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Modelling of Magnetostrictive Thin Films, Application to a Micromembrane

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Abstract. — The development of micro-actuators depends on the improvement, availability and use of “intelligent materials” such as piezoelectric or magnetostrictive materials. They allow a direct conversion of electromagnetic energy into mechanical energy. This article first presents a “state of the art” in the field of magnetostrictive materials in thin films. In a second step, a nonlinear magneto-mechanical strong coupling technique is developed to model such materials, it is applied to the example of a micromembrane in order to show the potential applications of such materials.

Résumé. — Le développement de micro-actionneurs dépend principalement du perfectionnement, de la disponibilité et de l'utilisation des matériaux intelligents comme les piézoélectriques ou les matériaux magnétostrictifs. Ils permettent une conversion directe de l'énergie électromagnétique en énergie mécanique. Dans un premier temps, cet article présente un état de l'art dans le domaine de la recherche sur les matériaux magnétostrictifs en couches minces. Dans un second temps, une technique de couplage fort magnéto-mécanique non linéaire est développée pour modéliser de tels matériaux. Elle est appliquée à la modélisation d'un exemple de micromembrane magnétostrictive, dans le but de montrer l'utilisation potentielle d'un tel phénomène dans le domaine des micro-actionneurs.

1. Introduction

New magnetostrictive materials, such as terfenol-D are mainly characterized by a strong coupling between magnetic and mechanical phenomena. The application of a magnetic field to these materials leads to important mechanical effects. The relations between their magnetic and mechanical characteristics are strongly nonlinear, reversible and may present some hysteresis.

Among the physical principles that enable to create a mechanical motion, the magnetostrictive effect introduces new fields for micro-actuators. Indeed, due to their strong magneto-mechanical coupling, magnetostrictive materials allow to develop electromagnetic devices with

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high energy density, fast response and fine precision. Recent developments in micromachining techniques of 3D microstructures enable to obtain significant forces with magnetic materials. Thus, at low scale, magnetostrictive materials have grown to be competitive to piezoelectric microtechnologies. As magnetostrictive thin films are metallic, they offer the advantage of being easily deposited on a substrate. They play the active role, for instance, to bend cantilevers or membranes. Moreover, they are sensitive to a magnetic field, which allows remote control.

In order to design magnetostrictive actuators an accurate modelling of their characteristics, including the nonlinearities and the magneto-mechanical coupling, is necessary. A finite element method computing process has been developed and adapted to magnetostrictive thin film problems. The coupling is achieved by a "strong coupling" method, which solves simultaneously the magnetic and the mechanical parts. This method is applied on a simple test device. In a first part some fundamentals about magnetostriction and materials for thin films are reviewed. The modelling process and the formulations used are then developed. At last some results about the modelling of a magnetostrictive membrane driven by a magnetic field are presented: the membrane bending *versus* the magnetic field and the influence of the film's thickness are studied.

2. Magnetostrictive Thin Films

2.1. INTRODUCTION. — Magnetostriction phenomenon has been discovered by the English scientist Joule in 1842. He first observed that an iron rod lengthens under the effect of a magnetic field directed following its axis. Other effects have then been observed, such as the Guillemin effect, the Wiedeman effect or the ΔE -effect. In general, magnetostriction can be defined as the relations between a body's mechanical characteristics and its magnetic characteristics [6,9].

Since the research for giant magnetostrictive materials has started, most of the work has been conducted about bulk materials of RE-Fe intermetallics. Nevertheless, giant magnetostrictive intermetallics applications and commercialization is still limited by their high production cost. The research and development of new giant magnetostrictive materials for thin films has recently been increased and may open new fields of application. Their production process is rather simple with respect to controlling composition and structure compared with those of bulk, moreover the two dimensional magneto-elastic interactions in giant magnetostrictive films will induce new applications. They can be deposited on conventional semiconductor devices, they may be very useful in order to develop a new generation of microsensors or micro-actuators. They are magnetically driven or sensitive, and able to integrate the mechanical part as well as the electronic part (command) in microdevices below the millimetre dimension [15,19,26,34]

2.2 THE PHENOMENON OF MAGNETOSTRICTION. — The crystalline network induces some easier magnetizing directions, reciprocally it is sensitive to the variations of the external magnetic field. Indeed, a magnetic field disturbs the electronic equilibrium in the network and then modifies the mechanical characteristics of the material: the crystal deforms spontaneously in order to lower the anisotropy energy. On the other hand, a mechanical stress modifies the crystalline network, and then changes the magnetic characteristics of the material.

