



Symmetries and asymmetries of multi-dimensional cheap talk games

Stéphan Sémirat

► To cite this version:

Stéphan Sémirat. Symmetries and asymmetries of multi-dimensional cheap talk games. Economics and Finance. Université Grenoble Alpes, 2018. English. NNT : 2018GREAE008 . tel-01947039v2

HAL Id: tel-01947039

<https://hal.science/tel-01947039v2>

Submitted on 9 May 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

THÈSE

Pour obtenir le grade de

DOCTEUR DE LA COMMUNAUTE UNIVERSITE GRENOBLE ALPES

Spécialité : **Sciences Économiques**

Arrêté ministériel : 25 mai 2016

Présentée par

Stéphan SÉMIRAT

Thèse dirigée par

Christophe BRAVARD

Professeur à l'Université Grenoble Alpes

préparée au sein du Laboratoire

Grenoble Applied Economics Lab

dans l'École Doctorale Sciences Économiques

Symétries et asymétries des jeux de conversation gratuite multi-dimensionnels

Symmetries and asymmetries of multi-dimensional cheap talk games

Thèse soutenue publiquement le 3 décembre 2018,
devant le jury composé de :

Monsieur Christophe BRAVARD

Professeur à l'Université Grenoble Alpes, Directeur de thèse

Monsieur Frédéric DEROÏAN

Directeur de recherche au CNRS à l'École d'Économie d'Aix-Marseille, Examineur

Madame Françoise FORGES

Professeure à l'Université Paris Dauphine, Présidente

Monsieur Olivier GOSSNER

Professeur à l'École Polytechnique, Rapporteur

Madame Jeanne HAGENBACH

Professeure à Sciences Po Paris, Examinatrice

Monsieur Frédéric KOESSLER

Directeur de recherche au CNRS à l'École d'Économie de Paris, Rapporteur



Remerciements

Cette thèse n'aurait pu se faire sans l'appui des équipes de direction de la Faculté d'Économie de Grenoble et du laboratoire GAEL, qui a permis et autorisé une décharge de la moitié de mon service d'enseignement. Je dois ce soutien à Alexis Garapin, Alain Laurent et Stéphane Lemarié que je remercie vivement.

Merci à tous les membres du laboratoire GAEL pour l'accueil, la logistique et le cadre de travail non seulement performant mais aussi sympathique.

Merci aux chercheurs avec qui j'ai pu interagir, notamment Pascal Billand, Paolo Crosetto, Benoît Crutzen, Jacques Durieu, Ani Guerdjikova, Jürgen Kamphorst, Nadine Massard, Sudipta Sarangi, Laurent Simula et Michal Urdanivia, pour leur disponibilité et les nombreux conseils qui m'ont été donnés.

Merci à Frédéric Deroïan, Françoise Forges, Olivier Gossner, Jeanne Hagenbach et à Frédéric Koessler, qui m'honorent en acceptant de prendre part au jury.

Merci à Frédéric Deroïan, Françoise Forges et Sabrina Teyssier qui ont accepté de participer aux comités de suivi de thèse. Leur expertise a guidé mes efforts.

Un grand merci à mon directeur de thèse, Christophe Bravard, pour son encadrement sans faille, sa disponibilité sans limite et pour le partage généreux de sa culture scientifique, économique, et sociale.

Enfin, merci sans économie à Florence, Zoé et Manon, qui donnent corps aux idées.

Table des matières

Introduction générale	1
1 Multi-dimensional cheap talk with complementarities	15
1.1 Introduction	16
1.2 Model setup	20
1.3 Analysis	21
1.3.1 Strategies	21
1.3.2 Influential equilibrium conditions	22
1.3.3 Influential equilibria	25
1.4 Robustness	27
1.4.1 A qualitative effect	27
1.4.2 Exogenously asymmetric contributions	29
1.5 Selection	30
1.5.1 Stability	31
1.5.2 Efficiency	31
1.6 Conclusion	33
Proofs	35
1.A Proof of Proposition 1.1	37
1.B Proof of Proposition 1.2	43
1.C Proof of Proposition 1.3	45
1.D Proof of Proposition 1.4	46
1.E Proof of Proposition 1.5	46
2 Uniform quadratic binary two-dimensional cheap talk	49
2.1 Introduction	50
2.2 Model setup	54
2.3 Analysis and examples	55
2.3.1 Strategies	55
2.3.2 Symmetric equilibria and their deviations	58
2.3.3 Asymmetric equilibria	60
2.4 Influence in the context of any conflict	64
2.5 Characterization of the equilibrium strategies	68

2.6	Conclusion	71
Proofs		73
2.A	Proof of Example 2.1	75
2.B	Proof of Example 2.2	75
2.C	Proof of Example 2.3	76
2.D	Proof of Lemma 2.1	76
2.E	Proof of Lemma 2.2	76
2.F	Proof of Lemma 2.3	79
2.G	Proof of Proposition 2.1	82
2.H	Proof of Lemma 2.5	85
3	Better or not that bad? Asymmetric comparative cheap talk	87
3.1	Introduction	88
3.2	Related literature	91
3.3	Model setup	93
3.3.1	Setting	93
3.3.2	Strategies and concept solution	94
3.3.3	Babbling, influential, comparative, or symmetric equilibria	95
3.4	Comparative equilibria	96
3.5	Asymmetric comparisons	99
3.5.1	Strongly asymmetric comparisons	102
3.5.2	Weakly asymmetric comparisons	103
3.5.3	Credible asymmetric comparisons	105
3.5.4	A stability criterion	107
3.6	Conclusion	109
Proofs		111
3.A	Proof of Proposition 3.1	113
3.B	Proof of Lemma 3.1	114
3.C	Proof of Lemma 3.2	120
3.D	Proof of Corollary 3.3	122
3.E	Proof of Proposition 3.3	122
3.F	Example	125
Conclusion générale		127

Introduction générale

Transmission stratégique d'information

Une information de mauvaise qualité peut entraîner une prise de décision aux conséquences désastreuses. À titre d'exemple, la crise des subprimes de 2007 fut caractéristique d'une information de mauvaise qualité sur les actifs échangés. Ainsi, l'information est un facteur clé de toute prise de décision. Les agents économiques sont donc demandeurs d'informations.

Sur un marché parfaitement concurrentiel, les prix des biens échangés, accessibles à tous les acteurs, sont porteurs de toute l'information pertinente. En effet, à l'équilibre, le prix égalise l'offre et la demande. Ainsi, le prix rend compte à la fois de l'utilité d'un bien, et de son coût. Or dans de nombreuses situations, il semble plus réaliste de supposer que des agents n'ont pas accès à toute l'information pertinente concernant les biens échangés. Certains agents peuvent être détenteurs d'informations qui leur sont privées. Ainsi, par exemple le vendeur d'une voiture d'occasion est mieux informé qu'un acheteur potentiel sur la qualité de la voiture, un individu qui souscrit une assurance est mieux informé sur ses comportements à risque que ne l'est la compagnie d'assurance, et un employé connaît mieux que l'employeur l'effort qu'il investit dans une tâche. Dans ce cas, le prix ne révèle pas toute l'information existante.

L'équilibre du marché peut être modifié par l'existence d'asymétries d'information entre les agents. Sous certaines conditions, le marché peut même disparaître. Dans ce cas, le cadre théorique du marché concurrentiel ne semble pas adapté pour rendre compte des interactions économiques. Plus précisément, l'hypothèse que chaque agent, pris individuellement, n'a pas d'influence sur les modalités de ses interactions économiques, semble peu réaliste. Au contraire, dans de nombreuses situations, il est plus plausible de supposer que les frictions informationnelles peuvent permettre aux agents mieux informés d'influencer les choix des agents avec lesquels ils interagissent. Dans cette perspective, les mécanismes sous-jacents aux interactions économiques peuvent être mieux identifiés à travers l'analyse des stratégies des agents selon leurs intérêts individuels et selon l'information privée dont ils peuvent être détenteurs.

L'analyse stratégique, en prenant en considération l'asymétrie d'information d'une situation donnée, se pose la question de la possibilité du partage de l'information privée. En effet, si par exemple l'agent détenteur d'une information privée a la possibilité et l'intérêt de révéler son information privée, alors l'asymétrie d'information est automatiquement

supprimée de l'interaction par le calcul stratégique. Ainsi, un candidat à l'embauche révèle sa forte adéquation à l'emploi proposé s'il en a la possibilité. Un individu accusé à tort d'un crime révèle son innocence s'il en a la possibilité.

Outre des déterminants contextuels, comme la possibilité matérielle de délivrer un message, la transmission effective d'information dans un cadre stratégique ne peut se faire qu'à la condition que le message délivré soit crédible. C'est cette question de la crédibilité de l'information transmise entre des agents aux intérêts individuels potentiellement divergents qui constitue le sujet d'étude de cette thèse. Nous examinons cette question dans un cadre d'analyse spécifique, qualifié de jeu de conversation gratuite par la littérature.

Jeu de conversation gratuite

La littérature distingue plusieurs cadres d'analyse pour la transmission stratégique d'information.

Dans un jeu de signal, la transmission d'information est coûteuse pour l'agent informé. Ce dernier encourt différents coûts selon son type (c'est à dire en fonction de son information privée). Dans ce cas, l'agent non informé peut observer la dépense réalisée par l'agent informé, qui est alors un signal de son type. Par exemple, un individu qui s'est engagé dans des études supérieures signale au marché de l'emploi sa capacité de travail, qui est *a priori* non observable par les employeurs.

Dans un jeu à information vérifiable, l'agent informé ne peut pas mentir à l'agent non informé concernant son information privée. Il peut toutefois révéler une partie de son information. Par exemple, une version allégée d'un logiciel commercial peut être mise à disposition d'un potentiel acheteur. Un média partisan ne rend compte que des retombées positives des décisions politiques de son parti.

Enfin, dans un jeu de persuasion, l'agent informé s'engage sur la procédure de transmission d'information avant d'être lui-même informé. Par exemple, un enseignant peut révéler l'habileté d'un étudiant à travers une évaluation dont les règles ont été précisées à l'avance. Un expert scientifique met en place une expérience encadrée par un protocole. Un interrogatoire judiciaire doit se conformer à certaines procédures.

Dans cette thèse, nous traitons du cas où la transmission d'information s'effectue sans coût, sans possibilité de vérification, et sans engagement. C'est ce cadre d'analyse que la littérature qualifie de jeu de conversation gratuite. Ce type de jeu se rencontre par exemple lorsqu'un agent informé conseille verbalement un autre agent. Les relations de mentor à mentoré, telles qu'un manager s'adressant à un employé concernant la conduite de son travail, ou un enseignant conseillant un étudiant à propos de la poursuite de ses études, peuvent être appréhendées par de tels jeux. Les relations d'expert à décideur politique, ou l'information délivrée par les groupes de pressions, peuvent être aussi abordées par ces jeux.¹ D'une manière générale, une phase de conversation gratuite entre deux agents peut

1. Citons, par exemple, le prochain vote au Parlement Européen concernant le projet de directive intitulé *Copyright*. Ce projet vise la mise en place, d'une part, d'un droit d'auteur sur l'information diffusée, et d'autre part, d'un engagement des plates-formes de téléchargement à faire respecter les contrats

être considérée au préalable de toute interaction économique avec asymétrie d'information.

Le modèle de jeu de conversation gratuite que nous étudions dans cette thèse a été initié par CRAWFORD et SOBEL (1982).

Présentons les principales caractéristiques du jeu de CRAWFORD et SOBEL. L'information privée de l'agent informé S est la réalisation θ d'une variable aléatoire distribuée sur un sous-ensemble Θ , compact et convexe, de l'espace euclidien \mathbb{R}^k , $k \geq 1$. L'agent informé S émet un message m , support de l'éventuelle information transmise. L'agent non informé R observe le message m délivré par l'agent informé, et choisit alors son action. L'action de R est représentée par un élément a de \mathbb{R}^k . À l'issue de leur interaction, les agents S et R obtiennent respectivement une utilité $U^S(a, \theta)$ et $U^R(a, \theta)$, où $(a, \theta) \mapsto U^P(a, \theta)$ est la fonction d'utilité de von Neumann-Morgenstern de l'agent $P \in \{S, R\}$.

Notons que dans ce modèle, les messages potentiellement délivrés n'ont pas de signification exogène au jeu. En quelque sorte, le langage est réduit à ses déterminants économiques. Plus précisément, la signification d'un message est déduite de son utilisation par les agents. Formellement, les messages m potentiellement délivrés appartiennent à un ensemble abstrait M de messages, et la seule condition sur M est que ses éléments peuvent être distingués les uns des autres par les agents.

Décrivons à présent les principaux enjeux stratégiques de ce modèle. Tout d'abord, puisque l'émission d'un message est non coûteuse et sans engagement, et puisque l'éventuelle information transmise est non vérifiable, aucune contrainte n'est imposée sur les messages délivrés par l'agent informé. En particulier, tout message peut être *a priori* délivré par l'agent informé. Dans ces conditions, tout message délivré rend nécessairement compte de l'intérêt de l'agent informé à délivrer ce message plutôt que n'importe quel autre message.

C'est la reconnaissance, par l'agent non informé, de l'intérêt de l'agent informé dans le message délivré, qui peut permettre la transmission d'information. En effet, si l'intérêt de l'agent informé dépend de son type, alors le message délivré, choisi par intérêt, rend potentiellement compte d'un ensemble restreint de types.

Par exemple, supposons qu'un expert militaire recommande ou ne recommande pas l'engagement dans la guerre selon son évaluation des forces militaires de son armée et de l'armée adverse, et que cette information soit son information privée. À partir de la recommandation de l'expert de s'engager ou de ne pas s'engager dans la guerre, le décideur politique peut alors lui-même estimer l'état des forces militaires, qu'il ne connaît pas *a priori*, et prendre la décision qui est conforme à ses propres intérêts, au-delà des intérêts de l'expert militaire.

Ainsi, il y a transmission stratégique d'information lorsque le message délivré permet à l'agent non informé de qualifier d'une certaine manière le type de l'agent informé. Par exemple, la transmission est totalement informative si chaque message est associé à un

associés aux contenus protégés. Les parlementaires européens sont intéressés par le degré de mobilisation des internautes. Cette information est détenue de manière privée par les grandes entreprises d'internet, qui peuvent observer à grande échelle les activités des internautes. Concernant la transmission d'informations des groupes de pression, le journal *Le Monde* du mardi 4 septembre 2018 rend compte de la transmission, par les grandes entreprises d'internet et à destination des parlementaires, d'informations faisant état d'une forte mobilisation contre la mise en place de la directive.

unique type. La transmission est non informative si le message délivré ne permet pas de distinguer les types. La transmission est partiellement informative si un message délivré permet à l'agent non informé d'exclure certaines réalisations potentielles du type.

Une fois le message reçu, l'agent *a priori* non informé extrait l'information éventuellement portée par le message afin de prendre sa décision. Le point important, d'un point de vue stratégique, est que le message délivré prend aussi en compte, par anticipation, la décision de l'agent non informé. Par exemple, si le décideur politique ne tient pas compte de la recommandation de l'expert militaire, alors l'expert n'a aucun intérêt à choisir une recommandation plutôt qu'une autre. Réciproquement, si l'expert militaire a des préférences pacifiques et recommande toujours le non engagement dans la guerre, indépendamment de sa connaissance des forces armées, alors le décideur politique n'extrait pas d'information du message de l'expert, ce qui n'incite pas l'expert à changer de message. Il y a alors absence de communication.

Plus généralement, dans un jeu de conversation gratuite, les messages délivrés par l'agent informé sont non seulement déterminés par son type, mais sont aussi guidés par l'utilisation du message transmis par l'agent non informé. En particulier, ils rendent non seulement compte de l'intérêt de l'agent informé relativement à son type et à l'action induite par le message (qu'il anticipe), mais aussi de l'intérêt de l'agent non informé à choisir précisément l'action anticipée par l'agent informé compte tenu de l'information sur le type qu'il déduit du message reçu.

Finalement, la transmission stratégique d'information est une situation d'équilibre entre les différents choix des agents, au sens où les choix des agents se prennent en compte et simultanément se déterminent l'un par rapport à l'autre.

Motivations théoriques

L'objectif de l'analyse des jeux de conversation gratuite est alors d'une part, d'identifier les conditions sur les paramètres du jeu permettant d'atteindre des équilibres informatifs, et d'autre part, lorsqu'il y a effectivement possibilité de transmission d'information, d'analyser la nature de l'information transmise en lien avec ses conséquences économiques.

Avant de présenter notre contribution, et afin de mieux la circonscrire, présentons les principaux résultats de la littérature qui ont motivé notre analyse.

Selon CRAWFORD et SOBEL (1982), une condition nécessaire et suffisante à la transmission d'information est l'alignement des intérêts des agents quant à l'action \mathbf{a} à décider, par l'agent non informé, étant donné le type θ de l'agent informé. Plus précisément, l'analyse de CRAWFORD et SOBEL permet de mettre en valeur un lien entre les différences d'intérêt des agents, et la quantité d'information potentiellement transmise à l'équilibre.

Donnons quelques détails formels de leur analyse. CRAWFORD et SOBEL supposent que l'information privée de l'agent informé S est la réalisation $\theta = \theta$ d'une variable aléatoire distribuée sur $[0, 1]$, et que l'action $\mathbf{a} = a$ de l'agent non informé R est un élément de \mathbb{R} . Les fonctions d'utilité U^S et U^R des deux agents sont deux fois continuellement différentiables, concaves, sur-modulaires (formellement : $\frac{\partial^2 U^P}{\partial a \partial \theta} > 0$) et, quel que soit $\theta \in [0, 1]$, atteignent

un maximum selon a , en $a^S(\theta)$ et $a^R(\theta)$ respectivement. Le conflit d'intérêt entre les agents est alors représenté par la distance $|a^S(\theta) - a^R(\theta)|$ entre leur action optimale, selon le type réalisé. L'hypothèse de concavité des fonctions d'utilité permet de garantir un conflit d'intérêt fini, et l'hypothèse de sur-modularité permet de rendre compte d'un intérêt commun des deux agents pour décider d'une action plus grande étant donné un type plus grand.

Sous ces conditions, dans un premier temps, les auteurs caractérisent les équilibres du jeu. Ils montrent que si, quel que soit $\theta \in [0, 1]$, $a^S(\theta) \neq a^R(\theta)$, alors :

- (i) les équilibres sont, à un ensemble de mesure nulle près, obtenus à travers une partition finie de l'ensemble des types $[0, 1]$ en N intervalles, $N \geq 1$, telle que
 - chaque intervalle est associé à un unique message m_j , et en particulier m_j est délivré par l'agent informé si et seulement si son type est élément de l'intervalle ;
 - chaque intervalle est associé à une unique action, $a(m_j)$, solution de la maximisation de l'espérance d'utilité de R , sachant que θ est élément de l'intervalle ;
- (ii) les partitions d'équilibre peuvent être partiellement ordonnées, selon la quantité d'information transmise, avec :
 - la partition $[0, 1]$, composée d'un unique intervalle, qui est non informative ;
 - une partition en $\bar{N} \geq 1$ intervalles, \bar{N} dépendant de U^S , U^R , et de la distribution de θ , qui correspond à la partition la plus informative ;
 - l'existence d'au moins une partition d'équilibre en N intervalles, pour tout N compris entre 1 et \bar{N} .

Ensuite, étant donnée la caractérisation des équilibres, CRAWFORD et SOBEL mettent en lumière le lien entre le conflit d'intérêt et la transmission d'information. À cette fin, ils considèrent, U^R étant fixée, des familles de fonctions $U^S = U_b^S$ pour l'utilité de l'agent informé, paramétrées par $b \in \mathbb{R}^+$ et satisfaisant certaines conditions de monotonie relativement à b (en particulier, $U_0^S = U^R$, $\frac{\partial^2 U_b^S}{\partial a \partial b} > 0$, et une certaine Condition (M) d'ordonnancement des partitions en intervalles de $[0, 1]$ telles que les types correspondants aux bornes des intervalles soient indifférents aux actions adjacentes). En particulier, la condition $\frac{\partial^2 U_b^S}{\partial a \partial b} > 0$ impose qu'un accroissement de b induit une préférence de S pour une plus grande action. Un accroissement de b représente donc un plus grand conflit d'intérêt entre R et S . Les auteurs montrent qu'étant donnée une telle famille, le nombre maximum d'éléments d'une partition d'équilibre $\bar{N} = N(b)$ est non décroissant avec b , et que pour tout N compris entre 1 et $N(b)$, il existe une unique partition d'équilibre à N éléments. Ainsi, le résultat important de CRAWFORD et SOBEL (1982) est qu'à un plus petit conflit est associé une possibilité de transmission d'information plus informative, tandis qu'un important conflit peut entraîner l'impossibilité de transmettre de l'information.

À titre d'exemple, les auteurs examinent des fonctions d'utilité quadratiques pour les agents, données par $U^R(a, \theta) = -(a - \theta)^2$ et $U_b^S(a, \theta) = -(a - (\theta + b))^2$. Dans ce cas, l'action préférée de R s'identifie au type θ , avec $a^R(\theta) = \theta$, tandis que l'action préférée de S est, relativement à celle de R , biaisée par b , avec $a^S(\theta) = \theta + b$. Lorsque la distribution des types est uniforme sur $[0, 1]$, une partition d'équilibre informative existe si et seulement si $b < \frac{1}{4}$. Au-delà, le conflit d'intérêt est tel qu'il interdit toute possibilité de transmission

d'information entre les agents.

Le modèle de CRAWFORD et SOBEL (1982) considère que l'information privée $\theta = \theta$ de l'agent informé est de nature uni-dimensionnelle. Or, il existe de nombreuses situations où l'information privée d'un agent peut être multi-dimensionnelle. Par exemple, les caractéristiques d'un bien échangé lors d'une interaction marchande peuvent être multiples (citons la durée de vie du bien, sa facilité d'utilisation, son empreinte écologique). De même, un employé sur le marché du travail peut disposer de plusieurs aptitudes à faire valoir à un employeur potentiel (citons par exemple ses capacités à travailler en équipe, ses aptitudes informatiques, son dynamisme). Un projet de loi possède plusieurs articles à potentiellement amender.

CHAKRABORTY et HARBAUGH (2007) considèrent une extension du modèle de CRAWFORD et SOBEL (1982) à deux dimensions. Les auteurs illustrent l'idée qu'un important conflit d'intérêt des agents le long de chaque dimension n'interdit pas la possibilité d'une transmission d'information. Plus précisément, ils supposent que l'information privée de l'agent informé S est représentée par $\theta = (\theta_1, \theta_2)$, et l'action de l'agent non informé R est représentée par $\mathbf{a} = (a_1, a_2) \in [0, 1]$. Les types θ_1 et θ_2 sont issus de deux variables aléatoires indépendantes et identiquement distribuées sur $[0, 1]$. Chaque agent $P \in \{S, R\}$ possède une fonction d'utilité additivement séparable, et symétrique relativement aux dimensions, $U^P(\mathbf{a}, \theta) = u^P(a_1, \theta_1) + u^P(a_2, \theta_2)$. Les auteurs montrent que si, en chaque dimension, les agents préfèrent associer une plus grande action à un plus grand type (si u^S et u^R sont sur-modulaires), alors la comparaison des deux types θ_1 et θ_2 constitue une stratégie d'équilibre de l'agent informé. En particulier,

- les actions induites par les messages m_1 et m_2 , données par $\mathbf{a}(m_1) = (a_1(m_1), a_2(m_1))$ et $\mathbf{a}(m_2) = (a_1(m_2), a_2(m_2))$ sont symétriques relativement aux messages et aux dimensions (elles satisfont $a_1(m_1) = a_2(m_2)$ et $a_1(m_2) = a_2(m_1)$) ;
- étant données des actions symétriques relativement aux messages et aux dimensions, l'agent informé a intérêt à délivrer le message m_1 lorsque $\theta_1 \geq \theta_2$, et à délivrer le message m_2 lorsque $\theta_1 < \theta_2$ (à un ensemble de types de mesure nulle près).

Ainsi, CHAKRABORTY et HARBAUGH caractérisent la comparaison symétrique des types de l'agent informé comme équilibre d'un jeu de conversation gratuite, dans lequel les agents possèdent potentiellement un important conflit sur chacun des types. Toutefois, l'hypothèse de symétrie de la distribution des types et des fonctions d'utilité des agents relativement aux deux dimensions semble cruciale au résultat.

En effet, LEVY et RAZIN (2007) montrent que la prise en compte d'une dimension supplémentaire pour l'information asymétrique peut au contraire interdire aux agents toute transmission d'information, même le long d'une dimension où ils n'ont pas de conflit. À cette fin, ils considèrent une distribution conjointe des types θ_1 et θ_2 qui ne vérifie pas nécessairement l'hypothèse d'indépendance des variables aléatoires sous-jacentes. Dans ce cas, la révélation d'une information sur θ_2 peut entraîner, par liaison, une révélation d'information sur θ_1 . Si les agents possèdent un important conflit concernant θ_1 , tel que toute transmission d'information sur θ_1 soit impossible, alors, quel que soit leur conflit sur θ_2 , par liaison, il est possible qu'aucune révélation d'information sur θ_2 ne puisse être obtenue à l'équilibre.

LEVY et RAZIN (2007) illustrent ce mécanisme en faisant l'hypothèse d'une fonction d'utilité lexicographique pour S , telle que S préfère l'action (a_1, a_2) à toute action (a'_1, a'_2) où $a_1 > a'_1$, et en ne faisant aucune hypothèse sur les préférences de S relativement aux actions de même première composante. De telles préférences peuvent être interprétées comme faisant état, d'une part, d'un biais infini de S relativement à l'action correspondant à son type θ_1 , et d'autre part d'une complémentarité de θ_1 et a_1 . En particulier, elles peuvent être déduites de préférences quadratiques, données par $-(a_1 - (\theta_1 + b))^2 - (a_2 - \theta_2)^2$, en faisant tendre b vers l'infini. De telles préférences se distinguent particulièrement du modèle de CHAKRABORTY et HARBAUGH (2007) par la forte asymétrie de l'utilité de S relativement aux deux dimensions de l'espaces des types. Pour l'agent non informé R , une fonction d'utilité quadratique $U^R = -\|\mathbf{a} - \boldsymbol{\theta}\|^2$ est considérée.

Sous ces conditions, toute révélation d'information qui induit deux actions $\mathbf{a}(m_1) = (a_1(m_1), a_2(m_1))$ et $\mathbf{a}(m_2) = (a_1(m_2), a_2(m_2))$ doit nécessairement vérifier $a_1(m_1) = a_1(m_2)$. En effet, si par exemple $a_1(m_1) > a_1(m_2)$, étant données les préférences de S , le message m_2 n'est jamais délivré. Ainsi, toute transmission d'information ne peut concerner que θ_2 . Or, si par exemple la distribution des types (θ_1, θ_2) est telle que $\theta_1 = \theta_2$, alors la forte liaison entre les types entraîne qu'aucune information ne peut être révélée concernant θ_2 . Plus généralement, la transmission d'information portant sur θ_2 est ainsi limitée par la distribution conjointe des deux types, indépendamment du conflit d'intérêt des agents concernant θ_2 .

Notons que d'après le résultat de CRAWFORD et SOBEL (1982), si les agents ont un faible conflit le long d'une dimension, une indépendance des types entre les deux dimensions autorise la révélation d'information concernant cette dimension. Par ailleurs, selon CHAKRABORTY et HARBAUGH (2007), la symétrie du conflit et l'indépendance des types autorisent une révélation d'information multi-dimensionnelle. Ainsi, dans le mécanisme décrit par LEVY et RAZIN (2007), la combinaison d'une forte dépendance des types et de l'asymétrie du conflit est cruciale au résultat.

Les analyses de CHAKRABORTY et HARBAUGH (2007) et LEVY et RAZIN (2007) circonscrivent de manière relativement générale l'impact de la considération d'une deuxième dimension au modèle de CRAWFORD et SOBEL (1982) sur la possibilité de transmission d'information. Dans une étude plus spécifique, appliquée au contenu informatif des décisions à l'intérieur de l'entreprise, KAMPHORST et SWANK (2016) contribuent à l'analyse en mettant en lumière un autre aspect de l'extension du modèle de CRAWFORD et SOBEL (1982) à deux dimensions.

KAMPHORST et SWANK supposent une distribution uniforme sur $[0, 1]^2$ pour le type $\boldsymbol{\theta} = (\theta_1, \theta_2)$ de S , représentant l'information privée d'un manager concernant l'habileté de deux employés. Compte tenu de cette information, le manager assigne un des deux employés à une occupation majeure, et l'autre à une occupation mineure. La décision du manager est représentée par un message m_j délivré aux employés, telle que l'employé $i = j$ est assigné à l'occupation majeure. Dans son occupation, chaque employé choisit une action $a_i \in \mathbb{R}$, $i \in \{1, 2\}$, représentant un niveau d'effort, pour laquelle il encourt le coût $a_i^2/2$, et produit $\eta\theta_i a_i$ s'il est assigné à l'occupation majeure, et $\theta_i a_i$ sinon, avec $\eta \geq 1$. Le niveau d'effort optimal d'un employé est donc égal à $a_i^R(\theta_i) = \eta\theta_i$ ou $a_i^R(\theta_i) = \theta_i$ selon son

assignation. La production commune est donnée par $U^S((a_1, a_2), (\theta_1, \theta_2)) = \eta a_j \theta_j + a_{-j} \theta_{-j}$. Les employés, a priori non informés de leur habileté, choisissent un niveau d'effort $a_1(m_j)$ et $a_2(m_j)$ selon l'information qu'ils extraient concernant leur habileté à partir de la décision m_j du manager.

Notons que le manager préfère un effort aussi grand que possible de la part des employés, tandis que les employés préfèrent ajuster leur effort à leur habileté. Le manager et chaque employé possèdent donc un important conflit d'intérêt concernant le niveau d'effort.

Les auteurs étudient dans un premier temps le cas $\eta = 1$. Dans ce cas, la production commune est donnée par $U^S((a_1, a_2), (\theta_1, \theta_2)) = a_1 \theta_1 + a_2 \theta_2$, et le jeu est un jeu de conversation gratuite, dont les hypothèses satisfont les hypothèses de sur-modularité du modèle de CHAKRABORTY et HARBAUGH (2007). Dans un deuxième temps, ils identifient une condition sur $\eta \geq 1$ qui ne modifie pas leurs conclusions. Lorsque $\eta = 1$, KAMPHORST et SWANK montrent que le manager peut fonder sa décision sur la comparaison symétrique des habiletés, selon l'équilibre symétrique mis en lumière par CHAKRABORTY et HARBAUGH (2007). Selon cet équilibre, la décision m_1 est prise lorsque $\theta_1 \geq \theta_2$, et sinon m_2 est décidée. Lorsque m_j est décidée, l'employé $i = j$ choisit un effort $a_i(m_j) = 2/3$, et l'autre employé choisit un effort $a_{-i}(m_j) = 1/3$.

La contribution de KAMPHORST et SWANK concerne l'existence d'un deuxième équilibre potentiellement atteint par les agents. Selon cet équilibre, la décision m_1 est prise lorsque $\theta_1 \geq \theta_2/2$, et sinon m_2 est décidée. Dans ce cas, les efforts des employés ne sont pas symétriquement distribués relativement aux dimensions et aux décisions. Si la décision m_1 est prise, les employés 1 et 2 choisissent des efforts $a_1(m_1) = 11/18$ et $a_2(m_1) = 4/9$ respectivement, et si la décision m_2 est prise, $a_1(m_2) = 1/6$ et $a_2(m_2) = 2/3$ sont choisis. En particulier, le manager confirme la croyance des employés concernant l'asymétrie de sa décision. Plus précisément, si les employés pensent que la décision est prise selon la comparaison de θ_1 à $\theta_2/2$, alors il coûteux pour le manager de dévier de cette règle de décision. Par exemple, par complémentarité à θ_1 , l'effort de l'employé 1 induit par la décision m_2 et donné par $a_1(m_2) = 1/6$ est très préjudiciable à la production commune lorsque θ_1 est relativement grand. Ainsi, compte tenu des efforts induits par la décision m_1 , et relativement aux efforts induits par la décision alternative m_2 , et compte tenu des croyances des employés, il est plus profitable pour le manager de décider m_2 uniquement $\theta_1 < \theta_2/2$. Lorsque " $\theta_1 < \theta_2/2$ " est révélé conformément à la connaissance du manager (c'est à dire lorsque $\theta_1 < \theta_2/2$), l'effort $a_1 = 1/6$ est moins préjudiciable à la production commune par complémentarité à la faible habileté θ_1 observée.

KAMPHORST et SWANK interprètent cet équilibre comme une discrimination potentielle et endogène au jeu. Notons en effet que sous l'équilibre asymétrique, lorsque $\theta_2 > \theta_1 \geq \theta_2/2$, alors m_1 est décidée, et l'employé 1 est assigné à l'occupation majeure, alors que son habileté est inférieure à celle de l'employé 2. En particulier, l'employé 2, le plus habile, est perçu comme le moins habile, et exerce un moindre effort, tandis que l'employé 1, le moins habile, est perçu comme le plus habile, et exerce le plus grand effort. Ces conclusions sont obtenues tant que que $\eta < \sqrt{3}/2 \simeq 1.23$.

Finalement, étant donnée la multiplicité des équilibres, KAMPHORST et SWANK proposent deux critères de sélection.

Un critère de stabilité asymptotique, qui distingue les équilibres selon la convergence vers les stratégies d'équilibre, relativement à une déviation initiale associée à une comparaison asymétrique quelconque révélant ou ne révélant pas $\theta_1 \geq t\theta_2$, $t \in (0, 1)$. Ce critère sélectionne l'équilibre asymétrique.

Un critère d'efficacité, qui examine l'espérance d'utilité d'un employé ou du manager avant la connaissance des habiletés des employés par le manager. Ce critère sélectionne l'équilibre symétrique pour chacun des employés et pour le manager.

Notons que dans le contexte de la transmission stratégique d'information, l'équilibre asymétrique mis en lumière par KAMPHORST et SWANK montre en particulier que dans le jeu symétrique, tel que celui investi par CHAKRABORTY et HARBAUGH (2007), l'unicité de la comparaison symétrique des deux composantes du type de l'agent informé n'est pas en général garantie. Ainsi, d'après KAMPHORST et SWANK, il existe au moins une situation où plusieurs comparaisons des types sont possiblement atteintes à l'équilibre.

Contributions

En résumé, les études citées précédemment nous disent que la prise en considération d'une deuxième dimension au modèle de CRAWFORD et SOBEL (1982) peut, sous des hypothèses fortes de symétrie et d'indépendance des paramètres du jeu, rendre possible la transmission d'information, alors qu'aucune transmission n'est possible dans chacune des dimensions considérée isolément (CHAKRABORTY et HARBAUGH, 2007). De plus, sous des hypothèses plus spécifiques, la symétrie des paramètres du jeu ne garantit pas la symétrie *ex-post* des stratégies des agents. En particulier, il existe au moins une situation où plusieurs équilibres peuvent être atteints, dont des équilibres asymétriques d'asymétrie endogène (KAMPHORST et SWANK, 2016). Enfin, les résultats de ces études ne sont pas robustes à l'introduction de fortes inter-dépendances entre les dimensions dans les paramètres du jeu (LEVY et RAZIN, 2007).

Les résultats de ces analyses soulèvent plusieurs questions théoriques. En particulier, les études des CHAKRABORTY et HARBAUGH (2007) et LEVY et RAZIN (2007) portent sur des situations extrêmes relativement aux facteurs multi-dimensionnels considérés (symétries ou liaisons inter-dimensionnelles fortes), avec des résultats opposés concernant la possibilité de transmission d'information. Ainsi, la question de la possibilité de transmission d'information dans le cadre multi-dimensionnel reste largement ouverte dans les situations intermédiaires. À travers cette question, la détermination de conditions générales d'existence d'un traitement asymétrique et, comme souligné par KAMPHORST et SWANK (2016), potentiellement injuste, est particulièrement motivante.

Dans les trois chapitres de cette thèse, nous explorons dans trois directions la caractérisation des équilibres du jeu de conversation gratuite dans le cadre multi-dimensionnel, en questionnant les modalités de symétrie et d'inter-dépendance des paramètres du jeu.

Impact de la complémentarité

Dans le Chapitre 1, nous explorons, dans un cadre spécifique, la question des effets d'inter dépendance sur la possibilité de transmission d'information. Alors que LEVY et RAZIN (2007) considèrent une forte dépendance de nature informationnelle (entre les types réalisés) et une forte asymétrie des préférences, nous examinons l'impact d'une forte dépendance entre les termes des fonctions d'utilité, tout en conservant la symétrie du jeu, ainsi que l'indépendance des types. Plus précisément, nous étendons formellement le modèle de KAMPHORST et SWANK (2016), en prenant en compte une éventuelle complémentarité entre les dimensions de la production. À cette fin, nous reprenons les hypothèses du modèle de KAMPHORST et SWANK, mais en supposant que la production agrégée est donnée par une fonction à élasticité de substitution constante de chacune des productions unidimensionnelles.

Formellement (pour $\eta = 1$), nous supposons que la fonction d'utilité de l'agent informé est donnée par $U^S((a_1, a_2), (\theta_1, \theta_2)) = \left(\frac{(a_1\theta_1)^r + (a_2\theta_2)^r}{2} \right)^{1/r}$, où $r \in (-\infty, 1]$, représente le degré de complémentarité des productions. Cette hypothèse permet d'analyser les cas d'une strict complémentarité (avec, lorsque $r \rightarrow -\infty$, $U^S((a_1, a_2), (\theta_1, \theta_2)) \rightarrow \min\{a_1\theta_1, a_2\theta_2\}$), d'une complémentarité de type Cobb-Douglas (avec, lorsque $r \rightarrow 0$, $U^S((a_1, a_2), (\theta_1, \theta_2)) \rightarrow \sqrt{a_1\theta_1 \times a_2\theta_2}$), et enfin d'une parfaite substituabilité ($U^S((a_1, a_2), (\theta_1, \theta_2)) = \frac{a_1\theta_1 + a_2\theta_2}{2}$ en $r = 1$). Concernant la fonction d'utilité de l'agent non informé, et la distribution des types, nous reprenons les hypothèses de KAMPHORST et SWANK (2016).

Par ailleurs, nous modifions le contexte d'application du modèle. Nous supposons que le modèle représente la situation d'un unique agent non informé, dont l'occupation possède deux dimensions productives distinctes, de productivités respectives θ_1 et θ_2 . Par exemple, en lien avec nos hypothèses, CUNHA, HECKMAN et SCHENNACH (2010) estiment la production de capital humain à différentes périodes de la vie, à travers la considération d'une fonction à élasticité de substitution constante entre les aptitudes cognitives et non cognitives des individus, d'une période à l'autre. Le modèle peut ainsi représenter la transmission d'information vers un étudiant, qui préfère investir davantage dans le champ le plus productif, qu'il ne connaît pas, depuis un conseiller d'éducation qui, contrairement à l'étudiant, connaît l'importance relative des dimensions de l'apprentissage, et prend en compte leur complémentarité. Le modèle augmente ainsi d'un caractère stratégique l'étude empirique de CUNHA, HECKMAN et SCHENNACH.

Nous montrons que la possibilité de transmission d'information est obtenue si et seulement si le degré de complémentarité considéré est suffisamment faible ($r \in (0, 1]$). Une forte complémentarité incite l'agent informé à associer, à travers sa révélation d'information, une faible productivité à un grand effort, afin de maximiser le minimum des contributions unidimensionnelles. Cette incitation est à l'encontre de celle de l'agent non informé, qui, par sur-modularité de sa fonction d'utilité, préfèrent associer une action plus grande à un type plus grand. Ainsi, une forte complémentarité est un facteur de rétention d'information.

Lorsque le degré de complémentarité est suffisamment faible, les résultats de KAMPHORST et SWANK (2016) sont maintenus et les effets mis en lumière exacerbés. En par-

ticulier, l'équilibre symétrique et un unique équilibre asymétrique (à une permutation des dimensions près) existent simultanément pour tout $r \in (0, 1]$. De plus, un degré de complémentarité plus fort (r plus près de 0) augmente le degré d'asymétrie de l'équilibre asymétrique, et par conséquent l'éventuel favoritisme associé à la décision de l'agent informé. Un degré de complémentarité plus fort augmente aussi la vitesse de convergence vers l'équilibre asymétrique, l'efficacité de l'équilibre asymétrique relativement à l'équilibre symétrique, du point de vue de l'agent informé, et le domaine de validité des résultats relativement à η (qui tend à couvrir $[1, +\infty)$ lorsque $r \rightarrow 0$). En particulier, un degré de complémentarité suffisamment fort rend *ex ante* l'équilibre asymétrique le plus efficace pour l'agent informé. En revanche, la plus faible quantité d'information délivrée *a priori* en cet équilibre entraîne une diminution de l'espérance d'utilité de l'agent non informé.

Le résultat donne ainsi une justification non seulement rationnelle mais aussi *a priori* efficace, du point de vue de l'agent informé, au potentiel décalage d'appariement entre les efforts de l'agent non informé et leur productivité uni-dimensionnelle. Le degré de complémentarité pris en considération par un conseiller d'éducation peut inciter le conseiller à recommander de façon crédible à un étudiant d'investir un plus grand effort dans la dimension la moins productive de son apprentissage. Le bénéfice de cette recommandation repose sur le trop faible investissement dans l'autre dimension, qui serait obtenu suite à la recommandation alternative, et compte tenu des croyances de l'étudiant concernant les recommandations (et des croyances du conseiller sur ces croyances).

Impact de l'asymétrie du conflit

Dans le second chapitre, nous explorons l'influence d'un certain type d'asymétrie des paramètres du jeu sur la possibilité de transmission d'information. Plus précisément, nous considérons une distribution uniforme du type $\boldsymbol{\theta} = (\theta_1, \theta_2)$ de l'agent informé, et des fonctions d'utilités quadratiques $U^R(\mathbf{a}, \boldsymbol{\theta}) = -\|\mathbf{a} - \boldsymbol{\theta}\|^2$ et $U^S(\mathbf{a}, \boldsymbol{\theta}) = -\|\mathbf{a} - (\boldsymbol{\theta} + \mathbf{b})\|^2$ pour les agents, où le vecteur $\mathbf{b} = (b_1, b_2) \in \mathbb{R}^2$ représente le biais bi-dimensionnel de l'agent informé. Ces hypothèses étendent ainsi l'exemple central de CRAWFORD et SOBEL (1982) à deux dimensions.

Ces hypothèses nous permettent d'explorer l'influence des différentes symétries du jeu sur la possibilité de transmission d'information. Plus précisément, un conflit $\mathbf{b} = (b_1, b_2)$ est nécessairement symétrique par rapport à l'axe qui le supporte, et il est asymétrique relativement à tout autre axe. Par ailleurs, la distribution uniforme des types sur $[0, 1]^2$ possède quatre axes de symétrie. Ainsi, le jeu est symétrique lorsque l'axe du conflit correspond à l'un des axes de symétrie de $[0, 1]^2$, et n'est pas symétrique sinon. En particulier, les hypothèses permettent d'explorer l'impact sur la possibilité de transmission d'information, d'un conflit non symétrique, relativement à une symétrie de l'ensemble $[0, 1]^2$ des types, ou d'un conflit symétrique, relativement à une distribution asymétrique des types.

Nous caractérisons l'ensemble des révélations d'information du jeu utilisant deux messages à l'équilibre. Lorsque $\mathbf{b} = (0, 0)$, la comparaison symétrique des types, transmettant la réalisation ou la non réalisation de $\theta_1 \geq \theta_2$, est une stratégie d'équilibre. C'est celle mise en lumière par CHAKRABORTY et HARBAUGH (2007). Mais lorsque $\mathbf{b} \neq (0, 0)$, elle n'est

pas unique. Les stratégies transmettant la réalisation ou la non réalisation de $\theta_1 \geq 1/2$, de $\theta_2 \geq 1/2$, ou encore de $\theta_1 + \theta_2 \geq 1$ sont aussi des stratégies d'équilibre de l'agent informé. Nous montrons que le nombre d'équilibres diminue avec l'étendue $\|\mathbf{b}\|$ de \mathbf{b} , mais qu'il existe toujours au moins deux couples de stratégies d'équilibre des agents quel que soit l'étendue de \mathbf{b} , et quelle que soit sa direction. En particulier, quel que soit $\mathbf{b} \in \mathbb{R}^2$, il existe une possibilité de transmission stratégique d'information.

Par rapport au cas $\mathbf{b} = (0, 0)$, lorsque l'étendue de \mathbf{b} augmente, la révélation d'information tend à concerner la dimension orthogonale à \mathbf{b} . Par exemple, si $\mathbf{b} = (b, b)$, avec b très grand, la nature de l'information transmise à l'équilibre tend à concerner $\theta_1 - \theta_2$ (par exemple en révélant ou en ne révélant pas $\theta_1 \geq \theta_2$). Par symétrie des paramètres du jeu, si $\mathbf{b} = (b, -b)$, avec b très grand, la révélation d'information tend à concerner $\theta_1 + \theta_2$. De manière générale, nous mettons en lumière des familles continues d'équilibres selon \mathbf{b} , prolongeant par continuité les équilibres potentiellement atteints lorsque $\mathbf{b} = (0, 0)$.

L'existence d'un équilibre dans toutes les directions étend le résultat de CHAKRABORTY et HARBAUGH (2007) à toutes les directions du plan euclidien (compte tenu de nos hypothèses plus restrictives sur les paramètres du jeu). La multiplicité des équilibres, elle aussi obtenue pour toutes les directions de \mathbf{b} , confirme également le résultat de KAMPHORST et SWANK (2016), associé à une direction spécifique du conflit des agents, alignée à une symétrie de $[0, 1]^2$. En revanche, en ce qui concerne le résultat de LEVY et RAZIN (2007), une importante asymétrie des préférences, relativement à deux dimensions quelconques de $[0, 1]^2$, n'interdit jamais la transmission d'information. Intuitivement, la géométrie de la distribution uniforme des types sur $[0, 1]^2$ permet d'obtenir une faible dépendance des types relativement à la dimension du conflit des agents, quelles que soient la direction et l'étendue de ce conflit. Cette relative indépendance permet la révélation d'information autour de la dimension orthogonale au conflit, où précisément le conflit des agents est faible.

Déterminants et enjeux de la nature comparative de la transmission d'information

Enfin, dans le troisième chapitre, nous étudions plus spécifiquement la nature comparative des équilibres du jeu de conversation gratuite multi-dimensionnel utilisant deux messages à l'équilibre, en lien avec les analyses de CHAKRABORTY et HARBAUGH (2007) et KAMPHORST et SWANK (2016).

Nous considérons des types θ_1 et θ_2 issus de deux variables aléatoires indépendantes et identiquement distribuées sur $[0, 1]$ selon une probabilité de distribution f . Nous supposons une fonction quadratique $U^R(\mathbf{a}, \boldsymbol{\theta}) = -\|\mathbf{a} - \boldsymbol{\theta}\|^2$ pour l'utilité de R , et une fonction additivement séparable et symétrique $U^S(\mathbf{a}, \boldsymbol{\theta}) = u(a_1, \theta_1) + u(a_2, \theta_2)$ pour l'utilité de S , avec u sur-modulaire. En particulier, dans ce modèle, les hypothèses du modèle de CHAKRABORTY et HARBAUGH (2007) sont satisfaites. Par conséquent, dans ce modèle, la comparaison symétrique des types est une condition suffisante pour la révélation d'information.

Dans un premier temps, nous montrons qu'un important conflit dans chaque dimension, représenté par la croissance de u par rapport à a quel que soit θ (obtenue à partir des

hypothèses de sur-modularité et de croissance de $a \mapsto u(a, 0)$, rend nécessaire la nature comparative de toute transmission d'information à l'équilibre. La nature comparative de la transmission d'information est représentée par l'existence d'une fonction φ croissante, et telle qu'à l'équilibre l'information transmise porte sur la réalisation ou la non réalisation de $\theta_1 \geq \varphi(\theta_2)$. De telles révélations d'information caractérisent donc les équilibres du jeu dans le cas d'un important conflit des agents.

Dans un second temps, en lien avec KAMPHORST et SWANK (2016), nous étudions la possibilité d'une révélation asymétrique d'information, d'asymétrie endogène, étant donnée la symétrie des paramètres du jeu. Nous donnons des conditions d'existence d'une telle possibilité, dans le cas où la sur-modularité de u s'exprime simplement par une séparabilité multiplicative, en sorte que $u(a, \theta) = h(a)g(\theta)$ pour des fonctions h et g de même variation. Nous montrons qu'un équilibre asymétrique est potentiellement atteint lorsque la convexité d'une des deux fonctions h et g est atténuée par une concavité suffisante de l'autre fonction. Plus spécifiquement, dans le cas où h est linéaire, il existe un équilibre asymétrique, d'asymétrie endogène, lorsque $\text{Cov}\left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta\right) > \frac{\mathbb{E}_f[\theta_1|\theta_1 \geq \theta_2] - \mathbb{E}_f[\theta_1|\theta_1 < \theta_2]}{8}$. Cette condition est satisfaite lorsque la dérivée logarithmique de g est suffisamment décroissante relativement à la distribution des types f , en sorte que $\theta \mapsto \frac{f(\theta)}{g'(\theta)/g(\theta)}$ soit croissante.

Enfin, nous donnons des conditions de stabilité asymptotique de l'équilibre symétrique, des équilibres asymétriques (le cas échéant), ainsi que de l'équilibre non informatif. Nous présentons en particulier une paramétrisation de u telle que l'équilibre asymétrique, lorsqu'il existe, est le seul équilibre asymptotiquement stable du jeu.

Les comparaisons sont largement utilisées dans les échanges informatifs des agents économiques. Un manager compare les employés pour promouvoir, un vendeur compare différents biens pour orienter le choix de l'acheteur potentiel, un enseignant compare les aptitudes d'un étudiant pour le guider dans ses choix d'étude. Dans ces contextes, le premier résultat de notre analyse rationalise la nécessaire nature comparative de toute transmission d'information à deux messages, à partir de l'hypothèse de croissance de l'utilité de l'agent informé relativement à l'action de l'agent non informé. Le second résultat illustre la possibilité d'un traitement comparatif asymétrique, sous des conditions simples sur la fonction d'utilité de l'agent informé. Notons que le traitement asymétrique est moins informatif que ne l'est le traitement symétrique. Il est donc, pour un agent non informé cherchant à ajuster son action au type de l'agent informé, *ex ante* moins efficace que l'équilibre symétrique. En particulier, contrairement à l'équilibre symétrique, l'existence d'un tel équilibre est conditionnée par l'existence d'un conflit d'intérêt entre les agents.

Chapitre 1

Multi-dimensional cheap talk with complementarities

Abstract. We analyze a cheap talk game in a two-dimensional framework, with complementarities between the dimensions. A Sender makes one of two recommendations to a Receiver concerning a production. The Receiver's contributions rely on a realized two-dimensional profile of abilities, and a chosen two-dimensional profile of efforts. The profile of abilities is the Sender's private information. Output is a constant elasticity of substitution function of the Receiver's contributions. The Sender prefers to maximize output but the Receiver prefers to adjust his efforts to his abilities. A credibility constraint imposes that the Sender contrasts the dimensions which leads the Receiver to specialize. However, with more complementarities, the Sender has more interest in the mismatch between the Receiver's perception of his abilities and their realization. Then, strong complementarities preclude the possibility of credible recommendations. Weaker complementarities allow the players to either treat the dimensions symmetrically, or favor one of them. Sufficient complementarities make the latter *ex-ante* more productive.

