

Photorealistic Surface Rendering with Microfacet Theory

Supplemental Document

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Summary

This document presents the exhaustive results of an experiment conducted to qualitatively validate the output of our filtering scheme. The experiment consisted in rendering a sphere with different isotropic materials that is displaced at the subpixel level with a random process exhibiting Beckmann (Gaussian) slope statistics under a directional light. Our filtering scheme predicts that such subpixel geometries can be simplified into a flat surface whose appearance is determined by the microfacet BRDF equation

$$f_r = \frac{F \cdot D \cdot G}{4 \cdot \cos \theta_i \cdot \cos \theta_o},$$

where F denotes the Fresnel term of the base BRDF of the surface, G denotes the Smith geometric attenuation factor, and D is the microfacet normal distribution function (NDF), defined as the convolution the microfacet slope probability density function (PDF) of the Beckmann surface and the base BRDF (if ever the base BRDF is not a microfacet BRDF, we convert it using our inversion procedure)

$$D(\mathbf{h}) = (P_{\mathbf{X}} * P_{\mathbf{Y}})(\tilde{\mathbf{h}}) \sec^4 \theta_h.$$

Since the displacements have Beckmann slope statistics, we have

$$P_{\mathbf{X}}(\tilde{\mathbf{h}}) = P_{\phi}(\tilde{\mathbf{h}}; \alpha_{\theta}, \alpha_{\phi}),$$

where $\alpha_{\theta} > 0$ and $\alpha_{\phi} > 0$ respectively denote the roughness parameters along the elevation and azimuthal angle on the sphere, and

$$P_{\phi}(\tilde{\mathbf{h}}; \alpha_{\theta}^2, \alpha_{\phi}^2) = \frac{1}{\alpha_1 \alpha_2} P_{std.\phi} \left(\frac{\tilde{x}_h}{\alpha_1}, \frac{\tilde{y}_h}{\alpha_2} \right),$$

having $P_{std.\phi} \geq 0$ denote the standard Beckmann microfacet slope PDF

$$P_{std.\phi}(\tilde{\mathbf{h}}) = \frac{1}{\pi} \exp(-\tilde{x}_h^2 - \tilde{y}_h^2).$$

In addition, we approximate $P_{\mathbf{Y}}$ with a Beckmann mixture model of size $N > 0$

$$P_{\mathbf{Y}}(\tilde{\mathbf{h}}) \approx \sum_{i=1}^N w_i \cdot P_{\phi}(\tilde{\mathbf{h}}; \alpha_i^2),$$

where we retrieve the (isotropic) roughness parameters $\alpha_1 > 0, \dots, \alpha_N > 0$ from the base BRDF microfacet slope PDF with an expectation maximization algorithm, and

$$\sum_{i=1}^N w_i = 1, \quad w_{1,\dots,N} \geq 0.$$

This approximation allows us to get a closed form expression for $P_{\mathbf{Z}}$

$$P_{\mathbf{Z}}(\tilde{\mathbf{h}}) = \sum_{i=1}^N w_i \cdot P_{\phi}(\tilde{\mathbf{h}}; \alpha_{\phi}^2 + \alpha_i^2, \alpha_{\phi}^2 + \alpha_i^2),$$

from which we can derive an analytic microfacet BRDF. For convenience, we provide the monostatic Smith shadowing term

$$\begin{aligned} G_1(\mathbf{k}) &= \frac{\cos \theta_k}{\int_{\Omega_+} D(\mathbf{h}) \cdot \underline{\mathbf{kh}} \cdot d\omega_h} \\ &= \frac{\cos \theta_k}{\int_{\Omega_+} \sum_{i=1}^N w_i \cdot P_{\phi}(\tilde{\mathbf{h}}; \alpha_{1,i}^2, \alpha_{2,i}^2) \sec^4 \theta_h \cdot \underline{\mathbf{kh}} \cdot d\omega_h} \\ &= \frac{\cos \theta_k}{\sum_{i=1}^N w_i \int_{\Omega_+} P_{\phi}(\tilde{\mathbf{h}}; \alpha_{1,i}^2, \alpha_{2,i}^2) \sec^4 \theta_h \cdot \underline{\mathbf{kh}} \cdot d\omega_h} \\ &= \frac{\cos \theta_k}{\sum_{i=1}^N w_i \cdot \frac{\cos \theta_k}{G_{1,\phi}(\mathbf{k}; \alpha_{1,i}^2, \alpha_{2,i}^2)}} \\ &= \sum_{i=1}^N \frac{G_{1,\phi}(\mathbf{k}; \alpha_{1,i}^2, \alpha_{2,i}^2)}{w_i} \\ &= \sum_{i=1}^N \frac{G_{1_std,\phi}(\mathbf{k}'_i)}{w_i}, \end{aligned}$$

where

$$G_{1_std,\phi}(\mathbf{k}) = \frac{2}{1 + \operatorname{erf}(\mathbf{v}) + \frac{\exp(-\mathbf{v}^2)}{\mathbf{v}\sqrt{\pi}}}$$

$$\mathbf{v} = \cot \theta_k,$$

and

$$\mathbf{k}'_i = \frac{1}{\sqrt{\alpha_{1,i}^2 x_k^2 + \alpha_{2,i}^2 y_k^2 + z_k^2}} \begin{bmatrix} \alpha_{1,i} x_k \\ \alpha_{2,i} y_k \\ z_k \end{bmatrix}.$$

The Tables

Each page displays the renderings obtained for a given material; it is divided into three sections (tables), in the following configuration (each image is referred to with an identifier with the format: *table#.row#.column#*).

1.1.1	1.1.2	1.1.3	1.1.4	1.1.5	1.1.6	1.1.7	1.1.8	1.1.9	1.1.10
1.2.1	1.2.2	1.2.3	1.2.4	1.2.5	1.2.6	1.2.7	1.2.8	1.2.9	1.2.10
1.3.1	1.3.2	1.3.3	1.3.4	1.3.5	1.3.6	1.3.7	1.3.8	1.3.9	1.3.10

2.1.1	2.1.2	2.1.3	2.1.4	2.1.5	2.1.6	2.1.7	2.1.8	2.1.9	2.1.10
2.2.1	2.2.2	2.2.3	2.2.4	2.2.5	2.2.6	2.2.7	2.2.8	2.2.9	2.2.10
2.3.1	2.3.2	2.3.3	2.3.4	2.3.5	2.3.6	2.3.7	2.3.8	2.3.9	2.3.10

3.1.1	3.1.2	3.1.3	3.1.4	3.1.5	3.1.6	3.1.7	3.1.8	3.1.9	3.1.10
3.2.1	3.2.2	3.2.3	3.2.4	3.2.5	3.2.6	3.2.7	3.2.8	3.2.9	3.2.10
3.3.1	3.3.2	3.3.3	3.3.4	3.3.5	3.3.6	3.3.7	3.3.8	3.3.9	3.3.10

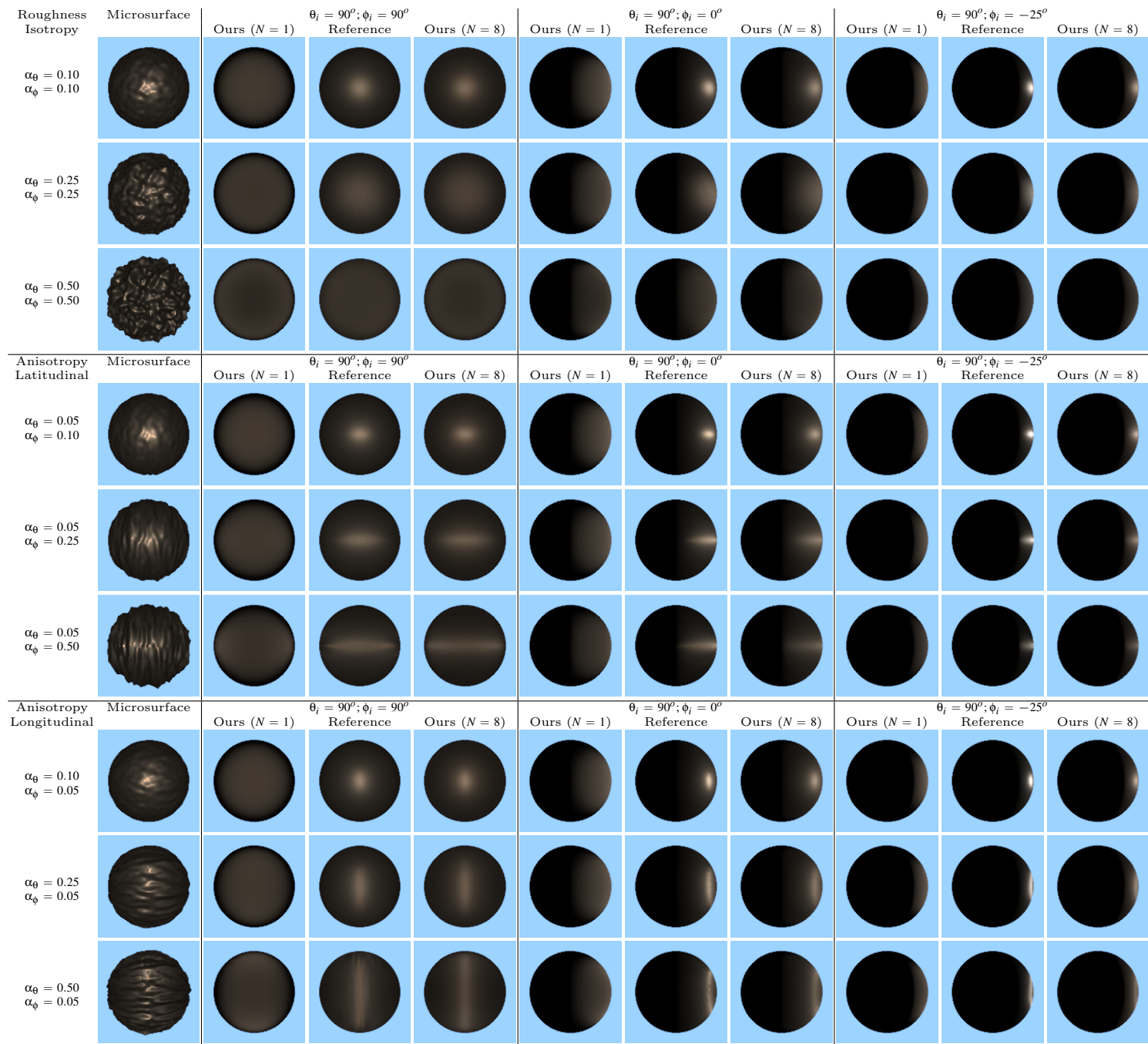
The first (top) table shows an arrangement of 10×3 spheres of the given material, in front of a homogeneous (blue) background, and illuminated by a unique directional white light source from three different directions; the directional light source was chosen over more complex lighting configurations to better emphasize the high frequencies exhibited by the base material. In polar coordinates, the three directions correspond to, for each row: $\theta_i = 90^\circ, \phi_i = 90^\circ$ (from the camera direction, first row), $\theta_i = 90^\circ, \phi_i = 0^\circ$ (from the right of the sphere, second row), $\theta_i = 90^\circ, \phi_i = -25^\circ$ (from the right of the sphere, behind it at 25° , third row).

Each row corresponds to a specific roughness configuration, and the leftmost image shows an exaggerated (scaled up) version of the displacements in order to give a rough estimate of the appearance of the shading, as well as of the orientations of the microfacet normals.

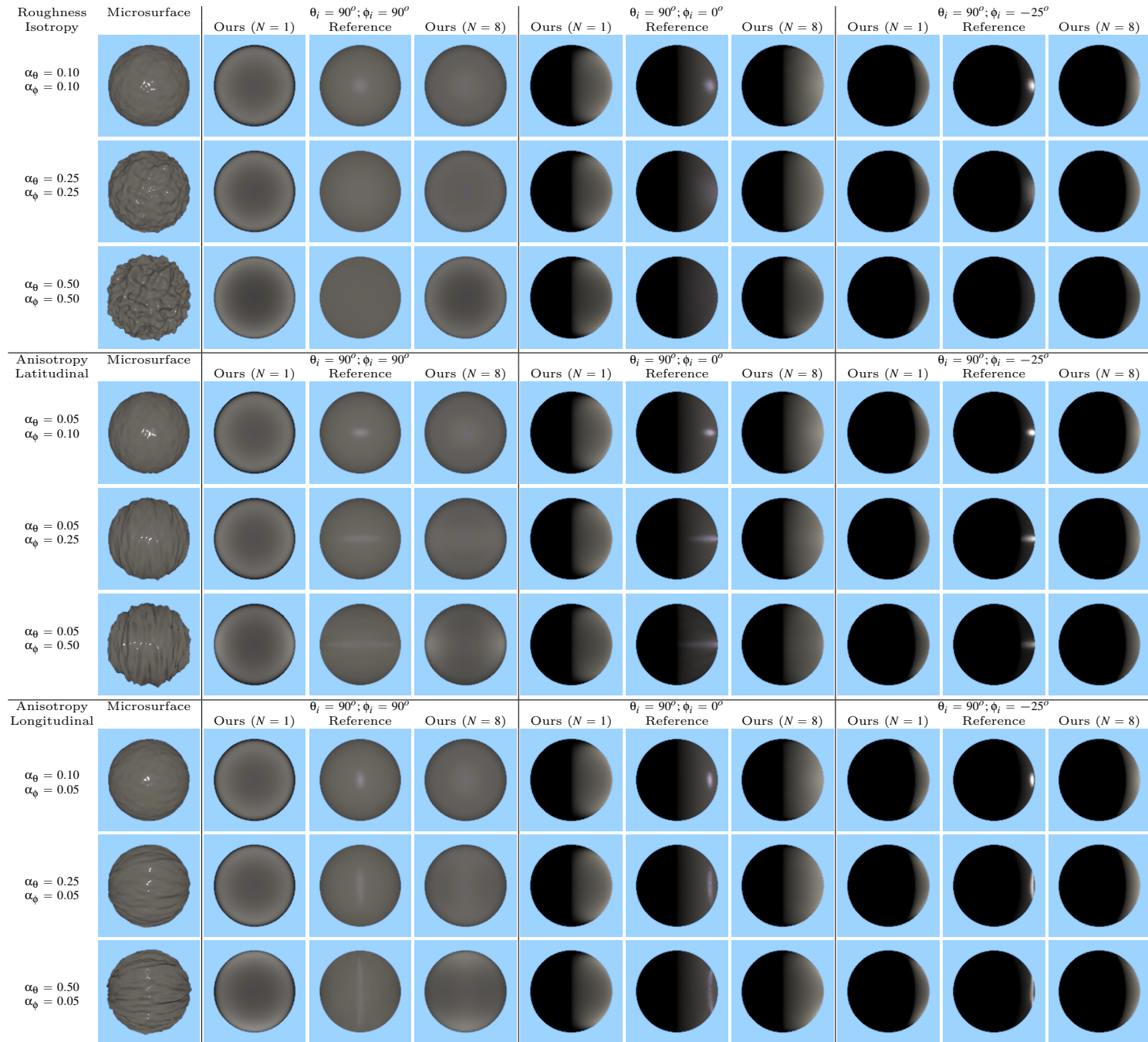
We computed results for three isotropic roughnesses ($\alpha_\theta = \alpha_\phi$). The following lines show similar results, but for latitudinal and longitudinal roughnesses.

All images were computed at a 128×128 resolution. We evaluated 256×256 samples per pixel for the reference images, and one sample per pixel with our mixture model with $N = 1$ and $N = 8$ lobes. Note that the configuration $N = 1$ corresponds to the results obtained with the filtering scheme described in the thesis.

