Draw Connecting Line” , “Draw Parallel Line” , and “Move”  and action buttons for “Undo” , “Give a Hint” , and “Restart Exercise” . The students are then asked to construct a parallelogram, in which $\overline{AB}$ is a base.

A Cinderella file and an html file for the interactive exercise are named “constructing_a_parallelogram” and placed on the /files/constructing_a_parallelogram/interactive_exercise/ folder on the accompanying CD-ROM. Have the students open the html file, start solving the exercise on their own, and let them press Give a Hint button “Give a Hint”  when they need help. The figure below shows the first screen of the interactive exercise.



`/files/constructing_a_parallelogram/interactive_exercise`
`/constructing_a_parallelogram`

The students should end up with a dynamic configuration for a parallelogram in which A , B , and C are free points and D is a fixed point that controls the parallelogram $ABCD$, that can be used to introduce the first portion of the activity as we will see. However, here is a screen shot that shows the last screen of the interactive exercise.



`/files/constructing_a_parallelogram/interactive_exercise`
`/constructing_a_parallelogram`

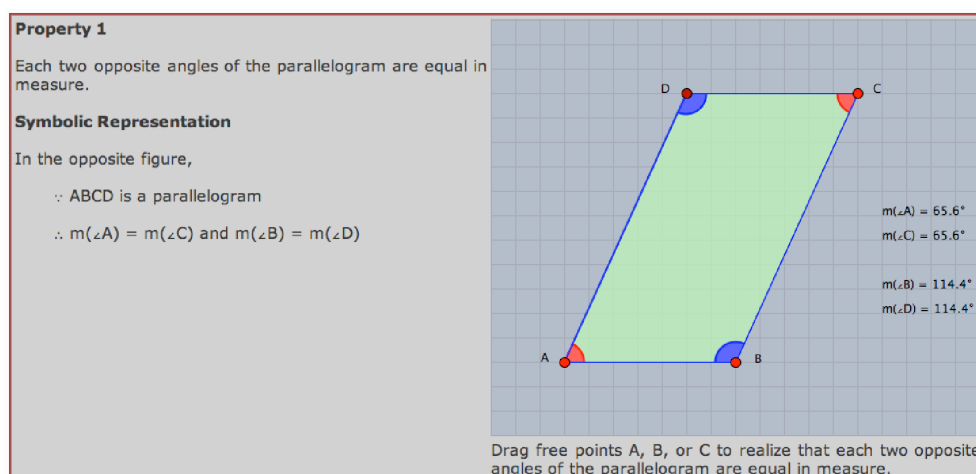
The handout of “Constructing a Parallelogram” activity consists of three pages. The first page is designed to help the students to produce many geometric generalizations related to the parallelogram in order to develop their verbal fluency as a component of geometric creativity. For doing so, have the students use the parallelogram dynamic configuration they constructed in the interactive exercise with “**Move**” mode and en-

courage them to verbally express themselves to come up with as many generalizations as possible related to the parallelogram. In the first few minutes let them write what they can deduce from the parallelogram configuration; later on it may be helpful to mention some hints to promote them to continue, such as:

- Think about the relationship between each two opposite sides in the parallelogram.
- Think about the relationship between each two opposite angles.
- Think about the relationship between each two consecutive angles.

Discuss the generalizations, which have been figured out by the students. With the students, decide which of these generalizations are right and which are wrong. Afterwards, let the students try to classify them into theorems, definitions, properties, and corollaries.

There are three pre-made applets, which are placed on the /files/constructing_a_parallel-gram/parallelogram_properties/ folder on the accompanying CD-ROM that can be used to present the parallelogram properties to the students. The applets show the parallelogram properties in three different forms: verbal form, symbolic form, and dynamic form.



/files/constructing_a_parallel-gram/parallelogram_properties/property_1

Applet 1 shows the first property of the parallelogram, which is: “Each two opposite angles in the parallelogram are equal in measure”. Have the students open the html

file for property 1, then drag free elements in the parallelogram configuration to realize that: In the parallelogram each two opposite angles are equal in measure.

Property 2

The sum of measures of each two consecutive angles in the parallelogram are supplementary.

Symbolic Representation

In the opposite figure,

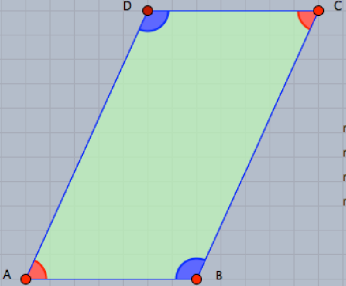
$\therefore$ ABCD is a parallelogram

$\therefore m(\angle A) + m(\angle B) = 180^\circ$ (two consecutive angles)

$m(\angle B) + m(\angle C) = 180^\circ$ (two consecutive angles)

$m(\angle C) + m(\angle D) = 180^\circ$ (two consecutive angles)

$m(\angle D) + m(\angle A) = 180^\circ$ (two consecutive angles)



$m(\angle A) = 65.6^\circ$
 $m(\angle B) = 114.4^\circ$
 $m(\angle C) = 65.6^\circ$
 $m(\angle D) = 114.4^\circ$

$m(\angle A) + m(\angle B) = 65.6^\circ + 114.4^\circ = 180^\circ$
 $m(\angle B) + m(\angle C) = 114.4^\circ + 65.6^\circ = 180^\circ$
 $m(\angle C) + m(\angle D) = 65.6^\circ + 114.4^\circ = 180^\circ$
 $m(\angle D) + m(\angle A) = 114.4^\circ + 65.6^\circ = 180^\circ$

Drag free points A, B, or C to realize that the sum of measures of each two consecutive angles in the parallelogram are supplementary.

/files/constructing_a_parallelogram/parallelogram_properties/property_2

Applet 2 shows the second property of the parallelogram which is: “The sum of measures of each two consecutive angles in a parallelogram is 180° ”. Have the students open the html file for property 2, then drag free elements in the parallelogram configuration to realize that: In the parallelogram, the sum of measures of each two consecutive angles is 180° .

Property 3

The opposite sides of the parallelogram are equal in length and its diagonals bisect each other.

Symbolic Representation

In the opposite figure,

$\therefore$ ABCD is a parallelogram and

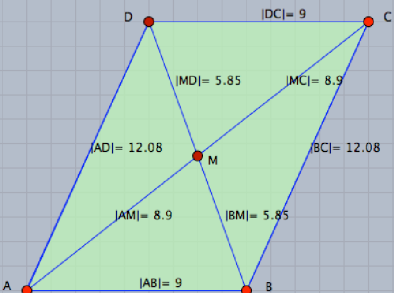
M is the intersection point of its diagonals.

$\therefore AB = DC$ (two opposite sides)

$BC = AD$ (two opposite sides)

$AM = MC$ (M is the midpoint of AC)

$BM = MD$ (M is the midpoint of BD)



$|AB| = 9$
 $|DC| = 9$
 $|AD| = 12.08$
 $|BC| = 12.08$
 $|AM| = 8.9$
 $|MC| = 8.9$
 $|BM| = 5.85$
 $|MD| = 5.85$

Drag free points A, B, or C to realize that the opposite sides of the parallelogram are equal in length and its diagonals bisect each other.

/files/constructing_a_parallelogram/parallelogram_properties/property_3

Applet 3 shows the third property of the parallelogram, which is: “The opposite sides

of a parallelogram are equal in length and its diagonals bisect each other”.

Concerning the second page of the activity handout, it is designed to help the students come up with as many various and different methods as they can to construct a parallelogram. This is assumed to help in developing the students' originality and flexibility components of geometric creativity through getting the opportunity to try novel and unusual methods and approaches to construct a parallelogram, vary the approaches of construction, and justify the construction algorithm order.

So, the students should be encouraged to use the generalizations they have figured out to construct a parallelogram using Cinderella by as many different methods as they can. In each construction method ask the students to list construction algorithm, explain the theoretical background of the construction algorithm, and logically justify the construction algorithm order.

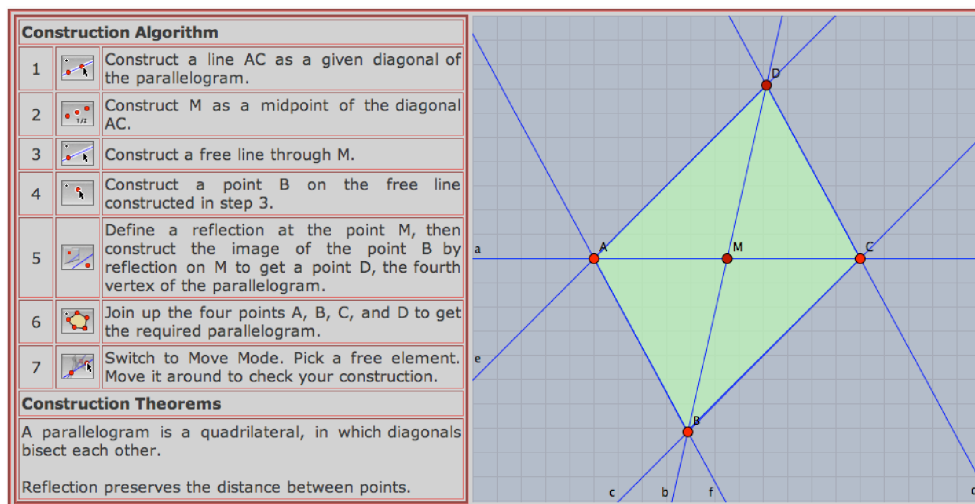
The third page of the activity is designed to help the students classify the different methods of construction. In this stage, the students should be asked to find out what criteria of classification can be used to classify the different methods of construction. Encourage the students to have their own criteria of classification.

There is a variety of classification criteria that can be considered for the developed constructions. The developed constructions could be developed according to the number of free points in the developed construction, or according to starting the construction with a base, a diagonal, or a point (the center of a parallelogram).

There are 20 prepared applets to show 10 different methods to construct a parallelogram on the /files/constructing_a_parallelogram/parallelogram_constructions/ folder on the accompanying CD-ROM. In these applets the construction algorithm and the theorem used on construction are written. After the students develop their own constructions and when they run out of ideas to develop further constructions, the teacher would pick some of the pre-made constructions, which are not constructed in the activity session, to present to the students and use them for further discussion.

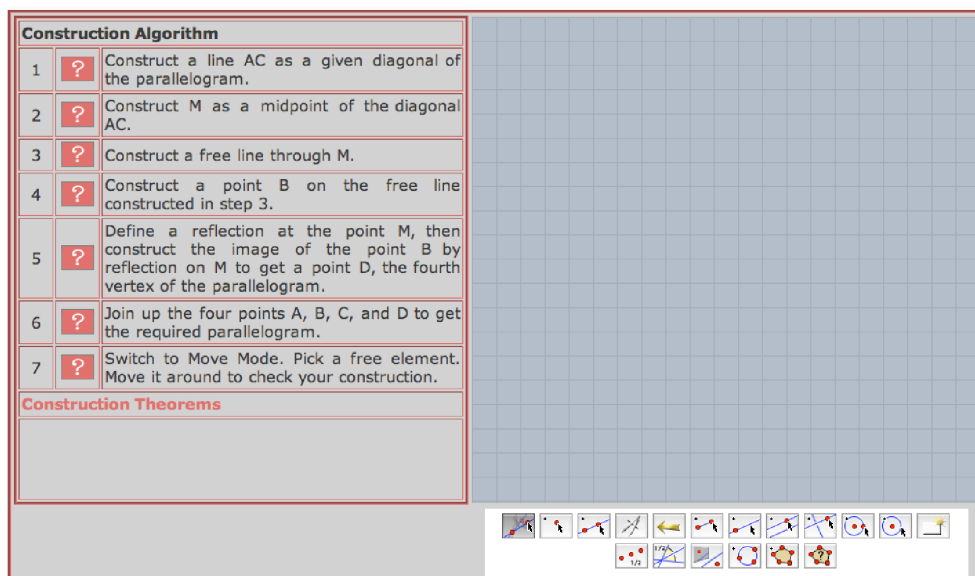
The applets are designed in two groups; one can be used to present the construction

method to the students (as an example, see the screen shot [below](#) for construction method 7).