The magnetostriction coefficient λ is usually defined by the relative length variation of the material magnetized at saturation under a direction β , the reference is the demagnetized state where the magnetostriction coefficient is zero. For a polycrystalline and isotrope sample magnetized at saturation.

$$\lambda(\theta) = \frac{3}{2} \lambda_s \left(\cos^2 \theta - \frac{1}{3} \right) \quad (1)$$

Table I. — Curie Temperature, magnetostriction of some amorphous and polycrystalline magnetostrictive materials (Data from [9, 10, 33]).

Material	λ_s (ppm)	T_C (°C)	λ_s (ppm)	T_C (°C)
	polycrystal	polycrystal	amorphous	amorphous
TbFe ₂	1735	711	475	425
DyFe ₂	433	635	50	362
SmFe ₂	-1560	676	-300	403
Tb _{0.3} Dy _{0.7} Fe ₂	1100	380	375	—

where: θ is the angle between magnetization direction and observed direction, λ_s is the isotropic magnetostriction coefficient. λ_s is usually measured with the parallel ($\lambda_{//}$) and perpendicular (λ_{\perp}) magnetostriction coefficients at saturation:

$$\lambda_s = \frac{2}{3}(\lambda_{//} - \lambda_{\perp}). \quad (2)$$

It is important to note that magnetostriction depends on the orientation of the magnetization with respect to the crystal axes, but, due to symmetry, does not depend on the sign of the direction [12, 24].

2.3. MAGNETOSTRICTIVE MATERIALS. — The development of giant magnetostrictive materials comes from the researches done in the Naval Ordnance Laboratory (NSWC) by the US Navy in the sixties. Working about magnetostrictive transducers, Iowa State University and N.O.L. discovered the “giant magnetostriction” of the rare-earth Tb and Dy [28, 44].

A basic knowledge about magnetostrictive materials may be useful for a better understanding of the phenomenon; a brief overview of their remarkable characteristics is proposed as following.

2.3.1. *Rare-Earth*. — Rare-earth are characterized by their 4f electron charge cloud which has a non-spherical shape. This shape progresses in a regular way through the lanthanide serie as electron are added. Sm is “egg-shaped”, Tb more like a discus. Thus, TbFe₂ and SmFe₂ are showing the most important magnetostriction, respectively positive and negative, at ambient temperature.

These materials exhibit magnetostrictive strains of two orders higher than Nickel, Cobalt or Iron, but at a cryogenic temperature because of their very low Curie temperature. For this reason, the research of materials owing a giant magnetostriction at ambient temperature has focused on RE-Fe alloys, where R represents a rare-earth. Indeed, the presence of iron in the alloy increases its Curie temperature. In Table I, the Curie temperature of different amorphous RE-Fe alloys are given, the magnetostriction curves of some of them are plotted in Figure 1 [9, 10, 38].

Polycrystalline RE-Fe present a higher Curie temperature. One main disadvantage is their strong anisotropy which disturbs the field rotation process. Indeed, in polycrystalline RE-Fe materials, the easy magnetizing directions are oriented at random, a field of around 1 T is thus needed to fully develop magnetostriction. In Figure 2, the compared magnetostriction of an amorphous and a polycrystalline TbDyFe alloy is plotted.

In order to get a low anisotropy two RE-Fe alloys having the same magnetostriction sign but opposed anisotropy constants have been combined, the best alloy obtained is known as Terfenol-D, and its composition is Tb_{1-x}Dy_xFe₂, with generally $x = 0.7$ [11, 22].

