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## Numerical results and Biot theory in anisotropic porous/fibrous media

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Biot theory [1] is a basic tool when studying the propagation of elastic waves in porous/fibrous solids. Due to the simplicity of the assumptions, isotropic materials are most often utilized [2,3]. However, in some cases, anisotropy has to be included in the modelization, e.g. with some fibrous materials. In this work, we have developed routines, based on the standard Biot model, which compute the various characteristic surfaces (slowness, phase and group velocities) [4] in a given plane for anisotropic porous media. By increasing the weight of the coupling constants, one can continuously vary from the limit case of anisotropic solids with no porosity to a number of different porous media. When the coupling terms are not equal to zero, one finds four instead of three propagating modes, the two other eigenvalues being complex, meaning in turns that the two remaining solutions do correspond to some evanescent modes. The slow transverse mode is unchanged. The fourth mode often called the Biot mode [5], which is very similar to the pseudo-thermal wave [6,7] observed in dynamic thermoelasticity, has always a very small wavespeed and is very highly damped.

### BIOT THEORY AND ANISOTROPY

One starts from the linear elasticity behaviour equation written down in tensorial form:

$$\begin{bmatrix} \sigma_{ij} \\ \sigma \end{bmatrix} = \begin{bmatrix} C_{ijkl} & Q_{ij}\delta_{ij} \\ Q_{kl}\delta_{kl} & R \end{bmatrix} \begin{bmatrix} \varepsilon_{kl} \\ \varepsilon \end{bmatrix}, \quad (1)$$

where  $i, j, k, l = 1, 2, 3$ , and where  $\sigma_{ij}$  and  $\sigma$  represents the stresses in the solid and in the fluid,  $\varepsilon_{kl}$  and  $\varepsilon$  being the corresponding strains. The elasticity tensor is denoted by  $C_{ijkl}$ , the

compressibility coefficient of the fluid by R, and Q is the diagonal coupling coefficient. Next, the equations of motion respectively of the solid and fluid phases are written as:

$$\begin{aligned} \rho_{11} \frac{\partial^2 u_i^s}{\partial t^2} + \rho_{12} \frac{\partial^2 u_i^f}{\partial t^2} &= C_{ijkl} \frac{\partial^2 u_i^s}{\partial x_j \partial x_k} + Q_{ii} \frac{\partial^2 u_i^f}{\partial x_i^2} \\ \rho_{12} \frac{\partial^2 u_i^s}{\partial t^2} + \rho_{22} \frac{\partial^2 u_i^f}{\partial t^2} &= Q_{ii} \frac{\partial^2 u_i^s}{\partial x_i^2} + R \frac{\partial^2 u_i^f}{\partial x_i \partial x_i} \end{aligned} \quad (2)$$

where  $u$  represent the displacement fields of the solid phase (S) and of the fluid (f), and where  $\rho_{11}$ ,  $\rho_{22}$  et  $\rho_{12}$  respectively are the densities of the solid, of the fluid and a coupling term such that  $\rho_{11}\rho_{11} - \rho_{12}^2 > 0$ . The next step deals with searching solutions in the form of plane waves, i.e.  $u_i = \exp j(\omega t - kx)$ .

Adding dissipative terms to equations (2) results in introducing complex densities (e.g.  $\tilde{\rho}_{11k} = \rho_{11k} - j(b_k/\omega)$ , where  $b_k$  are the components of the viscosity tensor). In such a case, the wavespeeds are complex and dispersive. We restrict here to the simpler case of real densities, which is however sufficient to describe the main features of related to anisotropy. Writing down the calculations yields the following compatibility equation :

$$\det = \begin{vmatrix} \Gamma_{11} - \rho_{11}V^2 & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} - \rho_{12}V^2 & \Gamma_{15} & \Gamma_{16} \\ \Gamma_{12} & \Gamma_{22} - \rho_{11}V^2 & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} - \rho_{12}V^2 & \Gamma_{26} \\ \Gamma_{13} & \Gamma_{23} & \Gamma_{33} - \rho_{11}V^2 & \Gamma_{34} & \Gamma_{35} & \Gamma_{36} - \rho_{12}V^2 \\ \Gamma_{14} - \rho_{12}V^2 & \Gamma_{24} & \Gamma_{34} & \Gamma_{44} - \rho_{22}V^2 & \Gamma_{45} & \Gamma_{46} \\ \Gamma_{15} & \Gamma_{25} - \rho_{12}V^2 & \Gamma_{35} & \Gamma_{45} & \Gamma_{55} - \rho_{22}V^2 & \Gamma_{56} \\ \Gamma_{16} & \Gamma_{26} & \Gamma_{36} - \rho_{12}V^2 & \Gamma_{46} & \Gamma_{56} & \Gamma_{66} - \rho_{22}V^2 \end{vmatrix} = 0. \quad (3)$$

