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Propagation of Adiabatic Shear Bands

S. Mercier and A. Molinari

Laboratoire de Physique et Mécanique des Matériaux, URA N°1215 du CNRS, I.S.G.M.P., Université de Metz, Ile du Saulcy, 57045 Metz, France

Abstract. The dynamic propagation of a shear band is assessed in mode II. A layer of finite thickness is subjected to simple shearing. The band propagates in the shear direction under steady state conditions. A variational approach is developed to calculate the shear band velocity and the length of the process zone. We analyse the roles of inertia, of loading conditions and of material properties such as yield stress, strain (or thermal) softening, strain hardening, strain rate sensitivity. The effect of elastic energy release is shown to be important in general.

Résumé. On considère la propagation d'une bande de cisaillement en mode II. Une couche de largeur finie et de longueur infinie est soumise à un cisaillement simple. La bande se propage dans la direction de cisaillement en régime stationnaire. La vitesse de propagation ainsi que la largeur de la zone de transition sont calculées par une approche variationnelle. Nous analysons les rôles respectifs de l'inertie, des conditions de chargement et des paramètres rhéologiques. On montre l'importance de la restitution d'énergie élastique dans ce processus.

1 Introduction

Adiabatic shear bands are narrow layers of intense shearing, observed in metals sustaining high rates of deformation. The mechanism of formation of these bands, as a result of a thermomechanical instability, has been analysed in details. Most of the analyses in the literature consider a shear band as a one dimensional phenomenon. However the mechanism of propagation of a band involves three (or at least two) dimensional aspects. Experimental studies, carried out by Marchand and Duffy [1] and Zhou *et al.* [2], provide informations on dynamic propagation of a shear band. Some analyses have been developed to model shear band propagation [3,4].

In this paper, we consider a layer of finite thickness, sustaining simple shearing. A shear band propagates in the shear direction under steady-state conditions. The material is elastic-viscoplastic and presents a thermal softening. Assuming incompressibility of the flow and adiabatic conditions, a variational approach is developed. Admissible velocity fields are defined in terms of three parameters: the shear band velocity, the characteristic length of the process zone and the width of the shear band. The analysis allows the calculation of the shear band celerity and of the length of the process zone, for a given shear band width. The effect of material parameters on the propagation process is analysed.

2 Kinematic of a propagating band

We consider an infinitely long layer with uniform properties and uniform thickness $2h$. Constant velocities $\pm V$ are prescribed at the boundaries $y = \pm h$. The shear band thickness is supposed to be

well characterized and has the value $2h_o$. The material far ahead of the tip is taken to be sheared uniformly, while the medium far behind the tip is pulled at velocity $\pm V$ outside the shear band ($|y| \geq h_o$), and undergoes an uniform shear strain rate within the band ($|y| \leq h_o$).

A cartesian coordinate system (x, y, z) is used as reference. This system is placed at the tip of the band and is translated with it along the specimen axis at the constant tip velocity C . Plane strain conditions are assumed. To represent the velocity and the strain rate fields in the specimen, assuming elastic and plastic incompressibility, we use an Eulerian formulation and introduce a stream function ϕ . This leads to

$$\begin{cases} \dot{\epsilon}_{xx} = \phi_{,xy} & \dot{\epsilon}_{yy} = -\phi_{,xy} & \dot{\epsilon}_{xy} = \frac{1}{2}(\phi_{,yy} - \phi_{,xx}) \\ v_x = \phi_{,y} & v_y = -\phi_{,x} \end{cases} \quad (1)$$

The kinematic of the flow is characterized by the celerity of the band C , the width of the band $2h_o$ and the process zone, of characteristic length λ , which is the region at the shear band tip, where a rapid change in the velocity profile is observed from linear to piecewise linear (see Fig. 1).