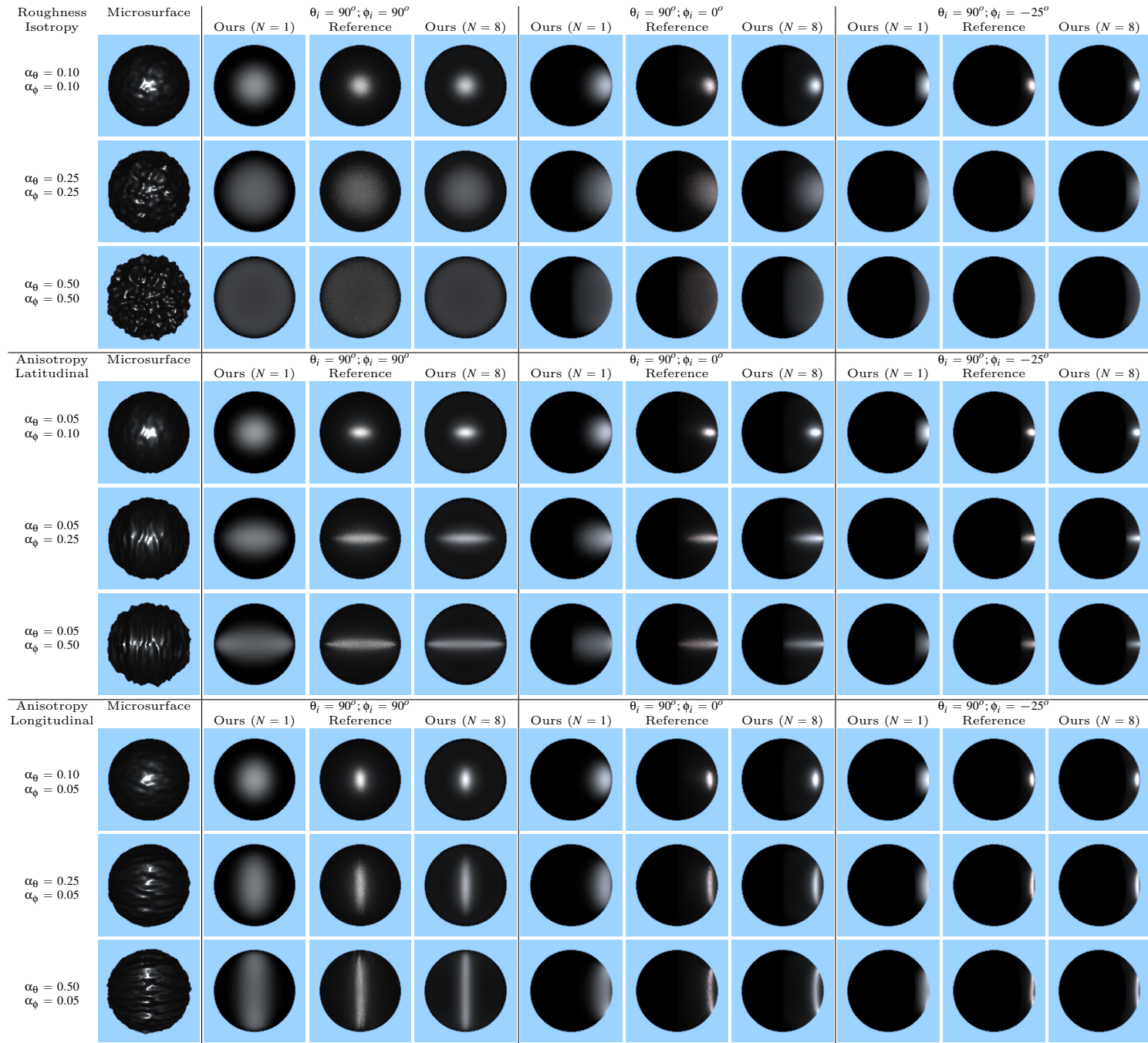
1 alum-bronze



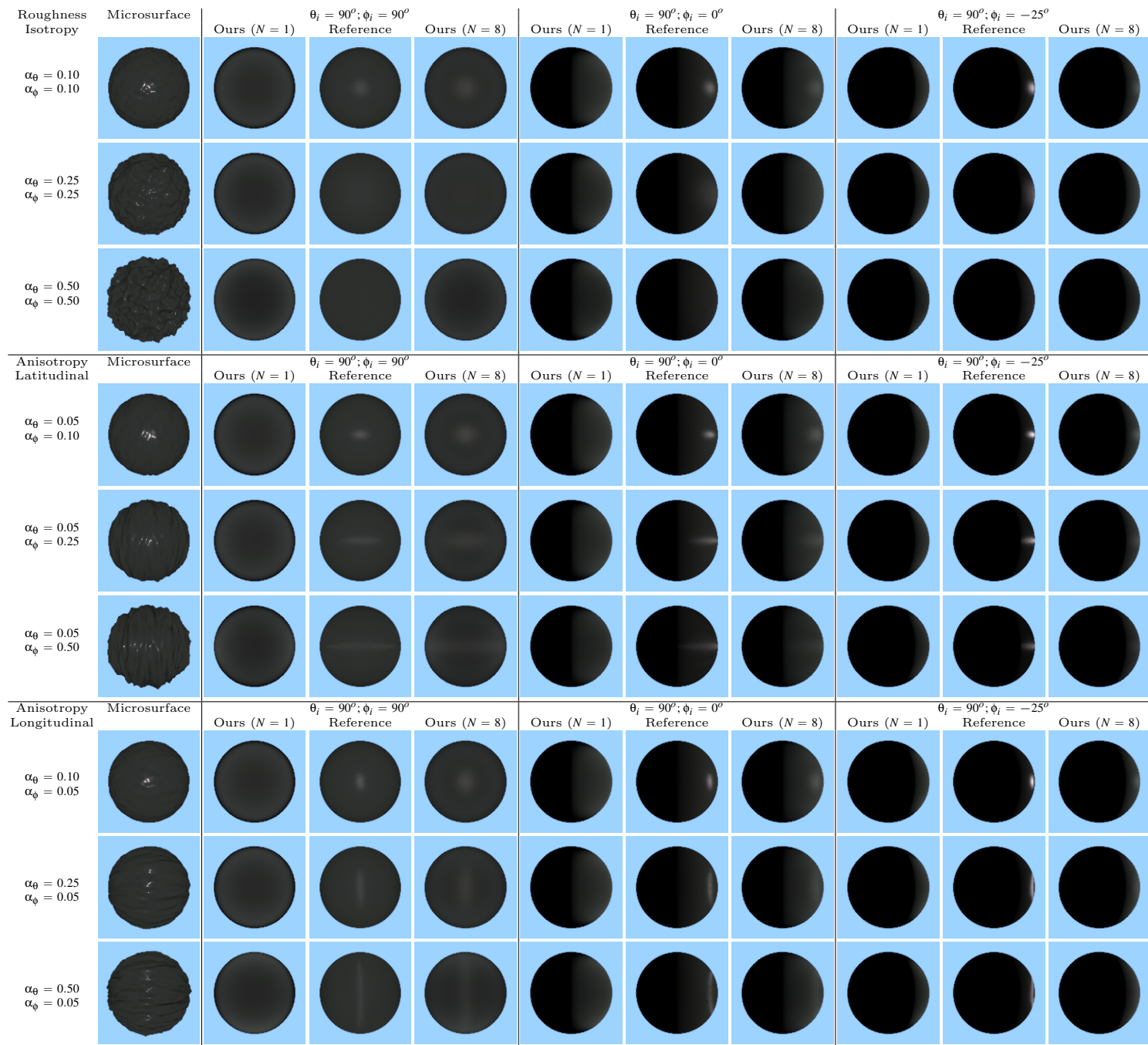
2 alumina-oxide



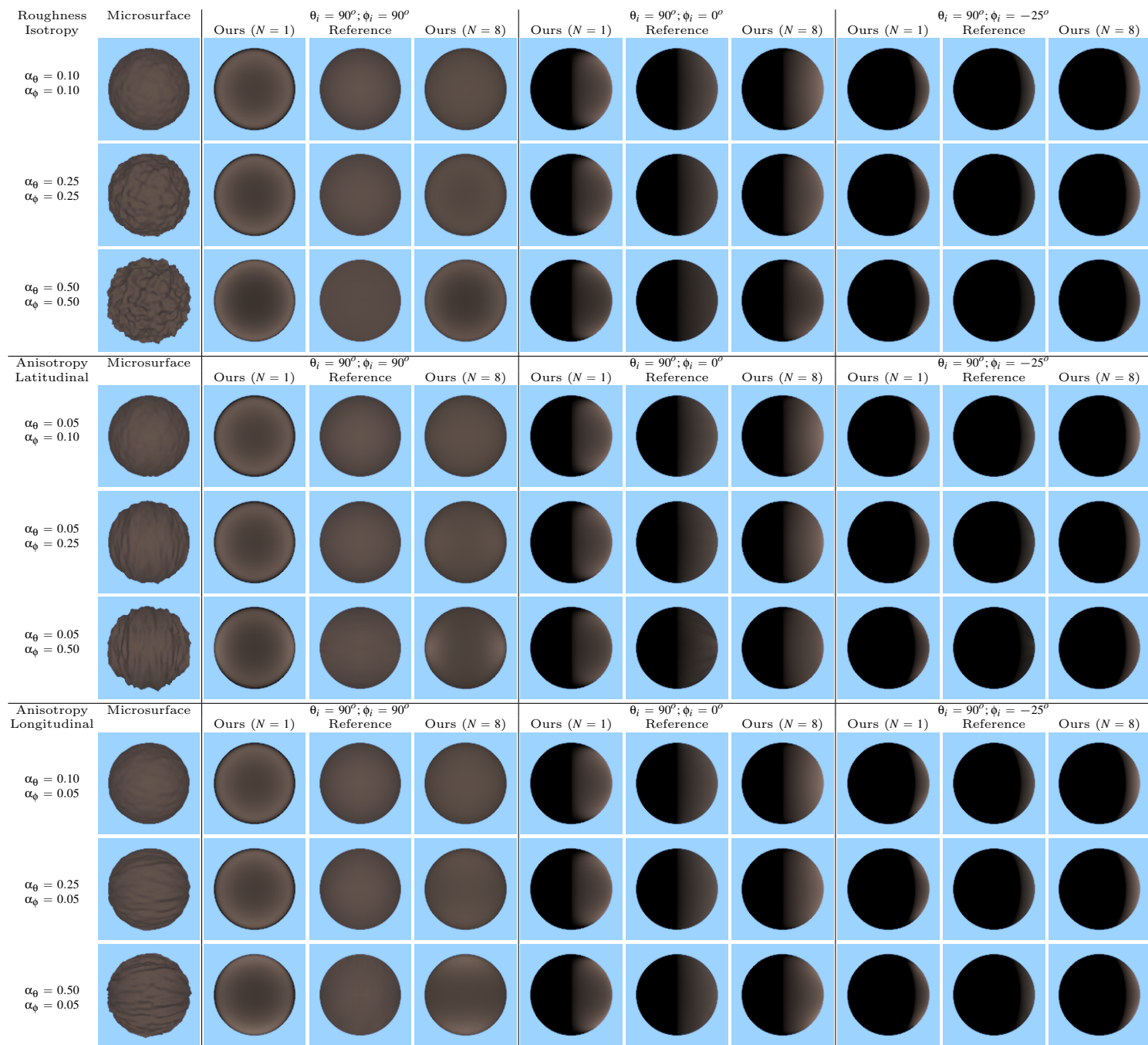
3 aluminium



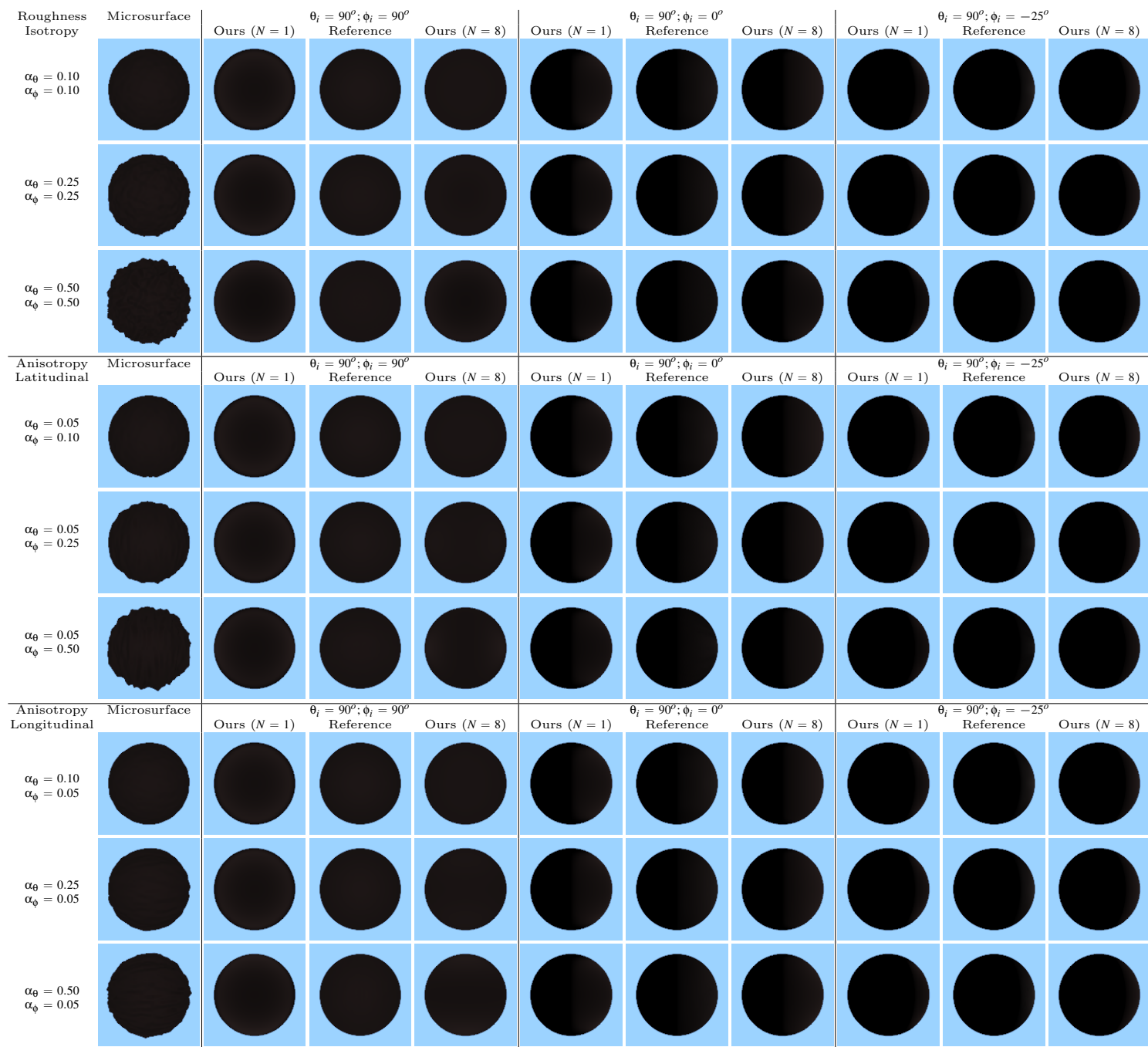
4 aventurnine



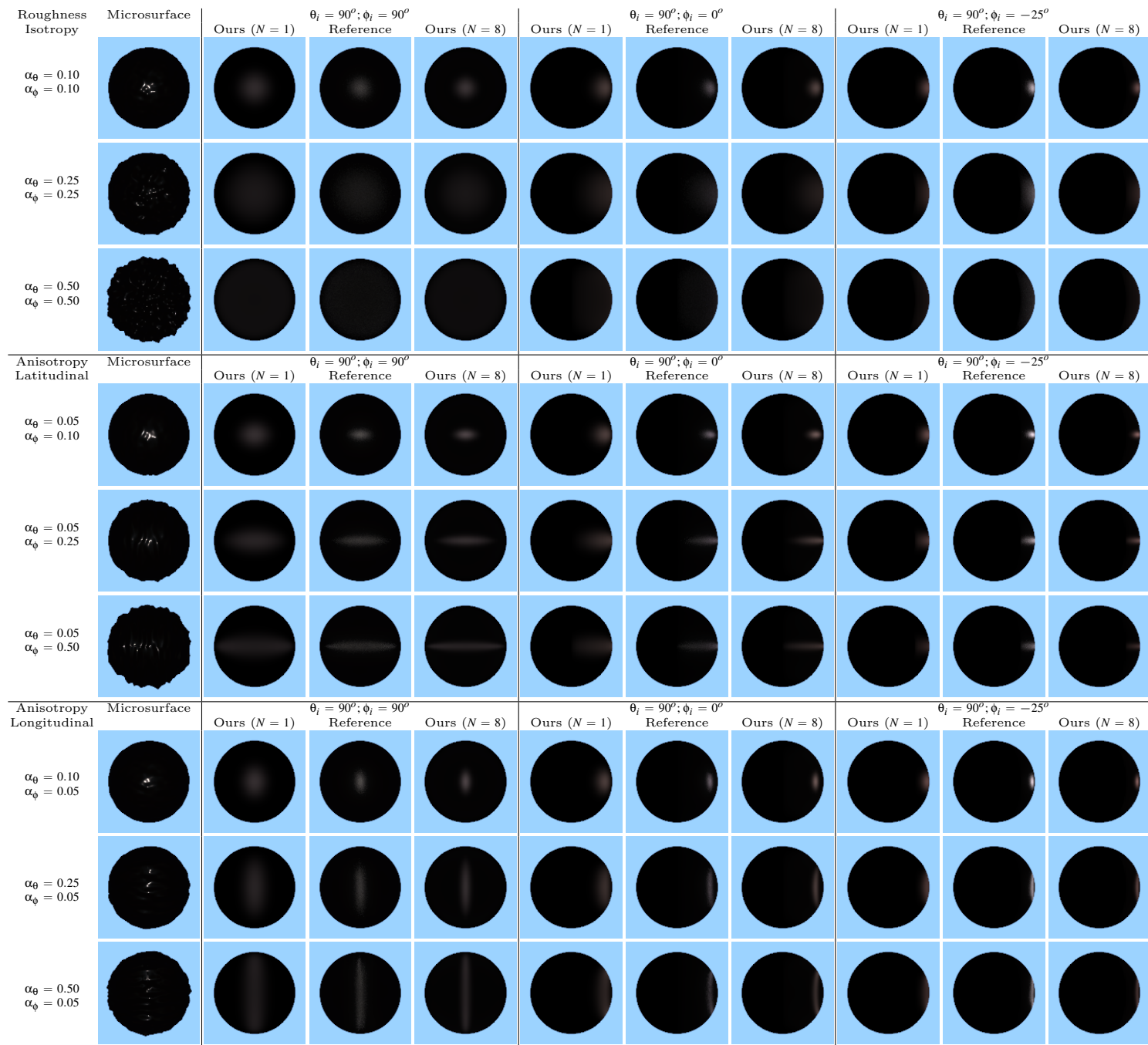
5 beige-fabric



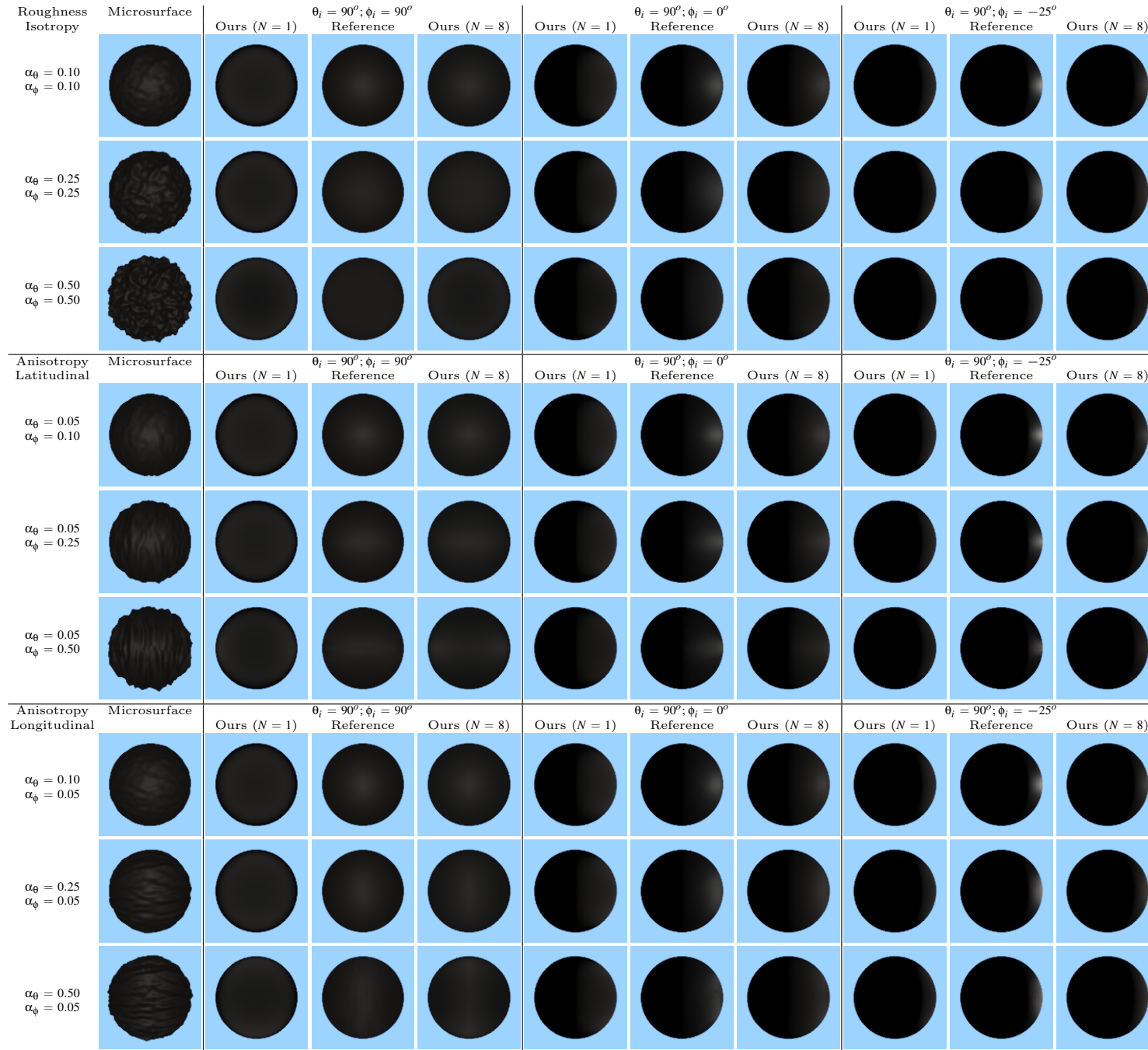
6 black-fabric



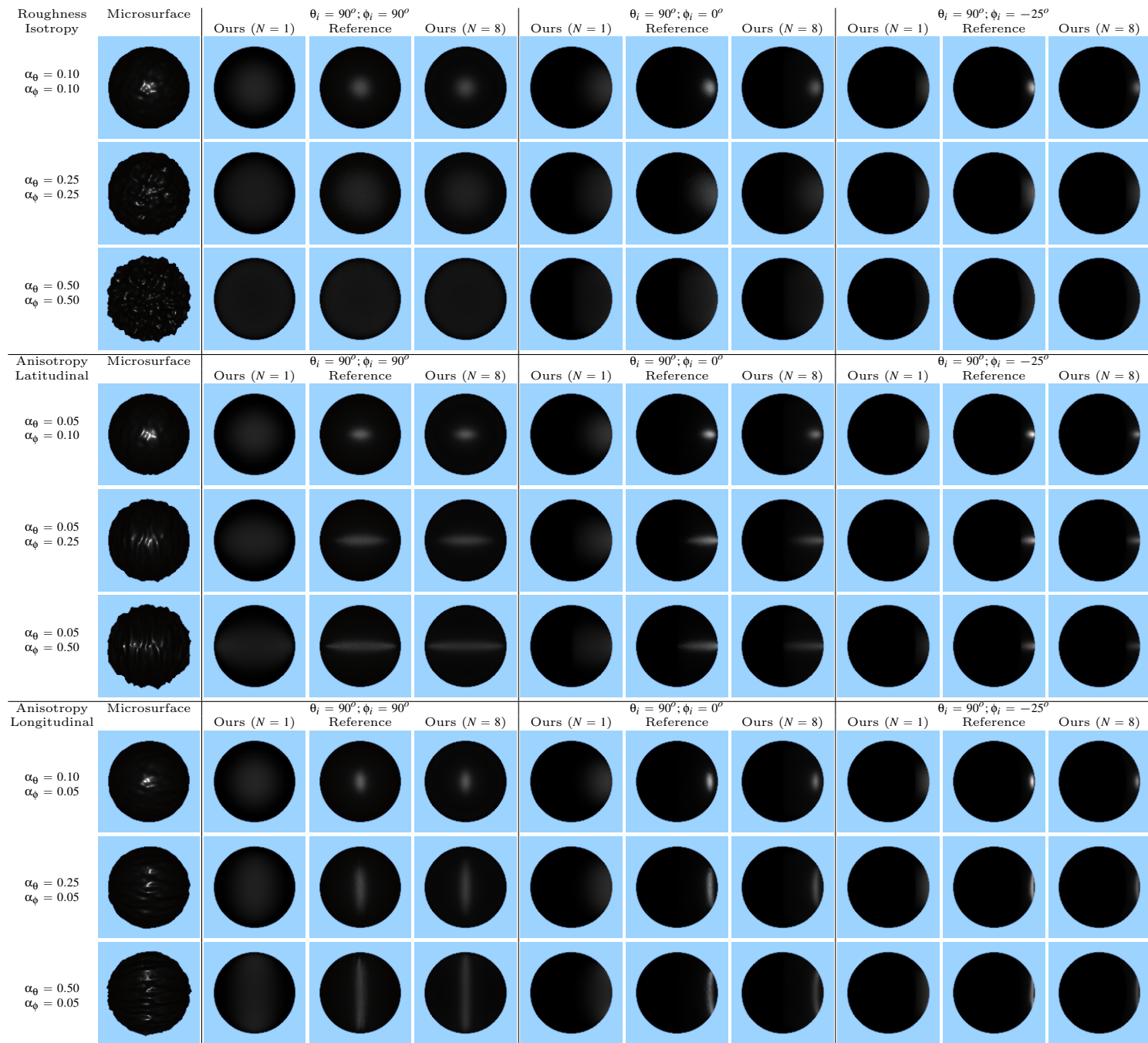
7 black-obsidian



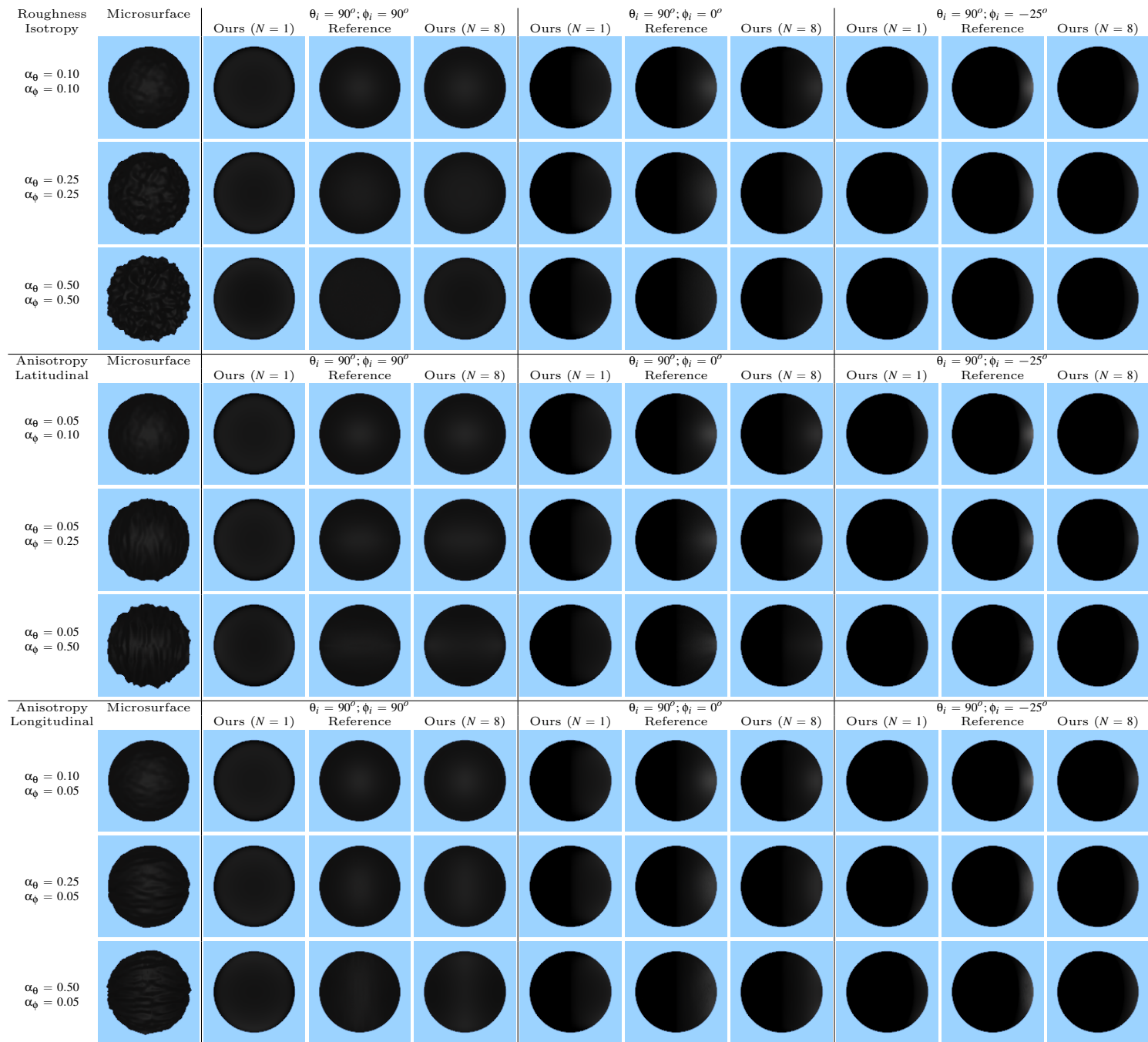
8 black-oxidized-steel



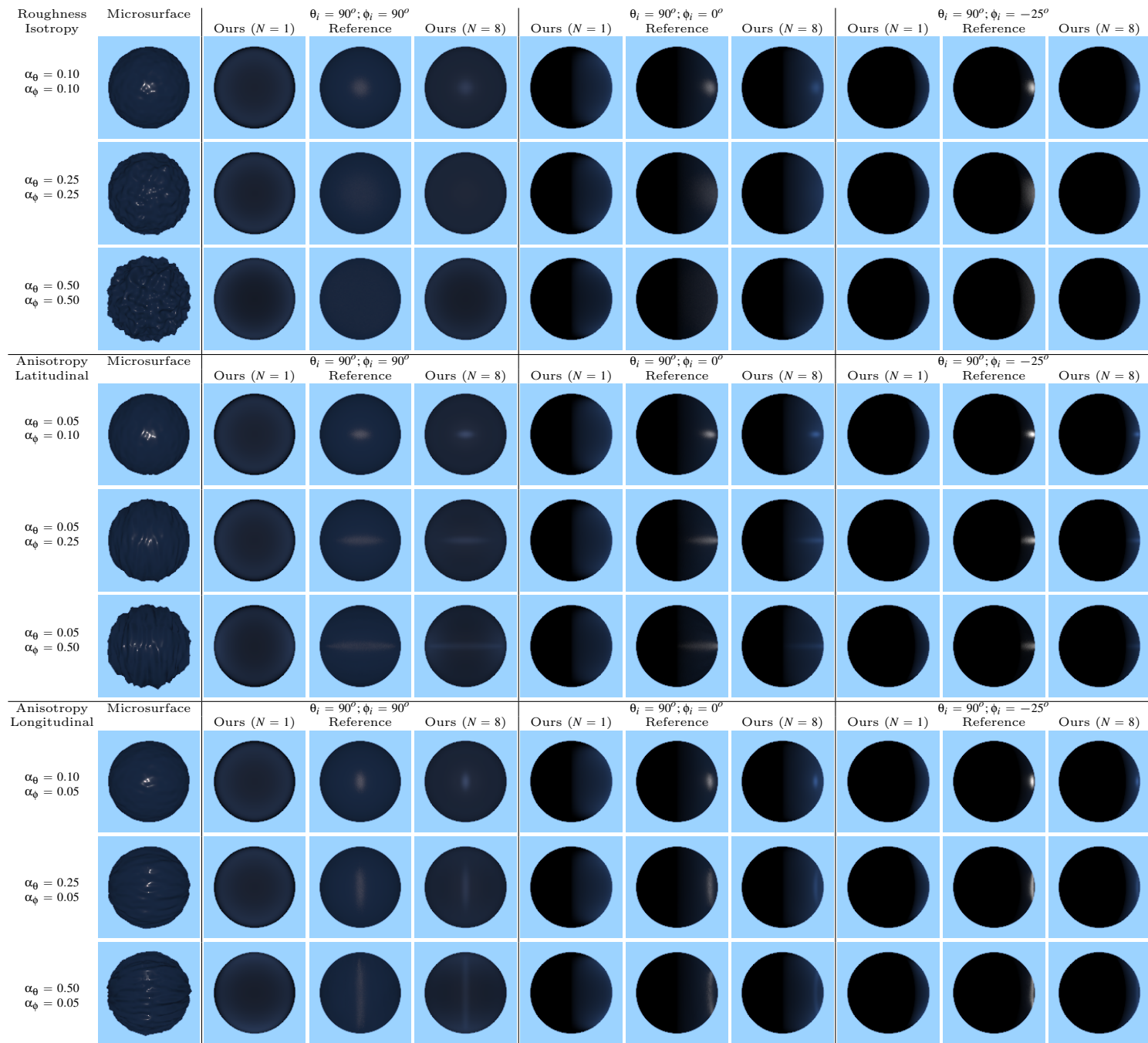
9 black-phenolic



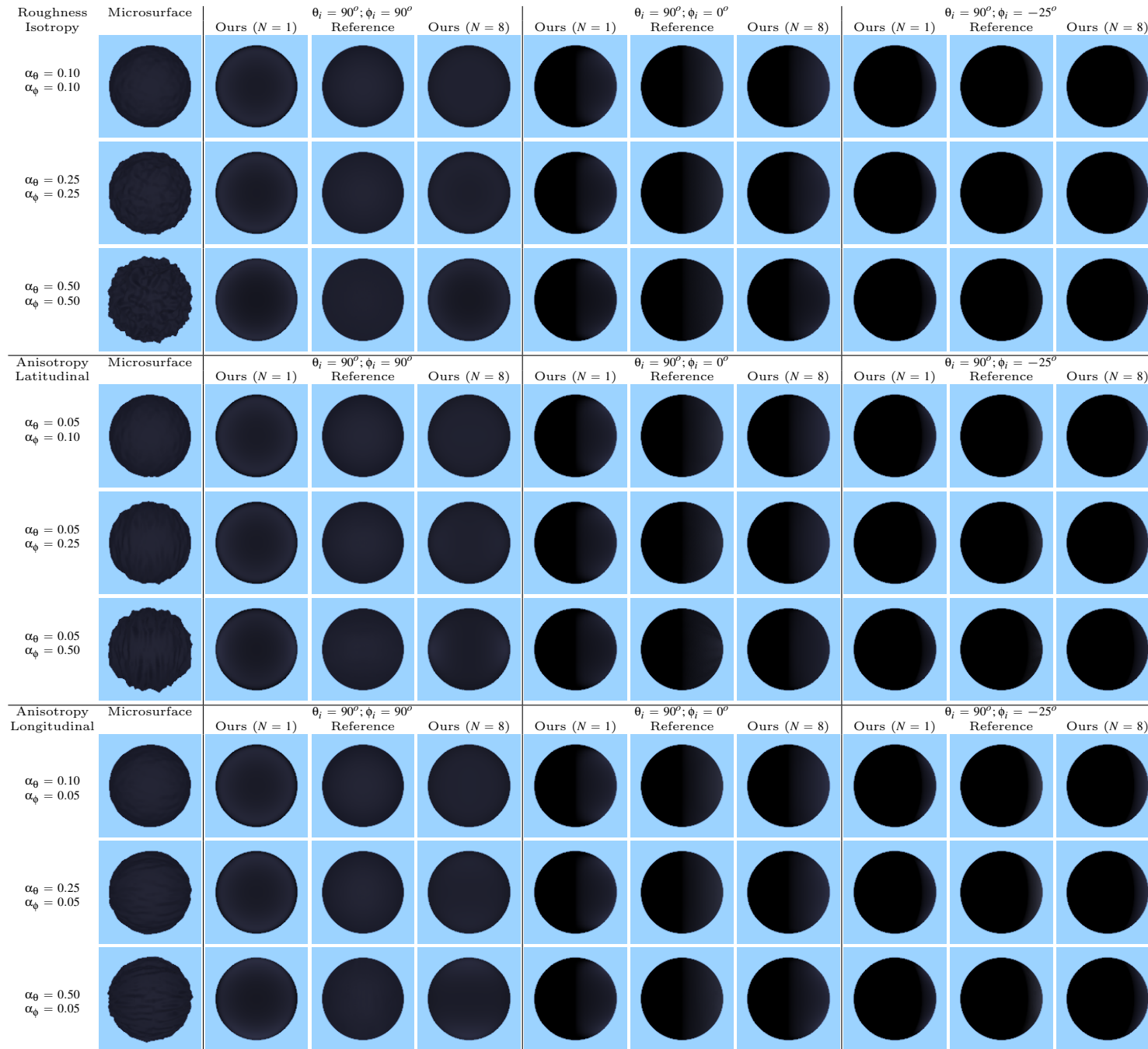
10 black-soft-plastic



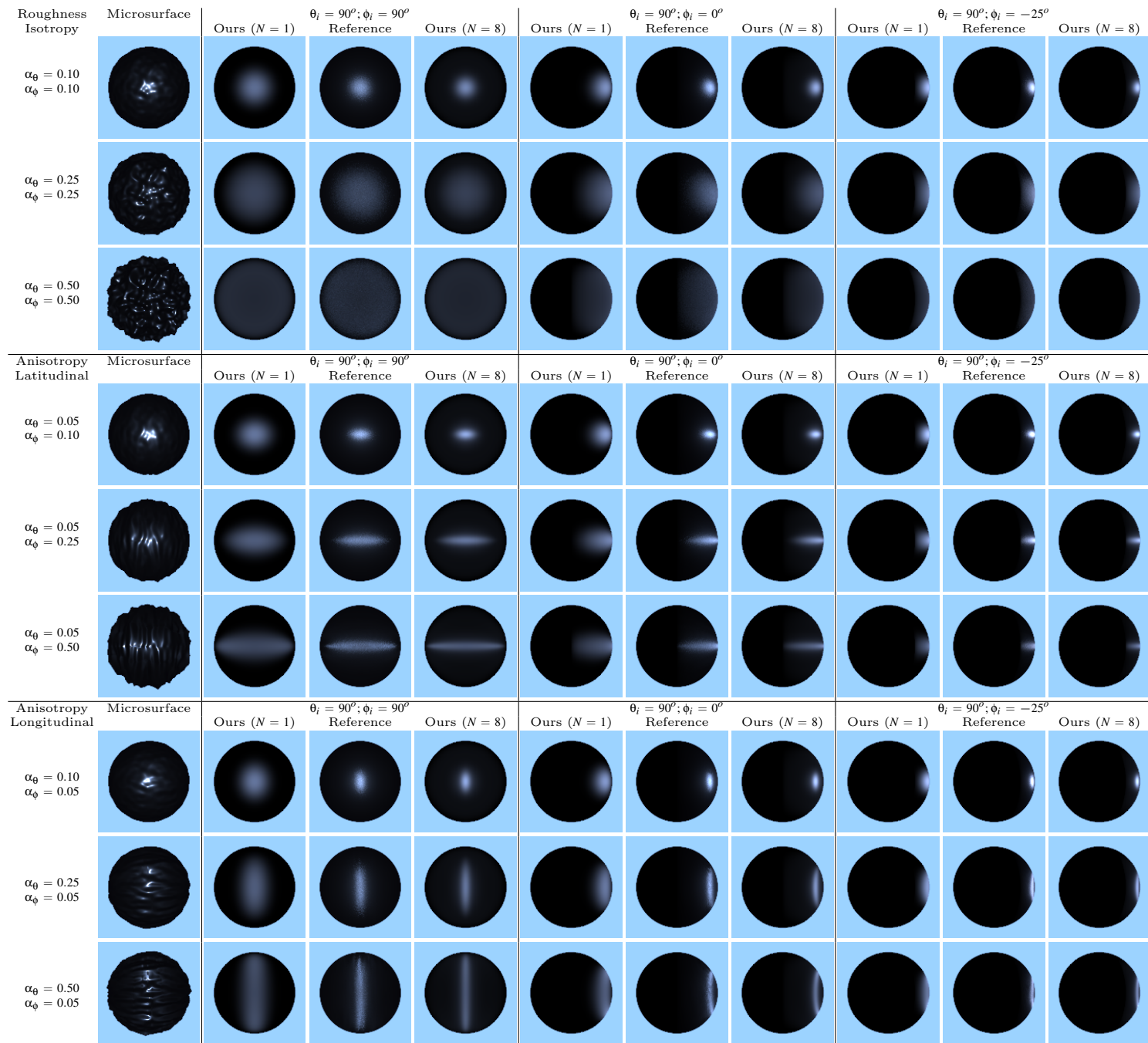
11 blue-acrylic



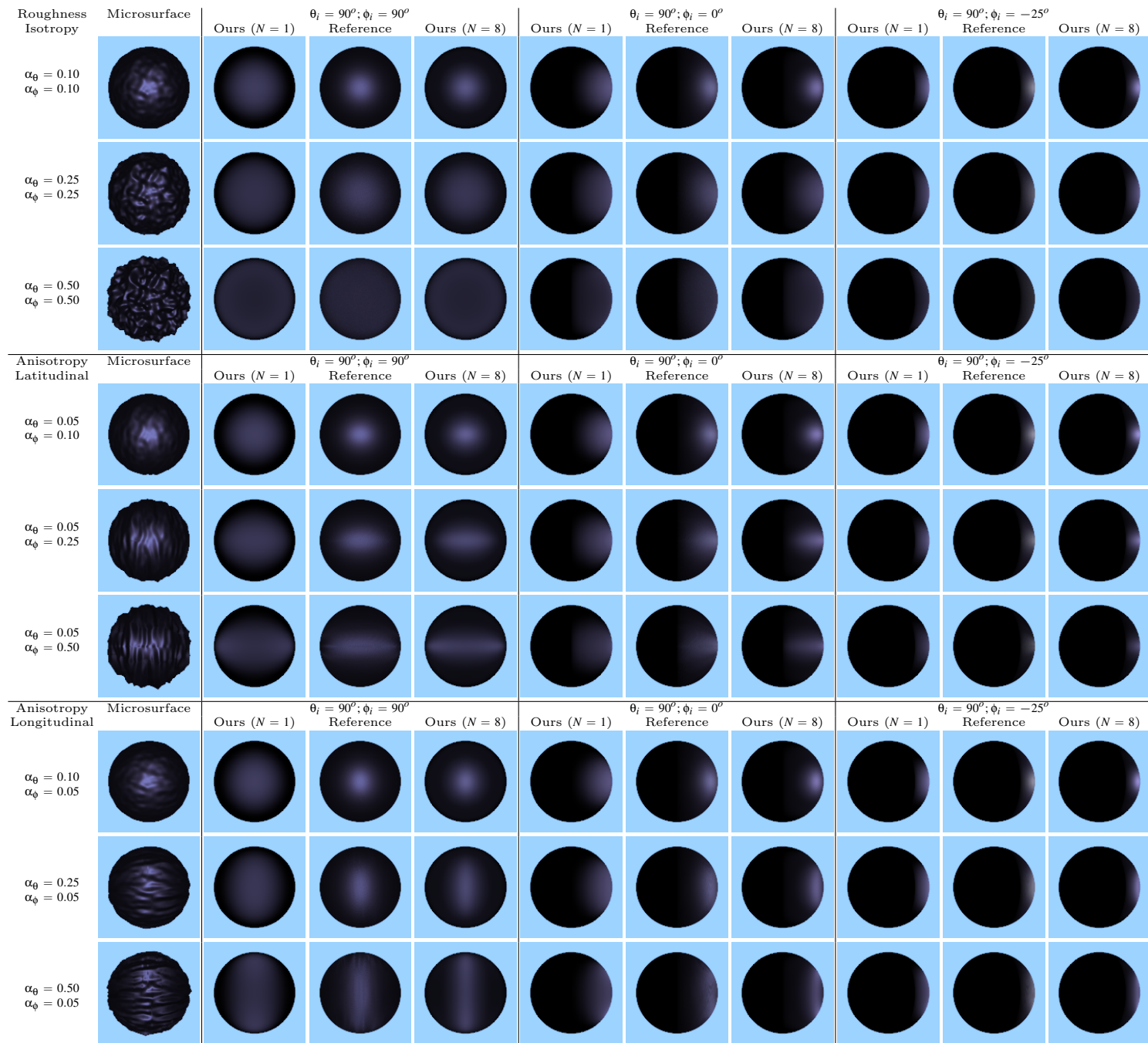
12 blue-fabric



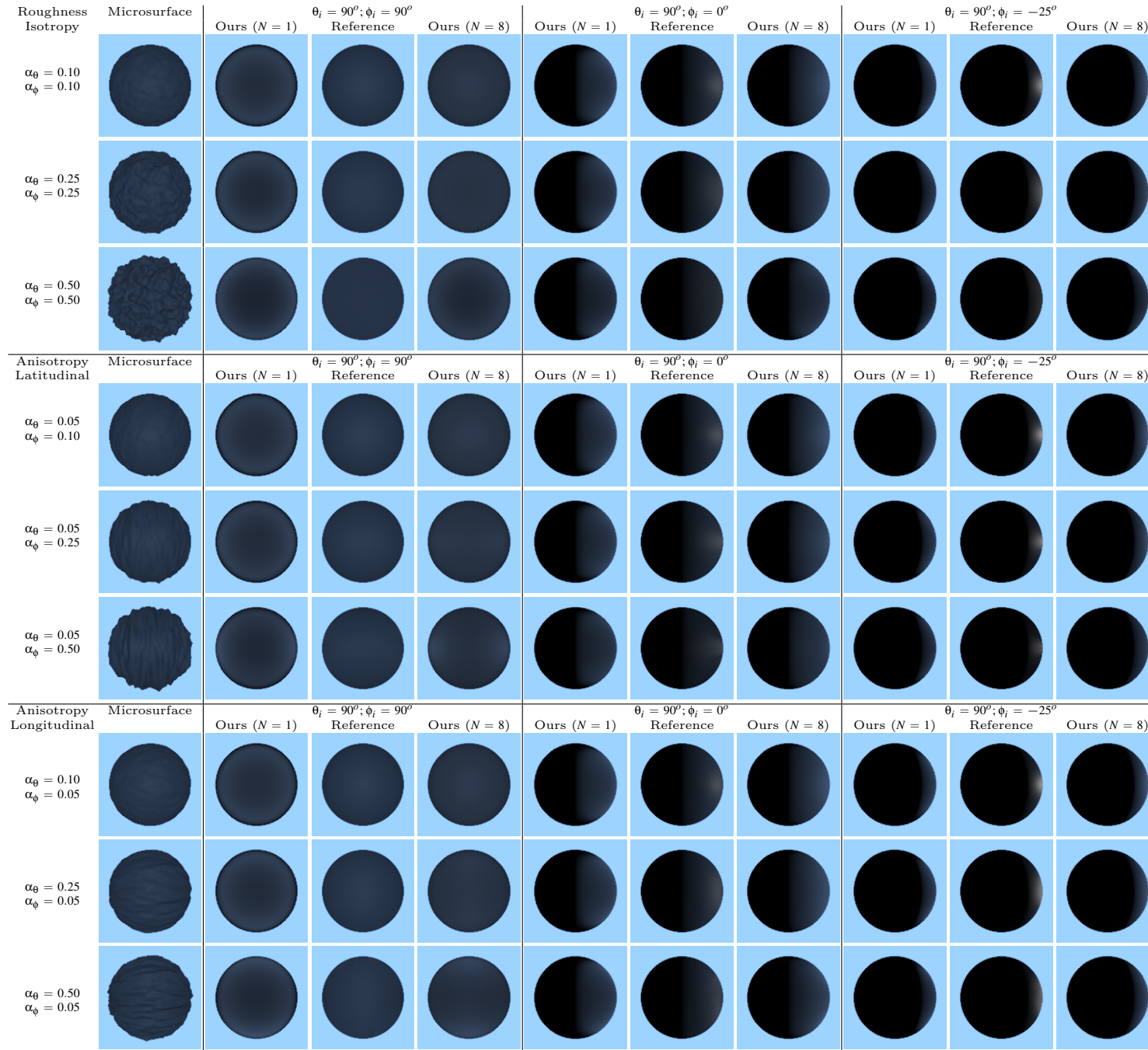
13 blue-metallic-paint2



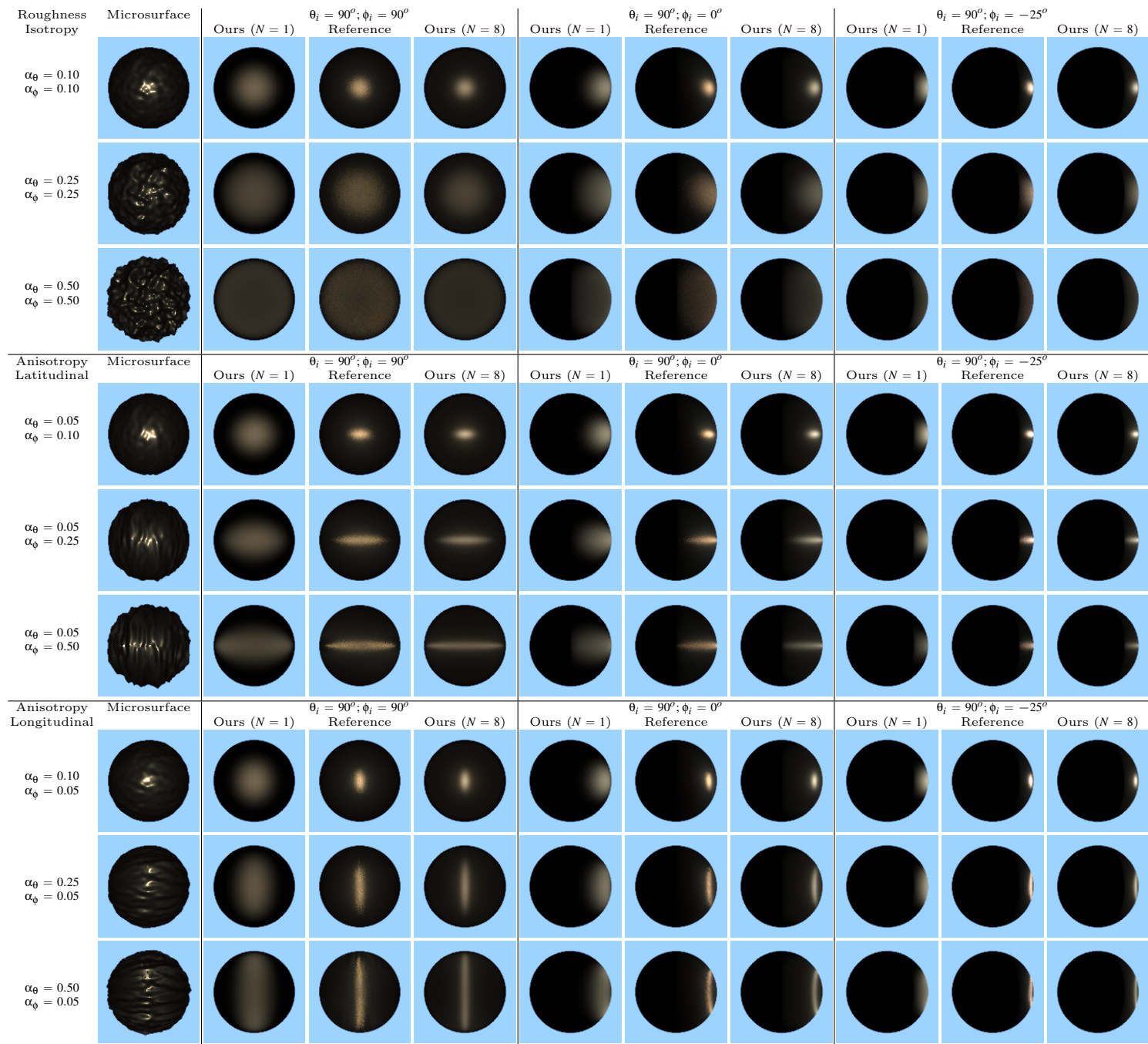
14 blue-metallic-paint

