

Energy management strategies for smart grids

Ph.D. thesis presented by:

Peter Pflaum

Supervised by:

Mazen Alamir

Mohamed-Yacine Lamoudi & Claude Le Pape

January 9th 2017

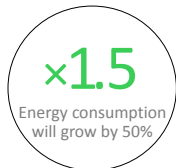
Schneider
Electric



Introduction



Challenge by 2050...



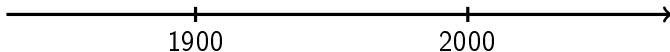
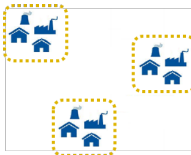
Note: Forecast for 2050 compared to 2009 levels

Introduction



The emergence of smart-/micro grids

Production islands
operated by
cities/industries

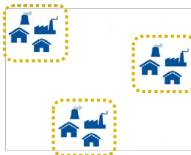


Introduction

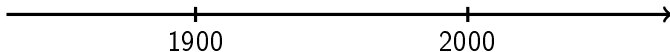


The emergence of smart-/micro grids

Production islands
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Centralization for
better availability

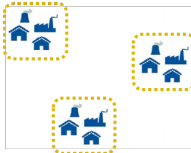


Introduction



The emergence of smart-/micro grids

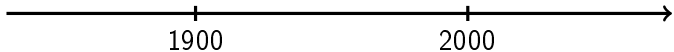
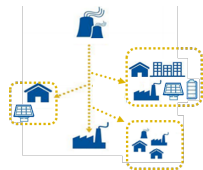
Production islands
operated by
cities/industries



Centralization for
better availability



Decentralization for
"territory" services



The AMBASSADOR project

European FP7 project dedicated to the development of **autonomous energy management systems at building- and district level**

"Develop efficient energy optimization systems for smart districts"

- 15 partners
- 11 different EU countries
- 4 years (2012 - 2016)
- 9.9 M€



Thesis objectives

- Design **control algorithms** able to improve energy management in smart grids
 - 1 Increase **energy efficiency** at district level
 - 2 Improve the **robustness** of the electricity grid
 - 3 Design **generic, scalable** and **modular** solutions

Overview of works

Distributed MPC
for smart grids

MPC for district heating
networks

EV charging station
control under
uncertainties

Battery sizing for PV
power plants

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P. Pflaum, M. Alamir, and M.Y. Lamoudi (2014). “Comparison of a primal and a dual decomposition for distributed MPC in smart districts”. In: *IEEE International Conference on Smart Grid Communications*

P. Pflaum, M. Alamir, and M.Y. Lamoudi (2015). “Scalability study for a hierarchical NMPC scheme for resource sharing problems”. In: *European Control Conference (ECC)*, pp. 1468–1473

F. Bourry et al. (2015). “Simulation for the evaluation of energy management algorithms at the district level - Example of use case from the AMBASSADOR project”. In: *PowerTech, 2015 IEEE Eindhoven*, pp. 1–6

P. Pflaum and M. Alamir (2015b). “Procédé de gestion en énergie d'une station de recharge de véhicules électriques”. [Patent](#)

P. Pflaum and M. Alamir (2015a). “Procédé de contrôle d'une centrale de production électrique”. [Patent](#)

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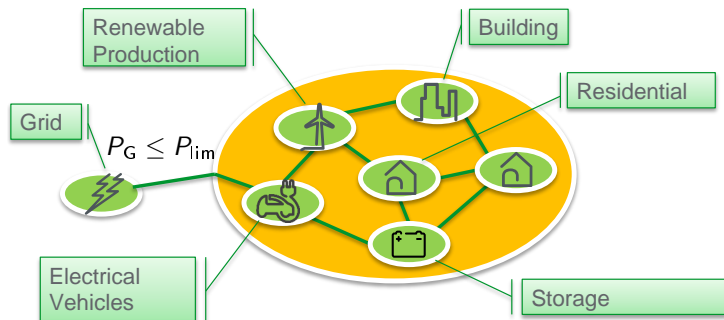
Outline

- 1 Distributed MPC for smart grids
- 2 Randomized algorithm for EV charging stations
- 3 General conclusion

Outline

- 1 Distributed MPC for smart grids
 - System model
 - Model Predictive Control
 - The centralized control problem
 - Primal and dual decomposition
 - Bundle method
 - Validation
- 2 Randomized algorithm for EV charging stations
- 3 General conclusion

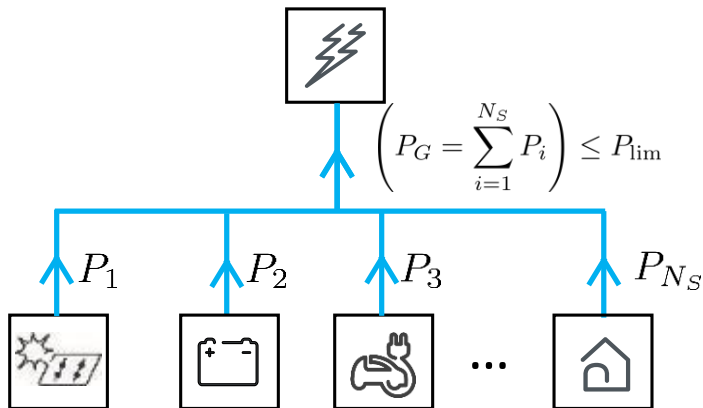
The smart grid



- Common grid connection with a limited capacity P_{lim}
- No consideration of the distribution grid topology and of stability issues (voltage-, frequency constraints,...)

The smart grid

Simplified view:



Building model

Linear first-order model

$$\vartheta^{t+1} = \vartheta^t + \alpha_1(\vartheta_{\text{ext}} - \vartheta^t) + \alpha_2 P_{\text{Bui}}$$



| | |
|-------------------------------------|--|
| $\vartheta, \vartheta_{\text{ext}}$ | indoor- and outdoor temperature |
| P_{Bui} | heating/cooling power of the HVAC system |
| α_1, α_2 | model coefficients |

Building model

Linear first-order model



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- Wide literature on building modeling & control available
- Significant energy and cost savings are reported

Mohamed Yacine Lamoudi, Mazen Alamir, and Patrick Béguery (2012). “Model predictive control for energy management in buildings. Part 1: Zone Model Predictive Control”. In: *Nonlinear Model Predictive Control*. Vol. 4. 1, pp. 21–26

Battery model

Linear first-order battery model

$$E^{t+1} = \begin{cases} E^t + \tau \cdot \eta^+ \cdot P_{\text{Bat}} & \text{if } P_{\text{Bat}} \geq 0 \\ E^t + \tau \cdot 1/\eta^- \cdot P_{\text{Bat}} & \text{otherwise} \end{cases}$$



| | |
|------------------|-----------------------------------|
| E | stored energy |
| P_{Bat} | charging/discharging power |
| η^+, η^- | charging/discharging efficiencies |
| τ | time step |

Electric vehicle charging station model

EVCS consisting of N_{CP} charging points

$$P_{EVCS} = \sum_{v=1}^{N_{CP}} P_{Bat,v}$$

$$E_v^{t+1} = E_v^t + \eta \cdot \tau \cdot P_{Bat,v} \quad ; \quad P_{Bat,v} \geq 0$$



| | |
|-------------|--|
| P_{EVCS} | power consumption of the EVCS |
| E_v | energy delivered to EV at v -th charging point |
| $P_{Bat,v}$ | charging power at v -th charging point |
| η | charging efficiency |
| τ | time step |

Renewable energy source model



- Non-controllable renewable energy sources
- Represented through their predicted production profile $\hat{\mathbf{P}}_{\text{RES}}$ only



Model Predictive Control

MPC - An intuitive concept

Anticipate the future to find the optimal control at each control instant.

Model

$$\frac{\partial \theta}{\partial \theta} \ln f_{\theta}(s) = \frac{\partial}{\partial \theta} \int_{\mathcal{X}} \pi(x, \theta) \ln \pi(x, \theta) dx$$

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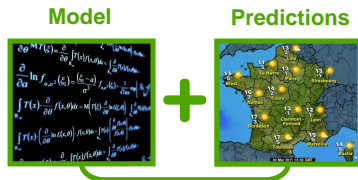
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Source of illustration: DMPC for energy management in buildings,
Mohamed-Yacine Lamoudi, 2012

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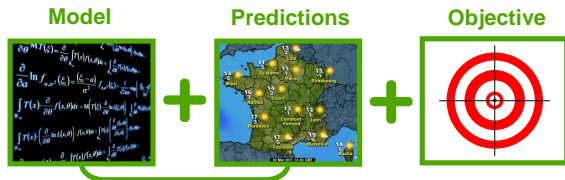


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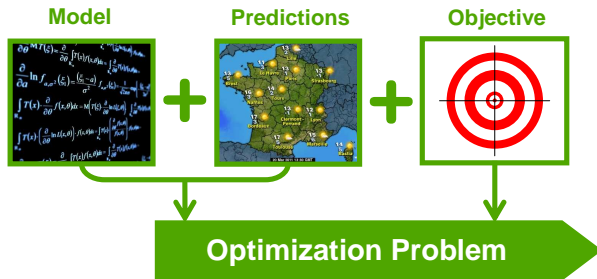


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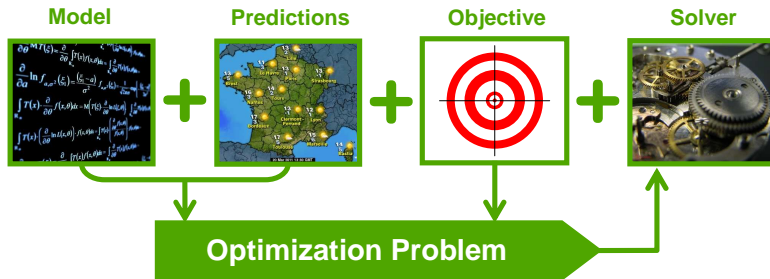


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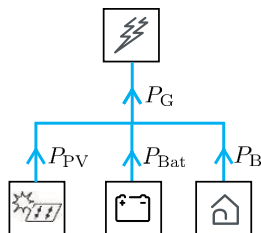
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Smart grid control objective

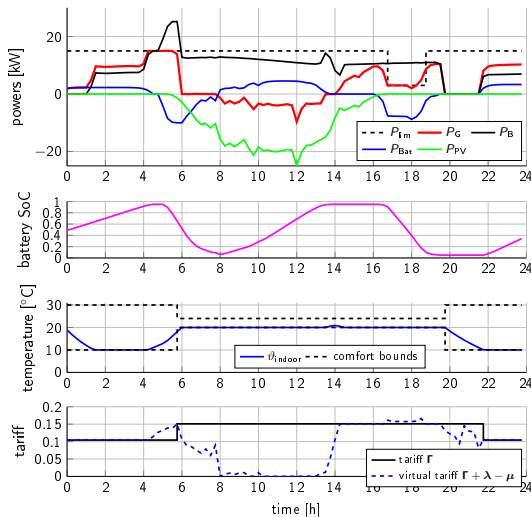
- Minimize energy costs
- Respect the global power limitation $P_G \leq P_{\text{lim}}$