1.1 Introduction

In many important situations, an agent chooses his actions conditional on the information revealed by a better informed agent. For instance, an employee does this when he takes into account his supervisor's recommendation concerning the conduct of his work. A pupil does it by acknowledging his teacher's expertise concerning his study choices. A child does it when conforming to parental rules. However, managers, teachers and parents have strong *biases* concerning the contributions of their "mentees". They prefer a higher contribution. According to the literature on strategic information transmission, such bias precludes the possibility of influential recommendations (Crawford and Sobel, 1982), unless agents take advantage of the multiple dimensions for communication (Chakraborty and Harbaugh, 2007).

This paper addresses the impact of the complementarities among the dimensions of the uninformed agent's contribution. In a productive context, the role of complementarities is particularly relevant. For instance, in the workplace, a bricklayer needs force *and* meticulousness to build a wall. At school or at home, skill formation involves complementarities among the child's skill dimensions (Cunha, Heckman, and Schennach, 2010).¹

We show that if the uninformed agent prefers to adjust his multi-dimensional effort to the multi-dimensional realized state (of which he is not aware), then in order to be credible, the transmission of information must contrast the dimensions of the state.² However, the inclusion of complementarities among the dimensions makes the information provider prefer to induce a cross-dimensional mismatch between state perception and state realization. Therefore, strong complementarities between dimensions preclude a possible agreement among the players. Limited complementarities allow players to agree either to compare the state components symmetrically, or to alter the recommendations in favor of one of the dimensions. Conditional on sufficient complementarities, the asymmetric treatment is preferred *ex-ante* by the informed agent.

Our setting follows the standard Sender-Receiver setting in the cheap talk literature, with the following specifications. The Sender (she) possesses private information concerning the productivity of the Receiver (he). We designate the Sender's private information the Receiver's *abilities*. We focus on two abilities dimensions.³ Given her private information, the Sender discloses one of two abstract messages. The meanings of the messages are defined endogenously in equilibrium. Note that the messages might represent explicit verbal communications from the Sender but might also represent the way the Receiver interprets the Sender's decisions or actions.⁴ Given the message he observes, the Receiver

1. And teachers do not consider the dimensions of knowledge as perfect substitutes.

2. In equilibrium, the disclosed information must be truthful, so that the uninformed agent's posterior beliefs correctly represent the expected state. Otherwise, the uninformed agent will have a profitable deviation.

3. For instance, in line with Cunha, Heckman, and Schennach (2010), the two dimensions might represent the cognitive and non-cognitive dimensions of abilities.

4. Notice that even child would proceed to Bayesian inferences to model the world around (Gopnik, 2012).

chooses his efforts along the two dimensions of his profile of abilities. We assume that in each dimension, the Receiver's contribution equals the product of his effort and his realized unknown ability. The Receiver's aggregate production is given as a constant elasticity of substitution (CES) function of his two contributions. The CES specification permits investigation of the equilibrium conditions as a function of the degree of complementarity between the Receiver's contributions. The Sender prefers to maximize the CES output. In contrast, due to the cost of his effort, in each dimension the Receiver prefers to adjust his effort to his ability.

Strategically, the Sender chooses the recommendation that maximizes the CES output, given her observation of abilities, and given the Receiver's contribution she anticipates. Reciprocally, the Receiver contributes according to his perceived abilities which are derived from the way the Sender chooses her recommendation. An influential equilibrium is sustained if the Sender correctly anticipates the Receiver's contribution, and the Receiver's perception of his ability correctly represents the Sender's private information.

First, we describe two necessary conditions for such an equilibrium.

The first condition rules out the existence of a recommendation that would induce a higher effort in both dimensions relative to the other recommendation. In equilibrium the Sender will always prefer to deviate to that recommendation, irrespective of her observation of the abilities. As a consequence, any influential recommendation necessarily induces a high effort in one dimension and a low effort in the other dimension. This imposes the disclosure of the relative abilities. The intuition is that since the Sender is biased toward a higher effort along each dimension, communication must occur orthogonal to that direction, *i.e.* by contrasting the dimensions.

The second condition rules out the case of too high a degree of complementarity involved in the CES function. Suppose for instance, that the Sender wants to maximize a minimum contribution along the dimensions. She then will prefer to match the lowest effort to the highest ability, and the highest effort to the lowest ability. However, since the Receiver adjusts his effort to his perceived ability, the highest effort must be derived from the Receiver's highest expectation concerning his ability which precludes a possible agreement.

Then we can show that if the cross-dimensional complementarities are limited, the players have multiple ways to agree.⁵ The information is transmitted along a unique cross-dimension, supported by the two induced actions but the multi-dimensionality of the framework permits players to identify multiple dimensions along which they might agree.

The first way to agree is symmetric relative to the two dimensions. The Sender symmetrically compares the Receiver's abilities, and the Receiver exerts symmetrically balanced efforts across dimensions and recommendations (which in turn determines the symmetric comparison). The second way is asymmetric relative to the two dimensions. It alters the recommendations and the subsequent effort investments in favor of one of those di-

5. For instance, concerning parental influence, Tenenbaum (2009) shows that when parents make recommendations concerning their children's course choices, the language they use to speak to their daughters tends to be more discouraging than that they employ to talk to their sons, whatever the study domain. This suggests that parents and children will play a different equilibrium conditional on the child's gender.

mensions.⁶ One recommendation is *ex-ante* more likely disclosed than the other, and accordingly, one dimension is *ex-ante* more likely invested in by the Receiver.

Moreover, the degree of complementarity exacerbates the asymmetry of the asymmetric equilibrium (it has no effect on the symmetric equilibrium). This is because more complementarity makes the expected recommendation even more profitable at the interim stage.

Finally, we show that the asymmetric equilibrium is *ex-ante* more productive with stronger complementarities,⁷ which is detrimental of the Receiver's accuracy of his relative abilities. The intuition is that more complementarities increase the Sender's interest in preventing the Receiver from neglecting one of the dimensions of his abilities. This is precisely the effect of the *ex-ante* lower informativeness of the asymmetric equilibrium.⁸ This illustrates how a cross-dimensional conflict of interest between the players arises with the introduction of complementarities on the Sender's side of the information transmission channel.

An extensive literature on strategic information transmission has emerged triggered by the seminal paper by Crawford and Sobel (1982) (see Sobel (2013) for a review). Crawford and Sobel (1982) characterize the impact of a conflict of interest between a Sender and a Receiver, on the Sender's influence on a Receiver's action in case of a unique information dimension.

In the case of multiple dimensions, theoretical results are scarce.

Chakraborty and Harbaugh (2007) show that symmetric comparison of the two dimensions of information defines an equilibrium in a context of a symmetric prior, and separable and super-modular conditions on the players' utility functions.⁹ This illustrates that the players might reach agreement if the information is revealed orthogonally to their conflict. In our setting, the sufficient condition is necessary also, because of the major level of conflict within each dimension. However, we show that players might find multiple ways to communicate along this orthogonal dimension. Along similar lines, Sémirat (2017) con-

6. Given the symmetry of the game, there are two asymmetric equilibria, one derived from the other by switching the dimensions labels.

7. Relative to the symmetric equilibrium, the asymmetric equilibrium is lower productive with weaker complementarities; the effect is reversed with stronger complementarities. Therefore, in both cases the model illustrates the possibility of a detrimental influence of parents' or teachers' superior information concerning children's abilities. As Heckman and Mosso (2014) argue, it is not budget constraints but parental attitudes, determined by their objective function, which are the major cause of observed low educational attainment children from low income families (Hackman, Farah, and Meaney, 2010; Knudsen et al., 2006; Heckman, 2008). Along the same lines, the parents' degree of risk aversion has been shown to be inversely related to the child's education attainment (Wölfel and Heineck, 2012; Brown, Ortiz-Nuñez, and Taylor, 2012; Checchi, Fiorio, and Leonardi, 2014).

8. According to Wang and Degol (2017), women are more likely to have a homogeneous abilities profile, which would give them more possibilities concerning choice of occupation. In contrast, men will be more specialized toward math-intensive fields, and will have a restricted occupational choice. This suggests also that a different equilibrium will be played during the skill formation process, based on the gender of the child.

9. In contrast, if the setting includes strong asymmetries, Levy and Razin (2007) show that the possibility of influence is precluded.

siders a uniform state space, a binary messages set, and separable quadratic preferences for both players. The author characterizes the equilibria of the game with respect to a varying bias, and derive the same multiplicity of equilibria for any extent and direction of the players' conflict.

The present study rules out the separable assumption of the Sender's utility function, and exploits the CES functional form to investigate the degree of complementarity in the Receiver's contribution as a cross-dimensional conflict between the players. The impact of payoff interdependencies between the dimensions has so far not been investigated under more general conditions. We find that the impact of the degree of complementarity on the equilibrium depends on the played equilibrium. It has no impact on the symmetric equilibrium, but exacerbates the asymmetry of the asymmetric equilibrium.

From a productive perspective, *ex-ante* the Sender's preferred equilibrium is the symmetric equilibrium *iff* the degree of complementarity involved is low. Chakraborty and Harbaugh (2010) note also that a Sender with quasi-convex preferences relative to the multi-dimensional Receiver's action would *ex-ante* prefer to be uninformative. However, Chakraborty and Harbaugh (2010) assume state independent preferences for the Sender. We show that the uninformative babbling equilibrium is never preferred to the asymmetric equilibrium which always displays relative better informativeness from the Sender's perspective.

The economic motivation for our study originates from a recent strand of work on confidence management which investigates the informational content of firm decisions, and how they might affect the employees' efforts.

Crutzen, Swank, and Visser (2013) study the impact of the revelation or withholding of firms' rankings of employees'. Their study shows that in general, the firm has an interest in withholding such an information. They focus on symmetric equilibria. Asymmetric equilibria are investigated in Kamphorst and Swank (2016), which considers the promotion decisions of managers, derived from their private information concerning the one-dimensional abilities of their employees. Similar to our study, the authors derive symmetric and asymmetric equilibria. In asymmetric equilibria, managers discriminate in favor of the employee that the players expect to be promoted.

While Kamphorst and Swank (2016) consider two Receivers, each of whom contributes along a unique dimension, we consider a unique Receiver who contributes along two dimensions. In formal terms, our model extends the Kamphorst and Swank's (2016) model. Kamphorst and Swank's (2016) study is restricted to the case where employees' contributions are perfect substitutes for aggregate team production. We include the possibility of complementarities which are prevalent in the workplace. Characterization of multiple equilibria according to the degree of complementarity includes Kamphorst and Swank's (2016) main result as a polar case. Our results highlight a more important issue related to asymmetric equilibria in that context. We show that a high degree of complementarity induces the Sender to *ex-ante* prefer the asymmetric over the symmetric treatment. This contrasts with the case of perfect substitution. We show also that the asymmetric equilibrium results from a qualitative distribution of effort across decisions, which makes the asymmetric equilibrium robust to changes in the prior distribution that would preclude any

asymmetric treatment in the case of perfect substitution. Finally, while the introduction of slight exogenous asymmetries between dimensions precludes the asymmetric equilibrium in case of perfect substitution, we show that it is resistant to large exogenous asymmetries if the complementarities involved are sufficient.

The paper is organized as follows. Section 1.2 describes the model set up and Section 1.3 presents the results. Section 1.3.1 and 1.3.2 discuss the strategies and necessary equilibrium conditions. Section 1.3.3 derives the existence of the symmetric and asymmetric equilibria for each degree of complementarity, and associated degree of complementarity to the asymmetry of the asymmetric equilibrium. In Section 1.4, we provide arguments related to the robustness of the effect of complementarity on the asymmetric equilibrium, and Section 1.5 investigate selection criteria. We find the asymmetric equilibrium to be stable and more productive provided sufficient complementarity. Section 1.6 concludes. Proofs are provided in the appendix.

1.2 Model setup

A Receiver (he) produces an output within a working environment. Production is decomposed along two dimensions $i = 1$ and $i = 2$. The Receiver contributes in each dimension $i \in \{1, 2\}$ at level y_i . Each contribution y_i relies on a realized level of ability a_i , and a chosen effort level e_i through

$$y_i = a_i e_i. \quad (1.1)$$

The Receiver derives utility from his contributions but suffers a quadratic effort cost along each dimension.¹⁰

His utility is given by

$$U(e_1, e_2) = \sum_{i=1,2} \left(y_i - \frac{1}{2} e_i^2 \right). \quad (1.2)$$

The Receiver is not aware of his abilities. He has a uniform prior on $[0, 1]$ in each dimension.

A Sender (she) is aware of (a_1, a_2) . Given her observation, she provides one of two recommendations to the Receiver. The recommendation takes the form of a message $\mathbf{m} \in \{\mathbf{m}_1, \mathbf{m}_2\}$. The Sender's utility derived from the Receiver's contributions is given by the CES function

$$Y_r = \left(\frac{y_1^r + y_2^r}{2} \right)^{\frac{1}{r}} \quad (1.3)$$

10. We do not integrate spillover effects between effort costs, and assume a quadratic and additively separable utility function for the Receiver. This is the assumption in most of the literature, and allows us to focus on the informational aspect of the equilibrium conditions (action identifies to the expected state in equilibrium). It represents situations where a change of effort in one dimension has no effect on the effort in the other dimension. This might be due to the independence of the dimensions considered (*e.g.* cognitive and non-cognitive effort), or to efforts exerted at sufficiently distant moments.

where $r \in (-\infty, 1]$ represents the degree of complementarity between the Receiver's contributions.¹¹

The Sender makes her recommendation in order to maximize (1.3).

Given the Sender's recommendation $\mathbf{m} \in \{\mathbf{m}_1, \mathbf{m}_2\}$, the Receiver chooses his effort levels to maximize his expected utility

$$\mathbb{E}[U|\mathbf{m}].$$

The timing of the game is as follows:

1. Nature draws abilities a_1 and a_2 , and reveals them to the Sender, but not to the Receiver, who has a uniform prior;
2. the Sender sends message $\mathbf{m} \in \{\mathbf{m}_1, \mathbf{m}_2\}$ to the Receiver;
3. the Receiver observes the Sender's message \mathbf{m} , and updates his beliefs about his abilities;
4. the Receiver chooses his level of efforts $e_i(\mathbf{m})$ according to his posterior beliefs;
5. payoffs are realized.

We look for perfect Bayesian equilibria of the game, *i.e.*: (i) the Receiver's effort strategy is optimal given his beliefs about his abilities, (ii) the Sender's disclosure strategy is optimal, given the employee's efforts strategy and beliefs, (iii) whenever possible, beliefs are updated according to Bayes's rule.

1.3 Analysis

1.3.1 Strategies

Given the message $\mathbf{m} \in \{\mathbf{m}_1, \mathbf{m}_2\}$ he observes, the Receiver exerts his efforts to maximize his expected utility at

$$\begin{aligned} (e_1(\mathbf{m}), e_2(\mathbf{m})) &= \arg \max_{(e_1, e_2) \in \mathbb{R}^2} \mathbb{E}[U(e_1, e_2)|\mathbf{m}] \\ &= \arg \max_{(e_1, e_2) \in \mathbb{R}^2} \mathbb{E} \left[a_1 e_2 - \frac{1}{2} e_1^2 + a_2 e_2 - \frac{1}{2} e_2^2 \middle| \mathbf{m} \right] \\ &= \arg \max_{(e_1, e_2) \in \mathbb{R}^2} e_2 \mathbb{E}[a_1|\mathbf{m}] - \frac{1}{2} e_1^2 + e_2 \mathbb{E}[a_2|\mathbf{m}] - \frac{1}{2} e_2^2 \\ &= (\mathbb{E}[a_1|\mathbf{m}], \mathbb{E}[a_2|\mathbf{m}]). \end{aligned} \tag{1.4}$$

11. Recall that for any $r \leq 1$, the elasticity of substitution between the contributions is given by $\rho = \frac{1}{1-r}$. In particular, at $r = 1$, we have $Y_r = \frac{y_1 + y_2}{2}$ and perfect substitution between the contributions, at $r \rightarrow 0$, Y_r tends to the constant return to scale symmetric Cobb-Douglas function $Y_r = \sqrt{y_1 y_2}$, and at $r \rightarrow -\infty$, Y_r tends to the Leontief function $Y_r = \min\{y_1, y_2\}$.

Reciprocally, given the Receiver's effort strategy $\mathbf{m} \mapsto (e_1(\mathbf{m}), e_2(\mathbf{m}))$, the Sender utility derived from the recommendation \mathbf{m} is given by

$$Y_r(\mathbf{m}) = \left(\frac{a_1^r e_1^r(\mathbf{m}) + a_2^r e_2^r(\mathbf{m})}{2} \right)^{\frac{1}{r}}.$$

Then the Sender chooses \mathbf{m}_1 or \mathbf{m}_2 conditional on $Y_r(\mathbf{m}_1) \geq Y_r(\mathbf{m}_2)$ or $Y_r(\mathbf{m}_1) \leq Y_r(\mathbf{m}_2)$ respectively.

Notice that if $e_1(\mathbf{m}_1) = e_1(\mathbf{m}_2)$ and $e_2(\mathbf{m}_1) = e_2(\mathbf{m}_2)$, then she will be indifferent between \mathbf{m}_1 and \mathbf{m}_2 , whatever (a_1, a_2) . Reciprocally, in the case that her choice does not depend on (a_1, a_2) , then the Receiver's posterior beliefs (1.4) would be equal to his prior beliefs, and we would have $e_1(\mathbf{m}_1) = e_1(\mathbf{m}_2)$ and $e_2(\mathbf{m}_1) = e_2(\mathbf{m}_2)$. These strategies define a *babbling equilibrium*.

Henceforth, we focus on *influential equilibria*, i.e. equilibria for which $e_1(\mathbf{m}_1) \neq e_1(\mathbf{m}_2)$ or $e_2(\mathbf{m}_1) \neq e_2(\mathbf{m}_2)$.

1.3.2 Influential equilibrium conditions

Let us distinguish the cases $r < 0$ and $r \in (0, 1]$.

Case $r \in (0, 1]$. If $r \in (0, 1]$, then we obtain (up to a null measure set of abilities) the following Sender's strategy:

$\mathbf{m}(a_1, a_2) = \mathbf{m}_1$ iff

$$\begin{aligned} & \left(\frac{a_1^r e_1^r(\mathbf{m}_1) + a_2^r e_2^r(\mathbf{m}_1)}{2} \right)^{\frac{1}{r}} \geq \left(\frac{a_1^r e_1^r(\mathbf{m}_2) + a_2^r e_2^r(\mathbf{m}_2)}{2} \right)^{\frac{1}{r}} \\ \iff & a_1^r e_1^r(\mathbf{m}_1) + a_2^r e_2^r(\mathbf{m}_1) \geq a_1^r e_1^r(\mathbf{m}_2) + a_2^r e_2^r(\mathbf{m}_2) \\ \iff & a_1^r (e_1^r(\mathbf{m}_1) - e_1^r(\mathbf{m}_2)) \geq a_2^r (e_2^r(\mathbf{m}_2) - e_2^r(\mathbf{m}_1)). \end{aligned}$$

Then in equilibrium, we must have $e_1^r(\mathbf{m}_1) - e_1^r(\mathbf{m}_2) \neq 0$ and $e_2^r(\mathbf{m}_2) - e_2^r(\mathbf{m}_1) \neq 0$ and of equal sign, otherwise the same recommendation would always be made. Then we derive the following conditional strategies:

if $e_1^r(\mathbf{m}_1) > e_1^r(\mathbf{m}_2)$ and $e_2^r(\mathbf{m}_2) > e_2^r(\mathbf{m}_1)$, that is $e_1(\mathbf{m}_1) > e_1(\mathbf{m}_2)$ and $e_2(\mathbf{m}_2) > e_2(\mathbf{m}_1)$, then

$$\mathbf{m}(a_1, a_2) = \mathbf{m}_1 \text{ iff } a_1 \geq \left(\frac{e_2^r(\mathbf{m}_2) - e_2^r(\mathbf{m}_1)}{e_1^r(\mathbf{m}_1) - e_1^r(\mathbf{m}_2)} \right)^{\frac{1}{r}} \times a_2,$$

or, if $e_1(\mathbf{m}_1) < e_1(\mathbf{m}_2)$ and $e_2(\mathbf{m}_2) < e_2(\mathbf{m}_1)$, then

$$\mathbf{m}(a_1, a_2) = \mathbf{m}_1 \text{ iff } a_1 \leq \left(\frac{e_2^r(\mathbf{m}_2) - e_2^r(\mathbf{m}_1)}{e_1^r(\mathbf{m}_1) - e_1^r(\mathbf{m}_2)} \right)^{\frac{1}{r}} \times a_2.$$

Notice that the above two disclosure rules and the corresponding conditions are equal up to a relabeling of the messages \mathbf{m}_1 and \mathbf{m}_2 .

Case $r < 0$. If $r < 0$, then we obtain the Sender's strategy:
 $\mathbf{m}(a_1, a_2) = \mathbf{m}_1$ iff

$$\begin{aligned} & \left(\frac{a_1^r e_1^r(\mathbf{m}_1) + a_2^r e_2^r(\mathbf{m}_1)}{2} \right)^{\frac{1}{r}} \geq \left(\frac{a_1^r e_1^r(\mathbf{m}_2) + a_2^r e_2^r(\mathbf{m}_2)}{2} \right)^{\frac{1}{r}} \\ \iff & a_1^r e_1^r(\mathbf{m}_1) + a_2^r e_2^r(\mathbf{m}_1) \leq a_1^r e_1^r(\mathbf{m}_2) + a_2^r e_2^r(\mathbf{m}_2) \\ \iff & a_1^r (e_1^r(\mathbf{m}_1) - e_1^r(\mathbf{m}_2)) \leq a_2^r (e_2^r(\mathbf{m}_2) - e_2^r(\mathbf{m}_1)). \end{aligned}$$

Then we have: if $e_1^r(\mathbf{m}_1) > e_1^r(\mathbf{m}_2)$ and $e_2^r(\mathbf{m}_2) > e_2^r(\mathbf{m}_1)$, that is $e_1(\mathbf{m}_1) < e_1(\mathbf{m}_2)$ and $e_2(\mathbf{m}_2) < e_2(\mathbf{m}_1)$, then

$$\mathbf{m}(a_1, a_2) = \mathbf{m}_1 \text{ iff } a_1 \geq \left(\frac{e_2^r(\mathbf{m}_2) - e_2^r(\mathbf{m}_1)}{e_1^r(\mathbf{m}_1) - e_1^r(\mathbf{m}_2)} \right)^{\frac{1}{r}} \times a_2,$$

or, if $e_1(\mathbf{m}_1) > e_1(\mathbf{m}_2)$ and $e_2(\mathbf{m}_2) > e_2(\mathbf{m}_1)$, then

$$\mathbf{m}(a_1, a_2) = \mathbf{m}_1 \text{ iff } a_1 \leq \left(\frac{e_2^r(\mathbf{m}_2) - e_2^r(\mathbf{m}_1)}{e_1^r(\mathbf{m}_1) - e_1^r(\mathbf{m}_2)} \right)^{\frac{1}{r}} \times a_2.$$

Again, the two rules and the corresponding conditions are equivalent up to a relabeling of the messages. However, notice for instance that if $\mathbf{m}(a_1, a_2) = \mathbf{m}_1$ iff $a_1 \geq ta_2$ for some $t > 0$, then we would have in equilibrium

$$e_1(\mathbf{m}_1) = \mathbb{E}[a_1 | \mathbf{m}_1] = \mathbb{E}[a_1 | a_1 \geq ta_2] > \mathbb{E}[a_1 | a_1 < ta_2] = \mathbb{E}[a_1 | \mathbf{m}_2] = e_1(\mathbf{m}_2),$$

and similarly, $e_2(\mathbf{m}_2) < e_2(\mathbf{m}_1)$. Therefore in the case that $r < 0$, the above disclosure rules and the corresponding conditions are not compatible. In other words, if $r < 0$ there is no influential equilibrium. Thus, we obtain the following necessary condition for an influential equilibrium to occur.

Lemma 1.1. *If $\mathbf{m} \mapsto (e_1(\mathbf{m}), e_2(\mathbf{m}))$ and $(a_1, a_2) \mapsto \mathbf{m}(a_1, a_2)$ define an influential equilibrium profile of strategies, then $r \in (0, 1]$, and up to a relabeling of the messages,*

$$e_1(\mathbf{m}_1) > e_2(\mathbf{m}_1), \text{ and } e_2(\mathbf{m}_2) > e_1(\mathbf{m}_2), \quad (1.5)$$

and

$$\mathbf{m}(a_1, a_2) = \begin{cases} \mathbf{m}_1, & \text{if } a_1 \geq ta_2, \\ \mathbf{m}_2, & \text{if } a_1 < ta_2, \end{cases} \quad (1.6)$$

where

$$t = \left(\frac{e_2^r(\mathbf{m}_2) - e_2^r(\mathbf{m}_1)}{e_1^r(\mathbf{m}_1) - e_1^r(\mathbf{m}_2)} \right)^{\frac{1}{r}}. \quad (1.7)$$

Notice that according to (1.4), the effort represents the Receiver's perception of his ability. Hence, the sign condition (1.5) implies that the Sender's disclosure rule must contrast the Receiver's perception of his abilities. For instance, if \mathbf{m}_1 induces a higher perception concerning the first dimension ($e_1(\mathbf{m}_1) > e_1(\mathbf{m}_2)$), then it necessarily induces a lower perception concerning the second dimension ($e_2(\mathbf{m}_1) < e_2(\mathbf{m}_2)$), and *vice versa*. Clearly, strategy (1.6) satisfies this requirement. Therefore, Conditions (1.4), (1.6) and (1.7) also are sufficient conditions for the existence of an equilibrium.

Finally, notice that the game is fully symmetric with respect to the dimensions of abilities and efforts. This symmetry implies that any existing strategy profile in equilibrium induces another equilibrium strategy profile as stated in the following lemma.

Lemma 1.2. *If for some $t \in (0, +\infty)$,*

$$\mathbf{m} \mapsto \begin{cases} (e_1(\mathbf{m}_1), e_2(\mathbf{m}_1)) & \text{if } \mathbf{m} = \mathbf{m}_1, \\ (e_1(\mathbf{m}_2), e_2(\mathbf{m}_2)) & \text{if } \mathbf{m} = \mathbf{m}_2, \end{cases} \quad \text{and } (a_1, a_2) \mapsto \begin{cases} \mathbf{m}_1, & \text{if } a_1 \geq ta_2, \\ \mathbf{m}_2, & \text{if } a_1 < ta_2, \end{cases}$$

define an influential equilibrium profile of strategies, then

$$\mathbf{m} \mapsto \begin{cases} (e_2(\mathbf{m}_2), e_1(\mathbf{m}_2)) & \text{if } \mathbf{m} = \mathbf{m}_1, \\ (e_2(\mathbf{m}_1), e_1(\mathbf{m}_1)) & \text{if } \mathbf{m} = \mathbf{m}_2, \end{cases} \quad \text{and } (a_1, a_2) \mapsto \begin{cases} \mathbf{m}_1, & \text{if } a_1 \geq \frac{1}{t}a_2, \\ \mathbf{m}_2, & \text{if } a_1 < \frac{1}{t}a_2, \end{cases}$$

define an influential equilibrium profile of strategies too.

In particular, Lemma 1.2 permits us to w.l.o.g. restrict to $t \in (0, 1]$ for the possible Sender's strategies (1.6) in equilibrium. Given the uniform prior distribution of abilities, this allows explicit expressions for the Receiver's derived efforts. According to (1.4), given (1.6), we have

$$\begin{aligned} e_1(\mathbf{m}_1) &= \mathbb{E}[a_1 | a_1 \geq ta_2] = \frac{\int_{a_2=0}^1 \int_{a_1=ta_2}^1 a_1 da_1 da_2}{\int_{a_2=0}^1 \int_{a_1=ta_2}^1 da_1 da_2} = \frac{1}{3} \frac{3-t^2}{2-t}, \\ e_2(\mathbf{m}_1) &= \mathbb{E}[a_2 | a_1 \geq ta_2] = \frac{\int_{a_2=0}^1 \int_{a_1=ta_2}^1 a_2 da_1 da_2}{\int_{a_2=0}^1 \int_{a_1=ta_2}^1 da_1 da_2} = \frac{1}{3} \frac{3-2t}{2-t}, \\ e_1(\mathbf{m}_2) &= \mathbb{E}[a_1 | a_1 < ta_2] = \frac{\int_{a_2=0}^1 \int_{a_1=0}^{ta_2} a_1 da_1 da_2}{\int_{a_2=0}^1 \int_{a_1=0}^{ta_2} da_1 da_2} = \frac{t}{3}, \\ e_2(\mathbf{m}_2) &= \mathbb{E}[a_2 | a_1 < ta_2] = \eta \frac{\int_{a_2=0}^1 \int_{a_1=0}^{ta_2} a_2 da_1 da_2}{\int_{a_2=0}^1 \int_{a_1=0}^{ta_2} da_1 da_2} = \frac{2}{3}. \end{aligned} \tag{1.8}$$

Then, according to (1.7), the equilibria of the game associated to $r \in (0, 1]$, if any, are determined by the solutions $t \in (0, 1]$ of

$$t = \left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}. \tag{1.9}$$

Next we characterize these solutions and how they depend on r .

Figure 1.1 represents a disclosure rule (1.6) associated to some $t \in (0, 1)$, and the induced effort levels $e_i(\mathbf{m}_j)$.

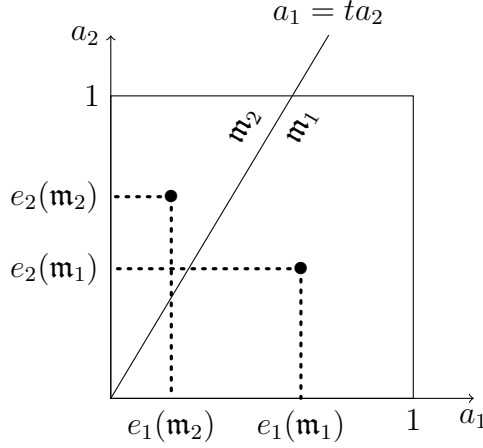


Figure 1.1 – A Sender's disclosure rule, and the subsequent Receiver's optimal efforts

1.3.3 Influential equilibria

Notice first that for any $r \in (0, 1]$, $t = 1$ is a solution of (1.9). The corresponding equilibrium strategies are given by

$$\mathbf{m}(a_1, a_2) = \begin{cases} \mathbf{m}_1 & \text{if } a_1 \geq a_2, \\ \mathbf{m}_2 & \text{if } a_1 < a_2, \end{cases} \quad \text{and } (e_1(\mathbf{m}), e_2(\mathbf{m})) = \begin{cases} \left(\frac{2}{3}, \frac{1}{3}\right) & \text{if } \mathbf{m} = \mathbf{m}_1, \\ \left(\frac{1}{3}, \frac{2}{3}\right) & \text{if } \mathbf{m} = \mathbf{m}_2. \end{cases}$$

Relative to the dimensions, they define a *symmetric equilibrium* in which the Sender reveals to the Receiver his highest ability (and thereby, also his lowest ability). Then, the Receiver symmetrically exerts his effort, with a higher exerted effort in the dimension corresponding to his highest ability, and a lower effort in the other dimension.

The intuition is as follows. Since effort and ability are complementary in each contribution $y_i = a_i e_i$, and since $r \in (0, 1]$, the Sender has an interest in encouraging effort in the dimension where ability is higher. Reciprocally, a higher effort is derived from a higher inferred ability. Hence, if the Sender truthfully reveals the highest ability, it will induce a corresponding preferred higher effort. This makes her recommendation credible in equilibrium.

The next proposition asserts that alongside the symmetric equilibrium, information might be revealed and processed asymmetrically.

Proposition 1.1. *For each $r \in (0, 1]$, there is a unique $t_r \in (0, 1)$ such that*

$$\mathbf{m}(a_1, a_2) = \begin{cases} \mathbf{m}_1 & \text{if } a_1 \geq t_r a_2, \\ \mathbf{m}_2 & \text{if } a_1 < t_r a_2, \end{cases} \quad \text{and } (e_1(\mathbf{m}), e_2(\mathbf{m})) = \begin{cases} \left(\frac{1}{3} \frac{3-t_r^2}{2-t_r}, \frac{1}{3} \frac{3-2t_r}{2-t_r}\right) & \text{if } \mathbf{m} = \mathbf{m}_1, \\ \left(\frac{t_r}{3}, \frac{2}{3}\right) & \text{if } \mathbf{m} = \mathbf{m}_2. \end{cases}$$

define an equilibrium profile of strategies.

Proposition 1.1 and Lemma 1.2 ensure that for each $r \in (0, 1]$, on both sides of the symmetric equilibrium that occurs at $t = 1$, there is another equilibrium. Furthermore, these equilibria fully characterize the game's equilibria.

In an asymmetric equilibrium, the Sender is more likely to induce a higher effort in an *ex-ante* determined dimension. In particular, she might encourage a high effort in a certain dimension despite a higher ability in the other dimension. This would induce a mismatch between abilities and efforts that is not in the Receiver's interest. For an intuitive interpretation, consider the case depicted in Figure 1.1. The Sender truthfully reveals whether the Receiver is

$$\begin{aligned} & \text{"}\mathbf{m}_1\text{: less worse concerning } a_1\text{"}, \text{ or} \\ & \text{"}\mathbf{m}_2\text{: much worse in concerning } a_1\text{"}. \end{aligned} \tag{1.10}$$

Given this, the Receiver will

- (a) exert medium efforts $e_1(\mathbf{m}_1)$ and $e_2(\mathbf{m}_1)$ conditional on the "less worse" recommendation \mathbf{m}_1 because he does not retrieve much information concerning his profile of abilities upon the observation of \mathbf{m}_1 , or
- (b) exert a very low effort $e_1(\mathbf{m}_2)$ and a medium effort $e_2(\mathbf{m}_2)$ conditional on the "much worse" recommendation, because it is revealed upon a very low ability a_1 , relative to a_2 .

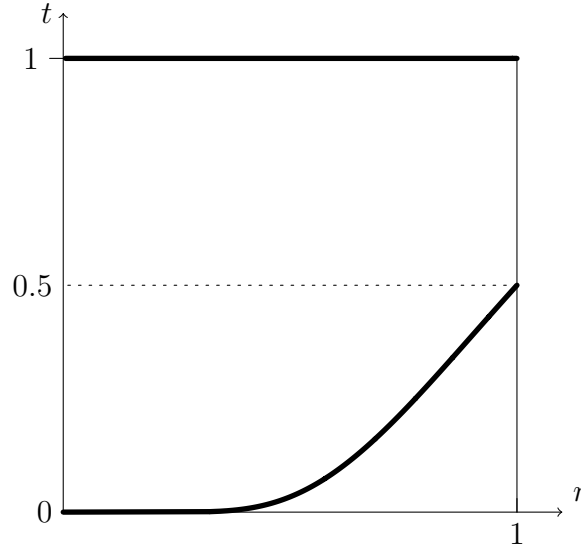
In particular, efforts are similar on \mathbf{m}_1 , and starkly different on \mathbf{m}_2 . According to the degree of complementarity involved in the CES function, this is likely to imply the disclosure of $\mathbf{m}_1 : Y_r(\mathbf{m}_1) > Y_r(\mathbf{m}_2)$ rather than the alternative recommendation. The alternative recommendation \mathbf{m}_2 will be revealed only in the case that a_1 is much worse than a_2 , because then any effort in the first dimension will be useless. This precisely confirms (1.10).¹²

Notice that the above argument shows that greater the complementary, the more asymmetric the Sender's decision-making rule in equilibrium. Indeed, greater complementarity exacerbates the detrimental effect on the production of the different efforts on \mathbf{m}_2 , so that the expected recommendation \mathbf{m}_1 is even more likely.

Proposition 1.2. *The asymmetry of the equilibrium asymmetric recommendations is exacerbated by the degree of complementarity of the Receiver's contributions. In particular, in the asymmetric equilibrium, the Sender tends to babble as r tends to 0.*

Figure 1.2 depicts the two equilibria occurring at $t = 1$ (the symmetric equilibrium) and at $t_r \in (0, 1)$ (the asymmetric equilibrium), with respect to the degree of complementarity $r \in (0, 1]$. At $r = 1$ (contributions are perfectly substitutable), we have $t_r = \frac{1}{2}$, and as r decreases to 0, so does t_r . Notice that a Sender's rule associated to $t = 0$ corresponds to a babbling rule (\mathbf{m}_1 will always be disclosed).

12. Proposition 1.1 states also that an asymmetric equilibrium occurs with perfect substitute contributions (when $r = 1$). This is the case investigated by Kamphorst and Swank, 2016. In Section 1.4 we show that the effect is due to the cardinality, rather the similarity or difference in the different levels of effort. We argue that it is closely related to the uniform prior, and we provide a different prior that would rule out the asymmetric equilibrium in the condition of low complementarity. The intuitive mechanism based on complementarities is more robust to such a change.

Figure 1.2 – $t = 1$, and $t = t_r$ as a function of r

1.4 Robustness

1.4.1 A qualitative effect

The effect shown in the previous section, based on the complementarity of the Receiver's contributions, does not explain the asymmetric equilibria in the case of perfect substitution between contributions ($r = 1$). Notice that in this case, the asymmetric equilibrium derives from the existence of the solution $t_1 = \frac{1}{2}$ of (1.9), *i.e.* on the fact that $e_2(\mathbf{m}_2) - e_2(\mathbf{m}_1) = \frac{1}{2}(e_1(\mathbf{m}_1) - e_1(\mathbf{m}_2))$ when the Sender reveals whether or not $a_1 \geq \frac{1}{2}a_2$. Therefore, it derives from the lack of compensation of $e_2(\mathbf{m}_2)$ relative to the reduction in effort $e_1(\mathbf{m}_2)$. In this equilibrium, a Receiver who is instructed to be worse in the first dimension does not infer that he will be much better in the second dimension. Next, we show that this effect relies crucially on the prior distribution of abilities that we have specified. In particular, specify a prior distribution such that the asymmetric equilibrium is precluded for perfect substitution, but persists in the presence of complementarity.

Consider a uniform prior on the set of abilities (a_1, a_2) such that

$$a_1 + a_2 \leq 1, a_1 \geq 0, a_2 \geq 0,$$

as illustrated in Figure 1.3.

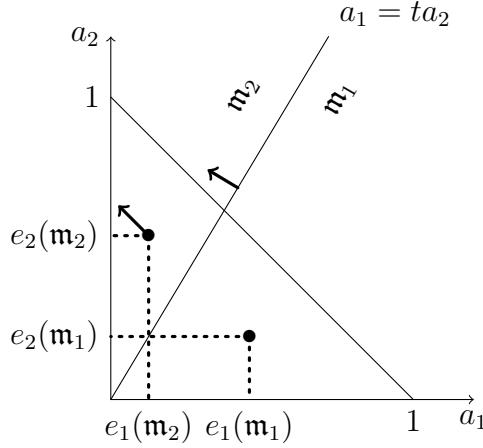


Figure 1.3 – The decision-making rule and subsequent effort strategy with dependent prior abilities

With such a prior, a Receiver who is instructed to be worse in the first dimension infers he is much better in the second one. We compute, for any $t \geq 0$, as in (1.8):

$$e_1(\mathbf{m}_1) = \frac{1}{3} \frac{2t+1}{1+t}, \quad e_2(\mathbf{m}_1) = \frac{1}{3} \frac{1}{1+t}, \quad e_1(\mathbf{m}_2) = \frac{1}{3} \frac{t}{1+t}, \quad e_2(\mathbf{m}_2) = \frac{1}{3} \frac{t+2}{1+t}.$$

In particular, independently of t , we have:

$$e_1(\mathbf{m}_1) - e_1(\mathbf{m}_2) = e_2(\mathbf{m}_2) - e_2(\mathbf{m}_1) \iff \frac{e_2(\mathbf{m}_2) - e_2(\mathbf{m}_1)}{e_1(\mathbf{m}_1) - e_1(\mathbf{m}_2)} = 1.$$

This implies that under perfect substitution between the Receiver's contributions, the symmetric equilibrium ($t = 1$) is the unique equilibrium.¹³ However, one can easily verify that given any $t \geq 0$, the equilibrium condition

$$t = \left(\frac{e_2^r(\mathbf{m}_2) - e_2^r(\mathbf{m}_1)}{e_1^r(\mathbf{m}_1) - e_1^r(\mathbf{m}_2)} \right)^{\frac{1}{r}}$$

is satisfied for a sufficiently low r . This implies the existence of an asymmetric equilibrium whenever r is sufficiently small. Figure 1.4 illustrates the solutions $t = 1$ and $t = t_r$ as a function of r in this setting. The symmetric equilibrium is the only equilibrium with low complementarity ($r \cong 1$) but for any r that is sufficiently low, the asymmetric equilibrium still occurs.

13. Moreover, any asymmetric disclosure rule used by the Sender (corresponding to a $t \neq 1$) would induce effort levels for which a Sender's best response would be the symmetric rule (corresponding to $t = 1$).

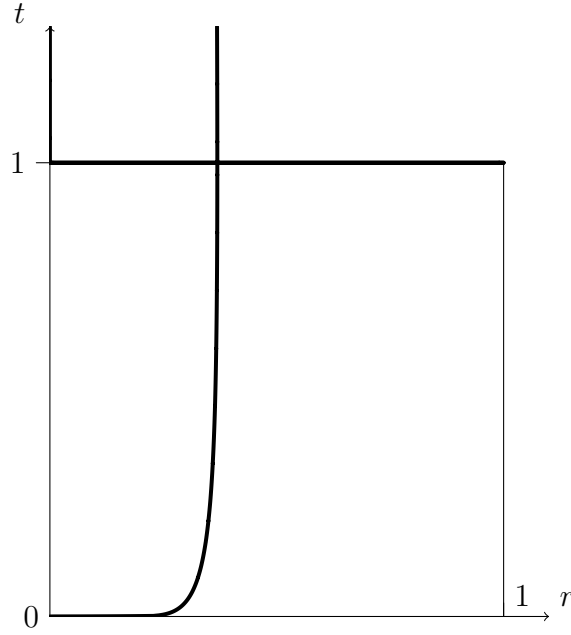


Figure 1.4 – $t = 1$, and $t = t_r$ as a function of r in case of dependent abilities

1.4.2 Exogenously asymmetric contributions

In this section, we assume that each recommendation represents an assignment to an activity, and that activities differ according to the exogenous relative importance (productivity) of the Receiver's contributions.

Activities 1 and 2 correspond to $\mathbf{m} = \mathbf{m}_1$ and $\mathbf{m} = \mathbf{m}_2$ respectively, with respective production functions

$$Y_r(\mathbf{m}_1) = (\eta(a_1 e_1(\mathbf{m}_1))^r + (a_2 e_2(\mathbf{m}_1))^r)^{\frac{1}{r}},$$

$$Y_r(\mathbf{m}_2) = ((a_1 e_1(\mathbf{m}_2))^r + \eta(a_2 e_2(\mathbf{m}_2))^r)^{\frac{1}{r}},$$

for some $\eta > 1$ that represents the relative importance of the most important dimension of the activity.

The differentiation among dimensions puts increasing importance on ability and effort in the most important dimension of an activity. The higher the relative importance of the contribution, the more it should prevent the Sender's mismatching of the highest ability to the highest effort in equilibrium. Therefore the Sender's incentive *ex-ante* to favor a certain dimension given asymmetric efforts, and the driver of the asymmetry vanish. However, complementarity severely mitigates this counter-effect. A high degree of complementarity requires an extreme differentiation of the dimensions in order to rule out the asymmetric equilibrium.

Proposition 1.3. *For any level of differentiation of the contributions η , the asymmetric equilibrium occurring at t_r will persist provided there is sufficient complementarities between the Receiver's contributions.*

Figure 1.5 illustrates the t_r values for different values of η .

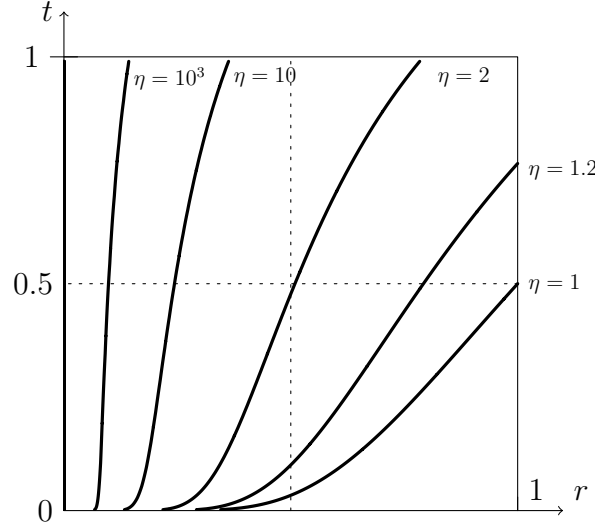


Figure 1.5 – Equilibrium t_r as a function of r and η

For a fixed level of complementarity, the more differentiated the dimensions, the less asymmetric the asymmetric equilibrium (if any). However, complementarity requires a higher level of differentiation to preclude the asymmetric equilibrium. Suppose for instance, that $\eta = 10^3$ so that the important dimension of an activity is a thousand times more productive than its least important dimension. Then, according to Figure 1.5, given sufficient complementarities, the asymmetric equilibria in which the Sender might not match the Receiver's highest ability to his highest effort will still occur. In addition, the extent of the asymmetry might still be very high ($t_r \rightarrow 0$ as $r \rightarrow 0$) in this setting.

In contrast, if the Receiver's contributions are perfect substitutes, then at $r = 1$, a slight differentiation ($\eta \geq 1.5$) will rule out the asymmetric equilibrium.¹⁴

1.5 Selection

In this section, we investigate two arguments related to selecting between the symmetric and the asymmetric equilibrium. Stability points to the asymmetric equilibrium. Moreover,

14. In a meta analysis of 172 studies, Lytton and Romney (1991) show that a gender bias does not emerge clearly within parent-child interactions except in the case of “encouragement of gendered activities”. According to our model, such gendered differentiation would be rational in order to increase the production of human capital if the corresponding dimensions of the child's activities show a high degree of complementarity. In the next section, we show that it would also be efficient.

the Receiver is *ex-ante* more productive in the asymmetric equilibrium if his production dimensions show a high degree of complementarity.

1.5.1 Stability

We say that an equilibrium is stable if, given beliefs close to the equilibrium's beliefs, the best responses of the agents lead to beliefs closer to the equilibrium's beliefs. It is said to be unstable otherwise. In this section we investigate this stability through belief deviations concerning the solution t corresponding to the Sender's disclosure rule in equilibrium.

Proposition 1.4. *For each $r \in (0, 1]$, the symmetric equilibrium is unstable and the asymmetric equilibrium is stable. Furthermore, the greater the degree of complementary, the more important the amplification of an anticipated small degree of asymmetry from the agents.*

The symmetric equilibrium is unstable due to confirmation and amplification by the Sender's rule, of any anticipated small asymmetry from the Receiver.

Set $t < 1$, with $t \cong 1$, an anticipated degree of asymmetry of the Receiver concerning the Sender's disclosure rule, with $e_i(\mathbf{m}_j)$, $i, j \in \{1, 2\}$, the corresponding effort exertions, and set

$$\hat{t}(t) = \left(\frac{e_2^r(\mathbf{m}_2) - e_2^r(\mathbf{m}_1)}{e_1^r(\mathbf{m}_1) - e_1^r(\mathbf{m}_2)} \right)^{\frac{1}{r}}$$

the Sender's best response. We find that if $t_r < t < 1$, then \hat{t} satisfies $t_r < \hat{t}(t) < t < 1$, so that the Sender's would confirm and amplify any anticipated asymmetry that is lower than the equilibrium asymmetry corresponding to t_r , and in contrast, if $0 < t < t_r$, then $0 < t < \hat{t}(t) < t_r$ so that her best response would decrease any extra level of asymmetry relative to the equilibrium asymmetry.

For instance, if $r = \frac{3}{4}$, then we find $\frac{\partial \hat{t}}{\partial t}(1) \cong 3$ at the symmetric equilibrium $t = 1$, and $\frac{\partial \hat{t}}{\partial t}(t_r) \cong 0.35$ at the asymmetric equilibrium $t_r \cong \frac{1}{4}$. For instance this implies $|\hat{t} - t_r| \cong 0.35 |t - t_r|$, so that the Sender's best response \hat{t} is almost three times closer to the equilibrium value t_r relative to an anticipated $t \neq t_r$.

1.5.2 Efficiency

In this section we investigate the relative efficiency of the equilibria. This is given by the expected output $\mathbb{E}[Y_r]$, *i.e.* before the abilities are observed.

Proposition 1.5. *From the Sender's perspective, with more complementarities, the asymmetric equilibrium becomes more productive relative to the symmetric equilibrium. It is efficient as $r \rightarrow 0$.*

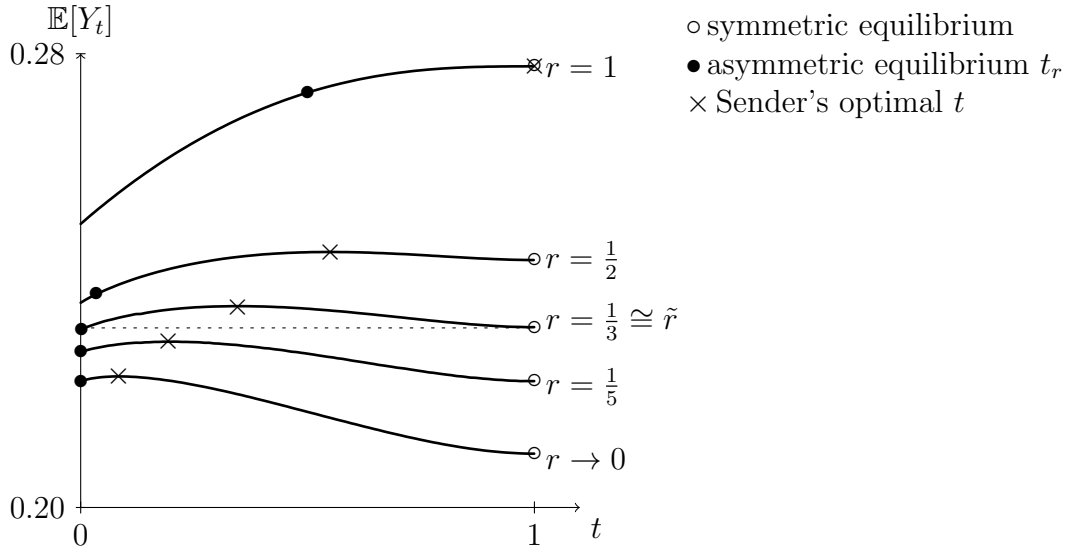
The intuition of Proposition 1.5 relies on the relative informativeness of the equilibria, and its impact on the CES output according to the degree of complementarity involved.

The symmetric equilibrium is symmetrically informative concerning abilities. It induces a matching of high ability with high effort, and low ability with low effort. Therefore, it is highly productive conditional on a low degree of complementarity. In contrast, the informativeness of the asymmetric equilibrium decreases with its asymmetry, and hence, according to Proposition 1.2, with the degree of complementarity. The more asymmetric the equilibrium, the more that exertion of medium efforts (close to the prior $\frac{1}{2}$) can be expected. Therefore, with complementarity, this equilibrium can be expected to be more productive than the symmetric equilibrium.

