MOJETTE ERASURE CODE FOR DISTRIBUTED STORAGE CODE À EFFACEMENT MOJETTE POUR LE STOCKAGE DISTRIBUÉ

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Hot data

- Performance-oriented storage
- Expensive hardware
- High Performance Computing (instant access)



Cold Data

- Durability-oriented storage
- $\, \odot \,$ Commodity hardware, strong availability
- \bigcirc Archiving¹ (write once, read in few hours)

 $^{^{1}}$ Amazon Glacier guarantees < 1 sec unavailability every 3000 years





[Weatherspoon et al., 2002]



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Replication for Hot Data

○ no computation



Erasure coding limited to Cold Data [André et al., 2014]

• extra computations (encoding, decoding)

 \bigcirc delays during reads & writes

Problem: Today, no storage system can use erasure coding with hot data

Question: How can we design an efficient erasure code to build a storage system that can manage both hot and cold data?



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Problem: Today, no storage system can use erasure coding with hot data Question: How can we design an efficient erasure code to build a storage system that can manage both hot and cold data?



- ANR project (Emergence)
- Favors erasure codes (EC) over plain replication for distributed storage
- Promotes EC designed in French labs
- Explores approaches based on discrete geometry









- 2. Systematic Mojette Erasure Code
- 3. Reprojection without reconstruction
- 4. Distributed Storage System: RozoFS
- 5. Conclusion

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Outline

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- 1.1 Distributed storage
- 1.2 Erasure Coding

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- Scalability
- Fault-tolerance:
 - creation • over the time
- Performance
- \bigcirc Financial cost



²Network Distributed Storage Systems [Oggier et al., 2012]

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NDSS model with input/output and storage supports



Packets of ${\mathcal M}$ by tes is fragmented and redundant blocks are distributed



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A subset of blocks can rebuild the original packet



A subset of blocks can rebuild the original packet

(n, k) MDS codes are optimal



MDS codes properties:

- compute n = 6 encoded blocks from k = 4 data blocks
 generate 6 × 1 kB from M = 4 kB
- **optimal**: decode from any set of k = 4 encoded blocks among n = 6 (each block is $\frac{M}{k}$)

Maximum Distance Separable (MDS) codes [Singleton, 1964]

Systematic vs Non-systematic



Systematic

0	$\operatorname{compute}$	(n -	- k)	parity	blocks
---	--------------------------	------	------	--------	--------

- + better encoding
- + no decoding if no erasure

Systematic vs Non-systematic





Systematic

- \bigcirc compute (n k) parity blocks
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Non-systematic

- \bigcirc computes *n* encoded blocks
- + encoded blocks not readable
- + same weight for each block

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(n = 3, k = 1) MDS systematic erasure code (repetition code)



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(n = 5, k = 4) MDS systematic erasure code (parity code)







(n = 5, k = 4) MDS systematic erasure code (parity code)



(n = 6, k = 4) MDS systematic erasure code (Reed-Solomon code)



[Reed et al., 1960]



[Reed et al., 1960]

•

REED-SOLOMON codes:

- $\bigcirc\,$ generic parameters and minimum redundancy (50%)
- $\bigcirc\,$ but more complicated encoding and decoding processes



(n = 6, k = 4) MDS systematic erasure code (Reed-Solomon code)

- 1. is MDS (or near-MDS), minimizing the redundancy
- 2. is systematic to enhance encoding and decoding operations

Design a mechanism to compute extra encoded blocks Embed its erasure code in a practical distributed storage system

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- 2.1 Mojette erasure code
- 2.2 Systematic version
- 2.3 Evaluations
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3	1	4
2	0	5
4	2	1

f(k, l)





























Redundant representation



Redundant representation



Non-systematic (n, k) Mojette encoding:

- \bigcirc consider k = Q lines in the discrete grid
- given the following constraint: $(p_i, q_i = 1)_{i \in \mathbb{Z}_n}$ [Parrein, 2001]
- \bigcirc compute *n* projections

Non-systematic (n, k) Mojette decoding:

- $\odot\,$ reconstruction is unique given any k projections among the n (Katz' criterion)
- $\odot\,$ apply reconstruction algorithm [Normand et al., 2006]

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Non-systematic vs systematic Mojette erasure code



(6,3) Non-systematic

- $\bigcirc~n$ encoded blocks: 6 projections
- \bigcirc size grows with value $|p_i|$
Non-systematic vs systematic Mojette erasure code



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(6,3) Non-systematic

- $\bigcirc~n$ encoded blocks: 6 projections
- \bigcirc size grows with value $|p_i|$

(6,3) Systematic

- $\bigcirc~k$ data blocks: 3 lines from grid
- \bigcirc n-k parity blocks: 3 projections

