ÉQUATIONS DISPERSIVES ET DISSIPATIVES ET QUELQUES APPLICATIONS EN ÉCOLOGIE

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Équations dispersives et dissipatives et quelques applications en écologie

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Equations dispersives et dissipatives et quelques applications en écologie

Ce mémoire d'habilitation présente mes principaux résultats en trois chapitres distincts.
Le premier concerne les équations aux dérivées partielles non-linéaires dispersives de la forme

$$u_t + f(u)_x - (-\partial_x^2)^{\alpha/2} u = 0.$$

Le problème de Cauchy y est étudié à travers une méthode de forme normale et des inégalités de dispersion. Deux propriétés de prolongement unique sont quant à elles établies via une inégalité de Carleman et une estimée de Bourgain.
La prise en compte de la viscosité et de la visco-élasticité fait l'objet du second chapitre. Décrite mathématiquement par un opérateur $\mathscr{A}$ tel que $\langle \mathscr{A}u, u\rangle > 0$, on y démontre la dissipation ainsi que la stabilité de l'énergie. La convergence vers la solution entropique de la loi de conversion hyperbolique

$$u_t + f(u)_x = 0,$$

est également prouvée.
Dans le troisième chapitre, complétement indépendant des deux premiers, on se focalise sur la modélisation des plantes et de leur environnement. Les interactions entre les plantes, les pathogènes et les ravageurs y sont dressées.

Dispersive and dissipative equations and some applications in ecology

This report presents my main results in three separate chapters.
The first one is about nonlinear dispersive partial differential equations of the form

$$u_t + f(u)_x - (-\partial_x^2)^{\alpha/2} u = 0.$$

The Cauchy problem is studied through a method of normal form and dispersion inequalities. Two unique continuation properties are established via a Carleman inequality and an estimate of Bourgain.
The consideration of the viscosity and viscoelasticity is the subject of the second chapter. Described mathematically by an operator $\mathscr{A}$ such that $\langle \mathscr{A}u, u\rangle > 0$, one shows the dissipation and the stability of the energy. The convergence to the entropy solution of the hyperbolic conversion law

$$u_t + f(u)_x = 0,$$

is also proved.
In the third chapter, completely independent of the first two, we focus on the modeling of plants and their environment. The interactions between plants, pathogens and pests are drawn up.

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Candeia que vai á frente alumia duas vezes.
La chandelle qui va devant éclaire deux fois
Proverbe portugais.

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Enfin, il y a dix ans, je rencontrais celle qui reste, aux côtés de mes parents et de mes frères, mon plus grand soutien, Lila.

CONTENTS

INTRODUCTION

1. Nonlinear dispersives PDEs

We are interested in dispersive equations of Korteweg-de Vries type of the form

$$u_t + f(u)_x - A^{\alpha/2}u_x = 0, \tag{0.1}$$

where $A := -\partial_x^2$, $\alpha \geq 1$. For $f(u) = u^2/2$, this includes the well-known Korteweg-de Vries equation ($\alpha = 2$) and the Benjamin-Ono one ($\alpha = 1$).
Due to the nonlinearity $f(u)_x$ depending on a spatial derivative, and therefore the difficulty of proving the locally Lipschitz character, to study the Cauchy problem, more particularly global existence and continuity with respect to initial data, remains a challenging problem.
One approach is to use dispersion inequalities as

$$||u(t)||_{L^\infty(\mathbb{R})} \lesssim \frac{||u_0||_{L^1\cap H^s}}{(1+|t|)^{1/(\alpha+1)}}.$$

This reflects the decay of solutions, due to the dispersion, *i.e.* the separation of Fourier modes which travel at different speeds [Mam08, Mam10a, Mam14, CGM15b]. Such inequality is deduced from the dispersion relation: the solution of the linearized equation around 0 is written in the form of an oscillating integral (via the Fourier transform in space)

$$u(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i(\xi^{\alpha+1}t + x\xi)}\hat{u}_0(\xi)d\xi,$$

and a stationary phase lemma leads to the conclusion.
When the solution is defined on a bounded domain, it becomes difficult to obtain such inequality. A change of viewpoint can be useful. For example,

when $f(u) = u^2/2$, there exists a change of variables $v = \Lambda u$ locally invertible, from which the equation (0.1) becomes

$$v_t - A^{\alpha/2} v_x = \tilde{f}(v),$$

and a study of this new equation allows extending the existence of solutions of (0.1) [Mam09a, Mam12b, Mam14]. This method is based on Poincaré's theory of normal form.

Another important question about the well-posedness concerns the uniqueness of solutions. It can be written as follows:

let $u(x,t) = 0$ for all $(x,t) \in \Omega \times I$ where $\Omega \times I \subset \mathbb{R} \times [0,T]$, then $u \equiv 0$?

When $I = [0,T]$ and Ω compact, an appropriate analytical expansion was proposed by Bourgain. Thanks to the Paley-Wiener theorem, it is possible to write

$$\hat{u}(\xi + i\sigma, t) = \mathrm{e}^{i(\xi+i\sigma)^{\alpha+1}t} \hat{u}_0(\xi + i\sigma),$$

and decay conditions on the initial datum u_0 lead to a positive conclusion [Mam09b, Mam10b]. This assumption can be weakened by taking $I \times \Omega$ an open subset of $[0,T] \times \mathbb{R}$. It is then necessary to determine a Carleman estimate of the form

$$\int |u|^2 \mathrm{e}^{\psi(x,t)} dx dt \lesssim \frac{1}{\tau^{\alpha+1}} \int |u_t + f(u)_x - A^{\alpha/2} u_x|^2 \mathrm{e}^{\psi(x,t)} dx dt,$$

where ψ is a convex function. Thus, when the right hand side is bounded, it is enough that τ goes to infinity so that u vanishes [Mam12a, Mam13a].

2. Damped nonlinear dispersive PDEs

Taking account of dissipation of energy in the equations reflects the damping of the waves due, for example, to the Coriolis force, or mangrove forests... From a mathematical point of view, the dissipation can be written

$$u_t + f(u)_x - \delta A^{\alpha/2} u_x + \varepsilon \mathscr{A} u = 0,$$

with $\langle \mathscr{A} u, u \rangle > 0$. Thus, multiplying the equation by u and integrating over space, it comes

$$\frac{d}{dt} ||u||_{L^2}^2 = -\varepsilon \langle \mathscr{A} u, u \rangle \leq 0,$$

and the L^2—norm is decreasing while it was preserved in the purely dispersive case (0.1). The study of the space

$$\mathscr{H} := \{u : \mathbb{R} \to \mathbb{R}; \langle \mathscr{A} u, u \rangle < +\infty\}$$

is then essential. Relevant choices of damping, for example $\mathscr{A}(u) = -u_{xx}$, provide continuous and compact injection properties in L^∞, for which $\mathscr{H}$ is an algebra. This enables to clearly define the nonlinearity $f(u)_x$. Therefore, one can build operators, *e.g.*

$$\widehat{\mathscr{A}(u)}(\xi) = \mathbb{1}_{\{\xi \leq N\}} \xi^2 \hat{u}(\xi), \text{ or } \mathscr{A}(u) = -\int_0^{+\infty} g(s) u_{xx}(t-s) ds,$$

which, in addition to damp the L^2—norm, stabilize the energy [CGM15a, DMP15].

Formally, when ε and δ go to 0, the following hyperbolic conservation law is found

$$u_t + f(u)_x = 0$$

for which there is a unique entropy solution. It is then possible to prove this convergence when the nonlinearity f is convex and when the dissipation is large compared to the dispersion ($\delta \ll \varepsilon$) [BCM14, BCM15b, BCM15a].

3. Multiscale modeling in botany

Current European prerogatives aim to reduce by 50% the amount of pesticides used to control pathogens and pests. A more respectful management of the environment through a better understanding of how plants and its epidemics at different scales of time and space is high-priority. For this purpose, we develop a plant-pathogen-pest modeling methodology at different spatial and temporal scales. While at the scale of a plant, a precise architectural view can be used, it becomes far more costly at a larger scale. Advection-reaction-diffusion equations then translate densities. These models are particularly well suited to the anthropic crops, such as vines.
It raises significant questions in the context of a more sustainable agricultural practice:

- how to improve the accuracy of models, including how nutrients, once collected, are redistributed throughout the plant? Thus, according to the osmotic transport assumption, the distribution of sucrose (C) via the pressure of the phloem (P) is driven by a nonlinear transport equation [MS15]
$$C_t = \nabla \cdot (C(d \nabla P)) + f(C);$$
- how the dynamics of the plant interacts with airborne diseases? At the local scale, the spread of spores (U) occurs along a cone of radius r whose generator is the direction (d) of the wind [MBC+10]
$$U(t, \alpha) \simeq \exp(-\beta d) \frac{\alpha - r(\alpha)}{\alpha}$$

whereas at the plot level, a reaction-diffusion equation is used [BCLM12, MBLC14]

$$U_t = \nabla \cdot (d \nabla U) - \delta U + f\,;$$

- how pests invade the fields? The adult pests (A) move across the landscape according to complex perceptions mechanisms, taking into account the resources (energy U, egg laying P) [BPM+15]

$$A_t = \nabla \cdot (d \nabla A) - v \nabla A - e A_U + f(P,A).$$

4. Numerical schemes and softwares development

We developed several computer codes as well with the aim to be easily reusable by non-specialists.

Due to their dispersive nature, which generates oscillations, and nonlinearity, which generates blow-up in finite time, we favor spectral methods combined with adaptive time schemes and fixed point iterations [Mam07, HMM09, Mam13b, MZ14]. These codes are used to study instabilities of Kadomtsev-Petviashvili solutions, and to illustrate the effects of dampings on Korteweg-de Vries solutions.

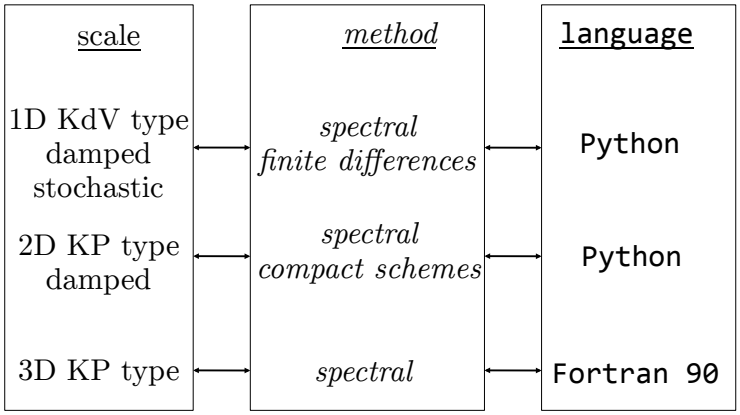

Figure 1: Numerical development environment to solve nonlinear dispersive equations.

Designed to be used by biologists, we develop a multiscale paradigm to simulate plant growth and the interactions with pathogens and pests. Friendly graphical interfaces were developed and the plant model was ported in the plateform OpenAlea [MBC+10, BCLM12, MBLC14, BPM+15, MS15].

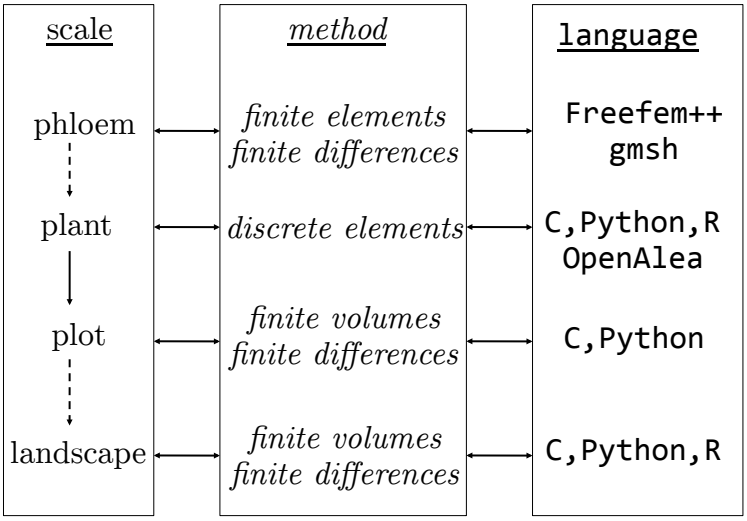

Figure 2: Numerical development environment of the multiscale ecological modeling.

CHAPTER 1

NONLINEAR DISPERSIVE PDES

Consider an irrotationnal flux of homogeneous incompressible fluid filling a volume $-h \leq z \leq \eta(x,y,t)$ [MT99, Whi74, Lan03, LS06, Mam07]. From Newton's second law, there exists φ a velocity potential such that

$$\Delta\varphi = 0 \quad \text{if} \quad -h < z < \eta(x,y,t) \tag{1.1}$$

satisfying the dynamic boundary conditions at the free surface

$$\begin{cases} \varphi_t + \dfrac{1}{2}|\nabla\varphi|^2 \\ -\dfrac{T}{\rho_0}\dfrac{\eta_{xx}(1+\eta_y^2)+\eta_{yy}(1+\eta_x^2)-2\eta_{xy}\eta_x\eta_y}{(1+\eta_x^2+\eta_y^2)^{3/2}} = 0 \text{ if } z=\eta(x,y,t) \\ \eta_t + \varphi_x\eta_x + \varphi_y\eta_y - \varphi_z = 0 \text{ if } z=\eta(x,y,t) \end{cases}$$

and the no-penetration condition at the bottom

$$\varphi_z = 0 \quad \text{if} \quad z=-h. \tag{1.2}$$

Assume that $\eta = \mathcal{O}(a)$, where a is the typical wave amplitude. Weakly transverse, small amplitude, long wavelength regime is expressed as

$$\alpha = \frac{a}{h} \ll 1,\ \beta = \frac{h^2}{k^2} \ll 1,\ \gamma = \frac{k^2}{l^2} \ll 1 \tag{1.3}$$

and is described by the change of variables

$$\begin{aligned} \varphi(x,y,z,t) &= \frac{gka}{c}\widetilde{\varphi}(\frac{x}{k},\frac{y}{l},\frac{z}{h}+1,\frac{c}{k}t), \\ \eta(x,y,t) &= a\widetilde{\eta}(\frac{x}{k},\frac{y}{l},\frac{c}{k}t), \end{aligned}$$

where $c=\sqrt{gh}$ denotes the typical velocity.

To solve the Laplace equation (1.1), we look for φ as a series expansion

$$\varphi(x,y,z,t) = \sum_{n=0}^{\infty} f_n(x,y,t)z^n.$$

If we denote $f := f_0$, an asymptotic expansion of φ with respect to β gives

$$\begin{aligned}\varphi(x,y,z,t) &= f-\left(\frac{\beta}{2}f_{xx}+\frac{\beta\gamma}{2}f_{yy}\right)z^2\\ &\quad+\left(\frac{\beta^2}{24}f_{xxxx}+\frac{\beta^2\gamma+\beta\gamma^2}{24}f_{xxyy}+\frac{\beta^2\gamma^2}{24}f_{yyyy}\right)z^4+\mathcal{O}(\beta^3)\end{aligned}$$

Then, for $w := f_x$, the following Boussinesq system is derived [Bou71]

$$\left\{\begin{array}{rcl}\eta_t+((1+\alpha\eta)w)_x-\dfrac{\beta}{6}w_{xxx}+\gamma w_{yy} &=& \mathcal{O}\left(\alpha^2+\beta^2+\gamma^2\right)\\ \eta_x+w_t+\alpha w w_x-\beta\left(\dfrac{1}{2}w_{xxt}+\sigma\eta_{xxx}\right) &=& \mathcal{O}\left(\alpha^2+\beta^2+\gamma^2\right).\end{array}\right.$$

If we restrict ourself to waves moving from left to right w.r.t. the $x-$direction, this system can be simplified to the Kadomtsev-Petviashvili equations [KP70]

$$\left(\eta_t+\eta_x+\frac{3}{2}\alpha\eta\eta_x+(\frac{1}{6}-\frac{\sigma}{2})\beta\eta_{xxx}\right)_x+\frac{\gamma}{2}\eta_{yy} = \mathcal{O}\left(\alpha^2+\beta^2+\gamma^2\right), \quad (1.4)$$

or its regularized version

$$\left(\eta_t+\eta_x+\frac{3}{2}\alpha\eta\eta_x-(\frac{1}{6}-\frac{\sigma}{2})\beta\eta_{xxt}\right)_x+\frac{\gamma}{2}\eta_{yy} = \mathcal{O}\left(\alpha^2+\beta^2+\gamma^2\right). \quad (1.5)$$

Equations (1.4) are called KP-I if $\sigma > 1/3$, KP-II if $\sigma < 1/3$, and equation (1.5) is called regularized KP-II (or KP-BBM-II) if $\sigma < 1/3$.

Remark 1.1 *In 1−dimension, we find the well-known Korteweg-de Vries equation [KdV95]*

$$\eta_t+\eta_x+\frac{3}{2}\alpha\eta\eta_x+(\frac{1}{6}-\frac{\sigma}{2})\beta\eta_{xxx}=0,$$

and the Benjamin-Bona-Mahony equation [BBM72]

$$\eta_t+\eta_x+\frac{3}{2}\alpha\eta\eta_x-(\frac{1}{6}-\frac{\sigma}{2})\beta\eta_{xxt}=0.$$

In the following, we present some results about the nonlinear dispersive equations. To simplify the readings, we consider the Cauchy problem

$$\begin{aligned}u_t+A^{\alpha/2}u_t+u_x+\left(\frac{u^{\rho+1}}{\rho+1}\right)_x &= 0 \quad (1.6)\\ u(x,t=0) &= u_0(x), \quad (1.7)\end{aligned}$$

where $A=-\partial_x^2$ and $\alpha \geq 1$.

1.1 Summary of well-posedness

The free evolution $(S_t)_{t\in\mathbb{R}}$ is given by

$$S_t u = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\xi x + i\frac{\xi}{1+\xi^\alpha}t}\widehat{u}(\xi)d\xi$$

and satisfies thanks to the Sobolev embedding:

Lemma 1.2 *For $s > 1/2$, and $u \in H^s(\mathbb{R})$, we have*

$$||S_t u||_s \lesssim ||u||_s,$$

and

$$||S_t\left((1+A^{\alpha/2})^{-1}(u^\rho u_x)\right)||_s \lesssim ||u||_s^{\rho+1}.$$

Then it is enough to prove that the solution of the initial value problem is the unique fixed point of the mild formulation [LP09, Tao05]

$$\Phi u(t) = S_t u_0 - \int_0^t S_{t-\tau}\left((1+A^{\alpha/2})^{-1}(u^\rho u_x)\right)(\tau)d\tau.$$

Theorem 1.3 *Let $\rho \geq 1, s > 1/2$ and $u_0 \in H^s(\mathbb{R})$. Then there exist $T \simeq 1/||u_0||_s^\rho > 0$ and a unique solution $u \in \mathscr{C}([-T,T];H^s(\mathbb{R}))$ solution of the initial value problem (1.6)-(1.7) with u_0 as initial datum.*
Moreover, the solutions u and v starting from u_0 and v_0 respectively, satisfies for $t \in [-T,T]$,

$$||u(t)-v(t)||_s \lesssim ||u_0 - v_0||_s.$$

To extend the time of existence, we notice by multiplying equation (1.6) by u and integrating over space, that the following quantity is conserved: for all $t \in \mathbb{R}$,

$$||u(t)||_{\alpha/2} = ||u_0||_{\alpha/2}.$$

Theorem 1.4 *Let $\alpha \geq 1$ and $u_0 \in H^{\alpha/2}(\mathbb{R})$. Then the initial value problem (1.6)-(1.7) is globally well-posed.*

Indeed, the Duhamel formulation is written

$$\begin{aligned} ||u(t)||_s &= \left|\left|S_t u_0 - \int_0^t S_{t-\tau}\left((1+A^{\alpha/2})^{-1}(u^\rho u_x)\right)d\tau\right|\right|_s \\ &\lesssim ||u_0||_s + \int_0^t ||u||_\infty^\rho ||u||_s d\tau, \end{aligned}$$

then Brezis-Gallouët's inequality [BG80] gives

$$||u(t)||_s \lesssim ||u_0||_s + \int_0^t \left(1+\sqrt{\log(1+||u||_s)}\right)||u||_s d\tau.$$

Gronwall's lemma implies that for $t \in \mathbb{R}$

$$||u(t)||_s \leq e^{Ce^{Ct}}, \tag{1.8}$$

whereas for $t \in [-T,T]$

$$||u(t)||_s \leq C||u_0||_s. \tag{1.9}$$

Remark 1.5 *To solve the standard initial value problem*

$$u_t - A^{\alpha/2}u_x + u^\rho u_x = 0, \tag{1.10}$$

a regularization is needed. Either parabolic [Iór90, Tem97, TM71]

$$u_t + u^\rho u_x - A^{\alpha/2}u_x - \varepsilon u_{xx} = 0,$$

in such way that the free evolution $(S_t)_{t\in\mathbb{R}}$ is given by

$$S_t u = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\xi x + i\xi^{\alpha+1}t - \varepsilon\xi^2 t}\widehat{u}(\xi)d\xi$$

and satisfies:

Lemma 1.6 *For $r,s \geq 0$, and $u \in H^s(\mathbb{R})$, we have*

$$||S_t u||_{r+s} \lesssim \left(1+\left(\frac{1}{2\varepsilon t}\right)^r\right)^{1/2}||u||_s,$$

and

$$||S_t u^\rho u_x||_s = ||S_t u^\rho u_x||_{(s-1)+1} \lesssim \left(1+\left(\frac{1}{2\varepsilon t}\right)\right)^{1/2}||u||_s^{\rho+1}.$$

Or, another way is to use the Bona-Smith regularization [BS75]

$$u_t + (1-\varepsilon\partial_x^2)^{-1}\left(u^\rho u_x - A^{\alpha/2}u_x\right) = 0.$$

In this case, the semi-group is written as

$$S_t u = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\xi x + i\frac{\xi^{\alpha+1}t}{1+\varepsilon\xi^2}}\widehat{u}(\xi)d\xi.$$

Lemma 1.7 *For $s \geq 0$, and $u \in H^s(\mathbb{R})$, we have*

$$||S_t u||_s \lesssim ||u||_s,$$

and

$$||S_t(1-\varepsilon\partial_x^2)^{-1} f(u)_x||_s \lesssim ||f(u)||_s.$$

The unique fixed point of the mild formulation is obtained for a time independent of ε and we take the limit as ε goes to 0.
By means of Bourgain's spaces and bilinear smoothing estimates, the well-posedness can be extended (e.g. $\alpha = 2, H^s(\mathbb{R})$ with $s > -3/4$) [Bou93b, CKS+03, KPV96].
Global well-posedness needs the two first conserved quantities: for all $t \in \mathbb{R}$,

$$N(t) := ||u(t)||_{L^2}^2 = N_0,$$

and

$$E(t) := \int_{-\infty}^{+\infty} (A^{1/2}u)^2 - \frac{u^{\rho+2}}{(\rho+1)(\rho+2)} dx = E_0.$$

Theorem 1.8 *Suppose $\rho < 4$. Let $u_0 \in H^1(\mathbb{R})$. Then the initial value problem (1.10) is globally well-posed.*

Indeed, we deduce from the Gagliardo-Nirenberg inequality that

$$\begin{aligned} ||u||_1^2 &= ||u||_{L^2}^2 + ||A^{1/2}u||_{L^2}^2 = N_0 + E_0 + \int_{-\infty}^{+\infty} \frac{u^{\rho+2}}{(\rho+1)(\rho+2)} dx \\ &\lesssim 1 + ||A^{1/2}u||_{L^2}^{\rho/2} ||u||_{L^2}^{(\rho+4)/2} \lesssim 1 + ||A^{1/2}u||_{L^2}^{\rho/2}, \end{aligned}$$

which is finite while $\rho < 4$.