For instance, let us compute the optimal level of asymmetry from the CES output perspective. At any $t \leq 1$, and level $r = \frac{1}{n}$ with $n \in \mathbb{N}^*$, the binomial expansion formula gives

$$\begin{aligned} \mathbb{E}[Y_t] &= \Pr(\mathbf{m}_1) \iint_{a_1 \geq ta_2} \left(\frac{(a_1 e_1(\mathbf{m}_1))^{\frac{1}{n}} + (a_1 e_2(\mathbf{m}_1))^{\frac{1}{n}}}{2} \right)^n da_1 da_2 \\ &\quad + \Pr(\mathbf{m}_2) \iint_{a_1 < ta_2} \left(\frac{(a_1 e_1(\mathbf{m}_2))^{\frac{1}{n}} + (a_1 e_2(\mathbf{m}_2))^{\frac{1}{n}}}{2} \right)^n da_1 da_2 \\ &= \frac{1}{2^n} \sum_{k=0}^{k=n} \left(\frac{n!}{k!(n-k)!} \frac{1}{3(1 + \frac{k}{n})} \times \right. \\ &\quad \left. \left(e_1^{k/n}(\mathbf{m}_1) e_2^{1-k/n}(\mathbf{m}_1) \frac{3 - t^{1+\frac{k}{n}}(2 - \frac{k}{n})}{2 - \frac{k}{n}} + e_1^{k/n}(\mathbf{m}_2) e_2^{1-k/n}(\mathbf{m}_2) t^{1+\frac{k}{n}} \right) \right) \end{aligned}$$

Figure 1.6 shows that the most productive disclosure strategy corresponds to the symmetric equilibrium strategy ($t = 1$) in the case only of perfect substitution between the Receiver's contributions. With complementarity, if the Sender commits to a strategy associated to t , she will prefer an out-of-equilibrium t which involves asymmetry. We find a threshold \tilde{r} for the degree of complementarity ($\tilde{r} \cong \frac{1}{3}$) below which the asymmetric equilibrium is *ex-ante* more productive relative to the symmetric equilibrium. The value $t = 0$ corresponds to a babbling equilibrium, which is non-informative, and is preferred to the symmetric equilibrium with stronger complementarities. However, notice that the optimal t , as $r \rightarrow 0$, does not tend to $t = 0$. In particular, the Sender will always prefer *ex-ante* to deliver some information, but this amounts will decrease with greater complementarity.

Figure 1.6 – Sender's *ex-ante* utility, with respect to t and r

1.6 Conclusion

We have shown that multiple treatments can arise in equilibrium when an agent is credibly recommended to make specific efforts concerning a production that involves multiple dimensions with complementarities. Note that the treatments differ in their informativeness and their efficiency. A symmetric equilibrium is the most informative and also the most productive in the presence of low degree of complementarity between the dimensions. An asymmetric equilibrium is the least informative and can result in a mismatch by the uninformed agent between his relative perceived abilities and his relative realized abilities. However, it is the most productive equilibrium in the presence of a high degree of complementarity.

Proofs

1.A Proof of Proposition 1.1

Proof. We first prove the existence of each equilibrium, and then their uniqueness.

Existence. Let $r \in (0, 1]$. According to Equation (1.9) in the main text, we need to prove that there exists (at least) a $t_r \in (0, 1)$ such that

$$t_r = f_r(t_r),$$

where f_r is given by

$$f_r(t) = \left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}.$$

Since f_r is continuous, and $[0, \frac{1}{2}]$ is compact and convex, we derive the result from Brouwer's fixed point theorem, if we prove that the range $[0, \frac{1}{2}]$ is stable under f_r , and ensure that $t = 0$ is not a solution of $t = f_r(t)$ (we would obtain $0 < t_r \leq \frac{1}{2} < 1$). For any $r \in (0, 1]$ and any $t \in [0, \frac{1}{2}]$, we have $f_r(t) > 0$. So we need: for any $0 \leq t \leq \frac{1}{2}$,

$$f_r(t) \leq \frac{1}{2}. \quad (1.11)$$

To obtain (1.11), we show first

$$f_r(t) \leq f_r\left(\frac{1}{2}\right), \quad (1.12)$$

and then we show

$$f_r\left(\frac{1}{2}\right) \leq \frac{1}{2}, \quad (1.13)$$

for any $t \in [0, \frac{1}{2}]$ and $r \in (0, 1]$.

Proof of (1.12). We show that f_r is increasing. From the increase in $x \mapsto x^{\frac{1}{r}}$, it is enough to show that $t \mapsto \frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}$ increases. To achieve this, we establish that:

$$t \mapsto \left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \text{ increases with } t, \quad (1.12a)$$

while

$$t \mapsto \left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r \text{ decreases with } t \quad (1.12b)$$

for any $r \in (0, 1]$.

Proof of (1.12a). The derivative of $t \mapsto \left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r$ is $t \mapsto r \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^{r-1} \frac{1}{(3-2t)(2-t)}$, which is positive for $t \in (0, \frac{1}{2}]$.

Proof of (1.12b). The derivative of $t \mapsto \left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r$ is $t \mapsto r \left(\frac{1}{3}\right)^r \left(\left(\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} - \frac{1}{t^{1-r}} \right)$. It is negative if and only if

$$\left(\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} \leq \frac{1}{t^{1-r}} \iff \left(\frac{3-t^2}{t(2-t)}\right)^r \leq \frac{(3-t^2)(2-t)}{t(1-t)(3-t)}. \quad (1.14)$$

For $t \in (0, \frac{1}{2}]$, we have $\frac{3-t^2}{t(2-t)} > 1$, hence $\left(\frac{3-t^2}{t(2-t)}\right)^r \leq \frac{3-t^2}{t(2-t)}$. Then (1.14) derives from $\frac{3-t^2}{t(2-t)} \leq \frac{(3-t^2)(2-t)}{t(1-t)(3-t)}$. This completes the proof of (1.12).

Proof of (1.13). We show that for any $r \in (0, 1]$, $\left(\frac{(\frac{2}{3})^r - (\frac{4}{9})^r}{(\frac{11}{36})^r - (\frac{1}{12})^r}\right)^{\frac{1}{r}} \leq \frac{1}{2}$, that is

$$\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r \leq \left(\frac{11}{36}\right)^r - \left(\frac{1}{12}\right)^r. \quad (1.15)$$

The mean value theorem yields

$$\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r = \frac{r}{a^{1-r}} \left(\frac{2}{3} - \frac{4}{9}\right) = \frac{2r}{9} \frac{1}{a^{1-r}}$$

for some $a \in [\frac{4}{9}, \frac{2}{3}]$, and

$$\left(\frac{11}{36}\right)^r - \left(\frac{1}{12}\right)^r = \frac{r}{b^{1-r}} \left(\frac{11}{36} - \frac{1}{12}\right) = \frac{2r}{9} \frac{1}{b^{1-r}}$$

for some $b \in [\frac{1}{12}, \frac{11}{36}]$. Since $\frac{4}{9} > \frac{11}{36}$, we have $a > b$, and thus $\frac{2r}{9} \frac{1}{a^{1-r}} \leq \frac{2r}{9} \frac{1}{b^{1-r}}$, which gives (1.15).

Uniqueness. To prove the uniqueness of an asymmetric equilibrium we need to prove the uniqueness of a solution of $t = f_r(t)$ on $(0, 1)$. It derives from the following fact (proved below):

$$\text{For each } r \in (0, 1], \text{ if } t \text{ solves } t = f_r(t) \text{ then } f'_r(t) < 1. \quad (1.16)$$

Indeed, since $f_r \neq \text{Id}$, then as illustrated in Figure 1.7, if there were multiple solutions $t \in (0, 1)$ to $t = f_r(t)$, then at least one of them would satisfy $f'_r(t) \geq 1$.¹⁵

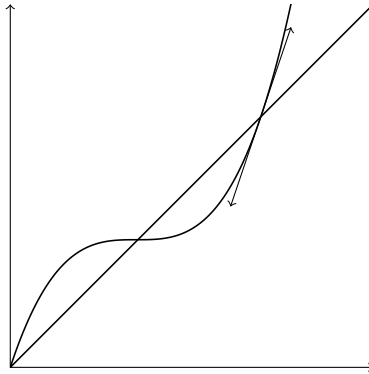


Figure 1.7 – $f'_r(t) \geq 1$ at some solution of $t = f_r(t)$

15. A formal proof is as follows. Let t_1 and t_2 be two solutions to $t = f_r(t)$ with $t_1 < t_2$, and such that $t \mapsto f_r(t) - t$ does not vanish in (t_1, t_2) . Suppose that $f'_r(t_1) < 1$ and $f'_r(t_2) < 1$. Then $t \mapsto f_r(t) - t$ would decrease at t_1 , and there would exist $t_1^+ > t_1$ such that $f_r(t_1^+) - t_1^+ < 0$, and it would also decrease at t_2 and there would exist $t_2^- < t_2$ such that $f_r(t_2^-) - t_2^- > 0$. Then by continuity $f_r(t) - t$ would vanish in $(t_1^+, t_2^-) \subset (t_1, t_2)$ which is impossible.

We relegate the proof of (1.16) to a standalone proof below. \square

Proof of (1.16). The proof is as follows. First, we compute an upper bound $f'_r(t) \leq A_r(t) + B_r(t)$ (*Step 1*), then we derive upper bounds $A_r(t) \leq A(r)$ and $B_r(t) \leq B(r)$ independent of t (*Steps 2A* and *2B* respectively), and finally we derive upper bounds $A(r) \leq A$ and $B(r) \leq B$ such that $A + B < 1$ (*Steps 3A* and *3B* respectively).

Step 1. The derivative of $t \mapsto f_r(t)$ is

$$\begin{aligned} f'_r(t) &= \left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}} \left(\frac{\left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \frac{1}{(3-2t)(2-t)}}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r} - \frac{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} - \frac{1}{3} \left(\frac{t}{3}\right)^{r-1}}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right) \\ &= f_r(t) \left(\frac{\left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \frac{1}{(3-2t)(2-t)}}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r} - \frac{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)}}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} + \frac{\frac{1}{t} \left(\frac{t}{3}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right) \end{aligned}$$

and since

$$\frac{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)}}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \geq 0,$$

we have

$$f'_r(t) \leq f_r(t) \left(\frac{\left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \frac{1}{(3-2t)(2-t)}}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r} + \frac{\frac{1}{t} \left(\frac{t}{3}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right).$$

Thus we have

$$f'_r(t) \leq A_r(t) + B_r(t), \quad (1.17)$$

with

$$A_r(t) = \frac{\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r} \left(\frac{1}{3} \frac{3-2t}{2-t} \right)^r \frac{1}{(3-2t)(2-t)},$$

and

$$B_r(t) = \frac{\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \frac{1}{t} \left(\frac{t}{3} \right)^r.$$

Step 2A. From (1.12a) and (1.12b), we have, for any $0 < t \leq \frac{1}{2}$,

$$\frac{\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r} = \frac{\left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \right)^{\frac{1}{r}-1}}{\left(\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r \right)^{\frac{1}{r}}} \leq \frac{\left(\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r \right)^{\frac{1}{r}-1}}{\left(\left(\frac{11}{18}\right)^r - \left(\frac{1}{6}\right)^r \right)^{\frac{1}{r}}}.$$

Similarly, $t \mapsto \frac{1}{3} \frac{3-2t}{2-t}$ and $t \mapsto \frac{1}{(3-2t)(2-t)}$ decrease, and thus for any $0 < t \leq \frac{1}{2}$, we have $\left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \frac{1}{(3-2t)(2-t)} \leq \left(\frac{1}{2}\right)^r \frac{1}{3}$. Then we get for any $0 < t \leq \frac{1}{2}$,

$$A_r(t) \leq A(r) \quad (1.18)$$

with

$$A(r) = \frac{1}{3} \left(\frac{1}{2}\right)^r \frac{\left(\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r\right)^{\frac{1}{r}-1}}{\left(\left(\frac{11}{18}\right)^r - \left(\frac{1}{6}\right)^r\right)^{\frac{1}{r}}}.$$

Step 2B. Recall that t satisfies

$$\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}\right)^{\frac{1}{r}} = t,$$

and thus we have

$$\begin{aligned} B_r(t) &= \frac{\left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}\right)^{\frac{1}{r}}}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \frac{1}{t} \left(\frac{t}{3}\right)^r = \frac{\left(\frac{t}{3}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} = \frac{1}{\left(\frac{3-t^2}{2-t}\right)^r \left(\frac{1}{t}\right)^r - 1} \\ &= \frac{1}{\left(\frac{3-t^2}{2-t}\right)^r \frac{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r} - 1} \end{aligned} \quad (1.19)$$

Next we need a lower bound for $\left(\frac{3-t^2}{2-t}\right)^r \frac{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}$. To do this, let us prove that it is decreasing with t . Its derivative with respect to t is equal to

$$\begin{aligned} &\frac{r \left(\frac{3-t^2}{2-t}\right)^r}{\left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r\right)^2} \left(\frac{(1-t)(3-t)}{(3-t^2)(2-t)} \left(\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r \right) \left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \right) \right. \\ &\quad + \left(\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} - \left(\frac{t}{3}\right)^r \frac{1}{t} \right) \left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \right) \\ &\quad \left. - \left(\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r \right) \left(\left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \frac{3-2t}{2-t} \right) \right) \end{aligned}$$

the sign of which is equal to the sign of the second factor, which expresses as a sum of three terms. The sign of the second term $\left(\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} - \left(\frac{t}{3}\right)^r \frac{1}{t} \right) \left(\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \right)$ is that of $\left(\left(\frac{3-t^2}{2-t}\right)^r \frac{(1-t)(3-t)}{(3-t^2)(2-t)} - \left(\frac{t}{3}\right)^r \frac{1}{t} \right)$. It is negative from $\left(\frac{3-t^2}{2-t}\right)^r \leq \frac{3-t^2}{t(2-t)}$ (because $\frac{3-t^2}{t(2-t)} > 1$), $t^r \geq t$ (because $t < 1$), and $\frac{(1-t)(3-t)}{(2-t)^2} \leq 1$.

The sign of the sum of the first and third terms is negative *iff*

$$\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r \frac{(3-t^2)(3-2t)}{(1-t)(3-t)} \leq 0.$$

Now $r \mapsto \left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r$ increases, and $r \mapsto -\left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r$ also, and thus so does the sum as a function of r . At $r = 1$ it is equal to $-\frac{2}{3}\frac{2t^3-2t^2-5t+6}{(1-t)(3-t)}$, which is negative (since $2t^3 - 2t^2 - 5t + 6 > 0$ if $t \geq 0$). This completes the proof that $t \mapsto \left(\frac{3-t^2}{2-t}\right)^r \frac{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r}$ decreases. It is thus greater than its value at $t = \frac{1}{2}$, hence:

$$\left(\frac{3-t^2}{2-t}\right)^r \frac{\left(\frac{1}{3}\frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\frac{3-2t}{2-t}\right)^r} \geq \left(\frac{11}{6}\right)^r \frac{\left(\frac{11}{18}\right)^r - \left(\frac{1}{6}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r} = \frac{\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r}. \quad (1.20)$$

From (1.19), we obtain

$$B_r(t) \leq B(r)$$

with $B(r) = \frac{1}{\frac{\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r} - 1}$.

Step 3. In order to get upper bounds of $A(r)$ and $B(r)$, we need accurate upper and lower bounds of expressions of the form $b^r - a^r$, with $b > a > 0$. Notice that $b^r - a^r = \int_a^b \frac{1}{r} x^{r-1} dx$, and as Figure 1.8 shows, the convexity of $x \mapsto \frac{1}{r} x^{r-1}$ allows the integral to be bound by two trapeze areas, such that

$$r(b-a) \left(\frac{a+b}{2}\right)^{r-1} \leq b^r - a^r \leq r(b-a) \frac{a^{r-1} + b^{r-1}}{2}. \quad (1.21)$$

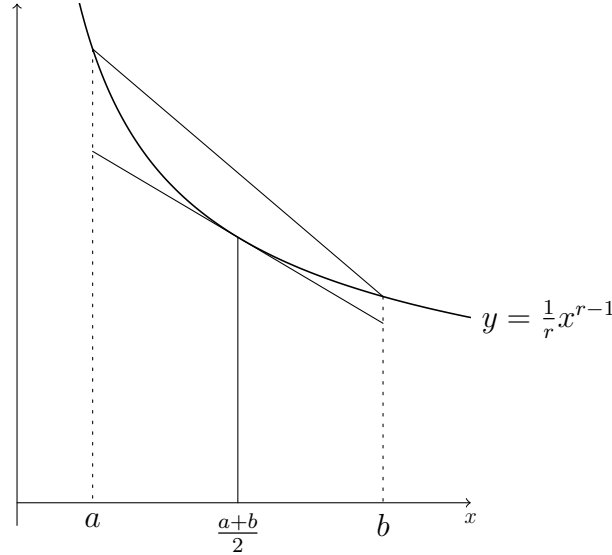


Figure 1.8 – Trapezoidal minoration and majoration of $b^r - a^r$

Step 3A. We need an upper bound for $A(r)$. From (1.21), we have

$$\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r = \left(\frac{2}{3}\right)^r \left(1^r - \left(\frac{2}{3}\right)^r\right) \leq \left(\frac{2}{3}\right)^r r \left(1 - \frac{2}{3}\right) \frac{1 + \left(\frac{2}{3}\right)^{r-1}}{2}$$

with $\left(\frac{2}{3}\right)^{r-1} \leq \frac{3}{2}$, so that

$$\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r \leq \left(\frac{2}{3}\right)^r \frac{5r}{12}.$$

From (1.21), we also have

$$\left(\frac{11}{18}\right)^r - \left(\frac{1}{6}\right)^r \geq \frac{8r}{7} \left(\frac{7}{18}\right)^r.$$

Then we obtain

$$A(r) \leq \frac{1}{r} \frac{48}{35} \left(\frac{3}{4}\right)^r \left(\frac{35}{96}\right)^{\frac{1}{r}},$$

which right hand side has a derivative equal to

$$\frac{48}{35} \left(\frac{3}{4}\right)^r \left(\frac{35}{96}\right)^{\frac{1}{r}} \frac{1}{r} \left(\frac{-1}{r} + \ln\left(\frac{3}{4}\right) + \frac{-1}{r^2} \ln\left(\frac{35}{96}\right)\right)$$

which sign is that of $\frac{-1}{r} + \ln\left(\frac{3}{4}\right) + \frac{-1}{r^2} \ln\left(\frac{35}{96}\right) = \frac{1}{r^2} (r^2 \ln\left(\frac{3}{4}\right) - r - \ln\left(\frac{35}{96}\right))$ which is positive when $0 < r < \alpha$ with $\alpha = \frac{1 - \sqrt{1 + 4 \ln\left(\frac{3}{4}\right) \ln\left(\frac{35}{96}\right)}}{2 \ln\left(\frac{3}{4}\right)}$ and negative when $\alpha < r \leq 1$. Therefore, for any $r \in (0, 1]$, we have

$$A(r) \leq \frac{1}{\alpha} \frac{48}{35} \left(\frac{3}{4}\right)^\alpha \left(\frac{35}{96}\right)^{\frac{1}{\alpha}} < 0.39. \quad (1.22)$$

Step 3B. In order to get an upper bound for $B(r) = \frac{1}{\frac{\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r - 1}}$, we need to get a

lower bound for $\frac{\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r}$. From (1.21), we get

$$\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r \geq r \frac{22}{27} \left(\frac{77}{108}\right)^{r-1},$$

and

$$\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r \leq r \frac{1}{9} \left(\left(\frac{2}{3}\right)^{r-1} + \left(\frac{4}{9}\right)^{r-1} \right),$$

so that

$$\frac{\left(\frac{121}{108}\right)^r - \left(\frac{11}{36}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{4}{9}\right)^r} \geq \frac{22}{3} \frac{1}{\left(\frac{72}{77}\right)^{r-1} + \left(\frac{48}{77}\right)^{r-1}} \geq \frac{22}{3} \frac{1}{\frac{77}{72} + \frac{77}{48}} = \frac{96}{35}$$

(notice that $\frac{x^r}{x} \leq \frac{1}{x}$ if $0 < x \leq 1$). This gives

$$B(r) \leq \frac{1}{\frac{96}{35} - 1} = \frac{35}{61}. \quad (1.23)$$

Finally, (1.23) and (1.22) show that, for each $r \in (0, 1]$, and each $t \in [0, \frac{1}{2}]$ such that $t = f_r(t)$, we have the upper bound

$$f'_r(t) \leq 0.39 + \frac{35}{61} < 1$$

as expected. \square

1.B Proof of Proposition 1.2

Proof. First, we show that $r \mapsto t_r$ increases. It does so *iff* for each $r, r' \in (0, 1]$:

$$\text{if } t_{r'} < t_r, \text{ then } r' < r. \quad (1.24)$$

Let us establish first that for any $r \in (0, 1]$ we have

$$t < f_r(t) \text{ for any } t \in [0, t_r) \quad (1.25)$$

(we also have $t > f_r(t)$ for any $t \in (t_r, \frac{1}{2}]$).

Indeed: $t \mapsto f_r(t) - t$ does not vanish in $[0, t_r)$, since t_r is the unique solution to $t = f_r(t)$ on $(0, 1)$, and $f_r(0) - 0 = f_r(0) > 0$.

Now let us consider $r, r' \in (0, 1]$, and suppose $t_{r'} \in [0, t_r)$. Then from (1.25), we have $t_{r'} < f_r(t_{r'})$, which, since $t_{r'} = f_{r'}(t_{r'})$, can be written as:

$$f_{r'}(t_{r'}) < f_r(t_{r'}). \quad (1.26)$$

It remains to show that this implies $r' < r$ in order to get (1.24). The implication is a consequence of the increasing $r \mapsto f_r(t)$ which we prove next.

For any $0 < x < 1$, the function $r \mapsto x^{\frac{1}{r}}$ is increasing, and therefore it is enough to prove that

$$r \mapsto \frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \quad (1.27)$$

is increasing for any $t \in [0, \frac{1}{2}]$.

Let $t \in (0, \frac{1}{2}]$. For any $r \in (0, 1]$, the sign of the derivative of (1.27) is that of

$$\begin{aligned} g_t(r) = & \left(\left(\left(\frac{2}{3} \right)^r \ln \left(\frac{2}{3} \right) - \left(\frac{1}{3} \frac{3-2t}{2-t} \right)^r \ln \left(\frac{1}{3} \frac{3-2t}{2-t} \right) \right) \left(\left(\frac{1}{3} \frac{3-t^2}{2-t} \right)^r - \left(\frac{t}{3} \right)^r \right) \right. \\ & \left. - \left(\left(\frac{2}{3} \right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t} \right)^r \right) \left(\left(\frac{1}{3} \frac{3-t^2}{2-t} \right)^r \ln \left(\frac{1}{3} \frac{3-t^2}{2-t} \right) - \left(\frac{t}{3} \right)^r \ln(t) \right) \right). \end{aligned}$$

Now notice that $x^r \ln(x) = \frac{1}{r} x^r \ln(x^r)$ so that $g_r(r)$ can be written

$$g_t(r) = \frac{1}{r} \left(\left(\left(\frac{2}{3} \right)^r \ln \left(\left(\frac{2}{3} \right)^r \right) - \left(\frac{1}{3} \frac{3-2t}{2-t} \right)^r \ln \left(\left(\frac{1}{3} \frac{3-2t}{2-t} \right)^r \right) \right) \left(\left(\frac{3-t^2}{2-t} \right)^r - \left(\frac{t}{3} \right)^r \right) - \frac{1}{r} \left(\left(\left(\frac{2}{3} \right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t} \right)^r \right) \left(\left(\frac{1}{3} \frac{3-t^2}{2-t} \right)^r \ln \left(\left(\frac{1}{3} \frac{3-t^2}{2-t} \right)^r \right) - \left(\frac{t}{3} \right)^r \ln \left(\left(\frac{t}{3} \right)^r \right) \right) \right).$$

Let us set $h(x) = x \ln(x)$, $a = \left(\frac{1}{3} \frac{3-2t}{2-t} \right)^r$, $b = \left(\frac{2}{3} \right)^r$, $c = \left(\frac{t}{3} \right)^r$ and $d = \left(\frac{1}{3} \frac{3-t^2}{2-t} \right)^r$. Then we obtain

$$g_t(r) = \frac{1}{r} (b-a)(c-d) \left(\frac{h(b)-h(a)}{b-a} - \frac{h(d)-h(c)}{d-c} \right),$$

the sign of which is equal to the sign of $\left(\frac{h(b)-h(a)}{b-a} - \frac{h(d)-h(c)}{d-c} \right)$. This is the difference between the slope of (AB) and the slope of (CD) , where A, B, C and D are the points on the graph of h with respective abscissa a, b, c and d (see Figure 1.9). The convexity of h , and

$$\frac{t}{3} < \frac{1}{3} \frac{3-2t}{2-t} < \frac{1}{3} \frac{3-t^2}{2-t} < \frac{2}{3},$$

so that

$$c < a < d < b,$$

allows us to derive the positivity of $g_t(r)$ from the Three Chords Lemma¹⁶ applied first, to the triangle CAB , where it shows that the slope of (AD) is steeper than the slope of (CD) , and second, to the triangle ADB , where it shows that the slope of (AB) is steeper than the slope of (AD) , therefore, the slope of (AB) is steeper than the slope of (CD) .

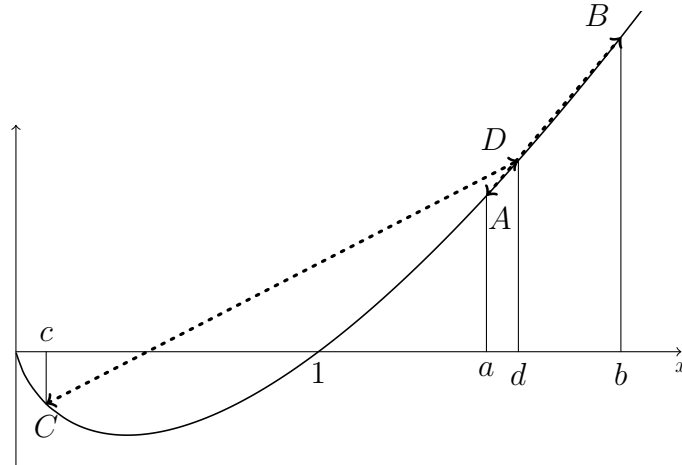


Figure 1.9 – The slopes of (AB) and (CD) with A, B, C, D on $y = x \ln(x)$

16. The Three Chords Lemma states that if X, Z, Y are three points on the graph of a convex function with abscissa $x_X < x_Z < x_Y$, then the slope of (XZ) will be less steep than the slope of (XY) , which will be less steep than the slope of (YZ) .

b) The result relies on $\lim_{r \rightarrow 0} t_r = 0$, which we prove next. We have to prove precisely that, if $r \rightarrow 0$, $t = 0$ is a solution of

$$t = \left(\frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} \right)^{\frac{1}{r}}.$$

From l'Hôpital's Rule

$$\lim_{r \rightarrow 0} \frac{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r} = \frac{\ln\left(\frac{2}{3}\right) - \ln\left(\frac{1}{3} \frac{3-2t}{2-t}\right)}{\ln\left(\frac{1}{3} \frac{3-t^2}{2-t}\right) - \ln\left(\frac{t}{3}\right)},$$

and since $0 < \frac{\ln\left(\frac{2}{3}\right) - \ln\left(\frac{1}{3} \frac{3-2t}{2-t}\right)}{\ln\left(\frac{1}{3} \frac{3-t^2}{2-t}\right) - \ln\left(\frac{t}{3}\right)} < 1$ for any $t \in (0, \frac{1}{2}]$, we have

$$\lim_{r \rightarrow 0} \left(\frac{\ln\left(\frac{2}{3}\right) - \ln\left(\frac{1}{3} \frac{3-2t}{2-t}\right)}{\ln\left(\frac{1}{3} \frac{3-t^2}{2-t}\right) - \ln\left(\frac{t}{3}\right)} \right)^{\frac{1}{r}} = 0.$$

□

1.C Proof of Proposition 1.3

Proof. We prove that at $t = 0$, $f_r(0) > 0$, and for each $t \in (0, 1]$, there exists a sufficiently small r such that $f_r(t) < t$. Thus, the result is a consequence of Bolzano's Theorem.

For each $\eta \geq 1$, for each $r \in (0, 1]$,

$$f_r(0) = \left(\left(\frac{4}{3}\right)^r - \left(\frac{1}{\eta^2}\right)^r \right)^{\frac{1}{r}} > 0.$$

Now fix $t \in (0, 1]$. Then

$$\begin{aligned} \lim_{r \rightarrow 0} \left(\frac{f_r(t)}{t} \right)^r &= \lim_{r \rightarrow 0} \frac{\left(\eta^2 \frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{t^r \left(\left(\eta^2 \frac{1}{3} \frac{3-t^2}{2-t}\right)^r - \left(\frac{t}{3}\right)^r \right)} = \lim_{r \rightarrow 0} \frac{\left(\eta^2 \frac{2}{3}\right)^r - \left(\frac{1}{3} \frac{3-2t}{2-t}\right)^r}{\left(\eta^2 \frac{1}{3} \frac{t(3-t^2)}{2-t}\right)^r - \left(\frac{t^2}{3}\right)^r} \\ &= \frac{\ln\left(\eta^2 \frac{2}{3}\right) - \ln\left(\frac{1}{3} \frac{3-2t}{2-t}\right)}{\ln\left(\eta^2 \frac{1}{3} \frac{3-t^2}{2-t}\right) - \ln\left(\frac{t}{3}\right)} = \frac{\ln\left(\eta^2 \frac{2(2-t)}{3-2t}\right)}{\ln\left(\eta^2 \frac{3-t^2}{t(2-t)}\right)}. \end{aligned}$$

and since for all $t \in (0, 1]$, $\frac{2(2-t)}{3-2t} < \frac{3-t^2}{t(2-t)}$ we obtain $\lim_{r \rightarrow 0} \left(\frac{f_r(t)}{t} \right)^r < 1$. So for a sufficiently small r , $f_r(t) < t$. □

1.D Proof of Proposition 1.4

Proof. Suppose that the Receiver anticipates a level \hat{t} of asymmetry of the Sender's disclosure rule. Efforts $e_i(m)[\hat{t}]$ correspond to \hat{t} , and Sender's response is a disclosure rule based on a level of asymmetry equal to $\left(\frac{e_2^r(2)[\hat{t}] - e_2^r(1)[\hat{t}]}{e_1^r(1)[\hat{t}] - e_1^r(2)[\hat{t}]}\right)^{\frac{1}{r}} = f_r(\hat{t})$. For an equilibrium occurring at t^* , it is stable if $|f_r(\hat{t}) - t^*|$ is less than $|\hat{t} - t^*|$. Therefore, stability is measured by the derivative $f'_r(t^*)$: if $f'_r(t^*) < 1$, then the equilibrium is stable, and if $f'_r(t^*) > 1$, it is unstable.

Consider the symmetric equilibrium. We have $f'_r(t^*) = f'_r(1) = \frac{\left(\frac{1}{3}\right)^r}{\left(\frac{2}{3}\right)^r - \left(\frac{1}{3}\right)^r} = \frac{2}{2^r - 1} > 2$ since $2^r - 1 < 2 - 1 = 1$. Moreover, $f'_r(1)$ increases with r , and $\lim_{r \rightarrow 0} f'_r(1) = +\infty$: the lower r , the more the symmetric equilibrium is unstable.

For an asymmetric equilibrium, we state $f_r(t_r) < 1$ in Equation (1.16). The proof follows the statement. \square

1.E Proof of Proposition 1.5

Proof. We need to compare the expected value $\mathbb{E}[Y_t]$ of the CES production function between the symmetric and the asymmetric equilibria. Let us set $t^* = 1$ or $t^* = t_r$ the solution corresponding to the equilibrium. We have

$$\begin{aligned} \mathbb{E}[Y_{t^*}] &= \Pr(\mathbf{m} = \mathbf{m}_1)E[Y_{t^*}(\mathbf{m}_1)] + \Pr(\mathbf{m} = \mathbf{m}_2)E[Y_{t^*}(\mathbf{m}_2)] \\ &= \int_{a_1 \geq t^* a_2} Y_{t^*}(\mathbf{m}_1) da_1 da_2 + \int_{a_1 < t^* a_2} Y_{t^*}(\mathbf{m}_2) da_1 da_2 \\ &= \int_{a_2=0}^1 \int_{a_1=t^* a_2}^1 \left(\frac{(E_{11}(t^*)a_1)^r + (E_{21}(t^*)a_2)^r}{2} \right)^{\frac{1}{r}} da_1 da_2 \\ &\quad + \int_{a_2=0}^1 \int_{a_1=0}^{t^* a_2} \left(\frac{(E_{12}(t^*)a_1)^r + (E_{22}(t^*)a_2)^r}{2} \right)^{\frac{1}{r}} da_1 da_2 \end{aligned} \tag{1.28}$$

with E_{ij} 's given by (1.8).

If $r = 1$, we find for the symmetric equilibrium $\mathbb{E}[Y_{t^*=1}] = \frac{5}{18} \cong 0.278$, and for the asymmetric equilibrium, occurring at $t^* = t_1 = \frac{1}{2}$, we find $\mathbb{E}[Y_{t^*=\frac{1}{2}}] = \frac{59}{216} \cong 0.273$. Therefore, the asymmetric equilibrium is *ex-ante* less productive than the symmetric equilibrium.

As r tends to 0, t_r tends to 0 and at $t^* = 0$, we have $E_{11}(0) = \frac{1}{2}$, $E_{21}(0) = \frac{1}{2}$, $E_{12}(0) = 0$, and $E_{22}(0) = \frac{2}{3}$. Moreover, $\left(\frac{(y_1)^r + (y_2)^r}{2}\right)^{\frac{1}{r}}$ tends to $\sqrt{y_1 y_2}$.

Hence as $r \rightarrow 0$, $t_r \rightarrow 0$, for the symmetric equilibrium we obtain

$$\begin{aligned}\mathbb{E}[Y_{t^*=1}] &= \int_{a_1 \geq a_2} \sqrt{\frac{2}{3}a_1 \frac{1}{3}a_2} da_1 da_2 + \int_{a_1 < a_2} \sqrt{\frac{1}{3}a_1 \frac{2}{3}a_2} da_1 da_2 \\ &= \frac{\sqrt{2}}{3} \int_{a_1, a_2} \sqrt{a_1 a_2} da_1 da_2 = \frac{2\sqrt{2}}{27} \cong 0.21,\end{aligned}$$

and for the asymmetric equilibrium we obtain

$$\mathbb{E}[Y_{t^* \rightarrow 0}] = \int_{a_1, a_2} \sqrt{\frac{1}{2}a_1 \frac{1}{2}a_2} da_1 da_2 = \frac{1}{2} \int_{a_1, a_2} \sqrt{a_1 a_2} da_1 da_2 = \frac{2}{9} \cong 0.22.$$

Therefore, the asymmetric equilibrium is *ex-ante* more productive than the symmetric equilibrium. \square

Chapitre 2

Uniform quadratic binary two-dimensional cheap talk

Abstract. We analyze a standard Crawford and Sobel's (1982) cheap talk game in a two-dimensional framework, with uniform prior, quadratic preferences and a binary disclosure rule. Information might be credibly revealed by the Sender to the Receiver when players are able to strategically set aside their conflict. We exploit the few symmetries of the game parameters to derive multiple continua of equilibria, when varying the Sender's bias over the entire euclidean space. In particular, credible information might be revealed whatever the bias. Then we show that the equilibria exhibited characterize the game's full set of pure strategy equilibria.

2.1 Introduction

Conflicts of interest prevail in most economic activities: for instance, employees and managers conflict over the employee's optimal effort, managers and shareholders conflict over the optimal allocation of capital, government and firms conflict over policy (hence the existence of lobbyists) and firm objectives (hence the existence of regulators), and so on. In this context, Barnard (1938)¹ distinguishes two ways of doing things:

An organization can secure the efforts necessary to its existence, then, either by the objective inducements it provides or by changing states of mind. [...] We shall call the processes of offering objective incentives “the method of incentives”; and the processes of changing subjective attitudes “the method of persuasion”.

Though monetary transfers are central to the design of incentives in organizations, there are a significant number of decisions that do not involve *direct* costs or benefits. In many situations, information is at the core of the decision-making process.

In this perspective the seminal work of Crawford and Sobel (1982) offers some guidelines related to the strategic aspects of information transmission between economic agents.² Crawford and Sobel (1982) study the effect of a conflict of interest on the influence that an informed agent (the Sender, she) might have on the action of an uninformed agent (the Receiver, he). The Sender possesses private information on a state of the world. She sends a message to the Receiver, who then takes an action. Players conflict over their preferred action. The difference is the Sender's *bias*. It provides reasons for the Receiver to be skeptical about the information disclosed. So the disclosed information might not influence his action. Crawford and Sobel (1982) show that multiple influential equilibria are associated with a limited bias. These authors characterize all equilibria. Equilibria are differentiated by their informativeness. An increased bias diminishes both the number of equilibria and their informativeness. A large bias precludes the possibility of influence.

In Crawford and Sobel's (1982) model, states and actions are one-dimensional. However many real-life interactions are multi-dimensional. In that case communication might not be restricted to communicating in each dimension separately. Spillover effects may arise across several dimensions, making analysis more complex. So in the multi-dimensional framework, results in the literature only refer to existence conditions. In the present paper we study an extension of Crawford and Sobel's (1982) model to include two dimensions under sharp assumptions which enable equilibria to be fully characterized. In particular, we consider two messages for the Sender's disclosure strategy,³ and we assume quadratic preferences for the players, and a uniform distribution over $[0, 1]^2$ for the states. These assumptions rule out the one-dimensional multiplicity of equilibria in each dimension, make

1. Cited by Laffont and Martimort (2002, p. 12).

2. See for instance Sobel (2013) for a review of the literature.

3. Many information disclosures are binary, for instance: endorsement, labeling, hiring, promotions, voting, court decisions. Moreover, the bit is the elementary element of information. So one could consider breaking down complex information disclosures into multiple binary ones.

the analysis tractable, and in our view provide non-trivial results.

To support the economic relevance of our study, consider for instance a manager who holds private information (θ_1, θ_2) concerning two tasks an employee has to carry out. Conditional on her private information, the manager makes a recommendation to the employee. The recommendation takes the form of a message m , chosen from a set of two alternatives m_1 and m_2 . Upon observing m , the employee makes an inference concerning (θ_1, θ_2) , and exerts efforts a_1 and a_2 on the respective tasks.⁴ Let us assume that the employee prefers to adjust his efforts to be as close as possible to the state (θ_1, θ_2) , whereas the manager prefers the employee to exert efforts as close as possible to $(\theta_1 + b_1, \theta_2 + b_2)$, where $(b_1, b_2) \in \mathbb{R}^2$ is the manager's bias.⁵ We are interested in characterizing the associations of states (θ_1, θ_2) with messages m_1 or m_2 that influence the efforts. Strategically recommendation m_i results from the manager's anticipation of the efforts obtained, which in turn result from the information revealed.

For instance, if b_1 and b_2 are substantial, then the manager is not able credibly to differentiate between "low" (θ_1, θ_2) s and "high" (θ_1, θ_2) s. She always prefers the employee to perceive a high value of (θ_1, θ_2) in order to induce the greatest efforts. Hence, regardless of the state, it is always in her interest to deviate from the lower recommendation to the higher one. Yet, as shown by Chakraborty and Harbaugh (2007), comparative statements between θ_1 and θ_2 might be credible. If the manager recommends " $\theta_1 \geq \theta_2$ " (m_1), then the employee simultaneously perceives a high value for θ_1 and low one for θ_2 . This induces him to make a great effort $a_1|m_1 = \mathbb{E}[\theta_1|m_1]$ and a lower one $a_2|m_1 = \mathbb{E}[\theta_2|m_1]$. The symmetry of the state distribution implies that the alternative symmetric recommendation " $\theta_1 < \theta_2$ " (m_2) induces symmetric efforts $a_1|m_2 = a_2|m_1$ and $a_2|m_2 = a_1|m_1$. The Sender prefers recommendation m_1 to m_2 whenever $(a_1, a_2)|m_1$ is closer than $(a_1, a_2)|m_2 = (a_2, a_1)|m_1$ to the her most highly preferred action $(\theta_1 + b_1, \theta_2 + b_2)$. So she prefers m_1 whenever (θ_1, θ_2) satisfies $\theta_1 + b_1 \geq \theta_2 + b_2$. In particular, if $b_1 = b_2$, then she has incentives to reveal m_1 : " $\theta_1 \geq \theta_2$ " or m_2 : " $\theta_1 < \theta_2$ " truthfully. This makes her recommendation to the employee credible.

We start by noting that the result obtained by Chakraborty and Harbaugh (2007) can be extended to cover multiple directions. For instance, if $b_1 = -b_2$, the manager might truthfully reveal whether $\theta_1 + \theta_2 \geq 1$ or not. More generally if Sender's preferences are symmetric relative to a symmetry of the distribution of states, then the symmetry of the two Receiver's posterior beliefs (given by $\mathbb{E}[(\theta_1, \theta_2)|m_i]$, $i \in \{1, 2\}$) results from and conditions the symmetry of the Sender's recommendations. So when the state distribution has multiple symmetries, each one might support the possibility of credible recommendations, conditional on a corresponding alignment with the bias. Given the four axial symmetries of the uniform distribution over $[0, 1]^2$, we obtain four *symmetric equilibria*, with each one

4. Alternatively the manager makes one of two *decisions*, which only rely on her private information and the different efforts they potentially induce.

5. For instance, if $b_1 > b_2$, the manager has an exogenous asymmetric interest concerning the tasks, and *ceteris paribus* prefers a higher effort in the first task relative to the second. Such preferences could represent some favoritism from the manager concerning one of the tasks, in line with the literature on multi-tasks in organizations (Dewatripont, Jewitt, and Tirole, 2000).

associated with a family of symmetric bias.

In any symmetric equilibrium, information is revealed and processed strictly orthogonally to the dimension of the bias (*e.g.* when $b_1 = b_2$ the revealed sign of $\theta_1 - \theta_2$ says nothing about $\theta_1 + \theta_2$). So the bias has no effect in the informative dimension, and no information is revealed in the bias dimension. However, according to Crawford and Sobel (1982), the existence of a limited bias in a specific dimension allows information revelation in that dimension. If players play a one-dimensional game with a limited bias, they have a strategy which aligns their choices. This mechanism does not simply extend to the multi-dimensional setup insofar as any adjustment of a player's strategy in one dimension prompts the other player's best response to vary in multiple dimensions. We show that in a symmetric equilibrium, this cross dimensional effect does not arise. The uniform distribution over $[0, 1]^2$ renders a statistical independence relation between the informative and uninformative dimensions of a symmetric equilibrium. Accordingly, players might play a one-dimensional information-transmission game in the informative dimension of a symmetric equilibrium. We thus obtain four continua of *deviated symmetric equilibria*. They substantially extend the set of biases that allow players to reach agreement. The set of biases is extended in the orthogonal dimension of each of the symmetric biases associated with the four symmetric equilibria.

Crucial to the existence of these deviated symmetric equilibria is the possibility of being informative along a dimension, while remaining silent orthogonally to that dimension. The uniform distribution over $[0, 1]^2$ lacks the independence required for this to occur for any random pair of orthogonal dimensions. However, we show next that given *any* dimension, players have the opportunity to strategically choose *some* information to be revealed along this dimension so that the recommendations are uninformative in its orthogonal dimension. Hence the uniform distribution presents a sufficiently weak dependence between the dimensions of a random pair of orthogonal dimensions. For instance, if the manager reveals asymmetrically and truthfully whether or not $\theta_1 \geq \theta_2/2$, then the Receiver infers $(\mathbb{E}[\theta_1|m_1], \mathbb{E}[\theta_2|m_1]) = (11/18, 4/9)$ and $(\mathbb{E}[\theta_1|m_2], \mathbb{E}[\theta_2|m_2]) = (1/6, 2/3)$, so that no information is revealed concerning $\theta_1/2 + \theta_2$ (we have $\mathbb{E}[\theta_1|m_i]/2 + \mathbb{E}[\theta_2|m_i] = 3/4$ for $i \in \{1, 2\}$). This defines a profile of equilibrium strategies when the Sender's bias provides her with the incentives to reveal the corresponding asymmetric information. Then, we obtain a continuum of *asymmetric equilibria*, associated with a large family of specific non-zero biases.

The asymmetric equilibria also extend the set of biases that allow players to reach agreement. Then we show that the multiple continua of equilibria permit *any bias* to be associated with an influential equilibrium. In organizations, given our assumptions, any conflict might “sneak” through the dimensions, to become an influential factor in the decisions and actions of the economic agents.

We also observe that among the family of biases associated with an asymmetric equilibrium, there are four symmetric biases, associated with the four symmetries of the distribution of the states. So the corresponding equilibrium strategies are asymmetric despite the symmetry of all of the parameters of the corresponding game. This shows the possibility of an *endogenously asymmetric* information transmission. In organizations, despite an *a*

priori symmetric dependence of the agents' payoffs with respect to the tasks, the agents' conflict provides them with incentives to potentially treat the dimensions of their work asymmetrically.

Finally, we show that the equilibria exhibited cover the full set of the game's pure strategy equilibria, which results in the characterization of this set.

Extensions of Crawford and Sobel's (1982) model to include multiple dimensions have been investigated in the literature. However, results concern the possibility of influence, using either full revelation or binary disclosure rules, and do not seek to characterize the equilibria. When players do not conflict, a large number of messages and an equal number of actions might occur in equilibrium, up to full revelation.⁶ Conflict introduces spillover effects between the multiple dimensions that make a general characterization beyond reach.

Battaglini (2002) notes that Sender and Receiver always have a dimension of agreement. Then when the state space is \mathbb{R}^2 , the Receiver might extract full information from two senders, through an equilibrium alignment of her and each of the sender's interest in their dimension of agreement. The author also notes that with quadratic preferences, a possibility of communication occurs in the dimension which is strictly orthogonal to the Sender's bias. In contrast, Ambrus and Takahashi (2008) show that full extraction may be impossible if states are restricted to belong to a closed subset of \mathbb{R}^d . We focus on binary disclosure rules, which are necessarily not fully revealing when the state space is infinite. Our result establishes that communication is possible if the state space is restricted, whatever the bias. Moreover, we show that communication does not necessarily occur strictly orthogonally to the bias. Indeed, in an asymmetric equilibrium, the bias does impact communication.

Chakraborty and Harbaugh (2007) show the existence of the symmetric comparative equilibrium, conditional on a symmetric distribution of states and additively separable and super-modular utility functions of the players.⁷ We note that Chakraborty and Harbaugh's (2007) result can be extended by symmetry to cover any direction such that the symmetric conditions are fulfilled with respect to that direction.

In a paper that complements Chakraborty and Harbaugh (2007), Levy and Razin (2007) investigate the large-conflict case, by assuming lexicographic preferences for the Sender. Note that when the Sender's preferences are quadratic, they tend to be lexicographic when the bias tends to infinity in a given direction. When the Sender has lexicographic preferences, Levy and Razin (2007) establish a necessary condition on the distribution of the states for communication to occur with k messages. We show that the uniform distribution over $[0, 1]^2$ satisfies this condition in any direction when $k = 2$. Hence when states are uniformly distributed on $[0, 1]^2$, Levy and Razin's (2007) result does not allow to establish that communication is impossible, whatever the direction of a Sender's arbitrary large

6. In the multi-dimensional framework, these conflict-free equilibria occur as so called *centroidal Voronoi tessellations*, see Du, Faber, and Gunzburger (1999) for a review. The work of Jäger, Metzger, and Riedel (2011) investigates the Voronoi tessellations with many cells, in a communication framework. In Jäger, Metzger, and Riedel (2011) players do not conflict.

7. Chakraborty and Harbaugh (2010) derive the existence of an influential equilibrium whatever the distribution of states. However, they assume that the Sender's preferences do not depend on the state.

bias. On the contrary, we find that for any extent and any direction of the bias, multiple possibilities of communication are possible. Moreover we find a continuum of equilibria such that when the bias tends to infinity, equilibrium conditions of the “finite” game (in which preferences are quadratic) do not necessarily converge to equilibrium conditions of the “infinite” game (in which preferences are lexicographic). This occurs although players’ preferences in the former game do converge to players’ preferences in the latter one. This result limits the scope of application of the large-conflict situation.

More generally, Levy and Razin (2007) show that multi-dimensionality might preclude the possibility of communication if game parameters lack sufficient symmetry relative to a specific direction. In contrast, Chakraborty and Harbaugh (2007) show that communication is possible whenever conflict is aligned with a symmetry of the state distribution. We exploit the simultaneous existence of the few symmetries of the state distribution. This precludes strong asymmetries in any direction. Combined with the flexibility of the one-dimensional framework, we obtain a possibility of communication whatever the direction and extent of the conflict.

Asymmetric equilibria have not been investigated in the literature in the context of more general distributions of states or more general utility functions. Using a slightly different setup, Kamphorst and Swank (2016) exhibit an endogenous asymmetric equilibrium, assuming a uniform distribution, a binary disclosure rule,⁸ quadratic preferences for the Receiver, and symmetric linear preferences for the Sender. The linear preferences assumption implies that Sender always prefers Receiver to take the highest possible action. We explore any fixed differences for the players’ preferred action. Kamphorst and Swank (2016) interpret the asymmetric equilibrium as a possible discriminatory practice by the players in organizations. Our result suggests that the discrimination considered by Kamphorst and Swank (2016) is precisely the consequence of the multiple symmetries of the game parameters. It allows players to agree in multiple ways. It might occur without conflict of interest, with an exogenous asymmetric conflict, or, in line with Kamphorst and Swank (2016), be endogenously driven by a symmetric conflict.

Section 2.2 presents the formal model. Section 2.3 establishes the equilibrium conditions and provides examples and the vocabulary to state the results. Section 2.4 extends the examples to cover many directions, and shows the existence of influential equilibrium strategies for any bias. Section 2.5 characterizes the equilibria as the full set of pure strategy equilibria of the game. Section 2.6 concludes. The proofs are provided in the Appendix.

2.2 Model setup

We consider an agent, the Sender (S , she), who is informed about a state of the world θ , and an agent, the Receiver (R , he), who is not. The state of the world $\theta = (\theta_1, \theta_2)$ is the realization of a random variable uniformly distributed over the compact convex set

8. Crutzen, Swank, and Visser (2013) show that in this setup, the number of messages cannot exceed 3.

$\Theta = [0, 1]^2$. The Sender sends a message m , one of two alternatives m_1 and m_2 . The Receiver observes the Sender's message, and takes an action $\mathbf{a} \in \mathbb{R}^2$.⁹ Both players' preferences rely on the state of the world $\boldsymbol{\theta}$ and on the Receiver's action \mathbf{a} , but not on the message m , which is purely informative. The conflict of interest between the players occurs as a constant difference in their preferred action \mathbf{a} . The Receiver prefers his action to be as close as possible to the state of the world. His utility U^R decreases with the (Euclidean) distance between \mathbf{a} and $\boldsymbol{\theta}$. We set

$$U^R(\mathbf{a}, \boldsymbol{\theta}) = -\|\mathbf{a} - \boldsymbol{\theta}\|^2.$$

The Sender prefers the Receiver's action to be as close as possible to a shift in the state of the world. The Sender's utility U^S decreases with the distance between \mathbf{a} and $\boldsymbol{\theta} + \mathbf{b}$, where $\mathbf{b} \in \mathbb{R}^2$ denotes the Sender's bias relative to the Receiver's preferred action, and represents the conflict of interest between the players. We set

$$U_b^S(\mathbf{a}, \boldsymbol{\theta}) = -\|\mathbf{a} - (\boldsymbol{\theta} + \mathbf{b})\|^2.$$

The game is denoted $\Gamma_{\mathbf{b}}$. Its timing is as follows:

1. Nature draws the state of the world $\boldsymbol{\theta} = (\theta_1, \theta_2)$, and reveals it to the Sender, but not to the Receiver (he has a uniform prior over $[0, 1]^2$, which is common knowledge);
2. the Sender sends a message $m \in \{m_1, m_2\}$ upon her observation of $\boldsymbol{\theta}$;
3. the Receiver observes the Sender's message m and updates his prior belief about $\boldsymbol{\theta}$;
4. the Receiver chooses his action $\mathbf{a} \in \mathbb{R}^2$ according to his posterior belief;
5. payoffs are realized.