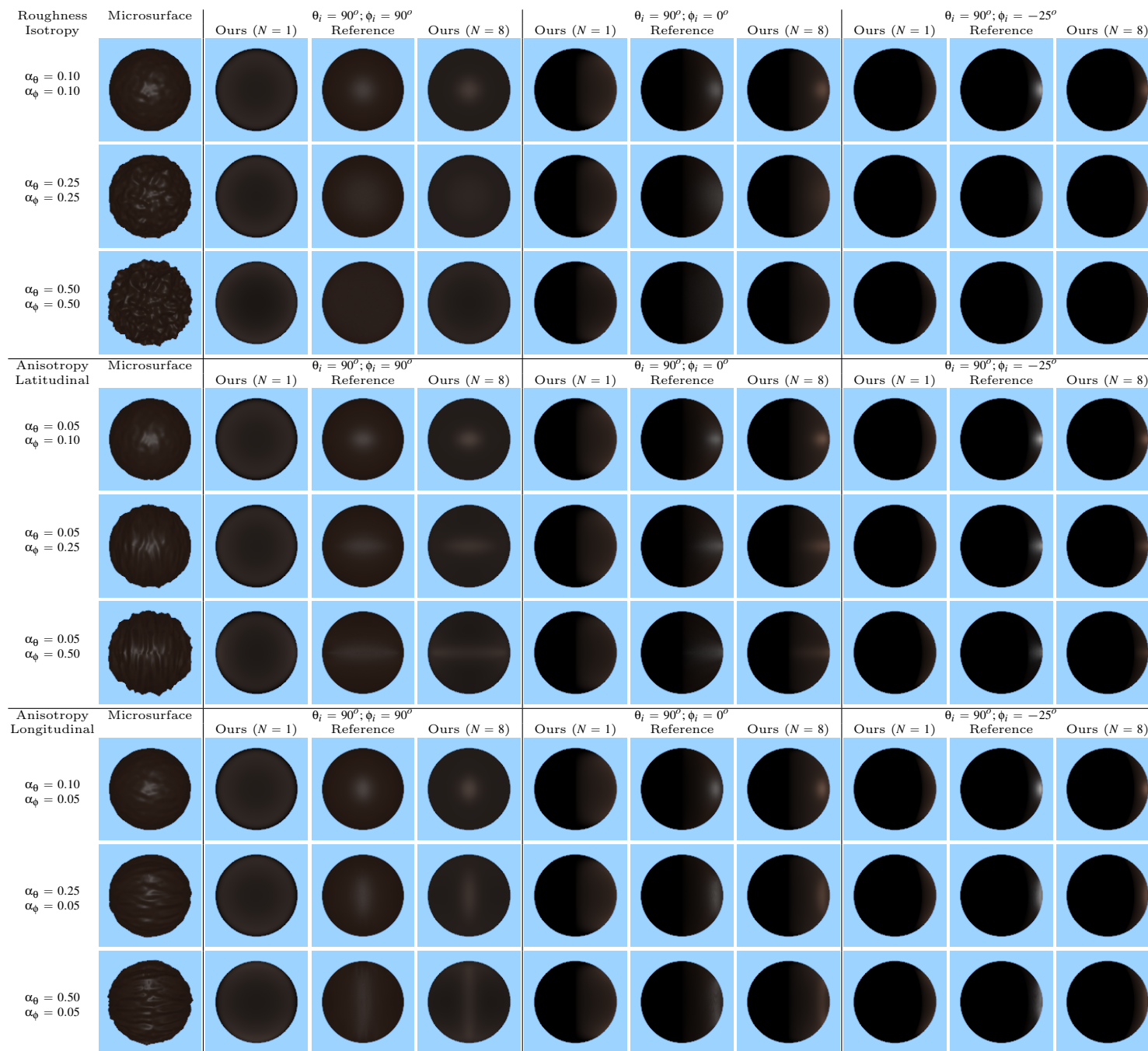


15 blue-rubber

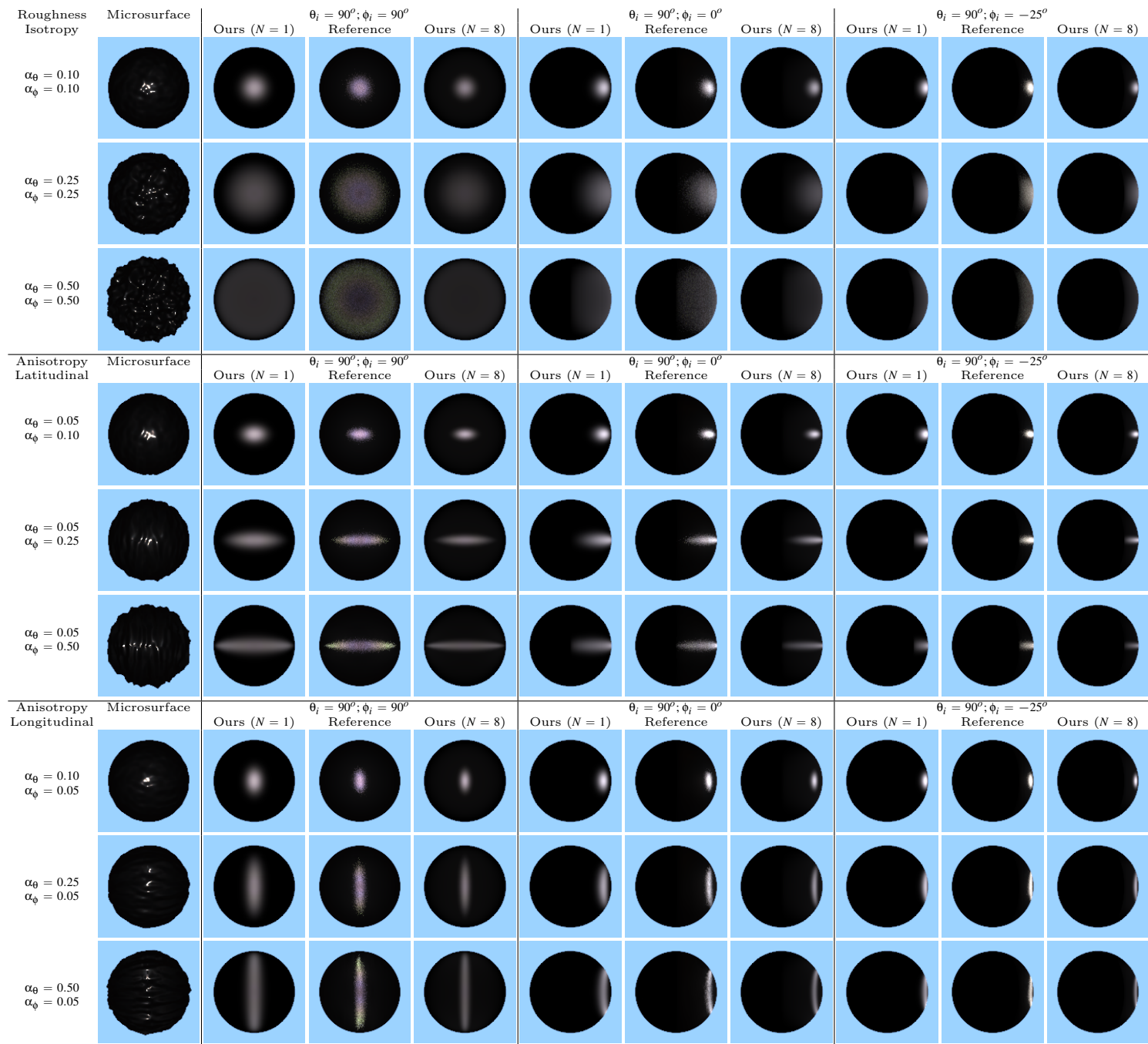


16 brass

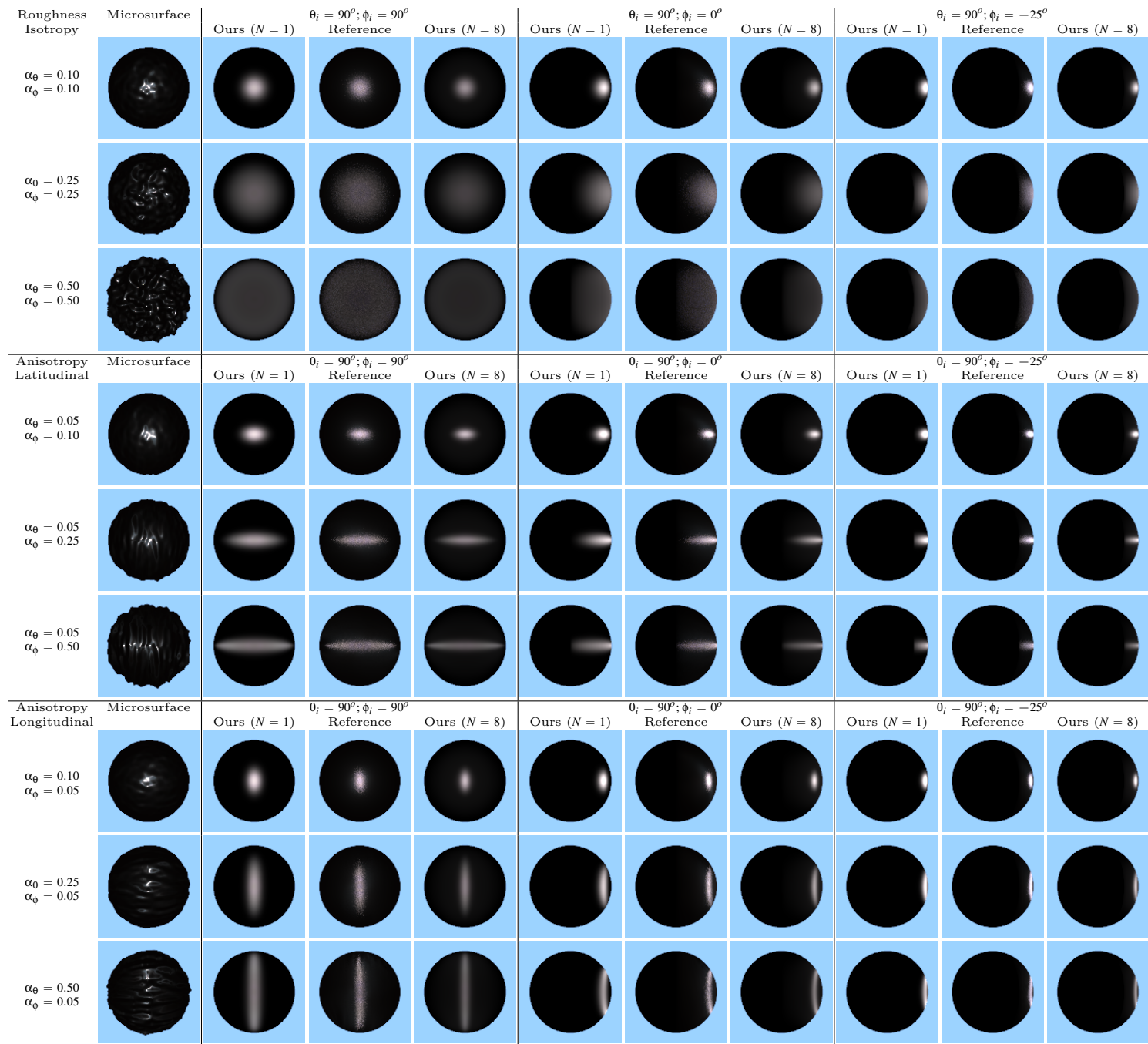


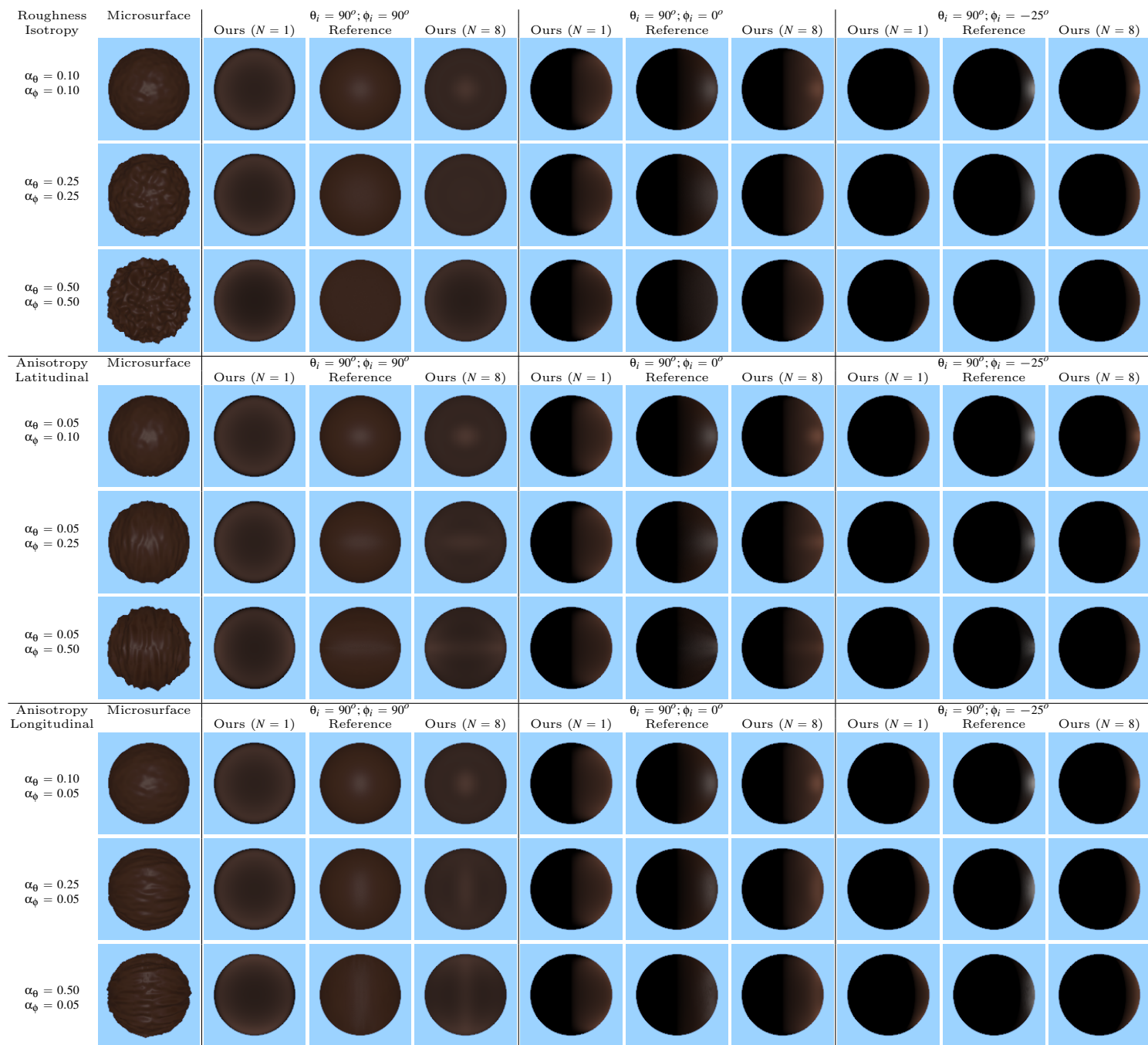


18 chrome

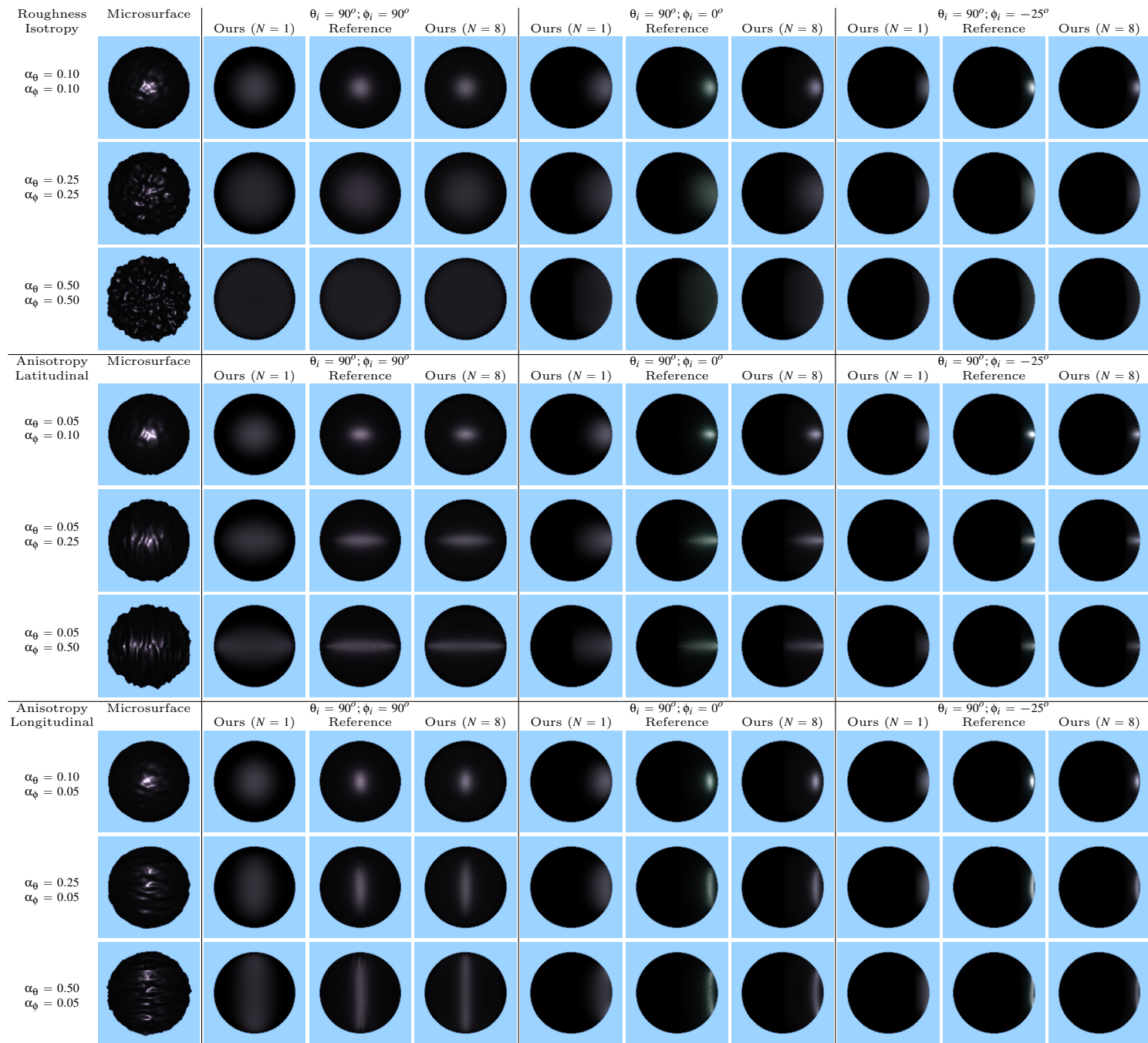


19 chrome-steel

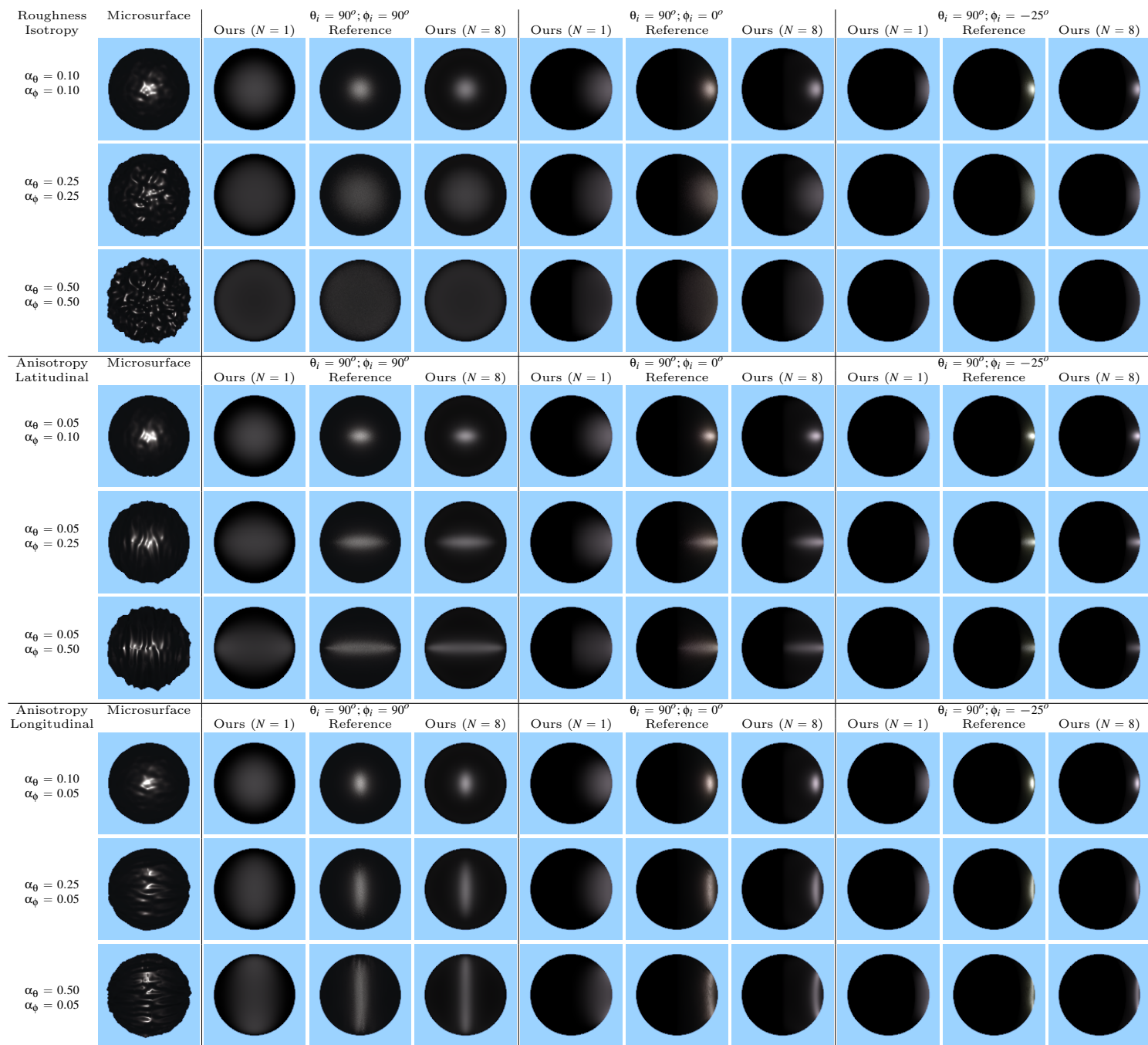




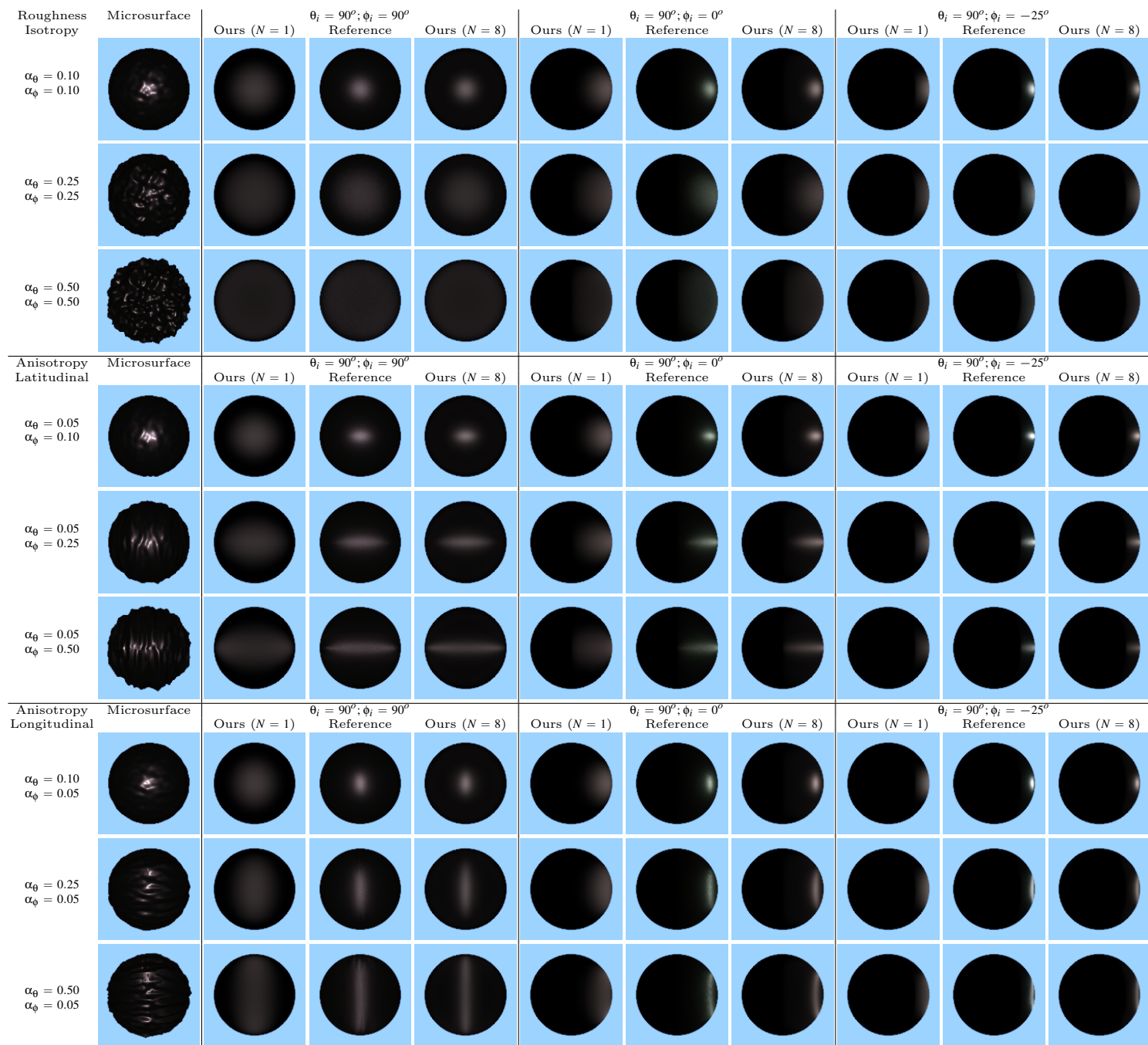
21 color-changing-paint1



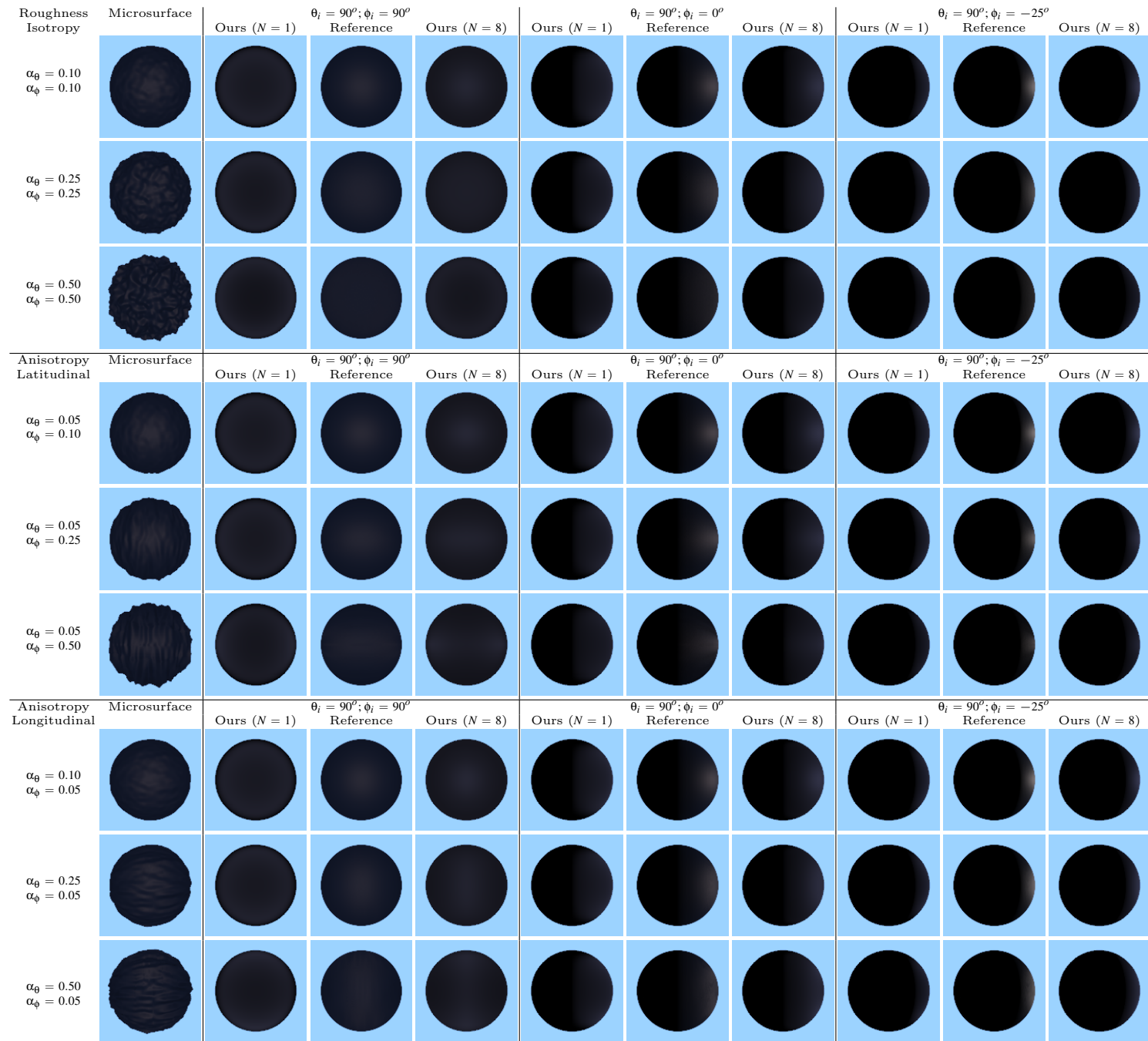
22 color-changing-paint2



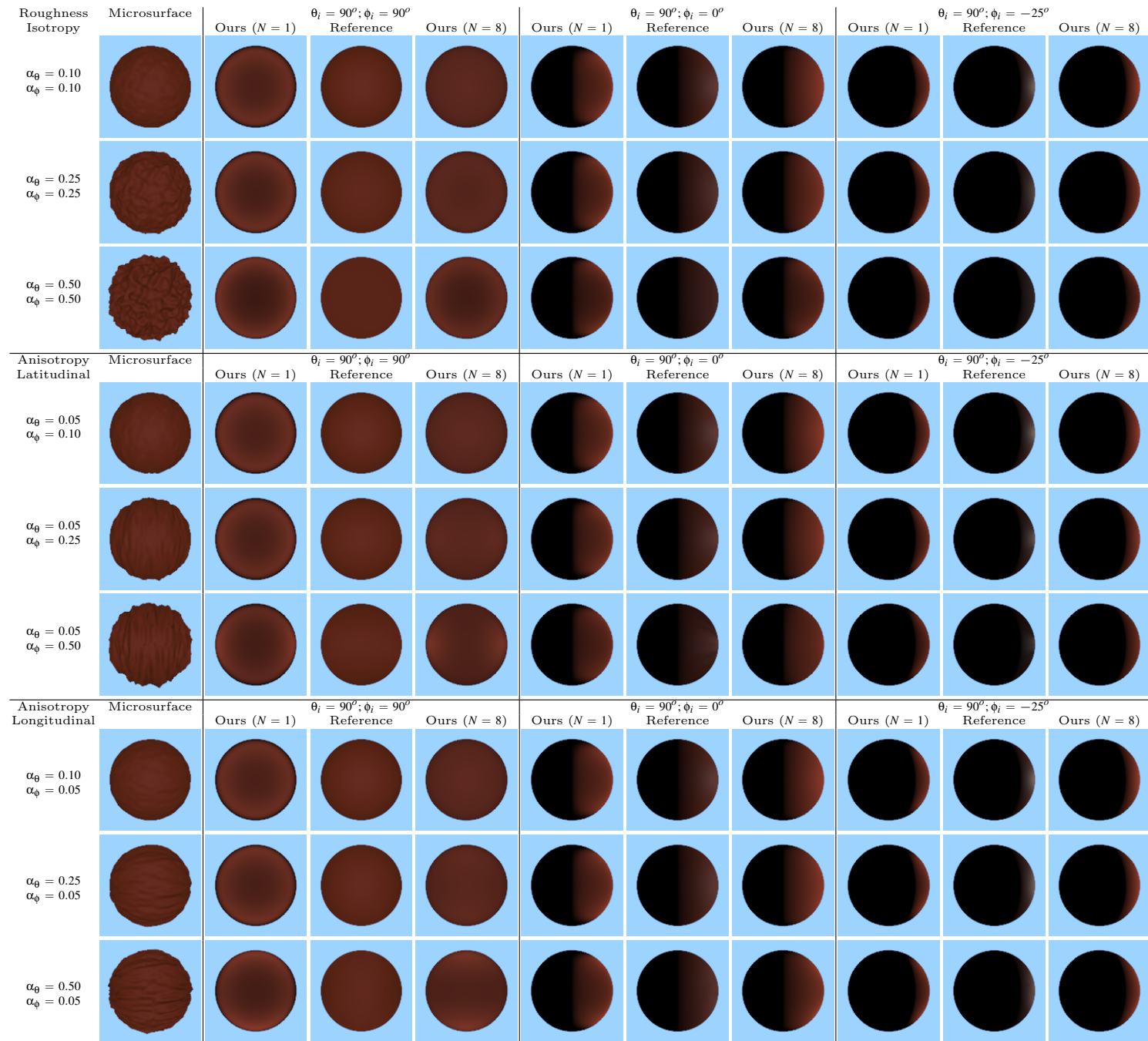
23 color-changing-paint3



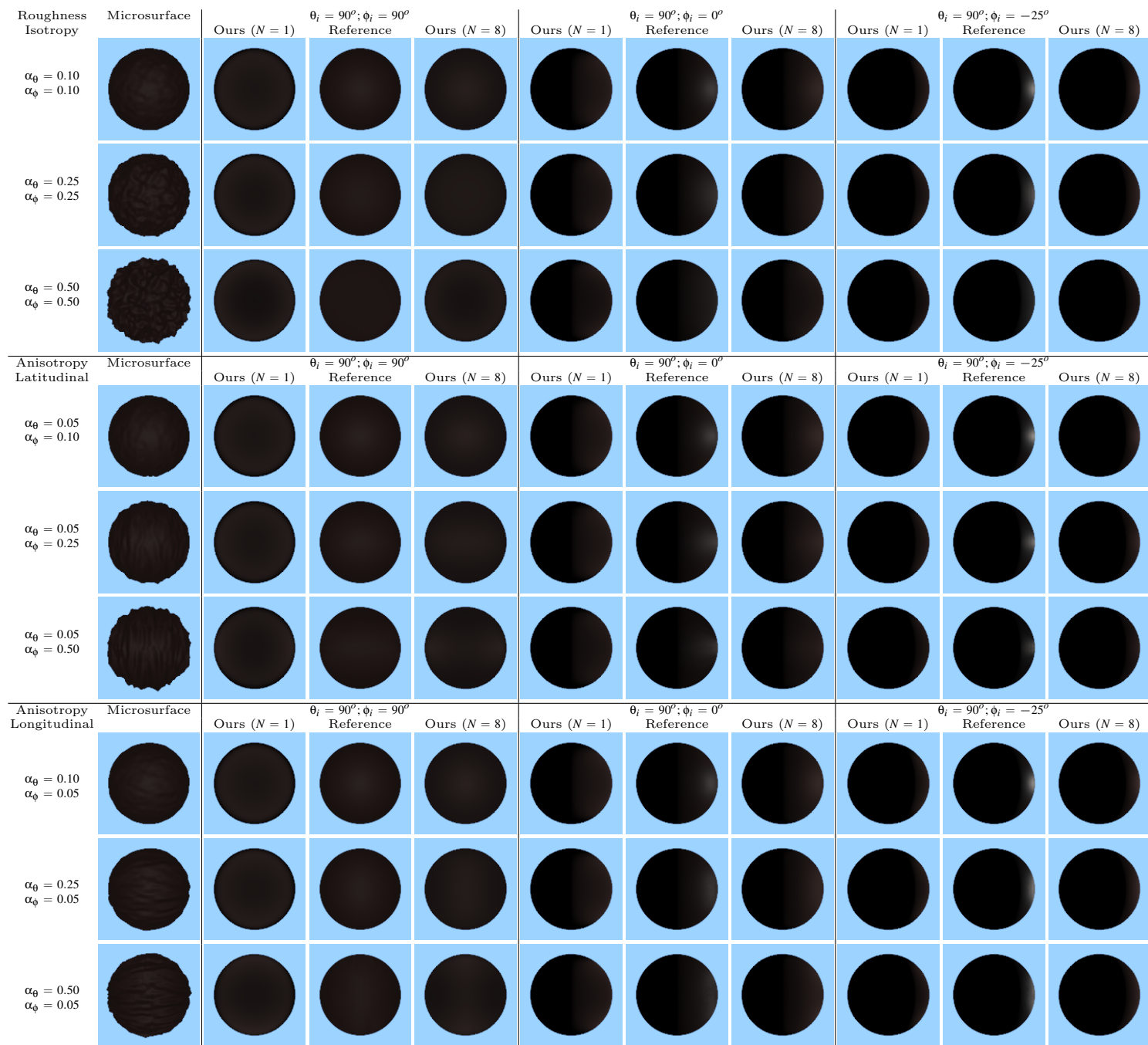
24 dark-blue-paint

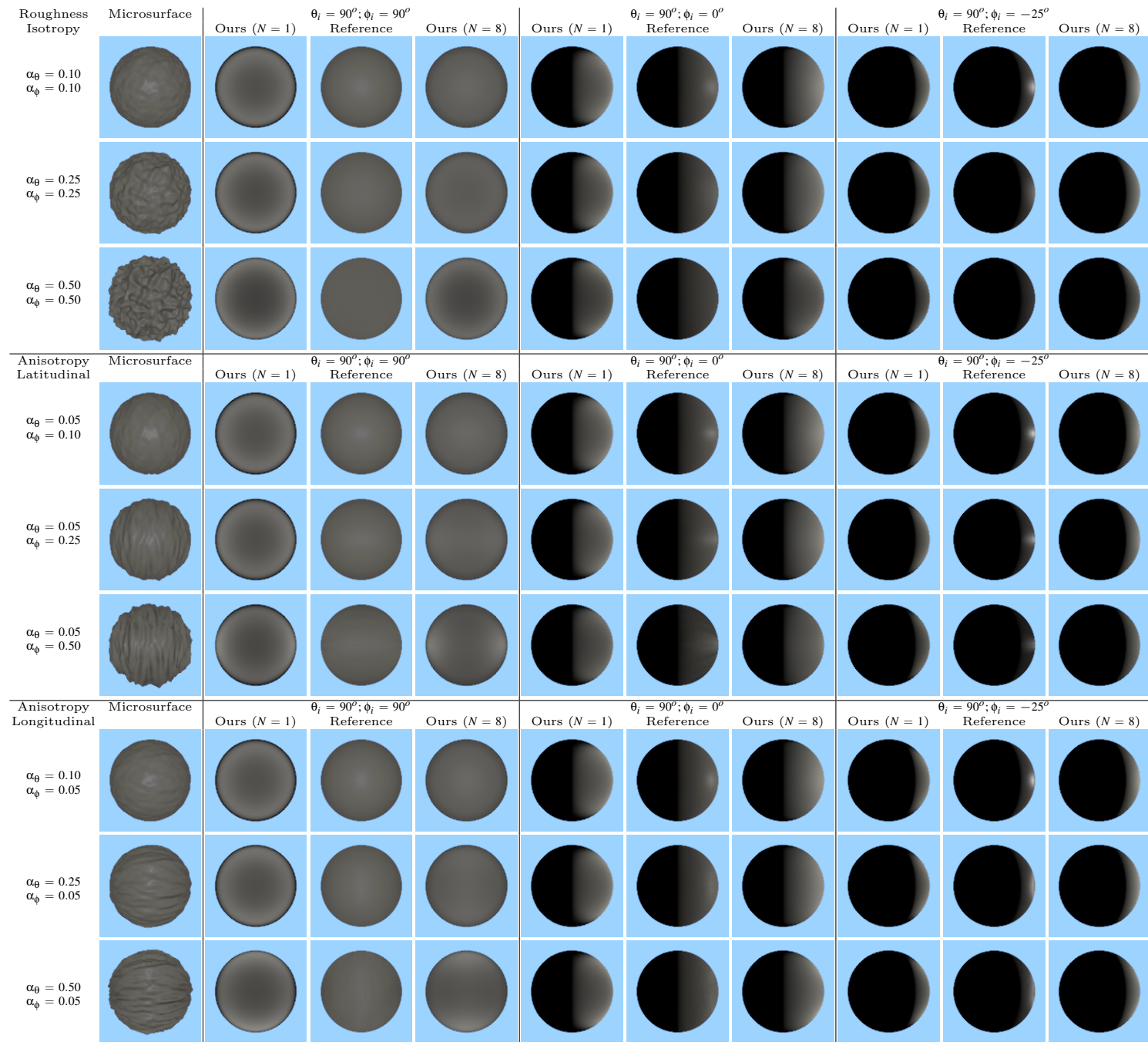


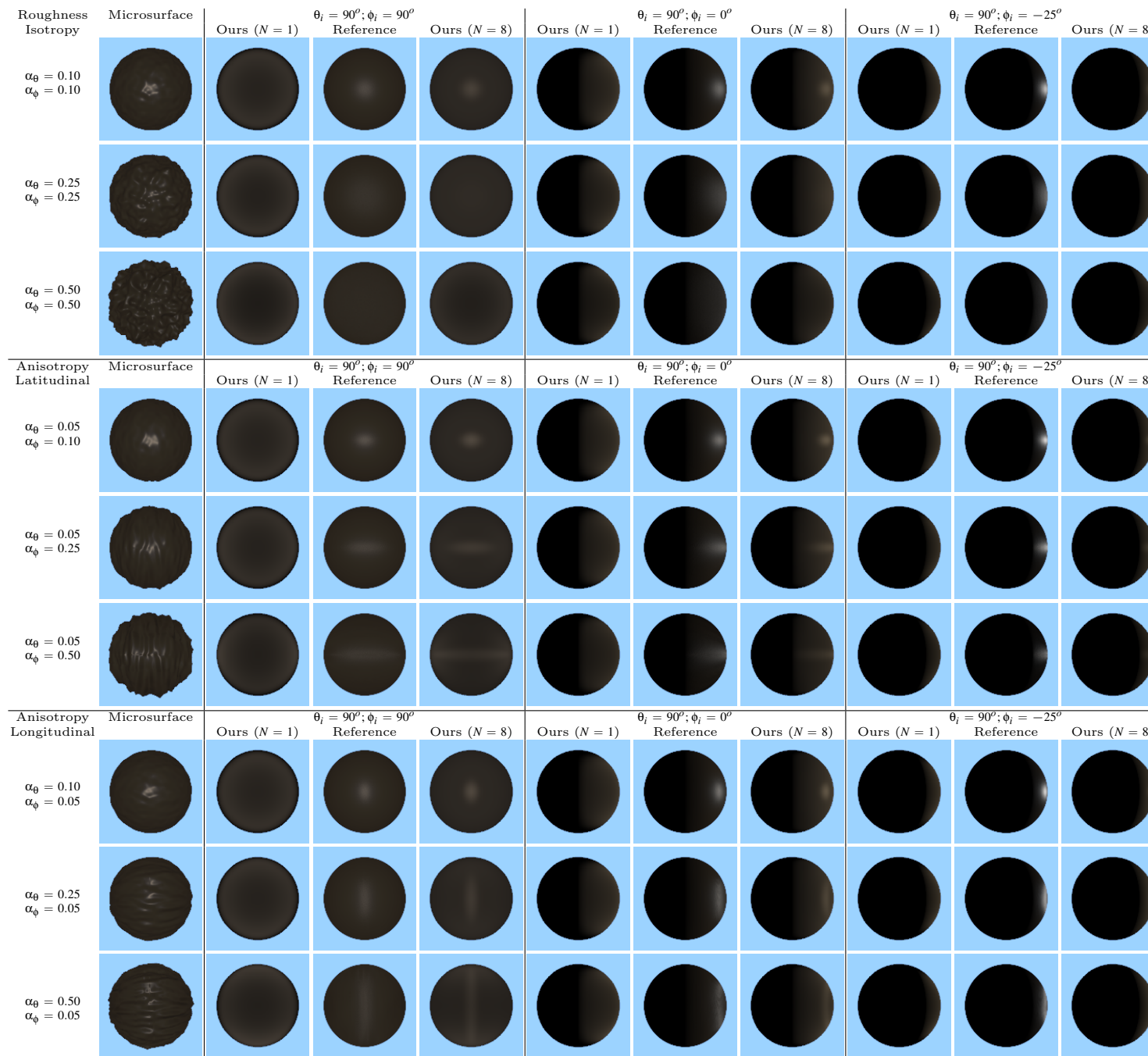
25 dark-red-paint



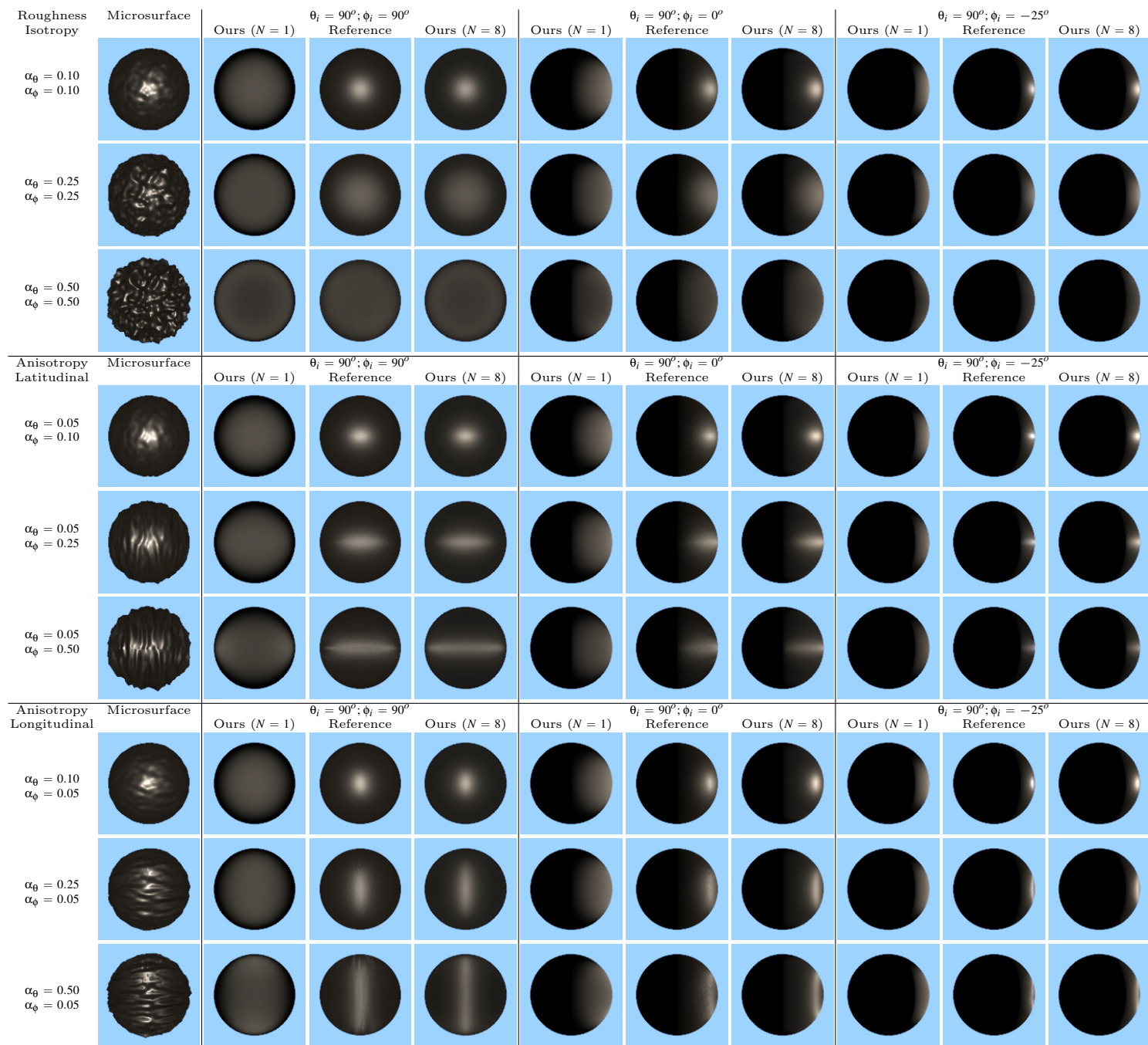
26 dark-specular-fabric



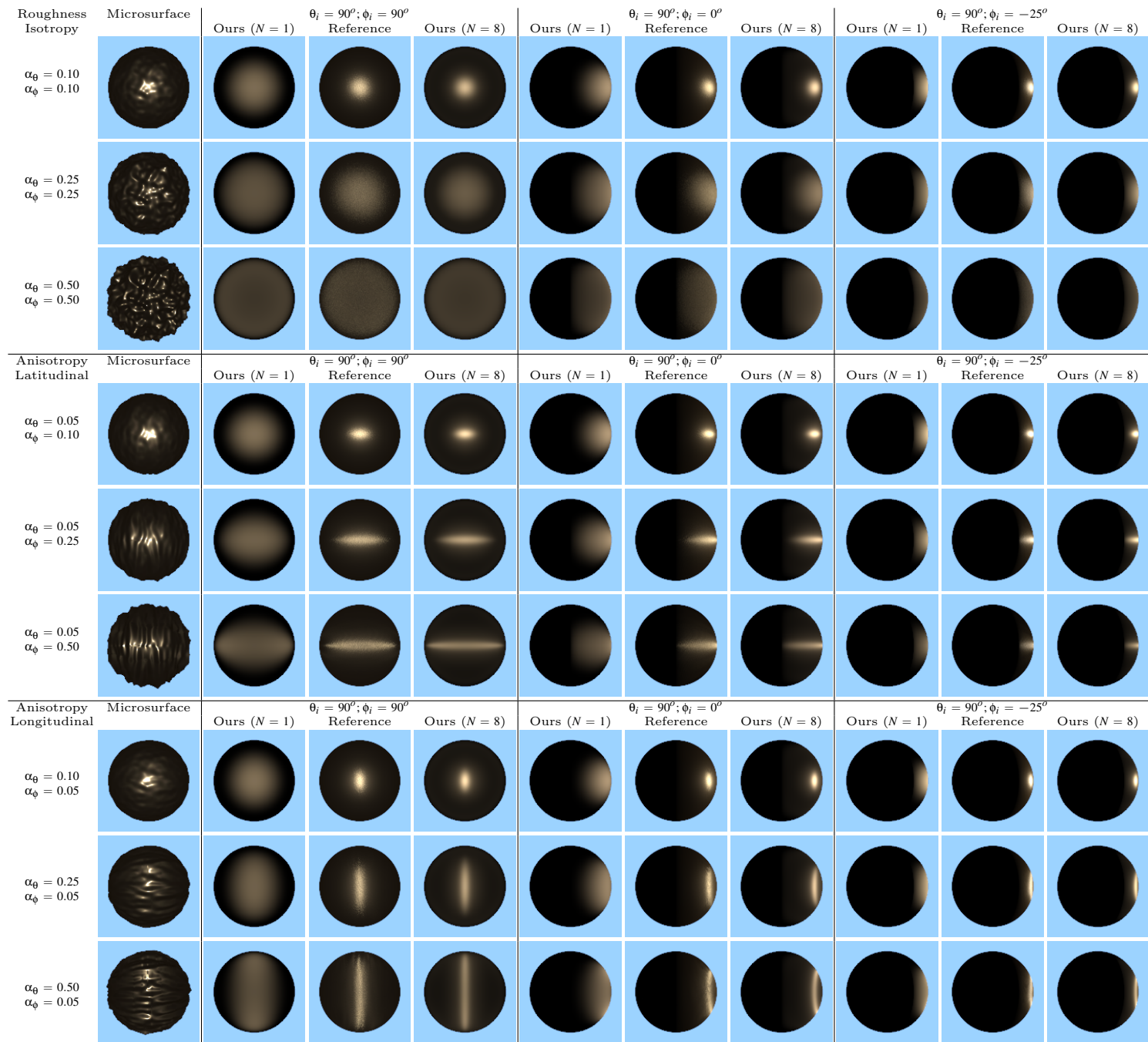




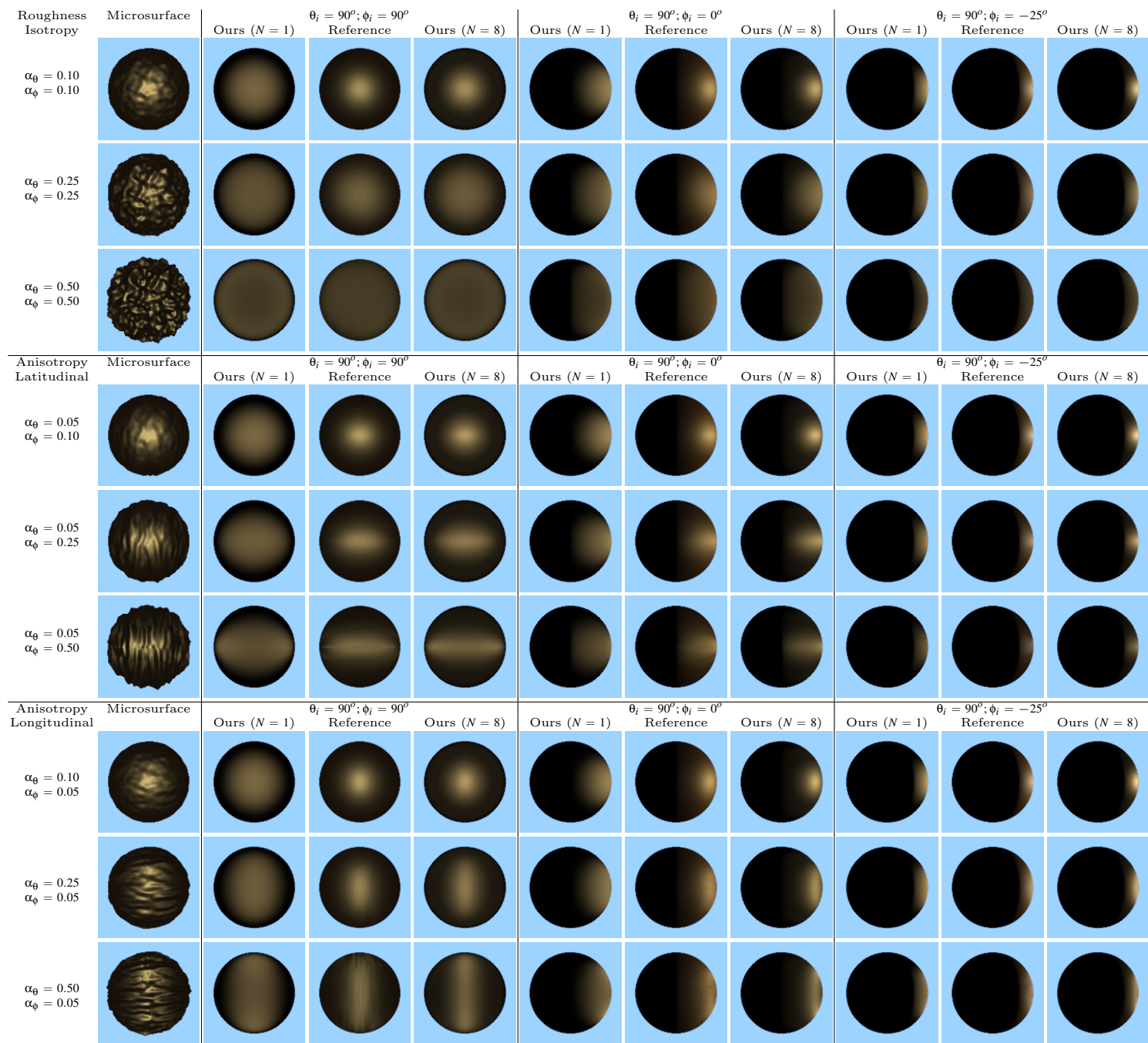