1.2 Normal form and large time existence

Based on Poincaré's theory of normal form, we can find a change of variable improving inequality (1.8) [Arn83, Sha85, Tzv04]. Assume $\rho = 1$ and $x \in \mathbb{T}$, we define $v = \Lambda u$ as

$$\Lambda u := u - \frac{1}{2} \sum_{k \neq k_1 \in \mathbb{Z}^*} e^{ikx} \frac{ik/(1+|k|^\alpha)}{\sigma(k_1) + \sigma(k-k_1) - \sigma(k)} \widehat{u}(k_1)\widehat{u}(k-k_1) = u + \Lambda_1(u,u),$$

and $\sigma(k) = ik/(1+k^\alpha)$ is the relation dispersion.

Proposition 1.9 *[Mam09a, Mam12b, Mam14] Assume $\alpha \geq 1$.*

i. For all $k \neq k_1 \in \mathbb{Z}^*$, we have

$$|\sigma(k_1)+\sigma(k-k_1)-\sigma(k)| \gtrsim |kk_1(k-k_1)|.$$

ii. There exists $\delta, \delta' > 0$ (small enough), such that for all $v \in B_\delta$, there exists a unique $u \in B_{\delta'}$ with $v = \Lambda u$.

The inverse function theorem is applied to define this local change of variable since for all $\varphi \in \mathscr{C}^\infty(\mathbb{T})$

$$\langle d\Lambda(u)\,,\,\varphi\rangle = \varphi + 2\Lambda_1(u,\varphi),$$

and $d\Lambda$ is continuous from $H^s(\mathbb{T})$ to itself and $d\Lambda(0) = Id$.
This new unknown v is solution of the nonlinear PDE

$$v_t + (1+A^{\alpha/2})^{-1}v_x = g(v),$$

where g is a trilinear operator obtained by direct computation

$$\begin{aligned} v_t + (1+A^{\alpha/2})^{-1}v_x &= (\Lambda u)_t + (1+A^{\alpha/2})^{-1}(\Lambda u)_x \\ &= -\frac{1}{2}\sum_{k\neq k_1\neq k_2\in\mathbb{Z}^*} e^{ikx}\frac{ik/(1+|k|^\alpha)\,k_1/(1+|k_1|^\alpha}{\sigma(k_1)+\sigma(k-k_1)-\sigma(k)}\,\widehat{u}(k_2)\widehat{u}(k_1-k_2)\widehat{u}(k-k_1). \end{aligned}$$

Note that

$$u^2(x) = \sum_{k\neq k_1\in\mathbb{Z}^*} e^{ikx}\widehat{u}(k_1)\widehat{u}(k-k_1).$$

A study of the mild solution, *i.e.*

$$\psi v(t) = S_t v_0 - \int_0^t S_{t-\tau}g(v)d\tau,$$

allows to conclude.

Theorem 1.10 *[Mam09a, Mam12b, Mam14] Let $s > 1/2$. There exists $\varepsilon > 0$ small enough such that if $u_0 \in H^s(\mathbb{T})$ with $||u_0||_s \leq \varepsilon$, then the unique solution $u \in \mathscr{C}([-T^2, T^2]; H^s(\mathbb{T}))$ of the Cauchy problem (1.6)-(1.7) with $T \simeq 1/||u_0||_s = \varepsilon^{-1}$, satisfies for $|t| \leq T^2$,*

$$||u(t)||_s \lesssim ||u_0||_s.$$

Remark 1.11 *This method remains true to the standard equation with $\alpha \geq 2$*

$$u_t - A^{\alpha/2}u_x + uu_x = 0,$$

but does not apply when $\alpha < 2$. In this case, for $k \neq k_1 \in \mathbb{Z}^*$

$$\left|\frac{kk_1}{\sigma(k_1)+\sigma(k-k_1)-\sigma(k)}\right|$$

is not bounded from above.

1.3 Dispersion and global well-posedness

Let us come back to $x \in \mathbb{R}$. In this case, it is possible to have help from decreasing properties of the free evolution. The decay of this semi-group, given by an oscillating integral, is estimated using Van der Corput's lemma and Riesz-Thorin's interpolation [Alb86, HN99, Ste93, Ste86, Str74].

Lemma 1.12 *[Mam08, Mam10a, Mam14] Let $s = 2\alpha - 1/2$. Then for $u_0 \in L^p(\mathbb{R}) \cap H^s(\mathbb{R})$, we have for all time $t \in \mathbb{R}$*

$$||S_t u_0||_{L^q} = \left|\left| \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\xi x + i\frac{\xi}{1+\xi^\alpha} t} \widehat{u_0}(\xi) d\xi \right|\right|_{L^q} = \left|\left| \mathscr{F}^{-1}(e^{i\frac{\xi}{1+\xi^\alpha} t}) * u_0 \right|\right|_{L^q} \lesssim \frac{||u_0||_{L^p \cap H^s}}{(1+|t|)^{\frac{1-2/q}{\alpha+1}}},$$

where $1/p + 1/q = 1$ and

$$1 \le p \le 2 \text{ if } \alpha \ge 2$$
$$1 < p \le 2 \text{ if } \alpha < 2.$$

This last condition is deduced from the Mikhlin-Hörmander theorem, the operator $(1 + A^{\alpha/2})^{-1}\partial_x$ being bounded in $L^p(\mathbb{R})$ with $1 \le p \le 2$, resp. $1 < p \le 2$ if $\alpha \ge 2$, resp. $\alpha < 2$.

Remark 1.13 *For the standard equation*

$$u_t - A^{\alpha/2} u_x + u^\rho u_x = 0,$$

we have for $\alpha \ge 1$, and $u_0 \in L^p(\mathbb{R})$, for all $t \in R$,

$$||S_t u_0||_{L^q} = \left|\left| \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\xi x + i\xi^{\alpha+1} t} \widehat{u_0}(\xi) d\xi \right|\right|_{L^q} \lesssim \frac{||u_0||_{L^p}}{(1+|t|)^{\frac{1-2/q}{\alpha+1}}}.$$

When $\alpha \ge 1$, the $(\alpha+1)$—th derivative of the function $\sigma(\xi) = \xi^{\alpha+1} + \xi x/t$ is uniformly bounded from below whereas the $(\alpha+1)$—th derivative of the regularized function $\sigma(\xi) = \xi/(1+\xi^\alpha) + \xi x/t$ goes to 0 as $\xi \to \infty$.

To take care of the nonlinearity, we write according to Duhamel's formulation

$$||u(t)||_{L^q} = \left|\left| S_t u_0 - \int_0^t S_{t-\tau}\left((1 + A^{\alpha/2})^{-1}(u^\rho u_x)\right) d\tau \right|\right|_{L^q} \lesssim \frac{||u_0||_{L^p \cap H^s}}{(1+|t|)^{\frac{1-2/q}{\alpha+1}}} + \int_0^t \frac{||u^{\rho+1}||_{L^p \cap H^s}}{(1+|1+t-\tau|)^{\frac{1-2/q}{\alpha+1}}} d\tau.$$

Let $N_T = \sup_{t\in[0,T]}(1+|t|)^{\frac{1-2/q}{\alpha+1}}||u(t)||_{L^q}$. Thus, according to the Leibniz rule,

$$N_T \lesssim ||u_0||_{L^p\cap H^s} + N_T^\rho \int_0^T \frac{(1+|t|)^{\frac{1-2/q}{\alpha+1}}}{(1+|1+t-\tau|)^{\frac{1-2/q}{\alpha+1}}(1+|\tau|)^{\rho\frac{1-2/q}{\alpha+1}}}d\tau,$$

The time integral is uniformly bounded as soon as $\rho > \alpha+1$.

Theorem 1.14 *[Mam08, Mam10a, Mam14] Let $\rho \geq \alpha+2$. Then there exists $\varepsilon > 0$ such that for $u_0 \in L^p\cap H^s(\mathbb{R})$, of norm smaller than ε, there exists a unique global in time solution $u \in \mathscr{C}(\mathbb{R}, L^p\cap H^s(\mathbb{R}))$ of (1.6)-(1.7) and $\forall t\in\mathbb{R}$,*

$$||u(t)||_{L^q} \lesssim \frac{||u_0||_{L^p\cap H^s}}{(1+|t|)^{\frac{1-2/q}{\alpha+1}}}.$$

Remark 1.15 *Concerning the KP-BBM equation [Mam08] (or the KP equations [HNS99])*

$$u_t - u_{xxt} + u_x + u^\rho u_x + \partial_x^{-1}u_{yy} = 0$$

the 2D—oscillatory integral

$$S_t u = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{i\xi_1 x + i\xi_2 y + i\frac{\xi_1+\xi_2^2/\xi_1}{1+\xi_1^\alpha}t}\widehat{u_0}(\xi_1,\xi_2)d\xi_1 d\xi_2$$

is reduced to the 1D—oscillatory integral using the change of variables $\xi_2 = \widetilde{\xi_2}\sqrt{\frac{|\xi_1|(1+\xi_1^2)}{|t|}}$,

$$\begin{aligned}\left|\int_{-|t|^\delta}^{|t|^\delta}\int_{-\infty}^{+\infty} e^{i\xi_1 x + i\xi_2 y + i\frac{\xi_1+\xi_2^2/\xi_1}{1+\xi_1^\alpha}t}\widehat{u_0}(\xi_1,\xi_2)d\xi_1 d\xi_2\right| &\\ = \left|\frac{\sqrt{\pi}e^{i\pi/4}}{|t|^{1/2}}\int_{-|t|^\delta}^{|t|^\delta}\sqrt{|\xi_1|(1+\xi_1^2)}e^{ix\xi_1 - i\frac{\xi_1}{1+\xi_1^2}t + i\frac{y^2}{4t^2}(\xi_1+\xi_1^3)}d\xi_1\right| &\\ \lesssim |t|^{\frac{-1+5\delta}{2}}.&\end{aligned}$$

Note that the operator $(1+A^{\alpha/2})^{-1}\partial_x$ is not bounded in $L^1(\mathbb{R}^2)$ but in $L^p(\mathbb{R}^2)$ with $1 < p < +\infty$.

Stochasticity in the dispersion was recently introduced for nonlinear Schrödinger equations [ASS13, dBD10, DT11, CG15]. It models the case where the linear oscillations is driven by a Brownian motion instead of linearly with respect to time. Let $W(t)$ be a standard real valued Wiener process associated to

the probability space $(\Omega, \mathscr{F}, \mathbb{P})$ and the stochastic basis $(\Omega, \mathscr{F}, \mathbb{P}, (\mathscr{F}_t)_{t\geq 0})$. Let u_s being a random $\mathscr{F}_s-$ variable valued in $H^1_x(\mathbb{R})$. For $t > s$, the solution u of

$$\begin{aligned} du - dA^{\alpha/2}u + u_x \circ dW &= 0 \\ u(s) &= u_s, \end{aligned}$$

is written in Fourier variable as

$$\widehat{u}(t,\xi) = e^{\frac{-i\xi}{1+\xi^\alpha}(W(t)-W(s))}\widehat{u}_s(\xi).$$

Due to the definition of the Wiener process being such that

$$W(t) - W(s) \sim \sqrt{t-s}\mathcal{N}(0,1) \text{ and independent,}$$

the average decay rate of the solution of the stochastic dispersive equation is twice as slow as those for equation with deterministic dispersion (see Fig. 1.1). Indeed, we have

$$\mathbb{E}\left(||S_t u_0||_{L^\infty}\right) = \mathbb{E}\left(||u_0||_{L^1}\right)\mathbb{E}\left(|W(t)-W(s)|^{-\frac{1}{\alpha+1}}\right) \lesssim \frac{\mathbb{E}\left(||u_0||_{L^1}\right)}{|t-s|^{\frac{1}{2(\alpha+1)}}}.$$

Theorem 1.16 *[CGM15b] Let $\rho \geq 2(\alpha+1)+2$. Then there exists $\varepsilon > 0$ such that for $u_0 \in L^p \cap H^s(\mathbb{R})$, of norm smaller than ε, there exists a unique global in time solution $u \in L^1\left(\Omega; \mathscr{C}(\mathbb{R}, L^p \cap H^s(\mathbb{R}))\right)$ of*

$$du - dA^{\alpha/2}u + u_x \circ dW + u^\rho u_x dt = 0$$

and $\forall t \in \mathbb{R}$,

$$\mathbb{E}(||u(t)||_{L^q}) \lesssim \frac{\mathbb{E}(||u_0||_{L^p\cap H^s})}{(1+|t|)^{\frac{1-2/q}{2(\alpha+1)}}}.$$

1.4 Unique continuation property

We wonder if, given two solutions $u(x,t) = v(x,t)$ on a domain Ω of $\mathbb{R}\times\mathbb{R}_+$, then $u \equiv v$. In other words, if $u(x,t) = 0$ on a domain Ω of $\mathbb{R}\times\mathbb{R}_+$, do $u \equiv 0$? Suppose Ω is an open subset of $\mathbb{R}\times[0,T]$. Then we prove:

Theorem 1.17 *[Mam12a, Mam13a] Let $u \in \mathscr{C}(0,T;H^s(\mathbb{R}))$ with $s \geq 4$ such that $u \equiv 0$ in Ω. Then $u \equiv 0$ in $\mathbb{R}\times[0,T]$.*

The proof is based on the following Carleman estimates [Car33, SS87]:

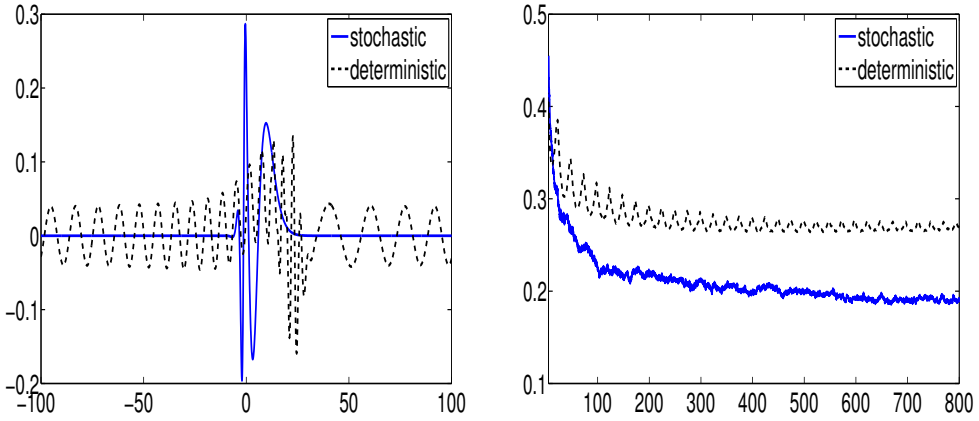

Figure 1.1: On the left, comparison of the solution of the deterministic (- -) and stochastic (—) BBM equation starting from the same initial datum at time $t = 800$. On the right, decay rate of the deterministic L^∞—norm (- -) and expectation of the L^∞—norm (—) with respect to time.

Lemma 1.18 *Let us denote*

$$\mathscr{L} := \partial_t - cA\partial_x + f(x,t)\partial_x,$$

with $c \in \mathbb{R}$ *and* $f \in L^\infty(\mathbb{R}\times[0,T])$, *and*

$$B_\delta := \left\{(x,t)\in\mathbb{R}^2; x^2+t^2<\delta^2\right\}.$$

Then for all $\Phi \in \mathscr{C}^\infty(B_\delta)$, $\tau > 0$ *such that*

$$\frac{||f||_{L^\infty}}{\tau^\alpha\delta^2} \leq \frac{1}{2},$$

we have

$$\tau^{\alpha+1}\int_{B_\delta}|\Phi|^2e^{2\tau\Psi}dxdt + \tau^\alpha\delta^2\int_{B_\delta}|\Phi_x|^2e^{2\tau\Psi}dxdt \lesssim \int_{B_\delta}|\mathscr{L}\Phi|^2e^{2\tau\Psi}dxdt,$$

where $\Psi(x,t) = (x-\delta)^2 + \delta^2t^2$.

Assuming that the solution u of $\mathscr{L}u = 0$ is such that $u \equiv 0$ in $\{x < |t|\}\cap B_\delta$, then using the fact that Ψ describes an ellipse, we have

$$\Psi(0,0) = \delta^2 \text{ and } \Psi(x,t) < \delta^2 \text{ elsewhere }.$$

Thus there exist $\varepsilon > 0$ such that

$$\Psi(x,t) \leq \delta^2 - \varepsilon \text{ in } B_\delta$$
$$\Psi(x,t) > \delta^2 - \varepsilon \text{ in } B_\eta \subset B_\delta,$$

and the Carleman estimate becomes, for $(u_n)_n$ a regularizing sequence,

$$\tau^{\alpha+1} e^{2\tau(\delta^2-\varepsilon)} \int_{B_\eta} |u_n|^2 dxdt \lesssim e^{2\tau(\delta^2-\varepsilon)} \int_{B_\delta} |\mathscr{L} u_n|^2 dxdt.$$

It is enough that τ goes to $+\infty$ to conclude. This local result remains true for any curvature $\{x < \mu(t)\}$ thanks to the change of variables

$$\widetilde{x} = x - \mu(t) + |t|, \ \widetilde{t} = t,$$

and it is extended using moving plane method [DM98, Hör76, Nir57].