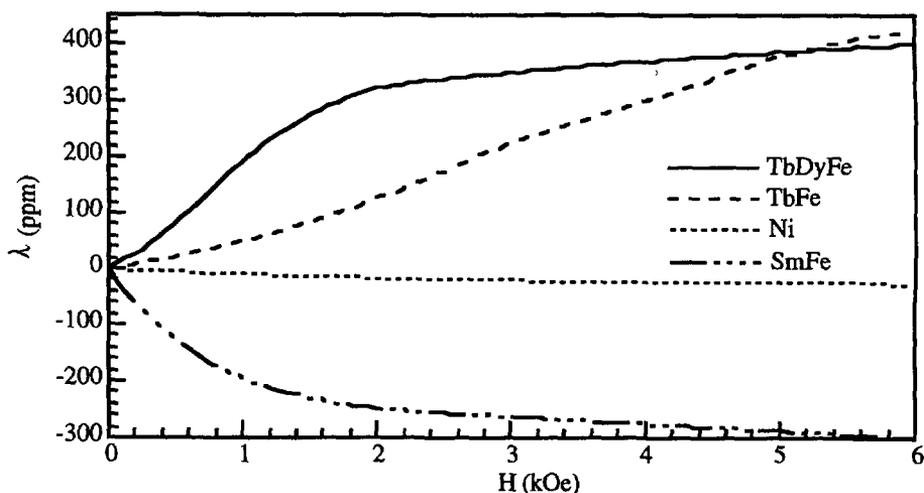


Fig. 1. — Magnetostriction of some amorphous films *vs.* the applied field in the film plane [33].