We look for the perfect Bayesian equilibria of this game, that is: (i) the Receiver's action strategy is optimal, given his posterior belief about the state of the world; (ii) the Sender's disclosure strategy is optimal, given the Receiver's action strategy and belief updating; (iii) whenever possible, beliefs are updated according to Bayes's rule.

2.3 Analysis and examples

2.3.1 Strategies

Players' pure strategies are as follows. The Sender chooses $\mathbf{m}(\boldsymbol{\theta}) \in \{m_1, m_2\}$ based on her observation of the state of the world $\boldsymbol{\theta}$. Let $\mathbf{a}(m_i) \in \mathbb{R}^2$ be the action played by the Receiver, given his posterior belief upon receiving m_i . For $i \in \{1, 2\}$, let $\mathbf{m}^{-1}(m_i)$ be the set of $\boldsymbol{\theta}$ for which message m_i is disclosed. If $\mathbf{m}^{-1}(m_i) \neq \emptyset$, then based on his observation

9. The multidimensionality of the game also represents the situation in which a single Sender sends a public message to two Receivers who each takes a one-dimensional action.

of m_i , the Receiver updates his information according to Bayes's rule to $\boldsymbol{\theta} \in \mathbf{m}^{-1}(m_i)$ and subsequently takes the action that maximizes his expected utility, at

$$\mathbf{a}(m_i) = \arg \max_{\mathbf{a} \in \mathbb{R}^2} \int_{\boldsymbol{\theta} \in \mathbf{m}^{-1}(m_i)} U^R(\mathbf{a}, \boldsymbol{\theta}) d\boldsymbol{\theta} = \mathbb{E}[\boldsymbol{\theta} | \mathbf{m}(\boldsymbol{\theta}) = m_i]. \quad (2.1)$$

Reciprocally, given the Receiver's belief updating and the corresponding action strategy $\mathbf{a} : m \mapsto \mathbf{a}(m)$, the Sender sends the message that maximizes her utility based on her observation of $\boldsymbol{\theta}$, and so

$$\mathbf{m}(\boldsymbol{\theta}) = \arg \max_{m \in \{m_1, m_2\}} U_b^S(\mathbf{a}(m), \boldsymbol{\theta}) = \{m_i, U_b^S(\mathbf{a}(m_i), \boldsymbol{\theta}) \geq U_b^S(\mathbf{a}(m_{-i}), \boldsymbol{\theta})\}, \quad (2.2)$$

where $-i$ represents the element of $\{1, 2\} \setminus \{i\}$.

In equilibrium, strategies $\mathbf{m} : \boldsymbol{\theta} \mapsto \mathbf{m}(\boldsymbol{\theta})$ and $\mathbf{a} : m \mapsto \mathbf{a}(m)$ are each player's best response.

The followings points are worth noting.

First, there are always equilibria in which no information is revealed, *i.e.* *babbling equilibria*. The construction is as follows: the Sender sends the same message m_i , for some $i \in \{1, 2\}$, regardless of $\boldsymbol{\theta} \in \Theta$; the Receiver always takes his action at $\mathbf{a}(m_i) = \mathbb{E}[\boldsymbol{\theta} | \mathbf{m}(\boldsymbol{\theta}) = m_i] = \mathbb{E}[\boldsymbol{\theta}] = (\frac{1}{2}, \frac{1}{2})$, regardless of the message received, and his off-path belief on receiving m_{-i} is the same as his belief on receiving m_i (in line with Farrell (1993, Section 3), m_1 and m_2 mean the same to the Receiver). We say that an equilibrium is *influential* if on the contrary, $\mathbf{a}(m_i) \neq \mathbf{a}(m_{-i})$.

Second, given a Receiver strategy \mathbf{a} , for any $\boldsymbol{\theta} \in \Theta$, we have $U_b^S(\mathbf{a}(m_i), \boldsymbol{\theta}) \geq U_b^S(\mathbf{a}(m_{-i}), \boldsymbol{\theta})$ iff

$$\begin{aligned} & -\|\mathbf{a}(m_i) - (\boldsymbol{\theta} + \mathbf{b})\|^2 \geq -\|\mathbf{a}(m_{-i}) - (\boldsymbol{\theta} + \mathbf{b})\|^2, \\ \text{iff } & \|(\mathbf{a}(m_i) - \mathbf{b}) - \boldsymbol{\theta}\|^2 \leq \|(\mathbf{a}(m_{-i}) - \mathbf{b}) - \boldsymbol{\theta}\|^2, \end{aligned}$$

so that the equilibrium condition of the Sender's strategy might be stated geometrically as: the Sender necessarily discloses m_i or m_{-i} conditional on $\boldsymbol{\theta}$ belonging to one side of the *perpendicular bisector* of the line that supports $\mathbf{a}(m_1) - \mathbf{b}$ and $\mathbf{a}(m_2) - \mathbf{b}$ (see Figure 2.1 for an illustration of this point). Given \mathbf{a} and \mathbf{b} , this bisector represents the set of states for which the Sender is indifferent between the two induced actions. In particular, a Sender's non-babbling strategy, up to the set of indifferent states, is fully characterized by a line that partitions Θ , and by the association of a message with either side of it. Now given a Sender's strategy characterized in this way, from (2.1), the Receiver's actions are represented by the mass centres of the partition elements.¹⁰

10. It is easy to show that given his belief, the Receiver has no interest in mixing his responses, so that the Sender discloses m_i or m_{-i} with probability 1 for the states that do not belong to the perpendicular bisector of $\mathbf{a}(m_1) - \mathbf{b}$ and $\mathbf{a}(m_2) - \mathbf{b}$. In particular, any mixed strategy equilibrium outcome is equivalent to a pure strategy equilibrium outcome. See Levy and Razin (2004, Proposition 1) for a generalization of this result.

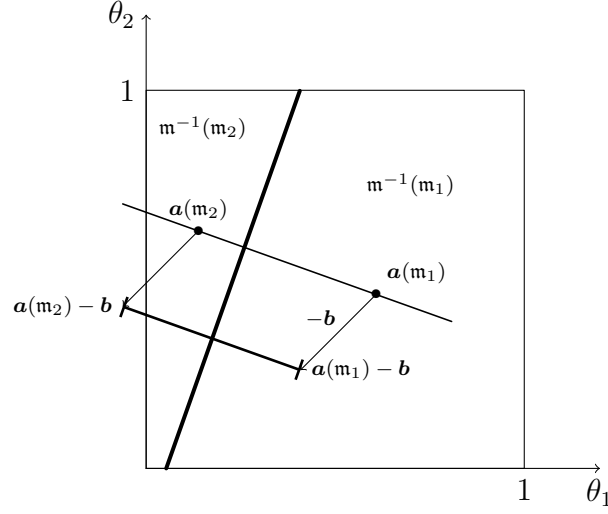


Figure 2.1 – Sender and Receiver's strategies

Third, given equilibrium strategies \mathbf{m} and \mathbf{a} , strategies \mathbf{m}' and \mathbf{a}' respectively given by $\mathbf{m}'(\boldsymbol{\theta}) = m_1$ if $\mathbf{m}(\boldsymbol{\theta}) = m_2$ and $\mathbf{m}'(\boldsymbol{\theta}) = m_2$ if $\mathbf{m}(\boldsymbol{\theta}) = m_1$, and $\mathbf{a}'(m_i) = \mathbf{a}(m_{-i})$, $i \in \{1, 2\}$, also are in equilibrium. Profiles of strategies (\mathbf{m}, \mathbf{a}) and $(\mathbf{m}', \mathbf{a}')$ are symmetric relative to the messages. We say that they are *message-symmetric*.

Fourth, let us consider potential *information symmetries*, i.e. relative to the way information is disclosed and processed. Let us consider orthogonal symmetries $s_{\boldsymbol{\tau}}$ with respect to an affine line of \mathbb{R}^2 with direction vector $\boldsymbol{\tau} = (\tau_1, \tau_2) \in \mathbb{R}^2$. Given a set $X \subseteq \mathbb{R}^2$ (which could be a (set of) state(s), action(s), or bias(es)), we say that X is *$\boldsymbol{\tau}$ -symmetric* if $s_{\boldsymbol{\tau}}(X) = X$.

Note that the uniform distribution of states over $\Theta = [0, 1]^2$ is *$\boldsymbol{\tau}$ -symmetric* for any $\boldsymbol{\tau} \in \{(-1, 1), (0, 1), (1, -1), (1, 0)\}$. Hence all the parameters of the game $\Gamma_{\mathbf{b}}$ are *$\boldsymbol{\tau}$ -symmetric* if the bias \mathbf{b} is *$\boldsymbol{\tau}$ -symmetric*, i.e. if it is collinear to $\boldsymbol{\tau}$ for some $\boldsymbol{\tau} \in \{(-1, 1), (0, 1), (1, -1), (1, 0)\}$. In that case, we say that *the game is $\boldsymbol{\tau}$ -symmetric*, and that it is symmetric if it is symmetric for some $\boldsymbol{\tau}$.

We extend the notion of symmetry to strategies, and say that Sender's strategy $\mathbf{m} : \boldsymbol{\theta} \mapsto \mathbf{m}(\boldsymbol{\theta})$ is *$\boldsymbol{\tau}$ -symmetric* if the Sender discloses her information *$\boldsymbol{\tau}$ -symmetrically*, i.e. if $s_{\boldsymbol{\tau}}(\mathbf{m}^{-1}(m_1)) = \mathbf{m}^{-1}(m_2)$. Similarly, Receiver's strategy \mathbf{a} is *$\boldsymbol{\tau}$ -symmetric* if the Receiver acts *$\boldsymbol{\tau}$ -symmetrically*, i.e. if $s_{\boldsymbol{\tau}}(\mathbf{a}(m_1)) = \mathbf{a}(m_2)$. If players' strategies are *$\boldsymbol{\tau}$ -symmetric* and in equilibrium, then so is the equilibrium. Given a *$\boldsymbol{\tau}$ -symmetric game*, we say that *an equilibrium is symmetric* if it is *$\boldsymbol{\tau}$ -symmetric*; otherwise it is *asymmetric*.

In the next two sub-sections we give examples of equilibrium strategies in which symmetry plays a crucial role. These examples are the basis for addressing the issue of the existence of an influential equilibrium in $\Gamma_{\mathbf{b}}$, for any $\mathbf{b} \in \mathbb{R}^2$, in Section 2.4.

2.3.2 Symmetric equilibria and their deviations

Chakraborty and Harbaugh (2007) show that regardless of the extent of the conflict of interest, if the conflict and prior are both $(1, 1)$ -symmetric, then comparison of the state components is influential. Basically the $(1, 1)$ -symmetry implies that the $(1, 1)$ -symmetric comparison induces $(1, 1)$ -symmetric posterior beliefs, and a consequently $(1, 1)$ -symmetric action strategy \mathbf{a} . Reciprocally, a $(1, 1)$ -symmetric conflict implies that the Sender's best response to a Receiver's $(1, 1)$ -symmetric strategy is a $(1, 1)$ -symmetric disclosure rule, which necessarily compares θ_1 and θ_2 .

The conditions of existence of such an equilibrium might not be restricted to the $(1, 1)$ -direction, but could be extended to any $\boldsymbol{\tau}$ for which the symmetric conditions are fulfilled. Specifically, given the quadratic preferences of the Sender, it needs the bias \mathbf{b} to be $\boldsymbol{\tau}$ -symmetric for some $\boldsymbol{\tau} \in \{(1, 1), (0, 1), (1, 0), (1, -1)\}$ of the $\boldsymbol{\tau}$ -symmetric state space. We obtain the following four *symmetric equilibria*.

Example 2.1. *Conditional on the bias $\mathbf{b} \in \mathbb{R}^2$, the following strategies are equilibrium strategies:*

- *if the bias $\mathbf{b} = (b, b)$, $b \in \mathbb{R}$ is $(1, 1)$ -symmetric, comparative $(1, 1)$ -symmetric strategies are*

$$\mathbf{m}_C(\boldsymbol{\theta}) = \begin{cases} m_1 & \text{if } \theta_1 \geq \theta_2, \\ m_2 & \text{if } \theta_1 < \theta_2, \end{cases} \quad \mathbf{a}_C(m) = \begin{cases} (\frac{2}{3}, \frac{1}{3}) & \text{if } m = m_1 \\ (\frac{2}{3}, \frac{1}{3}) & \text{if } m = m_2; \end{cases}$$

- *if the bias $\mathbf{b} = (b, -b)$, $b \in \mathbb{R}$, is $(1, -1)$ -symmetric, aggregative $(1, -1)$ -symmetric strategies are*

$$\mathbf{m}_A(\boldsymbol{\theta}) = \begin{cases} m_1 & \text{if } \theta_1 + \theta_2 \geq 1, \\ m_2 & \text{if } \theta_1 + \theta_2 < 1, \end{cases} \quad \mathbf{a}_A(m) = \begin{cases} (\frac{2}{3}, \frac{2}{3}) & \text{if } m = m_1 \\ (\frac{1}{3}, \frac{1}{3}) & \text{if } m = m_2; \end{cases}$$

- *if the bias $\mathbf{b} = (0, b)$ (resp. $\mathbf{b} = (b, 0)$), $b \in \mathbb{R}$, is $(0, 1)$ -symmetric (resp. $(1, 0)$ -symmetric), half-babbling $(1, 0)$ -symmetric (resp. $(0, 1)$ -symmetric) strategies are*

$$\mathbf{m}_H(\boldsymbol{\theta}) = \begin{cases} m_1, & \text{if } \theta_1 \geq \frac{1}{2} \text{ (resp. } \theta_2 \leq \frac{1}{2}), \\ m_2, & \text{if } \theta_i < \frac{1}{2} \text{ (resp. } \theta_2 > \frac{1}{2}), \end{cases} \quad \mathbf{a}_H(m) = \begin{cases} (\frac{3}{4}, \frac{1}{2}) \text{ (resp. } (\frac{1}{2}, \frac{1}{4})) & \text{if } m = m_1 \\ (\frac{1}{4}, \frac{1}{2}) \text{ (resp. } (\frac{1}{2}, \frac{3}{4})) & \text{if } m = m_2. \end{cases}$$

Chakraborty and Harbaugh (2007) obtain the equilibrium strategies $(\mathbf{m}_C, \mathbf{a}_C)$ from the symmetry of the state space, and the symmetry and super-modularity of the utility functions. By symmetry, all strategies exhibited in Example 2.1 could easily be derived under these more general conditions. However, Example 2.1 helps to highlight an important relationship between the different strategies. Specifically, if $\mathbf{b} = (0, 0)$, then strategies \mathbf{m}_C , \mathbf{m}_A and \mathbf{m}_H simultaneously occur as a potential Sender' equilibrium strategy. This highlights a new multiplicity of equilibria relative to the one-dimensional framework, which arises *across the dimensions*.¹¹ Strategies exhibited in Example 2.1 do not depend on the

11. In the one-dimensional framework, different sets of messages are associated with different equilibria. This multiplicity of equilibria also occurs in the framework considered here. Example 2.2 specifies the one-dimensional aspect of the strategies presented in Example 2.1.

extent $\|\mathbf{b}\| = b$ of the bias \mathbf{b} , but rather on its direction. When the direction is well defined, *i.e.* when $b \neq 0$, Example 2.1 points to a unique equilibrium strategy of $\Gamma_{\mathbf{b}}$. In particular, conflict might rule out the above-mentioned conflict-free multiplicity. In the next section, we show that this is not the case. When b continuously increases from 0 to $+\infty$, multiple strategies move continuously and remain in equilibrium.

Now, let us highlight that the strategies in Example 2.1 derive from a one-dimensional game of information transmission, relative to a specific dimension. For instance, let us decompose the Sender's utility in the $(1, -1)$ -directed "dimension of comparison" and its orthogonal $(1, 1)$ -directed dimension:

$$- \|\mathbf{a}(m_i) - (\boldsymbol{\theta} + \mathbf{b})\|^2 = - \left(((1, 1) \cdot (\mathbf{a}(m_i) - (\boldsymbol{\theta} + \mathbf{b})))^2 + ((1, -1) \cdot (\mathbf{a}(m_i) - (\boldsymbol{\theta} + \mathbf{b})))^2 \right), \quad (2.3)$$

where \cdot denotes the (Euclidean) scalar product. Note that for any $(1, 1)$ -symmetric action $\mathbf{a} = \mathbf{a}_C$, we have $(1, 1) \cdot \mathbf{a}_C(m_1) = (1, 1) \cdot \mathbf{a}_C(m_2)$, so that the first term of (2.3) is *independent* of $i \in \{1, 2\}$. This shows that the $(1, -1)$ -dimension fully determines the Sender's utility, and so it fully determines her strategy. Conversely, let \mathbf{m} be a Sender's strategy which is fully determined by the $(1, -1)$ -dimension. Then we necessarily have $(1, 1) \cdot \mathbf{a}(m_1) = (1, 1) \cdot \mathbf{a}(m_2)$. Let $\mathbf{a}(m_i) = (\mathbb{E}[\theta_1|m_i], \mathbb{E}[\theta_2|m_i])$ denotes the Receiver's expectation given his observation of the message m_i associated with \mathbf{m} . From $\theta_1 - \theta_2 = (1 - \theta_2) - (1 - \theta_1)$ and the $(1, -1)$ -symmetry of $\Theta = [0, 1]^2$, we have $\mathbf{m}((\theta_1, \theta_2)) = m_i$ *iff* $\mathbf{m}((1 - \theta_2, 1 - \theta_1)) = m_i$. This implies $\mathbb{E}[\theta_1|m_i] = \mathbb{E}[1 - \theta_2|m_i]$, *i.e.* $\mathbb{E}[\theta_1 + \theta_2|m_i] = 1$, regardless of i .

Therefore, the revealed information is restricted to the $(1, -1)$ -dimension *iff* actions are aligned with the $(1, -1)$ -dimension. This means that players might choose to play a one-dimensional game in the $(1, -1)$ -dimension in order to reach agreement.¹² Then according to the one-dimensional framework of Crawford and Sobel (1982), players might reach agreement if the bias $b_1 - b_2 = \Delta b$ in this dimension is limited. In that case, an agreement is obtained regardless of the bias in the uninformative $(1, 1)$ -dimension, *i.e.* regardless of $b_1 + b_2$.

Example 2.2. Consider the following Sender's strategy, denoted $\mathbf{m}_{C(c)}$, with $c \in (-1, 1)$:¹³

$$\mathbf{m}_{C(c)}(\boldsymbol{\theta}) = \begin{cases} m_1, & \text{if } \theta_1 \geq \theta_2 + c \\ m_2, & \text{if } \theta_1 < \theta_2 + c. \end{cases}$$

Let the bias be given by $\mathbf{b} = (b_1, b_2) \in \mathbb{R}^2$, and set $\Delta b = |b_2 - b_1|$. Then there exists $c \in (-1, 1)$ such that $\mathbf{m}_{C(c)}$ is an equilibrium strategy of $\Gamma_{\mathbf{b}}$ *iff* $\Delta b < \frac{1}{2}$.

12. A by-product is that the binary nature of our setting in the present case is very restrictive. According to the one dimensional framework, if the bias is sufficiently limited in the dimension of agreement, more than two messages might be used in equilibrium. However, investigating the corresponding model is beyond the scope of the present study.

13. Notice that when $|c| \rightarrow 1$, strategy $\mathbf{m}_{C(c)}$ tends to babble: either m_1 or m_2 would always be revealed, whatever $\boldsymbol{\theta}$.

Figure 2.2 illustrates the Sender's corresponding strategies $\mathbf{m}_{C(c)}$.

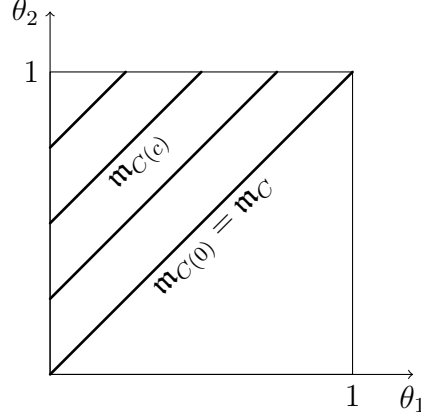


Figure 2.2 – Deviations of the comparative $(1, 1)$ -symmetric disclosure rule \mathbf{m}_C , due to bias $\mathbf{b} = (b_1, b_1 + \Delta b)$

Note that each symmetric equilibrium of Example 2.1 results from a single symmetry of the distribution of states. In contrast, the deviated equilibria of Example 2.2 are the result of the two $(1, 1)$ and $(1, -1)$ -symmetries of $[0, 1]^2$, and of their orthogonality. Similar deviated equilibrium strategies hold with regard to the aggregative \mathbf{m}_A and half-babbling \mathbf{m}_H strategies. Let $\mathbf{m}_{A(c)}$ and $\mathbf{m}_{H(c)}$ denote the corresponding strategies.

2.3.3 Asymmetric equilibria

In previous examples, τ -asymmetric equilibria of a τ -asymmetric game are interpreted as deviations of a τ -symmetric equilibria of a τ -symmetric game. They are obtained from a limited τ -asymmetry of the corresponding τ -symmetric bias. The four symmetries of $\Theta = [0, 1]^2$ condition the existence of these equilibria. The next example exhibits equilibrium strategies not based on these symmetries. It is depicted in Figure 2.3.

Example 2.3. *Players' strategies*

$$\mathbf{m}(\boldsymbol{\theta}) = \begin{cases} m_1 & \text{if } \theta_1 \geq \frac{1}{2}\theta_2, \\ m_2 & \text{if } \theta_1 < \frac{1}{2}\theta_2, \end{cases} \quad \mathbf{a}(m) = \begin{cases} (\frac{11}{18}, \frac{4}{9}) & \text{if } m = m_1, \\ (\frac{1}{6}, \frac{2}{3}) & \text{if } m = m_2, \end{cases}$$

are equilibrium strategies of $\Gamma_{\mathbf{b}(b)}$, where $\mathbf{b}(b) = (\frac{4}{45} + \frac{1}{2}b, \frac{-2}{45} + b)$, $b \in \mathbb{R}$.

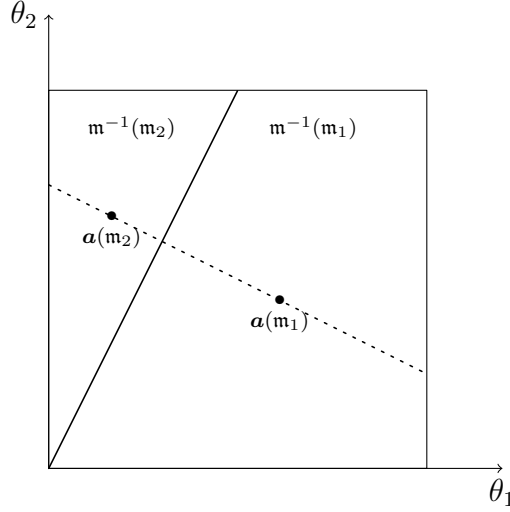


Figure 2.3 – Asymmetric equilibrium strategies

In Example 2.3 the revealed information concerns $\theta_1 - \frac{1}{2}\theta_2$ and thus it is revealed in the $(1, -\frac{1}{2})$ -dimension. The corresponding induced actions align with that dimension with $(\frac{1}{2}, 1) \cdot \mathbf{a}(m_1) = (\frac{1}{2}, 1) \cdot \mathbf{a}(m_2)$, i.e. $\frac{1}{2}\mathbb{E}[\theta_1|m_1] + \mathbb{E}[\theta_2|m_1] = \frac{1}{2}\mathbb{E}[\theta_1|m_2] + \mathbb{E}[\theta_2|m_2]$. Hence the *specific* information revealed about $\theta_1 - \frac{1}{2}\theta_2$ (whether it is positive or not) is not informative about $\frac{1}{2}\theta_1 + \theta_2$, i.e. in the dimension which is orthogonal to the informative dimension. Players exploit a *sufficiently weak dependence*¹⁴ between two orthogonal dimensions of the distribution of states in order to reach agreement. In particular, the Sender is uninformative in the $(\frac{1}{2}, 1)$ -dimension, so that her bias is irrelevant in that dimension. Then any $(\frac{1}{2}b, b)$, $b \in \mathbb{R}$, could be included added to the Sender's bias, without it changing the equilibrium conditions. In contrast, in the informative $(1, -\frac{1}{2})$ -dimension, information is revealed asymmetrically. This imposes a non-zero (and limited) bias. In that dimension the bias must match the asymmetry of the disclosed information. We find $(\frac{4}{45}, \frac{-2}{45})$ for the adequate $(1, -\frac{1}{2})$ -dimensional component of \mathbf{b} .

Note that if $b = \frac{4}{45}$, then $\mathbf{b} = (\frac{2}{9}, \frac{2}{9})$ is $(1, 1)$ -symmetric. So the game is $(1, 1)$ -symmetric. Therefore, strategies in Example 2.3 are *endogenously asymmetric*. Similarly if $b = \frac{-8}{45}$ (resp. $b = \frac{2}{45}$, $b = \frac{-4}{135}$), then $\mathbf{b} = (0, -\frac{2}{9})$ (resp. $\mathbf{b} = (\frac{1}{9}, 0)$, $\mathbf{b} = (\frac{2}{27}, \frac{-2}{27})$) and the strategies are asymmetric strategies of a $(0, 1)$ -symmetric (resp. $(1, 0)$ -symmetric, $(1, -1)$ -symmetric) game. In the context of organizations, this result means that despite *a priori* symmetric dependencies of the agents' payoffs with respect to a specific dimension, asymmetric treatments relative to that dimension are possible in equilibrium.

Strategies in Example 2.3 are no exception. Figure 2.4 illustrates a continuum of $(1, 1)$ -asymmetric Sender's strategies \mathbf{m} of the $(1, 1)$ -symmetric game $\Gamma_{\mathbf{b}}$, with $\mathbf{b} = (b, b)$. In

14. Note that $\frac{1}{2}\theta_1 + \theta_2$ is not, in general, independent of $\theta_1 - \frac{1}{2}\theta_2$. For instance, if the Sender reveals $\theta_1 - \frac{1}{2}\theta_2 = 1$, then the Receiver infers $\theta_1 = 1$ and $\theta_2 = 0$ so that $\frac{1}{2}\theta_1 + \theta_2 = \frac{1}{2}$. If the Sender reveals $\theta_1 - \frac{1}{2}\theta_2 = \frac{-1}{2}$, then the Receiver infers $\theta_1 = 0$ and $\theta_2 = 1$ so that $\frac{1}{2}\theta_1 + \theta_2 = 1 \neq \frac{1}{2}$. This contrasts with the independence between the informative and uninformative *dimensions* of a symmetric equilibrium. Here, the independence is derived from the equilibrium *issues* in these dimensions.

particular conflict does not rule out the multiplicity of conflict-free equilibria referred to in the previous section. As $\mathbf{b} = (b, b)$ increases to infinity, strategies $(\mathbf{m}_{A(c)}, \mathbf{a}_{A(c)})$ and $(\mathbf{m}_{H(c)}, \mathbf{a}_{H(c)})$ are ruled out but the asymmetric strategies depicted in Figure 2.4 are not, and neither are the symmetric strategies $(\mathbf{m}_C, \mathbf{a}_C)$, so that multiple equilibria remain.

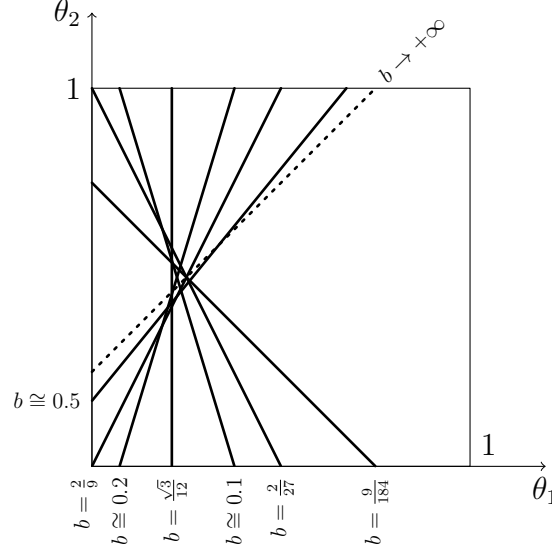


Figure 2.4 – Endogeneous $(1, 1)$ -asymmetric disclosure rules associated to $(1, 1)$ -symmetric bias $\mathbf{b} = (b, b)$

Finally, let us discuss the impact of the extent of conflict on the players' interaction. In a symmetric equilibrium, no information is revealed in the (symmetric) bias dimension. Since players' conflict is restricted to that dimension, the extent of the conflict has no impact on the outcomes. In contrast, as Figure 2.4 shows, the extent of the underlying symmetric conflict of an asymmetric equilibrium impacts the outcomes. In Figure 2.4, the dimension of agreement $(\mathbf{a}(m_1)\mathbf{a}(m_2))$ is aligned with the dimension of the symmetric bias $= (b, b)$ when b is close to 0, and tends to be orthogonal to it as b tends to infinity.

Let us provide some details on the convergence of the equilibrium conditions when the bias tends to infinity, in relation with Levy and Razin's (2004; 2007) important contributions. As argued by Levy and Razin (2007), when $b \rightarrow \infty$, quadratic preferences of the Sender converge to lexicographic preferences. To see this, consider for instance $\mathbf{b} = (b, b) \in \mathbb{R}^2$. Then given $\boldsymbol{\theta} = (\theta_1, \theta_2)$, the Sender prefers $\mathbf{a} = (a_1, a_2)$ to $\mathbf{a}' = (a'_1, a'_2)$ iff $-(a_1 - (\theta_1 + b))^2 - (a_2 - (\theta_2 + b))^2 \geq -(a'_1 - (\theta_1 + b))^2 - (a'_2 - (\theta_2 + b))^2$, i.e. iff

$$(a_1 - a'_1) \left(\theta_1 - \frac{a_1 + a'_1}{2} \right) + b(a_1 + a_2) \geq (a'_2 - a_2) \left(\theta_2 - \frac{a_2 + a'_2}{2} \right) + b(a'_1 + a'_2). \quad (2.4)$$

If b is sufficiently large relative to $a_1 + a_2 - (a'_1 + a'_2)$, so that in (2.4) the difference between the terms $b(a_1 + a_2)$ and $b(a'_1 + a'_2)$ dominates the difference of the other terms, then inequality (2.4) holds iff $a_1 + a_2 \geq a'_1 + a'_2$. In that case the Sender's preferred action is

based on the comparison of $a_1 + a_2$ and $a'_1 + a'_2$. This is a one-dimensional comparison, in the $(1, 1)$ -dimension of \mathbb{R}^2 . The Sender has lexicographic preferences because in that dimension, she always prefers the element with the highest value, and if values are equal, (2.4) might be written $\theta_1 - \frac{a_1 - a_2}{2} \geq \theta_2 - \frac{a'_1 - a'_2}{2}$, in which only the $(1, -1)$ -dimension is involved.

However the Sender's lexicographic preferences only concern actions such that b is sufficiently large relative to $(a_1 + a_2) - (a'_1 + a'_2)$. When b tends to $+\infty$, this holds for almost all actions. Thus when b tends to $+\infty$, Sender's preferences tend to be lexicographic over the full action space. Levy and Razin (2007) specifically examine the game in which Sender has lexicographic preferences, interpreting it as a game with quadratic preference and an "infinite bias".

Now, let us show that the outcomes of such an "infinite game" do not necessarily correspond to the limit, when b tends to $+\infty$, of the outcomes of game Γ_b , $\mathbf{b} = (b, b)$. To do this, let us examine the convergence of the equilibrium conditions of the finite games in which preferences converge to the $(1, 1)$ -oriented lexicographic preferences. When the Sender has $(1, 1)$ -oriented lexicographic preferences, then any information revelation in equilibrium necessarily induces $a_1(m_1) + a_2(m_1) = a_1(m_2) + a_2(m_2)$, since otherwise the message that induces the highest sum is always sent. Now if $a_1(m_1) + a_2(m_1) = a_1(m_2) + a_2(m_2)$, then inequality (2.4) is an equality at $\theta = \frac{a(m_1) + a(m_2)}{2}$. In particular, in equilibrium, the Sender is necessarily indifferent between the induced actions when the state equals their mid-point. However, for instance the family of equilibria depicted in Figure 2.4 is such that when $b \rightarrow \infty$ the limit of the set of the Sender's indifferent states does not contain the mid-point of the limit of the induced actions.¹⁵ Hence when the conflict tends to infinity, the limit of an equilibrium in the game with finite conflict (in which Sender's preferences are quadratic) does not necessarily converge to an equilibrium in the game with infinite conflict (in which Sender's preferences are lexicographic).

For a formal explanation, it should be noted that according to (2.4), $\theta = \frac{a(m_1) + a(m_2)}{2}$ is a Sender's indifferent state *iff* $b(a_1(m_1) + a_2(m_1)) = b(a_1(m_2) + a_2(m_2))$. The induced actions of the family of equilibria depicted in Figure 2.4 is such that $\lim_{b \rightarrow +\infty} (a_1(m_1) + a_2(m_1)) = \lim_{b \rightarrow +\infty} (a_1(m_2) + a_2(m_2))$, so that the dimension of the induced actions tends to align orthogonally to the conflict. This does tend to be similar to the condition associated with the lexicographic preferences case, in which in equilibrium $a_1(m_1) + a_2(m_1) = a_1(m_2) + a_2(m_2)$ is necessary. However $\lim_{b \rightarrow +\infty} (a_1(m_1) + a_2(m_1)) = \lim_{b \rightarrow +\infty} (a_1(m_2) + a_2(m_2))$ does not necessarily imply $\lim_{b \rightarrow +\infty} (b(a_1(m_1) + a_2(m_1))) = \lim_{b \rightarrow +\infty} (b(a_1(m_2) + a_2(m_2)))$.¹⁶ If $\lim_{b \rightarrow +\infty} (b(a_1(m_1) + a_2(m_1))) \neq \lim_{b \rightarrow +\infty} (b(a_1(m_2) + a_2(m_2)))$, then the mid-point of the limit of the induced actions does not correspond to an indifferent state. Therefore, the mid-point of the limit

15. This is easily derived by equation $\theta_2 = \theta_1 + 1/4$ which characterize the set of indifferent states when $b \rightarrow +\infty$. A proof of this characterization is provided in the appendix, in proving Lemma 2.3.

16. Stated in words, two vectors might tend to be orthogonal, but their scalar product does not necessarily converge to 0. For instance, when $b \rightarrow \infty$, vectors $(0, b)$ and $(1, 1/b)$ tend to be orthogonal, in the $(0, 1)$ and $(1, 0)$ respective directions, but for any $b \in \mathbb{R}$, we have $(0, b) \cdot (1, 1/b) = 1$.

of the induced actions does not necessarily correspond to an indifferent state of the limit strategies.¹⁷

2.4 Influence in the context of any conflict

Examples 2.1, 2.2 and 2.3 describe equilibrium strategies associated with some particular types of biases. In this section, we extend and exploit these strategies to derive the existence of equilibrium strategies for any bias.

Consider the correspondence

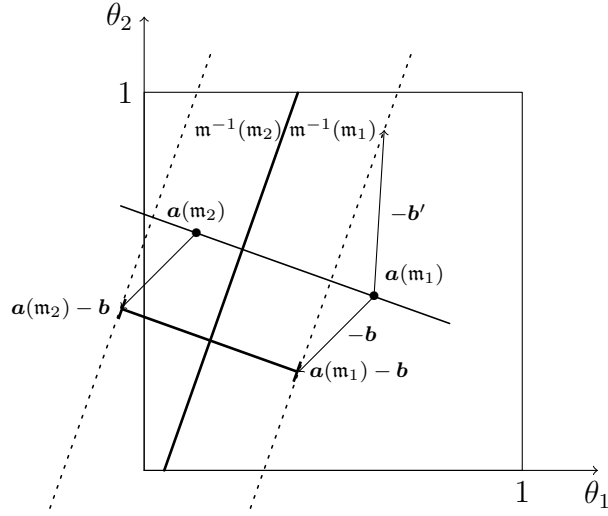
$$\mathcal{E} : \mathbf{b} \rightrightarrows (\mathbf{m}, \mathbf{a}),$$

which associates bias \mathbf{b} with the set of strategies (\mathbf{m}, \mathbf{a}) that occur as an influential equilibrium strategy in the game $\Gamma_{\mathbf{b}}$. We seek to show that for any $\mathbf{b} \in \mathbb{R}^2$, $\mathcal{E}(\mathbf{b})$ is not empty. To do this, we consider the inverse correspondence, denoted \mathcal{E}^{-1} , which associates strategies (\mathbf{a}, \mathbf{m}) with the set of biases $\mathbf{b} \in \mathbb{R}^2$ for which $(\mathbf{m}, \mathbf{a}) \in \mathcal{E}(\mathbf{b})$. Clearly we have $\mathcal{E}(\mathbf{b}) \neq \emptyset$ iff $\mathbf{b} \in \mathcal{E}^{-1}((\mathbf{m}, \mathbf{a}))$ for some (\mathbf{m}, \mathbf{a}) . Hence it is sufficient to provide many strategies (\mathbf{m}, \mathbf{a}) such that the sets of biases $\mathcal{E}^{-1}((\mathbf{m}, \mathbf{a}))$ cover \mathbb{R}^2 . First, we characterize the set $\mathcal{E}^{-1}((\mathbf{m}, \mathbf{a}))$, given a profile (\mathbf{m}, \mathbf{a}) (Lemma 2.1). Then we extend Example 2.3 continuously to cover a large number of directions (Lemmas 2.2 and 2.3), and then to cover all directions (Lemma 2.4). We then show that the strategies exhibited yield the result (Proposition 2.1).

Lemma 2.1. *Let (\mathbf{m}, \mathbf{a}) be such that $(\mathbf{m}, \mathbf{a}) \in \mathcal{E}(\mathbf{b})$ for some $\mathbf{b} \in \mathbb{R}^2$. Then for any $\mathbf{b}' \in \mathbb{R}^2$, $(\mathbf{m}, \mathbf{a}) \in \mathcal{E}(\mathbf{b}')$ iff $\mathbf{b}' \cdot (\mathbf{a}(m_1) - \mathbf{a}(m_2)) = \mathbf{b} \cdot (\mathbf{a}(m_1) - \mathbf{a}(m_2))$.*

From Lemma 2.1, the set of biases \mathbf{b}' for which strategies $(\mathbf{m}, \mathbf{a}) \in \mathcal{E}(\mathbf{b})$ are equilibrium strategies of $\Gamma_{\mathbf{b}'}$ is given by those \mathbf{b}' that project orthogonally as does \mathbf{b} onto the line $(\mathbf{a}(m_1)\mathbf{a}(m_2))$. Geometrically speaking (see Figure 2.5), a Sender's strategy \mathbf{m} is characterized by the association of a message m_i with each side of the perpendicular bisector of $\mathbf{a}(m_1) - \mathbf{b}$ and $\mathbf{a}(m_2) - \mathbf{b}$. Therefore $\mathcal{E}^{-1}(\mathbf{m}, \mathbf{a})$ corresponds to bias \mathbf{b}' such that $\mathbf{a}(m_1) - \mathbf{b}'$ and $\mathbf{a}(m_2) - \mathbf{b}'$ induce the same perpendicular bisector.

17. In particular, Levy and Razin (2004, Corollary 1(iii)), and the statement of Problem A) are not correct when a family of asymmetric equilibria such as the one depicted in Figure 2.4 is considered.

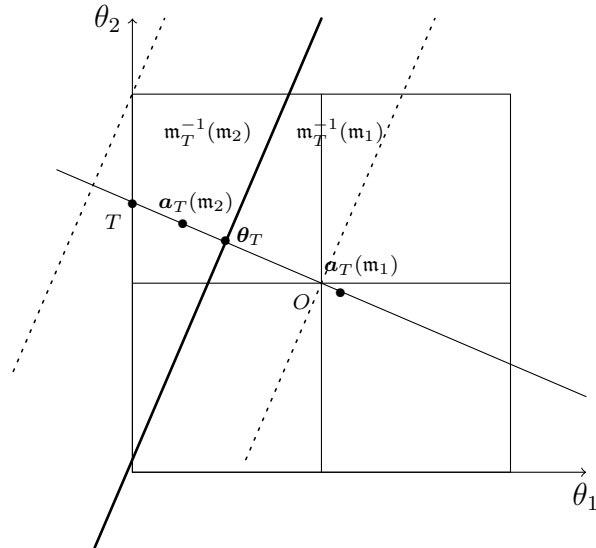

 Figure 2.5 – $\mathbf{b}' \in \mathcal{E}^{-1}((\mathbf{m}, \mathbf{a}))$

The following lemma extends Example 2.3 to cover a large number of directions. It is depicted in Figure 2.6.

Lemma 2.2. *For any line (OT) , with $O = \mathbb{E}[\Theta] = (\frac{1}{2}, \frac{1}{2})$ and $T \in \{0\} \times (\frac{1}{2}, 1)$, there exists a strategy equilibrium profile $(\mathbf{m}_T, \mathbf{a}_T)$ of the game $\Gamma_{\mathbf{b}_T}$, with $\mathbf{a}_T(m_1) \in (OT)$, $\mathbf{a}_T(m_2) \in (OT)$, and*

$$\mathbf{b}_T = \frac{\mathbf{a}_T(m_1) + \mathbf{a}_T(m_2)}{2} - \boldsymbol{\theta}_T, \quad (2.5)$$

for some $\boldsymbol{\theta}_T \in (OT)$.


 Figure 2.6 – Existence of an equilibrium strategy given T

We derive Lemma 2.2 by showing that the equilibrium conditions of Example 2.3 extend in all directions (OT) , where $T \in \{0\} \times (\frac{1}{2}, 1)$. In particular, for all these directions, $\Theta = [0, 1]^2$ always allows specific information to be disclosed in the (OT) -dimension so that no information is revealed in the corresponding orthogonal dimension.¹⁸

Lemma 2.3 provides us with the limit of the equilibrium strategies $(\mathbf{m}_T, \mathbf{a}_T)$ exhibited in Lemma 2.2, when T moves continuously to points $(0, \frac{1}{2})$ or $(0, 1)$. It allows us to extend the strategies $(\mathbf{m}_T, \mathbf{a}_T)$ *continuously*¹⁹ as influential strategies in all directions of the plane. Strategies $(\mathbf{m}_{C(c)}, \mathbf{a}_{C(c)})$ are defined in Example 2.2 as deviated comparative symmetric strategies. Strategies $(\mathbf{m}_{H(c)}, \mathbf{a}_{H(c)})$ are defined similarly with regard to symmetric half-babbling strategies.

Lemma 2.3. *Strategies $(\mathbf{m}_T, \mathbf{a}_T)$, $T \in \{0\} \times (\frac{1}{2}, 1)$ extend continuously at $T = (0, \frac{1}{2})$ and $T' = (0, 1)$ to asymmetric strategies $(\mathbf{m}_{H(c)}, \mathbf{a}_{H(c)})$ and $(\mathbf{m}_{C(c')}, \mathbf{a}_{C(c')})$ respectively, for some $c, c' \in (0, 1)$.*

Lemma 2.4 allows us to further extend the set of equilibrium strategies $(\mathbf{m}_T, \mathbf{a}_T)$ to cover all directions in the plane by considering symmetric transformations of $(\mathbf{m}_T, \mathbf{a}_T)$ with respect to the symmetries of Θ . Let ρ be one of the four axial symmetries of Θ , or a composition of them. From $\rho(\Theta) = \Theta$, strategy

$$\rho(\mathbf{m}) : \theta \mapsto \rho(\mathbf{m})(\theta) = m_i \text{ iff } \theta \in \rho^{-1}(\mathbf{m}^{-1}(m_i)),$$

associated with a given Sender's strategy \mathbf{m} , is well defined. Similarly, given a Receiver's strategy \mathbf{a} , we define $\rho(\mathbf{a})$ by $\rho(\mathbf{a})(m_i) = \rho(\mathbf{a}(m_i))$, $i \in \{1, 2\}$. Since ρ does not change measure and orthogonality, and since players' preferences are characterized as Euclidean distances, equilibrium conditions are invariant if all the parameters are transformed through ρ . Equilibrium strategies of $\Gamma_{\mathbf{b}}$ are transformed via ρ to equilibrium strategies of the transformed game $\Gamma_{\rho(\mathbf{b})}$. Since ρ^{-1} is also a symmetry of Θ , we obtain the equivalence stated in Lemma 2.4.

Lemma 2.4. *Let ρ be one of the axial symmetries of Θ , or a composition of them. Let $\mathbf{b} = (b_1, b_2) \in \mathbb{R}^2$, and let \mathbf{m} (resp. $\rho(\mathbf{m})$) and \mathbf{a} (resp. $\rho(\mathbf{a})$) be any strategies of the game $\Gamma_{\mathbf{b}}$ (resp. $\Gamma_{\rho(\mathbf{b})}$). Then strategies (\mathbf{m}, \mathbf{a}) are equilibrium strategies of $\Gamma_{\mathbf{b}}$ iff $(\rho(\mathbf{m}), \rho(\mathbf{a}))$ are equilibrium strategies of $\Gamma_{\rho(\mathbf{b})}$.*

For example, Figure 2.7 represents equilibrium strategies $\rho(\mathbf{m})$ and $\rho(\mathbf{a})$ associated with bias $\rho(\mathbf{b}) = (-b, b)$ derived from equilibrium strategies \mathbf{m} and \mathbf{a} depicted in Figure 2.3, associated with bias $\mathbf{b} = (b, b)$. The strategies are derived from the composition of the $(0, 1)$ and $(1, 1)$ -symmetries of Θ , *i.e.* the rotation $\rho_{\frac{\pi}{2}}$ of angle $\frac{\pi}{2}$ (in the trigonometric

18. In what follows, we show that this extends in all directions. Therefore the uniform distribution over $\Theta = [0, 1]^2$ renders an *overall weak dependence* of the states relative to orthogonal dimension pairs. Reciprocally, the construction might provide a way of measuring the independence of the θ s over several dimensions, *i.e.* to provide an overall measure of the symmetry of Θ . There is a substantial literature on this question, including Toth (2015).

19. The continuity of the strategies is with regard to actions $\mathbf{a}_T(m_i)$ and set of states $\mathbf{m}_T^{-1}(m_i)$, $i \in \{1, 2\}$.

sense) and centre $\mathbb{E}[\Theta] = (\frac{1}{2}, \frac{1}{2})$. Strategies $(\mathbf{m}_{A(c)}, \mathbf{a}_{A(c)})$ might be similarly derived from $(\mathbf{m}_{C(c)}, \mathbf{a}_{C(c)})$, and one of the two half-babbling strategy profile $(\mathbf{m}_{H(c)}, \mathbf{a}_{H(c)})$ might be similarly derived from the other.

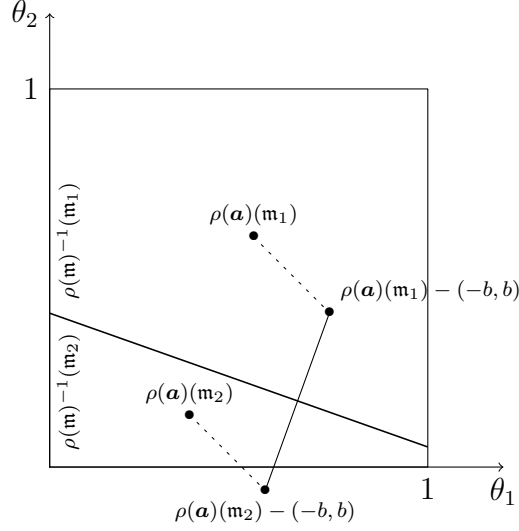


Figure 2.7 – Equilibrium strategies associated with the bias $\mathbf{b} = (-b, b)$

We can now derive the existence of an influential equilibrium for any bias as follows. First, if T spans the full border of Θ , strategies $(\mathbf{m}_T, \mathbf{a}_T)$ exhibited in Lemma 2.2 extend continuously in all directions as influential asymmetric strategies (Lemma 2.3 and 2.4). This provides us with an equilibrium strategy for all biases \mathbf{b}_T given by (2.5). Since strategies $(\mathbf{m}_T, \mathbf{a}_T)$ are asymmetric, the extent of these biases is necessarily bounded from below by some strictly positive fixed number. Second, Lemma 2.1 allows the existence of equilibrium strategies to be extended to all biases \mathbf{b} which are orthogonally projected as \mathbf{b}_T on the (OT) s. Then we obtain equilibrium strategies for biases the extent of which is bounded from below in one of the (OT) s-dimensions and may be arbitrary large in the orthogonal dimension. Since the (OT) s cover all directions, we obtain equilibrium strategies for all biases which are bounded from below in at least one of their components. Finally, for the remaining biases \mathbf{b} whose extent is small in both of their components, the symmetric strategies and their deviations yield the result.

Proposition 2.1. *For any $\mathbf{b} \in \mathbb{R}^2$, there exists a profile of strategies (\mathbf{m}, \mathbf{a}) that defines an influential equilibrium of the game $\Gamma_{\mathbf{b}}$.*

Therefore, whatever the conflict of interest between the Sender and the Receiver, there are grounds for an agreement. Given our assumptions, any conflict of interest in organizations could influence decisions and actions.

2.5 Characterization of the equilibrium strategies

In Section 2.4, we associated a set of biases to specific equilibrium strategies, and showed that the set of strategies reached the entire bias space. Although we did not determine the set of pure strategies equilibria for a given game $\Gamma_{\mathbf{b}}$, we did exhibit at least one. In this section, we derive the necessary part of the equilibrium conditions by showing that the strategies exhibited are the only ones that are in equilibrium.

Let $\mathbf{b} \in \mathbb{R}^2$, and let (\mathbf{m}, \mathbf{a}) be equilibrium strategies of $\Gamma_{\mathbf{b}}$. The law of iterated expectations gives $\mathbb{E}[\Theta] = |\mathbf{m}^{-1}(m_1)|\mathbf{a}(m_1) + |\mathbf{m}^{-1}(m_2)|\mathbf{a}(m_2)$, so that the line $(\mathbf{a}(m_1)\mathbf{a}(m_2))$ necessarily passes through $O = \mathbb{E}[\Theta]$. In particular, we necessarily have

$$(\mathbf{a}(m_1)\mathbf{a}(m_2)) = (OT) \text{ for some } T \text{ on the border of } \Theta. \quad (2.6)$$

Given such a T , the set of the Sender's indifferent state \mathcal{L} is the perpendicular bisector of $\mathbf{a}(m_1) - \mathbf{b}$ and $\mathbf{a}(m_2) - \mathbf{b}$, and thus, it is necessarily one of the lines perpendicular to (OT) . The following lemma characterizes lines \mathcal{L}_T , derived from strategies $(\mathbf{m}_T, \mathbf{a}_T)$, as the only perpendicular with this property.