29 gold-metallic-paint2



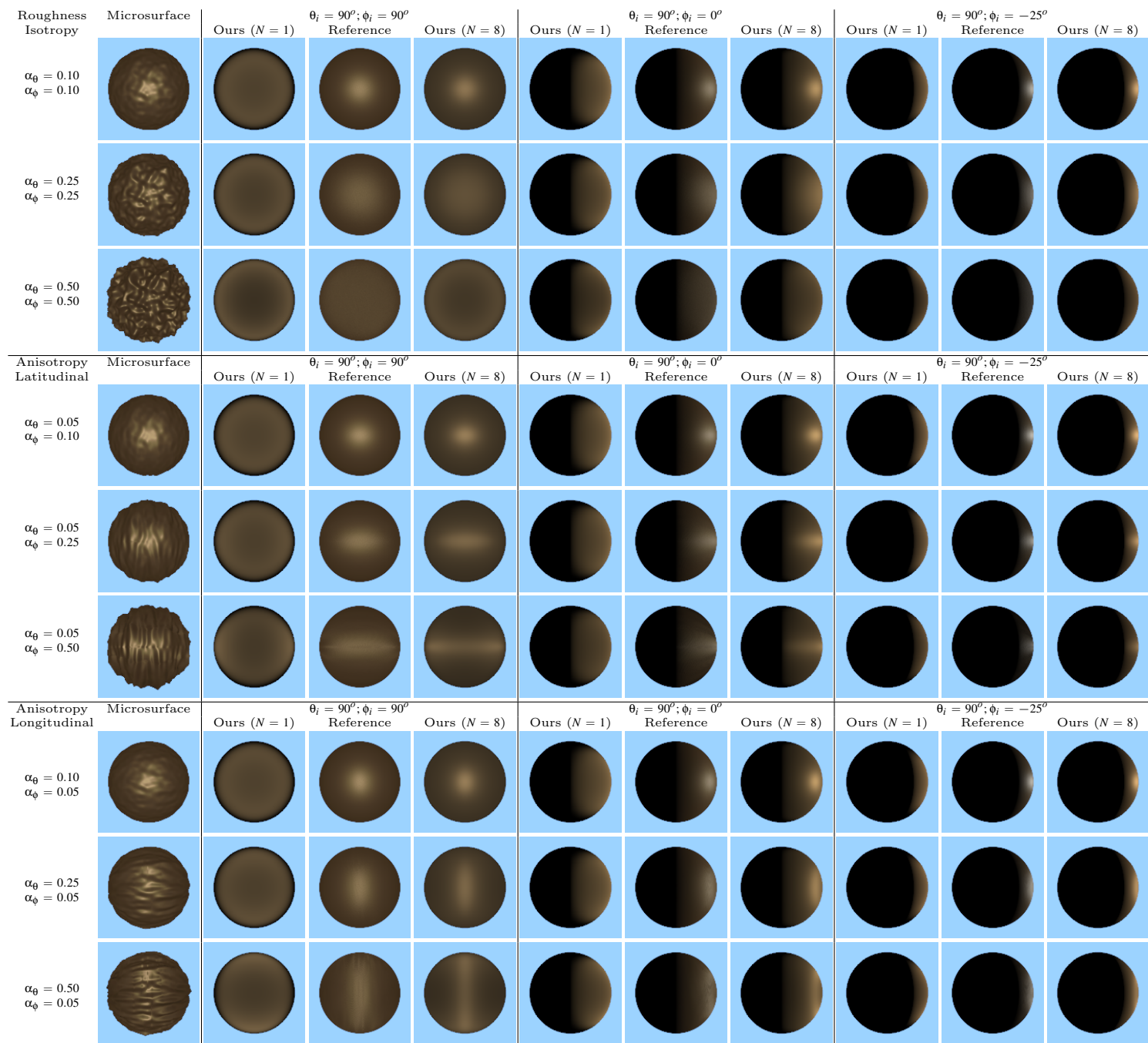
30 gold-metallic-paint3



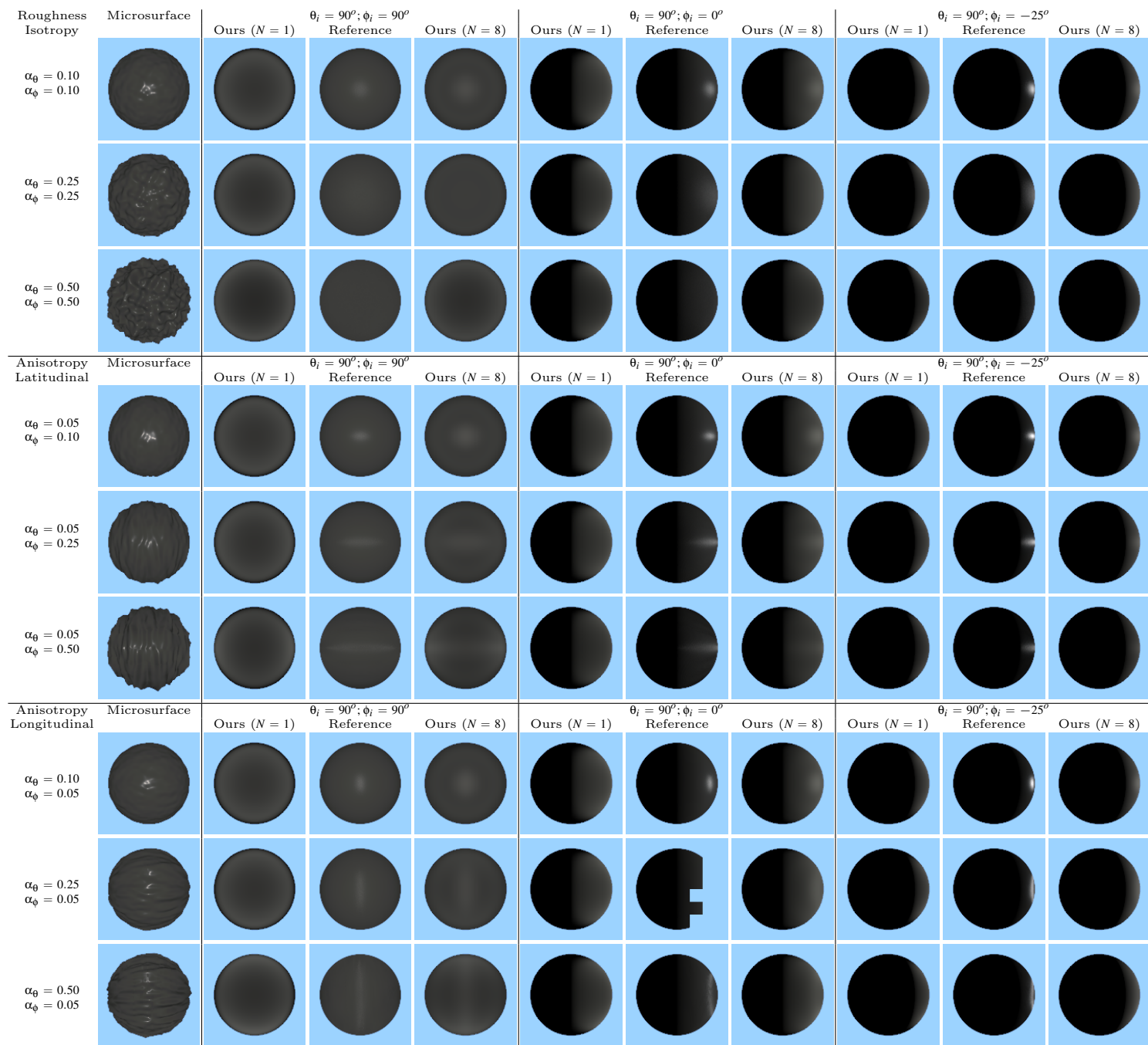
31 gold-metallic-paint



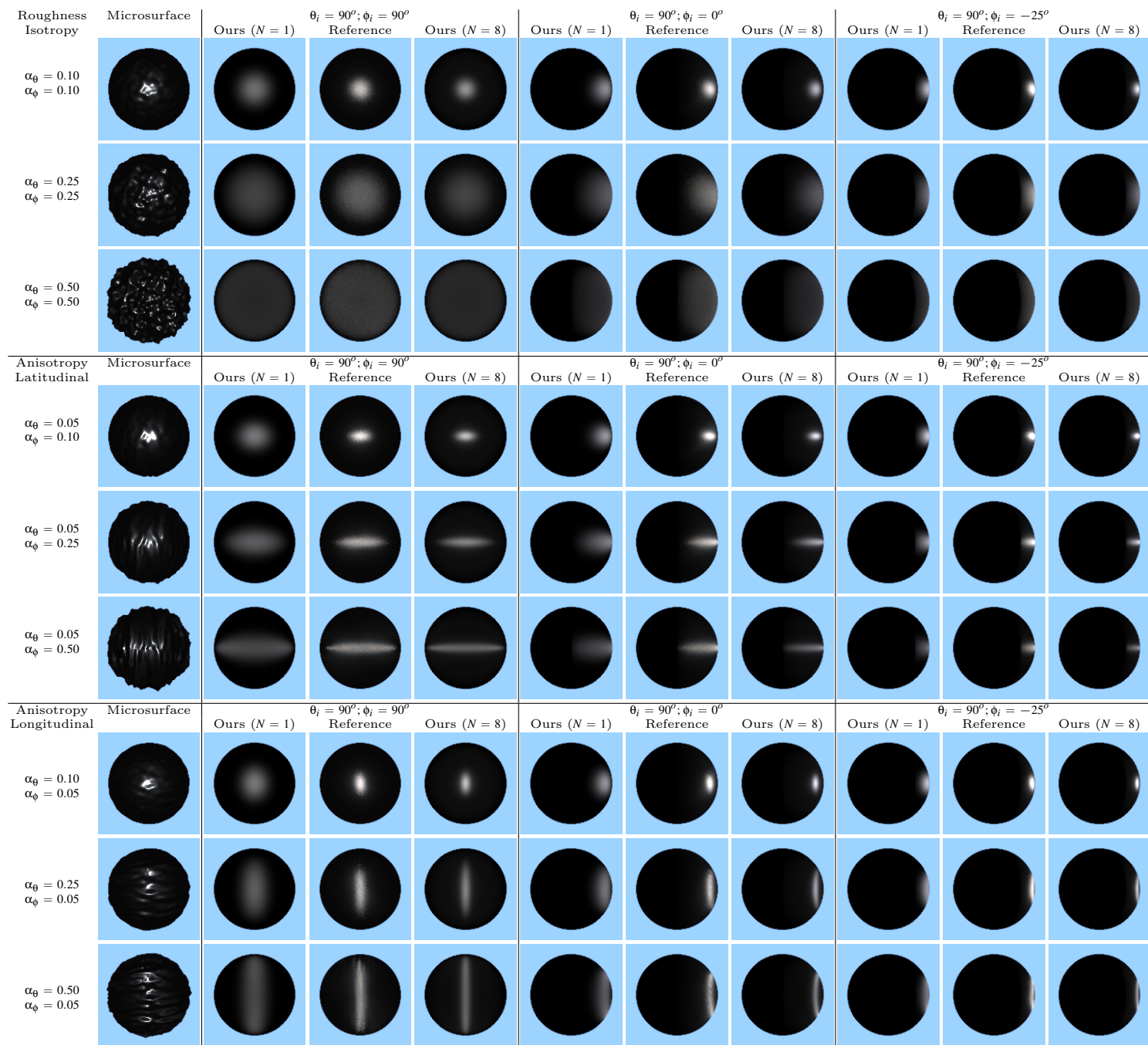
32 gold-paint



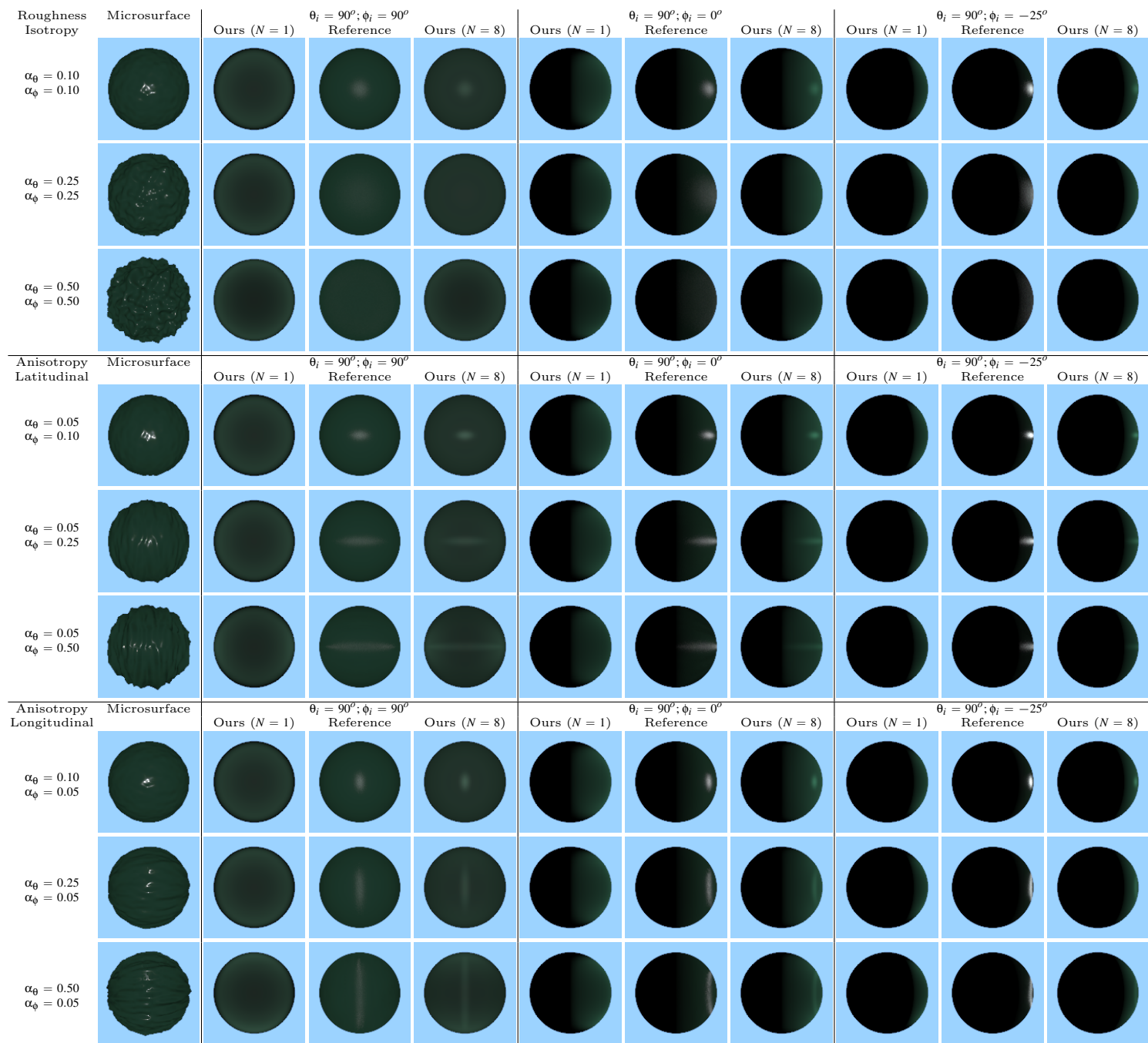
33 gray-plastic



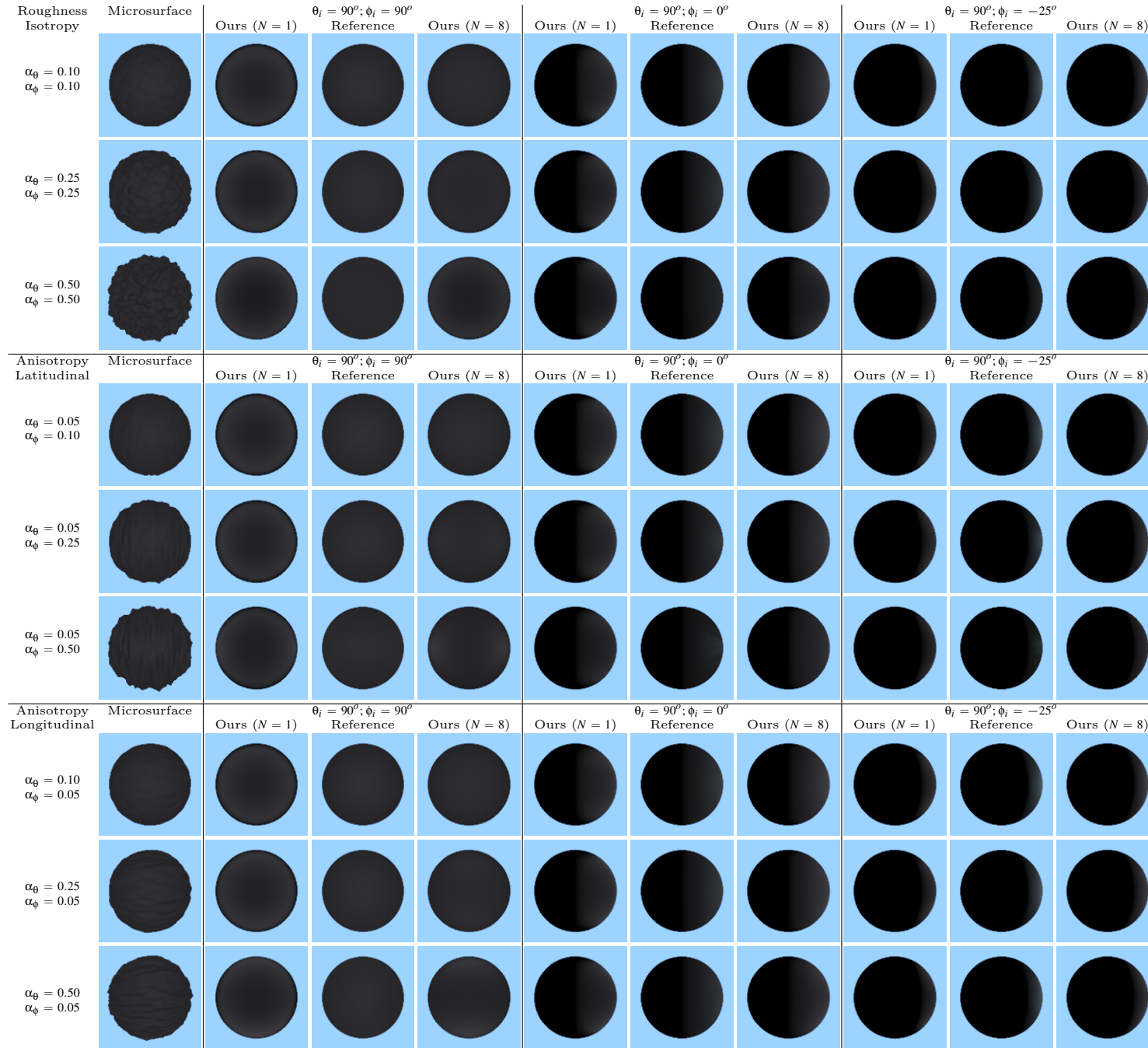
34 grease-covered-steel

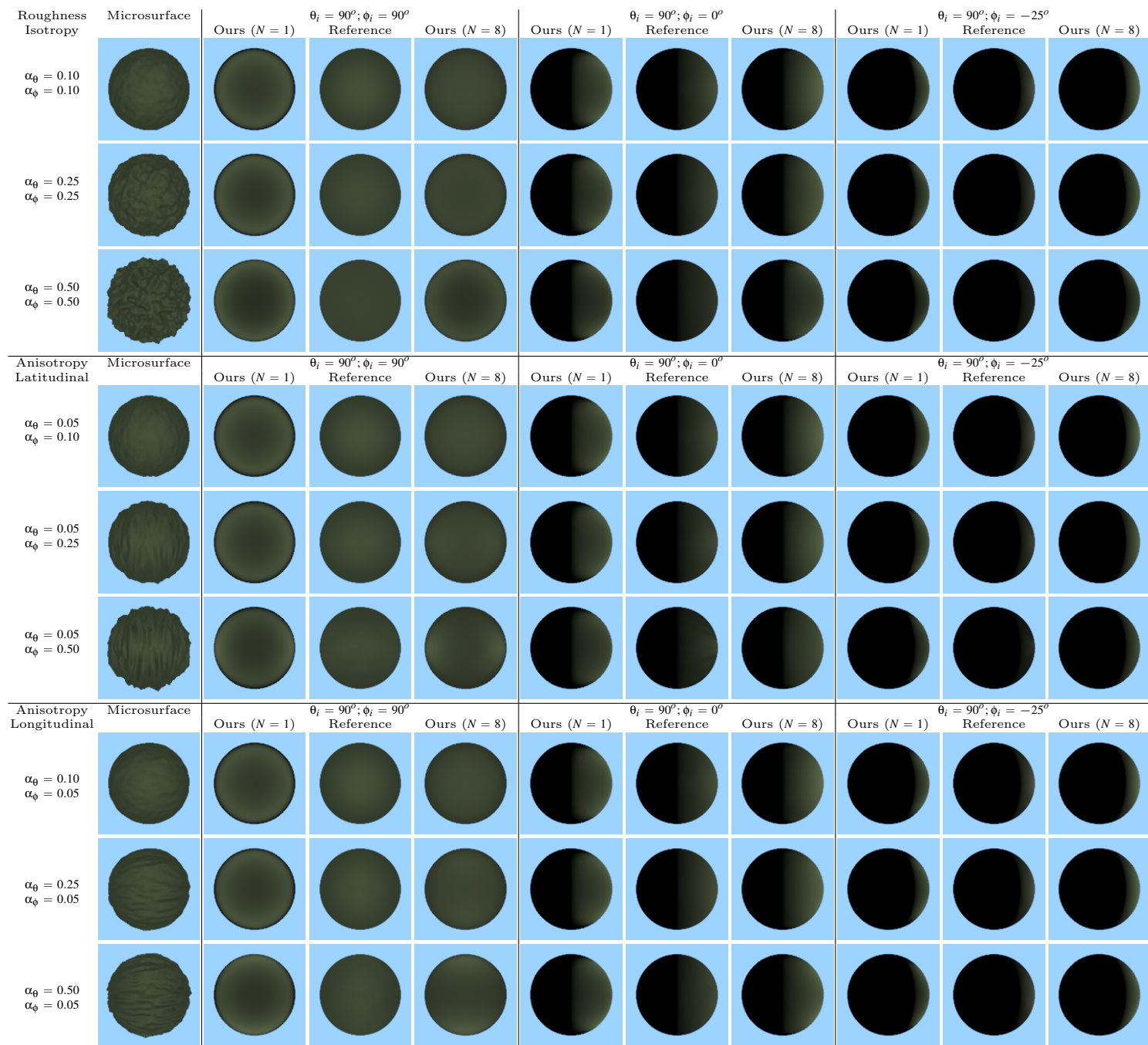


35 green-acrylic

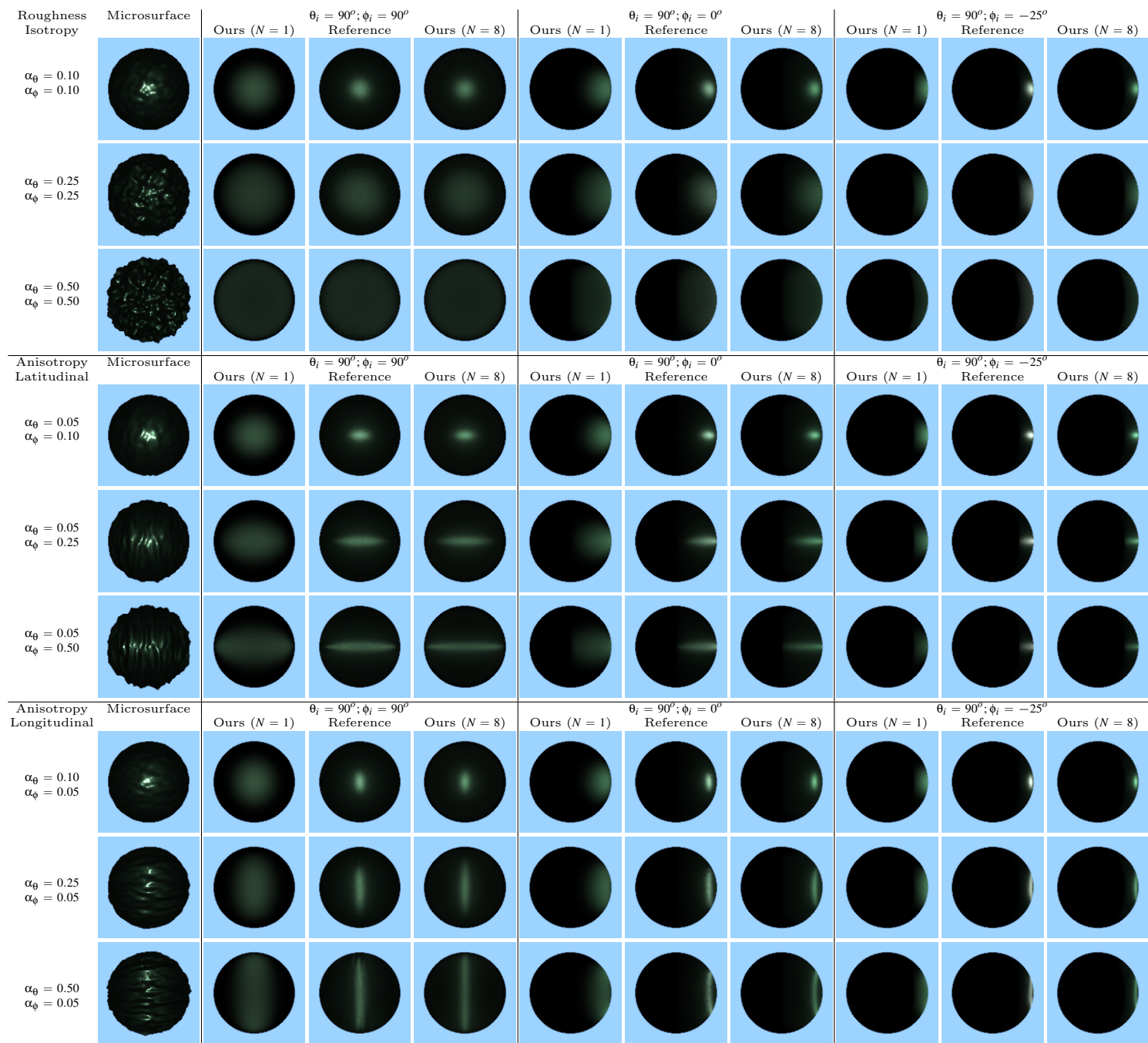


36 green-fabric

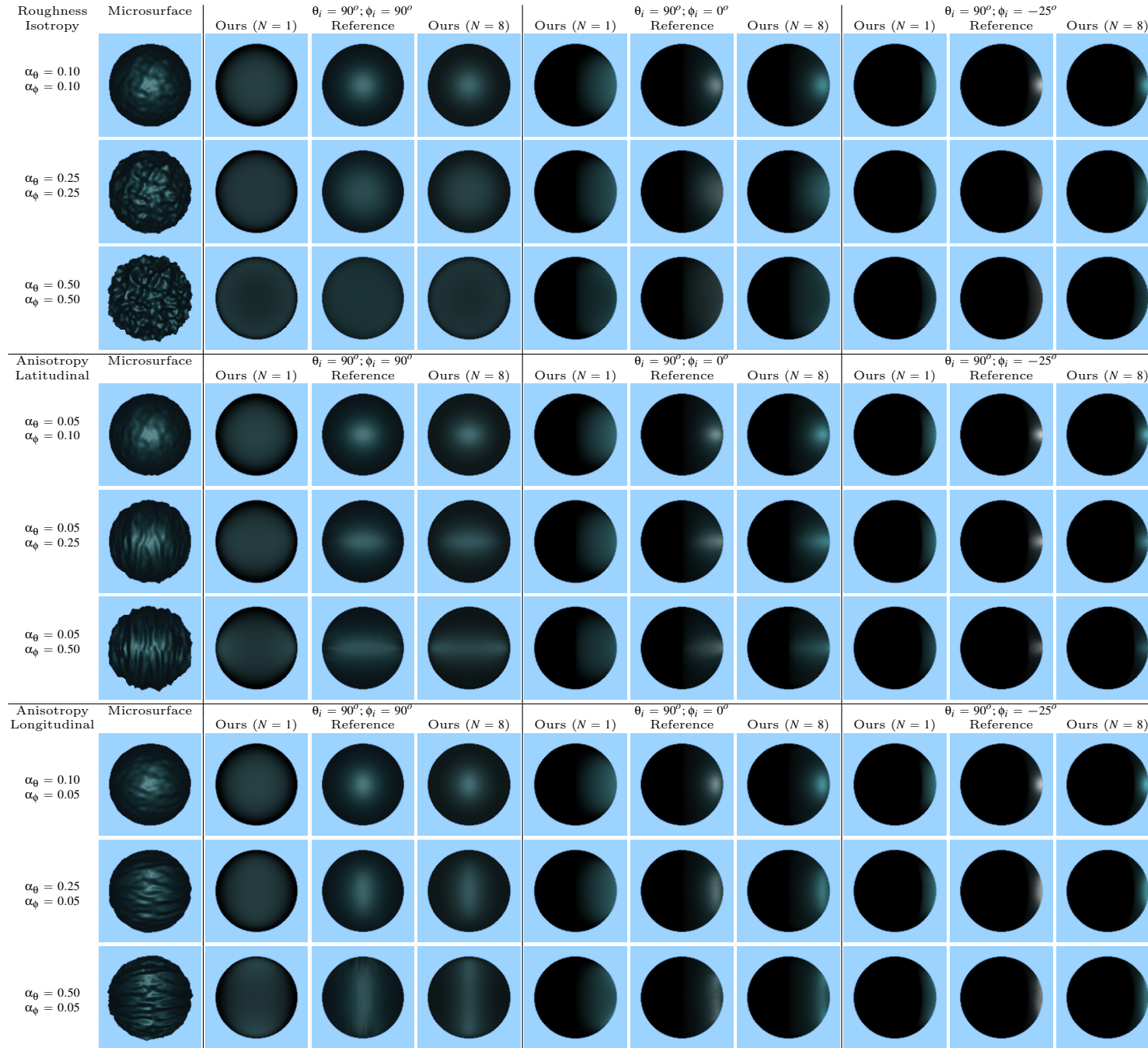




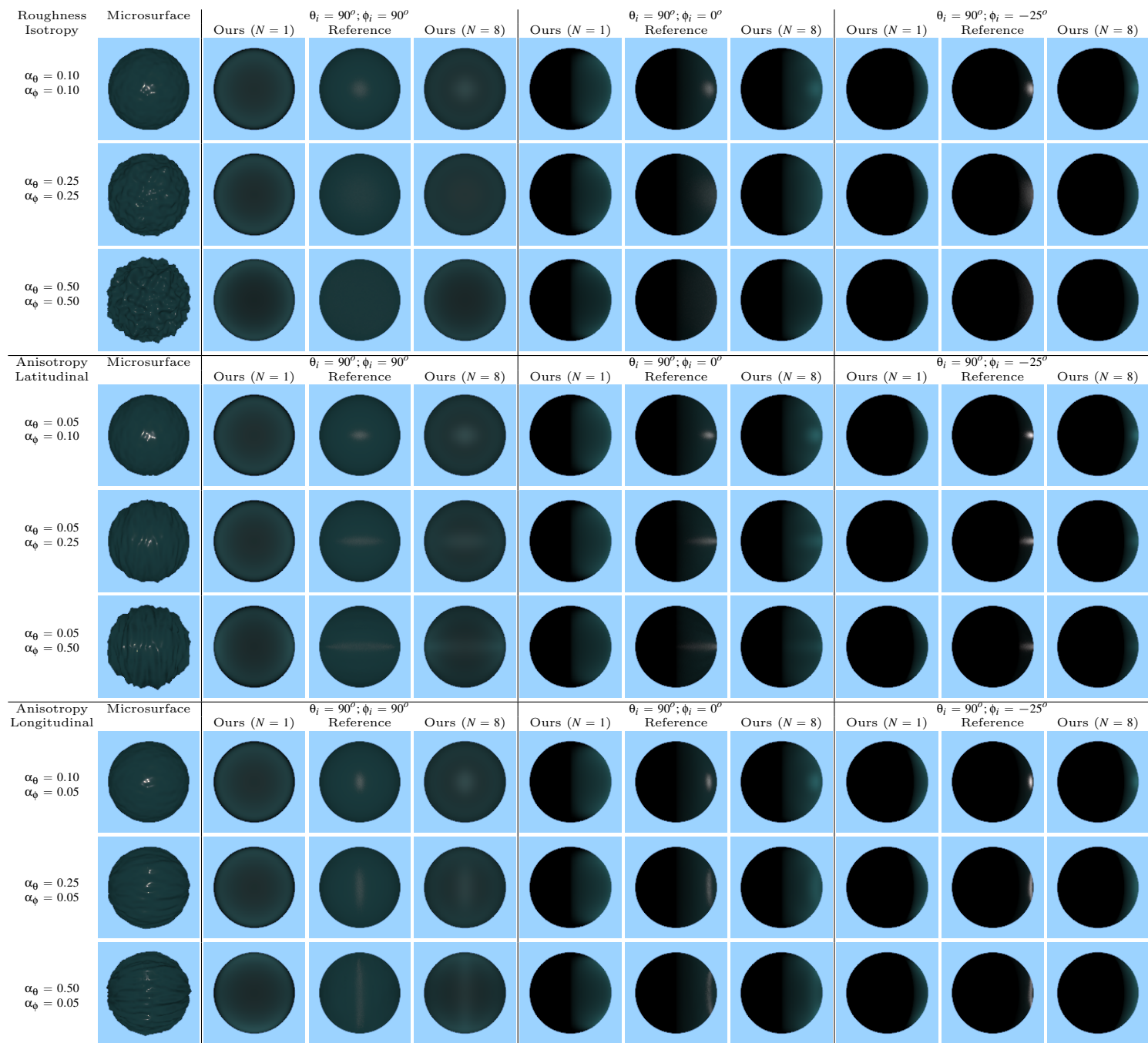
38 green-metallic-paint2



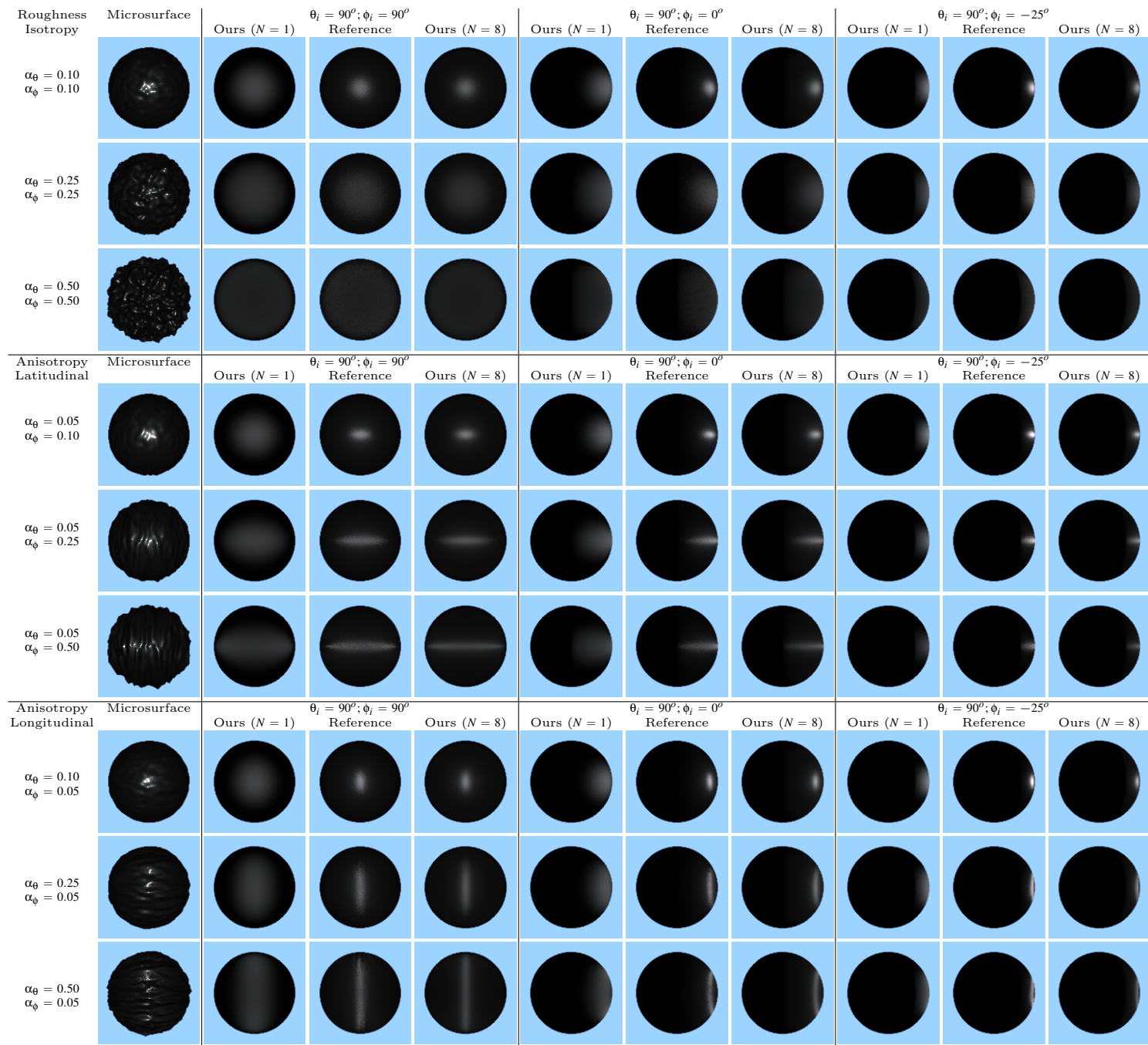
39 green-metallic-paint



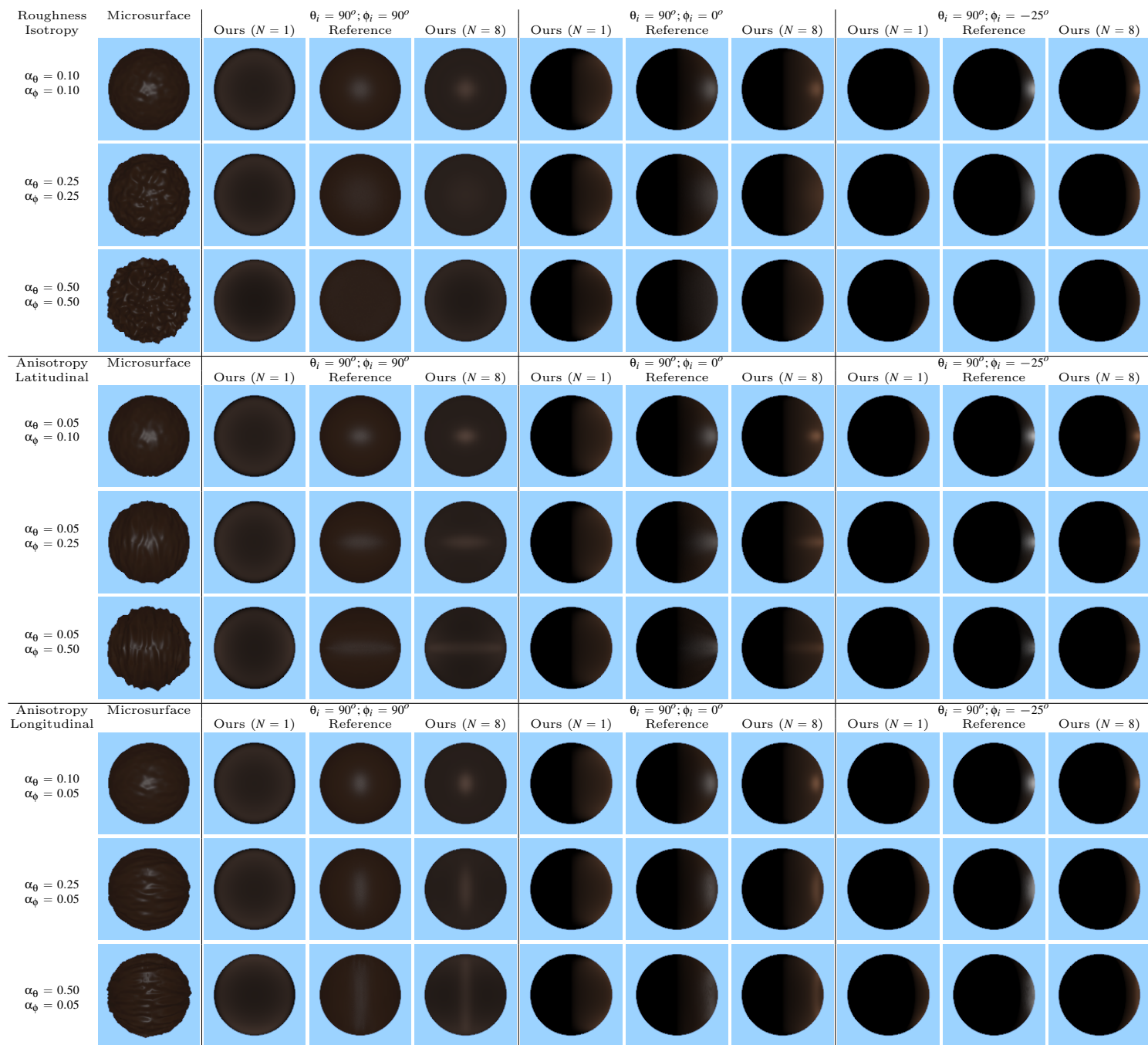
40 green-plastic



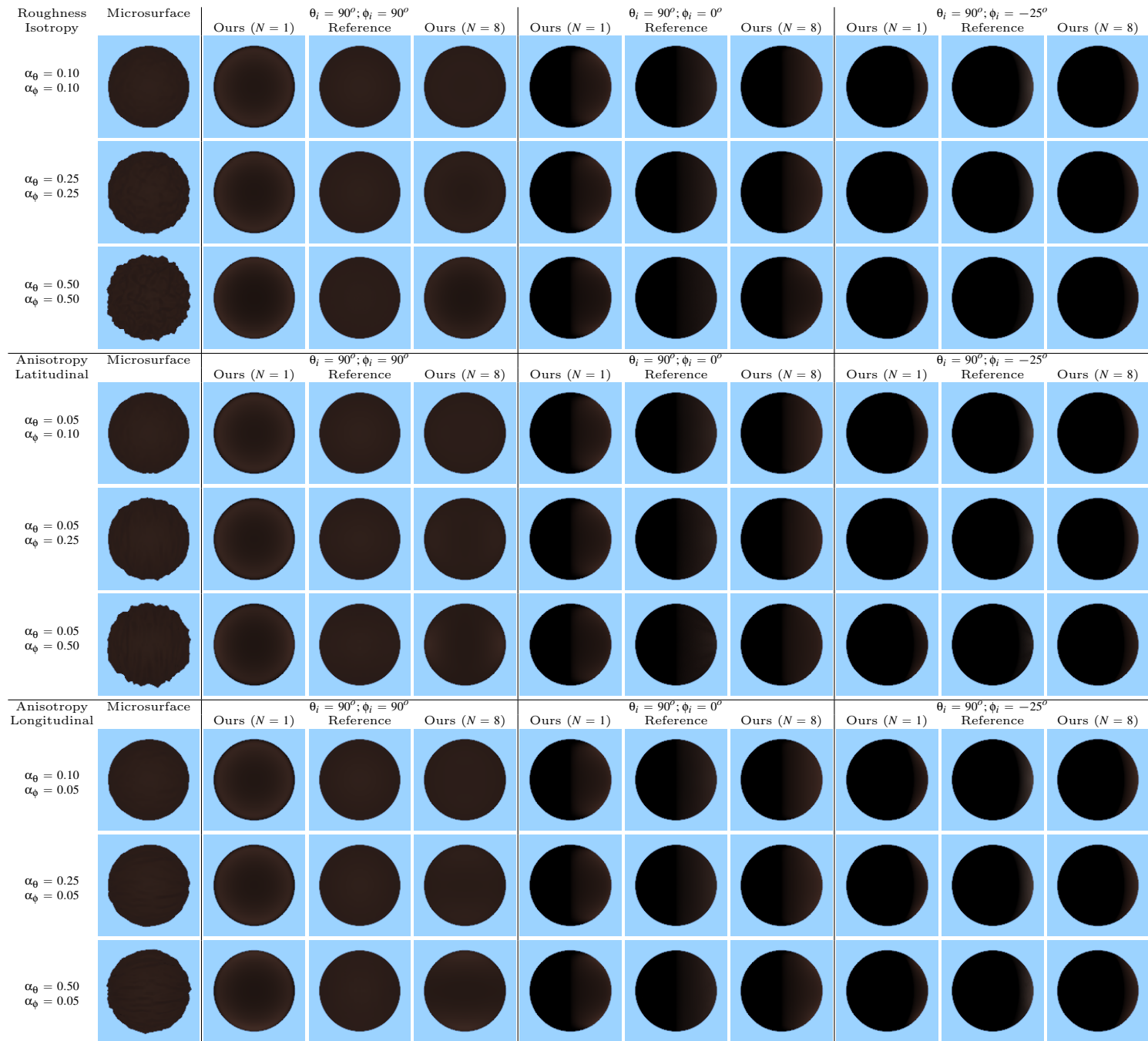
41 hematite



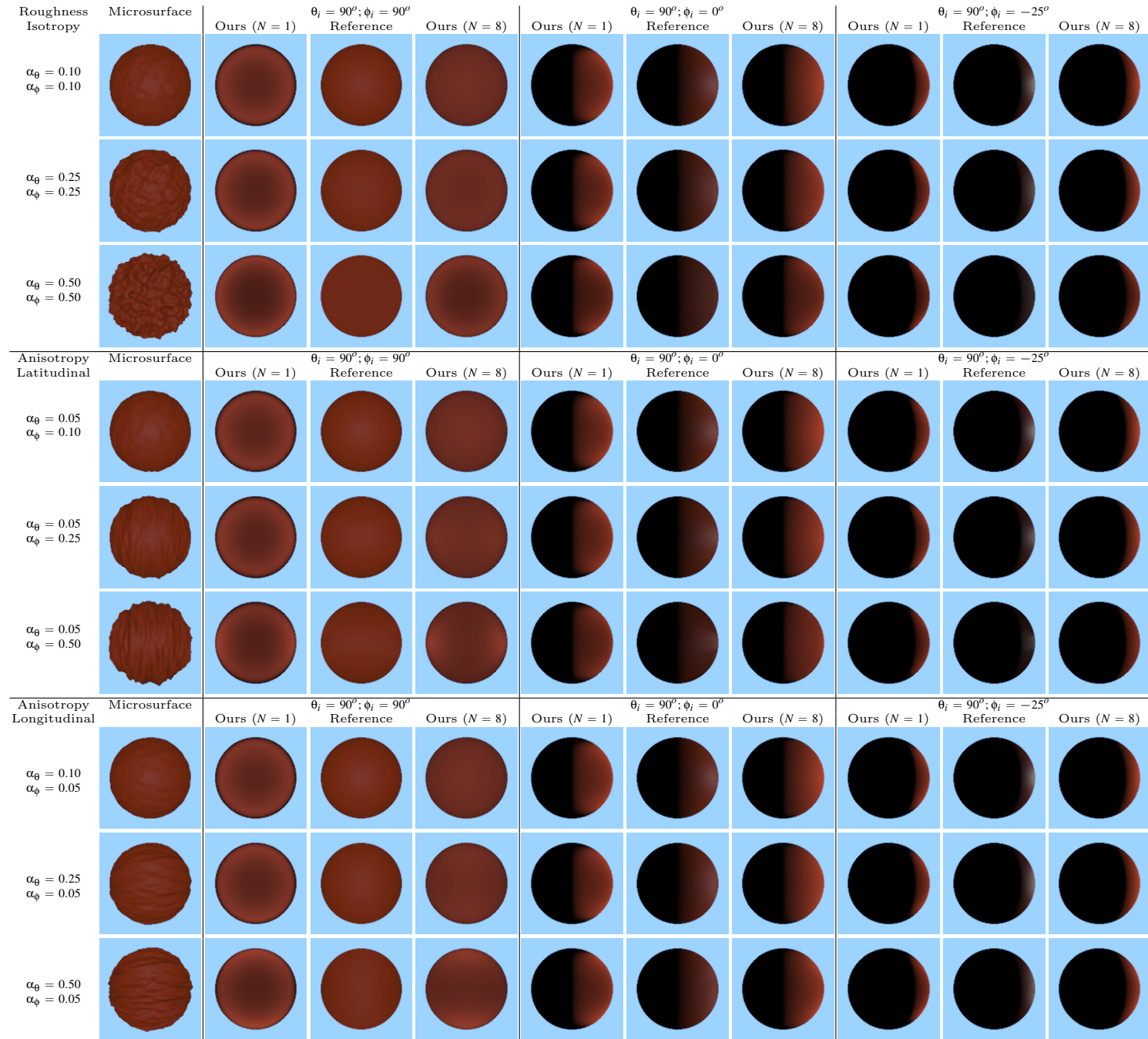
42 ipswich-pine-221



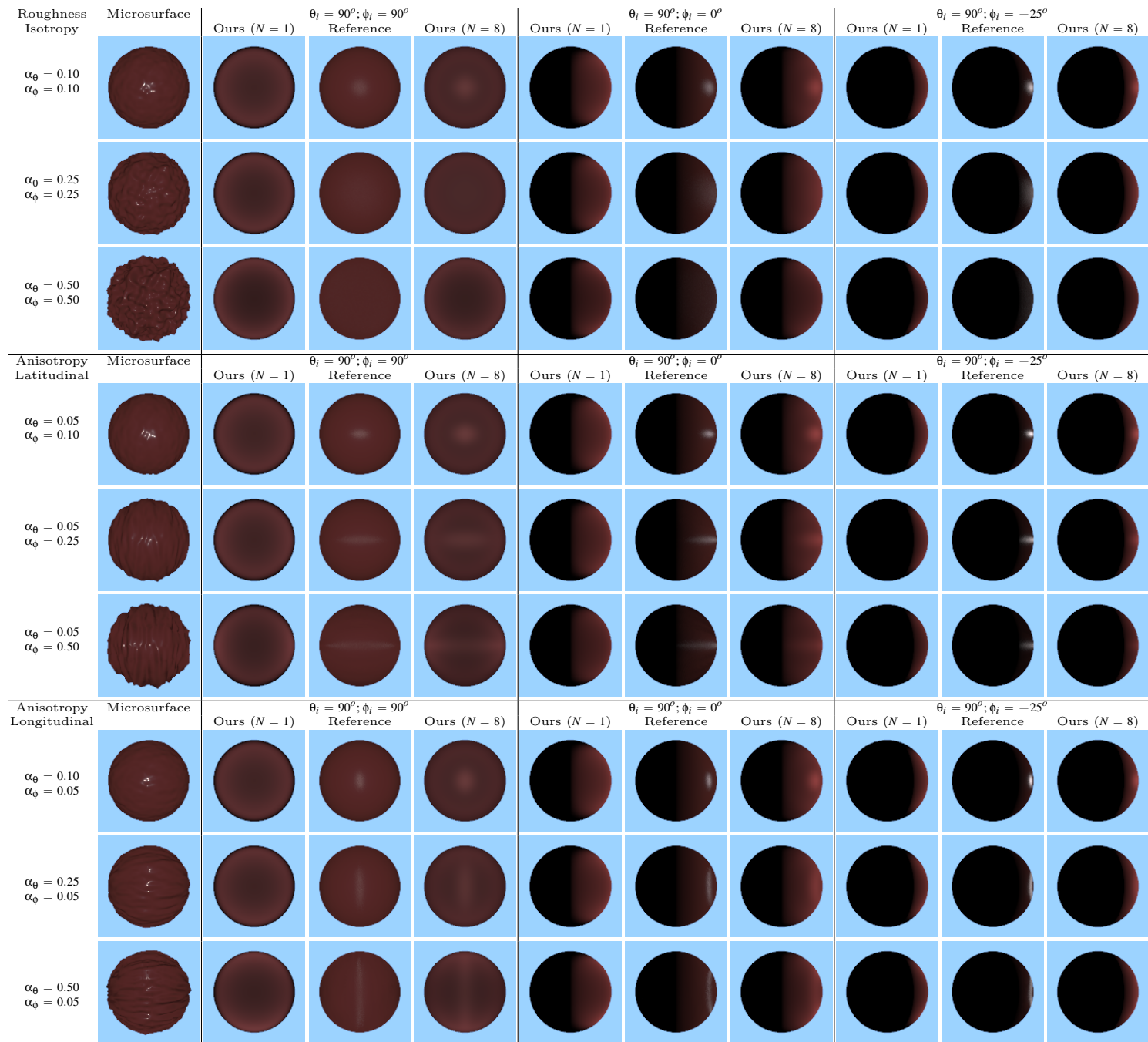