/files/constructing_a_parallelogram/parallelogram_constructions
/construction_method_7

The other group of applets provides the students with a construction area and construction tools construct the parallelogram themselves (as an example, see the screen shot [below](#) for construction method 7 construct it yourself).



/files/constructing_a_parallelogram/parallelogram_constructions
/construct_it_yourself_7

The applets of the other 9 construction methods are available on the /files/constructing_a_parallelogram/parallelogram_constructions/ folder on the accompanying CD-ROM.

Enrichment Activity 8, 9, and 10

Constructing a Rhombus, a Rectangle, and a Square

Activities 8, 9, and 10 are designed the same way as activity 7 and even share the same objective to develop and train the mathematically gifted students' skills for constructing many various and different dynamic configurations for an assigned figure. So, to avoid repetition in the teacher's guide, the instructions given in activity 7 might be appropriate to the activities 8, 9 and 10 with careful considerations to change what is required.

The locations of the developed applets and cdy files for the activities 8, 9, and 10 on the accompanying CD-ROM are as follows:

Rhombus developed applets on the `/files/constructing_a_rhombus/` folder.

Rectangle developed applets on the `/files/constructing_a_rectangle/` folder.

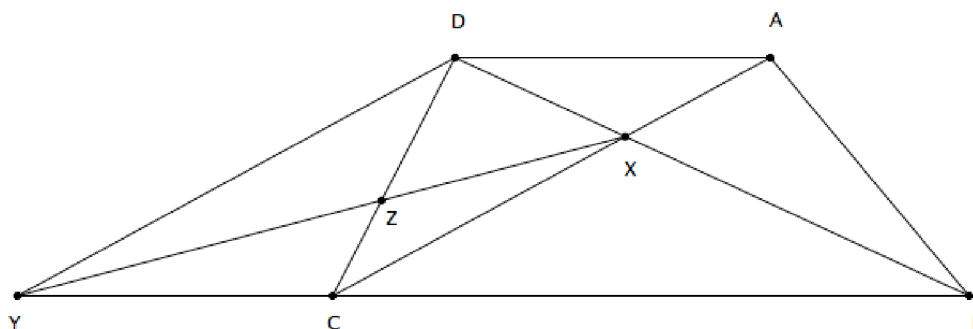
Square developed applets on the `/files/constructing_a_square/` folder.

Enrichment Activity 11

Posing Geometric Problems

This activity is designed to encourage the mathematically gifted students to **pose** as many various and different **geometric problems** as possible. These problems could be answered or deduced, in direct or indirect ways, from the given information in the situation using different facilities of Cinderella – constructing, dragging, measuring, and calculating facilities – in the framework of developing their geometric creativity components.

The Activity Problem¹⁴



In the figure above, $ABCD$ is a trapezium whose parallel sides are $\overline{AD}$ and $\overline{BC}$. X is the point where the diagonals $\overline{AC}$ and $\overline{BD}$ intersect. $\overrightarrow{DY}$ is drawn so that $\overrightarrow{DY} \parallel \overrightarrow{AC}$ and cuts $\overline{BC}$ at Y .

Use different facilities of Cinderella – constructing, dragging, measuring, and calculating facilities – to **pose** as many various and different **geometric problems** as possible, which could be answered either in direct or indirect ways based on the given information (see Student's Handout 14).

¹⁴Resources used in designing this activity:

El-Rayashy, H. A. M. and Al-Baz, A. I. (2000). A proposed strategy on group mastery learning approach in developing geometric creativity and reducing problem solving anxiety among preparatory stage students. *Journal of Mathematics Education in Faculty of Education - Benha, Zagazig University*, **3**, 65-207.

Activity Content Analysis

Concepts

▪ Parallelism	▪ Congruency of two triangles
▪ Area	▪ Similarity of two triangles
▪ Equivalent of two triangles	▪ Ratio & Proportional

Generalizations

- Triangles, which have the same base and vertices opposite to this base lying on the same straight line parallel to the base, have the same areas.
- Area of the triangle is equal to half area of the parallelogram, which have the same base and the vertex opposite to this base are on the same straight line.
- If a straight line is drawn parallel to one side of a triangle cutting the other two sides, then it divides them into four segments of proportional lengths.
- A parallel to a side of a triangle that intersects the other two sides cuts off a triangle similar to the original triangle.
- Two triangles are similar when the measures of the angles of the first triangle are equal to the measures of the corresponding angles of the second.
- Two triangles are similar when the measure of one angle in one of them equals the measure of one angle in the other and the lengths of the sides about these two angles are proportional.
- Two triangles are similar when the lengths of the sides of the first triangle are proportional to the lengths of the sides of the second.
- The ratio between the surface areas of two similar triangles is equal to the square of the ratio between the lengths of two corresponding sides.

- The ratio between the surface areas of two triangles, which have the same height, equals the ratio between the lengths of their bases.

Skills

- Skill of constructing dynamic figures using Cinderella application.
- Skill of posing and investigating geometric problems using different facilities of Cinderella application.

Objectives

The activity is designed to help the mathematically gifted students to:

- Construct a dynamic configuration using Cinderella application.
- Pose many various and different geometric problems, which could be answered from the given information in the geometric situation using different facilities of Cinderella application.
- Investigate the posed problems using different facilities of Cinderella application.
- Produce mathematical proofs for the investigated conjecture about the area of the four triangles obtained by joining the midpoints of the sides of a triangle.
- Produce mathematical proofs for the posed problems.

Materials

▪ Computers	▪ Handout 14
▪ Cinderella application	▪ CD-ROM of the program
▪ LCD projector	

Vocabulary

- Two similar triangles
- Two equivalent triangles
- Ratio & Proportion
- Surface area

Prerequisites

The students should know how to use the following tools before they start working in this activity:



Launch Cinderella or open new Cinderella window.



Use “**Move**” mode to move free elements by dragging the mouse.



Construct a point.



Construct a line connecting two points.



Construct parallel lines.



Generate the intersection of two geometric objects.



Measure the length of line segments.



Measure angles.



Define polygons.



Measure areas.



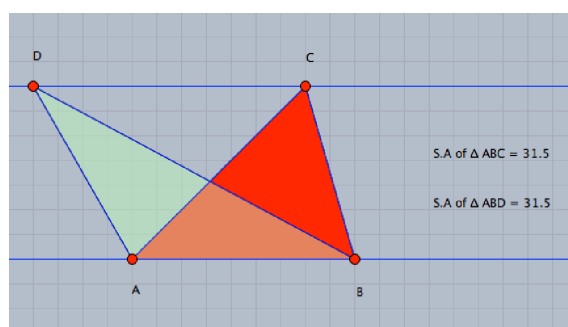
Use CindyScript language through the “**Define a Function**” tool

Teaching and Learning Strategies

Warm up

Two Cinderella files named “**warm_up_1**” and “**warm_up_2**” which are prepared and placed on the /files/posing-geometric-problems/ folder on the accompanying CD-ROM,

present a situation that asks the students to identify the necessary conditions and the necessary and sufficient conditions, which make the two triangles have the same areas in the given situation. Have the students open the Cinderella file, then using “Move” mode, drag free elements and freely discover the necessary conditions and the necessary and sufficient conditions, which make the two triangles have the same areas.



/files/posing-geometric-problems/warm-up_1

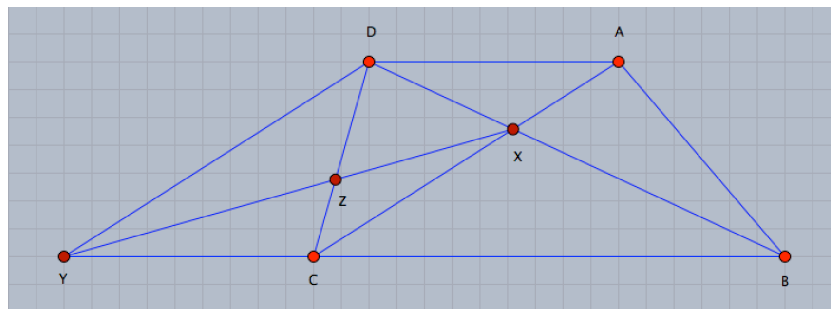
In this situation, there are two necessary conditions:

1. The two triangles ABC and ABD have a common base $\overline{AB}$.
2. The vertices C and D are on a straight line that is parallel to the common base.

Each one of them represents a necessary condition that makes the two triangles have the same areas and when one of them does not exist this implies that the two triangles do not have the same areas. Namely, the existence of both of them is necessary for the two triangles to have the same areas, and the existence of one alone is not sufficient for the two triangles to have the same areas. The union of the two conditions represents a necessary and sufficient condition, which is: “The two triangles have a common base and the other two vertices are on a straight line, which is parallel to the common base”. This necessary and sufficient condition is enough in its own rights to indicate that the two triangles have the same areas. Cinderella can effectively be used to visually testify these conditions. Encourage the students to design their endeavors using the two prepared cdy files – warm_up_1.cdy and warm_up_2.cdy – to visually testify each condition using different facilities of Cinderella.

Construction

Have the students use Cinderella's constructing facilities to come up with a dynamic configuration for the given situation, as shown in the figure below. According to the figure A , B , and C are arbitrary points that can be freely moved, D is a semi-free point that can be moved along $\overline{AD}$, and X , Y and Z are fixed points that cannot be moved.



/files/posing-geometric-problems/the_activity_problem

Posing and Investigating Problems

In the given situation, the students should be invited to pose many various and different problems, which can be answered or deduced either in direct ways or indirect ways from the given information in the situation. Actually, there are many various and different problems that connect several different basic areas in elementary geometry such as parallelism, measurement quadrilaterals, areas, similarity, ratio and proportional. Presenting situations like this one provides the students with opportunities to see mathematical connections among different areas in geometry as well as connect their previous and current geometric knowledge to construct their own understanding.

Problems Related to Measurements of Angles

Based on the parallelism theorem which states that “If a straight line intersects two parallel lines, then: (1) any two alternative angles are equal in measure, (2) any two corresponding angles are equal in measure, (3) any two interior angles on the same side

of the transversal are supplementary”, many problems can be easily posed pertaining to angles’ measurement and can be answered from the given information. Presented here are three examples with their reasoning:

1. $\because \overline{DY} \parallel \overline{AC}$ and $\overline{YB}$ is their transversal.

$$\therefore m(\angle BCA) = m(\angle CYD) \quad (\text{Corresponding angles})$$

2. $\because \overline{AD} \parallel \overline{BC}$ and $\overline{AC}$ is their transversal.

$$\therefore m(\angle DAC) = m(\angle BCA) \quad (\text{Alternative angles})$$

3. $\because \overline{AD} \parallel \overline{BC}$ and $\overline{AB}$ is their transversal.

$$\therefore m(\angle BAD) + m(\angle ABC) = 180^\circ \quad (\text{Interior angles})$$

Many other pairs of angles, either they have the same measure or they are supplementary, could also be posed using the parallelism theorem. Have the students use Cinderella’s “**Measure Angle**” tool  to pose and investigate other problems related to measurement of angles

Problems Related to Quadrilaterals

Two problems could be posed related to quadrilaterals, which are:

Show that:

1. $ACYD$ is a parallelogram.
2. $XCYP$ is a trapezium.
3. $XADY$ is a trapezium.