- Sylvain David et al. "Procédé et appareil permettant de reconstruire un bloc de données". Centre National de la Recherche Scientifique (CNRS) Université de Nantes. WO Patent App. PCT/EP2014/071,310. Oct. 18, 2013
- Dimitri Pertin et al. "Comparison of RAID-6 Erasure Codes". In: The third Sino-French Workshop on Information and Communication Technologies. SIFWICT. Nantes, France, June 2015
 - we designed an algorithm based on [Normand et al., 2006]
 - we implemented and evaluated our solution

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Experimentation setup

- Mojette vs Reed-Solomon (INTEL ISA-L)
- \bigcirc Small block sizes (i.e. $\mathcal{M} = 4$ KB)
- $\odot\,$ FEC4Cloud platform: Intel Xeon 1.80 GHz, RAM 16 GB

Encoding performance











29/63







29/63

Storage overhead evaluation



$$\begin{array}{l} \bigcirc \ \mu = \frac{\text{size of } n \ \text{encoded blocks}}{\text{size of } k \ \text{data blocks}} \\ \circ \ \mu_{\text{MDS}} = \frac{\pi}{k} \\ \circ \ \mu_{\text{Mojette}} = \frac{\#_{\text{pixel and bins}}}{\#_{\text{pixels}}} \end{array} \end{array}$$

Storage overhead evaluation



$$\bigcirc \mu = \frac{\text{size of } n \text{ encoded blocks}}{\text{size of } k \text{ data blocks}}$$
$$\circ \mu_{\text{MDS}} = \frac{n}{k}$$
$$\circ \mu_{\text{Mojette}} = \frac{\#_{\text{pixel and bins}}}{\#_{\text{pixels}}}$$

- 1. systematic version outperforms the non-systematic version
- 2. ...and Reed-Solomon codes from ISA-L (factor $\times 3$)
- 3. cost only 3% extra data compared to optimal codes (near-MDS)
- Dimitri Pertin et al. The Mojette erasure code for distributed file systems. Session poster. Amsterdam, The Netherlands: EuroSys'14, Apr. 2014
- Dimitri Pertin et al. "Performance evaluation of the Mojette erasure code for fault-tolerant distributed hot data storage". In: CoRR abs/1504.07038 [Apr. 2015]

2. Systematic Mojette Erasure Code

3. Reprojection without reconstruction

- 3.1 The reprojection problem
- 3.2 Distributed reprojection
- 3.3 Evaluation
- 4. Distributed Storage System: RozoFS

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Reprojection problem



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Distributed Reprojection



Distributed Reprojection



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- 1. distribute the reprojection process
- 2. ...without reconstructing original data (grid)
- 3. ...is faster by a factor $\mathbf{2}$
- Dimitri Pertin et al. "Re-projection without Reconstruction". In:
 9ème Journées du Groupe de travail de Géométrie Discrète, Reims Image 2014. Reims, France, Nov. 2014, p. 43

- 2. Systematic Mojette Erasure Code
- 3. Reprojection without reconstruction

4. Distributed Storage System: RozoFS

- 4.1 Distributed File System
- 4.2 RozoFS Architecture
- 4.3 Evaluations

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inodes

UNIX FS^3

- unique number
- \bigcirc POSIX metadata
 - size
 - permissions
 - timestamps
 - o ...
- \bigcirc pointer to data blocks

RozoFS

- unique file identifier
- POSIX metadata
 - size
 - permissions
 - \circ timestamps
 - o ...
- extended attributes
 - storage node ids
 - Mojette code parameters
 - o ...

³FS: File System

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Data Input (writes) and Output (reads)



RozoFS architecture using (3, 2) systematic Mojette code










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- \bigcirc RozoFS (Mojette(6,4)) vs CephFS (3-rep): 8 storage nodes
- IOzone (read/write, random/sequential)
- GRID'5k econome platform:

 $\circ~$ Intel Xeon 2.20 GHz, RAM 64 GB, 10 GbE, 7200 RPM disks

Pros:

- \bigcirc very popular
- $\bigcirc\,$ General purpose file system (HDFS is not)
- \bigcirc fault-tolerance by replication (Lustre is not)

Cons:

- \bigcirc based on objects
- \bigcirc file system interface is still in development

Possible competitors:

 \bigcirc GlusterFS

 \bigcirc Tahoe-LAFS

HDFS: Hadoop Distributed File System









5

Number of clients

6 7

3 4

8

9





(b) Random write



CephFS architecture using triplication and journal



CephFS architecture using triplication and journal



CephFS architecture using triplication and journal



5

4

3

2

1

3

4





5

Number of clients

6

7 8 9

RozoFS

CephFS

(b) Random read.



5

4

3

2

3

4

∎∎RozoFS

CephFS





5

Number of clients

6 7

ead.