Smart grid control objective

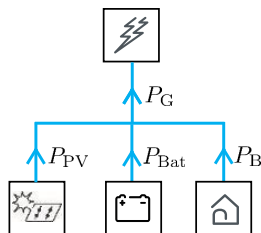
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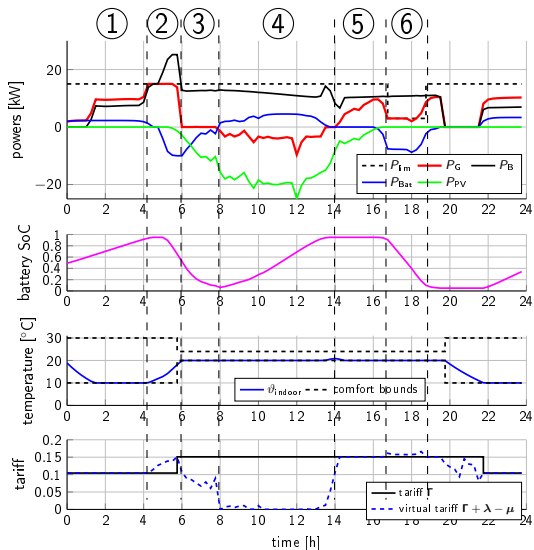
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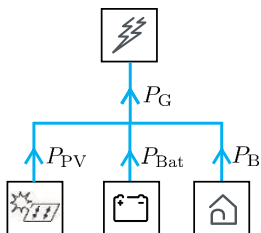
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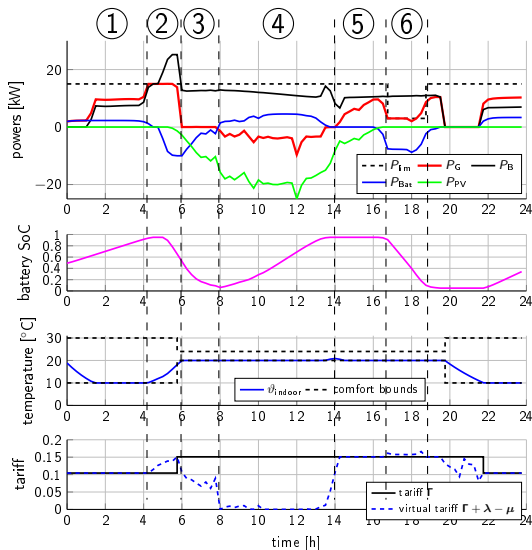
Smart grid control objective



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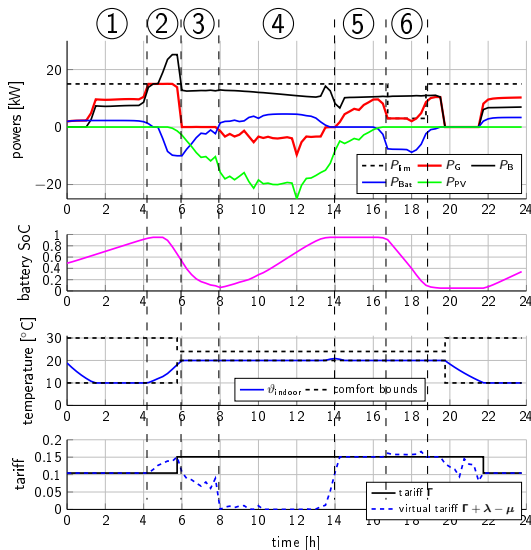
Smart grid control objective



- Minimize energy costs
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1 charge battery while energy is cheap

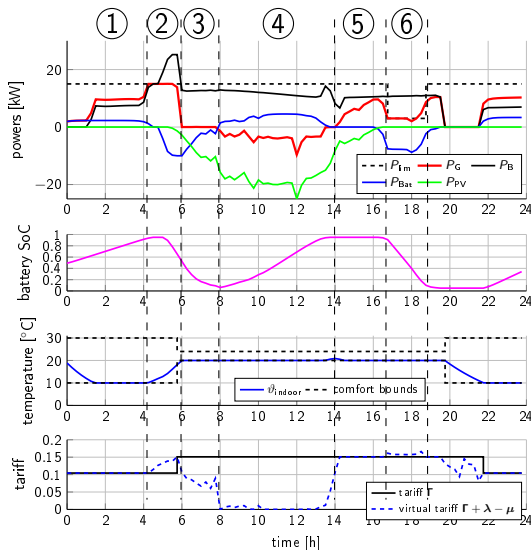
Smart grid control objective



- Minimize energy costs
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- 1 charge battery while energy is cheap
- 2 Battery supplies to building so that $P_G \leq P_{lim}$

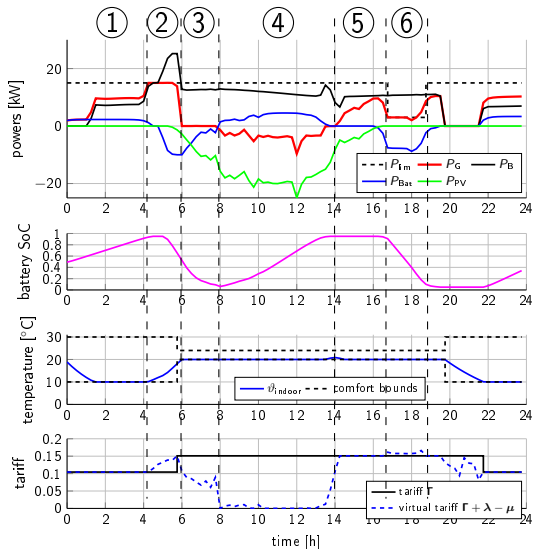
Smart grid control objective



- Minimize energy costs
- Respect the global power limitation $P_G \leq P_{lim}$

- ① charge battery while energy is cheap
- ② Battery supplies to building so that $P_G \leq P_{lim}$
- ③ $P_G = 0$

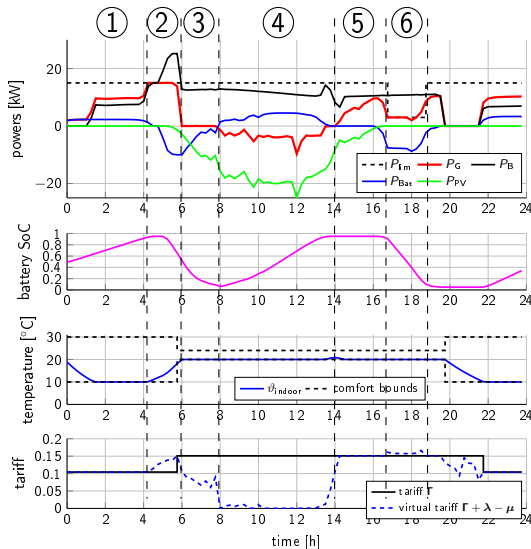
Smart grid control objective



- Minimize energy costs
- Respect the global power limitation $P_G \leq P_{lim}$

- 1 charge battery while energy is cheap
- 2 Battery supplies to building so that $P_G \leq P_{lim}$
- 3 $P_G = 0$
- 4 battery charges from PV

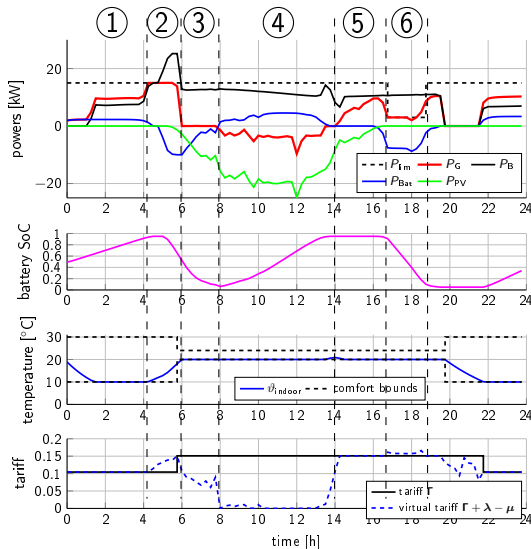
Smart grid control objective



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- Respect the global power limitation $P_G \leq P_{lim}$

- 1 charge battery while energy is cheap
- 2 Battery supplies to building so that $P_G \leq P_{lim}$
- 3 $P_G = 0$
- 4 battery charges from PV
- 5 buy expensive energy from grid to keep the battery charged

Smart grid control objective



- Minimize energy costs
- Respect the global power limitation $P_G \leq P_{lim}$

- 1 charge battery while energy is cheap
- 2 Battery supplies to building so that $P_G \leq P_{lim}$
- 3 $P_G = 0$
- 4 battery charges from PV
- 5 buy expensive energy from grid to keep the battery charged
- 6 Respond to a planned load shedding event

Resource sharing problem

- System composed of many **dynamically uncoupled** sub-systems
- Coupling through a **limited shared resource**

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$$\text{Minimize}_{\{\mathbf{r}_\ell, \mathbf{x}_\ell\}_{\ell \in \mathcal{S}}} \sum_{\ell \in \mathcal{S}} L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell)$$

L_ℓ local objective fc.

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L_ℓ local objective fc.
 \mathbf{x}_ℓ local variables
(indoor temperatures,...)

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| \mathbf{r}_ℓ | local resource profile (power consumption,...) |

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Subj. to:

$$\mathbf{h}_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \leq 0 \quad \forall \ell \in \mathcal{S}$$

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| \mathbf{R}_{lim} | global resource limit |
| $\boldsymbol{\Gamma}$ | energy tariff |

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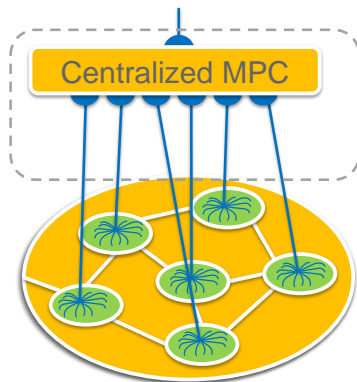
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| \mathbf{R}_{lim} | global resource limit |
| $\boldsymbol{\Gamma}$ | energy tariff |
| | $\boldsymbol{\Gamma}^+ = \text{buying}, \boldsymbol{\Gamma}^- = \text{selling}$ |

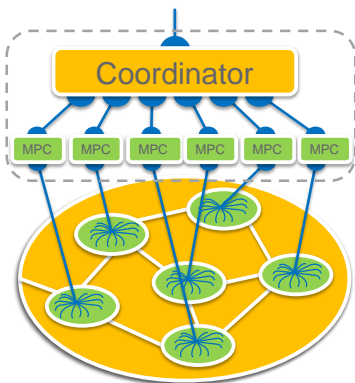
Centralized MPC controller

Drawbacks of a centralized MPC controller:

- Scalability
- Robustness
- Modularity
- Privacy



Distributed MPC controller



- Split centralized controller into several sub-problems and a coordinator
- Iterative process involving some communication between the coordinator and the sub-problems
- Convergence to the global optimum