43 light-brown-fabric

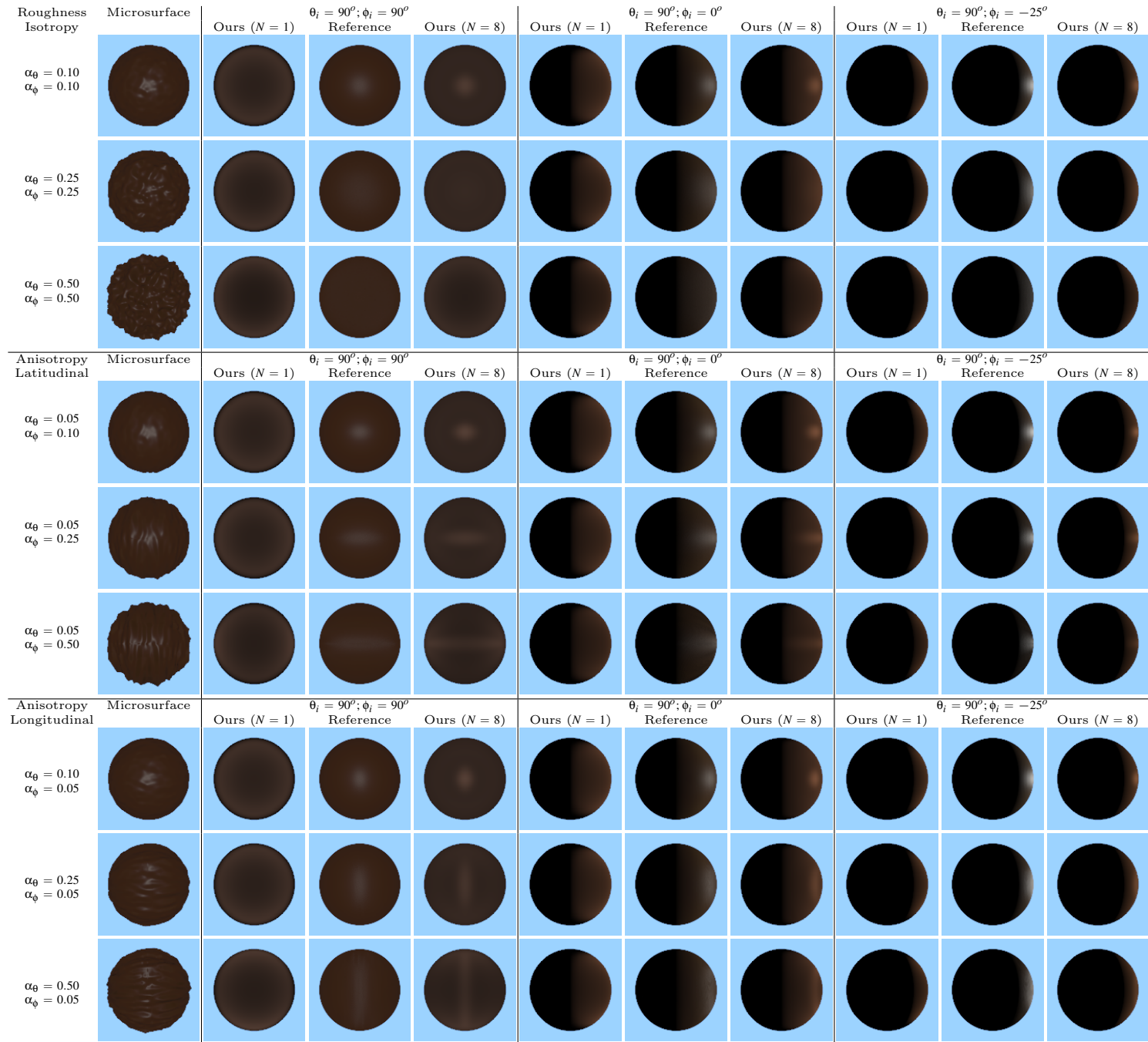


44 light-red-paint

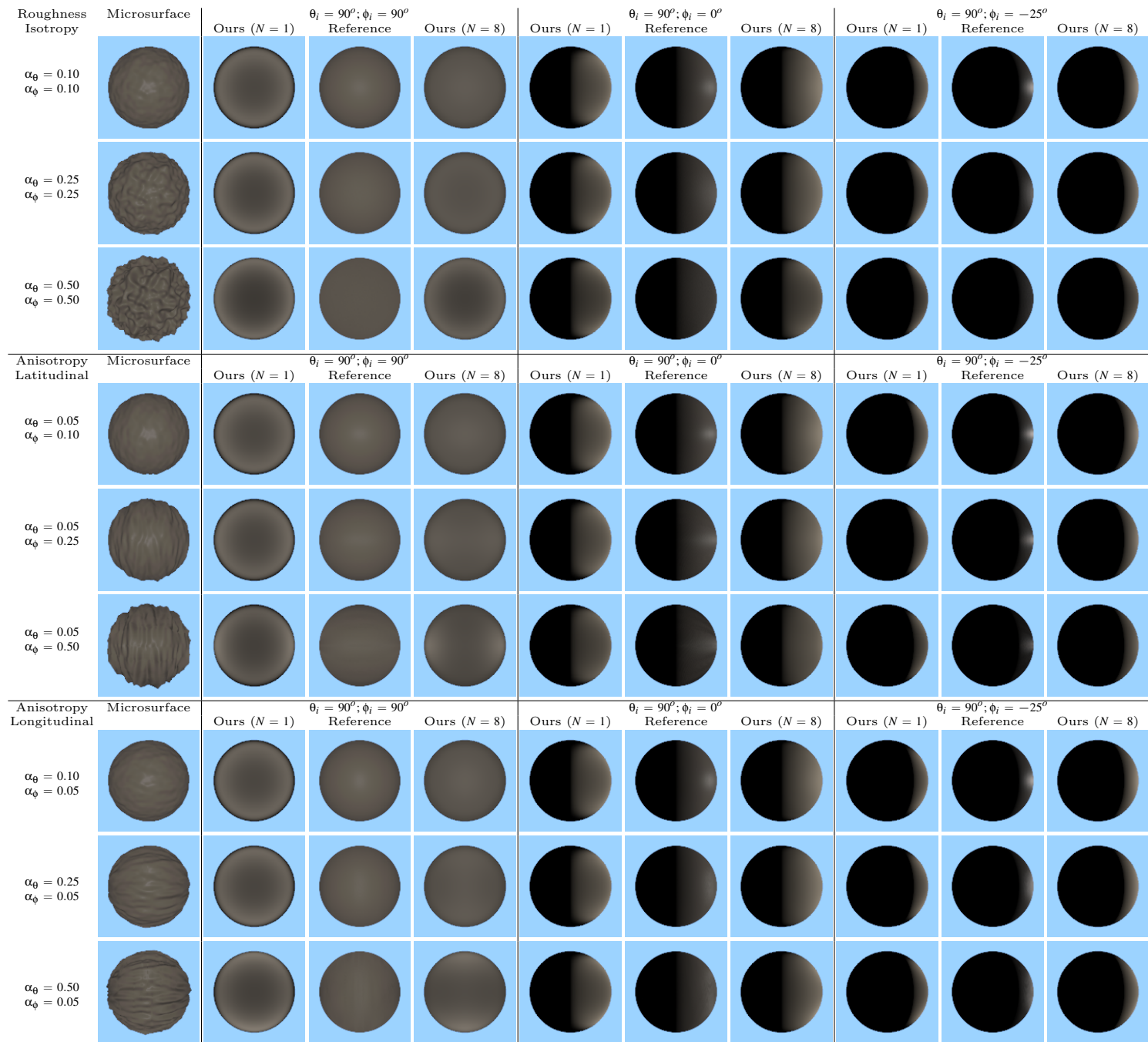


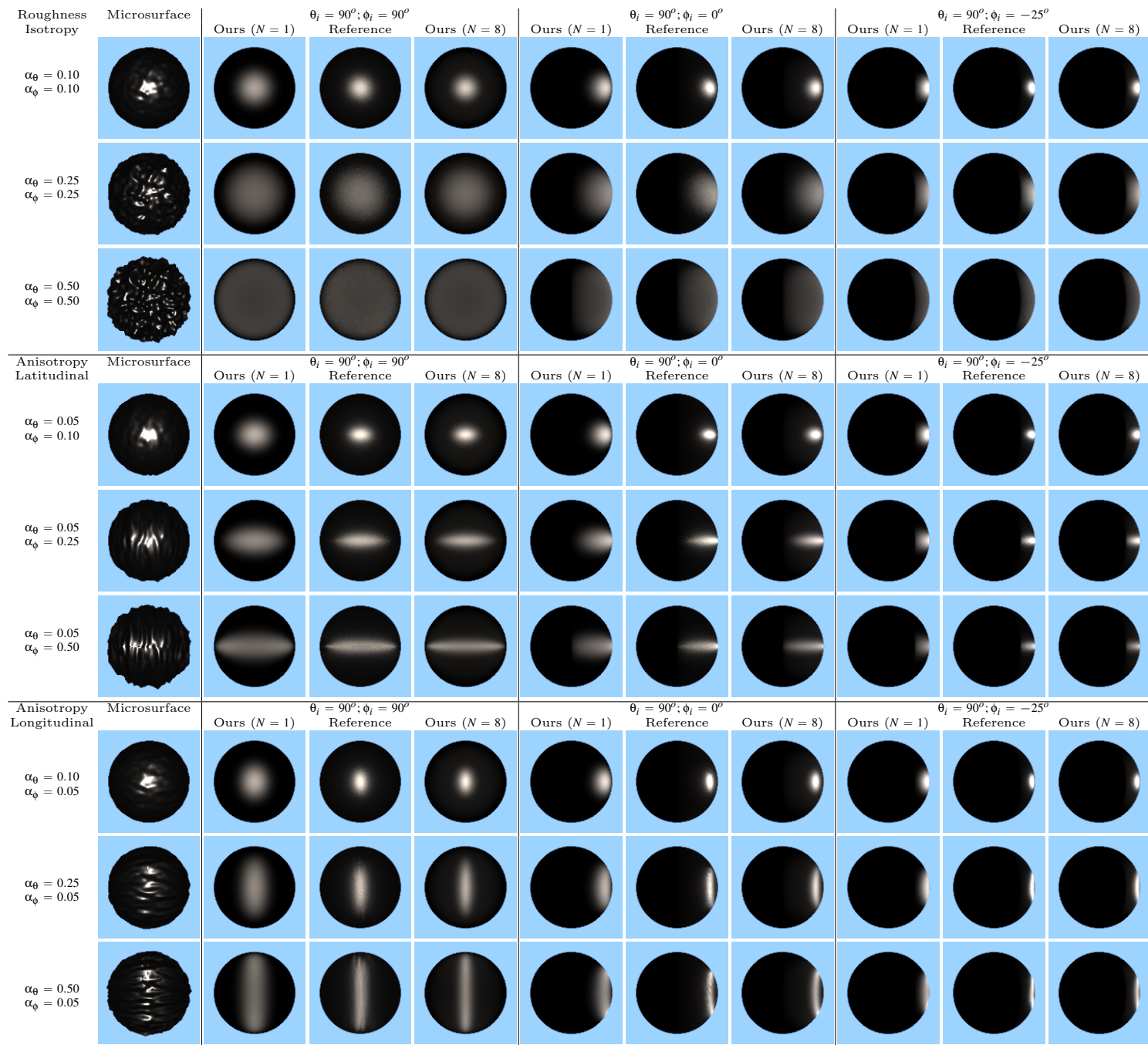
45 maroon-plastic



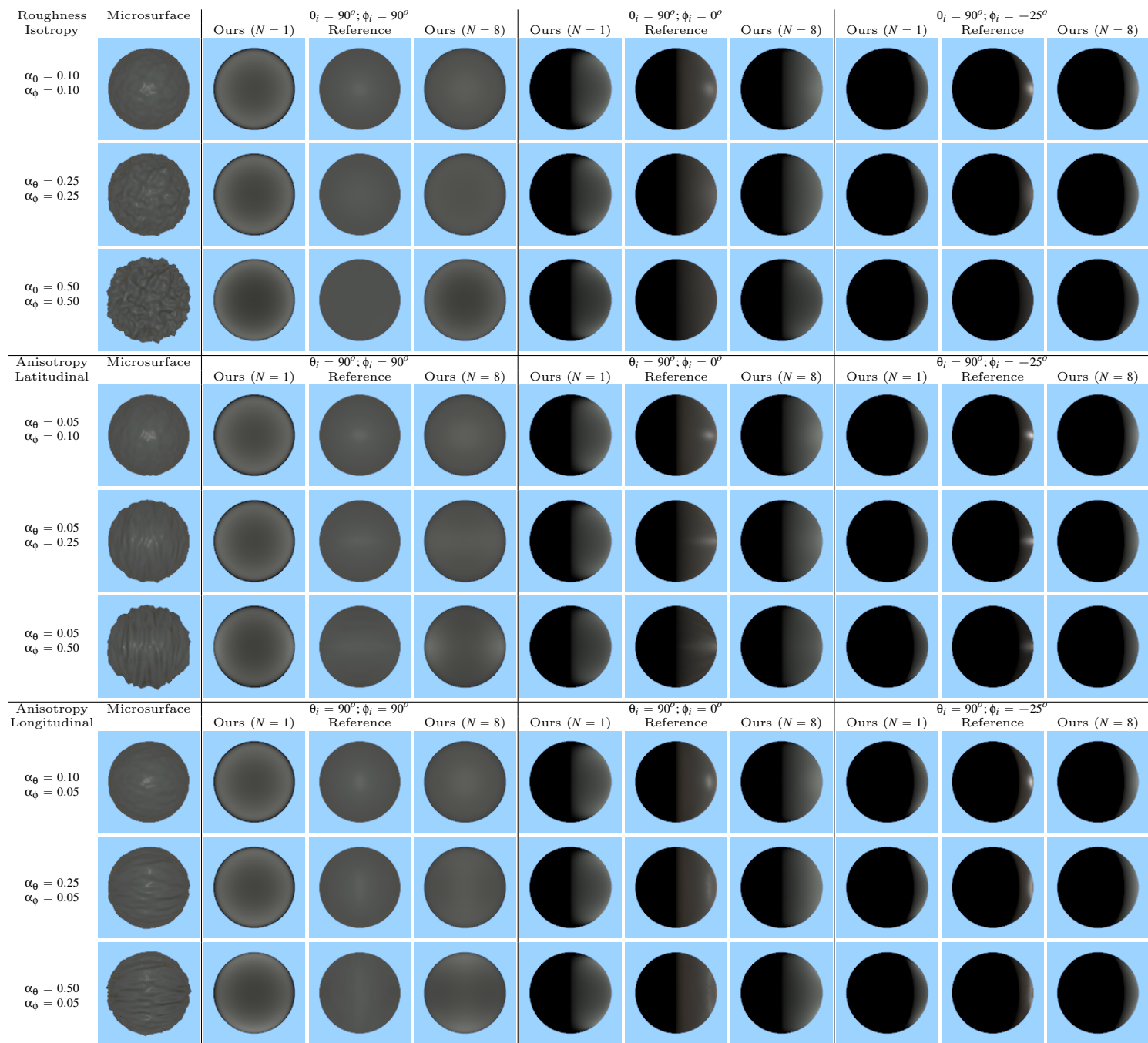


47 neoprene-rubber

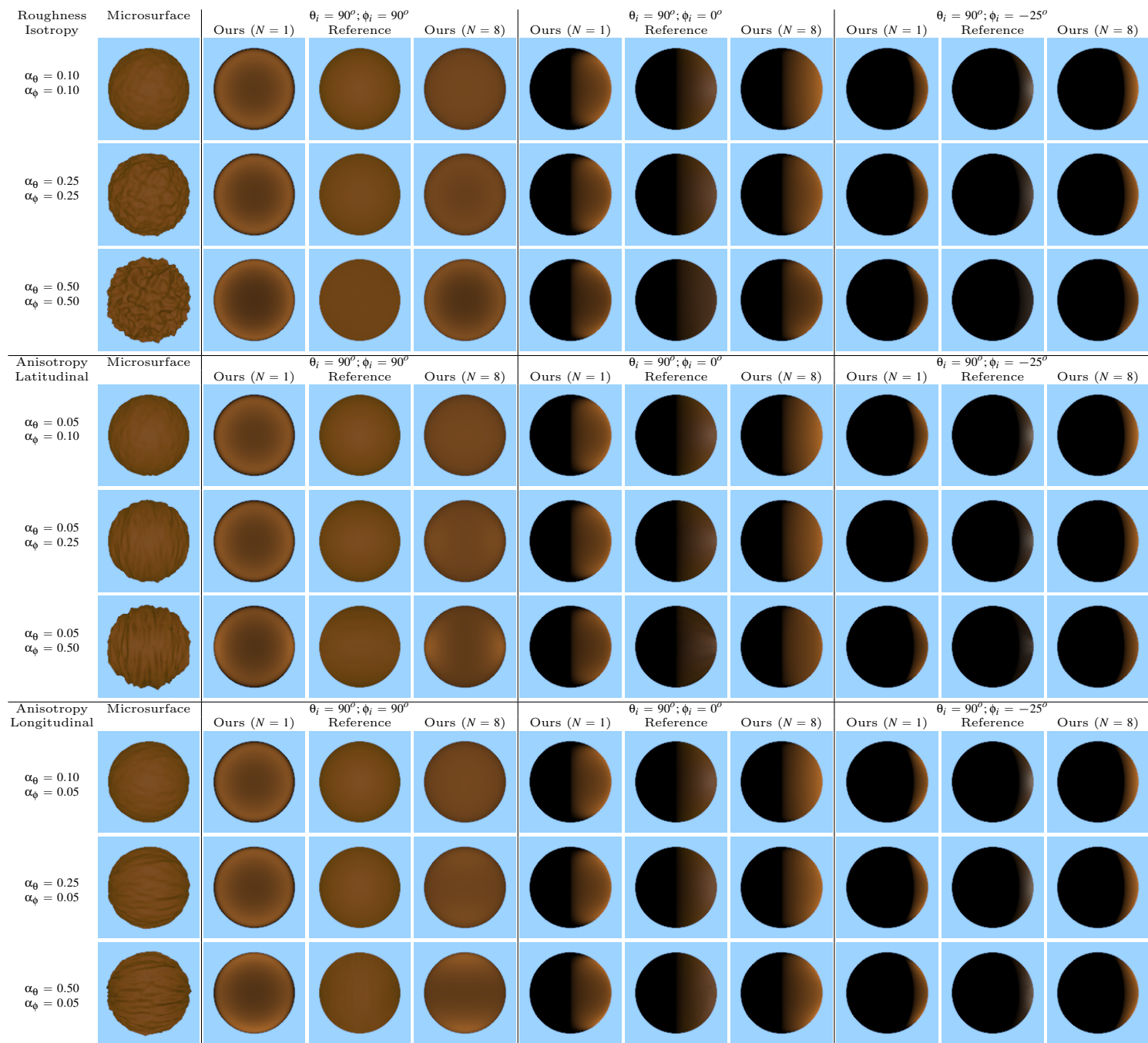




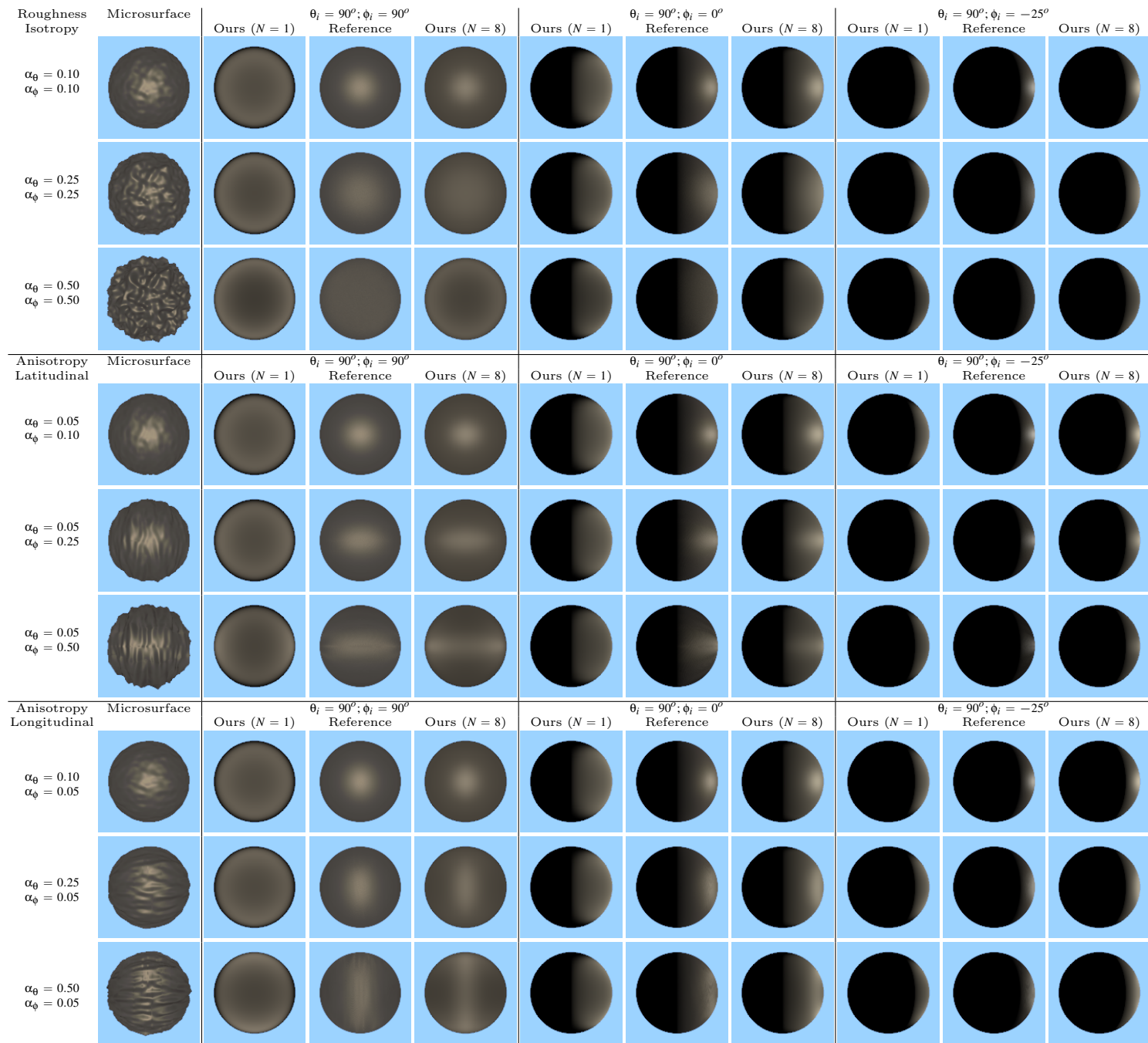
49 nylon



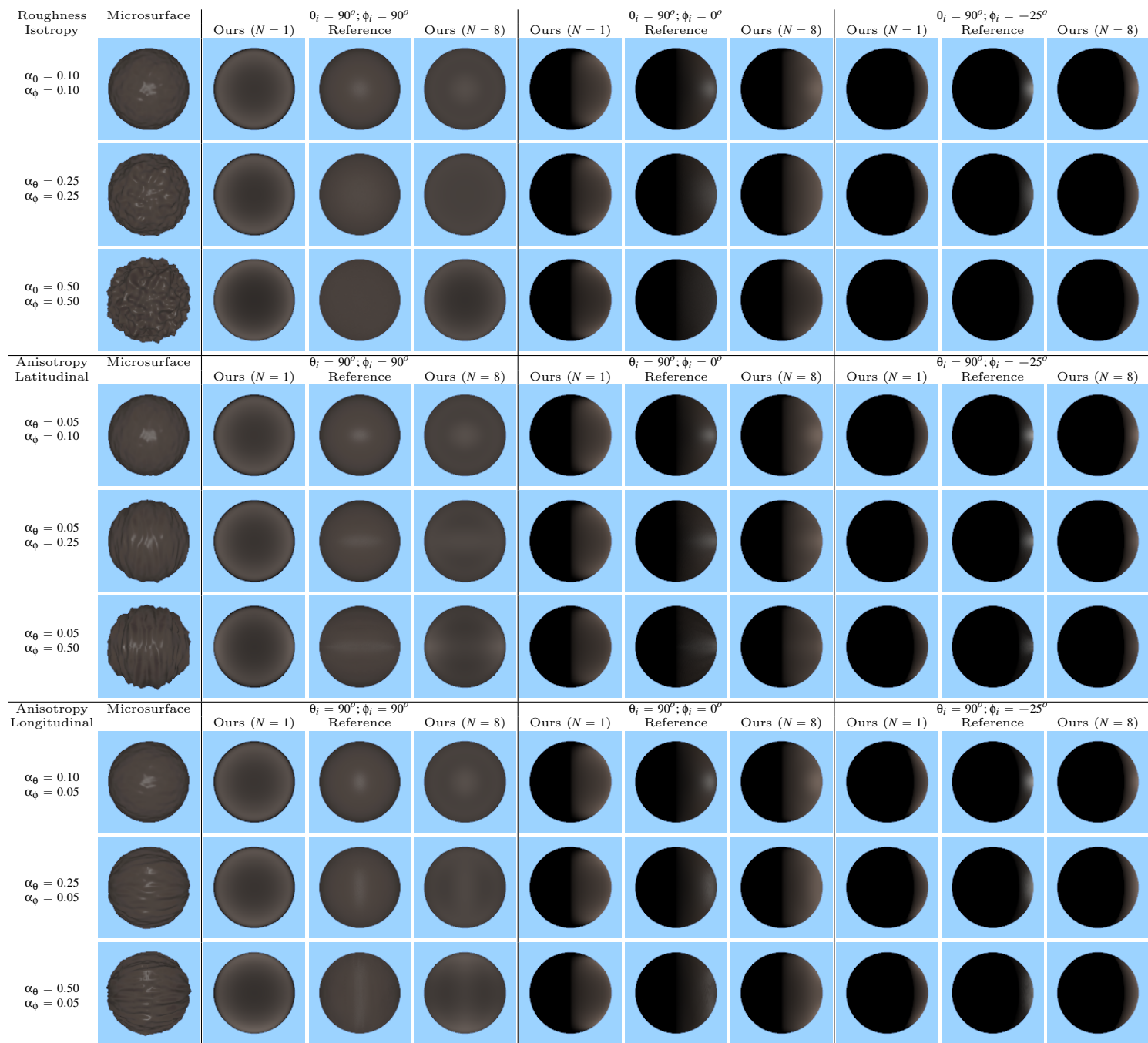
50 orange-paint



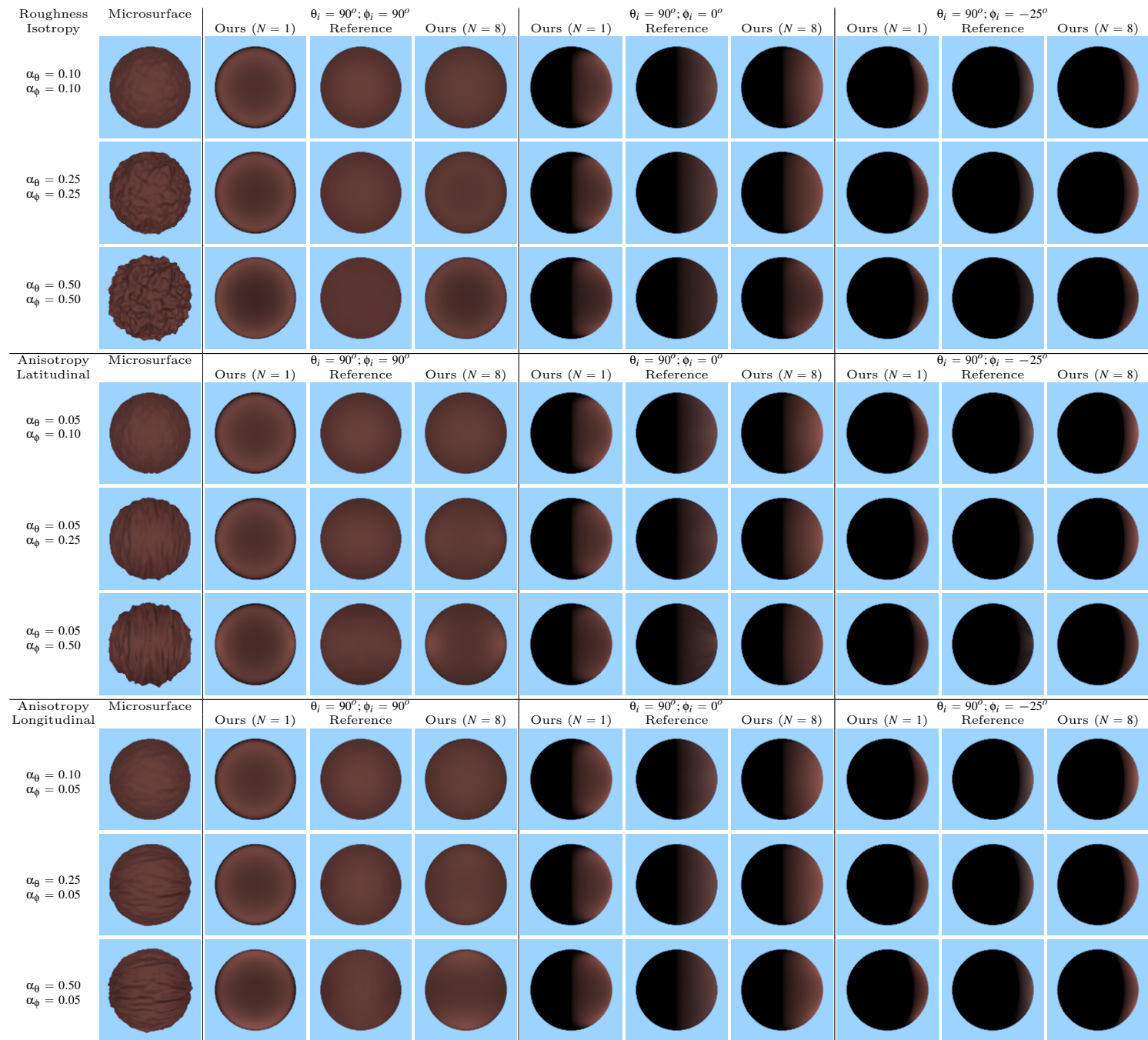
51 pearl-paint



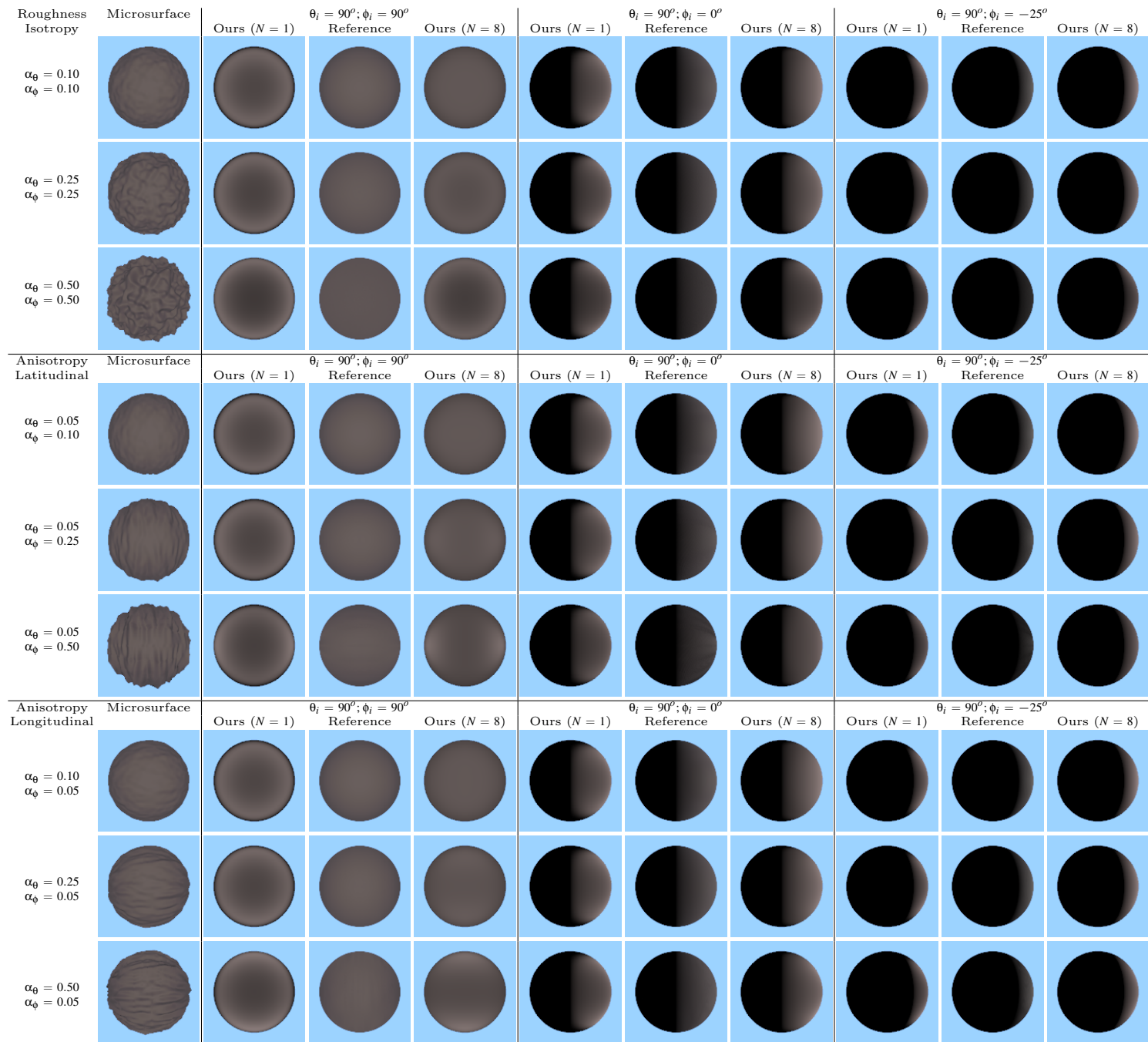
52 pickled-oak-260

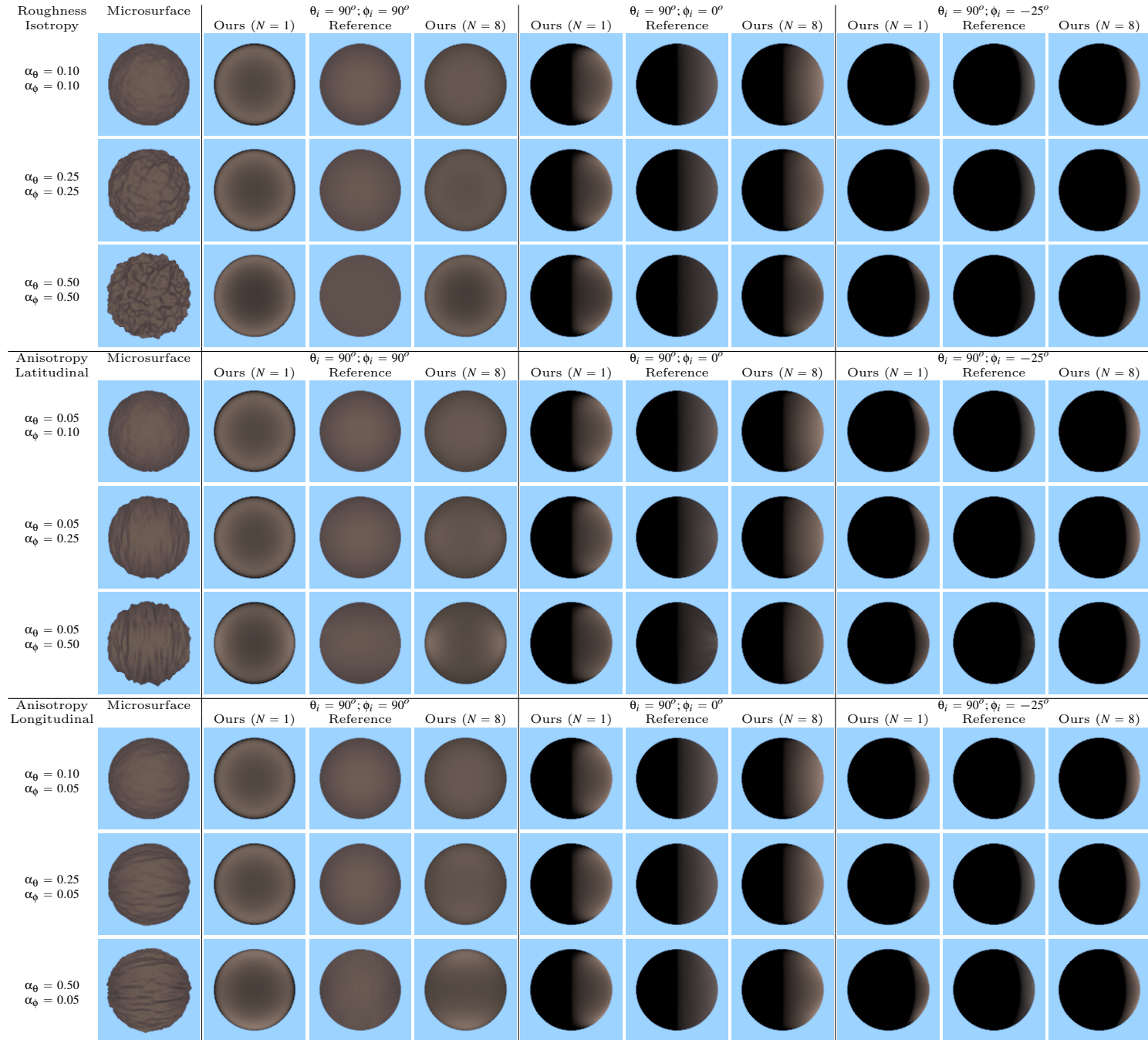


53 pink-fabric2

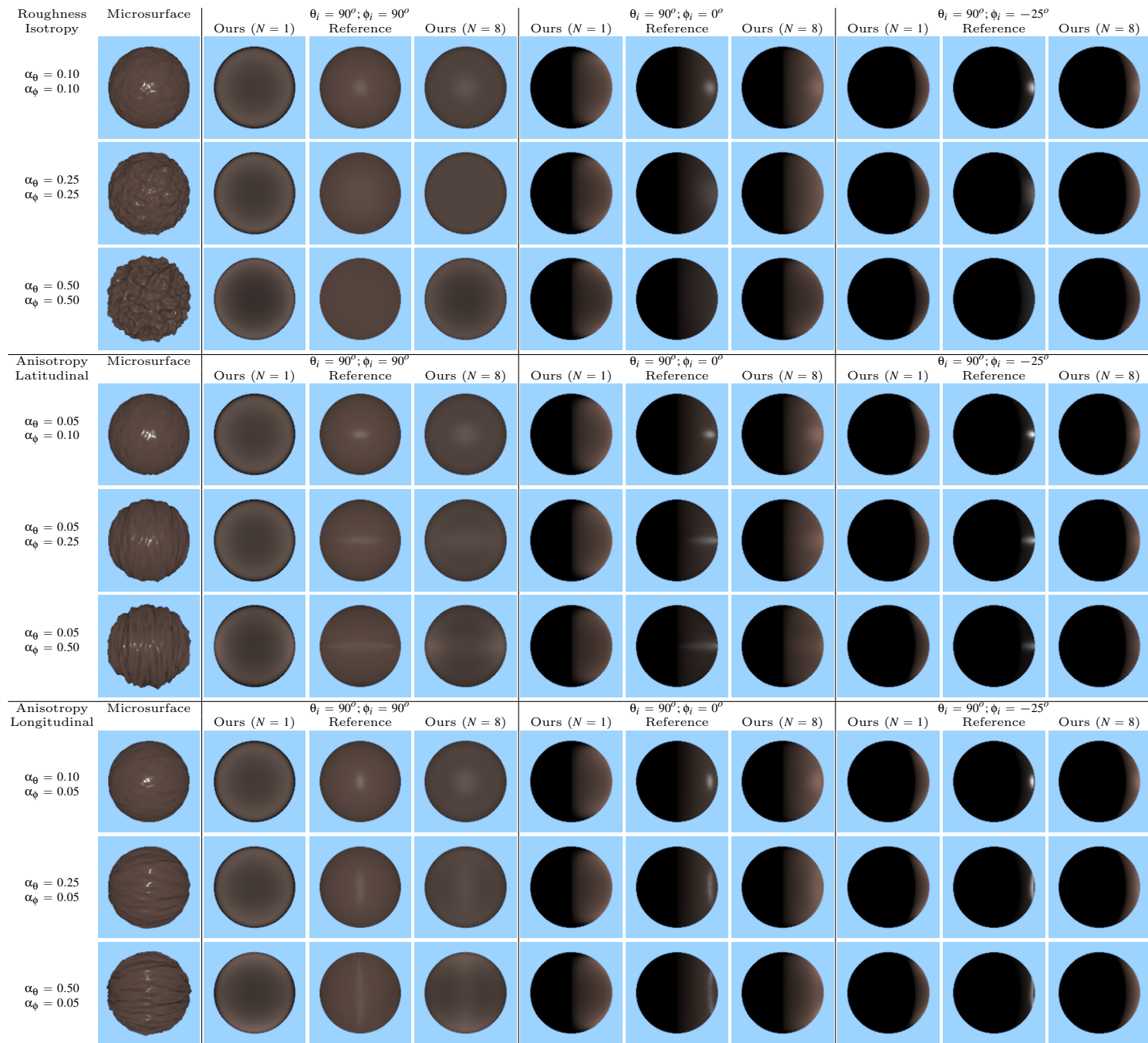


54 pink-fabric

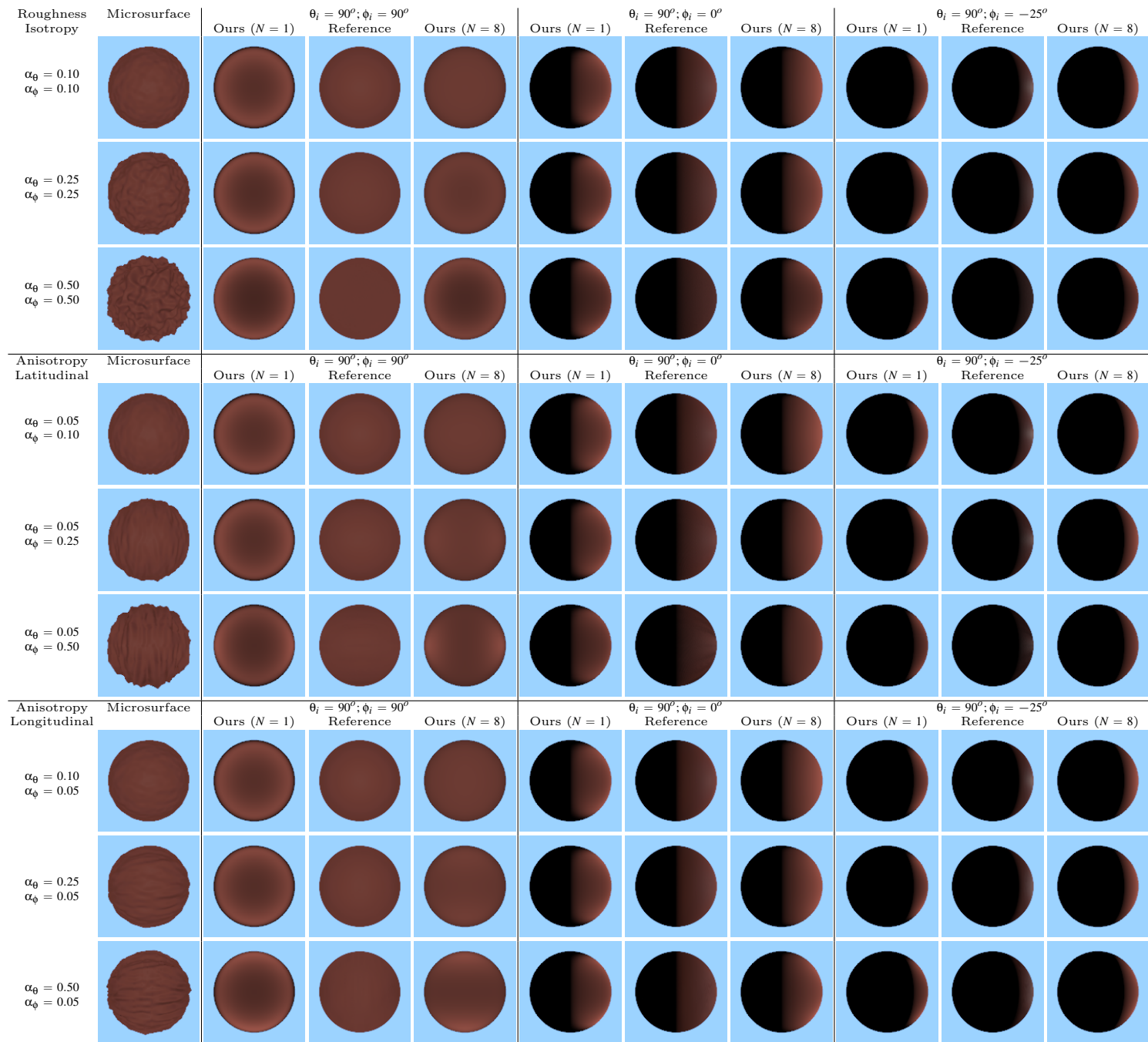




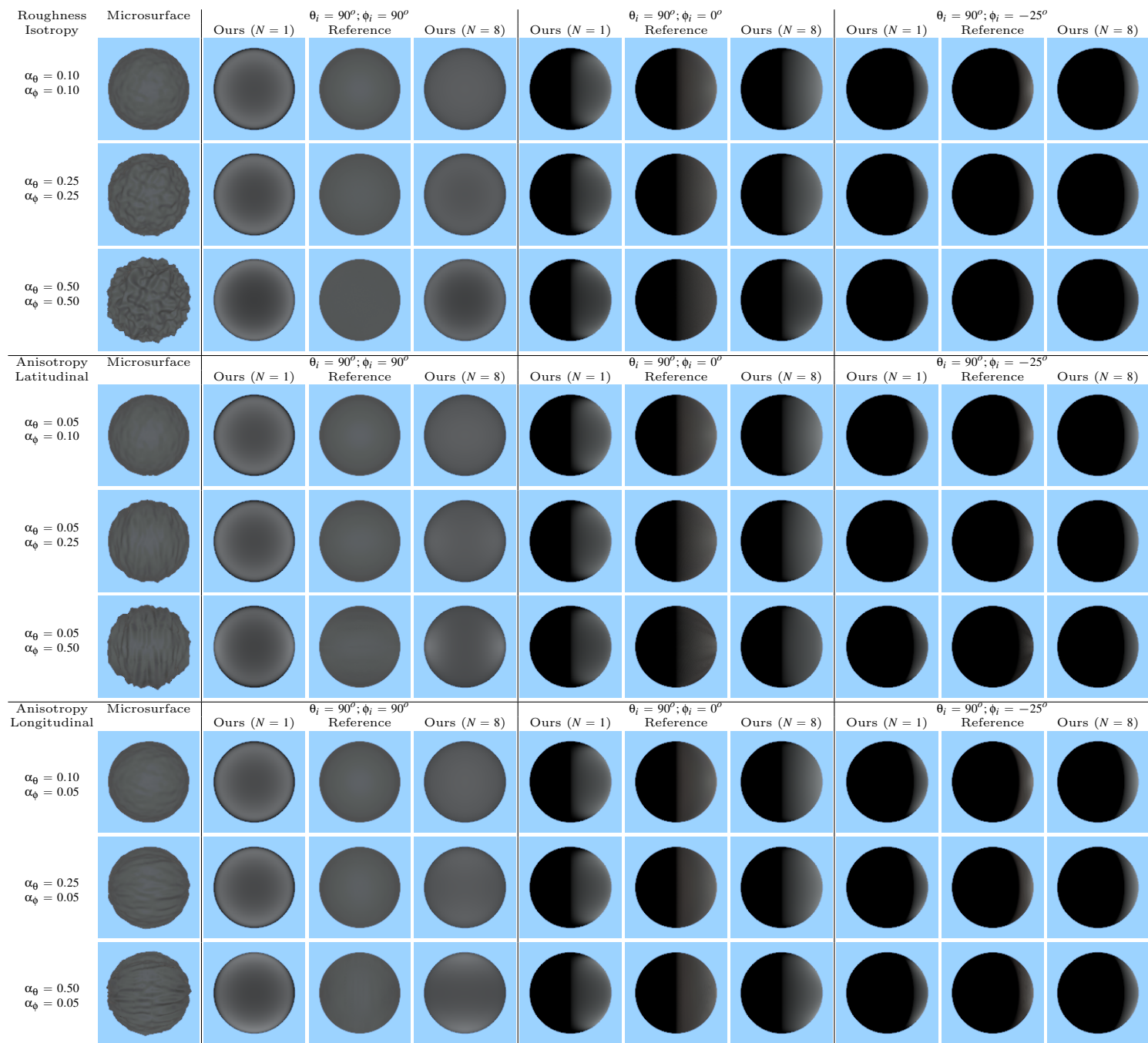
56 pink-jasper



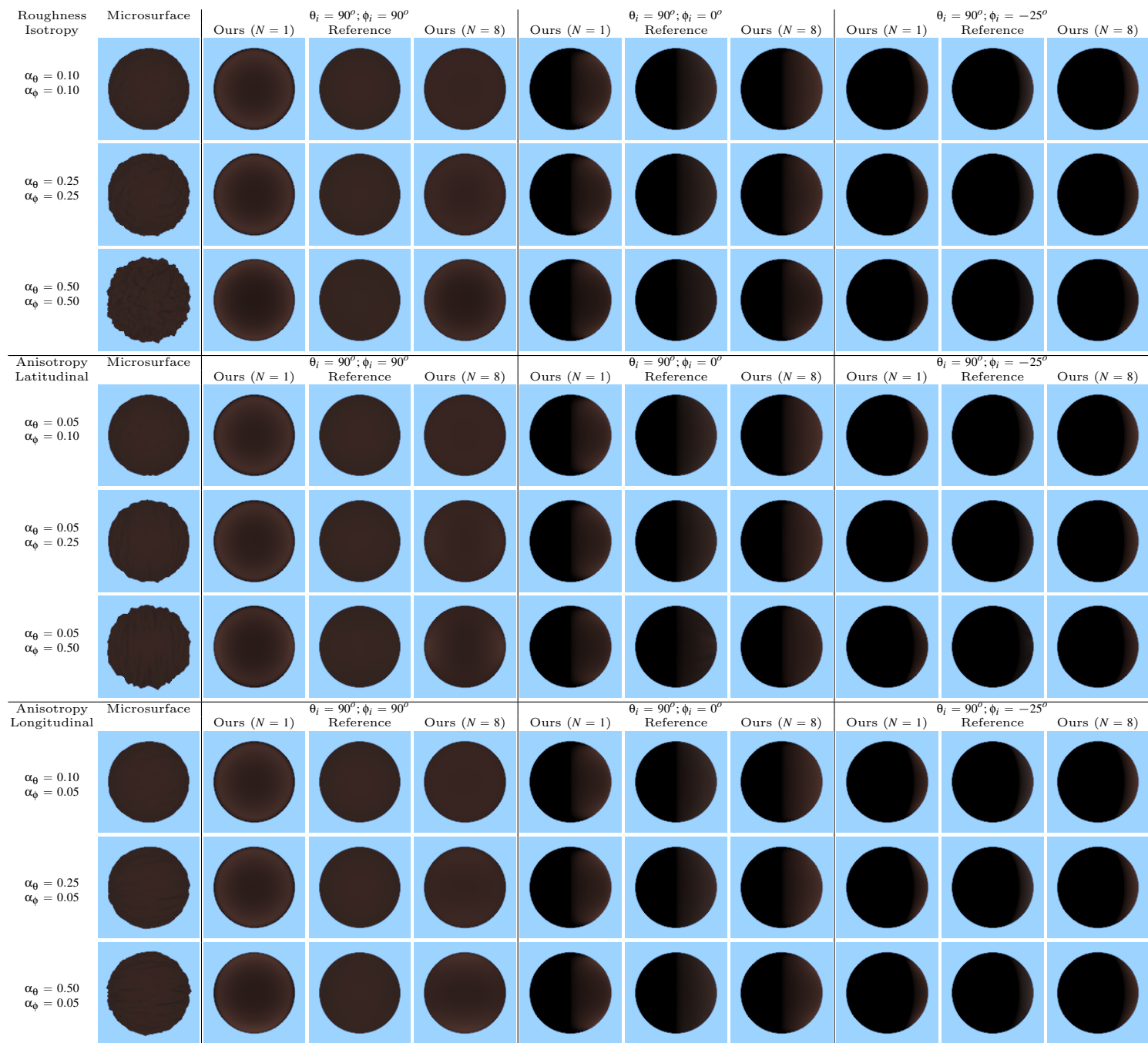
57 pink-plastic



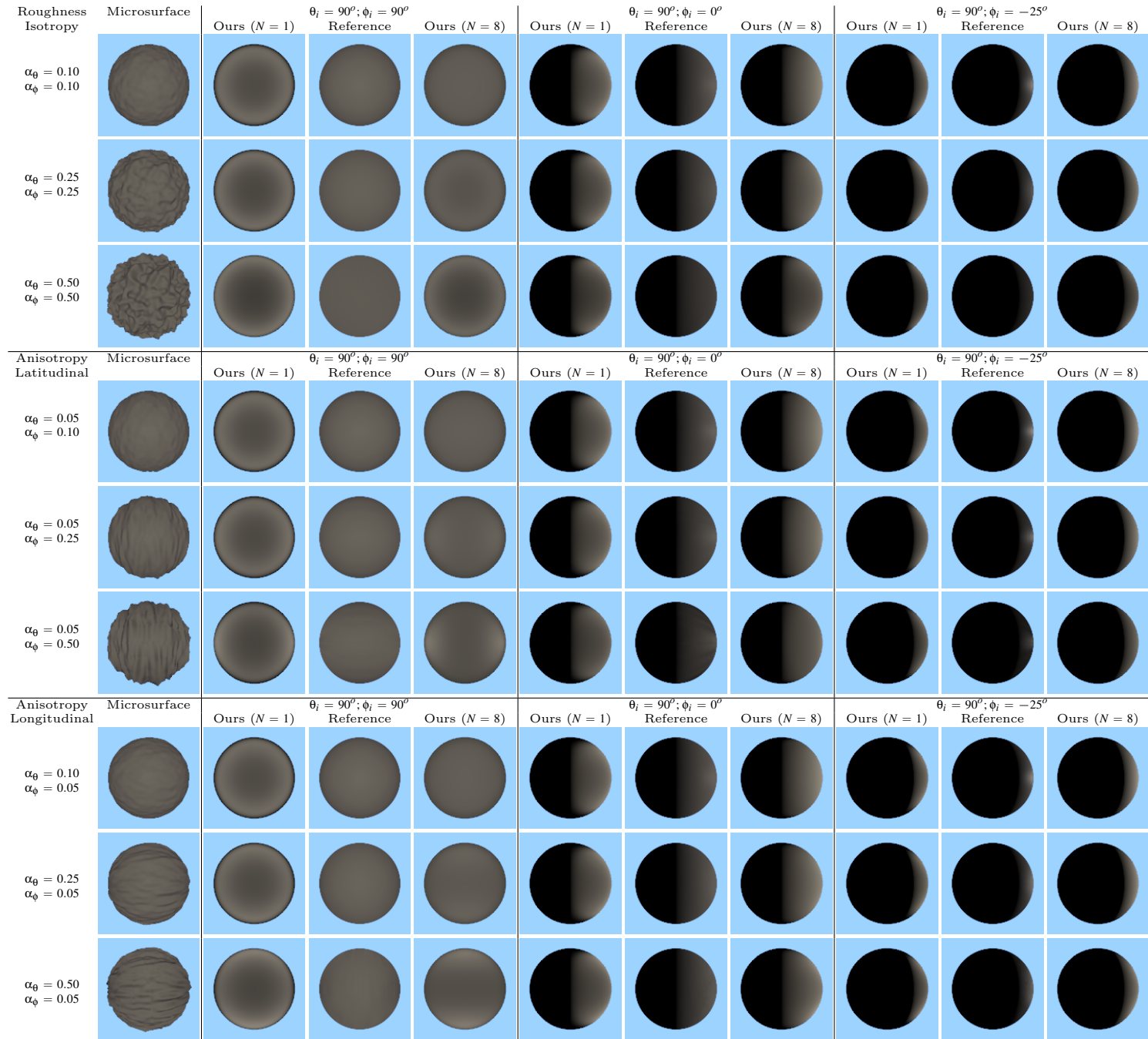