8

(b) Random read.



	\mathbf{s}_1	\mathbf{s}_2	s_3	s_4	s_5	\mathbf{s}_{6}	s_7	\mathbf{s}_8	Total
RozoFS	5.2	5.2	5.1	5.1	5.2	5.2	5.2	5.2	41.4
CephFS	14	9.6	8.5	12	12	9.8	12	12	89.9

Table: Evaluation of the storage consumption (in GB) for RozoFS and CephFS. Each of the 8 nodes in the cluster is depicted as s_i .

- 1. RozoFS can manage both hot and cold data (up to 60 kIOPS)
- 2. ...while providing fault-tolerance by the Mojette erasure code
- 3. ...and cuts by half the storage volume compared to CephFS.
- Dimitri Pertin et al. "Distributed File System Based on Erasure Coding for I/O-Intensive Applications". In: Proceedings of the 4th International Conference on Cloud Computing and Services Science. CLOSER. Barcelona, Spain, Apr. 2014, pp. 451–456
- Dimitri Pertin et al. "RozoFS: an erasure-coded distributed file system for I/O intensive workloads". Submitted to ACM Transactions on Storage (Dec. 2015)

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- 1. The systematic Mojette erasure code:
 - \bigcirc we designed an algorithm based on [Normand et al., 2006]
 - \bigcirc we showed our implementation in C outperforms:
 - the non-systematic version of Mojette-NS
 - $\circ\,$ up to 3× (encoding and decoding) RS codes from ISA-L

 \bigcirc and only 3% overhead compared to optimal codes (near-MDS)

- 2. Reprojection without reconstruction:
 - \bigcirc we designed a distributed method to compute new projections
 - \bigcirc ...without rebuilding the original data
 - we implemented this method in C
 - \bigcirc we showed it **outperforms** by a factor of 2 the classic method

- 3. Application in a distributed file system RozoFS:
 - we embedded the systematic Mojette erasure code in RozoFS (client-side) to provide fault-tolerance
 - \odot we showed RozoFS can manage both cold and hot data (60 kIOPS)
 - \bigcirc and outperforms CephFS by a factor 3 in random accesses
 - $\, \odot \,$...while dividing the volume of stored data by 2

- 1. Explore the links with other codes:
 - relations between Mojette and LDPC codes [Gallager, 1962]
 - $\bigcirc\,$ iterative reconstruction algorithm
- 2. Analysis of parity relations between bins of different projections:
 - $\bigcirc\,$ we discovered that groups of bins can sum to zero
 - $\bigcirc\,$ reconstruct erroneous bins without transferring whole projections
 - $\bigcirc\,$ ongoing work with Şuayb Arslan

- 1. Going further on RozoFS experimentations:
 - \bigcirc active benchmarking [Gregg, 2013]
 - $\bigcirc\,$ impact of an increasing number of storage servers
 - \bigcirc impact of errors on performances
- 2. Distributed management of metadata:
 - $\bigcirc\,$ metadata server is a single point of failure
 - $\bigcirc\,$ distribute metadata as Mojette projections
 - $\bigcirc\,$ ongoing work in the thesis of Bastien Confais (Oct 2015)

Thank you.



1. International journal (w/ committee)

Dimitri Pertin et al. "RozoFS: an erasure-coded distributed file system for I/O intensive workloads". Submitted to ACM Transactions on Storage (Dec. 2015).

2. International conferences (w/ committee)

Dimitri Pertin and Nicolas Normand. "Re-projection without Reconstruction". In: 9ème Journées du Groupe de travail de Géométrie Discrète, Reims Image 2014. Reims, France, Nov. 2014, p. 43.

Dimitri Pertin et al. "Comparison of RAID-6 Erasure Codes". In: The third Sino-French Workshop on Information and Communication Technologies. SIFWICT. Nantes, France, June 2015.

Dimitri Pertin et al. "Distributed File System Based on Erasure Coding for I/O-Intensive Applications". In: Proceedings of the 4th International Conference on Cloud Computing and Services Science. CLOSER. Barcelona, Spain, Apr. 2014, pp. 451–456.

Dimitri Pertin et al. "Spatial Implementation for Erasure Coding by Finite Radon Transform". In: International Symposium on signal, Image, Video and Communication. Valenciennes, France, July 2012.

Dimitri Pertin et al. The Mojette erasure code for distributed file systems. Session poster. Amsterdam, The Netherlands: EuroSys'14, Apr. 2014.

3. International communications (w/o acts)

Benoît Parrein et al. FEC4Cloud: a research project for promoting erasure codes in Cloud storage architectures. Bordeaux, France: Algebra, Codes and Networks (ACN), June 2014.

