



Étude théorique et numérique des modes propres acoustiques dans un conduit avec écoulement et parois absorbantes

Javier Rodríguez Sánchez

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THÈSE

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Étude théorique et numérique des modes propres acoustiques dans un conduit avec écoulement et parois absorbantes

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Abstract

The study presented in this thesis is within the domain of modal acoustics of lined ducts with grazing flow. We consider an upstream source of noise with a fixed frequency, within a lined duct. From this, we study the eigenmodes in terms of wavenumber that are present in this system.

With this study, we contribute to the better understanding of sound propagation in the described configuration. Within its main applications, we can find the noise reduction from aeroengines.

A numerical analysis with the pseudospectral collocation method, based on Chebyshev polynomials was used to obtain the spectrum of modes within the duct, in a domain transversal to the mean flow. For this, two programs were used: On one hand, within the frame of this thesis, the program FiEStA was developed. It solves the linearized Euler Equations, considering either one or two dimensions of the transversal plane. On the other hand, the already existing program MAMOUT was used for verification and to solve also the linearized Navier-Stokes Equations to observe the effects of viscosity.

With these tools, the first result was to notice the effects of three parameters: When the aspect ratio grows, the density of modes in the spectrum grows also. In particular, we have more propagative modes. As the mean flow Mach number grows, we observe these effects on the eigenvalues: a displacement to the negative real part, a slight amplification of their absolute value and a displacement towards the modes of lower index. The difference in mean flow profile induces another displacement in modes, not easily predictable. It changes also the shape of eigenfunctions, which is clearly seen for the planewave mode. The impedance changes induce a cyclic exchange of eigenvalues from their hard wall value to the hard wall value of a consecutive mode. The change of eigenfunction is gradually change in wavelength, to obtain the shape of the destination mode. With some impedance values, a pair of modes, called the acoustic surface modes arise. They are characterized by the exponential shape of their eigenfunctions.

Besides these acoustic surface modes, there are also a pair of hydrodynamic surface modes which come to light with some values of impedance and shape and Mach number of the mean flow. With a benchmark data, these modes were studied. The impedance was considered from the model of a measured liner while the mean flow profile was taken from experimental values. With this, the hydrodynamic mode was found. With specific values of frequency, the set of parameters gives rise to an instability. Using the Briggs-Bers criterion for stability, the instability was found to be absolute for a given frequency.

From the comportment of modes with different values of impedance, and in accordance with published results, we defined the condition that the spectrum has to fulfill to reduce as much as possible the upstream noise. This is what we called the optimal impedance. We obtained it for several flow profiles and frequencies, in both 1D and 2D domains.

Résumé

L'étude présentée dans cette thèse se situe dans le domaine de l'acoustique modale des conduits avec des parois absorbantes et un écoulement moyen. Nous considérons une source de bruit en amont avec une fréquence fixe. Avec cela, nous étudions les modes propres acoustiques du conduit en terme de nombre d'onde qui sont présents.

Avec cette étude, nous contribuons à la meilleure compréhension de la propagation du son dans ce type de configuration. Parmi les applications, il y a la réduction du bruit des moteurs des aéronefs.

Une analyse numérique par la méthode pseudospectrale de collocation, sur la base de polynômes de Chebyshev, a été mise en oeuvre pour obtenir le spectre des modes, dans un domaine transversal. Pour cela, deux programmes ont été utilisés : le programme FiEStA, qui a été développé dans le cadre de cette thèse, et qui résout les équations d'Euler linéarisées, en considérant un problème à une ou deux dimensions. D'autre part, le programme MAMOUT, a été utilisé pour résoudre les équations de Navier-Stokes linéarisées, pour étudier plus spécifiquement les effets de la viscosité.

Avec ces outils, on a constaté les effets de trois paramètres : lorsque le rapport d'aspect augmente, la densité des modes, en particulier des modes propagatifs, se développe également. Quand le nombre de Mach de l'écoulement moyen augmente, on observe les effets suivants sur les valeurs propres : un déplacement vers la partie réelle négative, une amplification de leur valeur absolue et un déplacement vers les modes d'indice inférieur. Le profil d'écoulement moyen induit aussi un déplacement dans les valeurs propres, pas facilement prévisible. Il modifie également la forme des fonctions propres ; ce qui est notamment visible pour le mode d'onde plane. Les changements d'impédance induisent un échange cyclique de valeurs propres entre les valeurs de parois rigides des modes consécutifs. Avec certaines valeurs d'impédance, les modes acoustiques de paroi apparaissent. Ils sont caractérisés par la forme exponentielle de leurs fonctions propres.

En plus des modes acoustiques, il existe des modes hydrodynamiques de surface qui se sont révélés avec quelques valeurs d'impédance et forme et nombre de Mach de l'écoulement moyen. Pour un ensemble de données de référence, ces modes ont été étudiés. L'impédance a été considérée avec un modèle basé sur des données de la littérature, tout comme le profil d'écoulement moyen. Un mode hydrodynamique a été trouvé. Avec certaines valeurs de la fréquence, l'ensemble des paramètres donne lieu à une instabilité. En utilisant le critère Briggs Bres pour la stabilité, l'instabilité a été jugée absolue.

À partir du comportement des modes avec différentes valeurs de l'impédance, et conformément aux résultats publiés, nous avons défini la condition que le spectre doit remplir pour réduire autant que possible le bruit. C'est cela qu'on appelle l'impédance optimale. Nous avons calculé cette valeur pour différents fréquences et écoulements moyens.

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¹N.B. Cette partie de la thèse comporte un résumé étendu en français. Celui-ci conserve la même structure en chapitres du texte principal contenu dans la première partie. L’information et les résultats les plus importants sont contenus ici. Pour plus de détails sur les développements et les résultats, j’invite les lecteurs à se diriger vers la première partie, rédigée en anglais.

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Part I

English text

Introduction

The study presented in this thesis is within the domain of modal acoustics of lined ducts with grazing flow. In this sense, we consider a monochromatic upstream source of noise, and we study the propagation of sound in terms of eigenmodes.

The main objective of this work is to know better about sound propagation and reduction in lined ducts with flow. Its aimed application is to help with the reduction of noise nuisance in this kind of configurations. This problem is found in the aeronautic industry, as well as in air conditioning systems and even in railroad tunnels.

To reduce noise in ducts, they are lined with acoustic absorbent materials. The acoustic behaviour of these materials are usually characterised by a complex number, the acoustic impedance. This value depends on the construction of the material, but also in some characteristics of the sound field and the flow themselves. Besides the issue of being an implicit problem, it is also hard to solve exactly with an arbitrary shape for the mean flow profile and an acoustic liner.

As a result, there is not a complete theory to predict in detail the behaviour of present modes in the studied configuration. A theory of this class would help to identify different kinds of modes and their characteristics. Also, it would be useful to find the absorbent material that leads to the best noise reduction, given the characteristics of the flow and the noise.

Rienstra [94] studied the different kind of modes present in ducts, with emphasis on surface waves. Marx et. al. [76] have found evidence that some of these surface modes can be unstable. These modes are of particular interest because, in spite of being hard to find in flight conditions, if they do happen, they would increase the sound level supposed to be lowered and could even damage some structures.

To model the acoustic impedance of liners, several models are used. Rienstra [96] studied some of them in time domain.

Cremer [38] with an analytical development, found an expression for the optimal impedance in function of frequency. However it is only valid without mean flow. Tester [113] extended the formula from Cremer and found a relation with uniform mean flow for high frequency modes.

The study I present aims at identifying the spectrum of modes present in lined ducts, subject to a mean flow. From this, we study both acoustic and hydrodynamic eigenmodes, with special focus on their stability. Moreover, we aim at identifying, for any flow profile and sound frequency, the impedance that gives the best noise attenuation.

To achieve the stated goals, I developed and validated a spectral collocation code to solve the linearized Euler equations as an eigenvalue problem. This code was used to get the spectrum for different sets of parameters; and also to make sensibility analysis with respect to certain parameters such as the impedance, the flow profile or the Mach number. It was also compared to

an already existing program, used within the laboratory.

From the comportement of modes with different values of impedance, and in accordance with Cremer and Tester, we defined the condition to fulfill for the optimal impedance. From this, we obtained it for several flow profiles and frequencies, in both 1D and 2D domains.

With both programs, and a set of parameters studied also by Primus [91] with data from Jones [60], an hydrodynamic mode was studied. It was found to be an unstable mode, present in a narrow range of impedance and frequency values.

Chapter 1

Context

1.1 Noise mitigation

According to the World Health Organization, one in three individuals is annoyed at daytime and one in five has disturbed sleep at night because of traffic noise. Also, repetitive exposure to high-level noise can decrease performance at school or workplace and increase aggressiveness. Besides these annoyances, noise can produce or accelerate health issues as diverse as hearing impairment or tinnitus, hypertension, myocardial infarction and cognitive impairment in children [123, 122, 121]. Because of these risks and by the sake of peace, its mitigation has always been an important subject of study, particularly in contemporary life since we are surrounded by many noise sources.

On this modern-life noise sources, we can highlight aircraft by its impact on life, both from an economic and human connectivity point of view.

Noise mitigation in aircraft results always from a compromise between production costs and flight efficiency. Since most of noise is generated by the propulsion systems, they are closely related in the final noise fingerprint of aeroplanes. The Advisory Council for Aeronautics Research in Europe (ACARE) established in 2001, among other goals, the reduction of external aircraft noise in one half for 2020, and by a 65% for 2050 from their FlightPath 2050 goals [36]. These objectives will help to get financial support for studies devoted to noise reduction. They will also reflect economic benefits, since removing hearing disturbances beyond airport limits may permit to schedule flights all day long.

1.1.1 Noise sources and mitigation in aeronautics

Modern high bypass ratio aeroplanes have several noise sources. Figure 1.1 shows the perceived level of most of these sources, according to the German Aerospace Centre (DLR) and the National Aeronautics and Space Administration of the United States (NASA). Most of them can be attributed either to airframe or jet or fan noise.

Next we give a general overview of these sources and some techniques that have been employed to face the noise they generate, as well as some research trends. For a more detailed study on this subject, we invite the reader to the bibliographic review of Casalino [29] and its references.

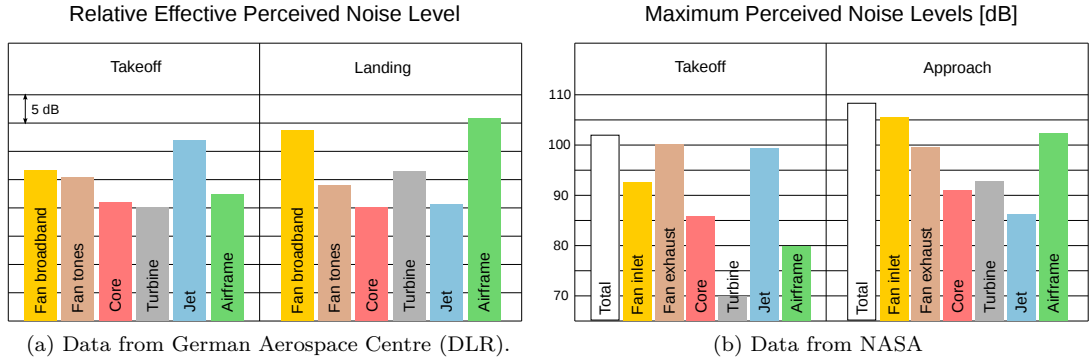


Figure 1.1 – Level of aircraft noise sources.

1.1.1.1 Airframe noise

Airframe noise correspond to the noise due to the structure of the aeroplane and not to its propulsive components. It is the principal contribution in landing approach phases, mostly because the undercarriage and the high-lift devices are spread out [71].

To reduce the noise produced by this source, passive control methods can be used to modify the surface mechanical impedance close to the edges of rigid surfaces to make a coupling between the hard wall and the air. This is intended to be done in the trailing edge, flaps or slats. This can be done by the means of brushes, sawtooth edges or porous surfaces. There is also a research to install acoustic liners in the gap between the slat and the wing. Active control methods are also investigated trough more long-term studies such as the use of plasma actuators or by blowing air trough the surfaces [71].

For the undercarriage, one strategy is the use of add-on fairings and covers to increase its aerodynamics [31].

1.1.1.2 Jet noise

Jet noise is produced by the small and large scale turbulent structures of flow. The first jet aeroplanes emitted very high noise levels mostly because of the high speed of the exhaust flow. The inclusion of the bypass engines, allowed a reduction in flow speed by moving bigger masses of air. Bypass ratio is the proportion of air that goes trough the bypass with respect to the air that crosses the turbine. Even with bypass ratios of the order of unity, i. e. as much air passing by the turbine as through the bypass, the sound level is reduced by about $10dB$ [107]. This is because the acoustic power radiated by a jet is proportional to the jet speed raised to the power of 8. With nowadays high bypass ratio aircraft, at takeoff, the jet noise contributes to almost one half of the noise radiated by aircraft.

This noise is further reduced by the use of nozzle tabs and chevrons. There is also the research for distributed nozzles, to split the exhaust plume into an array of smaller plumes with higher frequency components, more effectively absorbed by the atmosphere. One technique of active control is the injection of water microjets to affect turbulent structures [62]. Furthermore, as for the jet noise, research in active control are currently undertaken for example with Plasma Synthetic Jet (PSJ) or Dielectric Barrier Discharges (DBD) [30, 63, 28].

1.1.1.3 Fan noise

Fan noise is produced by interactions between the flow, the rotor and the stator; another contribution is due to supersonic blade tip rotation. This noise has broadband and high frequency tonal components [41]. Fan is dominant in turbo-machinery noise, and it is a major contributor to whole aircraft noise [2, 3]. At takeoff, it is a noise source as important as the jet [117].

The Tyler-Sofrin selection rule is used to adjust the number of rotor blades and stator vanes in a way that the tones resulting from their interaction be evanescent (cut-off) and thus they don't propagate outside the nacelle [99].

The main strategy for fan noise reduction is the use of absorbing materials to line the engines [2, 3], leading the linings to be a very important ingredient in determining the acoustic footprint of aircraft. The innovation in these materials are the use of zero-splice liners [48] and the hybrid passive-active treatments [47].

The negatively scarfed intake, as shown in image 1.2 also aids in noise reduction because it radiates less noise downwards and it gives more surface for acoustic lining [81].



Figure 1.2 – Negatively scarfed intake, picture taken from [81].

Besides the noise of main engines, the auxiliary power unit is another source of noise, specially while taxiing; it is called “ramp-noise”. This noise is also reduced by the means of acoustic liners [84].

In the present thesis, the noise that will be considered is this last type, as it is described in next section.

1.1.2 Techniques of noise mitigation in ducts

Fan noise in high bypass ratio aeroplanes is produced and propagated within a duct. This kind of configuration of noise in ducts with grazing flow can also be found in other systems such as air conditioning systems and even railway tunnels. This problem of noise in ducts with flow is the main objective that will be addressed in this thesis.

Noise mitigation in these configurations can be done by active actuators, passive materials or some combinations of them.

1.1.2.1 Passive techniques

The main strategy to mitigate noise within ducts is the use of passive absorbing materials as acoustic liners, this will be discussed next in section 1.2.

Another strategy of sound attenuation is the introduction of waveguides called Herschel-Quincke tubes to make destructive interference passively by difference of length of paths; they are used in series with liners. Bandwidth can be modified with the diameter of the tubes [89].

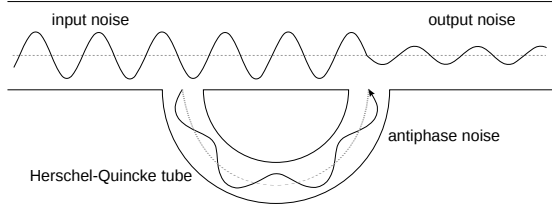


Figure 1.3 – Schema of a duct with a sound reduction by destructive interference done by Herschel-Quincke tubes.

For low frequencies, where neither liners nor Herschel-Quincke tubes are practical, other types of means to decrease noise are being investigated. One configuration that is being studied experimentally as well as numerically is the flute-like silencer. It consists in a cavity filled with a different gas, to have a different speed of sound, contiguous to the main duct and communicated with it by a membrane in the inlet and in the exit [53]. It has two resonant frequencies, one due to the difference of phase between ends of the cavity and another one obtained from the tension of the membrane and inertia of fluids and membrane.

1.1.2.2 Active techniques

Active systems can be used for any frequency. They are specially used for low frequencies, where passive materials are not reliable. They consist mainly in recreating the sound field, or some tones present on it, but in antiphase, i.e. with a difference of phase of a half-period. In this way, noise is annihilated due to destructive interference. A schema of an active controlled duct is shown in figure 1.4. These systems need an input signal to feed the generating device. The most common input signal is the noise itself to be annihilated. However, the resulting signal after the actuator can also be used. In both cases, they are backed by electronics and algorithms to obtain the desired effect [44].

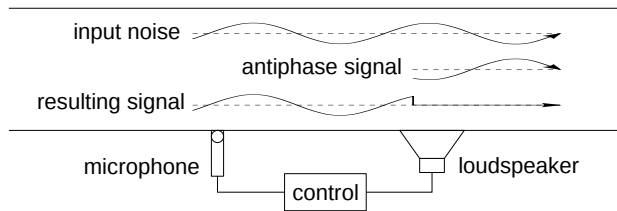


Figure 1.4 – Schema of a duct with a sound annihilated by destructive interference of an active system. Back propagation of sound from the secondary source is not considered.

There are two main strategies; one of them, based on the feed-forward control principle, takes either a prescribed input source, normally monochromatic, or an input close to noise source. It

is widely used to validate the technique on controlled experiments [45]. The other strategy is supported on feedback, on which the input signal is not known beforehand but it is measured directly, adapting the system in real-time. It is more used in real life cases where sources are unknown or change from time to time, and where they don't use to be monochromatic [10].

A problem present in systems with active actuators, is that sound from loudspeaker propagates also in opposite direction. This induces an unwanted feedback from secondary source to primary input, specially when standing waves are formed in ducts. This can corrupt the reference signal and render inefficient the noise mitigation. To solve this issue, there are several techniques, as the use of a secondary source to cancel the feedback signal itself or of dual reference microphones to discriminate primary and feedback signals [102].

A second technique is the active control of impedance, it is conceived to aim at some impedance for a wall. On one of these techniques, the pressure and velocity information are gathered close to the surface of a loudspeaker whose impedance is changed to a prescribed value. This method is used without mean flow in closed pipe [114], with normal incidence and in anechoic chamber [45] in normal and oblique incidence, giving good attenuation in a wide spectrum of low frequencies.

Another active technique consists on placing micro-electro-mechanical systems in the walls of a duct to reduce the sound transmission along it by annihilating the sound reflection. It has been modelled numerically and tested in a closed pipe, reduction of sound has been observed by about 5 dB in a bandwidth between 700 and 1300 Hz [35, 34].

1.1.2.3 Hybrid techniques

Hybrid sound absorbers may be a combination of both passive materials and active actuators. It uses each behaviour in the frequency range for which it is the most adapted. In low frequency, the active components are turned on, while active control is useless in high frequency, when reaction time becomes important. For high frequencies, the passive compartment dominates, while passive materials for low frequencies absorption are burdensome and become unrealisable.

One configuration widely studied is a single orifice or perforate plate backed with a mobile piston to tune the depth of the rear cavity [33, 64]. This technique can be seen as making resonators targeted to any frequency. This is equivalent to tune the acoustic impedance of a passive absorber in order to make it more reliable to some frequencies. These systems make use also of a reference signal. This signal, differently from purely active methods, can be the output noise. In this technique, the system tunes the depth of the cavity [64].

Hybrid systems can also be conceived to aim at some impedance value for the input noise. As impedance depends on frequency, the noise is analysed to know which are the most prominent frequencies. Some of these techniques rely on a single-perforation backed by variable-depth cavities [64] or on a perforated plate [11, 105] with piezoelectric actuators inside the cavities. Sometimes, the targeted value is a purely real impedance, but it can also achieve a prescribed value or a function of noise characteristics [10, 47, 105]. However, with the materials used as liners, not any value can be attained, and sub-optimal solutions are frequently used [11].

1.2 Acoustic liners

The passive materials that line ducts to mitigate noise must be adapted according to the situation, for example, in air conditioning systems, foams can be used as they are lightweight and cheap to produce. While in aeroengines, the material has to be as thin and lightweight as possible and

it must be tolerant to wear, to avoid damage to other components and to postpone as much as possible their replacement. They have to be reliable in flight conditions; either because of heat near the combustion, where ceramic materials are often used, or low temperatures in the intake, where usually structured metallic materials are placed.

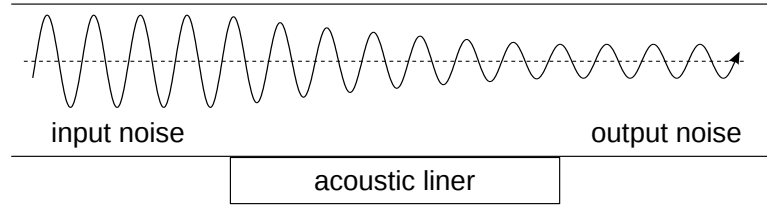


Figure 1.5 – Illustration of the placement of a liner.

In general, these materials may have sound propagation inside them by some sort of communication, as is the case of drainage slots [18, 106]. When this is the case, the liner is called non-locally reacting or bulk reacting, since its behaviour depends on local conditions of the field, but also on how sound is propagated within the material. As an example we can find porous materials [83, 55]. Nevertheless, in most of liners used nowadays we can neglect this and assume that they only allow sound propagation normal to the wall [109]. This way, modifications to sound field are point-wise. In this case we are talking of a locally reacting liner. In the rest of this work, unless otherwise specified, we will refer to this kind of liners, and we will omit to call them locally reacting.

In real-life applications, structured materials have small perforations that communicate the cavities for water drainage [18]. However, this is normally neglected in the analysis and they are assumed as locally reacting.

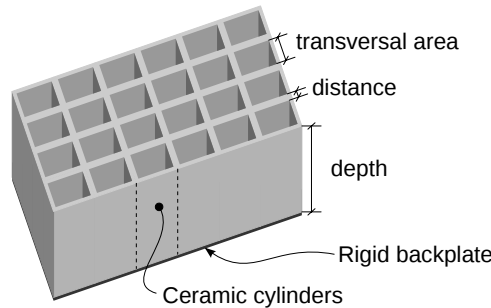


Figure 1.6 – Schema of a ceramic tubular liner.

These materials are usually made of arrays of resonant cavities, they can be either ceramic cylinders, as those shown in figure 1.6 or metal structured materials. The most used of these structured materials consist of a panel of honeycomb cells with a rigid back-plate topped with a perforated plate and sometimes also with a wiremesh; these materials are called single degree of freedom (SDOF for short). They are illustrated in figure 1.7. Liners with multiple layers of honeycomb panels, separated by perforated plates are also frequently used. When there are two layers, they are called double degree of freedom (DDOF). They are represented in figure 1.8.

The dissipation mechanism of these kind of materials is due to acoustic energy being dissipated by viscous friction because of the movement of air through the perforated plate. Besides, the

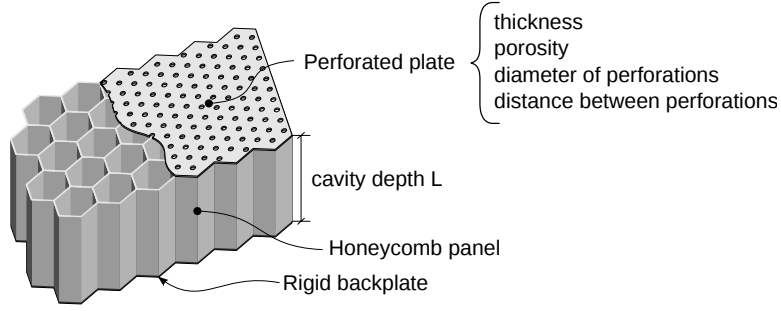


Figure 1.7 – Schema of a SDOF liner.

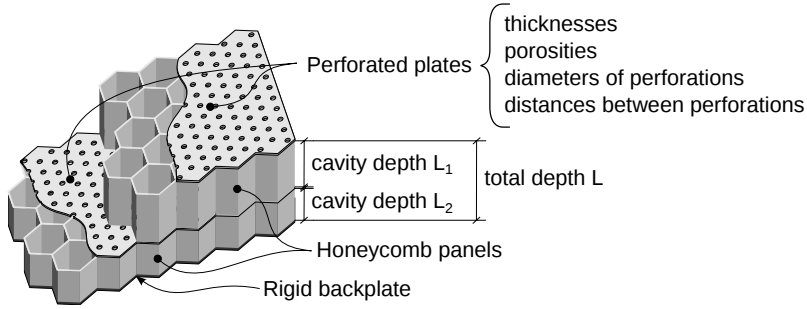


Figure 1.8 – Schema of a DDOF liner.

dimension of the cavities can induce destructive interference for some frequencies for which the maximal attenuation takes place.

1.2.1 Characteristics

To fully characterize a liner, each kind of liner has to be treated separately.

The parameters for ceramic tubular liners are the depth of cylinders and the surface porosity. This quantity can be obtained with the separation between cavities and their transverse area.

For a perforated plate we need to know its thickness, the size of holes and the porosity, defined as the proportion of perforated surface to total area. Studies made by Ingard [58] show a nonlinear behaviour at high amplitudes of the incident acoustic wave for single perforations. However, the interaction between holes has to be taken into account [111], since they are generally close enough so that they can not be considered under the assumption of widely separated holes.

For a SDOF material, besides the aforementioned properties of the perforated plate, there are also other parameters as the depth and volume of cavities. For DDOF materials in addition, we need to know the depth and volume of the second cavities and the total depth; as well as the characteristics of the plate separating the two layers [42].

As it can be induced, for non-locally reacting liners, there would be many other parameters depending likewise on their construction, but also on the full acoustic field and on the way sound is propagated within them [5].

1.2.2 Acoustic impedance

While all these parameters are required to characterize a material, the need of a simpler way to know its acoustic behaviour arises. Without the need to know the structure or material of a liner, it can globally be described by its acoustic impedance $\mathcal{Z}$. This quantity is defined as the quotient between acoustic pressure p and particle velocity $\underline{u}$ normal to the surface of the liner, oriented towards the liner itself; this direction is denoted by $\underline{n}$. In general, impedance can depend on both frequency and wavenumber [14], however, most of studies are made only taking into account the dependence on frequency. In the work presented here, when impedance has a dependence, this will be the path to follow.

$$\mathcal{Z}(\omega) = \frac{p(\omega)}{\underline{u}(\omega) \cdot \underline{n}} \quad (1.1)$$

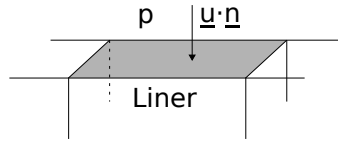


Figure 1.9 – Pressure and normal velocity incident to the liner.

It is a complex quantity whose real and imaginary parts are called acoustic resistance R and reactance χ respectively. Its multiplicative inverse is the acoustic admittance $\mathcal{Y}$:

$$\mathcal{Z} = R + i\chi = \mathcal{Y}^{-1} \quad (1.2)$$

Hereafter, to name these quantities we will omit the adjective “acoustic”.

The resistance is related to the energy flow to the wall; if R is positive, the wall absorbs energy, and the material is passive; in this case, the acoustic field should decay when propagating along a lined duct. If R is negative, the wall is active and produces energy, so there are acoustic sources beyond the interface. Because of this, to describe a passive material, this quantity needs to be positive.

A fluid has a characteristic impedance $\mathcal{Z}_0$, defined as the product of its mean density ρ_0 and the speed of sound c : $\mathcal{Z}_0 = \rho_0 c$. When the sound attacks an interface within this fluid, we use the specific impedance defined as $\frac{\mathcal{Z}}{\rho_0 c}$ [99]. It has the advantage to be a dimensionless quantity.

The reactance for a tubular liner is essentially determined by the depth while the resistance by the porosity of the surface. For honeycomb structured materials, the whole resistance of a liner depends mostly on the perforated plate meanwhile the dimensions of cavities chiefly determine the reactance of the liner.

1.2.3 Measurement

To determine the impedance of a material, several techniques are employed, according to the flow and the location of the material [74].

If we achieve to determine the impedance of a liner in the absence of flow, we can not be sure of the behaviour of the material under grazing flow. For instance, when a flow is present, we have

to account for the Doppler shift to know the actual frequency of sound at a fixed point, due to convection of sound by the mean flow. This shift moreover is not the same for all the canal with a shear flow [24, 21]. The acoustic proprieties of the lining change this in the presence of a flow. These variations are not easy to predict due to nonlinear interactions between the flow and the acoustic modes present in the surface of the liner [58, 73]. The effect of flow is more important for higher speeds [75]. Another way to determine impedance with flow has consequently, to be investigated.

1.2.3.1 Direct methods

For locally reacting liners, an ideal way to determine the impedance is to directly measure acoustic pressure and normal particle velocity at the wall. These quantities can be directly determined by different methods; some of them are exposed next [68].

For acoustic pressure, microphones are very commonly used since they are a low cost and reliable device. However, they can present some inconveniences for measurements with flow, specially if the flow is incoming normal to the membrane of the microphone, since the latter would remain in constant tension.

Velocity amplitude can be measured by a hot-wire anemometer, which consists on a fine metallic wire heated by an electric current. When air passes by, the cable temperature lowers thus changing its resistance with a calibration, these resistance variations determine the velocity [54]. This device can also be used in the presence of a grazing flow [6].

To determine the velocity in a given direction, the device called microflown or μ -flown can be used. It is based in the principle of a double hot wire with a small separation of tens of microns. The difference in resistance between the wires determines the velocity in the direction of the separation of both wires [40]. This device has been successfully used to determine the impedance of a material without a mean flow, by determining directly the particle velocity normal to the liner [115, 72, 67].

Optical methods may also be employed to determine the velocity of a flow. Laser Doppler Velocimetry (LDV) is based on the principle that a particle moving at some speed will act as a moving light source when illuminated. The frequency of emission of this moving source depends on its velocity and the frequencies of illumination. When light is monochromatic, the change in frequency can be known by the interference stripes pattern between the original source and the moving one. This change in frequency gives the information of velocity of the particle [124]. If we use two or three light sources, we can determine the corresponding number of particle velocity components [68]. This procedure is used to determine aerodynamic velocity. To use it for acoustic measurements, the flow speed component has to be rejected, this can be done by correlation with an acoustic signal [86].

Another optical method is the particle image velocimetry (PIV). This method follows the path of a set of particles in two consecutive images, taken with a known lapse. The displacement of particles determines the velocity field in one plane. To enlighten the particles, a laser sheet is often used [1]. Even this technique is used since the eighties, it has been used only in 2008 to determine an acoustic field in a turbulent flow, thanks to the work of Fischer [43]. This is done by calculating the velocity in two consecutive instants when the acoustic wave has undergone a half-period shift. This procedure requires the previous knowledge of frequency of the sound to synchronise measurements. This method has been successfully employed to characterise an acoustic liner [51].

It is not always possible to use these methods because microphones and hot-wires have to be installed at the same wall to determine its impedance and their effect could be not negligible in the behaviour of the flow and the acoustic field. For optical measurements, a transparent duct is obviously needed.

So, some other ways of determining the impedance of a material are used.

1.2.3.2 Impedance tube

The impedance tube is a device that serves to measure the impedance of a material, see figure 1.10. It is based on the relation of the impedance with the reflection coefficient r . The reflection coefficient is the proportion of an incident wave that is reflected back at an interface: if a sound wave of amplitude A arrives normal to an interface, there is a part of it that is back-reflected; this reflected wave has an amplitude equal to $A \cdot r$. This proportion depends on the impedance, if we had a perfectly rigid wall, with infinite impedance, the reflection would be total and $r = 1$; on the other hand, for a completely acoustic transparent wall, with impedance equal to the intrinsic of the medium $\rho_0 c$, there is no reflection and $r = 0$. The relation between the impedance and the reflection coefficient is in general:

$$\mathcal{Z} = \frac{\rho_0 c}{\cos(\vartheta)} \frac{1 + r}{1 - r} \quad (1.3)$$

Where ϑ is the incident angle. In an impedance tube, this angle ϑ is assumed to be zero [65]. Only plane waves are thus considered; this establishes a superior limit to frequency, based on duct size.

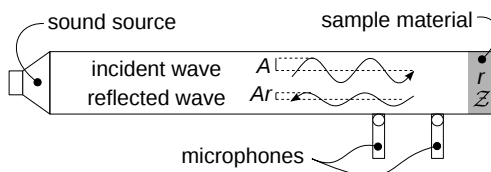


Figure 1.10 – Schema of an impedance tube.

Inverting relation (1.3) for normal incidence, leads to:

$$r = \frac{\mathcal{Z} - \rho_0 c}{\mathcal{Z} + \rho_0 c}$$

The role of the impedance in reflection already mentioned becomes evident:

$$\mathcal{Z} = \rho_0 c \Rightarrow r = 0$$

$$\mathcal{Z} \rightarrow \infty \Rightarrow r = 1.$$

In this device, impedance can be determined also with only one mobile microphone. There is also a way to measure impedance of a material in a tube with only one microphone placed at two different locations at different times. It has the advantage to avoid the simultaneity in measures, with the possible error of phase mismatching, at the small cost of using a periodic pseudo-random noise source [32].

As the impedance tube is a closed device, it is impossible to make measures of the impedance of materials with flow with it unless it undergoes some modifications, as explained below.

1.2.3.3 Impedance tube with flow

One technique to determine the impedance of a resistive layer in presence of tangential flow is with a modified impedance tube with a vein put behind the material, perpendicular to the tube [108]. This configuration is shown in figure 1.11.

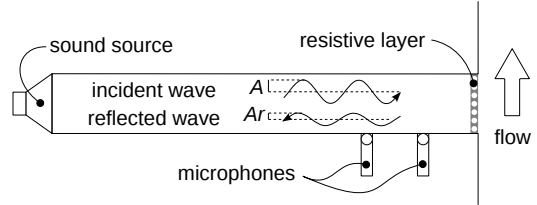


Figure 1.11 – Schema of an impedance tube with flow behind a resistive layer.

1.2.3.4 *In-situ* method

One way to measure the impedance in SDOF liners is the called *in-situ* method, which uses two microphones, one at the plate and the other at the bottom of resonating cavities [73] as shown in figure 1.12. Velocity can be obtained assuming normal modes shape of pressure and velocity and the relations between them as obtained from solutions of Euler equations 1.3.2. This method can be modified to measure velocity from one of the methods already exposed; this way can calculate directly the impedance by making and we don't need any more the knowledge of the relations between pressure and velocity.

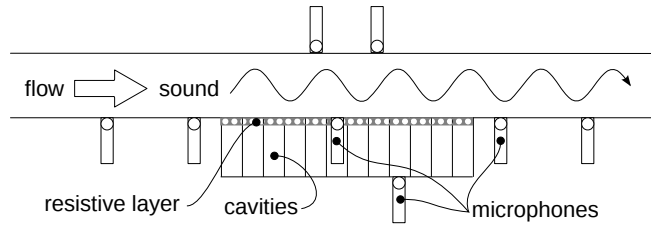


Figure 1.12 – Schema of the *in-situ* method to determine the impedance of a SDOF material.

One main bother of this method is that it is destructive for the material, since it has to be perforated to install the microphones. Another one is that it can not be used for porous materials.

1.2.3.5 Purely LDV method

The velocity can, for instance, be directly measured by LDV, which reduces notably the disturbance since only one microphone is needed at the plate. If we assume again the relations of acoustic variables, from Euler equations, pressure can be deduced from velocity to obtain the impedance by using only LDV [69, 87]. In this way, we proceed inversely compared to the *in-situ* method, where velocity is obtained from pressure.

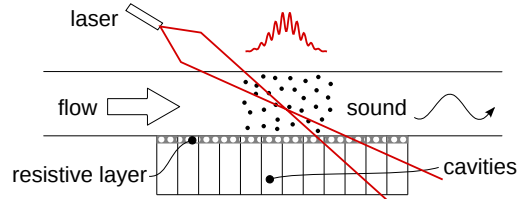


Figure 1.13 – LDV method to measure velocity in the center of the vein.

1.2.3.6 Indirect methods

There are also indirect methods of measurement based on the called “eduction method”, pioneered at NASA [118, 60, 61]. Here we will only give a brief overview of some of them.

The method developed at NASA consists in a finite element method for acoustic propagation with an inverse search for impedance. The search is based on minimising the difference between the signal measured and a signal predicted for a given value of impedance. This signal measured is the pressure at the opposite side of the liner, minimising the local effects of the microphones at the impedance wall. However, many microphones are needed, specially when we get far from resonant frequencies of the liner.

At Onera [90, 88] a similar method has been developed, but it uses as signal the particle velocity measured with LDV over the liner. This method has the advantage to obtain data just over the liner without modifying the local field.

Another variation to this method, used at LAUM and UTC, consists in modelling the propagation via a multimodal method [5, 110]. Its main advantage with respect to the previous one is that it needs less microphones and that by appealing to normal modes, the calculation of the propagation is simpler and, therefore, faster.

Some variation of this method include different quantities for the study. For example, at NLR and AleniaAermacchi they calculate insertion loss contours with a finite element method and then the impedance is back-calculated from the measured insertion loss [104]. Meanwhile, at DLR they assume very low reflections to calculate the transmission coefficient. From it, with an in-house algorithm they calculate the wavenumber of modes over the lined section of the duct [19]. From the wavenumber, knowing also the wavenumber of free field and the Mach number of flow, they compute directly the impedance.

These two methods, as well as the one developed at LAUM, use as signal the pressure detected from microphones but instead of placing microphones on the lined section, they are located upstream and downstream, since it is either attenuation or transmission what is needed to calculate the impedance.

1.2.4 Impedance models

Once determined the impedance of a material, to use this data within analytic or computational studies, a model to describe it in function of frequency is desirable.

The impedance of a material can be represented with complicated functions of frequency. In experimental, analytic and computational studies, it is very common to use a fixed frequency source, so it is important for the impedance model to use frequency as a variable.

There are also time-dependent formulations for impedance, but they are not of interest in this study, a good synthesis of frequency and time dependent models can be found in [96].

The conditions that must be met for an impedance model to be physically representative of a passive material, are condensed in the called *Theorem 1* from [96] which we show here:

$$\left. \begin{aligned} \mathcal{Z}(\omega) \text{ is analytic in } \text{Im}(\omega) < 0 \\ \mathcal{Z}(\omega) \neq 0 \text{ for } \text{Im}(\omega) < 0 \\ \mathcal{Z}(\omega) = \overline{\mathcal{Z}(-\omega)} \quad \forall \omega \in \mathbb{R} \\ \text{Re}(\mathcal{Z}(\omega)) \geq 0 \quad \forall \omega \in \mathbb{R} \end{aligned} \right\} \quad (1.4)$$

where the upper-bar $\overline{\mathcal{Z}}$ indicates the complex conjugate, Re and Im stand for the real and imaginary part respectively.

The first three conditions are required to comply with causality and the fourth comes from the need of the passive material to be unable to produce energy.

In following sections, we will introduce some of the models that are used to describe the impedance in liners.

1.2.4.1 Mass-spring-damper

This model consists in a frequency dependence of the impedance with an analogy of a damped mechanical resonator, described by the formula:

$$\mathcal{Z}(\omega) = R + i m \omega - i \frac{K}{\omega} \quad (1.5)$$

where R is identified with the damping, m with the mass and K with the springiness of the associated mechanical system.

This model fulfils the conditions given by (1.4), if all parameters are positive.

1.2.4.2 Helmholtz resonator

A material made up by an array of Helmholtz resonators is modelled by:

$$\mathcal{Z}(\omega) = \mathcal{Z}_L - i \cot\left(\frac{\omega L}{c}\right) \quad (1.6)$$

here, $\mathcal{Z}_L$ is the face-sheet impedance, normally modelled, as for the previous model, by $R + i \omega m$. The term $-\cot\left(\frac{\omega L}{c}\right)$ is the cavity reactance, with L the cell depth.

As for the MSD model, conditions (1.4) are achieved if all parameters are positive.

1.2.4.3 Double array Helmholtz resonator

A liner of type DDOF, when both layers behave as Helmholtz resonators, is described by [42]:

$$\mathcal{Z}(\omega) = \mathcal{Z}_1 + \frac{\mathcal{Z}_2 \frac{\cos(\omega L_1/c) \sin(\omega L_2/c)}{(\sin \omega L/c)} - i \cot(\omega L/c)}{1 + i \mathcal{Z}_2 \frac{\sin(\omega L_1/c) \sin(\omega L_2/c)}{(\sin \omega L/c)}} \quad (1.7)$$

where the depth of the cavities are: L_1 for the cavities at top, L_2 for those at bottom and their addition is the total depth $L = L_1 + L_2$. Also the impedance of the top resistive layer is $\mathcal{Z}_1$ and that of the layer separating the two layers of cavities is given by $\mathcal{Z}_2$. These impedances can be described as above, for the previous models as $\mathcal{Z}_n = R_n + im_n\omega$.

1.2.4.4 Extended Helmholtz resonator

The previous model can be generalized to be more realistic:

$$\mathcal{Z}(\omega) = \mathcal{Z}_L - i\beta_{\mathcal{Z}} \cot\left(\frac{\omega L}{c} - i\frac{\varepsilon_{\mathcal{Z}}}{2}\right) \quad (1.8)$$

where the introduced parameters $\beta_{\mathcal{Z}}$ and $\varepsilon_{\mathcal{Z}}$ represent respectively a scaling of the cavity reactance and the damping of the fluid within the cavity.

In terms of matching with the experiments, both the mass-spring-damper and the extended Helmholtz resonator are in good agreement with measured impedances with a slight advantage of the later [113, 94, 13].

1.2.4.5 Based on physical features

From the studies of the impedance of resonant cavities and perforated plates, several models of impedance have been obtained. They can be based either on theoretical developments or on empirical observations. This way, formal dependence on frequency, acoustic velocity or mean flow tangent to the interface have been obtained. A detailed study is made in the second chapter of the PhD thesis of Malmay [74].

For SDOF liners, these models use as parameters the physical attributes such as:

- ν : viscosity of the air
- e : covering plate thickness
- d : diameter of holes
- σ : porosity of the plate, i.e. portion of perforated surface

besides the already mentioned depth of cavity L .

One of these impedance models, given by Guess [50] recovers the general form of impedance of Helmholtz resonator (1.6), but for the face-sheet impedance it gives:

$$\mathcal{Z}_L = R_L + i\chi_L \quad (1.9)$$

where R_L , the layer resistance; and χ_L , the layer reactance are derived from physical features of the plate as follows:

$$R_L = \frac{\sqrt{8\nu\omega}}{\sigma c} \left(1 + \frac{e}{d}\right) + \frac{1}{8\sigma} \left(\frac{\omega d}{c}\right)^2$$

$$\chi_L = \frac{\omega}{\sigma c} \left[e + \frac{8d}{3\pi}(1 - 0.7\sqrt{\sigma}) + \sqrt{\frac{8\nu}{\omega}} \left(1 + \frac{e}{d}\right) \right]$$

This formulation does not account for a grazing flow. In case, a correction is done for both the resistance and the reactance [50].

$$R_L = \frac{\sqrt{8\nu\omega}}{\sigma c} \left(1 + \frac{e}{d}\right) + \frac{1}{8\sigma} \left(\frac{\omega d}{c}\right)^2 + \boxed{0.3 \frac{1 - \sigma^2}{\sigma} M}$$

$$\chi_L = \frac{\omega}{\sigma c} \left[e + \boxed{\frac{1}{1 + 305M^3}} \cdot \frac{8d}{3\pi} (1 - 0.7 \sqrt{\sigma}) + \sqrt{\frac{8\nu}{\omega}} \left(1 + \frac{e}{d}\right) \right]$$

This is only one of many models that exist to describe the liner based on its geometry. As these models are obtained from formal analysis, one would hope to apply them to many different liners. Unfortunately, there are some assumptions made on their development and construction. Besides, there are also some terms obtained semi-empirically. This entails these models to perform quite well only for some liners, related to those on which they were constructed. But unfortunately there exists not yet a global model to be applicable to all liners in order to predict their impedance for any frequency.

1.3 Model of duct aeroacoustics

Up to here, we have been concerned by the description of the absorbing material only, mainly in terms of its impedance, which is already a simplification. However, the whole acoustic field inside an aircraft aeroengine or any other real-world configuration of duct with flow depends on many other aspects. To describe the details of an aeroengine, from an acoustic point of view, we would need to acknowledge the rotation of the fan and the combustion in the turbine with the associated multiphysics phenomena. In present work, we will be interested neither in thermal nor in combustion nor in moving parts.

The physics of the duct has to be put in a model to recover most of the acoustic behaviour. From this, we do not take into account the fan or any other sound sources. We are only interested in the features of the sound field and not on how it is generated. From that, the study of aeroacoustics in lined ducts is based on a model which describes sound propagation of the acoustic field or on acoustic modal analysis.

Besides isolating the acoustics of the problem, the model has to overcome other simplifications to achieve an approximate description of a general system. This way, we do not pretend to get a detailed description of a limited system. Rather we obtain a model easier to solve, which permits to analyse a wider spectra of cases.

1.3.1 Geometry simplifications

With respect to the duct, the first simplification we have to account for is its geometry. To simplify the geometry of a whole turbofan engine, there are two main kinds of geometry used. Each one of them exposes advantages and inconveniences.