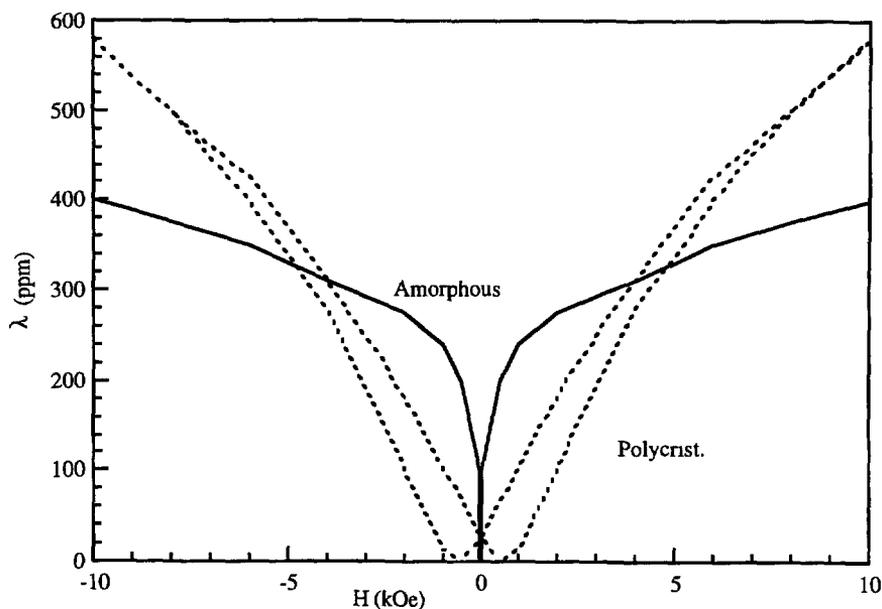


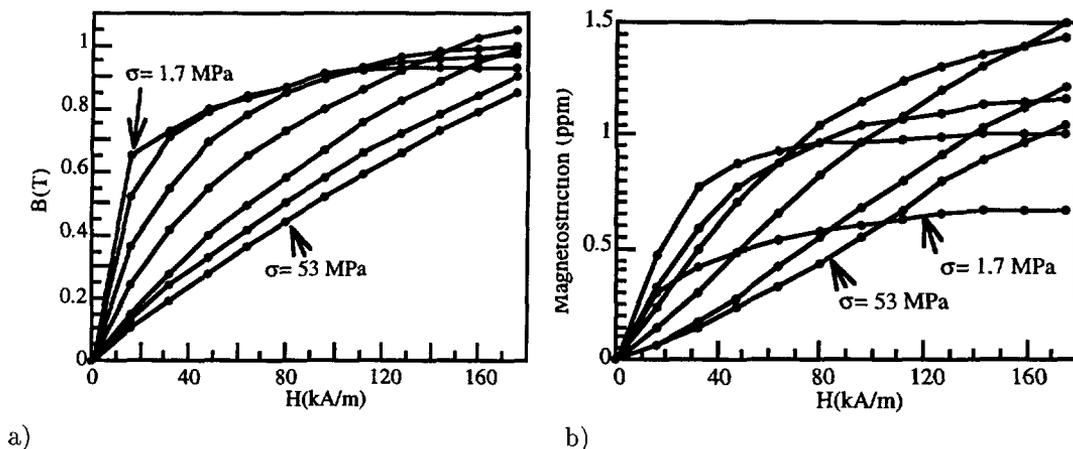
Fig. 2. — Compared magnetostriction *vs.* external field of an amorphous and a polycrystalline Tb-DyFe film [15] ($1 \text{ Oe} = 10^3/4\pi \text{ A m}^{-1}$, $1 \text{ ppm} = 10^{-6}$).

Recent research aims to improve low field large magnetostriction of rare-earth materials in order to use them for microdevices applications.

2.3.2. Elastic Characteristics of RE-Fe Components. — Table II gives densities and Young moduli for polycrystalline TbFe₂, ErFe₂, Fe₂, TbFe₃ and Ni. The quantity: $(1/2 E \lambda_s^2)$ represents the energy density, which is the amount of magnetic energy which can be transformed into elastic energy per unit volume of the material.

Table II. — *Elastic characteristics of polycrystalline TbFe₂, ErFe₂, Fe₂, TbFe₃ and Ni [9].*

	TbFe ₂	ErFe ₂	TbFe ₃	Ni
ρ (g cm ⁻³)	9.1	9.7	9.4	-
$E \times 10^{-10}$ (N m ⁻²)	9 × 4	12.1	13.1	21
$E\lambda_s^2/2$ (×10 ⁻³ Jm ⁻³)	145	3.2	32	0.1

Fig. 3. — Experimental curves: a) $B(H, \sigma)$, b) $\varepsilon(H, \sigma)$. For $\sigma = 1.7, 4.7, 9, 19, 30, 44, 53$ MPa.

Very large changes in elastic moduli with magnetic field (ΔE effect) have been observed in the highly magnetostrictive RE-Fe compounds. The modulus continues to change far above technical magnetic saturation and can be associated with domain wall motion, as well as with an intrinsic softening of the lattice due to local atomic magneto-elastic interactions [39].

2.3.3. Measurement of the Characteristics of Terfenol-D. — Highly magnetostrictive materials are nonlinear. Nonlinear effects play a significant part in the performance of a device based on magnetostrictive materials. Therefore these materials have to be characterized in a systematic way for the various conditions [29, 31].

A set-up device has been developed at the L.E.G. for measuring the magnetostriction of Terfenol-D rods. The typical magnetostriction curves obtained with this experimental set-up are presented in Figure 3 (all measurements are done along the rod axis) [41].

Magnetostrictive rods have been used in such diverse applications as underwater sound transducers, linear and rotary motors, active vibration control, actuators, high or low pressure pumps, etc. [8, 16].

2.4. MAGNETOSTRICTIVE THIN FILMS. — The fabrication and design of thin films is now well known. Many devices are adapted to the different materials that have to be deposited. The main challenge is to control the deposition stoichiometry in the case of alloy, as well as the film growth process in order to get an homogeneous and uniform film.

Both the deposition process and the temperature variation induce an internal stress in the film which may dramatically alter its performances. This internal stress increases with the film's thickness and is a serious limitation to the deposition of thick films.

Usually, it is not easy to obtain the mechanical characteristics of thin films: the film has to be separated from the substrate and numerous experimental problems have to be solved [32].

Low field magnetostriction is related to magnetic anisotropy, and thus depends on the composition of the film. As well as anisotropy, large magnetostriction are found in the amorphous state. The room temperature magnetostriction of amorphous TbFe_2 , $\text{Tb}_{0.3}\text{Dy}_{0.7}\text{Fe}_2$ and DyFe_2 are given in Table I.