Lemma 2.5. *For any $T \in \{0\} \times (0, \frac{1}{2})$, the perpendicular bisector \mathcal{L}_T of $\mathbf{a}_T(m_1) - \mathbf{b}_T$ and $\mathbf{a}_T(m_2) - \mathbf{b}_T$ is the only line \mathcal{L} that partitions Θ into two sets Θ_1 and Θ_2 , with $|\Theta_i| > 0$, such that the line going through $\mathbb{E}[\Theta_1]$ and $\mathbb{E}[\Theta_2]$ is perpendicular to \mathcal{L} .*

By symmetry and according to the equivalence stated in Lemma 2.4, Lemma 2.5 extends to all T s such that (OT) is not the horizontal, vertical nor any diagonal axes of Θ . Therefore, according to (2.6), out of these cases, strategies $(\mathbf{m}_T, \mathbf{a}_T)$ cover all possible equilibrium strategies (\mathbf{m}, \mathbf{a}) of a given game $\Gamma_{\mathbf{b}}$. In the case in which $(\mathbf{a}(m_1)\mathbf{a}(m_2))$ is the horizontal, vertical or one of the diagonal axes of Θ , strategies $(\mathbf{m}_{C(c)}, \mathbf{a}_{C(c)})$ and $(\mathbf{m}_{H(c)}, \mathbf{a}_{H(c)})$, $c \in (-1, 1)$ and their transformations through one of the four symmetries of Θ cover all lines perpendicular to $(\mathbf{a}(m_1)\mathbf{a}(m_2))$. Hence, it necessarily includes any line that defines a Sender's indifferent set of states.

Proposition 2.2. *Given $\mathbf{b} \in \mathbb{R}^2$, a profile of equilibrium strategies of $\Gamma_{\mathbf{b}}$ is either*

- *an asymmetric profile of strategies $(\mathbf{m}_T, \mathbf{a}_T)$, for some $T \in \{0\} \times (0, \frac{1}{2})$ (as defined in Lemma 2.2), or*
- *a symmetric profile of strategies $(\mathbf{m}_{C(c)}, \mathbf{a}_{C(c)})$, or $(\mathbf{m}_{H(c)}, \mathbf{a}_{H(c)})$, for some $c \in (-1, 1)$ (as defined in Example 2.2 and hereafter), or*
- *a transformation of one of these profiles of strategies by a composition of the four axial symmetries of $\Theta = [0, 1]^2$.*

Next we provide a geometric representation of influential strategies (\mathbf{m}, \mathbf{a}) of a given game $\Gamma_{\mathbf{b}}$, $\mathbf{b} \in \mathbb{R}^2$. According to Lemma 2.1, $(\mathbf{m}, \mathbf{a}) \in \mathcal{E}(\mathbf{b})$ is conditioned by the projection \mathbf{b}^π of \mathbf{b} onto the support $(\mathbf{a}(m_1)\mathbf{a}(m_2))$ of the induced actions (see Figure 2.5). For instance, $(\mathbf{m}_T, \mathbf{a}_T)$ is an equilibrium profile of strategies of $\Gamma_{\mathbf{b}}$ iff $-\mathbf{b}$ projects as $-\mathbf{b}_T$ on $(\mathbf{a}_T(m_1)\mathbf{a}_T(m_2))$. Figures 2.8, 2.9 and 2.10 depict the inverse map of the corresponding projections associated with specific continua of equilibrium strategies (\mathbf{m}, \mathbf{a}) .

The construction is as follows. Given strategies (\mathbf{m}, \mathbf{a}) which are equilibrium strategies of some game, we first represent the projected bias $-\mathbf{b}^\pi \in \mathcal{E}^{-1}((\mathbf{m}, \mathbf{a}))$, which is aligned with $(\mathbf{a}(m_1)\mathbf{a}(m_2))$. Then for any $\mathbf{b} \in \mathbb{R}^2$, we have $\mathbf{b} \in \mathcal{E}^1((\mathbf{m}, \mathbf{a}))$ iff $-\mathbf{b}$ projects as $-\mathbf{b}^\pi$ on $(\mathbf{a}(m_1)\mathbf{a}(m_2))$, i.e. on the $-\mathbf{b}^\pi$ -directed line.

For instance, in Figure 2.8, the bold $(1, -1)$ -directed segment corresponds to projected bias $-\mathbf{b}^\pi$ for $(\mathbf{m}, \mathbf{a}) = (\mathbf{m}_{C(c)}, \mathbf{a}_{C(c)})$, for all $c \in (-1, 1)$. Then bias $\mathbf{b} = (b_1, b_2)$ for which $(\mathbf{m}_{C(c)}, \mathbf{a}_{C(c)}) \in \mathcal{E}(\mathbf{b})$ for some $c \in (-1, 1)$ corresponds to the points $(-b_1, -b_2)$ that project orthogonally at $-\mathbf{b}^\pi$, onto the line starting from the origin, i.e. in the corresponding $(1, -1)$ -directed segment. A bias \mathbf{b} projects orthogonally onto any segment iff the corresponding strategies are equilibrium strategies of $\Gamma_{\mathbf{b}}$.

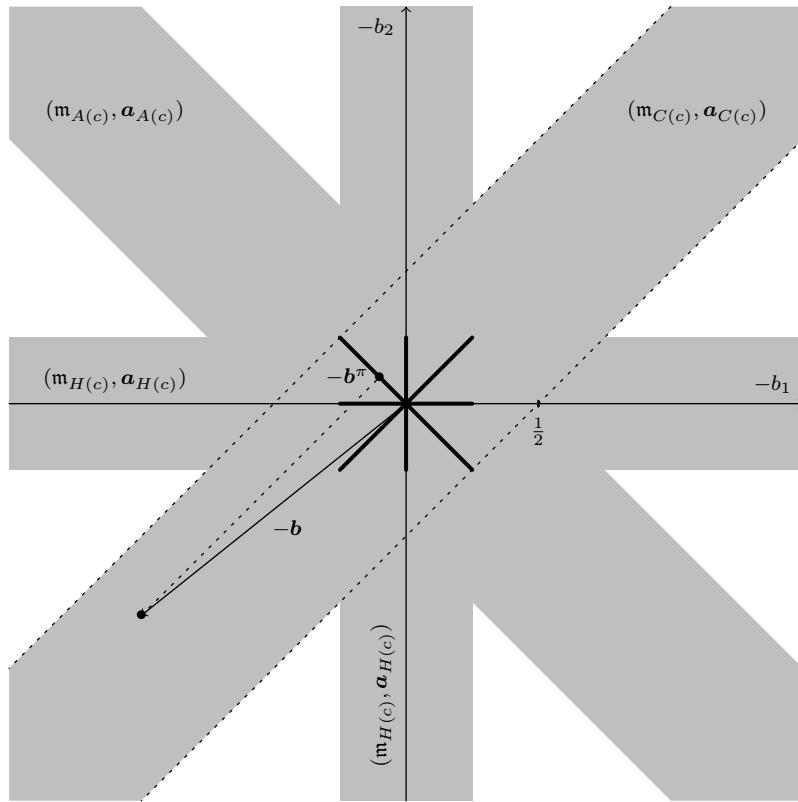


Figure 2.8 – Deviated symmetric equilibrium strategies (\mathbf{m}, \mathbf{a}) associated with bias \mathbf{b}

In Figure 2.9, the same construction is used to derive bias \mathbf{b} for which $(\mathbf{m}, \mathbf{a}) = (\mathbf{m}_T, \mathbf{a}_T) \in \mathcal{E}(\mathbf{b})$, for some $T \in \{0\} \times [\frac{1}{2}, 1]$. Given $-\mathbf{b}^\pi$ (corresponding to a bold point in the figure) a bias $-\mathbf{b}$ projects orthogonally at $-\mathbf{b}^\pi$ onto the line starting from the origin iff the corresponding strategies are equilibrium strategies of $\Gamma_{\mathbf{b}}$ (then the line from the origin characterizes the direction $(\mathbf{a}(m_1)\mathbf{a}(m_2))$ of the dimension of the players' agreement). The representation is extended by symmetry for T spanning the full border of Θ in Figure 2.10.

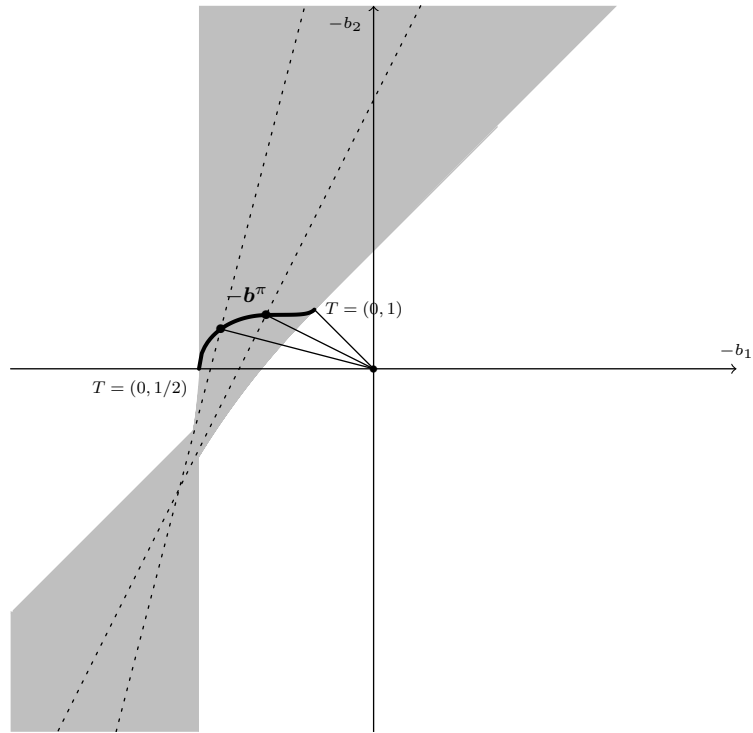


Figure 2.9 – Asymmetric equilibrium strategies $(\mathbf{m}_T, \mathbf{a}_T)$ associated with bias \mathbf{b} , for $T \in \{0\} \times [\frac{1}{2}, 1]$

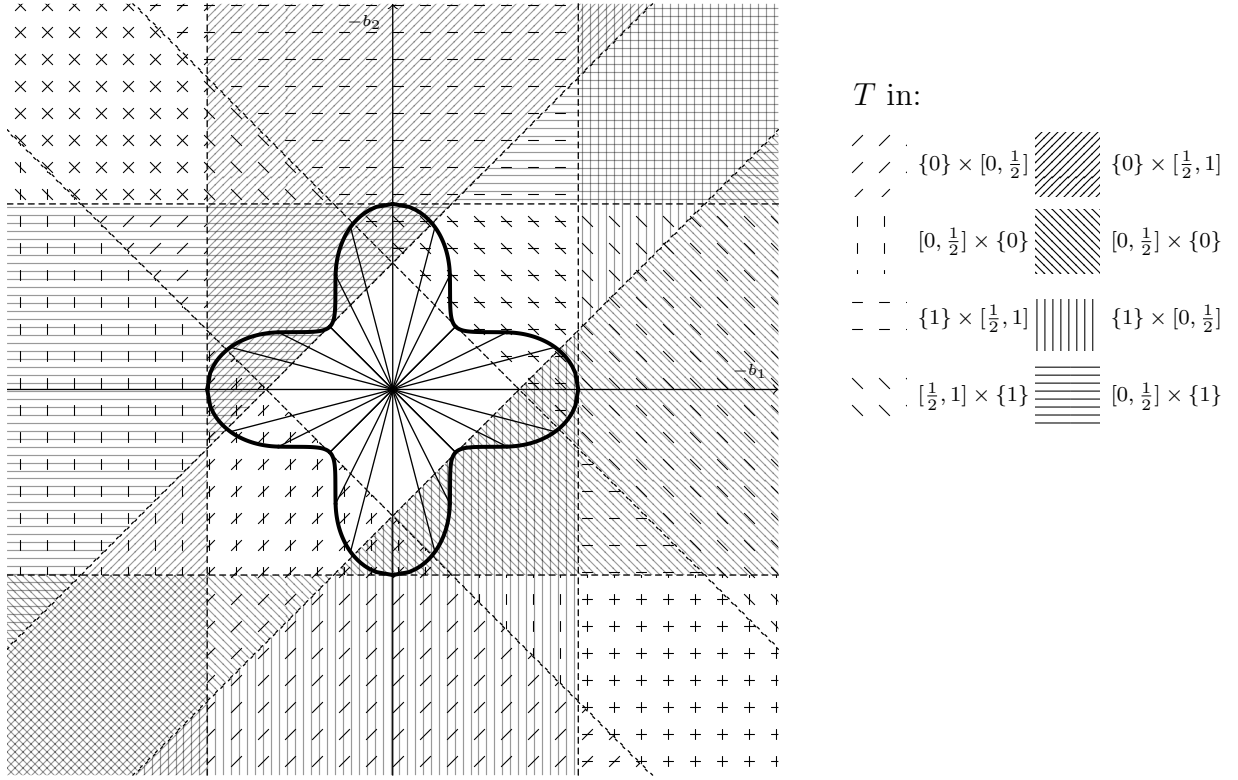


Figure 2.10 – Asymmetric equilibrium strategies $(\mathbf{m}_T, \mathbf{a}_T)$ associated with bias \mathbf{b} , for T on the border of Θ

2.6 Conclusion

In this paper, we characterize the set of equilibria of an extension of the Crawford and Sobel's (1982) model of information transmission. We consider two dimensions, and we assume a uniform prior, quadratic preferences, and a binary disclosure rule. To do so we show the existence of continua of equilibria in the various games $\Gamma_{\mathbf{b}}$, $\mathbf{b} \in \mathbb{R}^2$, enabling us to associate equilibria with biases. In particular, we show that at least one influential equilibrium might be associated with any bias.

The symmetries of the game parameters underpin our results. In particular, the utility functions we consider make it possible to derive equilibrium conditions in simple geometric terms. We relate these conditions to the symmetries of the uniform distribution over $[0, 1]^2$. The equilibria result from the multiplicity of dimensions along which information may be disclosed, combined with the flexibility of communication in any of these dimensions.

While the existing literature suggested the possibility of communication based on strict symmetries and any conflict, and the impossibility of communication based on large asymmetries and infinite conflict, our result shows that adding dimensions to the communication environment introduces other bases for agreement between players with finite conflict.

Proofs

2.A Proof of Example 2.1

Given the Sender's strategy \mathbf{m}_C , we derive from (2.1) the induced actions

$$\mathbf{a}_C(m_1) = \left(\frac{2}{3}, \frac{1}{3}\right), \text{ and } \mathbf{a}_C(m_2) = \left(\frac{1}{3}, \frac{2}{3}\right).$$

For \mathbf{m}_A (resp. the two \mathbf{m}_H), we obtain $\mathbf{a}_A(m_1) = \left(\frac{2}{3}, \frac{2}{3}\right)$ and $\mathbf{a}_A(m_2) = \left(\frac{1}{3}, \frac{1}{3}\right)$ (resp. $\mathbf{a}_H(m_1) = \left(\frac{3}{4}, \frac{1}{2}\right)$ and $\mathbf{a}_H(m_2) = \left(\frac{1}{4}, \frac{1}{2}\right)$, or $\mathbf{a}_H(m_1) = \left(\frac{1}{2}, \frac{3}{4}\right)$ and $\mathbf{a}_H(m_2) = \left(\frac{1}{2}, \frac{1}{4}\right)$).

Reciprocally, given the Receiver's strategy $\mathbf{a} : m \mapsto \mathbf{a}(m)$, for instance $\mathbf{a}(m_1) = \mathbf{a}_C(m_1) = \left(\frac{2}{3}, \frac{1}{3}\right)$, and $\mathbf{a}(m_2) = \mathbf{a}_C(m_2) = \left(\frac{1}{3}, \frac{2}{3}\right)$ for the game associated to the bias $\mathbf{b} = (b, b)$, $b \in \mathbb{R}$, from (2.2), we have $\mathbf{m}(\boldsymbol{\theta}) = m_1$ if and only if (up to a null measure set)

$$\begin{aligned} & -\|\mathbf{a}_C(m_1) - (\boldsymbol{\theta} + \mathbf{b})\|^2 \geq -\|\mathbf{a}_C(m_2) - (\boldsymbol{\theta} + \mathbf{b})\|^2 \\ \iff & -\left(\frac{2}{3} - (\theta_1 + b)\right)^2 - \left(\frac{1}{3} - (\theta_2 + b)\right)^2 \geq -\left(\frac{1}{3} - (\theta_1 + b)\right)^2 - \left(\frac{2}{3} - (\theta_2 + b)\right)^2 \\ \iff & \theta_1 \geq \theta_2 \end{aligned}$$

and similarly $\mathbf{m}(\boldsymbol{\theta}) = m_2$ if and only if $\theta_1 < \theta_2$. Therefore, strategies \mathbf{m}_C and \mathbf{a}_C are best response in equilibrium. Proofs for the profiles of strategies $(\mathbf{m}_A, \mathbf{a}_A)$ and $(\mathbf{m}_H, \mathbf{a}_H)$ are similar.

2.B Proof of Example 2.2

Consider the Sender's strategy

$$\mathbf{m}_{C(c)}(\boldsymbol{\theta}) = \begin{cases} m_1, & \text{if } \theta_1 \geq \theta_2 + c, \\ m_2, & \text{if } \theta_1 < \theta_2 + c, \end{cases}$$

for some $c \leq 0$. Then from (2.1), we obtain, for $c > -1$,

$$\mathbf{a}(m_1) = \mathbb{E}[\boldsymbol{\theta} | \theta_1 \geq \theta_2 + c] = \left(\frac{1}{3} \frac{c^3 + 3c^2 + 3c - 2}{c^2 + 2c - 1}, \frac{1}{3} \frac{1 - c^3 + 3c - 1}{c^2 + 2c - 1}\right),$$

and

$$\mathbf{a}(m_2) = \mathbb{E}[\boldsymbol{\theta} | \theta_1 < \theta_2 + c] = \left(\frac{c+1}{3}, \frac{2-c}{3}\right).$$

Reciprocally, given the Receiver's strategy $\mathbf{a}_{C(c)}$ defined by the above actions, from (2.2), message m_1 is disclosed if and only if $-\|\mathbf{a}_{C(c)}(m_1) - (\boldsymbol{\theta} + \mathbf{b})\|^2 \geq -\|\mathbf{a}_{C(c)}(m_2) - (\boldsymbol{\theta} + \mathbf{b})\|^2$, where $\mathbf{b} = (b_1, b_2) \in \mathbb{R}^2$. Set $\mathbf{b} = (b_1, b_1 + \Delta b)$ with $\Delta b = b_2 - b_1 \in \mathbb{R}$. Then in equilibrium, we must have

$$\theta_1 \geq \theta_2 + \frac{1}{3} \frac{c(c+2)(2c-1)}{c^2 + 2c - 1} + \Delta b,$$

so that c must solve $\Delta b = \frac{1}{3} \frac{c^3 + 3c^2 - c}{c^2 + 2c - 1}$. This defines a strictly increasing function $\Delta b \mapsto c(\Delta b)$ from $\Delta b \in (-\frac{1}{2}, 0]$ to $(-1, 0]$. A similar bijection is obtained in case $c \geq 0$ and $\Delta b \in [0, \frac{1}{2})$.

2.C Proof of Example 2.3

It is straightforward to verify that each strategy of the profile of strategies (\mathbf{m}, \mathbf{a}) stated in the example is the best response of the other, *i.e.* satisfies (2.1) and (2.2).

2.D Proof of Lemma 2.1

Let (\mathbf{m}, \mathbf{a}) be such that $(\mathbf{m}, \mathbf{a}) \in \mathcal{E}(\mathbf{b})$ for some $\mathbf{b} \in \mathbb{R}^2$. Let $\mathbf{b}' \in \mathbb{R}^2$. From $\|\mathbf{x}\|^2 = \mathbf{x} \cdot \mathbf{x}$, given \mathbf{a} , we obtain for instance $\mathbf{m}'(\boldsymbol{\theta}) = m_1$ *iff*

$$\begin{aligned} & -\|\mathbf{a}(m_1) - (\boldsymbol{\theta} + \mathbf{b}')\|^2 \geq -\|\mathbf{a}(m_2) - (\boldsymbol{\theta} + \mathbf{b}')\|^2 \\ \iff & -\|(\mathbf{a}(m_1) - \mathbf{b} - \boldsymbol{\theta}) - (\mathbf{b}' - \mathbf{b})\|^2 \geq -\|(\mathbf{a}(m_2) - \mathbf{b} - \boldsymbol{\theta}) - (\mathbf{b}' - \mathbf{b})\|^2 \\ \iff & -\|\mathbf{a}(m_1) - (\boldsymbol{\theta} + \mathbf{b})\|^2 \geq -\|\mathbf{a}(m_2) - (\boldsymbol{\theta} + \mathbf{b})\|^2 - 2(\mathbf{b}' - \mathbf{b}) \cdot (\mathbf{a}(m_1) - \mathbf{a}(m_2)). \end{aligned}$$

Therefore given \mathbf{a} , the Sender of $\Gamma_{\mathbf{b}'}$ uses the same disclosure strategy \mathbf{m}' as the Sender of $\Gamma_{\mathbf{b}}$ who uses \mathbf{m} for any observed $\boldsymbol{\theta} \in \Theta$ *iff* $(\mathbf{b}' - \mathbf{b}) \cdot (\mathbf{a}(m_1) - \mathbf{a}(m_2)) = 0$. Reciprocally, strategies \mathbf{m}' and \mathbf{m} induce the same actions.

2.E Proof of Lemma 2.2

Consider a line (OT) with $O = \mathbb{E}[\Theta] = (\frac{1}{2}, \frac{1}{2})$ and $T \in \{0\} \times (\frac{1}{2}, 1)$ (see Figure 2.6 in the main text). We seek to show that for any such line, there exists a game $\Gamma_{\mathbf{b}_T}$ such that (OT) supports the two induced actions of the Receiver of $\Gamma_{\mathbf{b}_T}$, for some profile $(\mathbf{m}_T, \mathbf{a}_T)$ of equilibrium strategies.

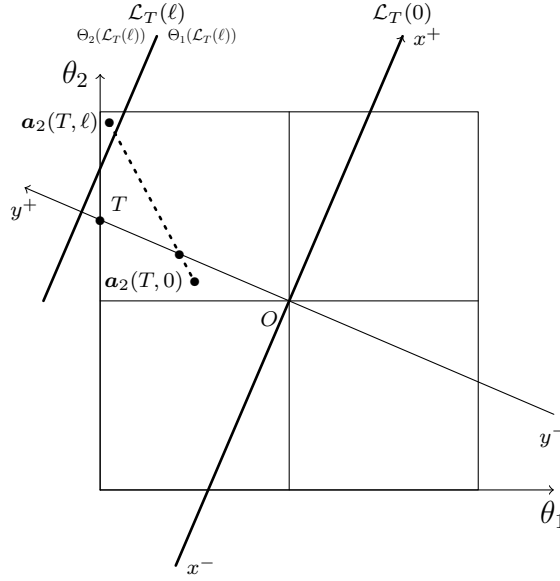
Since for any profile (\mathbf{m}, \mathbf{a}) of equilibrium strategies, line $(\mathbf{a}(m_1)\mathbf{a}(m_2))$ is perpendicular to the Sender's set of indifferent states $\mathcal{L} = \{\boldsymbol{\theta}, U_S(\boldsymbol{\theta}, \mathbf{a}(m_1)) = U_S(\boldsymbol{\theta}, \mathbf{a}(m_2))\}$, a necessary equilibrium condition is the orthogonality of \mathcal{L} and (OT) . Given \mathcal{L}_T perpendicular to (OT) , let us denote $\Theta_1(\mathcal{L}_T)$ and $\Theta_2(\mathcal{L}_T)$ the two regions situated on the sides of \mathcal{L}_T , with for instance $O \in \Theta_1(\mathcal{L}_T)$.²⁰ Then we have to show that the players' strategies \mathbf{m}_T and \mathbf{a}_T given by

$$\mathbf{m}_T(\boldsymbol{\theta}) = \begin{cases} m_1 & \text{if } \boldsymbol{\theta} \in \Theta_1(\mathcal{L}_T), \\ m_2 & \text{if } \boldsymbol{\theta} \in \Theta_2(\mathcal{L}_T), \end{cases} \quad \text{and } \mathbf{a}_T(m) = \begin{cases} \mathbb{E}[\Theta_1(\mathcal{L}_T)] & \text{if } m = m_1, \\ \mathbb{E}[\Theta_2(\mathcal{L}_T)] & \text{if } m = m_2, \end{cases}$$

define a profile of equilibrium strategies which satisfies $\mathbf{a}_T(m_i) \in (OT)$ for $i \in \{1, 2\}$. Note that it is sufficient to show that $\mathbf{a}_T(m_2) \in (OT)$, since we necessarily have $\mathbf{a}_T(m_1) \in (O\mathbf{a}_T(m_2)) = (OT)$.

To do this, let us consider the orthogonal coordinate system $\mathcal{R}_T(O, x, y)$, with $(y^- Oy^+)$ supported by (OT) (see Figure 2.11).

20. By setting $O \in \Theta_2(\mathcal{L}_T)$, we can derive the message symmetric profile of strategies $(\mathbf{m}'_T, \mathbf{a}'_T)$ such that for any $\boldsymbol{\theta} \in \Theta$, $\mathbf{m}'_T(\boldsymbol{\theta}) = m_1$ *iff* $\mathbf{m}_T(\boldsymbol{\theta}) = m_2$, and $\mathbf{a}'_T(m_1) = \mathbf{a}_T(m_2)$, $\mathbf{a}'_T(m_2) = \mathbf{a}_T(m_1)$.


 Figure 2.11 – $\mathcal{L}_T(\ell)$ and subsequent $\mathbf{a}_2(T, \ell)$

Given $\ell \in \mathbb{R}$, let $\mathcal{L}_T(\ell)$ be the line perpendicular to (OT) and situated at a distance ℓ of O , with $O \notin \Theta_2(\mathcal{L}_T(\ell))$, and let

$$\mathbf{a}_2(T, \ell) = \mathbb{E}[\boldsymbol{\theta} | \boldsymbol{\theta} \in \Theta_2(\mathcal{L}_T(\ell))]$$

denote the corresponding expectation. Let $\bar{\ell}$ be such that $(0, 1) \in \mathcal{L}_T(\bar{\ell})$. For any $T \in \{0\} \times (\frac{1}{2}, 1)$, if ℓ is sufficiently close to $\bar{\ell}$, then $\Theta_2(\mathcal{L}_T(\ell)) \subset \{(x, y), x > 0\}$, and thus $x(\mathbf{a}_2(T, \ell)) > 0$ for any such ℓ . Therefore, by continuity, if there exists $\underline{\ell}$ such that

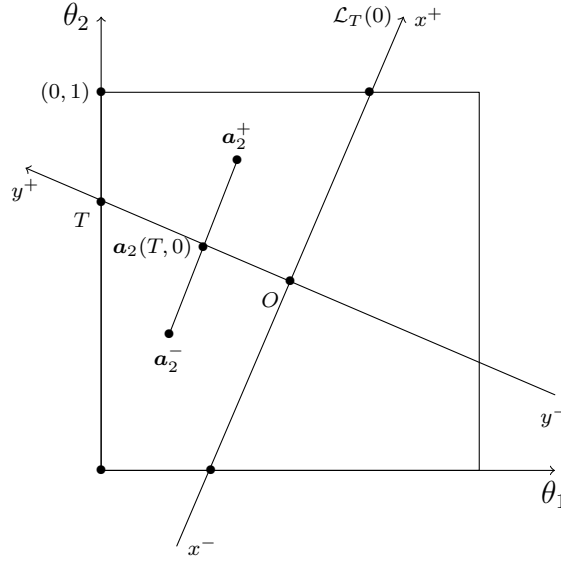
$$x(\mathbf{a}_2(T, \underline{\ell})) < 0, \quad (2.7)$$

then there is some $\ell^* \in (\underline{\ell}, \bar{\ell})$ such that $x(\mathbf{a}_2(T, \ell^*)) = 0$. Let us show that for any $T \in \{0\} \times (\frac{1}{2}, 1)$,

$$x(\mathbf{a}_2(T, 0)) < 0, \quad (2.8)$$

i.e. $\underline{\ell} = 0$ satisfies Condition (2.7).

Let us set $\Theta^+ = \{\boldsymbol{\theta} \in \Theta, x(\boldsymbol{\theta}) > 0, y(\boldsymbol{\theta}) > 0\}$ and $\Theta^- = \{\boldsymbol{\theta} \in \Theta, x(\boldsymbol{\theta}) < 0, y(\boldsymbol{\theta}) > 0\}$, and $\mathbf{a}_2^+ = \mathbb{E}[\Theta^+]$, $\mathbf{a}_2^- = \mathbb{E}[\Theta^-]$ (see Figure 2.12).


 Figure 2.12 – $\mathbf{a}_2(T, 0) = \frac{\mathbf{a}_2^+ + \mathbf{a}_2^-}{2}$

Then from the law of iterated expectations, we have

$$(|\Theta^+| + |\Theta^-|)\mathbf{a}_2(T, 0) = |\Theta^+|\mathbf{a}_2^+ + |\Theta^-|\mathbf{a}_2^-,$$

where $|\cdot|$ denotes the Lebesgue measure. Now note that rotation $\rho_{\frac{\pi}{2}}$ of angle $\frac{\pi}{2}$ and centre O transforms Θ^+ to Θ^- point wise, so that for any $\boldsymbol{\theta} \in \Theta$, we have $(x(\boldsymbol{\theta}), y(\boldsymbol{\theta})) \in \Theta^+$ iff $(-y(\boldsymbol{\theta}), x(\boldsymbol{\theta})) \in \Theta^-$. Then on the one hand, we have $|\Theta^+| = |\Theta^-|$, which gives

$$\mathbf{a}_2(T, 0) = \frac{\mathbf{a}_2^+ + \mathbf{a}_2^-}{2},$$

and on the other hand, we have $\rho_{\frac{\pi}{2}}(\mathbf{a}_2^+) = \mathbf{a}_2^-$, which gives

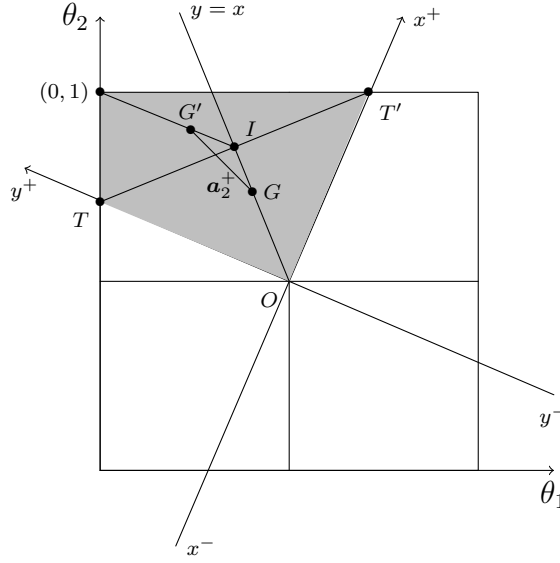
$$\begin{cases} x(\mathbf{a}_2^+) = y(\mathbf{a}_2^-), \\ y(\mathbf{a}_2^+) = -x(\mathbf{a}_2^-). \end{cases}$$

Consequently, Condition (2.8) is satisfied iff $\frac{x(\mathbf{a}_2^-) + x(\mathbf{a}_2^+)}{2} < 0$, i.e.

$$x(\mathbf{a}_2^+) < y(\mathbf{a}_2^+). \quad (2.9)$$

Now let T' be the point such that $\rho_{\frac{\pi}{2}}(T') = T$ (see Figure 2.13), and let us partition the set Θ^+ according to triangles $\Delta = (OTT')$ and $\Delta' = \Theta^+ \setminus (OTT')$. Again, from the law of iterated expectations, \mathbf{a}_2^+ is situated on line (GG') , where $G = \mathbb{E}[\boldsymbol{\theta} | \boldsymbol{\theta} \in \Delta]$ and $G' = \mathbb{E}[\boldsymbol{\theta} | \boldsymbol{\theta} \in \Delta']$ are the respective mass centres of triangles Δ and Δ' .²¹

21. Bearing in mind that the mass centre of a triangle is situated at the intersection of the median lines.


 Figure 2.13 – $y(\mathbf{a}_2^+) > x(\mathbf{a}_2^+)$

Note that from $OT = OT'$, median (OI) of (OTT') has equation $x = y$ in $\mathcal{R}(0, x, y)$. Therefore $x(G) = y(G)$ and Condition (2.9) holds iff

$$x(G') < y(G'). \quad (2.10)$$

Finally, note that G' is situated on the median line of Δ' which passes through I and the point $(0, 1)$, between I and $(0, 1)$ which satisfy $y(I) = x(I)$ and $x((0, 1)) < y((0, 1))$ respectively. Therefore $G' \in \{(x, y), y > x\}$ and Condition (2.10) holds.

Since (2.10) hold, so do Conditions (2.9), (2.8) and (2.7). Therefore we have $x(\mathbf{a}_2(T, 0)) < 0$, and we obtain the existence of $\ell^* \in (0, \ell)$ such that $\mathbf{a}_2(T, \ell^*) \in (OT)$ as wanted.

Next, we define the game $\Gamma_{\mathbf{b}_T}$. We have to find some $\mathbf{b}_T \in \mathbb{R}^2$ such that strategies \mathbf{m}_T and \mathbf{a}_T given by

$$\mathbf{m}_T(\boldsymbol{\theta}) = \begin{cases} m_1 & \text{if } \boldsymbol{\theta} \in \Theta_1(\mathcal{L}_T(\ell^*)), \\ m_2 & \text{if } \boldsymbol{\theta} \in \Theta_2(\mathcal{L}_T(\ell^*)), \end{cases} \quad \text{and } \mathbf{a}_T(m) = \begin{cases} \mathbf{a}_1(T, \ell^*) & \text{if } m = m_1, \\ \mathbf{a}_2(T, \ell^*) & \text{if } m = m_2, \end{cases}$$

are equilibrium strategies of $\Gamma_{\mathbf{b}_T}$. A sufficient condition is that $\mathcal{L}_T(\ell^*)$ is the perpendicular bisector of $\mathbf{a}_T(m_1) - \mathbf{b}_T$ and $\mathbf{a}_T(m_2) - \mathbf{b}_T$. Let us set $\mathbf{b}_T = \frac{\mathbf{a}_T(m_1) + \mathbf{a}_T(m_2)}{2} - \boldsymbol{\theta}_T$, where $\boldsymbol{\theta}_T = (OT) \cap \mathcal{L}_T(\ell^*)$, so that $\boldsymbol{\theta}_T$ is the mid-point of $\mathbf{a}_T(m_1) - \mathbf{b}_T$ and $\mathbf{a}_T(m_2) - \mathbf{b}_T$. Since $\mathcal{L}_T(\ell^*) \perp (OT)$, $\mathcal{L}_T(\ell^*)$ has the desired property.

2.F Proof of Lemma 2.3

We show that strategies $(\mathbf{m}_T, \mathbf{a}_T)$ extend continuously to one of the symmetric deviated strategies. By construction, when T spans continuously the open segment $\{0\} \times (\frac{1}{2}, 1)$, line

\mathcal{L}_T , and therefore²² sets $\mathbf{m}_T^{-1}(m_i)$, $i \in \{1, 2\}$, actions $\mathbf{a}_T(m_1)$ and $\mathbf{a}_T(m_2)$, and bias

$$T \mapsto \mathbf{b}_T = \frac{\mathbf{a}_T(m_1) + \mathbf{a}_T(m_2)}{2} + \boldsymbol{\theta}_T \in (OT) \quad (2.11)$$

can all be chosen to move continuously.

When $T \rightarrow T_1 = (0, 1)$, we have: (a) actions tend to be onto the diagonal line (OT_1) , and (b) Sender's set of indifferent states \mathcal{L}_T tends to intersect the diagonal line orthogonally at some $\boldsymbol{\theta}_{T_1} = (\theta_{T_1}, 1 - \theta_{T_1})$, for some $0 \leq \theta_{T_1} \leq \frac{1}{2}$.

If $\theta_{T_1} > 0$, this necessarily identifies a $(1, 1)$ -symmetric (deviated if $\theta_{T_1} < \frac{1}{2}$) comparative profile of strategies $(\mathbf{m}_{C(c)}, \mathbf{a}_{C(c)})$, $c \in (0, 1)$, since, as shown in Example 2.2, when c spans $(0, 1)$, the support of the induced actions and the sets of indifferent states associated with profiles $(\mathbf{m}_{C(c)}, \mathbf{a}_{C(c)})$ span all such orthogonal lines.

If $\theta_{T_1} = 0$, this identifies a babbling equilibrium. Let us show that this case does not occur.

Let us set $T(0, \frac{1}{2} + t)$, where $t > 0$ is sufficiently close to T_1 , and let us denote $(z(t), 1)$, with $0 \leq z(t) \leq \frac{1}{2}$, the coordinates of the intersection of \mathcal{L}_T and the upper border of Θ (see Figure 2.14).

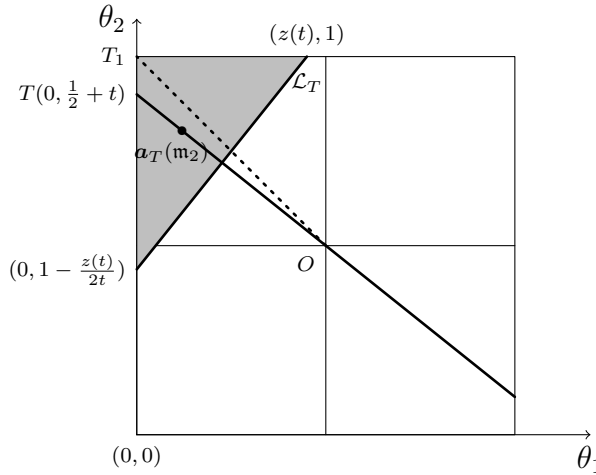


Figure 2.14 – $\Theta_2(\mathcal{L}_T)$ as $T \rightarrow T_1$

Suppose $\theta_{T_1} = 0$. This implies that if T is sufficiently close to T_1 , then line \mathcal{L}_T intersects the diagonal line (OT_1) at some $\boldsymbol{\theta}_T = (\theta_T, 1 - \theta_T)$ with $\lim_{T \rightarrow T_1} \theta_T = \theta_{T_1} = 0$, so that $\lim_{t \rightarrow \frac{1}{2}} z(t) = 0$.

Let us show that $\lim_{t \rightarrow \frac{1}{2}} z(t) > 0$, so that $\theta_{T_1} = 0$ does not hold.

22. The rule applied for choosing which region on the side of $\mathcal{L}_T(\ell^*)$ is associated with m_1 and which one is associated with m_2 must be consistent for all T s. For instance, following the choice made in the proof of Lemma 2.2, we have to choose $O \in \Theta_1(\mathcal{L}_T)$ for every T .

From $\mathcal{L}_T \perp (OT)$, it is easy to derive that the vertex of $\Theta_2(\mathcal{L}_T) = \mathbf{m}_T^{-1}(m_2)$ are given by $(0, 1)$, $(z(t), 1)$ and $(0, 1 - \frac{z(t)}{2t})$, as depicted in Figure 2.14. The coordinates of the mass centre $\mathbf{a}(m_2)$ of $\Theta_2(\mathcal{L}_T)$ are given by $\mathbf{a}(m_2) = (\frac{1}{3}z(t), 1 - \frac{1}{3}\frac{z(t)}{2t})$. From the equilibrium condition $\mathbf{a}(m_2) \in (OT)$, with $(OT) = \{(\theta_1, \theta_2) \in \Theta, \theta_2 = -2t\theta_1 + \frac{1}{2} + t\}$, we obtain, for any t sufficiently close to $\frac{1}{2}$,

$$1 - \frac{1}{3} \frac{z(t)}{2t} = -2t \frac{1}{3} z(t) + \frac{1}{2} + t.$$

This gives $z(t) = \frac{t - \frac{1}{2}}{\frac{-1}{6t} + \frac{2}{3}t} = \frac{6t}{2(2t+1)}$, and in particular, $\lim_{t \rightarrow \frac{1}{2}} z(t) = \frac{3}{4} > 0$, as expected.

Let us now look at the other limit of $(\mathbf{m}_T, \mathbf{a}_T)$, *i.e.* when $T \rightarrow T_0 = (0, \frac{1}{2})$. The arguments are similar, but $\Theta_2(\mathcal{L}_T)$ has a more complex shape.

We have: (a) actions $\mathbf{a}_T(m_1)$ and $\mathbf{a}_T(m_2)$ tend to be supported on the *horizontal line* (OT_0) , and (b) line \mathcal{L}_T of the Sender's indifferent states tends to intersect the horizontal line orthogonally at some $\theta_{T_0} = (\theta_{T_0}, \frac{1}{2})$, for some $0 \leq \theta_{T_0} \leq \frac{1}{2}$.

Again, if $\theta_{T_0} > 0$, this necessarily identifies a $(0, 1)$ -symmetric (deviated if $\theta_{T_0} < \frac{1}{2}$) half-babbling profile of strategies $(\mathbf{m}_{H(c)}, \mathbf{a}_{H(c)})$, $c \in (0, 1)$, whereas if $\theta_{T_0} = 0$, this identifies a babbling equilibrium. We show that this cannot occur.

Suppose $\theta_{T_0} = 0$. This implies that if T is sufficiently close to T_0 , the set of indifferent states \mathcal{L}_T associated with strategies $(\mathbf{m}_T, \mathbf{a}_T)$ intersects the horizontal line (OT_0) at $\theta_T = (\theta_T, \frac{1}{2})$, with $\lim_{T \rightarrow T_1} \theta_T = \theta_{T_1} = 0$. Since \mathcal{L}_T is orthogonal to (OT) , and since (OT) tends to the horizontal line (OT_0) , we have that \mathcal{L}_T tends to the vertical line supported by $(0, 0) - (0, 1)$.

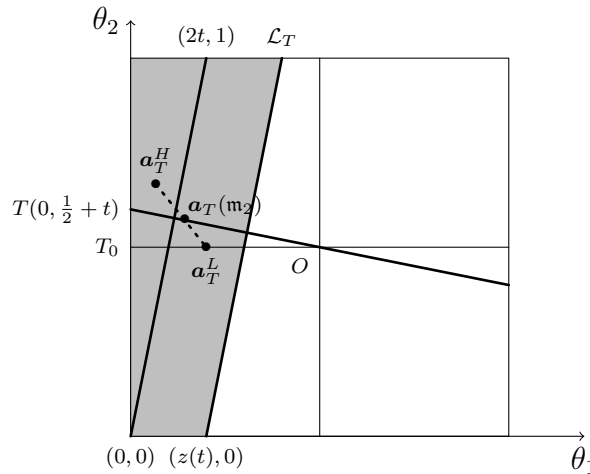


Figure 2.15 – Partition of $\Theta_2(\mathcal{L}_T)$ and subsequent expectations

Let us consider $T(0, \frac{1}{2} + t)$, with $t > 0$, sufficiently close to T_0 , and let us parameterize the set $\Theta_2(\mathcal{L}_T) = \mathbf{m}_T^{-1}(m_2)$ through a partition consisting of a triangle with $(0, 0)$ as vertex

and hypotenuse parallel to \mathcal{L}_T , and a parallelogram of width $z(t)$ for some $z(t) \in [0, \frac{1}{2})$, as depicted in Figure 2.15. Equality $\theta_{T_0} = 0$ implies $\lim_{t \rightarrow 0} z(t) = 0$ and thus we seek to show $\lim_{t \rightarrow 0} z(t) > 0$ in order to obtain a contradiction.

From the law of iterated expectations, $\mathbf{a}(m_2)$ might be written $\mathbf{a}_T(m_2) = (1 - \lambda(z(t), t))\mathbf{a}_T^H + \lambda(z(t), t)\mathbf{a}_T^L$, with $\lambda(z(t), t) = \frac{V(z(t))}{V(z(t)) + t} \in [0, 1]$, where $V(z(t))$ and t are the respective Lebesgue measure of the parallelogram and the triangle. The coordinates of their respective mass centre are given by $\mathbf{a}_T^L = \left(\frac{2t+z(t)}{2}, \frac{1}{2}\right)$ and $\mathbf{a}_T^H = \left(\frac{2}{3}t, \frac{2}{3}\right)$ respectively. From $\mathbf{a}_T(m_2) \in (OT)$, with $(OT) = \{(\theta_1, \theta_2) \in \Theta, \theta_2 = -2t\theta_1 + \frac{1}{2} + t\}$, we obtain, for any t sufficiently close to 0,

$$(1 - \lambda(z(t), t))\frac{2}{3} + \lambda(z(t), t)\frac{1}{2} = -2t \left((1 - \lambda(z(t), t))\frac{2}{3}t + \lambda(z(t), t)\frac{2t + z(t)}{2} \right) + \frac{1}{2} + t. \quad (2.12)$$

Let us set $\lambda = \lim_{t \rightarrow 0} \lambda(z(t), t) \in [0, 1]$. Then when $t \rightarrow 0$, Equation (2.12) gives $(1 - \lambda)\frac{2}{3} + \lambda\frac{1}{2} = \frac{1}{2}$, so that

$$\lambda = \lim_{t \rightarrow 0} \lambda(z(t), t) = 1. \quad (2.13)$$

Now for $t > 0$, Equation (2.12) is

$$z(t) = \frac{-1}{6\lambda(z(t), t)} \frac{1 - \lambda(z(t), t)}{t} + \frac{1}{\lambda(z(t), t)} - \frac{4}{3}t \frac{(1 - \lambda(z(t), t))}{\lambda(z(t), t)} - 2t, \quad (2.14)$$

and note that

$$\frac{1 - \lambda(z(t), t)}{t} = \frac{1 - \frac{V(z(t))}{V(z(t)) + t}}{t} = \frac{t}{t(V(z(t)) + t)} = \frac{1}{V(z(t)) + t}.$$

If $\lim_{t \rightarrow 0} z(t) = 0$, then we have $\lim_{t \rightarrow 0} V(z(t)) = 0$, and thus

$$\lim_{t \rightarrow 0} \frac{1 - \lambda(z(t), t)}{t} = \infty.$$

Now (2.13) and (2.14) give the impossible $0 = \infty$. Therefore $\lim_{t \rightarrow 0} z(t) > 0$ as expected.

2.G Proof of Proposition 2.1

We show that the strategies $(\mathbf{m}_T, \mathbf{a}_T)$ exhibited in Lemma 2.2 allows any bias $\mathbf{b} \in \mathbb{R}^2$ to be associated with an influential equilibrium.

According to Lemma 2.3, maps

$$T \mapsto \mathbf{b}_T = \frac{\mathbf{a}_T(m_1) + \mathbf{a}_T(m_2)}{2} + \boldsymbol{\theta}_T,$$

and $T \mapsto \mathbf{a}_T(m_i)$, $i \in \{1, 2\}$, can be extended continuously to the *closed* segment $\{0\} \times [\frac{1}{2}, 1]$, with *non babbling* strategies $(\mathbf{m}_T, \mathbf{a}_T)$ for each T . According to the four symmetries of Θ , and according to Lemma 2.4, these maps extend continuously to the full border of Θ . Note that this needs a suitable choice of *message assignment* across the T s. W.l.o.g., we assume $O = \mathbb{E}[\Theta] \in \mathbf{m}_T^{-1}(m_1)$ for all T .

Now let us consider the continuous maps

$$b_\Theta : T \mapsto \|\mathbf{b}_T\|,$$

and, given $\mathbf{b} \in \mathbb{R}^2$,

$$\pi_{\mathbf{b}} : T \mapsto \left\| \mathbf{b} \cdot \frac{\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2)}{\|\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2)\|} \right\|.$$

Each map is defined on the full border of Θ . Map $\pi_{\mathbf{b}}$ is the normalized norm of the projection of \mathbf{b} onto (OT) . Note that

$$b_\Theta(T) = \pi_{\mathbf{b}}(T) \text{ iff } \|\mathbf{b}_T\| \|\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2)\| = \|\mathbf{b} \cdot (\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2))\|,$$

and that $\|\mathbf{b}_T\| \|\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2)\| = \mathbf{b}_T \cdot (\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2))$ according to our choice $O \in \mathbf{m}_T^{-1}(m_1)$ (the alternative choice would have given $\|\mathbf{b}_T\| \|\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2)\| = -\mathbf{b}_T \cdot (\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2))$). Then we obtain:

$$b_\Theta(T) = \pi_{\mathbf{b}}(T) \text{ iff } \begin{cases} \mathbf{b}_T \cdot (\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2)) = \mathbf{b} \cdot (\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2)), \text{ or} \\ \mathbf{b}_T \cdot (\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2)) = -\mathbf{b} \cdot (\mathbf{a}_T(m_1) - \mathbf{a}_T(m_2)). \end{cases}$$

Now consider the central symmetry ρ of Θ . Strategies $\rho(\mathbf{m}_T)$ and $\rho(\mathbf{a}_T)$ are given by strategies $\mathbf{m}_{\rho(T)}$ and $\mathbf{a}_{\rho(T)}$, associated with bias $\mathbf{b}_{\rho(T)} = \rho(\mathbf{b}_T) = -\mathbf{b}_T$, and we have $\mathbf{a}_T(m_1) = \mathbf{a}_{\rho(T)}(m_2)$, $\mathbf{a}_T(m_2) = \mathbf{a}_{\rho(T)}(m_1)$. Then the second equation above might be written

$$\mathbf{b}_{\rho(T)} \cdot (\mathbf{a}_{\rho(T)}(m_1) - \mathbf{a}_{\rho(T)}(m_2)) = \mathbf{b} \cdot (\mathbf{a}_{\rho(T)}(m_1) - \mathbf{a}_{\rho(T)}(m_2)).$$

From Lemma 2.1, we obtain:

$$b_\Theta(T) = \pi_{\mathbf{b}}(T) \text{ iff } [(\mathbf{m}_T, \mathbf{a}_T) \in \mathcal{E}(\mathbf{b}), \text{ or } (\mathbf{m}_{\rho(T)}, \mathbf{a}_{\rho(T)}) \in \mathcal{E}(\mathbf{b})].$$

This means: if \mathbf{b} projects as \mathbf{b}_T or $-\mathbf{b}_T$ onto (OT) , then respectively $(\mathbf{m}_T, \mathbf{a}_T)$ or $(\mathbf{m}_{\rho(T)}, \mathbf{a}_{\rho(T)})$ are profiles of equilibrium strategies of $\Gamma_{\mathbf{b}}$. In particular, we have:

$$\text{if } b_\Theta(T) = \pi_{\mathbf{b}}(T) \text{ for some } T, \text{ then } \mathcal{E}(\mathbf{b}) \neq \emptyset. \quad (2.15)$$

Next, we apply Bolzano's Theorem to show that there is always such a T , unless \mathbf{b} is small. Let $[b_\Theta, \bar{b}_\Theta] \subset \mathbb{R}^+$ denotes the set of values spanned by b_Θ (continuous) when T spans the border of Θ (compact). We have $b_\Theta \geq 0$, and according to Example 2.3, $\bar{b}_\Theta > 0$. Concerning $\pi_{\mathbf{b}}$, since (OT) spans every directions when T spans the border of Θ , $\pi_{\mathbf{b}}(T)$ spans $[0, \|\mathbf{b}\|]$ ($\pi_{\mathbf{b}}$ has been normalized).