Remark 1.19 *Unique continuation properties can be proved for the KP equations using the following Carleman estimates:*

Lemma 1.20 *Let $\delta > 0$ and $B_\delta := \{(x,y,t) \in \mathbb{R}^3; x^2 + y^2 + t^2 < \delta^2\}$. Then for all $\Phi \in \mathscr{C}^\infty(B_\delta)$, $\Psi(x,y,t) = (x-\delta)^2/2 + (y-\delta)^2/2 + \delta^2 t^2$ and $\tau > 0$ with*

i.

$$\frac{||f_1||_\infty^2}{\tau^3\delta^2} + \frac{||f_2||_\infty^2}{\tau^2\delta^2}\left(1 + \frac{1}{\tau^2}\right) \leq \frac{1}{4},$$

we have

$$\tau^4\delta^2 \int_{B_\delta} |\Phi|^2 e^{2\tau\Psi} dxdydt \lesssim \int_{B_\delta} |\mathscr{L}_{rKP}\Phi|^2 e^{2\tau\Psi} dxdydt,$$

for $\mathscr{L}_{rKP} := \partial_{xt} - \partial_{xxxt} + c_1\partial_{xx} + c_2\partial_{xxxx} + c_3\partial_{xy} + c_4\partial_{xxxy} + \partial_{yy} + f_1(x,y,t)\partial_x + f_2(x,y,t)\partial_{xx}$.

ii.

$$\frac{||f_1||_\infty^2}{\tau\delta^2} + \frac{||f_2||_\infty^2(1 + c_1^2/\delta^2)}{\tau^2} \leq \frac{1}{4},$$

we have

$$\tau^2\delta^2 \int_{B_\delta} |\Phi|^2 e^{2\tau\Psi} dxdydt \lesssim \int_{B_\delta} |\mathscr{L}_{KP}\Phi|^2 e^{2\tau\Psi} dxdydt,$$

for $\mathscr{L}_{KP} := \partial_{xt} + \gamma\partial_{xxxx} + c_1\partial_{xx} + c_2\partial_{xy} + \partial_{yy} + f_1(x,y,t)\partial_x + f_2(x,y,t)\partial_{xx}$.

The rest of the proof is dealt with similarly in three steps: first in the subset $\{(x<|t|, y<|t|)\}$*, then to any curvature* $\{(x<\mu_1(t), y<\mu_2(t))\}$*, and finally extended to the whole domain.*

Let us suppose now that the solution is compactly supported. Then:

Theorem 1.21 *[Bou97, Pan05, Mam09b, Mam10b] Let* $u \in \mathscr{C}(0,T;H^s(\mathbb{R}))$ *with* $s \geq 4$ *such that for all* $t \in [0,T]$*,* $u(t)$ *is compactly supported. Then* u *vanishes identically.*

To explain the proof, let us consider the linear equation

$$u_t - A^{\alpha/2}u_x = 0.$$

Since u is compactly supported, the Fourier transform in space has a bounded analytic expansion thanks to the Paley-Wiener theorem, given by

$$|\widehat{u}(\xi+i\sigma,t)| = \left|e^{i(\xi+i\nu)^{\alpha+1}t}\widehat{u_0}(\xi+i\nu)\right| = e^{t\mathrm{Im}\sigma}|\widehat{u_0}(\xi+i\nu)| < +\infty. \qquad (1.11)$$

The fundamental estimate lies on the fact that the Fourier transform of a function with compact support is decreasing but not as fast as a exponential function. More precisely, we have:

Lemma 1.22 *[Bou97] Let* $\Phi \in \mathscr{C}_0^\infty(\mathbb{R})$ *with*

$$\mathrm{supp}\Phi \subseteq [-B,B]$$

For all $Q>0, R>0$*, there exists* $|\xi|>R$ *such that for all* $|\nu| \leq B^{-1}(1+|\log(|\Phi(\xi)|)|)^{-1}$

$$\widehat{\Phi}(\xi) \gtrsim \widehat{\Phi} * \widehat{\Phi} * \cdots * \widehat{\Phi}(\xi+i\nu),$$

and

$$|\widehat{\Phi}(\xi+i\nu)| > \frac{1}{2}|\widehat{\Phi}(\xi)| > \frac{1}{2}e^{-|\xi|/Q}.$$

This last inequality can be written as

$$|\xi| \geq R + Q|\log(|\widehat{\Phi}(\xi)|)|,$$

thus, we can choose ξ large enough, *i.e.* $R, Q > B$, and $\nu \simeq 1/\xi$ so that, as soon as $\alpha > 1$,

$$t\mathrm{Im}\sigma = t\left(\xi^\alpha \nu - \xi^{\alpha-2}\nu^2 + \ldots\right) \longrightarrow_{\xi\to+\infty} +\infty$$

and the boundedness of (1.11) is contradicted if u is not zero.

Remark 1.23 *Actually, it is enough that there exists one time $t_1 \in [0,T]$ such that $u(t_1)$ is compactly supported.*

In dimension greater than 2, the last lemma can be written as follows:

Lemma 1.24 *Let $\Phi \in \mathscr{C}_0^\infty(\mathbb{R}^n)$ with*

$$\text{supp}\Phi \subseteq [-B,B]^n$$

For all $Q>0, R>0$, there exists $\xi \in \mathbb{R}^n$ with for $1 \leq j \leq n$, $|\xi_j| > R$ such that for all $\nu \in \mathbb{R}^n$ with for $1 \leq j \leq n$, $|\nu_j| \leq B^{-1}(1+|\log(|\Phi(\xi)|)|)^{-1}$

$$|\widehat{\Phi}(\xi + i\nu)| > \frac{1}{2}|\widehat{\Phi}(\xi)| > \frac{1}{2}e^{-\sum_{j=1}^n |\xi_j|/Q}.$$

Applying to the Kadomtsev-Petviashvili equations

$$(u_t + u_{xxx} + uu_x)_x + su_{yy} = 0,$$

it comes

$$|\widehat{u}(\xi_1 + i\nu_1, \xi_2 + i\nu_2, t)| = \mathrm{e}^{(3\xi_1^2\nu_1 - \nu_1^3) + s\frac{2\xi_1\xi_2\nu_2}{\xi_1^2+\nu_1^2}} \left|\widehat{u_0}(\xi_1 + i\nu_1, \xi_2 + i\nu_2)\right|.$$

Only if $s=1$, it is possible to choose ξ_1 and ξ_2 large and $\nu_1 \simeq 1/\xi_1$, $\nu_2 = 0$, so that $\widehat{u}$ goes to infinity [Pan05]. At the opposite, if $\nu_1 = 0$, it remains

$$\frac{\xi_2\nu_2}{\xi_1}$$

which can be large if $\xi_2 \gg \xi_1$ with $|\xi_1|$ and $|\xi_2|$ greater than R [EP11]. To prove the unique continuation property, it has to be showed that one direction can be favored.

1.5 Perspectives

- It seems possible to generalize the fundamental lemma of Bourgain for functions without compact support. In particular, we could favor one direction by adjusting the rate of decay of the solution, and thereby justify this unique continuation property of KP. To deal with compactly supported solutions, we can consider pseudo-dispersion [RH93, Ros94].
- Recent works [LPS14, LPS15] are interesting in the behavior of low-dispersive effects, *i.e.* $0 < \alpha < 1$. It seems interesting to see when these dispersive effects occur and how they can converge to the hyperbolic conservation law.

- As continuation of the work done for the stochastic BBM equation, we focus on the dispersive properties of stochastic Korteweg-de Vries equation, and try to show how the dispersion is affected. Note that the initial value problem for Korteweg-de Vries equations with modulated dispersion was addressed recently in [CG15].

CHAPTER 2

DAMPED NONLINEAR DISPERSIVE PDES

In some situations, the viscosity, which can influence the wave propagation and dissipate the wave energy, has to be included in the model description. When the viscous effects are taking into account, the Newton second law provides the following Navier-Stokes equation, for $u=(u_1,u_2,u_3)$

$$\left\{\begin{array}{rcll}\rho(u_t+u\cdot\nabla u)-\nu\Delta u+\nabla p &=& -\rho g\overrightarrow{z} & \text{if } -h\le z\le\eta(x,y,t)\\ \nabla\cdot u &=& 0 & \text{if } -h\le z\le\eta(x,y,t)\end{array}\right.$$

with the dynamic boundary conditions at the free surface

$$\left\{\begin{array}{ll}(-pI+\nu(\nabla u+(\nabla u)^t))\cdot\overrightarrow{n}= & \\ \qquad T\dfrac{\eta_{xx}(1+\eta_y^2)+\eta_{yy}(1+\eta_x^2)-2\eta_{xy}\eta_x\eta_y}{(1+\eta_x^2+\eta_y^2)^{3/2}} & \text{if } z=\eta(x,y,t)\\ \eta_t+u_1\eta_x+u_2\eta_y-u_3=0 & \text{if } z=\eta(x,y,t)\\ u_3=0 & \text{if } z=-h.\end{array}\right.$$

One difficulty lies in the fact that there is no potential function φ solution of the Laplace equation such that $u=\nabla\varphi$. If, as in Chapter 1 we restrict ourself to small amplitude long waves (see 1.3), we can obtain the following viscous KdV equations

$$\eta_t+\eta_x+\widetilde{\alpha}\eta\eta_x+\widetilde{\beta}\eta_{xxx}-\widetilde{\gamma}\mathscr{A}\eta=0,$$

where

$$\mathscr{A}\eta=\eta_{xx}\quad[\text{Whi74}],$$

or

$$\mathscr{A}\eta=\int_0^{t/\alpha}\frac{\eta_x(x+y,t)}{\sqrt{y}}dy\quad[\text{LM15}],$$

or

$$\mathscr{A}\eta=\int_0^{t}\frac{\eta_t(x,s)}{\sqrt{t-s}}ds\quad[\text{Dut09, KM75}].$$

In the following, we present some results concerning the initial value problem

$$u_t + f(u)_x - u_{xxt} - \mathscr{A}u = 0 \tag{2.1}$$
$$u(x,0) = u_0(x), \tag{2.2}$$

with $\langle \mathscr{A}u, u\rangle > 0$.

2.1 Frequency damping

Define $\mathscr{A}$ by its Fourier symbol $\widehat{\mathscr{A}u}(\xi) = \gamma(\xi)\widehat{u}(\xi)$, with $\gamma(\xi) \geq 0$, for all $\xi \in \mathbb{R}$ [OS70]. Then, we note by multiplying (2.1) by u and integrating over space that

$$\frac{d}{dt}||u||^2_{L^2} = -2||u||^2_{\mathscr{H}^1_\gamma},$$

where

$$\mathscr{H}^s_\gamma := \left\{u : \mathbb{R} \to \mathbb{C}; ||\gamma(\xi)^{s/2}\widehat{u}(\xi)||_{L^2} < +\infty\right\}$$

is the natural space of study of the Cauchy problem. This space satisfies some properties of injection:

Proposition 2.1 *[CGM15a] Let $s > 0$.*

i. If $1/\gamma^s \in L^1(\mathbb{R})$, then $\mathscr{H}^s_\gamma(\mathbb{R}) \hookrightarrow L^\infty(\mathbb{R})$ continuously.

ii. If $\gamma_1(\xi) \geq \gamma_2(\xi)$, then $\mathscr{H}^s_{\gamma_1}(\mathbb{R}) \hookrightarrow \mathscr{H}^s_{\gamma_2}(\mathbb{R})$ compactly.

iii. If $1/\gamma^s \in L^1(\mathbb{R})$ and for all $\xi, \xi_1 \in \mathbb{R}$, $\sqrt{\gamma(\xi)} \leq \sqrt{\gamma(\xi-\xi_1)} + \sqrt{\gamma(\xi_1)}$ then $\mathscr{H}^s_\gamma(\mathbb{R})$ is an algebra.

Remark 2.2 *When $\gamma(\xi) = \xi^2$, the space $\mathscr{H}^s_\gamma(\mathbb{R})$ is the classical Sobolev space $H^s(\mathbb{R})$.*

In the same way the well-posedness of the initial value problem (1.6) is proved, if the damping is larger enough, *i.e.* $\gamma(\xi) \geq \xi^2$, the semi-group becomes regularizing and a fixed point of the mild formulation can be done.

Lemma 2.3 *[CGM15a] For $r, s \geq 0$, and $u \in H^s(\mathbb{R})$, we have*

$$||S_t u||_{\mathscr{H}^{r+s}_\gamma} \lesssim \left(1 + \left(\frac{1}{2\varepsilon t}\right)^r\right)^{1/2} ||u||_{\mathscr{H}^s_\gamma},$$

where

$$S_t u = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\xi x + i\xi^{\alpha+1}t - \gamma(\xi)t}\widehat{u}(\xi)d\xi.$$

On the other hand, parabolic or Bona-Smith regularization can be performed to prove the well-posedness in $H^1(\mathbb{R})$ for any functions γ.

Theorem 2.4 *[CGM15a] Let $u_0 \in H^1(\mathbb{R}) \cap \mathscr{H}^1_\gamma(\mathbb{R})$. Then there exist $T \simeq \frac{1}{||u_0||_{H^1 \cap \mathscr{H}^1_\gamma}}$ and a unique solution $u \in \mathscr{C}([-T,T]; H^s(\mathbb{R}) \cap \mathscr{H}^1_\gamma(\mathbb{R}))$ solution of the initial value problem (2.1)-(2.2) with u_0 as initial datum.*

Remark 2.5 *Smoothing effects using Bourgain's spaces can be proved to obtain well-posedness when $\mathscr{A} = (-\Delta)^s$ [Ven11, Ven10].*

The decay rate can be estimated as the quotient of the two norms $H^1(\mathbb{R})$ and $\mathscr{H}^1_\gamma(\mathbb{R})$:

Proposition 2.6 *We define*

$$G(t) := \frac{||u(t)||_{\mathscr{H}^1_\gamma}}{||u(t)||_{H^1}}.$$

Then

i. $||u(t)||^2_{H^1} = e^{-2\int_0^t G^2(s)ds} ||u_0||^2_{H^1}$,

ii. $||u(t)||^2_{\mathscr{H}^1_\gamma} = G^2(t) e^{-2\int_0^t G^2(s)ds} ||u_0||^2_{H^1}$.

iii. In particular, $||u(t)||_{H^1} \to_{t\to+\infty} 0$ *iff* $t \to G(t) \notin L^2_t(0,+\infty)$.

The advantage of this frequency approach is the opportunity to build suitable dampings, in the sense that since we are focusing on long waves problem, it is necessary to reduce low frequencies (see Fig. 2.1).

2.2 Dissipation with memory

The disadvantage of the preceding method lies in the instantaneous regularization of the solution. To avoid this non-physical phenomenon, using memory effect offers a finite speed of regularization without loss of stability [Daf70]. Consider the Burgers-BBM equation [Wan97, Wan14, WY97], for $x \in \Omega$, a bounded domain of $\mathbb{R}$,

$$u_t + u_x - u_{xxt} - u_{xx} = 0 \tag{2.3}$$

complemented with the Dirichlet boundary condition

$$u_{|\partial\Omega} = 0.$$

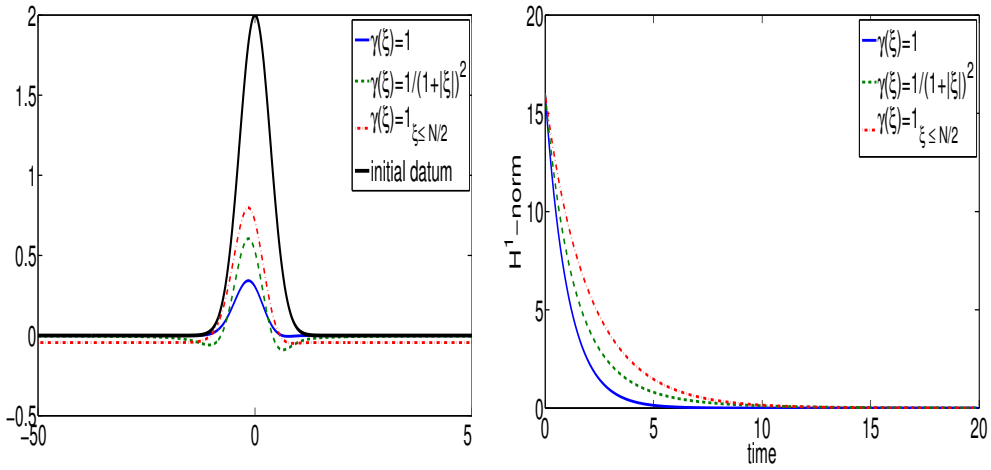

Figure 2.1: On the left, solution of the damped equation with $\gamma(\xi) = 1$ (—), $\gamma(\xi) = 1/(1+|\xi|)^2$ (- -) and $\gamma(\xi) = \mathbb{1}_{\xi \le N/2}$ (-·-) starting from the same initial datum (—) at time $t = 20$. On the right, decay of the H^1—norm with respect to time.

Then

$$\frac{d}{dt}||u||^2_{H^1} = -2||u||^2_{H^1},$$

and the Poincaré inequality gives

$$||u(t)||^2_{H^1} \le e^{-\kappa t}||u_0||^2_{H^1},\ \kappa > 0.$$

Define now the equation with memory

$$u_t + u_x - u_{xxt} - \int g(s)u_{xx}(t-s)ds + u^\rho u_x = 0. \tag{2.4}$$

Here $\rho \in \mathbb{N}$ is a fixed constant, while g is a bounded convex summable function on $[0,\infty)$ of total mass 1. This function g has the explicit form

$$g(s) = \int_s^\infty \mu(y)\,dy,$$

where $\mu : \mathbb{R}^+ = (0,\infty) \to [0,\infty)$, the so-called memory kernel, is a non-increasing absolutely continuous summable function of total mass $g(0) > 0$. Equation (2.4) can be interpreted as a memory relaxation of the Burgers-BBM model (2.3), which is formally recovered when $\rho = 1$ and the kernel g collapses into the Dirac mass at zero. It is also worth noting that the memory

term provides a more realistic description of the Fick law. In particular, it prevents the infinite propagation speed of regularization [Daf70].
In order to explain the difficulties encountered, an enlightening example is provided by a comparison between the classical heat equation

$$u_t - u_{xx} = 0$$

and its memory relaxation, *i.e.* the Gurtin-Pipkin equation [GP68]

$$u_t - \int_0^\infty g(s) u_{xx}(t-s)\, ds = 0.$$

In the first case, the exponential stability is almost trivial, whereas the exponential stability of the Gurtin-Pipkin model has been proved only in recent years [GNP01, GP06]. Similarly, we obtain for the equation (2.4):

Theorem 2.7 *[DMP15] Let μ be the memory kernel with*

$$\mu'(s) + \delta\mu(s) \leq 0, \tag{2.5}$$

for some $\delta > 0$ and almost every $s \in \mathbb{R}^+$. There exist a strictly positive constant ω, depending on μ and the domain Ω, and an increasing positive function $\mathcal{Q}_\rho$, depending on μ, Ω and ρ such that

$$\|u(t)\|^2_{\mathscr{H}} \leq \mathcal{Q}_\rho(E(0)) e^{-\omega t}.$$

The result is based on the energy identity

$$\frac{d}{dt}\|u(t)\|_{H^1} + \frac{\delta}{2}\|\eta\|^2_{\mathscr{M}} - \frac{1}{2}\int_0^\infty \mu'(s)\|\eta_x(s)\|^2\, ds \leq 0,$$

where the phase space $\mathscr{H} = H^1_0(I) \times \mathscr{M}$ is endowed with the norm

$$\|(u,\eta)\|^2_{\mathscr{H}} = \|u\|^2_{H^1} + \|\eta\|^2_{\mathscr{M}},$$

and the so-called memory space is $\mathscr{M} = L^2_\mu(\mathbb{R}^+; H^1_0(I))$ of square summable H^1_0-valued functions on $\mathbb{R}^+$ with respect to the measure $\mu(s)ds$.

2.3 Hyperbolic convergence

The dispersion and the dissipation can be viewed as a correction of the hyperbolic conservation law

$$u_t + f(u)_x = 0. \tag{2.6}$$

To solve (2.6) with a first order Euler scheme

$$\frac{u(x+h)-u(x)}{h} = u_x + \frac{h}{2}u_{xx} + \mathcal{O}(h^2)$$

is equivalent to solve

$$u_t^\varepsilon + f(u^\varepsilon)_x = \varepsilon u_{xx}^\varepsilon, \text{ with } \varepsilon = \frac{h}{2}.$$

And, it is known [Kru70, Whi74] that the sequence $(u^\varepsilon)_\varepsilon$ converges, when ε goes to 0, to the unique entropy solution defined by

$$\eta(u)_t + q(u)_x \leq 0 \text{ in } \mathscr{D}'(\mathbb{R}).$$

To increase the consistency, we can use the following second order scheme

$$\frac{u(x+h)-u(x-h)}{h} = u_x - \frac{h^2}{6}u_{xxx} + \mathcal{O}(h^4),$$

and the equivalent equation is

$$u_t^\delta + f(u^\delta)_x = -\delta u_{xxx}^\delta \text{ with } \delta = \frac{h^2}{6}.$$

Unfortunately, this equation is dispersive, which produces oscillations and therefore the sequence $(u_\delta)_\delta$ does not converge to the entropy solution as δ goes to 0 [LL83]. Combining both methods, we obtain

$$u_t^{\varepsilon,\delta} + f(u^{\varepsilon,\delta})_x = -\varepsilon u_{xx}^{\varepsilon,\delta} - \delta u_{xxx}^{\varepsilon,\delta}.$$

Three cases are possible:

i. $\delta \gg \varepsilon^2$, oscillations are prevailing and there is no convergence [LL83];

ii. $\delta \ll \varepsilon^2$ (or $\delta \ll \varepsilon$ if one contents oneself to progressive wave [PR07]), the sequence $(u^{\varepsilon,\delta})_{\varepsilon,\delta}$ converges to the entropy solution as ε, δ go to 0 [Sch82];

iii. $\delta = C\varepsilon^2$, the sequence converges to a solution, not necessarily entropy [HL97].

