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## Measurement of Energy Reflection Coefficient Using Stress Waves

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**ABSTRACT.** To make clear the reflection characteristics at fillet parts, the energy reflection coefficient from the fillets is measured. The fillet is composed of the same material as the structure. A semiconductor strain gauge is used to detect single stress pulses. Different from the conventional method where the energy reflection coefficient is the summation of the transfer function obtained by the Discrete Fourier Transform, the coefficient is calculated using strain energy measured by changing the duration of a single stress pulse and the curvature of the fillet. The effectiveness of the new method is confirmed by comparing the results obtained by conventional and new methods.

**Résumé :** Les caractéristiques de la réflexion d'une onde sur un épaulement sont mesurées. L'épaulement est composé du même matériau que la structure. Une jauge à fil résistant semiconducteur est utilisée pour détecter l'impulsion de contrainte. Le coefficient de réflexion est calculé directement par l'énergie de déformation en changeant la durée de l'impulsion et la courbure de l'épaulement. Cette méthode est différente de la méthode conventionnelle où le coefficient de réflexion est obtenu par sommation de la fonction de transfert par la transformation de Fourier. L'efficacité de la nouvelle méthode est confirmée en comparant les résultats obtenus par les deux méthodes.

### 1. INTRODUCTION

A stress wave which propagates through a solid material exhibits complex behavior due to the reflection and transmission. Detailed investigation of the behavior of stress waves is essential to design structural members which receive impact load. To obtain basic information regarding dynamic strength of structural members, it is important to obtain reflection coefficient. Several studies have been done on the reflection characteristics of the stress waves at the parts with discontinuities in shapes[1]-[5] and also with inhomogeneity in materials[6].

A Fourier transform has been used so far to examine the reflection coefficient of a stress wave. However the Fourier Transform is a rather complicated method. Therefore the development of an analytical method without Fourier Transform is expected.

In this study, we propose a simple method for measuring reflection coefficients using a single pulse. In the simple method, only the energy of incident and reflected waves is taken into consideration without caring out the Fourier analysis. As an example, the reflection coefficients of a stress pulse propagating through a strip plate connected to a fillet was obtained experimentally. A single stress pulse was generated using a compressed air gun: a striker accelerated by the gun hit the surface of the strip plate with the fillet. To detect the pulse, a semiconductor strain gauge with high sensitivity and responsibility to wide range frequencies was used. Stress pulses of different duration were applied to the fillets with various cross-sections, and reflected pulses were measured. Next, reflection coefficients were calculated from the ratio of reflection pulse energy to incident pulse energy, i.e. the strain energy ratio. The reflection coefficient obtained by the proposed method was compared with the efficient by the Fourier Transform.

### 2. MEASUREMENT OF THE ENERGY REFLECTION COEFFICIENT USING A STRESS PULSE

When the incident longitudinal wave I and the reflected wave R are measured separately, the energy reflection coefficient is calculated as the ratios of incident energy to reflection energy. Figure 1 shows the schematic diagram of the stress pulse. In Figure 1,  $\epsilon(t)$  is the strain at the time  $t$ .  $I_s$  and  $I_e$  represent the start and the end points of the incident pulse respectively, and  $R_s$  and  $R_e$  are the equivalent points for the reflected wave. The  $N$  is the number of sampling points for the wave data used for Discrete Fourier Transform ( DFT ).

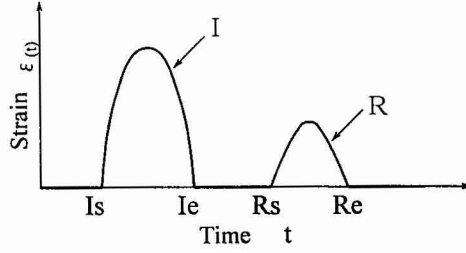


Figure 1: Schematic diagram of stress pulses

The power spectra of the incident wave and the reflected waves are expressed as

$$I(k) * I(k) = \left| \sum_{n=Is}^{Ie} \varepsilon(n) \exp\left(-j2\pi \frac{n}{N} k\right) \right|^2, \quad \dots(1)$$

$$R(k) * R(k) = \left| \sum_{n=Rs}^{Re} \varepsilon(n) \exp\left(-j2\pi \frac{n}{N} k\right) \right|^2, \quad \dots(2)$$

where  $j^2 = -1$ ,  $k$  is the discrete frequency,  $n$  is the discrete time, and  $I(k)^*$  and  $R(k)^*$  are the complex conjugates of  $I(k)$  and  $R(k)$ , respectively. The  $k$  is an integer between 1 and  $N/2$  when  $fs/N$  is used as a unit and  $fs$  is the sampling frequency. Here,  $fs = 256$  and  $N = 1024$ .