Note that if T is such that $(OT) \perp \mathbf{b}$, then $\pi_{\mathbf{b}}(T) = 0$, and thus we have

$$b_{\Theta}(T) - \pi_{\mathbf{b}}(T) = b_{\Theta}(T) - 0 \geq 0$$

for any such T . Therefore, by continuity and Bolzano's Theorem, if there exists T such that

$$b_{\Theta}(T) - \pi_{\mathbf{b}}(T) \leq 0 \tag{2.16}$$

then there exists T onto the border of Θ such that $b_{\Theta}(T) - \pi_{\mathbf{b}}(T) = 0$, which implies $\mathcal{E}(\mathbf{b}) \neq \emptyset$ according to (2.15).

First, let us consider the case of a large bias. If $\|\mathbf{b}\| \geq \bar{b}_{\Theta}$, choose T such that (OT) and \mathbf{b} have the same direction, so that $\pi_{\mathbf{b}}(T) = \|\mathbf{b}\|$. Then we have

$$b_{\Theta}(T) - \pi_{\mathbf{b}}(T) \leq \bar{b}_{\Theta} - \|\mathbf{b}\| \leq 0,$$

and (2.16) holds for such a T . Thus $\mathcal{E}(\mathbf{b}) \neq \emptyset$ for all biases \mathbf{b} with $\|\mathbf{b}\| \geq \bar{b}_{\Theta}$.

Next, consider the case of a small bias. Suppose that (2.16) does not hold when T spans the full border of Θ . In particular, it does not hold if $T_1 = (0, 1)$ or $T_2 = (0, 0)$, so that for $i \in \{1, 2\}$, $b_{\Theta}(T_i) - \pi_{\mathbf{b}}(T_i) > 0$, *i.e.*

$$\pi_{\mathbf{b}}(T_i) < b_{\Theta}(T_i). \tag{2.17}$$

Note that T_1 and T_2 are such that $(\mathbf{m}_{T_1}, \mathbf{a}_{T_1}) = (\mathbf{m}_{C(c)}, \mathbf{a}_{C(c)})$ for some $c \in (-1, 0)$, and $(\mathbf{m}_{T_2}, \mathbf{a}_{T_2}) = (\mathbf{m}_{A(c')}, \mathbf{a}_{A(c')}) = \rho_{\frac{\pi}{2}}(\mathbf{m}_{C(c')}, \mathbf{a}_{C(c')})$ for some $c' \in (-1, 0)$.²³ In particular, according to Example 2.2, we have $\mathbf{b}_{T_i} = (b_1, b_2)$ for some $(b_1, b_2) \in \mathbb{R}^2$ such that $|b_2 - b_1| < \frac{1}{2}$. Furthermore, since \mathbf{b}_{T_1} is supported on (OT_1) , we have $b_2 = -b_1$. Hence $|b_2 - b_1| = 2|b_1|$ so that $|b_1| < \frac{1}{4}$. Then we obtain:

$$b_{\Theta}(T_1) = \|\mathbf{b}_{T_1}\| = \sqrt{b_1^2 + b_2^2} = |b_1|\sqrt{2} < \frac{\sqrt{2}}{4}.$$

Similarly, we have

$$b_{\Theta}(T_2) = \|\mathbf{b}_{T_2}\| < \frac{\sqrt{2}}{4}.$$

Consequently, from (2.17), \mathbf{b} projects onto (OT_1) and (OT_2) as vectors the norm of them is less than $\frac{\sqrt{2}}{4}$. Since (OT_1) and (OT_2) are orthogonal, we can deduce

$$\|\mathbf{b}\| = \sqrt{\pi_{\mathbf{b}}(T_1)^2 + \pi_{\mathbf{b}}(T_2)^2} < \sqrt{b_{\Theta}(T_1)^2 + b_{\Theta}(T_2)^2} < \sqrt{\frac{1}{4}} = \frac{1}{2}.$$

Then, setting $\mathbf{b} = (b'_1, b'_2) \in \mathbb{R}^2$, it is necessary that $|b'_1| < \frac{1}{2}$ and $|b'_2| < \frac{1}{2}$, so that $|b'_2 - b'_1| \leq |b'_2| + |b'_1| < \frac{1}{2}$. Again, according to Example 2.2, there is some c'' such that $(\mathbf{m}_{C(c'')}, \mathbf{a}_{C(c'')})$ is an equilibrium strategy of $\Gamma_{\mathbf{b}}$, and thus $\mathcal{E}(\mathbf{b}) \neq \emptyset$.

23. By construction we derive $c' = c$ from the symmetries, but this is not useful for the proof.

2.H Proof of Lemma 2.5

We show that given $T(0, \frac{1}{2} + t) \in \{0\} \times (\frac{1}{2}, 1)$, for some $t \in (0, \frac{1}{2})$, there is a unique line \mathcal{L}_T such that (i) \mathcal{L}_T is orthogonal to (OT) , (ii) \mathcal{L}_T partitions Θ through $\Theta = \Theta_1 \cup \Theta_2$, with $|\Theta_i| > 0$, $i \in \{1, 2\}$ and w.l.o.g. $O \in \Theta_1$, and (iii) $\mathbb{E}[\Theta_i] \in (OT)$, $i \in \{1, 2\}$. Notice that the existence of \mathcal{L}_T has already been established and it is sufficient to show that if conditions (i), (ii) and (iii) are satisfied for some \mathcal{L} , then \mathcal{L} is uniquely determined.

Let us parameterize any line \mathcal{L} satisfying these conditions through

$$\mathcal{L} = \{\theta \in \Theta, \theta_1 = 2e\theta_2 + c - e\},$$

so that it passes through the points $(c, \frac{1}{2})$, with $c \leq \frac{1}{2}$, and through $(c + e, 1)$ and $(c - e, 0)$ for some $e > 0$ (see Figure 2.16).

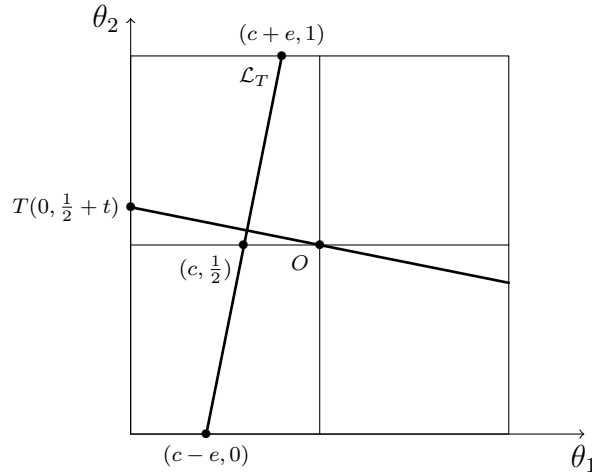


Figure 2.16 – Parametrization of \mathcal{L}_T

Notice that Condition (i) gives $((c + e, 1) - (c, \frac{1}{2})) \cdot ((\frac{1}{2}, \frac{1}{2}) - (0, \frac{1}{2} + t)) = 0$, *i.e.* $e = t$, and in particular,

$$e \in (0, \frac{1}{2}). \quad (2.18)$$

Condition (ii) implies

$$c + e < 1, \text{ and } c - e < \frac{1}{2}, \quad (2.19)$$

since otherwise, we would have $O \notin \Theta_1$. To derive Condition (iii) with respect to c and e , let us compute the expectations with respect to each side of \mathcal{L}_T and with respect to

parameters c and e . We compute

$$\begin{aligned} E_{11}(c, e) &= \frac{\iint_{\theta_1 > 2e\theta_2 + c - e} \theta_1 d\boldsymbol{\theta}}{\iint_{\theta_1 > 2e\theta_2 + c - e} d\boldsymbol{\theta}} = \frac{3c^2 + e^2 - 3}{6(c-1)}, & E_{12}(c, e) &= \frac{\iint_{\theta_1 < 2e\theta_2 + c - e} \theta_1 d\boldsymbol{\theta}}{\iint_{\theta_1 < 2e\theta_2 + c - e} d\boldsymbol{\theta}} = \frac{3c^2 + e^2}{6c}, \\ E_{21}(c, e) &= \frac{\iint_{\theta_1 > 2e\theta_2 + c - e} \theta_2 d\boldsymbol{\theta}}{\iint_{\theta_1 > 2e\theta_2 + c - e} d\boldsymbol{\theta}} = \frac{3c + e - 3}{6(c-1)}, & E_{22}(c, e) &= \frac{\iint_{\theta_1 < 2e\theta_2 + c - e} \theta_2 d\boldsymbol{\theta}}{\iint_{\theta_1 < 2e\theta_2 + c - e} d\boldsymbol{\theta}} = \frac{3c + e}{6c}. \end{aligned}$$

Now we have that \mathcal{L} is orthogonal to $(\mathbf{E}_1 \mathbf{E}_2)$, with $\mathbf{E}_1(E_{11}(c, e), E_{21}(c, e))$ and $\mathbf{E}_2(E_{12}(c, e), E_{22}(c, e))$, iff $(E_{11} - E_{12}, E_{21} - E_{22}) \cdot (e, \frac{1}{2}) = 0$, which gives

$$e^2 = 3c - 3c^2 - \frac{1}{2}. \quad (2.20)$$

According to (2.18), e^2 spans $(0, \frac{1}{4})$ and therefore (2.20) has two solutions $c_1 = \frac{3 - \sqrt{3}\sqrt{1-4e^2}}{6}$ and $c_2 = \frac{3 + \sqrt{3}\sqrt{1-4e^2}}{6}$. Now in particular, (2.19) gives $c_2 + e < 1$, which implies $e < \frac{1}{4}$. It also gives $c_2 - e < \frac{1}{2}$, which implies $e > \frac{1}{4}$. Hence, for any $e \in (0, \frac{1}{2})$, c_2 does not satisfy (2.19), and c_1 is the only solution to (2.20). Hence, there is a unique \mathcal{L} that satisfies conditions (i), (ii) and (iii).

Chapitre 3

Better or not that bad? Asymmetric comparative cheap talk

Abstract. Comparisons are a prevalent ingredient of the circulation of information among economic agents. However, there are multiple ways to compare. A symmetric comparison reveals whether or not one attribute is better than the other. An asymmetric comparison reveals whether one attribute is “not that bad” or “much worse” than the other. If an asymmetric comparison is credible, then the revealed “not that bad” attribute is perceived better, even if it is worse. We investigate a symmetric setting, in which a Sender issues one of two messages about a two-dimensional private type, in order to influence the two-dimensional action of another agent. The Sender’s incentive to exaggerate in each dimension leads any credible message to be comparative. The symmetry of the setting guarantees the credibility of the symmetric comparison. Conditions on the agents’ utility functions are given such that it also allows the existence of a credible (endogenously) asymmetric comparison. Then a stability criterion is established, which eventually points to the asymmetric comparison, if any, and also excludes the most informative symmetric comparison, and any uninformative treatment.

3.1 Introduction

Economic agents often use comparisons as a tool for communication, *e.g.* when an advertiser contrasts the features of a product to influence the buyer's choices, when a manager compares employee's skills to a promote and increase efforts, or when an attorney compares the circumstantial aspects of a defendant's situation to influence the verdict. Note that there are multiple ways to compare. For instance, messages such as "that attribute is *better* than this one" and "that attribute is *not that bad* relative to this one" are both comparative, but transmit different informations. While the former message is issued from a *symmetric comparison*, the latter message is issued from an *asymmetric comparison*. In an asymmetric comparison, one of the message is *ex ante favored*. Indeed, an attribute is not revealed "not that bad" only if it is "much worse", and so it is eventually revealed "not that bad" even if it is worse. Thus, if it is believed, an asymmetric comparison potentially misleads the receiver of the information concerning the relative value of the attributes. This may lead a buyer to invest more in the least valuable product, an employee to exert higher effort in the least productive task, or a prosecutor to under-evaluate a psychosocial dimension.

In this paper, we propose a simple setting that allows to understand the prevalence and the diversity of such *comparative cheap talk*. In particular, we provide relationship between the agents' utility functions and the different ways to compare. We highlight that an *ex ante* symmetric situation sustains the possibility of an *ex post* asymmetric treatment. Moreover, given the multiple ways to compare, we propose a stability criterion, and exhibit situations such that the asymmetric comparison is the unique stable credible information revelation.

We model information transmission from strategic considerations, following the cheap talk literature initiated by Crawford and Sobel's (1982) model. In this model, information is transmitted from a *Sender* (she) to a *Receiver* (he) through abstract messages. Each agent's utility relies on the Sender's private information (her type), and the Receiver's decision (his action). The model addresses equilibria in which the Sender's incentives to reveal some information are generated by the Receiver's subsequent choice of action and reciprocally. Then, the messages used in equilibrium are endogenously determined by the agents' interests. In particular, the model allows to derive the nature of the revealed information from the agents' utility functions.

We investigate a two-dimensional version of the model. We exclude exogenous asymmetries between the dimensions, which would unsurprisingly result in an asymmetric treatment, and focus on *ex ante* symmetric situations. More precisely, we consider two one-dimensional Sender's types θ_1 and θ_2 , realized from two independently and identically distributed random variables over $[0, 1]$. The Receiver takes two one-dimensional actions a_1 and a_2 in \mathbb{R} , after his observation of a message m issued by the Sender. We focus on binary messaging rules, and restrict the messages space to two alternatives m_1 and m_2 (this corresponds, for instance, to the situation where the Sender either approves or disapproves an anticipated profile of actions of the Receiver). As a benchmark, we assume that, in each

dimension, the Receiver prefers to adjust his action to the Sender's type.¹ This allows us to simplify the agents' conflict of interest wrt the Receiver's actions to the Sender's incentives to differentiate her private information from its perception by the Receiver. The Sender's utility function is assumed to be additively separable wrt the two dimensions.

As an illustration, consider an extension of Meade's (1952) example of positive externalities. An apple farmer (the Sender) has two distinct lands 1 and 2, and a beekeeper (the Receiver) produces honey from the trees' blooming. The apple farmer uses two distinct pesticides on land 1 and 2, with respective quality θ_1 and θ_2 . A lower quality hurts the ability of bees to pollinate. The beekeeper invest an amount a_i in beehives near land $i \in \{1, 2\}$, at cost (*e.g.* a cost of effort) $-a_i^2/2$, and then produces honey at level $a_i\theta_i$. Thus, he optimally adjusts his investments at $(a_1, a_2) = (\theta_1, \theta_2)$. However, he is not aware of (θ_1, θ_2) . Given (θ_1, θ_2) , the apple farmer issues a message m_j , among two alternatives m_1 and m_2 . In equilibrium, the issued message m_j reveals credible information on (θ_1, θ_2) , which induces specific investments $(a_1(m_j), a_2(m_j))$ in beehives. The apple farmer's utility function is symmetric and additively separable wrt to the two lands and relies on the qualities of the pesticides and the beekeeper's investments. It is given by $U((a_1, a_2), (\theta_1, \theta_2)) = u(a_1, \theta_1) + u(a_2, \theta_2)$ with u twice continuously differentiable. A message is credible *iff* it is in the Sender's interest to issue it, given the induced investments, and given the qualities of the pesticides.

First, we show that if, whatever her types, the Sender has incentive to exaggerate the values of her types², then any credible message takes the form of a *comparative statement* between the Sender's types (Proposition 3.1). Formally, the two messages issued in equilibrium are necessarily given by m_1 : " $\theta_1 \geq \varphi(\theta_2)$ " and m_2 : " $\theta_1 < \varphi(\theta_2)$ ", for some continuously increasing function $\theta \mapsto \varphi(\theta)$. Function φ characterizes the Sender's information revelation about her types, and its processing by the Receiver. Figure 3.1 depicts such a Sender's φ -comparison.

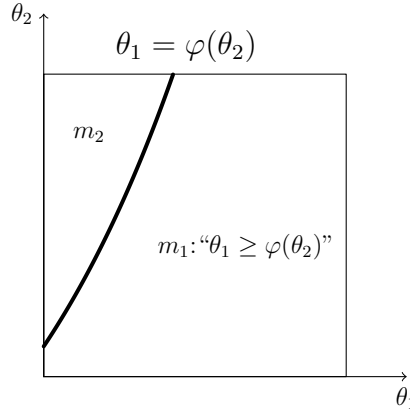


Figure 3.1 – A potential credible information revelation

1. This is obtained when the Receiver has a quadratic utility function wrt the difference of type and action in each dimension. This assumption is usual in the literature.

2. Formally, the conditions are: $\frac{\partial^2 u}{\partial \theta \partial a} > 0$ and for any $a \in [0, 1]$, $\frac{\partial u}{\partial a}(a, 0) \geq 0$.

For instance, the apple farmer has incentive to exaggerate the qualities of the pesticides in order to induce the beekeeper to invest more. Then she might influence the beekeeper if and only if she (implicitly or explicitly) compares the two pesticides. Such an information revelation induces a higher investment near one land and a lower one near the other land, relative to the alternative profile of investment induced by the alternative message. The intuition is straightforward. Relative to the alternative message, a message cannot influence the beekeeper to invest more in the two lands. Indeed, given the Sender's incentives to induce higher actions whatever her types, such a message is always preferred by the apple farmer, and is non credible.

Concerning the existence of φ -profiles of strategies in equilibrium, first note that the full symmetry of the situation wrt the two dimensions guarantees that a symmetric comparison of the Sender's type is credible. Indeed, given the symmetry of the distribution of types, messages $m_1: " \theta_1 \geq \theta_2 "$ and $m_2: " \theta_1 < \theta_2 "$ induces symmetric actions such that $a_1(m_1) = a_2(m_2)$ and $a_1(m_2) = a_2(m_1)$. And reciprocally, given symmetric actions, since the Sender benefits more from higher action with higher type, she has incentives to associate the highest type with the highest action and thereby to symmetrically compare her type.

However, the symmetry of the treatment is not necessary. Suppose that the beekeeper expects m_1 to be issued, unless quality θ_1 is *much worse* than quality θ_2 . Such a belief leads the beekeeper to decide a poor investment near land 1 in case m_2 is unexpectedly issued. The detrimental effect on the production of apples in land 1 precisely provides the apple farmer with the incentives to reveal m_1 whenever θ_1 is "not that bad" relative to θ_2 , and eventually even if it is worse. In particular, the apple farmer confirms the beekeepers' asymmetric expectations. Thus, even if the apple farmer benefits more from higher investment with higher quality, she may have incentives to induce the association of the highest investment with the lowest quality of pesticide.

We guarantee the possibility of a credible asymmetric comparison in case the Sender's utility function is given by $U^S((a_1, a_2), (\theta_1, \theta_2)) = a_1g(\theta_1) + a_2g(\theta_2)$, for functions g such that $g' > 0$, $g(0) = 0$ and g'/g is sufficiently decreasing relative to the distribution of types^{3 4} (Proposition 3.2).

Finally, given the multiplicity of equilibria, we propose a simple stability criterion in order to remove the indeterminacy of the agent's behavior. In case $u(a, \theta) = ag(\theta)$, the set of best response of a player is characterized by a family of functions φ_t , where parameter t reports on the overall *degree of symmetry* of the player's strategy. An equilibrium is stable if, whenever a player deviates from the equilibrium degree of symmetry, the degree of symmetry of the other player's best response reduces the extent of the deviation. It is unstable if the extent of the deviation is exacerbated. Thus, our stability criterion reports on an asymptotic stability of the players' strategies.⁵ We examine the stability

3. The precise condition is $\text{Cov}\left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta\right) > \frac{\mathbb{E}[\theta_1 \geq \theta_2] - \mathbb{E}[\theta_1 < \theta_2]}{8}$.

4. We more generally investigate the case where the Sender's utility function is given by $U^S((a_1, a_2), (\theta_1, \theta_2)) = h(a_1)g(\theta_1) + h(a_2)g(\theta_2)$, and obtain that a credible asymmetric comparison exists if the concavity of h is counterbalanced by the convexity of g or *vice versa*.

5. See for instance Fudenberg and Tirole (1991, section I.2.5) for a discussion of this stability.

of the symmetric equilibrium, of the uninformative (babbling) equilibrium, and of the potential asymmetric equilibria wrt to the Sender's utility function. In particular, we exhibit situations such that the existence of a credible asymmetric comparison ensures its stability, and implies the instability of the other equilibria (Proposition 3.3). For instance, if the Sender's utility function is $u(a, \theta) = a\theta^r$, then there is a unique credible asymmetric comparison *iff* $r < 2$. Whenever it exists, the asymmetric comparison is stable, and the other equilibria are unstable. And the symmetric comparison is stable *iff* there is no asymmetric equilibrium (*iff* $r \geq 2$).

The paper is organized as follows. Section 3.2 presents the related literature. Section 3.3 describes the model setup. Section 3.4 derives the necessity of the comparative nature of the Sender's strategy. Section 3.5 exhibits the possibility of credible asymmetric comparison investigates in case of a multiplicatively separable Sender's utility function, and investigate the stability of the different equilibria. Section 3.6 concludes. Proofs are given in the appendix.

3.2 Related literature

A large literature on strategic information transmission has emerged from the paper of Crawford and Sobel (1982).⁶ Crawford and Sobel (1982) present a framework where the agents communicate on a single dimension. The authors show that costless and unverifiable information transmission is limited by the agents' conflict of interest. In particular, there is no information transmission if conflict is sufficiently large.

However, if agents communicate on two dimensions, Chakraborty and Harbaugh (2007) provide sufficient conditions for allowing the Sender to transmit credible information. These conditions impose symmetries of the game parameters wrt to the two dimensions.⁷ When these conditions hold, Chakraborty and Harbaugh (2007) show that the symmetric comparison of the Sender's types is a credible information revelation. While the authors only consider the symmetric comparison, we extend the investigation to any binary information revelation. We show that if the Sender has incentive to exaggerate the values of her types, then the comparative nature of any credible information revelation is also necessary. Moreover, we establish that the uniqueness of a binary credible information revelation is not guaranteed. Beside the symmetric comparison, the Sender potentially credibly reveals her information asymmetrically.

Chakraborty and Harbaugh (2010) also assumes that the Receiver prefers to adjust his action to the state, and explore the polar case where the Sender's utility function is independent of her types. The authors show that for any such utility function, and for any prior distribution of the types, the Sender might influence the Receiver's action by using two messages. The authors do not address the question of multiple possibilities of

6. See for instance Sobel (2013) for a review of the literature.

7. Levy and Razin (2007) complement Chakraborty and Harbaugh (2007). They show that strong spillover effects across the dimensions might preclude the possibility of any credible information transmission, even on a dimension where agent do not conflict.

communication, and the related question of the nature of the transmitted information. Chakraborty and Harbaugh (2010) also note that the Receiver is always *ex ante* interested in retrieving information from the Sender, while the Sender has interest to communicate on her type *iff* her preferences are quasi-convex wrt the Receiver's action. Indeed, in that case, she prefers mean preserved contrasted actions, which are induced by her information revelation, instead of the actions taken without communication, that correspond to the prior expectations. Our contribution highlights the influence of the convex preferences on the nature of communication. In particular, with convex preferences wrt actions, not only the Sender prefers communication, she is also potentially more informative. More precisely, we show conditional on the concavity of her preferences wrt types, if her preferences wrt actions are sufficiently convex, then the symmetric comparison, which is the most informative equilibrium, is also the unique informative equilibrium. This occurs with more concave preferences wrt to types given more convex preferences wrt to actions and *vice versa*.

Che, Dessein, and Kartik (2013) specifically investigate the role of asymmetries in the Receiver's prior beliefs across the different dimensions. They consider a Sender who recommends one of two projects to a Receiver, who also has the possibility to choose an outside option. The authors show that the Sender's information revelation is comparative, and that she has incentives to "pander" toward the project which has an higher posterior expectation. In particular, she may recommend a project which is *ex ante* stochastically ranked lower than the other project. In our setting, the Receiver does not choose between projects, but chooses different levels of investment in the two dimensions. However, in an asymmetric treatment, the Sender's incentives to asymmetrically compare her types are precisely generated by the different posteriors of the Receiver it determines. Then, in line with Che, Dessein, and Kartik (2013), she may recommend an higher investment in the lowest profitable dimension. In particular, given the full symmetry of the situation, the study illustrates that the pandering effect might also be endogenously obtained, and might occur beside a symmetric treatment. Moreover, in that case, we show that the asymmetric treatment is potentially more stable than the symmetric one.

In line with Crawford and Sobel's (1982) leading example, Sémirat (2017) consider the case of quadratic preferences for both players, and a uniform distribution of types. The players' conflict is represented by a finite constant bias of the Sender relative to the Receiver's preferred action. The author shows that whatever the extent and the direction of the bias, the Sender has multiple ways to credibly reveal information with two messages. The Sender's incentives to exaggerate that we consider report on an "infinite bias" in the positive direction. Our result on the multiple equilibria confirms Sémirat's (2017) result in case of extremely large bias, based on the qualitative aspect of the players' conflict.

Our contribution is also related to the literature on equilibria selection in cheap talk models. The multiplicity of equilibria is a common feature of these models. Sobel (2013, Section 4) decomposes the indeterminacy concerning the player's behavior in three kinds: multiple off-path responses to new messages, multiple meanings of message, and multiple associations between types and actions. The multiplicity of equilibria in our setting is of the third kind of indeterminacy. In the literature, this kind of indeterminacy has only

been investigated in the one-dimensional case, where the criteria select the equilibria according to the number of messages used on the equilibrium path. In the literature, the most informative equilibrium is usually selected, *e.g.* in Chen, Kartik, and Sobel (2008). In our setting, there exist multiple equilibria when the Sender uses two messages. Hence, the criteria proposed in the previous literature do not apply. The criterion that we define is not a general criterion. We specifically exploit the characterization of the equilibria that we obtain. However, it reports on the convergence of the players' strategies to the selected equilibrium. Interestingly, we exhibit situations such that neither the most informative equilibrium, nor the least informative equilibrium is selected, but the in-between asymmetric comparison.

Crawford and Sobel's model (1982) has generated many applications (for instance Sobel (2013) reviews the theoretical literature). Our setting extends Kamphorst and Swank's (2016) application in personnel economics. In Kamphorst and Swank (2016), the authors investigate the information provision of the promotion decision of a manager (the Sender) concerning two employees (two one-dimensional Receivers). Employees choose an effort level after their observation of the manager's decision, and the manager bases her decision on her private observation of the employees' abilities. Kamphorst and Swank (2016) consider a uniform distribution for the Sender's type, and a Sender's linear utility function $u(a, \theta) = a\theta$ in each dimension. The authors interpret the asymmetric treatment as a potential discriminatory practice of the firm. Our result on the existence of a credible asymmetric comparison extends Kamphorst and Swank's (2016) investigation to non uniform priors, and more general preferences for the Sender.

3.3 Model setup

3.3.1 Setting

A Sender possesses private informations concerning two issues. Each private information θ_i , $i \in \{1, 2\}$, is the realization of a random variable, with continuous density f with full support $[0, 1]$ and cumulative distribution function F . A typical realization is the Sender's type, denoted by $\boldsymbol{\theta} = (\theta_1, \theta_2) \in [0, 1]^2$. We let $f(\boldsymbol{\theta}) = f(\theta_1)f(\theta_2)$ denote the probability that $\boldsymbol{\theta}$ is realized.

A Receiver takes actions $a_1 \in \mathbb{R}$ and $a_2 \in \mathbb{R}$ on the respective issues. A typical profile of actions is denoted by $\mathbf{a} = (a_1, a_2)$. Before taking his actions, the Receiver observes a message $m_j \in \{m_1, m_2\}$ delivered by the Sender.⁸

The Receiver's prefers to adjust his action (a_1, a_2) to the Sender's type (θ_1, θ_2) . His utility is given by

$$U^R(\mathbf{a}, \boldsymbol{\theta}) = -\|\mathbf{a} - \boldsymbol{\theta}\|^2.$$

8. More precisely, messages m_1 and m_2 are taken from the infinite set of Borel sets of $[0, 1]^2$. The meaning of messages m_1 and m_2 will be endogenous to the equilibrium.

The Sender's utility is given by

$$U^S(\mathbf{a}, \boldsymbol{\theta}) = u(a_1, \theta_1) + u(a_2, \theta_2),$$

where u is twice continuously differentiable in each of its arguments.

The timing of the game is as follows.

1. Nature draws the state of the world $\boldsymbol{\theta} = (\theta_1, \theta_2) \in [0, 1]^2$, and reveals it to the Sender (it is her type), but not to the Receiver (he has a prior belief);
2. the Sender sends a message $m_j \in \{m_1, m_2\}$ to the Receiver;
3. the Receiver observes the message m_j and updates his prior belief about the Sender's type $\boldsymbol{\theta}$;
4. the Receiver chooses his action $\mathbf{a} = (a_1, a_2) \in \mathbb{R}^2$ according to his posterior belief;
5. payoffs are realized.

3.3.2 Strategies and concept solution

When information transmission is costless and unverifiable, any message can be used. This introduces a credibility requirement on the transmitted information. In equilibrium, the Receiver finds a guaranty on the transmitted information in the Sender's incentives for truth telling. Reciprocally, the Sender finds the incentives for truth telling given the Receiver's subsequent actions that truthful information revelation potentially induces.

Formally, players' pure strategies are as follows. A Sender's pure strategy \mathbf{m} associates to each of her type ($\boldsymbol{\theta}$) and information set ($\{\boldsymbol{\theta}\}$) a message:

$$\begin{aligned} \mathbf{m} : [0, 1]^2 &\rightarrow \{m_1, m_2\} \\ \boldsymbol{\theta} &\mapsto \mathbf{m}(\boldsymbol{\theta}). \end{aligned}$$

A Receiver's pure strategy \mathbf{a} associates to each of her type (\emptyset) and information set (m_j) an action:

$$\begin{aligned} \mathbf{a} : \{m_1, m_2\} &\rightarrow \mathbb{R}^2 \\ m_j &\mapsto \mathbf{a}(m_j) = (a_1(m_j), a_2(m_j)) \end{aligned}$$

Our concept solution is the Bayes-Nash Equilibrium. It relies on the following assumptions.

- (i) Each Receiver's (type contingent) strategy maximizes his (interim) expected payoff, given his posterior belief $f(\cdot|m_j)$, given the received message $m_j \in \{m_1, m_2\}$, given the Sender's strategy \mathbf{m} :

$$\begin{aligned} \mathbf{a}^m(m_j) &= \arg \max_{\mathbf{a} \in \mathbb{R}^2} \iint_{\boldsymbol{\theta} \in [0, 1]^2} U^R(\mathbf{a}, \boldsymbol{\theta}) f(\boldsymbol{\theta}|m_j) d\boldsymbol{\theta}. \\ &= \mathbb{E}_{f(\cdot|m_j)}[\boldsymbol{\theta}] \end{aligned} \tag{R}$$

- (ii) Receiver's prior belief f is updated to the posterior $f(\cdot|m_j)$ according to Bayes's rule whenever it is possible.

- (iii) Each Sender's type contingent strategy maximizes her payoff given the Receiver's strategy \mathbf{a} :

$$\mathbf{m}_{\mathbf{a}}(\boldsymbol{\theta}) = \arg \max_{m_j \in \{m_1, m_2\}} U^S(\mathbf{a}(m_j), \boldsymbol{\theta}). \quad (\text{S})$$

Then an equilibrium profile of strategies $(\mathbf{a}^*, \mathbf{m}^*)$ is such that $\mathbf{a}^* = \mathbf{a}^{\mathbf{m}^*}$ is the best response to \mathbf{m}^* , and $\mathbf{m}^* = \mathbf{m}_{\mathbf{a}^*}$ is the best response to \mathbf{a}^* .

3.3.3 Babbling, influential, comparative, or symmetric equilibria

In cheap talk games, *babbling* equilibria always exists. In pure strategies, they are given by the following strategies. Suppose that the Sender always sends the same message m_j . Then from Bayes's rule, the Receiver's posterior belief $f(\cdot|m_j)$ equals his prior f . Assume that the Receiver's posterior belief $f(\cdot|m_{-j})$ is also equal to f . Then the Receiver takes identical actions $\mathbf{a}(m_1) = \mathbf{a}(m_2)$. Given such actions, for any type, the Sender is indifferent between the two messages. Hence, she has (weak) incentives to always send the same message.

Following Sobel (2013), we define an *influential equilibrium* when the Receiver's action strategy $\mathbf{a} : m_j \mapsto \mathbf{a}(m_j)$ is not constant on the equilibrium path. This requires that m_1 and m_2 are both on the equilibrium path. If a message m_j is on the equilibrium path, then Bayes's rule implies, for any $\boldsymbol{\theta} \in [0, 1]^2$,

$$f(\boldsymbol{\theta}|m_j) = \begin{cases} \frac{f(\boldsymbol{\theta})}{\Pr(\mathbf{m}(\boldsymbol{\theta})=m_j)} & \text{if } \mathbf{m}(\boldsymbol{\theta}) = m_j, \\ 0 & \text{otherwise,} \end{cases}$$

and thus the best response of the Receiver upon the reception of m_j is to act at

$$\mathbf{a}^{\mathbf{m}}(m_j) = \mathbb{E}[\boldsymbol{\theta}|\mathbf{m}(\boldsymbol{\theta}) = m_j],$$

where $\mathbb{E} = \mathbb{E}_f$ denotes the expectation according to f .

Since we are interested in comparisons, let us define the comparative way to transmit information. An information revelation \mathbf{m} is said *comparative* if the message sent contrasts the two types. Formally, up to a relabeling of the messages, \mathbf{m} is comparative if whenever the Sender is indifferent between m_1 and m_2 at some $(\theta_1, \theta_2) \in (0, 1)^2$, then she (weakly) prefers to send m_1 at any higher θ_1 or lower θ_2 , and she (weakly) prefers to send m_2 at any lower θ_1 or higher θ_2 . For instance, messages m_1 : " $\theta_1 \geq 2\theta_2$ " and m_2 : " $\theta_1 < 2\theta_2$ " define a comparative information revelation. Messages m_1 : " $\theta_1 + \theta_2 \geq 1$ " and m_2 : " $\theta_1 + \theta_2 < 1$ " do not. More generally, messages m_1 : " $\psi(\theta_1, \theta_2) \geq 0$ " and m_2 : " $\psi(\theta_1, \theta_2) < 0$ " define a comparative information revelation iff $(\psi(\theta_1, \theta_2) = 0 \iff \theta_1 = \varphi(\theta_2))$ for some non decreasing function φ .

Finally, a profile of strategies is said *symmetric* (wrt the dimensions $i = 1$ and $i = 2$ of the state space $[0, 1]^2$ and action space \mathbb{R}^2) if $\mathbf{m}(\theta_1, \theta_2) = m_1$ iff $\mathbf{m}(\theta_2, \theta_1) = m_2$, and if $a_1(m_1) = a_2(m_2)$, $a_1(m_2) = a_2(m_1)$.

3.4 Comparative equilibria

In this section, we give conditions on the Sender's utility function which characterize the information revelations that she may credibly use as comparative ones. Chakraborty and Harbaugh (2007) show that in a symmetric setting, if the players have a minimal commonality of interest (if they both prefer to associate higher action with higher type), then the Sender might credibly and symmetrically compare her two types. Chakraborty and Harbaugh's (2007) conditions are $\frac{\partial^2 u^P}{\partial \theta \partial a} > 0$, $P \in \{S, R\}$, where u^P is the utility function of player P in each dimension $i \in \{1, 2\}$. In our setting, since $\frac{\partial^2 u^R}{\partial \theta \partial a} = \frac{\partial^2 (-(a-\theta)^2)}{\partial \theta \partial a} = 2 > 0$, the remaining condition is that the Sender's utility function satisfies $\frac{\partial^2 u}{\partial \theta \partial a} > 0$. This implies, according to Chakraborty and Harbaugh (2007, Proposition 1), that messages m_1 : " $\theta_1 \geq \theta_2$ ", m_2 : " $\theta_1 < \theta_2$ ", and actions $\mathbf{a}(m_1) = \mathbb{E}[\boldsymbol{\theta} | \theta_1 \geq \theta_2]$, $\mathbf{a}(m_2) = \mathbb{E}[\boldsymbol{\theta} | \theta_1 < \theta_2]$, define an equilibrium profile of strategies. Note that the defined strategies are symmetric wrt dimensions $i = 1$ and $i = 2$. Thus, in our game, if $\frac{\partial^2 u}{\partial \theta \partial a} > 0$, the (symmetric) comparative way to transmit and process information is a sufficient condition for credibility. The next proposition shows that if whatever her types, the Sender has incentives to exaggerate their values, then the comparative nature of any binary information revelation is also necessary.

Proposition 3.1. *If for any $(a, \theta) \in [0, 1]^2$,*

$$\frac{\partial^2 u}{\partial \theta \partial a}(a, \theta) > 0 \text{ and } \frac{\partial u}{\partial a}(a, 0) \geq 0,$$

then a profile of strategies $(\mathbf{m}^, \mathbf{a}^*)$ defines an influential equilibrium profile of strategies iff, up to a relabeling of the dimensions, and up to a relabeling of the messages, there exists a non decreasing continuous function $\varphi : [0, 1] \rightarrow [0, 1]$, and equilibrium strategies \mathbf{m}_φ and \mathbf{a}^φ defined by, for any $\boldsymbol{\theta} = (\theta_1, \theta_2) \in [0, 1]^2$,*

$$\mathbf{m}_\varphi(\boldsymbol{\theta}) = \begin{cases} m_1 & \text{if } \theta_1 \geq \varphi(\theta_2), \\ m_2 & \text{if } \theta_1 < \varphi(\theta_2), \end{cases}$$

and, for any $m \in \{m_1, m_2\}$,

$$\mathbf{a}^\varphi(m) = \begin{cases} \mathbb{E}[\boldsymbol{\theta} | \theta_1 \geq \varphi(\theta_2)] & \text{if } m = m_1, \\ \mathbb{E}[\boldsymbol{\theta} | \theta_1 < \varphi(\theta_2)] & \text{if } m = m_2, \end{cases}$$

such that $(\mathbf{m}^, \mathbf{a}^*) = (\mathbf{m}_\varphi, \mathbf{a}^\varphi)$. In particular, it is necessary and sufficient that \mathbf{m}^* is a credible comparative information revelation, with best response \mathbf{a}^* .*

If $\frac{\partial^2 u}{\partial \theta \partial a} > 0$ and $\frac{\partial u}{\partial a}(a, 0) \geq 0$ holds for any $a \in [0, 1]$, then Proposition 3.1 identifies the set of equilibria of the game with the set of continuously increasing functions $\varphi : [0, 1] \rightarrow [0, 1]$ such that \mathbf{m}_φ is a best response to \mathbf{a}^φ , and reciprocally. Figure 3.2 depicts a non credible information revelation. In Figure 3.2, function φ is not increasing. Figure

3.3 depicts a potential credible information revelation. In Figure 3.3, message m_1 is sent whenever θ_1 is “not so low” relative to θ_2 . If instead θ_1 is “much worse” than θ_2 , then message m_2 is sent.

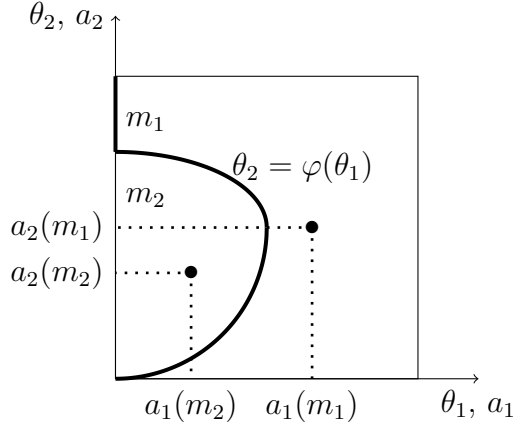


Figure 3.2 – A non credible non comparative information revelation

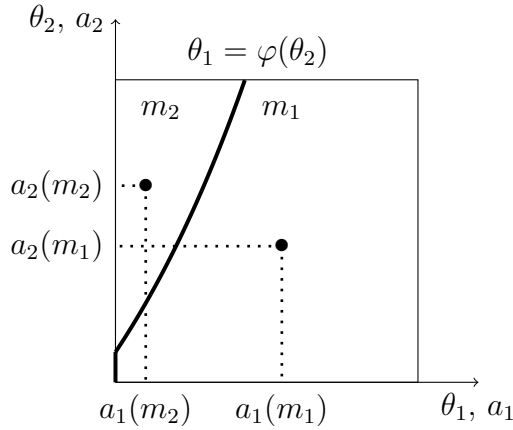


Figure 3.3 – A potentially credible comparative information revelation

The following illustrates Proposition 3.1 with specific functional form of the Sender’s utility function.

Example 3.1. Suppose that $f = 1$ is uniform, and consider the symmetric comparative strategies $(\mathbf{m}_C, \mathbf{a}^C)$, defined by

$$\mathbf{m}_C(\boldsymbol{\theta}) = \begin{cases} m_1 & \text{if } \theta_1 \geq \theta_2, \\ m_2 & \text{if } \theta_1 < \theta_2, \end{cases} \quad \text{and } \mathbf{a}^C(m_j) = \begin{cases} (\frac{2}{3}, \frac{1}{3}) & \text{if } m_j = m_1, \\ (\frac{1}{3}, \frac{2}{3}) & \text{if } m_j = m_2, \end{cases}$$

and the non comparative strategies $(\mathbf{m}_A, \mathbf{a}^A)$, defined by

$$\mathbf{m}_A(\boldsymbol{\theta}) = \begin{cases} m_1 & \text{if } \theta_1 + \theta_2 \geq 1, \\ m_2 & \text{if } \theta_1 + \theta_2 < 1, \end{cases} \quad \text{and } \mathbf{a}^A(m_j) = \begin{cases} (\frac{2}{3}, \frac{2}{3}) & \text{if } m_j = m_1, \\ (\frac{1}{3}, \frac{1}{3}) & \text{if } m_j = m_2. \end{cases}$$

Then the following points are straightforward to verify.

- Assume $U^S(\mathbf{a}, \boldsymbol{\theta}) = a_1\theta_1 + a_2\theta_2$, so that the Sender has strong incentives to exaggerate the values of her types. Then we have $\frac{\partial^2 u}{\partial \theta \partial a} = 1 > 0$ and for any $a \in \mathbb{R}$, $\frac{\partial u}{\partial a}(a, 0) = 0 \geq 0$, so that both assumptions in Proposition 3.1 hold. The symmetrically comparative $(\mathbf{m}_C, \mathbf{a}^C)$ is an equilibrium profile of strategies, but $(\mathbf{m}_A, \mathbf{a}^A)$ is not.
- Suppose $U^S(\mathbf{a}, \boldsymbol{\theta}) = U^R(\mathbf{a}, \boldsymbol{\theta}) = -(a_1 - \theta_1)^2 - (a_2 - \theta_2)^2$, so that players do not conflict. In particular, the Sender has no incentive to exaggerate the values of her types. Then we have $\frac{\partial^2 u}{\partial \theta \partial a} = 2 > 0$, and for any $a \in [0, 1]$, $\frac{\partial u}{\partial a}(a, 0) = -2a < 0$. Therefore, only the first assumption in Proposition 3.1 holds. The symmetrically comparative $(\mathbf{m}_C, \mathbf{a}^C)$ is an equilibrium profile of strategies, but so is $(\mathbf{m}_A, \mathbf{a}^A)$. In particular, a credible binary information revelation is not necessarily comparative.

Let us provide details on the mechanism that sustains Proposition 3.1. Given (θ_1, θ_2) , the Sender prefers to send m_1 rather than m_2 iff $u(a_1(m_1), \theta_1) + u(a_2(m_1), \theta_2) \geq u(a_1(m_2), \theta_1) + u(a_2(m_2), \theta_2)$. Equivalently, her messaging rule is:

$$\mathbf{m}(\theta_1, \theta_2) = m_1 \iff u(a_1(m_1), \theta_1) - u(a_1(m_2), \theta_1) \geq u(a_2(m_2), \theta_2) - u(a_2(m_1), \theta_2). \quad (3.1)$$

Assume $\frac{\partial^2 u}{\partial \theta \partial a} > 0$ and for any $a \in [0, 1]$, $\frac{\partial u}{\partial a}(a, 0) \geq 0$. Suppose that $a_2(m_2) > a_2(m_1)$ (up to a relabeling of the messages, this is wlog). Let us show that this necessarily implies $a_1(m_1) > a_1(m_2)$. From the assumptions, for any $\theta \in [0, 1]$, we have

$$u(a_2(m_2), \theta) - u(a_2(m_1), \theta) > u(a_2(m_2), 0) - u(a_2(m_1), 0) \geq 0,$$

so that along dimension 2, the Sender has incentives to exaggerate the value of her type $\theta = \theta_2$ whatever its value. However, in an influential equilibrium, there exists $(\theta_1, \theta_2) \in [0, 1]^2$ such that the Sender nevertheless prefers to send m_1 rather than m_2 . According to (3.1), and the above inequality applied to $\theta = \theta_2$, we obtain

$$u(a_1(m_1), \theta_1) - u(a_1(m_2), \theta_1) \geq u(a_2(m_2), \theta_2) - u(a_2(m_1), \theta_2) > 0.$$

This precisely implies $a_1(m_1) > a_1(m_2)$, since otherwise, again from the assumptions, we would have $u(a_1(m_1), \theta_1) - u(a_1(m_2), \theta_1) < u(a_1(m_1), 0) - u(a_1(m_2), 0) \leq 0$.

Now from $a_2(m_2) > a_2(m_1)$ and $a_1(m_1) > a_1(m_2)$, the Sender's messaging rule (3.1) issues the result of a comparison between two “net utilities”. More precisely, in each dimension i , when she choses message $m_{j=i}$, she obtains the “revenue” $u(a_i(m_{j=i}), \theta_i)$, diminished by the “opportunity cost” $u(a_i(m_{j \neq i}), \theta_i)$. Moreover, in each dimension i , from $\frac{\partial^2 u}{\partial \theta \partial a} > 0$, the net utility is strictly increasing with the corresponding type θ_i . Hence,

given the profiles of actions, when the Sender compares the two net utilities, she implicitly compares her two types θ_1 and θ_2 .

Note that in case the induced actions are symmetric, *i.e.* $a_1(m_1) = a_2(m_2)$ and $a_1(m_2) = a_2(m_1)$, the sides of the inequality in (3.1) are identically equal as functions of θ_i . This implies that the Sender symmetrically compares her types, *i.e.* function φ of Proposition 3.1 is $\varphi = \text{Id}$. Given the symmetry of the types distribution, such a comparison induces the symmetry of the induced actions. This ensures the credibility of the symmetric comparison, as exhibited by Chakraborty and Harbaugh (2007). However, in case actions are not symmetric, *i.e.* $a_1(m_1) \neq a_2(m_2)$ or $a_1(m_2) \neq a_2(m_1)$, then we have $\varphi \neq \text{Id}$. In that case, Proposition 3.1 does not guarantee the existence of a $\varphi \neq \text{Id}$ such that $(\mathbf{m}_\varphi, \mathbf{a}^\varphi)$ are equilibrium strategies. We address this question in the next question.

Before that, let us note the following points concerning Proposition 3.1.

First, Proposition 3.1 also characterizes the mixed strategies of the players, up to a zero measure set. Indeed, concerning the Receiver, note that for any $j \in \{1, 2\}$, $\mathbf{a}^m(m_j) = \mathbb{E}_{f(\cdot|m_j)}[\theta]$ is uniquely determined by $f(\cdot|m_j)$. Therefore, given his observation of m_j , the Receiver's preferred action is unique, and he never uses non degenerate mixed strategies in equilibrium. Concerning the Sender, she has (weak) interest to mix only at the set of types where she is indifferent between the two messages. *Whatever* $\mathbf{a}(m_1) \neq \mathbf{a}(m_2)$, this set is characterized by a one dimensional equality that relates θ_1 and θ_2 . In particular, it is a zero measure set.

Second, from the full symmetry of the game parameters, if messages m_1 : " $\theta_1 \geq \varphi(\theta_2)$ " and m_2 : " $\theta_1 < \varphi(\theta_2)$ " are credible, then so are m_1 : " $\theta_2 \geq \varphi(\theta_1)$ " and m_2 : " $\theta_2 < \varphi(\theta_1)$ ". In particular, the set of credible asymmetric comparisons, if not empty, has an even number of elements, which are paired wrt to the symmetry of the setting.

3.5 Asymmetric comparisons

According to Proposition 3.1, when the Sender has incentives to exaggerate the values of her types, the set of equilibria of the game is identified with the set of continuously increasing functions $\varphi : [0, 1] \mapsto [0, 1]$ such that \mathbf{m}_φ is a best response to \mathbf{a}^φ and reciprocally. The case $\varphi = \text{Id}$ recovers the symmetric comparative equilibrium exhibited by Chakraborty and Harbaugh (2007). A credible asymmetric comparison is obtained if there exists an equilibrium profile of strategies $(\mathbf{m}_\varphi, \mathbf{a}^\varphi)$ such that $\varphi \neq \text{Id}$. Note that given the full symmetry of the game parameters, the asymmetry of any credible asymmetric comparison is endogenous.

In this section, we provide conditions on the Sender's utility function that guarantee the existence of a credible asymmetric comparison. To this end, we assume that there exist twice continuously differentiable functions g and h such that for any $(a, \theta) \in \mathbb{R} \times [0, 1]$,

$$u(a, \theta) = h(a)g(\theta). \quad (3.2)$$

We assume that the condition $\frac{\partial^2 u}{\partial \theta \partial a} > 0$ in Proposition 3.1 holds, *i.e.* $h'g' > 0$. We furthermore assume that $g(0) = 0$, so that for any $a \in \mathbb{R}$, $u(a, 0) = 0$. In particular, the

second condition $\frac{\partial u}{\partial a}(a, 0) \geq 0$ in Proposition 3.1 (weakly) holds. In words, the Sender has weak (resp. strict) incentives to exaggerate the value of her type in each dimension if her type is 0 (resp. > 0).⁹ Note that the assumptions imply that for any $\theta \in [0, 1]$, $g'(\theta) \neq 0$, and thus g is monotonous. From $g(0) = 0$, we have that either g is strictly increasing and positive, or g is strictly decreasing and negative. Also note that h is increasing in the former case, and decreasing in the latter case.