To regularize the numerical scheme, Von Neumann [vNR50, LN99, CL98, KW08] proposed a pseudo-dissipation. In the same spirit, Brenier and Levy [BL00] suggested a pseudo-dispersion. We can combine both as follows

$$u_t + f(u)_x = \varepsilon h(u_x)_x + \delta g(u_{xx})_x, \tag{2.7}$$

where $h(u)u \geq 0$.

Theorem 2.8 *[BCM14, BCM15b] Assume that*

$$\begin{aligned} |f^{(i)}(u)| &\lesssim |u|^{\alpha_f+1-i} \text{ for } 0 \le i \le 2 \\ |g^{(j)}(u)| &\lesssim |u|^{\alpha_g+1-j} \text{ for } 0 \le j \le 7 \\ |h^{(k)}(u)| &\lesssim |u|^{\alpha_h+1-k} \text{ for } 0 \le k \le 6, \end{aligned}$$

with

$$h'(u) \ge C_0 > 0.$$

Define

$$\alpha := \begin{cases} \min(\alpha_f, \alpha_g, \alpha_h) \text{ if } ||u||_4 < 1 \\ \max(\alpha_f, \alpha_g, \alpha_h) \text{ if } ||u||_4 \ge 1 \end{cases}$$

with $\alpha_g \ge 1$. *Let* $u_0 \in H^4(\mathbb{R})$ *with*

$$||u_0||_4^{\alpha_g} \lesssim \frac{\varepsilon}{\delta}. \tag{2.8}$$

There exists $T = T(1/||u_0||_4) > 0$ *such that there exists a unique solution* $u \in \mathscr{C}([-T,T], H^4(\mathbb{R}))$ *of the initial value problem (2.7) with* u_0 *as initial datum.*

Restrictions on u_0 and h come from the computation of the Sobolev norm:

$$\begin{aligned} \frac{d}{dt}||u||_4 &\le ||u||_4^{\alpha+2} + \int_{-\infty}^{+\infty} u_{5x}^2 \left(-\varepsilon h'(u_x) + \delta u_{xxx} g''(u_{xx})\right) dx \\ &\le ||u||_4^{\alpha+2} + \int_{-\infty}^{+\infty} u_{5x}^2 \left(-\varepsilon C_0 + \delta ||u||_4^{\alpha_g}\right) dx. \end{aligned} \tag{2.9}$$

If the right-hand-side of (2.9) is negative, Gronwall's lemma implies that the H^4—norm is bounded on $[0,T]$. It is enough that the right-hand-side of (2.9) is negative for the initial datum, which is equivalent to inequality (2.8), to be preserved with respect to time.

As for linear dissipation and dispersion, there exist regimes for which the convergence to the entropy solution is true.

Theorem 2.9 *[BCM14, BCM15a] Suppose* $h(u) = u$. *Let* $\varepsilon > 0$ *and* $f : \mathbb{R} \to \mathbb{R}$ *be a convex flux function with* $f''(u) \lesssim (1 + |u|^\beta)$.

i. If $g(u) = -u^2$, *we choose* $\delta = o(\varepsilon^{5/2})$ *and* $0 \le \beta < 1/2$.

ii. If $g(u) = -|u|$, *we choose* $\delta = o(\varepsilon^2)$ *and* $0 \le \beta < 3$.

Then the family solutions $(u_{\varepsilon,\delta})$ *of* (2.9) *converges to the entropy solution of* (2.6).

We prove that there exists a bounded measure $\mu \leq 0$ [DiP85] such that

$$\eta(u)_t + q(u)_x \longrightarrow \mu \quad \text{in } D'(\mathbb{R}\times(0,T))$$

for an arbitrary convex function η and $\eta' f' = q'$ (we assume η', η'', η''' to be bounded functions on $\mathbb{R}$). The main trick is to rewrite the entropy equality in the form

$$\eta(u)_t + q(u)_x = \mu_1 + \mu_2 + \mu_3 + \mu_4^- + \mu_4^+$$

where

$$\begin{aligned} \mu_1 &:= \varepsilon\,(\eta'(u)\,u_x)_x,\ \mu_2 := -\varepsilon\,\eta''(u)\,u_x^2,\ \mu_3 := -\delta\,(\eta'(u)\,u_{xx}^2)_x, \\ \mu_4^- &:= -\delta\ \eta''(u)\,u_x^-\,u_{xx}^2,\ \mu_4^+ := \delta\ \eta''(u)\,u_x^+\,u_{xx}^2. \end{aligned}$$

Thus we can prove:

Lemma 2.10 *The solution* $\{u_{\varepsilon,\delta}\}$ *satisfies*

i. $\{\varepsilon\,u_x^2\}$ *is bounded in* $L^1([0,T]\times\mathbb{R})$.

ii. $\{\varepsilon\,u_x\} \to 0$ *when* $\varepsilon \to 0$ *in* $L^2([0,T]\times\mathbb{R})$.

iii. $\{\delta\,u_x^-\,u_{xx}^2\}$, *where* $u_x^- = max(0,-u_x)$, *is bounded in* $L^1([0,T]\times\mathbb{R})$.

iv. $\{\delta\,u_x^+\,u_{xx}^2\} \to 0$, *where* $u_x^+ = max(0,u_x)$ *when* $\varepsilon \to 0$ *in* $L^1([0,T]\times\mathbb{R})$.

v. $\{\delta\,u_{xx}^2\} \to 0$ *when* $\varepsilon \to 0$, *in* $L^1([0,T]\times\mathbb{R})$.

Application to combustion. The understanding of the small-scales mechanisms occurring during the shock front formation is one of the major challenges in gas combustion. For such flows in which the ram pressure dominates thermal quantities, the gas equations reduce to the viscous Burgers equation [Whi74, ZZMR02, TD10].
We devote ourselves to investigating the following modified Burgers equation with space-and time-dependent viscosity [TMB15]

$$u_t + uu_x = (\nu u_x)_x.$$

The viscosity $\nu(x,t)$ depends explicitly on an active scalar field $\theta(x,t)$ that obeys an advection-diffusion equation

$$\theta_t + u\theta_x = D\theta_{xx}.$$

The scalar θ refers to the local mass-fraction of the heaviest component in the binary gas mixture and satisfies $0 \leq \theta \leq 1$, and D refers to the diffusion coefficient. The coupling between these two equations is operated through the

relationship $\nu = \sum_{j=0}^{3} a_j \theta^j$, where the coefficients a_j depends on the gas type. It is found that surprisingly for all conditions at $R_\nu > 1$, where the initial kinematic viscosity ratio is defined by $R_\nu = \nu_{max}/\nu_{min}$, *i.e.* for waves becoming more and more viscous in average (Fig. 2.2):

i. a steeper maximum gradient in the shock transition zone,

ii. velocity spectra extended toward the finest small-scales,

iii. a enhanced energy dissipation rate at the time of peak energy dissipation.

These results will be useful to the understanding of small-scales dynamics for one-dimensional shocks propagating in multi-component gas mixtures where noticeable active scalar effects are present.

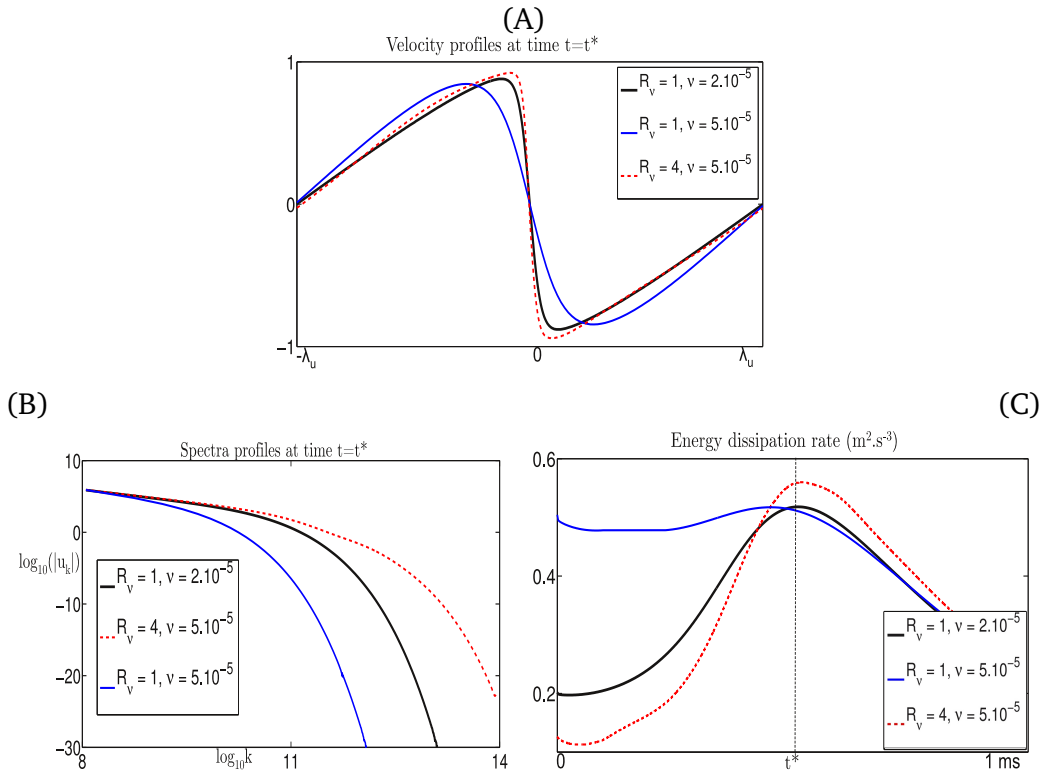

Figure 2.2: (A) Shock profiles at time t^*, (B) velocity spectra at time t^* and (C) energy dissipation rate with respect to time.

2.4 Perspectives

- Damping can be viewed as a control term to stabilize the system. We are interested in the manner in which one can build its damping (wavemaker, porous domains...).
- The study about the memory kernel we carried out is a first step. It is important to show that the equation with memory converges to the one without memory as the kernel goes to the Dirac mass. The third step consists of studying the forced equation [CR04, Ghi88, Ghi94, Gou00]. To do this it is necessary to add a weighted distribution between the classical diffusion and the memory to control the nonlinear force terms.
- Up to now, there is no or few numerical methods (except Gear schemes for fractional derivatives) to solve PDE with memory. It seems crucial to develop efficient methods that can simulate many memory kernels.
- One of our objectives is to prove the convergence to the entropy solution of the hyperbolic conservation law with nonlinear dissipation. In this case, it is necessary to add a weak linear compensation. In the same spirit, we are dealing with the convergence in the case of a non-convex nonlinearity.

CHAPTER 3

MULTISCALE MODELING IN BOTANY

In a context of climate change and respect for the environment, it is important to develop mathematical modeling of plants and their habitats. Scales of time and space being often incompatible with our way of life, it is essential to develop appropriate models. The interactions between plant, climate and man are more relevant than ever. Modeling is a key approach to improve the understanding of host-pathogen-pest dynamical interactions, to rank known factors that initiate the development of an epidemic and to test strategies to control and reduce its spread.

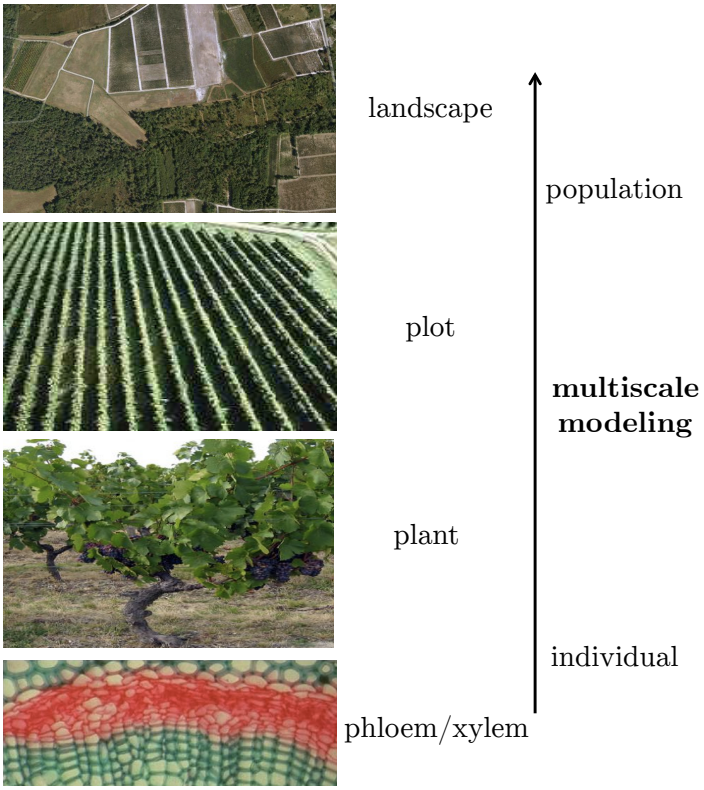

Figure 3.1: Multiscale paradigm.

3.1 Plant growth

Our approach is based on the existence of a dynamic discrete model coupling the growth of a vine stock and the development of powdery mildew. The 3D plant architecture is described as a multiscale object [GC98] made of a trunk, branches, themselves made of secondary branches ... themselves made of leaves and fruits (Fig. 3.2).

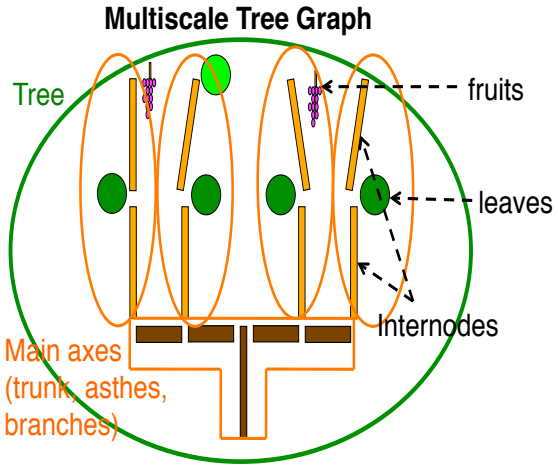

Figure 3.2: Multiscale profile of a plant.

Like all living beings, the plant growth is based on available resources and climate. As soon as the threshold, depending on accumulated resources, is reached, new branches, leaves or fruits are born. Both shoots growth and area of leaf surface satisfy a logistic equation function of resources (Fig. 3.3). The appearance and growth of secondary shoots is highly dependent on the vigor of the plant. The development of secondary shoots is stimulated by shoot topping [CCN+08, CSR+08, DBP07, VGGC+11, BCLM12].

The description can be more accurate, especially for the accumulation and transport of resources across the plant. From simple summation of fresh and warm temperatures [BCLM12], the distribution via the phloem–xylem is more complex. According to the osmotic hypothesis [M̈26, M̈27, M̈30, Hor58], the mass balance conservation of the sucrose concentration C is written, for $x \in \Omega$ and $t > 0$ [CF73, FTC75, SH14, TH03, MS15]

$$\begin{cases} \dfrac{e}{E}\partial_t P - \nabla\cdot\left(\dfrac{e}{\mu}(k\nabla P)\right) - L_R(\psi - P + RTC) - V_s U &= 0 \\ e\partial_t C - \nabla\cdot\left(\dfrac{e}{\mu}C(k\nabla P)\right) - U &= 0, \end{cases}$$

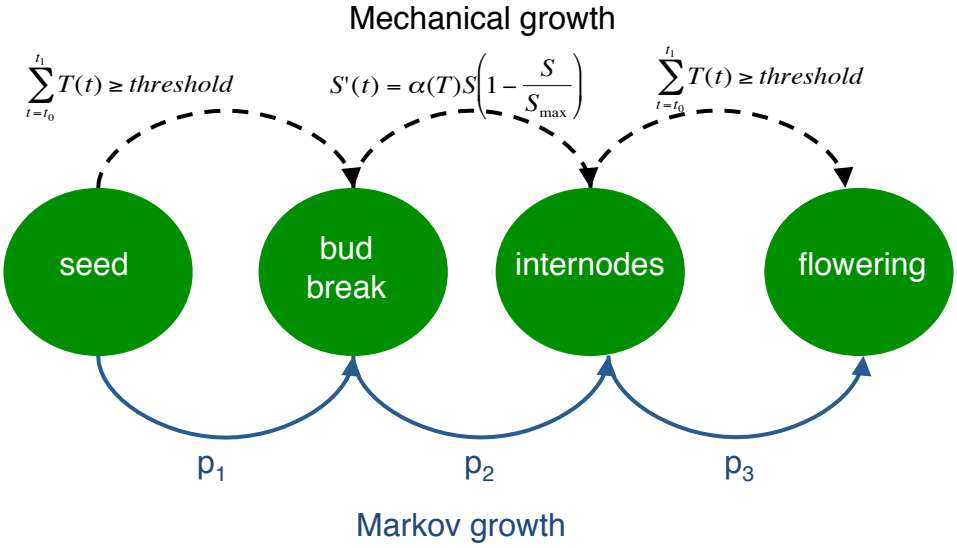

Figure 3.3: Schematic representation of the plant growth: mechanical (above) and stochastic (below) .

with Neumann conditions on the domain boundary

$$\partial_n P(t,x) = 0, \tag{3.1}$$

and on the transverse boundaries

$$\partial_n C(t,x) = 0, \tag{3.2}$$

with initial data

$$P(0,x) = P_0(x) \text{ and } C(0,x) = C_0(x). \tag{3.3}$$

Boundary conditions on entrance and exit are translated by source and sink terms

$$U = \begin{cases} \widetilde{U} & \text{in } \Omega_l \text{ the loading area (source)} \\ -\dfrac{\widetilde{U}}{C^*}C & \text{in } \Omega_u \text{ the unloading area (sink)} \\ 0 & \text{elsewhere.} \end{cases}$$

Here $\widetilde{U}$ denotes a loading rate and C^* a reference sucrose concentration. This system is integrated from Darcy's law over phloem thickness e and pressure P with non constant viscosity μ. L_R denotes the radial hydraulic conductivity, ψ the xylem hydrostatic pressure, R the gas constant T the temperature, V_s the partial molal volume of sucrose and phloem Young's modulus is E (Fig. 3.4).

Theorem 3.1 *[MS15] Assume*

$$0 < d_{\min} < \frac{k(x,t)}{\mu(x,t)} < d_{\max} < +\infty.$$

Let $(P_0, C_0) \in H^4(\Omega) \times H^4(\Omega)$ *be positive initial datum. Then there exists* $T > 0$ *and a unique weak solution* $(P, C) \in \mathscr{C}(0,T;H^4(\Omega)) \times \mathscr{C}(0,T;H^4(\Omega))$ *of initial-boundary values problem.*
Moreover if parameters $E, e, L_R, T, \psi, V_s, \widetilde{U}, C^*$ *are positive and chosen such that*

$$L_R RT - \frac{V_s \widetilde{U}}{C^*} > 0.$$

Then, for all time $t \geq 0$ *and a.e.* $x \in \Omega$,

$$P(t,x) \geq 0 \text{ and } C(t,x) \geq 0.$$

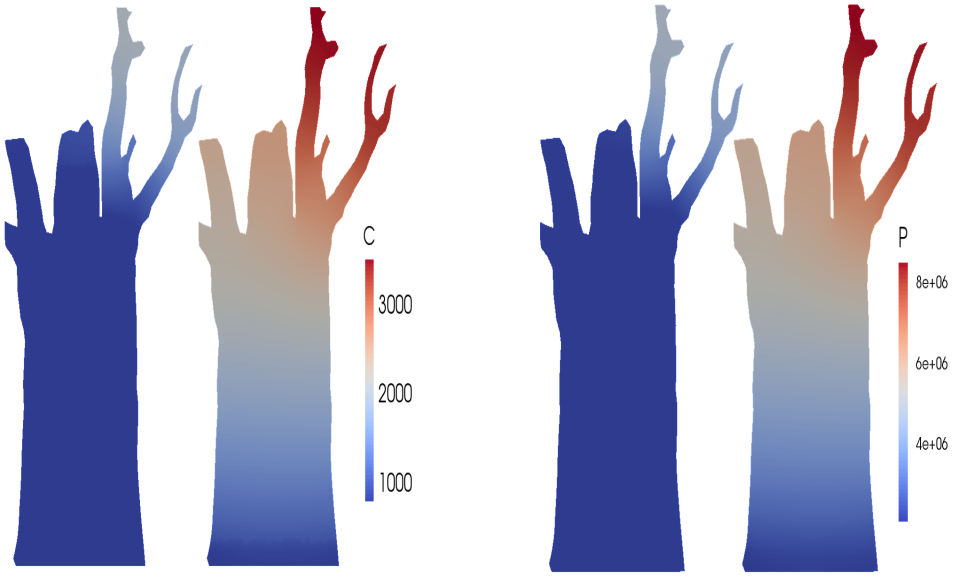

Figure 3.4: Evolution with respect to time of the sucrose concentration (C) at left and the pressure (P) at right.