From these simplifications, the more realistic shape, in the sense of approximation to a real aeroengine, is a cylinder, either annular or not. It can be with constant or varying section [116, 93, 94]. While for numeric studies it does not pose any trouble, for experiments it is quite difficult to bend a test material to make it fit the cylindrical wall.

The other simplification is the use of a rectangular geometry. In this configuration, liners can be placed in one or more walls [75]. In fact, it uses to be placed only in a limited part of a wall. This shape is easier to use or compare with experiments, since it is much easier to find a slice of any material to make it fit in a flat wall.

1.3.1.1 B2A

At Onera, the Aero-thermo-Acoustic Bench, shorted B2A from french “Banc Aéro-thermo-Acoustique” is used to study these materials. It is shown in figure 1.14. It has a square 50 mm side section. Its testing cell has 150 mm long with transparent walls to be able to make LDV measurements. The sample holder is located below the testing cell. It covers a 30 mm wide central band and allows materials with a depth of up to 100 mm. These dimensions imply that the duct in the testing cell of B2A has all walls fully reflective but for the lower one which is lined in the central 30 mm.

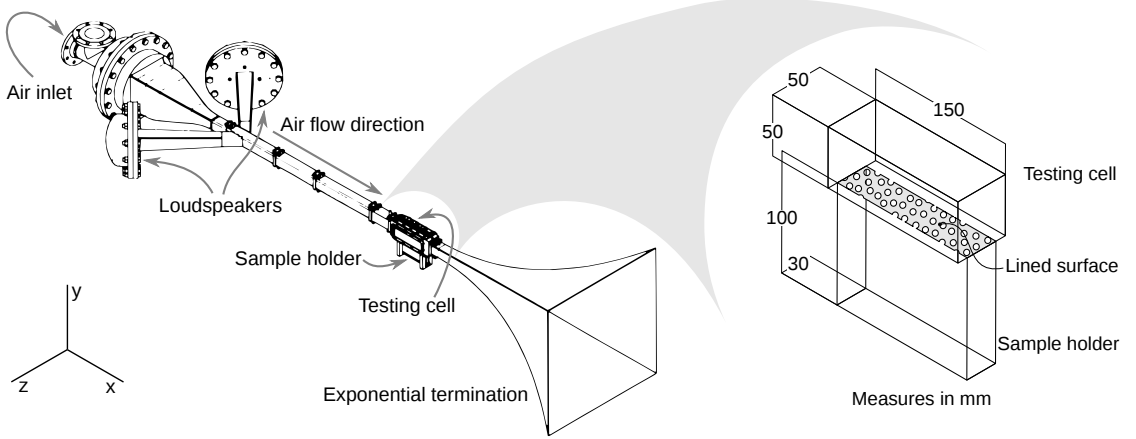


Figure 1.14 – Schema of the Aero-thermo-Acoustic Bench “B2A” showing its main components, with a close-up to the testing cell and sample holder with their dimension.

Another geometric simplification is related to the portion of walls covered by the absorbing material. Not all the inside of aeroengines are covered with liners. This renders the attenuation less effective. Besides that, discontinuities in impedance, between rigid and lined walls, introduce new families of modes that have also to be attenuated [92, 85]. Until recently [29], all of aeroengines had liners sectioned with rigid joints or splices between bands of lined material. This was the reason to develop the zero-splice liners.

1.3.2 Governing equations

Besides the geometry, the whole physics of the problem requires the equations that model the acoustic behaviour. In general, a physical system needs as much equations as variables. But, as it has been said, some of the later, as temperature, are not considered for the present purely aeroacoustic problem. The remaining variables need a system of equations to be solved. To compose this system, we assume an isentropic ideal gas, described by the equation of state (1.10) and the equation for entropy (1.11). The rest of the problem is characterised by the continuity

(1.12) and momentum equations, whose most general form are given by the Navier-Stokes equation (1.13) for fluid mechanics problems.

$$p = \rho RT \quad (1.10)$$

$$\frac{ds}{dt} = 0 \quad (1.11)$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0} \quad (1.12)$$

$$\boxed{\rho \frac{\partial \underline{u}}{\partial t} + \rho (\underline{u} \cdot \nabla \underline{u}) + \nabla p - \nabla \cdot \boldsymbol{\tau} = 0} \quad (1.13)$$

The last equation is homogeneous since the external forces as gravity are not taken into consideration. The viscous stress tensor is represented by $\boldsymbol{\tau}$. When the fluid is at local thermodynamical equilibrium, it is related to dynamic viscosity μ of the fluid by:

$$\boldsymbol{\tau} = \mu \left(\nabla \underline{u} + (\nabla \underline{u})^T \right) - \frac{2}{3} \mu (\nabla \cdot \underline{u}) \mathbf{I} \quad (1.14)$$

The super-index T indicates the tensor transposition. Equation (1.13) can often be simplified. For instance, for an incompressible fluid (1.13) becomes:

$$\rho \left(\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} \right) = -\nabla p + \mu \nabla^2 \underline{u} \quad (1.15)$$

On the other hand, since the acoustic field is practically unaffected by air viscosity, it is very common to neglect the term related to viscous stresses in equation (1.13). This gives the Euler equation for momentum:

$$\boxed{\rho \frac{\partial \underline{u}}{\partial t} + \rho (\underline{u} \cdot \nabla) \underline{u} = -\nabla p} \quad (1.16)$$

1.3.3 Small perturbation technique

To solve the acoustic field, physical quantities are assumed as mean values with a perturbation superimposed. It is a widespread procedure in aeroacoustics to solve the problem for mean values independently. Then, the characteristics of mean field can be considered as given parameters to calculate only on the perturbation field.

In our notation, perturbation is marked with a prime and the mean value with a zero sub-index:

$$q = q_0 + q' \quad (1.17)$$

for any quantity q .

The amplitude of mean values is of order of the unity, meanwhile the perturbation has an amplitude of order ε , with $\varepsilon \ll 1$. This condition is used to determine if a quantity can be negligible or comparable in magnitude to another one.

Equations (1.12), (1.13) and (1.16) can be linearized by expanding all variables as in (1.17) and neglecting terms of order greater than unity in ε , this will be used to solve them, as it will be exposed in chapter 2. What is really solved is thus the linear version of those equations.

Another linearized equation can be regarded when mean flow velocity is low and constant at all points. This way, variations of mean pressure and density can be neglected. This leads to describe the perturbation field with the use of the convected wave equation [52]:

$$\boxed{\frac{1}{c^2} \left(\frac{\partial}{\partial t} + \underline{u}_0 \cdot \nabla \right)^2 p' + \nabla^2 p' = 0} \quad (1.18)$$

1.3.4 Modal form

If the characteristics of the problem remain constant along the duct, the shape of the perturbation can be represented as a superposition of normal modes [95, 117, 84]. However, this idea can be extrapolated also to varying ducts [93, 21, 37]. To write these modes in duct aeroacoustics, we introduce the frequency ω and the axial wavenumber α . In the most general case, these two quantities are complex:

$$\omega = \omega_r + i \cdot \omega_i \quad , \quad \alpha = \alpha_r + i \cdot \alpha_i \quad (1.19)$$

We can introduce a modal form for the temporal dependence of the perturbation only as:

$$q' = \hat{q} e^{i(\omega t)} \quad (1.20)$$

where the amplitude term $\hat{q}$ includes the spatial dependence. With this, the convected wave equation (1.18) can be turned into the convected Helmholtz equation:

$$\boxed{\frac{1}{c^2} (i\omega + \underline{u}_0 \cdot \nabla)^2 p + \nabla^2 p = 0} \quad (1.21)$$

However, the full modal form that will be used to solve equations in this work writes:

$$q' = \hat{q} e^{i(\omega t - \alpha x)} \quad (1.22)$$

Here, as the longitudinal dependence is already given, the amplitude term $\hat{q}$ depends only on the transverse coordinates of the pipe, either (y, z) in rectangular or (r, ϑ) in cylindrical coordinate systems.

From a physical point of view, each mode represent a characteristic oscillatory movement of a system. In a duct, this means a characteristic wave sustained along it. From a mathematical point of view, modes correspond to the terms of a Fourier decomposition [66] of the acoustic field if either $\alpha_i = 0$ or $\omega_i = 0$.

1.3.5 Mean flow

Flow in an aeroengine has a complex, fully three-dimensional pattern. Close to the inlet, the boundary layer is very thin and the flow can be laminar. But in most of the engine duct, the

flow is turbulent and highly sheared [117, 76]. In experiments the flow is turbulent with fully developed boundary layers [60, 12], and when the duct is rectangular, corner vortices exist that make the flow three-dimensional.

However, when acoustic propagation in a flow duct is studied, such complexity is neglected and simplifications are made. First, the mean flow is supposed to go only on the longitudinal direction, with a dependence on the transverse coordinates only. The strongest simplification is neglecting all viscous effects on the mean flow, and thus considering a uniform or “plug” flow [113, 5]. Recently, people began to study acoustic propagation in a flow duct with a 2D sheared flow profile, modelled either with a parabolic shape [24] or a linear one [25] or piecewise linear [77]. The flow profile can also be fitted on measurements either by the means of analytic functions [60, 75], by a model profile [78] or with a hyperbolic tangent profile [100] which accounts for an almost uniform flow with a thin boundary layer close to the wall.

1.3.6 Boundary condition

The boundary of the system (Γ) to account for is divided in three zones. They are illustrated in figure 1.15 for a rectangular duct with the lower wall lined. Taking the notation already introduced in the definition of impedance (1.1), these zones are:

- ◀ At the rigid walls (Γ_0), normal movement of particles is prohibited, so acoustic velocity is imposed to zero:

$$\underline{u}'(\underline{r}) \cdot \underline{n} = 0 \quad \forall \underline{r} \in \Gamma_0 \quad (1.23)$$

- ◀ At the lined wall (Γ_Z), the normal velocity is linked to pressure by the definition of impedance:

$$Z \underline{u}'(\underline{r}) \cdot \underline{n} = p'(\underline{r}) \quad \forall \underline{r} \in \Gamma_Z \quad (1.24)$$

- ◀ At the input and output of flow (Γ_{io}), there must not be reflection for perturbations. As it was seen in subsection 1.2.2, it is equivalent to input an impedance equal to the characteristic one of the fluid:

$$\rho_0 c \underline{u}'(\underline{r}) \cdot \underline{n} = p'(\underline{r}) \quad \forall \underline{r} \in \Gamma_{io} \quad (1.25)$$

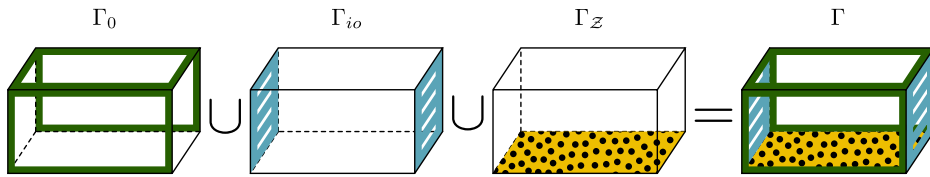


Figure 1.15 – Boundary of a rectangular duct (Γ) with bottom wall lined. Zones indicated: Γ_{io} input and output of flow. Γ_0 rigid walls. Γ_Z Lower wall lined.

These conditions are used for a fluid mechanics problem. This means that the boundary condition on a lined wall can only be written as in (1.24) if the grazing flow is zero at the wall, as it is when the mean flow is solution of Navier Stokes equations. Indeed, when we are solving the

linearized Euler equations, the mean flow profile can be taken as an input parameter which can be modelled either as a sheared flow with a no-slip condition at the lined wall, or as a uniform flow slipping at the wall. In that case, the acoustic boundary condition of the lined wall has to be modelled in a specific way.

1.3.6.1 Ingard - Myers boundary condition

The most used model for slipping mean flow profile is the Myers [82] or Ingard-Myers [57] boundary condition. This condition takes into account a vortex sheet close to the lined wall with characteristic width δ much smaller than the acoustic wavelength. But as sound doesn't distinguish between a finite boundary layer and a vortex sheet, it is taken the limit $\delta \rightarrow 0$ [97]. This condition allows continuity of particle displacement across the vorticity sheet. It is described by:

$$\omega \hat{v}(\Gamma_Z)Z = (\alpha u_0(\Gamma_Z) - \omega) \hat{p}(\Gamma_Z) \quad (1.26)$$

In the particular case of a sheared flow, where $u_0(\Gamma_Z) = 0$, the boundary condition (1.24) is recovered.

This boundary condition has, however, brought a serious problem for the analysis of stability of modes [13]. It allows an unbounded maximal exponential temporal growth for the solutions. The situation on which this problem is solved with an uniform flow and the Myers boundary condition is then called *ill-posed*.

Two boundary conditions have recently been developed to overcome this ill-posedness.

1.3.6.2 Rienstra boundary condition

Rienstra [97, 98], via an incompressible analysis, determined a value for a critical vortex sheet width δ_c . Below this value, the flow becomes absolutely unstable, while for higher δ it is either stable or convectively unstable. Rienstra gives an analytic approximation for δ_c for a mass-spring-damper model 1.2.4.1. This value is found to be very small.

In the same paper, Rienstra gives a shape that should have a boundary condition, called by him “corrected” or “regularised” Myers boundary condition. This condition uses δ as a parameter and recovers the original Myers boundary condition when $\delta \rightarrow 0$.

1.3.6.3 Brambley boundary condition

In a parallel study, Brambley [14] developed a similar formulation for a well-posed modified Myers boundary condition. This new condition is compared with a sheared flow with a hyperbolic tangent shape [100]. This condition uses also a boundary layer width δ that converts the condition in the original Myers boundary condition when it approaches zero.

These two conditions have been compared by Brambley [14] and Gabard [46]. They state that they are mainly similar, but with slight changes mostly due to the difference in compressibility and dimensions of the analysis.

1.4 Modes

According to the dimensions and boundary conditions of the domain, there is a whole family of modes that can exist. The modes that can be found as solutions of the linearized Euler equations, are typically acoustic modes. The description of the acoustic field in a flow in terms of modes is useful to discriminate relevant information that would be hidden within a complete solution of the acoustic field. This is useful, for example, when we are only interested in wavenumbers and we can avoid to calculate the shape of eigenfunctions.

1.4.1 Acoustic modes

To introduce acoustic modes, we write the system in one of the simplest cases. This is the case of a 2D duct with fully reflective bottom and upper walls and no mean flow. We will consider only one transverse coordinate y . This system is modelled by the Helmholtz equation with boundary conditions as those in (1.23):

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - \left(\frac{\omega}{c}\right)^2 p = 0 \quad (1.27)$$

These solutions are obtained by separation of variables as:

$$p(x, y, t) = \hat{p}_\beta e^{i\omega t} e^{-i\alpha x} e^{-i\beta y} \quad (1.28)$$

where we have introduced $\hat{p}_\beta$, the complex amplitude term and β , the wavenumber along y , which is obtained from the height of the duct H and the boundary conditions:

$$\beta_n = \frac{n\pi}{2H} \quad , \quad n = 0, 1, 2, 3, \dots \quad (1.29)$$

The n index is used to identify a mode. If we introduce the solution (1.28) into equation (1.27) we obtain the dispersion relation:

$$\alpha^2 + \beta^2 - \frac{\omega^2}{c^2} = 0 \quad \Rightarrow \quad \alpha = \pm \sqrt{\frac{\omega^2}{c^2} - n^2 \frac{\pi^2}{4H^2}} \quad (1.30)$$

If, for a real frequency and some value of β we have: $\omega^2 > c^2 \beta^2$, we obtain for α a couple of real values; one positive and one negative. These modes with a real axial wavenumber are propagative, they are called “cut-on” modes. They propagate without being damped along the duct, since $\alpha_i = 0$. The positive root corresponds to a right running mode, whereas the negative does it to a left running one. If we input this shape $\alpha = \alpha_r$ in the modal form (1.22) we obtain:

$$q' = \hat{q} e^{i(\omega t - \alpha_r x)} = \hat{q} (\cos(\omega t - \alpha_r x) + i \sin(\omega t - \alpha_r x)) \quad (1.31)$$

which are sustained waves propagating without damping. For β sufficiently large, we may have $\omega^2 < c^2 \beta^2$. Here the dispersion relation has two purely imaginary roots, again, in pairs of one positive and one negative. These modes are evanescent, they do not propagate and are called “cut-off” modes. As infinite values of n lead to such case, there are infinite cut-off solutions of the dispersion relation. Again, when input in the modal form, we obtain:

$$\begin{aligned} q' &= \hat{q} e^{i(\omega t - i\alpha_i x)} \\ &= \hat{q} e^{i\omega t + \alpha_i x} \\ &= \hat{q} e^{\alpha_i x} (\cos(\omega t) + i \sin(\omega t)) \\ &= \hat{q} e^{-|\alpha_i x|} (\cos(\omega t) + i \sin(\omega t)) \end{aligned} \quad (1.32)$$

which are waves with an amplitude decaying exponentially in either direction of x . We recall that, for these modes, the product $\alpha_i x$ has always a negative sign. This is because when x goes in the positive direction, α_i is negative and viceversa. This way, the value of frequency determines how many propagative modes there will be. However, corresponding to $\alpha = \pm\omega/c$ the mode for which the index n is zero, and then also β , will always be present. This mode is called plane wave, because of in the absence of a mean flow with hard walled duct, its wave front has a flat shape. In figure 1.16 is shown the spectrum obtained for a frequency $\omega = 3.5 \frac{H}{c}$ for a hard walled

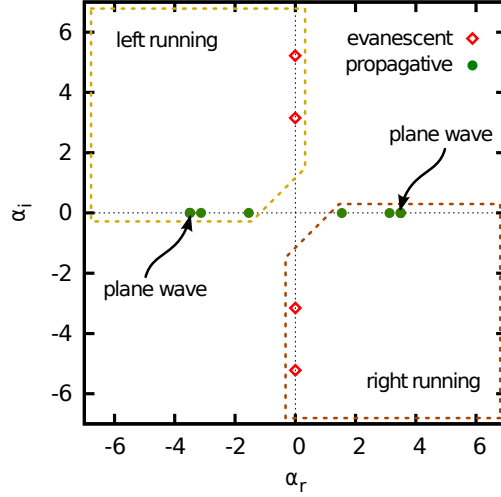


Figure 1.16 – Spectrum obtained for a duct with hard walls and no mean flow. Frequency $\omega = 3.5 \frac{H}{c}$. Different kinds of modes are marked.

duct without mean flow. It shows the different kind of modes in terms of sense of propagation and whether they are cut-on or cut-off modes, as well as the plane wave.

1.4.2 Surface modes

Besides the aforementioned acoustic modes, which are always present in ducts, there are also other families of modes that can appear. These are the surface modes, called this way because their amplitude is several orders of magnitude larger close to the wall and it diminishes exponentially while moving away [94].

For some values of impedance, two new family of modes can appear. The acoustic surface modes show up without the need of a grazing flow and its phase velocity is about the same as for the other acoustic modes. While the hydrodynamic surface modes need the presence of the mean flow and their phase velocity is considerably lower than those of the acoustic modes.

1.4.3 Hydrodynamic instability

In general, all modes are expected to reduce their amplitude while propagating [100]. However, in some cases this simply does not happen. This is, for example, the case of the acoustic propagative modes in non-dissipating media, when $\mathcal{Z}$ is either purely imaginary or infinity. This is the only case on which acoustic modes don't decay. If we input a finite impedance, all acoustic modes decay.

It can happen also that a hydrodynamic mode grows while propagating [94, 76], that means it becomes a hydrodynamic instability.

1.4.3.1 Experimental evidence

The evidence of this instability has been found in some configurations:

Meyer *et. al.* [79], made experiments with liners consisting of Helmholtz resonators or rockwool. In their results, they obtained for Helmholtz resonators an amplification of sound reported as negative attenuation. The behaviour of this sound amplification is really that of an instability, since the amplitude increases exponentially with distance. They ascribed this behaviour to the turbulent air stream on the duct. Later, Bauer [7] observed experimentally that the perforated plates of acoustic liners can generate noise, induced by the turbulent layer above them. He called that the liner self-noise. More recently, Aurégan and Marx [4, 76] found transmission coefficients larger than one, achieving values up to three. It has also been proved, that this increased noise is not classical whistling, which has a characteristic sharp peak spectrum. It is actually due to true sound amplification [4], found for frequencies slightly higher than the resonance frequency of the resonators which compose the liner.

This unstable behaviour is said to be linked to an aeroacoustical convective instability [76]. However, neither the setting off elements nor the precise conditions under which this instability is found are completely clear.

1.4.3.2 Evidence in numerical studies

In numerical studies, some unstable behaviour has also been found:

Studying the different kind of modes present in this configuration, Rienstra found that for some values of impedance, a hydrodynamic mode can become unstable [94]. However, his analysis is based on the already exposed Ingard - Myers boundary condition which permits unbounded growth. Some works mention the instability to be artificially introduced by the use of Myers boundary condition [100, 13]. However, this does not necessarily means that the instability does not exist. In later works, Rienstra recovered the analysis with a non-uniform, no-slip mean flow profile, thus using the full impedance boundary condition (1.24) and he obtained again the unstable behaviour [12, 100].

Marx *et. al.* [75] studied this instability also with a full profile; i.e. not a slipping plug flow. They compare their results with experimental data [76] and they obtain a good agreement in the magnitude of wavelength and growth rate. In a more recent work, [78], same authors include the effects of viscosity. On this study, they state that the instability found in inviscid analysis is of absolute kind while its behaviour changes with viscosity into a convective one.

Common circumstances that lead to an amplification of sound by the liner are: plane waves, low resistance liners and high Mach number [75, 100]. It has also been reported that the boundary layer must be very thin for this instability to occur [100]. Most of these conditions are met in aeronautical industry liners, but resistances are not that low. This fact explains that the instability has not yet been observed in industrial cases.

1.4.4 Group velocity

As it has been said, to distinguish between an instability and a decaying mode in opposite direction, the sense of propagation of the propagation has to be known. The group velocity is a value that can indicate this sense. It is defined as $v_g = \frac{\partial \omega}{\partial \alpha}$. This quantity has the clear meaning, in conservative media, of the velocity of a wave packet, when it has a real magnitude, also identified with the velocity of a signal or of energy propagation [16].

For dissipative media, group velocity becomes a complex quantity and it has not a clear meaning, it has even been mentioned as a mathematical artefact of purely abstract interest, without a physical meaning [39]. However, its real part has been identified with the velocity of energy propagation in some specific cases while its imaginary part is related with dissipation [49]. Its magnitude, or its real and imaginary parts can have some strange values as negative, infinite or zero, in neither of these cases, group velocity represents the velocity of the concerned mode [80].

These conclusions have emerged from studies in electromagnetic fields. However, we assume these statements to remain valid because of the similarities in equations and concepts between acoustic and optic waves. Also given that strange values for group velocity are found in both cases.

So, when a mode with a possible unstable behaviour is present, its propagation direction has to be determined by other means; as it will be explained hereafter.

1.4.5 Stability criterion

To verify the stability of modes, for deciphering their propagation direction, the Briggs-Bers [8] criterion has to be used. Developed originally to study the stability in plasmas, it is widely used nowadays in aeroacoustics.

This criterion consists in a double integral transform, Fourier in space and Laplace in time; to be applied, it needs one inversion contour for each transform.

The Laplace transform $t \rightarrow i\omega$ has to respect causality condition; for this to be done, we need a contour that splits the ω complex plane in two halves, the lower one without any mode and the upper one with all of them. To achieve this, the model needs that all temporal modes have a bounded growth, this means they have to be limited by their negative imaginary part. When a model fails to have a bounded growth, it is said to be “ill-posed”, in the sense used in section 1.3.6.1

The Fourier transform $x \rightarrow \alpha$ distinguishes the modes according to their propagation direction; to do so, the contour has to divide the α complex plane in such a way that all left-propagating modes remain in the upper part and all the right-propagating ones in the lower one.

To know the direction of propagation of a mode, the imaginary part of the frequency has to be increased from highly negative to zero; the modes that follow a trajectory coming from the lower half are right-propagating and those who come from up, go to the left.

If at the real frequency all modes remain in the same half-plane from which they came, the system is stable. If a mode crosses the real axis, it becomes convectively unstable. If, for some value of ω , two spatial modes from opposite sides of the real axis coincide, we are dealing with an absolute instability, then the long-time behaviour of the system is determined by this frequency and wavenumber, in both senses, upwards and downwards.

This review has shown the basis for the study of duct aeroacoustics. Several studies have been exposed, specially conducing to the identification of the hydrodynamic instability. These studies, however, up to author's knowledge, have only been done with one transverse independent variable, either the normal coordinate y for the rectangular geometries or the radial one r for cylindrical.

It is of interest to question the behaviour of such instability in a bidimensional analysis. This will be done in a rectangular geometry with coordinates y and z taken into account.

This will be analysed numerically with the help of two codes that will be presented in the next chapter.

Chapter 2

Modes calculation

In the present chapter we will introduce the calculation of modes for the problem of a lined duct with flow. Most part of it will be devoted to the description of two cases that will be recovered in the following chapters: on one hand, a bidimensional viscous analysis and, on the other hand, a three-dimensional inviscid study. Each case is modelled by a different set of equations from those exposed in section 1.3.2. Here, when we refer to a n -dimensional problem, we account for the dimension of the equations. The domain is actually of one less dimension, since the x coordinate dependence is always assumed in modal shape (1.22). This is why in the following, we can refer to the 2D case in (x, y) also as “unidimensional” and to the three-dimensional case as “transverse”, since the domain lies in the plane transverse to the main flow (y, z); which is aligned in the u direction.

We will start with the governing system of equations that has to be solved. This will put them in the shape that will be solved. Then we will show analytic results for some simple cases. In a third part, we will present the two codes used to find the modes, each specially aimed to one of the two cases mentioned above. Finally, we will show the influence of some parameters.

We have chosen a rectangular geometry to obtain results comparable to the experiments performed at Onera-DMAE. We consider only the bottom wall lined as in figure 1.15. We set it up this way also to match with the experimental configuration. This is a good approximation even that B2A 1.14 has not completely lined the lower wall. The main geometry used is shown in figure 2.1. It shows the coordinate system used, with the x axis along the duct, and y and z axes are the normal (height) and transverse (width) coordinates. The height of the duct is $2H$ and its width $2L$. From these quantities, we define the aspect ratio as the proportion of length to height: $\mathcal{A} = \frac{L}{H}$.

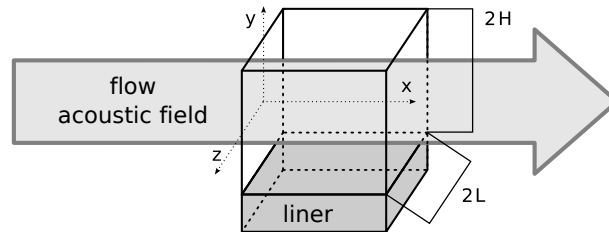


Figure 2.1 – Main geometry used for the study presented here.

2.1 Development of equations

The problem studied in this work can be described by either the linearized Navier-Stokes, if we consider a viscous fluid, or by the linearized Euler equations if we consider an inviscid flow. These equations are developed from their full form that was exposed in section 1.3.2.

2.1.1 2D linearized Navier-Stokes equations

To undertake the bidimensional viscous problem, described by the linearized Navier-Stokes equations (LNSE), the program MAMOUT was used. It will be presented in section 2.4. Here, we will show the development done by Jean-Philippe Brazier to the equations to get them as they are written in the code.

First, we rewrite continuity (1.12) and momentum (1.13) in x and y equations for this geometry,

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial v}{\partial y} + v \frac{\partial \rho}{\partial y} = 0 \quad (2.1)$$

$$\begin{aligned} \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= \frac{4}{3} \frac{\partial \mu}{\partial x} \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial \mu}{\partial x} \frac{\partial v}{\partial y} + \frac{4}{3} \mu \frac{\partial^2 u}{\partial x^2} + \frac{1}{3} \mu \frac{\partial^2 v}{\partial y \partial x} \\ &\quad - \frac{\partial p}{\partial x} + \frac{\partial \mu}{\partial y} \frac{\partial u}{\partial y} + \frac{\partial \mu}{\partial y} \frac{\partial v}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \end{aligned} \quad (2.2a)$$

$$\begin{aligned} \rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} &= \frac{4}{3} \frac{\partial \mu}{\partial y} \frac{\partial v}{\partial y} - \frac{2}{3} \frac{\partial \mu}{\partial y} \frac{\partial u}{\partial x} + \frac{4}{3} \mu \frac{\partial^2 v}{\partial y^2} + \frac{1}{3} \mu \frac{\partial^2 u}{\partial y \partial x} \\ &\quad - \frac{\partial p}{\partial y} + \frac{\partial \mu}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial \mu}{\partial x} \frac{\partial v}{\partial x} + \mu \frac{\partial^2 v}{\partial x^2} \end{aligned} \quad (2.2b)$$

Then, even if we will not deal with thermal effects in the presentation, the way to develop equations for MAMOUT considers the energy equation, whose full form is presented next:

$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} + \nabla \cdot (k \nabla T) + \mu \Phi \quad (2.3)$$

with:

$$\begin{aligned} \Phi &= 2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 + 2 \left(\frac{\partial w}{\partial z} \right)^2 \\ &\quad + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 - \frac{2}{3} (\nabla \cdot \underline{u})^2 \end{aligned} \quad (2.4)$$

This introduces new variables: T is the absolute temperature, h the enthalpy and k the thermal conductivity. Equation (2.3), written for the bidimensional (x, y) case gives:

$$\begin{aligned} \rho \frac{\partial h}{\partial t} + \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} &= \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + \frac{\partial k}{\partial x} \frac{\partial T}{\partial x} + k \frac{\partial^2 T}{\partial x^2} + \frac{\partial k}{\partial y} \frac{\partial T}{\partial y} + k \frac{\partial^2 T}{\partial y^2} \\ &\quad + \left(-\frac{2}{3} \mu \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2 \mu \frac{\partial u}{\partial x} \right) \frac{\partial u}{\partial x} + \mu \left(\frac{\partial u}{\partial y} \frac{\partial v}{\partial x} \right) \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\ &\quad + \left(-\frac{2}{3} \mu \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2 \mu \frac{\partial v}{\partial y} \right) \frac{\partial v}{\partial y} \end{aligned} \quad (2.5)$$

Temperature is related to enthalpy thanks to the hypothesis of an ideal ideal gas:

$$T = \frac{h}{C_p} \quad (2.6)$$

where C_p is the specific heat capacity at constant pressure. On the other hand, thermal conductivity k is cleared out in terms of viscosity by the definition of Prandtl number Pr :

$$Pr = \frac{\mu C_p}{k} \Rightarrow k = \frac{\mu C_p}{Pr} \quad (2.7)$$

The viscosity is assumed to follow Sutherland's law:

$$\mu = \mu_{ref} \sqrt{\frac{h}{h_{ref}}}$$

where μ_{ref} and h_{ref} are reference values. If we take both viscosity and enthalpy as a mean value with small perturbations, so they take the form:

$$\mu = \mu_0 + \mu' h = h_0 + h'$$

we can rewrite Sutherland's law as:

$$\mu_0 + \mu' = \mu_{ref} \sqrt{\frac{h_0 + h'}{h_{ref}}}$$

this shape can be expanded as a Taylor series about h_0 up to first order in ε :

$$\mu_0 + \mu' = \mu_{ref} \sqrt{\frac{h_0}{h_{ref}}} + h' \frac{\mu_{ref}}{2} \frac{1}{\sqrt{h_0 h_{ref}}}$$

As this is valid for any value of h' , in particular for $h' = 0$ we have:

$$\mu_0 = \mu_{ref} \sqrt{\frac{h_0}{h_{ref}}} \Rightarrow \mu' = h' \frac{\mu_{ref}}{2} \frac{1}{\sqrt{h_0 h_{ref}}}$$

If we define $K(y) = \frac{\mu_{ref}}{2} \frac{1}{\sqrt{h_0 h_{ref}}}$, we can write:

$$\mu' = h' K(y) \quad (2.8a)$$

$$\frac{\partial \mu_0}{\partial y} = K(y) \frac{\partial h_0}{\partial y} \quad (2.8b)$$

Next we take the shape of the mean flow velocity to be only in x direction and with y dependence:

$$\underline{u}_0 = (u_0(y), 0) \quad (2.9)$$

After substitution of (2.6), (2.7), (2.8) and (2.9) in equations (2.1), (2.2) and (2.5), and neglecting terms of $\mathcal{O}(\varepsilon^2)$ we obtain:

$$0 = \frac{\partial \rho'}{\partial t} + u_0 \frac{\partial \rho'}{\partial x} + \rho_0 \frac{\partial u'}{\partial x} + v' \frac{\partial \rho_0}{\partial y} + \rho_0 \frac{\partial v'}{\partial y} \quad (2.10a)$$

$$\begin{aligned} \rho_0 \frac{\partial u'}{\partial t} + \rho_0 u_0 \frac{\partial u'}{\partial x} + \rho_0 v' \frac{\partial u_0}{\partial y} &= \mu_0 \frac{\partial^2 u'}{\partial y^2} + \frac{4}{3} \mu_0 \frac{\partial^2 u'}{\partial x^2} + \frac{1}{3} \mu_0 \frac{\partial^2 v'}{\partial y \partial x} \\ &+ \left(h' \frac{dK(y)}{dy} + K(y) \frac{\partial h'}{\partial y} \right) \frac{\partial u_0}{\partial y} \\ &- \frac{\partial p'}{\partial x} + \frac{\partial v'}{\partial x} \frac{\partial \mu_0}{\partial y} + \frac{\partial u'}{\partial y} \frac{\partial \mu_0}{\partial y} + K(y) h' \frac{\partial^2 u_0}{\partial y^2} \end{aligned} \quad (2.10b)$$

$$\begin{aligned} \rho_0 \frac{\partial v'}{\partial t} + \rho_0 u_0 \frac{\partial v'}{\partial x} &= \mu_0 \frac{\partial^2 v'}{\partial y^2} + \frac{4}{3} \mu_0 \frac{\partial^2 v'}{\partial y^2} + \frac{1}{3} \mu_0 \frac{\partial^2 u'}{\partial y \partial x} + K(y) \frac{\partial h'}{\partial x} \frac{\partial u_0}{\partial y} - \frac{\partial p'}{\partial y} \\ &+ \frac{4}{3} \frac{\partial v'}{\partial y} \frac{\partial \mu_0}{\partial y} - \frac{2}{3} \frac{\partial u'}{\partial x} \frac{\partial \mu_0}{\partial y} \end{aligned} \quad (2.10c)$$

$$\begin{aligned} \rho_0 \frac{\partial h'}{\partial t} + \rho_0 u_0 \frac{\partial h'}{\partial x} + \rho_0 v' \frac{\partial h_0}{\partial y} &= \frac{\mu_0}{Pr} \frac{\partial^2 h'}{\partial y^2} + K(y) h' \left(\frac{\partial u_0}{\partial y} \right)^2 + \frac{\partial p'}{\partial t} + \frac{1}{Pr} \frac{\partial \mu_0}{\partial y} \frac{\partial h'}{\partial y} \\ &+ 2\mu_0 \frac{\partial v'}{\partial x} \frac{\partial u_0}{\partial y} + \frac{\mu_0}{Pr} \frac{\partial^2 h'}{\partial x^2} + 2\mu_0 \frac{\partial u_0}{\partial y} \frac{\partial u'}{\partial y} + \frac{K(y) h'}{Pr} \frac{\partial^2 h_0}{\partial y^2} \\ &+ \frac{1}{Pr} \left(h' \frac{dK(y)}{dy} + K(y) \frac{\partial h'}{\partial y} \right) \frac{\partial h_0}{\partial y} + u_0 \frac{\partial p'}{\partial x} \end{aligned} \quad (2.10d)$$

Meanwhile, the equation of state for mean values reads:

$$h_0 = \frac{p_0 C_p}{\rho_0 R}$$

which, for perturbations and clearing for ρ' gives:

$$\rho' = \frac{\rho_0 p'}{p_0} - \frac{\rho_0 h'}{h_0}$$

We assume now a modal shape (1.22) for the four remaining variables (u, v, p, h) with a small abuse of notation by using the same representation for the amplitude of modes as it was for the full variable. In following equations, q will refer to the modal amplitudes.

Introducing the modal shape into equations (2.10) we obtain the system of equations coded

into MAMOUT:

$$0 = -i\alpha\rho_0 u + \rho_0 \frac{\partial v}{\partial y} + \frac{\partial \rho_0}{\partial y} v + i(\omega - u_0\alpha) \frac{\rho_0}{p_0} p - i(\omega - u_0\alpha) \frac{\rho_0}{h_0} h \quad (2.11a)$$

$$0 = -\mu_0 \frac{\partial^2 u}{\partial y^2} - \frac{\partial \mu_0}{\partial y} \frac{\partial u}{\partial y} + \left(i(\omega - u_0\alpha)\rho_0 + \frac{4}{3}\alpha^2\mu_0 \right) u + \frac{i\alpha\mu_0}{3} \frac{\partial v}{\partial y} + \left(\rho_0 \frac{\partial u_0}{\partial y} + i\alpha \frac{\partial \mu_0}{\partial y} \right) v - i\alpha p - \frac{\partial}{\partial y} \left(K(y) \frac{\partial u_0}{\partial y} h \right) \quad (2.11b)$$

$$0 = \frac{1}{3}i\alpha\mu_0 \frac{\partial u}{\partial y} - \frac{2}{3}i\alpha \frac{\partial \mu_0}{\partial y} u + \frac{4}{3}\mu_0 \frac{\partial^2 v}{\partial y^2} + (i(\omega - \alpha u_0)\rho_0 + \alpha^2\mu_0) v - \frac{4}{3} \frac{\partial \mu_0}{\partial y} \frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} - i\alpha K(y) \frac{\partial u_0}{\partial y} h \quad (2.11c)$$

$$0 = -2\mu_0 \frac{\partial u_0}{\partial y} \frac{\partial u}{\partial y} + \left(\rho_0 \frac{\partial h_0}{\partial y} + 2i\alpha\mu_0 \frac{\partial u_0}{\partial y} \right) v - i(\omega - u_0\alpha)p - \frac{\mu_0}{Pr} \frac{\partial^2 h}{\partial y^2} - \frac{2}{Pr} \frac{\partial \mu_0}{\partial y} \frac{\partial h}{\partial y} + \left(i(\omega - u_0\alpha)\rho_0 - K(y) \left(\frac{\partial u_0}{\partial y} \right)^2 + \frac{1}{Pr} \left(\alpha^2\mu_0 - \frac{\partial^2 \mu_0}{\partial y^2} \right) \right) h \quad (2.11d)$$

If we assume constant mean values for viscosity, density and enthalpy, previous system of equations (2.11a), (2.11b), (2.11c) and (2.11d) can be condensed into one matricial equation, using $\omega^* = (\omega - u_0\alpha)$:

$$\begin{pmatrix} -i\alpha\rho_0 & \rho_0 D_y & i\omega^* \frac{\rho_0}{p_0} & -i\omega^* \frac{\rho_0}{h_0} \\ -\mu_0 D_y^2 + i\omega^* \rho_0 + \frac{4}{3}\alpha^2\mu_0 & \frac{i\alpha\mu_0}{3} D_y + \rho_0 u_0^y & -i\alpha & -K u_0^y D_y - K u_0^{yy} \\ \frac{1}{3}i\alpha\mu_0 D_y & \frac{4}{3}\mu_0 D_y^2 + i\omega^* \rho_0 + \alpha^2\mu_0 & D_y & -i\alpha K D_y \\ -2\mu_0 u_0^y D_y & 2i\alpha\mu_0 u_0^y & -i\omega^* & -\frac{\mu_0}{Pr} D_y^2 + i\omega^* \rho_0 - K (u_0^y)^2 + \frac{\alpha^2\mu_0}{Pr} \end{pmatrix} \begin{pmatrix} u \\ v \\ p \\ h \end{pmatrix} = 0 \quad (2.12)$$

Here we note the derivative operator with respect to y as D_y and the second derivative as D_y^2 . The derivatives of the mean flow $\frac{\partial u_0}{\partial y}$ and $\frac{\partial^2 u_0}{\partial y^2}$ are written as u_0^y and u_0^{yy} . This is made because mean flow velocity profile and its derivatives are considered as given parameters for the calculation of perturbations.

2.1.2 3D Linearized Euler equations

The inviscid case, described by the linearized Euler equations (LEE), was studied using both programs. On one hand, MAMOUT was used, mostly to compare some cases with the viscous study to evaluate the effects of viscosity, in this case, equations were only developed for a 2D study. On the other hand, the code FiEstA was developed in order to make a global or bi-global stability analysis with Euler equations. In this work, bi-global makes reference to a global stability analysis in a bidimensional spatial domain. The program itself will be presented in section 2.5. Next we will show the development done to Euler equations in order to obtain the shape on which they are implemented in FiEstA. As it was exposed in sections 1.3.3 and 1.3.4, convected Helmholtz equation and convected wave equation consider a mean flow profile constant at all points. Since we are interested in a full flow profile, we choose LEE in favour of said equations for the inviscid study. Now we will develop Euler equations to obtain the linearized form. Up to here, we worked with physical dimensional quantities. A different way to proceed, very extended

in many fields of physics, is to take dimensionless variables. This has some advantages: the main one is that some variables that would have very different order of magnitude in some system of units, end with comparable sizes. This can be useful in particular for numerical calculations, since computers store numbers in a fixed size mantissa and dealing with numbers of very different orders of magnitude can lead to a lost of information. This is also useful when comparing two set-ups with the same shape but different sizes. Even if the input data and results of the programs can be given in dimensional quantities, the codes themselves work in nondimensionalised units. Here we will expose the way the nondimensionalisation was done into FiEstA to develop LEE. To work with nondimensional variables, we have to chose a set of physical quantities as reference. The variables of the problem described by LEE have unities that are written in terms of three basic physical dimensions [59]: length, time and mass. This implies that we have to choose three quantities with some combination of these dimensions to render the system nondimensional. The variables chosen are the speed of sound in the medium c , the mean density of the fluid ρ_0 and the half height of the duct H . We take the half height since our coordinate system has the origin centred in the duct, this way the domain is bounded for $y \in (-1, 1)$ and for $z \in (-\mathcal{A}, \mathcal{A})$. Whith $\mathcal{A}$ the aspect ratio that. This choice of variables, already found in literature [75], has several advantages, since it is based in well known characteristics of the problem and it has a simple way to recover variables in physical dimensions. The variables to render nondimensional in this study, besides the spatial coordinates (x, y, z) and time (t) , are the velocity $\underline{u} = (u, v, w)$, the pressure p and the density of the flow ρ . The physically dimensional variables are notated with a tilde over them ($\tilde{q}$), while the nondimensional ones are written simply (q). By a dimensional analysis, the latter can be written in terms of the former this way:

$$(x, y, z) = \frac{1}{H} (\tilde{x}, \tilde{y}, \tilde{z}) \quad , \quad (u, v, w) = \frac{1}{c} (\tilde{u}, \tilde{v}, \tilde{w}) \quad , \quad p = \frac{\tilde{p}}{\rho_0 c^2} \quad , \quad t = \frac{c}{H} \tilde{t} \quad , \quad \rho = \frac{\tilde{\rho}}{\rho_0} \quad (2.13)$$

Needless to way that here we have taken again the q nomenclature for the full nondimensional variable. Nondimensional frequency and wavenumbers are then rewritten as:

$$\omega = \frac{H}{c} \tilde{\omega} \quad , \quad \alpha = H \tilde{\alpha} \quad , \quad \beta = H \tilde{\beta} \quad , \quad \gamma = H \tilde{\gamma} \quad (2.14)$$

Here, in addition to the already known α and β , we use γ , the wavenumber along z . From these three wavenumbers, we can define the wavevector $\underline{k}$ as (α, β, γ) . With previous relations (2.13) and (2.14), we can recover any variable from the nondimensional form to the dimensional one. We continue next with the linearization of equations, using the nondimensionalised units. To do so, we start with the description of sound speed for an isentropic ideal fluid [99]:

$$c^2 = \frac{\partial p}{\partial \rho} \quad (2.15)$$

which in reduced units and after linearization, is a proportion relation for density and pressure fluctuations. We use it to obtain:

$$p' = \rho' \quad (2.16)$$

Which allows us to reduce one variable in the system.

Introducing this relation into the continuity equation (1.12), with the linearization and the modal form, we obtain:

$$i\omega p' - i\alpha u_0 p' - i\alpha u' + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \quad (2.17)$$

Proceeding in a similar way with the components of Euler equation for momentum (1.16) we get:

$$i\omega u' - u_0 i\alpha u' + \frac{\partial u_0}{\partial y} v' + \frac{\partial u_0}{\partial z} w' = i\alpha p' \quad (2.18a)$$

$$i\omega v' - u_0 i\alpha v' = -\frac{\partial p'}{\partial y} \quad (2.18b)$$

$$i\omega w' - u_0 i\alpha w' = -\frac{\partial p'}{\partial z} \quad (2.18c)$$

All terms in equations (2.17) and (2.18) include one of the fluctuation variables (u', v', w', p') . As these four variables are considered in modal form (1.22), they are proportional to $e^{i(\omega t - \alpha x)}$, so we can take out the exponential terms from the equations, leading to an identical system, but with a replacement of variables from q' to their y and z amplitude term. As it was done before in section 2.1.1, this term will next be simply called q . We can regroup the resulting equations into a single matrix equation:

$$\begin{pmatrix} i\omega - i\alpha u_0 & u_0^y & u_0^z & -i\alpha \\ 0 & i\omega - i\alpha u_0 & 0 & D_y \\ 0 & 0 & i\omega - i\alpha u_0 & D_z \\ -i\alpha & D_y & D_z & i\omega - i\alpha u_0 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \\ p \end{pmatrix} = 0 \quad (2.19)$$

Here, similar to what we did for LNSE, we note the derivative operators D_y and D_z for y and z derivatives, respectively. Also, the derivatives of the mean flow $\frac{\partial u_0}{\partial y}$ are written as u_0^y and $\frac{\partial u_0}{\partial z}$ as u_0^z . This is done, again, because in these equations mean flow and its derivatives are parameters to solve the perturbation field.