Primal- and dual decomposition

Iterative information exchange in the two methods

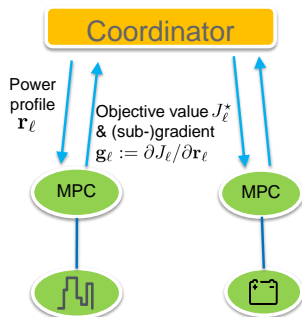
Primal decomposition

Dual decomposition

Primal- and dual decomposition

Iterative information exchange in the two methods

Primal decomposition

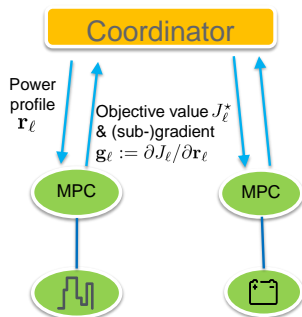


Dual decomposition

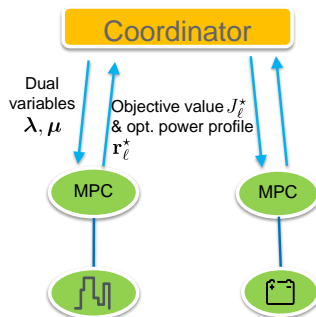
Primal- and dual decomposition

Iterative information exchange in the two methods

Primal decomposition



Dual decomposition



Qualitative comparison

Primal decomposition:

- Feasibility at each iteration
($\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \leq \mathbf{R}_{\text{lim}}$)
- The centralized problem only has to be **convex**
- nb. of decision variables **proportional** to N_S

Dual decomposition:

- Feasibility only after convergence
- The centralized problem must be **strongly convex**
- nb. of decision variables is **independent** of N_S

Primal decomposition

Initial centralized problem:

$$\begin{aligned}
 & \text{Minimize}_{\{\mathbf{r}_l, \mathbf{x}_l\}_{l \in \mathcal{S}}} \sum_{l \in \mathcal{S}} L_l(\mathbf{x}_l, \mathbf{r}_l) + \boldsymbol{\Gamma}^T \cdot \left(\sum_{l \in \mathcal{S}} \mathbf{r}_l \right) \\
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 \end{aligned}$$

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$$\text{Subj. to: } \sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \leq \mathbf{R}_{\text{lim}}$$

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Sub-problem ℓ
(for given \mathbf{r}_ℓ):

$$\boldsymbol{\Gamma} = \begin{cases} \boldsymbol{\Gamma}^+ & \text{if } \sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \geq 0 \\ \boldsymbol{\Gamma}^- & \text{otherwise} \end{cases}$$

$$\begin{aligned} J_\ell(\mathbf{r}_\ell) &:= \text{Minimize}_{\mathbf{x}_\ell} L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \\ \text{Subject to: } & \mathbf{h}_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \leq 0 \end{aligned}$$

Primal decomposition

Coordinator problem:

$$\begin{aligned} \text{Minimize}_{\{\mathbf{r}_\ell\}_{\ell \in \mathcal{S}}} \quad & \sum_{\ell \in \mathcal{S}} J_\ell(\mathbf{r}_\ell) + \boldsymbol{\Gamma}^T \cdot \left(\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \right) \\ \text{Subj. to:} \quad & \sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \leq \mathbf{R}_{\text{lim}} \end{aligned}$$

Sub-problem ℓ
(for given \mathbf{r}_ℓ):

$$\boldsymbol{\Gamma} = \begin{cases} \boldsymbol{\Gamma}^+ & \text{if } \sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \geq 0 \\ \boldsymbol{\Gamma}^- & \text{otherwise} \end{cases}$$

$$\begin{aligned} J_\ell(\mathbf{r}_\ell) := \text{Minimize}_{\mathbf{x}_\ell} \quad & L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \\ \text{Subject to:} \quad & \mathbf{h}_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \leq 0 \end{aligned}$$

Primal decomposition

Difficulty: $J_\ell(\mathbf{r}_\ell)$ can only be evaluated *point-wise*!

Coordinator problem:

$$\begin{aligned} \text{Minimize}_{\{\mathbf{r}_\ell\}_{\ell \in \mathcal{S}}} \quad & \sum_{\ell \in \mathcal{S}} J_\ell(\mathbf{r}_\ell) + \boldsymbol{\Gamma}^T \cdot \left(\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \right) \\ \text{Subj. to:} \quad & \sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \leq \mathbf{R}_{\text{lim}} \end{aligned}$$

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Dual decomposition

Initial centralized problem:

$$\begin{aligned}
 & \text{Min}_{\{\mathbf{r}_\ell, \mathbf{x}_\ell\}_{\ell \in \mathcal{S}}} \sum_{\ell \in \mathcal{S}} L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) + \boldsymbol{\Gamma}^T \cdot \left(\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \right) \\
 \text{Subj. to: } & \mathbf{h}_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \leq 0 \quad \forall \ell \in \mathcal{S} \\
 & \sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \leq \mathbf{R}_{\text{lim}} \\
 & \boldsymbol{\Gamma} = \begin{cases} \boldsymbol{\Gamma}^+ & \text{if } \sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \geq 0 \\ \boldsymbol{\Gamma}^- & \text{otherwise} \end{cases}
 \end{aligned}$$

Dual decomposition

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 & \text{Min}_{\{\mathbf{r}_\ell, \mathbf{x}_\ell\}_{\ell \in \mathcal{S}}} \sum_{\ell \in \mathcal{S}} L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) + \boldsymbol{\Gamma}^T \cdot \left(\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \right) \\
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 \end{aligned}$$

Dual decomposition

Step 1: Build Lagrangian $\mathcal{L}(\mathbf{x}_l, \mathbf{r}_l, \boldsymbol{\lambda})$

Step 2: Dual problem

$$\text{Max}_{\boldsymbol{\lambda}} \left[\text{Min}_{\{\mathbf{r}_l, \mathbf{x}_l\}_{l \in \mathcal{S}}} \sum_{l \in \mathcal{S}} L_l(\mathbf{x}_l, \mathbf{r}_l) + \boldsymbol{\Gamma}^T \cdot \left(\sum_{l \in \mathcal{S}} \mathbf{r}_l \right) + \boldsymbol{\lambda}^T \cdot \left(\sum_{l \in \mathcal{S}} \mathbf{r}_l - \mathbf{R}_{\text{lim}} \right) \right]$$

Subj. to: $\mathbf{h}_l(\mathbf{x}_l, \mathbf{r}_l) \leq 0 \quad \forall l \in \mathcal{S}$

$$\boldsymbol{\Gamma} = \begin{cases} \boldsymbol{\Gamma}^+ & \text{if } \sum_{l \in \mathcal{S}} \mathbf{r}_l \geq 0 \\ \boldsymbol{\Gamma}^- & \text{otherwise} \end{cases}$$

$$\boldsymbol{\lambda} \geq 0$$

Dual decomposition

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NOT decomposable yet

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Dual decomposition

Step 1: Build Lagrangian $\mathcal{L}(\mathbf{x}_\ell, \mathbf{r}_\ell, \boldsymbol{\lambda})$

Step 2: Dual problem

$$\begin{aligned} \text{Max}_{\boldsymbol{\lambda}} \left[\text{Min}_{\{\mathbf{r}_\ell, \mathbf{x}_\ell\}_{\ell \in \mathcal{S}}} \sum_{\ell \in \mathcal{S}} L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) + \boldsymbol{\Gamma}^T \cdot \left(\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \right) + \boldsymbol{\lambda}^T \cdot \left(\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell - \mathbf{R}_{\text{lim}} \right) \right] \\ \text{Subj. to: } \mathbf{h}_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \leq 0 \quad \forall \ell \in \mathcal{S} \end{aligned}$$

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$$\boldsymbol{\lambda} \geq 0$$

Step 3: Introduce additional variable $\boldsymbol{\mu}$

Dual decomposition

Step 1: Build Lagrangian $\mathcal{L}(\mathbf{x}_\ell, \mathbf{r}_\ell, \boldsymbol{\lambda})$

Step 2: Dual problem

$$\begin{aligned} \text{Max}_{\boldsymbol{\lambda}, \boldsymbol{\mu}} \left[\text{Min}_{\{\mathbf{r}_\ell, \mathbf{x}_\ell\}_{\ell \in \mathcal{S}}} \sum_{\ell \in \mathcal{S}} L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) + (\boldsymbol{\Gamma}^+ - \boldsymbol{\mu})^T \cdot \left(\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \right) + \boldsymbol{\lambda}^T \cdot \left(\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell - \mathbf{R}_{\text{lim}} \right) \right] \\ \text{Subj. to: } \mathbf{h}_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \leq 0 \quad \forall \ell \in \mathcal{S} \\ 0 \leq \boldsymbol{\mu} \leq \boldsymbol{\Gamma}^+ - \boldsymbol{\Gamma}^- \\ \boldsymbol{\lambda} \geq 0 \end{aligned}$$

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Dual decomposition

Step 1: Build Lagrangian $\mathcal{L}(\mathbf{x}_\ell, \mathbf{r}_\ell, \boldsymbol{\lambda})$

Step 2: Dual problem

$$\begin{aligned} \text{Max}_{\boldsymbol{\lambda}, \boldsymbol{\mu}} \left[\text{Min}_{\{\mathbf{r}_\ell, \mathbf{x}_\ell\}_{\ell \in \mathcal{S}}} \sum_{\ell \in \mathcal{S}} L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) + (\boldsymbol{\Gamma}^+ - \boldsymbol{\mu})^T \cdot \left(\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell \right) + \boldsymbol{\lambda}^T \cdot \left(\sum_{\ell \in \mathcal{S}} \mathbf{r}_\ell - \mathbf{R}_{\text{lim}} \right) \right] \\ \text{Subj. to: } \mathbf{h}_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \leq 0 \quad \forall \ell \in \mathcal{S} \\ 0 \leq \boldsymbol{\mu} \leq \boldsymbol{\Gamma}^+ - \boldsymbol{\Gamma}^- \\ \boldsymbol{\lambda} \geq 0 \end{aligned}$$

DECOMPOSABLE

Step 3: Introduce additional variable $\boldsymbol{\mu}$

Dual decomposition

Step 2: Dual problem

$$\begin{aligned}
 & \text{Max}_{\lambda, \mu} \left[\text{Min}_{\{\mathbf{r}_\ell, \mathbf{x}_\ell\}_{\ell \in \mathcal{S}}} \sum_{\ell \in \mathcal{S}} \left(L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) + (\boldsymbol{\Gamma}^+ + \boldsymbol{\lambda} - \boldsymbol{\mu})^T \cdot \mathbf{r}_\ell \right) - \boldsymbol{\lambda}^T \cdot \mathbf{R}_{\text{lim}} \right] \\
 & \text{Subj. to: } \mathbf{h}_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \leq 0 \quad \forall \ell \in \mathcal{S} \\
 & \quad 0 \leq \boldsymbol{\mu} \leq \boldsymbol{\Gamma}^+ - \boldsymbol{\Gamma}^- \\
 & \quad \boldsymbol{\lambda} \geq 0
 \end{aligned}$$

Dual decomposition

Step 2: Dual problem

Sub-problem ℓ (for given λ, μ)

$$J_\ell(\lambda, \mu) := \text{Min}_{\mathbf{r}_\ell, \mathbf{x}_\ell} L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) + (\Gamma^+ + \lambda - \mu)^T \cdot \mathbf{r}_\ell$$

$$\text{Subj. to: } \mathbf{h}_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \leq 0$$

$$\text{Max}_{\lambda, \mu} \left[\text{Min}_{\{\mathbf{r}_\ell, \mathbf{x}_\ell\}_{\ell \in \mathcal{S}}} \sum_{\ell \in \mathcal{S}} \left(L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) + (\Gamma^+ + \lambda - \mu)^T \cdot \mathbf{r}_\ell \right) - \lambda^T \cdot \mathbf{R}_{\text{lim}} \right]$$