3.2 Pathogen spread

Current strategies for crop protection generate complex and controversial issues both on the human health and the environment. Growers are now constrained to significantly reduce the use of fungicides (Directive 1107/2209/

EU). The high success of controlling plant pathogens through pesticide applications has limited the amount of attention paid to influence of cultural practices or alternative control methods. Over the past 30 years, strategies for crop protection relying on the use of pesticides have generated complex and controversial issues both concerning the human health and the environment. Consequently, low-pesticide systems based on innovative control methods need to be developed and their performance to be evaluated [AGB+13].

The temperature and the leaf age are recognized as the driving force for the development of the pathogen, while wind factors dictate spore dispersal. In our model, the dynamics of the pathogen development on leaves is split into infection, colony growth, and dispersion. At each time step, a quantity of spores is released, depending on wind intensity, colony size, and dispersed within a cone given by wind direction. The spore density decreases, first as an exponential dilution with axial distance to the source, and then linearly according to the radial position within the cone. The quantity of spores caught by a leaf depends on its distance from the source, and is proportional to its surface area (Fig. 3.5).

This very detailed Individual Based Model (IMB) is a complex discrete mechanistic architectural model at the plant scale, which explicitly incorporates both the host growth and the development and dispersion of the pathogen.

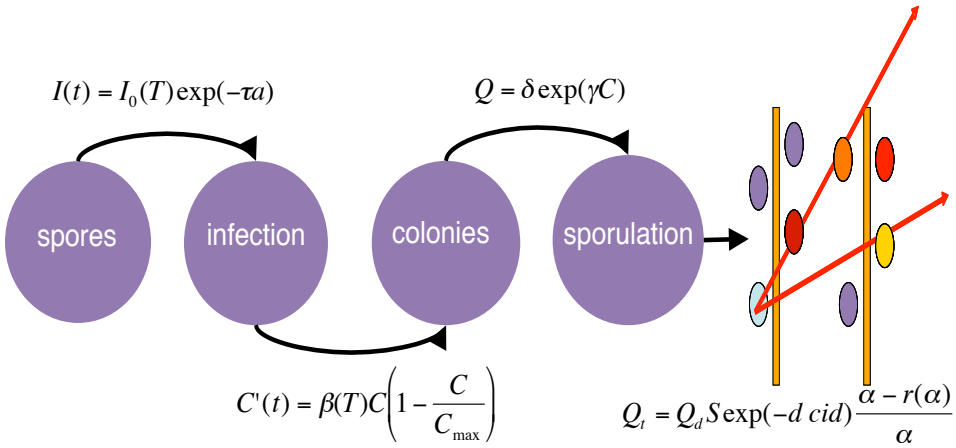

Figure 3.5: Schematic representation of the pathogen propagation.

One control method could rely on modification of plant growth and architecture by using architectural diversity and cultural practices. Evidences of epidemic variations attributed to modifications of canopy porosity and susceptibility in main pathosystems and on their effects on pathogen processes such as infection and dispersion have recently been reviewed. Indeed, a sensitivity anal-

ysis identifies the factors favoring or restricting the spread of disease. This sensitivity analysis, with respect to various crop practices (e.g. shoot topping day, cut height), plant architecture (organs length, vigor, distance between shoots), powdery mildew properties (infection day, dispersal coefficients) and climate (year, temperature, wind) indicates the interest of these parameters variability. Among other things, plant vigor and climatic conditions can strongly modify the disease dynamics. Also, to reduce the disease, it is better to lengthen the branches and to treat the plant as soon as possible (Fig. 3.6).

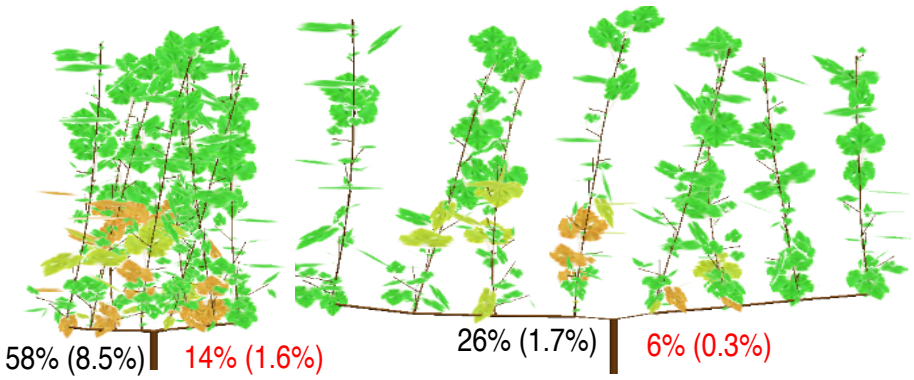

Figure 3.6: Number (surface) of diseased leaves in a not treated plant (in black) and a plant with one fungicide treatment at flowering (in red).

The model strengthens observed experimental results about the effects of the rate of leaf emergence and of the number of leaves at flowering on the severity of the disease. But it also underlines strong variations of the dynamics of the disease depending on the vigor and indirectly on the climatic scenarii by altering the synchronism between the disease and the production of susceptible organs [DMRC10, DBP07, CMB15].

3.3 The plot scale

Epidemiological models taking into account the crop growth and susceptibility are particularly important to study perennial plants for which experiments are difficult to set up. The host population shows a high degree of spatial structure at the field level (culture in rows or individual vine, topped or not) and at individual plant level (various pruning types) exhibits rapid changes of susceptibility over time and is subjected to a high degree of human interference during its development [CCC09, VGGC+11].

At first glance, the easiest way would be to describe each plant of the plot as in Sections 3.1 and 3.2. To obtain accurate observation, one has to know every

leaf, every source of infection and every spore. This count becomes very expensive and this method is not reasonable. The model presented here includes ontogenic resistance (organ development age-related resistance) as well as biologically relevant parameters for plant growth and pathogen development based on the IBM model. The unit considered for the description of the pathological state of the leaves surface is the leaf area index (LAI) defined here as the leaf area (expressed in terms of square metres) in a one metre square section of ground area. The disease cycle is the following (see Fig. 3.7 for a schematic representation of the model): susceptible leaves (denoted by S) inoculated with spores first become latent (L), then turn infectious (I) and produce spores during some infectious period after which they are removed (R) as they cannot be infected again. In addition, susceptible leaves become resistant (T) to inoculation because of their age. This leads to the following compartmental submodel for LAIs, defined by a system of ODEs [MBLC14]

$$\begin{cases} \dfrac{d}{dt}S(x,t) &= -a\left(e_S\delta_S U_S + e_L\delta_L U_L\right)\dfrac{S(x,t)}{N(x,t)} \\ & \quad +\alpha N(x,t)\left(1-\dfrac{N(x,t)}{k(x,t)}\right)-\dfrac{1}{m}S(x,t) \\ \dfrac{d}{dt}L(x,t) &= a\left(e_S\delta_S U_S + e_L\delta_L U_L\right)\dfrac{S(x,t)}{N(x,t)}-\dfrac{1}{j}L(x,t) \\ \dfrac{d}{dt}I(x,t) &= \dfrac{1}{j}L(x,t)-\dfrac{1}{i}I(x,t) \\ \dfrac{d}{dt}R(x,t) &= \dfrac{1}{i}I(x,t) \\ \dfrac{d}{dt}T(x,t) &= \dfrac{1}{m}S(x,t). \end{cases}$$

Parameter a is defined as the average size of infected area created by a single spore. We notice that the total LAI is denoted by $N = S+L+I+R+T$ follows a logistic law

$$\frac{d}{dt}N(x,t) = \alpha N(x,t)\left(1-\frac{N(x,t)}{k(x,t)}\right).$$

The airborne spores are torn off from the colonies on the leaves as the consequence of air turbulence and travel in the air according to a diffusion process. Hence it is assumed that the distribution of the airborne spores emitted by a single lesion at the origin obeys a Gaussian density with a variance increasing linearly in time [Die79, KMK27, Thi77, VdBZM88b, VdBZM88c, VdBZM88a]. Once emitted from a colony, spores may remain trapped within the canopy and disperse at short range or escape the canopy and disperse at long range

[Ayl99]. Therefore, the concentration of airborne spores U_S and U_L obey the following advection-reaction-diffusion equations

$$\begin{cases} \partial_t U_S(x,t) &= \nabla\cdot(D_S\nabla U_S(x,t)) - \delta_S U_S(x,t) + \gamma f I(x,t) \\ \partial_t U_L(x,t) &= \nabla\cdot(D_L\nabla U_L(x,t)) - \delta_L U_L(x,t) + \gamma(1-f)I(x,t) - V\cdot\nabla U_L \end{cases}$$

where the right-hand terms are the spores source terms with V the velocity of the daily dominant wind on the spores above the canopy. Spores are produced by unit of infectious leaves area I at rate $\gamma > 0$. Each emitted spore has a probability $f(N)$ in $[0,1]$ to be short-range dispersed and thus a probability $(1-f(N))$ to be long-range dispersed.

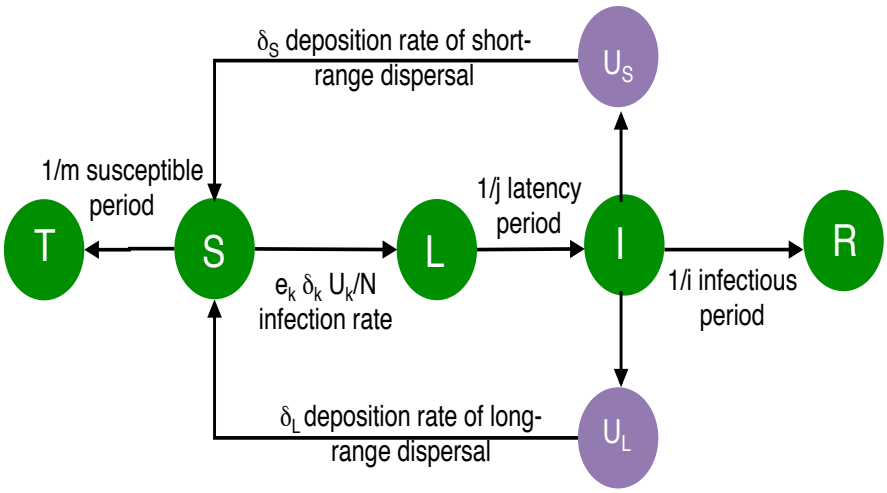

Figure 3.7: Flow chart of the plant-pathogen system.

Theorem 3.2 *[BCLM12, MBLC14] Under compatibility conditions for U_S and U_L at the each boundaries of Ω, i.e. the continuity of spores densities and fluxes $[U_S]=0, [U_L]=0$ and $[D_S\nabla U_S]=0, [D_L\nabla U_L]=0$, the system is globally well-posed.*
Moreover $(U_S, U_L, S, L, I, R, T)$ converges a.e. to $(0,0,0,0,0,R^*,T^*)$ with $R^*(x)+T^*(x)=k(x)$.

Spatial heterogeneity can be generated with changes over time. The model allows generating plots or patches within plots which differ in phenology, growth rate, crop management and training system for various climatic scenarii which can differently impact plant and pathogen growth. The effects of individual plant and crop heterogeneities on pathogen or disease spread have rarely been explicitly taken into account in epidemiological models.

Remark 3.3 *Spatial heterogeneity was introduced into a reaction-diffusion on a periodic monodimensional domain with alternating favourable and unfavourable patches [SKT86, KKTS03, KKS10]. Numerical simulations showed then that the expansion again converges towards a travelling periodic wave with a speed depending on the direction of propagation and maximal along the rows. Decreasing the scale of fragmentation without changing the pattern of heterogeneity decreases the spreading speed. These results have been confirmed by a mathematical analysis [BHR05].*

The model is able to retrieve the main characteristics of the system:

i. a host growing during the whole season with time evolution in susceptibility;

ii. a crop highly structured in rows with potential heterogeneities of plant growth and susceptibility within and between plots.

These characteristics can be modified by cultural management. Simulations are performed to test the effect of crop spatial heterogeneities, within and between plots, on the disease spread. Heterogeneities considered here are the plant growth (vigour, earliness), susceptibility (susceptible vs resistant, treated vs untreated) and the spatial arrangements (patches vs rows). The main effect on disease reduction is obtained by arrangement in rows of susceptible and fully resistant plants, and applying an early fungicide treatment (Fig. 3.8) [AGB+13, STSA10, MBLC14].

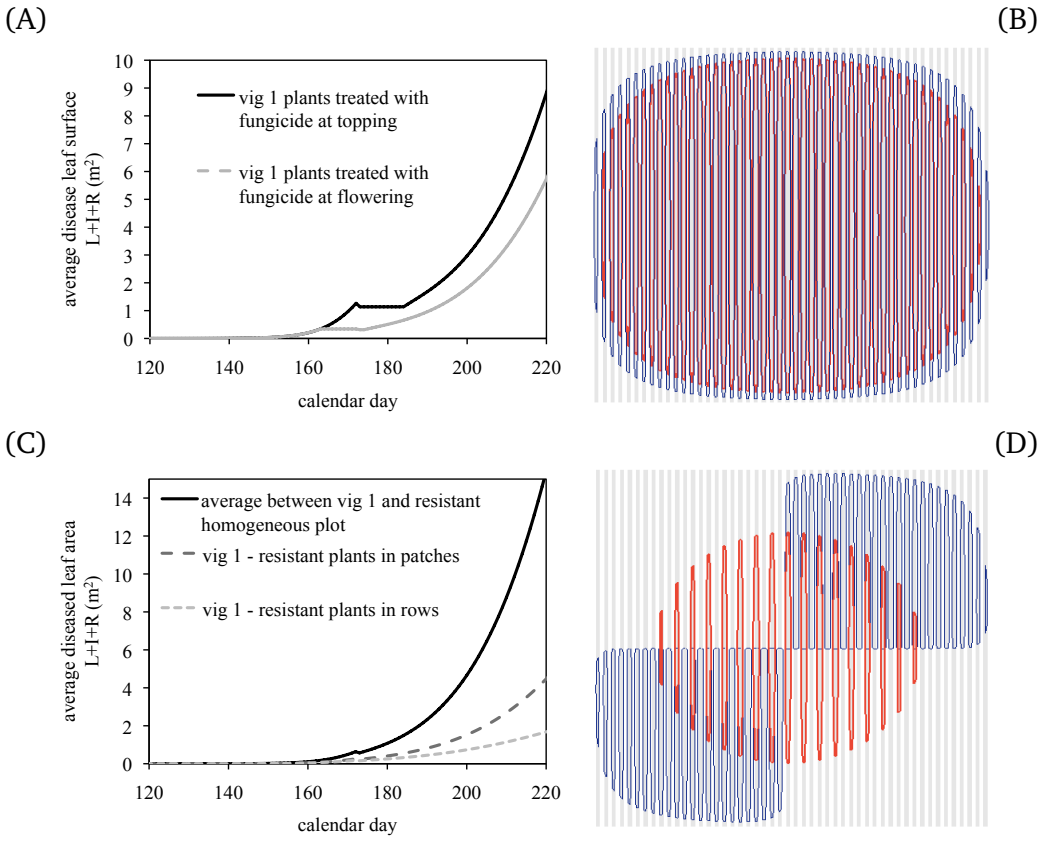

Figure 3.8: Temporal evolution of the total diseased surface area (A) and (C) for early inoculated plot (day 119, year 2004). Fungicide treatment depends on the date of application (flowering or topping) and distance covered by the disease when fungicide is applied at topping (blue) or at flowering (red) (B). Vines are distributed either in patches or in rows according to levels of susceptibility (susceptible vs. resistant) (C and D). Disease spread (D) indicates the distance covered by the disease depending on the structure in patches (blue) or in rows (red).

3.4 Pest dynamics

Regarding pest management, developing alternative methods requires extensive knowledge of pest colonisation processes and persistence strategies. Because insect pest populations are known to be greatly influenced by landscape features [JF97], experimenting crop protection strategies carries the ecological engineering endeavors at the agro-ecosystem scale. At this scale, levers for action consist in landscape modifications concerning, *e.g.* crop rotation pat-

terns, fields size and geometry or semi-natural habitat networks. Modeling approaches enable us to test in silico the modifications of the landscape structure and composition in order to assess its putative relationship with the population dynamics. From a theoretical point of view, we consider pest distribution across the agricultural mosaic as an emerging pattern resulting from interactions between the pest and landscape features. Some of these interactions take place at the population scale and result in landscape dependent growth and death rates, carrying capacities or migration rates [SK97]. Still, other interactions such as resource perception, energy allocation, spatial memory or optimal foraging strategies, inherently occur at the individual scale and can be determinant of the pest population distribution. We propose to account for foraging processes and their determinants in order to ensure the population model a strong dependence on the resource distribution on the landscape. In heterogeneous environments, foraging relies on informed movements requiring both perceptive and navigational abilities. It determines, among other things, how individuals aggregate in space, *i.e.* habitat selection depending on resource distribution [HM14, NGR+08]. The type of resources searched is often closely related to the energy supply of the forager, dealt with as a latent metabolic variable. Here we focus on feeding and laying resources.

The cabbage root fly Delia radicum L. is an important pest of Brassicaceous crops, very common species in temperate regions [Fin89]. This insect, which develops through distinctive larval, pupal, and adult stages, is active from April to October. In April, adult flies emerge from diapausing pupae, the resistant stage for the cold season. Adults oviposit on the ground close to plant stems or directly on stems, and after hatching, larvae develop inside the root causing damage. Before ovipositing in Brassica crops, adults need to mate and to feed on owers from hedgerows and field-banks. Both resources affect the energy supply either positively (*i.e.* feeding) or negatively (*i.e.* ovipositing) and hence, impact population dynamics (Fig. 3.9).

We built a stage-structured demographical model composed of four ODEs and one PDE spatialised to account for agricultural landscape heterogeneity and adult fly ability to disperse through a diffusion term. Landscape dependence is described via two individual-scale processes of resource use. First we introduce perception kernels of landscape features to define a spatially heterogeneous diffusion. Secondly, we add an energetic dimension, denoted by U, in which adult flies evolve depending on the landscape element (*i.e.* feeding or oviposition sites). Therefore two kinds of individual-landscape interactions, namely resource perception and energy supply management, were included as additional driving forces of the spatial population dynamics. The population model is characterised by five developmental stages: diapausing pupae (P_D), non-diapausing pupae (P_N), adults (A), eggs (E) and larvae (L) and the

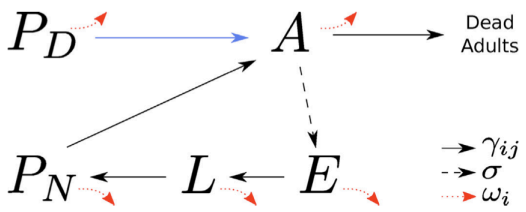

Figure 3.9: Synthetic view of the pest model: diapausing pupae (P_D), non-diapausing pupae (P_N), eggs (E), larvae (L) and adults (A). The solid arrows represent conversion between state variables (with associated coefficients $\gamma_{i,j}$). The black arrows cycle occurs 2-3 times during the season. The solid blue arrow conversion occurs only once a year, at the beginning of the season. The dashed black arrow represents new individuals creation by eggs laying (with associated coefficient σ). The red dotted arrows represent mortality rates (ω_i).

population dynamics is governed by [BPM+15]:

$$\begin{cases} \dfrac{d}{dt}P_D(x,t) &= -\gamma_{P_D,A}(\theta)P_D - \omega_{P_D}P_D \\ \dfrac{\partial}{\partial t}A(x,t,U) &= \gamma_{P_D,A}(\theta)P_D + \gamma_{P_N,A}(\theta)P_N\gamma_{P_D,A}(\theta)P_D - \gamma_{A,\cdot}A - \omega_A A \\ &\quad +\nabla_x \cdot (D\nabla_x A) - C \cdot \nabla_U A \\ \dfrac{d}{dt}E(x,t) &= \sigma(x,\theta)A - \gamma_{E,L}(\theta)E - \omega_E E \\ \dfrac{d}{dt}L(x,t) &= \gamma_{E,L}(\theta)E - \gamma_{L,P_D}(\theta)L - \omega_L L \\ \dfrac{d}{dt}P_N(x,t) &= \gamma_{L,P_N}(\theta)L - \gamma_{P_N,A}P_N - \omega_{P_N}P_N. \end{cases}$$

A global sensitivity analysis evaluates the impact of both the individual-scale processes on the population dynamics. This exploration demonstrated the biological relevance of the model according to field observations and theoretical expectations. Our key finding is that resource perception and energy supply management appear as significant as the demographic component regarding the resulting dynamics of the pest. Building on its acute multiscale landscape dependence, this model reveals particularly efficient to investigate the putative relationships between agricultural landscape features and pest outbreaks.