2.1.3 Generalized eigenvalue problem

This problem described by the LEE or LNSE will be tackled via the solution of a discretised generalized eigenvalue problem [94] of the shape:

$$\mathbf{A}\varphi = \lambda \mathbf{B}\varphi \quad (2.20)$$

where $\mathbf{A}$ and $\mathbf{B}$ are matrices, λ the eigenvalue and $\varphi = (u \ v \ w \ p)^T$ the eigenfunction. Each component of φ is a vector with the values of the variable at the chosen collocation points.

To solve the equations as a generalized eigenvalue problem, there are two possible ways to proceed: one is a temporal analysis, for which a real wavenumber α is fixed and we search for a spectrum in complex frequencies. One problem of this case arises if a frequency model of the impedance is used, since the boundary condition has to be defined for the statement of the problem. But as it depends on the eigenvalues, this leads to a non-standard eigenvalue problem. However, if impedance is a fixed value a temporal analysis is possible.

This kind of analysis will be used in chapter 5 to validate the well-posedness of the stability criterion. From the programs used in this thesis, up to now, only MAMOUT is able to perform this kind of calculation.

The other way to proceed is a spatial analysis, for which the real frequency is fixed, while the complex wavenumber is sought as of eigenvalues. This way, even in if the impedance has a frequency dependence, it remains fixed and we can recover a generalized eigenvalue problem.

This analysis is also supported by experiments in which a single frequency sound is fed to the loudspeakers [75, 76]. This is the kind of analysis for which we will develop the system.

The full spatial generalized eigenvalue problem for LEE of system (2.19) is then written as the general system that will be treated in this work:

$$\begin{pmatrix} i\omega & u_0^y & u_0^z & 0 \\ 0 & i\omega & 0 & D_y \\ 0 & 0 & i\omega & D_z \\ 0 & D_y & D_z & i\omega \end{pmatrix} \begin{pmatrix} u \\ v \\ w \\ p \end{pmatrix} = \alpha \begin{pmatrix} iu_0 & 0 & 0 & i \\ 0 & iu_0 & 0 & 0 \\ 0 & 0 & iu_0 & 0 \\ i & 0 & 0 & iu_0 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \\ p \end{pmatrix} \quad (2.21)$$

To write LNSE as a spatial eigenvalue problem, either the full version (2.11) or the simplified version (2.12), it can be written as:

$$\mathbf{A}\varphi + \lambda\mathbf{B}\varphi + \lambda^2\mathbf{C}\varphi = 0$$

Note the sign change in $\mathbf{B}$ with respect to (2.20). We have to deal with the squared alpha and rewrite the system to put it in the shape of (2.20). To do so, we count on an auxiliary variable: $\psi = (\lambda\varphi, \varphi)^T$ and we write the system as:

$$\begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \psi + \lambda \begin{pmatrix} \mathbf{B} & \mathbf{A} \\ -\mathbf{1} & \mathbf{0} \end{pmatrix} \psi = 0 \quad (2.22)$$

where, for the simplified system (2.12):

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} 0 & \rho_0 D_y & i\omega \frac{\rho_0}{p_0} & -i\omega \frac{\rho_0}{h_0} \\ -\mu_0 D_y^2 + i\omega \rho_0 & \rho_0 u_0^y & 0 & -K u_0^y D_y - K u_0^{yy} \\ 0 & \frac{4}{3}\mu_0 D_y^2 + i\omega \rho_0 & D_y & 0 \\ -2\mu_0 u_0^y D_y & 0 & -i\omega & -\frac{\mu_0}{Pr} D_y^2 + i\omega \rho_0 - K (u_0^y)^2 \end{pmatrix} \\ \mathbf{B} &= \begin{pmatrix} -i\rho_0 & 0 & iu_0 \frac{\rho_0}{p_0} & -iu_0 \frac{\rho_0}{h_0} \\ iu_0 \rho_0 & \frac{i\mu_0}{3} D_y & -i & 0 \\ \frac{1}{3}i\mu_0 D_y & iu_0 \rho_0 & 0 & -iK D_y \\ 0 & 2i\mu_0 u_0^y & -iu_0 & iu_0 \rho_0 \end{pmatrix} \\ \mathbf{C} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{4}{3}\mu_0 & 0 & 0 & 0 \\ 0 & \mu_0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mu_0}{Pr} \end{pmatrix} \end{aligned}$$

In next parts, we will develop the approach only for LEE equations, since for the LNSE this can be done in a similar way, although being more laborious.

To be introduced into the codes, the eigenvalue system is discretized using the collocation points that will be introduced in section 2.3.1.

2.1.3.1 Boundary condition

Boundary conditions (1.24) and (1.23) are rewritten in this domain as:

$$\mathcal{Z}v(-1, z) = p(-1, z) \quad (2.23a)$$

$$v(1, z) = 0 \quad (2.23b)$$

$$w(y, -\mathcal{A}) = 0 \quad (2.23c)$$

$$w(y, \mathcal{A}) = 0 \quad (2.23d)$$

These conditions have to be introduced in the corresponding location of the discretized system (2.21). This means that for the system (2.20), the entry corresponding to the point $(-1, z)$ should read in the equation (2.23a), and so on for the remaining conditions.

2.1.3.2 Shooting method

To follow the behaviour of a particular mode, a shooting method was also included in the program. This method requires to proceed with the equations in a slightly different way.

As mean flow is already known, equation (2.18a) is a purely algebraic equation. This is used to solve u in terms of v , w and p :

$$u = \frac{\alpha p + iu_0^y v + iu_0^z w}{\omega - \alpha u_0} \quad (2.24)$$

after substitution of (2.24) into system (2.19) we obtain:

$$\begin{pmatrix} D_y + \frac{\alpha u_0^y}{\omega - \alpha u_0} & D_z + \frac{\alpha u_0^z}{\omega - \alpha u_0} & i\omega - i\alpha u_0 + \frac{\alpha^2}{\omega - \alpha u_0} \\ i\omega - i\alpha u_0 & 0 & D_y \\ 0 & i\omega - i\alpha u_0 & D_z \end{pmatrix} \begin{pmatrix} v \\ w \\ p \end{pmatrix} = 0 \quad (2.25)$$

Once discretised, this system requires to be inhomogeneous in order to be solved as a linear system such as $A\underline{x} = \underline{b}$. To achieve this, the system is modified to introduce the normalization condition $p(1, \mathcal{A}) = 1$, actually assuming that the pressure fluctuation is not zero at the chosen location $(1, \mathcal{A})$. Meanwhile, the condition to shoot for is $v(1, 1) = 0$, required by the boundary conditions. In practice, the system is solved iteratively as the eigenvalue is searched in such a way that $v(1, 1) = 0$.

2.2 Some analytic solutions

System (2.21) with boundary conditions (2.23), does not always have explicit solutions. However, for some simple cases, solutions are easily obtained.

2.2.1 Hard wall, no flow

The easiest way to set the problem has already been discussed in section 1.4.1 while introducing the acoustic modes in two dimensions. We recall that this case consists in a hard-walled duct, without mean flow. Here, modes are those of any rectangular homogeneous medium with fully

reflective boundary wall. These modes are easily obtained from system (2.21) setting u_0 , u_0^y and u_0^z equal to zero and the boundary condition (2.23a) simply becoming $\hat{v}(-1, z) = 0$, we obtain:

$$\left[\begin{pmatrix} i\omega & 0 & 0 & 0 \\ 0 & i\omega & 0 & D_y \\ 0 & 0 & i\omega & D_z \\ 0 & D_y & D_z & i\omega \end{pmatrix} - \alpha \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} u \\ v \\ w \\ p \end{pmatrix} = 0 \quad (2.26)$$

From the first three equations, we write u , $D_y v$ and $D_z w$ in terms of p and we introduce them into the last equation to obtain:

$$D_y^2 p + D_z^2 p + \omega^2 p - \alpha^2 p = 0 \quad (2.27)$$

which is a normalised Helmholtz equation for the modal form, i.e., equation (1.21) with $c = 1$, $\underline{u}_0 = 0$ and introducing (1.22) for the x dependence. Its solution, obtained by separation of variables, is:

$$p(y, z) = p_{\beta\gamma} e^{i\beta y} e^{i\gamma z} \quad (2.28)$$

where $p_{\beta\gamma}$ is a complex amplitude term. The wavenumbers β and γ are obtained from physical domain dimensions and boundary conditions as:

$$\beta_n = \frac{n\pi}{2}, \quad \gamma_m = \frac{m\pi}{2\mathcal{A}}, \quad n, m = 0, 1, 2, 3, \dots \quad (2.29)$$

In this case, the couple (n, m) of indices are needed to identify a mode. For this case, the dispersion relation is written as:

$$\alpha^2 + \beta^2 + \gamma^2 - \omega^2 = 0 \quad \Rightarrow \quad \alpha_{n,m} = \pm \sqrt{\omega^2 - \frac{n^2\pi^2}{4} - \frac{m^2\pi^2}{4\mathcal{A}^2}} \quad (2.30)$$

It can be seen that the solution for bidimensional acoustic modes shown in section 1.4.1 can be easily recovered by lowering one dimension. This is achieved simply by setting $m = 0$.

2.2.2 Hard wall, uniform flow

With the introduction of a uniform flow, the shape of equations changes somehow. For this case, the system is:

$$\left[\begin{pmatrix} i\omega & 0 & 0 & 0 \\ 0 & i\omega & 0 & D_y \\ 0 & 0 & i\omega & D_z \\ 0 & D_y & D_z & i\omega \end{pmatrix} - \alpha \begin{pmatrix} iu_0 & 0 & 0 & i \\ 0 & iu_0 & 0 & 0 \\ 0 & 0 & iu_0 & 0 \\ i & 0 & 0 & iu_0 \end{pmatrix} \right] \begin{pmatrix} u \\ v \\ w \\ p \end{pmatrix} = 0 \quad (2.31)$$

using fact that u_0 has a constant value, and then, its y and z derivatives are zero. We recall the boundary conditions to be the same as in the previous solution. If we proceed as the case without flow, we arrive to a solution equal to that of the previous case, but with a different shape for the dispersion relation:

$$\alpha_{n,m} = \frac{-\omega u_0 \pm \sqrt{\omega^2 - (1 - u_0^2) \cdot (\beta_n^2 + \gamma_m^2)}}{1 - u_0^2} \quad (2.32)$$

As system dimensions and boundary conditions remain the same as before, the values of β and γ stay as they were in (2.29).

2.2.3 Absorbent wall, no flow

If we proceed differently, and instead of input a flow, we introduce an absorbent lower wall, most part of the solution remains unchanged. For instance the dispersion relation is the same as (2.30) and the shape of pressure is also given by (2.28). However, the change that arises is substantial. The collection of normal wavenumbers β , and from them also the eigenvalue α results from a transcendental equation:

$$2\beta \tan(2\beta) = 2i \frac{\omega}{Z} \quad (2.33)$$

that can only be solved by series or numerically.

2.2.4 Absorbent wall, uniform flow

When we have the combination of both, a uniform flow profile and a finite impedance, the shape of α remains the same in terms of β , γ and ω as in equation (2.32). However, there are some changes for β , given by the difference in boundary condition introduced in section 1.3.6. If we take the rather simple Ingard - Myers boundary condition (1.26), β is obtained from:

$$2\beta \tan(2\beta) = 2i \frac{(\alpha u_0 - \omega)^2}{Z\omega} = \frac{2i}{Z\omega} \left[\left(\frac{\omega u_0 \mp \sqrt{\omega^2 - (1 - u_0^2) \cdot (\beta^2 + \gamma^2)}}{u_0^2 - 1} \right) u_0 - \omega \right]^2 \quad (2.34)$$

which complicates the calculations further. A series or numerical solution is needed again. We have used the Ingard - Myers boundary condition as an example. If we take a more physically meaningful and elaborate shape as those presented by Rienstra or Brambley, the resulting equation would be significantly more complicated.

This happens with the simplest flow profile. Other mean flow profiles have been studied for whose analytic solutions have been obtained in terms of series: the linear velocity profile [25], the hyperbolic tangent [22], the exponential velocity profile [26] and the parabolic profile [24].

For a general profile, even series solutions can become impossible to obtain. This includes experimentally measured profiles, which can be approximated by complicated or piece-wise functions and calculated profiles which may not even have a closed analytic form. This orientates most of studies, as our own work, towards numerical solutions.

2.3 Numerical calculation

As it has been exposed, the acoustic field present in a duct with absorbent walls can be obtained in a general case only by numerical methods. In the work presented here modes were calculated using two different programs: MAMOUT and FiEStA. Both programs calculate modes in the same way, but each of them was used to study a particular case. These cases are, on one hand, a bidimensional viscous analysis and, on the other hand, a three-dimensional inviscid study.

2.3.1 Collocation method

Both codes use the pseudospectral collocation method to find the eigenvalues and functions. It is a variation of spectral methods which consists in making a decomposition of functions in an orthonormal basis.

The collocation method consists in discretising the domain into a set of points $\underline{x}_j$. The boundary condition is imposed in the points that lie in the border, while the differential equation is written for the remaining points. Then a solution is sought as a projection into a basis of functions. This way, the solution obtained is a sum of functions which fulfil simultaneously the boundary condition for the points on which it was imposed and the differential equation for the remaining points.

One set of functions widely used as a base, and that is employed here, is the set of Chebyshev polynomials T_n . Their shape is given by the recurrence relation:

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_{n+1}(x) &= 2xT_n(x) - T_{n-1}(x) \end{aligned}$$

With a change of variable $x = \cos \vartheta$, we obtain an alternative way to write these polynomials as:

$$T_n(\cos(\vartheta)) = \cos(n\vartheta) \Rightarrow T_n(x) = \cos(n \arccos(x))$$

Chebyshev polynomials up to degree five are illustrated in figure 2.2 for the interval $[-1, 1]$.

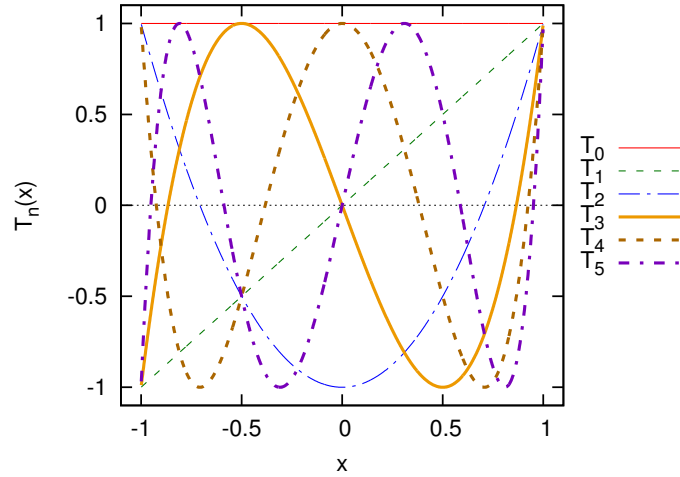


Figure 2.2 – First six Chebyshev polynomials in the interval $[-1, 1]$.

Chebyshev polynomials are defined for any $x \in \mathbb{R}$. And, by their construction, is evident that T_n is a polynomial of order n . From figure 2.2 we can see that all their roots are real and lie within the interval $(-1, 1)$.

The points used are the Gauss - Lobatto points. They are given as those points in the interval $(-1, 1)$ at which the extrema of the Chebyshev polynomial of order N are attained. This gives $N + 1$ points, including both -1 and 1 . These points are written as:

$$\xi_j = \cos\left(\frac{\pi j}{N}\right) \quad \text{for } j = 0, 1, \dots, N$$

where ξ are the collocation points and N is the order of the maximal polynomial considered in the base.

In the bidimensional domain considered to solve Euler equations by FiEStA, the points are redimensionalised in z to fit the full non-dimensional domain as:

$$(y_j, z_k) = \left(\cos\left(\frac{\pi j}{N_y}\right), \mathcal{A} \cos\left(\frac{\pi k}{N_z}\right) \right) \quad \text{for } j = 0, 1, \dots, N_y \wedge k = 0, 1, \dots, N_z$$

where N_y and N_z are the maximum order of points in each direction.

Derivation in this method is done by matrix multiplication. These matrices are developed from the properties of Chebyshev polynomials [27]. For the Gauss - Lobatto points they take the general following form:

$$\begin{aligned} D_{\Xi_{j,k}} &= \frac{c_j}{c_k} \frac{(-1)^{j+k}}{\xi_j - \xi_k} & j \neq k \\ D_{\Xi_{j,j}} &= -\frac{\xi_j}{2(1 - \xi_j^2)} & j = 1, \dots, N-1 \\ D_{\Xi_{0,0}} &= \frac{2N^2 + 1}{6} \\ D_{\Xi_{N,N}} &= -\frac{2N^2 + 1}{6} \end{aligned}$$

with:

$$\begin{aligned} c_j &= 2 & j = 0, N \\ c_j &= 1 & j = 1, \dots, N-1 \end{aligned}$$

Here N corresponds either to N_y or to N_z depending on the coordinate on which the derivative matrix operates. These matrices are re-scaled by the corresponding factor this way:

$$D_y = D_{\Xi} \quad \text{and} \quad D_z = \frac{D_{\Xi}}{\mathcal{A}} \quad (2.35)$$

These derivation matrices are introduced into system (2.21) and (2.25) in the corresponding places of the derivation matrices. The derivatives of mean flow are obtained by matrix multiplication with the velocity vector.

2.4 MAMOUT program

One of the codes used in this study is called MAMOUT from the french acronym “Module d’Analyse MOdale Unidimensionnelle avec les polynômes de Tchebychev” [15] which states for: “Unidimensional modal analysis module with Chebyshev polynomials”. This code was developed by Jean-Philippe Brazier at Onera/DMAE. This code contains actually two programs:

SPECTRE. This program calculates the whole spectrum for a duct with given parameters, by solving the eigenvalue problem.

MODITER. With this program one can follow a single mode by variation of frequency with a shooting method.

The set of parameters that have to be prescribed are given by a directives file. The main structure of this file is as follows:

- ◀ Physical parameters of problem:
 - ◀ incompressibility
 - ◀ viscosity
 - ◀ geometry: unidimensional, transverse cylindrical, transverse rectangular
- ◀ Mean flow profile: besides no mean flow and uniform profile, it can consider, among others:
 - ◀ linear with or without slip in bottom wall
 - ◀ parabolic
 - ◀ Poiseuille with or without a symmetry axis
 - ◀ hyperbolic tangent with or without a mixing layer
 - ◀ double hyperbolic tangent for double mixing layer
 - ◀ flow profile read from a file
- ◀ Nondimensional parameters:
 - ◀ Mach number (top value)
 - ◀ Reynolds number if viscous
 - ◀ Prandtl number if viscous and compressible
- ◀ Quantities for dimensional or nondimensional analysis:
 - ◀ height of duct
 - ◀ reference speed = sound speed times Mach number
 - ◀ mean density of fluid
- ◀ Impedance condition at top and bottom wall, recognising:
 - ◀ hard wall
 - ◀ free wall
 - ◀ feed impedance
 - ◀ Enhanced Helmholtz Resonator model for impedance
- ◀ Type of calculation between temporal and spatial.
- ◀ Frequency or range to sweep.
- ◀ Axial wavenumber, or interval.
- ◀ Mesh and domain dimensions.
- ◀ Chosen eigenfunctions for output.
- ◀ For MODITER, the iteration steps.

The results obtained with MAMOUT depend on the program run. Both programs output the domain characteristics in one file called `pocol.dat`. It includes collocation points, mean flow profile and its derivatives.

Besides previous file, SPECTRE program outputs one or two more files:

valprop.dat is always output. It contains the eigenvalues being either the set of frequencies *omega* for a temporal analysis, or the set of wavenumbers *alpha* for a spatial analysis.

vecprop.dat includes the shape of selected eigenfunctions, if demanded. They are identified by an index. This index can only be known after a first run of SPECTRE, and it may change after the modification of a parameter.

On the other hand, MODITER outputs two files:

vpmodit.dat has the tracking of wavenumber, frequency and impedance regardless if it is a temporal or spatial analysis.

vecprop.dat contains the eigenfunction associated to the mode at beginning and end of iterations.

2.5 FiEStA

The name of this code states for the acronym of “Finder of Eigenmodes for Study in Aeroacoustics”. It was fully developed in the frame of this thesis, and it will be more deeply discussed. Differently from MAMOUT, FiEStA is one single program with various modules to perform different studies, which are selected by the input file. The main structure of this file, which exposes needed parameters, is presented next as it was done for MAMOUT. These options will be further explained.

- ◀ Dimension variables (set to one for nondimensional analysis):
 - ◀ Half height of duct
 - ◀ Speed of sound
 - ◀ Mean density of fluid
- ◀ Geometry of the domain: Unidimensional or 2D transverse
- ◀ Aspect ratio
- ◀ Mean flow profile, besides no flow and uniform flow:
 - ◀ Parabolic flow
 - ◀ Poiseuille flow
 - ◀ Flow profile read from a file
- ◀ Mach number (top or mean value)

- ◀ Frequency
- ◀ Impedance
 - ◀ Hard wall
 - ◀ Given impedance or admittance
 - ◀ Enhanced Helmholtz Resonator
 - ◀ Optimal impedance* as by Cremer
 - ◀ Optimal impedance* as by Tester
- ◀ Numerical method to solve
 - ◀ Full spectrum
 - ◀ Multiple spectra
 - ◀ Shooting method on one or two modes
 - ◀ Optimal impedance* search
- ◀ Only for iterative methods (multispectra, shooting)
 - ◀ Wavenumber of mode to follow (one or two)
 - ◀ Parameter to vary among:
 - Frequency
 - Aspect ratio
 - Mach number
 - Impedance or admittance
 - ◀ step size and total steps for varying parameter

* Note: The optimal impedance will be discussed in chapter 4.

The first items are self explained: the dimension variables, the geometry of the domain and aspect ratio. For the mean flow profile, we will deepen the discussion slightly.

The Poiseuille flow is solved from the Poisson equation:

$$\nabla^2 u_0 = f \quad (2.36)$$

where f is a fixed value, proportional to the pressure gradient. However, this gradient doesn't have a meaningful value, since the resulting flow is normalised according to the Mach number either for the top or mean value.

When reading the flow profile from a file, splines approximation is considered, but with the tested profile, from the benchmark data obtained thankfully from M. Jones from NASA [60], two problems arose. First, with the module used for interpolating, that was already coded into MAMOUT, before testing in FiEStA, the no-slip condition had to be forced by trial and error, since the interpolating routine allowed the curve to pass close to the interpolating nodes and not through them in order to gain in softness of the curve. Second, strong oscillations in the curve between interpolating nodes appeared near the walls, due to the fast-changing shape of the profile.

To deal with the problem of slipping profile at walls, the weight of border points was increased with respect to non border points. This workaround was shown useful leaving however a small slipping flow at walls. To solve both problems at once, the interpolation was made via the Chebyshev polynomials. As this interpolation ensures to pass by given points, we had to input the zero flow at walls once for all. This interpolation could present the oscillation problem which was solved by adjusting the quantity of interpolation polynomials.

Next entries in the parameters file are, again, easily understood; Mach number, frequency and impedance. It can be either a fixed value or the parameters for the enhanced Helmholtz resonator model. The optimal impedance, will be discussed later in chapter 4.

The choice of numerical method is also clear, between the full spectrum for a set of parameters or, for one varying parameter, either multiple spectra or a shooting method to follow one or two modes. The search of optimal impedance will be discussed later, as already stated.

2.5.1 Structure of the code FiEstA

The program FiEstA was written in Fortran95 using LAPACK libraries to solve the linear problem for the shooting method and the eigenvalue problem of the full spectrum. The program consists on several modules in order to split tasks and save time when compiling, testing, modifying and reusing the code. These modules are named hereafter next with a short description:

FiEstA.f90 The assembling code.

parametros.f90 Module containing global parameters and variables

entrar.f90 Module for reading the instructions file and setting up parameters. Here, the physical and numerical parameters are read, and the arrays are allocated at the beginning and deallocated at the end of calculations.

salir.f90 Module to output calculated data. A new directory is created to store the calculations. The flow profile, the eigenspectrum, the eigenfunctions and the set of parameters are stored into this directory.

dominio.f90 Generation of discrete domain, derivative matrices and mean flow.

eigenlapack.f90 Set-up and solution of the eigenvalue problem using LAPACK subroutines. The solution is obtained with the subroutine ZGGEV (*complex double precision general eigenvalue solver*).

disparar.f90 This module sets the matrices and solves the shooting method. The shooting is calculated using the ZGESV (*complex double precision general linear system solver*) subroutine also from the LAPACK library.

2.5.2 Comparison between MAMOUT and FiEstA

As two different codes were used to perform calculation in this doctoral work, one could wonder why not to use only one code. The answer is, in part academical and practical. From an academic point of view, as one of the purposes of a PhD thesis is the formation of human resources, it was considered desirable to ask the author to develop a code by himself to fully understand the internal workaround of the solution of the problem. That is why the program FiEstA was conceived.

The practical response is that, in the initial stages of the development of FiEstA, it was intended to solve only academical cases, but it derived in a different way than MAMOUT, so, at a given time it was easily for the author to keep coding into FiEstA than to change the mental structure to fit into MAMOUT, thus ending with two complimentary pieces of software.

Main differences between these programs are that MAMOUT, besides solving LEE, deals with the LNSE, allowing the viscosity to be taken into account. On the other hand FiEstA considers non viscous cases only. However FiEstA can consider a bidimensional transverse domain to make the bi-global analysis.

As exposed in 2.1.3, MAMOUT has the possibility to deal with temporal analysis while FiEstA is not yet capable. This will be used in chapter 5 to verify the validity of the stability criterion.

Another difference between both programs is the normalisation of eigenfunctions. While MAMOUT sets to one the top value of pressure, FiEstA set to one the value at the top wall, i.e. the wall opposite to the liner.

2.6 Validation of FiEstA

Now we will show some of the first results obtained with FiEstA. We validate the code against known solutions, MAMOUT output or published results. Unless otherwise stated, solutions in this section have been obtained with FiEstA.

To validate FiEstA, we will compare its solutions against three different kinds of solution. First, we will consider the simple cases for whom there are exact analytic solutions, as those exposed in section 2.2. Then we will compare the results with those obtained with MAMOUT for more general cases, since it had already been validated as for the work of Boyer et. al. [12]. Lastly, we will compare with a hydrodynamic mode published by Marx et. al. [75].

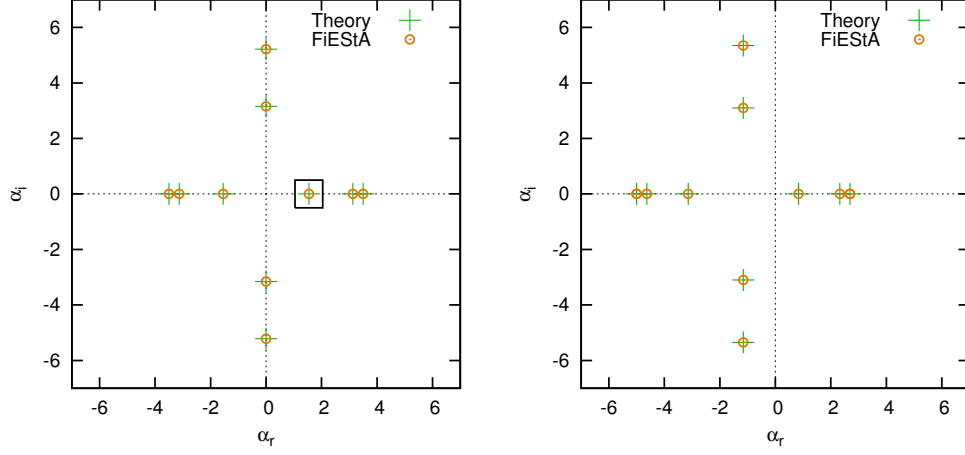
2.6.1 Validation with analytic solutions

The first cases for validation are with fully reflective walls. We will show the validation for both unidimensional and transverse domains, since the subroutines to construct and calculate the eigenmodes are independent.

The spectra obtained are compared to theoretical values from (2.30) and (2.32) in figure 2.3 for the unidimensional case and in figure 2.4 for the transverse one. For the last case, we chose an aspect ratio $\mathcal{A} = 1.1$ to distinguish between transverse and normal wavenumbers. Each figure shows the spectrum without mean flow and with a uniform flow at a Mach number of 0.3 to clearly observe the effect introduced by the mean flow. We pick a non dimensional frequency equal to 3.5, as that presented in figure 1.16.

In figures 2.3 and 2.4, a perfect match between theory and eigenvalues obtained with FiEstA is clearly shown. We can see that the eigenvalues are displaced to the left in the presence of a mean flow.

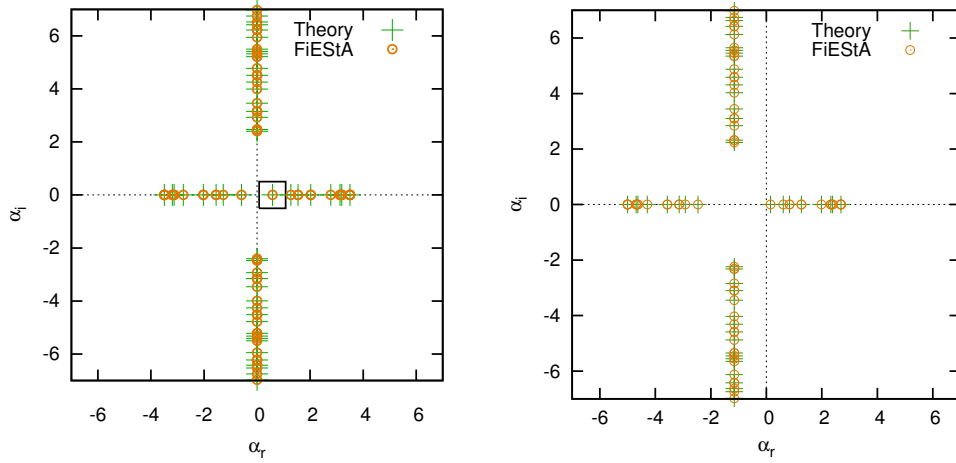
Besides the eigenspectra, we compare the eigenfunctions with theory. We focus on the amplitude of pressure perturbation since it is the variable for which solutions were obtained in (2.28). In figure 2.5 we show the absolute value of pressure fluctuation. This is shown for the 1D and transverse cases, with the same set of parameters as before. We present the function for $n = 2$ in the unidimensional case and for $n = 2, m = 1$ for the transverse one, which correspond



(a) No mean flow.

(b) Uniform mean flow, Mach = 0.3.

Figure 2.3 – Spectra obtained with FiEStA compared to theory from section 2.2. Unidimensional domain, fully reflective walls, non dimensional frequency $\omega = 3.5$. The eigenvalue for right-running mode $n = 2$ is marked.



(a) No mean flow.

(b) Uniform mean flow, Mach = 0.3.

Figure 2.4 – Spectra obtained with FiEStA compared to theory from section 2.2. Transverse domain. Fully reflective walls, non dimensional frequency $\omega = 3.5$, aspect ratio $\mathcal{A} = 1.1$. The wavenumber inside the square is for mode $(n, m) = (2, 1)$.

to the eigenvalues inside squares from figures 2.3 and 2.4.

Figure 2.5 allows to confirm a perfect agreement between the pressure eigenfunctions obtained with FiEStA and the theory, for both the unidimensional and the transverse cases.

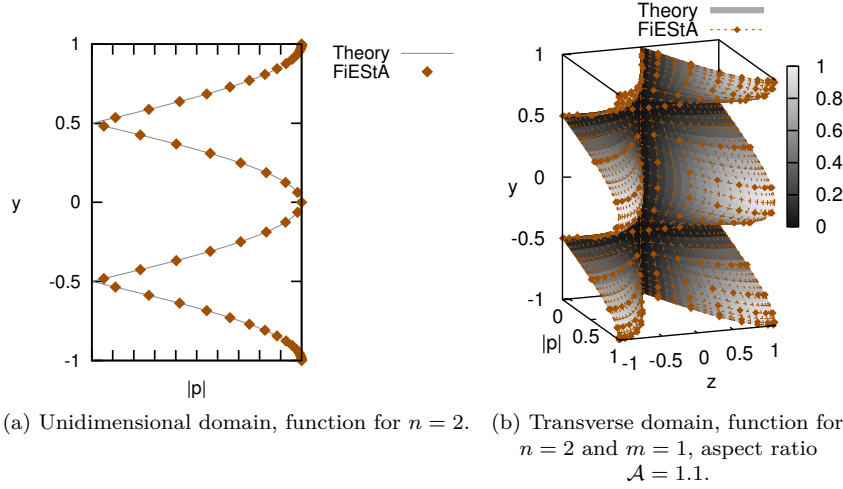


Figure 2.5 – Selected eigenfunctions, highlighted in figures 2.3 and 2.4 obtained with FiEstA compared to theory from section 2.2 Fully reflective walls, non dimensional frequency $\omega = 3.5$.

2.6.2 Validation with solutions from MAMOUT

For those cases where analytic solutions are difficult or impossible to obtain, the validation of FiEstA is done with respect to results obtained with MAMOUT. Because of the construction of MAMOUT, this can be done only for the unidimensional cases.

To compare with MAMOUT, we chose the Poiseuille profile, since it is a widespread case and the shape of the mean flow is solved in a different way inside the programs; while MAMOUT uses the known parabolic solution, FiEstA solves the Poisson equation to be able to do the same way for the transverse case. In addition, MAMOUT normalises flow speed with respect to the maximum velocity value, FiEstA can either do the same way or normalise with the mean flow speed.

First, in figure 2.6 we show how both flow profiles superpose perfectly. Half of points were removed for clarity in the curves. The top value Mach number is observed at 0.3.

In figure 2.7 are shown the spectra with a Poiseuille flow profile and a top value Mach number of 0.3 with again a frequency $\omega = 3.5$. Two cases are shown, one with hard walls and the other one with an impedance of $1 + i$. The eigenfunction for pressure for the mode $n = 2$ is shown in figure 2.8. In both cases, either hard or absorbent wall, functions superpose exactly. However, as normalisation is not made the same way in both programs, the results obtained with MAMOUT were renormalised to match those of FiEstA.

2.6.3 Validation with hydrodynamical mode from Marx

Another comparison is shown in figures 2.9 and 2.10. They provide the spectrum and eigenfunction published by Marx [75] compared to those got with same set of parameters in FiEstA. This case is again a unidimensional analysis. For this comparison, the flow profile used by Marx was directly coded into FiEstA.

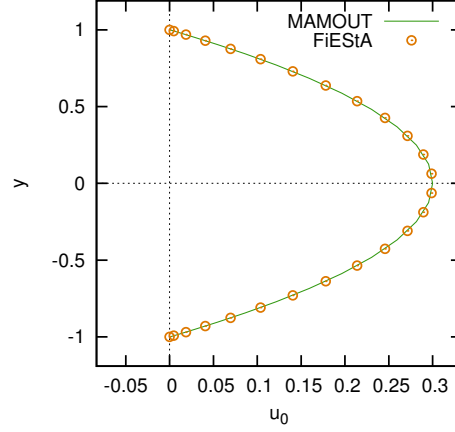


Figure 2.6 – Poiseuille flow profile with top value Mach number 0.3. Comparison between MAMOUT and FiEstA.

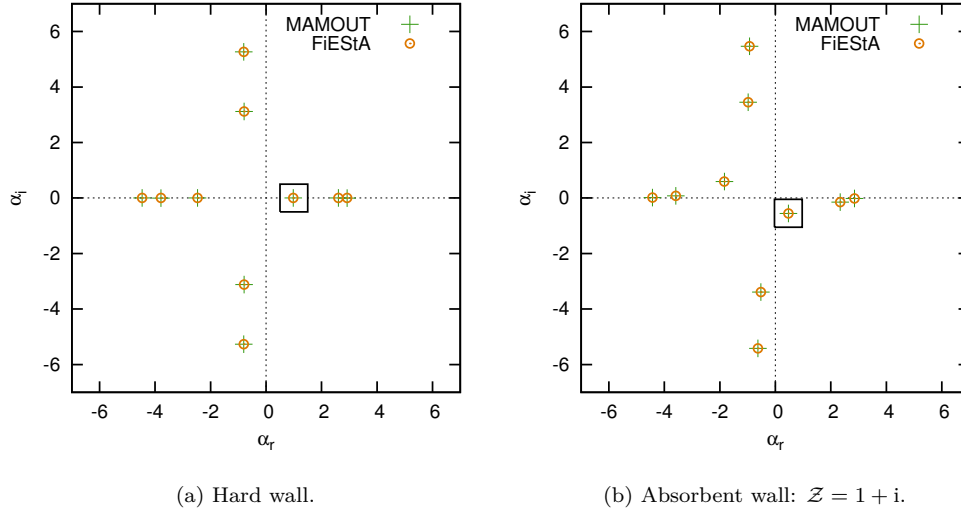


Figure 2.7 – Spectra obtained with FiEstA compared to results from MAMOUT. Unidimensional domain. Non dimensional frequency $\omega = 3.5$, mean flow Poiseuille profile, top value Mach number 0.3. Marked modes are plotted in figure 2.8

As before, we compare one set of eigenfunctions, in this case we focus on the hydrodynamic mode. Its eigenvalue is marked in figure 2.9. The eigenfunctions are shown in figure 2.10. In this case we plot the eigenfunctions shown in [75]: longitudinal and normal velocities.

Figure 2.9 shows that for the general case of a more sheared flow profile and an absorbent wall, we get good agreement between FiEstA and published results for the eigenspectrum. In figure 2.10 is shown that eigenfunctions also coincide with published results from Marx.

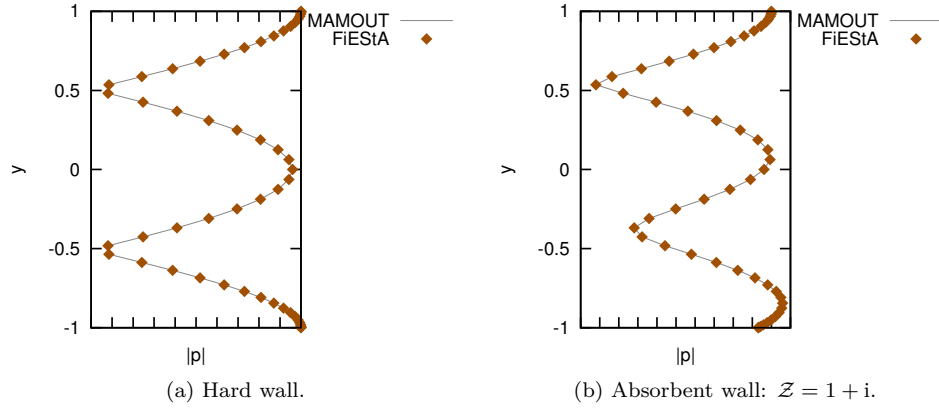


Figure 2.8 – Eigenfunction obtained for pressure for the mode $n = 2$, marked in the spectra of figure 2.7.

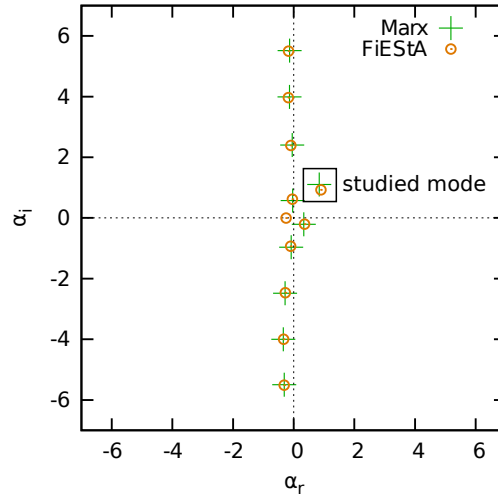


Figure 2.9 – Eigenspectrum obtained with FiEstA compared to that of Marx [75], highlighting the hydrodynamic mode.

In this chapter we exposed the development and validation of the code FiEstA with respect to analytic solutions, the already validated code MAMOUT and published data.

As shown in this last section, the shape and Mach number of the flow profile, as well as the impedance, modify the spectrum and eigenfunctions obtained.

In next chapter, we will see separately the impact of these two parameters as well as the aspect ratio.

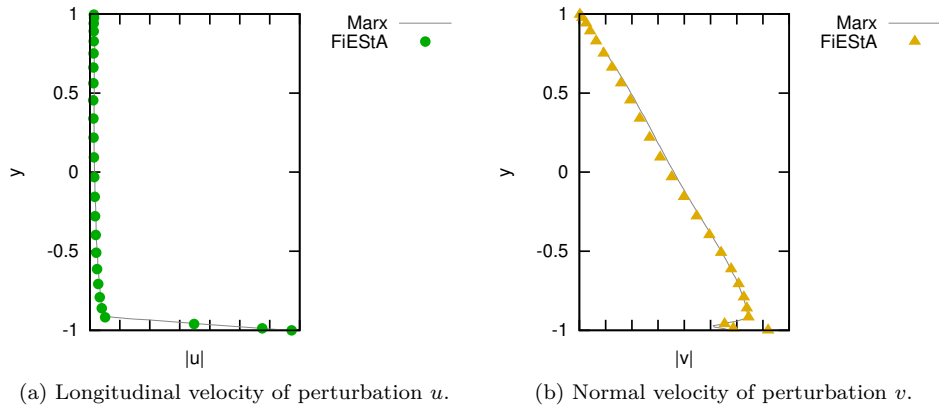


Figure 2.10 – Comparison of eigenfunctions of the hydrodynamic mode marked in 2.9 between the results from Marx [75] and FiEstA.

Chapter 3

Parameters influence

In this chapter we show how the eigenmodes in the lined duct are modified by three parameters: the aspect ratio, the mean flow, and the impedance.

3.1 Aspect ratio

The aspect ratio is considered for the transversal analysis only, since it has no meaning otherwise. As the stability problem has been written in nondimensional units, the height of the duct is fixed. The variation of aspect ratio can thus be seen as changing the width of the duct. We will start the sensitivity analysis of this parameter for a hard-walled duct without mean flow. Solutions are obtained directly from the analytic dispersion relation (2.30).

The effect induced by the aspect ratio can be seen in figures 3.1 and 3.2. In these figures we show the full spectra for four different aspect ratios. Modes for which $n = 0$ are highlighted to show the displacement of eigenvalues with higher m . In the limit where $\mathcal{A} \rightarrow \infty$, we loose the z dependence as $\gamma_m \rightarrow 0$ for all values of m , and we recover the 1D solutions, as it could be expected. These solutions are also shown as reference; they are the same values as those in figure 2.3a. They can be obtained also as those modes for which $m = 0$.

With hard walls, and a flow velocity profile with the same shape in y and z , for some values of the aspect ratio, there are also a superposition of eigenvalues, which induces that some points coincide in the spectrum. These values are those for which $n^2 = (\frac{m}{\mathcal{A}})^2$. This is why in figure 3.1, the spectrum for $\mathcal{A} = 1$ looks much cleaner than that one for $\mathcal{A} = 1.1$ even that both values are close. In the former case, all modes (i, j) have the same eigenvalue than modes (j, i) . And that is why the values of α with $n = 0$ and those of unidimensional modes coincide. They can, however, be distinguished by the shape or their eigenfunctions. All what has been shown about the influence of the aspect ratio is just an illustration of equation (2.30). When the mean flow profile can be decomposed as a product of two functions, one with y dependence only and the other one with z dependence, then, the resulting equations are separable. This implies that wavenumbers β and γ can be calculated by considering only the factor function of the mean flow that corresponds with its coordinate. In a more general case, as with a Poisuille flow profile, it is not possible to separate variables and both wavenumbers β and γ have to be obtained otherwise. If we consider some finite values of impedance in the bottom wall, the coincidence of modes does not exist any more, since β and γ of same index are not equal in general. This is illustrated in

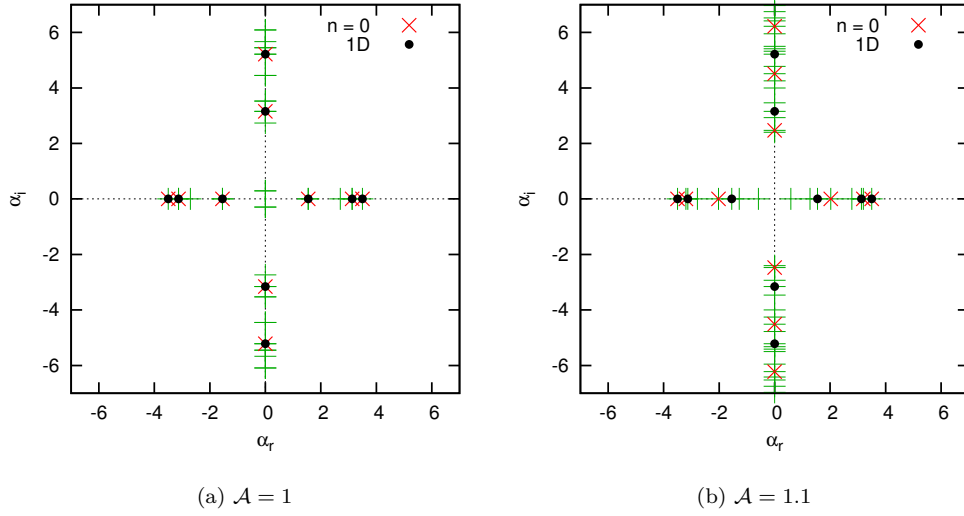


Figure 3.1 – Spectra for two close values of aspect ratio. Frequency 3.5, hard walls, no mean flow. Wavenumbers for $n = 0$ are shown in red crosses. The eigenvalues of the unidimensional problem are shown as reference in black circles.