The reflection coefficient  $A(k)$  is calculated from equations (1) and (2).

$$A(k) = R(k)^* R(k) / I(k)^* I(k). \quad \dots(3)$$

When a dynamic load is applied to an elastic body with a length of  $\ell$ , the body deforms and the strain energy is stored in the body. Because the strain changes with time, the strain energy is also changed. In this case, the strain energy for incident and reflected pulses are given by the following equations.

$$I_u = \frac{1}{2} E \ell A \sum_{n=Is}^{Ie} |\varepsilon(n)|^2, \quad \dots(4)$$

$$R_u = \frac{1}{2} E \ell A \sum_{n=Rs}^{Re} |\varepsilon(n)|^2, \quad \dots(5)$$

where  $A$  is the cross-sectional area, and  $E$  is the Young's modulus. The reflection coefficient of the strain energy  $A_u$  is

$$A_u = R_u / I_u. \quad \dots(6)$$

Substituting eqs. (4) and (5) into eq. (6), we obtain

$$A_u = \left\{ \frac{1}{2} E \ell A \sum_{n=Rs}^{Re} |\varepsilon(n)|^2 \right\} / \left\{ \frac{1}{2} E \ell A \sum_{n=Is}^{Ie} |\varepsilon(n)|^2 \right\} = \frac{\sum_{n=Rs}^{Re} |\varepsilon(n)|^2}{\sum_{n=Is}^{Ie} |\varepsilon(n)|^2}. \quad \dots(7)$$

Therefore, the reflection coefficient of the strain energy can be obtained simply by calculating the ratio of  $R_u/I_u$  shown by the eq. (7). That is, if a single stress pulse is used, the reflection coefficient can be obtained

without using a DFT.

### 3. REFLECTION CHARACTERISTICS OF A STRESS PROPAGATION IN STRIP PLATE WITH A FILLET

Using the above-mentioned simple method, the reflection coefficient in a strip plate with a fillet was obtained experimentally.

#### 3.1 Shape of strip plate with fillet and measurement system

Figure 2 shows a schematic diagram of the experimental setup for a measurement system including a strip plate connected to the fillet, the position of a semiconductor strain gauge and a compressed air gun used in this experiment. The fillet is shown as a shadowed portion in the figure. An isotropic DAP (Diallylphtalate) resin is used as the material for the strip plate. The  $\rho$  is the density,  $c_1, c_2$  and  $E_1, E_2$  are the propagation velocity and of a pulse and the Young's modulus in the strip plate I and II respectively. The mechanical properties of the resin are as follows:  $E_1, E_2 = 3.09 \text{ G Pa}$ , Poisson's ratio  $\nu = 0.37$ ,  $\rho = 1.21 \times 10^3 \text{ kg/m}^3$ .

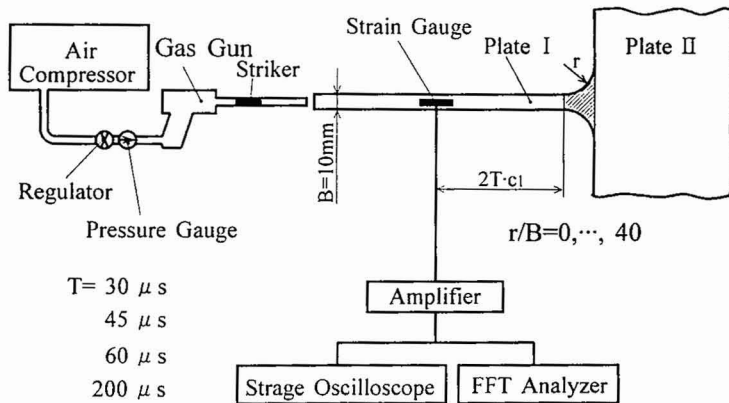


Figure 2: Schematic diagram of strip plate with fillet and experimental setup.