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$$0 \leq \mu \leq \Gamma^+ - \Gamma^-$$

$$\lambda \geq 0$$

Dual decomposition

Step 2: Dual problem

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$$0 \leq \mu \leq \Gamma^+ - \Gamma^-$$

$$\lambda \geq 0$$

Coordinator problem

$$\text{Max}_{\lambda, \mu} \left[\sum_{\ell \in \mathcal{S}} J_\ell(\lambda, \mu) - \lambda^T \cdot \mathbf{R}_{\text{lim}} \right]$$

$$\text{Subj. to: } 0 \leq \mu \leq \Gamma^+ - \Gamma^-$$

$$\lambda \geq 0$$

Dual decomposition

Step 2: Dual problem

Sub-problem ℓ (for given λ, μ)

$$J_\ell(\lambda, \mu) := \text{Min}_{\mathbf{r}_\ell, \mathbf{x}_\ell} L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) + (\Gamma^+ + \lambda - \mu)^T \cdot \mathbf{r}_\ell$$

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$$\text{Max}_{\lambda, \mu} \left[\text{Min}_{\{\mathbf{r}_\ell, \mathbf{x}_\ell\}_{\ell \in \mathcal{S}}} \sum_{\ell \in \mathcal{S}} \left(L_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) + (\Gamma^+ + \lambda - \mu)^T \cdot \mathbf{r}_\ell \right) - \lambda^T \cdot \mathbf{R}_{\text{lim}} \right]$$

$$\text{Subj. to: } \mathbf{h}_\ell(\mathbf{x}_\ell, \mathbf{r}_\ell) \leq 0 \quad \forall \ell \in \mathcal{S}$$

$$0 \leq \mu \leq \Gamma^+ - \Gamma^-$$

$$\lambda \geq 0$$

Coordinator problem

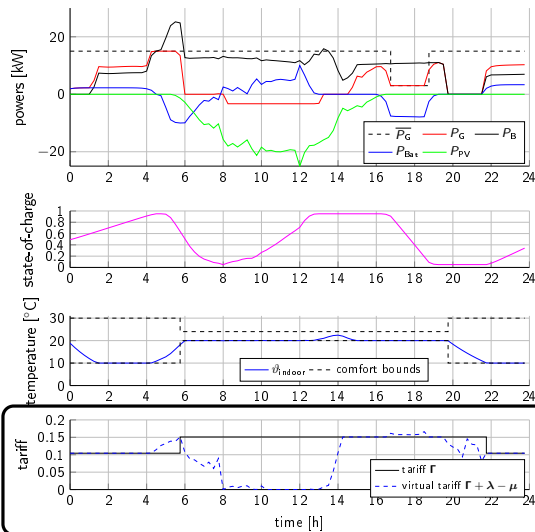
$$\text{Max}_{\lambda, \mu} \left[\sum_{\ell \in \mathcal{S}} J_\ell(\lambda, \mu) - \lambda^T \cdot \mathbf{R}_{\text{lim}} \right]$$

$$\text{Subj. to: } 0 \leq \mu \leq \Gamma^+ - \Gamma^-$$

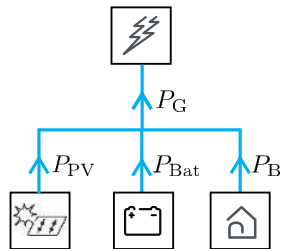
$$\lambda \geq 0$$

Difficulty: $J_\ell(\lambda, \mu)$ can only be evaluated *point-wise*!

Recall...



λ, μ act like modifications of the energy tariff !



Bundle method to solve the coordinator problem

- Iteratively build cutting plane approximations of the sub-problems' cost functions

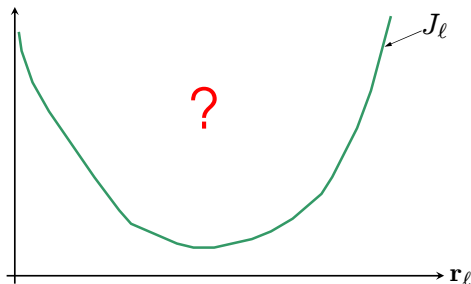
Primal coordinator problem:

$$\begin{aligned} \text{Minimize}_{\{\mathbf{r}_\ell\}_{\ell \in S}} \quad & \sum_{\ell \in S} J_\ell(\mathbf{r}_\ell) + \boldsymbol{\Gamma}^T \cdot \left(\sum_{\ell \in S} \mathbf{r}_\ell \right) \\ \text{Subj. to:} \quad & \sum_{\ell \in S} \mathbf{r}_\ell \leq \mathbf{R}_{\text{lim}} \\ & \boldsymbol{\Gamma} = \begin{cases} \boldsymbol{\Gamma}^+ & \text{if } \sum_{\ell \in S} \mathbf{r}_\ell \geq 0 \\ \boldsymbol{\Gamma}^- & \text{otherwise} \end{cases} \end{aligned}$$

Antonio Frangioni (2002). "Generalized bundle methods". In: *SIAM Journal on Optimization* 13.1, pp. 117–156

Bundle method to solve the coordinator problem

- Iteratively build cutting plane approximations of the sub-problems' cost functions



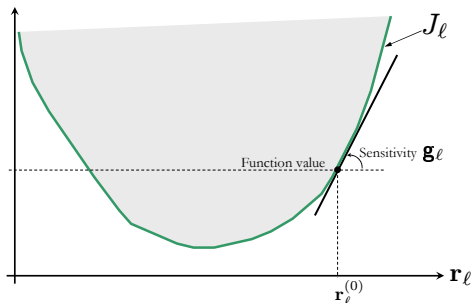
$$\begin{aligned}
 &\text{Minimize}_{\{r_\ell\}_{\ell \in S}} \sum_{\ell \in S} J_\ell(r_\ell) + \Gamma^T \cdot \left(\sum_{\ell \in S} r_\ell \right) \\
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 &\Gamma = \begin{cases} \Gamma^+ & \text{if } \sum_{\ell \in S} r_\ell \geq 0 \\ \Gamma^- & \text{otherwise} \end{cases}
 \end{aligned}$$

Source of illustration: DMPC for energy management in buildings, Mohamed-Yacine Lamoudi, 2012

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Bundle method to solve the coordinator problem

- Iteratively build cutting plane approximations of the sub-problems' cost functions



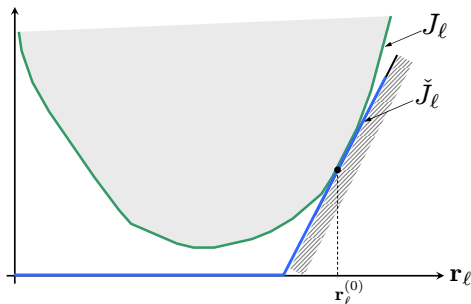
$$\begin{aligned} & \text{Minimize}_{\{r_\ell\}_{\ell \in S}} \sum_{\ell \in S} J_\ell(r_\ell) + \Gamma^T \cdot \left(\sum_{\ell \in S} r_\ell \right) \\ & \text{Subj. to:} \quad \sum_{\ell \in S} r_\ell \leq R_{\text{lim}} \\ & \Gamma = \begin{cases} \Gamma^+ & \text{if } \sum_{\ell \in S} r_\ell \geq 0 \\ \Gamma^- & \text{otherwise} \end{cases} \end{aligned}$$

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Bundle method to solve the coordinator problem

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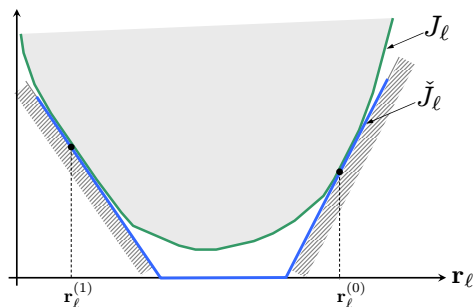
$$\begin{aligned} \text{Minimize} \quad & \sum_{\{r_\ell\}_{\ell \in S}} J_\ell(r_\ell) + \Gamma^T \cdot \left(\sum_{\ell \in S} r_\ell \right) \\ \text{Subj. to:} \quad & \sum_{\ell \in S} r_\ell \leq R_{\text{lim}} \\ & \Gamma = \begin{cases} \Gamma^+ & \text{if } \sum_{\ell \in S} r_\ell \geq 0 \\ \Gamma^- & \text{otherwise} \end{cases} \end{aligned}$$

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Bundle method to solve the coordinator problem

- Iteratively build cutting plane approximations of the sub-problems' cost functions



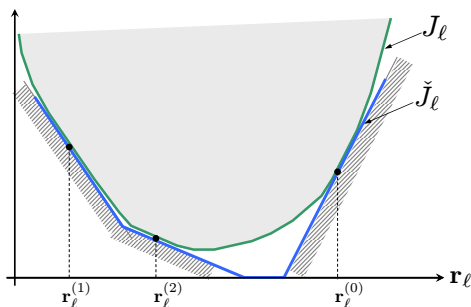
$$\begin{aligned}
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Bundle method to solve the coordinator problem

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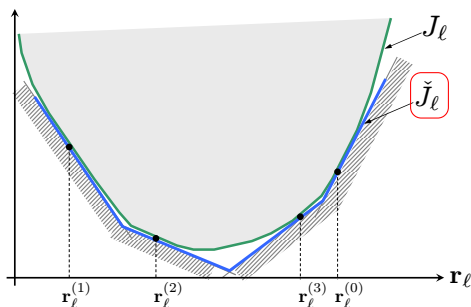
$$\begin{aligned}
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Bundle method to solve the coordinator problem

- Iteratively build cutting plane approximations of the sub-problems' cost functions



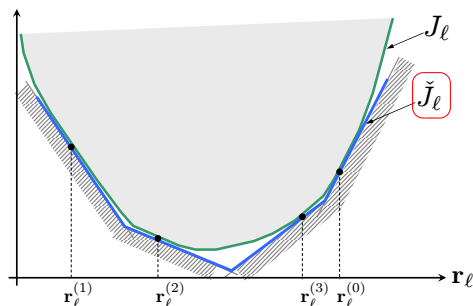
$$\begin{aligned}
 & \text{Minimize} && \sum_{\ell \in S} \check{J}_\ell(\mathbf{r}_\ell) + \boldsymbol{\Gamma}^T \cdot \left(\sum_{\ell \in S} \mathbf{r}_\ell \right) \\
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 \end{aligned}$$

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Bundle method to solve the coordinator problem

- Iteratively build cutting plane approximations of the sub-problems' cost functions



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 & \text{Minimize}_{\{r_\ell\}_{\ell \in S}} \sum_{\ell \in S} \check{J}_\ell(r_\ell) + \Gamma^T \cdot \left(\sum_{\ell \in S} r_\ell \right) \\
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 \end{aligned}$$

→ can be solved as a LP

Source of illustration: DMPC for energy management in buildings, Mohamed-Yacine Lamoudi, 2012

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Solving the coordinator problem