Sputtered alloys of composition RE-Fe are both structurally and magnetically amorphous. As seen previously, Curie temperature ranges from below room temperature to ~ 400 K. The most striking feature of these alloys is their large coercivity at low temperature [36].

Giant magnetostrictive thin films are of high interest for active mechanical actuators. Research focuses on materials exhibiting low field magnetostriction and soft-magnetic properties. In order to improve the low field magnetostrictive properties of amorphous RE-Fe films, solutions have been proposed:

- 1) to increase the total rare-earth content compared to that of the crystalline phase,
- 2) to alloy different rare-earth which compensate their anisotropy respectively,
- 3) to add small quantities of Bore in order to modify the amorphous state of the film,
- 4) to induce tensile stress by controlling the preparation conditions.

Latter point is capital for thin films fabrication: deposition rate, bias voltage, sputtering pressure for instance are of great influence for the structure of the film.

Point 2 may be illustrated by two different methods used in order to reduce the magnetocrystalline anisotropy: one is to alloy positive and negative magnetocrystalline materials such as $(\text{Tb-Dy})\text{Fe}_2$, the other is to prepare amorphous state alloys. since amorphous thin films are usually easily obtained by sputtering techniques.

Amorphous films with a giant magnetostriction of about 250 ppm at 0.1 Tesla and 400 ppm at 0.5 Tesla for the TbDyFe with an in-plane magnetic easy axis have been successfully prepared

Other structures have been studied such as the combination of two amorphous thin films, one (TbFe) having a positive magnetostriction and the other (SmFe) having a negative one, this "sandwich" multilayered film enables large deflection in low magnetic field. Some other kind of multi-layered systems have been studied with R-Co alloys: the rare-earth differs from one layer to another. If the composition is well chosen these multilayers may exhibit some specific properties [18, 33, 45].

2.4.1. Magnetostriction Measurement. — The deflection of a cantilever is measured; it consists of a non magnetic substrate and a magnetic thin film, and is assumed to be under the influence of an isotropic stress due to thermal expansion and an uniaxial magnetic field (Joule magnetostriction). A formula has been proposed by Klockholm in 1976 [47], which gives a relation between the deflection and the magnetostriction coefficient in the film, depending on different parameters such as the thickness of the film or its Young modulus.

This formula may unfortunately predict, under certain circumstances, magnetostrictive strains about twice larger than the one actually observed, so a corrected formula has been proposed which takes into account the anisotropic character of the Joule magnetostriction [25]:

$$\lambda_s = \frac{2 (\Delta_{//} - \Delta_{\perp})}{9} \frac{E_s h_s^2}{E_f h_f} \frac{1 + \nu_f}{1 + \nu_s} \quad (3)$$

where:

- $\Delta_{//}$ is the deflection measured when the field is applied along the measuring direction, Δ_{\perp} along the perpendicular direction.
- λ_s is the magnetostriction coefficient at saturation.
- E_s and E_f are the Young modulus of the substrate and the film respectively .
- h_s and h_f are the thickness of the substrate and the film respectively.
- ν_s and ν_f are the Poisson ratio of the substrate and the film respectively.

This equation is valid only for isotropic magnetostrictive films sufficiently thick, where surface magnetostriction is negligible ($h_f \geq 50$ nm), deposited on substrates much thicker than the films ($h_s \geq 100 h_f$). Indeed, surface effects have an important influence in the case of very thin magnetostrictive films; these effects may induce a significant difference between the coefficients characterizing the material in thin film and those of bulk materials. Furthermore, due to symmetrical considerations, it is not possible to describe the magnetostriction of thin films correctly with only one coefficient λ_s .

2.4.2. Measurement of ΔE -Effect. — The deflection of a cantilever is used to determine the ΔE -effect, as well as the Young modulus of the film. Parameters are then calculated by a theoretical analysis using the measured data [17, 40].