Dimitri Pertin. RozoFS: A Distributed File System based on Erasure Coding for I/O Intensive Workloads. Workshop on Storage and Processing of Big Data, WOS 4. Technicolor, Cesson-Sévigné, France, Dec. 4, 2014. Dimitri Pertin. RozoFS: A High-Performance Encoded-based Distributed Filesystem. Future Cloud Symposium. Session poster. Inria Conference Centre, Rennes, France: EIT Digital, 19 10, 2015.

Benoît Parrein et al. "FEC4Cloud: a research project promoting erasure coding for Cloud storage architectures". In: Rendez-vous de la Recherche et de l'Enseignement de la Sécurité des Systèmes d'Information (RESSI). Troyes, France, May 2015.

Benoît Parrein et al. RozoFS: a fault tolerant I/O intensive distributed file system based on Mojette erasure code. Toulouse, France: Workshop Autonomic, Oct. 2014.

Dimitri Pertin and Nicolas Normand. Re-projection without reconstruction. Mojette Day 2015. Polytech Nantes, Nantes, France, Feb. 2015.

4. Demonstrations

Benoît Parrein et al. Video streaming over RozoFS with fault tolerance. NEM SUMMIT. Cité de Congrès, Nantes, France, Oct. 28, 2013.

Dimitri Pertin et al. RozoFS: a fault tolerant distributed file system based on the Mojette transform. Third Workshop On Storage and Cloud Computing, WOS 3. Technicolor, Cesson-Sevigné, France, Nov. 2013.

5. Vulgarisations

Dimitri Pertin. Reduce the storage consumption of your storage clusters with RozoFS. Free and Open Source Software Developers' European Meeting FOSDEM'14. Université Libre de Bruxelles, Belgium, Feb. 2014.

Dimitri Pertin. Stockage distribuée à la Mojette : Gastronomie, Tomographie, Transmission et Stockage. Pas Sage En Seine. Paris, France, Apr. 2014.

Dimitri Pertin and Nicolas Normand. RozoFS: Le Système de Fichiers Distribués basé sur un code à effacement. Forum de Médias. Montaigu, France: Lycée Léonard de Vinci, Feb. 2014.



RozoBox: PRED with Olivier Blin & Quentin Lebourgois

$$B(P, Q, p_i, q_i) = (Q - 1)|p_i| + P$$
(1)

$$\mu = \frac{\sum\limits_{i=0}^{n-1} B(P, Q, p_i, q_i)}{P \times Q}.$$
(2)



Dependence graph for a (6×4) Mojette code





Layout	k	п	storage nodes	robustness	
0	2	3	4	1	
1	4	6	8	2	
2	8	12	16	4	

If Katz is invalid, it is not possible to reconstruct uniquely the grid

$$f = f_{SC} + \sum_{i} a_i g_i \tag{3}$$

- $\bigcirc\,$ start rebuilding grid
- $\bigcirc f_{SC}$ is the rebuilt image when zone erodée is null
- $\bigcirc~f$ is decomposed into f_{SC} and convolution of ghosts and coefficients

- $\bigcirc\,$ fix projection values to zero
- $\bigcirc f_{SC} = 0$ (so the reconstruction algorithm ends)
- $\bigcirc\,$ since we consider only one projection: height of zone erodée is 1

$$f_{S}^{\{(p_{i},q_{i}=1)\}} = h * g_{S \setminus \{(p_{i},q_{i})\}}$$
(4)

$$M_{\{(p_k,q_k)\}}\left[f_{S}^{\{(p_i,q_i)\}}\right] = \underbrace{M_{\{(p_k,q_k)\}}[h]}_{h} *M_{\{(p_k,q_k)\}}\left[g_{S\setminus\{(p_i,q_i)\}}\right]$$
(5)



Convolution of f with the ghost $g_{\{-1,1\},\{1,1\}}$. The result gives an image whose related projections are null. Operations are done modulo 6.
$$M_{\{(p_i,q_i)\}}\left[f_S^{\{(p_i,q_i)\}}\right] = h * M_{\{(p_i,q_i)\}}\left[g_{S \setminus \{(p_i,q_i)\}}\right]$$
(6)

$$M_{\{(p_k,q_k)\}}\left[f_S^{\{(p_i,q_i)\}}\right] = h * M_{\{(p_k,q_k)\}}\left[g_{S \setminus \{(p_i,q_i)\}}\right]$$
(7)

$$M_{\{(p_k,q_k)\}}\left[f_S^{\{(p_i,q_i)\}}\right] = M_{\{(p_i,q_i)\}}\left[f\right] \\ *^{-1}M_{\{(p_i,q_i)\}}\left[g_{\{S\setminus\{p_i,q_i\}\}}\right]$$
(8)
$$*M_{\{(p_k,q_k)\}}\left[g_{\{S\setminus\{p_i,q_i\}\}}\right].$$