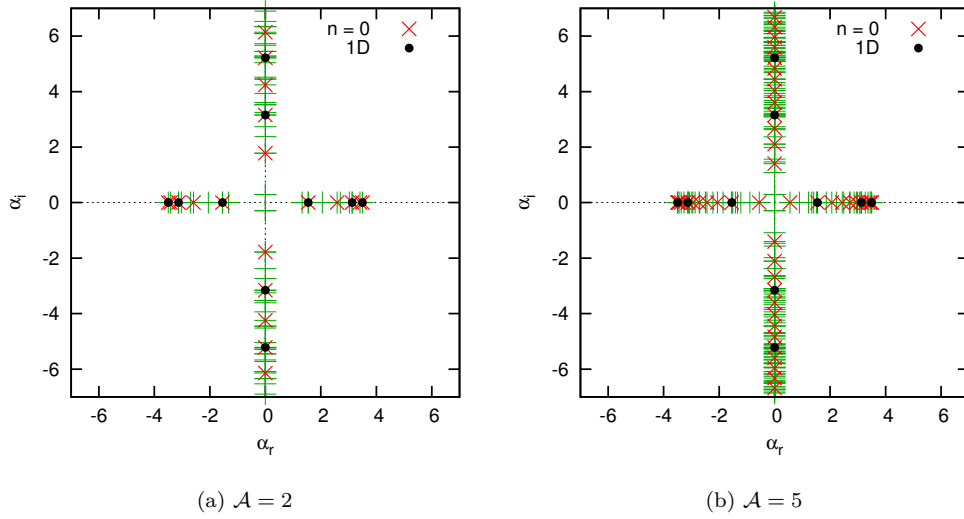


Figure 3.2 – Spectra for two large aspect ratios. Frequency 3.5, hard walls, no mean flow. Wavenumbers for $n = 0$ are shown in red crosses. The eigenvalues of the unidimensional problem are shown as reference in black circles.

figure 3.3. Where the modes with $n = 0$ and the 1D modes are not coincident.

Eigenvalues of same index in y and z can be different even with hard walls when the flow profile is not symmetric in y and z . This is shown in figure 3.4a for an aspect ratio equal to 1. Here, we use a profile provided by Jones and collaborators from their benchmark data [60]. We

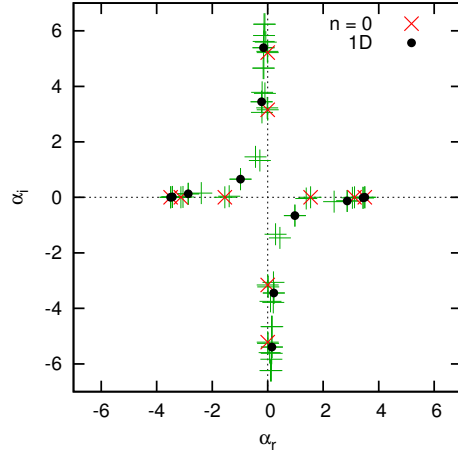


Figure 3.3 – Spectrum for an aspect ratio $\mathcal{A} = 1$ with a frequency $\omega = 3.5$ without mean flow. Lined wall of impedance $\mathcal{Z} = 1 + i$. Wavenumbers for $n = 0$ in red crosses and eigenvalues of the unidimensional problem in black circles.

show no longer neither the $n = 0$ nor $1D$ eigenvalues. However, if we use an impedance, as in figure 3.4b, the spectrum looks more crowded because the eigenvalues are more scattered.

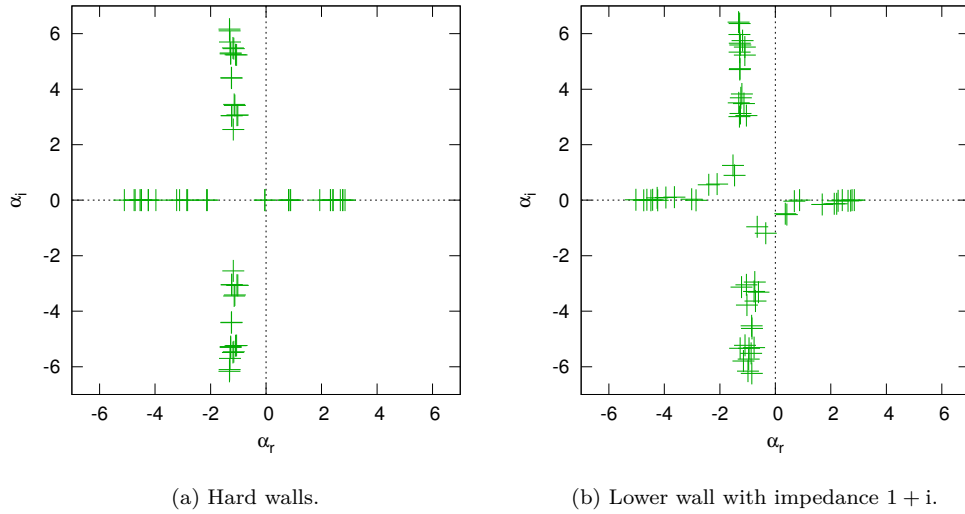


Figure 3.4 – Spectrum with an aspect ratio $\mathcal{A} = 1$ with a frequency $\omega = 3.5$. Non symmetric flow profile from Jones [60] with bulk Mach number 0.3.

In general, for any impedance value and any flow profile, the effects of the aspect ratio are the same as those exposed above but for the coincidence of modes at certain values of $\mathcal{A}$.

To study the influence of next parameters, for the sake of clearness, we will consider the unidimensional analysis only.

3.2 Mean flow

For the variation of mean flow, there are two different features that influence the eigenmodes: the Mach number and the shape of the flow profile.

For the Mach number, either way it is calculated, the main effects on the spectra are those seen in the dispersion relation with uniform flow (2.32). First, the eigenvalues are displaced toward the negative real part by the term $-\omega u_0$. As previously observed in figures 2.3, 2.4 and 2.7. Then, their absolute value is slightly amplified because of the factor $\frac{1}{1-u_0^2}$. Finally, they are displaced towards the modes of lower index by the $(1 - u_0^2)$ term inside the root.

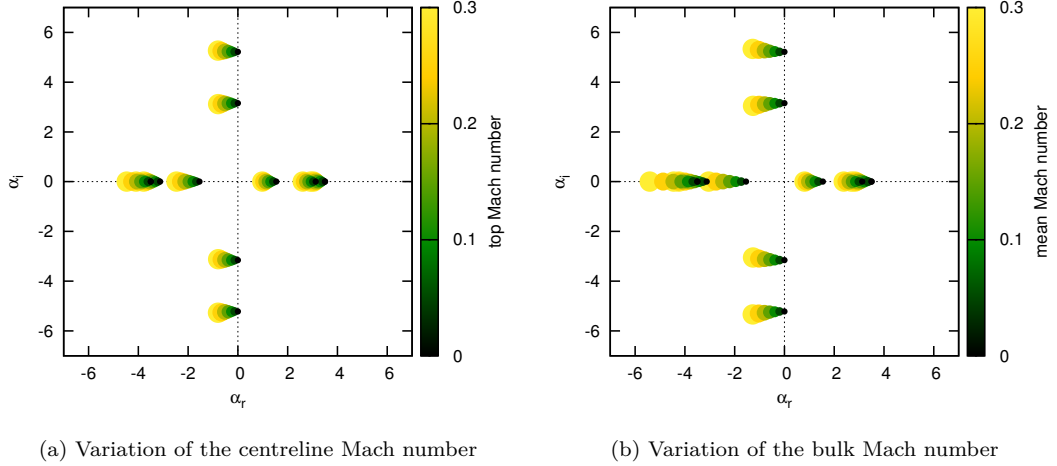


Figure 3.5 – Influence of Mach number on the spectrum. Unidimensional analysis with Poiseuille mean flow profile. Mach number between 0 and 0.3 indicated by the colour palette and the size of points, to see overlapping values.

In figure 3.5, a sheared laminar Poiseuille flow was used to illustrate how the effects described before are present also in this case. In regards of Mach number, we have to take care on the way it is considered. As we already said, in MAMOUT, it is defined as the centreline value while FiEstA can consider either the centreline or the bulk flow value. In both cases, changing the Mach number induces a renormalisation of the flow only. However, while this change of definition is transparent for the uniform mean flow, it is not trivial to know beforehand in case of more complicated profiles. That is why in figure 3.5 we have taken both normalisations to compare them. We can see that changing the centreline Mach number has a larger impact than changing the bulk Mach number, as commonly expected. It must be noted also that with the bulk Mach number, the propagative upstream eigenvalues are more separated than the rest.

To study the effect of the shape of the profile itself, we have picked four different profiles plotted in figure 3.6, chosen either with the same centreline or bulk Mach number.

In tables 3.1 and 3.2 we show the right running eigenvalues for each normalisation of Mach number. The full spectra are shown in figures 3.7 and 3.8. The general shape of spectra does not change, but values are a bit displaced with respect to each other.

When we look for cut-off modes, we can see that sheared flow profiles always give a displacement from a vertical line obtained with a uniform flow. This is shown in figure 3.8 for values with the

n	uniform	Poiseuille	Δ	Marx	Δ	Jones	Δ
0	2.692+0.000i	2.919+0.000i	0.227	2.754+0.000i	0.061	2.793+0.000i	0.100
1	2.322+0.000i	2.608+0.000i	0.286	2.402+0.000i	0.080	2.445+0.000i	0.123
2	0.833+0.000i	0.983+0.000i	0.150	0.859+0.000i	0.026	0.900+0.000i	0.067
3	-1.154-3.100i	-0.794-3.119i	0.360	-1.094-3.055i	0.075	-0.994-3.087i	0.160
4	-1.154-5.347i	-0.802-5.269i	0.360	-1.148-5.249i	0.098	-1.041-5.262i	0.141

Table 3.1 – First five eigenvalues for right running modes for four flow profiles. Absolute displacement with respect to values with uniform flow profile shown in columns with Δ . Top value Mach number considered.

n	uniform	Poiseuille	Δ	Marx	Δ	Jones	Δ
0	2.692+0.000i	2.688+0.000i	0.005	2.688+0.000i	0.004	2.690+0.000i	0.003
1	2.322+0.000i	2.381+0.000i	0.059	2.337+0.000i	0.015	2.342+0.000i	0.020
2	0.833+0.000i	0.806+0.000i	0.027	0.810+0.000i	0.023	0.823+0.000i	0.010
3	-1.154-3.100i	-1.262-3.051i	0.119	-1.235-3.025i	0.110	-1.210-3.050i	0.075
4	-1.154-5.347i	-1.277-5.339i	0.123	-1.296-5.257i	0.169	-1.267-5.278i	0.133
5	-1.154-7.280i	-1.275-7.296i	0.122	-1.353-7.169i	0.228	-1.327-7.191i	0.194
6	-1.154-9.100i	-1.271-9.138i	0.123	-1.409-8.963i	0.289	-1.383-8.996i	0.251
7	-1.154-10.866i	-1.266-10.924i	0.126	-1.460-10.696i	0.351	-1.453-10.720i	0.333
8	-1.154-12.599i	-1.261-12.678i	0.133	-1.504-12.390i	0.408	-1.510-12.416i	0.400
9	-1.154-14.312i	-1.256-14.411i	0.142	-1.535-14.061i	0.456	-1.555-14.091i	0.458
10	-1.154-16.011i	-1.252-16.130i	0.154	-1.551-15.719i	0.493	-1.595-15.744i	0.516
11	-1.154-17.700i	-1.248-17.839i	0.168	-1.554-17.373i	0.517	-1.626-17.387i	0.567
12	-1.154-19.382i	-1.245-19.540i	0.183	-1.545-19.028i	0.528	-1.646-19.013i	0.615
13	-1.154-21.058i	-1.242-21.237i	0.199	-1.529-20.688i	0.527	-1.646-20.637i	0.647
14	-1.154-22.730i	-1.240-22.928i	0.216	-1.508-22.354i	0.516	-1.637-22.266i	0.669
15	-1.154-24.398i	-1.238-24.616i	0.234	-1.486-24.025i	0.500	-1.620-23.902i	0.681

Table 3.2 – Eigenvalues for right running modes for four flow profiles. Absolute displacement with respect to values with uniform flow profile shown in columns with Δ . Bulk value Mach number considered.

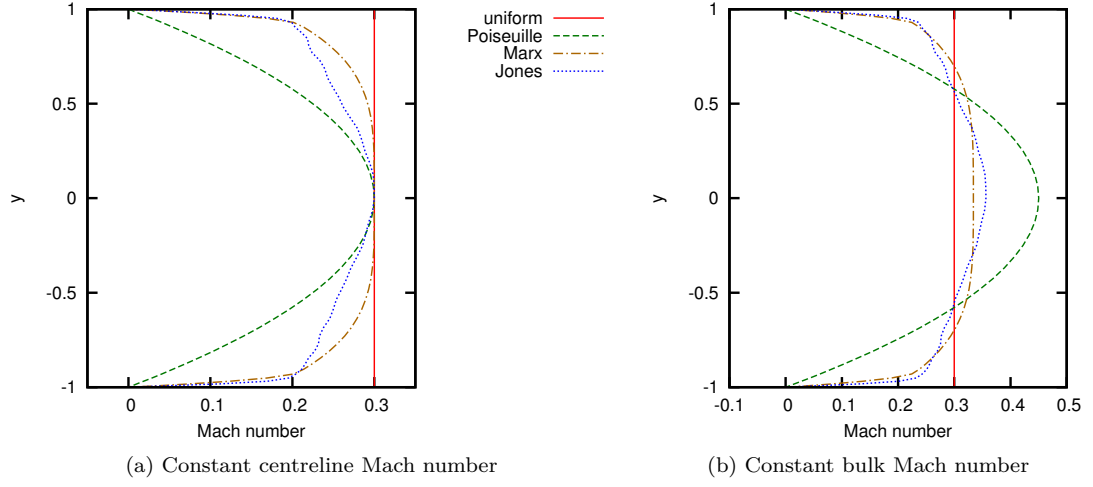


Figure 3.6 – Four flow profiles used to compare their influence on the solution. $M = 0.3$.

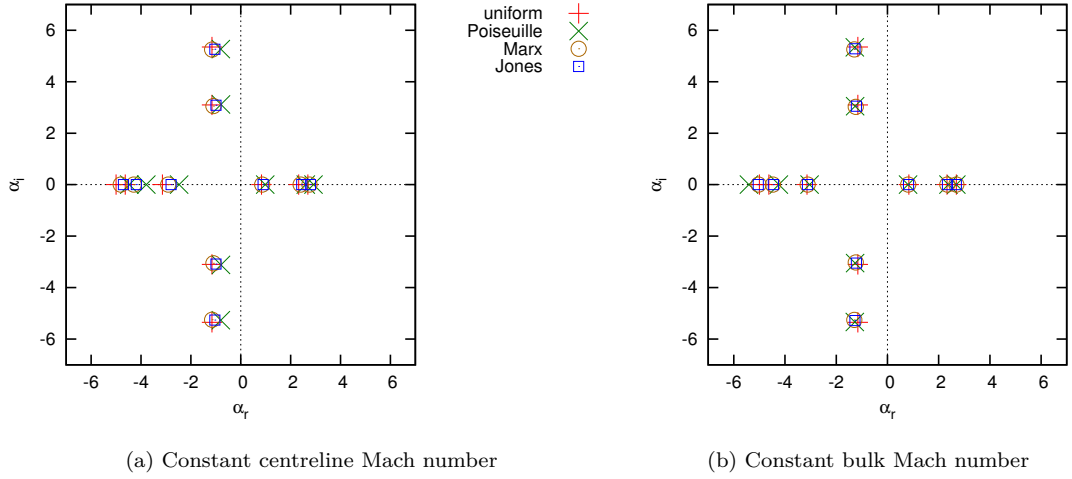


Figure 3.7 – Spectra obtained with each one of the flow profiles of figure 3.6. $M = 0.3$.

mean value Mach number considered. This is also true for the Poiseuille profile, even if it is hard to notice in the figure.

The information obtained in tables 3.1 and 3.2, and their illustration in figure 3.7 show the relevance of considering a fixed bulk flow value rather than a fixed centreline value. We can see that for downstream propagative modes, the bulk flow rate gives a better accord than the centreline Mach number. Meanwhile for evanescent modes, the shape of the mean flow profile acquires more relevance as eigenvalues tend to be displaced from uniform values; this effect becomes more pronounced as mode index increases. However, the upstream propagative modes are more influenced by the shape of the flow profile either way the Mach number is considered.

In most applications, the understanding of propagative modes is more relevant, particularly

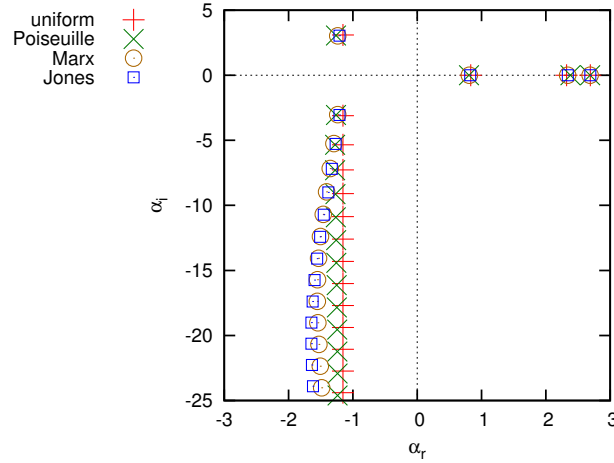


Figure 3.8 – Zoom out of the spectrum to show displacement of the eigenvalues of higher index. $M = 0.3$ Mean value Mach number. Vertical axis is five times bigger.

for noise mitigation. Because of this, for cases where uniform flow profile or convected Helmholtz equation is used in analysis to compare with experiments, bulk Mach number suits better. Besides, in many experimental facilities, as it is the case of B2A at Onera or the iterations of the GIT at NASA, in the range of operational frequencies, only the planewave mode is propagative. However, it must be noted that it is not usually the case in industrial applications.

Besides the wavenumbers, the flow profile also modifies the shape of eigenfunctions. We show this for the planewave mode in figures 3.9 and 3.10. We consider this mode, since all others oscillate and slight changes would be harder to notice. In figure 3.9 we show the 1D case for the same four profiles and for without mean flow. While figure 3.10 displays the transversal case for Poiseuille flow and for the profile from Jones; mean flow profiles are also presented in the figure. In figure 3.9 we can see that, as expected, with a uniform flow profile the function preserves the flat shape it has in the no-flow case. The figure also makes evident that for all other profiles, the eigenfunction is not any more a straight line. We recall that eigenfunctions are normalized in a way such that the hard wall has always a pressure equal to 1.

In both unidimensional and transversal cases we can note that profiles with a more uniform comportment in the central part, as those from Marx or Jones, make the function closer to a straight line than the Poiseuille profile, which lacks a central plateau and induces the greatest variation from those observed. Only for the measured profile from Jones, it can be noted that pressure is of lower amplitude the side of the lined wall, even if we do not consider actually the impedance of the wall. This is seen in both figures 3.9 and 3.10.

3.3 Impedance

In real-life conditions, one can not simply change the aspect ratio of a duct or the flow profile passing through it at will. However, one can normally choose a material for lining the internal walls of the duct. We may not be able to actually select a material with a desired impedance value, but we can pick the best suited one among those available.

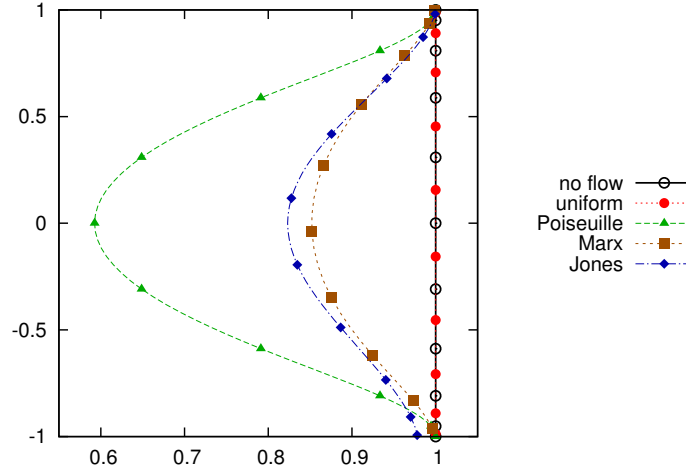


Figure 3.9 – Pressure eigenfunction for planewave mode with five different flow profiles. Unidimensional case. Bulk Mach number $M = 0.3$.

In this section, we will study the impact of impedance. As it is a complex quantity, we proceeded in a particular way explained next.

3.3.1 Reactance sweep

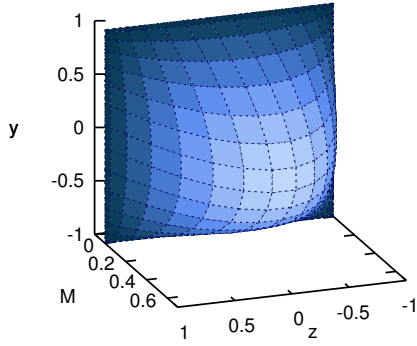
To observe the effects of impedance, we fix the resistance at different values and we sweep the reactance from highly negative values to highly positive ones. With this we obtain what is seen in figure 3.11 for a frequency $\omega = 3.5$ without mean flow.

We can see that for large values of resistance, while changing the reactance, eigenvalues make small loops that start and finish in the hard-wall values. If the resistance is lowered, these loops grow until those from lower order modes collide. After this coincidence, involved modes exchange their position between extrema of reactance, following a clockwise rotation for growing reactance. If we keep lowering the resistance, these exchanges include more and more modes always in a cyclic way. In a manner such that at the end of the reactance sweep, modes of index n end at index $n - 1$ and the plane wave mode ends at the highest index mode involved in the exchange.

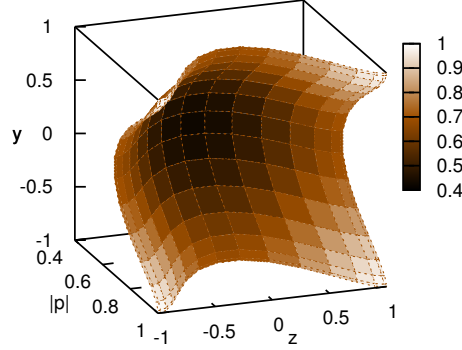
The comportment exhibited can turn an evanescent mode into propagative and viceversa. In fact, for any finite value of impedance, most modes lie in the second and fourth quadrants, thus having a mixed comportment propagative and evanescent, as they are described in equations (1.31) and (1.32) in section 1.4.1. This means that they are propagating waves with an exponential damping, with a general shape:

$$q' = \hat{q}e^{i(\omega t - \alpha_r x - i\alpha_i x)} = \hat{q}e^{\alpha_i x} (\cos(\omega t - \alpha_r x) + i \sin(\omega t - \alpha_r x)) \quad (3.1)$$

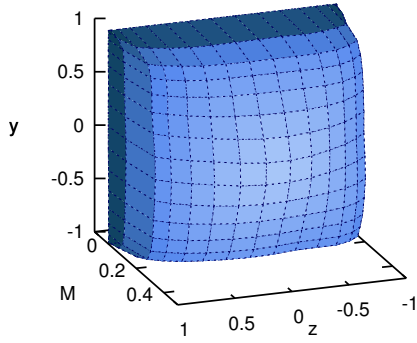
When modes exchange places, there are values of the impedance for whose it is not clear the index of a mode, not even while watching the eigenfunctions. This is particularly true for the plane wave which passes through the called acoustic surface mode [94] as it was exposed in section 1.4. In the transversal domain, each mode from the unidimensional analysis is present as a whole family, with a sinusoidal shape for the z dependence, this also happens to the surface



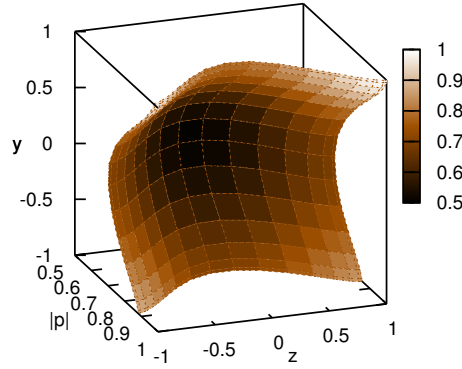
(a) Poiseuille flow profile



(b) Planewave mode, Poiseuille flow profile



(c) Flow profile from Jones



(d) Planewave mode, flow profile from Jones

Figure 3.10 – Flow profile and pressure eigenfunction for the planewave mode with different mean flow profiles. Transversal analysis. Bulk Mach number $M = 0.3$.

mode. For this mode, the whole family generated has the y dependence with the characteristic exponential shape, and the z behaviour has the oscillating shape of classic acoustic modes.

To observe the effect of the impedance with respect to in the eigenfunctions, we have picked three modes. These modes are those that start their trajectories, as hard wall modes 1, 3 and 0. Given the change in modes they make, they will be labelled $1 \rightarrow 0$, $3 \rightarrow 2$ and $0 \rightarrow S \rightarrow 4$. Here, the 'index' S indicates the acoustic surface mode.

We will show only the case for a resistance of 0.5, as it is the more ample curve in figure 3.11 and the surface mode is clear. For this, we have selected reactance values that give representative steps in the shape of eigenfunctions, specially for that of mode $0 \rightarrow S \rightarrow 4$.

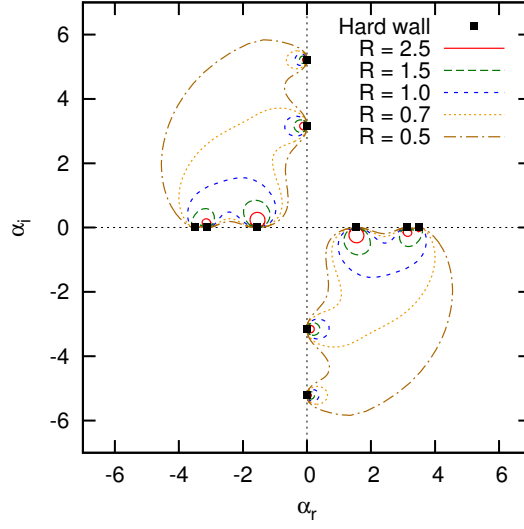


Figure 3.11 – Eigenvalues followed for different values of resistance indicated. Reactance sweep between -20 and 20. Hard wall, values shown as reference.

For a resistance of 0.5, as in the last curve of figure 3.11 and different values of reactance. The evolution of the pressure eigenfunction of this mode is shown in figure 3.12c. There, the value of reactance increases from left to right, these values, as well as the eigenvalues of the three modes are condensed in table 3.3. It must be kept in mind that the eigenfunctions are normalised so that $p = 1$ at the hard wall $y = 1$. The same information about the transition of modes from figure 3.12c is shown in figure 3.12d in logarithmic scale. This is done to emphasise on the exponential behaviour [94] of the surface mode at the lined wall.

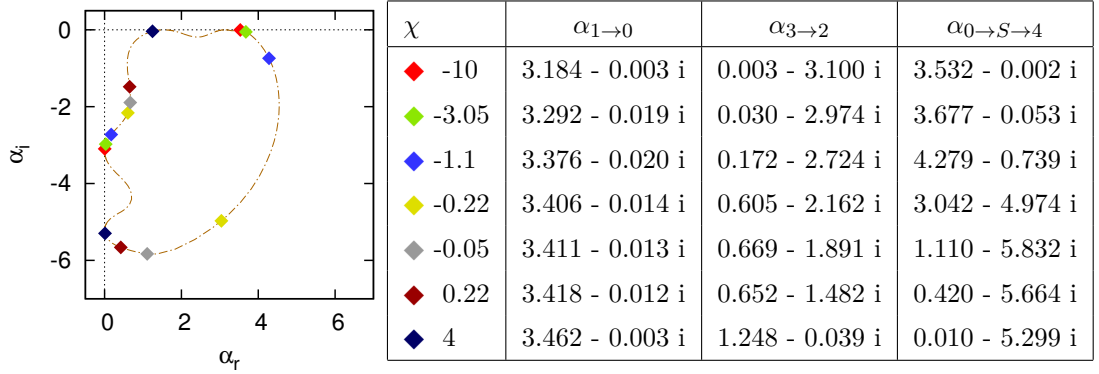


Table 3.3 – Eigenvalues of selected modes shown in figure 3.12 with respect to the reactance value.

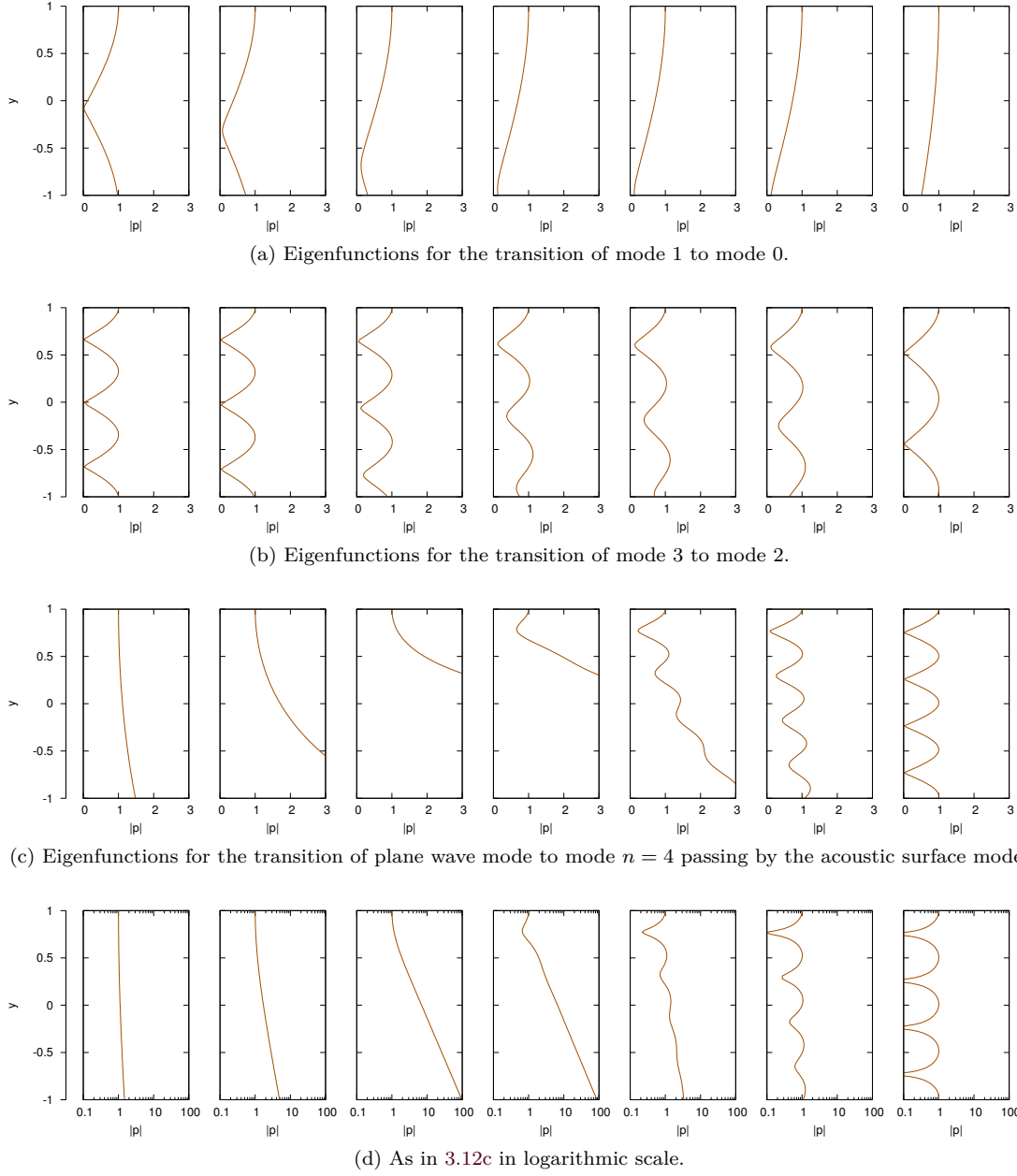


Figure 3.12 – Eigenfunctions for the transition of several modes with a fixed resistance = 0.5 and for some values of reactance shown in table 3.3.

3.3.2 Sensibility to impedance

To know how sensible are the eigenvalues to variations of impedance, we pick an impedance and four close values with positive and negative changes of resistance and reactance. We have chosen an impedance of $0.50 - 0.27i$ since we are also interested in the acoustic surface mode.

The variation introduced for the real and imaginary parts of the impedance is set to 0.05.

Mode	Impedance 0.50-0.27i ○	Impedance 0.55-0.27i ▲	Impedance 0.45-0.27i ▼	Impedance 0.50-0.22i ▲	Impedance 0.50-0.32i ▼
S	3.486-4.550i	3.102-4.162i	3.970-4.944i	3.042-4.974i	3.830-4.121i
0	3.405-0.014i	3.405-0.016i	3.405-0.013i	3.406-0.014i	3.403-0.015i
1	2.514-0.180i	2.518-0.200i	2.511-0.160i	2.534-0.176i	2.494-0.183i
2	0.572-2.232i	0.640-2.270i	0.507-2.201i	0.604-2.162i	0.536-2.294i
3	0.471-4.711i	0.502-4.778i	0.427-4.657i	0.538-4.664i	0.416-4.745i
4	0.390-6.719i	0.387-6.770i	0.380-6.663i	0.452-6.719i	0.341-6.723i

Table 3.4 – Wavenumbers of acoustic modes with changes of impedance. Index of modes correspond with their order without considering the surface mode. Surface mode is labeled 'S'.

Mode	Impedance 0.55-0.27i ▲	Impedance 0.45-0.27i ▼	Impedance 0.50-0.22i ▲	Impedance 0.50-0.32i ▼
S	0.5459	0.6236	0.6135	0.5508
0	0.0015	0.0015	0.0015	0.0015
1	0.0203	0.0196	0.0197	0.0201
2	0.0780	0.0716	0.0769	0.0717
3	0.0735	0.0698	0.0821	0.0648
4	0.0518	0.0567	0.0622	0.0491

Table 3.5 – Absolute variation of eigenmodes with respect to those with the mean impedance from table 3.4.

This gives us five spectra shown in figure 3.13, with the data of table 3.4. In table 3.5 we show the absolute displacement of modes for given impedance with respect to central values, obtained with $\mathcal{Z} = 0.50 - 0.27i$. This shows that classical acoustic modes are much less displaced with variations of impedance, while the surface mode is ostensibly more displaced. With respect to acoustic modes, the modes with the highest variation are those who have to travel the longest distances in their loop from one hard wall mode to another, as can be seen in figure 3.11. This is the case here for mode 2. Only for the positive variation of reactance, mode 3 has a more important change than mode 2. This evidences also a dependence not easily predicted with respect to impedance values even at low variations as those introduced here. However, within these impedance values all present modes preserve their behaviour and can not be confused. When impedance is close to the value at which a collision of eigenvalues happen, involved eigenvalues can change very rapidly. In most of cases, if we are not close of such a case, we can consider that small variations in impedance value will lead to a system with modes behaving the same way.

The overall behaviour of the mixed influence of impedance and mean flow can be deduced

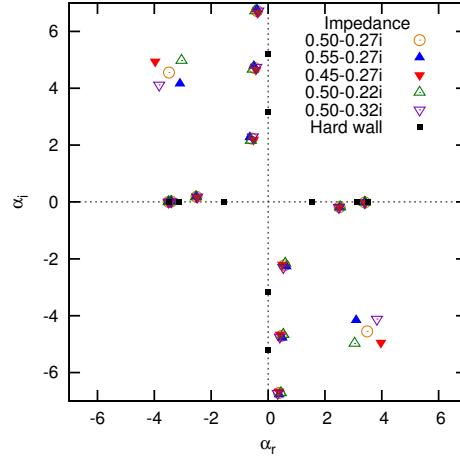


Figure 3.13 – Spectra for five different values of impedance. Central value is $\mathcal{Z} = 0.50 - 0.27i$. The rest are positive or negative variations of 0.05 either in real or imaginary part.

mostly from what has been seen independently for both. An example of this can be seen in figure 3.4b for the transversal domain. However, for some configurations there is the pair of hydrodynamic surface modes that were mentioned in 2.6.3 that arises. In chapter 5 we will study a particular case on which these hydrodynamic modes appear.

For a given mean flow and noise frequency, there must exist an impedance value that gives the best noise attenuation. This will be presented in next chapter.

In this chapter we showed the influence of the aspect ratio, the flow profile and the impedance with respect to the eigenmodes of the square duct with flow.

For the bidimensional analysis, higher aspect ratios make the spectra more crowded and the quantity of cut-on modes increases. For some values, with hard walls, some modes have the same eigenvalue, but they can be distinguished by their eigenfunctions. This coincidence does not happen if an absorbent material is put on one wall.

The influence of flow profile and Mach number was also studied, with emphasis on the spectra and the shape of the planewave mode. The consideration of the plug flow can induce variations of the wavenumbers and eigenfunctions with respect to physical profiles. It is important to consider the bulk Mach number to have better approximations, specially for cut-on modes. Centreline value Mach number leads to more important differences.

Impedance may induce the appereance of the acoustic surface mode. Small variations of impedance value gives very similar results, unless we deal with the acoustic surface mode. Sensitivity analysis of a case for which a hydrodynamic surface modes is present will be studied in detail in chapter 5.

In next chapter, the optimal impedance is studied.

Chapter 4

Optimal impedance

This chapter is based on a communication made by the author at Acoustics2012 congress [101]. The main idea behind it is the optimal value of impedance that reduces as much as possible the noise.

Given the objective of acoustic liners, since their first use in the sixties of previous century to reduce aircraft noise, there is a lot of research done oriented to improve the sound reduction made by them.

It is known that, in the absence of flow, the minimum of transmission is given at the resonance frequency of the liner. One could expect to obtain similar results with flow, however, acoustic propagation in this case is not easily predictable due to the interaction between the noise and the flow. This is especially true close to the liner, where the flow is highly vortical [76].

In a real world lined duct with a complex sound source, as an aeroengine, there are many parameters that can influence the way a liner modifies the acoustic field. Since the goal of acoustic lining in ducts is noise reduction, the main interest on studying the acoustic behaviour inside these ducts, is the attainment of the best noise reduction. The way to achieve this goal is determined by the properties of each duct and the noise characteristics.

4.1 Improvement of liners

As it was said in section 1.2, each liner has a particular behaviour depending on several aspects. Some of these aspects are its construction, the geometry and characteristics of the whole domain and the flow profile. Given this, an optimal liner should be constructed case-wise considering all particularities of each configuration. This can be achieved from an empiric way, on which noise mitigation is measured in real conditions for available materials. Then, the best suited one is selected. These materials could have been previously inspected to know their impedance at some frequencies and wind speeds or not. Their impedance can even never be known.

From another perspective, an optimal liner can be obtained based on theoretical models. From this perspective, under the configuration of a system we search for the impedance that best reduces noise either in analytic or numerical ways. Then a liner is either built-up aiming this impedance according to models based on physical characteristics as those described in section 1.2.4.5 or selected from materials with measured impedance [29].

Many strategies used to ameliorate the noise reduction from liners are based in two objectives: either to increase the bandwidth of the peak of absorption or further augment the attenuation around some frequencies. We will briefly comment few of those techniques found in recently published records.

Astley et al. [3], based on impedance models to describe liners, determine uniform value of impedance, from the different physical characteristics of the liner. With this, they make CAA calculations with the finite element method with different liners. They estimate the propagation and radiation obtained after the absorption by the liner. These data are gathered into some attenuation curve levels to determine the impedance that yields to best noise mitigation.

Law and coworkers [70] sought to optimise the attenuation of broadband noise. This is done by using consecutively three different liners in a cylindrical duct. They also draw upon impedance models on industrial liners. They considered the impedance and length of each segment as variables in the optimisation process. However, they conclude that the improvement is small compared with an optimal liner of uniform impedance.

The work of Campos and Oliveira [23] aims to get a non uniform liner impedance in annular ducts. They optimise it by acquiring the maximum possible attenuation in axial direction for the least attenuated mode.

Sanada and Tanaka made a liner as a DDOF with the perforated plate between cavities replaced by a flexible panel, to increase the bandwidth of absorption [103].

In their work, Tayong and collaborators present a relation between separation of microperforations and resistance. He shows also that there exists a critical Reynolds number since which the absorption coefficient of perforated plates starts to decrease. He introduces an expression that can be used to design optimal perforated plates for larger Reynolds number range [111].

4.2 Quantification of noise mitigation

Anyway the improvement of a noise reduction device is done, its performance has to be somehow determined. This can be quantified by the means of different quantities [56]:

Transmission loss: The logarithmic ratio between the signal before and after the silencer. It can be measured between two contiguous reverberation rooms separated by a wall containing the silencer, or between the extrema of two pipes. The first way is not used with duct silencers, and the second has the problem of reflecting signals at the silencer and at the end of pipe.

Insertion loss: The change of signal level at a given point with the insertion of the absorbing material. It is simpler to measure than transmission loss. This quantity is the most used to verify the performance of an absorbing material, since it directly represents its effect.

Noise reduction: Difference between signal levels at both extrema of silencer. This depends strongly on system characteristics and not only on the silencer itself.

Attenuation rate: Decrease of amplitude of the signal per unit length. It is complicated to measure, since the attenuation can not be constant all along the duct. Its mean value can be deduced from the transmission loss.

In these definitions, the measured signal can either be sound pressure level or acoustic power. In this work, to keep consistency with previously presented results, if needed, we will consider

the sound pressure level as the signal. It is obtained from the amplitude of the eigenfunction for pressure of the perturbation modes.

Transmission loss, insertion loss and noise reduction need measurements done at both ends of a limited noise reduction device. Since our analysis is made in a domain supposed infinite along the propagation coordinate, the attenuation rate is the only quantity on which we can rely to quantify the performance of a liner. From this quantity, if we considered a finite length of the liner we could retrieve the transmission loss.

The attenuation rate is obtained from the calculations made with either MAMOUT or FiEstA, by developing the form of the modes from equation (1.22), obtained from the spatial analysis, where the frequency is real and the wavenumber is complex, which is the general shape of equation (3.1) for modes that have the mixed behaviour between cut-on and cut-off.

This equation evidences the role of the imaginary part of wavenumber in the growth or decay of each mode. In fact, this number is closely related to the attenuation rate. This relation can be seen by comparing the amplitude of pressure at a given point $p(0)$ with itself a unit distance away $p(1)$. We obtain for attenuation in dB:

$$20 \log \left(\frac{p(0)}{p(1)} \right) = -20 \log \left(\frac{pe^{\alpha_i}}{pe^0} \right) = -20 \log (e^{\alpha_i}) = -20 \frac{\ln(e^{\alpha_i})}{\ln(10)} = -\frac{20}{2.3} \alpha_i = -8.7 \alpha_i \quad (4.1)$$

Here, the attenuation rate is positive if the mode decreases in amplitude. This is why it has the difference of sign with α_i , which is negative for decreasing right running modes.

In following parts, we will only consider the proportionality and we will look for the value of α_i , knowing that for right running modes, they are more attenuated as α_i has lower values. In this way, for the full spectre, the maximal attenuation is obtained when all wavenumbers from right running modes have their imaginary part as low as possible and, viceversa, the left running modes have it as big as possible.

4.3 Deduction of optimal impedance

Given that the decay of modes is exponential, the least attenuated mode, i.e. with the highest α_i for right running modes, is the one that contributes more to the noise in the far regime. So the eigenvalue of this mode has to be as below as possible of the real axis to obtain the best noise reduction of the whole spectrum.

To identify the spectrum of the optimal impedance, we recall the observations made in section 3.3, about the effect of the impedance in the spectrum when sweeping reactance from highly negative to highly positive values: With high resistances, we obtain loops that grow as resistance lowers. These loops make the initially propagative modes to get more attenuated up to some reactance value from which they start to be less attenuated until they reach again the hard wall values. If resistance is lowered, we have the coincidence of modes and the successive exchange of eigenvalues in a cyclic way with each time more modes involved, as they were described in section 3.3.1.

In this section we will consider a frequency $\omega = 0.924$, which corresponds to $2kHz$ in B2A. For this frequency, with hard walls, there is only one propagative mode in each direction, while the rest are evanescent. In figure 4.1 we show the characteristic loops and exchange of modes from the swept of reactance at some resistance values for this case.

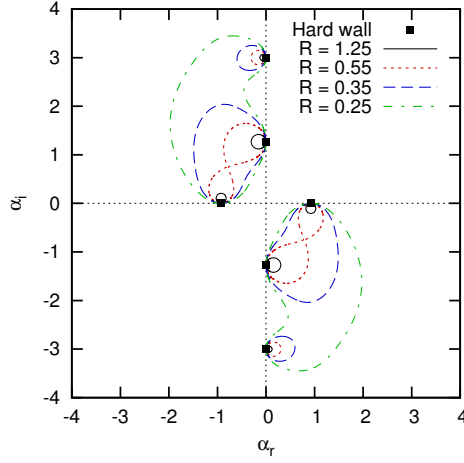


Figure 4.1 – Spectrum without mean flow for different values of resistance and large intervals of reactance. $\omega = 0.924$.