The  $r$  was the radius of the fillet. Thickness of the plate  $t$  was 6 mm. To obtain the reflection coefficient which depends on the pulse duration and the shape of the fillet, the width and the length of the plate II were semi-infinite. The radius  $r$  was changed keeping the plate width  $B = 10 \text{ mm}$  of the plate I. The  $r/B$  was varied by 8 steps from 0 to 40.0.

In order to generate the longitudinal stress pulse, the compression gas gun was employed due to the easiness to control the amplitude and duration time of stress pulse [7]. A circular striker bar made of an acrylic resin had a diameter of 4.5 mm and the impact edge was rounded to the radius of 2.3 mm. The length of the striker was changed from 5.5 to 235.0 mm to obtain the duration  $T$  of 30, 45, 60 and 200  $\mu\text{s}$ .

To detect the stress pulses, a bridge circuit which incorporated semiconductor strain gauges with a gauge length of 2 mm was used. The position of the strain gauge was determined to separate the incident and reflected pulses and also to prevent the distortion of the waveform during the propagation. As shown in Figure 2, the strain gauge was pasted at the position  $2T \cdot c_1$  from the fillet. The  $c_1$  was 1830 m/s, and the positions were 110, 165, 220, and 732 mm from the fillet for the duration of 30, 45, 60 and 200  $\mu\text{s}$  respectively. The waveform was observed using a storage oscilloscope, and the recording of the waveform and the frequency analysis were conducted using a Fast Fourier Transform (FFT) analyzer.

### 3.2 Measurement of the energy reflection coefficient from the fillet

For the comparison with the energy reflection coefficients  $A(k)$  and  $A_u$ , the  $A(k)$  was obtained by the frequency analysis using DFT.

Figure 3 shows a typical example of the waveforms of the incident pulse I and reflected pulse R obtained in the experiment. This is the case for  $r/B = 2.5$ , and  $T = 45 \mu s$ .

Figure 4 shows the relationship between the  $r/B$  and the  $A(k)$  calculated by eq. (3) using experimental data in terms of duration of pulses.

The  $c_1$  and  $c_2$  represent the propagation velocity of a one-dimensional and a two-dimensional stress waves respectively, and the acoustic impedance takes different values in each plate, which generates a reflected wave at the fillet. In addition, the sign of the reflected wave is the same as that of the incident wave, the reflection coefficient becomes negative.

In the case of  $r/B \approx 0$ , that is, the cross section changes suddenly at the fillet, the pulse begin to propagate two dimensionally. The reflection coefficient is determined by the width of the transmitted-side [8]. Accordingly, the reflection coefficient has a maximum.

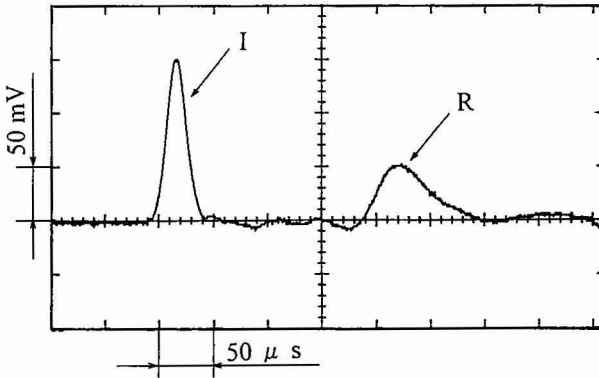


Figure 3: Waveform of the stress pulse .  
( I : incident, R: Reflected,  $r/B=2.5$ ,  $T=45 \mu s$  )

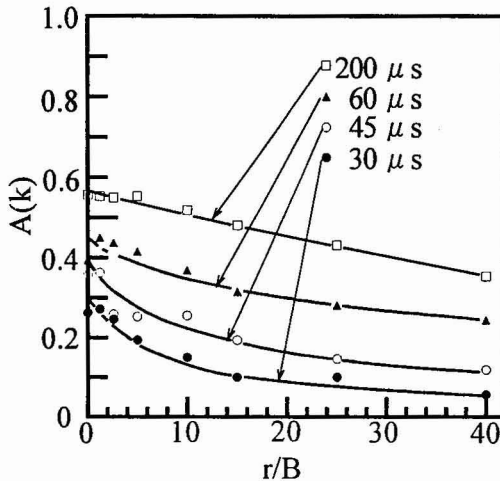


Figure 4: Reflection coefficients  $A(k)$  derived from the power spectrum of stress pulse at the fillet.