2.4.3. Measurement of Internal Stress. — Magnetostriction coefficients dependence with internal stress is generally negligible, but has to be considered when a coefficient is zero or near zero, since its relative variation may become important. Nevertheless, stress can modify magnetic characteristics a lot, as shown in Sections 2.2 and 2.3.3 [24]. In Figure 4 the magnetostriction *versus* external magnetic field is plotted for a $(\text{Tb}_{0.27}\text{Dy}_{0.73})_{0.42}\text{Fe}_{0.58}$ amorphous film: depending on the substrate internal stress will be tensile (Si substrate) or compressive (CuBe substrate), thus changing a lot magnetostrictive characteristics.

The deflection of the cantilever is used to calculate the residual isotropic stress inside the active thin film, thanks to the Stoney formula [27, 42].

3. Magnetostriction Modelling Methods

Most of the magnetostriction modelling theory related to magnetostrictive rods consider a linear range and a dynamic excitation.

Two types of classical models for magnetostrictive transducers exist, the lumped parameters or the distributed parameters model. Both of them are unidimensional and may be derived, leading either to analytical formula or to a transfer matrix treatment, a numerical method currently used for piezoelectric transducers [8, 13]

Finite element models have been developed based on an original variational principle in order to be able to model any type of magnetostrictive devices without any heavy restriction on their shape and on materials involved, or any assumptions on the magnetic and displacement fields.

3.1. MODELLING THE CHARACTERISTICS OF MATERIAL. — The first step for a nonlinear Finite Element modelling is to define a good model for the magnetic and mechanical characteristics of the material. A few is known about the characteristics of magnetostrictive materials in thin films. Due to this lack of data, the assumption was done, in a first stage, that the characteristics of magnetostrictive thin films remain the same as the one obtained for bulk materials. A simple and quickly implemented analytic interpolation is proposed. Experimental data are used as reference in order to get the material's global behaviour.

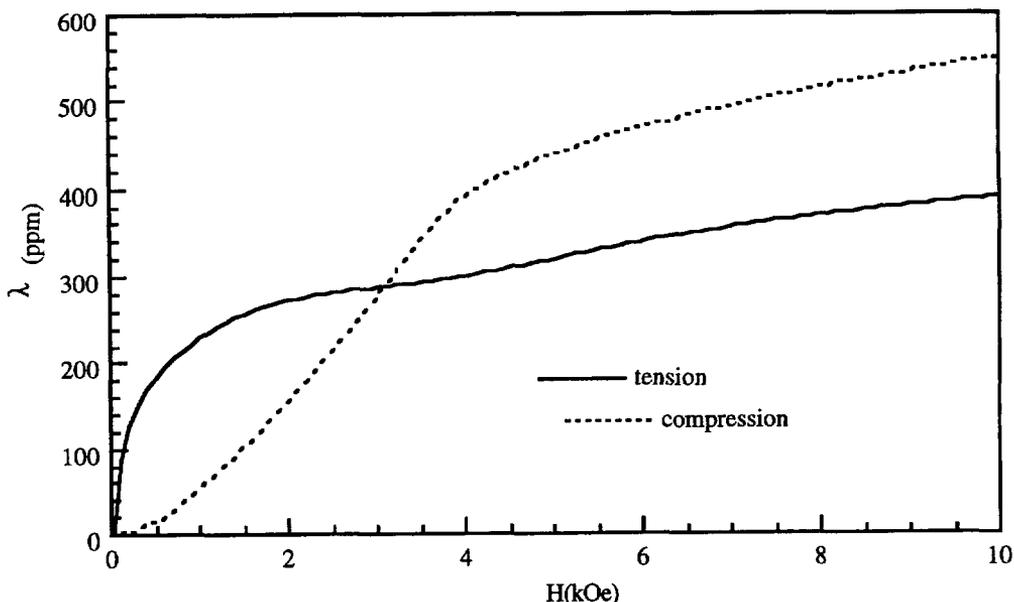


Fig. 4 — Magnetostriction vs. external field for a film with tensile or compressive stress [15] ($1 \text{ Oe} = 10^3/4\pi \text{ A m}^{-1}$, $1 \text{ ppm} = 10^{-6}$)

3.1.1. $H(B, \sigma)$ Curves. — The proposed interpolation is an analytic formula using an exponential:

$$H(B, \sigma) = p_H(\sigma)B + \exp[n(B - a_H(\sigma))] \quad (4)$$

with

$$p_H(\sigma) = p_1 + p_2\sigma \quad (5)$$

$$a_H(\sigma) = a_1 + a_2\sigma. \quad (6)$$

Parameters p_1 , p_2 , a_1 , a_2 , n are optimized in order to fit as good as possible the experimental data. These parameters have no real physical meaning, they are just calculated in order to fit the experimental curves as good as possible.