Problems Related to Lengths of Sides

Since in the parallelogram $ACYD$ each two opposite sides are equal in length, then the following two problems related to lengths of sides can be posed:

1. $AD = CY$

2. $AC = DY$

Have the students use Cinderella's **"Measure Distance"** tool  to come up with and investigate the two problems above.

Problems Related to Areas

Once one student comes up with a problem related to areas, make use of this response by generating a brainstorming question around it to encourage the students to come up with many various and different problems related to areas. In fact, there are many various and different problems related to areas that could likely be posed. With respect to the theorem which states that: "Triangles, which have the same base and vertices opposite to this base lying on the same straight line parallel to the base, have the same areas", the following problems can be posed:

1. $S.A \text{ of } \triangle ADB = S.A \text{ of } \triangle ADC$
2. $S.A \text{ of } \triangle AXB = S.A \text{ of } \triangle DXC$
3. $S.A \text{ of } \triangle ABC = S.A \text{ of } \triangle DBC$
4. $S.A \text{ of } \triangle DXC = S.A \text{ of } \triangle YCX$
5. $S.A \text{ of } \triangle DZX = S.A \text{ of } \triangle YZC$
6. $S.A \text{ of } \triangle YDX = S.A \text{ of } \triangle DYC$
7. $S.A \text{ of } \triangle YDX = S.A \text{ of } \triangle ADC$
8. $S.A \text{ of } \triangle AXB = S.A \text{ of } \triangle YCX$
9. $S.A \text{ of } \triangle DBC = S.A \text{ of } \triangle XBY$
10. $S.A \text{ of } \triangle YCX = S.A \text{ of } \triangle AXB = S.A \text{ of } \triangle DXC$
11. $S.A \text{ of } \triangle DZY = S.A \text{ of } \triangle CXZ = S.A \text{ of } \triangle AXD$

As for the theorem which states that: "Area of the triangle is equal to half the area of the parallelogram, which have the same base and the vertices opposite to this base

are on the same straight line”, another set of problems related to areas can be posed as follows:

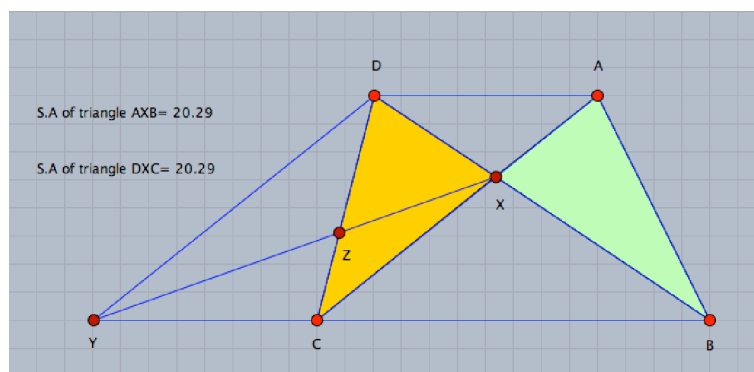
1. $S.A \text{ of } \triangle ADC = \frac{1}{2} S.A \text{ of Parallelogram } ACYD$
2. $S.A \text{ of } \triangle CYD = \frac{1}{2} S.A \text{ of Parallelogram } ACYD$

Consequently, two more problems could be posed:

1. $S.A \text{ of } \triangle ADC = S.A \text{ of } \triangle CYD$
2. $S.A \text{ of } \triangle YDX = S.A \text{ of } \triangle ADC = S.A \text{ of } \triangle CYD = S.A \text{ of } \triangle ADB$

Investigating such problems related to areas using Cinderella is quite easy. Encourage the students to use Cinderella's measuring facilities to calculate the areas of the figures included in each suggested problem, then use the dragging facilities of Cinderella to visually and dynamically testify each problem.

For example, to investigate that $S.A \text{ of } \triangle AXB = S.A \text{ of } \triangle DXC$, have the students switch to the **“Define a Polygon Mode”** to define the two triangles AXB and DXC as two polygons: *Poly0* and *Poly1*, then, switch to the **“Measure Area of a Polygon Mode”** and just click inside each triangle (Polygon) to get its area. Now, have the students use Cinderella's dragging facilities to visually and dynamically observe the area of the two triangles. When free points are dragged, the areas of the two triangles remain the same, as shown in the figure below.



/files/posing_geometric_problems/two_equivalent_triangles

After investigating the problems using different facilities of Cinderella, encourage the students to produce logical mathematical proofs for each problem using the theorems mentioned above.

Problems Related to Similarity

With respect to similarity, there are only three pairs of similar triangles, which could be deduced from the given information.

In triangle BDY , since $\overline{XC} \parallel \overline{DY}$, then $\overline{XC}$ cuts triangle BDY into two similar triangles. That is, two triangles BXC and BDY are similar.

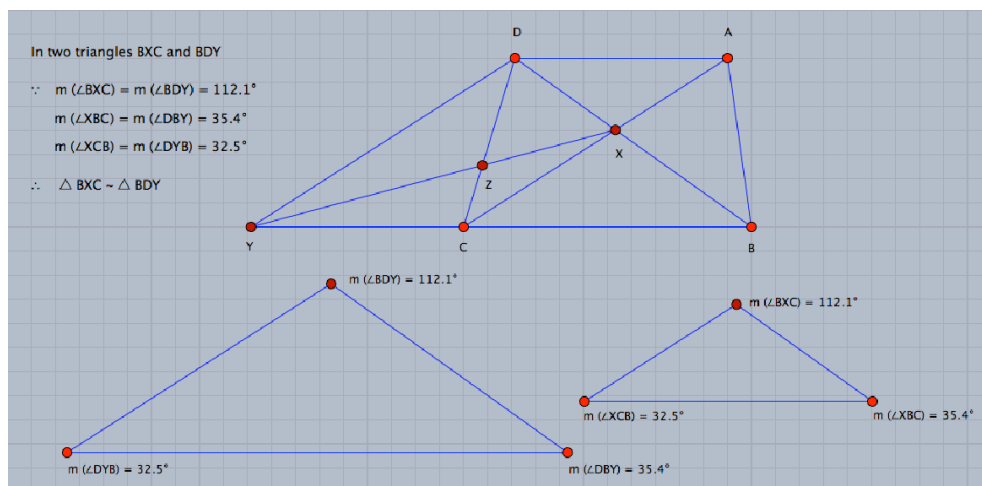
1. $\triangle BXC \sim \triangle BDY$

The other two pairs of similar triangles are:

2. $\triangle AXD \sim \triangle CXB$

3. $\triangle XZC \sim \triangle YZD$

To investigate the similarity of the two triangles using Cinderella, have the students use the constructing facilities of Cinderella to construct dynamic copies of the two triangles BXC and BDY and use measuring facilities to measure the angles of the two triangles then use the dragging facilities to testify whether the measures of the angles of triangle BXC still equal to the measures of the angles of triangle BDY under dragging.



When free points are dragged, the measures of corresponding angles in the two triangles BXC and BDY remain equal, as shown in the figure below, and this shows that the two triangles BXC and BDY are similar based on the theorem which states that: “If the measures of the angles of one triangle are equal to the measures of the angles of another triangle, then the two triangles are similar”.

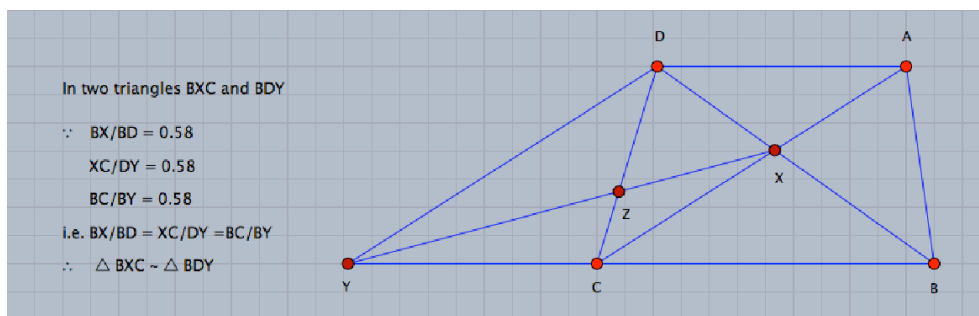
There is another possibility to investigate that $\triangle BXC$ and $\triangle BDY$ are similar. This possibility lies in using the dragging and calculating facilities of Cinderella, considering the theorem which states that: “Two triangles are similar when the lengths of the sides of the first triangle are proportional to the lengths of the sides of the second”. That is, to examine whether $\triangle BXC$ and $\triangle BDY$ are similar using Cinderella, the interactive ratio between each two corresponding side lengths of the two triangles should be calculated. For doing so, either CindyScript language in “**Define a Function**” mode or referencing text in “**Add Text**” mode can be used. Then again, to calculate the interactive ratio between the corresponding side lengths, have the students switch to “**Add Text**” mode by clicking the button  and enter the following referencing text in “**Add Text**” window:

$$BX/BD = @\{|B,X|/|B,D|\}$$

$$XC/DY = @\{|X,C|/|D,Y|\}$$

$$BC/BY = @\{|B,C|/|B,Y|\}$$

Then, have the students click on the OK button. This will calculate the interactive ratio between each two corresponding side lengths of the two triangles, as shown in the figure below:



/files/posing_geometric_problems/two_similar_triangles_2

Now, have the students switch to **“Move”** mode and alter one of the configuration free points (A , B , C , or D) and observe the interactive ratio under dragging. When free points are dragged, the interactive ratio between each two corresponding side lengths of the two triangles $\triangle BXC$ and $\triangle BDY$ remain the same and this shows that the lengths of the sides of $\triangle BXC$ are proportional to the lengths of the sides of $\triangle BDY$, and for that reason $\triangle BXC \sim \triangle BDY$.

It should be mentioned, here, that Cinderella's **“Similarity”** mode can be used as a way to investigate the similarity of the two triangles; since using such mode affords the students the opportunity to instantly and automatically check the similarity of the two triangles. For doing so, in the given situation, have the students switch to **“Similarity”** mode by choosing the menu item **“Modes/Transformation/Similarity”** then define a similarity transformation that will be used to examine the similarity of the two triangles BXC and BDY by specifying the pre-image and image of two pairs of points, let's say $X \rightarrow D$ and $C \rightarrow Y$. After the students specified the four points, the similarity is completely defined and a transformation button appears in Cinderella window. Now to examine the similarity of the two triangles BXC and BDY , have the students select the vertices of $\triangle BXC$ and then generate the similarity transformation by pressing the button . This action will generate the image of $\triangle BXC$ under the defined similarity transformation by highlighting three vertices. In this case, they (the vertices) will be B , D , and Y , which means that $\triangle BXC$ is similar to $\triangle BDY$.

After the students investigate the similarity of the two triangles in different situations using Cinderella, encourage them to generate formal mathematical proofs to show the similarity of the two triangles BXC and BDY . Proving that $\triangle BXC$ is similar to $\triangle BDY$ could be achieved in many different ways, here is one of them:

In triangle BDY ,

$$\therefore \overline{XC} \parallel \overline{DY}$$

$$\therefore m(\angle BXC) = m(\angle BDY) \text{ and } m(\angle XBC) = m(\angle DBY) \quad (\text{Corresponding angles})$$

Now, since two triangles BXC and BDY have:

1. $m(\angle BXC) = m(\angle BDY)$
2. $m(\angle XBC) = m(\angle DBY)$
3. $\angle B$ is a common angle for the two triangles.

then $\triangle BXC \sim \triangle BDY$.