As the  $r/B$  ( $B$ : constant) increases, the cross-sectional area at the fillet increased gradually from the plate

I to the plate II. The velocity of the incident pulse is constant. However, the velocity at the fillet changes gradually depending on the shape of the fillet, that is, the acoustic impedance changes gradually. The reflection coefficient depends on the acoustic impedance. Therefore, if the  $r/B$  increases more, the reflection coefficient decreases.

As the duration becomes shorter, the reflection coefficient becomes smaller. Figure 5 (a) and (b) show representative power spectra calculated by eqs. (1) and (2) where the results of the impact test for various  $r/B$  and duration are used. The (a) shows the result for  $r/B = 2.5$ , and  $T = 45 \mu s$ , and (b) for  $T = 200 \mu s$ . The figure (a) shows that the incident pulse for a short duration contains fairly high frequency components, on the other hand, the high frequency components are lost in the reflected pulse. Reduction of the power may be caused by transmission of the high frequency components. However, in case of (b), the incident pulse contains low frequency components mainly, and little reduction of the power is observed. Low frequency components reflect and the reflection coefficient becomes high.

Next, the value of the  $A_u$  was obtained by eq. (6) changing the  $r/B$  similar to the case of  $A(k)$  (Figure 4). Figure 6 shows the relationship between  $r/B$  and  $A_u$ . The  $A_u$  and  $A(k)$  coincide with each other. This result shows that we can obtain reflection coefficient by using the pulse energy instead of complicated DFT procedure.

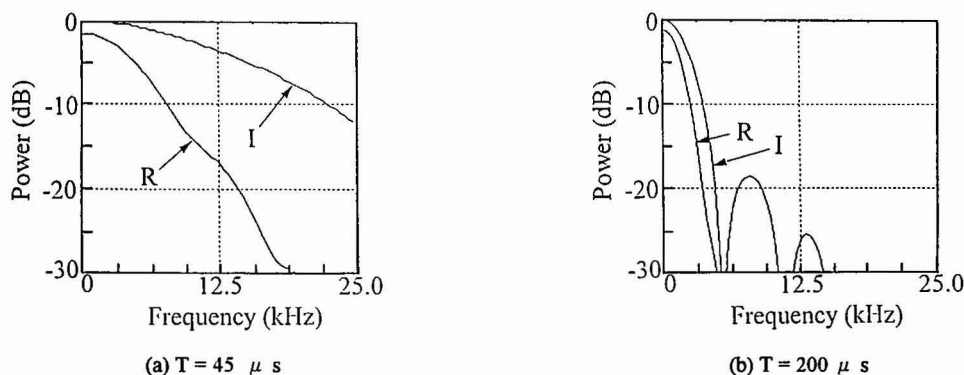


Figure 5: Power spectra of incident and reflected pulse.

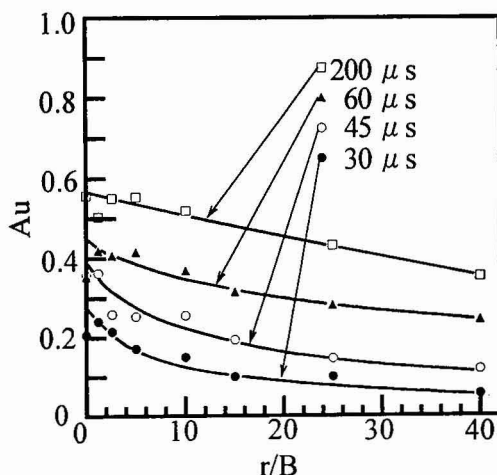


Figure 6: Reflection coefficients  $A_u$  obtained by a simple method.

#### 4. CONCLUSION

In order to clarify the reflection characteristics of the stress waves in the strip plate with fillet, the reflection coefficient of stress pulse was measured. We measured the energy reflection coefficient by a simple method using single pulse. The reflection coefficient obtained by the simple method coincides with the efficient by the conventional Fourier Transform.

As the result, the following conclusions are obtained.

- 1) As the radius of the fillet increases, reflection coefficient decreases.
- 2) As the duration of the incident pulse becomes shorter, the reflection coefficient becomes smaller because of transmission of the high frequency components.
- 3) Using the stress pulses, the energy reflection coefficients can easily be obtained by the strain energy without performing the DFT.

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