From said image, one can glimpse which is the optimal impedance for this frequency: At a resistance of 0.55, for a reactance equal to -0.44 we obtain a coincidence of modes 0 and 1. This equality of eigenvalues comes from a true convergence of both modes into one with double multiplicity, It is not due only to a fortuity coincidence of eigenvalues, as those seen for some values of $\mathcal{A}$ in section 3.1. This fact can be observed by looking at the eigenfunctions.

This coincidence corresponds to a branch point, which will be called the double point. We will show that this corresponds to the optimal impedance, in the sense of the most attenuated spectrum for this configuration.

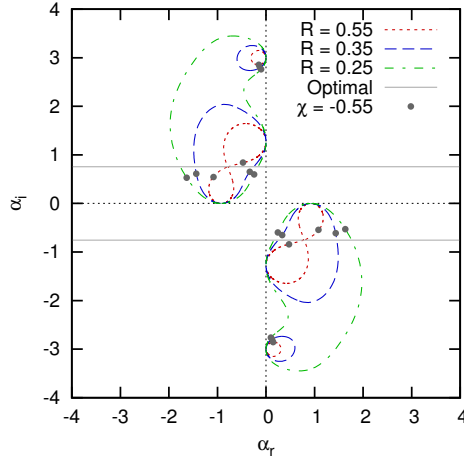


Figure 4.2 – Modes followed for varying reactance and three fixed resistances. Optimal attenuation indicated and reactance $\chi = -0.55$. $\omega = 0.924$.

If we take resistances higher than the 0.55 value, as is the case for $R = 1.25$, we can not

expect to have a better attenuation than this, since the loop for the plane wave mode remains less attenuated. If, instead, we take lower resistances, one could think that modes involved in the big loop are more attenuated, however, as it is shown in figure 4.2 for $R = 0.35$ and 0.25 , there is not a reactance value for which both modes be more attenuated. This is seen from the reactance value shown $\chi = -0.55$, for which both modes have lower attenuation than that obtained at the double point. Also because when one eigenvalue moves downwards, the other one moves upwards.

4.4 Optimal impedance as a function of frequency

To obtain the maximal attenuation rate, Cremer [38] defined in his study an optimal impedance and he gave a formula to obtain it. His calculations are done without mean flow and for low non-dimensional frequencies, where only the plane wave is a propagative mode. He states that it is obtained when the “least attenuated” pair of modes achieves a maximum attenuation; with the low frequency assumption, this pair is made up from modes of index $n = 0, 1$, as we observed in figure 4.2. According to him, the formula for the optimal impedance is:

$$\mathcal{Z}_C = (0.91 - 0.76i) \frac{2\omega}{\pi} \quad (4.2)$$

Tester [112], for his part observed that the definition with the “least attenuated pair of modes” was only right in the exposed case of low frequencies. This is because modes of higher order can be less attenuated than the given pair, when they become propagative. He says that the optimal impedance as given by Cremer is the maximum attenuation of the “most propagative” pair of modes. He also corrected the complex constant in the formula. Besides, he gives the corresponding coincidence of several pairs of modes and a formulation for any pair $n, n + 1$ for big n .

He further extended the formula to consider a uniform mean flow [113]. In this case, he considers only right-running modes, given the break of symmetry by the introduction of the mean flow as it is seen in section 3.2.

The development done by Tester considers what he calls *well-cut-on* modes, which fulfil:

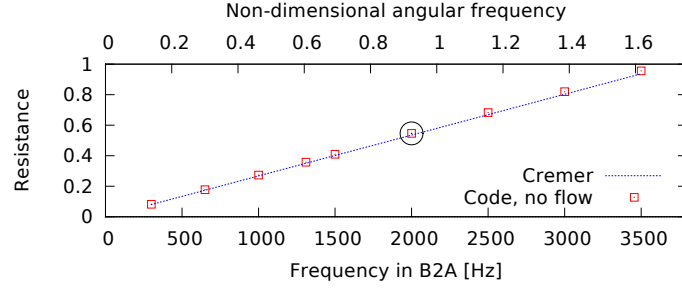
$$\omega > |2\beta H| \sqrt{1 - M^2} \quad (4.3)$$

So it is best suited for low order modes, high frequencies or high Mach numbers. The formula for the optimal impedance by Tester, with a uniform flow profile is:

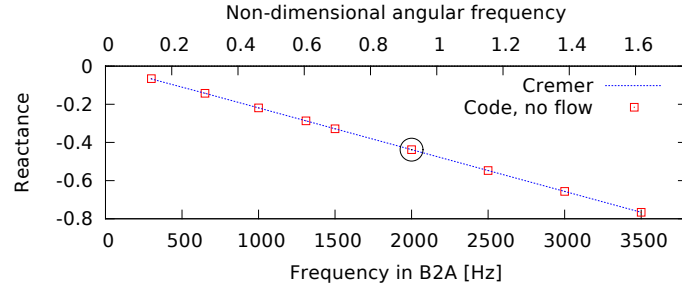
$$\mathcal{Z}_T = (0.929 - 0.744i) \frac{2\omega}{\pi} \frac{1}{(1 + M)^2} \quad (4.4)$$

The impedance for the double point was obtained by trial and error for different frequencies. In figure 4.3 the frequency evolution of optimal resistance and reactance without flow are shown compared to Cremer’s values. A good agreement is obtained. The barely noticeable differences in the resistance are due to the inaccuracy in the formula of Cremer, corrected by Tester; taking the corrected value, these differences vanish.

Figure 4.4 shows the same kind of plot for a uniform flow and a Poiseuille shear flow, now compared to Tester’s formula. The bulk Mach number is the same in both cases.



(a) Optimal specific resistance



(b) Optimal specific reactance

Figure 4.3 – Optimal impedance without flow ($\square$), compared to Cremer values (---), the frequency $\omega = 0.924$, case previously studied is distinguished by a black circle ($\circ$). Dimensional frequencies correspond to the *B2A* set-up.

We can see that the trends obtained are very close to the formula of Tester. However some differences are observed, especially in the low frequency range. These differences have been observed before[9]. They arise because in this case we are not in the regime of “well-cut-on” modes exposed above in (4.3).

Results with a shear and uniform flow profiles are closer between them than to those obtained with the formula.

4.5 Group velocity

The study of the group velocity for the modes involved in the double point is now presented. The group velocity is defined as the derivative of the frequency with respect to the wavenumber:

$$v_{gr} = \frac{\partial \omega}{\partial \alpha} \quad (4.5)$$

Without absorption, it is a real number which represents the velocity of energy propagation. For the cases with absorption, as those presented here, it is a complex number whose real part is associated in particular cases with the energy propagation and its imaginary part with the absorption of energy [49].

This is only a preliminary study; to understand the stability properties of the modes, the Briggs-Bers criterion will have to be implemented.

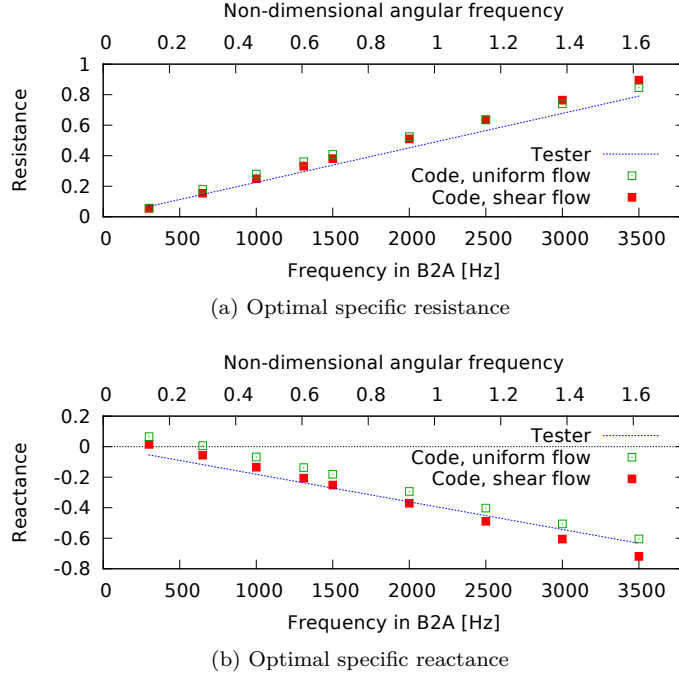


Figure 4.4 – Optimal impedance with uniform flow ($\square$) and shear flow ($\blacksquare$), compared to Tester values (---). Mach = 0.1. Dimensional frequencies correspond to the *B2A* set-up.

4.5.1 Reactance changes

The evolution of the group velocity with changes in impedance is investigated close to the double point. The resistance is fixed and the impedance is swept around the optimal value.

Without mean flow, figure 4.5 shows the group velocity of modes 0 and 1 for resistances equal to 1.25 and 0.75. Both cases are far from the double point. Labels '0' and '1' correspond to their hard-wall classification for negative reactance values. The asterisk '*' indicates the value of optimal reactance. It can be seen that group velocities for both modes are always positive. As the resistance is decreased, the group velocity of mode 0 starts to rise close to the optimal reactance. At a certain value it becomes infinite and reappears negative.

The group velocities for resistances close to the optimal one are shown in figure 4.6 without mean flow, and in figure 4.7 with a shear mean flow with bulk Mach number of 0.1. When the resistance is slightly lower than the optimal one (subfigures (b)), there is a change in branch at the optimal reactance, which does not exist when the resistance is slightly higher than the optimal one (subfigures (a)).

It must be noticed that close to the optimal impedance value, a pitch on the group velocity can be observed: the group velocity of both modes approach zero, one being negative and the other positive. These negative values or those divergent ones were not expected. It is important to note that the sign of v_{gr} does not tell the direction of propagation of a mode. To be certain about this, one must rely on other tools as the Briggs-Bers criterion that is shown in section 5.3.1. When we apply this criterion to selected modes, we get the certainty that they are right running modes.

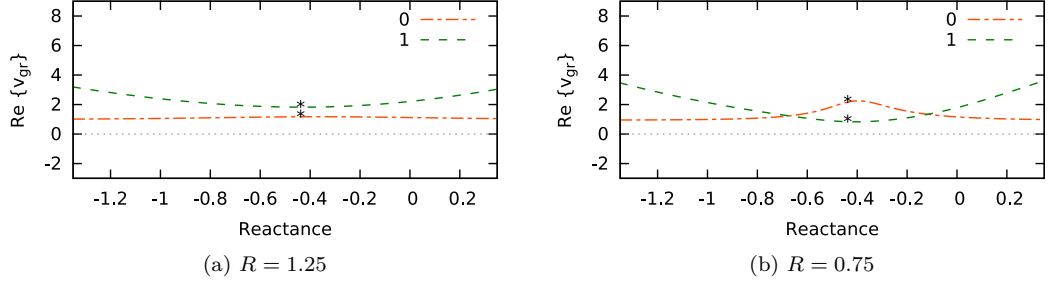


Figure 4.5 – Group velocity vs reactance for the two modes whose eigenvalues coincide without mean flow; resistance is fixed at a value much larger than the optimal one. $\omega = 0.924$.

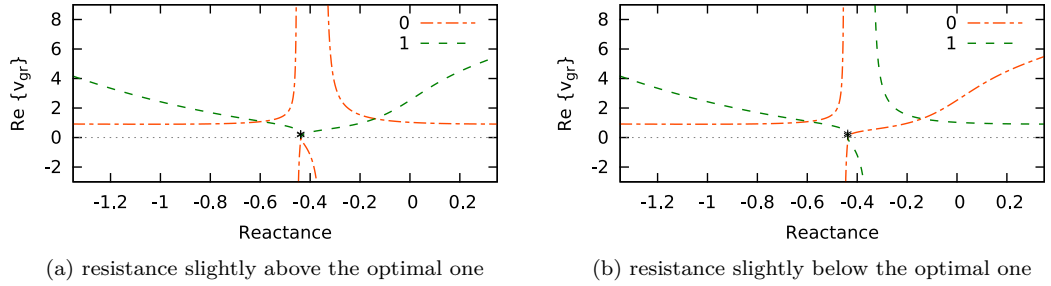


Figure 4.6 – Group velocity vs reactance for the two modes whose eigenvalues coincide without mean flow.

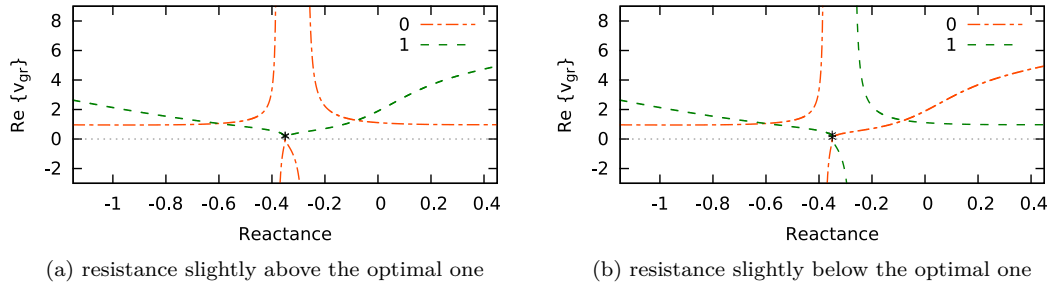


Figure 4.7 – Group velocity vs reactance for the two modes whose eigenvalues coincide with a shear mean flow $M = 0.1$; resistance slightly above and below the optimal one.

With a shear mean flow, the value of the optimal impedance is shifted but the mode behaviour is the same as without flow.

4.6 Extension to 2D modes

Previous exposition for optimal impedance was developed for a unidimensional domain. The main idea can be extended to account for the transverse coordinate also. We recall that in our configuration, only bottom wall is considered to be lined.

For the β corresponding to the double point, we have a set of wavenumbers α , one for each γ . So, all the modes $(0, m)$ and $(1, m)$ are coincident. And, as expected from the results from above, the best attenuation for each pair is obtained at these coincidences. This is shown in figure 4.8 for the same characteristics as above: $\omega = 0.924$, $\mathcal{Z} = 0.55 - 0.44i$, and with an aspect ratio $\mathcal{A} = 1$

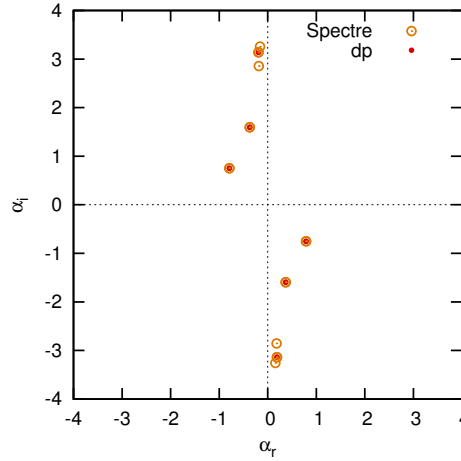


Figure 4.8 – Spectrum without mean flow for the optimal impedance in a bidimensional domain with $\omega = 0.924$. Double points are marked in full circles and labeled ‘dp’.

From figure 4.8 we can see that there are three double points, corresponding with the pairs with index for γ , $m = 0, 1, 2$. The two remaining eigenvalues that do not correspond to double points are modes $(2, 0)$ and $(2, 1)$.

In figure 4.9 we can see the characteristic loops of big intervals of reactance changes with resistance fixed at the optimal value. This shows effectively the double points at the optimal impedance. We can see also the coincidence of modes $(0, 1)$ and $(1, 0)$ for the hard wall values, as expected for $\mathcal{A} = 1$. The eigenvalues of double points imply the best attenuation for the pair, as it was for the unidimensional analysis.

So, the general idea behind the optimal impedance for the unidimensional analysis remains completely valid when growing one more dimension. This emphasises the validity to make the unidimensional studies, taking into account that for complex flow profiles, the whole transverse domain has to be considered.

We could go even further if we put a liner with the optimal impedance in the wall along the $z = -\mathcal{A}$ wall. In this case, all pair of modes $(n, 0)$ and $(n, 1)$ would also coincide. This would make the four modes in figure 4.9: $(0, 0)$, $(0, 1)$, $(1, 0)$ and $(1, 1)$ to be the same, further improving the noise reduction. In the present state of FiEStA, we are not able to verify this, since it only considers fully reflective walls in said position, however it can be modified to study this case.

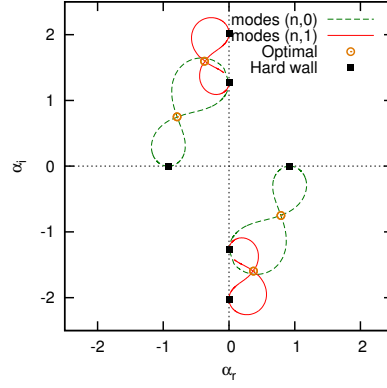


Figure 4.9 – Path of the modes involved in the two first double points for varying reactance at optimal resistance.

In the exposed case, where $\mathcal{A}$ is equal to one, the optimal impedance would be the same as for the wall at $y = -1$, however, in the general case, the optimal impedance may be adjusted to have the double point. We assume that this value should be tweaked considering a nondimensional frequency normalised by $\mathcal{A}$, this way:

$$\omega_z = \mathcal{A}\omega = \frac{L}{H} \frac{H}{c} \tilde{\omega} = \frac{L}{c} \tilde{\omega}$$

and from here, calculate the optimal impedance with the formulas. This should be valid only if the flow profile allows separation of variables. With a general flow profile, this would only give an approximate value.

In this chapter, we started with the quantification of noise reduction. This information was used to identify the spectrum with the best sound attenuation. This spectrum contains always a coincidence of two modes, the called double point.

From there, we defined the optimal impedance, and found its value for some cases. We compared these values with existing formulas and we saw that they correspond well within the hypotheses needed.

We found that the group velocity of the modes of the double point is zero. The behaviour of both modes is exchanged after the optimal resistance between extrema of reactance.

In a 2D analysis, we found the same coincidence.

In next chapter, a configuration for which a hydrodynamic surface mode is present will be presented.

Chapter 5

Hydrodynamic Modes

In this chapter we will show the particular case of the hydrodynamic modes. As it was said when we presented the context in section 1.4.2, for some values of impedance, with some flow profiles, a couple of hydrodynamic modes can arise. These modes have been studied based with the benchmark data provided by Jones and coworkers [60]. The first section 5.1, about the acoustic propagation and the impedance eduction, is a work done by Julien Primus that is exploited here to deepen on the hydrodynamic instability.

In section 5.1, the impedance eduction model is presented and used to find the impedance. The eigenmode analysis is in section 5.2 which is deepened in section 5.3 with the stability analysis. The effects of viscosity on the stability of modes are studied on section 5.5. Finally, conclusions are given.

5.1 Acoustic propagation simulation and impedance eduction

This section gives a comparison between numerical simulations performed at ONERA and benchmark data published by Jones et al. [60]. The case of the NASA grazing incidence tube (GIT) treated with a ceramic tubular liner is studied, in the presence of a grazing flow. A scheme of the test section of the GIT is given in figure 5.1. Sound pressure level (SPL) and phase at the rigid wall opposite the liner sample are acquired with 31 microphones. The duct is lined with a ceramic tubular material referenced CT57, made of parallel cylinders embedded in a ceramic matrix. This locally reacting liner, which is nearly linear with respect to incident acoustic level and mean flow velocity, shows a porosity of $\sigma = 57\%$. The diameter of the cells is 0.6 mm, and their 85.6 mm-depth leads to a resonance frequency around 1 kHz.

Benchmark data provided by Jones et al. [60] are successively processed with two distinct approaches. The *direct simulations* compute the acoustic propagation inside the duct by using the liner impedance educed by Jones et al. as boundary condition, as well as the measured exit impedance for the termination section. Results are then compared to the acoustic pressure experimentally acquired at the rigid wall opposite the liner. In the *impedance eduction* approach, an inverse method is applied to the parietal acoustic pressure measurements to find the impedance of the liner by an optimization technique. This impedance is then compared to the impedance educed by Jones et al. Both direct and inverse aspects have been validated in the configuration

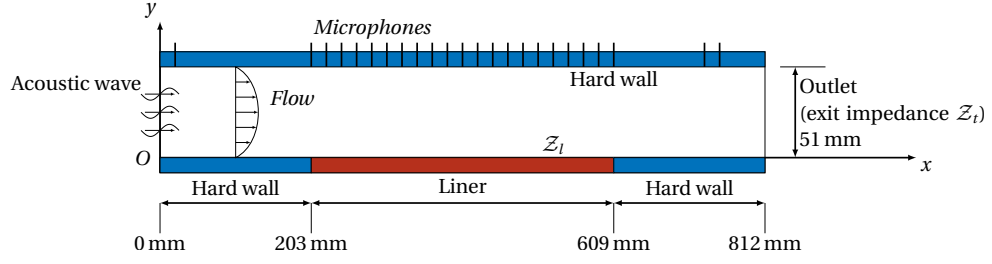


Figure 5.1 – 2D scheme of the test section of the NASA Grazing Incidence Tube.

of the GIT without flow in a previous paper [91].

5.1.1 Direct simulations of acoustic propagation

In the following, a tilde over the variable $\tilde{q}$ denotes a physical quantity, the normalized quantity is $q = \tilde{q}/\tilde{q}_{ref}$. The physical values used to normalize are: the speed of sound c_0 , the mean density of the fluid ρ_0 and the height of the duct H . From these, the normalization values for pressure is $\frac{1}{\rho_0 c_0^2}$ and for time $\frac{H}{c_0}$.

5.1.1.1 Governing equations and numerical method

The computational model relies on the two-dimensional linearized Euler equations, for a harmonic wave with a $e^{i\omega t}$ convention (ω is the angular frequency). As the numerical method is described in details in a previous paper [91], only its essential features are reminded here. Let u (resp. v) be the streamwise (resp. transversal) acoustic velocity, and p the acoustic pressure. The 2D linearized Euler equations for an acoustic perturbation $\varphi = (u, v, p)^T$ (T being the transpose) of a subsonic steady mean flow with velocity U_0 along x and V_0 along y read

$$i\omega\varphi + \mathbf{A}^i \partial_i \varphi + \mathbf{B}\varphi = 0 \quad (5.1)$$

with $\partial_1 = \partial/\partial x$ and $\partial_2 = \partial/\partial y$. The repetition of index i denotes an implicit sum over its possible values, following Einstein summation convention. In the case of an incompressible mean flow, matrices $\mathbf{A}^i$ and $\mathbf{B}$ are defined by:

$$\mathbf{A}^1 = \begin{pmatrix} U_0 & 0 & 1 \\ 0 & U_0 & 0 \\ 1 & 0 & U_0 \end{pmatrix}, \quad \mathbf{A}^2 = \begin{pmatrix} V_0 & 0 & 0 \\ 0 & V_0 & 1 \\ 0 & 1 & V_0 \end{pmatrix} \quad (5.2)$$

and

$$\mathbf{B} = \begin{pmatrix} \partial_1 U_0 & \partial_2 U_0 & 0 \\ \partial_1 V_0 & -\partial_1 U_0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (5.3)$$

As a shear flow profile with a no-slip condition at the wall will be considered in the present study, the impedance boundary condition can be written:

$$\mathcal{Z} = \frac{p}{v^i n_i} = \frac{p}{v_n} \quad (5.4)$$

with $\mathcal{Z}$ the normalized specific impedance and $v_n = v^i n_i$ the normal acoustic velocity.

The equations are spatially discretized with a discontinuous Galerkin (DG) scheme. In this method, the quantities are allowed to be discontinuous across element boundaries, the inter-cell connection being ensured by continuity of the fluxes through an upwind numerical scheme. The numerical fluxes describing the boundary conditions are detailed in [91].

5.1.1.2 Illustration of a numerical flow instability

The acoustic propagation in the GIT bench is simulated in the presence of a grazing flow with a bulk Mach number $M_b = 0.335$ (centerline Mach number of 0.4). The flow profile $M(y)$ has been measured with Pitot tubes by Jones et al. [60]. In the present paper the velocity profile acquired at $(x = 356 \text{ mm} ; z = 25.4 \text{ mm})$ is considered ; these coordinates approximately correspond to the middle of the lined section along x , and to the center of the test cell width. Together with a no-slip condition at the wall, the flow profile is approximated by cubic splines as shown in figure 5.2. A harmonic plane wave is prescribed at the inlet, for test frequencies varying from 0.5

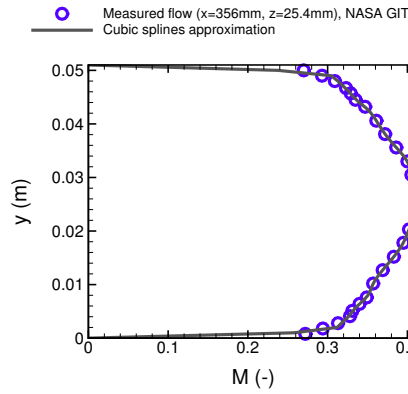


Figure 5.2 – Measured and approximated flow profile. $M_b = 0.335$.

to 3.0 kHz in steps of 0.5 kHz. The physical domain is meshed with Lagrange $\mathbb{P}_1$ elements. A common mesh is used for all frequencies, with a resolution reaching $\lambda/20$ at 3 kHz where λ is the wavelength. This has been found sufficient to avoid numerical diffusion and dispersion. The impedance boundary condition applied to the lined section uses the impedance values identified by Jones et al. [60]. An impedance boundary condition is prescribed at the termination of the duct as well, with impedance values measured by a two-microphone method in the reference paper of Jones et al. [60].

Figure 5.3 shows the comparison between measurements and numerical simulations performed with the discontinuous Galerkin solver. The acoustic pressure at the rigid wall opposite the liner is plotted in terms of sound pressure level and phase along the streamwise coordinate x . As the acoustic wave in the DG code has an arbitrary complex amplitude, computed values are calibrated to match the measurements at $x = 0 \text{ mm}$. Computed SPL and phase compare very favorably to the experimental data. At 1.0 kHz however a numerical instability arised with the original mesh. It has been represented by a dotted line in figure 5.3. At this particular frequency the mesh resolution had to be reduced to $\lambda/12$ in order to introduce numerical diffusion and remove the instability. The result, shown in solid lines, then agrees well with the measured data. The same kind of instability has been observed at 0.5 kHz, but is not plotted in figure 5.3 for the

sake of clarity. The whole field at 1 kHz is plotted in figure 5.4 for the real part of the acoustic pressure, with particularly high contour levels. The figure reveals that the instability appears in the form of spatially growing structures with a wavelength close to 105 m^{-1} , and is initiated at the leading edge of the liner ($x = 203 \text{ mm}$). As already pointed out by Burak et al. [17], the growth of the instability stops at the trailing edge of the material sample ($x = 609 \text{ mm}$).

5.1.2 Impedance eduction

An inverse method for liner impedance eduction is now applied to the configuration described above. The eduction process consists in iterating on the unknown parameters until the calculated acoustic pressure p_{calc} at the rigid wall opposite the liner gives the best prediction of the measured acoustic pressure p_{meas} . The objective function is consequently defined as follows:

$$\mathcal{J} = \sum_{m=1}^N \|p_{\text{calc}}^m - p_{\text{meas}}^m\|^2 \quad (5.5)$$

where $N = 31$ is the total number of microphone probes. Although the liner impedance is the search parameter in which we are mostly interested, other parameters are included to the search process. As an arbitrary amplitude is given to the acoustic perturbation in the DG solver, a complex calibration coefficient C is taken into account. Moreover, the exit impedance can be included as a search parameter to take into account reflections occurring at the duct termination. Two identification cases have been treated:

- ◀ in the “NRp” case (for “Non-Reflective” eduction case from pressure measurements), the termination is assumed to be perfectly anechoic. The exit impedance is set to $\mathcal{Z}_t = 1.0 + 0.0i$, and four search parameters are considered by the inverse process (real part and imaginary part of the liner impedance $\mathcal{Z}_l$ and of the calibration coefficient C).
- ◀ in the “EIp” case (for “Exit impedance” eduction case from pressure measurements), the exit impedance is part of the search process and six search parameters are considered by the inverse process (real part and imaginary part of the liner impedance $\mathcal{Z}_l$, of the exit impedance $\mathcal{Z}_t$ and of the calibration coefficient C). Since the termination is expected to be nearly anechoic, the initial value chosen for the exit impedance $\mathcal{Z}_t$ is the one of a non-reflecting termination:

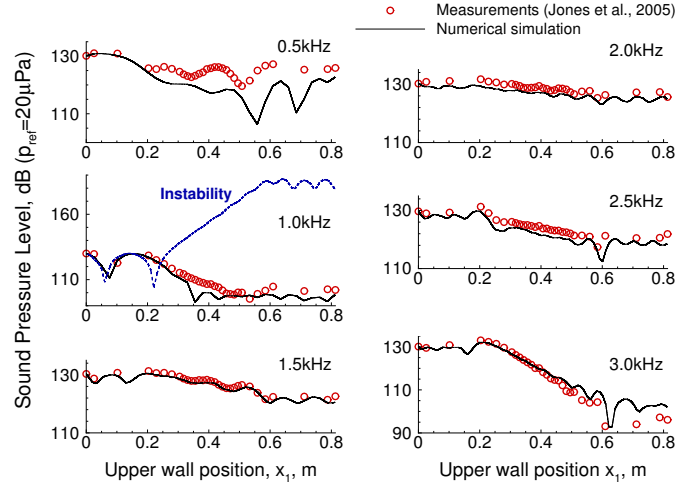
$$\mathcal{Z}_t^{\text{init}} = 1.0 + 0.0i \quad (5.6)$$

In both cases the initial value for the liner impedance $\mathcal{Z}_l$ is fixed at $\mathcal{Z}_l^{\text{init}} = 1.0 + 1.0i$. The impedances are held into the following bounds:

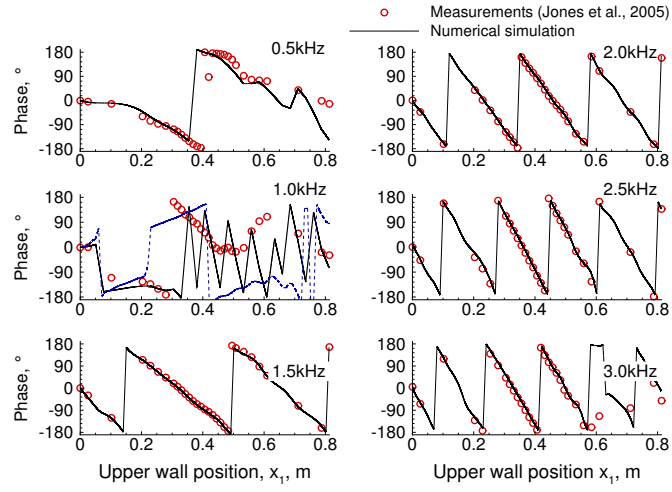
$$\begin{cases} 0.0 \leq r \leq 5.0 \text{ and } -5.0 \leq \chi \leq 5.0 & \text{for the liner impedance} \\ 0.5 \leq r \leq 1.5 \text{ and } -0.5 \leq \chi \leq 0.5 & \text{for the exit impedance} \end{cases} \quad (5.7)$$

where $r = \text{Re}(\mathcal{Z})$ is the normalized specific resistance and $\chi = \text{Im}(\mathcal{Z})$ the normalized specific reactance. The calibration coefficient C is not bounded. The same meshes as in the previous section are used, the mesh being released at 0.5 and 1.0 kHz.

A limited memory version of the BFGS (Broyden, Fletcher, Goldfarb, Shanno) algorithm for bounded parameters [20] is used to find the minimum of $\mathcal{J}$. The gradient of the objective function required by this quasi-Newton optimization method is calculated analytically thanks to the resolution of the adjoint equations, which allows to increase the number of parameters without



(a) Sound pressure level (dB)



(b) Phase (°)

Figure 5.3 – Sound pressure level and phase at the rigid wall opposite the liner, in the NASA GIT with a flow of bulk Mach number $M_b = 0.335$. Measured values (circles) and simulated values (lines).

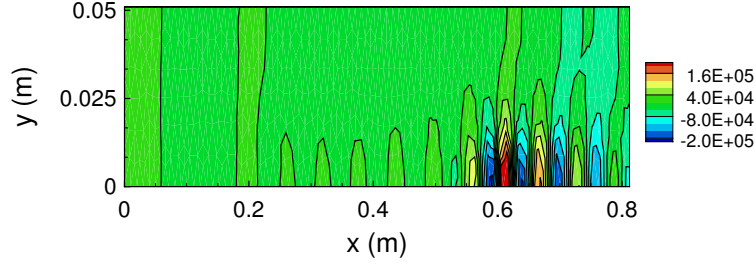


Figure 5.4 – Acoustic pressure.

increasing the computational cost as occurs with classical methods such as finite differences. Further details on the inverse process can be found in reference [91].

The impedances identified with the DG method in the “NRp” and “EIp” cases are compared to the one deduced by Jones et al. [60] in figure 5.5, showing a good agreement between the two methods. Note that at the end of the inverse procedure, none of the resulting resistances or reactances is at the lower or upper bound defined by equation (5.7). A small discrepancy appears at $f = 2$ kHz ; at this frequency, the absorption effects of the liner are particularly weak, leading to convergence difficulties in the optimization algorithm. This behavior has already been noticed and discussed in the case without flow [91]. Another disagreement between the impedances found with the two methods emerges at the lowest frequency $f = 0.5$ kHz. This difference could be attributed to the form of the flow profile, which is assumed to be uniform by Jones et al. [60] whereas a shear profile is assumed in the present method. Watson and Jones [119, 120] actually mentioned anomalous behaviours at low frequencies under uniform flow assumption, concluding on the necessity to take into account boundary layer effects for a correct estimation of the liner impedance. Nevertheless a significant disparity also emerges depending on whether the “NRp” or the “EIp” case is looked upon, even though they both use the same flow profile. The results at $f = 0.5$ kHz therefore remain unclear.

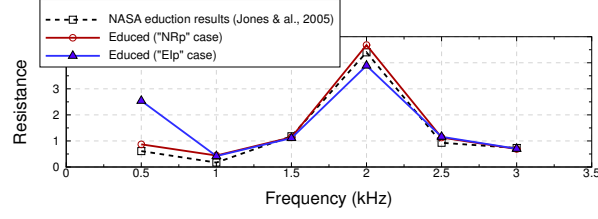
It is worth noting that in spite of the small mesh resolution at 1 kHz, the present solver converged to the same impedance as Jones et al. However, the presence of the numerical instability revealed in section 5.1.1 raises the question of the meaning of this impedance. Two cases may arise:

- ◀ if the instability is purely numerical then the fact to remove it by releasing the mesh is legitimate, and the deduced impedance is representative of the physical behavior of the liner ;
- ◀ if the instability is linked to a hydrodynamic mode, the deduced impedance may be meaningful only if the mode has no effect on the parietal pressure.

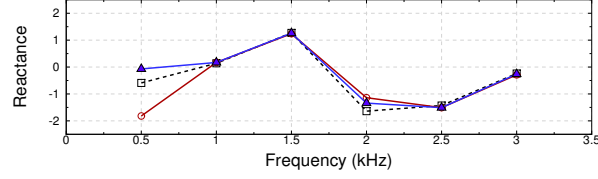
In this context, it is essential to have an accurate knowledge of the eigenmodes of the treated duct, in this configuration of frequency and flow profile. This is the goal of the next section.

5.2 Eigenmode Analysis

In this section, we perform an analysis of the spatial eigenvalue problem to identify unstable modes. This study is made along the dimension normal to the lined wall (y) and at a streamwise



(a) Liner resistance



(b) Liner reactance

Figure 5.5 – Liner impedance identified from pressure measurements in the NASA GIT, with a bulk Mach number $M_c = 0.335$. Values educed by Jones et al. [60] (dashed lines and rectangles), and educed with our method in the “NRp” case (solid lines and circles) and in the “Elp” case (solid lines and triangles).

position centered in the domain; this way we suppose to be placed far enough from the ends of the liner to ignore border effects, and we can focus on the behaviour of the local modes.

As we are searching for flow instabilities, the choice of the mean flow to be considered is of primary importance. To get rid of any liner-induced modification of the flow profile, and to be comparable with the numerical simulations of section 5.1, we decided to consider only the mean flow profile measured upstream the lined section as the mean flow for the linear stability analysis. However, it must be kept in mind that results of this analysis could be sensitive to the mean flow profile. This will be investigated in section 5.4.1

Figure 5.6 shows the experimental measurements of the two different mean flow profiles, upstream and midstream compared, with the interpolation by Chebyshev polynomials. The variation between both profiles can be due to the non-linear interaction with the modes generated over the liner or, more probably, to the roughness effect of the liner. The apparent difference between bulk flow-rates is compensated near the corners of the duct.

5.2.1 Governing equations

In order to write the linearized Euler equations as an eigenvalue problem, from equations (5.1), (5.2) and (5.3), the flow and pressure fields are written as the sum of a steady base value plus a small-amplitude unsteady perturbation:

$$\frac{\tilde{q}}{\tilde{q}_{ref}} = q = Q_0 + q'$$

where Q_0 stands for the base value and q' for the disturbance which is then sought under the harmonic form:

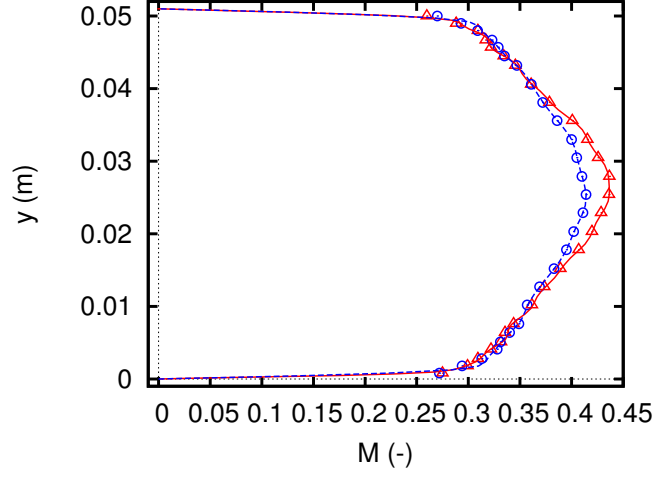


Figure 5.6 – Comparison of mean flow profiles at center-line. Upstream profile with red triangles and full line, midstream profile in blue circles and dashed line. Experimental data from Jones et al. [60] in symbols. Chebyshev interpolation with lines.

$$q' = \hat{q} \cdot e^{i\omega t - i\alpha x} \quad (5.8)$$

With this, equation (5.1) reads:

$$\mathbf{R}(U, \omega)\varphi = \alpha \mathbf{S}\varphi \quad (5.9)$$

where α , the eigenvalue is the complex streamwise wavenumber and operators $\mathbf{R}$ and $\mathbf{S}$ are defined by:

$$\mathbf{R} = i\omega \mathbf{I} + \mathbf{A}^2 \frac{\partial}{\partial y} + \mathbf{B} \quad , \quad \mathbf{S} = i\mathbf{A}^1 \quad (5.10)$$

keeping in mind that $\frac{\partial}{\partial x}\varphi = -i\alpha\varphi$.

As the base flow considered is the red curve of figure 5.6, which is parallel to the x axis and has only dependence on y , then $\mathbf{R}$ and $\mathbf{S}$ matrices read:

$$\mathbf{R} = \begin{pmatrix} i\omega & U_y & 0 \\ 0 & i\omega & D_y \\ 0 & D_y & i\omega \end{pmatrix} \quad , \quad \mathbf{S} = \begin{pmatrix} iU & 0 & i \\ 0 & iU & 0 \\ i & 0 & iU \end{pmatrix} \quad (5.11)$$

where U and U_y denote the mean flow and its y derivative.

These equations are similar to equation (30) of [78] with the coice of $+i\omega$ convention for the time dependence of the modal form.

5.2.2 Impedance model

Boundary conditions are described at the top by a perfectly reflective wall condition and at the bottom by the impedance condition as given in equation 5.4. The frequency dependence of this impedance is described by the enhanced Helmholtz resonator model [96]. This model is defined by a set of parameters linked to physical characteristics of the liner:

$$\mathcal{Z} = r + id\omega - i\beta_{\mathcal{Z}} \cot\left(\omega \frac{L}{c_0} - i\frac{\varepsilon_{\mathcal{Z}}}{2}\right) \quad (5.12)$$

where $r + id\omega$ is the face-sheet impedance, L is the depth of the resonators, $\beta_{\mathcal{Z}}$ is a parameter for scaling the cavity reactance, $\varepsilon_{\mathcal{Z}}$ is related to the damping by the fluid within the cavity. To be physically meaningful, all parameters are required to have positive values[96].

The experimental values of the impedance, obtained by Jones et al. [60] with a bulk Mach number of 0.33, are shown in figure 5.7, compared with the EHR model. The values used for these parameters, based on the best fitting of the data from the GIT, under the physical constraints of positivity, are:

$$r = 10^{-6} , \quad d = 10^{-6} , \quad \frac{L}{c_0} = 2.677 \times 10^{-4} , \quad \beta_{\mathcal{Z}} = 0.58 , \quad \varepsilon_{\mathcal{Z}} = 1.4$$

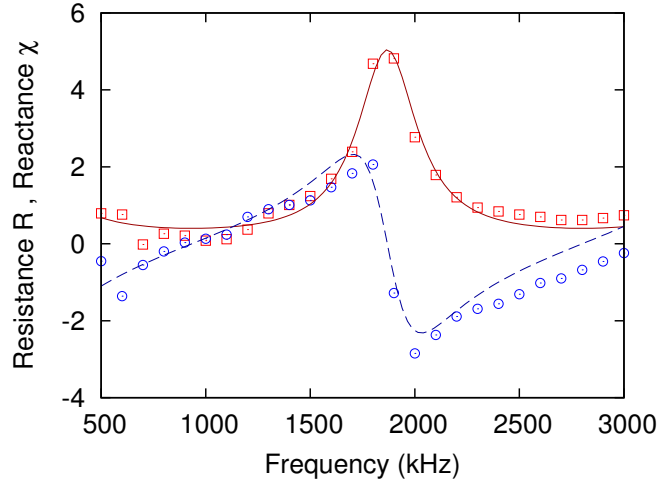


Figure 5.7 – EHR Impedance model in lines, compared to data from Jones et al. [60] in symbols. Resistance in red solid line and squares; reactance in blue dashed line and circles.

5.2.3 Numerical scheme

The eigenvalue problem 5.9 is solved by a spectral collocation method based on Chebyshev polynomials, as in [12] where more details can be found.

The mesh convergence is verified by progressively increasing the quantity of points until the eigenvalues stop moving to a precision of 10^{-6} . After testing the convergence, we take 20% extra points to avoid possible changes in different cases.

The eigenfunctions are normalized in such a way that the maximum of pressure fluctuation is always set equal to 1.

5.2.4 Examples of eigenmodes

The spectrum obtained for a frequency of 1 kHz is shown in figure 5.8. The modes of the continuous branch are shown by small diagonal crosses; in the following of this paper, this modes will not be shown anymore for the sake of clarity. Acoustic modes are represented in red circles and hydrodynamic modes in green full squares. Labels are used for one acoustic mode (Ac2) and for both hydrodynamic modes (H1 and H2).

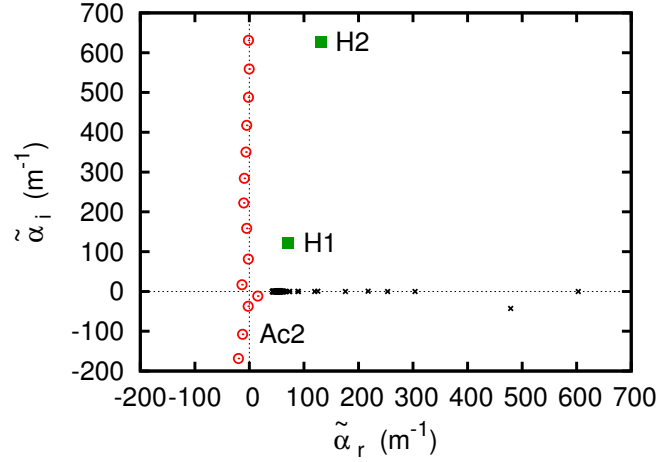


Figure 5.8 – Acoustic modes in red circles $\circ$. Hydrodynamic modes in green full square $\blacksquare$. Labeled modes for future reference.

Figure 5.9 shows the amplitude of the eigenfunctions for the right-running acoustic mode labeled Ac2 in figure 5.8 and the two hydrodynamic modes H1 and H2. The shape of both hydrodynamic modes are equivalent and they correspond to surface modes, given that their amplitude is the highest close to the lined wall, while the acoustic mode show the classic oscillatory shape. This is observed for the pressure and both components of the velocity.

A hydrodynamic instability is supposed to exist for this configuration, that would explain the unstable behaviour observed in figure 5.3. A similar behaviour has been found in the works of Boyer et. al. [12] and Marx et. al. [75] for another mean flow profiles and liners. This will be deepen in next section.