Problems Related to Ratio & Proportion

Using Talis Theorem which states that: “If two transversal cut several parallel lines, so the lengths of the corresponding segments on the two transversal are proportional”, the following problem can be posed:

$$1. \frac{BX}{XD} = \frac{BC}{CY}$$

As $(\because \overleftrightarrow{XC} \parallel \overleftrightarrow{DY}; \overleftrightarrow{BD}$ and $\overleftrightarrow{BY}$ are two transversals $\therefore \frac{BX}{XD} = \frac{BC}{CY}$)

Two more problems could be posed as a result of similarity – the similarity of the two triangles BXC and BDY – one related to the ratio between their corresponding side lengths and the other related to the ratio between their surface areas. With respect to the ratio between their corresponding sides, the ratio between each pair of corresponding side lengths are equal. In other words, since the two triangles BXC and BDY are similar, so the side lengths of $\triangle BXC$ are proportional to their corresponding side lengths in $\triangle BDY$.

$$2. \frac{BX}{BD} = \frac{BC}{BY} = \frac{CX}{DY}$$

As for the ratio between their surface areas, the ratio between the surface areas of the two triangles is equal to the square of the ratio between the lengths of two corresponding sides.

$$3. \frac{S.A \text{ of } \triangle BCX}{S.A \text{ of } \triangle BYD} = \left[\frac{XC}{DY} \right]^2$$

In addition to the previously-posed problems, by considering the lemma which states that: “The ratio between the surface areas of two triangles which have the same height equals the ratio between the lengths of their bases”, many other problems related to

ratio and proportional could be posed. Among others, the following problems could be posed:

$$4. \frac{S.A \text{ of } \triangle XBC}{S.A \text{ of } \triangle XCY} = \frac{BX}{XD}$$

$$5. \frac{S.A \text{ of } \triangle DBC}{S.A \text{ of } \triangle DCY} = \frac{BX}{XD}$$

$$6. \frac{S.A \text{ of } \triangle ABX}{S.A \text{ of } \triangle AXD} = \frac{BC}{CY}$$

$$7. \frac{S.A \text{ of } \triangle CBX}{S.A \text{ of } \triangle CXD} = \frac{BC}{CY}$$

To complete this section, the students should be encouraged to use Cinderella not only to pose problems similar to the previously-posed problems related to ratio and proportional but also to use the calculating facilities of Cinderella through either “**CindyScript**” language or “**Referencing Text**” to investigate the posed problems as explained in previous sections.

Produce Mathematical Proofs

After the students use Cinderella to pose and investigate the problems, the next step would be to provide them with opportunities to use the formal logical and deductive reasoning steps to produce mathematical proofs for the posed problems.

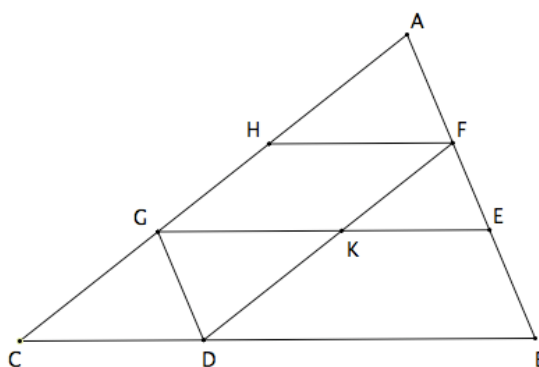
Enrichment Activity 12

Finding Geometric Relationships

The activity is designed to help the mathematically gifted students to **find** as many various and different **geometric relationships** as possible that could be deduced, in direct or indirect ways, from the given geometric situation using different facilities of Cinderella – constructing, dragging, measuring, and calculating facilities – in the framework of developing their geometric creativity components.

The Activity Problem

In the following figure, $AF = BE$, $\overline{FH} \parallel \overline{EG} \parallel \overline{BC}$ and $\overline{GD} \parallel \overline{AB}$.



Use different facilities of Cinderella – constructing, dragging, measuring, and calculating facilities – to **find** as many various and different **geometric relationships** as possible, which could be deduced in direct or indirect ways based on the given information (see Student's Handout 15).

Activity Content Analysis

Concepts

- Parallelism
- Ratio & Proportional
- Equivalent of two triangles
- Congruency of two triangles
- Similarity of two triangles
- Area

Generalizations

- The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is equal to half of it.
- Two triangles are congruent if the length of each side of the first triangle is equal to the length of the corresponding side in the second triangle.
- Talis Theorem: "If two transversal cut several parallel lines, so the lengths of the corresponding segments on the two transversal are proportional".
- A parallel to a side of a triangle that intersects the other two sides cuts off a triangle similar to the original triangle.
- The ratio between the surface areas of two similar triangles is equal to the square of the ratio between the lengths of two corresponding sides.
- Parallelograms, which have the same base and the opposite sides to this base are on the same straight line, have the same area.

Skills

- Skill of constructing dynamic figures using Cinderella application.
- Skill of finding and investigating geometric relationships using different facilities of Cinderella application

Objectives

The activity is designed to help the mathematically gifted students to:

- Construct dynamic configurations using Cinderella application.
- Find many various and different geometric relationships that could be deduced from the given information in the situation using different facilities of Cinderella application.

- Investigate the found geometric relationships using different facilities of Cinderella application.
- Produce mathematical proofs for the found relationships.

Materials

- Computers
- Cinderella application
- LCD projector
- Handout 15
- CD-ROM of the program

Vocabulary

- Two congruent triangles
- Two similar triangles
- Two equivalent triangles
- Ratio & Proportion
- Surface area

Prerequisites

The students should know how to use the following tools before they start working in this activity:



Launch Cinderella or open new Cinderella window.



Use “**Move**” mode to move free elements by dragging the mouse.



Construct a point.



Construct a line connecting two points.



Construct parallel lines.



Generate the intersection of two geometric objects.



Measure the length of line segments.



Measure angles.



Define polygons.



Measure areas.

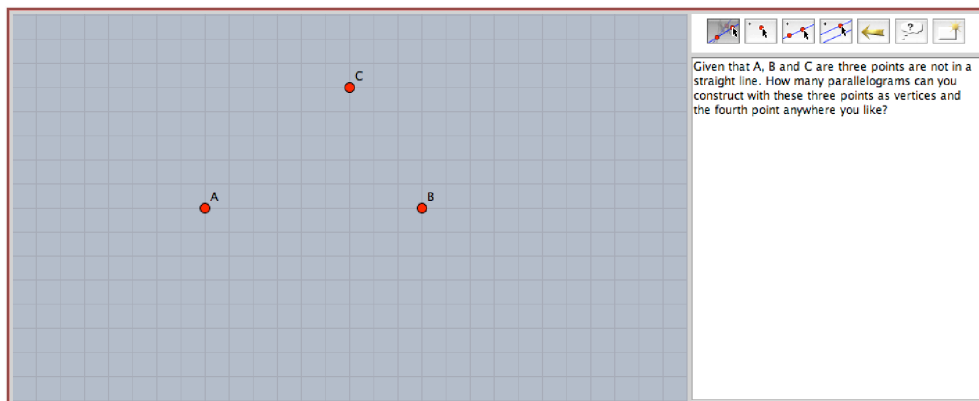


Use CindyScript language through the “**Define a Function**” tool

Teaching and Learning Strategies

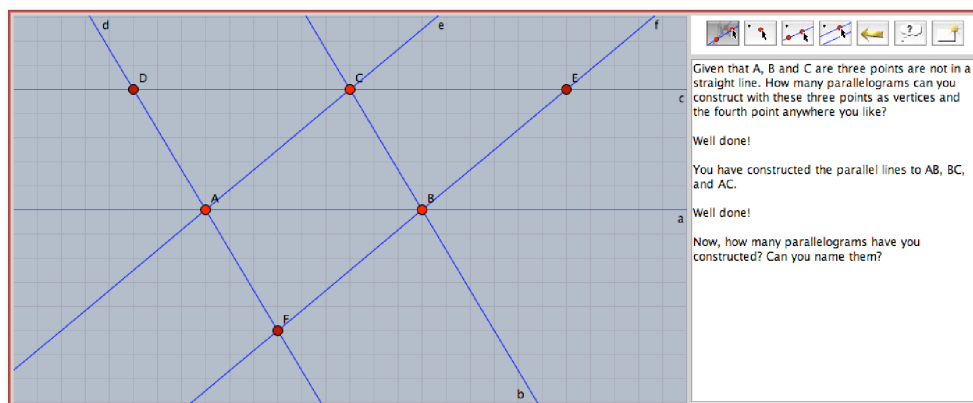
Warm up

A Cinderella interactive exercise is designed to get the students mentally ready to work in this activity. In the interactive exercise, the students are given three non-collinear points A , B , and C and asked to find the fourth vertex to complete the parallelogram and identify how many parallelograms can be constructed using these three points as vertices. A Cinderella file and an html file are named “constructing-parallelograms” and placed on the /files/finding_geometric_relationships/constructing-parallelograms/ folder on the accompanying CD-ROM for this exercise. Have the students open the html file, start solving the exercise on their own, and let them press “**Give a Hint**” button  when they need help. The figure below shows the first screen of the interactive exercise.



/files/finding_geometric_relationships/constructing-parallelograms
/constructing-parallelograms

The students should end up with a dynamic configuration that is exactly similar to the one below. In this configuration A , B , and C are free points and D , E , and F are fixed points that control the three constructed parallelograms.



/files/finding_geometric_relationships/constructing_parallelograms
/constructing_parallelograms

After the students solve the interactive exercise and the accompanying questions, which are: “How many parallelograms have you construct? And can you name them?” The teacher may take a few minutes to orally ask a few questions about the dynamic configuration of the interactive exercise. The teacher may ask:

- What is the relationship between the areas of the three constructed parallelograms? What is the area of each parallelogram in terms of the area of $\triangle ABC$?
- How many trapeziums are there in the configuration? Can you name them? What is the relationship between their areas? What is the area of each trapezium in term of the area of $\triangle ABC$?
- How many triangles are similar to triangle $\triangle ABC$? Explain your answer.
- How many triangles are congruent to triangle $\triangle ABC$? Explain your answer.
- What is the relationship between $\overline{AC}$ and $\overline{FE}$? Explain your answer.
- Are $\overline{DC}$, $\overline{DA}$, $\overline{CB}$ and $\overline{AF}$ proportional in length? Explain your answer.

ask them to use Cinderella to list as many relationships as they can that are related to the subject area of the proposed relationship.

Here are lists of relationships the students may come up with that are classified according to the basic areas of geometry mentioned before with some brainstorming questions the teacher may use. Each of these relationships can be easily approved in different ways and situations using different facilities of Cinderella and the students should be encouraged to do so.

Relationships Related to Measurement of Angles

Using the parallelism theorem, many relationships can easily be deduced in relation to to angles' measurements such as:

$m(\angle AFH) = m(\angle AEK) = m(\angle ABC)$	(Corresponding angles)
$m(\angle AHF) = m(\angle AGE) = m(\angle ACB)$	(Corresponding angles)
$m(\angle FKG) = m(\angle FDC)$	(Corresponding angles)
$m(\angle FKE) = m(\angle FDB)$	(Corresponding angles)
$m(\angle HFK) = m(\angle FKE)$	(Alternative angles)
$m(\angle GKD) = m(\angle KDB)$	(Alternative angles)
$m(\angle KGD) = m(\angle GDC)$	(Alternative angles)
$m(\angle HFE) + m(\angle FEG) = 180^\circ$	(Interior angles)
$m(\angle GEB) + m(\angle EBC) = 180^\circ$	(Interior angles)
$m(\angle FHG) + m(\angle HGE) = 180^\circ$	(Interior angles)
$m(\angle EGC) + m(\angle GCB) = 180^\circ$	(Interior angles)

Have the students use Cinderella's **“Measure Angle”** tool  to find relationships related to measurement of angles and examine their correctness.