Given these assumptions, Proposition 3.1 ensures that any credible binary information revelation is comparative. The assumption (3.2) allows us to furthermore characterize players' strategies $(\mathbf{m}_\varphi, \mathbf{a}^\varphi)$ in terms of a one dimensional parameter $t \in (0, 1]$ (instead of φ , which is an infinite dimensional parameter). To see this, recall that φ is the function such that the Sender's comparison between

$$u(a_1(m_1), \theta_1) - u(a_1(m_2), \theta_1) = g(\theta_1)(h(a_1(m_1)) - h(a_1(m_2))),$$

and

$$u(a_2(m_2), \theta_2) - u(a_2(m_1), \theta_2) = g(\theta_2)(h(a_2(m_2)) - h(a_2(m_1))),$$

identifies, up to a relabeling of dimensions or messages, with the comparison between θ_1 and $\varphi(\theta_2)$. Therefore, up to relabeling of dimensions or messages, φ is defined, for any $\theta \in [0, 1]$, by

$$\varphi(\theta) = g^{-1} \left(\frac{h(a_2^\varphi(m_2)) - h(a_2^\varphi(m_1))}{h(a_1^\varphi(m_1)) - h(a_1^\varphi(m_2))} g(\theta) \right), \quad (3.3)$$

with $\frac{h(a_2^\varphi(m_2)) - h(a_2^\varphi(m_1))}{h(a_1^\varphi(m_1)) - h(a_1^\varphi(m_2))} \in (0, 1]$. In particular, the Sender's best response to given profiles of action is fully determined by the ratio $\frac{h(a_2^\varphi(m_2)) - h(a_2^\varphi(m_1))}{h(a_1^\varphi(m_1)) - h(a_1^\varphi(m_2))}$. Therefore, any Sender's strategy \mathbf{m}_φ in equilibrium is characterized by some $t \in (0, 1]$ such that

$$\varphi = \varphi_t = g^{-1}(tg).$$

Reciprocally, any Receiver's strategy \mathbf{a}^φ in equilibrium is the best response to some Sender's strategy \mathbf{m}_{φ_t} , associated with some $t \in (0, 1]$. Hence, it is also determined by some $t \in (0, 1]$, as $\mathbf{a}^{\varphi_t} = \mathbf{a}^{\mathbf{m}_{\varphi_t}}$.

We obtain that $(\mathbf{m}^*, \mathbf{a}^*) = (\mathbf{m}_{\varphi^*}, \mathbf{a}^{\varphi^*})$ is an equilibrium profile of strategies *iff* there exists $t^* \in (0, 1]$ such that $\varphi^* = \varphi_{t^*} = g^{-1}(t^*g)$, and $(\mathbf{m}_{\varphi_{t^*}}, \mathbf{a}^{\varphi_{t^*}})$ is an equilibrium profile of strategies. Let us now characterize the corresponding t^* . Given $t \in (0, 1]$, let $\text{Br}^S(t)$ be such that $\mathbf{m}_{\varphi_{\text{Br}^S(t)}}$ is the sender's best response to the φ_t -profiles of actions $\mathbf{a}^{\varphi_t}(m_j)$, $j \in \{1, 2\}$. According to (3.3), $\text{Br}^S(t)$ is given by

$$\text{Br}^S(t) = \frac{h(a_2^{\varphi_t}(m_2)) - h(a_2^{\varphi_t}(m_1))}{h(a_1^{\varphi_t}(m_1)) - h(a_1^{\varphi_t}(m_2))}.$$

In equilibrium, the Sender's $\varphi_{\text{Br}^S(t^*)}$ -comparison precisely induces actions $\mathbf{a}^{\varphi_{t^*}}(m_j)$, $j \in \{1, 2\}$, and reciprocally. This implies the equilibrium condition $\varphi_{\text{Br}^S(t)} = \varphi_t$, *i.e.* $(\mathbf{m}_{\varphi_t^*}, \mathbf{a}^{\varphi_t^*})$

9. Formally, since $h'g' > 0$, for any $a^+ > a^-$, and any $\theta \in (0, 1]$, $u(a^+, \theta) - u(a^-, \theta) = g(\theta)(h(a^+) - h(a^-)) = (g(\theta) - g(0))(h(a^+) - h(a^-)) = g'(\tilde{\theta})h'(\tilde{a}) > 0$, for some $\tilde{\theta} \in (0, \theta]$ and $\tilde{a} \in [a^-, a^+]$.

is an equilibrium profile of strategies *iff*, up to relabeling of dimension or messages, $t^* \in (0, 1]$ is a solution of

$$\text{Br}^S(t) = t. \quad (3.4)$$

In words, the equilibrium condition is that the Sender's comparison, determined by t , and associated with $\text{Br}^S(t)$, precisely induces the Receiver's profiles of actions that determine it.

Note that if $t = 1$, then the Sender compares θ_1 and $\varphi_t(\theta_2) = g^{-1}(1g(\theta_2)) = \theta_2$. Therefore, $t = 1$ characterizes the symmetric credible comparison. Hence, as depicted in Figure 3.4, $t = 1$ is always a solution of (3.4). Let us show that more generally, t drives the *overall degree of symmetry* of the players' strategies wrt the two dimensions. If $0 < t_1 < t_2 < 1$, then $t_1 g < t_2 g$ if g is increasing and positive, and $t_1 g > t_2 g$ if g is decreasing and negative. Then in both cases, we obtain $g^{-1}(t_1 g) < g^{-1}(t_2 g)$, *i.e.* $\varphi_{t_1} < \varphi_{t_2}$. Then, given (θ_1, θ_2) , if $m_1: " \theta_1 \geq \varphi_{t_1}(\theta_2) "$ is issued from the φ_{t_1} -comparison, then $\theta_1 \geq \varphi_{t_2}(\theta_2)$, and thus $m_1: " \theta_1 \geq \varphi_{t_1}(\theta_2) "$ is also issued from the φ_{t_2} -comparison. In other words, as t decreases from 1 to 0, the player's strategies favor more and more message m_1 . For instance, when $t \simeq 0$, the Sender compares θ_1 and, from $g(0) = 0$, $\varphi_t(\theta_2) = g^{-1}(tg(\theta_2)) \simeq g^{-1}(0) = 0$. In that case, she almost always issues $m_1: " \theta_1 \geq 0 "$ and the comparison is strongly asymmetric in favor of m_1 .

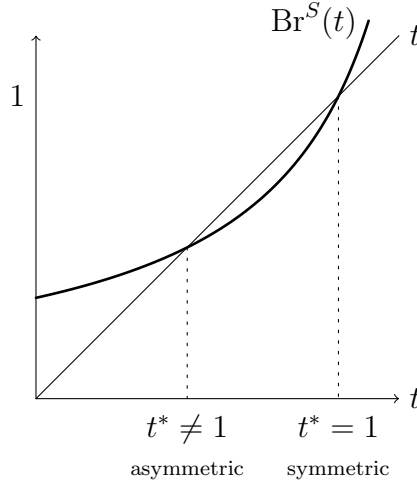


Figure 3.4 – Equilibrium conditions in terms of $t \in (0, 1]$

In the three next sections, we derive conditions on the existence of a solution $t \neq 1$ of (3.4), which corresponds to a credible asymmetric comparison. To do this, we use a continuity argument on the degree of symmetry t of the players' strategies.¹⁰ We give conditions that ensure the existence of a t such that $\text{Br}^S(t) > t$, and conditions that ensure the existence of a t such that $\text{Br}^S(t) < t$. The corresponding conditions are identified by

10. Since we are looking for a *second* solution to $\text{Br}^S(t) = t$, we cannot use general fixed point theorems such as Brouwer's Theorem, or, as in Chakraborty and Harbaugh (2010), Borsuk-Ulam's Theorem.

examining the two extreme cases $t > 0, t \simeq 0$, and $t < 1, t \simeq 1$. The two cases respectively correspond to strongly asymmetric comparisons in favor of m_1 , and weakly asymmetric comparisons in favor of m_1 .

3.5.1 Strongly asymmetric comparisons

Our goal is to examine the degree of symmetry $\text{Br}^S(t)$ of the Sender's best response to strongly asymmetric profiles of actions $\mathbf{a}^{\varphi_t}(m_j)$, $j \in \{1, 2\}$, $t > 0, t \simeq 0$. The mechanism is depicted in Figure 3.5. If t is sufficiently close to 0, then, if h is convex or linear, the Sender's best response asymmetry $\text{Br}^S(t)$ reduces the asymmetry t of the Receiver's profiles of actions, so that $\text{Br}^S(t) > t$, and if h is sufficiently concave, then the Sender's best response $\text{Br}^S(t)$ exacerbates the asymmetry t in favor of m_1 , so that $\text{Br}^S(t) < t$.

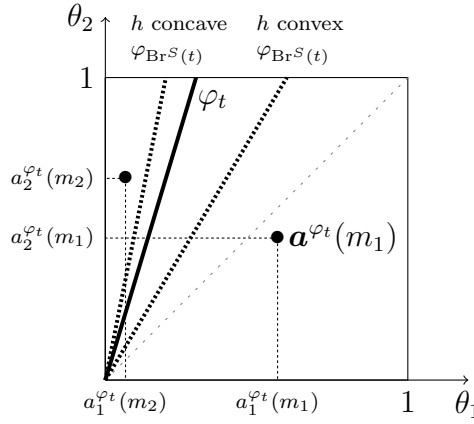


Figure 3.5 – The Sender's best response to strongly asymmetric profiles of actions

To get an intuition, let us first examine the profiles of induced actions if $t \simeq 0$, and then the corresponding Sender's incentives to issue m_1 or m_2 . If $t \simeq 0$, then for any $\theta \in [0, 1]$, $\varphi_t(\theta) = g^{-1}(tg(\theta)) \simeq g^{-1}(0)$, and since $g(0) = 0$, $\varphi_t(\theta) \simeq 0$. Therefore, a strongly asymmetric φ_t -comparison in favor of m_1 compares θ_1 and $\varphi_t(\theta_2) \simeq 0$. In particular, message m_1 mostly reveals m_1 : " $\theta_1 \geq 0$ ", so that it is almost uninformative. Therefore, whatever the distribution of types f , profile $\mathbf{a}^{\varphi_t}(m_1) = (a_1^{\varphi_t}(m_1), a_2^{\varphi_t}(m_1))$ is very close to the prior expectation $(\mathbb{E}, \mathbb{E}) = (\mathbb{E}_f[\theta_1], \mathbb{E}_f[\theta_2])$. In contrast, message m_2 : " $0 \simeq \varphi_t(\theta_2) > \theta_1$ " is highly informative on θ_1 . More precisely, it encompasses only very low values of θ_1 . Therefore, it induces a very low action $a_1^{\varphi_t}(m_2) \simeq 0$. Concerning $a_2^{\varphi_t}(m_2)$, as depicted in Figure 3.5, its value as $t \rightarrow 0$ relies on the increasing of $\theta_2 \mapsto \lim_{t \rightarrow 0} \varphi_t(\theta_2)$.¹¹ More precisely, from the independence of θ_1 and θ_2 , a slowly increasing φ (in Figure 3.5, a more vertical graph), induce action $a_2^{\varphi_t}(m_2)$ to get closer to $\mathbb{E} = \mathbb{E}_f[\theta_2]$. We obtain the following properties of the induced actions.

11. For instance, if f is uniform, and $\varphi_t = t \text{Id}$, then it is straightforward to verify that $\mathbb{E}_f[\theta_2 | t\theta_2 > \theta_1] = 2/3$ is independent of t .

Lemma 3.1. *If t is sufficiently close to 0, then*

$$\frac{a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1)}{a_1^{\varphi_t}(m_1) - a_1^{\varphi_t}(m_2)} > t, \quad (3.5)$$

and if furthermore $\mathbb{E} \left[\frac{f(0)}{g'(0)/g(\theta)} \right] < 1$, then

$$a_2^{\varphi_t}(m_2) > a_1^{\varphi_t}(m_1) > a_2^{\varphi_t}(m_1) > a_1^{\varphi_t}(m_2). \quad (3.6)$$

Given such profiles of actions, let us now examine the Sender's incentives to issue m_1 or m_2 . Note that $\text{Br}^S(t)$ decomposes through

$$\text{Br}^S(t) = \frac{h(a_2^{\varphi_t}(m_2)) - h(a_2^{\varphi_t}(m_1))}{h(a_1^{\varphi_t}(m_1)) - h(a_1^{\varphi_t}(m_2))} = \frac{a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1)}{a_1^{\varphi_t}(m_1) - a_1^{\varphi_t}(m_2)} \frac{\frac{h(a_2^{\varphi_t}(m_2)) - h(a_2^{\varphi_t}(m_1))}{a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1)}}{\frac{h(a_1^{\varphi_t}(m_1)) - h(a_1^{\varphi_t}(m_2))}{a_1^{\varphi_t}(m_1) - a_1^{\varphi_t}(m_2)}}. \quad (3.7)$$

Then for instance, if h is convex, inequality (3.5), ranking (3.6) and decomposition (3.7) yields $\text{Br}^S(t) > t$. In words, if h is convex, the Sender prefers to induce the contrasted profile of actions $\mathbf{a}^{\varphi_t}(m_2) = (a_1^{\varphi_t}(m_2), a_2^{\varphi_t}(m_2))$, where $a_1^{\varphi_t}(m_2) \simeq 0$ and $a_2^{\varphi_t}(m_2)$ is sufficiently greater than \mathbb{E} , rather than the profile $\mathbf{a}^{\varphi_t}(m_1)$ which is close to (\mathbb{E}, \mathbb{E}) . In that case, she is very likely to send m_2 : “ $\theta_1 < \varphi_t(\theta_2)$ ”, even when her type (θ_1, θ_2) is such that $\theta_1 \geq \varphi_t(\theta_2)$. This rules out the credibility of message m_2 .

Corollary 3.1. *If $\mathbb{E} \left[\frac{f(0)}{g'(0)/g(\theta)} \right] < 1$, h is convex or linear, then there exists $t_{g,h}^- \in (0, 1]$ such that for any $t \in (0, t_{g,h}^-)$, $\text{Br}^S(t) > t$, and in particular, strategies $(\mathbf{m}^{\varphi_t}, \mathbf{a}_{\varphi_t})$ are not equilibrium strategies.*

Note that if h is linear, then $\text{Br}^S(t)$ simplifies to $\frac{a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1)}{a_1^{\varphi_t}(m_1) - a_1^{\varphi_t}(m_2)}$. In that case, the same result is obtained from (3.5).

Corollary 3.2. *If h is linear, then if t is sufficiently close to 0, then $\text{Br}^S(t) > t$.*

Note that similarly, ranking (3.6) and the decomposition (3.7) precludes the credibility of message m_1 if h is sufficiently concave relative to the (finite) value of $\frac{a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1)}{a_1^{\varphi_t}(m_1) - a_1^{\varphi_t}(m_2)}$. In that case, we obtain $\text{Br}^S(t) < t$. In particular, given f , given g such that $\mathbb{E} \left[f(0) \frac{g(\theta)}{g'(0)} \right] < 1$, and given t sufficiently close to 0, there exists a specific degree of concavity of h such that $\text{Br}^S(t) = t$. Hence, a specific degree of concavity of h yields the existence of a credible asymmetric comparison.

3.5.2 Weakly asymmetric comparisons

Next, we investigate the degree of symmetry $\text{Br}^S(t)$ of the Sender's best response to “almost symmetric” profiles of actions \mathbf{a}^{φ_t} , with $t \simeq 1$, $t < 1$. The mechanism is depicted in Figure 3.6.

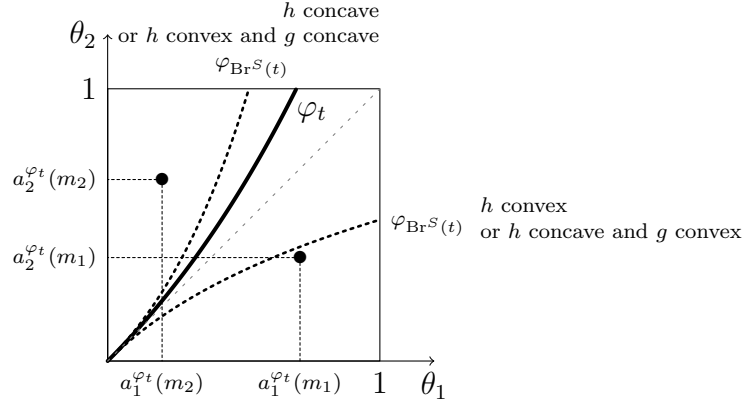


Figure 3.6 – The Sender’s best response to weakly asymmetric profiles of actions

For instance, if g is sufficiently concave, then $\varphi_t(\theta_2) = g^{-1}(tg(\theta_2))$ is substantially lower than θ_2 when θ_2 is high, and instead it is closer to θ_2 when θ_2 is low.¹² Hence, the more concave g , the more increasing $\varphi - \text{Id}$. Thus, the asymmetric treatment between θ_1 and θ_2 , which is driven by the comparison of θ_1 and $\varphi_t(\theta_2)$, is exacerbated when types are high. From the Receiver’s perspective, this implies that, with a more concave g , the favored message m_1 : “ $\theta_1 \geq \varphi_t(\theta_2)$ ” encompasses more high values of θ_1 and θ_2 . Then, induced actions $a_1^{\varphi_t}(m_1)$ and $a_2^{\varphi_t}(m_1)$ are increased when t slightly decreases from $t = 1$.¹³ By symmetry, we also obtain a lower $a_2^{\varphi_t}(m_2)$ and a lower $a_1^{\varphi_t}(m_2)$. Moreover, the effect is strengthened when $f(\theta)$ is more dense at high values.

Now, recall that $g(\theta_1)h(a_1^{\varphi_t}(m_1))$ represents the “revenue” part of the Sender’s net utility that she derives in dimension 1 from sending m_1 , and that $g(\theta_2)h(a_2^{\varphi_t}(m_1))$ represents the “opportunity cost” part of not sending m_1 . Thus, *ceteris paribus*, higher values for both actions, relative to the values associated with $t = 1$, leads the Sender to more prefer to send m_1 . By symmetry (at $t = 1$), a similar but reversed effect is applied to the m_2 -induced actions. Hence, the effect that leads the Sender to prefer to send m_1 is reinforced when the m_2 -induced actions are taken into her comparison of net utilities. In particular, the Sender exacerbates any slight asymmetry, associated with $t \simeq 1$, $t < 1$, of the φ_t -profiles of actions. This precisely implies $\text{Br}^S(t) < t$.

The next lemma formally gives the variation of the degree of symmetry $\text{Br}^S(t)$ of the Sender’s best response to a weakly asymmetric φ_t -profile of actions, wrt the variations $\left. \frac{\partial a_1^{\varphi_t}(m_1)}{\partial t} \right|_{t=1}$ and $\left. \frac{\partial a_1^{\varphi_t}(m_2)}{\partial t} \right|_{t=1}$ of the induced actions in dimension $i = 1$ at $t = 1$.

12. Formally, the derivative of φ_t is $\frac{tg'(\theta)}{g'(\varphi_t(\theta))}$. From $\varphi_t(\theta) < \theta$, we obtain $g'(\varphi_t(\theta)) < g'(\theta)$ if g is convex, and $g'(\varphi_t(\theta)) > g'(\theta)$ if g is concave. Thus, φ_t' is lower when g is concave, and thus φ_t is closer to Id when g is convex than when g is concave.

13. Formally, the derivative $\bar{\alpha} = \left. \frac{\partial a_1^{\varphi_t}(m_1)}{\partial t} \right|_{t=1}$ and $\underline{\alpha} = \left. \frac{\partial a_1^{\varphi_t}(m_2)}{\partial t} \right|_{t=1}$ are related to the shape of g through the equality $\bar{\alpha} - \underline{\alpha} = -2 \text{Cov} \left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta \right)$. For instance, if g'/g is sufficiently decreasing wrt to f , then $\text{Cov} \left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta \right) > 0$, which implies $\bar{\alpha} - \underline{\alpha} < 0$.

Lemma 3.2. *If*

$$h(\bar{a}) - h(\underline{a}) < 2(h'(\underline{a}) \times \underline{\alpha} - h'(\bar{a}) \times \bar{\alpha}), \quad (3.8)$$

(resp. $>$) then $\text{Br}^S(t) < t$ (resp. $> t$) for $t < 1$ sufficiently close to 1, where

$$\begin{aligned} \bar{a} &= \mathbb{E}[\theta_1 | \theta_1 \geq \theta_2], & \underline{a} &= \mathbb{E}[\theta_1 | \theta_1 < \theta_2], \\ \bar{\alpha} &= \left. \frac{\partial a_1^{\varphi_t}(m_1)}{\partial t} \right|_{t=1} = 2\mathbb{E} \left[\frac{f(\theta)}{g'(\theta)/g(\theta)} (\bar{a} - \theta) \right], \\ \underline{\alpha} &= \left. \frac{\partial a_1^{\varphi_t}(m_2)}{\partial t} \right|_{t=1} = 2\mathbb{E} \left[\frac{f(\theta)}{g'(\theta)/g(\theta)} (\theta - \underline{a}) \right]. \end{aligned}$$

Alternatively, the following corollary provides specific conditions on g , and on h , that allows the comparison of $\text{Br}^S(t)$ and t .

Corollary 3.3. (i) *If $\text{Cov} \left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta \right) > \frac{\bar{a}-\underline{a}}{8}$, h is concave or linear, and h' is convex, then if $t < 1$, t sufficiently close to 1, then $\text{Br}^S(t) < t$.*
(ii) *If $\text{Cov} \left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta \right) < 0$ and h is convex or linear, then if $t < 1$, t sufficiently close to 1, then $\text{Br}^S(t) > t$.*

Again, the condition is simplified if h is linear.

Corollary 3.4. *If h is linear, then if $t < 1$ is sufficiently close to 1, $\text{Cov} \left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta \right) > \frac{\bar{a}-\underline{a}}{8}$ implies $\text{Br}^S(t) < t$.*

3.5.3 Credible asymmetric comparisons

The existence of a credible asymmetric comparison is obtained if there exists $t^* \in (0, 1)$ such that $\text{Br}^S(t^*) = t^*$. It is guaranteed by a change of sign of $t \mapsto \text{Br}^S(t) - t$ between $t \simeq 0$, $t > 0$, and $t \simeq 1$, $t < 1$. Sufficient conditions for such a change of sign are obtained from the arguments of the two previous sections. More precisely, when h is non linear, there exists $t^* \in (0, 1)$ such that the φ_{t^*} -comparison is credible if $\mathbb{E} \left[\frac{f(0)}{g'(0)/g(\theta)} \right] < 1$, and either

- (i) h is sufficiently concave, and $h(\bar{a}) - h(\underline{a}) > 2(h'(\underline{a}) \times \underline{\alpha} - h'(\bar{a}) \times \bar{\alpha})$, or
- (ii) h is convex, and $h(\bar{a}) - h(\underline{a}) < 2(h'(\underline{a}) \times \underline{\alpha} - h'(\bar{a}) \times \bar{\alpha})$.

Point (i) and (ii) respectively correspond to $\text{Br}^S(t) < t$ at $t \simeq 0$ and $\text{Br}^S(t) > t$ at $t \simeq 1$, and $\text{Br}^S(t) > t$ at $t \simeq 0$ and $\text{Br}^S(t) < t$ at $t \simeq 1$.

Let us note that *roughly speaking*, point (i) holds only if h is sufficiently concave and g is sufficiently convex, while point (ii) holds only if h is sufficiently convex and g is sufficiently concave. Indeed, consider point (i). Suppose for instance that $\text{Cov} \left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta \right) > 0$, i.e. roughly $f/(g'/g)$ is increasing. Then we have $\bar{\alpha} < \underline{\alpha}$ (see Footnote 13), which implies $2(h'(\underline{a}) \times \underline{\alpha} - h'(\bar{a}) \times \bar{\alpha}) > 2\underline{\alpha}(h'(\underline{a}) - h'(\bar{a}))$. Therefore, the second condition of point (i) requires that $h(\bar{a}) - h(\underline{a}) > 2\underline{\alpha}(h'(\underline{a}) - h'(\bar{a}))$. Thus, from $\underline{\alpha} > 0$, this requires a *limited*

concavity of h . This implies that point (i), which requires a sufficient concavity of h , is likely to hold only if $\text{Cov}\left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta\right) < 0$, *i.e.* roughly $f/(g'/g)$ is decreasing. Now consider point (ii). Suppose that $\text{Cov}\left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta\right) < 0$, *i.e.* $f/(g'/g)$ is roughly decreasing. Then similarly, we have $\bar{\alpha} > \underline{\alpha}$, and thus $2(h'(\underline{a}) \times \underline{\alpha} - h'(\bar{a}) \times \bar{\alpha}) < 2\bar{\alpha}(h'(\underline{a}) - h'(\bar{a}))$. From $\bar{\alpha} > 0$, we obtain that the second condition in point (ii) requires a limited convexity of h to hold. This implies that point (ii), which requires the convexity of h , is likely to hold only if $\text{Cov}\left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta\right) > 0$, *i.e.* roughly $f/(g'/g)$ is increasing.

Thus, roughly speaking, a credible asymmetric comparison exists if the convexity of h is counterbalanced by the concavity of g , or *vice versa*. The following proposition gives precise conditions when h is linear. It is obtained from Corollaries 3.2 and 3.4, which respectively give the conditions such that $\text{Br}^S(t) > t$ at $t \simeq 0$, $t > 0$, and $\text{Br}^S(t) < t$ at $t \simeq 1$, $t < 1$.

Proposition 3.2. *If $U^S((a_1, a_2), (\theta_1, \theta_2)) = a_1g(\theta_1) + a_2g(\theta_2)$, with $g' > 0$ and $g(0) = 0$ then if $\text{Cov}\left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta\right) > \frac{\bar{a}-a}{8}$, then there exists $t^* \in (0, 1)$ such that the profile of (asymmetric) strategies $(\mathbf{m}^{\varphi_{t^*}}, \mathbf{a}_{\varphi_{t^*}})$ given by*

$$\mathbf{m}_{\varphi_{t^*}}(\theta) = \begin{cases} m_1 & \text{if } \theta_1 \geq \varphi_{t^*}(\theta_2), \\ m_2 & \text{if } \theta_1 < \varphi_{t^*}(\theta_2), \end{cases} \quad \text{and } \mathbf{a}_{\varphi_{t^*}}(\mathbf{m}) = \begin{cases} \mathbb{E}[\theta | \theta_1 \geq \varphi_{t^*}(\theta_2)] & \text{if } \mathbf{m} = m_1, \\ \mathbb{E}[\theta | \theta_1 < \varphi_{t^*}(\theta_2)] & \text{if } \mathbf{m} = m_2, \end{cases}$$

where $\varphi_{t^*} = g^{-1}(t^*g)$, is an equilibrium profile of strategies.

In a nutshell, when h is linear, the existence of a credible asymmetric comparison reports on two effects, which relative importance is reversed between the cases $t = 0$ and $t = 1$. First, if $t \simeq 0$, we have $\text{Br}^S(t) > t$, *i.e.* strongly asymmetric induced actions $\mathbf{a}^{\varphi_t}(m_j)$, $j \in \{1, 2\}$ rules out the credibility of message m_2 . Second, if $t \simeq 1$, we have $\text{Br}^S(t) < t$ if $f/(g'/g)$ is sufficiently increasing, *i.e.* weakly asymmetric actions $\mathbf{a}^{\varphi_t}(m_j)$, $j \in \{1, 2\}$ rules out the credibility of message m_1 . In case $t \simeq 0$, a linear h is “sufficiently convex” to give the Sender the incentives to induce the contrasted m_2 -profiles of actions rather than the actions induced by the uninformative m_1 , even if $\theta_1 \geq \varphi_t(\theta_2)$. In case $t \simeq 1$, if $f/(g'/g)$ is sufficiently increasing, the slightly favored issue m_1 encompasses more high values of both types. The linearity of h provides the Sender with the incentives to reveal m_1 , even if $\theta_1 < \varphi_t(\theta_2)$. Hence the effect in case $t \simeq 0$ is driven by the overall asymmetry of the treatment, driven by a $t \simeq 0$ which annihilates the role of g . The effect in case $t \simeq 1$ is driven by the differentiated asymmetric treatment of low and high types, obtained from the concavity of g .

The following example illustrates Proposition 3.2 with the family of power functions for g , and a uniform prior $f = 1$. In that case, the change of sign of $\text{Br}^S(t)$ between $t \rightarrow 0$ and $t = 1$ is necessary and sufficient to obtain the existence of a credible asymmetric comparison. In particular, the assumptions implies the uniqueness of an asymmetric comparison in favor of m_1 , if any.

Example 3.2. Let $r > 0$ and suppose that $U^S((a_1, a_2), (\theta_1, \theta_2)) = a_1\theta_1^r + a_2\theta_2^r$, and that $f = 1$ is uniform. Then up to a relabeling of the messages and the dimensions, there exists a unique credible asymmetric comparison iff $\text{Cov}\left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta\right) > \frac{\bar{a}-a}{8}$, i.e. iff $0 < r < 2$.

3.5.4 A stability criterion

According to Proposition 3.2, there are potentially multiple equilibria in which the players might find themselves. In that case, a stability criterion is worth of interest in order to select among the different equilibria. The characterization of the credible information revelations as φ_t -comparison allows to establish a simple stability criterion.

We capture stability as follows. Consider φ_{t^*} -equilibrium profile of strategies, associated with $t^* \in (0, 1]$. Let \hat{t} be sufficiently close to t^* , and suppose that a player $P \in \{R, S\}$ deviates to the corresponding $\varphi_{\hat{t}}$ -strategy. If the other player's best response, associated with $\text{Br}^{-P}(\hat{t}) \in \mathbb{R}$, is closer to t^* than \hat{t} , then the equilibrium strategies are attracting wrt the players' behavior.

Formally, we say that an equilibrium associated with $t^* \in (0, 1]$ is *stable* if for any \hat{t} sufficiently close to t^* , for any player $P \in \{R, S\}$,

$$|\text{Br}^{-P}(\hat{t}) - t^*| \leq |\hat{t} - t^*|,$$

with a strict inequality for at least one player.¹⁴

Let us first suppose that the Sender deviates from a credible φ_{t^*} -comparison to the $\varphi_{\hat{t}}$ -comparison, for \hat{t} sufficiently close to t^* . Then the Receiver's best response is given by actions $a_i(m_1) = \mathbb{E}[\theta_i | \theta_1 \geq \varphi_{\hat{t}}(\theta_2)]$ and $a_i(m_2) = \mathbb{E}[\theta_i | \theta_1 < \varphi_{\hat{t}}(\theta_2)]$, $i \in \{1, 2\}$. They exactly corresponds to the $\varphi_{\hat{t}}$ -profiles of actions. Hence we have $\text{Br}^R(\hat{t}) = \hat{t}$, so that $|\text{Br}^R(\hat{t}) - t^*| = |\hat{t} - t^*|$. Thus, from the Receiver's perspective, any equilibrium is (weakly) stable.

Next, suppose that the Receiver deviates from the φ_{t^*} -profiles of actions to the $\varphi_{\hat{t}}$ -profiles of actions, with \hat{t} sufficiently close to t^* . The Sender's best response is determined by

$$\text{Br}^S(\hat{t}) = \frac{h(a_2^{\varphi_{\hat{t}}}(m_2)) - h(a_2^{\varphi_{\hat{t}}}(m_1))}{h(a_1^{\varphi_{\hat{t}}}(m_1)) - h(a_1^{\varphi_{\hat{t}}}(m_2))}.$$

Then the stability of the equilibrium is derived from the variation of $\text{Br}^S(t)$ at $t = t^*$. Let us examine the stability of the different equilibria.

Stability of the babbling equilibria. Consider a babbling equilibrium, in which wlog the Sender always reveal m_1 . We may associate $t^* = 0$ to this equilibrium, by identifying the Sender's strategy to \mathbf{m}_{φ_0} , with $\varphi_0 = \lim_{t \rightarrow 0} \varphi_t = (\theta \mapsto 0)$.¹⁵ Then, if $\mathbb{E}\left[\frac{f(0)}{g'(0)/g(\theta)}\right] < 1$, from

14. Note that our stability criterion does not allow *any* small deviation of the players. Deviations are restricted to φ_t -comparison or φ_t -profiles of actions for some $t \in (0, 1)$. It can easily be established that the consideration of any small deviation does not permit to select among equilibria.

15. If we consider the Receiver's actions, we do not have $\lim_{t \rightarrow 0} \mathbf{a}^{\varphi_t}(m_j) = \mathbb{E}[\theta]$ for any $j \in \{1, 2\}$. We

decomposition (3.7) and Lemma 3.1, given \hat{t} , if h is sufficiently concave, then $0 < \text{Br}^S(\hat{t}) < \hat{t}$ so that $|\text{Br}^S(\hat{t}) - 0| < |\hat{t} - 0|$ and the babbling equilibrium is stable. If in contrast, h is convex or linear, we have $\text{Br}^S(\hat{t}) > \hat{t} > 0$, which implies $|\text{Br}^S(\hat{t}) - 0| > |\hat{t} - 0|$ and instability of the babbling equilibrium. Note that in that latter case, the Sender confirms and amplifies any slight information revelation anticipated by the Receiver.

Stability of the symmetric equilibrium. Now consider the symmetric equilibrium, associated with $t^* = 1$. From Lemma 3.2, if $\hat{t} < 1$ is sufficiently close to 1, then if $h(\bar{a}) - h(\underline{a}) < 2(h'(\underline{a}) \times \underline{a} - h'(\bar{a}) \times \bar{a})$, then $\text{Br}^S(\hat{t}) < \hat{t} < 1$. Thus, the symmetric equilibrium is unstable. The Sender confirms and amplifies any asymmetry anticipated by the Receiver. If instead $h(\bar{a}) - h(\underline{a}) > 2(h'(\underline{a}) \times \underline{a} - h'(\bar{a}) \times \bar{a})$, then $\text{Br}^S(\hat{t}) > \hat{t}$. In that case, the symmetric equilibrium is stable if $\text{Br}^S(t)$ is not higher than $1 + (1 - \hat{t})$. This is equivalent to $|\frac{\partial}{\partial t} \text{Br}^S(t)|_{t=1}| < 1$. Lemma 3.2 precisely reports on $\frac{\partial \text{Br}^S(t)}{\partial t}$ (see (3.18) in the Appendix). We obtain that the symmetric equilibrium is stable iff $h(\bar{a}) - h(\underline{a}) > 2(h'(\underline{a})\underline{a} - h'(\bar{a})\bar{a}) > -(h(\bar{a}) - h(\underline{a}))$.

Stability of the credible asymmetric comparisons. The stability of a credible asymmetric comparisons, if any, is derived from the variations of the Sender's best response $\text{Br}^S(t)$, as t ranges over $(0, 1)$. For instance, as depicted in Figure 3.4, from $\text{Br}^S(t) > t$ as $t \simeq 0$ (which might be obtained from Lemma 3.1), if Br^S is increasing on $(0, t_1^*)$, where t_1^* is the smallest solution of $\text{Br}^S(t) = t$, i.e. corresponds to the strongest asymmetric credible comparison, then the $\varphi_{t_1^*}$ -equilibrium strategies defines a stable equilibrium. Then if Br^S is further increasing on (t_1^*, t_2^*) , where t_2^* is the next solution $t_2^* < 1$ to $\text{Br}^S(t) = t$, if any, then the $\varphi_{t_2^*}$ -equilibrium strategies is an unstable equilibrium. The arguments goes on up to $t_\ell^* = 1$, for some $\ell \in \{2, 3, \dots\}$, which corresponds to the number of credible asymmetric comparison, if any. When h is linear, condition $\text{Cov}\left(\frac{f(\varphi_t(\theta))}{g'(\varphi_t(\theta))/g(\theta)}, \theta\right) \geq 0$ in the next proposition precisely reports on the variation of Br^S .

Proposition 3.3. *If $U^S((a_1, a_2), (\theta_1, \theta_2)) = h(a_1)g(\theta_1) + h(a_2)g(\theta_2)$, with $h'g' > 0$ and $g(0) = 0$, then*

- *if $\mathbb{E}\left[\frac{f(0)}{g'(0)/g(\theta)}\right] < 1$, a babbling equilibrium is stable iff h is sufficiently concave, and in particular, it is unstable if h is convex or linear;*
- *the symmetric equilibrium is stable (resp. unstable) if $|2(h'(\underline{a}) \times \underline{a} - h'(\bar{a}) \times \bar{a})| < h(\bar{a}) - h(\underline{a})$ (resp. $>$).*

If moreover h is linear, then

- *a babbling equilibrium is unstable*
- *the symmetric equilibrium is unstable iff $\text{Cov}\left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta\right) > \frac{\bar{a}-\underline{a}}{8}$*
- *if for any $t \in (0, 1)$, $\text{Cov}\left(\frac{f(\varphi_t(\theta))}{g'(\varphi_t(\theta))/g(\theta)}, \theta\right) \geq 0$, and if there are $\ell \geq 1$ credible asymmetric comparisons, associated with $0 < t_1^* \leq t_2^* < \dots t_\ell^* < 1$, then the $(\mathbf{m}_{\varphi_{t_k^*}}, \mathbf{a}^{\varphi_{t_k^*}})$ -*

have $\lim_{t \rightarrow 0} \mathbf{a}^{\varphi^t}(m_1) = \mathbb{E}[\theta]$ but for instance, $\lim_{t \rightarrow 0} \mathbf{a}_1^{\varphi^t}(m_2) = 0$. However, at the corresponding babbling equilibrium, m_2 is out of the equilibrium path. Thus, if we only consider actions taken on the equilibrium path, actions do converge to the corresponding actions of the babbling equilibrium.

profile of strategies is stable iff $k \in \{1, \dots, \ell\}$ is odd. Moreover, the symmetric equilibrium is stable iff ℓ is even.

Note that if Br^S is increasing on $(0, 1]$, and if there is a unique credible asymmetric comparison, then it is stable, and it is the unique stable equilibrium. The example below shows how an asymmetric credible comparison may be appealing whenever it exists and it is unique.

Example 3.3 (Continued). *Let $r > 0$ and suppose that $U^S((a_1, a_2), (\theta_1, \theta_2)) = a_1\theta_1^r + a_2\theta_2^r$, and that $f = 1$ is uniform. The symmetric comparison is stable iff it is the unique credible comparison (i.e. iff $r \geq 2$). Whenever a credible asymmetric comparison exists (when $0 < r < 2$), it is unique and it is the unique stable credible comparison.*

3.6 Conclusion

This paper analyzes a cheap talk game of information transmission in a two-dimensional framework. We assume a symmetric prior distribution, a binary disclosure rule, and a large symmetric conflict between the players. The large conflict imposes the comparative nature of any credible binary information revelation. The full symmetry of the game parameter does not guarantee the symmetry of an equilibrium treatment. On the contrary, potential asymmetric information revelations wrt to the two dimensions are possible, and are eventually more appealing than the most informative symmetric treatment or the uninformative treatment. An asymmetric treatment potentially misleads the receiver of the information wrt to the relative importance of the dimensions of his uncertainty.

Given the prevalence of conflict of interest between economic agents, and the importance of information transmission in organizations and markets, concerns for future research might be to determine how effective and general the exhibited endogenous asymmetric treatment is. In particular, it would be worth of interest to investigate whether or not the focal nature of the symmetric treatment prevails over the appealing nature of an asymmetric one. Some anecdotal endogenously asymmetric recommendations (*e.g.* toward boys and girls) might provide an illustration of the appealing nature of the asymmetric treatment.

Proofs

3.A Proof of Proposition 3.1

The sufficient part of Proposition 3.1 is straightforward: if there exists a function φ such that strategies \mathbf{m}_φ and \mathbf{a}^φ as defined in the proposition, in particular such that \mathbf{m}_φ is the Sender's best response to \mathbf{a}^φ and reciprocally, then $(\mathbf{m}_\varphi, \mathbf{a}^\varphi)$ defines an equilibrium profile of strategies. We have to show the necessary part, which is: if $(\mathbf{m}^*, \mathbf{a}^*)$ is an influential equilibrium profile of strategies, then there exists a function φ and strategies $(\mathbf{m}_\varphi, \mathbf{a}^\varphi)$ as defined in the proposition such that $(\mathbf{m}^*, \mathbf{a}^*) = (\mathbf{m}_\varphi, \mathbf{a}^\varphi)$.

Let $(\mathbf{m}^*, \mathbf{a}^*)$ be an influential equilibrium profile of strategies. Recall that it is influential if messages m_1 and m_2 are both on the equilibrium path, and $\mathbf{a}^*(\mathbf{m}_1) \neq \mathbf{a}^*(\mathbf{m}_2)$, i.e. $a_1^*(m_1) - a_1^*(m_2) \neq 0$ or $a_2^*(m_2) - a_2^*(m_1) \neq 0$. Let us first consider the strategy \mathbf{m}^* . Since \mathbf{m}^* is the Sender's best response to \mathbf{a}^* , for any $\theta = (\theta_1, \theta_2)$, up to her set of indifferent types, the Sender chooses to send m_1 iff

$$u(a_1^*(m_1), \theta_1) + u(a_2^*(m_1), \theta_2) \geq u(a_1^*(m_2), \theta_1) + u(a_2^*(m_2), \theta_2).$$

Therefore the Sender uses the messaging rule:

$$\mathbf{m}^*(\theta) = m_1 \iff u(a_1^*(m_1), \theta_1) - u(a_1^*(m_2), \theta_1) \geq u(a_2^*(m_2), \theta_2) - u(a_2^*(m_1), \theta_2). \quad (3.9)$$

Consider the sides of the inequality in (3.9) as functions of θ_1 and θ_2 respectively. From $\frac{\partial^2 u^S}{\partial \theta \partial a} > 0$, both sides are monotonous wrt the corresponding θ_i (or constant if $a_i(m_i) = a_i(m_{-i})$). Let us show that if $\frac{\partial u}{\partial a}(a, 0) \geq 0$ or $\frac{\partial u}{\partial a}(a, 1) \leq 0$, then either they are both non decreasing, or they are both non increasing. Suppose that the lhs of the inequality in (3.9) is non decreasing. Then we have $a_1^*(m_1) \geq a_1^*(m_2)$, and for any $\theta_1 \in [0, 1]$,

$$u(a_1^*(m_1), \theta_1) - u(a_1^*(m_2), \theta_1) \geq u(a_1^*(m_1), 0) - u(a_1^*(m_2), 0).$$

Suppose that the rhs of the inequality in (3.9) is non increasing. Then we have $a_2^*(m_2) \leq a_2^*(m_1)$, and for any $\theta_2 \in [0, 1]$,

$$u(a_2^*(m_2), \theta_2) - u(a_2^*(m_1), \theta_2) \leq u(a_2^*(m_2), 0) - u(a_2^*(m_1), 0).$$

Now if $\frac{\partial u}{\partial a}(a, 0) \geq 0$, then $u(a_1^*(m_1), 0) - u(a_1^*(m_2), 0) \geq 0$, and $u(a_2^*(m_2), 0) - u(a_2^*(m_1), 0) \leq 0$. Then we obtain, for any $(\theta_1, \theta_2) \in [0, 1]^2$, $u(a_1^*(m_1), \theta_1) - u(a_1^*(m_2), \theta_1) \geq u(a_2^*(m_2), \theta_2) - u(a_2^*(m_1), \theta_2)$. According to (3.9), this implies that message m_1 is always sent. This contradicts the requirement of influence of the equilibrium. If instead the lhs of the inequality in (3.9) is non increasing and the rhs is non decreasing, then we have $a_1^*(m_1) \leq a_1^*(m_2)$, for any $\theta_1 \in [0, 1]$,

$$u(a_1^*(m_1), \theta_1) - u(a_1^*(m_2), \theta_1) \leq u(a_1^*(m_1), 0) - u(a_1^*(m_2), 0),$$

(with a strict inequality if $a_1^*(m_1) \neq a_1^*(m_2)$), and $a_2^*(m_2) \geq a_2^*(m_1)$, and for any $\theta_2 \in [0, 1]$,

$$u(a_2^*(m_2), \theta_2) - u(a_2^*(m_1), \theta_2) \geq u(a_2^*(m_2), 0) - u(a_2^*(m_1), 0)$$

(with a strict inequality if $a_2^*(m_1) \neq a_2^*(m_2)$). Then $\frac{\partial u}{\partial a}(a, 0) \geq 0$ implies $u(a_1^*(m_1), 0) - u(a_1^*(m_2), 0) \leq 0$ and $u(a_2^*(m_2), 0) - u(a_2^*(m_1), 0) \geq 0$. Since either $a_1^*(m_1) \neq a_1^*(m_2)$ or $a_2^*(m_1) \neq a_2^*(m_2)$, we obtain, for any $(\theta_1, \theta_2) \in [0, 1]^2$, $u(a_1^*(m_1), \theta_1) - u(a_1^*(m_2), \theta_1) < u(a_2^*(m_2), \theta_2) - u(a_2^*(m_1), \theta_2)$. Then according to (3.9), message m_1 is never sent. This also contradicts the requirement of influence of the equilibrium. Hence both sides of the inequality in (3.9) are non decreasing, or both sides are non increasing if $\frac{\partial u}{\partial a}(a, 0) \geq 0$. The case $\frac{\partial u}{\partial a}(a, 1) \leq 0$ follows from a similar argument.

Then, up to a relabeling of the messages, we can assume that $a_1^*(m_1) \geq a_1^*(m_2)$ and $a_2^*(m_2) \geq a_2^*(m_1)$, so that both sides of (3.9) are non decreasing. Now up to a relabeling of the dimensions, we can assume that $u(a_1^*(m_1), 1) - u(a_1^*(m_2), 1) \geq u(a_2^*(m_2), 1) - u(a_2^*(m_1), 1)$ so that m_1 is sent at $(\theta_1, \theta_2) = (1, 1)$. Then since $\theta_2 \mapsto u(a_2^*(m_2), \theta_2) - u(a_2^*(m_1), \theta_2)$ is increasing, for any $\theta_2 \in [0, 1]$, $u(a_1^*(m_1), 1) - u(a_1^*(m_2), 1) \geq u(a_2^*(m_2), \theta_2) - u(a_2^*(m_1), \theta_2)$. In particular,

$$\varphi(\theta_2) = \min\{\theta_1 \in [0, 1], u(a_1^*(m_1), \theta_1) - u(a_1^*(m_2), \theta_1) \geq u(a_2^*(m_2), \theta_2) - u(a_2^*(m_1), \theta_2)\}$$

is well defined, and from this definition, the Sender's message rule (3.9) might be written

$$\mathbf{m}^*(\boldsymbol{\theta}) = m_1 \text{ iff } \theta_1 \geq \varphi(\theta_2).$$

Moreover, φ is continuous, and if $\theta'_2 \geq \theta_2$, then for any $\theta_1 \in [0, 1]$ such that $u(a_1^*(m_1), \theta_1) - u(a_1^*(m_2), \theta_1) \geq u(a_2^*(m_2), \theta'_2) - u(a_2^*(m_1), \theta'_2)$, we have $u(a_1^*(m_1), \theta_1) - u(a_1^*(m_2), \theta_1) \geq u(a_2^*(m_2), \theta_2) - u(a_2^*(m_1), \theta_2)$. This implies $\varphi(\theta'_2) \geq \varphi(\theta_2)$. Thus φ is non decreasing.

3.B Proof of Lemma 3.1

We want to show that if t is sufficiently close to 0, then

$$\frac{a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1)}{a_1^{\varphi_t}(m_1) - a_1^{\varphi_t}(m_2)} > t, \quad (3.10)$$

and

$$a_2^{\varphi_t}(m_2) > a_1^{\varphi_t}(m_1) > a_2^{\varphi_t}(m_1) > a_1^{\varphi_t}(m_2). \quad (3.11)$$

Let us first show that $a_1^{\varphi_t}(m_1) > a_1^{\varphi_t}(m_2)$ and $a_2^{\varphi_t}(m_2) > a_2^{\varphi_t}(m_1)$. Set, for any $t \in (0, 1]$, any $j \in \{1, 2\}$,

$$\Theta_j^{\varphi_t} = \mathbf{m}_{\varphi_t}^{-1}(m_j), \quad |\Theta_j^{\varphi_t}| = \Pr(\mathbf{m}_{\varphi_t}(\boldsymbol{\theta}) = m_j) = \iint_{\boldsymbol{\theta} \in \Theta_j^{\varphi_t}} f(\boldsymbol{\theta}) d\boldsymbol{\theta}.$$

From the Law of Iterated Expectations, for any $i \in \{1, 2\}$, we have

$$\begin{aligned} \mathbb{E} = \mathbb{E}[\theta_i] &= |\Theta_1^{\varphi_t}| \mathbb{E}[\theta_i | \mathbf{m}_{\varphi_t}(\boldsymbol{\theta}) = m_1] + |\Theta_2^{\varphi_t}| \mathbb{E}[\theta_i | \mathbf{m}_{\varphi_t}(\boldsymbol{\theta}) = m_2] \\ &= |\Theta_1^{\varphi_t}| a_i^{\varphi_t}(m_1) + |\Theta_2^{\varphi_t}| a_i^{\varphi_t}(m_2), \end{aligned}$$

so that

$$\begin{aligned} a_1^{\varphi_t}(m_1) - a_1^{\varphi_t}(m_2) &= \frac{\mathbb{E} - |\Theta_2^{\varphi_t}| a_1^{\varphi_t}(m_2)}{|\Theta_1^{\varphi_t}|} - a_1^{\varphi_t}(m_2) \\ &= \frac{\mathbb{E} - |\Theta_2^{\varphi_t}| a_1^{\varphi_t}(m_2) - (1 - |\Theta_2^{\varphi_t}|) a_1^{\varphi_t}(m_2)}{|\Theta_1^{\varphi_t}|} \\ &= \frac{\mathbb{E} - a_1^{\varphi_t}(m_2)}{|\Theta_1^{\varphi_t}|}, \end{aligned}$$

and similarly, $a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1) = \frac{a_2^{\varphi_t}(m_2) - \mathbb{E}}{|\Theta_1^{\varphi_t}|}$. Therefore, the sign of $a_1^{\varphi_t}(m_1) - a_1^{\varphi_t}(m_2)$ and $a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1)$ is given by the sign of $\mathbb{E} - a_1^{\varphi_t}(m_2)$ and $a_2^{\varphi_t}(m_2) - \mathbb{E}$ respectively.

Note that for any $\theta_2 \in (0, 1)$, we have $0 < \varphi_t(\theta_2) < 1$, so that $\Theta_1^{\varphi_t}$ and $\Theta_2^{\varphi_t}$ decompose respectively through

$$\Theta_1^{\varphi_t} = \bigcup_{\theta=(\theta_1, \theta_2) \in [0,1]^2, \theta_1 \geq \varphi_t(\theta_2)} \{\theta\} = \bigcup_{\theta_2 \in [0,1]} [\varphi_t(\theta_2), 1] \times \{\theta_2\},$$

and

$$\Theta_2^{\varphi_t} = \bigcup_{\theta=(\theta_1, \theta_2) \in [0,1]^2, \theta_1 < \varphi_t(\theta_2)} \{\theta\} = \bigcup_{\theta_2 \in [0,1]} [0, \varphi_t(\theta_2)) \times \{\theta_2\}.$$

Consequently, we have

$$\begin{aligned} a_2^{\varphi_t}(m_2) &= \mathbb{E}[\theta_2 | \theta_1 < \varphi_t(\theta_2)] = \frac{\iint_{\theta_1 < \varphi_t(\theta_2)} \theta_2 f(\theta_1) f(\theta_2) d\theta}{|\Theta_2^{\varphi_t}|} = \frac{\int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} \theta_2 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2}{|\Theta_2^{\varphi_t}|}, \\ a_1^{\varphi_t}(m_2) &= \mathbb{E}[\theta_2 | \theta_1 < \varphi_t(\theta_2)] = \frac{\iint_{\theta_1 < \varphi_t(\theta_2)} \theta_1 f(\theta_1) f(\theta_2) d\theta}{|\Theta_2^{\varphi_t}|} = \frac{\int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} \theta_1 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2}{|\Theta_2^{\varphi_t}|}. \end{aligned}$$

Then we obtain

$$\begin{aligned} |\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E}) &= \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} \theta_2 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 - \mathbb{E} \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 \\ &= \int_{\theta_2=0}^1 \theta_2 F(\varphi_t(\theta_2)) f(\theta_2) d\theta_2 - \mathbb{E} \int_{\theta_2=0}^1 F(\varphi_t(\theta_2)) f(\theta_2) d\theta_2 \\ &= \mathbb{E}[\theta F(\varphi_t(\theta))] - \mathbb{E}[\theta] \mathbb{E}[F(\varphi_t(\theta))] \\ &= \text{Cov}(F(\varphi_t(\theta)), \theta) > 0, \end{aligned}$$

since $F(\varphi_t)$ is increasing. Therefore $a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1) > 0$. In order to obtain $a_1^{\varphi_t}(m_1) -$

$a_1^{\varphi_t}(m_2) > 0$, it remains to show that $a_1^{\varphi_t}(m_2) - \mathbb{E} < 0$. We have

$$\begin{aligned} |\Theta_2^{\varphi_t}|(a_1^{\varphi_t}(m_2) - \mathbb{E}) &= \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} \theta_1 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 - \mathbb{E} \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 \\ &= \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} (\theta_1 - \mathbb{E}) f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2. \end{aligned}$$

Let us show that for any $t \in (0, 1]$, and any $\theta_2 < 1$, $\int_{\theta_1=0}^{\varphi_t(\theta_2)} (\theta_1 - \mathbb{E}) f(\theta_1) d\theta_1 < 0$, so

that the result follows. If θ_2 is such that $\varphi_t(\theta_2) \leq \mathbb{E}$, then $\int_{\theta_1=0}^{\varphi_t(\theta_2)} (\theta_1 - \mathbb{E}) f(\theta_1) d\theta_1 < 0$

$\int_{\theta_1=0}^{\varphi_t(\theta_2)} (\varphi_t(\theta_2) - \mathbb{E}) f(\theta_1) d\theta_1 \leq 0$. If instead $\varphi_t(\theta_2) > \mathbb{E}$, then the second term of the sum

$$\int_{\theta_1=0}^{\varphi_t(\theta_2)} (\theta_1 - \mathbb{E}) f(\theta_1) d\theta_1 + \int_{\theta_1=\varphi_t(\theta_2)}^1 (\theta_1 - \mathbb{E}) f(\theta_1) d\theta_1 = 0,$$

is strictly positive, which implies the strict negativity of the first term.