5.3 Stability analysis

Given the positive imaginary part of the eigenvalue got for both hydrodynamic modes shown in figure 5.8, taking into account the modal form (5.8); each one can be either a stable mode propagating to the left (negative x) or an unstable right running mode. Thus, to determine the stability of these modes, we need to determine their sense of propagation. To do this, we resort

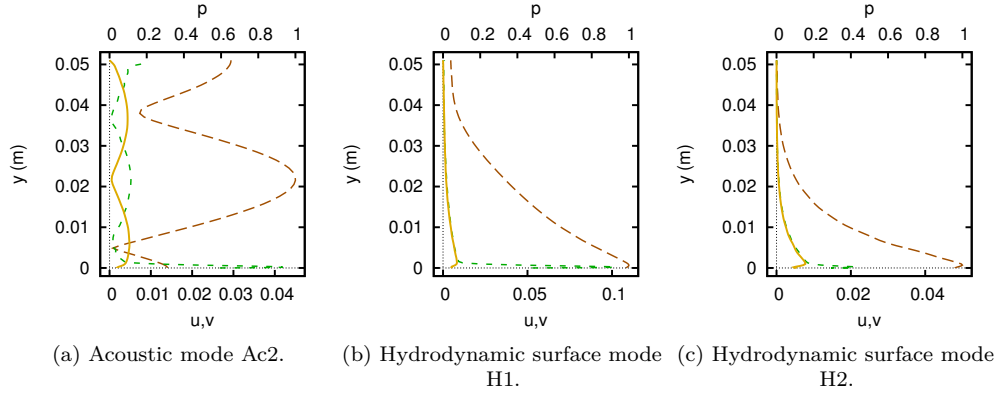


Figure 5.9 – Absolute values of eigenfunctions for selected modes.

— — pressure , - - - streamwise velocity , — normal velocity.

to Briggs-Bers [8] stability criterion, developed initially for plasmas and widely used nowadays in aeroacoustics. This criterion, as well as the needed requirements of the model for its validity are clearly explained in [13]. It is shortly explained hereafter.

5.3.1 Stability criterion

Briggs-Bers (BB) criterion is based on a double integral transform of the problem; Fourier in space and Laplace in time. To be applied, it needs one inversion contour for each transform.

The Laplace transform $t \rightarrow i\omega$ has to respect causality; for this to be done, we need a contour that splits the ω complex plane in two halves, the lower one without any mode and the upper one with all of them. To achieve this, the model needs that all temporal modes have a bounded growth, which means that their negative imaginary part has to be limited.

The Fourier transform $x \rightarrow \alpha$ distinguishes the modes according to their propagation direction; to do so, the contour has to divide the α complex plane in such a way that all left-propagating modes remain in the upper part and all the right-propagating ones in the lower one.

To know the direction of propagation of a mode, the imaginary part of the frequency has to be increased from highly negative values to zero; the modes that follow a trajectory coming from the lower half are right-propagating and those who come from up, go to the left.

If when the real frequency is reached, all modes remained in the same half-plane from which they are originated, the system is stable. If while making this trajectory, some mode crosses the real axis, it becomes convectively unstable. If, still with a negative ω_i , two spatial modes from opposite sides of the real axis coincide, we are dealing with an absolute instability. In this case the large-time behaviour of the system is determined by this frequency and wavenumber, in both senses, upwards and downwards. In figure 5.10 we find the application of Briggs-Bers criterion representing the three possible cases.

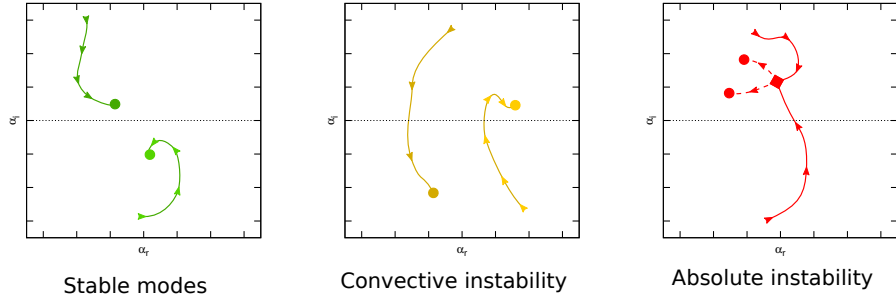


Figure 5.10 – Use of Briggs-Bers criterion to verify the sense of propagation and stability of modes.

5.3.1.1 Validity of Briggs-Bers criterion

To verify if this criterion is applicable and that it does not lead to an *ill-posed* problem as stated by Brambley [13], the system must have a bounded maximal temporal growth for real wavenumbers.

It will be shown later that only the mode H1 is implicated in this validation because the mode H2 is left-propagating, thus it is stable and it has a decrease rate and not a growth rate.

The limit of the growth rate is sought by following the H1 mode in the ω plane. For this, the eigenvalue problem 5.9 is rewritten in order to solve the temporal eigenvalue problem:

$$\mathbf{R}'(U, \alpha)\varphi = \omega \mathbf{S}'\varphi \quad (5.13)$$

Next, the imaginary part of the frequency is decreased until a pure real wavenumber is obtained. Then, in the temporal analysis, the path of ω is followed when α_r is swept. The imaginary part of ω has to remain limited in the negative half plane, so that the mode has a bounded temporal growth.

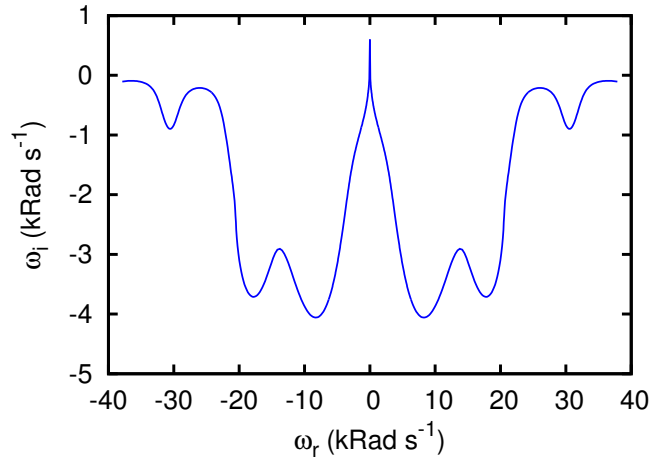


Figure 5.11 – Trajectory of the hydrodynamic mode H1 in temporal analysis. It shows a bounded temporal growth.

In figure 5.11 the trajectory for the temporal analysis of the hydrodynamic mode H1 for varying real wavenumbers is shown. It shows clearly that ω_i is bounded by its negative side, indicating the bounded temporal growth, thus allowing the use of Briggs-Bers criterion to analyse the stability of modes in this case.

This result is in agreement with Rienstra [96], who states that the EHR model of impedance, with given constraints for the parameters, does not lead to an ill-posed problem.

5.3.2 Stability of hydrodynamic modes

Once knowing that the BB criterion is valid for our case, we track the hydrodynamic modes in the α plane by changing ω_i .

The paths for the H1 and H2 modes are shown in figure 5.12. To obtain this plot, instead of increasing the imaginary part of ω as stated in the description of the stability criterion, we actually decrease ω_i from 0, for which the α values of both modes are already known. During the track, the minimum value of ω_i is $-10000 \text{ rad} \cdot \text{s}^{-1}$ which is well below the lower limit shown in figure 5.11 for the H1 mode. As the mode H2 doesn't cross the real axis, it is a left-running mode. For the mode H1, things are different, as it comes from the lower half of the plane and crosses the real axis: it is actually a mode propagating towards the right. This, combined with the positive sign of α_i , implies that it is a convectively unstable mode.

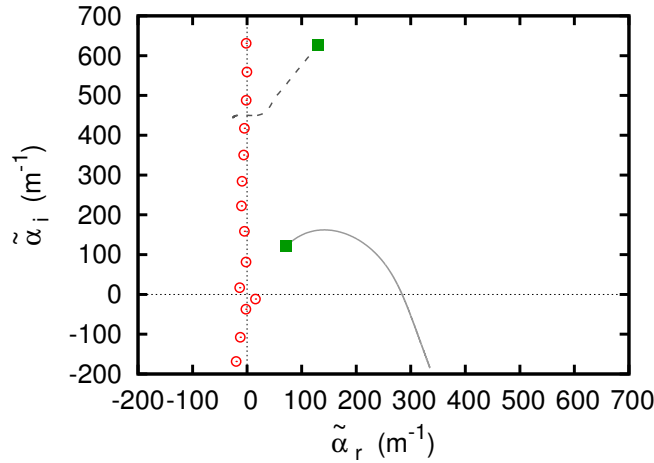


Figure 5.12 – Briggs-Bers at 1 kHz for the hydrodynamic modes. — H1 , - - - H2 , ■ $\omega_i = 0$. ω_i provenance for negative values, shows it is a right running mode.

5.3.3 Evolution in frequency

To know if this instability is present only for 1 kHz, as it is observed in section 5.1, a frequency sweep has been made between the limits of the impedance eduction performed by Jones [60], i.e. from 0.5 to 3.0 kHz.

This results are shown in figure 5.13, with emphasis on the values at 0.5, 1.0, 1.5 and 3.0 kHz. It shows that for some frequency between 1.0 and 1.5 kHz, the H1 mode reaches the acoustic

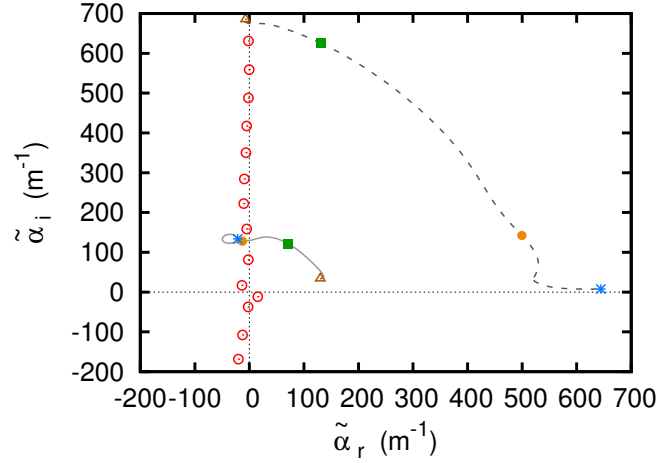


Figure 5.13 – Wavenumber evolution of H1 and H2 modes for different real frequencies: ■ 1.0 kHz full green square, △ 0.5 kHz brown empty triangle, ● 1.5 kHz yellow full circle, * 3.0 kHz blue asterisk.

branch, while the opposite happens to the mode H2, which is acoustic for 0.5 kHz, and transforms into a hydrodynamic one when the frequency increases.

When repeating the BB analysis for 1.5 kHz, the continuation of the mode H1, now an acoustic mode, behaves, as expected, as a left running mode, while the mode H2 becomes unstable. Only with this analysis we can not tell at which frequency modes change their propagation sense.

5.3.4 Absolute instability

If both modes interchange their direction at a negative imaginary part for ω , this is due to a point where an absolute instability takes place, as two modes form opposite propagation directions coincide. To be sure if this is the case here, the BB analysis was repeated for several real frequencies.

In figure 5.14, we superimpose to the frequency swept of figure 5.13 the path of modes H1 and H2 obtained with variations of ω_i for 1043 and 1045 Hz. By the BB criterion, this means that at 1043 Hz, H1 mode remains unstable and H2 mode remains stable. However, at 1045 Hz, both modes have acquired the behaviour that had the other at 1043 Hz. This shows that modes H1 and H2 do coincide in a point in this narrow interval of frequencies. As previously stated, this branch point in the spectre indicates an absolute instability, because both modes have different sense of propagation, and ω_i variations are from zero toward the negative part. This is observed for other case by Marx [78].

This coincidence is not dependent of the input frequency, and should be present whichever it is. And, since its frequency is quite close to the 1 kHz problem found in previous sections, it could have some relation with the unstable behaviour found in section 5.1.

As we are dealing with an absolute instability, and not only a convective one, the flow profile could already have been modified by it, even if we took the precaution of using the upstream flow profile. However, this could not be case, as in [78] given that viscosity, not taken into account in

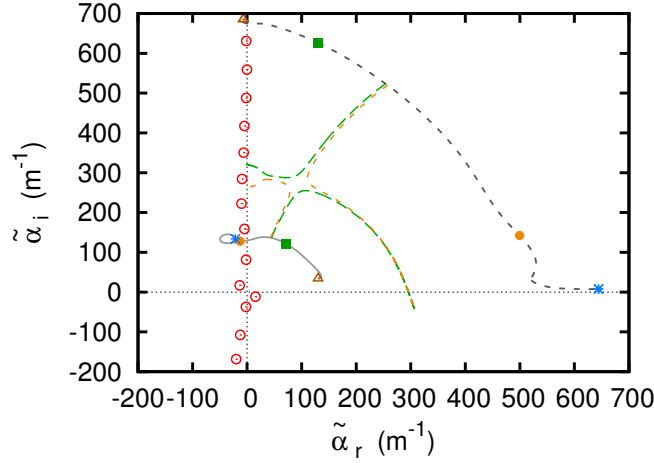


Figure 5.14 – Trajectories of H1 and H2 modes from 1043 Hz (— — —) and 1045 Hz (· · · · ·) showing an absolute instability.

Euler equations, can dampen this instability, this is studied in section 5.5.

5.4 Sensibility to parameters

In this section we will study the sensibility of the stability of modes with two physical modifications. On one side, we will repeat the previous analysis with the midstream flow profile. On the other side, impedance model will be varied by the modification of some parameters of the model.

5.4.1 Sensibility to mean flow profile

In latter sections, we have shown the presence of an hydrodynamic instability with the flow profile upstream the liner.

To verify the sensibility of our results to the shape of the flow profile, we have repeated the results with the midstream profile, i. e. the blue curve in figure 5.6.

The results are in accordance with that shown. There are also two hydrodynamic surface modes, one stable and one unstable at 1 kHz. There is also an absolute instability, in this case, the frequency is slightly lower; it lies between 1029 and 1031 Hz. Figure 5.15 shows the evidence of such instability. It is similar to figure 5.14 with the swept in frequencies in both real and imaginary parts.

5.4.2 Sensibility to parameters of the impedance model

As done in previous section with the flow profile, the sensibility with variation of two parameters, β_Z and ε_Z of the impedance model was also studied. In fact, the variation was done only for β_Z while ε_Z was adjusted to have the least difference between the model and the experimental points, while keeping r , d and L as before.

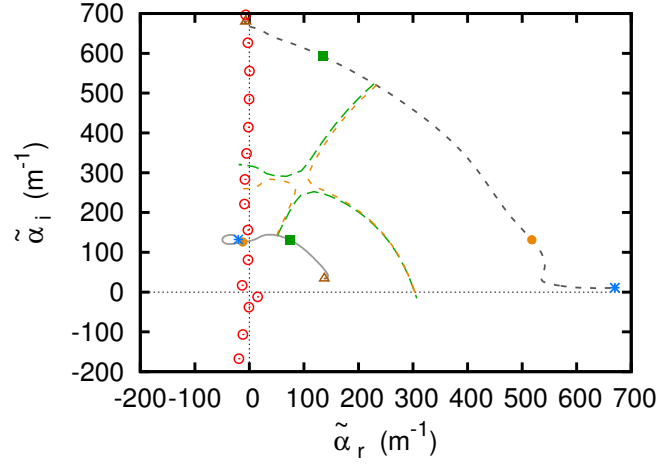


Figure 5.15 – Trajectories of H1 and H2 modes from 1029 Hz (— — —) and 1031 Hz (· · · · ·) showing an absolute instability with the midstream flow profile.

The values chosen for β_Z were 0.48 and 0.68. From those, ε_Z was adjusted to 1.3 and 1.55 respectively. We show the EHR model with those values in figure 5.16.

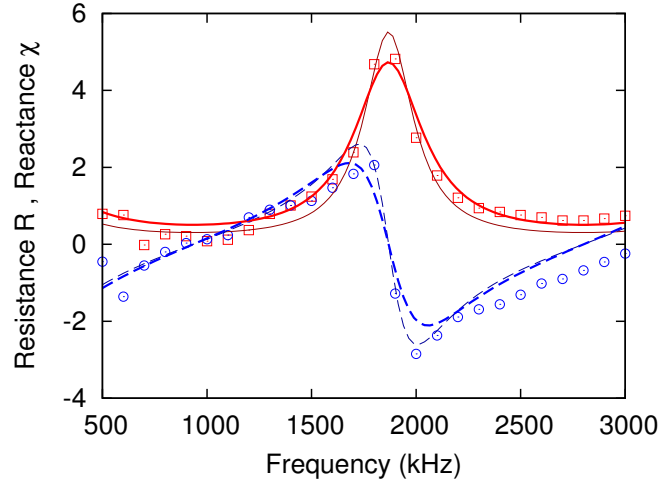


Figure 5.16 – EHR Impedance model with two values of β_Z and respective values of ε_Z , data from Jones et al. [60] in symbols. Resistance in red solid line and squares; reactance in blue dashed line and circles. Model with $\beta_Z = 0.48$, $\varepsilon_Z = 1.30$ in thin lines, and with $\beta_Z = 0.68$, $\varepsilon_Z = 1.55$ in wide lines.

In this case, the spectrum gets more modified than with the change of flow profile. In figure 5.17 we can see the spectre for the three cases compared. Note that the scale has changed from figure 5.15. Even that hydrodynamic modes have moved, they are still present in both cases. Acoustic modes, on the other hand remain close to those in the previous analysis, as expected.

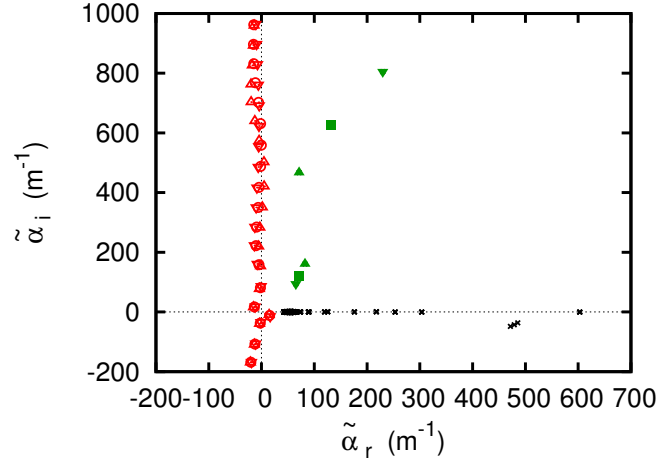


Figure 5.17 – Spectra obtained with the three set of parameters. Model with $\beta_Z = 0.48$ and $\varepsilon_Z = 1.3$ in upside-down triangles: $\blacktriangledown$ for H1 and H2 modes and $\blacktriangledown$ for acoustic modes. Model with $\beta_Z = 0.68$ and $\varepsilon_Z = 1.55$ in triangles: $\blacktriangle$ for H1 and H2 modes and $\blacktriangle$ for acoustic modes.

If we repeat the Briggs-Bers analysis with the hydrodynamic modes for these two cases, we obtain the same results as before. While mode H1 is unstable, mode H2 is stable. This is shown for both models together in figure 5.18.

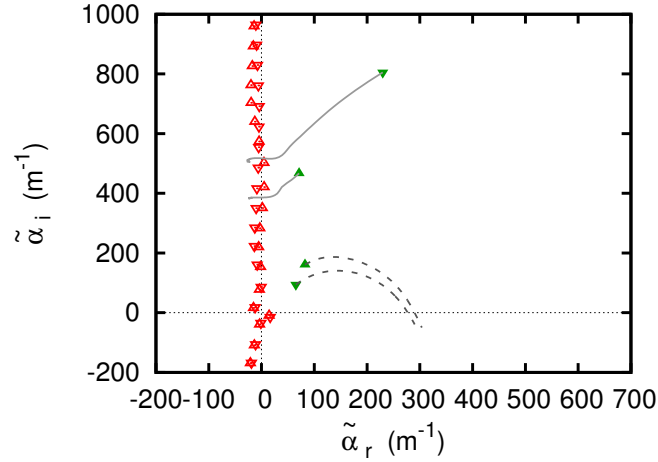


Figure 5.18 – Briggs-Bers stability analysis at 1 kHz. Made for the hydrodynamic modes for two set of parameters of the impedance model.

When varying the frequency, both modes follow similar paths from that of the original model. This is shown in figure 5.19. If we repeat the stability analysis at marked frequencies, we find that they have a change in stability between 1000 and 1500 Hz. If we narrow the interval at which this exchange takes place, we find that the frequency is not the same as before, but it lies

close. This is the evidence of an absolute instability. In figures 5.20a and 5.20b we show the Briggs-Bers path of both modes in each model. The exchange of behaviour takes place between 1065 and 1067 for the model with $\beta_Z = 0.48$ and between 1025 and 1027 when $\beta_Z = 0.68$.

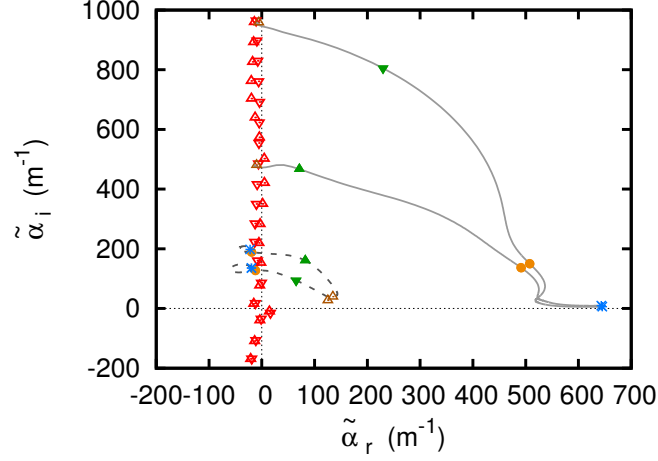


Figure 5.19 – Wavenumber evolution of H1 and H2 modes for different real frequencies: $\blacktriangle$ $\blacktriangledown$ 1 kHz full green triangle, $\triangle$ 0.5 kHz brown empty triangle, $\bullet$ 1.5 kHz yellow full circle, $*$ 3 kHz blue asterisk.

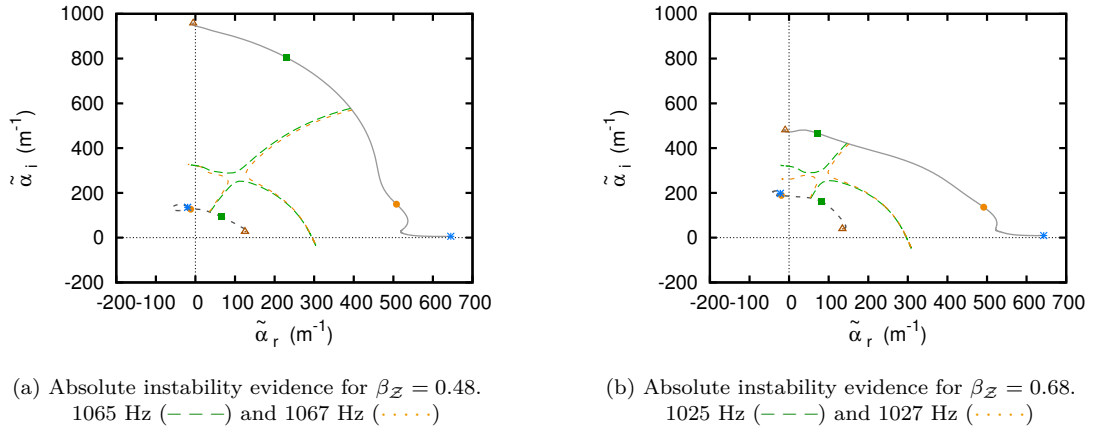


Figure 5.20 – Trajectories of H1 and H2 modes for two models of impedance. An absolute instability is shown at different but close frequencies.

The sensibility with respecto to the flow profile and the impedance model show that the behaviour remains the same as seen before. This indicates that the absolute instability is indeed present in this model.

5.5 Investigation of viscosity effects

To verify whether the unstable mode is damped by the viscosity or it is still present, the same case is now studied as an eigenvalue problem for the linearized Navier-Stokes equations.

5.5.1 Governing equations

If we develop the linearised Navier-Stokes equations with the modal shape (5.8), we put in one single matrix equation, and using $\omega^* = (\omega - U\alpha)$, the system reads:

$$\begin{pmatrix} -i\alpha\rho_0 & \rho_0 D_y & i\omega^* \frac{\rho_0}{Pr} & -i\omega^* \frac{\rho_0}{h_0} \\ -\mu_0 D_y^2 + i\omega^* \rho_0 + \frac{4}{3}\alpha^2 \mu_0 & \frac{i\alpha\mu_0}{3} D_y + \rho_0 U_y & -i\alpha & -KU_y D_y - KU_{yy} \\ \frac{1}{3}i\alpha\mu_0 D_y & \frac{4}{3}\mu_0 D_y^2 + i\omega^* \rho_0 + \alpha^2 \mu_0 & D_y & -i\alpha K D_y \\ -2\mu_0 U_y D_y & 2i\alpha\mu_0 U_y & -i\omega^* & -\frac{\mu_0}{Pr} D_y^2 + i\omega^* \rho_0 - K(U_y)^2 + \frac{\alpha^2 \mu_0}{Pr} \end{pmatrix} \begin{pmatrix} u \\ v \\ p \\ h \end{pmatrix} = 0 \quad (5.14)$$

Where μ_0 , h_0 are the mean values for viscosity and enthalpy, respectively. Pr is the Prandtl number, the ratio between viscous and thermal diffusion rates, and K is a function obtained from the law of Sutherland with shape: $K(y) = \frac{\mu_{ref}}{2} (h_0 h_{ref})^{-\frac{1}{2}}$ where the $(\cdot_{ref})$ subindex indicates reference values.

The LNSE (5.14) can be written with the shape:

$$\mathbf{R}^\mu(U, \omega)\varphi - \alpha \mathbf{S}^\mu \varphi + \alpha^2 \mathbf{Q}^\mu = 0 \quad (5.15)$$

To deal with the squared alpha, we rewrite the system to put it in the shape of a general eigenvalue problem. To do so, we account on an auxiliary variable: $\psi = (\alpha\varphi \ \varphi)^T$ and we write the system as:

$$\begin{pmatrix} \mathbf{Q}^\mu & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \psi + \alpha \begin{pmatrix} -\mathbf{S}^\mu & \mathbf{R}^\mu \\ -\mathbf{1} & \mathbf{0} \end{pmatrix} \psi = 0 \quad (5.16)$$

Where:

$$\mathbf{R}^\mu = \begin{pmatrix} 0 & \rho_0 D_y & i\omega \frac{\rho_0}{p_0} & -i\omega \frac{\rho_0}{h_0} \\ -\mu_0 D_y^2 + i\omega \rho_0 & \rho_0 U_y & 0 & -K U_y D_y - K U_{yy} \\ 0 & \frac{4}{3}\mu_0 D_y^2 + i\omega \rho_0 & D_y & 0 \\ -2\mu_0 U_y D_y & 0 & -i\omega & -\frac{\mu_0}{Pr} D_y^2 + i\omega \rho_0 - K (U_y)^2 \end{pmatrix}$$

$$\mathbf{S}^\mu = \begin{pmatrix} i\rho_0 & 0 & -iU \frac{\rho_0}{p_0} & iU \frac{\rho_0}{h_0} \\ -iU \rho_0 & -\frac{i\mu_0}{3} D_y & i & 0 \\ -\frac{1}{3}i\mu_0 D_y & -iU \rho_0 & 0 & iK D_y \\ 0 & -2i\mu_0 U_y & iU & -iU \rho_0 \end{pmatrix}$$

$$\mathbf{Q}^\mu = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{4}{3}\mu_0 & 0 & 0 & 0 \\ 0 & \mu_0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mu_0}{Pr} \end{pmatrix}$$

5.5.2 Influence of viscosity

Figure 5.21 shows the influence of the viscosity on the eigenvalues; this is made by solving the eigenvalue problem for several values of μ_0 up to the physical value $\mu_a = 1.81 \times 10^{-5} \text{Pa s}$.

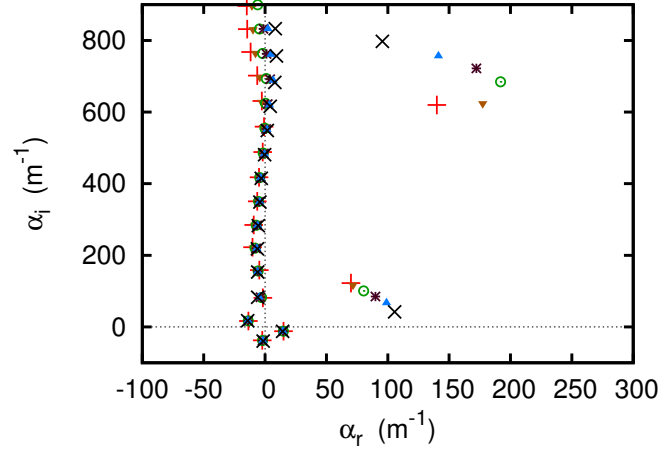


Figure 5.21 – Spectra at 1 kHz for several values of viscosity: $+$ $\mu_0 = 0$, $\blacktriangledown$ $\mu_0 = \frac{\mu_a}{100}$, $\circ$ $\mu_0 = \frac{\mu_a}{10}$, $*$ $\mu_0 = \frac{\mu_a}{4}$, $\blacktriangle$ $\mu_0 = \frac{\mu_a}{2}$, $\times$ $\mu_0 = \mu_a$.

The acoustic modes stay almost in the same location in the spectrum whichever the viscosity. On the contrary, both hydrodynamic modes are strongly influenced by the change of viscosity: α_i is lowered for the mode H1 and increased for H2.

As a consequence, the variation of α_i observed in figure 5.21 means that the spatial evolution of both hydrodynamic modes is damped by the increase of viscosity.

In this study, viscosity is considered constant in all the domain, so no viscosity model is used. In [78], viscosity is modelled and they state that results are alike to those made without a model but with a viscosity increased 30 times. When we repeat the calculations with that value of viscosity, the mode H1 lies in the range of the continuous branch, as shown in figure 5.22.

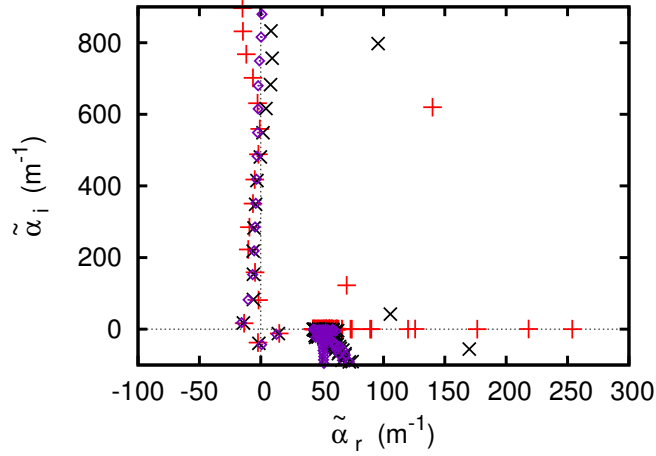


Figure 5.22 – Viscosity impact on spectra: $+$ $\mu_0 = 0$, $\times$ $\mu_0 = \mu_a$, $\diamond$ $\mu_0 = 30\mu_a$.

As the continuous branch is completely in the attenuation region, in this case it is not an instability. With respect to mode H2, it becomes an acoustic mode, erasing any possibility of unstable behaviour.

The rest of the viscous analysis will be made with the actual viscosity of air, i.e. not increased.

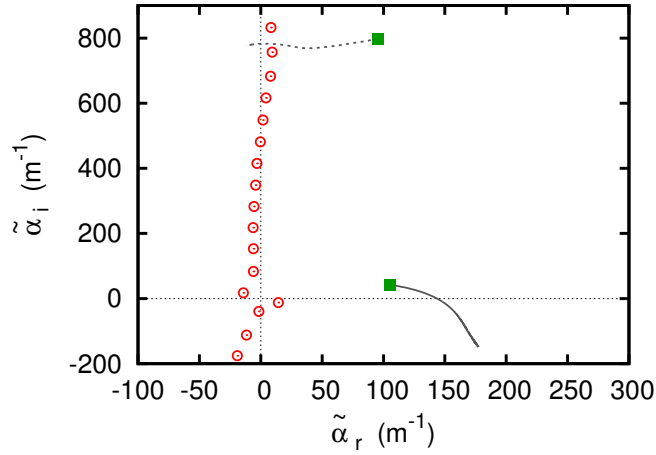


Figure 5.23 – BB at 1kHz with viscosity.

When we apply the BB criterion, shown in figure 5.23, to modes H1 and H2 in the case $\mu_0 = \mu_a$, we find that H1 is still a right-running convective instability, while mode H2, is a left running stable mode.

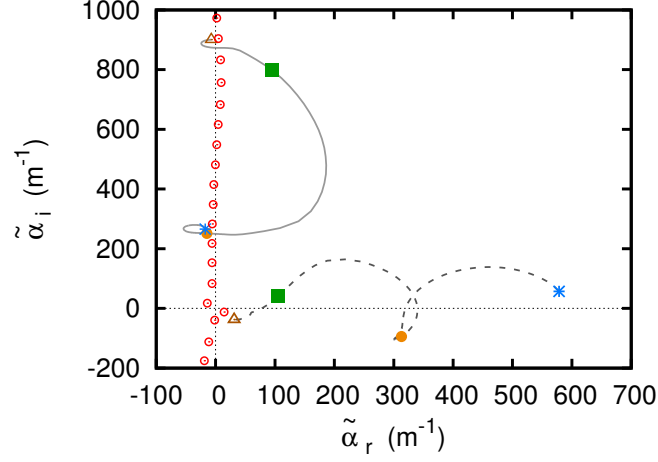


Figure 5.24 – Wavenumber evolution of H1 and H2 modes for different real frequencies with viscosity set to the viscosity of air: ■ 1 kHz full green square, △ 0.5 kHz brown empty triangle, ● 1.5 kHz yellow full circle, * 3 kHz blue asterisk.

Finally, when we track the H1 mode for several frequencies, in the case with $\mu_0 = \mu_a$ we can see that there is a range of frequencies at which the mode becomes stable, around 0.5 and 1.5 kHz, while the greatest amplification rates lie around 1 and 3 kHz. We can see also that in this case, there is not an exchange of behaviour between the two hydrodynamic modes, which implies that an absolute instability can not take place.

Conclusion

We have shown an impedance reduction method which evidences an unstable behaviour.

We found hydrodynamic modes both stable and unstable at 1 kHz. Their stability has been studied by the Briggs-Bers criterion. Given the flow profile and impedance model, there is an absolute instability at a frequency slightly above 1 kHz.

A sensibility study was done and the absolute instability is still present with variations of flow profile and impedance model. Although the model remains close to the provided data, and the flow profile is measured at other plane within the same experimental veine.

The damping of hydrodynamic modes due to the presence of viscosity, can explain the fact that this instabilities are rarely found in experiments. The absolute instability does not take place for this case when viscosity is considered.

Conclusion

Since high levels of noise are bothersome and can represent threats to health, reducing noise nuisances is an ever-challenging subject of study. In this work, we focus on acoustic modes travelling along ducts lined with absorbent materials to achieve a noise mitigation. The rectangular geometry used in this study is a simplification of an aeroengine. This geometry allows us to compare results with experiments in simple test veins such as B2A from Onera or GIT and its iterations from NASA.

To introduce the idea of noise reduction in ducts with flow, we outlined the notion of noise regarded as a collection of acoustic modes. Each mode has an associated eigenvalue. The spectrum obtained depends mostly on the acoustic impedance of the absorbent material that lines the bottom wall of the duct, and the flow characteristics.

We presented two pieces of software that were used to obtain the spectra of acoustic modes in this thesis. These are MAMOUT and FiEStA, the former is a code already validated, written by Jean-Philippe Brazier. This can take into account either the linearized Navier-Stokes equations or, without considering the viscosity, the linearized Euler equations. However it can only considerate a unidimensional domain. On the other hand, FiEStA was written in the frame of his thesis by the author. This code considers only the linearized Euler equations, but it can solve them in either a unidimensional domain or a bidimensional transversal domain.

The program FiEStA was validated against analytic solutions when they are obtainable and with results from MAMOUT and other published data. With it we showed the main modifications to the spectra and eigenfunctions with the modification of three characteristics of the problem. The modifications introduced were the aspect ratio for the transversal case, the value of the acoustic impedance and the shape of the flow profile and the Mach number. It was shown that even for more complicated cases with experimental flow profiles and absorbent wall, most of the changes induced by these modifications can be extrapolated from the analytic solutions. This is especially true for purely acoustic modes. However, the acoustic surface mode is more sensible to impedance values. This mode is not found in analytic solutions, since they do not take into account a finite impedance.

This research around the parameters influence led to search the impedance that gives the best noise reduction. To do so, we picked up the formulas stated by Cremer and Tester and the shape of spectra for different impedance values. With that, it was shown that the spectrum with the best noise reduction is obtained when there is a coincidence of the least attenuated pair of consecutive modes $n, n + 1$. In many cases only the plane wave is a propagative mode. Then, the coincident pair is this mode with the first cut-off mode 0, 1. This happens for the frequency range and dimensions of the experimental veins B2A and GIT.

The formula obtained by Cremer does not consider a mean flow, it has a value corrected by

Tester. This is why we considered only the formula by Tester, either with or without flow. We recall that this last formula only takes into account the uniform mean flow profile, and it is valid for what Tester calls “well-cut-on” modes, which means either low order modes, high frequencies or high Mach numbers. Both formulas are for a centred unidimensional domain, normal to the lined wall.

Even these formulas consider at best a uniform flow profile, they are good approximations mostly for low Mach numbers. With them, the optimal impedance is close for any flow profile, considering the same bulk Mach number. However, close to the optimal impedance, eigenvalues change faster and we can easily be far from the coincidence. Besides, with a mean flow, one has to choose if the optimal impedance will be calculated for right or left running modes, since the spectrum is not symmetric.

The observations done for the optimal impedance, are valid also for the bidimensional transversal domain. In this case, we have a whole family of coincident modes $(0, m), (1, m)$. Because of the two-dimensional characteristics of the flow profile, modes are modified from the unidimensional domain, and thus, the optimal impedance may not be the same.

The group velocity for the double point was also studied. In any case, at the optimal impedance, involved modes have a zero group velocity. For fixed resistance values higher than the optimal value, and sweeping reactances, both group velocities are positive. When the resistance approaches the optimal value, one mode descends to zero while the other diverges to $+\infty$ and appears from $-\infty$ to reach also zero. Both modes have a zero group velocity for the optimal reactance. After the optimal resistance, modes exchange their behaviour. This exhibits a global behaviour with stable modes.

Besides the acoustic modes, when we take into account a flow profile, a pair of hydrodynamic surface modes may also appear. The stability of these modes was studied with the Briggs-Bers criterion for the configuration of the GIT lined with a material named CT57. With a frequency of 1 kHz, one mode was shown to be stable whilst the other is unstable. An absolute instability was found for a frequency with real part slightly above 1 kHz, even with minor variations of the impedance model or the mean flow profile.

The study of hydrodynamic modes was repeated with the linearized Navier-Stokes equations, to take into account viscosity. It was shown that they remain one stable and one unstable, however both are damped. This effect can explain that the instability is rarely found in experiments. Besides, the absolute instability no longer takes place.

Perspectives

Some research that can be done beyond this thesis is exposed next.

In general, both for the instability and for the optimal impedance, the analysis should also be made in a bidimensional domain, longitudinal to the vein. This way, it can be compared with aeroacoustics propagation programs. Another modification that should be made to the domain is to include impedance discontinuities to better represent the placement of liners in B2A, for example.

With respect to the hydrodynamic instability, it would be useful to search for experimental evidence of its existence for the configuration used in this study. It can be extended to use another experimental flow profiles and liners to see the range of the existence of the instability; both in numerical and experimental studies.

Some ameliorations that could be done to FiEStA, to improve its research capabilities, are:

- ◀ Implement a subroutine to seek for the optimal impedance. This can be achieved by considering the fact that at the optimal impedance, the pair of modes have a zero group velocity. With this knowledge, starting from Tester's formula, a shooting method can be established aiming at the zero group velocity condition. The resulting impedance should be the optimal one.
- ◀ As well as it is for the studies in general, upgrade the code to consider a longitudinal domain, and discontinuities of the impedance along any wall. In a more evolved stage, it could consider also other geometries, as well as annular ducts. It could also deal with non linear effects.

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Deuxième partie

Résumé étendu en français ¹

¹N.B. Cette partie de la thèse comporte un résumé étendu en français. Celui-ci conserve la même structure en chapitres du texte principal contenu dans la première partie. L'information et les résultats les plus importants sont contenus ici. Pour plus de détails sur les développements et les résultats, j'invite les lecteurs à se diriger vers la première partie, rédigée en anglais.

Introduction

L'étude présentée dans cette thèse, réside dans le domaine de l'acoustique modale des conduits à parois absorbantes avec un écoulement porteur. Ici, on considère une source monochromatique de bruit et on étudie la propagation du son dans en se penchant sur modes propres du conduit.

Le principal objectif de cette thèse est de mieux comprendre la propagation et la réduction du son dans ce type de configuration. L'application visée est d'aider à la réduction des nuisances sonores dans des configurations de ce type. Ce problème se pose dans l'industrie aéronautique ainsi que dans des systèmes de climatisation ou dans le domaine automobile.

Pour arriver à ce but, les parois internes des conduits sont recouvertes de matériaux absorbants nommés avec le mot anglais *liner*. Ces matériaux sont caractérisés pour un nombre complexe, l'impédance acoustique. Actuellement, il n'existe aucune théorie capable de décrire complètement l'impact dans le champ acoustique de ces matériaux. Pour cela, il y a plusieurs études pré-existantes autour du sujet.

Rienstra [94] a étudié les différents types des modes dans la dite configuration, avec un accent sur les modes de paroi. Marx et. al. [76] ont mis en évidence évidence que quelques modes peuvent être instables.

Cremer [38] et Tester [113] avec des développements analytiques ont trouvé des formules pour obtenir une impédance optimale en fonction de la fréquence du bruit.

Les travaux menés dans cette thèse visent à étudier les modes acoustiques et hydrodynamiques présents dans les conduits traités acoustiquement soumis à un écoulement moyen ; avec un focus fait sur la stabilité. Une partie de la thèse s'attache aussi à trouver l'impédance optimale. Pour ce faire, j'ai développé un code de calcul et j'ai employé aussi un autre déjà existant au sein du laboratoire.

Chapitre 1

Contexte

1.1 Mitigation du bruit

D'après l'Organisation Mondiale de la Santé, un individu sur trois ressent une gêne pendant la journée et un sur cinq a des perturbations de sommeil pendant la nuit à cause du bruit engendré par le trafic. L'exposition répétitive aux hauts niveaux du bruit peut aussi entraîner différents problèmes tels que la baisse de performance à l'école et au travail, l'augmentation de l'agressivité, la déficience auditive ou les acouphènes, l'hypertension, l'infarctus du myocarde et des troubles cognitifs chez les enfants[123, 122, 121]. De ce fait, la réduction du bruit reste un sujet important d'étude ; particulièrement pour la vie contemporaine où nous sommes entourés par plusieurs sources de bruit.

1.1.1 Sources de bruit en l'aéronautique

Les avions modernes ont plusieurs sources de bruit. La figure 1.1 montre le niveau perçu de ces sources, selon le Centre Aérospatial Allemand (DLR) et l'Administration National de l'Aéronautique et de l'Espace des États Unis (NASA).

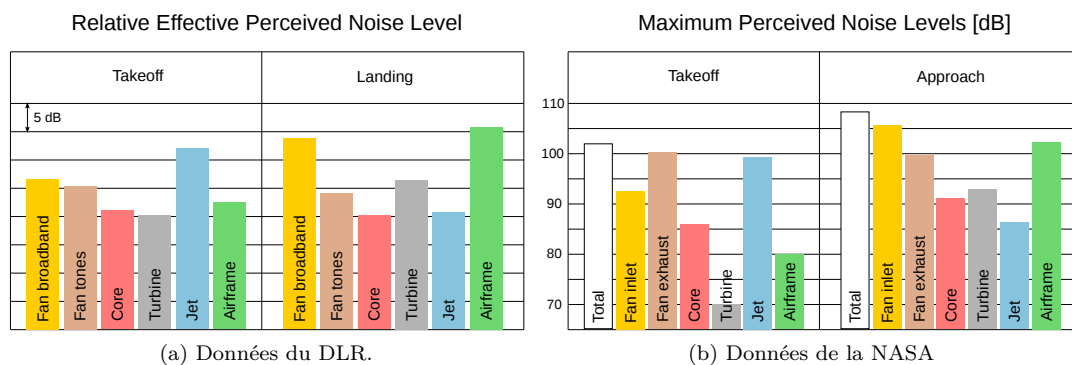


FIGURE 1.1 – Niveau de bruit pour les différents sources des avions.

L'étude présentée ici, traite le bruit de fan. Celui-ci se propage dans un conduit soumis à un écoulement moyen. Pour cela nous allons présenter les techniques pour réduire le bruit dans ce type de configurations.

1.1.2 Techniques de mitigation du bruit dans des conduits

À part le bruit de fan dans les avions, cette configuration peut se trouver aussi dans les systèmes de climatisation et les silencieux automobiles, parmi autres systèmes. La réduction du bruit dans cette configuration peut se faire avec des actionneurs actifs, matériaux passifs ou une combinaison des deux.

1.2 Traitements acoustiques : *liners*

Pour réduire le bruit dans les conduits, ils sont revêtus avec des traitements acoustiques ; pour nommer ces matériaux, on emploie le mot anglais *liner*. La figure 1.2 montre comment ces matériaux agissent. Les figures 1.3, 1.4 et 1.5, montrent la structure de trois types de liners.