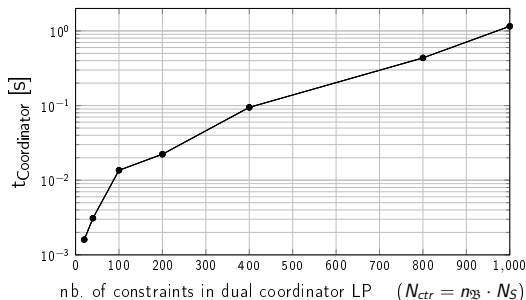
| | |
|-----------|--|
| N_{ctr} | number of constraints |
| N_S | number of sub-systems |
| n_{cg} | number of cuts per cutting plane model |

- each cut results in one constraint in the LP: $N_{ctr} = n_{cg} \cdot N_S$

Solving the coordinator problem

| | |
|--------------------|--|
| N_{ctr} | number of constraints |
| N_S | number of sub-systems |
| $n_{\mathfrak{B}}$ | number of cuts per cutting plane model |

- each cut results in one constraint in the LP: $N_{ctr} = n_{\mathfrak{B}} \cdot N_S$
- Problem: Computation time increases with N_S

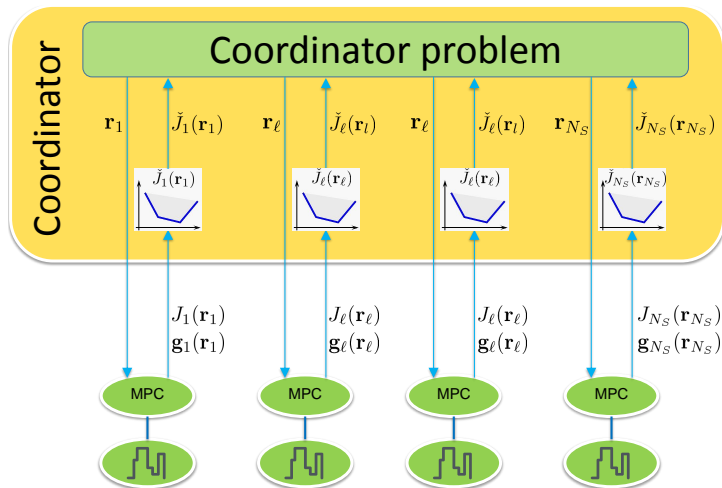


Disaggregated vs. Aggregated Bundle

Disaggregated bundle method

Precise, but high

$$N_{ctr} = N_S \cdot n_{\mathcal{S}}$$

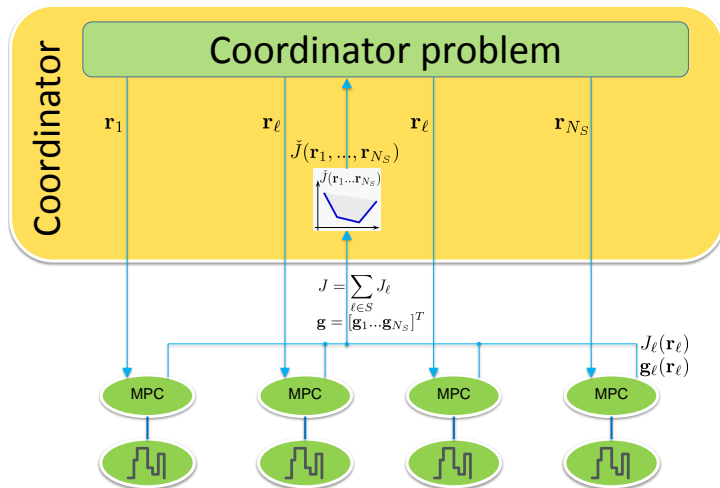


Disaggregated vs. Aggregated Bundle

Fully aggregated bundle method

Less precise, but

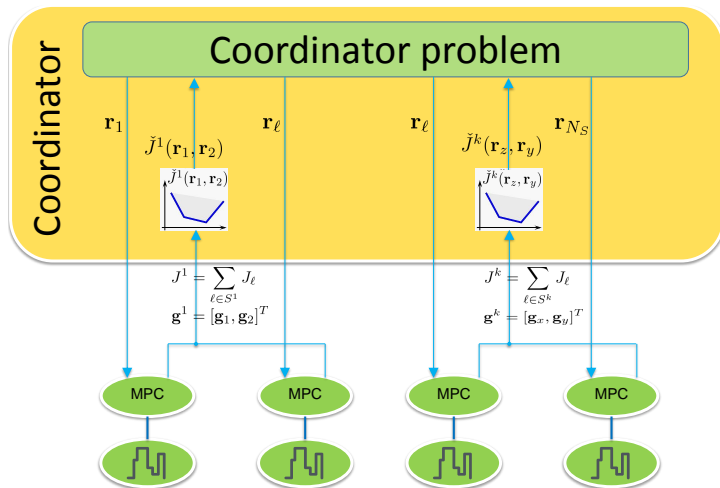
$$N_{ctr} = n_{\mathcal{S}}$$



Disaggregated vs. Aggregated Bundle

Semi-aggregated bundle method

Compromise between the other two



Optimal trade-off for the bundle aggregation

| | n_{iter} | $t_{Coordinator}$ |
|----------|------------|-------------------|
| dis-agg | low | high |
| semi-agg | | |
| agg | high | low |

Trade-off between n_{iter} and $t_{Coordinator}$!

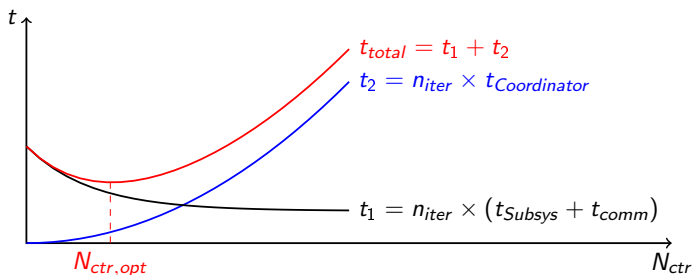
Computation time: $t_{total} = n_{iter} \cdot (t_{Coordinator} + t_{comm} + t_{Subsys})$

Optimal trade-off for the bundle aggregation

| | n_{iter} | $t_{Coordinator}$ |
|----------|------------|-------------------|
| dis-agg | low | high |
| semi-agg | medium | medium |
| agg | high | low |

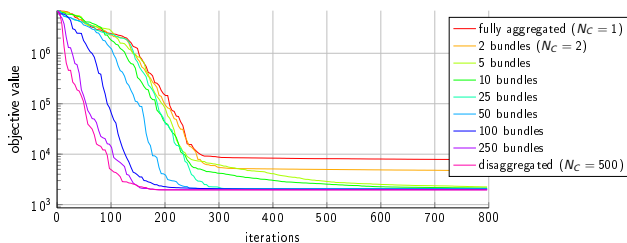
Trade-off between n_{iter} and $t_{Coordinator}$!

Computation time: $t_{total} = n_{iter} \cdot (t_{Coordinator} + t_{comm} + t_{Subsys})$



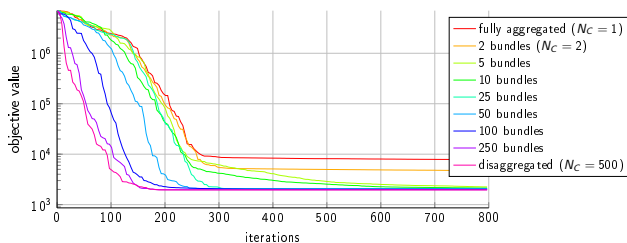
Optimal bundle aggregation level

n_{iter} with primal decomposition ($N_S = 500$ sub-systems)



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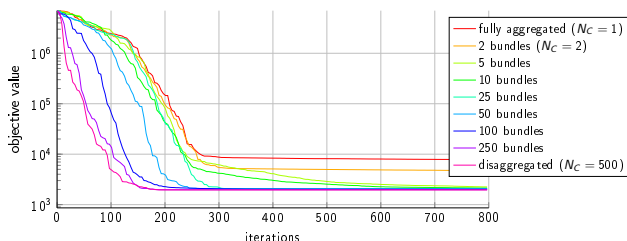


Optimal aggregation level: $N_C = 25$

$$t_{total} = n_{iter} \cdot (t_{Coordinator} + t_{comm} + t_{Subsys}) = 285 \cdot (1.77 + 0 + 0.1) = 533sec$$

Optimal bundle aggregation level

n_{iter} with primal decomposition ($N_S = 500$ sub-systems)



Optimal aggregation level: $N_C = 25$

$$t_{total} = n_{iter} \cdot (t_{Coordinator} + t_{comm} + t_{Subsys}) = 285 \cdot (1.77 + 0 + 0.1) = 533sec$$

- High **coordinator computation time** with primal decomposition when the number of sub-systems N_S is high

Dual decomposition – 1000 buildings

- District composed of **1000** buildings of different size and inertia
- Prediction horizon $t_{horizon} = 24\text{h}$, sampling period $\tau = 15\text{min}$
- Number of decision variables \approx **800000**
- Time to solve one building MPC problem: $t_{Subsys} = 75\text{ms}$
- $t_{Coordinator} =$ **79ms** (aggregated bundle)
- Around **30** iterations required in closed loop
- Building MPC problems can be solved in parallel

Total computation time to solve the problem:

$$t_{total} = n_{iter} \cdot (t_{Subsys} + t_{Coordinator}) = 30 \cdot (75[\text{ms}] + 79[\text{ms}]) = \mathbf{4.6[\text{s}]}$$

(Inter(R) Core(TM) i7-3540M CPU @ 3.00 GHz, 16,0 Go RAM, Gurobi 6.0 for LPs and QPs)

Real-world application

Implementation of the primal- and dual frameworks on two pilot sites:

- LTCP (Lavrion, Greece)



- 5 buildings
- 1 H_2 storage system
- 1 battery
- 1 PV installation
- Wind turbines

- INES (Chambéry, France)



- 4 buildings
- 2 batteries
- 1 EV charging station
- 1 PV installation

Real-world issues & cost savings

Real-world issues

- Data acquisition
- Interaction with low-level control systems
- Forecast errors/Availability of forecasts

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- Data acquisition
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Cost savings

- Annual cost savings (obtained in simulation): 0% - 30%
- Highly context dependent (sub-system types, tariff, climate,...)