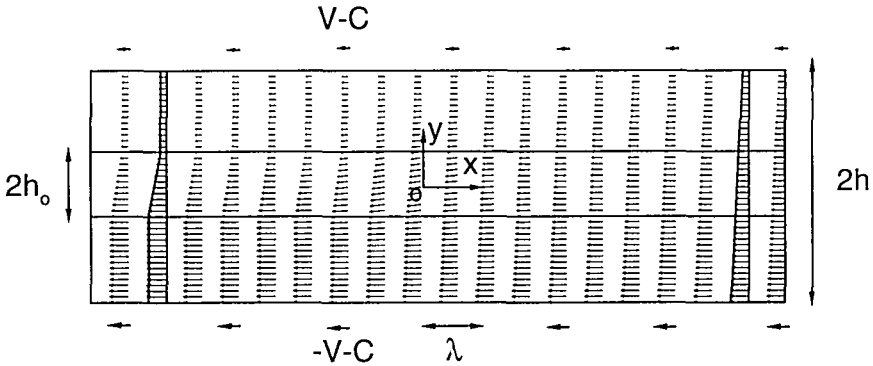


Figure 1: View of the modelled velocity field, measured with respect to the frame moving at the celerity C of the shear band tip. Note the change in the velocity profile, in the vicinity of the process zone, from linear to piecewise linear.

To satisfy incompressibility and boundary conditions, the stream function is defined as follows:

$$\phi(x, y) = \begin{cases} \phi_1(x, y) & \text{for } h_o \leq |y| \leq h \\ \phi_2(x, y) & \text{for } |y| \leq h_o \end{cases} \quad (2)$$

$$\phi_1(x, y) = \frac{Vh}{\alpha(x)} \left(\left| \frac{y}{h} \right|^{\alpha(x)} - 1 \right) - Cy \quad (3a)$$

$$\phi_2(x, y) = h \left[\frac{V}{\alpha(x)} \left(\left| \frac{h_o}{h} \right|^{\alpha(x)} - 1 \right) - \frac{Cy}{h} \right] - V \left| \frac{h_o}{h} \right|^{\alpha(x)-1} \frac{h_o^2 - y^2}{2h_o} \quad (3b)$$

with

$$\alpha(x) = \frac{3}{2} + \frac{1}{2} \tanh \left(\frac{x}{\lambda} \right) \quad (4)$$

3 Variational approach

An approximation of the real solution is obtained using a variational approach. The width of the band $2h_0$ is considered as given; thus only λ and C have to be optimized. Virtual velocity fields are defined by:

$$\delta v_i = \frac{\partial v_i}{\partial C} \delta C + \frac{\partial v_i}{\partial \lambda} \delta \lambda \quad (i = 1, 2) \quad (5)$$

The conditions satisfied by the virtual velocity field along all the boundaries of the layer ($-\infty \leq x \leq +\infty$ and $y = \pm h$, $-h \leq y \leq h$ and $x = \pm\infty$) are.

$$\delta v_x = -\delta C \quad , \quad \delta v_y = 0 \quad (6)$$

The principle of virtual work applied to the real stress field σ_{ij} and the virtual velocity fields δv_i can be expressed as

$$\delta I = \int_{-\infty}^{+\infty} \int_{-h}^h (s_{ij} \delta \dot{\epsilon}_{ij} + \rho v_{i,k} v_k \delta v_i) dx dy + \delta C \int_{-h}^h [\sigma_{xx}]_{-\infty}^{+\infty} dy + \delta C \int_{-\infty}^{+\infty} [s_{xy}]_{-h}^h dx = 0 \quad (7)$$

Considering that the relation (7) applies for any values of δC and $\delta \lambda$, two relationships are obtained:

$$\int_{-h}^h [\sigma_{xx}]_{-\infty}^{+\infty} dy + \int_{-\infty}^{+\infty} [s_{xy}]_{-h}^h dx - \int_{-\infty}^{+\infty} \int_{-h}^h \rho \left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} \right) dx dy = 0 \quad (8a)$$

$$\int_{-\infty}^{+\infty} \int_{-h}^h \left(x s_{ij} \frac{\partial \dot{\epsilon}_{ij}}{\partial x} + 2 s_{xx} \dot{\epsilon}_{xx} + 2 s_{xy} \frac{\partial v_y}{\partial x} \right) dx dy - \rho C \int_{-\infty}^{+\infty} \int_{-h}^h \left(\left(\frac{\partial v_x}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial x} \right)^2 \right) dx dy = 0 \quad (8b)$$

The equation (8a) is simply the equation of momentum balance for the entire layer. The last term in (8a) is easily calculated with use of the definition of the velocity fields:

$$\int_{-\infty}^{+\infty} \int_{-h}^h \rho \left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} \right) dx dy = -\frac{4}{3} \rho V^2 (h - h_0) \quad (9)$$

The calculation of the other terms which are stress dependent needs to introduce the constitutive law.