58 polyethylene



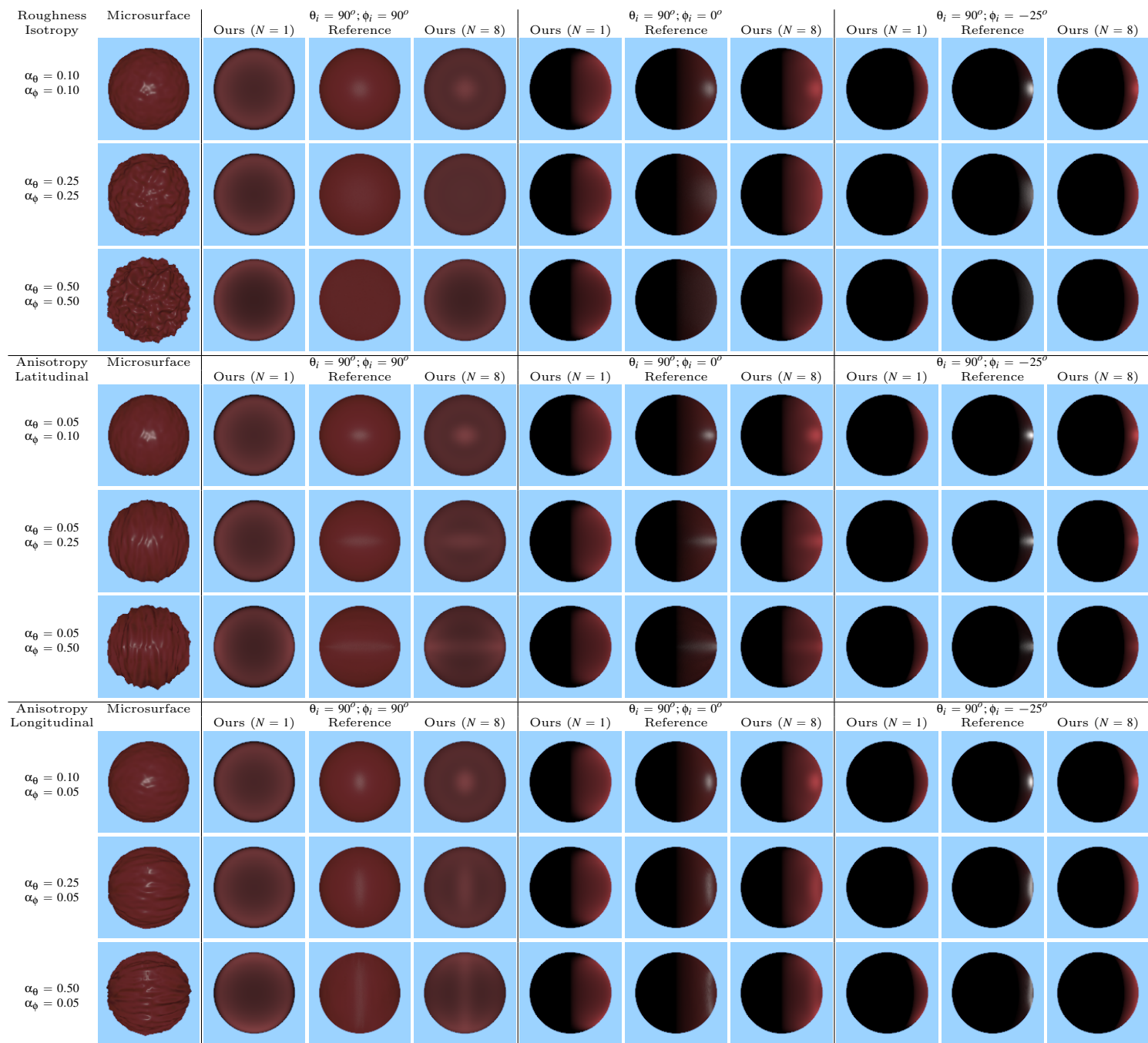
59 polyurethane-foam

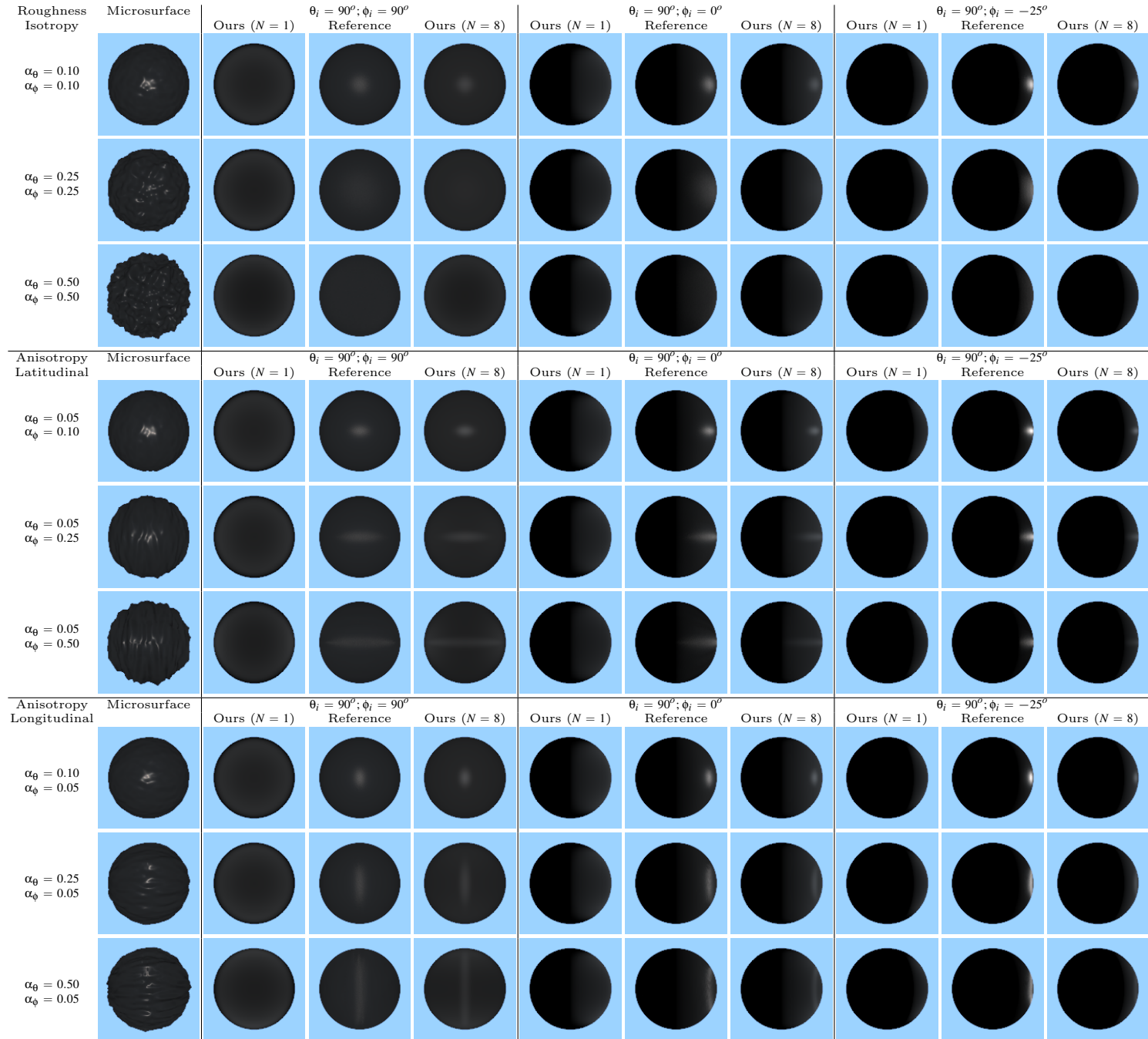


60 pure-rubber

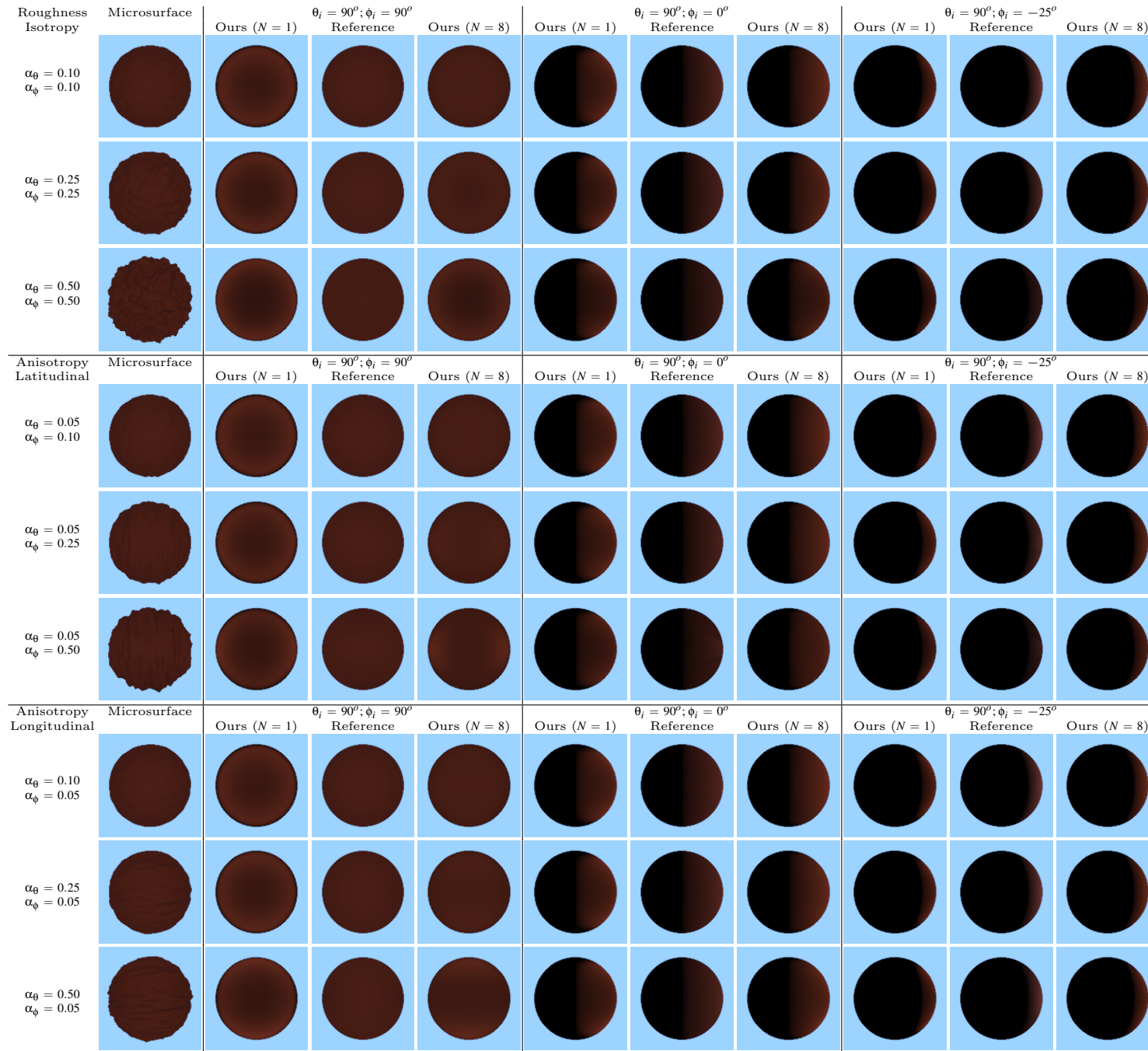


61 purple-paint

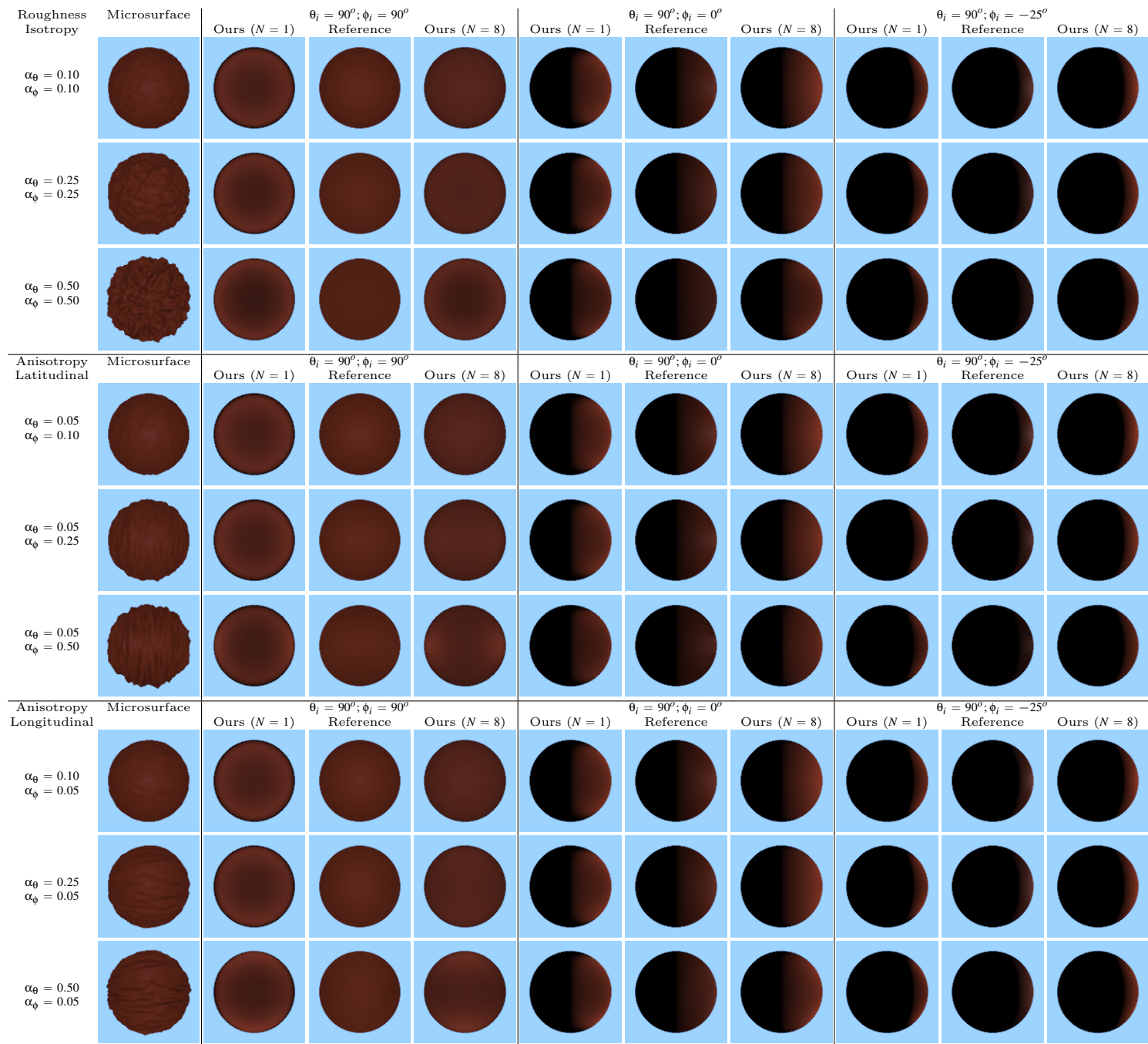




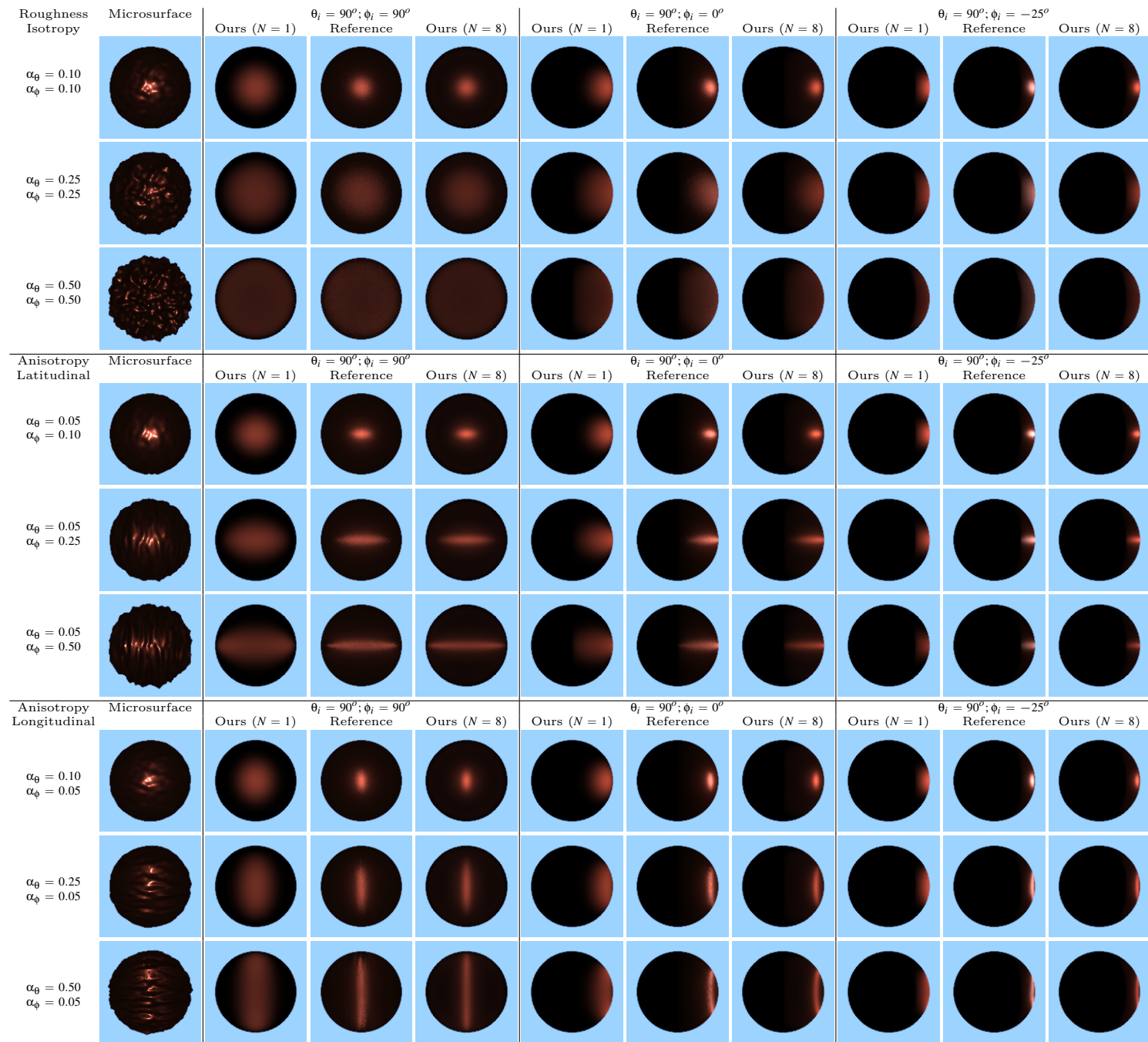
63 red-fabric2



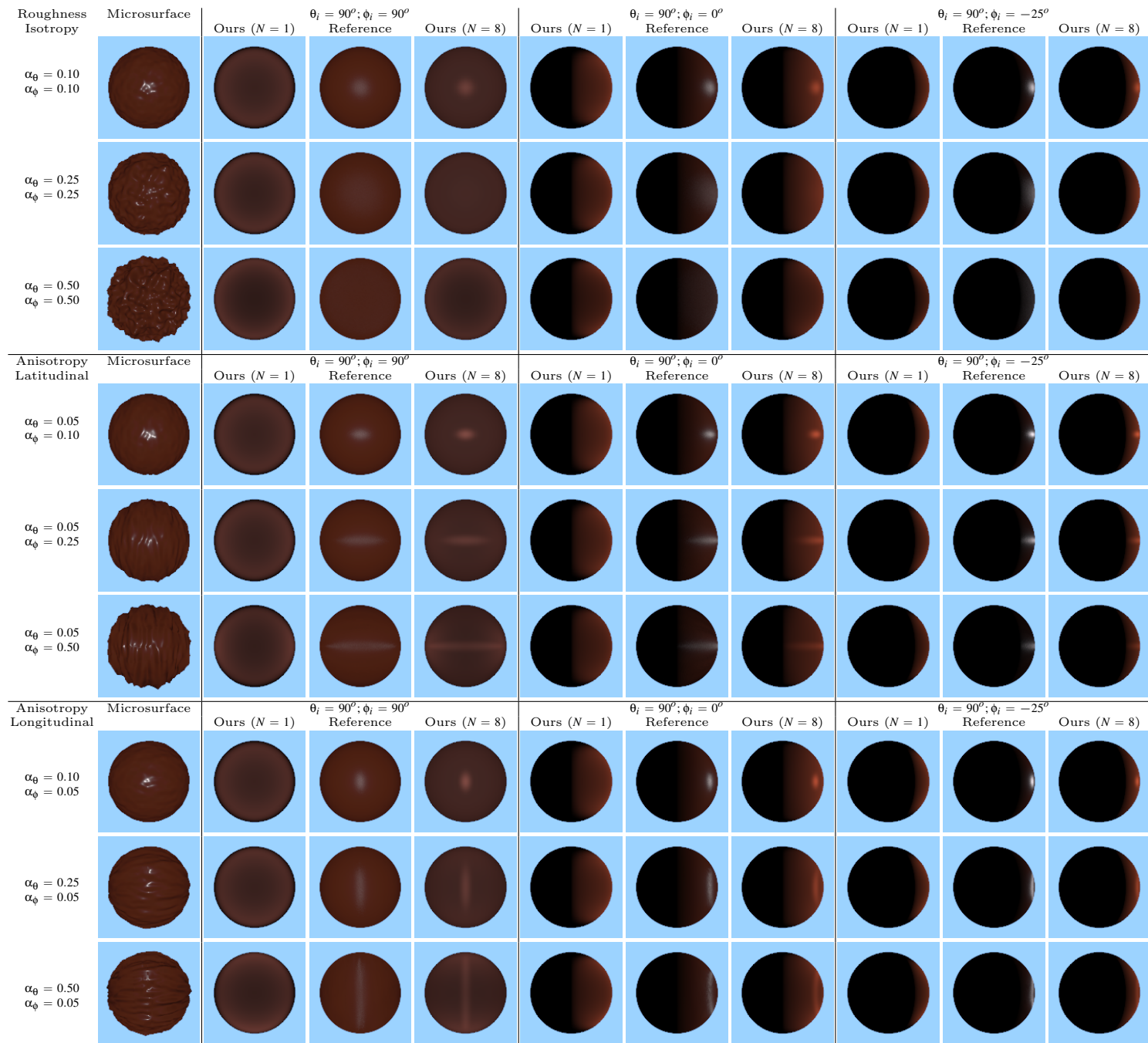
64 red-fabric



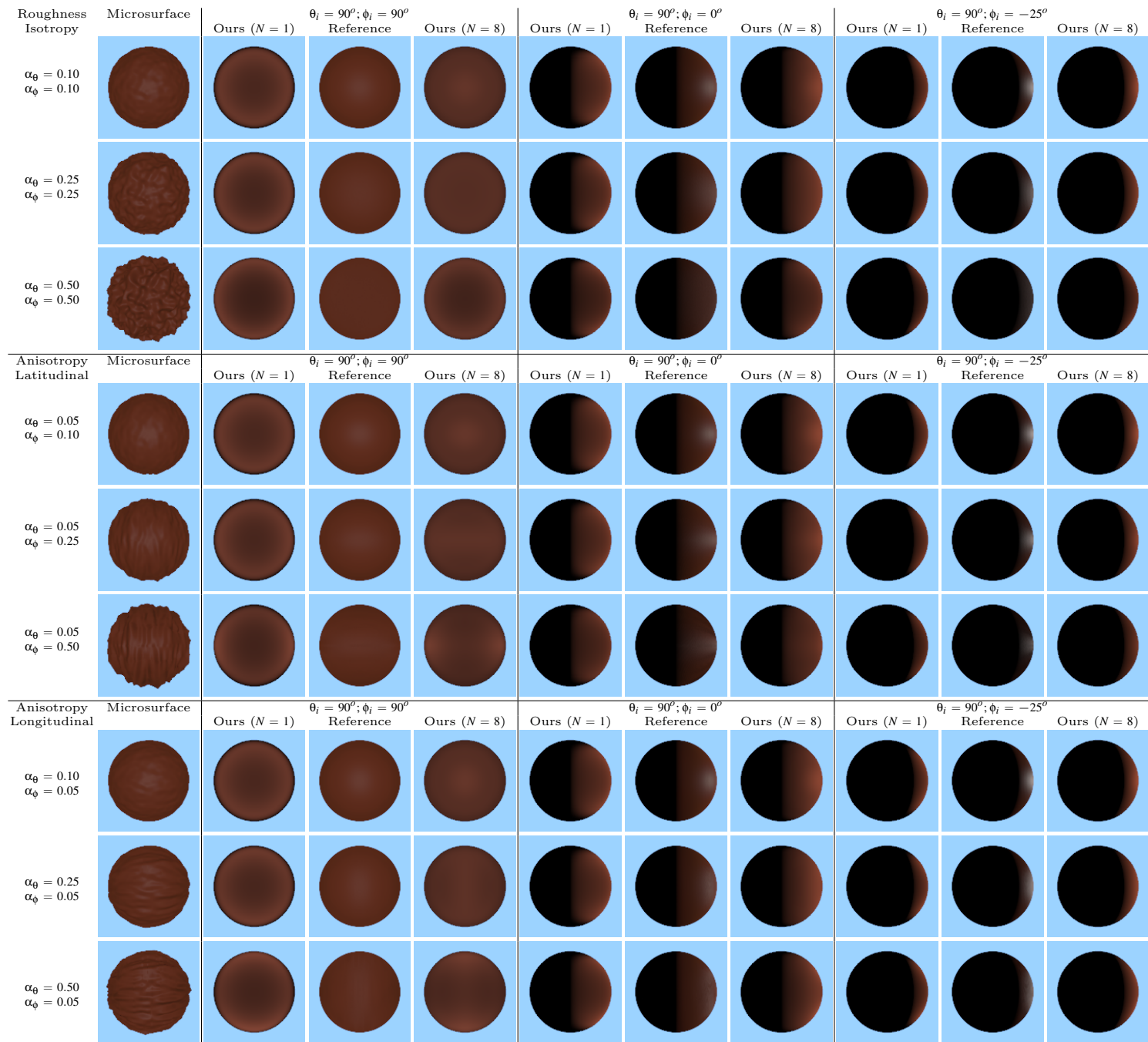
65 red-metallic-paint



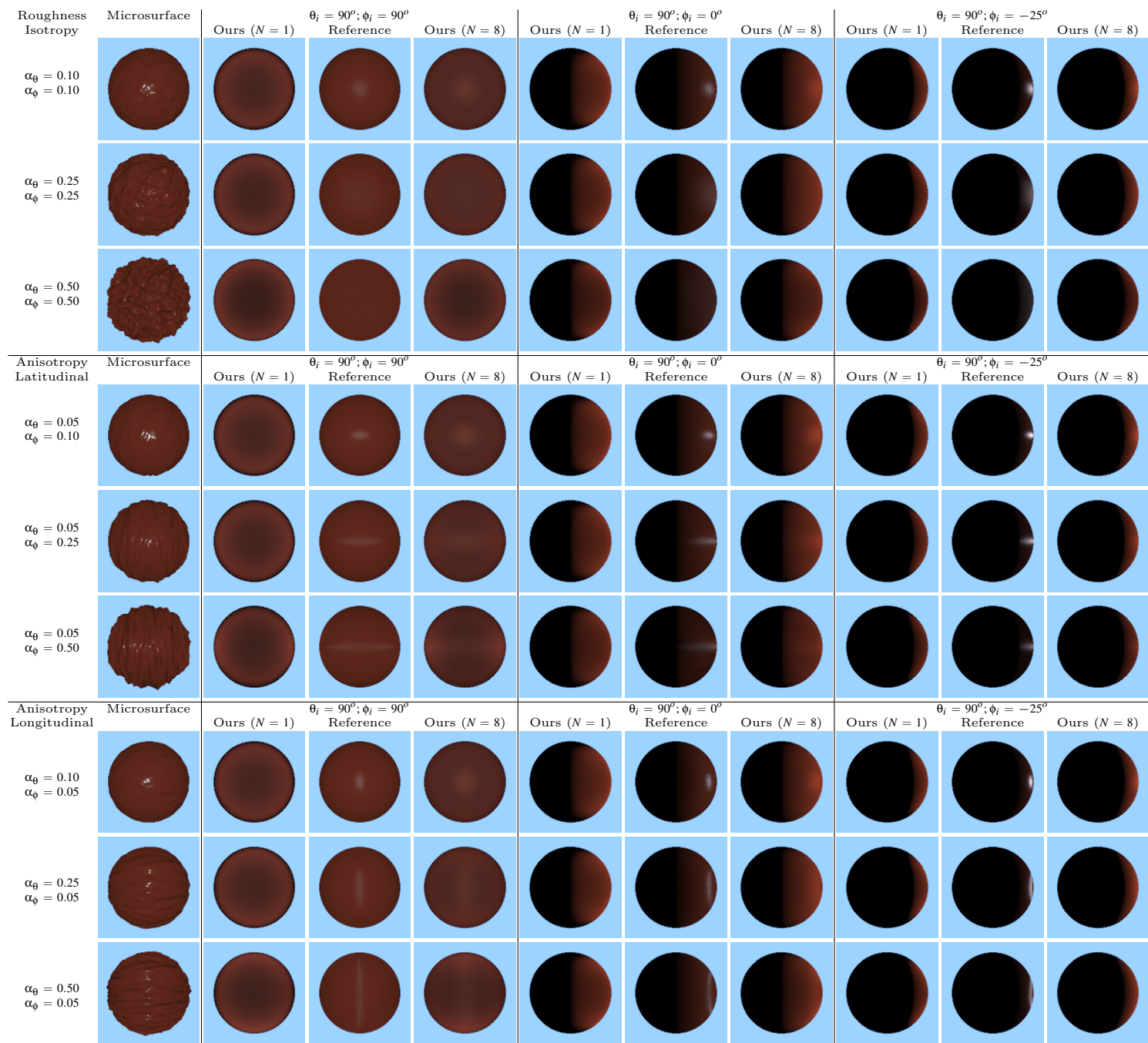
66 red-phenolic



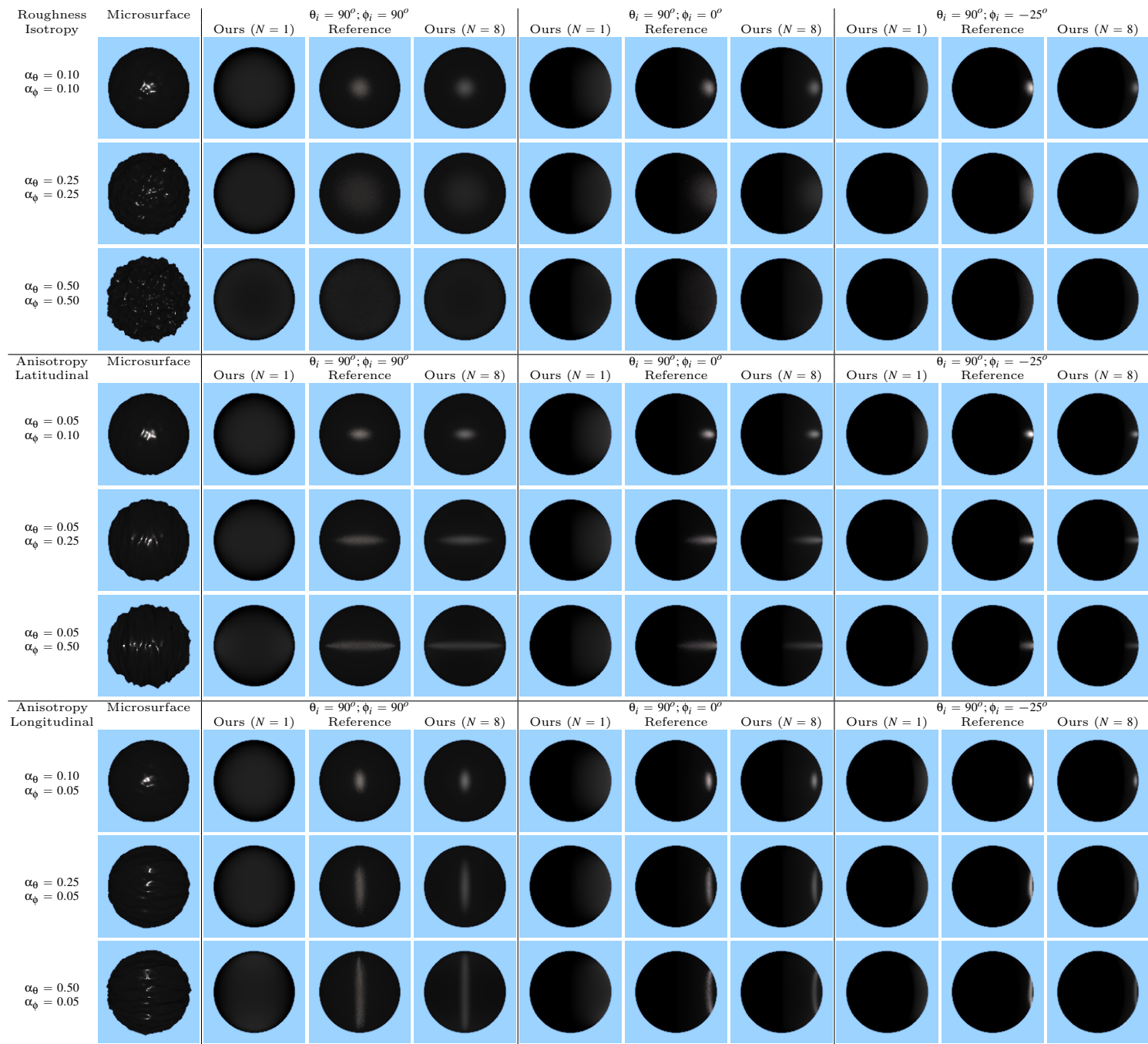
67 red-plastic



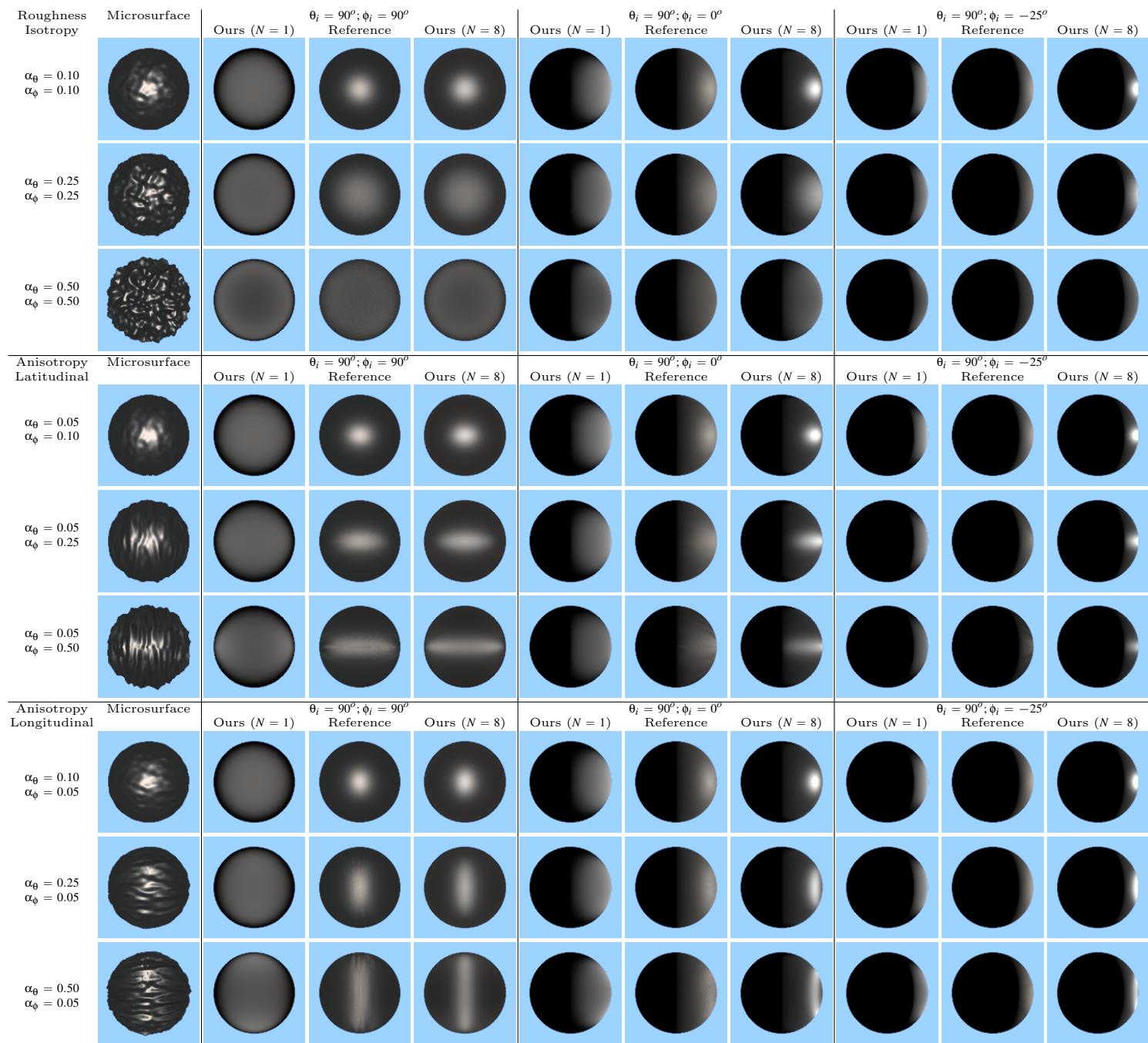
68 red-specular-plastic



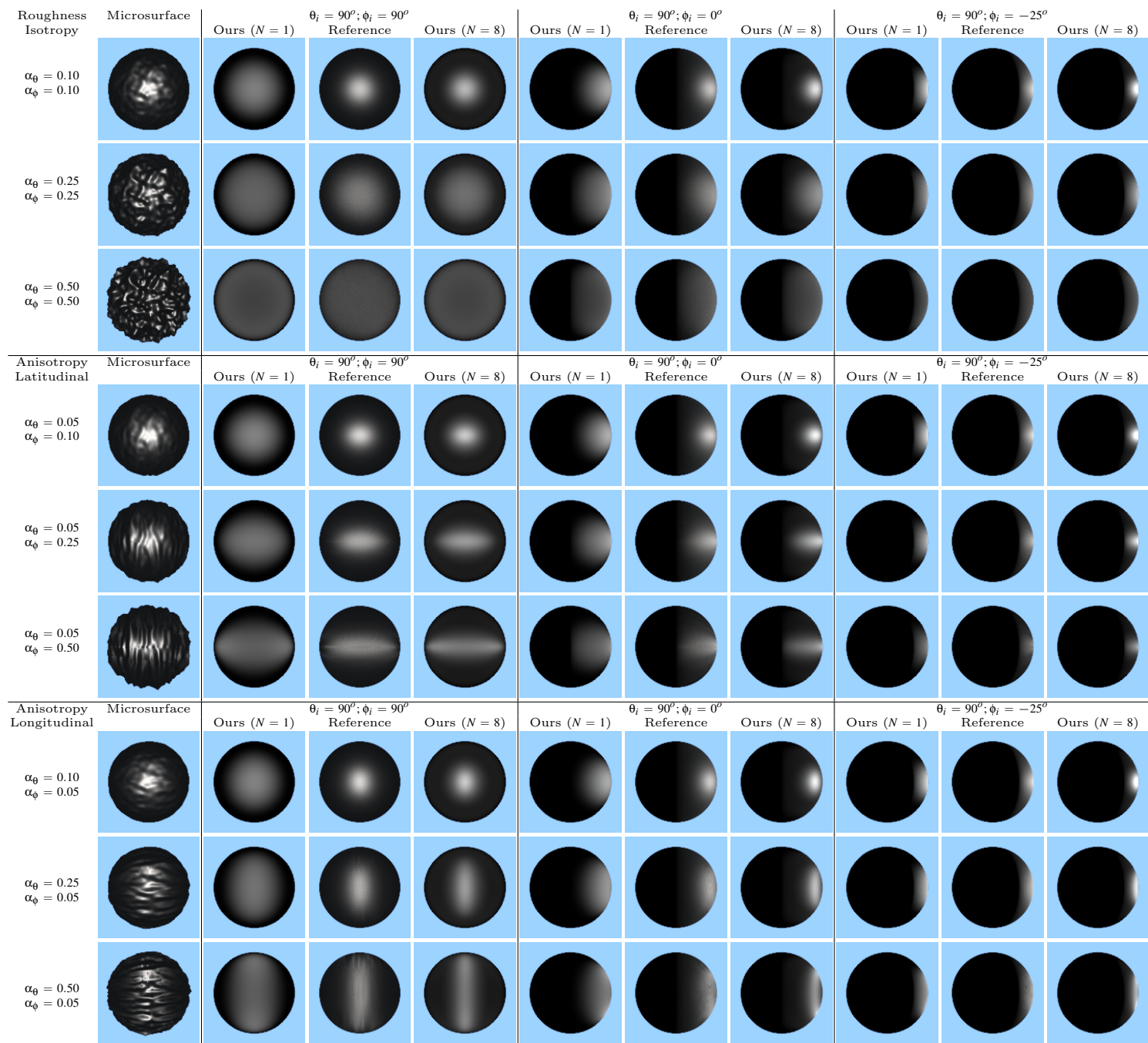
69 silicon-nitride



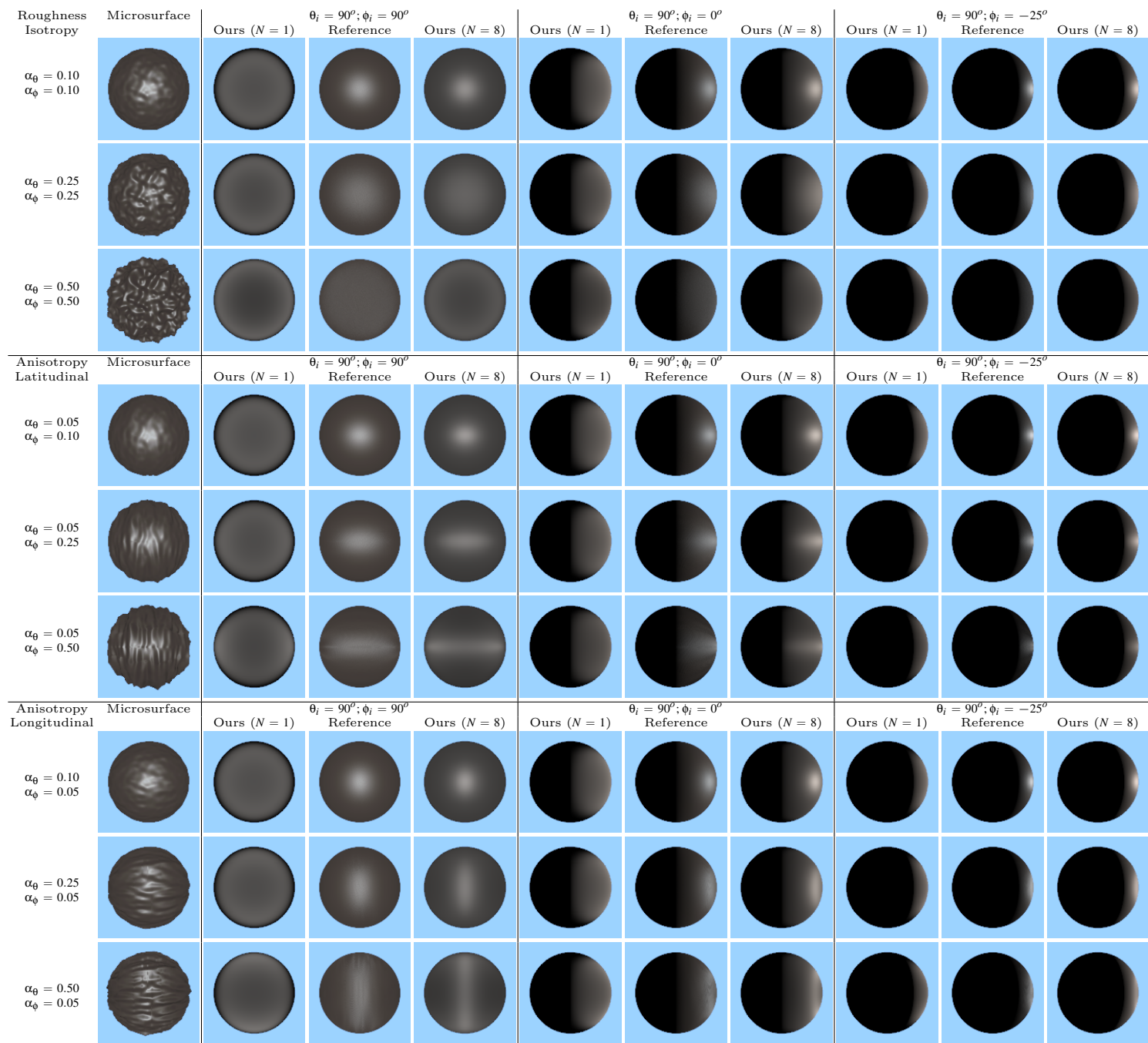
70 silver-metallic-paint2



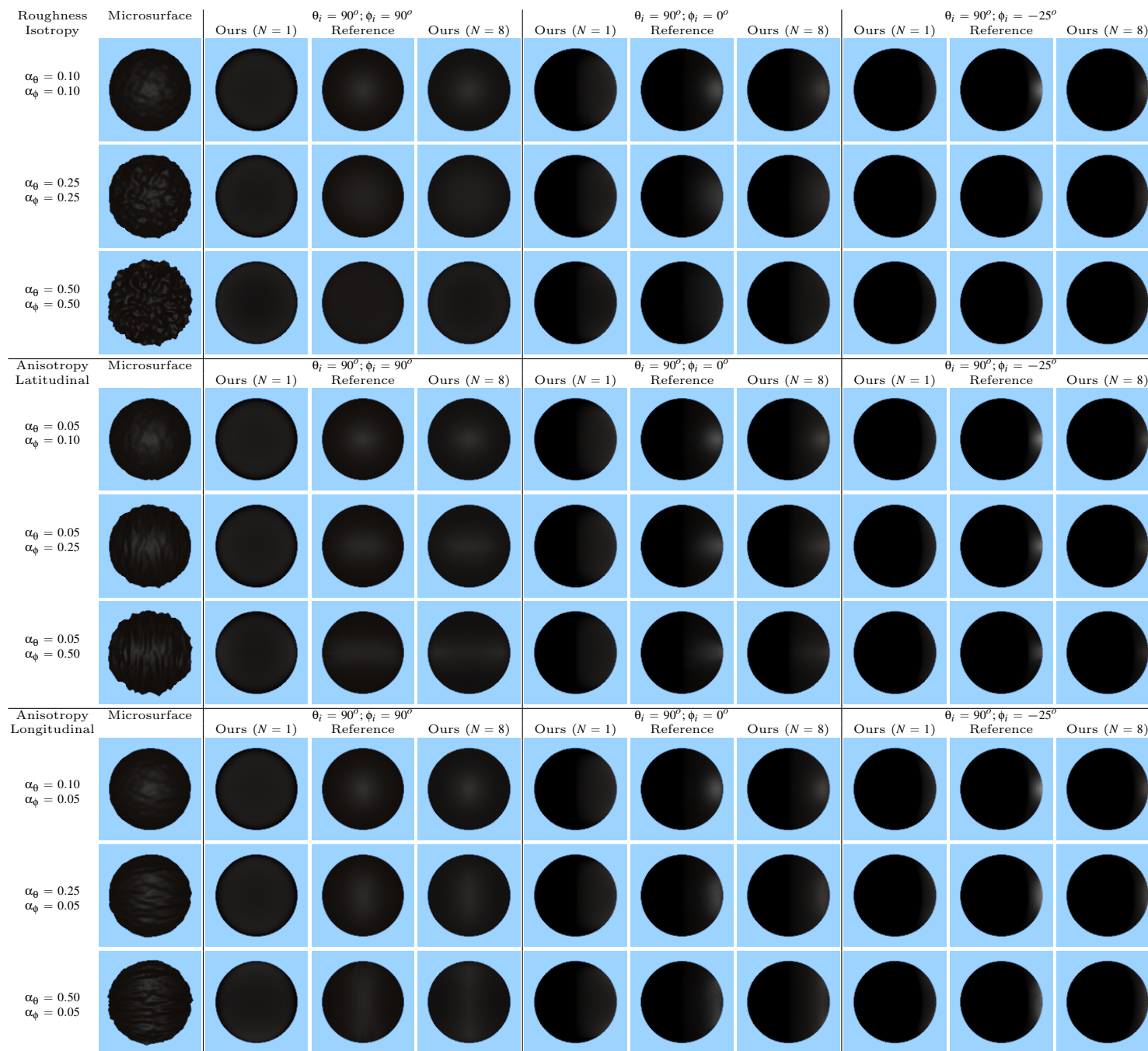
71 silver-metallic-paint