Figure 5 compares experimental data [23] and analytical interpolation obtained with an optimal set of parameters. The magnetic behaviour of magnetostrictive materials is well represented. Stress has a clear influence on the permeability.

3.1.2. $\sigma(\varepsilon, H)$ Curves. — These curves require more parameters. An exponential based analytical formula is used again:

$$\sigma(H, \varepsilon) = \exp[n(\varepsilon - a_\sigma(H))] - \exp[-n(\varepsilon - b_\sigma(H))] + E_\sigma(H)\varepsilon + T_\sigma(H) \quad (7)$$

Parameters are optimized in order to correspond with the experimental data.

The curves obtained by this formula for various values of the magnetic field are plotted in Figure 6. The behaviour of the magnetostrictive material is well represented. Values do not fit perfectly with experimental data, but their order of magnitude, the variations of the curve are representative.

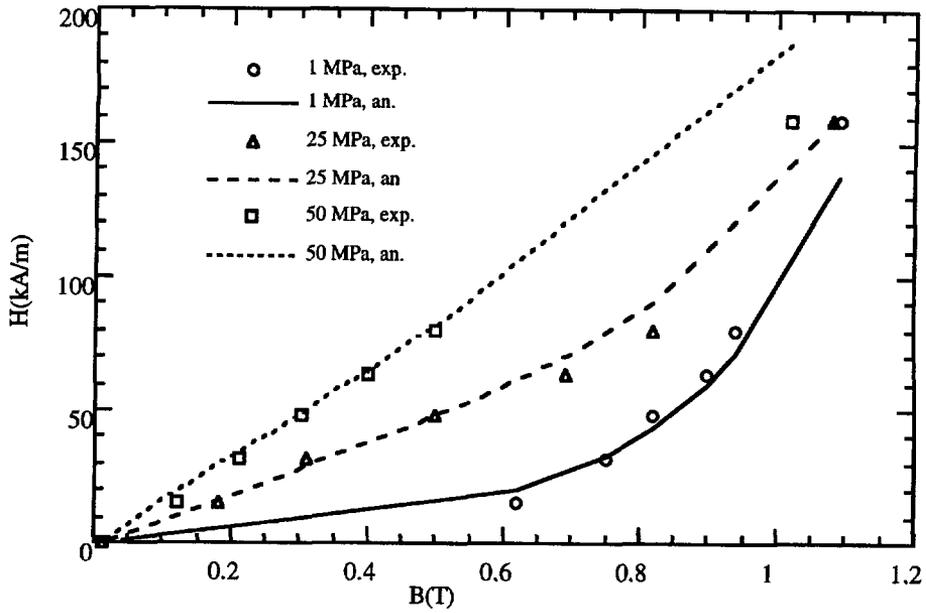


Fig. 5. — Example of interpolated curves $H(B)$ for several stress values (in MPa). The analytic interpolation (an) is compared with experimental data (exp.) (data from [23], p. 69)