Relationships Related to Quadrilaterals

In the given situation, there are 5 parallelograms, which are: $EBDG$, $AFDG$, $FKGH$, $KDCG$, and $FDCH$. Also, there are 4 trapeziums, which are: $EBDK$, $AFKG$, $FDGH$, and $FEGH$.

Brainstorming questions can be asked here to encourage the students to make conjectures related to quadrilaterals. The teacher may ask:

- How many parallelograms can you deduce from configuration? Can you name them?
- How many trapeziums can you deduce from the configuration? Can you name them?

Here, the teacher may encourage the students to use different facilities of Cinderella to examine whether the figures $EBDG$, $AFDG$, $FKGH$, $KDCG$, and $FDCH$ are parallelograms. Using Cinderella to show that a figure is a parallelogram in different ways is explained before in activity 4.

Relationships Related to Lengths of Sides

Since each two opposite sides of the parallelogram are equal in length, another set of relationships can be deduced pertaining to lengths of sides:

1. $AF = GD$	7. $EG = BD$
2. $AG = FD$	8. $HF = GF$
3. $HF = CD$	9. $GK = CD$
4. $FD = HC$	10. $FK = HG$
5. $AG = HC$	11. $KD = GC$
6. $EB = GD$	12. $AH = GC$

Have the students use Cinderella's "**Measure Distance**" tool  to investigate the validity of each of the relationships above.

Relationships Related to Congruency

Brainstorming questions:

- How many pairs of congruent triangle can you deduce from the situation?
- Name them?

In this situation, there are three pairs of congruent triangles, as follows:

1. $\triangle AFH \equiv \triangle DGK$
2. $\triangle AFH \equiv \triangle GDC$
3. $\triangle DGK \equiv \triangle GDC$

Relationships Related to Areas

Brainstorming questions:

- How many pairs of equivalent parallelograms can you find in the given situation?
- How many pairs of equivalent trapeziums can you find in the given situation?
- How many pairs of equivalent triangles can you find in the given situation?
- How many pairs of equivalent figures can you find in the given situation?

Relationships related to areas are:

Equivalent parallelograms

1. *S.A of Parallelogram EBDG = S.A of Parallelogram AFDG*
2. *S.A of Parallelogram AFDG = S.A of Parallelogram FDCH*
3. *S.A of Parallelogram EBDG = S.A of Parallelogram FDCH*

Equivalent trapeziums

4. *S.A of Trapezium EBDK = S.A of Trapezium AFKG*
5. *S.A of Trapezium AFKG = S.A of Trapezium FDGH*

$$6. \text{ S.A of Trapezium } EBCG = \text{ S.A of Trapezium } AFDC$$

Equivalent triangles

$$7. \text{ S.A of } \triangle AEG = \text{ S.A of } \triangle FBD$$

$$8. \text{ S.A of } \triangle AFH = \text{ S.A of } \triangle DGK$$

$$9. \text{ S.A of } \triangle AFH = \text{ S.A of } \triangle GDC$$

$$10. \text{ S.A of } \triangle GDC = \text{ S.A of } \triangle DGK$$

Relationships Related to Similarity

Regarding similarity, there are more than 15 pairs of similar triangles that can be deduced from the given situation using either the theorem which states that: “A parallel to a side of a triangle that intersects the other two sides cuts off a triangle similar to the original triangle”, or the definition of two similar triangles. Encourage the students to find as many pairs of similar triangles as they can by using the given information in the situation. Here is a list of pairs of similar triangles:

$$1. \triangle AFH \sim \triangle ABC$$

$$9. \triangle GDC \sim \triangle FBD$$

$$2. \triangle AFH \sim \triangle AEG$$

$$10. \triangle GDC \sim \triangle DGK$$

$$3. \triangle AFH \sim \triangle FEK$$

$$11. \triangle DGK \sim \triangle ABC$$

$$4. \triangle AFH \sim \triangle FBD$$

$$12. \triangle DGK \sim \triangle FEK$$

$$5. \triangle AFH \sim \triangle GDC$$

$$13. \triangle DGK \sim \triangle FBD$$

$$6. \triangle AFH \sim \triangle DGK$$

$$14. \triangle FEK \sim \triangle ABC$$

$$7. \triangle GDC \sim \triangle ABC$$

$$15. \triangle FEK \sim \triangle FBD$$

$$8. \triangle GDC \sim \triangle FEK$$

$$16. \triangle FBD \sim \triangle ABC$$

Each of the similarity relationships above could be investigated either using different facilities of Cinderella or “**Similarity**” mode of Cinderella’s geometric transformation as it is explained in the previous activity.

Relationships Related to Ratio & Proportion

Using Talis Theorem, we can deduce two relationships related to ratio and proportion as follows:

$$(\because \overleftrightarrow{FH} \parallel \overleftrightarrow{EG} \parallel \overleftrightarrow{BC}; \overleftrightarrow{AB} \text{ and } \overleftrightarrow{AC} \text{ are two transversals } \therefore \frac{AF}{AH} = \frac{FE}{HG} = \frac{EB}{GC})$$

$$(\because \overleftrightarrow{GD} \parallel \overleftrightarrow{AB}; \overleftrightarrow{CA} \text{ and } \overleftrightarrow{CB} \text{ are two transversals } \therefore \frac{CG}{GA} = \frac{CD}{DB})$$

Considering the found relationships related to similarity and as a result of this similarity, two more relationships related to ratio and proportion could be deduced for each pair of similar triangles; one related to lengths of their corresponding sides and the other related to their surface areas. Here is an example:

Since the two triangles $\triangle AFH$ and $\triangle ABC$ are similar, so the lengths of the sides of $\triangle AFH$ are proportional to the lengths of the sides of $\triangle ABC$. Symbolically, it can be expressed as follows:

$$(\because \triangle AFH \sim \triangle ABC \therefore \frac{AF}{AB} = \frac{AH}{AC} = \frac{FH}{BC})$$

The other relationship, which is related to surface areas of the two similar triangles, could be deduced using the theorem which states that: “The ratio between the surface areas of two similar triangles is equal to the square of the ratio between the lengths of two corresponding sides”. Consequently,

$$(\because \triangle AFH \sim \triangle ABC \therefore \frac{S.A \text{ of } \triangle AFH}{S.A \text{ of } \triangle ABC} = \left[\frac{FH}{BC} \right]^2)$$

As in the example above, the students should come up with two relationships related to ratio and proportion as well as use Cinderella's facilities to investigate them for each pair of similar triangles previously listed.

Produce Mathematical Proofs for the Relationships

After the students find and investigate the relationships, they should be encouraged to use the formal logical and deductive reasoning steps to build mathematical proofs for the investigated relationships.

Resources

Used in Developing the Suggested Enrichment Program

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Appendix D

**Experts Who Validated
the Enrichment Program**

A List of the Experts Who Validated the Suggested Enrichment Program

Name	Title
Dr. Albert Organian	LGH Schule Mathematics Teacher
Prof. Dr. Astrid Beckmann	Professor of Mathematics and Mathematics Education, President of University of Education Schwäbisch Gmünd
Dipl. Päd. Axel M. Blessing	Research Assistant in Mathematics and Informatics Institute, University of Education Schwäbisch Gmünd
Christian Dohrmann	Research Assistant in Mathematics and Informatics Institute, University of Education Schwäbisch Gmünd
M. A. Enas Mohamed Farahat	Assistant lecturer, Curriculum and Instruction Department, Dameitta Faculty of Education, Mansoura University, Egypt
Prof. Dr. Helmut Albrecht	Professor of Mathematics and Mathematics Education, Director of Mathematics and Informatics Institute, University of Education Schwäbisch Gmünd
Prof. Dr. Klaus-Peter Eichler	Professor of Mathematics and Mathematics Education, University of Education Schwäbisch Gmünd
Dr. Olga Lomonosova	LGH Schule Mathematics Teacher

Appendix E

A Letter to Experts for Judging on the Program Appropriateness

A Letter to the Experts for Judging on the Appropriateness of the Suggested Enrichment Program

Dear

The researcher is preparing Ph. D. dissertation entitled: “The effectiveness of an enrichment program using dynamic geometry software in developing mathematically gifted students’ geometric creativity in high schools”. Therefore, the researcher prepared a suggested enrichment program using dynamic geometry software, Cinderella application, as a mediation tool to hopefully develop the geometric creativity abilities among the mathematically gifted students in high schools. The suggested enrichment program is comprised of three interrelated portions: students handouts, a teacher guide, and a CD-ROM.

The researcher presents the suggested enrichment program in its three portions to you and hopes your assistance as an expert in the field of teaching and learning mathematics to decide on its appropriateness and suggest any changes to modify it in the framework of the following criteria:

- To what extent the suggested enrichment program is appropriate to the level of the mathematically gifted students in the high schools.
- To what extent the enrichment activities included in the suggested enrichment program are appropriate to develop the mathematically gifted students’ geometric creativity in high schools.
- To what extent the use of Cinderella application is appropriate to the learning actions adopted in the suggested enrichment program.
- To what extent the teacher’s guide is appropriate to guide the teaching and learning processes during the program’s sessions.
- To what extent the directions set in the teacher’s guide are clear.
- Further comments for development.

Appendix E. A Letter to Experts for Judging on the Appropriateness of the Program

Thank you very much in advance for your time and consideration in this program.

The Researcher

Mohamed El-Demerdash

Ph. D. Student

Institute of Mathematics and Informatics

Faculty II

University of Education Schwäbisch Gmünd

Appendix F

**A Letter to Judges for Validating
the Geometric Creativity Test**

A Letter to the Judges for Validating the Geometric Creativity Test

Dear ...

The researcher is preparing Ph. D. thesis entitled: “The effectiveness of an enrichment program using dynamic geometry software in developing mathematically gifted students’ geometric creativity in high schools”. The researcher prepared a test to assess the geometric creativity of the mathematically gifted students before and after administering the suggested enrichment program. The GCT has 12 items designed to measure the following creative thinking abilities in the field of geometry:

1. **Fluency:** the student’s ability to pose or come up with many geometric ideas or configurations related to a geometric problem or situation in a short time. It is objectively defined as the number of relevant responses toward a geometric problem or situation. It is intended to be measured by items 1, 2, 3, and 4.
2. **Flexibility:** the student’s ability to vary the approach or suggest a variety of different methods toward a geometric problem or situation. It is objectively defined as the number of different categories of relevant responses: answers, methods, or questions toward a geometric problem or situation. It is intended to be measured by items 5, 6, and 9.
3. **Originality/Novelty:** the student’s ability to try novel or unusual approaches toward a geometric problem or situation. It is objectively defined as the statistical infrequency of responses in relation to peer group. It is intended to be measured by items 7, 8, and 12.
4. **Elaboration:** the student’s ability to redefine a single geometric problem or situation to create others, which is not the geometric problem, situation itself, or even its solutions but rather the careful thinking upon the particular aspects that govern the geometric problem or situation, changing one or more of these aspects by substituting, combining, adapting, altering, expanding, eliminating,

rearranging, or reversing and then speculating on how this single change would have a ripple effect on other aspects of the problem or the situation at hand. It is objectively defined as the number of follow-up questions or problems that are posed by elaborating one or more aspects of the given geometric problem or situation. It is intended to be measured by items 10 and 11.