4 Constitutive law

The material is assumed to be incompressible and elastic-viscoplastic. The inelastic behaviour is described using a finite strain version of the classical J_2 flow theory. The elastic law has the following form:

$$s_{ij}^* = C_{ijkl} (\dot{\epsilon}_{ij} - \dot{\epsilon}_{ij}^p) \quad (10)$$

where C_{ijkl} are the elastic moduli of the material, $\dot{\epsilon}_{ij}$ is the total strain rate and $\dot{\epsilon}_{ij}^p$ is the plastic strain rate. In this expression, $s_{ij}^* = \dot{s}_{ij} - \omega_{jk} s_{ki} - \omega_{ik} s_{kj}$, denotes the Jaumann rate of the stress deviator. The plastic strain rate is given by the J_2 flow theory:

$$\dot{\epsilon}_{ij}^p = \frac{3}{2} \frac{\dot{\epsilon}}{\sigma_e} s_{ij} \quad (11)$$

where σ_e and $\dot{\bar{\epsilon}}^p$ represent the effective stress and the effective plastic strain rate, respectively given by

$$\sigma_e = \left(\frac{3}{2} s_{ij} s_{ij} \right)^{\frac{1}{2}} \quad \dot{\bar{\epsilon}}^p = \left(\frac{2}{3} \dot{\epsilon}_{ij}^p \dot{\epsilon}_{ij}^p \right)^{\frac{1}{2}} \quad (12)$$

The cumulated plastic deformation is obtained by integration of the effective plastic strain along streamlines. The material behaviour is further described by expressing the equivalent stress in terms of the cumulated plastic strain, effective plastic strain rate and temperature:

$$\sigma_e = K (\bar{\epsilon}^p)^n T^{-\nu} (\dot{\bar{\epsilon}}^p)^m \quad (13)$$

where the strain rate sensitivity exponent m is assumed positive, n being the strain hardening exponent ($n > 0$) and ν the thermal sensitivity exponent ($\nu > 0$).

The energy equation provides the evolution law for the temperature which, upon neglecting heat conduction, is

$$\rho c \dot{T} = \beta s_{ij} \dot{\epsilon}_{ij}^p = \beta \sigma_e \dot{\bar{\epsilon}}^p \quad (14)$$

where c is the heat capacity and β characterizes the proportion of plastic work converted into heat.

5 Shear band velocity and size of the process zone

At $x = +\infty$, simple shearing conditions are satisfied, and $s_{xx} = 0$ as a consequence of the flow law. For a strain rate sensitive material with softening, $s_{xx} = 0$ is also satisfied at $x = -\infty$. In addition the pressure $p = \frac{1}{3} tr \underline{\underline{\sigma}}$ is uniform at $x = \pm\infty$, therefore:

$$\int_{-h}^h [\sigma_{xx}]_{-\infty}^{+\infty} dy = 2h[p]_{-\infty}^{+\infty} \quad (15)$$

To calculate the pressure p , we use the principle of conservation of momentum for the upper half layer $-\infty \leq x \leq +\infty$, $0 \leq y \leq h$. This leads to:

$$h[p]_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} s_{xy}(x, h) dx - \int_{-\infty}^{+\infty} s_{xy}(x, 0) dx - \frac{1}{2} \rho (h - h_0) V (C - \frac{4}{3} V) = 0 \quad (16)$$

Equation (16) in conjunction with (8a) leads to the following relationship

$$\int_{-\infty}^{+\infty} [s_{xy}(x, h) + s_{xy}(x, -h) - 2s_{xy}(x, 0)] dx - \rho VC(h - h_0) = 0 \quad (17)$$