In such equation, V represents the phase wavespeed ( $V = k/\omega$ ), and the terms  $\Gamma_{ij}$  are linked to  $C_{ijkl}$ , R, et Q. For instance, the  $\Gamma$  components of the top left 3 x 3 sub-determinant is nothing else than the Christoffel tensor, i.e.  $\Gamma_{ij} = C_{ijkl} n_j n_k$ , where i, j, k, l = 1, 2, 3, where  $n$  is the wavevector direction. The full determinant has 4 real roots corresponding to propagating modes as well as two additional imaginary roots which do not contribute to the propagation. By supposing the propagation to take place in plane (1,3) of a transverse isotropic solid where  $n_1 = \cos\theta$ ,  $n_2 = 0$ , et  $n_3 = \sin\theta$ , then the above determinant has the simpler form :

$$\det = V^2 [(\rho_{11}\rho_{22} - \rho_{12}^2)V^2 - \rho_{22}\Gamma_{22}] \begin{vmatrix} \Gamma_{11} - \rho_{11}V^2 & \Gamma_{13} & \Gamma_{14} - \rho_{12}V^2 & \Gamma_{16} \\ \Gamma_{13} & \Gamma_{33} - \rho_{11}V^2 & \Gamma_{34} & \Gamma_{36} - \rho_{12}V^2 \\ \Gamma_{14} - \rho_{12}V^2 & \Gamma_{34} & \Gamma_{44} - \rho_{22}V^2 & \Gamma_{46} \\ \Gamma_{16} & \Gamma_{36} - \rho_{12}V^2 & \Gamma_{46} & \Gamma_{66} - \rho_{22}V^2 \end{vmatrix} = 0 \quad (4)$$

$$\begin{aligned}
 \Gamma_{11} &= C_{11}n_1^2 + C_{44}n_3^2 ; \Gamma_{14} = Mn_1^2 ; \Gamma_{16} = Mn_1n_3 \\
 \Gamma_{22} &= C_{66}n_1^2 + C_{44}n_3^2 ; \Gamma_{34} = Qn_1n_3 ; \Gamma_{36} = Qn_3^2 \\
 \Gamma_{33} &= C_{44}n_1^2 + C_{33}n_3^2 ; \Gamma_{44} = Rn_1^2 ; \Gamma_{46} = Rn_1n_3 , \\
 \Gamma_{13} &= (C_{13}+C_{44})n_1n_3 ; \Gamma_{66} = Rn_3^2
 \end{aligned}
 \tag{5}$$

with  $M = Q_{11} = Q_{22}$ , et  $Q = Q_{33}$ .

Accordingly, there exists a trivial root, i.e.  $V = \{\rho_{22}\Gamma_{22}/\rho_{11}\rho_{22}-\rho_{12}^2\}^{1/2}$ , which is identical to  $(\Gamma_{22}/\rho_{11})^{1/2}$  when  $\rho_{12}=0$ , which is the wavespeed of the pure transverse mode. Some significant numerical results obtained with the above procedure are plotted on the Figure. Further details are provided in reference [8].

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### CAPTION FOR THE ILLUSTRATION

*Figure : Characteristic surfaces (wavespeed curves) in plane (1,3) of a transversely isotropic porous material. The properties of the material are the following:*

*$C_{11} = 26.1$  GPa;  $C_{33} = 65.5$  GPa;  $C_{44} = 10.5$  GPa;  $C_{66} = 8.50$  GPa;  $C_{13} = 9.58$  GPa;  $C_{12} = C_{11}-2C_{66} = 9.31$  GPa;  $\rho_{11} = 2.108$  g/cm<sup>3</sup>;  $\rho_{22} = 1$  g/cm<sup>3</sup>;  $\rho_{12} = 0.1$  g/cm<sup>3</sup>.*

*a)  $M = Q = R = 0.5$  SI ; b)  $M = Q = 7.5$  SI ;  $R = 5$  SI.*

*The solid lines represent the group wavespeeds. The dashed lines represent the phase wavespeeds. The slow transverse mode nearly has identical phase and group wavespeeds.*