I hope that you can help me to validate the test items by reviewing and judging them in the framework of the following considerations:

- To what extent the test items are clear and readable.
- To what extent the test items are appropriate in measuring what they are designed to measure.
- To what extent the test items are appropriate to the level of the mathematically gifted students in the high schools.
- Further comments for development.

Thank you very much in advance for your time and consideration in this test.

The Researcher

Mohamed El-Demerdash

Ph. D. Student

Institute of Mathematics and Informatics

Faculty II

University of Education Schwäbisch Gmünd

Appendix G

**Judges Who Validated the
Geometric Creativity Test**

A List of the Judges Who Validated the Geometric Creativity Test

Name	Title
Dr. Albert Organian	LGH Schule Mathematics Teacher
Dr. Andreas Kittel	Lecturer in Mathematics and Informatics Institute, University of Education Schwäbisch Gmünd
Prof. Dr. Astrid Beckmann	Professor of Mathematics and Mathematics Education, President of University of Education Schwäbisch Gmünd
Dipl. Päd. Axel M. Blessing	Research Assistant in Mathematics and Informatics Institute, University of Education Schwäbisch Gmünd
Christian Dohrmann	Research Assistant in Mathematics and Informatics Institute, University of Education Schwäbisch Gmünd
M. A. Enas Mohamed Farahat	Assistant lecturer, Curriculum and Instruction Department, Dameitta Faculty of Education, Mansoura University, Egypt
Prof. Dr. Gerald Wittmann	Professor of Mathematics and Mathematics Education, University of Education Schwäbisch Gmünd
Prof. Dr. Helmut Albrecht	Professor of Mathematics and Mathematics Education, Director of Mathematics and Informatics Institute, University of Education Schwäbisch Gmünd

Appendix G. A List of the Judges Who Validated the Geometric Creativity Test

Prof. Dr. Jiansheng Bao	Professor of Mathematics Education, Mathematics Department, Soochow University, China

Prof. Dr. Klaus-Peter Eichler	Professor of Mathematics and Mathematics Education, University of Education Schwäbisch Gmünd

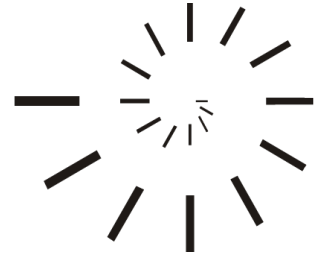
Dr. Olga Lomonosova	LGH Schule Mathematics Teacher

Dr. Silke Ladel	Lecturer in Mathematics and Informatics Institute, University of Education Schwäbisch Gmünd

Appendix H

The Geometric Creativity Test

(GCT – English Version)



The Geometric Creativity Test

(GCT – English Version)

Prepared by

Mohamed El-Sayed Ahmed El-Demerdash

Master of Education (Curriculum and Instruction)

Under Supervision of

Professor Dr. Ulrich Kortenkamp

Professor of Mathematics and Education

Institute of Mathematics and Computer Science

University of Education Karlsruhe

The Geometric Creativity Test

Directions

This geometric creativity test is a part of an educational research aiming at assessing your creativity in geometry. So write freely all you think of without fear or hesitation of your responses, cause that will help you to express your creative personality that is inside you and you may not know about it, and that also enables us to find how creative you are in geometry.

The items in the booklet provide you with opportunities to think freely in geometry, produce mathematical relationships, new geometric proofs, and solve non-routine geometric problems which have various and different methods of solution as well as give you the opportunity to pose some relevant problems toward a geometric situation. So try to respond to each item by the greatest number of unusual, various, and different ideas – things no one else in your class will think of. Let your mind go far and deep in thinking up ideas.

This geometric creativity test contains 12 items. You will have 100 minutes to complete the test. Make good use of your time and work as fast as you can without rushing. If you run out of ideas for a certain item go on to the next item. Respond to those items, mainly based upon your quick performance, recording your ideas, and use your mathematical knowledge in a creative (non-routine) fashion.

Record your ideas in the suitable place for each item. If you need more space to write, ask for extra copies of the question's paper. Do not erase any responses because we need to know everything about your geometric thinking. Do your best!

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Student's Data

Name	School
Teacher's Name	Grade
Birth Date (D/M/Y)	Boy/Girl?

Index of the Mathematical Symbols Used in the Test

$\overline{AB}$	The line segment whose endpoints A and B .
AB	The length of the line segment $\overline{AB}$.
$\overrightarrow{AB}$	The ray whose starting point A and passes through B .
$\leftrightarrow{AB}$	The straight line passes through A and B .
$\overline{AB} \parallel \overline{CD}$	$\overline{AB}$ is parallel to $\overline{CD}$.
$\overline{AB} \perp \overline{CD}$	$\overline{AB}$ is perpendicular to $\overline{CD}$.
$\angle ABC$	The angle ABC whose vertex at B and two arms $\overrightarrow{BA}$ and $\overrightarrow{BC}$.
$m(\angle ABC)$	The measurement of $\angle ABC$.
$F \equiv G$	F is congruent to G .
$F \sim G$	F is similar to G .

Item 1

Write down as many geometric concepts and terminologies as possible that start with letter p.

For example: Polygon.

If you need more space, write on the back of this page.

Item 2

Write down as many generalizations (theorems, definitions, properties, and corollaries) as you can that are related to the right-angled triangle.

For example: In the right-angled triangle, the length of the median from the vertex of the right angle is equal to half the length of the hypotenuse.

If you need more space, write on the back of this page.

Item 3

Suppose we (you and I) are playing a guessing game of determining the name of a geometric figure. In this game, I think of a geometric figure and you will ask me questions about the figure, I should answer, until you determine the figure.

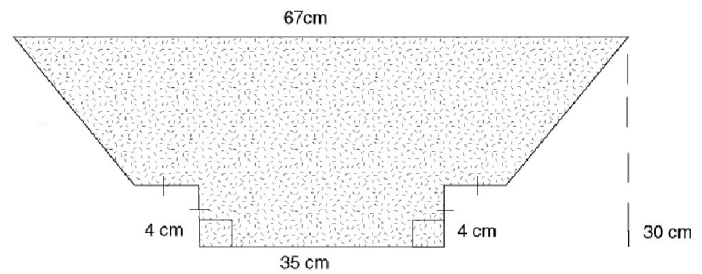
Your task is to list as many questions as you can which should be answered in order to determine the name of the figure.

For example: Is it a plane figure such as a rectangle? Is it a solid figure such as a sphere? Does it have vertices? How many?

If you need more space, write on the back of this page.

Item 4

Find by **all possible ways** the area of the opposite figure.

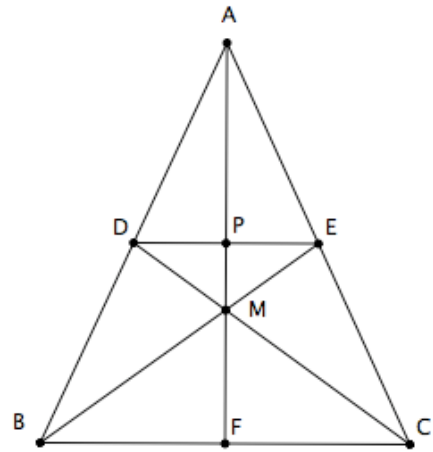


If you need more space, ask for extra papers of this test item.

Item 5

In the opposite figure, ABC is an isosceles triangle, in which $AB = AC$. D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$ and $\overline{BE}$ intersects $\overline{CD}$ at M . $\overrightarrow{AM}$ is drawn to cut $\overline{BC}$ at F , and $\overline{DE}$ is drawn to cut $\overrightarrow{AM}$ at P .

Imagine yourself a mathematician; try to pose the greatest number of various and different problems related to the opposite figure, which could be answered either in direct or indirect ways using the given information. You do not need to solve the problems you write.



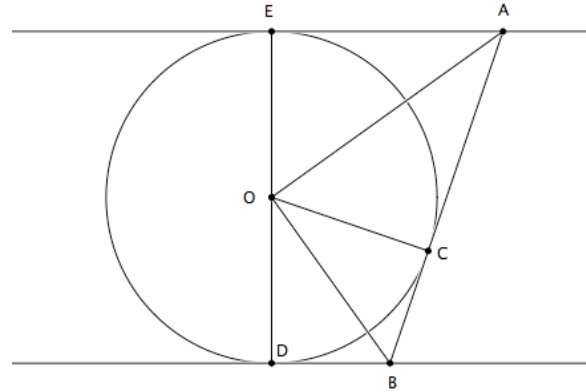
For example: Prove that: $\triangle DMB \cong \triangle EMC$; Show that $DECF$ is a parallelogram.

If you need more space to write problems, ask for extra papers of this test item.

Item 6

Two parallel lines are tangents to a circle with centre O , and a third line, also tangent to the circle, meets the two parallel lines at points A and B and the circle at C .

Pose as many various and different problems as possible that could be deduced from the given information. You do not need to solve the problems you write.



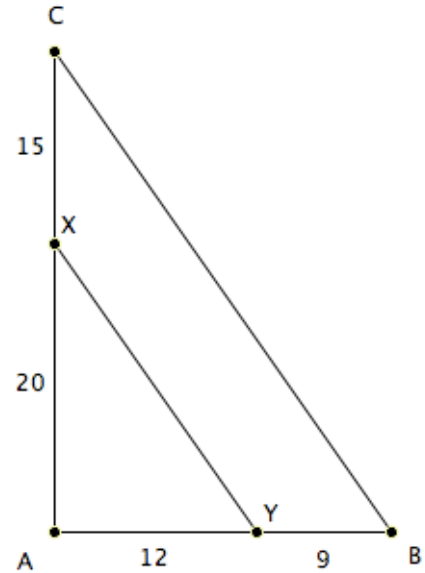
For example: Prove that: $ACOE$ is a cyclic quadrilateral; Show that $AE = AC$.

If you need more space to write problems, ask for extra papers of this test item.

Item 7

By using the information given on this figure, prove
by all possible ways that: $\overline{XY} \parallel \overline{CB}$.

You can construct any segment you think may help
you to get the proof.



If you need more space, ask for extra papers of this test item.

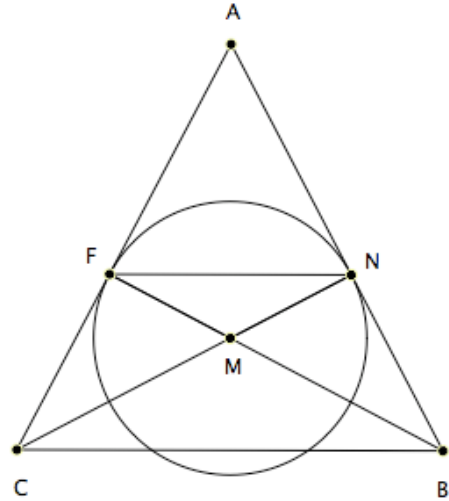
Item 8

In the opposite figure, ABC is a triangle.

$\overline{AB}$, $\overline{AC}$ touch a circle with centre M at N and F respectively, $\overline{BF} \cap \overline{CN} = \{M\}$.

Prove by all possible ways that:

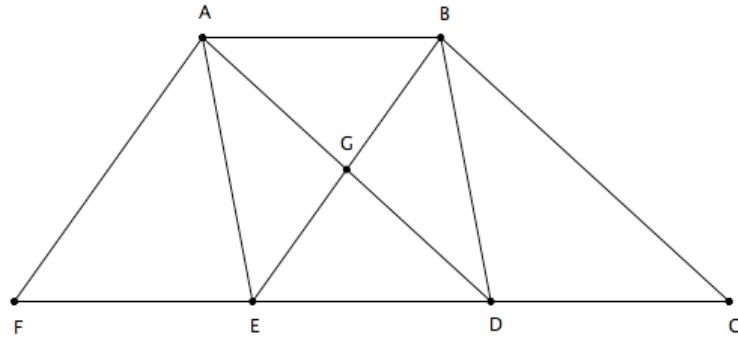
$\triangle ANF \sim \triangle ABC$.