From the flow law, we have $s_{xx} = -s_{yy}$. Thus, equation (8b) can be written as

$$2 \int_{-\infty}^{+\infty} \int_{-h}^h \left(x [s_{xx} \frac{\partial \dot{\epsilon}_{xx}}{\partial x} + s_{xy} \frac{\partial \dot{\epsilon}_{xy}}{\partial x}] + s_{xx} \dot{\epsilon}_{xx} + s_{xy} \frac{\partial v_y}{\partial x} \right) dx dy - \rho C \int_{-\infty}^{+\infty} \int_{-h}^h \left(\left(\frac{\partial v_x}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial x} \right)^2 \right) dx dy = 0 \quad (18)$$

The celerity of the shear band tip C and the characteristic length of the process zone λ are computed with the set of equations (17) and (18).

6 Elastic-viscoplastic material

The constitutive behaviour is that introduced in section 4, with elastic effects accounted by the Hooke's law (10). The hardening law (13) includes strain rate, strain hardening and temperature effects. The material parameters are representative of a C.R.S. 1018 steel, see Table 1. The thickness of the layer $2h = 2.5\text{mm}$, the shear band thickness $2h_0 = 100\mu\text{m}$ ($h_0 = 0.04h$) and the velocity $V = 2\text{m/s}$ applied at the boundaries are consistent with the work of Marchand and Duffy [1]. Note that a nominal shear strain rate of 1600s^{-1} is applied .

Table 1: Material parameters for CRS1018

Parameter	K	m	n	ν	c	ρ
CRS1018	$6,302.10^6$ Int. Syst.	0.019	0.015	0.38	$500 \text{ J}(\text{kg}\cdot\text{K})^{-1}$	$7,800 \text{ kg m}^{-3}$

The effects of the strain rate sensitivity parameter m are illustrated in Fig. 2. For the C.R.S. 1018 steel, we have $m = 0.019$, and the value of $C = 1250\text{m/s}^{-1}$ is obtained for the shear band velocity. Increasing the values of m has a stabilizing effect manifesting itself by a significant decrease of the celerity C . Note that the size of the process zone λ is decreased when m is increased.

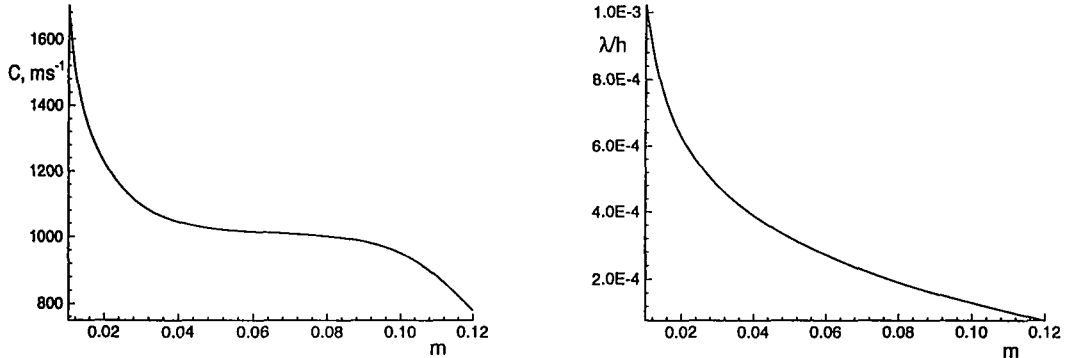


Figure 2: Stabilizing effect of the strain rate sensitivity m on the shear band celerity C and on the size of the process zone λ . The material is elastic viscoplastic and the following values have been used: $h = 1.25 \text{ mm}$, $h_0 = 0.04h$, $V = 2 \text{ ms}^{-1}$, together with the material parameters of Table 1, except for m .

Similarly, the influence of the hardening exponent n results in a smaller process zone and a smaller celerity C for increasing values of n . The celerity C is decreased from $C = 1250\text{m/s}$ to $C = 520\text{m/s}$ when n varies from 0.02 to 0.15.