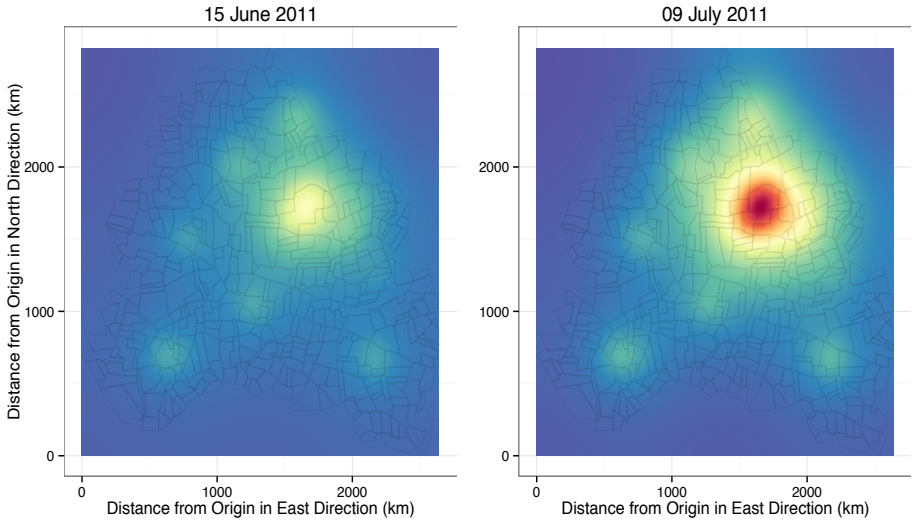

Figure 3.10: Simulated spatial distribution of the Delia radicum adults across the agricultural landscape at two dates in the activity season 2010-2011 (see Fig. 4.8).

3.5 Perspectives

- It is observed in nature that the wavefront of the disease has a tendency to speed up. A better integration of local turbulence (*i.e.* Navier-Stokes) and the capillary effects (*i.e.* dispersion) is essential to improve our models. Also more direct processes of foraging (*i.e.* advection) and the consideration on a random behavior of flies is examined.

- The implementation of other pathosystems (apple scab, wheat septoria, downy mildew) is another outlook.

CHAPTER 4

NUMERICAL SCHEMES AND SOFTWARES DEVELOPMENT

Through these various studies, we developed several computer codes as well with the aim to be easily reusable by non-specialists.

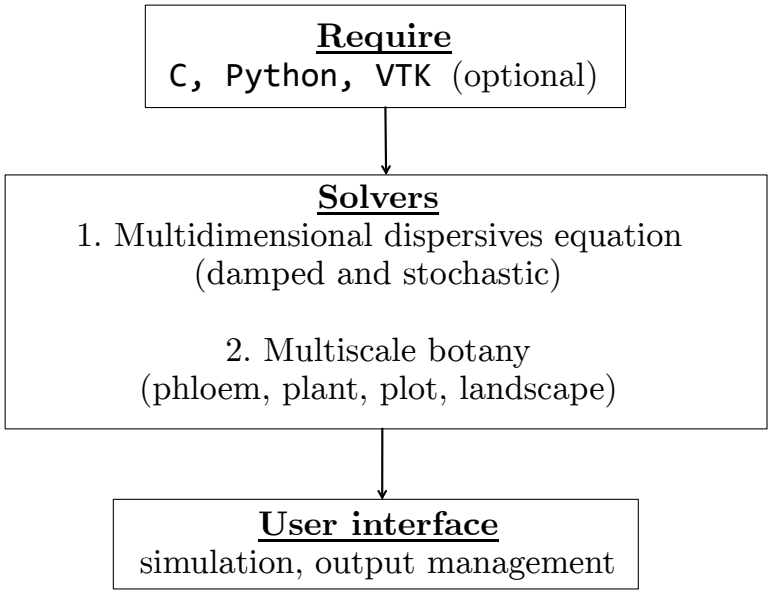

Figure 4.1: General structure of the development environment.

4.1 Dispersive and Dissipative waves

Spectral methods are very suitable to the discretization of such equations, the nonlinearity, the derivatives and the anti-derivative being easily treated [Waz01, KS12]. To preserve the conservation laws, high order schemes are required. The obtained nonlinear system is then solve using fixed point method [Mam07, HMM09, Mam13b, MZ14].
For example, let us consider the $3+1$—dimension KP equations

$$u_t - A^{\alpha/2}u_x + \mathscr{A}u + u^\rho u_x + a\partial_x^{-1}u_{yy} + b\partial_x^{-1}u_{zz} = 0.$$

Let $\Delta t > 0$ be the step of the time discretization, and set $t_n = n\Delta t$. For $n \in \mathbb{N}$, we denote by $\widehat{u}_n$ an approximation of $\widehat{u}(\xi_1,\xi_2,\xi_3,t_n)$ and by u_n an approximation of $u(x,y,z,t_n)$. By denoting by $m \geq 1$ a maximal number of iterations of the fixed point iterations, a predictor-corrector algorithm is encoded as follows:

- Set $\widehat{u}_{0,m} := \ldots := \widehat{u}_{0,1} := \widehat{u}_{0,0} := \widehat{u_0}$.
- For $n = 0, 1, \ldots,$ compute:
$$\widehat{u}_{n+1,0} := \widehat{u}_{n,m} + \Delta t \left(-\mathrm{i}\xi_1 \widehat{(\frac{u_{n,m}^{p+1}}{p+1})} + \mathrm{i}(\xi_1^{\alpha+1} - a\frac{\xi_2^2}{\xi_1} - b\frac{\xi_3^2}{\xi_1})\widehat{u}_{n,m} \right),$$
  - For $r = 0, 1, \ldots, m-1,$
$$\widehat{u}_{n+1,r+1} := \widehat{u}_{n,m} + \frac{\Delta t}{2}\left(-\mathrm{i}\xi_1 \widehat{(\frac{u_{n+1,r}^{p+1}}{p+1})} + \mathrm{i}(\xi_1^3 - \frac{\xi_2^2}{\xi_1}a - \frac{\xi_3^2}{\xi_1}b)\widehat{u}_{n+1,r} \right.$$
$$\left. -\mathrm{i}\xi_1 \widehat{(\frac{u_{n,m}^{p+1}}{p+1})} + \mathrm{i}(\xi_1^3 - \frac{\xi_2^2}{\xi_1}a - \frac{\xi_3^2}{\xi_1}b)\widehat{u}_{n,m} \right).$$

The iterations are stopped in one of the two following cases:

- when $\frac{\|u_{n+1,r+1} - u_{n+1,r}\|_{l^2(\Omega)}}{\|u_{n+1,0}\|_{l^2(\Omega)}} \leq \tau$, with $\tau > 0$ a fixed tolerance. We then set $u_{n+1} := u_{n+1,r+1}$;
- or when $r = m-1$. Here, we set $u_{n+1} := u_{n+1,m}$. We mention that the step Δt can be reduced in this case, in order to improve the previous relative error.

To capture various phenomena appearing in the evolution of these equations, *e.g.* blow-up in finite time, an adaptive time step is used to improve the relative error. A python interface was built to compute $(1+1)$ and $(2+1)$—dimension dispersive-dissipative equation. Concerning the $(3+1)$—D, the code is written in fortran to speed up the computation.

Remark 4.1 *[CGM15b] As regards stochastic equations*

$$du - dA^{\alpha/2}u + u_x \circ dW + u^\rho u_x dt = 0,$$

the definition of the Stratonovich integral has to be used for the time discretization. This integral $\int_0^T u(t) \circ dW_t$ *is defined as the limit in probability of the sum*

$$\sum_{n=1}^{N} \frac{u(t_{n+1}) + u(t_n)}{2} (W(t_{n+1}) - W(t_n)).$$

Thus, for consistent numerical computations on long time intervals, the Cranck-Nicolson method will be used:

$$(1+\xi^2)(\widehat{u}_{n+1} - \widehat{u}_n) + i\xi \frac{\widehat{u}_{n+1} + \widehat{u}_n}{2} \Delta W_n + \frac{i\xi\Delta t}{2} \left(\widehat{\frac{u_{n+1}^{p+1}}{p+1}} + \widehat{\frac{u_n^{p+1}}{p+1}} \right) = 0$$

where $\Delta W_n = (W(t_{n+1}) - W(t_n))$. *The Stratonovich integral is used since Itô's formulation is not suitable for the discretization of these stochastic pdes. Indeed, Itô's integral is explicitly defined as*

$$\sum_{n=1}^{N} u(t_n)(W(t_{n+1}) - W(t_n)),$$

and an explicit discretization of the equations implies a loss of conservation laws. Note also that, contrary to the deterministic case, the Crank-Nicolson scheme applied to stochastic differential equations is of order 1 (Fig. 4.2).

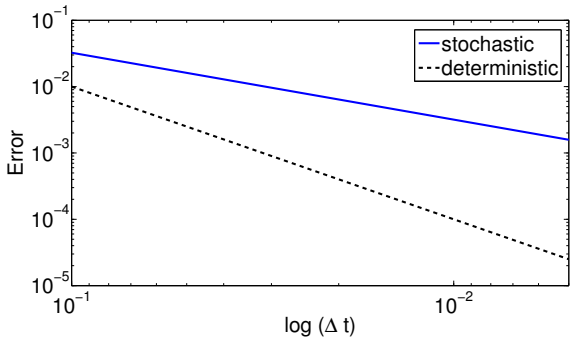

Figure 4.2: Order of convergence of the Crank-Nicolson scheme with respect to Δt for the determistic (- -) and stochastic (—) BBM equation.

Application to the study of KP instabilities. It consists of investigating, in diverse contexts, the blow-up in finite time, the dispersion, the solitonic behavior and the transverse instabilities [HMM09, Mam13b] of the solution of

$$(u_t + u_x + u^\rho u_x + u_{xxx})_x + a u_{yy} + b u_{zz} = 0,\ a = \pm 1, b = \pm 1.$$

The propagation of localized initial data

$$u_0(x) = \alpha(1 - 2sx^2)\mathrm{e}^{-s(x^2+y^2+z^2)},\ \alpha, s \in \mathbb{R}_+^*,$$

shows here that the effect of negative transverse directions is much less regularizing, the discrete solution of the Cauchy problem blows up in finite time for a weaker nonlinearity [dBS97, dBS96, Sau93, Sau95] (Fig. 4.3).

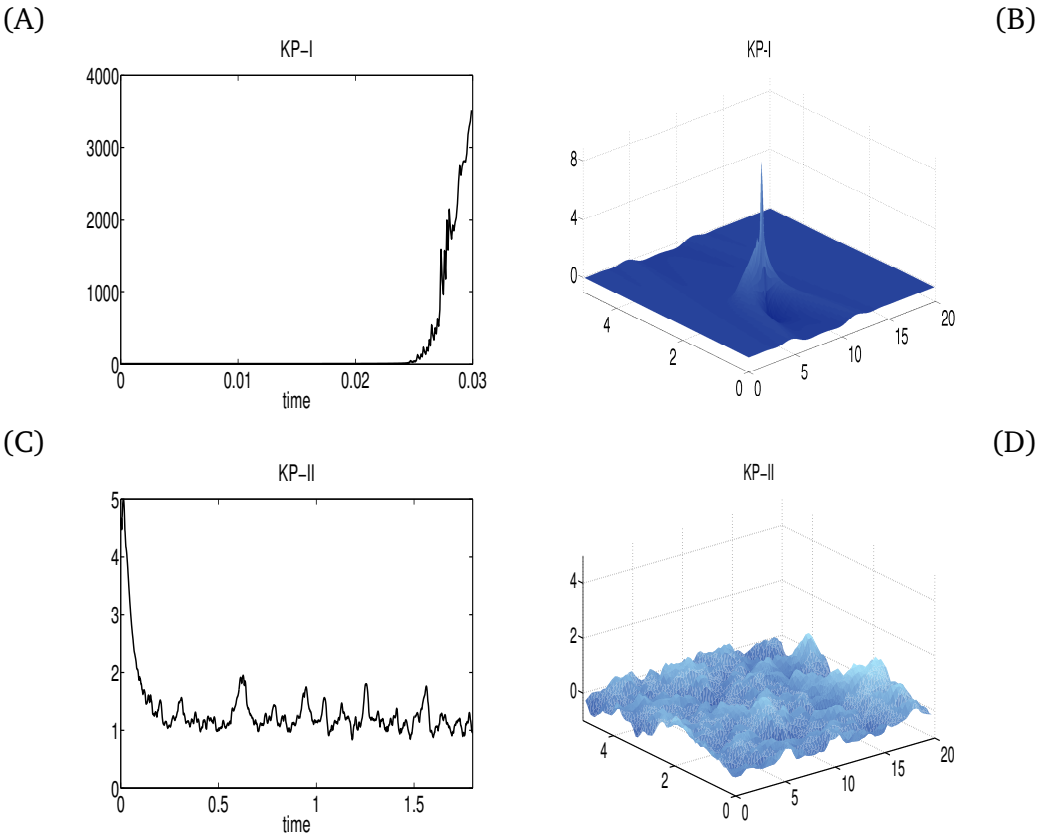

Figure 4.3: Blow-up of the solution of the KP-I equation (A and B) and dispersion of the solution of the KP-II equation (C and D) with a nonlinearity $u^2 u_x$. (A) and (C) represents the L^∞—norm of the solution.

The numerical transverse instabilities of the line-soliton

$$\Phi_c(x,t) = \left(\frac{(p+1)(p+2)}{2}c\right)^{1/p} \mathrm{sech}^{2/p}\left(\frac{p\sqrt{c}}{2}(x - ct)\right),$$

is investigating around a perturbation of the velocity

$$c(y,z) := c(1+\varepsilon\cos(\frac{2\pi y}{\lambda_y}+\frac{2\pi z}{\lambda_z}))$$

with $\lambda_y, \lambda_z > 0$, and $\varepsilon > 0$ is a fixed small value. We observe a stronger form of instability in the sense that solutions, initially close to solitary waves, becomes unstable [APS97, RT08, RT12] but may blow up in finite time [Liu02, Liu00]. Similarly, we deal with the transverse instability of the profile of the Zaitsev traveling waves [Zai83]

$$\psi_c(x,y,t) = 12\alpha^2\frac{1-\beta\cosh(\alpha x-\omega t)\cos(\delta y)}{(\cosh(\alpha x-\omega t)-\beta\cos(\delta y))^2}, \beta = \sqrt{\frac{\delta^2-3\alpha^4}{\delta^2}}$$
$$3\alpha^4 < \delta^2, \omega = \frac{\delta^2+\alpha^4}{\alpha},\ c = \frac{\omega}{\alpha}.$$

Once again, the negative transverse direction generates instabilities (Fig. 4.4).

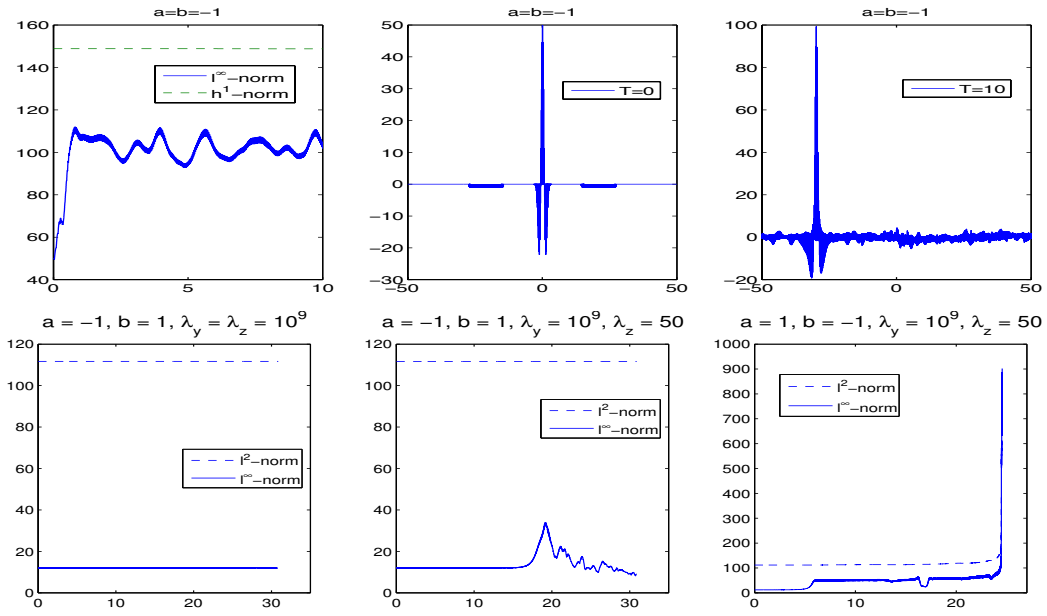

Figure 4.4: Solitonic to solitonic behavior of large amplitude solution of KP-I (A). Transverse instabilities of the Zaitsev soliton under the 3D-KP flow.

4.2 Plant-Pathogen-Pest interactions

Plant growth and airborne spread. We develop our own code (written in C and Python languages) which simulates the mechanistic plant growth and its

interaction with an airborne pathogen. However, to share it with the largest number (including biologists), the code was ported in OpenAlea [MBC+10]. This plateform offers efficient 3D visualization tools that are easily non-specialist user accessible and benefits from a user support service [PDKB+08]. Two modules, simulating the vine stock growth and the powdery mildew propagation, are integrated in OpenAlea. A friendly user interface is devised to facilitate numerical simulations of the model, and to facilitate the investigation of the spatial distribution of the disease (Fig. 4.5).

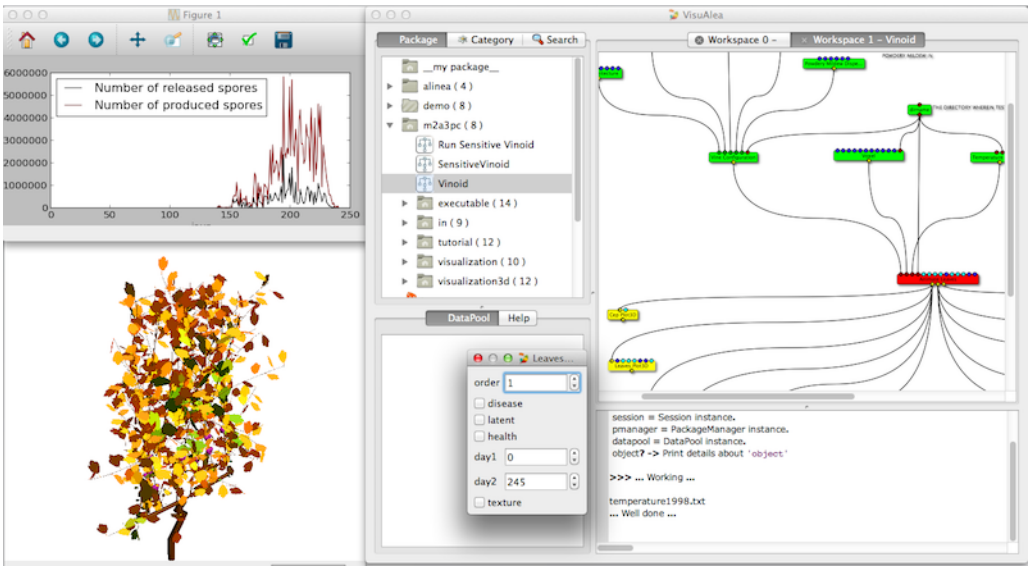

Figure 4.5: Vinoid interface under OpenAlea.