If you need more space, ask for extra papers of this test item.

Item 9

In the opposite figure, $\overline{AB} \parallel \overline{FC}$ and $AB = CD = DE = EF$.



Write down, as many names of pairs of equivalent geometric figures – equal in area – as possible, which are included in the figure opposite. You do not need to show why the two figures, you write, are equivalent.

For example: Triangle BCE and parallelogram ABDE is a pair of equivalent figures.

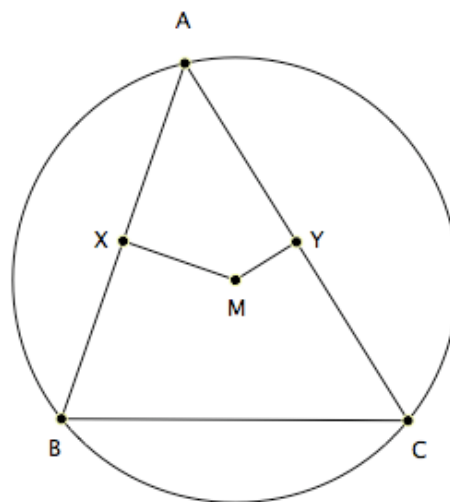
If you need more space, ask for extra papers of this test item.

Item 10

In the opposite figure, M is the centre of a circle, $\overline{MX} \perp \overline{AB}$, and Y is the mid-point of $\overline{AC}$.

Prove that $\overline{XY} \parallel \overline{BC}$.

In this problem, It is not required to give a logical proof as usual but to pose as many problems as possible by elaborating – substituting, adapting, altering, expanding, eliminating, re-arranging or reversing – the conditions of the given problem. You do not need to solve the problems you write.



For example: In the opposite figure, given that: M is the centre of a circle, Y is the mid-point of $\overline{AC}$, and $\overline{XY} \parallel \overline{BC}$. Prove that $\overline{MX} \perp \overline{AB}$.

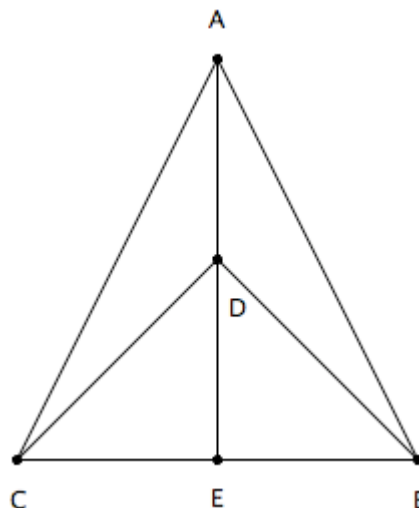
If you need more space to write problems, ask for extra papers of this test item.

Item 11

In the opposite figure, $AB = AC$ and $m(\angle ABC) = m(\angle DBC)$.

Prove that $\overleftrightarrow{AE}$ is the axis of $\overline{BC}$.

In this problem, it is not required to give a mathematical proof. Your task is to think carefully upon the particular aspects that govern the problem, re-define one or more of these aspects by substituting, adapting, altering, expanding, eliminating, rearranging or reversing to make up as many problems or situations as you can. You do not need to solve the problems you write.



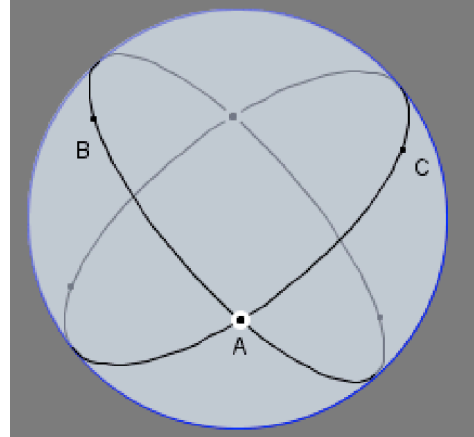
For example: In the opposite figure, given that $\overleftrightarrow{AE}$ is the axis of $\overline{BC}$. Prove that $m(\angle ABC) = m(\angle DBC)$.

If you need more space to write problems, ask for extra papers of this test item.

Item 12

¹⁵Write down as many ideas as you can on what could be happened as a result of doing Euclidean geometry on the spherical surface instead of doing it on a plane surface.

For example: If we start drawing two intersecting lines on the spherical surface, we will eventually end up with two intersecting points as shown in the accompanying figure. Let your mind go far and deep in thinking up possible ideas for this situation.



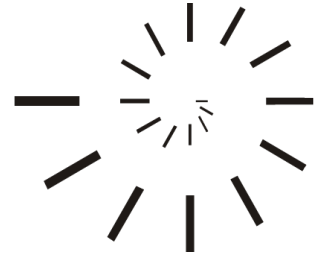
If you need more space to write ideas, write on the back of this page.

¹⁵Adapted from: Balka (1974b) in Mann, E. L. (2005). *Mathematical creativity and school mathematics: Indicators of mathematical creativity in middle school students*. Ph.D. thesis, Connecticut University, United States, pp. 79-80

Appendix I

The Geometric Creativity Test

(GCT – German Version)



Test zur Geometrischen Kreativität (GCT – DE)

Erstellt von

Mohamed El-Sayed Ahmed El-Demerdash

Master of Education (Curriculum and Instruction)

Übersetzung und Betreuung

Professor Dr. Ulrich Kortenkamp

Professor für Mathematik und ihre Didaktik

Institut für Mathematik und Informatik

Pädagogische Hochschule Karlsruhe

Test zur Geometrischen Kreativität

Anleitung

Dieser Test ist Teil eines mathematikdidaktischen Forschungsprojekts welches deine Kreativität in der Geometrie messen soll. Daher bitten wir dich, alle deine Gedanken frei und ohne Zurückhaltung aufzuschreiben, denn dies hilft dir, deine eigene Kreativität auszudrücken. Es kann gut sein, dass du viel kreativer bist, als du denkst, und wenn du alles aufschreibst, dann haben wir eine Chance das herauszufinden.

Die einzelnen Fragestellungen in diesem Heft geben dir die Gelegenheit, ganz frei geometrisch zu denken, mathematische Bezüge herzustellen, neue geometrische Beweise zu finden und außergewöhnliche Geometrieaufgaben zu lösen. Für alle Aufgaben gibt es nicht nur einen richtigen Weg, sondern viele. Zusätzlich möchten wir, dass du dir auch neue Fragen und Aufgaben ausdenkst, die zu den vorgestellten geometrischen Situationen passen. Bitte versuche also bei jeder Aufgabe so viele Antworten wie möglich zu geben – auch unübliche. Versuche, Antworten zu geben, auf die niemand anderes in deiner Klasse gekommen wäre. Lasse deine Gedanken schweifen und habe neue Ideen!

Dieser Test besteht aus 12 Aufgaben. Du hast 100 Minuten für die Bearbeitung des gesamten Tests. Teile deine Zeit gut ein und arbeite so schnell wie du kannst, aber dennoch gründlich. Wenn du zu einer Aufgabe keine Ideen mehr hast, dann bearbeite die nächste. Beantworte die Aufgaben zügig und intuitiv, verlasse dich auf deine mathematischen Kenntnisse und benutze diese auf kreative, also auch auf unübliche, Art und Weise.

Schreibe alle deine Ideen unter der Aufgabe auf dem jeweiligen zur Verfügung stehenden Platz auf. Wenn der Platz nicht ausreicht, dann frage nach mehr Papier. Bitte radriere oder lösche deine Antwort nicht weg – wir möchten alles über deine geometrischen Gedanken wissen. Gib alles!

BLÄTTERE ERST UM, WENN ES DIR GESAGT WIRD.

Schülerdate

Name		Schule	
Lehrer(in)		Klasse	
Geburtsdatum		Geschlecht (m/w)	

Verzeichnis der im Test verwendeten mathematischen Symbole

$\overline{AB}$	Strecke mit den Endpunkten A und B
$ \overline{AB} $	Länge der Strecke $\overline{AB}$.
$\overline{AB} \parallel \overline{CD}$	$\overline{AB}$ ist parallel zu $\overline{CD}$.
$\overline{AB} \perp \overline{CD}$	$\overline{AB}$ ist senkrecht zu $\overline{CD}$.
$\sphericalangle ASB$	Winkel mit Scheitel S und den Schenkeln $\overline{SB}$ und $\overline{SA}$.
$ \sphericalangle ABC $	Maß des Winkels $\sphericalangle ASB$
$F \equiv G$	F ist kongruent zu G .

Aufgabe 1

Gib so viele geometrische Konzepte und Begriffe an, die mit dem Buchstaben „P“ anfangen, wie dir einfallen.

Ein Beispiel: Polygon.

Wenn du mehr Platz benötigst, schreibe auf der Rückseite dieses Blattes weiter.

Aufgabe 2

Schreibe so viele Verallgemeinerungen (Sätze, Definitionen, Eigenschaften, Folgerungen) wie du kannst auf, die mit rechtwinkligen Dreiecken zu tun haben.

Ein Beispiel: In einem rechtwinkligen Dreieck ist die Seitenhalbierende der Hypotenuse halb so lang wie die Hypotenuse

Wenn du mehr Platz brauchst, kannst du auf der Rückseite schreiben.

Aufgabe 3

Nimm an, dass wir ein Ratespiel spielen, bei dem du den Namen einer geometrischen Figur herausfinden sollst.

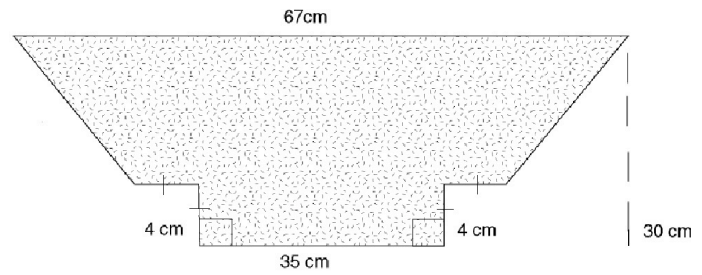
Schreibe so viele Fragen wie möglich auf, die dir helfen heraus zu bekommen, um welche Figur es sich handelt.

Ein Beispiel: Ist es eine ebene Figur wie ein Rechteck? Ist es ein Körper, wie zum Beispiel eine Kugel? Gibt es Eckpunkte? Wie viele

Wenn du mehr Platz brauchst, darfst du auf der Rückseite weiter schreiben.

Aufgabe 4

Finde **so viele Wege wie möglich**, die Fläche der nachstehenden Figur zu berechnen.

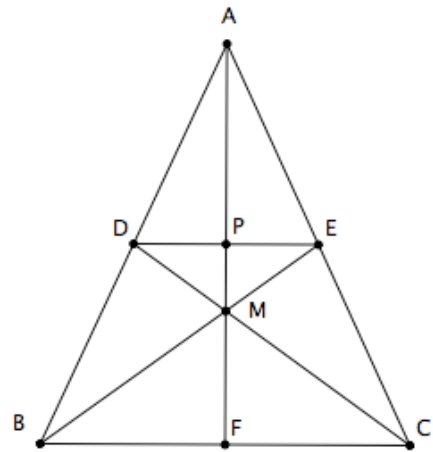


Wenn du mehr Platz brauchst, frage nach weiteren Aufgabenblättern.