When the value of the thermal sensitivity ν is increased, antagonistic effects are induced. In the one hand, thermal softening is enhanced, and this should favorize material instability. In the other hand, the stress level in front of the band is decreased in a significant way (from $\tau = 710\text{MPa}$ to $\tau = 360\text{MPa}$ when ν varies from 0.3 to 0.42). Therefore heat production is lowered and material instability is refrained. To really capture the role of the thermal sensitivity, we adjust the level of K in equation (13) so as to have the same flow stress in front of the band for all values of ν . The effects so obtained are solely due to material softening. We observe in Fig. 3 a strong increase of C and λ with ν .

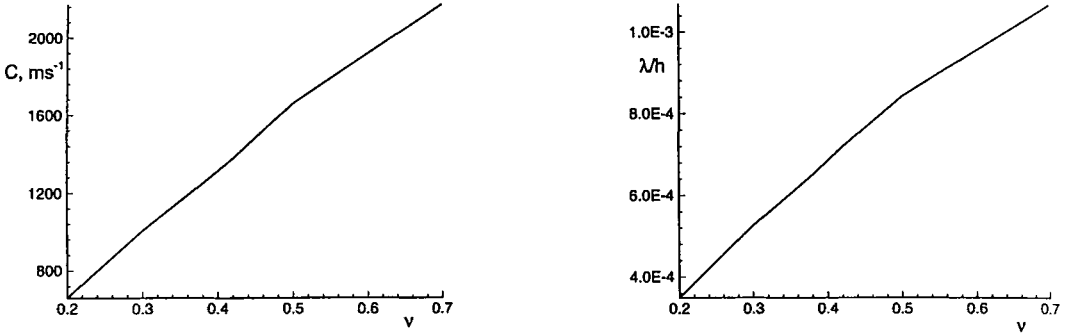


Figure 3: Effect of the thermal softening parameter ν on the shear band velocity C and on the size of the process zone λ . We have taken $m = 0.019$ and the same values of the parameters as those of Fig. 2.

The effect of elasticity is analysed in Fig. 4. Due to the incompressibility assumption, only the shear modulus μ enters in the formulation of the model. Elastic energy release, associated to the stress drop in the vicinity of the shear band tip, contributes to the propagation process in the same way that in classical fracture mechanics. Therefore, elastic effects are expected to promote instability. The smaller is the shear modulus, the more important is the elastic energy release, and the higher is the shear band velocity, see Fig. 4. The length of the process zone follows the variation of C . The celerity, for a rigid plastic material ($\mu \rightarrow +\infty$), has a value of 268ms^{-1} to be compared with the celerity of 1250ms^{-1} obtained for the C.R.S. 1018 steel including elastic effects. Elastic energy release appears to have an important role, and could lead, if neglected, to a large underestimation of the shear band velocity.

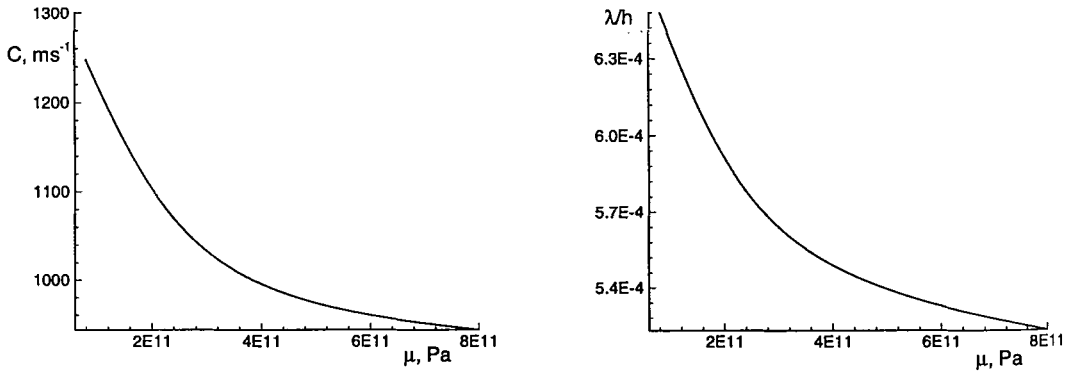


Figure 4: Effect of the elastic energy release on the shear band propagation. The smaller is the elastic shear modulus μ , the larger is the energy release and the higher is the shear band velocity. We have taken $m = 0.019$ and the same values of the parameters as those of Fig. 2.

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