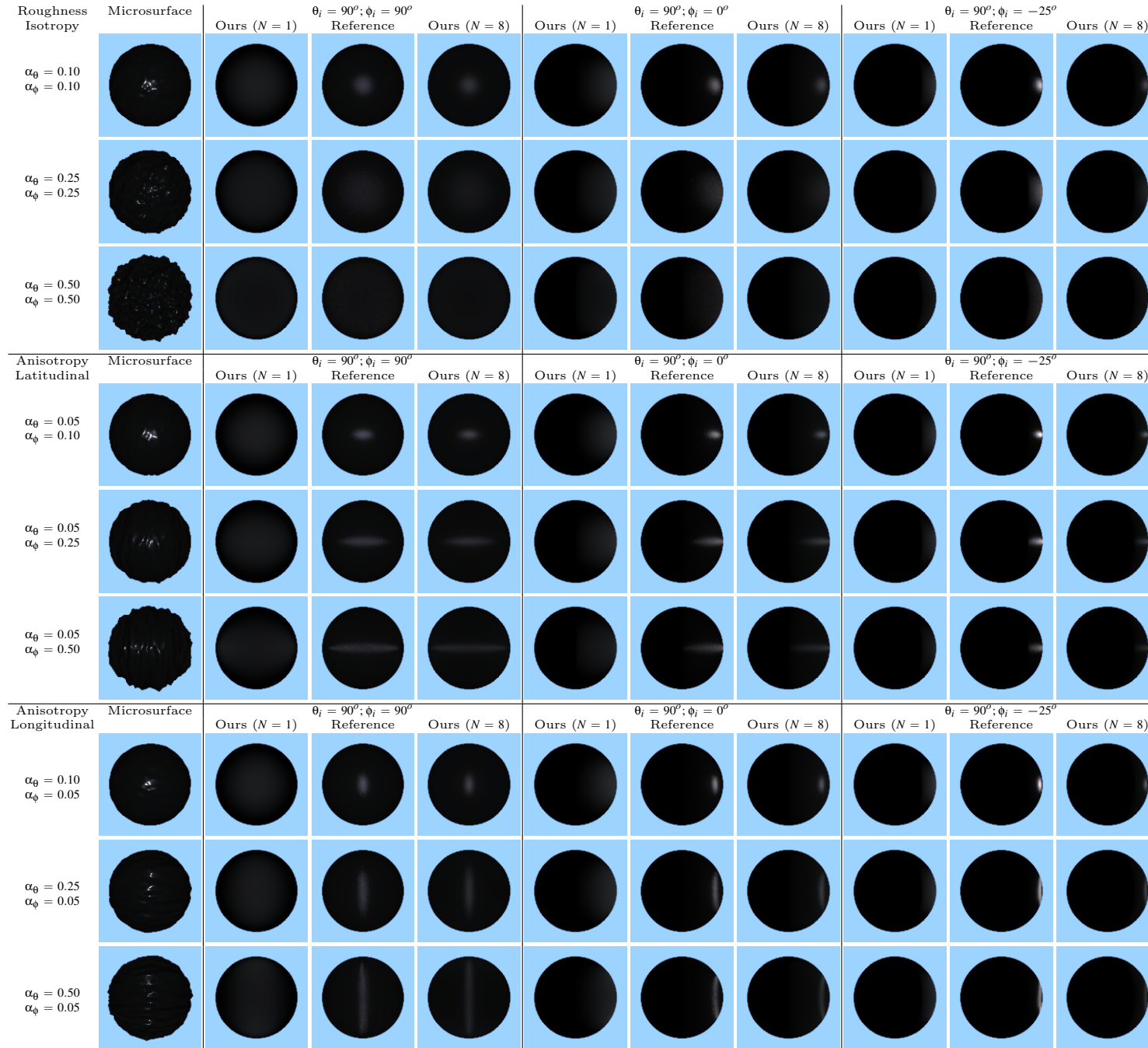
72 silver-paint



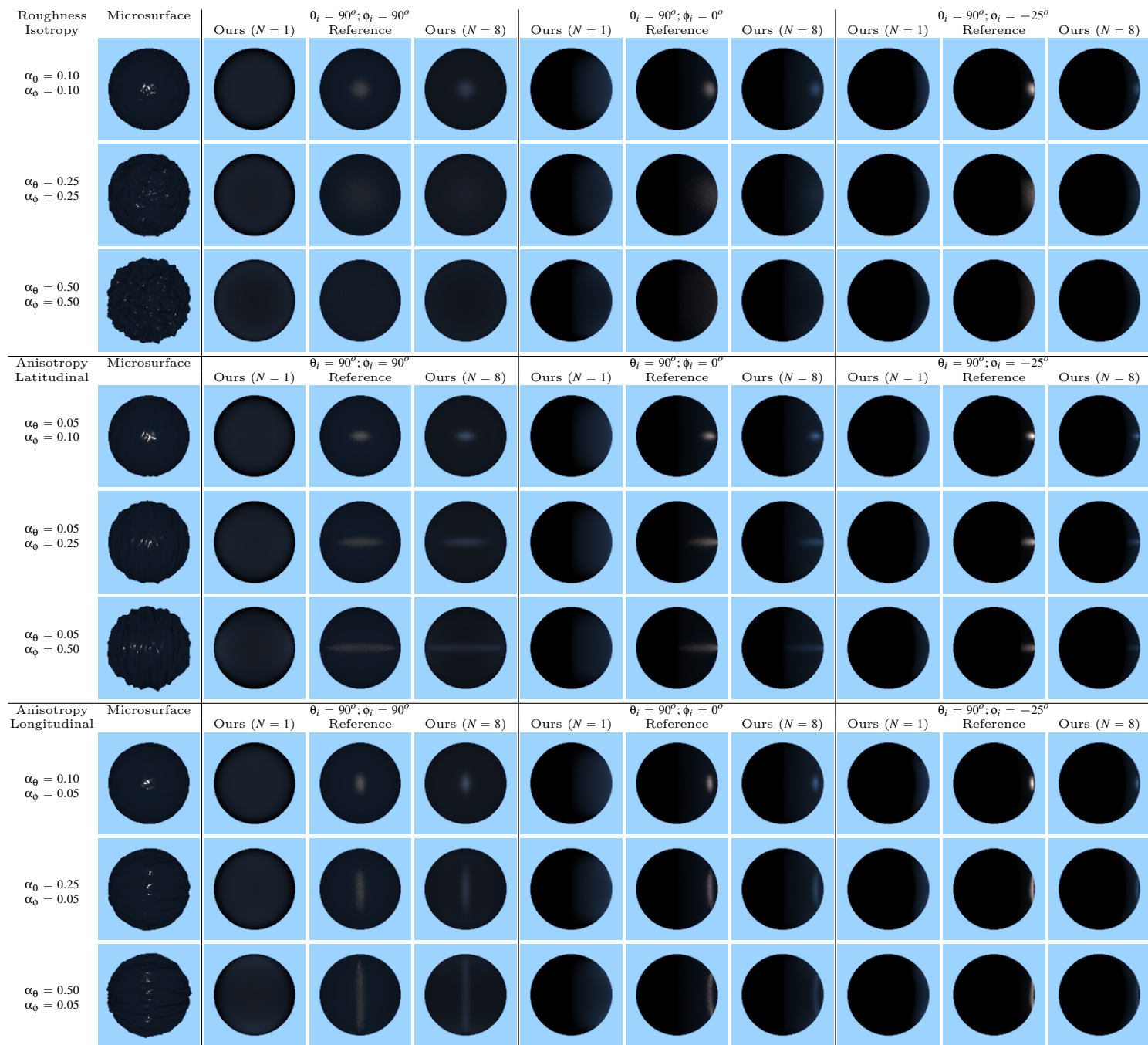
73 special-walnut-224



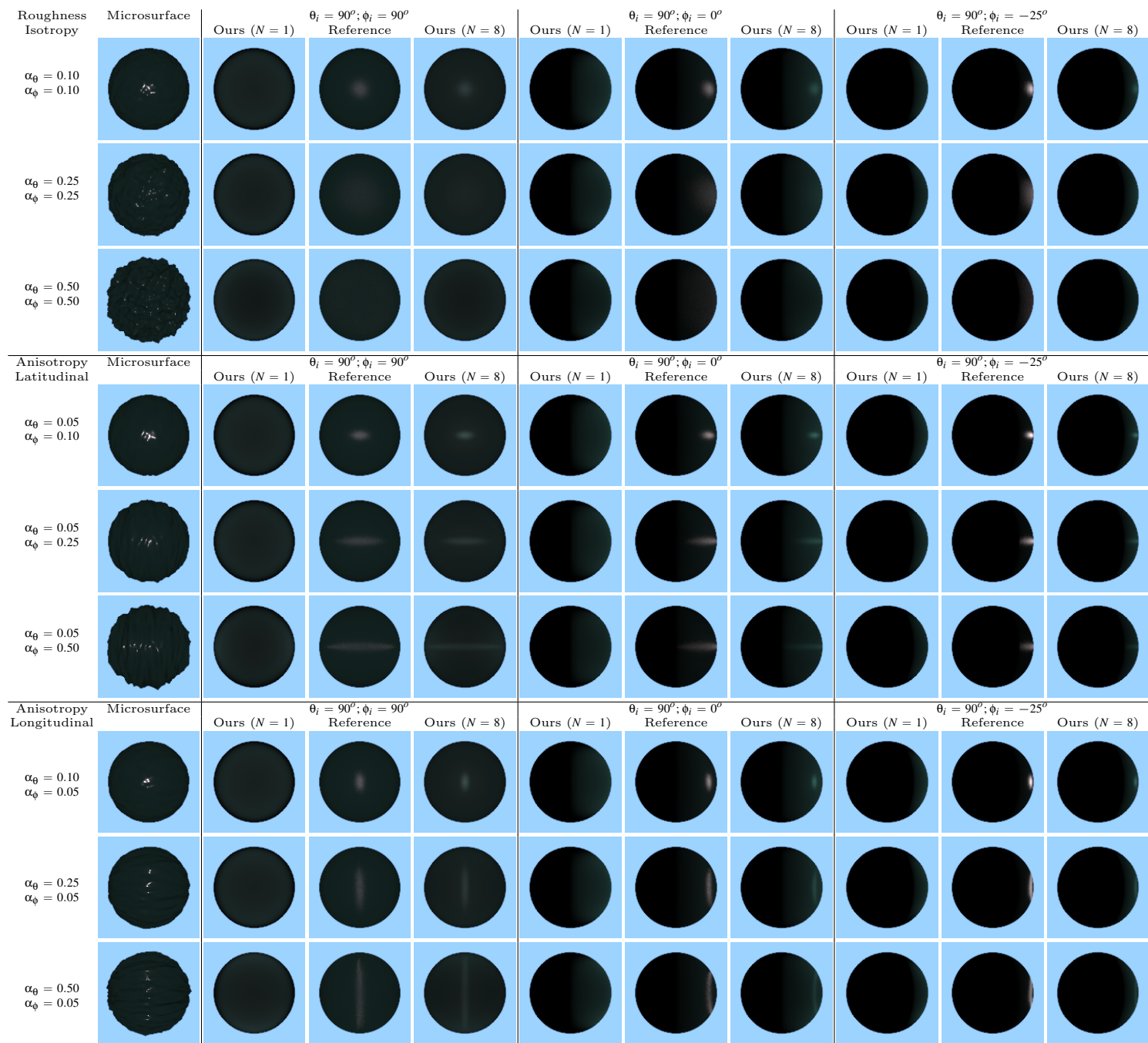
74 specular-black-phenolic



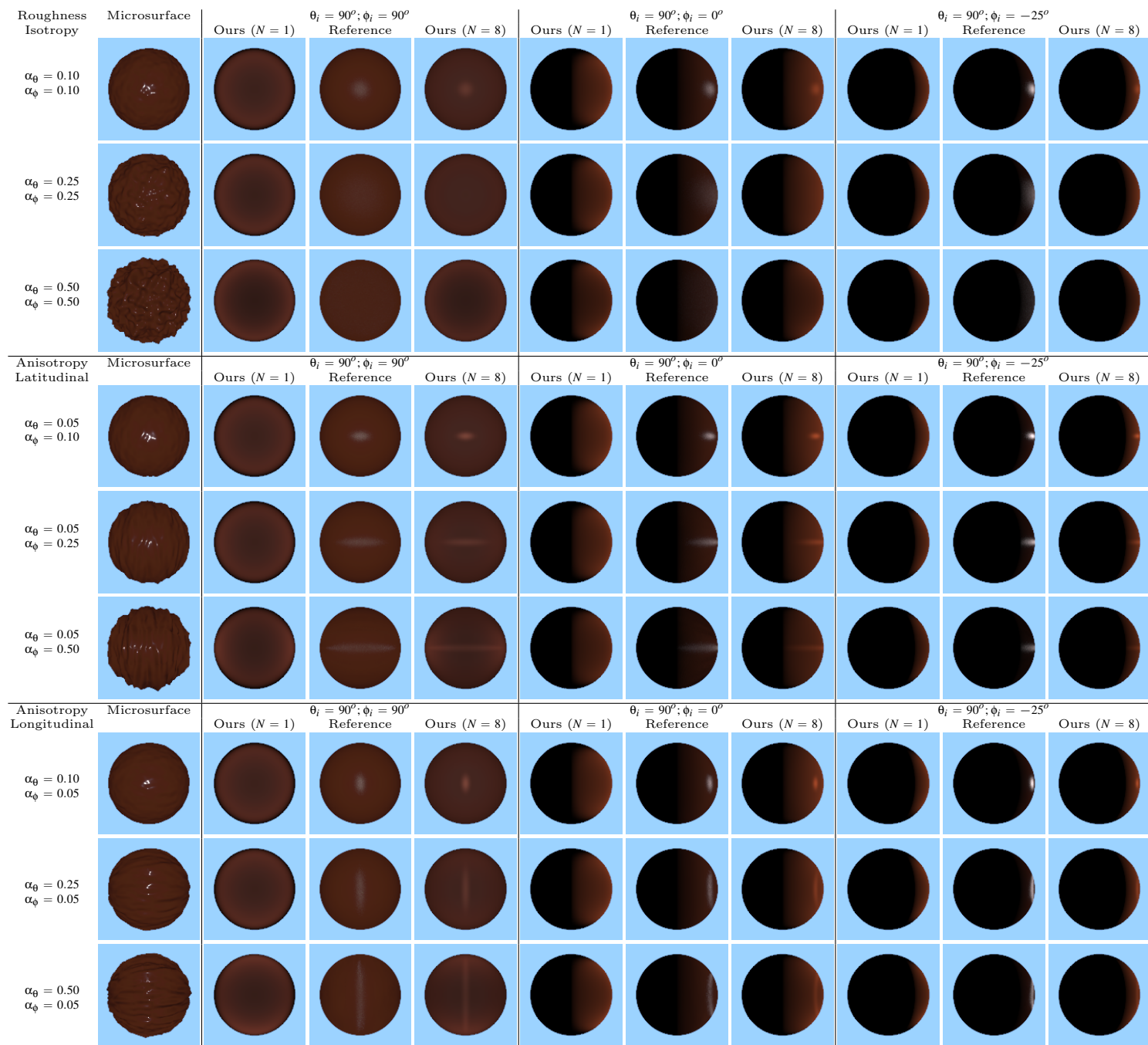
75 specular-blue-phenolic



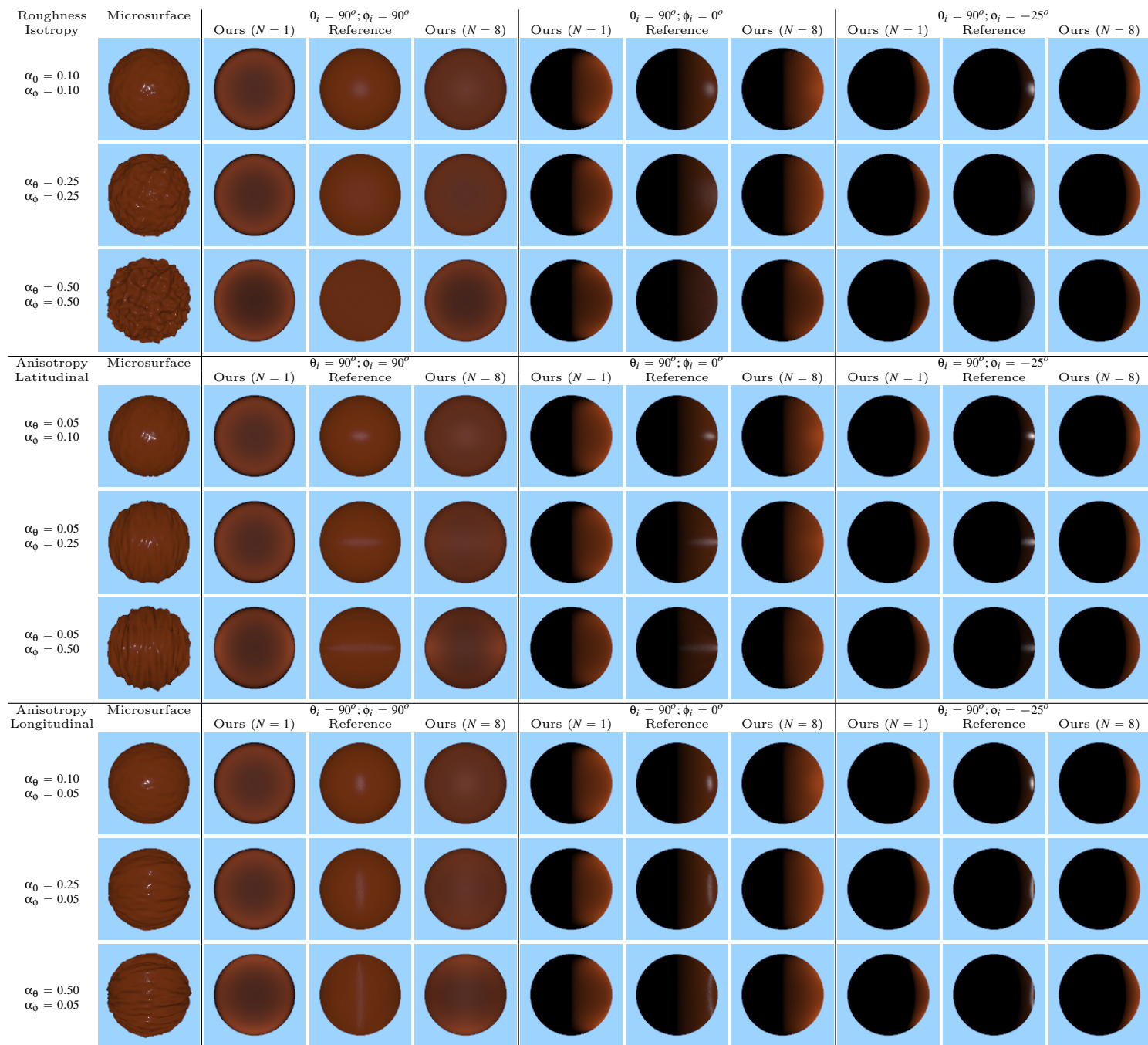
76 specular-green-phenolic



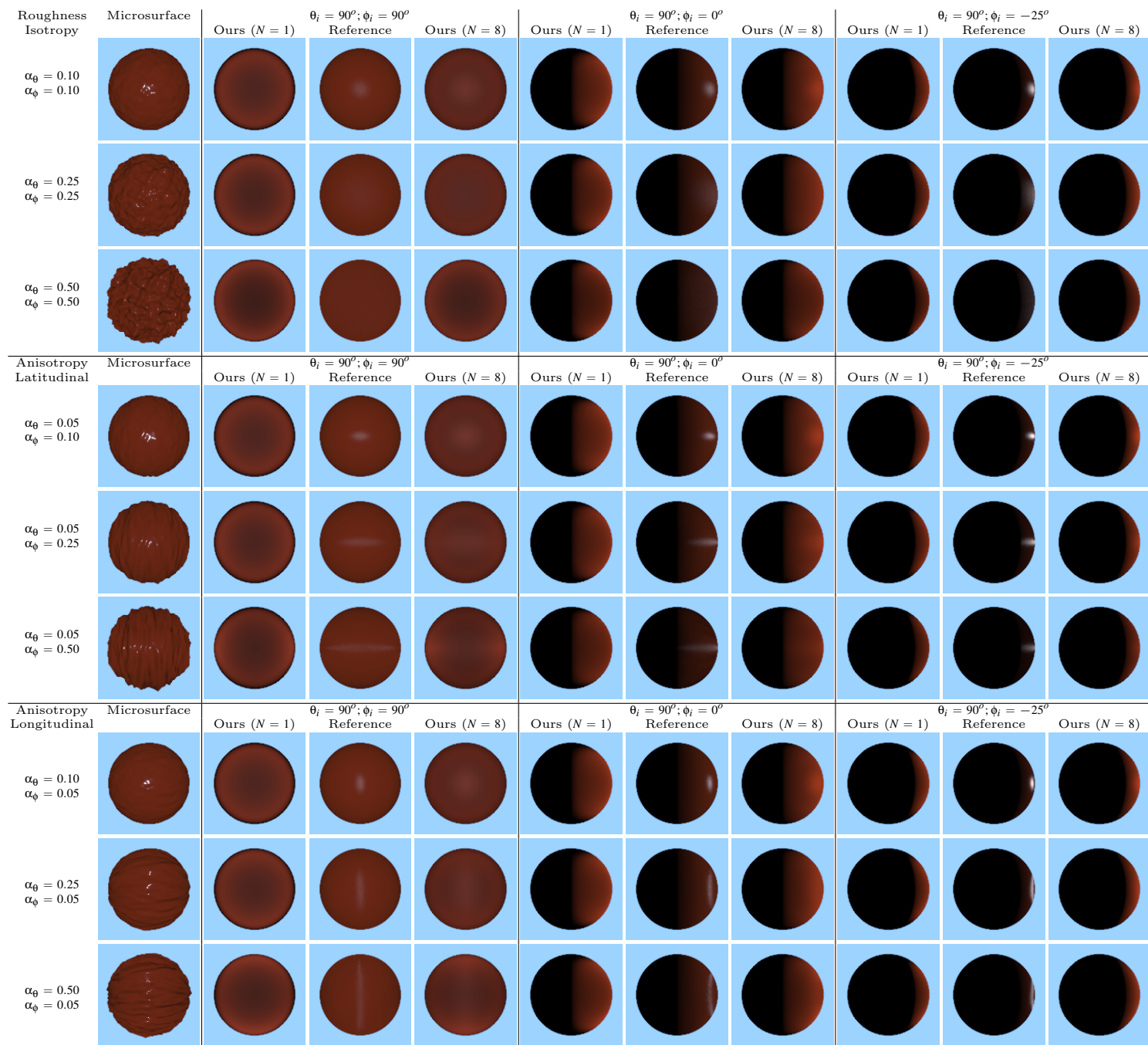
77 specular-maroon-phenolic



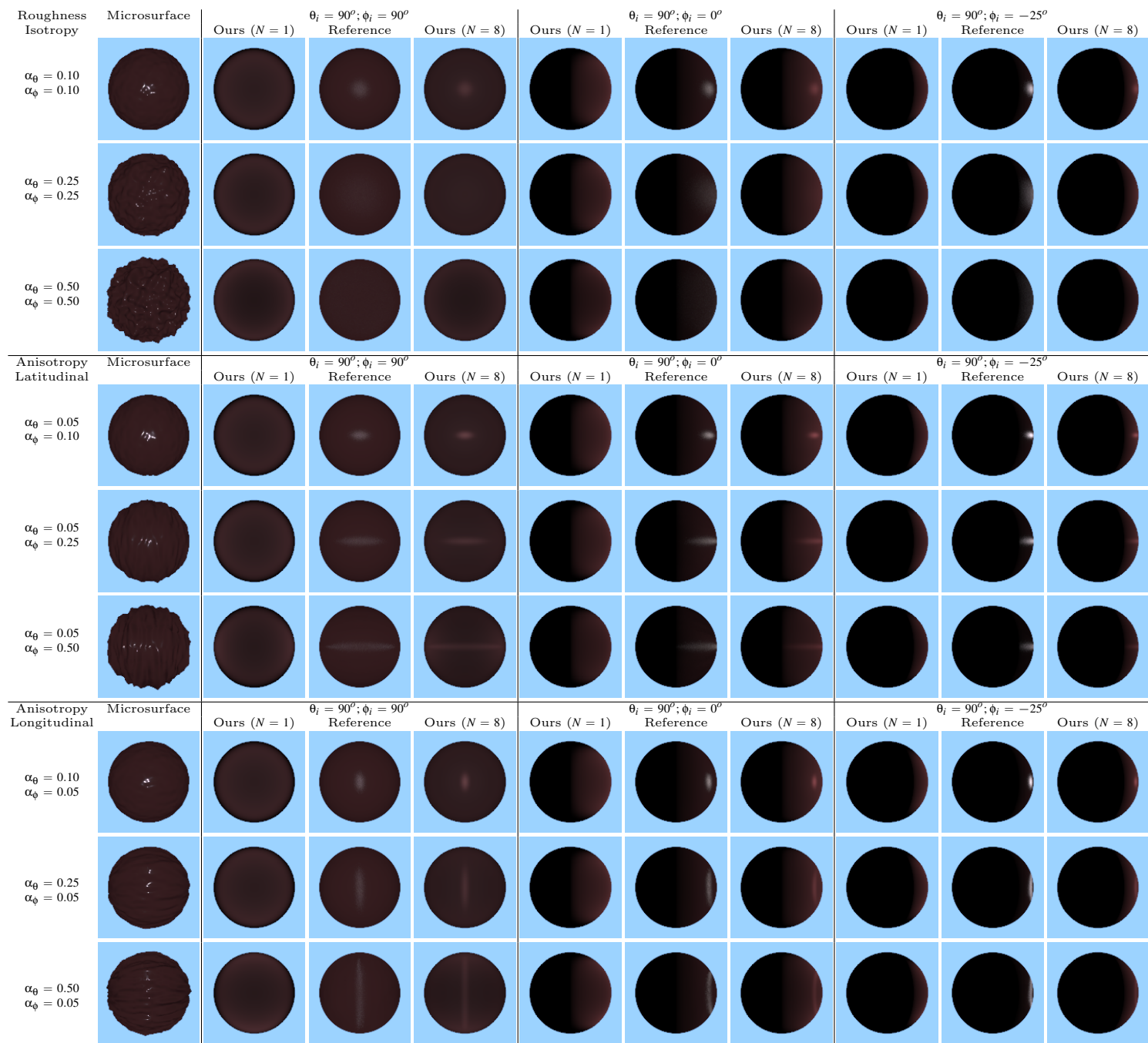
78 specular-orange-phenolic



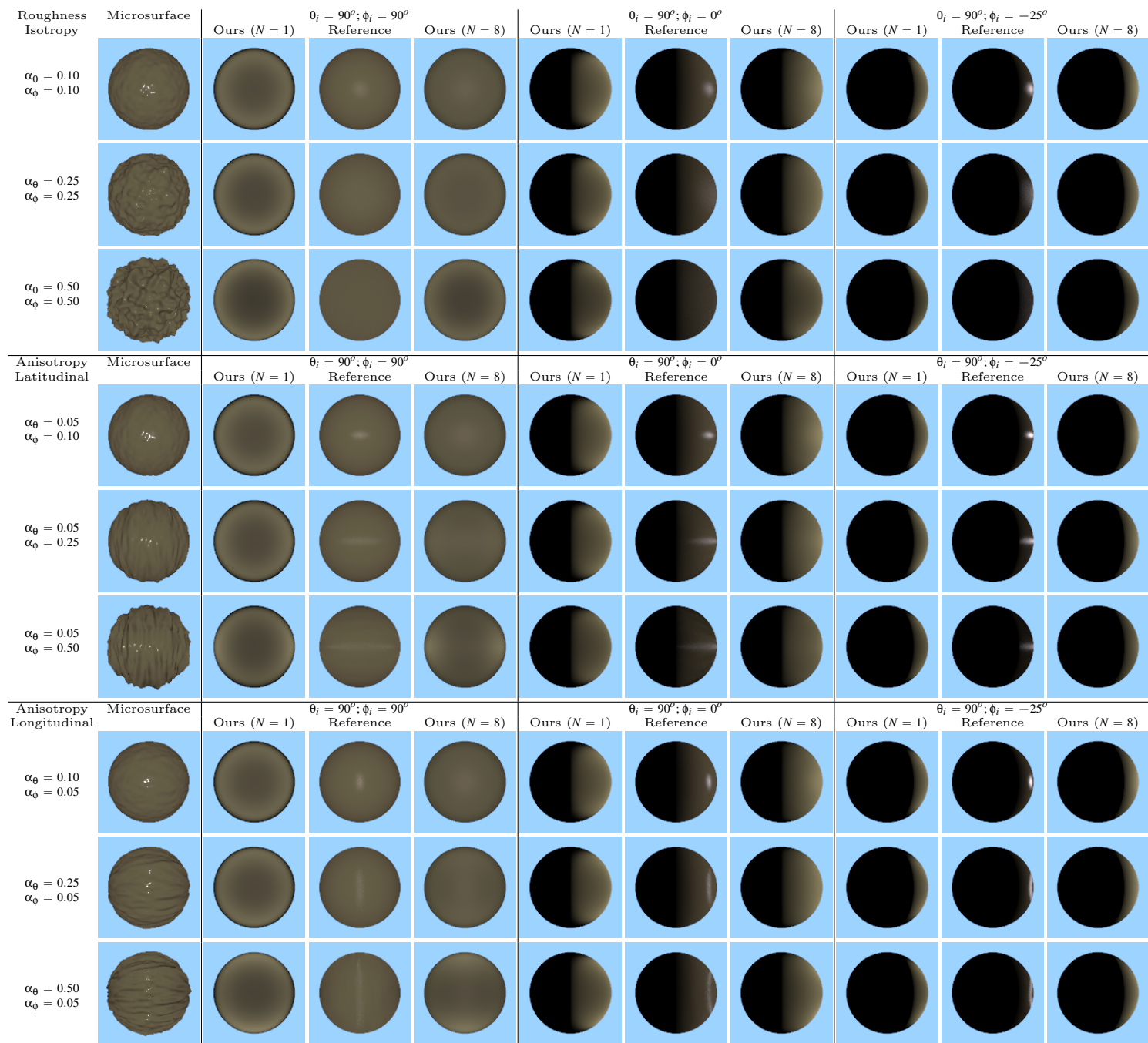
79 specular-red-phenolic



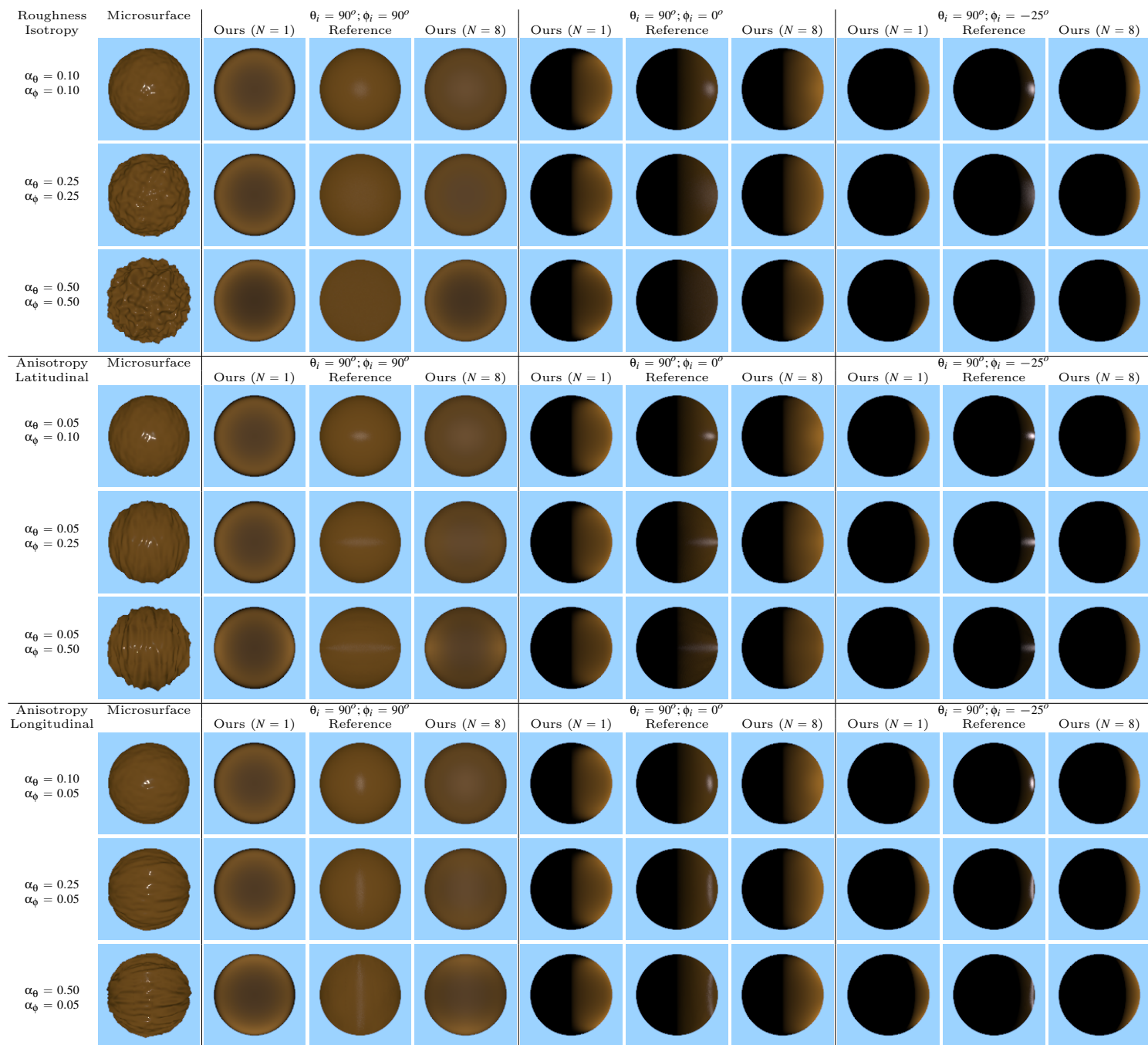
80 specular-violet-phenolic

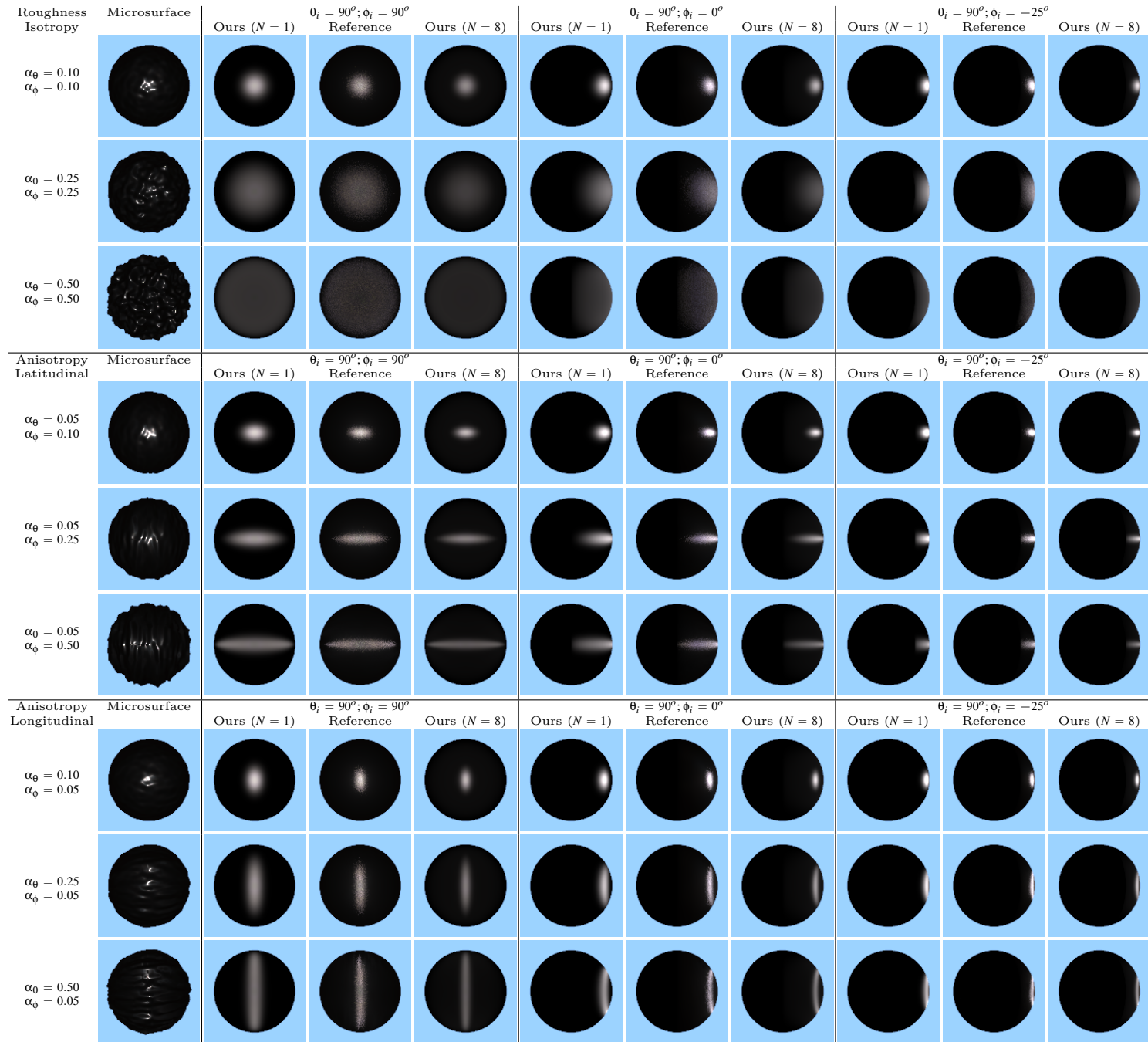


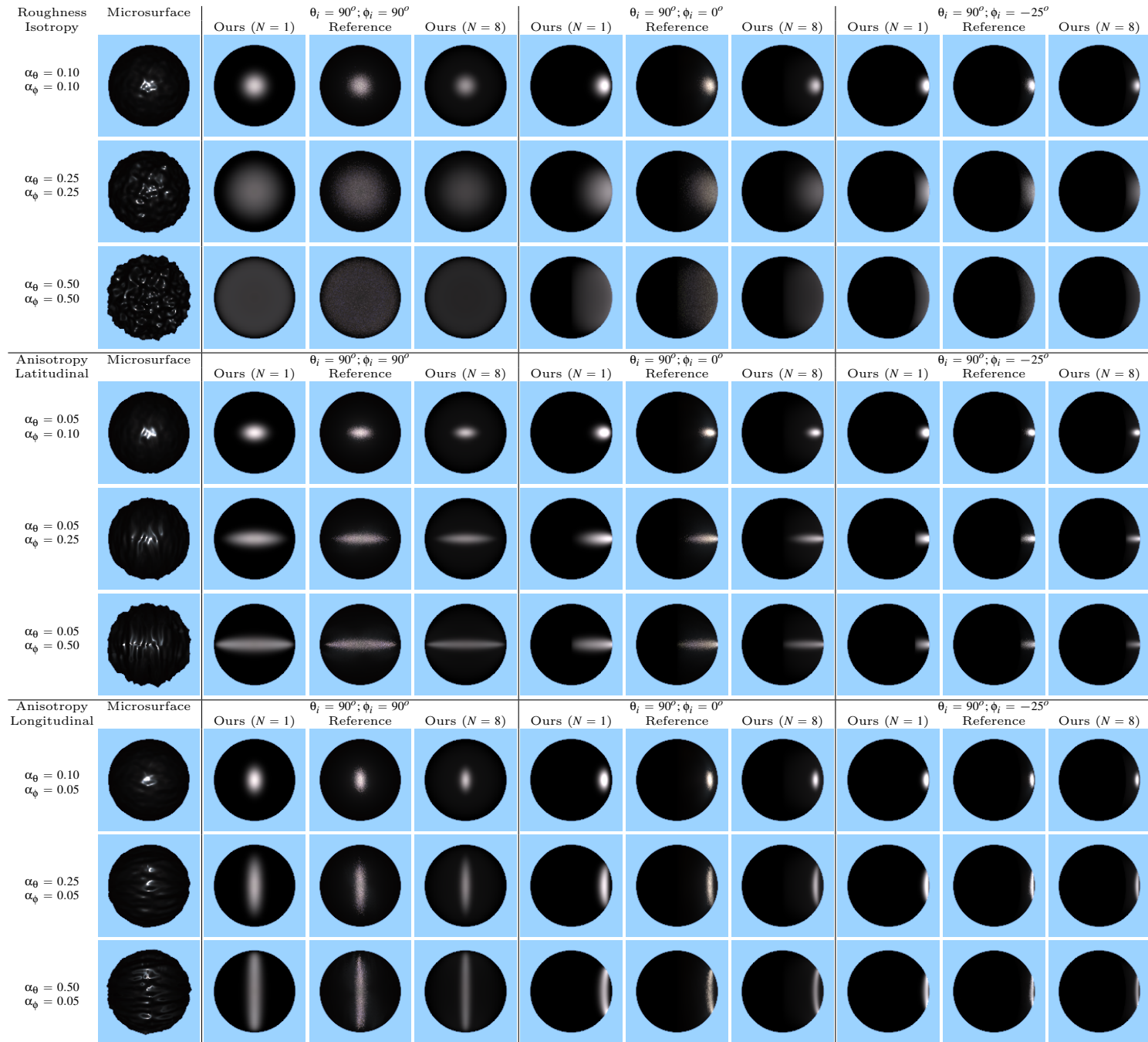
81 specular-white-phenolic



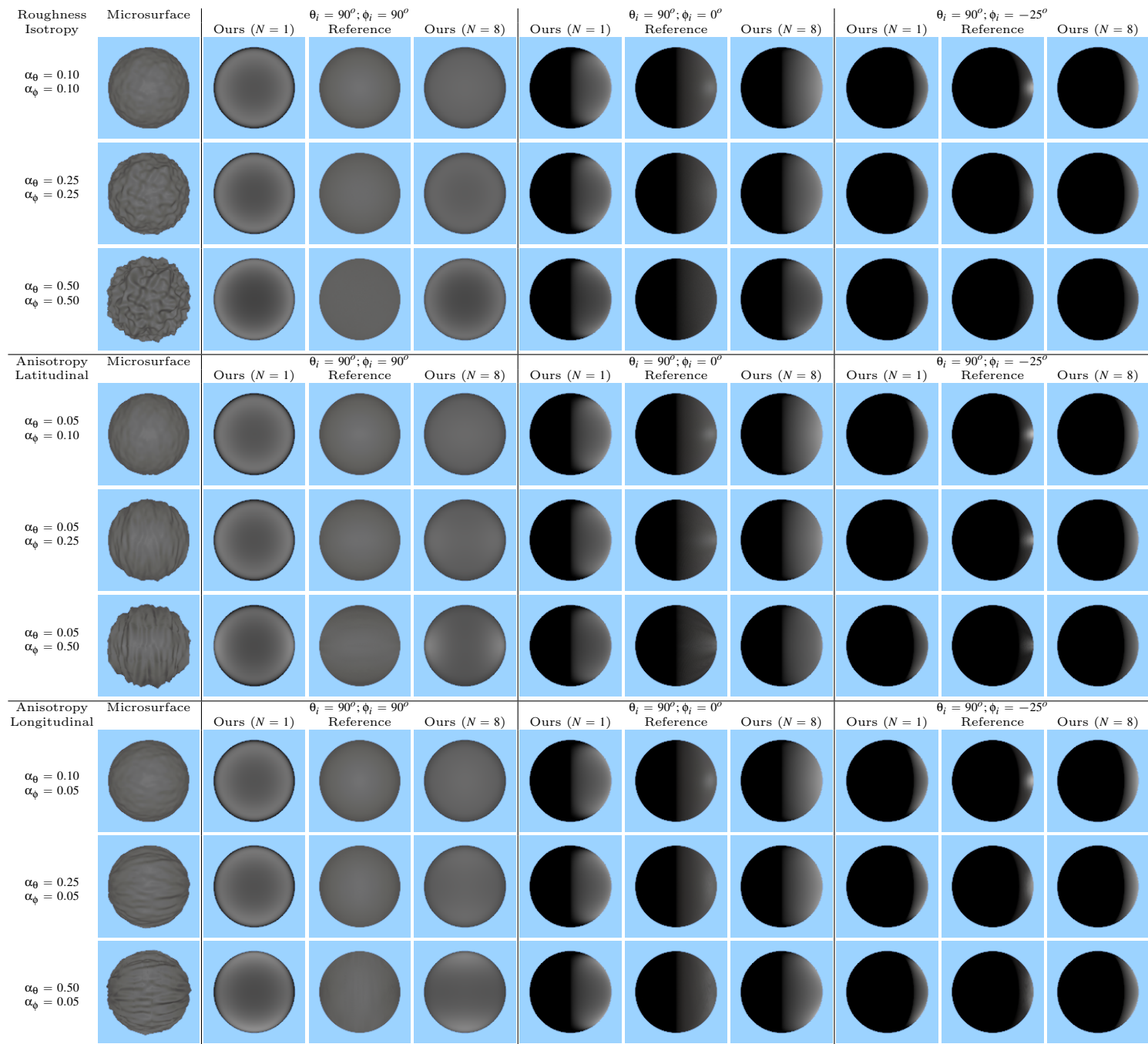
82 specular-yellow-phenolic



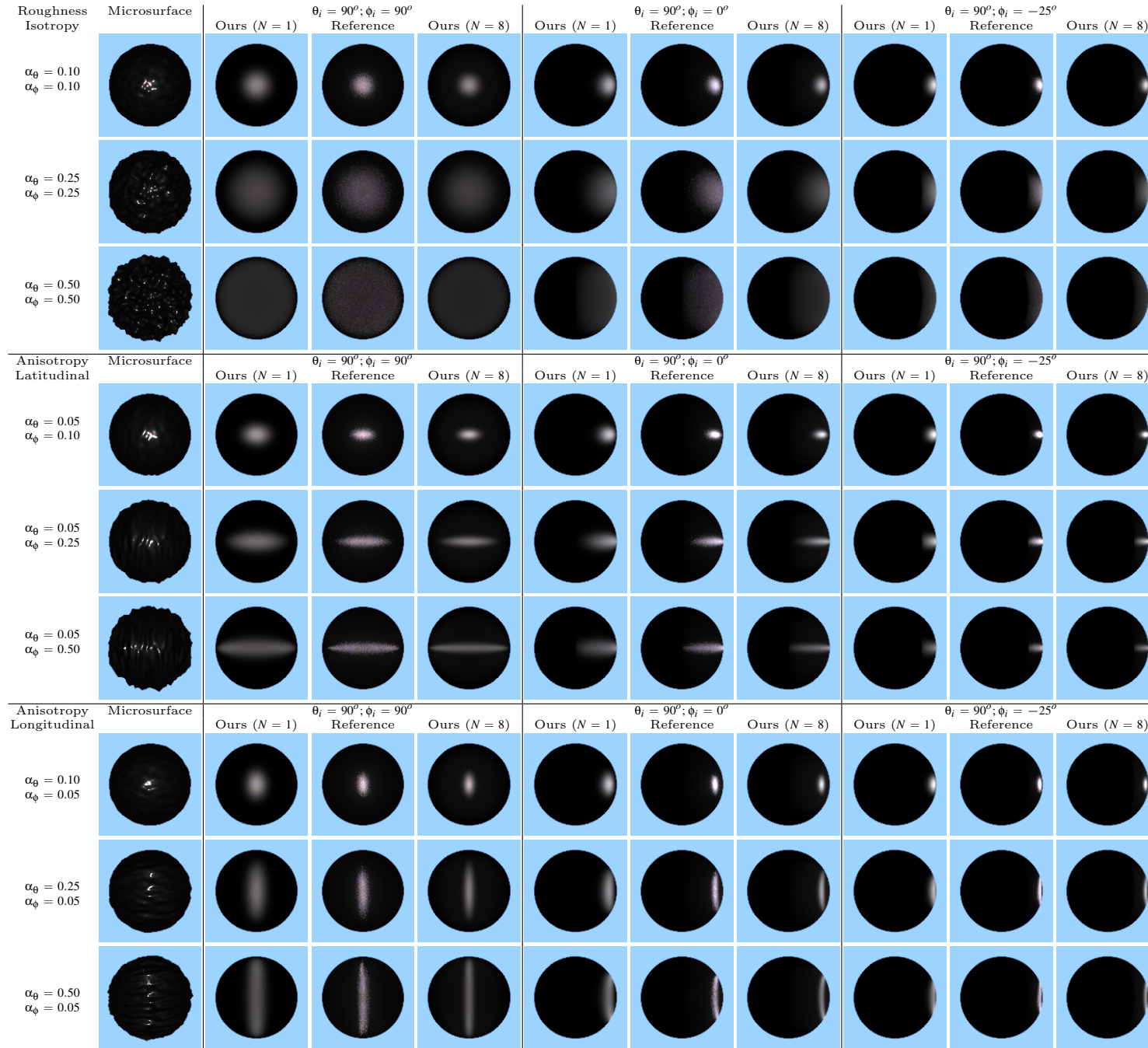