Phloem-Xylem transport. We describe the numerical discretization used to approximate the pressure and the carbohydrate concentration given by the equations (3.1)-(3.1) [MS15]. To simulate large space and time realistic multi-dimensional phloem trees, the CFL condition of the proposed scheme should not be too restrictive.
To deal with the nonlinearity, the equation (3.1)

$$\partial_t C - \nabla C \cdot \left(\frac{k}{\mu}\nabla P\right) - C\,\nabla \cdot \left(\frac{k}{\mu}\nabla P\right) - \frac{U}{e} = 0$$

is split in two parts, for $x \in \Omega$ and $t \in [0, T]$

$$\begin{cases} \partial_t C^{1/2} - \left(\frac{k}{\mu}\nabla P\right) \cdot \nabla C^{1/2} - \frac{U}{e} &= 0 \\ C^{1/2}(x,0) &= C_0(x). \end{cases}$$

$$\left\{\begin{array}{rcl} \partial_t C - C\,\nabla\cdot\left(\dfrac{k}{\mu}\nabla P\right) &=& 0 \\ C(x,0) &=& C^{1/2}(x,T), \end{array}\right.$$

and the change of variables $\widetilde{C} = \log(C)$ is applied to the second equation to obtain

$$\partial_t \widetilde{C} - \nabla\cdot\left(\frac{k}{\mu}\nabla P\right) = 0.$$

Let $n \in \mathbb{N}^*$. We denote P^n, respectively C^n the approximation at time $t_n = n\Delta t$ of the pressure P, respectively the carbohydrate concentration C. Let $\varphi_1, \varphi_2, \varphi_3$ be test functions in H^1. Finite elements method in space is used, while the time discretization is obtained with a semi-implicit scheme (Fig. 4.6):

- Set P_0, C_0 and $N \in \mathbb{N}^*$.
- For $n = 1$ to N

$$\begin{array}{rcl} \langle\dfrac{C^{n+1/2}-C^n}{\Delta t},\varphi_1\rangle - \langle\left(\dfrac{k}{\mu}\nabla P^n\right)\cdot\nabla C^n,\varphi_1\rangle - \langle\dfrac{U^n}{e},\varphi_1\rangle &=& 0 \\ \widetilde{C}^{n+1/2} = \log(C^{n+1/2}) & & \\ \langle\dfrac{\widetilde{C}^{n+1}-\widetilde{C}^{n+1/2}}{\Delta t},\varphi_2\rangle + \langle\dfrac{k}{\mu}\nabla P^{n+1},\nabla\varphi_2\rangle + \displaystyle\int_{\partial\Omega}\frac{k}{\mu}\partial_n P^{n+1}\varphi_2 &=& 0 \\ \langle\dfrac{P^{n+1}-P^n}{\Delta t},\varphi_3\rangle + \langle\dfrac{E}{\mu}k\nabla P^{n+1},\nabla\varphi_3\rangle - \displaystyle\int_{\partial\Omega}\frac{E}{\mu}k\partial_n P^{n+1}\varphi_3 & & \\ -\langle\dfrac{EL_R}{e}(\psi - P^n + RTC^n,\varphi_3\rangle - \langle\dfrac{V_s U^n}{e},\varphi_3\rangle &=& 0 \\ C^{n+1} = \exp(\widetilde{C}^{n+1}). & & \end{array}$$

The proposed semi-implicit scheme has the benefit of explicit and implicit schemes in the sense that it is uncondionnally stable without solving nonlinear iterations at each time step. It is implemented using the software FreeFem++.

Remark 4.2 *The finite element method is here preferred because our phloem and xylem database are in this format.*

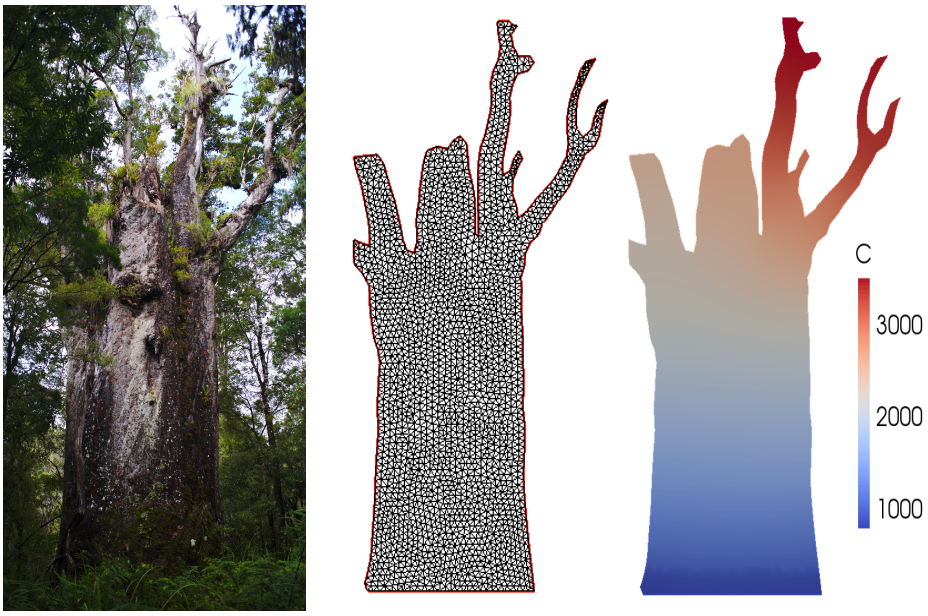

Figure 4.6: Finite elements mesh built from an image of the Te Matua Ngahere, symbol of New Zealand.

Pathogen spread at the plot scale. The numerical method has several constraints: it needs to preserve the positivity and the geometry of the domain (*e.g.* the row structure of the vineyards) and to simulate a large number of heterogeneous plants. Once again, we prefer unconditionally stable semi-implicit scheme combining an upwind method for the convection and a finite volume method for the diffusion [BPM+15].
A python interface was built to compute small plot. Nevertheless, a supercomputer was used for very large plot (> 6000 plants).

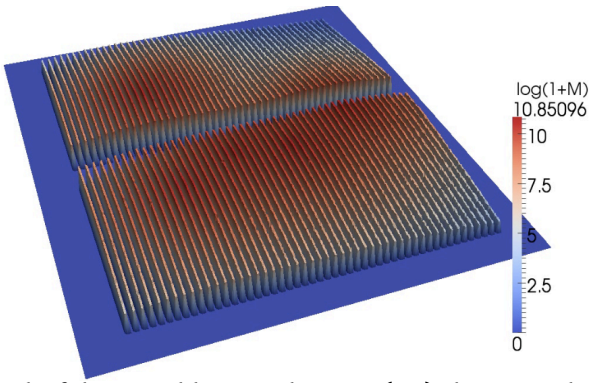

Figure 4.7: Spread of diseased leaves density (M) throw a plot made of $12OOO$ plants.

Pest landscape invasion. We proceed as above for the resolution of the system, except that the diffusion is solved by ADI method [MBLC14]. This choice is justified by the spatialised population data obtained from a geomatic two-year survey in an agricultural area of Brittany (Fig. 4.8).

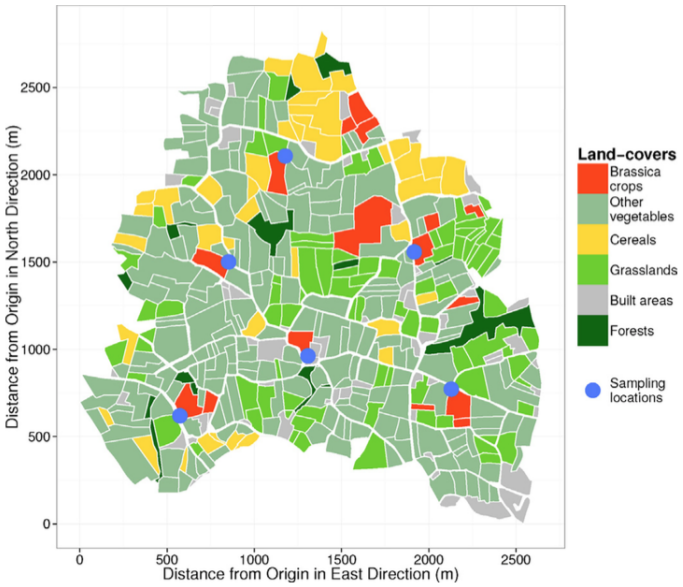

Figure 4.8: Map of the agricultural area with locations of Brassica crops.

AUTHOR'S PAPERS IN THIS DOCUMENT

[BCLM12] J.B. Burie, A. Calonnec, M. Langlais, and Y. Mammeri, *Modeling the spread of a pathogen over a spatially heterogeneous growing crop*, Proceedings of the International Workshop on Plant Models and Application, IEEE Press (Shanghai), 2012.

[BCM14] N. Bedjaoui, J.M.C. Correia, and Y. Mammeri, *On vanishing dissipative-dispersive perturbations of hyperbolic conservation laws*, Proceedings of the 5th International Conference on Fluid Mechanics and Heat & Mass Transfer (Lisbon), 2014.

[BCM15a] _____, *On a limit of perturbed conservation laws with diffusion and non-positive dispersion*, submitted for publication (2015), 20 pages.

[BCM15b] _____, *Well-posedness of the generalized Korteweg-de Vries-Burgers equation with nonlinear dispersion and nonlinear dissipation*, Int. J. Pure Math **2** (2015), 38–46.

[BPM+15] Y. Bourhis, S. Poggi, Y. Mammeri, A.-M. Cortesero, A. Le Rallec, and N. Parisey, *Perception-based foraging for competing resources: assessing pest population dynamics at the landscape scale from heterogeneous resource distribution*, Ecological Modelling **312** (2015), 211–221.

[CGM15a] J.P. Chehab, P. Garnier, and Y. Mammeri, *Long-time behavior of solutions of a BBM equation with generalized damping*, DCDS series B (2015), 25 pages.

[CGM15b] M. Chen, O. Goubet, and Y. Mammeri, *Generalized regularized long waves equations with white noise dispersion*, submitted for publication (2015), 17 pages.

[CMB15] A. Calonnec, Y. Mammeri, and C. Bruchou, *A sensitivity analysis of the vignoid model to highlight the complex interactions between the plant growth, the pathogen, the crop management and climate*, work in progress (2015).

[DMP15] F. Dell'oro, Y. Mammeri, and V. Pata, *The Benjamin-Bona-Mahony equation with dissipative memory*, NoDEA (2015), 13 pages.

[HMM09] F. Hamidouche, Y. Mammeri, and S.M. Mefire, *Numerical study of the solutions of the 3d generalized Kadomtsev-Petviashvili equations for long times*, Comm. Comput. Phys. **6** (2009), no. 5, 1022–1062.

[Mam07] Y. Mammeri, *Comparison between shallow water models in 2-dimension (in french)*, Math. Model. Numer. Anal. **42** (2007), no. 3, 513–542.

[Mam08] ———, *On the decay in time of solutions of some generalized regularized long waves equations*, Comm. Pure Appl. Ana. **7** (2008), no. 3, 513–532.

[Mam09a] ———, *Long time bounds for the periodic Benjamin-Ono-BBM equation*, Nonlin. Anal. TMA **71** (2009), no. 10, 5010–5021.

[Mam09b] ———, *Unique continuation property for the KP-BBM equation*, Diff. Int. Eq. **22** (2009), no. 3-4, 393–399.

[Mam10a] ———, *On the decay in time of solutions of the generalized regularized Boussinesq system*, Adv. Nonlin. Stud. **10** (2010), 349–387.

[Mam10b] ———, *Unique continuation property for Boussinesq-type systems*, Comm. Math. Anal. **9** (2010), no. 1, 121–127.

[Mam12a] ———, *Carleman estimates and unique continuation property for the Kadomtsev-Petviashvili equations*, Appl. Anal. (2012), 1–10.

[Mam12b] ———, *Continuation of time bounds for a regularized Boussinesq system*, Acta Appl. Math. **117** (2012), 1–13.

[Mam13a] ———, *A note on Carleman estimates and unique continuation property for the Boussinesq system*, Comm. Math. Anal. **15** (2013), no. 2, 29–38.

[Mam13b] ———, *Numerical study of the regularizing effect of the 3d weakly transverse BBM equations for long times*, Appl. Math. Comput. **219** (2013), 29–38.

[Mam14] ———, *Asymptotic behavior of small solution of the Benjamin-Ono equations with time-dependant coefficients*, J. Appl. Anal. (2014), 19.

[MBC+10] Y. Mammeri, J.B. Burie, A. Calonnec, T. Cokelaer, E. Costes, M. Langlais, and C. Pradal, *Modelling of the airborne dispersal of a pathogen over a structured vegetal cover*, Proceedings of the 6th International Workshop on Functional-Structural Plant Models (UC Davis), 2010.

[MBLC14] Y. Mammeri, J.B. Burie, M. Langlais, and A. Calonnec, *How changes in the dynamic of crop susceptibility and cultural practices can be used to better control the spread of a fungal pathogen at the plot scale?*, Ecological Modelling **290** (2014), 178–191.

[MS15] Y. Mammeri and D. Sellier, *A multi-dimensional model of the non-linear non-steady-state phloem transport*, submitted for publication (2015), 16 pages.

[MZ14] Y. Mammeri and Y. Zhang, *Comparison of solutions of Boussinesq systems*, Adv. Pure Appl Anal. **5** (2014), no. 2, 101–115.

[TMB15] B. Talbot, Y. Mammeri, and N. Bedjaoui, *Viscous shock anomaly in a variable-viscosity Burgers flow with an active scalar*, Fluid Dynamics Research (2015), 16.

REFERENCES

[ABS89] C.J. Amick, J.L. Bona, and M.E. Schonbek, *Decay of solutions of some nonlinear wave equations*, J. Differential Equations **81** (1989), no. 1, 1–49.

[AGB+13] D. Andrivon, C. Giorgetti, A. Baranger, A. Calonnec, P. Cartolaro, R. Faivre, S. Guyader, P.E. Lauri, F. Lescourret, L. Parisi, B. Ney, B. Tivoli, and I. Sache, *Defining and designing plant architectural ideotypes to control epidemics*, Euro. J. Plant Patho. **135** (2013), 611–617.

[Alb86] J. Albert, *Dispersion of low-energy waves for the generalized Benjamin-Bona-Mahony equation*, J. Differential Equations **63** (1986), no. 1, 117–134.

[APS97] J.C. Alexander, R.L. Pego, and R.L. Sachs, *On the transverse instability of solitary waves in the Kadomtsev-Petviashvili equation*, Phys. Lett. A **226** (1997), 187–192.

[Arn83] V.I. Arnold, *Geometric methods in the theory of ordinary differential equations*, Springer-Verlag, 1983.

[ASS13] P. Antonelli, J.-C. Saut, and C. Sparber, *Well-posedness and averaging of NLS with time-periodic dispersion management*, Adv. Diff. Eq. **18** (2013), no. 12, 49–68.

[Ayl99] D.E. Aylor, *Biophysical scaling and the passive dispersal of fungus spores: relationship to integrated pest management strategies*, Agricultural and Forest Meteorology **97** (1999), 275–292.

[BBM72] T.B. Benjamin, J.L. Bona, and J.J. Mahony, *Model equations for long waves in nonlinear dispersive systems*, Philos. Trans. Roy. Soc. London Ser. A **272** (1972), no. 1220, 47–78.

[BCL07] J.B. Burie, A. Calonnec, and M. Langlais, *Modeling of the invasion of a fungal disease over a vineyard. mathematical modeling of biological systems*, Modeling and Simulation in Science, Engineering and Technology II (2007), 11–21.

[BCS02] J.L. Bona, M. Chen, and J.-C. Saut, *Boussinesq equations and other systems for small-amplitude long waves in nonlinear dispersive media. I. Derivation and linear theory*, J. Nonlinear Sci. **12** (2002), no. 4, 283–318.

[BdBD15] R. Belaouar, A. de Bouard, and A. Debussche, *Numerical analysis of the nonlinear Schrödinger equation with white noise dispersion*, Stochastic PDE (2015), 29 pages.

[Ben67] T.B. Benjamin, *Internal waves of permanent form in fluids of great depth*, J. Fluid. Mech. **29** (1967), 559–592.

[Ben72] ———, *The stability of solitary waves*, Proc. Roy. Soc. (London) Ser. A **328** (1972), 153–183.

[BG80] H. Brezis and T. Gallouet, *Nonlinear Schrödinger evolution equations*, Nonlinear Anal. **4** (1980), no. 4, 677–681.

[BHR05] H. Berestycki, F. Hamel, and L. Roques, *Analysis of the periodically fragmented environment model: II – biological invasions and pulsating travelling fronts*, J. Math. Pures Appl. **84** (2005), 1101–1146.

[BL00] Y. Brenier and D. Levy, *Dissipative behavior of some fully non-linear KdV-type equations*, Phys. D **3–4** (2000), 277–294.

[BLC11] J.B. Burie, M. Langlais, and A. Calonnec, *Switching from a mechanistic model to a continuous model to study at different scales the effect of vine growth on the dynamic of a powdery mildew epidemic*, Ann. Botany **107** (2011), 885–895.

[Bou71] M.J. Boussinesq, *Théorie générale des mouvements qui sont propagés dans un canal rectangulaire horizontal*, C.R. Acad. Sci. Paris **73** (1871), 256–260.

[Bou93a] J. Bourgain, *Fourier transform restriction phenomena for certain lattice subsets and applications to nonlinear evolution equations. I. Schrödinger equations*, Geom. Funct. Anal. **3** (1993), no. 2, 107–156.

[Bou93b] ———, *Fourier transform restriction phenomena for certain lattice subsets and applications to nonlinear evolution equations. II. The KdV-equation*, Geom. Funct. Anal. **3** (1993), no. 3, 209–262.

[Bou93c] ———, *On the Cauchy problem for the Kadomtsev-Petviashvili equation*, Geom. Funct. Anal. **3** (1993), no. 4, 315–341.

[Bou97] ———, *On the compactness of the support of solutions of dispersive equations*, Internat. Math. Res. Notices (1997), no. 9, 437–447.

[BS75] J.L. Bona and R. Smith, *The initial-value problem for the Korteweg-de Vries equation*, Philos. Trans. Roy. Soc. London Ser. A **278** (1975), no. 1287, 555–601.

[Bur48] J.M. Burgers, *A mathematical model illustrating the theory of turbulence*, Adv. Appl. Mech. **1** (1948), 171.

[Car33] T. Carleman, *Sur les systèmes linéaires aux dérivées partielles du premier ordre à deux variables.*, C. R. Acad. Sci., Paris **197** (1933), 471–474 (French).

[CCC09] A. Calonnec, P. Cartolaro, and J. Chadoeuf, *Highlighting features of spatiotemporal spread of powdery mildew epidemics in the vineyard using statistical modeling on field experimental data*, Phytopath. **99** (2009), 411–422.

[CCDC06] A. Calonnec, P. Cartolaro, L. Deliere, and J. Chadoeuf, *Powdery mildew on grapevine: the date of primary contamination affects disease development on leaves and damage on grape*, IOBC/wbrs Bulletin **29** (2006), no. 11, 67–73.

[CCN+08] A. Calonnec, P. Cartolaro, J-M. Naulin, D. Bailey, and M. Langlais, *A host-pathogen simulation model: Powdery mildew of grapevine*, Plant Pathology **57** (2008), 493–508.

[CF73] A. L. Christy and J. M. Ferrier, *A mathematical treatment of Münch's pressure-flow hypothesis of phloem translocation*, Plant Physio. **52** (1973), 531–538.

[CG15] K. Chouk and M. Gubinelli, *Nonlinear pdes with modulated dispersion II: Korteweg-de Vries equation*, arXiv (2015), 1406.7675.

[CKS+03] J. Colliander, M. Keel, G. Staffilani, H. Takaoka, and T. Tao, *Sharp global well- posedness for KdV and modified KdV on R and T*, J. Amer. Math. Soc. **16** (2003), no. 3, 705–749.

[CL98] J. M. C. Correia and P. G. LeFloch, *Nonlinear diffusive-dispersive limits for multidimensional conservation laws*, Adv. in PDES and Related Areas (1998), 103–123.

[CR04] M. Cabral and R. Rosa, *Chaos for a damped and forced KdV equation*, Phys. D **192** (2004), no. 3–4, 265–278.

[CSR+08] E. Costes, C. Smith, M. Renton, Y. Guédon, P. Prusinkiewicz, and C. Godin, *MAppleT: simulation of apple tree development using mixed stochastic and biomechanical models*, Funct. Plant Bio. **35** (2008), 936–950.

[Daf70] C.M. Dafermos, *Asymptotic stability in viscoelasticity*, Arch. Rational Mech. Anal. **37** (1970), 297–308.

[dBD10] A. de Bouard and A. Debussche, *The nonlinear Schrödinger equation with white noise dispersion*, J. Func. Anal. **259** (2010), 1300–1321.

[DBP07] F. Didelot, L. Brun, and L. Parisi, *Effects of cultivar mixtures on scab control inapple orchards*, Plant Pathology **56** (2007), 1014–1022.

[dBS96] A. de Bouard and J.-C. Saut, *Remarks on the stability of generalized KP solitary waves*, Mathematical problems in the theory of water waves (Luminy, 1995), Contemp. Math., vol. 200, Amer. Math. Soc., Providence, RI, 1996, pp. 75–84.

[dBS97] ———, *Solitary waves of generalized Kadomtsev-Petviashvili equations*, Ann. Inst. H. Poincaré Anal. Non Linéaire **14** (1997), no. 2, 211–236.

[DD02] A. Debussche and L. DiMenza, *Numerical simulation of focusing stochastic nonlinear Schrödinger equations*, Phys. D **162** (2002), no. 3–4, 131–154.

[Die79] O. Diekmann, *Run for your life. a note on the asymptotic speed of propagation of an epidemic.*, J. Diff. Eq. **33** (1979), 58–73.

[DiP85] R.J. DiPerna, *Measure-valued solutions to conservation laws*, Arch. Rational Mech. Anal. **88** (1985), no. 3, 223–270.

[DM98] M. Davila and G. Perla Menzala, *Unique continuation for the Benjamin-Bona-Mahony and Boussinesq's equations*, Nonlinear Diff. Eq. Appl. **5** (1998), 367–382.