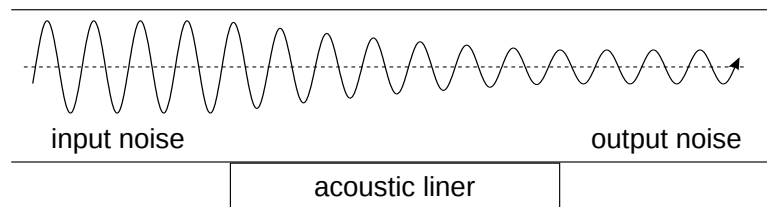


FIGURE 1.2 – Illustration de l'emploi d'un liner.

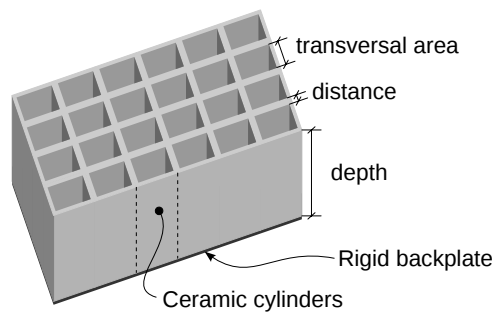


FIGURE 1.3 – Schéma d'un liner tubulaire céramique.

1.2.1 Impédance acoustique

Le paramètre qu'on emploie pour caractériser les liners est l'impédance acoustique Z . Elle est définie comme le rapport entre la pression acoustique p et la vitesse de particule u perpendiculaire à la surface du liner, dirigée vers le liner ; ceci est montré dans la figure 1.6. En général, l'impédance dépend de la fréquence entre autres paramètres. Dans ce travail on n'est intéressé qu'à la dépendance de la fréquence.

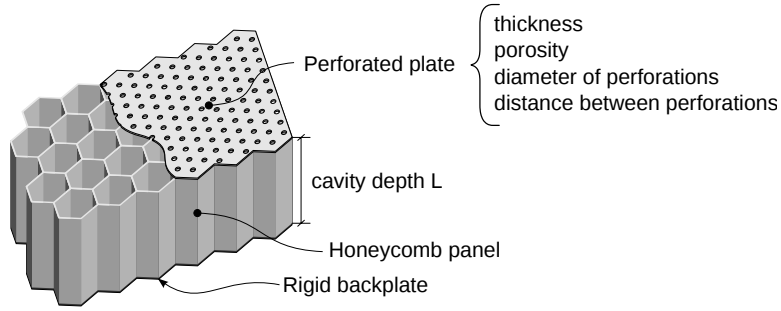


FIGURE 1.4 – Schéma d'un liner à un degré de liberté.

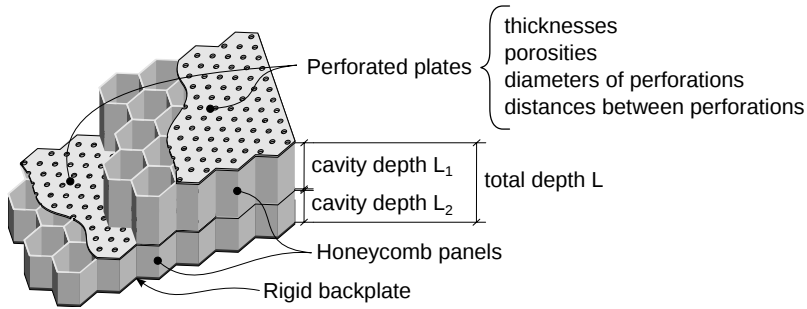


FIGURE 1.5 – Schéma d'un liner à deux degrés de liberté.

$$Z(\omega) = \frac{p(\omega)}{\underline{u}(\omega) \cdot \underline{n}} \quad (1.1)$$

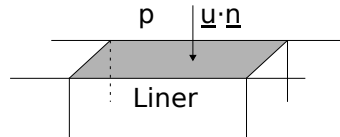


FIGURE 1.6 – Pression et vitesse incident au liner.

L'impédance est une quantité complexe, dont la partie réelle s'appelle résistance R et la partie imaginaire, réactance χ .

$$Z = R + i\chi \quad (1.2)$$

1.3 Modèle

Le problème est simplifié de plusieurs façons. Ceci est fait car on ne s'intéresse à décrire en détail qu'un système limité. Ainsi on obtient un modèle simple à résoudre lequel permet d'analyser un spectre plus ample des cas.

Le problème est simplifié en considérant seulement un son monochromatique sans s'intéresser à la source. La géométrie est considérée carrée, avec la paroi absorbante en bas. Cette géométrie est trouvée dans plusieurs bancs d'essai ; tel est le cas de celui de l'Onera, le Banc Aéro-thermo-Acoustique, raccourci B2A. Il est montrée dans la figure 1.7

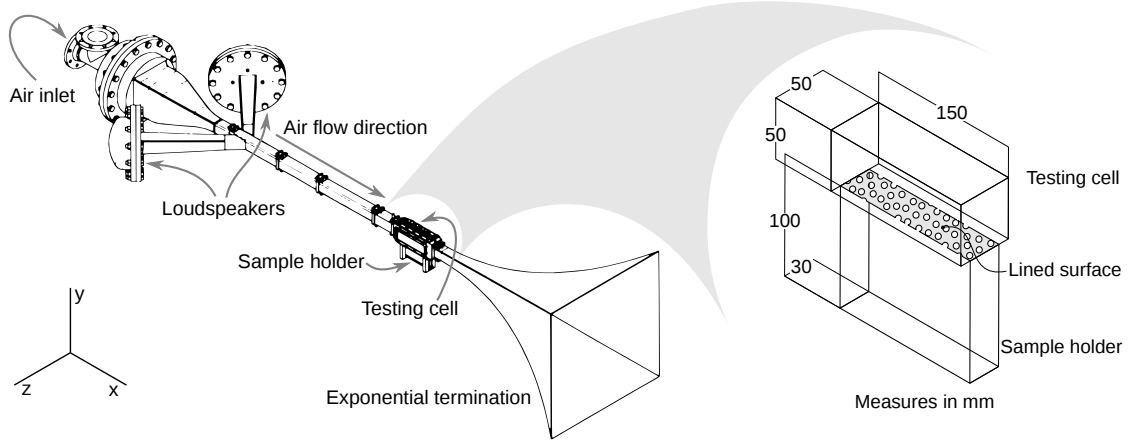


FIGURE 1.7 – Schéma du Banc Aéro-thermo-Acoustique “B2A”. Ses principaux composants sont montrés avec un plan serré sur la cellule d’essai et le porte-échantillon avec leur dimensions.

1.3.1 Équations.

En plus des simplifications géométriques, le système a besoin d’autant équations que de variables. Pour un gaz parfait isentropique en équilibre thermodynamique local, sans considérer la force gravitationnelle, ces équations sont : l’équation d’état (1.3), l’équation de l’entropie (1.4), l’équation de la continuité (1.5) et l’équation de quantité de mouvement, soit dans la forme des équations de Navier-Stokes (1.6) ou celles d’Euler si on ne s’intéresse pas aux effets de la viscosité.

$$p = \rho RT \quad (1.3)$$

$$\frac{ds}{dt} = 0 \quad (1.4)$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0} \quad (1.5)$$

$$\boxed{\rho \frac{\partial \underline{u}}{\partial t} + \rho (\underline{u} \cdot \nabla) \underline{u} + \nabla p - \nabla \cdot \left[\mu \left(\nabla \underline{u} + (\nabla \underline{u})^T \right) - \frac{2}{3} \mu (\nabla \cdot \underline{u}) \mathbf{I} \right] = 0} \quad (1.6)$$

$$\boxed{\rho \frac{\partial \underline{u}}{\partial t} + \rho (\underline{u} \cdot \nabla) \underline{u} = -\nabla p} \quad (1.7)$$

1.3.2 Technique des petites perturbations.

Pour résoudre le champ acoustique, les quantités physiques sont supposées comme une valeur moyenne avec la superposition d’une perturbation d’ordre beaucoup plus petit. Ici, la perturbation

est noté avec un prime, et la valeur moyenne avec un indice zéro :

$$q = q_0 + q' \quad (1.8)$$

pour toutes les quantités q .

De cette façon, les équations sont linéarisées en négligeant tous les termes d'ordre supérieur à l'unité en terme des perturbations.

1.3.3 Forme modale.

On suppose les caractéristiques constantes tout le long du conduit. Ainsi nous pouvons écrire les perturbations comme la superposition des modes normaux [95, 117, 84]. Pour ce faire, on introduit la fréquence ω et le nombre d'onde axial α . En général, ces quantités sont complexes.

$$\omega = \omega_r + i \cdot \omega_i \quad , \quad \alpha = \alpha_r + i \cdot \alpha_i \quad (1.9)$$

Avec cela, la forme modale des perturbations est :

$$q' = \hat{q} e^{i(\omega t - \alpha x)} \quad (1.10)$$

Où le terme d'amplitude $\hat{q}$ prend la dépendance des coordonnées spatiales transversales à l'axe du conduit.

1.3.4 Écoulement moyen.

Dans ce travail, on considère que l'écoulement moyen va seulement dans la direction axiale et qu'il ne dépend que des coordonnées normales.

1.3.5 Conditions à la frontière.

Les conditions à la frontière sont considérées en trois zones différentes, montrées dans la figure 1.8.

◀ Aux parois rigides (Γ_0), la vitesse normale est zéro :

$$\underline{u}'(\underline{r}) \cdot \underline{n} = 0 \quad \forall \underline{r} \in \Gamma_0 \quad (1.11)$$

◀ À la paroi traitée ($\Gamma_{\mathcal{Z}}$), la vitesse normale est liée à la pression par l'impédance :

$$\mathcal{Z} \underline{u}'(\underline{r}) \cdot \underline{n} = p'(\underline{r}) \quad \forall \underline{r} \in \Gamma_{\mathcal{Z}} \quad (1.12)$$

◀ À l'entrée et la sortie, l'impédance est considérée celle caractéristique de l'écoulement.

$$\rho_0 c \underline{u}'(\underline{r}) \cdot \underline{n} = p'(\underline{r}) \quad \forall \underline{r} \in \Gamma_{io} \quad (1.13)$$

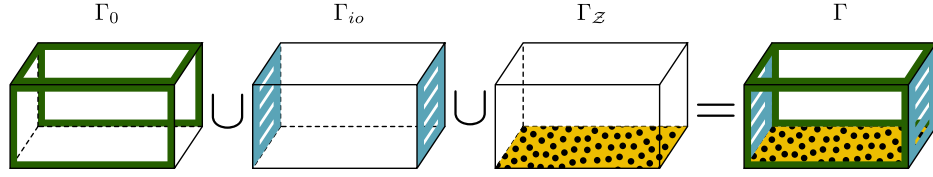


FIGURE 1.8 – Conditions à la frontière selon les différents zones.

1.4 Modes

Dans la configuration étudiée ici, il peut y avoir différentes familles de modes.

1.4.1 Modes acoustiques.

On trouve ces modes quelles que soient les conditions de frontière et d'écoulement moyen. Dans le cas des parois rigides sans écoulement moyen, la forme de la perturbation de pression prend la forme :

$$p(x, y, t) = \hat{p}_\beta e^{i\omega t} e^{-i\alpha x} e^{-i\beta y} \quad (1.14)$$

avec

$$\beta_n = \frac{n\pi}{2H} \quad , \quad n = 0, 1, 2, 3, \dots \quad (1.15)$$

où β_n est le nombre d'onde perpendiculaire et l'index n sert pour identifier un mode. À partir de cela, on obtient la relation de dispersion :

$$\alpha = \pm \sqrt{\frac{\omega^2}{c^2} - n^2 \frac{\pi^2}{4H^2}} \quad (1.16)$$

Pour le cas transversal, il faut aussi considérer le nombre d'onde transversal γ , ainsi, la fonction propre de la pression est :

$$p(x, y, z, t) = \hat{p}_\beta e^{i\omega t} e^{-i\alpha x} e^{-i\beta y} e^{-i\gamma z} \quad (1.17)$$

où :

$$\beta_n = \frac{n\pi}{2} \quad , \quad \gamma_m = \frac{m\pi}{2\mathcal{A}} \quad , \quad n, m = 0, 1, 2, 3, \dots \quad (1.18)$$

Dans ce cas, il faut avoir le couple (n, m) pour identifier un mode. Dans ce cas, la relation de dispersion prend la forme :

$$\alpha^2 + \beta^2 + \gamma^2 - \omega^2 = 0 \quad \Rightarrow \quad \alpha_{n,m} = \pm \sqrt{\omega^2 - \frac{n^2\pi^2}{4} - \frac{m^2\pi^2}{4\mathcal{A}^2}} \quad (1.19)$$

Les valeurs propres obtenues avec la racine positive correspondent aux modes qui se propagent vers la droite, pendant que ceux dont la valeur propre est obtenue avec la racine négative, sont des modes qui se propagent vers la gauche.

Quand $n = 0$ ou $(n, m) = (0, 0)$, selon le cas, on parle du mode plan. De plus, les modes pour lesquels la valeur de α est réelle, sont propagatifs, aussi appelés “*cut-on*” en anglais. Ceux qui ont une valeur imaginaire, sont évanescents ou “*cut-off*” aussi en anglais.

1.4.2 Modes de surface.

Ces modes sont appelés de surface car leur amplitude décroît de façon exponentielle quand on s'éloigne de la paroi avec le liner. Ils ne peuvent exister qu'avec une paroi à impédance finie. Cette famille est divisée en deux : les modes acoustiques de surface et les modes hydrodynamiques de surface. Ils peuvent être distingués par leur vitesse de phase, de l'ordre de la vitesse du son pour les modes acoustiques, mais notablement plus faibles pour les modes hydrodynamiques.

Parmi les modes de surface hydrodynamiques, il peut y avoir des modes instables. Un tel cas sera étudié dans le chapitre 5.

Ce chapitre montre la base pour l'étude des modes présents dans des conduits à parois traitées. On a parlé de l'existence des familles des modes et des modes instables. On s'intéresse au comportement des modes, lequel sera analysé dans la suite du manuscrit à l'aide des codes de calcul.

Chapitre 2

Calcul des Modes

Dans ce chapitre, on montre les calculs faits pour obtenir les modes. Deux cas sont pris en compte ; d'un côté, une analyse en régime visqueux bidimensionnel, pendant que de l'autre côté, une analyse en régime non visqueux tridimensionnel. Quand on parle d'un domaine n -dimensionnel, on considère les dimensions des équations. Depuis que la dépendance en x est donnée par la forme modale (1.10), les dimensions du domaine du problème sont en réalité d'un ordre en moins que celles des équations.

Pour la géométrie donnée, la figure 2.1 montre le système de coordonnées employé. À partir des dimensions, on définit le rapport d'aspect : $\mathcal{A} = \frac{L}{H}$.

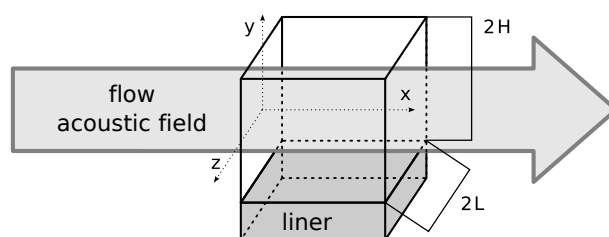


FIGURE 2.1 – Géométrie du problème.

2.1 Équations

Les équations considérées sont celles indiquées dans la section 1.3.1.

2.1.1 Équations de Navier-Stokes linéarisées en 2D

Pour entreprendre la résolution du problème visqueux bidimensionnel, les équations de Navier-Stokes ont été linéarisées pour les introduire dans le code MAMOUT. Elles sont considérées de la

façon suivante, en considérant $\omega^* = (\omega - u_0\alpha)$:

$$\begin{pmatrix} -i\alpha\rho_0 & \rho_0 D_y & i\omega^* \frac{\rho_0}{p_0} & -i\omega^* \frac{\rho_0}{h_0} \\ -\mu_0 D_y^2 + i\omega^* \rho_0 + \frac{4}{3}\alpha^2 \mu_0 & \frac{i\alpha\mu_0}{3} D_y + \rho_0 u_0^y & -i\alpha & -K u_0^y D_y - K u_0^{yy} \\ \frac{1}{3}i\alpha\mu_0 D_y & \frac{4}{3}\mu_0 D_y^2 + i\omega^* \rho_0 + \alpha^2 \mu_0 & D_y & -i\alpha K D_y \\ -2\mu_0 u_0^y D_y & 2i\alpha\mu_0 u_0^y & -i\omega^* & -\frac{\mu_0}{Pr} D_y^2 + i\omega^* \rho_0 - K (u_0^y)^2 + \frac{\alpha^2 \mu_0}{Pr} \end{pmatrix} \begin{pmatrix} u \\ v \\ p \\ h \end{pmatrix} = 0 \quad (2.1)$$

Ici, l'opérateur des dérivées par rapport à y est noté D_y et D_y^2 . Les dérivées de l'écoulement moyen $\frac{\partial u_0}{\partial y}$ et $\frac{\partial^2 u_0}{\partial y^2}$ sont écrites u_0^y et u_0^{yy} .

2.1.2 Équations d'Euler linéarisées en 3D

Le cas non visqueux, décrit par les équations d'Euler a été étudié avec les deux codes MAMOUT et FiEStA. Le premier, ne prend en compte que l'étude en 2D, pendant que le dernier prend aussi le cas 3D. Ce dernier cas est connu sous la dénomination d'analyse bi-globale. Pour mettre dans le code les équations, elles sont prises de la manière montrée ici :

$$\begin{pmatrix} i\omega - i\alpha u_0 & u_0^y & u_0^z & -i\alpha \\ 0 & i\omega - i\alpha u_0 & 0 & D_y \\ 0 & 0 & i\omega - i\alpha u_0 & D_z \\ -i\alpha & D_y & D_z & i\omega - i\alpha u_0 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \\ p \end{pmatrix} = 0 \quad (2.2)$$

Comme pour le système des équations de Navier-Stokes, les opérateurs de dérivation sont notés D_y et D_z pour les dérivées par rapport à y et z .

2.1.3 Problème généralisé de valeurs propres

Les systèmes d'équations conduisent à la résolution d'un problème généralisé des valeurs propres discret [94] avec cette forme :

$$\mathbf{A}\varphi = \lambda\mathbf{B}\varphi \quad (2.3)$$

où $\mathbf{A}$ et $\mathbf{B}$ sont des matrices, λ est la valeur propre et $\varphi = (u \ v \ w \ p)^T$ est la fonction propre.

Dans la plupart de ce travail, on fait une analyse spatiale. Ici, la valeur propre à chercher est le nombre d'onde α et la fréquence ω est un paramètre donné. Seulement pour le cas où nous voulons valider le critère de stabilité, il a fallu faire l'analyse temporelle, avec le nombre d'onde pris comme un paramètre et la fréquence cherchée comme la valeur propre.

2.2 Codes de calcul

Le problème a été résolu dans les codes de calcul par la méthode de collocation spectrale, avec le domaine de calcul discrétisé par les points de Gauss - Lobatto.

2.2.1 Programme MAMOUT

Un programme employé dans ce travail a été celui appelé MAMOUT de l'acronyme “Module d'Analyse MODale Unidimensionnelle avec les polynômes de Tchebychev” [15]. Ce code a été développé par Jean-Philippe Brazier au sein de l'Onera/DMAE. Celui peut considérer le cas sans ou avec viscosité, c'est à dire, les équations linéarisées de Navier-Stokes ou celles d'Euler, seulement pour un problème bidimensionnel. Il peut faire aussi bien une analyse temporelle que spatiale.

2.2.2 Programme FiEStA

Un second programme, développé dans le cadre de cette thèse est appelé FiEStA, pour l'acronyme “Finder of Eigenmodes for Study in Aeroacoustics” (Trouveur des modes propres pour l'étude en aéroacoustique). Ce code ne considère que les équations d'Euler linéarisées, donc le cas non visqueux. Il prend en compte les équations 2D ou 3D. Mais il ne fait que l'analyse spatiale.

FiEStA a été validé sur des cas analytiques pour les solutions simples et par rapport à MAMOUT ou la littérature dans d'autres cas.

2.2.2.1 Validation de FiEStA

Les figures 2.2 et 2.3 montrent les spectres obtenus avec FiEStA, comparés avec des cas obtenus avec la relation de dispersion (1.16). Ces spectres sont obtenus avec des parois complètement réfléchives, sans écoulement moyen, pour un domaine unidimensionnel et transversal avec une fréquence $\omega = 3.5$ et un rapport d'aspect $\mathcal{A} = 1.1$ pour le cas transversal.

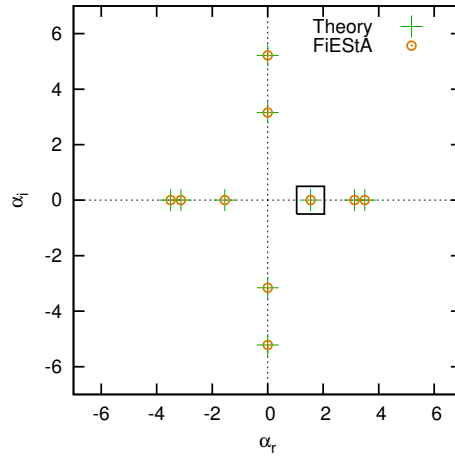


FIGURE 2.2 – Comparaison des spectres obtenus avec FiEStA et la théorie. Domaine unidimensionnel, parois réfléchives, $\omega = 3.5$.

Un accord parfait entre la théorie et FiEStA est montré dans les figures 2.2 et 2.3. Les fonctions propres ont aussi été validées de cette manière. Deux fonctions propres de la pression sont montrées dans la figure 2.4 ; où la coïncidence complète est évidente.

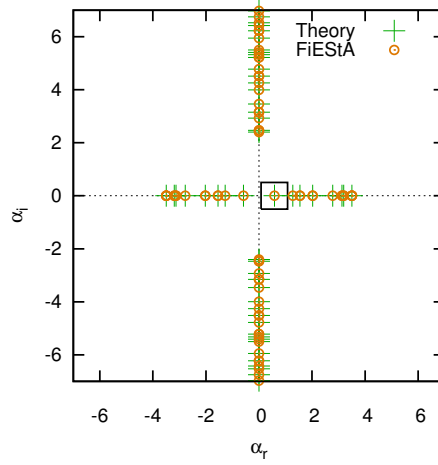


FIGURE 2.3 – Comparaison des spectres obtenus avec FiEstA et la théorie. Domaine unidimensionnel, parois réfléchives, $\omega = 3.5$, $\mathcal{A} = 1.1$.

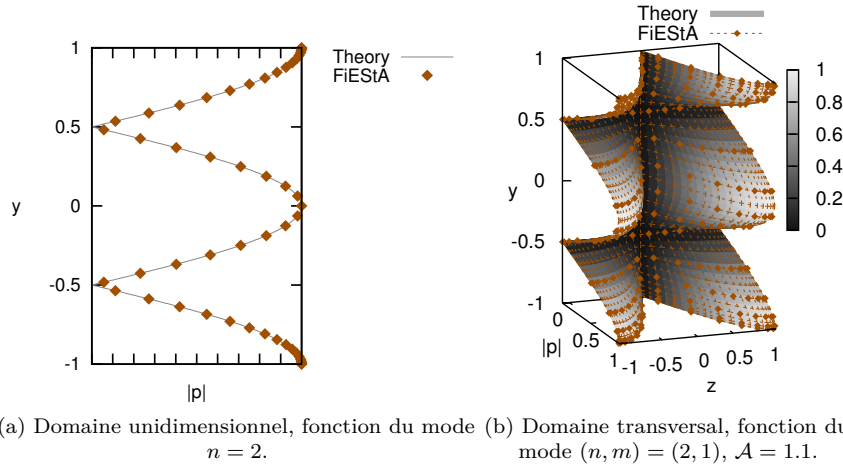
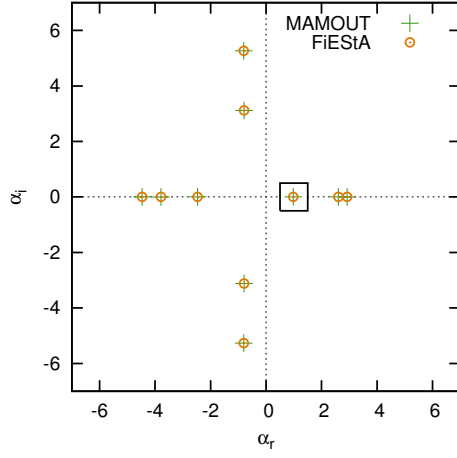
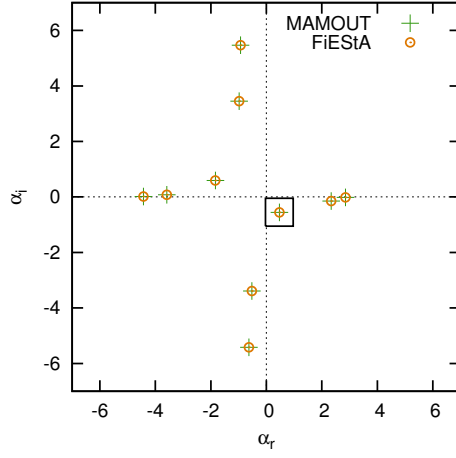


FIGURE 2.4 – Fonctions propres de la pression des modes marqués dans les figures 2.2 et 2.3. Comparaison des fonctions théoriques et de celles obtenues avec FiEstA. Parois réfléchives, $\omega = 3.5$.

Pour autres cas, où les solution analytiques sont difficiles ou impossibles à obtenir, FiEstA est validé par rapport a MAMOUT. On a choisi un cas avec un profil de Poiseuille avec un nombre de Mach central de 0.3 ; avec des parois rigides ou avec une impédance $\mathcal{Z} = 1 + i$. Les spectres sont montrés dans la figure 2.5, pendant que les fonctions propres de la pression sont dans la figure 2.6. Dans tout les cas, l'accord parfait est clair.

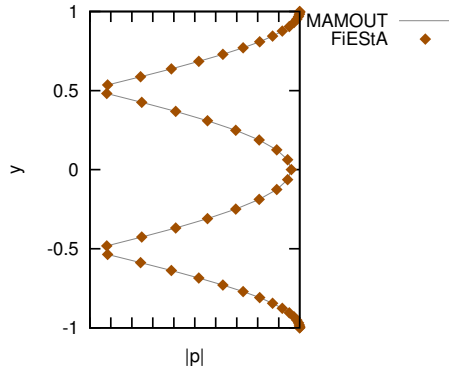


(a) Parois rigides.

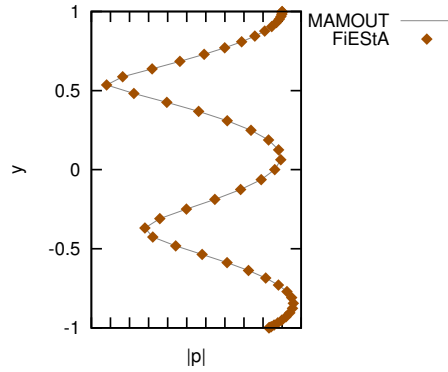


(b) Paroi absorbante : $\mathcal{Z} = 1 + i$.

FIGURE 2.5 – Comparaison des spectres obtenus avec FiESa et MAMOUT. Domaine unidimensionnel, $\omega = 3.5$, écoulement moyen avec un profil de Poiseuille, nombre de Mach central égal à 0.3. Les modes marqués de la figure 2.6



(a) Paroi rigide.



(b) Paroi absorbante : $\mathcal{Z} = 1 + i$.

FIGURE 2.6 – Fonction propre de la pression pour le mode $n = 2$.

Les principaux paramètres considérés dans les programmes qui modifient le spectre et les fonctions propres sont la fréquence, la valeur de l'impédance de la paroi traitée et le profil et nombre de Mach de l'écoulement moyen.

Dans le chapitre suivant, nous montrerons l'impact de l'impédance et l'écoulement moyen, ainsi que celui du rapport d'aspect.

Chapitre 3

Influence des paramètres

Dans ce chapitre, nous montrons l'influence sur les modes propres de trois paramètres : le rapport d'aspect, l'écoulement moyen et l'impédance.

3.1 Rapport d'aspect

Le rapport d'aspect est considéré seulement pour l'analyse transversale. La variation de ce paramètre peut être vue comme la variation de la largeur du conduit. Pour les cas sans écoulement moyen et avec parois rigides, nous pouvons voir son influence via la relation de dispersion (1.19). Quatre spectres avec différents valeurs de $\mathcal{A}$ sont montrés sur les figures 3.1 et 3.2.

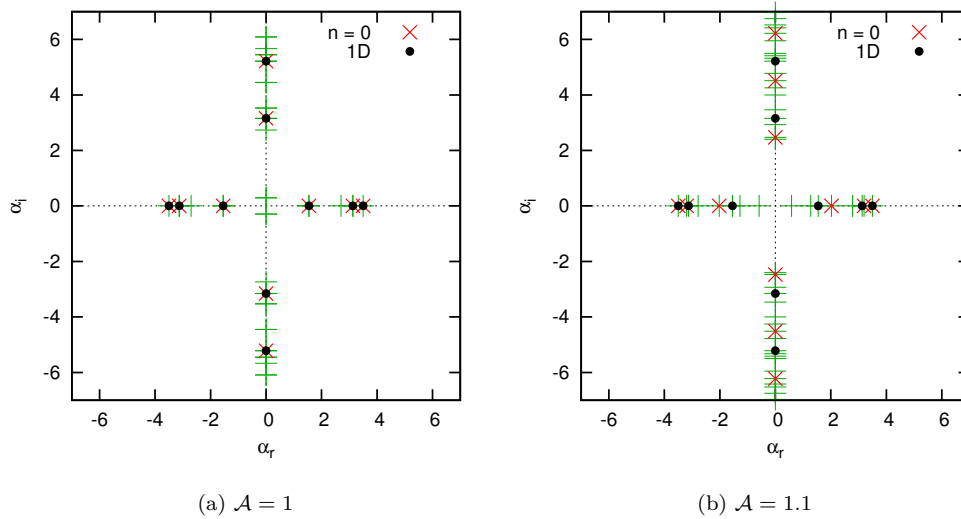


FIGURE 3.1 – Spectres pour rapports d'aspect 1 et 1.1 avec une fréquence de 3.5, parois rigides et sans écoulement moyen. Les nombres d'onde pour $n = 0$ sont montrés en croix rouges. Ceux du problème unidimensionnel sont en cercles noirs.

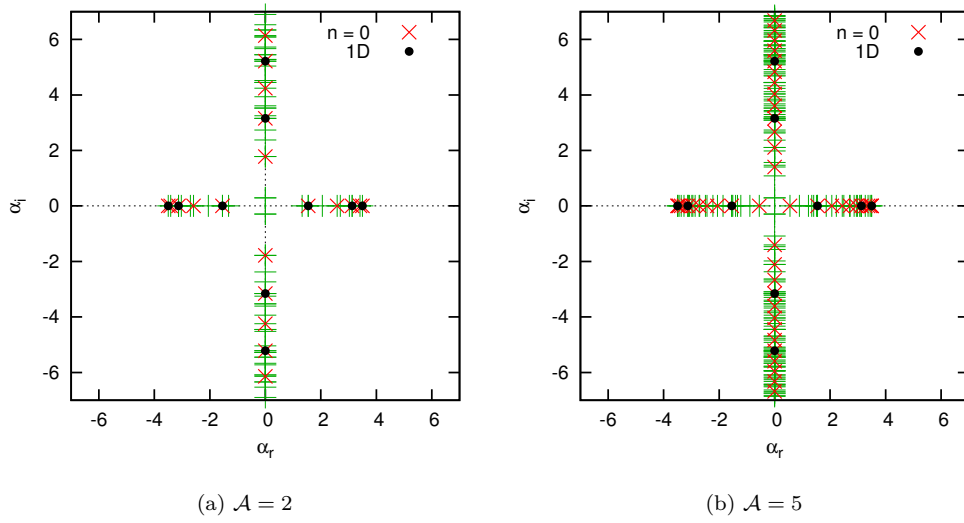


FIGURE 3.2 – Spectres pour rapports d’aspect 2 et 5 avec une fréquence de 3.5, parois rigides et sans écoulement moyen. Les nombres d’onde pour $n = 0$ sont montrés en croix rouges. Ceux du problème unidimensionnel sont en cercles noirs.

Le rapport d’aspect rend les spectres plus ou moins peuplés. Cela vient aussi du fait que pour certaines valeurs il existe une coïncidence de valeurs des modes différents.

La présence d’une paroi absorbante rend plus complexes les spectres. La figure 3.3 montre un spectre pour les mêmes paramètres que la figure 3.1a mais avec une impédance $\mathcal{Z} = 1 + i$. Ici, la coïncidence des modes unidimensionnels avec ces avec $n = 0$ n’existe plus.

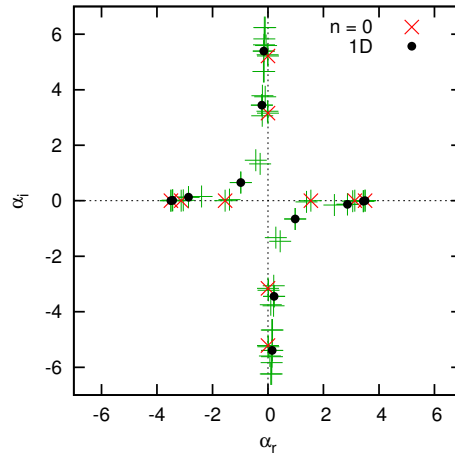


FIGURE 3.3 – Spectre pour les paramètres : $\mathcal{A} = 1$, $\omega = 3.5$, $\mathcal{Z} = 1 + i$; sans écoulement moyen. Nombres d’onde pour $n = 0$ sont montrés en croix rouges. Ceux du problème unidimensionnel sont en cercles noirs.

De manière similaire, quand un écoulement moyen est présent, le spectre est modifié. Dans la figure 3.4 sont montrés deux spectres avec un profil d'écoulement obtenu des travaux de Jones [60], avec un nombre de Mach débitant 0.3.

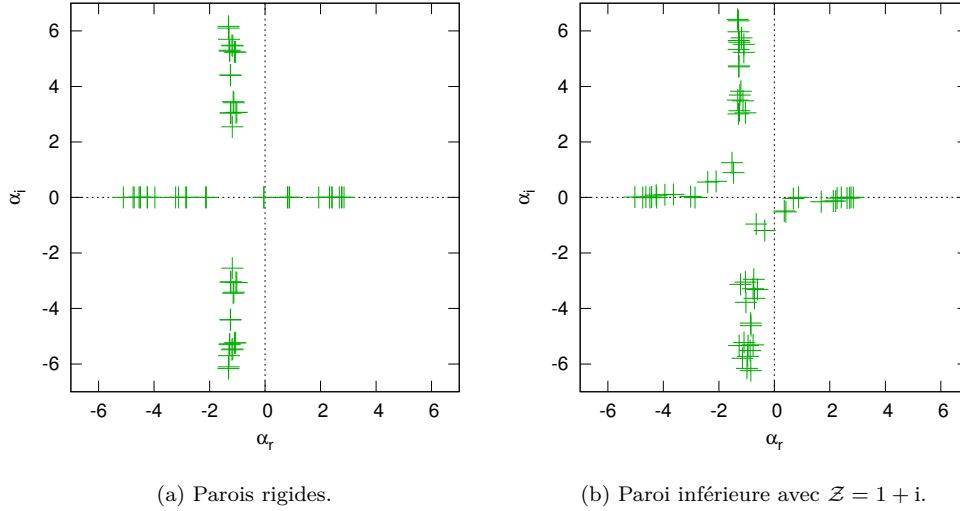


FIGURE 3.4 – Spectres pour $\mathcal{A} = 1$, $\omega = 3.5$. Cas à paroi rigide et absorbante. Profil d'écoulement de Jones [60] avec nombre de Mach débitant de 0.3.

Les paramètres suivants seront analysés pour un domaine unidimensionnels seulement.

3.2 Écoulement moyen

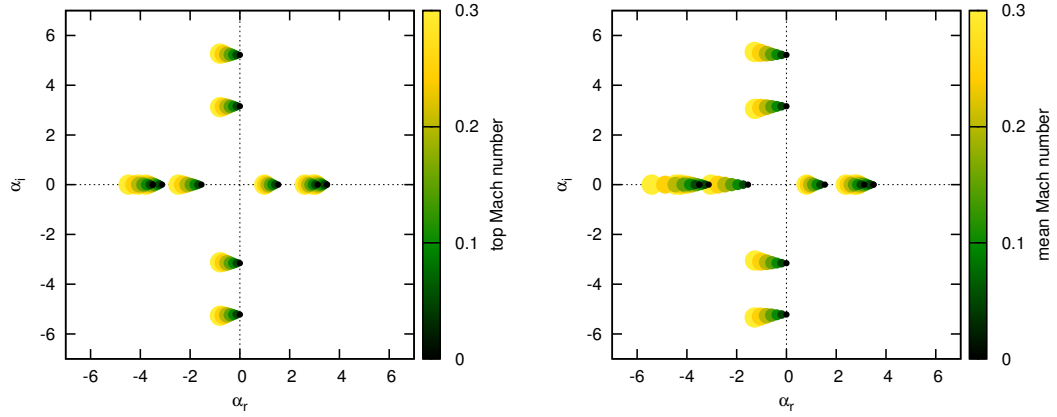
L'écoulement moyen est caractérisé par le nombre de Mach et la forme du profil. Le nombre de Mach est considéré de deux façons différentes, soit par la valeur au centre de la veine d'essai ou par la valeur débitante. La figure 3.5 montre les spectres selon le nombre de Mach défini par la valeur centrale et par la valeur débitante.

Pour étudier l'impact du profil de l'écoulement, quatre profils sont considérés. Ils sont montrés dans la figure 3.6 pour le nombre de Mach 0.3 central et débitant.

Dans la figure 3.7 sont montrés les spectres pour les différents profils de la figure 3.6.

Quand on prend en compte le nombre de Mach débitant, les spectres sont plus similaires pour les profils étudiés. Pour cette considération du nombre de Mach, les modes propagatifs dans le sens contraire à l'écoulement changent bien plus que les autres.

En plus des spectres, le profil d'écoulement modifie aussi les fonctions propres. Elles sont montrées dans la figure 3.8 pour le cas unidimensionnel et dans la figure 3.9 pour le cas bidimensionnel.



(a) Variation du nombre de Mach au centre de la veine (b) Variation du nombre de Mach débitant

FIGURE 3.5 – Influence du nombre de Mach entre 0 et 0.3 sur le spectre avec le profil de Poiseuille.

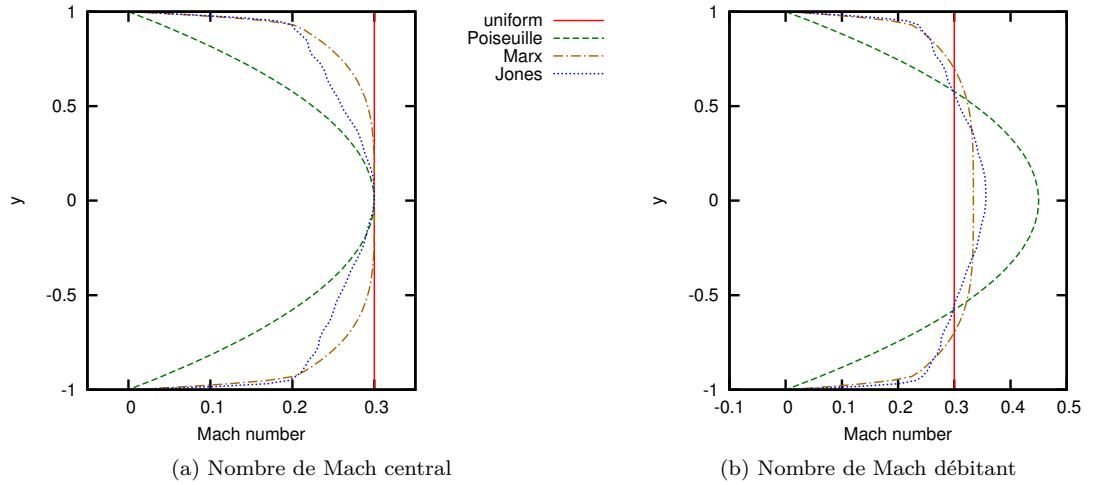


FIGURE 3.6 – Quatre profils d'écoulement. Nombre de Mach $M = 0.3$.

3.3 Impédance

Dans cette section, nous étudions l'influence de l'impédance. Pour cela nous fixons une valeur de la résistance et nous faisons balayer la réactance de valeurs très négatives jusqu'à des valeurs très positives. Dans la figure 3.10, les trajectoires que les modes font pendant le balayage pour cinq valeurs de la résistance sont montrées. Le cas considère une fréquence de 3.5 et un écoulement moyen nul.

Les boucles entre valeurs de paroi rigide sont claires. Pour des valeurs de la résistance plus basses, ces boucles grandissent jusqu'au moment où deux modes coïncident. Les boucles continuent

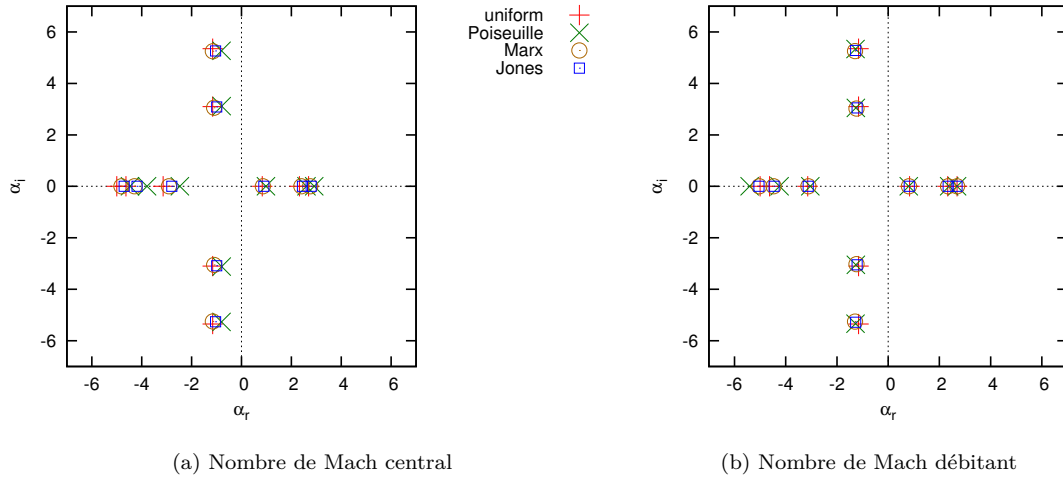


FIGURE 3.7 – Spectres obtenus avec les profils de la figure 3.6. $M = 0.3$.

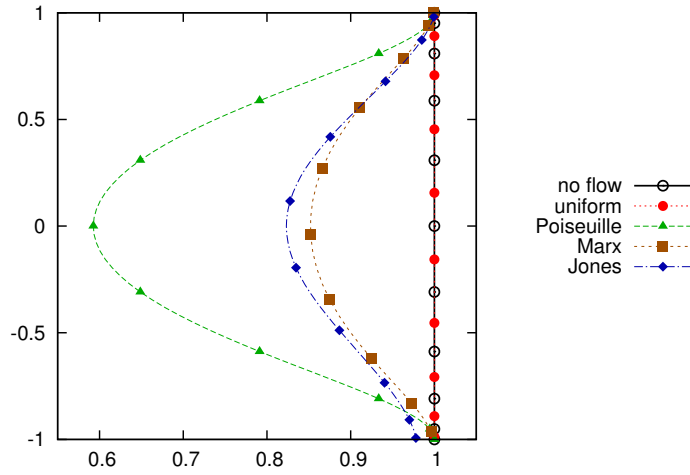
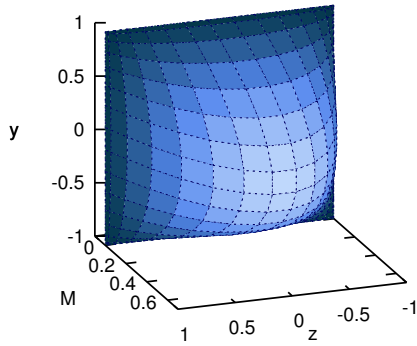


FIGURE 3.8 – Fonction propre de la pression pour le mode plan avec quatre profils d'écoulement. Nombre de Mach débitant $M = 0.3$.

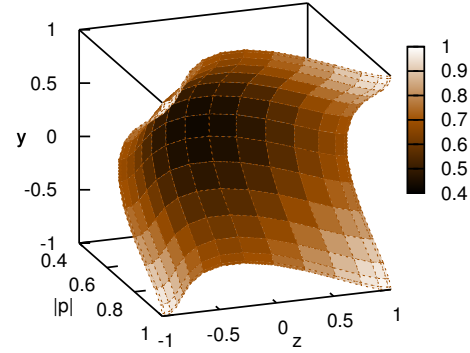
à grandir et chaque fois plus de modes font l'échange d'une façon cyclique. Pendant cet échange des modes, ils sont convertis entre modes de différents ordres. Cette conversion est montrée dans la figure 3.11 pour les fonctions propres de la pression de trois modes. Le mode acoustique de surface, avec sa forme exponentielle est montré sur les figures 3.11c et 3.11d.

Il doit exister une valeur de l'impédance qui donne la meilleure atténuation du son. Cela serait présenté dans le chapitre suivant.