Conclusions -DMPC

Summary

- Primal- and dual decomposition frameworks for problems with
 - limited shared resources
 - buying- and selling tariffs on the globally consumed resource
- Efficient implementation of the bundle method
- Comparison of the two approaches
 - Better scalability with dual decomposition
 - Primal is less restrictive (convexity required, but not strong convexity)
- Feasibility & efficiency shown in simulations and real-life experiments

Future works

- Asynchronous information exchange
- Benefit guarantee for each actor

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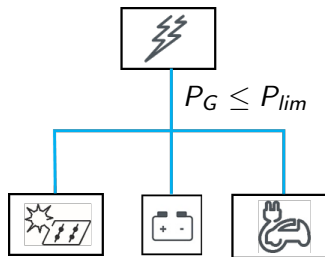
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Outline

- 1 Distributed MPC for smart grids
- 2 Randomized algorithm for EV charging stations
 - Motivation
 - Problem statement
 - The approach
 - Validation through simulation
 - Conclusion
- 3 General conclusion

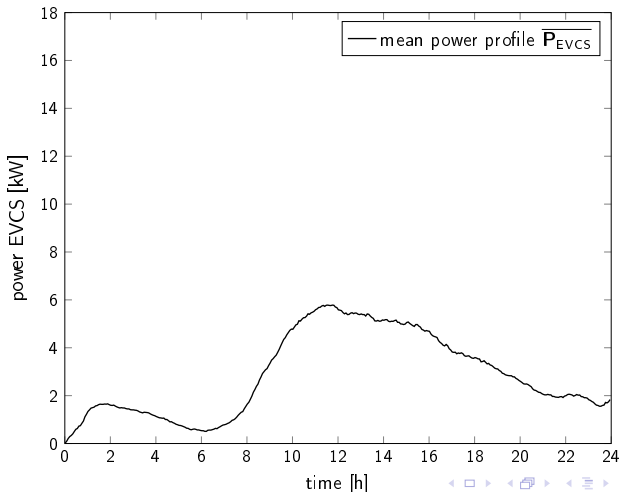
Motivation

- Model Predictive Control performs poorly when uncertainties are high
- This has been put into evidence at the Euref Campus in Berlin



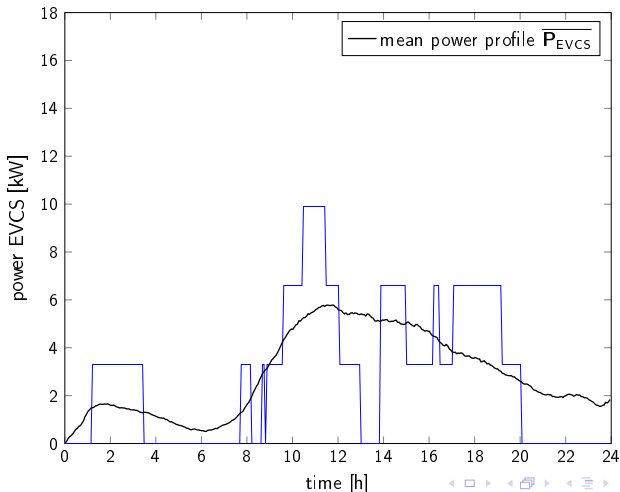
Motivation

Uncertain power consumption of the EVCS consisting of 10 charging points (10 scenarios)



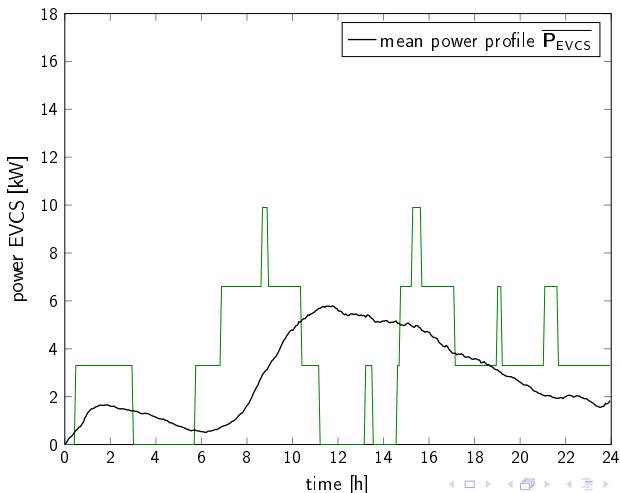
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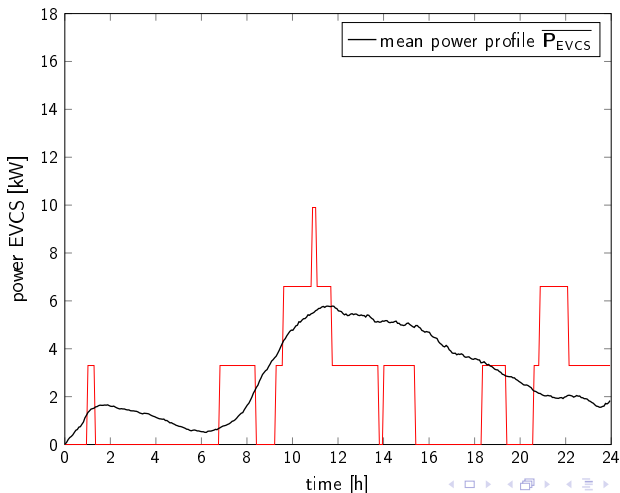
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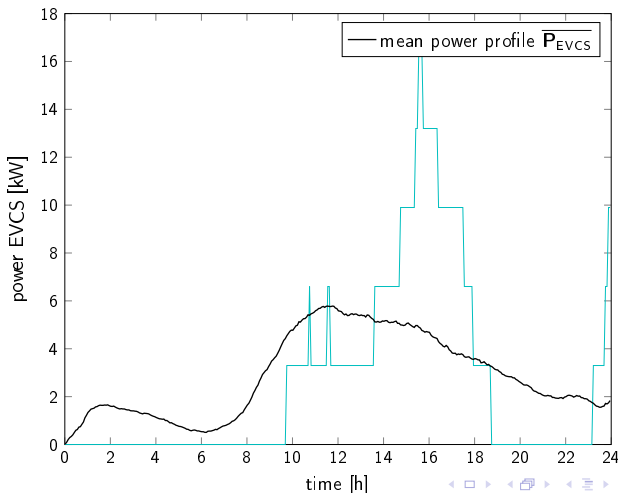
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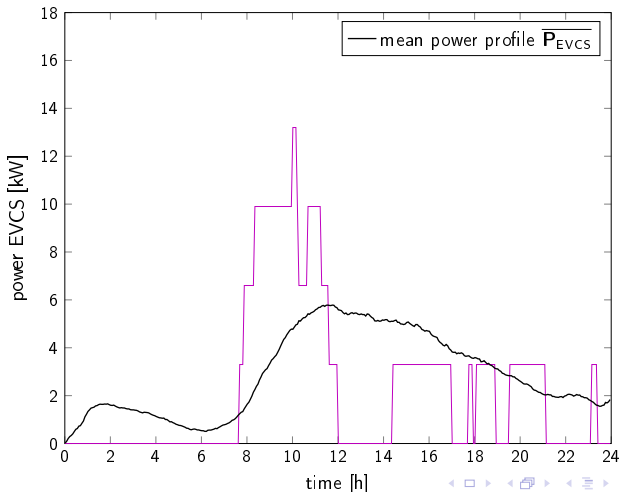
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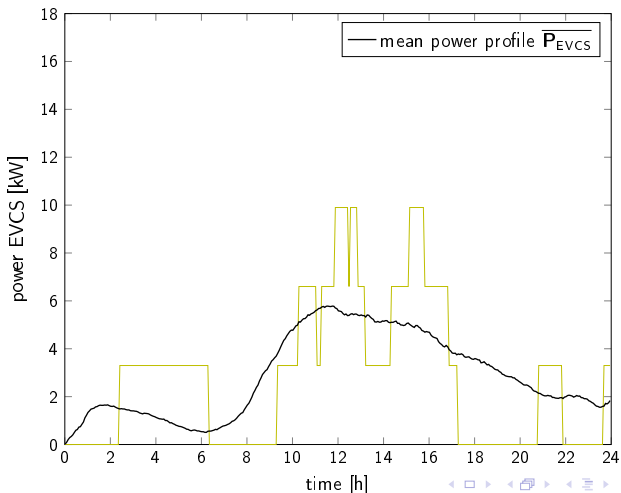
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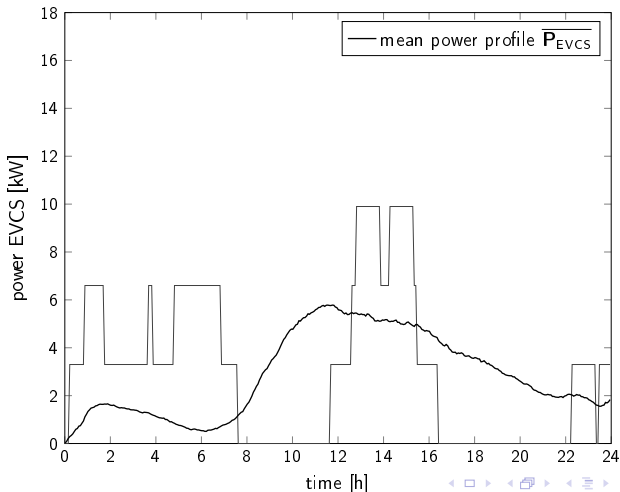
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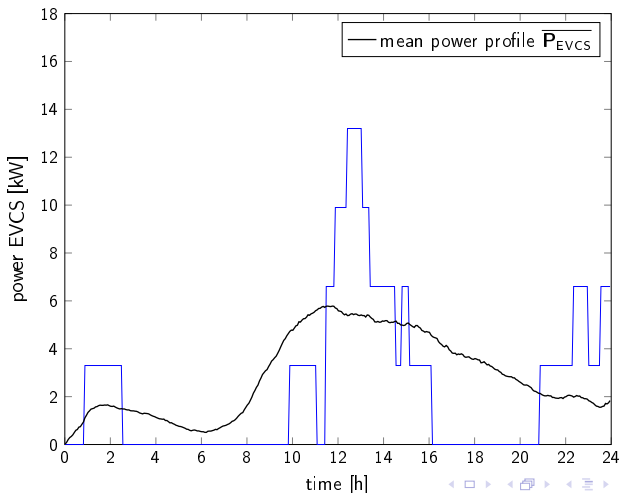
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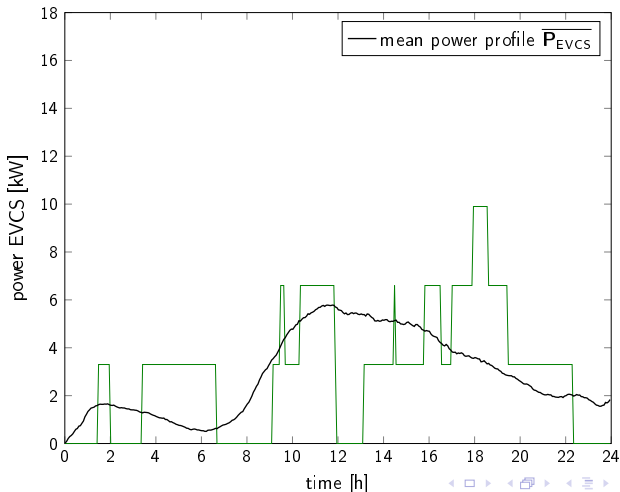
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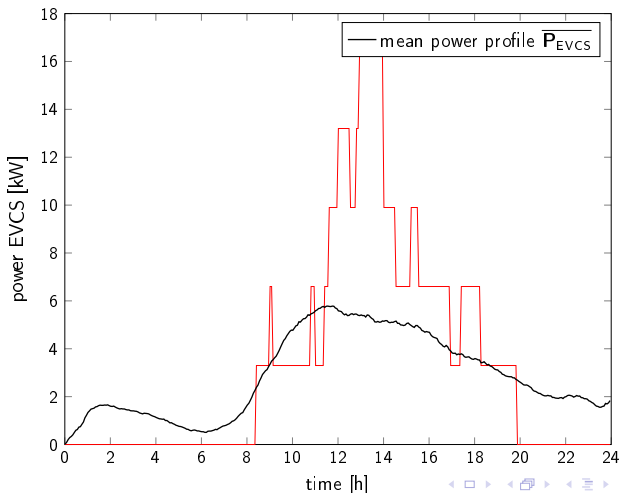
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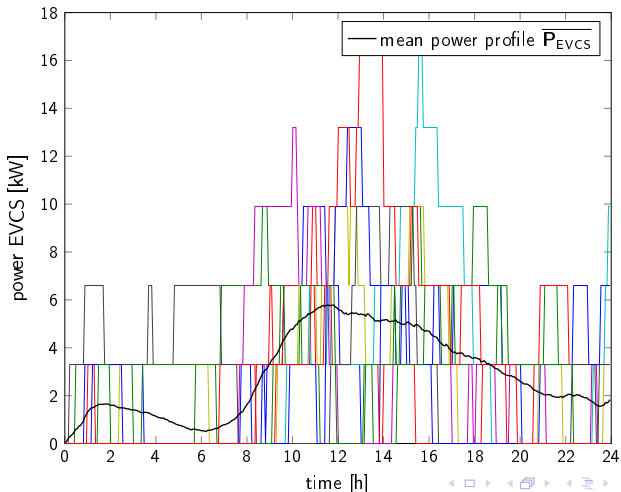
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EVCS control under uncertainties