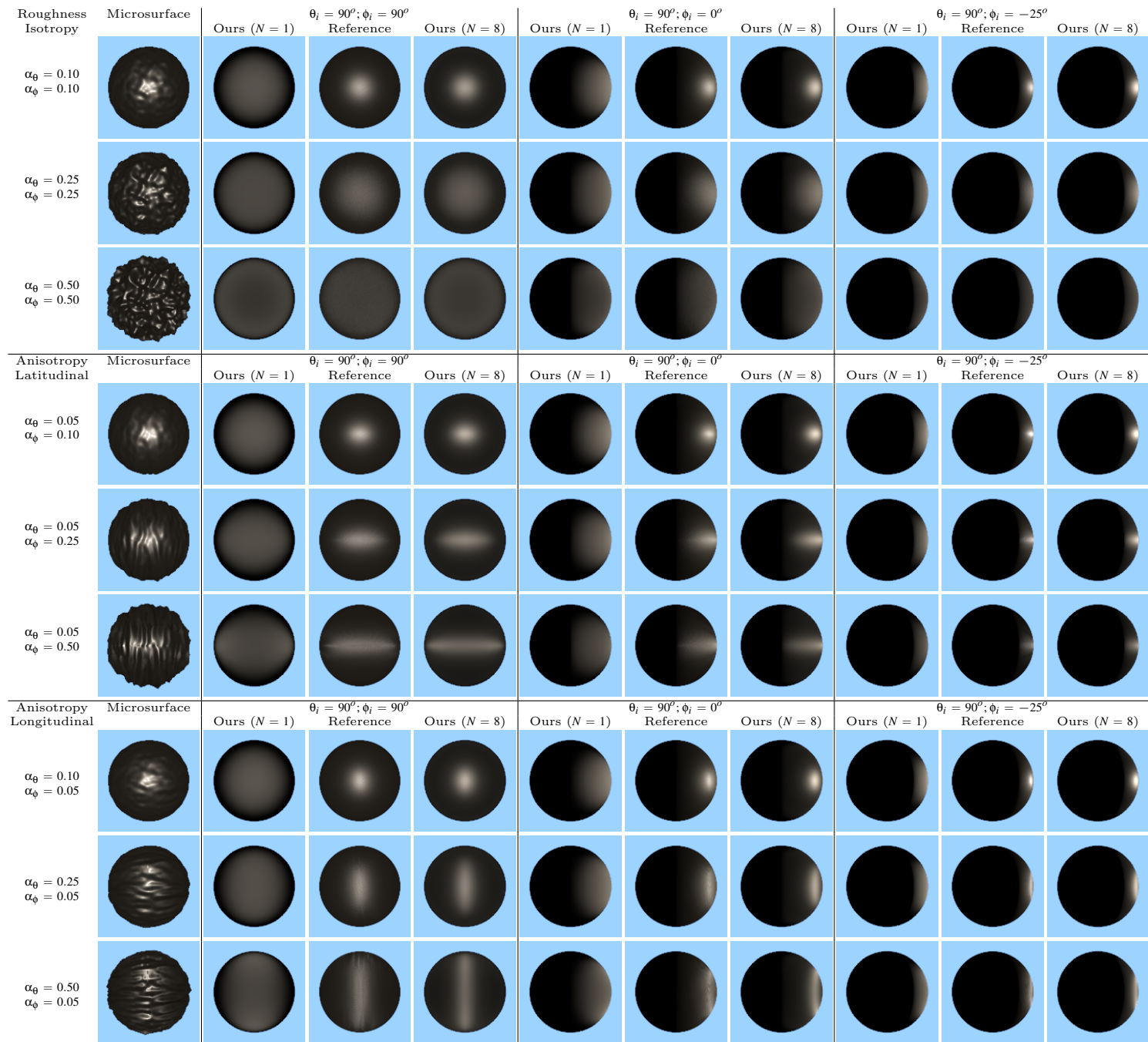
85 teflon



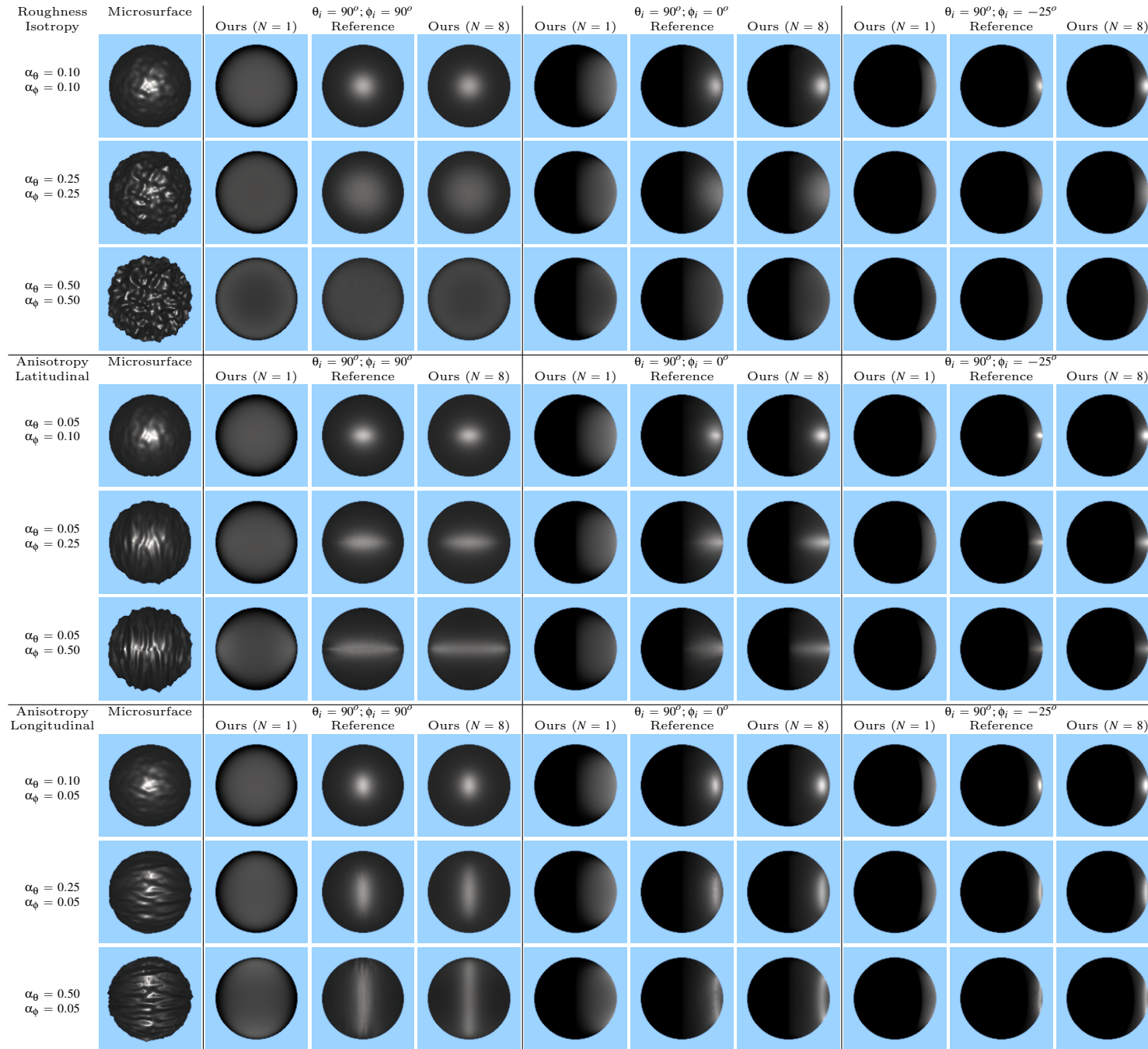
86 tungsten-carbide



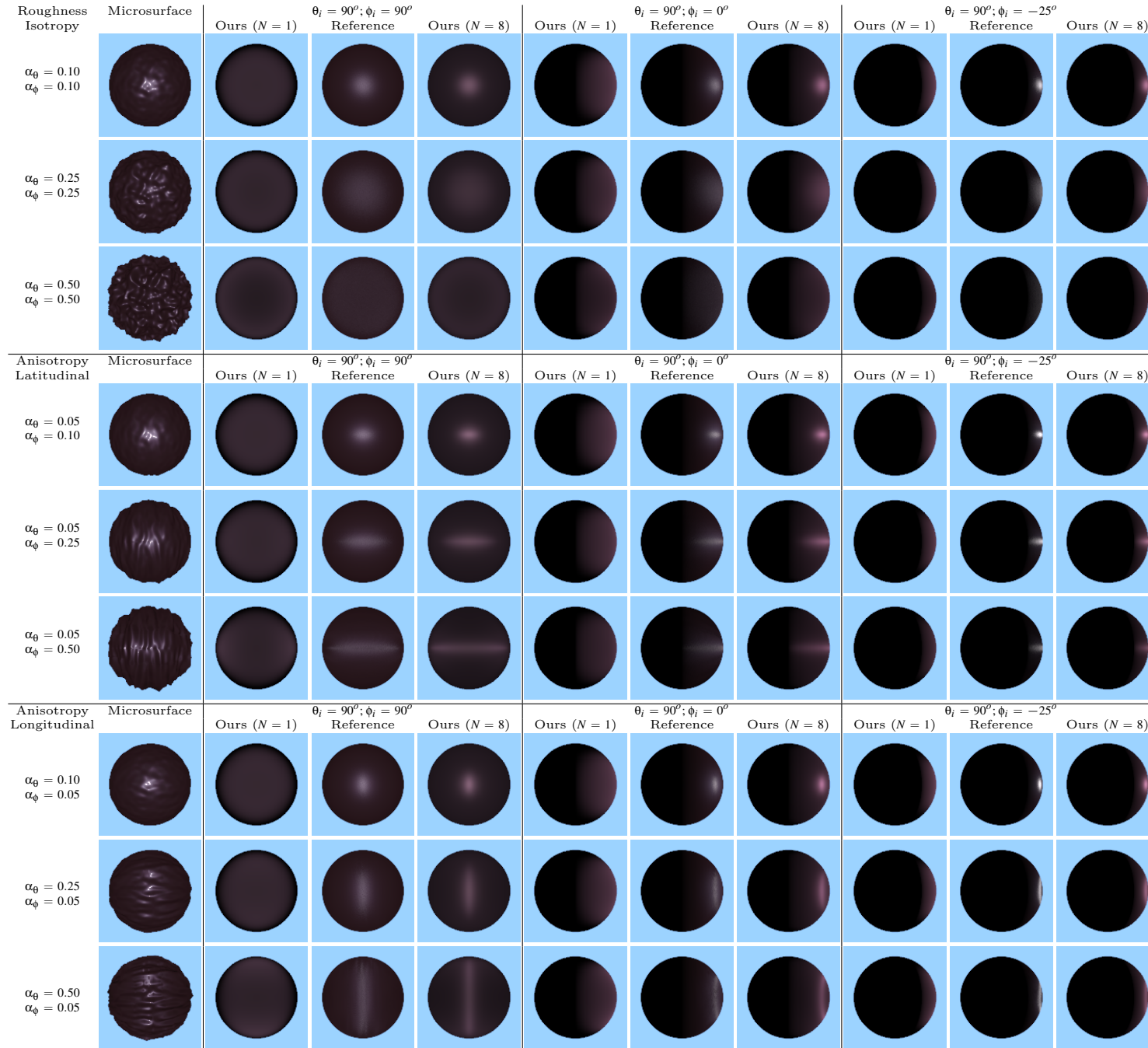
87 two-layer-gold



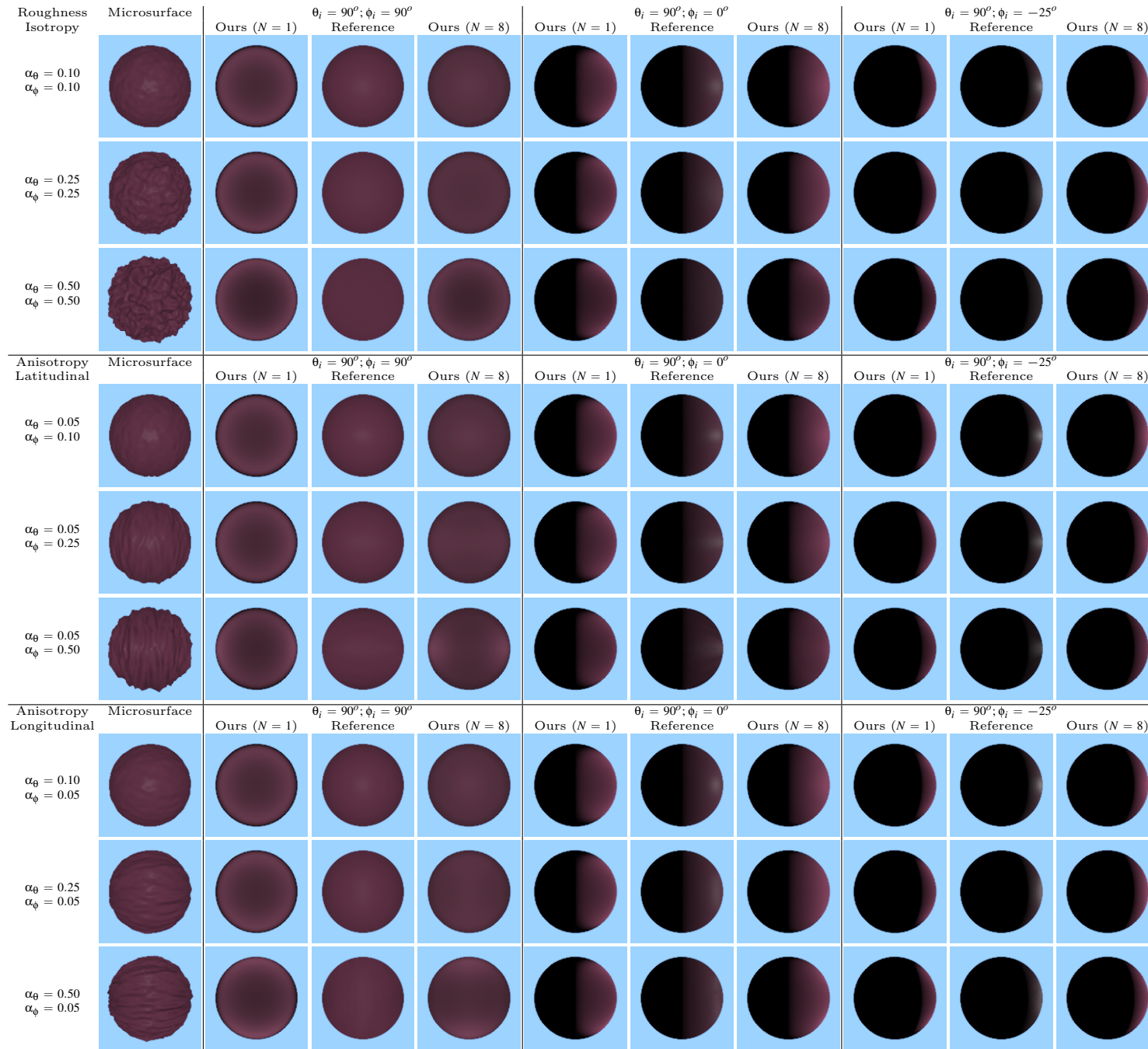
88 two-layer-silver



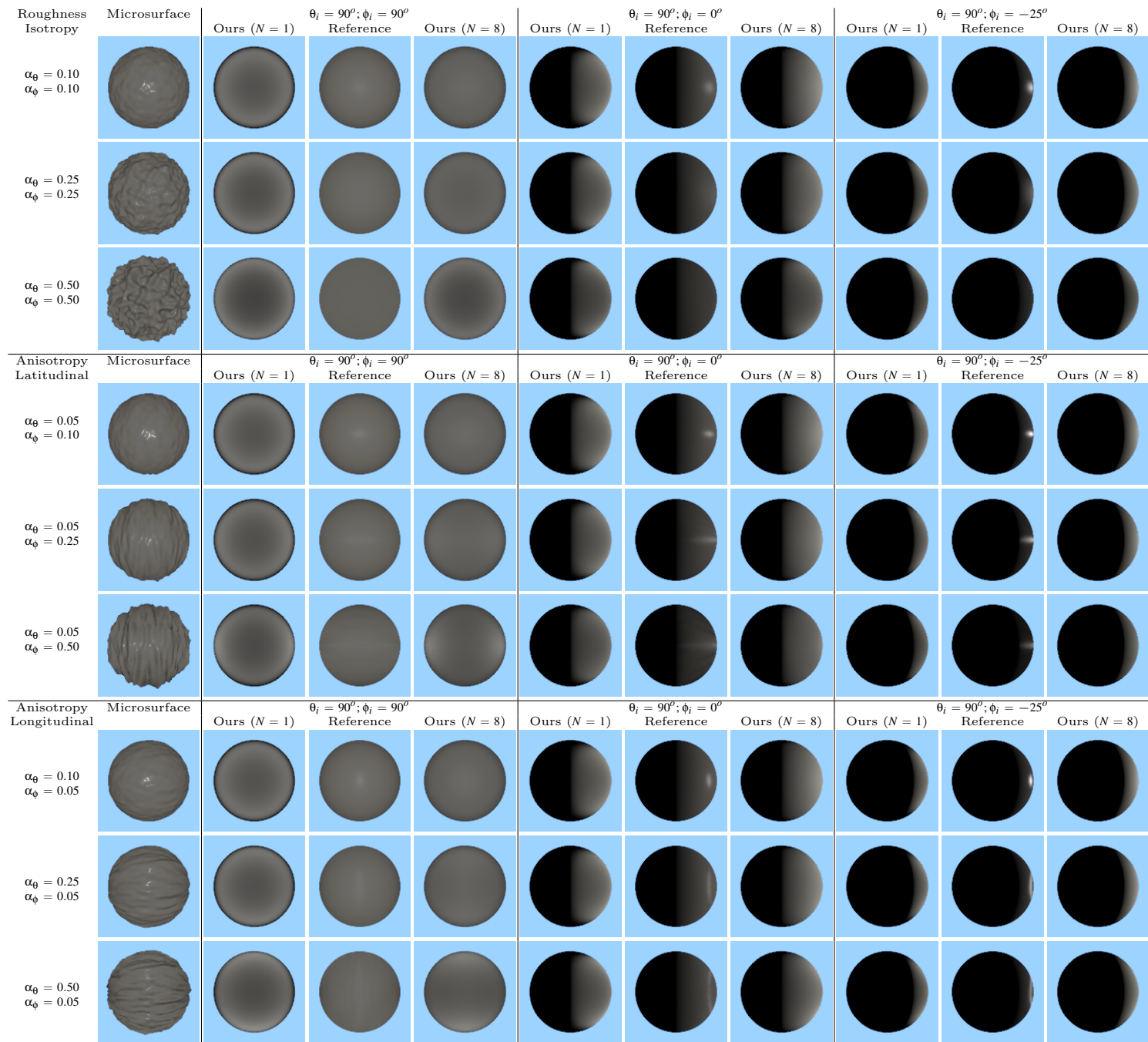
89 violet-acrylic



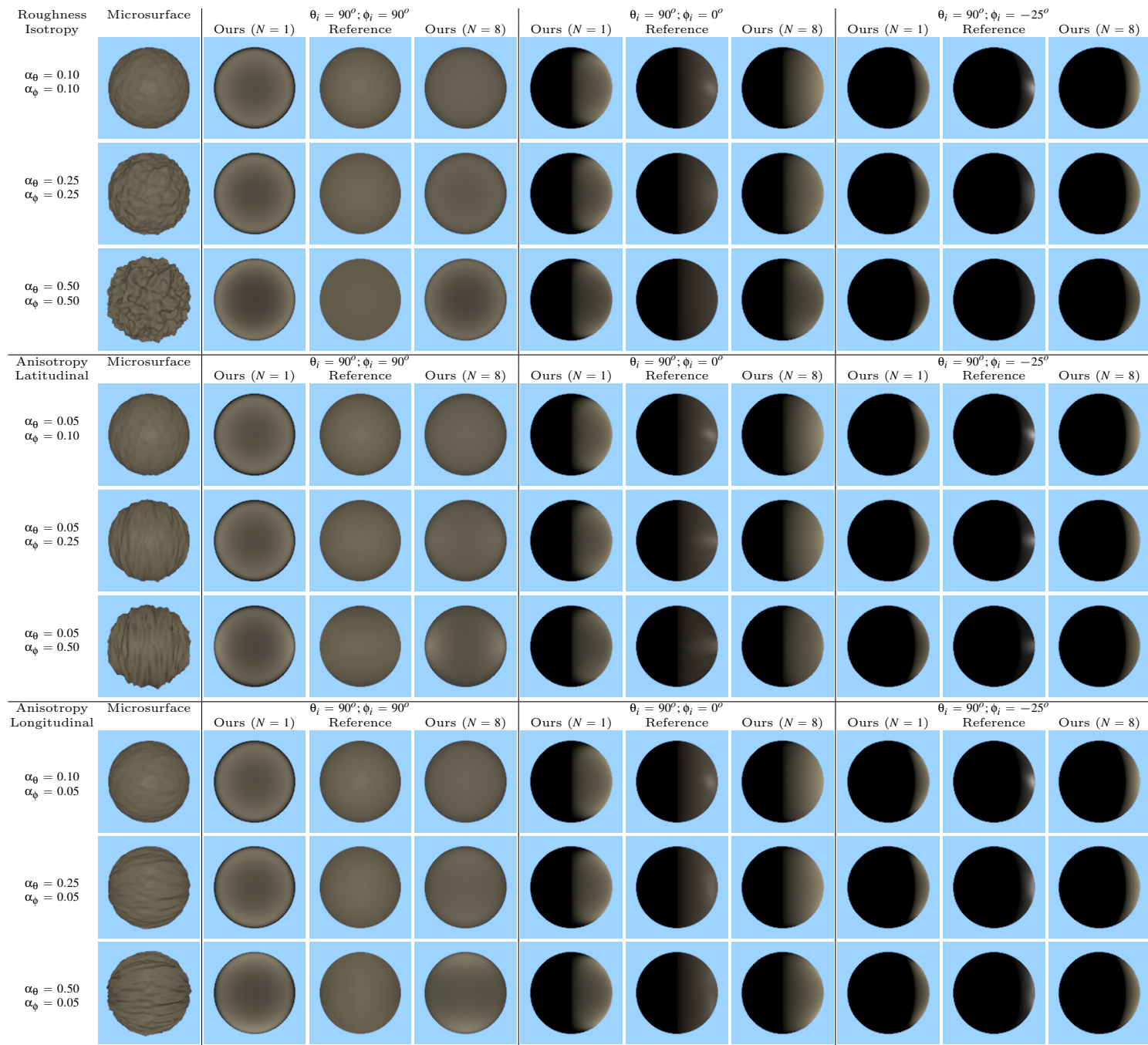
90 violet-rubber



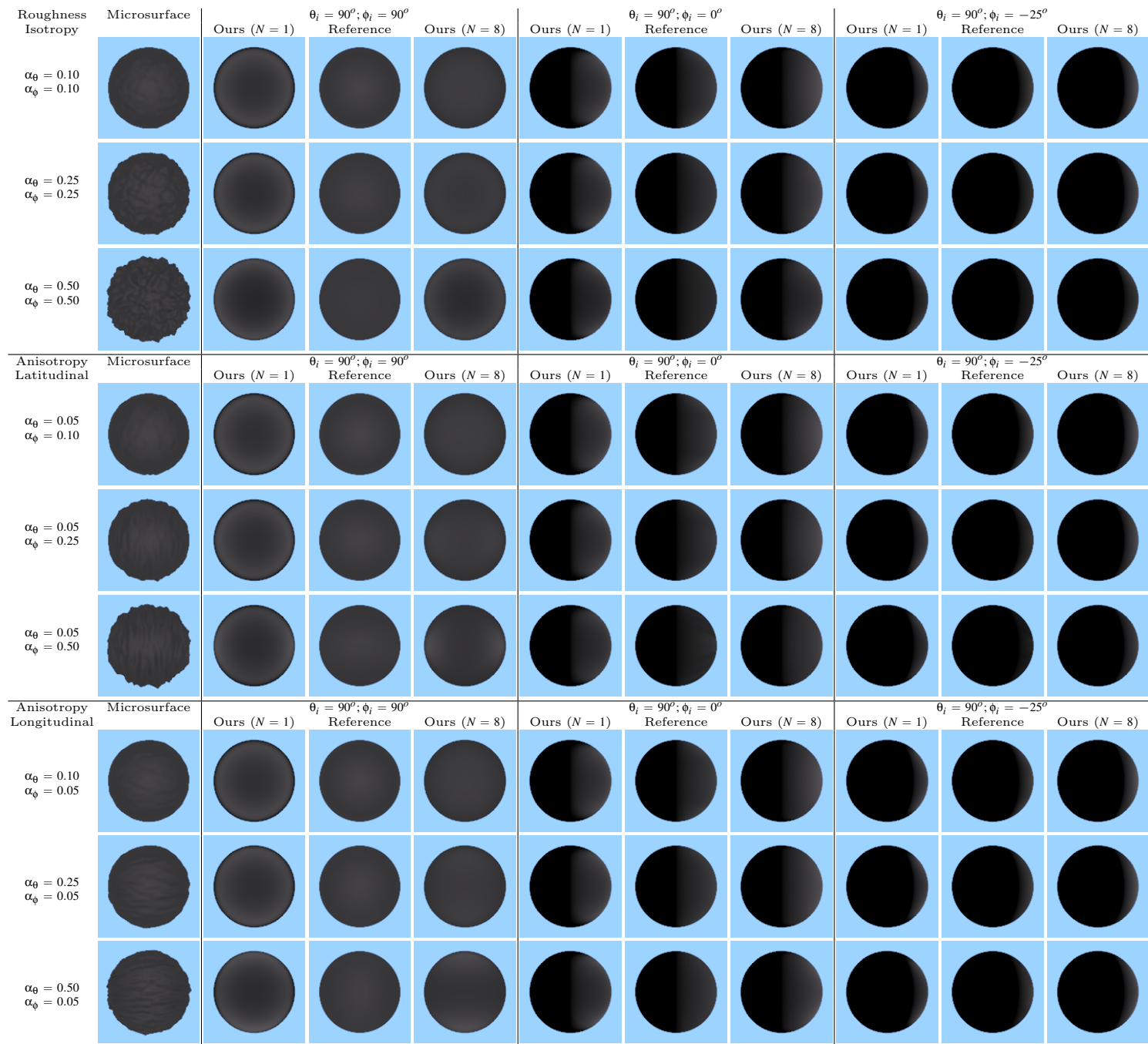
91 white-acrylic



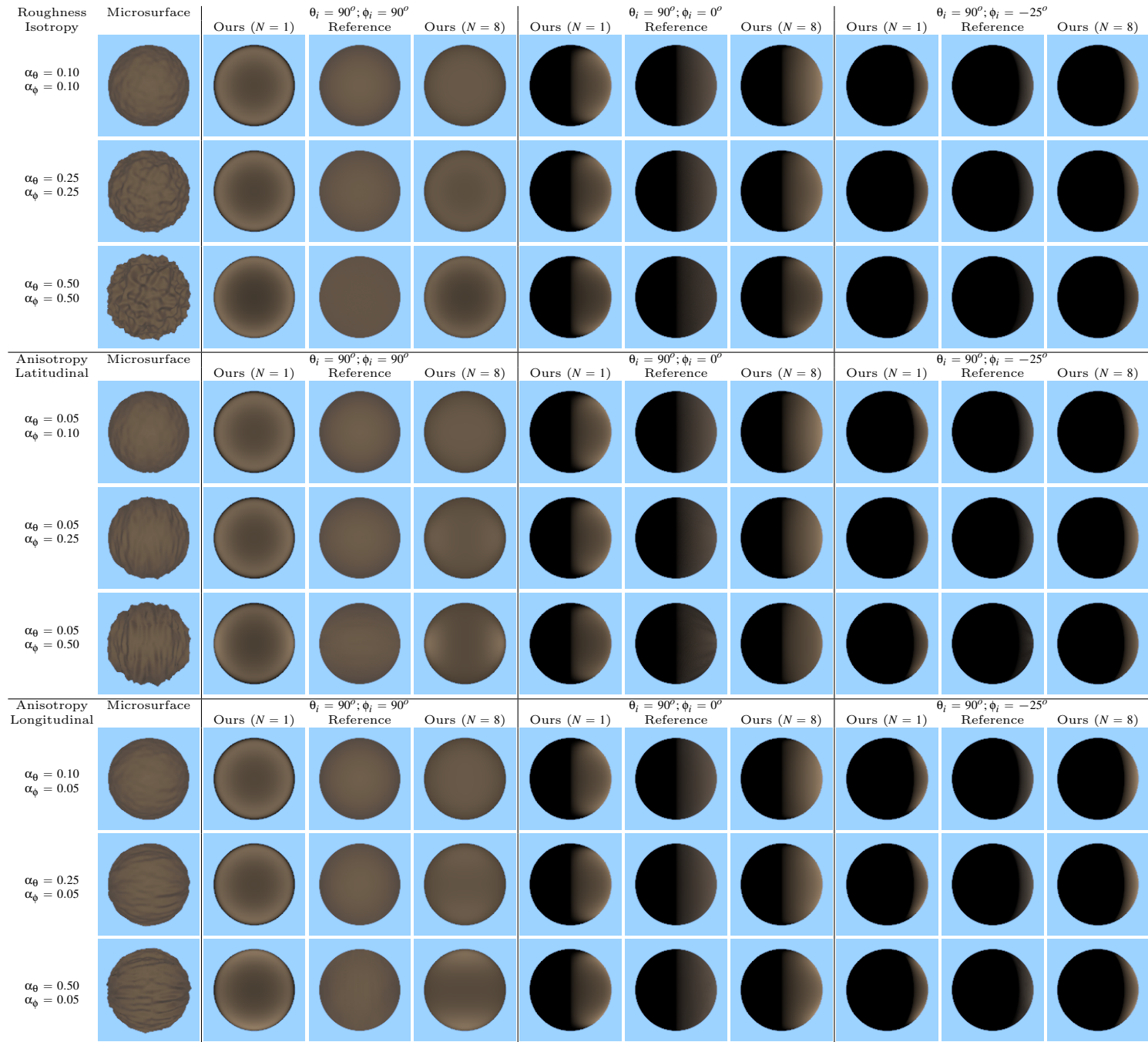
92 white-diffuse-bball



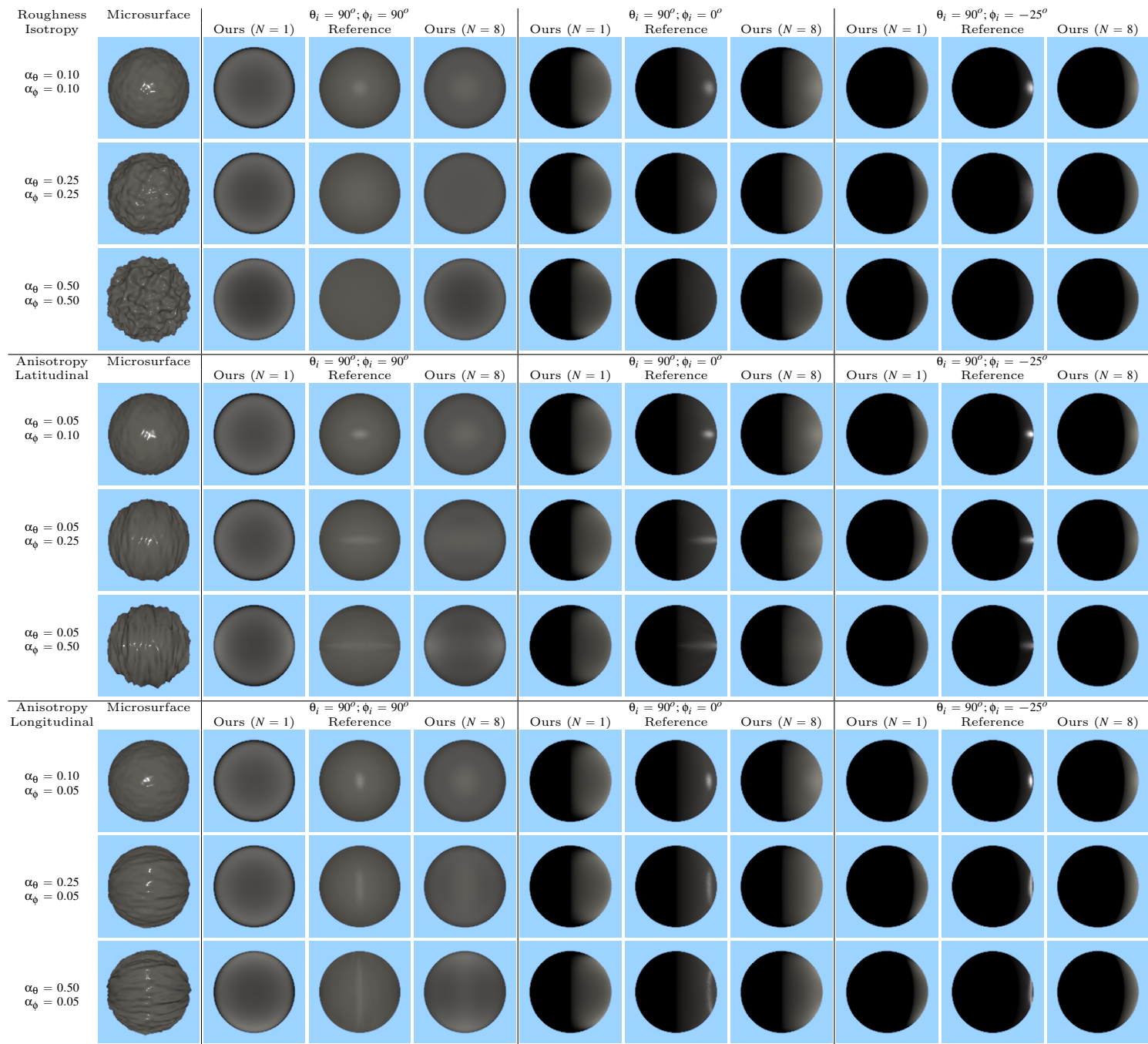
93 white-fabric2



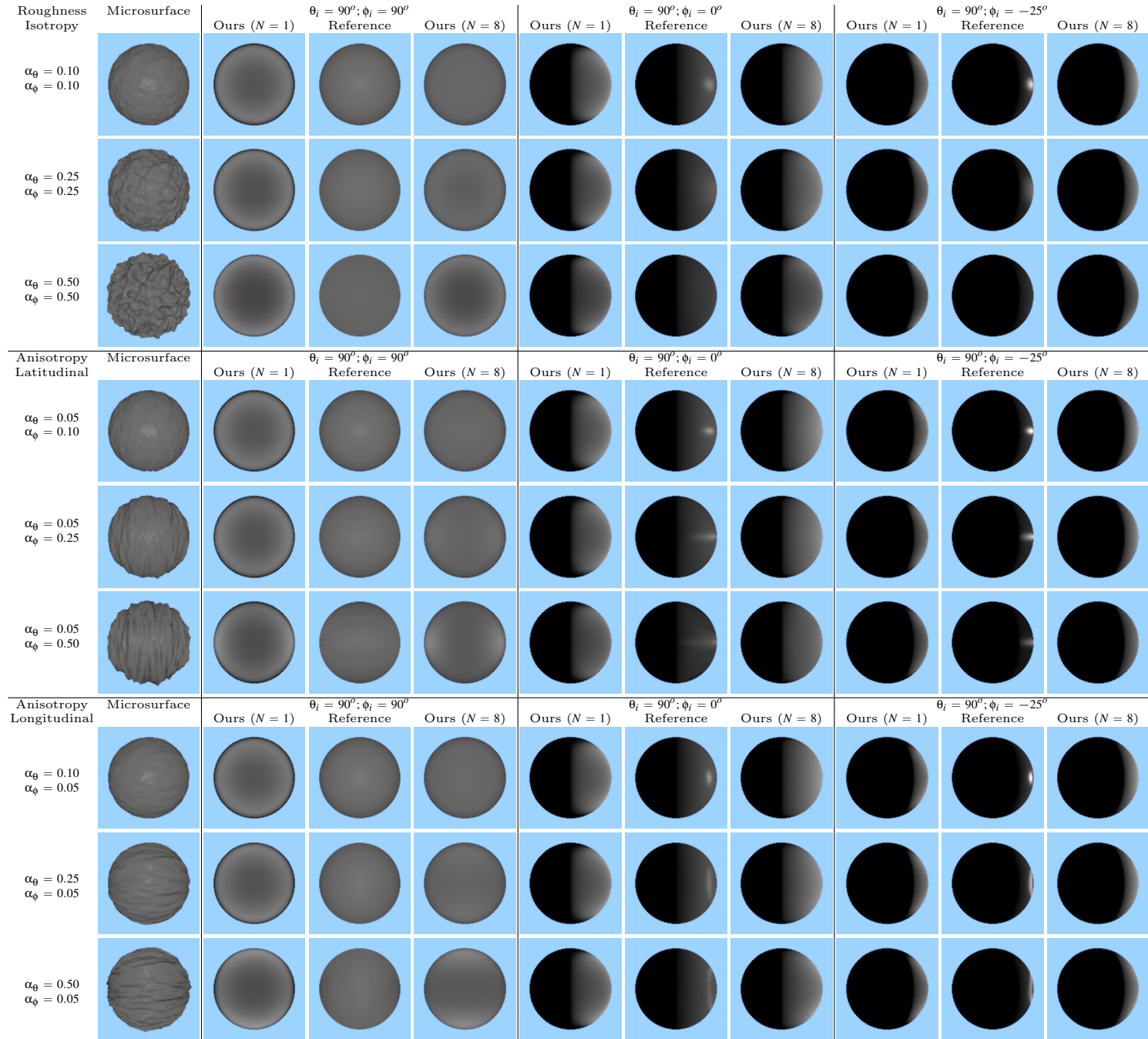
94 white-fabric



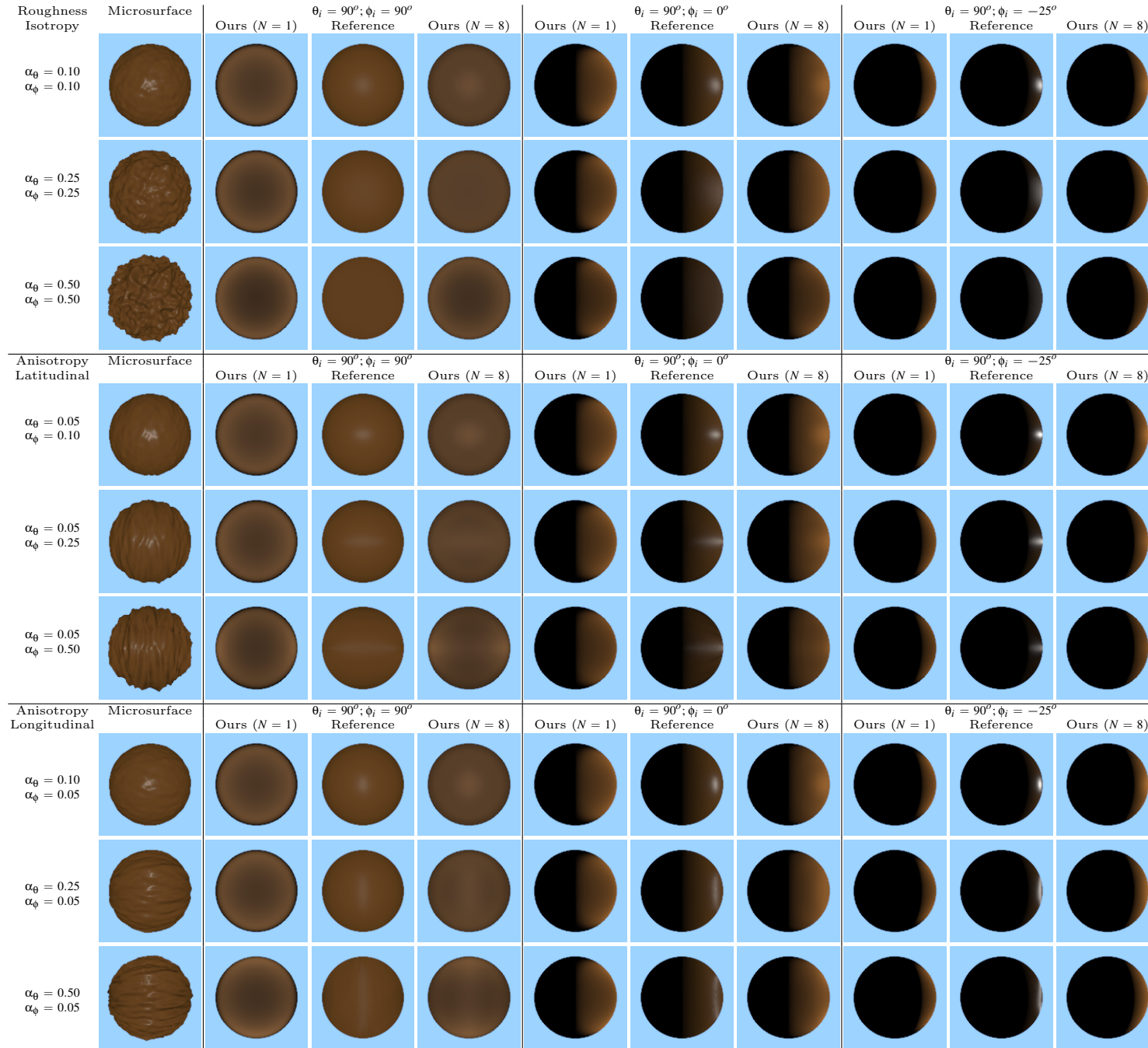
95 white-marble



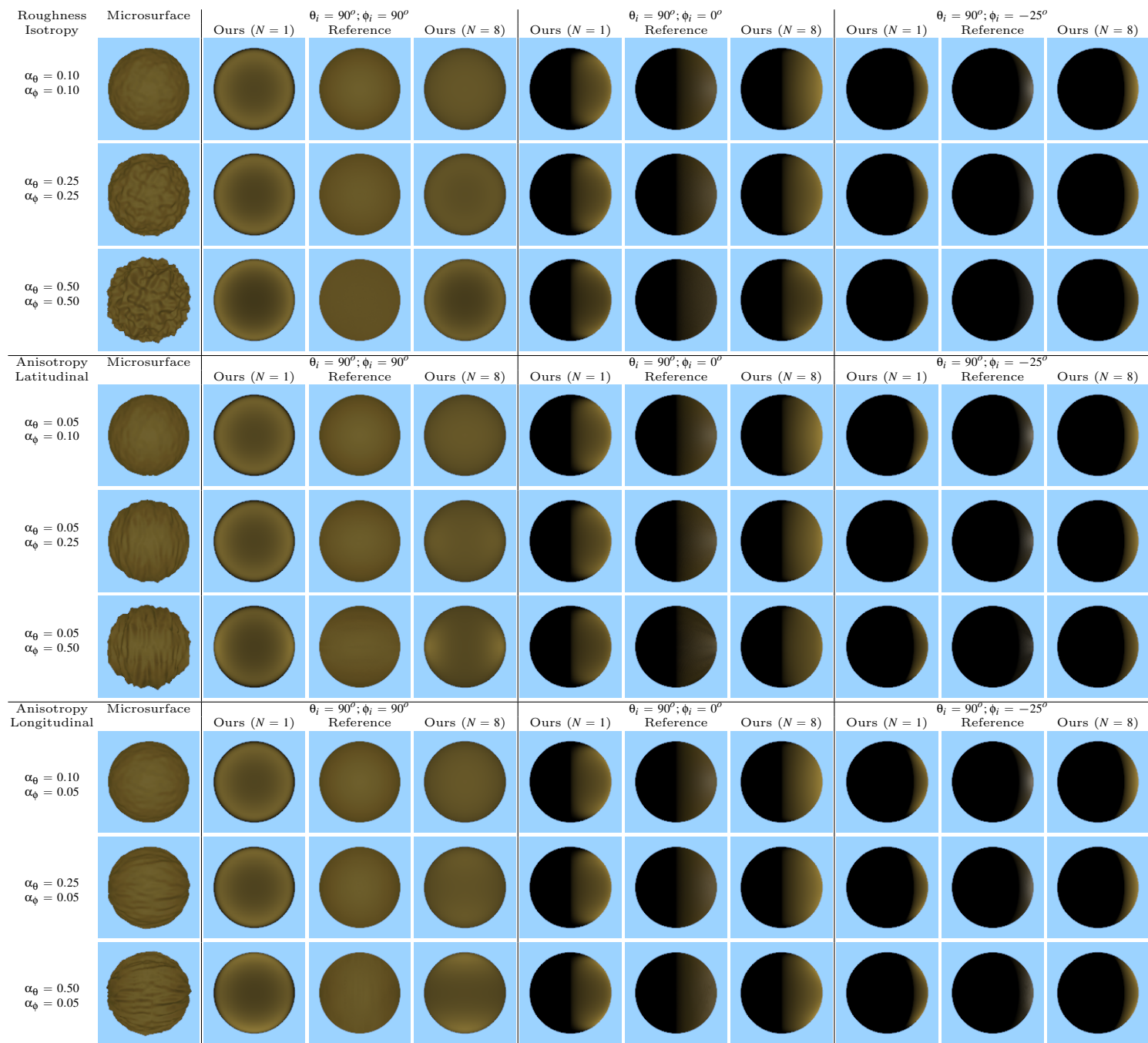
96 white-paint



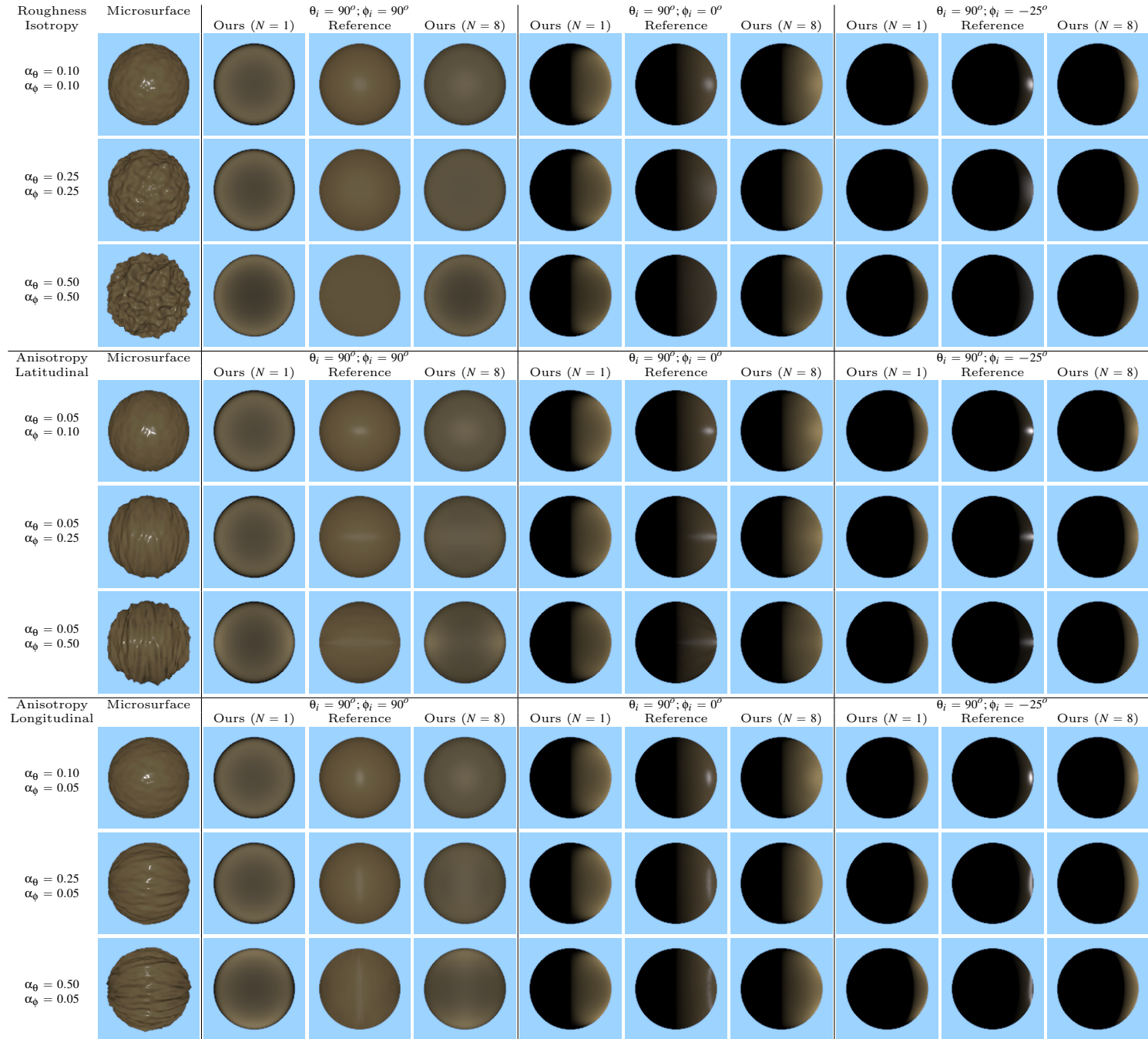
97 yellow-matte-plastic



98 yellow-paint



99 yellow-phenolic



100 yellow-plastic