Now let us look at (3.10). We have

$$\frac{a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1)}{a_1^{\varphi_t}(m_1) - a_1^{\varphi_t}(m_2)} = \frac{|\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E})}{|\Theta_2^{\varphi_t}|(\mathbb{E} - a_1^{\varphi_t}(m_2))},$$

with $a_2^{\varphi_t}(m_2) > \mathbb{E} > a_1^{\varphi_t}(m_2)$, so that (3.10) is equivalent to

$$t|\Theta_2^{\varphi_t}|(a_1^{\varphi_t}(m_2) - \mathbb{E}) + |\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E}) > 0. \quad (3.12)$$

As $t \rightarrow 0$, $\varphi_t \rightarrow (\theta \mapsto 0)$, and consequently $|\Theta_2^{\varphi_t}| \rightarrow 0$ and the above expression tends to 0. Therefore, (3.12) holds for $t > 0$ sufficiently close to 0 whenever

$$\frac{\partial (t|\Theta_2^{\varphi_t}|(a_1^{\varphi_t}(m_2) - \mathbb{E}) + |\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E}))}{\partial t} > 0$$

for any $t > 0$ sufficiently close to 0. Next, we compute the above derivative by using Leibniz Integral Rule.¹⁶ Recall that

$$|\Theta_2^{\varphi_t}|(a_1^{\varphi_t}(m_2) - \mathbb{E}) = \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} (\theta_1 - \mathbb{E}) f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2.$$

16. From the assumptions $h'g' > 0$ and $g(0) = 0$, all functions used in the computation are at least twice continuously differentiable and bounded above and below as function of t or θ , so that the rule can be used whenever it is needed.

We have, for any $\theta_2 \in [0, 1]$,

$$\frac{\partial \varphi_t}{\partial t}(\theta_2) = \frac{g(\theta_2)}{g'(\varphi_t(\theta_2))},$$

(recall that g' does not annihilate), and for any $\theta_2 \in [0, 1]$, the derivative of $t \mapsto \int_{\theta_1=0}^{\varphi_t(\theta_2)} (\theta_1 - \mathbb{E})f(\theta_1) d\theta_1$ is given by

$$(\varphi_t(\theta_2) - E)f(\varphi_t(\theta_2))\frac{\partial \varphi_t}{\partial t}(\theta_2) = (\varphi_t(\theta_2) - E)f(\varphi_t(\theta_2))\frac{g(\theta_2)}{g'(\varphi_t(\theta_2))}.$$

Then we have, at any $t_0 \in (0, 1]$,

$$\left. \frac{\partial (|\Theta_2^{\varphi_t}|(a_1^{\varphi_t}(m_2)) - \mathbb{E})}{\partial t} \right|_{t=t_0} = \int_{\theta_2=0}^1 (\varphi_{t_0}(\theta_2) - E[\theta])f(\varphi_{t_0}(\theta_2))\frac{g(\theta_2)}{g'(\varphi_{t_0}(\theta_2))}f(\theta_2) d\theta_2.$$

Similarly, from

$$|\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E}) = \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} (\theta_2 - \mathbb{E})f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2,$$

we compute, for any $t_0 \in (0, 1]$,

$$\left. \frac{\partial (|\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E}))}{\partial t} \right|_{t=t_0} = \int_{\theta_2=0}^1 (\theta_2 - \mathbb{E})f(\varphi_{t_0}(\theta_2))\frac{g(\theta)}{g'(\varphi_{t_0}(\theta_2))}f(\theta_2) d\theta_2.$$

Then we obtain

$$\begin{aligned} & \left. \frac{\partial (t|\Theta_2^{\varphi_t}|(a_1^{\varphi_t}(m_2) - \mathbb{E}) + |\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E}))}{\partial t} \right|_{t=t_0} \\ &= |\Theta_2^{\varphi_{t_0}}|(a_2^{\varphi_{t_0}}(m_2) - \mathbb{E}) + \int_{\theta_2=0}^1 (t_0(\varphi_{t_0}(\theta_2) - \mathbb{E}) + \theta_2 - \mathbb{E})f(\varphi_{t_0}(\theta_2))\frac{g(\theta)}{g'(\varphi_{t_0}(\theta_2))}f(\theta_2) d\theta_2. \end{aligned} \tag{3.13}$$

This gives

$$\begin{aligned}
& \lim_{t_0 \rightarrow 0} \int_{\theta_2=0}^1 (t_0(\varphi_{t_0}(\theta_2) - \mathbb{E}) + \theta_2 - \mathbb{E}) f(\varphi_{t_0}(\theta_2)) \frac{g(\theta)}{g'(\varphi_{t_0}(\theta_2))} f(\theta_2) d\theta_2 \\
&= \int_{\theta_2=0}^1 \lim_{t_0 \rightarrow 0} \left((t_0(\varphi_{t_0}(\theta_2) - \mathbb{E}) + \theta_2 - \mathbb{E}) f(\varphi_{t_0}(\theta_2)) \frac{g(\theta)}{g'(\varphi_{t_0}(\theta_2))} f(\theta_2) \right) d\theta_2 \\
&= \int_{\theta_2=0}^1 (\theta_2 - \mathbb{E}) f(0) \frac{g(\theta)}{g'(0)} f(\theta_2) d\theta_2 \\
&= f(0) \operatorname{Cov} \left(\frac{g(\theta)}{g'(0)}, \theta \right).
\end{aligned}$$

Now note that $f(0) > 0$, and $g/g'(0)$ is strictly increasing. Therefore $f(0) \operatorname{Cov} \left(\frac{g(\theta)}{g'(0)}, \theta \right)$ is strictly positive. It follows that the integral term of (3.13) is arbitrary close to $f(0) \operatorname{Cov} \left(\frac{g(\theta)}{g'(0)}, \theta \right) > 0$ whenever $t_0 > 0$ is sufficiently close to 0. Since the first term of (3.13) is positive and tends to 0 with t_0 , we get

$$\left. \frac{\partial (t|\Theta_2^{\varphi_t}|(a_1^{\varphi_t}(m_2) - \mathbb{E}) + |\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E}))}{\partial t} \right|_{t=t_0} > 0$$

whenever $t_0 > 0$ is sufficiently close to 0. This shows (3.10).

Next we show that if t is sufficiently close 0, then (3.11) holds. From $a_1^{\varphi_t}(m_1) > a_1^{\varphi_t}(m_2)$ and $a_2^{\varphi_t}(m_2) > a_2^{\varphi_t}(m_1)$, it is sufficient to show that

$$a_2^{\varphi_t}(m_1) > a_1^{\varphi_t}(m_2) \tag{3.14}$$

and

$$a_2^{\varphi_t}(m_2) > a_1^{\varphi_t}(m_1) \tag{3.15}$$

hold for t sufficiently close to 0. Let us first consider (3.14). On the one hand, from the increasing of φ_t , we have

$$\begin{aligned}
0 < a_1^{\varphi_t}(m_2) &= \frac{\int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} \theta_1 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2}{|\Theta_2^{\varphi_t}|} \\
&\leq \frac{\int_{\theta_2=0}^1 \varphi_t(\theta_2) \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2}{|\Theta_2^{\varphi_t}|} \\
&\leq \frac{\varphi_t(1) \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2}{|\Theta_2^{\varphi_t}|} = \varphi_t(1) \frac{|\Theta_2^{\varphi_t}|}{|\Theta_2^{\varphi_t}|} = \varphi_t(1) = g^{-1}(tg(1)).
\end{aligned}$$

Hence we have

$$\lim_{t \rightarrow 0} a_1^{\varphi_t}(m_2) = g^{-1}(0) = 0.$$

One the other hand,

$$\begin{aligned} a_2^{\varphi_t}(m_1) &= \frac{\int_{\theta_2=0}^1 \left(\int_{\theta_1=\varphi_t(\theta_2)}^1 \theta_2 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2}{|\Theta_1^{\varphi_t}|} \\ &= \frac{\int_{\theta_2=0}^1 \theta_2 (1 - F(\varphi_t(\theta_2))) f(\theta_2) d\theta_2}{\int_{\theta_2=0}^1 (1 - F(\varphi_t(\theta_2))) f(\theta_2) d\theta_2} \end{aligned}$$

tends to $\mathbb{E}[\theta_2] = \mathbb{E} > 0$ as $t \rightarrow 0$. Therefore, if t is sufficiently close to 0, then (3.14) holds.

Next, consider (3.15). From the Law of Iterated Expectations, we have

$$\begin{aligned} a_2^{\varphi_t}(m_2) - a_1^{\varphi_t}(m_1) &= a_2^{\varphi_t}(m_2) - \mathbb{E} + \mathbb{E} - a_1^{\varphi_t}(m_1) \\ &= \frac{|\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E})}{|\Theta_2^{\varphi_t}|} + \mathbb{E} - \frac{\mathbb{E} - a_1^{\varphi_t}(m_2)}{|\Theta_1^{\varphi_t}|} \\ &= \frac{|\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E})}{|\Theta_2^{\varphi_t}|} + \frac{(|\Theta_1^{\varphi_t}| - 1)\mathbb{E} + a_1^{\varphi_t}(m_2)}{|\Theta_1^{\varphi_t}|} \\ &= \frac{|\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E})}{|\Theta_2^{\varphi_t}|} - \frac{|\Theta_2^{\varphi_t}|(\mathbb{E} - a_1^{\varphi_t}(m_2))}{|\Theta_1^{\varphi_t}|}. \end{aligned}$$

It is positive *iff*

$$\frac{|\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E})}{|\Theta_2^{\varphi_t}|(\mathbb{E} - a_1^{\varphi_t}(m_2))} > \frac{|\Theta_2^{\varphi_t}|}{|\Theta_1^{\varphi_t}|}. \quad (3.16)$$

We have shown that if t is sufficiently close to 0, then

$$\frac{|\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E})}{|\Theta_2^{\varphi_t}|(\mathbb{E} - a_1^{\varphi_t}(m_2))} = \frac{a_2^{\varphi_t}(m_2) - a_2^{\varphi_t}(m_1)}{a_1^{\varphi_t}(m_1) - a_1^{\varphi_t}(m_2)} > t.$$

It follows that (3.16) holds at any $t > 0$ sufficiently close to 0 if $\frac{|\Theta_2^{\varphi_t}|}{|\Theta_1^{\varphi_t}|} < t$ holds at any $t > 0$ sufficiently close to 0, *i.e.* if

$$t|\Theta_1^{\varphi_t}| - |\Theta_2^{\varphi_t}| > 0 \quad (3.17)$$

holds at any $t > 0$ sufficiently close to 0. Since $\lim_{t \rightarrow 0} (t|\Theta_1^{\varphi_t}| - |\Theta_2^{\varphi_t}|) = 0$, it is sufficient to show that

$$\frac{\partial}{\partial t} (t|\Theta_1^{\varphi_t}| - |\Theta_2^{\varphi_t}|) > 0$$

holds at any $t > 0$ sufficiently close to 0. From

$$|\Theta_2^{\varphi_t}| = \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2$$

we deduce, for any $t_0 \in (0, 1)$,

$$\left. \frac{\partial |\Theta_2^{\varphi_t}|}{\partial t} \right|_{t=t_0} = \int_{\theta_2=0}^1 \left(f(\varphi_{t_0}(\theta_2)) \frac{g(\theta_2)}{g'(\varphi_{t_0}(\theta_2))} \right) f(\theta_2) d\theta_2$$

and from $|\Theta_2^{\varphi_t}| = 1 - |\Theta_1^{\varphi_t}|$, we obtain $\left. \frac{\partial |\Theta_1^{\varphi_t}|}{\partial t} \right|_{t=t_0} = - \left. \frac{\partial |\Theta_2^{\varphi_t}|}{\partial t} \right|_{t=t_0}$. Then, we have

$$\begin{aligned} & \left. \frac{\partial (t|\Theta_1^{\varphi_t}| - |\Theta_2^{\varphi_t}|)}{\partial t} \right|_{t=t_0} \\ &= |\Theta_1^{\varphi_{t_0}}| + t_0 \left. \frac{\partial |\Theta_1^{\varphi_t}|}{\partial t} \right|_{t=t_0} - \left. \frac{\partial |\Theta_2^{\varphi_t}|}{\partial t} \right|_{t=t_0} \\ &= |\Theta_1^{\varphi_{t_0}}| - (1 + t_0) \int_{\theta_2=0}^1 \left(f(\varphi_{t_0}(\theta_2)) \frac{g(\theta_2)}{g'(\varphi_{t_0}(\theta_2))} \right) f(\theta_2) d\theta_2, \end{aligned}$$

which tends to $1 - \mathbb{E} \left[f(0) \frac{g(\theta)}{g'(0)} \right]$ as $t_0 \rightarrow 0$. We conclude that if $\mathbb{E} \left[f(0) \frac{g(\theta)}{g'(0)} \right] < 1$, then if t is sufficiently close to 0, then (3.16) holds, and so does (3.11).

3.C Proof of Lemma 3.2

We want to compare $\text{Br}^S(t) = \frac{h(a_2^{\varphi_t}(m_2)) - h(a_2^{\varphi_t}(m_1))}{h(a_1^{\varphi_t}(m_1)) - h(a_1^{\varphi_t}(m_2))}$ and t , for $t < 1$, t sufficiently close to 1. Function $\text{Br}^S(t) - t$ annihilates at $t = 1$, and thus, if $\left. \frac{\partial \text{Br}^S(t)}{\partial t} \right|_{t=1} > 1$, then $\text{Br}^S(t) < t$ for $t < 1$ sufficiently close to 1, and if instead $\left. \frac{\partial \text{Br}^S(t)}{\partial t} \right|_{t=1} < 1$ then $\text{Br}^S(t) > t$ for $t < 1$ sufficiently close to 1. Let us compute $\left. \frac{\partial \text{Br}^S(t)}{\partial t} \right|_{t=1}$. To do this, we need $\left. \frac{\partial a_i(m_j)}{\partial t} \right|_{t=1}$ for any $i, j \in \{1, 2\}$. We have

$$\begin{aligned} \frac{\partial (|\Theta_1^{\varphi_t}| a_1^{\varphi_t}(m_1))}{\partial t} &= \frac{\partial}{\partial t} \int_{\theta_2=0}^1 \left(\int_{\theta_1=\varphi_t(\theta_2)}^1 \theta_1 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 \\ &= \int_{\theta_2=0}^1 \frac{\partial}{\partial t} \left(\int_{\theta_1=\varphi_t(\theta_2)}^1 \theta_1 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 \\ &= - \int_{\theta_2=0}^1 \varphi(\theta_2) f(\varphi(\theta_2)) \frac{\partial \varphi_t}{\partial t}(\theta_2) f(\theta_2) d\theta_2 \end{aligned}$$

which gives, at $t = 1$,

$$\left. \frac{\partial (|\Theta_1^{\varphi_t}| a_1^{\varphi_t}(m_1))}{\partial t} \right|_{t=1} = -\mathbb{E} \left[\frac{\theta f(\theta)}{g'(\theta)/g(\theta)} \right].$$

We compute similarly

$$\left. \frac{\partial |\Theta_1^{\varphi_t}|}{\partial t} \right|_{t=1} = -\mathbb{E} \left[\frac{f(\theta)}{g'(\theta)/g(\theta)} \right],$$

and from

$$\left. \frac{\partial (|\Theta_1^{\varphi_t}| a_1^{\varphi_t}(m_1))}{\partial t} \right|_{t=1} = \left. \frac{\partial |\Theta_1^{\varphi_t}|}{\partial t} \right|_{t=1} a_1^{\varphi_1}(m_1) + |\Theta_1^{\varphi_1}| \left. \frac{\partial a_1^{\varphi_t}(m_1)}{\partial t} \right|_{t=1},$$

with $a_1^{\varphi_1}(m_1) = \bar{a}$ and $|\Theta_1^{\varphi_1}| = \frac{1}{2}$, we obtain

$$\begin{aligned} \left. \frac{\partial a_1^{\varphi_t}(m_1)}{\partial t} \right|_{t=1} &= 2 \left(-\mathbb{E} \left[\frac{\theta f(\theta)}{g'(\theta)/g(\theta)} \right] - \left(-\mathbb{E} \left[\frac{f(\theta)}{g'(\theta)/g(\theta)} \right] \bar{a} \right) \right) \\ &= 2 \mathbb{E} \left[\frac{f(\theta)}{g'(\theta)/g(\theta)} (\bar{a} - \theta) \right]. \end{aligned}$$

Similarly, we obtain

$$\begin{aligned} \left. \frac{\partial a_1^{\varphi_t}(m_2)}{\partial t} \right|_{t=1} &= 2 \mathbb{E} \left[\frac{f(\theta)}{g'(\theta)/g(\theta)} (\theta - \underline{a}) \right], \\ \left. \frac{\partial a_2^{\varphi_t}(m_2)}{\partial t} \right|_{t=1} &= - \left. \frac{\partial a_1^{\varphi_t}(m_1)}{\partial t} \right|_{t=1}, \\ \left. \frac{\partial a_2^{\varphi_t}(m_1)}{\partial t} \right|_{t=1} &= - \left. \frac{\partial a_1^{\varphi_t}(m_2)}{\partial t} \right|_{t=1}. \end{aligned}$$

(that the two latter derivatives might be obtained by symmetry). Set $\bar{\alpha} = \left. \frac{\partial a_1^{\varphi_t}(m_1)}{\partial t} \right|_{t=1} = 2 \mathbb{E} \left[\frac{f(\theta)}{g'(\theta)/g(\theta)} (\bar{a} - \theta) \right]$, and $\underline{\alpha} = \left. \frac{\partial a_1^{\varphi_t}(m_2)}{\partial t} \right|_{t=1} = 2 \mathbb{E} \left[\frac{f(\theta)}{g'(\theta)/g(\theta)} (\theta - \underline{a}) \right]$. Then we have

$$\begin{aligned} \left. \frac{\partial \text{Br}^S(t)}{\partial t} \right|_{t=1} &= \left. \frac{\partial}{\partial t} \left(\frac{h(a_2^{\varphi_t}(m_2)) - h(a_2^{\varphi_t}(m_1))}{h(a_1^{\varphi_t}(m_1)) - h(a_1^{\varphi_t}(m_2))} \right) \right|_{t=1} \\ &= \left((h'(\bar{a})(-\bar{\alpha}) - h'(\underline{a})(-\underline{\alpha})) (h(\bar{a}) - h(\underline{a})) \right. \\ &\quad \left. - (h(\bar{a}) - h(\underline{a})) (h'(\bar{a})\bar{\alpha} - h'(\underline{a})\underline{\alpha}) \right) \\ &\quad / (h(\bar{a}) - h(\underline{a}))^2 \\ &= 2 \frac{h'(\underline{a})\underline{\alpha} - h'(\bar{a})\bar{\alpha}}{h(\bar{a}) - h(\underline{a})}. \end{aligned} \tag{3.18}$$

Consequently, $\left. \frac{\partial \text{Br}^S(t)}{\partial t} \right|_{t=1} > 1$ iff $h(\bar{a}) - h(\underline{a}) < 2(h'(\underline{a})\underline{\alpha} - h'(\bar{a})\bar{\alpha})$.

3.D Proof of Corollary 3.3

According to Lemma 3.2, $\text{Br}(t) < t$ iff

$$h(\underline{a}) + h'(\underline{a}) \times 4\mathbb{E} \left[\frac{f(\theta)(\theta - \underline{a})}{g'(\theta)/g(\theta)} \right] > h(\bar{a}) + h'(\bar{a}) \times 4\mathbb{E} \left[\frac{f(\theta)(\bar{a} - \theta)}{g'(\theta)/g(\theta)} \right].$$

This condition might be written

$$\frac{8}{\bar{a} - \underline{a}} \frac{\frac{h'(\bar{a}) + h'(\underline{a})}{2}}{\frac{h(\bar{a}) - h(\underline{a})}{\bar{a} - \underline{a}}} \text{Cov} \left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta \right) - 2 \frac{h'(\bar{a}) - h'(\underline{a})}{\frac{h(\bar{a}) - h(\underline{a})}{\bar{a} - \underline{a}}} \mathbb{E} \left[\frac{f(\theta)}{g'(\theta)/g(\theta)} \right] > 1. \quad (3.19)$$

— If h' is convex or linear (resp. concave or linear), then

$$\frac{h(\bar{a}) - h(\underline{a})}{\bar{a} - \underline{a}} \leq \frac{h'(\bar{a}) + h'(\underline{a})}{2}$$

(resp. \geq , see for instance Merkle (1998)). Then if furthermore $\text{Cov} \left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta \right) > \frac{\bar{a} - \underline{a}}{8}$, the first term of the lhs of (3.19) is greater than 1. If furthermore h is concave or linear, then $h'(\bar{a}) - h'(\underline{a}) \leq 0$ and the second term of the lhs of (3.19) is positive. Then, the three conditions imply that (3.19) holds.

— If $\text{Cov} \left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta \right) < 0$, and $h'(\bar{a}) - h'(\underline{a}) > 0$, then both terms of the lhs of (3.19) are negative and (3.19) does not hold.

3.E Proof of Proposition 3.3

We prove the fifth point in Proposition 3.3. If h is linear, then we have $\text{Br}^S(t) = \frac{a_2^{\varphi t}(m_2) - a_2^{\varphi t}(m_1)}{a_1^{\varphi t}(m_1) - a_1^{\varphi t}(m_2)}$. Suppose that $\text{Br}^S(t) = t$ has a solution in $(0, 1)$, and let t^* be the smallest one.

Let us first show that $\left. \frac{\partial \text{Br}^S(t)}{\partial t} \right|_{t=t^*} \leq 1$. Suppose that $\left. \frac{\partial \text{Br}^S(t)}{\partial t} \right|_{t=t^*} > 1$. Then we have $\text{Br}^S(t) < t^*$ for $t < t^*$ sufficiently close to t^* . However, according to Lemma 3.1, if t is sufficiently small then $\text{Br}^S(t) > t$. Then by continuity, there exists $t \in (0, t^*)$ such that $\text{Br}^S(t) = t$. This is impossible since t^* is the smallest solution of $\text{Br}^S(t) = t$.

Since we have $\left. \frac{\partial \text{Br}^S(t)}{\partial t} \right|_{t=t^*} \leq 1$, if moreover $\left. \frac{\partial \text{Br}^S(t)}{\partial t} \right|_{t=t^*} \geq -1$, then t^* is associated with a stable equilibrium. Let us show that if $\text{Cov} \left(\frac{f(\varphi_{t^*}(\theta))}{g'(\varphi_{t^*}(\theta))/g(\theta)}, \theta \right) \geq 0$, then $\left. \frac{\partial \text{Br}^S(t)}{\partial t} \right|_{t=t^*} \geq 0 > -1$. From the Law of Iterated Expectation, we may write

$$\text{Br}^S(t) = \frac{|\Theta_2^{\varphi t}| a_2^{\varphi t}(m_2) - |\Theta_2^{\varphi t}| \mathbb{E}}{|\Theta_2^{\varphi t}| \mathbb{E} - |\Theta_2^{\varphi t}| a_1^{\varphi t}(m_2)}.$$

Set

$$\begin{aligned}
 A(t) &= |\Theta_2^{\varphi_t}| a_2^{\varphi_t}(m_2) - |\Theta_2^{\varphi_t}| \mathbb{E} \\
 &= \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} \theta_2 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 - \mathbb{E} \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 \\
 &= \int_{\theta_2=0}^1 (\theta_2 - \mathbb{E}) \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 \\
 &= \int_{\theta_2=0}^1 (\theta_2 - \mathbb{E}) F(\varphi_t(\theta_2)) f(\theta_2) d\theta_2,
 \end{aligned}$$

and

$$\begin{aligned}
 B(t) &= |\Theta_2^{\varphi_t}| \mathbb{E} - |\Theta_2^{\varphi_t}| a_1^{\varphi_t}(m_2) \\
 &= \mathbb{E} \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 - \int_{\theta_2=0}^1 \left(\int_{\theta_1=0}^{\varphi_t(\theta_2)} \theta_1 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 \\
 &= \int_{\theta_2=0}^1 \left(\mathbb{E} F(\varphi_t(\theta_2)) - \int_{\theta_1=0}^{\varphi_t(\theta_2)} \theta_1 f(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2 \\
 &= \int_{\theta_2=0}^1 \left(\mathbb{E} F(\varphi_t(\theta_2)) - \left([\theta_1 F(\theta_1)]_0^{\varphi_t(\theta_2)} - \int_{\theta_1=0}^{\varphi_t(\theta_2)} F(\theta_1) d\theta_1 \right) \right) f(\theta_2) d\theta_2 \\
 &= \int_{\theta_2=0}^1 \left((\mathbb{E} - \varphi_t(\theta_2)) F(\varphi_t(\theta_2)) + \int_{\theta_1=0}^{\varphi_t(\theta_2)} F(\theta_1) d\theta_1 \right) f(\theta_2) d\theta_2.
 \end{aligned}$$

We have $\frac{\partial}{\partial t} \text{Br}^S(t) \Big|_{t=t^*} = \frac{\partial}{\partial t} \left(\frac{A(t)}{B(t)} \right) \Big|_{t=t^*} \geq 0$ iff

$$A'(t^*)B(t^*) + A(t^*)B'(t^*) \geq 0.$$

Note that $A(t^*) = t^*B(t^*)$ and $B(t^*) > 0$, so that $A'(t^*)B(t^*) + A(t^*)B'(t^*) \geq 0$ whenever

$$A'(t^*) + t^*B'(t^*) \geq 0.$$

From the Fundamental Theorem of Calculus, we have, for any $t \in (0, 1)$,

$$A'(t) = \int_{\theta_2=0}^1 (\theta_2 - \mathbb{E}) \frac{\partial}{\partial t} F(\varphi_t(\theta_2)) f(\theta_2) d\theta_2,$$

and

$$\begin{aligned}
 B'(t) &= \int_{\theta_2=0}^1 \left(\left(\left(-\frac{\partial}{\partial t} \varphi_t(\theta_2) \right) F(\varphi_t(\theta_2)) + (\mathbb{E} - \varphi_t(\theta_2)) \frac{\partial}{\partial t} F(\varphi_t(\theta_2)) \right) \right. \\
 &\quad \left. + \frac{\partial}{\partial t} \varphi_t(\theta_2) F(\varphi_t(\theta_2)) \right) f(\theta_2) d\theta_2 \\
 &= \int_{\theta_2=0}^1 (\mathbb{E} - \varphi_t(\theta_2)) \frac{\partial}{\partial t} F(\varphi_t(\theta_2)) f(\theta_2) d\theta_2.
 \end{aligned}$$

Then we obtain, at $t = t^*$,

$$\begin{aligned} A'(t^*) + t^* B'(t^*) &= \int_{\theta_2=0}^1 \left((\theta_2 - \mathbb{E}) \frac{\partial F(\varphi_t(\theta_2))}{\partial t} \Big|_{t=t^*} + t^* (\mathbb{E} - \varphi_{t^*}(\theta_2)) \frac{\partial F(\varphi_t(\theta_2))}{\partial t} \Big|_{t=t^*} \right) f(\theta_2) d\theta_2 \\ &= \int_{\theta_2=0}^1 (\theta_2 - t^* \varphi_{t^*}(\theta_2) - (1 - t^*) \mathbb{E}) \frac{\partial F(\varphi_t(\theta_2))}{\partial t} \Big|_{t=t^*} f(\theta_2) d\theta_2. \end{aligned}$$

Now note that for any $\theta_2 \in [0, 1]$, any $t \in (0, 1)$, $-\varphi_t(\theta_2) \geq -\theta_2$. Then we have

$$\begin{aligned} A'(t^*) + t^* B'(t^*) &\geq (1 - t^*) \int_{\theta_2=0}^1 (\theta_2 - \mathbb{E}) \frac{\partial F(\varphi_t(\theta_2))}{\partial t} \Big|_{t=t^*} f(\theta_2) d\theta_2 \\ &= (1 - t^*) \int_{\theta_2=0}^1 (\theta_2 - \mathbb{E}) f(\varphi_{t^*}(\theta_2)) \frac{g(\theta_2)}{g'(\varphi_{t^*}(\theta_2))} f(\theta_2) d\theta_2 \\ &= (1 - t^*) \text{Cov} \left(\frac{f(\varphi_{t^*}(\theta))}{g'(\varphi_{t^*}(\theta))/g(\theta)}, \theta \right). \end{aligned}$$

Therefore, as we claimed, if $\text{Cov} \left(\frac{f(\varphi_{t^*}(\theta))}{g'(\varphi_{t^*}(\theta))/g(\theta)}, \theta \right) \geq 0$, then $\frac{\partial \text{Br}^S(t)}{\partial t} \Big|_{t=t^*} \geq 0 > -1$ and t^* is associated with a stable equilibrium.

Now suppose that there are $\ell - 1 \geq 1$ asymmetric equilibria, associated with $0 < t^* = t_1^* \leq t_2^* < \dots < t_{\ell-1}^*$, and set $t_0^* = 0$, $t_\ell^* = 1$. We have shown that if $k = 1$, then $\text{Br}^S(t) > t$ on (t_{k-1}^*, t_k^*) , and that if $\text{Cov} \left(\frac{f(\varphi_{t_k^*}(\theta))}{g'(\varphi_{t_k^*}(\theta))/g(\theta)}, \theta \right) \geq 0$ then $\frac{\partial \text{Br}^S(t)}{\partial t} \Big|_{t=t_k^*} \geq 0$. Moreover, by definition $\text{Br}^S(t) < t$ on (t_k^*, t_{k+1}^*) . Consider any such t_k^* , *i.e.* such that $\text{Br}^S(t) > t$ on (t_{k-1}^*, t_k^*) , $\text{Br}^S(t) < t$ on (t_k^*, t_{k+1}^*) , and $\frac{\partial \text{Br}^S(t)}{\partial t} \Big|_{t=t_k^*} \geq 0$. Let us show that $\frac{\partial \text{Br}^S(t)}{\partial t} \Big|_{t=t_{k+1}^*} > 1$. Suppose $\frac{\partial \text{Br}^S(t)}{\partial t} \Big|_{t=t_{k+1}^*} \leq 1$. Then we have $\text{Br}^S(t) < t$ for $t < t_{k+1}^*$ sufficiently close to t_{k+1}^* . Then by continuity, there exists $t \in (t_k^*, t_{k+1}^*)$ such that $\text{Br}^S(t) = t$. This contradicts the definition of t_{k+1}^* .

Therefore $\frac{\partial \text{Br}^S(t)}{\partial t} \Big|_{t=t_{k+1}^*} > 1$, and the $\varphi_{t_{k+1}^*}$ -equilibrium is unstable. Now we have $\text{Br}^S(t) > t$ on (t_{k+1}^*, t_{k+2}^*) , from what we derive $\frac{\partial \text{Br}^S(t)}{\partial t} \Big|_{t=t_{k+2}^*} < 1$, and $\frac{\partial \text{Br}^S(t)}{\partial t} \Big|_{t=t_{k+2}^*} \geq 0$ whenever $\text{Cov} \left(\frac{f(\varphi_{t_{k+2}^*}(\theta))}{g'(\varphi_{t_{k+2}^*}(\theta))/g(\theta)}, \theta \right) \geq 0$. Thus, the $\varphi_{t_{k+2}^*}$ -equilibrium is stable.

Finally, if for any $t \in (0, 1)$, $\text{Cov} \left(\frac{f(\varphi_t(\theta))}{g'(\varphi_t(\theta))/g(\theta)}, \theta \right) \geq 0$, we obtain that the $\varphi_{t_1^*}$ -equilibrium is stable, the $\varphi_{t_2^*}$ -equilibrium is unstable, the $\varphi_{t_3^*}$ -equilibrium is stable, and so on, up to $t_\ell^* = 1$.

3.F Example

Proof of the existence. If $g(\theta) = g_r(\theta) = \theta^r$, we find $\frac{f(\theta)}{g'(\theta)/g(\theta)} = \frac{\theta}{r}$, and then

$$\text{Cov}\left(\frac{f(\theta)}{g'(\theta)/g(\theta)}, \theta\right) = \mathbb{E}\left[\frac{\theta}{r}\right] - \mathbb{E}[\theta] \mathbb{E}\left[\frac{\theta}{r}\right] = \frac{1}{12r}.$$

Moreover, $\bar{a} = \mathbb{E}[\theta_1 | \theta_1 \geq \theta_2] = \frac{2}{3}$ and $\underline{a} = \mathbb{E}[\theta_2 | \theta_1 \geq \theta_2] = \frac{1}{3}$. Then Corollary 3.2 gives the existence of an asymmetric equilibrium if $\frac{1}{12r} > \frac{\frac{2}{3} - \frac{1}{3}}{8} = \frac{1}{24}$, i.e. $0 < r < 2$. We show that an asymmetric equilibrium exists iff $0 < r < 2$.

From $g_r(\theta) = \theta^r$, we have $\varphi_t(\theta) = (t\theta^r)^{1/r} = \theta t^{1/r}$. Then we derive

$$\mathbf{a}^{\varphi_t}(m_1) = \left(\frac{1}{3} \frac{3 - t^{2/r}}{2 - t^{1/r}}, \frac{1}{3} \frac{3 - 2t^{1/r}}{2 - t^{1/r}}\right),$$

and

$$\mathbf{a}^{\varphi_t}(m_2) = \left(\frac{1}{3} t^{1/r}, \frac{2}{3}\right).$$

Then we have $|\Theta_2^{\varphi_t}|(a_1^{\varphi_t}(m_2) - \mathbb{E}[\theta_1]) = \frac{t^{1/r}}{2} \left(\frac{1}{3} t^{1/r} - \frac{1}{2}\right)$ and $|\Theta_2^{\varphi_t}|(a_2^{\varphi_t}(m_2) - \mathbb{E}[\theta_2]) = \frac{t^{1/r}}{2} \left(\frac{2}{3} - \frac{1}{2}\right)$, which gives

$$\text{Br}_r^S(t) = \frac{1}{3 - 2t^{1/r}}.$$

In particular, there exists $t \in (0, 1)$ such that $\text{Br}_r^S(t) = t$ iff there exists $t \in (0, 1)$ such that

$$z_r(t) = 2t^{(1+r)/r} - 3t + 1 = 0.$$

The derivative $z'_r(t) = 2\frac{1+r}{r}t^{1/r} - 3$ satisfies $z'_r(t) < 0$ iff $t < t(r)$, with

$$t(r) = \left(\frac{3}{2} \frac{r}{1+r}\right)^r,$$

and we have $t(r) < 1$ iff $\frac{r}{1+r} < \frac{2}{3}$, i.e. $r < 2$. Hence if $r < 2$, z_r attains its minimum value at $t(r)$, and in particular, z_r increases on $[t(r), 1)$, so that if $t(r) < t < 1$, then $z_r(t) < z_r(1) = 0$. From $z_r(0) = 1 > 0$, we obtain a unique $t \in (0, t(r))$ such that $z_r(t) = 0$. If in contrast $r \geq 2$, then z_r decreases on $(0, 1)$ from $z_r(0) = 1$ to $z_r(1) = 0$, and thus annihilates only at $t = 1$. \square

Proof of the stability. First, notice that at $t \rightarrow 0$, $\text{Br}_r^S(t) = \frac{1}{3 - 2t^{1/r}} \rightarrow 1$, so that the asymmetry of an anticipated strongly asymmetric comparison drives a Sender's best response $\text{Br}_r^S(t)$ close to $t = 1$. In particular, the babbling equilibrium is unstable relative to the anticipation of a small amount of information.

We compute for any $t \in (0, 1]$,

$$\frac{\partial \text{Br}_r^S}{\partial t}(t) = \frac{-2}{r} \frac{t^{(1-r)/r}}{(3 - 2t^{1/r})^2}.$$

At any solution $t^* \neq 0$ of $\text{Br}_r^S(t) = t$, we have

$$3 - 2t_*^{1/r} = \frac{1}{t_*}, \quad (3.20)$$

and therefore we have

$$\frac{\partial \text{Br}_r^S}{\partial t}(t_*) = \frac{-2}{r} t_*^{(1+r)/r}. \quad (3.21)$$

At $t_* = 1$, which corresponds to the symmetric equilibrium, we obtain $\left| \frac{\partial \text{Br}_r^S}{\partial t}(t_*) \right| = \frac{2}{r}$, and in particular, $\left| \frac{\partial \text{Br}_r^S}{\partial t}(t_*) \right| < 1$ iff if $r > 2$. There is an asymmetric equilibrium iff $0 < r < 2$. Therefore the symmetric equilibrium is a stable equilibrium iff it is the unique influential equilibrium.

Let us now look at $t_* = t_*(r)$, which corresponds to the asymmetric equilibrium. Using (3.20) again, we can furthermore rewrite (3.21) as

$$\frac{\partial \text{Br}_r^S}{\partial t}(t) = \frac{-2}{r} \left(\frac{1}{3 - 2t^{1/r}} \right)^{(1+r)/r}.$$

Now recall that $t_*(r) < t(r) = \left(\frac{3}{2} \frac{r}{1+r} \right)^r$, and notice that $t \mapsto \frac{-2}{r} \left(\frac{1}{3 - 2t^{1/r}} \right)^{(1+r)/r}$ is decreasing. This gives

$$0 > \frac{\partial \text{Br}_r^S}{\partial t}(t_*) > -\frac{2}{r} \left(\frac{1}{3 - 2 \left(\frac{3}{2} \frac{r}{1+r} \right)^{r/r}} \right)^{(1+r)/r} = -\frac{2}{r} \left(\frac{1+r}{3} \right)^{(1+r)/r} > -1,$$

and thus for any $r \in (0, 2)$, $|\text{Br}_r^S(t_*)| < 1$. Thus, as long as an asymmetric equilibrium is sustained, it is a stable equilibrium. \square

Conclusion générale

L'information est un facteur clé de la prise de décision. Dans cette thèse, nous avons abordé d'un point de vue stratégique la question de la transmission d'information entre agents aux intérêts potentiellement divergents. La transmission d'information que nous avons considérée est non coûteuse, non vérifiable et sans engagement.

Une caractéristique importante des modèles étudiés est de pouvoir décrire le contenu de l'information éventuellement transmise en la réduisant à ses déterminants rationnels. Cette caractéristique permet de rationaliser la communication entre agents, sans tenir compte du support de l'éventuelle information transmise. Il est alors possible d'analyser tout autant la recommandation d'un enseignant à un étudiant, que celle d'un média à un citoyen, d'un lobby à un décideur politique, ou encore le feedback d'un manager à un employé.

Dans le premier chapitre, nous avons illustré comment l'assignation d'un agent à une occupation, par un agent mieux informé sur la productivité des dimensions de l'occupation, pouvait être en partie déterminée par l'information transmise par cette assignation. Du point de vue de l'agent non informé, cette affectation peut alors se réaliser au détriment de son information, en appariant son plus grand effort à la dimension la moins productive, et son plus petit effort à la dimension la plus productive. La complémentarité des dimensions de la production est un paramètre clé du résultat et de l'importance de l'effet.

Concernant ce premier chapitre, une direction pour la recherche future pourrait concerner l'économie comportementale. En effet, d'une part la simplicité du résultat devrait permettre l'observation des choix des agents en situation. D'autre part, le caractère de statique comparative du résultat, selon le degré variable de la complémentarité considérée, pourrait permettre un contrôle de l'asymétrie de la révélation d'information éventuellement atteinte par les agents. Par exemple, une large littérature empirique concerne l'impact des attitudes des parents ou des enseignants sur le développement des aptitudes des individus. À notre connaissance, peu d'études concernent la nature de l'information transmise. En lien avec CUNHA, HECKMAN et SCHENNACH (2010), et plus généralement avec les sciences de l'éducation, une étude empirique pourrait être envisagée, à propos des recommandations des agents informés (parents, enseignants) concernant la production de capital humain des individus, selon le degré de complémentarité de ses dimensions, envisagé du point de vue informé, et selon les perceptions induites sur les individus. En particulier, la multiplicité d'équilibres du modèle théorique autorise précisément plusieurs types d'influence, qu'il pourrait être intéressant de mesurer.

Dans le second chapitre, nous avons caractérisé l'ensemble des équilibres du jeu de communication gratuite à deux dimensions, à deux messages, à distribution uniforme et à fonctions d'utilité quadratiques pour les deux agents. En particulier, nous avons montré qu'un agent informé, possédant un biais constant par rapport à un agent non informé, pouvait transmettre une partie de son information quelle que soit l'étendue et la direction de son biais. D'un point de vue théorique, le résultat donne ainsi des conditions suffisantes telles que l'ajout d'une dimension au modèle de CRAWFORD et SOBEL (1982) entraîne l'existence d'une possibilité de transmission stratégique d'information quel que soit le conflit d'intérêt

des agents. D'un point de vue plus appliqué, le résultat rend compte de la possibilité de trouver un compromis sur l'action à décider pour deux agents aux intérêts potentiellement très divergents. L'analyse géométrique donne la mesure du compromis selon l'étendue du conflit. En particulier, en rapport avec les résultats de CRAWFORD et SOBEL, un conflit faible autorise de nombreuses possibilités de partager l'information, tandis qu'un conflit important impose le partage d'une information orthogonale au conflit.

Le résultat du second chapitre se fonde sur des hypothèses très restrictives. Pour la recherche future, il serait intéressant de circonscrire le domaine de validité de ce résultat. L'analyse montre que les multiples symétries de la distribution uniforme sur $[0, 1]^2$ sont cruciales. Plus précisément, elles rendent compte d'une relative indépendance des types dans toutes les directions du plan, ce qui permet au conflit d'être suffisamment limité le long de sa dimension orthogonale, et ainsi y autorise une révélation d'information (uni-dimensionnelle et trans-dimensionnelle). Dans une recherche future, une mesure de la symétrie globale de l'espace des types pourrait être investie, ou, de manière équivalente, une mesure de l'indépendance des types selon toutes les directions de plan. Le résultat pourrait alors être éventuellement étendu aux distributions de types possédant une symétrie globale suffisante.

Enfin, dans le troisième chapitre, nous avons, dans un premier temps, identifié des conditions simples sur les fonctions d'utilité des agents qui caractérisent toute révélation d'information à deux messages comme comparative. Dans un second temps, nous avons montré que l'unicité de la comparaison symétrique n'était pas garantie en général. Le premier résultat permet de rationaliser la nature comparative de la transmission d'information, observée dans de nombreuses situations économiques. Le second résultat généralise le résultat de KAMPHORST et SWANK (2016) en proposant des aspects qualitatifs des paramètres du jeu qui le permettent. D'après ce second résultat, une révélation d'information peut, de manière crédible, induire un agent non informé à investir davantage là où il n'est pas le plus productif.

Là encore, étant donné l'importance du résultat (au moins au regard du nombre d'informations auxquelles les individus sont confrontés quotidiennement), il serait souhaitable, pour une recherche future, de tester l'effectivité du mécanisme dans les comportements des agents économiques en situation.

De manière plus théorique, il serait intéressant d'étendre le résultat à la prise en considération d'un nombre quelconque de messages. Nous conjecturons qu'un grand conflit dans chaque dimension impose, là aussi, la nature comparative de la révélation d'information. D'une manière plus importante, il serait souhaitable de déterminer, plus généralement, l'impact des conditions d'existence d'un équilibre *ex post* asymétrique sur la révélation d'information à un nombre quelconque de messages. En effet, il se pourrait que dans ce cadre, ces conditions rendent compte de la possibilité d'une révélation d'information plus informative.

Bibliographie

- AMBRUS, Attila et Satoru TAKAHASHI (2008). “Multi-sender cheap talk with restricted state spaces”. In : *Theoretical Economics* 3, p. 1-27.
- BARNARD, Chester Irving (1938). *The functions of the executive*. Cambridge : Harvard University Press.
- BATTAGLINI, Marco (2002). “Multiple referrals and multidimensional cheap talk”. In : *Econometrica* 70.4, p. 1379-1401.
- BROWN, Sarah, Aurora ORTIZ-NUÑEZ et Karl TAYLOR (2012). “Parental risk attitudes and children’s academic test scores : Evidence from the US panel study of income dynamics”. In : *Scottish Journal of Political Economy* 59.1, p. 47-70.
- CHAKRABORTY, Archishman et Rick HARBAUGH (2007). “Comparative cheap talk”. In : *Journal of Economic Theory* 132.1, p. 70-94.
- (2010). “Persuasion by Cheap Talk”. In : *The American Economic Review* 100.5, p. 2361-2382.
- CHE, Yeon-Koo, Wouter DESSEIN et Navin KARTIK (2013). “Pandering to persuade”. In : *American Economic Review* 103.1, p. 47-79.
- CHECCHI, Daniele, Carlo V FIORIO et Marco LEONARDI (2014). “Parents’ risk aversion and children’s educational attainment”. In : *Labour Economics* 30, p. 164-175.
- CHEN, Ying, Navin KARTIK et Joel SOBEL (2008). “Selecting Cheap-Talk Equilibria”. In : *Econometrica* 76.1, p. 117-136.
- CRAWFORD, Vincent P. et Joel SOBEL (1982). “Strategic information transmission”. In : *Econometrica*, p. 1431-1451.
- CRUTZEN, Benoît S.Y., Otto H. SWANK et Bauke VISSER (2013). “Confidence management : on interpersonal comparisons in teams”. In : *Journal of Economics & Management Strategy* 22.4, p. 744-767.
- CUNHA, Flavio, James J HECKMAN et Susanne M. SCHENNACH (2010). “Estimating the technology of cognitive and noncognitive skill formation”. In : *Econometrica* 78.3, p. 883-931.
- DEWATRIPONT, Mathias, Ian JEWITT et Jean TIROLE (2000). “Multitask agency problems : Focus and task clustering”. In : *European Economic Review* 44, p. 869-877.
- DU, Qiang, Vance FABER et Max GUNZBURGER (1999). “Centroidal Voronoi tessellations : Applications and algorithms”. In : *SIAM review* 41.4, p. 637-676.
- FARRELL, Joseph (1993). “Meaning and credibility in cheap-talk games”. In : *Games and Economic Behavior* 5.4, p. 514-531.

- FUDENBERG, Drew et Jean TIROLE (1991). *Game theory*. T. 393. 12, p. 80.
- GOPNIK, Alison (2012). “Scientific thinking in young children : Theoretical advances, empirical research, and policy implications”. In : *Science* 337.6102, p. 1623-1627.
- HACKMAN, Daniel A, Martha J FARAH et Michael J MEANEY (2010). “Socioeconomic status and the brain : mechanistic insights from human and animal research”. In : *Nature Reviews Neuroscience* 11.9, p. 651-659.
- HECKMAN, James J (2008). “Schools, Skills, and synapses”. In : *Economic Inquiry* 46.3, p. 289-324.
- HECKMAN, James J et Stefano MOSSO (2014). “The Economics of Human Development and Social Mobility”. In : *Annual Review of Economics* 6, p. 689-733.
- JÄGER, Gerhard, Lars P. METZGER et Frank RIEDEL (2011). “Voronoi languages : Equilibria in cheap-talk games with high-dimensional types and few signals”. In : *Games and Economic Behavior* 73.2, p. 517-537.
- KAMPHORST, Jurjen J.A. et Otto H. SWANK (2016). “Don’t demotivate, discriminate”. In : *American Economic Journal : Microeconomics* 8.1, p. 1-27.
- KNUDSEN, Eric I et al. (2006). “Economic, neurobiological, and behavioral perspectives on building America’s future workforce”. In : *Proceedings of the National Academy of Sciences* 103.27, p. 10155-10162.
- LAFFONT, Jean-Jacques et David MARTIMORT (2002). *The theory of incentives : the principal-agent model*. Princeton university press.
- LEVY, Gilat et Ronny RAZIN (mai 2004). *Multidimensional Cheap Talk*. Rapp. tech. 4393. CEPR Discussion Paper.
- (2007). “On the limits of communication in multidimensional cheap talk : a comment”. In : *Econometrica* 75.3, p. 885-893.
- LYTTON, Hugh et David M ROMNEY (1991). “Parents’ differential socialization of boys and girls : A meta-analysis.” In : *Psychological bulletin* 109.2.
- MEADE, James E (1952). “External economies and diseconomies in a competitive situation”. In : *The economic journal* 62.245, p. 54-67.
- MERKLE, Milan (1998). “Conditions for convexity of a derivative and some applications to the gamma function”. In : *Aequationes Mathematicae* 55.3, p. 273-280.
- SÉMIRAT, Stéphan (2017). *Uniform quadratic binary two-dimensional cheap talk*. Rapp. tech. Université Grenoble Alpes.
- SOBEL, Joel (2013). “Giving and receiving advice”. In : *Advances in Economics and Econometrics : Volume 1, Economic Theory : Tenth World Congress*. Sous la dir. de Daron ACEMOGLU, Manuel ARELLANO et Eddie DEKEL. T. 49. Cambridge University Press.
- TENENBAUM, Harriet R (2009). ““You’d Be Good at That” : Gender Patterns in Parent-Child Talk about Courses”. In : *Social Development* 18.2, p. 447-463.
- TOTH, G. (2015). *Measures of Symmetry for Convex Sets and Stability*. Universitext. Springer International Publishing.
- WANG, Ming-Te et Jessica L DEGOL (2017). “Gender gap in science, technology, engineering, and mathematics (STEM) : current knowledge, implications for practice, policy, and future directions”. In : *Educational psychology review* 29.1, p. 119-140.

WÖLFEL, Oliver et Guido HEINECK (2012). “Parental risk attitudes and children’s secondary school track choice”. In : *Economics of Education Review* 31.5, p. 727-743.