Context

- EVCS with M charging points
- No forecasts of the individual EV behavior available
- But known statistic model of the EV behavior

EVCS occupancy scenario

$$S = \{t_{arr,v}, t_{dep,v}, E_{req,v}\} \quad v = 1, \dots, n_{EV}$$

The scenario S is the set of EVs which arrive during a given day

Objective

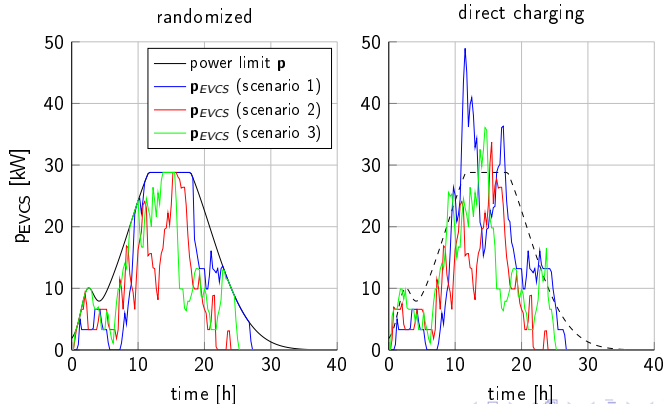
Design a controller for the EVCS that

- respects an a priori declared power limitation profile \mathbf{p}
- guarantees the QoS (Quality of Service) at a high probability ($\geq (1 - \eta)$)

Objective

Design a controller for the EVCS that

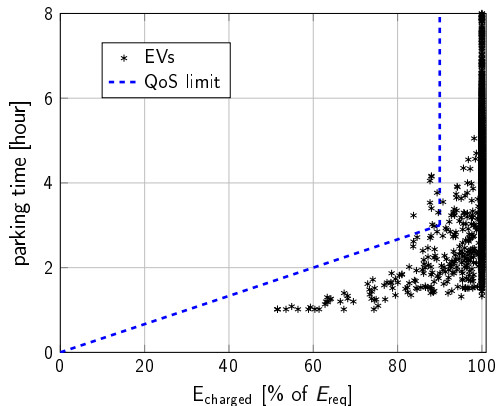
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Quality of Service

EV satisfaction metric:

$$\frac{E_{\text{charged}}}{E_{\text{req}}} \geq c_{\text{satis}} \times \min \left(1, \frac{t_{\text{park}}}{T_{\text{th}}} \right)$$



QoS indicator g

$$g(S) := \begin{cases} 0 & \text{if all EVs in a scenario } S \text{ are satisfied} \\ 1 & \text{otherwise} \end{cases}$$

Relaxation of the QoS constraint

| | |
|---------------|-----------------------------------|
| $g(S)$ | QoS indicator (0 := satisfied) |
| \mathcal{S} | uncertainty set |

Robust constraint:

$$g(S) = 0 \quad \forall S \in \mathcal{S}$$

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First relaxation:

$$\overbrace{Pr(g(S) = 1) \leq \eta} \quad (\eta := \text{precision})$$

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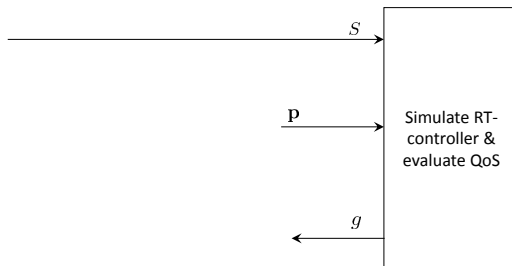
Randomized algorithm

If for no more than m out of N scenarios $g(S) = 1$, then the QoS is guaranteed with probability $> (1 - \eta)$ and confidence $> (1 - \delta)$.

Teodoro Alamo et al. (2015). "Randomized methods for design of uncertain systems: Sample complexity and sequential algorithms". In: *Automatica* 52, pp. 160–172

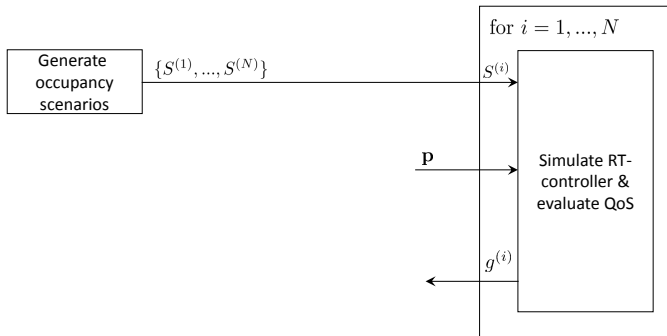
Outline of the approach

S occupancy scenario
 p power limit profile
 g QoS indicator



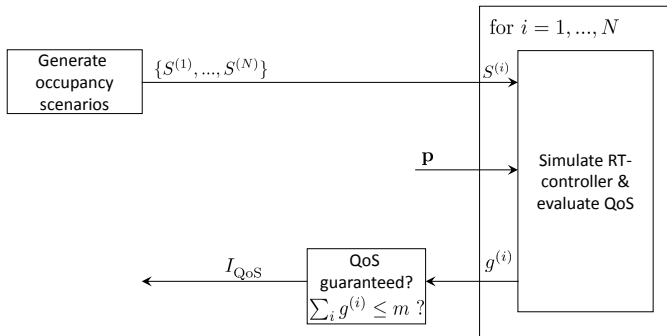
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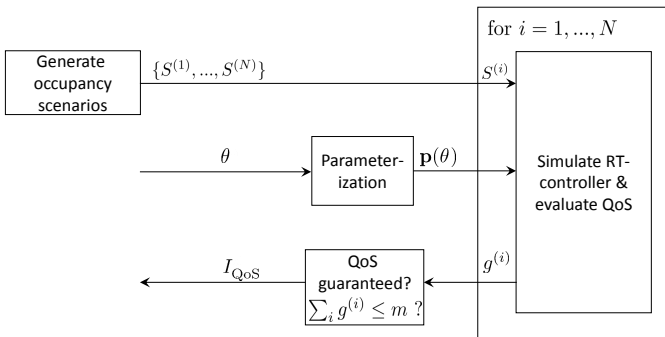
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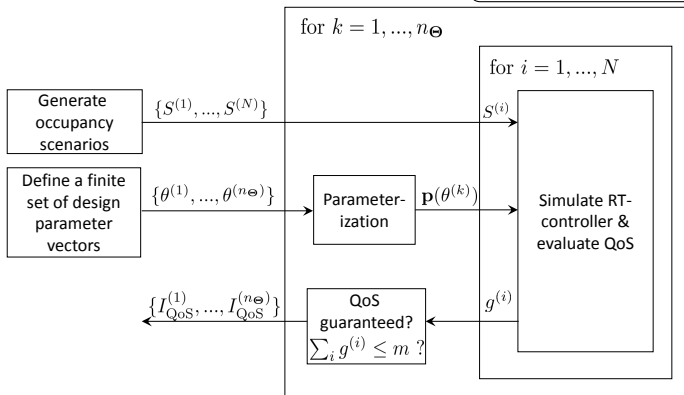
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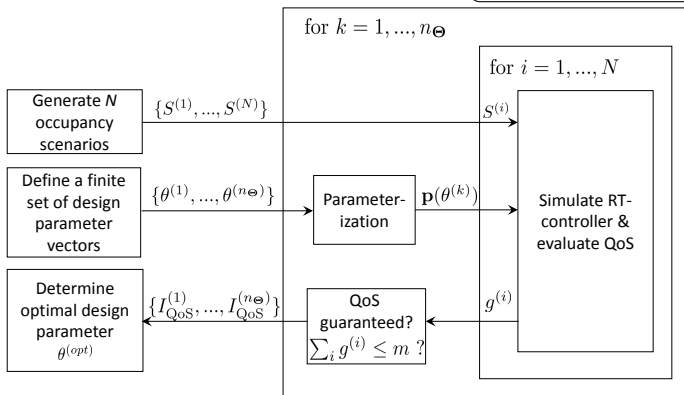
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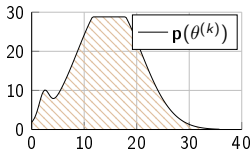
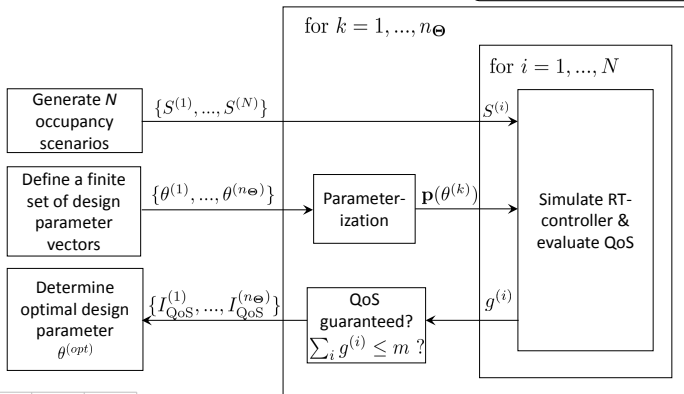
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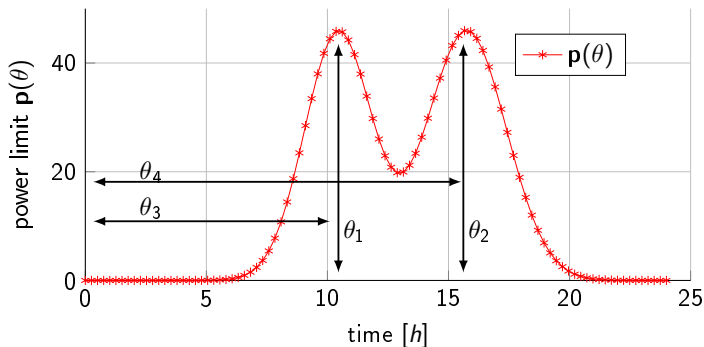
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$$\theta^{(opt)} := \text{Min}_{k \in \{1 \dots n_{\Theta}\}} \int \mathbf{p}(\theta^{(k)}) dt$$

Parameterization of the power limit profile $\mathbf{p}(\theta)$

- Low-dimensional design parameter vector $\theta := (\theta_1, \dots, \theta_4)^T$
- Allow sufficiently rich variations in the profile $\mathbf{p}(\theta)$ (e.g. two superposed Gaussian distributions)



Randomized algorithm

| | |
|--------------|-----------------------|
| η | precision |
| δ | confidence |
| m | positive integer |
| n_{Θ} | $\text{card}(\Theta)$ |

What is the necessary number of scenarios N ?