[DMRC10] L. Delière, A.S. Miclot, P. Rey, and A. Calonnec, *Efficacy of fungicides with various modes of action in controlling the early stages of an erysiphe necator induced epidemic*, Pest Management Sci. **66** (2010), 1367–1373.

[DP99] A. Debussche and J. Printems, *Numerical simulation of the stochastic Korteweg-de Vries equation*, Phys. D **134** (1999), 200–226.

[DT11] A. Debussche and Y. Tsutsumi, *1d quintic nonlinear Schrödinger equation with white noise dispersion*, J. Math. Pures Appli. **96** (2011), 363–376.

[Dut09] D. Dutykh, *Visco-potential free-surface flows and long wave modelling*, Eur. J. Mech. B Fluids **28** (2009), 430–443.

[EP11] A. Esfahani and A. Pastor, *On the unique continuation property for Kadomtsev–Petviashvili-I and Benjamin–Ono–Zakharov–Kuznetsov equations*, Bull. London Math. Soc. **43** (2011), no. 6, 1130–1140.

[Fin89] S. Finch, *Ecological considerations in the management of Delia pest species in vegetable crops?*, Annu. Rev. Entomol. **34** (1989), no. 1, 117–137.

[FTC75] J.M. Ferrier, M.T. Tyree, and A.L. Christy, *The theoretical time-dependent behavior of a Münch pressure-flow system: the effect of sinusoidal time variation in sucrose loading and water potential*, Can. J. Bot. **53** (1975), 1120–1127.

[GC98] C. Godin and Y. Caraglio, *A multiscale model of plant topological structures*, Journal of Theoretical Biology **191** (1998), 1–46.

[GDMD06] A.-C. Galucio, J.-F. Deü, S. Mengué, and F. Dubois, *An adaptation of the Gear scheme for fractionnal derivatives*, Comput. Methods Appl. Mech. Engrg. **195** (2006), 6073–6085.

[Ghi88] J.-M. Ghidaglia, *Weakly damped forced Korteweg-de Vries equations behave as a finite-dimensional dynamical system in the long time*, J. Differential Equations **74** (1988), no. 2, 369–390.

[Ghi94] ———, *A note on the strong convergence towards attractors of damped forced KdV equations*, J. Differential Equations **110** (1994), no. 2, 356–359.

[GNP01] C. Giorgi, M.G. Naso, and V. Pata, *Exponential stability in linear heat conduction with memory: a semigroup approach*, Comm. Appl. Anal. **5** (2001), 121–133.

[Gou00] O. Goubet, *Asymptotic smoothing effect for weakly damped forced Korteweg-de Vries equations*, Discrete Cont. Dyn. Syst. **6** (2000), no. 3, 625–644.

[GP68] M.E. Gurtin and A.C. Pipkin, *A general theory of heat conduction with finite wave speeds*, Arch. Rational Mech. Anal. **31** (1968), 113–126.

[GP06] M. Grasselli and V. Pata, *A reaction-diffusion equation with memory*, Discrete Cont. Dyn. Syst. **15** (2006), 1079–1088.

[HKN07] N. Hayashi, E.I. Kaikina, and P.I. Naumkin, *Large time asymptotics for the BBM-Burgers equation*, Ann. I.H.P. **8** (2007), no. 3, 485–511.

[HL97] B.T. Hayes and P.G. LeFloch, *Nonclassical shocks and kinetic relations: Scalar conservation laws*, Arch. Rational Mech. Anal. **139** (1997), 1–56.

[HM14] A.I. Houston and J.M. McNamara, *Foraging currencies, metabolism and behavioural routines*, J. Anim. Ecol. **83** (2014), no. 1, 30–40.

[HN99] N. Hayashi and P.I. Naumkin, *Large time asymptotics of solutions to the generalized Benjamin-Ono equation*, Trans. Amer. Math. Soc. **351** (1999), no. 1, 109–130.

[HNS99] N. Hayashi, P. I. Naumkin, and J.-C. Saut, *Asymptotics for large time of global solutions to the generalized Kadomtsev-Petviashvili equation*, Comm. Math. Phys. **201** (1999), no. 3, 577–590.

[Hor58] L. Horwitz, *Some simplified mathematical treatments of translocation in plants*, Plant Physio. **33** (1958), 81–93.

[Hör76] L. Hörmander, *Linear partial differential operators*, Springer Verlag, New York, 1976.

[IN98] R.J. Iório, Jr. and W.V.L. Nunes, *On equations of KP-type*, Proc. Roy. Soc. Edinburgh Sect. A **128** (1998), no. 4, 725–743.

[Ió90] R.J. Iório, Jr., *KdV, BO and friends in weighted Sobolev spaces*, Functional-analytic methods for partial differential equations (Tokyo, 1989), Lecture Notes in Math., vol. 1450, Springer, Berlin, 1990, pp. 104–121.

[JF97] I.D. Jonsen and L. Fahrig, *Response of generalist and specialist insect herbivores to landscape spatial structure?*, Landsc. Ecol. **12** (1997), no. 3, 185–197.

[KdV95] D.J. Korteweg and G. de Vries, *On the change of form of long waves advancing in a rectangular canal and on a new type of long stationnary waves*, Phil. Maj. **39** (1895), 422–443.

[KKS10] N. Kinezaki, K. Kawasaki, and N. Shigesada, *The effect of the spatial configurationof habitat fragmentation on invasive spread*, Theo. Pop. Bio. **78** (2010), 298–308.

[KKTS03] N. Kinezaki, K. Kawasaki, F. Takasu, and N. Shigesada, *Modeling biological invasions into periodically fragmented environments*, Theo. Pop. Bio. **64** (2003), 291–302.

[KM75] T. Kakutani and K. Matsuuchi, *Effect of viscosity of long gravity waves*, J. Phys. Soc. Japan **39** (1975), no. 1, 237–246.

[KMK27] W. Kermack and A. Mc Kendrick, *Contributions to the mathematical theory of epidemics, part 1*, Proc. Royal Soc. London **115** (1927), 700–721.

[KP70] B.B. Kadomtsev and V.I. Petviashvili, *Model equation for long waves in nonlinear dispersive systems*, Sov. Phys. Dokady. **15** (1970), 891–907.

[KP88] T. Kato and G. Ponce, *Commutator estimates and the Euler and Navier-Stokes equations*, Comm. Pure Appl. Math. **41** (1988), no. 7, 891–907.

[KPV96] C.E. Kenig, G. Ponce, and L. Vega, *A bilinear estimate with applications to the KdV equation*, J. Amer. Math. Soc. **9** (1996), no. 2, 573–603.

[KPV03] ———, *On the unique continuation of solutions to the generalized KdV equation*, Math. Res. Lett. **10** (2003), no. 5-6, 833–846.

[Kru70] S.N. Kruzkov, *First order quasilinear equations in several independent variables*, Mat. Sb., Math. USSR Sb. **81, 10** (1970), 285, 217.

[KS12] C. Klein and J.-C. Saut, *Numerical study of blow-up and stability of solutions to generalized Kadomtsev-Petviashvili equations*, J. Nonlinear Sci. **22** (2012), 763–811.

[KW08] C.I. Kondo and C.M. Webler, *The generalized BBM-Burger equations with non-linear dissipative term: existence and convergence results*, Appl. Anal. **87** (2008), no. 9, 1085–1101.

[Lan03] D. Lannes, *Consistency of the KP approximation*, Discrete Cont. Dyn. Syst. (2003), Suppl. 517–525.

[Liu00] Y. Liu, *Blow up and instability of solitary-wave solutions to a generalized Kadomtsev-Petviashvili equation*, Trans. Amer. Math. Soc. **353** (2000), no. 1, 191–208.

[Liu02] Y. Liu, *Strong instability of solitary-wave solutions to a Kadomtsev-Petviashvili equation in three dimensions*, J. Differential Equations **180** (2002), no. 1, 153–170.

[LL83] P.D. Lax and C.D. Levermore, *The small dispersion limit of the Korteweg-de Vries equation*, Comm. Pure Appl. Math. **36** (1983), I. 253, II 571, III 809.

[LM15] H. Le Meur, *Derivation of a viscous KP including surface tension, and related equations*, arXiv (2015), 16 pages.

[LN99] P.G. LeFloch and R. Natalini, *Conservation laws with vanishing nonlinear diffusion and dispersion*, Nonlinear Anal. **36** (1999), 213–230.

[LP09] F. Linares and G. Ponce, *Introduction to nonlinear dispersive equations*, Springer, 2009.

[LPS14] F. Linares, D. Pilod, and J.-C. Saut, *Dispersive perturbations of Burgers and hyperbolic equations I: local theory*, SIAM J. Math.Anal. **46** (2014), no. 2, 1505–1537.

[LPS15] ———, *Remarks on the orbital stability of ground state solutions of fKdV and related equations*, arXiv (2015), no. 1503.03084v1, 22 pages.

[LS06] D. Lannes and J.-C. Saut, *Weakly transverse Boussinesq systems and the KP approximation*, Nonlinearity **19** (2006), 2853–2875.

[M26] E. Münch, *über dynamik der saftströmungen*, Ber. Deut. Bot. Ges. **44** (1926), 68–71.

[M27] E Münch, *Versuche über den saftkreislauf*, Ber. Deut. Bot. Ges. **45** (1927), 340–356.

[M30] E. Münch, *Die stoffbewegungen in der planze*, Jena, Gustav Fischer (1930).

[MST02] L. Molinet, J.-C. Saut, and N. Tzvetkov, *Global well-posedness for the KP-I equation*, Math. Ann. **324** (2002), no. 2, 255–275.

[MST07] L. Molinet, J.-C. Saut, and N. Tzvetkov, *Remarks on the mass constraint for kp-type equations*, SIAM Journal on Mathematical Analysis **39** (2007), no. 2, 627–641.

[MT99] A. Miranville and R. Temam, *Mathematical modelling in continuum mechanics*, Cambridge University Press, 1999.

[NGR+08] R. Nathan, W.M. Getz, E. Revilla, M. Holyoak, R. Kadmon, D. Saltz, and P.E. Smouse, *A movement ecology paradigm for unifying organismal movement research*, Proc. Natl. Acad. Sci. **105** (2008), no. 49, 19052–19059.

[Nir57] L. Nirenberg, *Uniqueness of cauchy problems for differential equations with constant leading coecient*, Comm. Pure Appl. Math. **10** (1957), 89–105.

[Oks00] B. Oksendal, *Stochastic differential equations. an introduction with applications*, 5th edition, corrected 2nd printing, Springer, 2000.

[Ono75] H. Ono, *Algebraic solitary waves in stratified fluids*, J. Phys. Soc. Japan **39** (1975), no. 4, 1082–1091.

[OS70] E. Ott and R.N. Sudan, *Damping of solitary waves*, Phys. of Fluids **13** (1970), no. 6, 1432–1434.

[Pan05] M. Panthee, *Unique continuation property for the Kadomtsev-Petviashvili (KP-II) equation*, Electron. J. Differential Equations (2005), No. 59, 12 pp.

[PDKB+08] C. Pradal, S. Dufour-Kowalski, F. Boudon, C. Fournier, and C. Godin, *OpenAlea: A visual programming and component-based software platform for plant modeling*, Funct. Plant Bio. **35** (2008), no. 9-10, 751–760.

[PR07] B. Perthame and L. Ryzhik, *Moderate dispersion in conservation laws with convex fluxes*, Commun. Math. Sci. **5** (2007), no. 2, 473–484.

[RH93] P. Rosenau and J.M. Hyman, *Compactons: solitons with finite wavelengths*, Phys. Rev. Lett. **70** (1993), no. 5, 564–567.

[Ros94] P. Rosenau, *Nonlinear dispersion and compact structures*, Phys. Rev. Lett. **73** (1994), no. 13, 1737–1741.

[RT08] F. Rousset and N. Tzvetkov, *Transverse nonlinear instability of solitary waves for some hamiltonian pdes*, J. Math. Pures Appl. **90** (2008), no. 6, 550–590.

[RT12] ———, *Stability and instability of the KdV solitary wave under the KP-I flow*, Commun. Math. Phys. **313** (2012), no. 1, 155–173.

[Sau93] J.-C. Saut, *Remarks on the generalized Kadomtsev-Petviashvili equations*, Indiana Univ. Math. J. **42** (1993), no. 3, 1011–1026.

[Sau95] ———, *Recent results on the generalized Kadomtsev-Petviashvili equations*, Acta Appl. Math. **39** (1995), no. 1-3, 477–487, KdV '95 (Amsterdam, 1995).

[Sch82] M.E. Schonbek, *Convergence of solutions to nonlinear dispersive equations*, Comm. PDE **7** (1982), no. 8, 959–1000.

[SH14] D. Sellier and J.J. Harrington, *Phloem transport in trees: A generic surface model*, Eco. Mod. **290** (2014), 102–109.

[Sha85] J. Shatah, *Normal forms and quadratic nonlinear Klein-Gordon equations*, Comm. Pure Appl. Math. **38** (1985), 685–69.

[SK97] N. Shigesada and K. Kawasaki, *Biological invasions: Theory and practice*, Oxford University Press, 1997.

[SKT86] N. Shigesada, K. Kawasaki, and E. Teramoto, *Traveling periodic waves in hetero- geneous environments*, Theo. Pop. Bio. **30** (1986), 143–160.

[Sle13] M. Slemrod, *From Boltzmann to Euler: Hilbert's 6th problem revisited*, Comput. Math. Appl. **65** (2013), 1497–1501.

[SPH11] D. Sellier, M.J. Plank, and J. J. Harrington, *A mathematical framework for modelling cambial surface evolution using a level set method*, Ann. Botany **108** (2011), no. 6, 1001–1011.

[SS87] J.-C. Saut and B. Scheurer, *Unique continuation for some evolution equations*, J. Differential Equations **66** (1987), no. 1, 118–139.

[ST04] J.-C. Saut and N. Tzvetkov, *Global well-posedness for the KP-BBM equations*, AMRX Appl. Math. Res. Express (2004), no. 1, 1–16.

[Ste86] E. Stein, *Oscillatory integrals in Fourier analysis*, Beijing lectures in harmonic analysis (Beijing, 1984), Ann. of Math. Stud., vol. 112, Princeton Univ. Press, Princeton, NJ, 1986, pp. 307–355.

[Ste93] E. Stein, *Harmonic analysis : real-variable methods, orthogonality, and oscillatory integrals*, Princeton University Press, 1993.

[Str74] W.A. Strauss, *Dispersion of low-energy waves for two conservative equations*, Arch. Rational Mech. Anal. **55** (1974), 86–92.

[STSA10] N. Sapoukhina, Y. Tyutyunov, I. Sache, and R. Arditi, *Spatially mixed crops tocontrol the stratified dispersal of airborne fungal diseases*, Eco. Mod. **221** (2010), 2793–2800.

[Tao05] T. Tao, *Nonlinear dispersive equations - local and gloabl analysis*, Amer. Math. Soc., 2005.

[TD10] C.V. Tran and D.G. Dritschel, *Energy dissipation and resolution of steep gradients in one-dimensional burgers ows*, Phys. of Fluids **22** (2010), 7 pages.

[Tem97] R. Temam, *Infinite-dimensional dynamical systems*, Second edition, Springer-Verlag, 1997.

[TH03] M.V. Thompson and N.M Holbrook, *Application of a single-solute non-steady-state phloem model to the study of long-distance assimilate transport*, J. Theo. Bio. **220** (2003), 419–455.

[Thi77] H.R. Thieme, *A model for the spatial spread of an epidemic*, J. Math. Bio. **4** (1977), 337–351.

[TM71] M. Tsutsumi and T. Mukusa, *Parabolic regularizations for the generalized Korteweg-de Vries equation*, Funkcial. Ekvac. **14** (1971), 89–110.

[Tzv04] N. Tzvetkov, *Long time bounds for the periodic KP-II equation*, Int. Math. Res. Not. (2004), no. 46, 2485–2496.

[VdBZM88a] F. Van den Bosch, J.C. Zadoks, and J.A.J. Metz, *Focus expansion in plant disease I: The constant rate of focus expansion*, Phytopath. **78** (1988), 54–58.

[VdBZM88b] ———, *Focus expansion in plant disease II: Realistic parameter-sparse models*, Phytopath. **78** (1988), 59–64.

[VdBZM88c] ———, *Focus expansion in plant disease III: Two experimental examples*, Phytopath. **78** (1988), 919–925.

[Ven10] S. Vento, *Asymptotic behavior of solutions to dissipative Korteweg-de Vries equations*, Asymptot. Anal. **68** (2010), no. 3, 155–186.

[Ven11] ———, *Global well-posedness for dissipative Korteweg-de Vries equations*, Funkcial. Ekvac. **54** (2011), no. 1, 119–138.

[VGGC+11] H. Valdes-Gomez, C. Gary, P. Cartolaro, M. Lolas-Caneo, and A. Calonnec, *Powdery mildew development is positively influenced by grapevine vegetative growth induced by different soil management strategies*, Crop protection **30** (2011), 1168–1177.

[vNR50] J. von Neumann and R.D. Richtmyer, *A method for the numerical calculation of hydrodynamical shocks*, J. Appl. Phys. **21** (1950), 380.

[Wan97] B. Wang, *Strong attractors for the Benjamin-Bona-Mahony equation*, Appl. Math. Lett. **10** (1997), no. 2, 23–28.

[Wan14] M. Wang, *Long time dynamics for a damped Benjamin-Bona-Mahony equation in low regularity spaces*, Nonlinear Anal. **105** (2014), 134–144.

[Waz01] A. Wazwaz, *Computational approach to soliton solutions of the Kadomtsev-Petviashvili equation*, Appl. Math. Comput. **123** (2001), no. 2, 205–217.

[WCR+96] L. Willocquet, D. Colombet, M. Rougier, J. Fargues, and M. Clerjeau, *Effectsof radiation, especially ultraviolet b, on conidial germination and mycelialgrowth of grape powdery mildew*, Euro. J. Plant Patho. **102** (1996), 441–449.

[Whi74] G.B. Whitham, *Linear and nonlinear waves*, John Wiley & Sons, New York, 1974.

[WY97] B. Wang and W. Yang, *Finite-dimensional behaviour for the Benjamin-Bona-Mahony equation*, J. Phys. A: Math. Gen. **30** (1997), 4877–4885.

[Zai83] A.A. Zaitsev, *Formation of stationary waves by superposition of solitons*, Sov. Phys. Dokl. **28** (1983), no. 9, 720–722.

[ZZ92] M. Zawolek and J.C. Zadoks, *Studies in focus development: An optimum for the dual dispersal of plant pathogens*, Phytopath. **82** (1992), 1288–1297.

[ZZMR02] G.P. Zank, Y. Zhou, W.H. Matthaeus, and W.K.M. Rice, *The interaction of turbulence with shock waves: A basic model*, Phys. of Fluids **14** (2002), 3766–3774.

Equations dispersives et dissipatives et quelques applications en écologie

Ce mémoire d'habilitation présente mes principaux résultats en trois chapitres distincts.

Le premier concerne les équations aux dérivées partielles non-linéaires dispersives de la forme

$$u_t + f(u)_x - (-\partial_x^2)^{\alpha/2} u = 0.$$

Le problème de Cauchy y est étudié à travers une méthode de forme normale et des inégalités de dispersion. Deux propriétés de prolongement unique sont quant à elles établies via une inégalité de Carleman et une estimée de Bourgain.

La prise en compte de la viscosité et de la visco-élasticité fait l'objet du second chapitre. Décrite mathématiquement par un opérateur $\mathscr{A}$ tel que $\langle \mathscr{A}u, u\rangle > 0$, on y démontre la dissipation ainsi que la stabilité de l'énergie. La convergence vers la solution entropique de la loi de conversion hyperbolique

$$u_t + f(u)_x = 0,$$

est également prouvée.

Dans le troisième chapitre, complétement indépendant des deux premiers, on se focalise sur la modélisation des plantes et de leur environnement. Les interactions entre les plantes, les pathogènes et les ravageurs y sont dressées.

Dispersive and dissipative equations and some applications in ecology

This report presents my main results in three separate chapters.

The first one is about nonlinear dispersive partial differential equations of the form

$$u_t + f(u)_x - (-\partial_x^2)^{\alpha/2} u = 0.$$

The Cauchy problem is studied through a method of normal form and dispersion inequalities. Two unique continuation properties are established via a Carleman inequality and an estimate of Bourgain.

The consideration of the viscosity and viscoelasticity is the subject of the second chapter. Described mathematically by an operator $\mathscr{A}$ such that $\langle \mathscr{A}u, u\rangle > 0$, one shows the dissipation and the stability of the energy. The convergence to the entropy solution of the hyperbolic conversion law

$$u_t + f(u)_x = 0,$$

is also proved.

In the third chapter, completely independent of the first two, we focus on the modeling of plants and their environment. The interactions between plants, pathogens and pests are drawn up.