Aufgabe 5

Das Dreieck $\triangle ABC$ ist gleichschenkelig mit der Grundseite $\overline{BC}$. D ist der Mittelpunkt der Strecke $\overline{AB}$, E ist der Mittelpunkt der Strecke $\overline{AC}$ und die Strecke $\overline{BE}$ schneidet die Strecke $\overline{CD}$ in M . Der Strahl mit Anfangspunkt A durch M schneidet die Strecke $\overline{BC}$ in F und die Strecke $\overline{DE}$ schneidet den Strahl mit Anfangspunkt A durch M in P .

Versetze dich in die Rolle eines Mathematikers und versuche so viele verschiedene Aufgaben wie möglich zu formulieren, die zu dieser Figur gestellt werden könnten. Du musst die Aufgaben nicht selbst lösen!



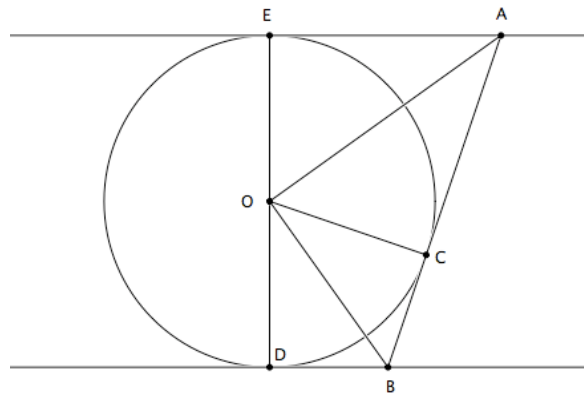
Beispiele: „Beweise dass $\triangle DMB \equiv \triangle EMC$ “ oder „Zeige, dass $DECF$ ein Parallelogramm ist“

Wenn du mehr Platz brauchst, frage nach zusätzlichen Aufgabenblättern für diese Aufgabe.

Aufgabe 6

In der nebenstehenden Figur sehen wir zwei parallele Tangenten an einen Kreis mit Mittelpunkt O sowie eine dritte Tangente mit Berhrpunkt C , die die beiden ersten in A und B schneidet.

Formuliere so viele verschiedene Aufgaben wie möglich, die zu dieser Figur gestellt werden könnten. Du musst diese nicht selbst lösen!



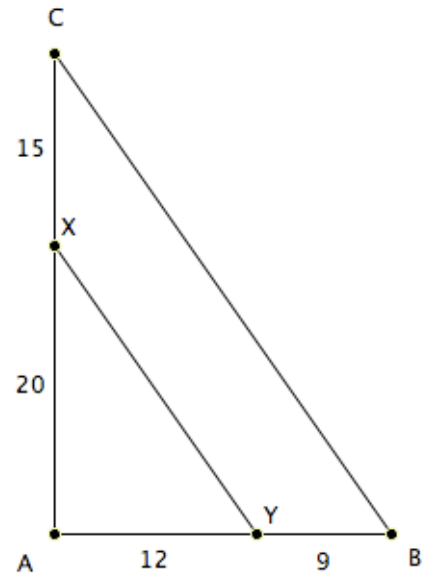
Beispiele: „Beweise dass A , C , O und E auf einem gemeinsamen Kreis liegen“ oder „Zeige, dass die Strecken $\overline{AE}$ und $\overline{AC}$ gleich lang sind.“

Wenn du mehr Platz brauchst, frage nach zusätzlichen Aufgabenblättern für diese Aufgabe.

Aufgabe 7

Finde so viele Beweise wie möglich dafür, dass in der nebenstehende Zeichnung $\overline{XY} \parallel \overline{CB}$ gilt.

Du darfst Hilfslinien einzeichnen, wenn sie dir dabei helfen.

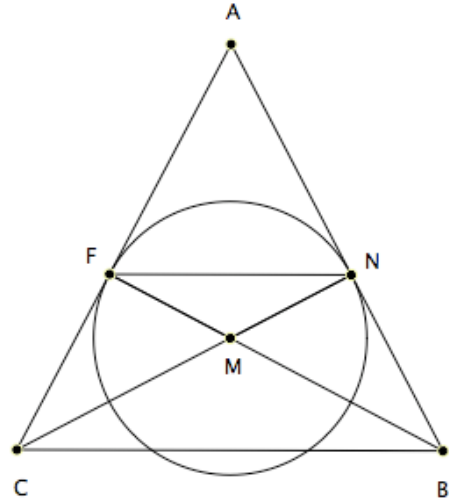


Wenn du mehr Platz brauchst, frage nach zusätzlichen Aufgabenblättern für diese Aufgabe.

Aufgabe 8

In der nebenstehenden Zeichnung ist ABC ein Dreieck. Die Strecke $\overline{AB}$ und die Strecke $\overline{AC}$ berühren den Kreis mit Mittelpunkt M in N und F , darüber hinaus schneiden sich die Strecken $\overline{BF}$ und $\overline{CN}$ in M .

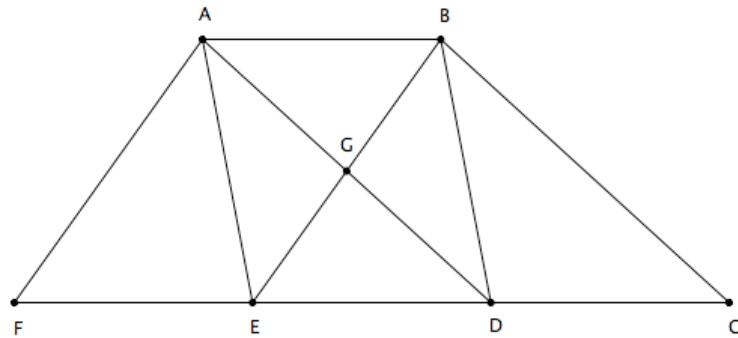
Finde **möglichst viele Beweise dafür**, dass die beiden Dreiecke $\triangle ANF$ und $\triangle ABC$ ähnlich sind.



Wenn du mehr Platz brauchst, frage nach zusätzlichen Aufgabenblättern für diese Aufgabe.

Aufgabe 9

In der nebenstehenden Zeichnung sind die Strecken $\overline{AB}$ und $\overline{FC}$ parallel, und die Strecken $\overline{AB}$, $\overline{CD}$, $\overline{DE}$ und $\overline{EF}$ sind alle gleich lang.



Finde in der Zeichnung so viele Paare von geometrischen Figuren, wie möglich, die den gleichen Flächeninhalt haben. Du brauchst dies nicht begründen.

Zum Beispiel: Das Dreieck BCE und das Parallelogramm ABDE haben den gleichen Flächeninhalt.

Wenn du mehr Platz brauchst, frage nach zusätzlichen Aufgabenblättern für diese Aufgabe.

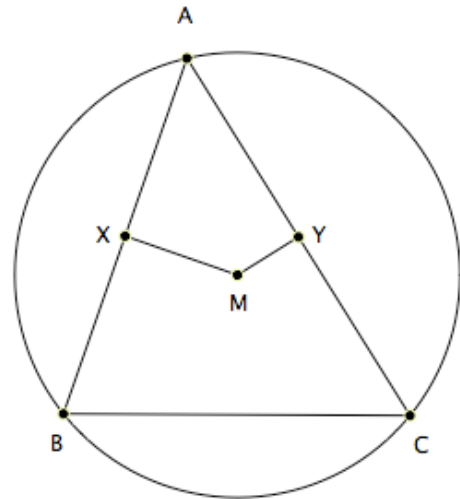
Aufgabe 10

Zu der nebenstehenden Zeichnung gehört die folgende Aufgabe:

M ist der Mittelpunkt des Kreises, $\overline{MX} \perp \overline{AB}$
und Y ist Mittelpunkt der Strecke $\overline{AC}$.

Beweise, dass $\overline{XY} \parallel \overline{BC}$.

Du sollst diesen Beweis nicht machen, sondern stattdessen aus der obigen Aufgabe so viele Variationen wie möglich herstellen. Dazu kannst du Teile ersetzen, anpassen, verändern, erweitern, entfernen, umstellen oder umkehren. Du musst die neuen Aufgaben nicht lösen!



Ein Beispiel: Gegeben ist, dass M Mittelpunkt des Kreises ist, Y Mittelpunkt der Strecke $\overline{AC}$ ist, und $\overline{XY} \parallel \overline{BC}$. Beweise: $\overline{MX} \perp \overline{AB}$.

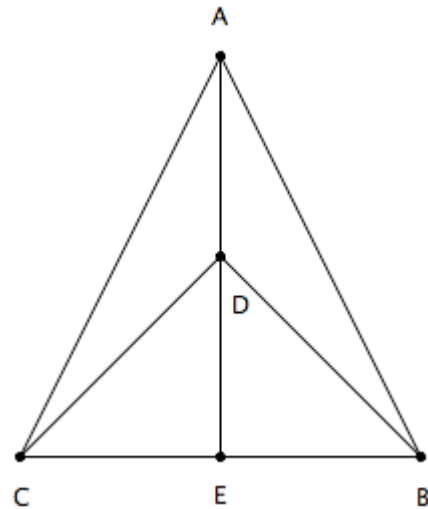
Wenn du mehr Platz brauchst, frage nach zusätzlichen Aufgabenblättern für diese Aufgabe.

Aufgabe 11

Zu der nebenstehenden Zeichnung geht die Aufgabe, Gegeben: $|\overline{AB}| = |\overline{AC}|$,
 $|\angle DCB| = |\angle DBC|$.

Beweise, dass die Gerade durch A und E Symmetrieachse der Strecke $\overline{BC}$ ist.

Du sollst diesen Beweis nicht ausführen, sondern stattdessen aus der obigen Aufgabe so viele Variationen wie möglich herstellen. Dazu kannst du Teile ersetzen, anpassen, verändern, erweitern, entfernen, umstellen oder umkehren. Du musst die neuen Aufgaben nicht lösen!



Zum Beispiel: Gegeben: die Gerade durch A und E ist Symmetrieachse der Strecke $\overline{BC}$

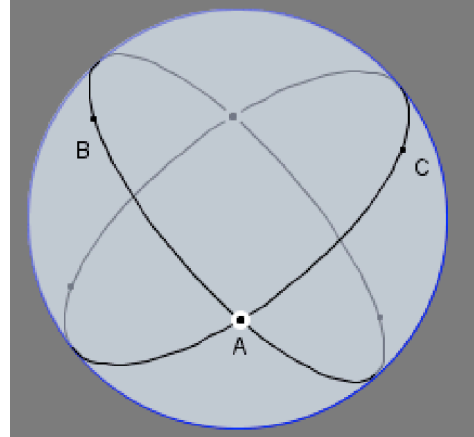
Beweise: $|\angle DCB| = |\angle DBC|$.

Wenn du mehr Platz brauchst, frage nach zusätzlichen Aufgabenblättern für diese Aufgabe.

Aufgabe 12

¹⁶Schreibe so viele Dinge wie möglich auf, die passieren könnten, wenn man Geometrie nicht in der Ebene, sondern auf einer Kugel macht.

Zum Beispiel: Wenn wir zwei sich schneidende Geraden auf der Kugel zeichnen, dann kann es passieren, dass sie sogar zwei Schnittpunkte haben, wie in der nebenstehenden Zeichnung. Denke gut nach, lasse deine Gedanken schweifen, damit du möglichst viele Ideen bekommst.



Wenn du mehr Platz benötigst, schreibe auf der Rückseite dieses Blattes weiter.

¹⁶Nach: Balka (1974b) in Mann, E. L. (2005). *Mathematical creativity and school mathematics: Indicators of mathematical creativity in middle school students*. Ph.D. thesis, Connecticut University, United States, pp. 79-80