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What is the necessary number of scenarios N ?

$$N \geq \frac{1}{\eta} \left(\frac{e}{e-1} \right) \left(\ln \frac{n_{\Theta}}{\delta} + m \right)$$

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Statistic model - Euref campus

Euref Campus

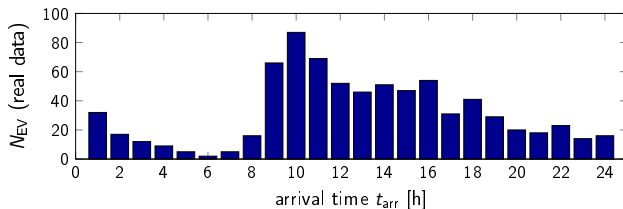
E-car sharing station located at the Euref Campus in Berlin



- 800 charging events
- Available information for each event: t_{arr} , t_{park} , E_{req}

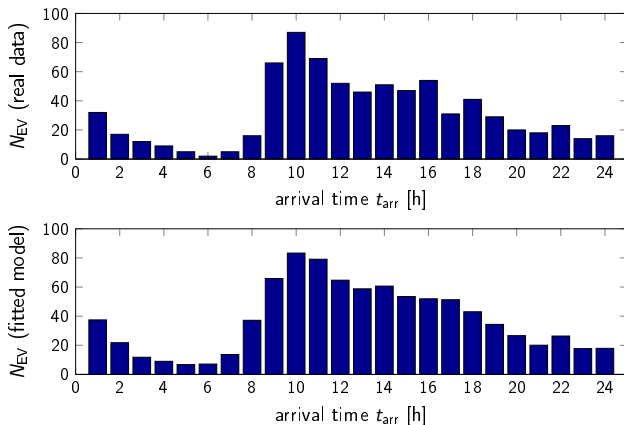
Statistic model - Euref campus

Fit non-parametric probability distributions to t_{arr} , t_{park} , E_{req}



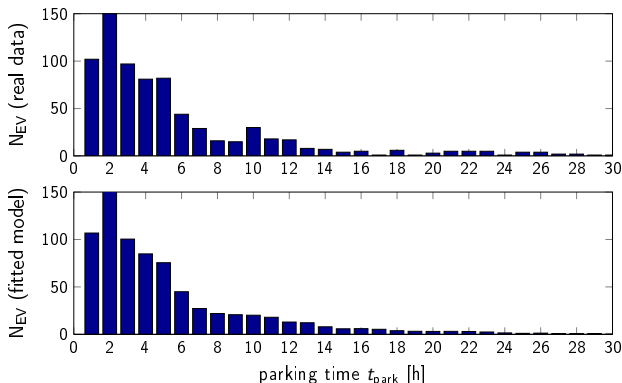
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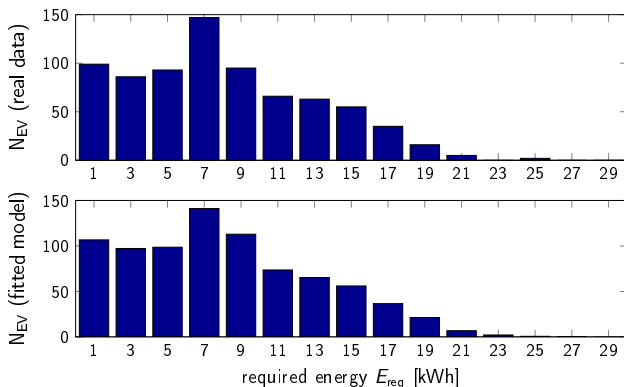
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Algorithm configuration

Algorithm parameters

| | |
|--------------|-----------------------------|
| η | 0.05 (accuracy) |
| δ | 0.05 (confidence) |
| n_{Θ} | 432 (design parameters) |
| m | 20 (unsatisfying scenarios) |
| N_{CP} | 50 (charging points) |

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$$N \geq \frac{1}{\eta} \left(\frac{e}{e-1} \right) \left(\ln \frac{n_{\Theta}}{\delta} + m \right) = 920$$

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Computation time using brute-force:

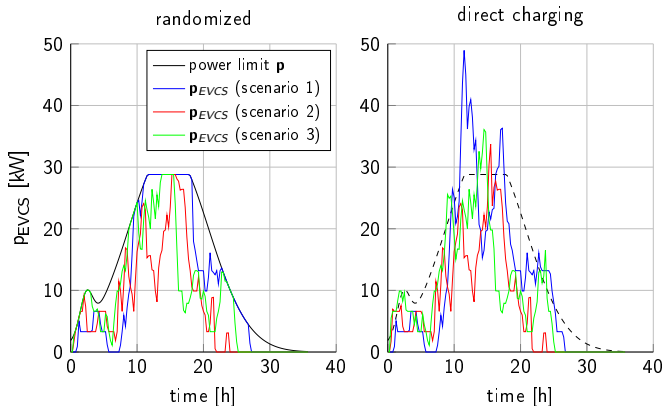
$$t_{\text{comp}} = t_{\text{iteration}} \cdot n_{\Theta} = 5.8 \text{sec} \cdot 432 = 42 \text{min}$$

($t_{\text{iteration}}$:= time to simulate the real-time controller for N scenarios)

(Inter(R) Core(TM) i7-3540M CPU @ 3.00 GHz, 16,0 Go RAM)

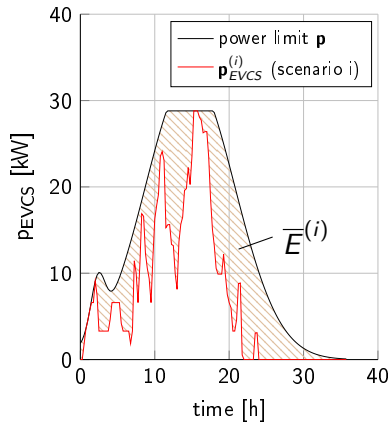
Validation through simulation (Euref Campus)

Comparison with a direct charging strategy



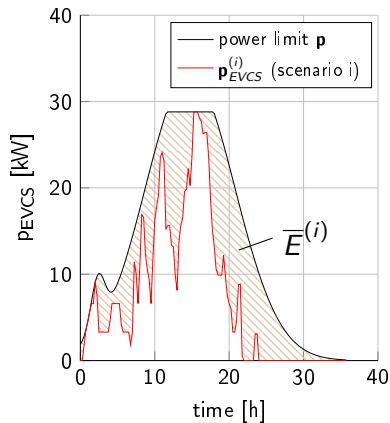
Validation through simulation (Euref Campus)

Mean over-estimated energy need



Validation through simulation (Euref Campus)

Mean over-estimated energy need



$$\bar{E} = \frac{1}{N} \sum_{i=1}^N \bar{E}^{(i)}$$

For a single scenario $S^{(i)}$:

$$\bar{E}^{(i)} = \int_{t=0}^{t_{end}} p(t) - p_{EVCS}^{(i)}(t) dt$$

Validation through simulation (Euref Campus)

Mean over-estimated energy need

Sensitivity of \bar{E} w.r.t. N_{CP} and η :

| | | η | | | |
|----------|-----|-------------|------|------|------|
| | | 0.05 | 0.1 | 0.15 | 0.2 |
| N_{CP} | 25 | 58.4 | 56.3 | 54.2 | 51.7 |
| | 50 | 37.5 | 34.1 | 32.9 | 31.2 |
| | 100 | 30.6 | 28.8 | 27.1 | 25.1 |

η probability of QoS violation
 N_{CP} number of charging points

$$\bar{E} = \frac{1}{N} \sum_{i=1}^N \bar{E}^{(i)}$$

For a single scenario $S^{(i)}$:

$$\bar{E}^{(i)} = \int_{t=0}^{t_{end}} \mathbf{p}(t) - \mathbf{p}_{EVCS}^{(i)}(t) dt$$

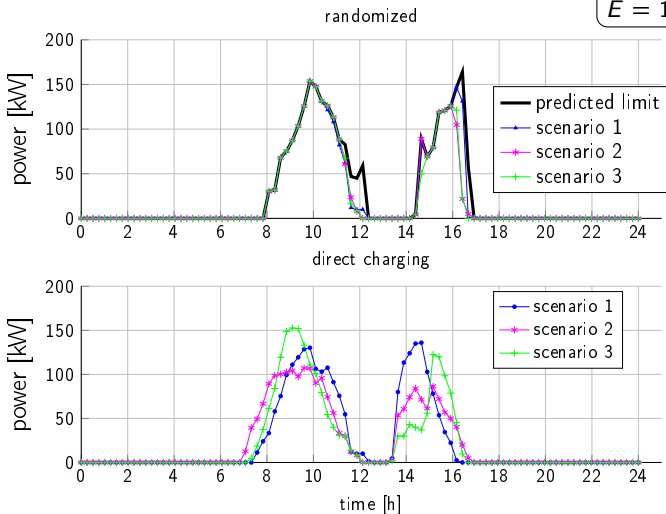
Validation through simulation (Company site)

Comparison with a direct charging strategy

$$\eta = 0.05$$

$$N_{CP} = 20$$

$$\bar{E} = 12.5\%$$



Conclusion

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- Energy management strategy for an EVCS allowing to
 - respect an upper bound on the power consumption
 - guarantee the QoS with a configurable probability
- Validation on statistic data from the Euref Campus in Berlin

Future extensions

- Additional battery to further tighten the upper bound profile
- Also compute a guaranteed lower bound profile

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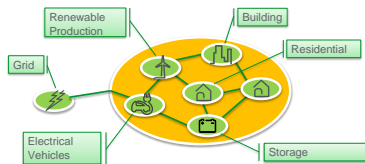
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General conclusion

Summary

1 Distributed MPC design for Large-scale systems

- Primal and dual decomposition
- Energy efficiency in smart grids
- Real-time implementation



General conclusion

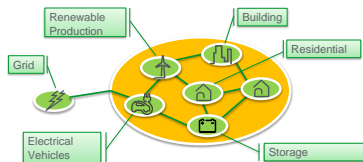
Summary

1 Distributed MPC design for Large-scale systems

- Primal and dual decomposition
- Energy efficiency in smart grids
- Real-time implementation

2 Randomized algorithms for EV charging stations

- Explicitly deal with uncertainty
- Upper bound power profile
- Probabilistic QoS guarantee
- Validation on real data



General conclusion

Future works...

- Optimization framework integrating a **flexibility market model**
- **MPC prototype** at the Learning Grid by Grenoble
- Integration of MPC into **micro grid design tools**

Thank you for your attention!

Questions ?