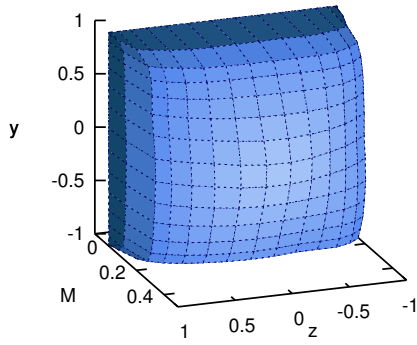
Dans ce chapitre, l'influence du rapport d'aspect, l'écoulement moyen et l'impédance a été montrée pour les modes du conduit à section rectangulaire.



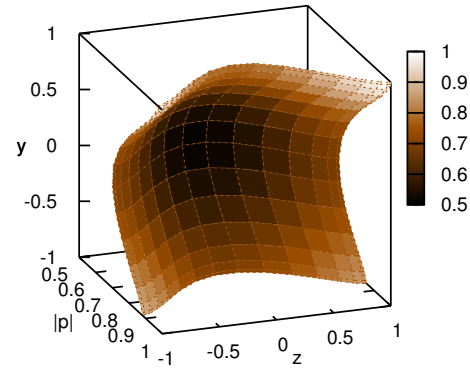
(a) Profil d'écoulement de Poiseuille



(b) Mode plan, écoulement de Poiseuille



(c) Profil d'écoulement de Jones



(d) Mode plan, écoulement de Jones

FIGURE 3.9 – Profils d'écoulement avec un nombre de Mach débitant 0.3 et fonction propre de la pression pour le mode plan pour chaque profil. Analyse transversale.

Le rapport d'aspect rend le spectre plus ou moins peuplé, on voit apparaître une coïncidence des valeurs propres de modes différents.

L'influence de l'écoulement moyen nous oriente à considérer le nombre de Mach débitant, spécialement pour les modes "cut-on". La valeur centrale donne des variations plus importantes.

La valeur de l'impédance fait l'échange des modes de façon cyclique. Pour des résistances assez basses, le mode acoustique de surface apparaît.

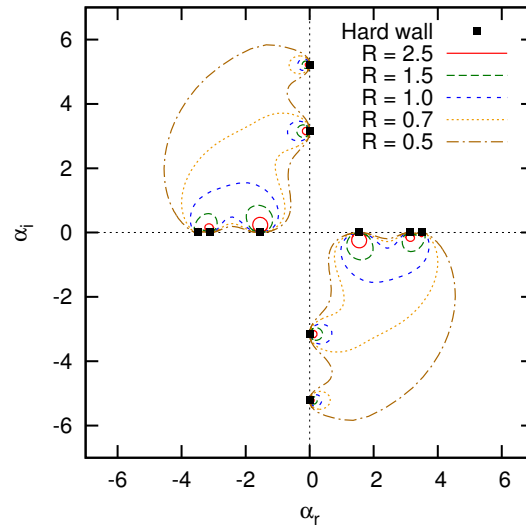
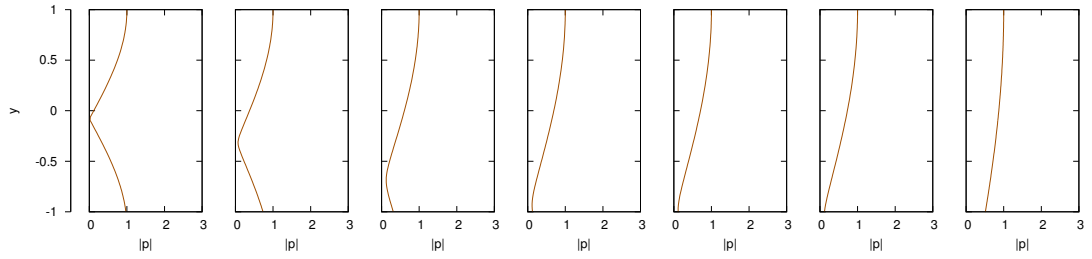
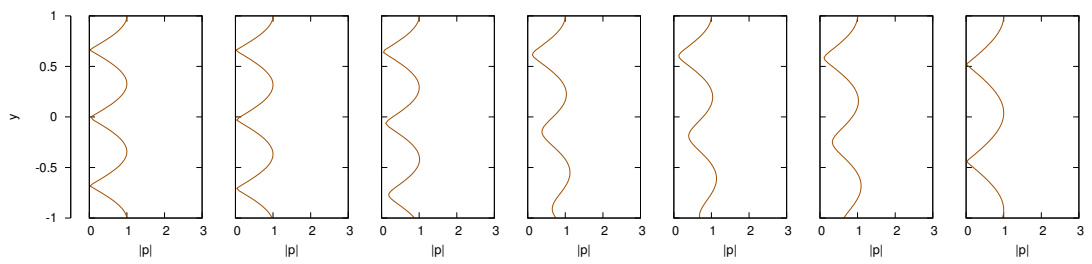


FIGURE 3.10 – Trajectoires des valeurs propres pour les résistances montrées. Balayage de la réactance entre -20 and 20. Modes de paroi rigide montrés pour référence.

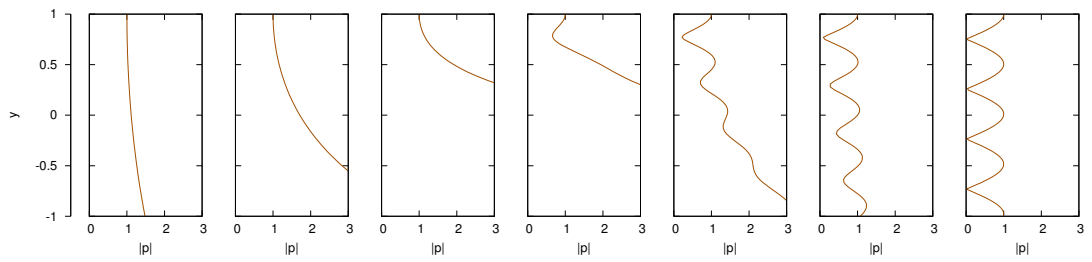
Dans le chapitre suivant, l'impédance optimale est étudiée.



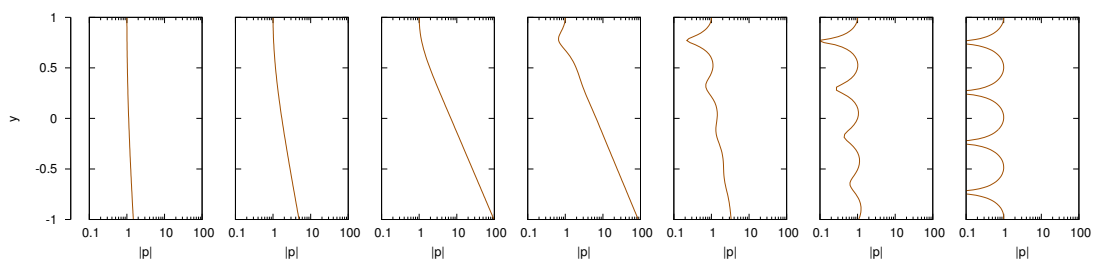
(a) Fonctions propres pour la transition du mode 1 au mode 0.



(b) Fonctions propres pour la transition du mode 3 au mode 2.



(c) Fonctions propres pour la transition du mode plan au mode 4 à travers le mode de surface.



(d) Figure 3.11c en échelle logarithmique.

FIGURE 3.11 – Fonctions propres pour la transition des modes à une résistance = 0.5.

Chapitre 4

Impédance optimale

Ce chapitre est basé sur une communication faite par l’auteur dans le congrès Acoustics2012 [101]. L’idée principale repose sur la valeur optimale pour l’impédance qui fait réduire le plus possible le bruit dans une configuration géométrique simplifiée où le conduit est de longueur infinie avec un traitement acoustique recouvrant toute une paroi.

Pour quantifier la réduction du bruit, on considère le taux d’atténuation. Elle est définie comment la diminution de l’amplitude d’un signal par unité de longueur. À partir de cette définition, on peut voir le lien entre cette valeur en dB et la partie imaginaire des valeurs propres des modes :

$$20 \log \left(\frac{p(0)}{p(1)} \right) = -20 \log \left(\frac{pe^{\alpha_i}}{pe^0} \right) = -20 \log (e^{\alpha_i}) = -20 \frac{\ln(e^{\alpha_i})}{\ln(10)} = -\frac{20}{2.3} \alpha_i = -8.7 \alpha_i \quad (4.1)$$

Ici, le taux d’atténuation est positif quand un mode diminue son amplitude ; cela explique la différence de signe avec α_i , laquelle est négative pour les modes décroissants propagatifs vers la droite. Pour cette raison, dans les parties suivantes, on ne verra que la valeur de α_i pour considérer le taux d’atténuation.

4.1 Dédution de l’impédance optimale

Le mode le moins atténué est celui pour lequel α_i est la plus grande, pour les modes qui se propagent vers la droite. Pour cela, pour avoir la meilleure atténuation du bruit, on se centre sur ce mode. Donc le mode propre doit être le plus loin possible de l’axe réel.

Pour identifier l’impédance optimale, on reprend les observations faites dans la section 3.3 sur les boucles des modes dans les spectres.

Dans cette section, on considère une fréquence $\omega = 0.924$, qui correspond à $2kHz$ dans B2A. Pour cette fréquence il n’y a qu’un mode propagatif dans chaque direction. La figure 4.1 montre les boucles pour les valeurs de résistance indiquées sans écoulement moyen.

À partir de cette figure, on peut voir que l’impédance optimale pour cette fréquence est celle pour laquelle il y a la coïncidence des modes 0 et 1. Cela arrive à une impédance $\mathcal{Z} = 0.55 - 0.44i$. Cette correspondance des modes consiste vraiment à un seul mode avec double multiplicité. Cela

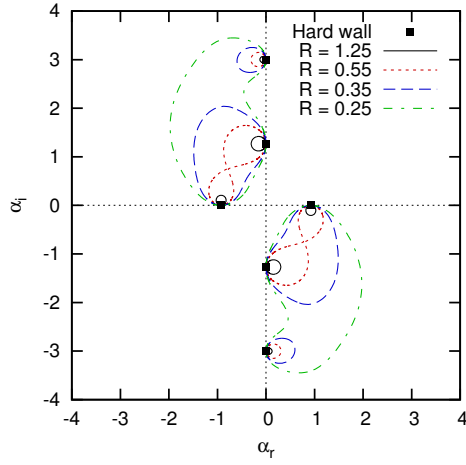


FIGURE 4.1 – Spectre sans écoulement moyen pour les valeurs de résistance indiqués et grands intervalles de réactance. $\omega = 0.924$.

peut se voir en regardant les fonctions propres des modes impliqués. La coïncidence correspond à un point de branchement qu'on appellera point double.

Dans la figure 4.2, on peut voir comment ce point est celui pour lequel les deux modes 0 et 1 sont plus atténués, c'est à dire, les plus éloignés de l'axe réel. Pour le montrer, il y a des points indiqués à une réactance de -0.55 où il y a toujours un mode moins atténué que ceux du point double.

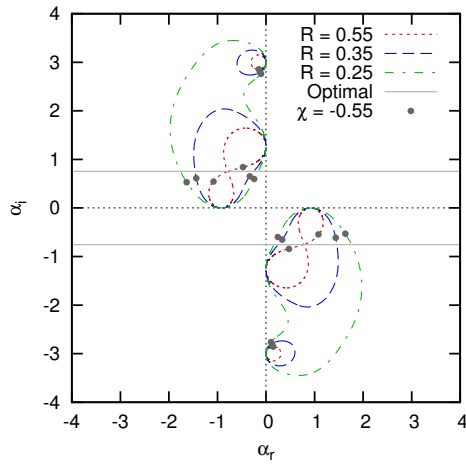


FIGURE 4.2 – Suivi des modes pour réactance variable et trois valeurs de la résistance. Atténuation optimale à $\chi = -0.55$. $\omega = 0.924$.

4.2 Impédance optimale en fonction de la fréquence

Pour obtenir l'atténuation la plus grande, Cremer [38] a défini la formule pour l'obtenir sans écoulement moyen et seulement avec un mode propagatif. Cette loi est :

$$\mathcal{Z}_C = (0.91 - 0.76i) \frac{2\omega}{\pi} \quad (4.2)$$

Tester a corrigé la constante de la loi [112] et a étendu la formule pour considérer un écoulement moyen uniforme [113]. Il fait ses calculs pour ce qu'il appelle des modes "très propagatifs" (*"well cut-on"*) qui respectent :

$$\omega > |2\beta H| \sqrt{1 - M^2} \quad (4.3)$$

La loi de Tester pour l'impédance optimale avec un écoulement uniforme est :

$$\mathcal{Z}_T = (0.929 - 0.744i) \frac{2\omega}{\pi} \frac{1}{(1 + M)^2} \quad (4.4)$$

L'impédance pour le point double a été obtenue par essais et erreurs pour différentes fréquences. Ces valeurs sont comparées avec la formule de Cremer corrigée par Tester pour le cas sans écoulement sur la figure 4.3. Pour un profil de Poiseuille et uniforme, ils sont comparés avec la loi de Tester sur la figure 4.4.

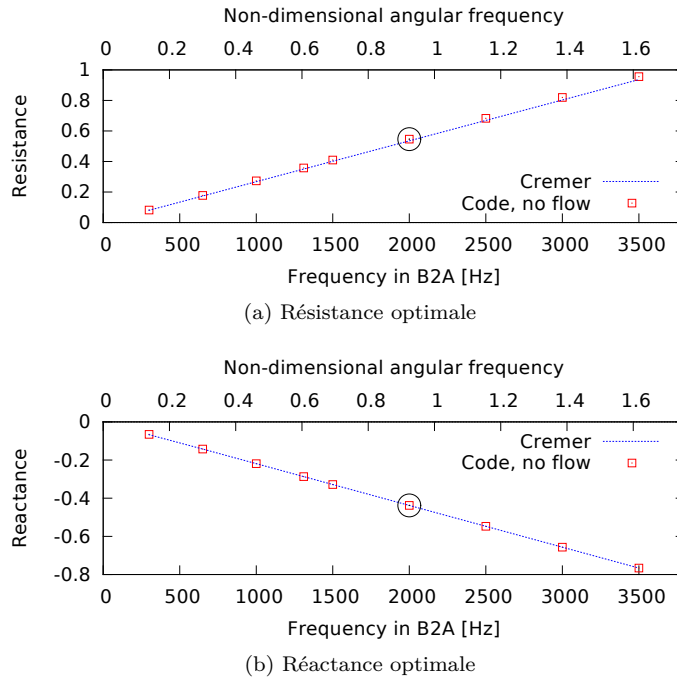


FIGURE 4.3 – Impédance optimale sans écoulement ($\square$), comparée aux valeurs de Cremer (---), $\omega = 0.924$ indique avec un cercle noir. Les fréquences dimensionnelles correspondent à $B2A$.

À partir des images 4.3 et 4.4 on peut voir que la loi de Cremer est respectée complètement, pendant que celle de Tester montre une tendance proche, même s'il y a des différences. Elles sont dues au fait que les modes montrés ne correspondent pas aux modes "très propagatifs".

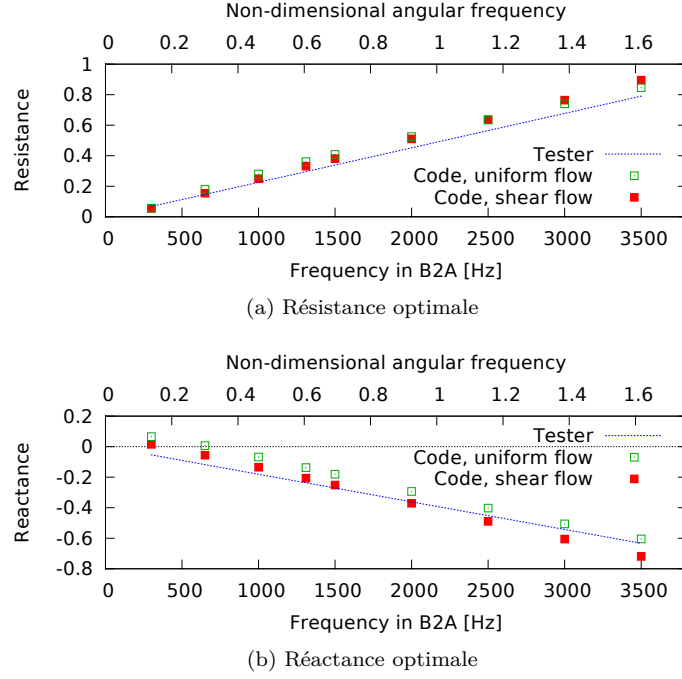


FIGURE 4.4 – Impédance optimale avec écoulement uniforme (□) et avec le profil de Poiseuille (■), comparée aux valeurs de Tester (---). Mach = 0.1. Les fréquences dimensionnelles correspondent à $B2A$.

4.3 Vitesse de groupe

L'étude de la vitesse de groupe pour les modes du point double est montrée.

La vitesse de groupe est définie comment :

$$v_{gr} = \frac{\partial \omega}{\partial \alpha} \quad (4.5)$$

4.3.1 Changements de la réactance

L'évolution de la vitesse de groupe pour des résistances fixes, et réactances variables est montrée dans les figures 4.5 et 4.6 pour une étude sans écoulement. Il est évident qu'à la valeur de l'impédance optimale, les deux modes coïncident en une vitesse de groupe nulle. Dans la figure 4.7 cette coïncidence à $v_{gr} = 0$ est montrée pour un cas avec un écoulement moyen à Mach = 0.1 et un profil cisailé.

Dans ce chapitre, nous avons identifié le spectre qui donne la meilleure atténuation du son. Ce spectre contient toujours une coïncidence de deux modes, le point double.

De là, nous avons défini l'impédance optimale, et trouvé sa valeur pour certains cas. Ces valeurs ont été comparées avec des formules existantes et nous avons vu qu'elles correspondent bien.

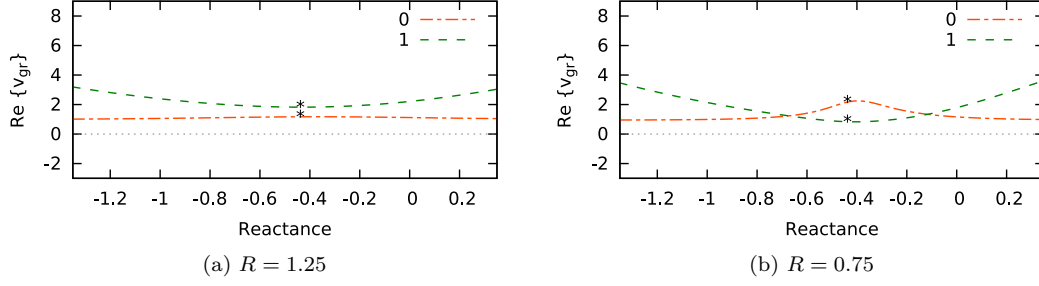


FIGURE 4.5 – Vitesse de groupe à réactance variable pour les modes coïncidents. Sans écoulement moyen. Résistance plus grande que l'optimale. $\omega = 0.924$.

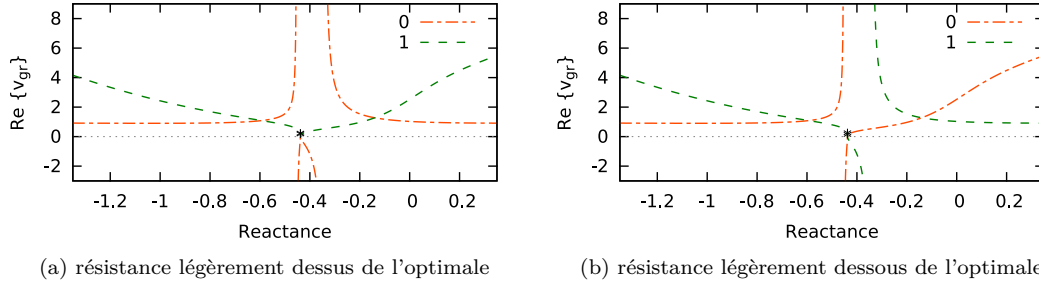


FIGURE 4.6 – Vitesse de groupe à réactance variable pour les modes coïncidents. Sans écoulement moyen. Résistance légèrement dessus et dessous de l'optimale. $\omega = 0.924$.

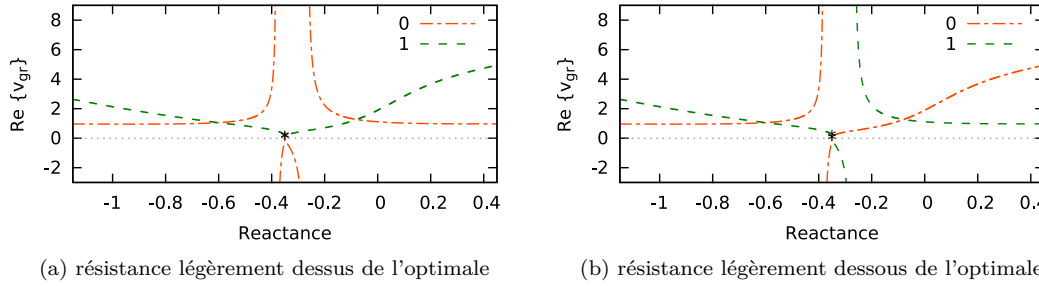


FIGURE 4.7 – Vitesse de groupe à réactance variable pour les modes coïncidents. Écoulement moyen à $M = 0.1$ cisailé. Résistance légèrement dessus et dessous de l'optimale. $\omega = 0.924$.

Nous avons constaté que la vitesse de groupe des modes du point double est égale à zéro. Le comportement des deux modes est échangé après la résistance optimale.

Dans le chapitre suivant, une configuration pour laquelle un mode de surface hydrodynamique est présent sera montrée.

Chapitre 5

Modes Hydrodynamiques

Dans ce chapitre nous montrons un cas particulier des modes hydrodynamiques. Cet étude est basée sur les données de Jones et collaborateurs [60]. Les travaux de Julien Primus sur l'identification d'impédance dans la section 5.1 donnent lieu à l'étude présentée ici.

5.1 Analyse des modes propres

Dans ce cas, on considère les deux profils d'écoulement mesurés montrés dans la figure 5.1.

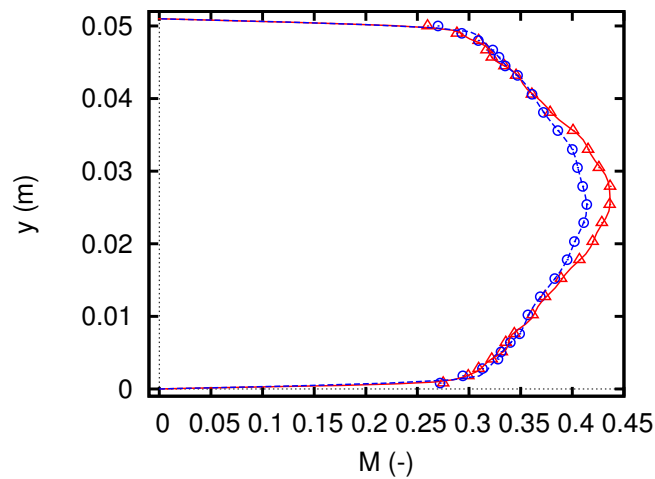


FIGURE 5.1 – Profils d'écoulement dans le centre de la veine. Profil amont le liner en triangles rouges et ligne solide. Profil sur le liner en cercles bleus et ligne pointillée. Données de Jones et al. [60] en symboles.

5.1.1 Modèle d'impédance

L'impédance de la paroi inférieure est décrite par le modèle étendu du résonateur de Helmholtz (nommé *EHR* pour son acronyme anglais) [96]. Celui prend la forme :

$$\mathcal{Z} = r + id\omega - i\beta_{\mathcal{Z}} \cot\left(\omega\frac{L}{c_0} - i\frac{\varepsilon_{\mathcal{Z}}}{2}\right) \quad (5.1)$$

où $r + id\omega$ est l'impédance de la feuille, L est la profondeur des résonateurs, $\beta_{\mathcal{Z}}$ est un paramètre d'échelle de la réactance de la cavité, $\varepsilon_{\mathcal{Z}}$ est lié à la viscosité de l'écoulement. Tous ces paramètres doivent être positifs [96].

La figure 5.2 montre les valeurs d'impédance obtenus par Jones et al. [60] pour un nombre de Mach débitant de 0.33. Elles sont comparées avec le modèle employé avec les paramètres :

$$r = 10^{-6}, \quad d = 10^{-6}, \quad \frac{L}{c_0} = 2.677 \times 10^{-4}, \quad \beta_{\mathcal{Z}} = 0.58, \quad \varepsilon_{\mathcal{Z}} = 1.4$$

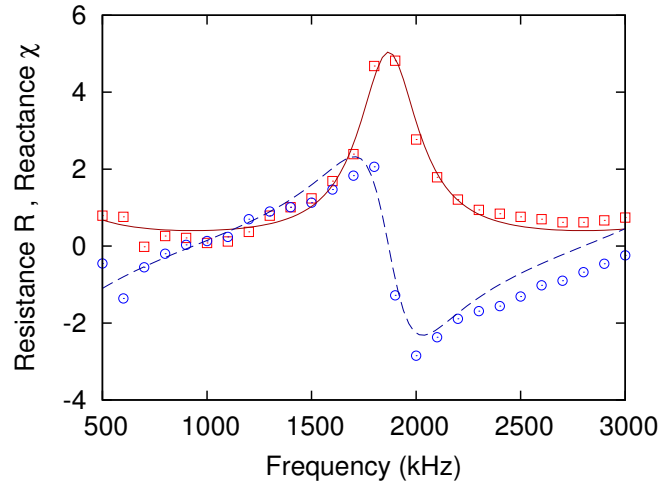


FIGURE 5.2 – Modèle EHR en lignes, comparé aux données de et al. [60] en symboles.

5.1.2 Exemples des modes propres

Pour 1 kHz, le spectre est montré dans la figure 5.3. La branche continue est montrée en croix petites ; ces modes ne seront plus tracés par la suite. Les modes acoustiques sont en cercles rouges et les hydrodynamiques en carrés verts. Des étiquettes sont mises sur le deuxième mode acoustique (Ac2) et pour les modes hydrodynamiques (H1 et H2).

Les fonctions propres des modes marqués sont montrés dans la figure 5.4.

Une instabilité hydrodynamique est supposée d'exister pour cette configuration.

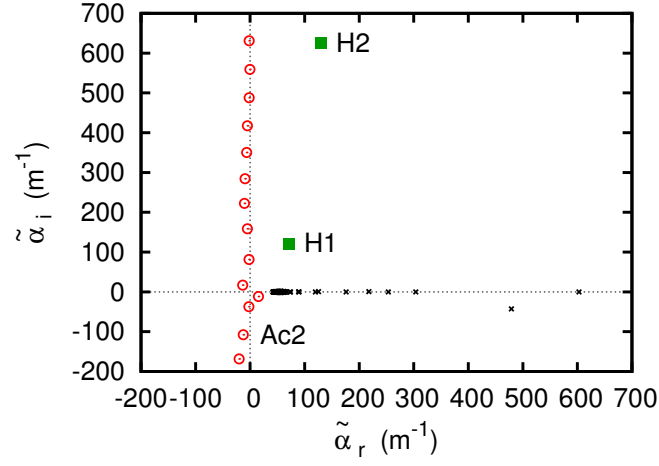


FIGURE 5.3 – Modes acoustiques en cercles rouges $\circ$, modes hydrodynamiques en carrés verts $\blacksquare$.

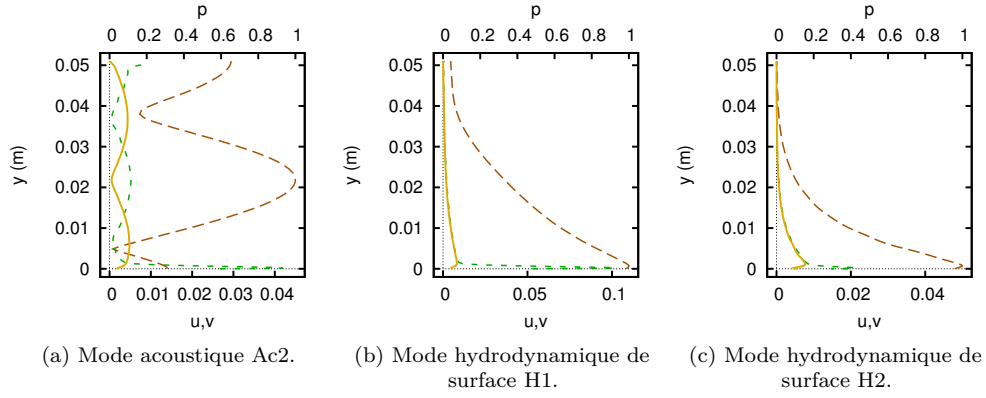


FIGURE 5.4 – Valeur absolue des fonctions propres des modes sélectionnés.
 — — pression , - - - vitesse longitudinale , — vitesse normale.

5.2 Analyse de stabilité

Comme ils ont une partie imaginaire positive, les deux modes hydrodynamiques peuvent être soit des modes stables qui se propagent vers la gauche ou des modes instables qui se propagent vers la droite. Pour connaître dans quel cas nous sommes, il faut connaître leur sens de propagation. Pour ce faire, on va vers le critère de stabilité de Briggs-Bers [8].

5.2.1 Critère de stabilité

Pour connaître le sens de propagation d'un mode avec le critère de Briggs-Bers, il faut augmenter la partie imaginaire de la fréquence dès valeurs très négatives jusqu'à zéro. Les modes qui viennent de la moitié inférieure sont propagatifs vers la droite, pendant que ceux qui proviennent de la supérieure, vont vers la gauche.

Si tous les modes restent dans la même moitié de provenance, le système est stable. S'il y a un mode qui traverse l'axe réel, il devient convectivement instable. Si, encore pour $\omega_i < 0$, deux modes de cotés opposés coïncident, on est dans le cas d'une instabilité absolue. Le système serait caractérisé au temps longs par cette fréquence et nombre d'onde. Dans la figure 5.5 ces cas sont illustrés.

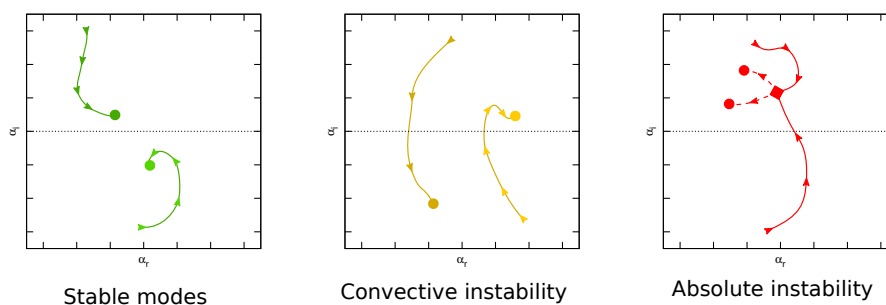


FIGURE 5.5 – Illustration du critère de Briggs-Bers pour vérifier le sens de propagation des modes.

5.2.2 Stabilité des modes hydrodynamiques

On fait le suivi des modes hydrodynamiques pour ω_i variable. Les trajectoires des modes H1 et H2 sont montrées dans la figure 5.6. D'après cette figure, on peut voir que le mode H2 reste dans la même moitié du plan complexe ; donc, il s'agit d'un mode stable. Par contre, le mode H1 traverse l'axe réel : il est un mode propagatif vers la droite. Cela veut dire qu'il est un mode convectivement instable.

5.2.3 Évolution en fréquence

Pour savoir si l'instabilité est présente pour d'autres fréquences en plus de 1 kHz, on a fait un balayage en fréquence entre 0.5 et 3.0 kHz ; les limites des données de Jones [60].

Ces résultats sont montrés dans la figure 5.7. Quand on fait à nouveau l'analyse BB pour 1.5 kHz, H1 est devenu un mode acoustique et se propage vers la droite. Pendant que le mode H2 a

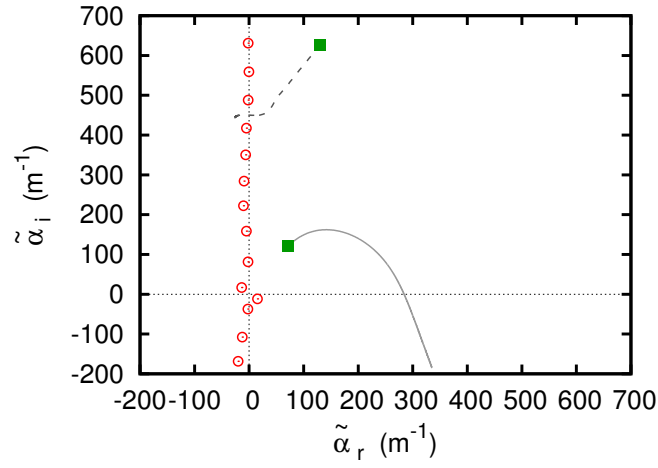


FIGURE 5.6 – Briggs-Bers à 1 kHz pour les modes hydrodynamiques. — H1 , - - - H2 ,
 ■ $\omega_i = 0$. ω_i provenance montre qu'ils sont des modes propagatifs vers la droite.

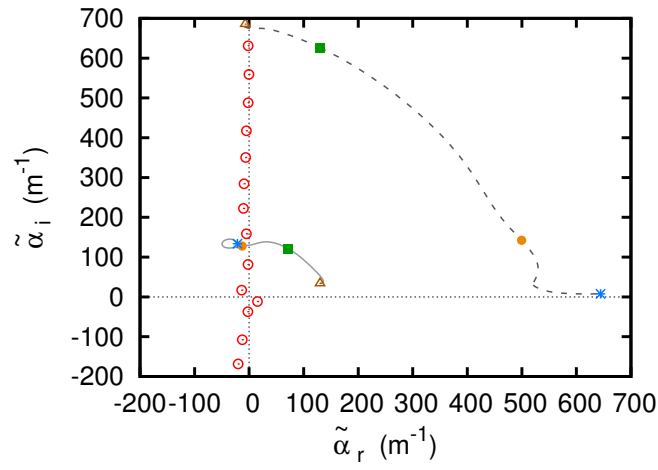


FIGURE 5.7 – Évolution en fréquence des nombres d'onde pour les modes H1 et H2 : ■ 1 kHz, △
 0.5 kHz, ● 1.5 kHz, * 3 kHz.

changé son sens de propagation et est donc devenu instable.

5.2.4 Instabilité absolue

Si les deux modes ont changé leur sens de propagation avec une valeur négative de ω_i , cela peut être le signe d'une instabilité absolue. Pour trouver si cela arrivait, l'analyse BB s'est faite pour plusieurs fréquences.

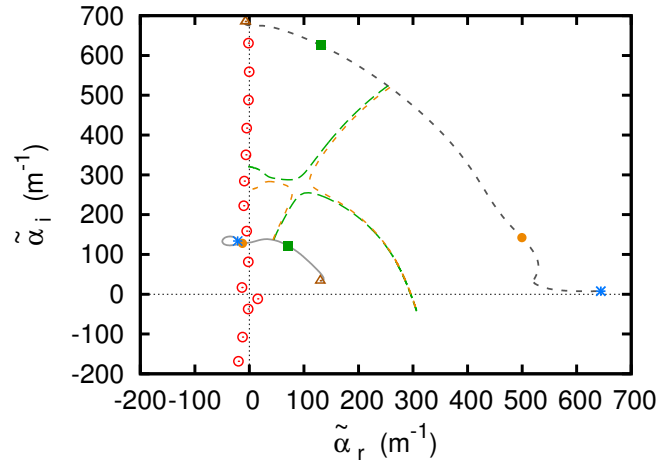


FIGURE 5.8 – Trajectoires des modes H1 et H2 à 1043 Hz (— — —) et 1045 Hz (· · · · ·) un instabilité absolue est évidente.

Dans la figure 5.8, la présence d'une instabilité absolue entre 1043 et 1045 Hz est claire car il y a un échange entre les modes H1 et H2. Il faut noter que cette instabilité absolue reste même avec le profil d'écoulement sur le liner et avec des petites modifications des paramètres du modèle d'impédance. Bien que les valeurs de la fréquence peuvent se modifier légèrement.

5.3 Investigation des effets de la viscosité

Pour étudier si le mode instable est atténué par la présence de la viscosité, l'étude est répétée avec les équations linéarisées de Navier-Stokes.

La figure 5.9 montre l'effet de la viscosité sur les valeurs propres. Plusieurs valeurs de l'impédance sont considérées pour pouvoir suivre les modes avant d'arriver à la valeur de la viscosité de l'air.

Les modes acoustiques restent dans les mêmes endroits pour n'importe quelle viscosité. Par contre, les modes hydrodynamiques sont fortement influencés, la valeur de α_i décroît pour H1 et augmente pour H2. Cela veut dire que les deux modes deviennent plus stables.

L'analyse BB est montrée dans la figure 5.10, pour les deux modes. On trouve que H1 continue à être une instabilité convective, pendant que H2 est un mode stable qui se propage vers la gauche.

Quand on fait le suivi des modes entre 0.5 et 3.0 kHz, on peut voir qu'il y a un grand intervalle

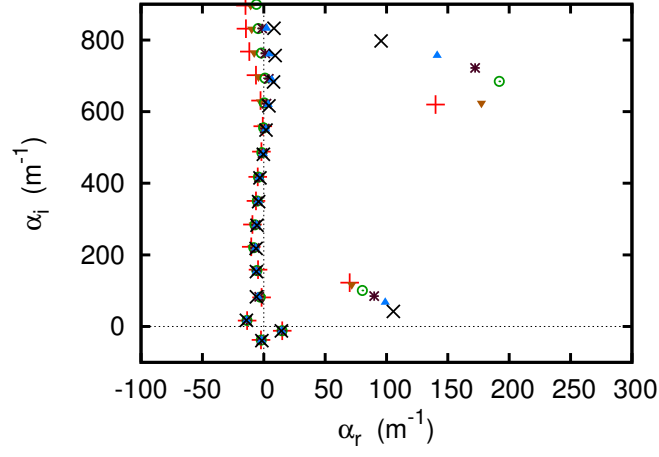


FIGURE 5.9 – Spectre à 1 kHz pour différents valeurs de la viscosité : $+$ $\mu_0 = 0$, $\blacktriangledown$ $\mu_0 = \frac{\mu_a}{100}$, $\circ$ $\mu_0 = \frac{\mu_a}{10}$, $*$ $\mu_0 = \frac{\mu_a}{4}$, $\blacktriangle$ $\mu_0 = \frac{\mu_a}{2}$, $\times$ $\mu_0 = \mu_a$.

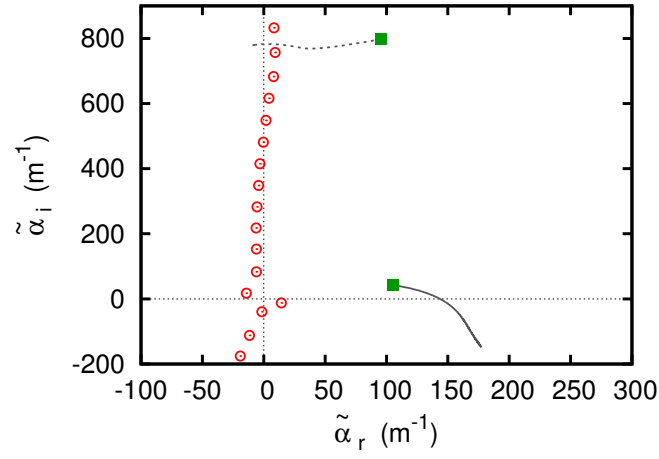


FIGURE 5.10 – BB à 1kHz avec viscosité.

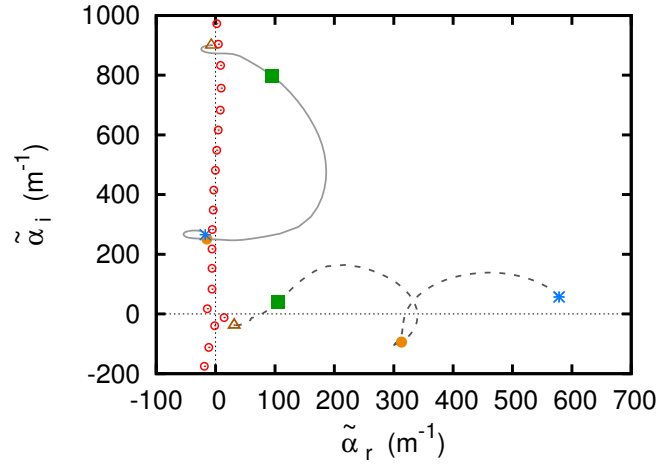


FIGURE 5.11 – Évolution en fréquence des nombres d'onde des modes H1 et H2 avec la viscosité de l'air : ■ 1 kHz, △ 0.5 kHz, ● 1.5 kHz, * 3 kHz.

de fréquences pour lesquelles H1 est stable. Dans ce cas il n'y a pas d'échange de comportement pour les modes. Cela veut dire que l'instabilité reste toujours convective.

Dans ce chapitre, on a montré l'existence de deux modes hydrodynamiques parmi lesquels un est instable convectivement.

Pour des valeurs proches à 1.0 kHz il y a aussi une instabilité absolue.

La viscosité rend plus atténués les deux modes et fait disparaître l'instabilité absolue.

Conclusion

Dans ce travail, nous faisons une étude sur les modes acoustiques présents dans des conduits revêtus de matériaux absorbants pour obtenir une atténuation du bruit. La géométrie employée est rectangulaire pour pouvoir comparer avec les expériences faites dans les veines d'essai comme B2A de l'Onera ou GIT et ses dérivés de la NASA.

Le champ acoustique est considéré comme une collection de modes. Chaque mode a une valeur propre associée. Le spectre obtenu dépend essentiellement de l'impédance acoustique du matériau absorbant et des caractéristiques de l'écoulement.

Deux programmes sont employés dans ce travail : MAMOUT, écrit par Jean-Philippe Brazier et FiEStA, développé dans le cadre de cette thèse. Celui-ci a été validé par rapport à des solutions analytiques et avec des résultats de Mamout et d'autres données publiées.

Avec FiEStA, nous avons montré les principales modifications du spectre et des fonctions propres introduites en fonction de trois caractéristiques du problème. Ces modifications sont : le rapport d'aspect pour le cas transversal, la valeur de l'impédance acoustique et la forme et le nombre de Mach du profil de l'écoulement moyen.

La recherche autour de l'influence de l'impédance conduit à trouver la valeur qui donne la meilleure réduction du bruit dans la configuration simplifiée d'un conduit infiniment long et uniformément traité acoustiquement. Pour ce faire, nous avons récupéré les formules énoncées par Cremer et Tester. Avec cela, on a montré que le spectre de la meilleure réduction du bruit est obtenue lorsqu'il y a une coïncidence d'une paire de modes. La formule obtenue par Cremer ne considère pas un écoulement moyen et Tester en propose un correctif. Tester obtient aussi une loi qui considère un écoulement moyen à profil uniforme qui n'est valable que pour ce qu'il appelle des modes "très propagatifs". Ces formules donnent un accord assez bon pour l'impédance optimale avec les résultats obtenus numériquement dans cette thèse.

La vitesse de groupe pour le point double a également été étudiée. Dans tous les cas, à l'impédance optimale, les modes impliqués ont une vitesse de groupe nulle. Après cette valeur, les modes échangent leur comportement. Cela présente un comportement global avec des modes stables.

En plus des modes acoustiques, une paire de modes de surface hydrodynamiques peut également apparaître avec un écoulement moyen. La stabilité de ces modes a été étudiée avec le critère Briggs Bers pour la configuration de GIT à partir des données obtenues de la littérature. Avec ces données, à une fréquence de 1 kHz, un mode a été trouvé stable pendant que l'autre est instable. Une instabilité absolue a été trouvée pour une fréquence réelle légèrement au-dessus de 1 kHz, même avec des variations du modèle d'impédance et du profil d'écoulement.

L'étude des modes hydrodynamiques a été répétée avec les équations de Navier-Stokes linéarisées, pour considérer la viscosité. Dans ce cas, le mode instable devient plus atténué et il

n'existe plus d'instabilité absolue.

Perspectives

Certaines recherches qui peuvent être faites au-delà de cette thèse sont exposées maintenant.

D'une manière générale, aussi bien pour l'étude de l'instabilité et l'impédance optimale, l'analyse doit également être faite dans un domaine bidimensionnel longitudinal à la veine. De cette façon, on pourrait comparer les résultats avec des programmes de propagation aéroacoustique. Une autre modification qui devrait être faite est d'inclure des discontinuités d'impédance pour mieux représenter le placement des liners dans le banc B2A, par exemple.

En ce qui concerne l'instabilité hydrodynamique, il serait utile de rechercher des preuves expérimentales de son existence, pour la configuration utilisée dans cette étude.

Quelques améliorations pourraient aussi être faites à FiEStA, pour approfondir ses capacités de recherche :

- ◀ Mettre en œuvre une sous-routine pour chercher l'impédance optimale. Ceci peut être réalisé car, à cette valeur, la paire de modes a une vitesse de groupe nulle. Cette condition peut être obtenue par une méthode de tir.
- ◀ De même que pour les études en général, considérer dans le code un domaine longitudinal et les discontinuités de l'impédance le long d'une paroi. Dans une étape plus évoluée, on pourrait aussi envisager d'autres géométries et de traiter des effets non-linéaires..