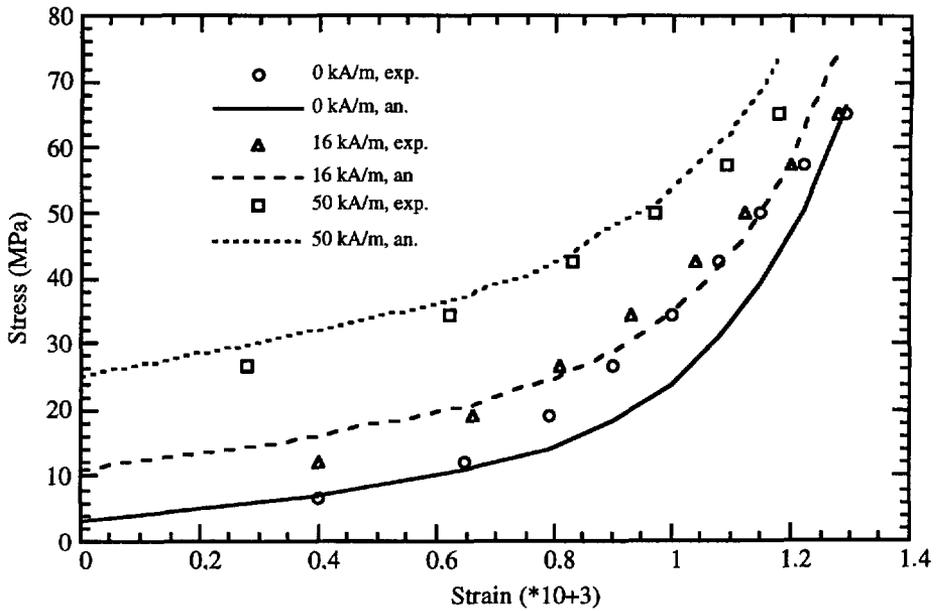


Fig. 6. — Example of interpolated stress-strain curves for several values of magnetic field (in kA m^{-1}). The analytic interpolation (an) is compared with experimental data (exp.) (data from [23], p. 41).

3.1.3. Conclusion. — This interpolation model will be used to provide the characteristics of magnetostrictive materials for the thin film's modelling. It is supposed to be close enough to reality to study the behaviour of magnetostrictive thin films, and to validate the nonlinear modelling process.

For a more accurate modelling, necessary for micropositioning devices or micro-actuators, a better interpolation method can easily be added.

3.2. MAGNETIC FIELD COMPUTING THROUGH MAGNETIC VECTOR POTENTIAL. — Two main methods exist to compute magnetic fields with the finite element theory: the vector potential method, using the fact that: $\text{div}\mathbf{B} = 0$ and then: $\mathbf{B} = \text{curl}\mathbf{A}$. or the scalar potential using the fact that: $\text{curl}\mathbf{H} = 0$ and so: $\mathbf{H} = -\text{grad}\phi$ (in the air) [37].

Remark:

- \mathbf{H} represents the magnetic field vector, in A m^{-1}
- \mathbf{B} is the induction vector, in Tesla

Both methods have their own advantages and drawbacks. The 2D approach is planned. The first method has been chosen as assumptions are less restrictive.

When the magnetic field \mathbf{H} is a nonlinear function of the induction \mathbf{B} , the nonlinear system has to be solved by an iterative way such as the Newton-Raphson method. This method has been successfully implemented in Finite Element Softwares and proved to be effective.

For magnetostrictive thin films, the magnetic characteristics are the interpolated nonlinear $H(B, \sigma)$ curves. In order to get the differential reluctivity and the associated coercivity field, it is necessary to know the applied stress. The stress σ_0 is thus calculated on each integration point, and then the other data are computed. The behaviour of the material around the point (B_0, ε_0) may then be linearized by a Taylor development to the first order:

$$H = H_{cd} + v_d B \quad (8)$$

where:

$$v_d = \frac{\partial H}{\partial B}(B_0, \sigma_0) = P_H(\sigma_0) + n \exp(n(B_0 - a_H(\sigma_0))) \quad (9)$$

is the differential reluctivity at constant stress and

$$\begin{aligned} H_{cd} &= H_0 - v_d B_0 \\ &= H(B_0, \sigma_0) - v_d B_0 \text{ is the coercivity field induced by the applied stress.} \end{aligned} \quad (10)$$

3.3. ELASTICITY. — The Finite Element Method has been widely used in the calculation of mechanical structures, using the elasticity assumptions. It has proved to be a powerful tool for the mechanical design of devices [2, 20, 43]. The elasticity theory is based on the assumption that displacements remain small and that the stress-strain relation is linear [43].

For isotropic materials, a linear relation between stress and strain tensors is used: the Hooke's law. It involves two parameters characterizing the material: the Young modulus E and the Poisson's ratio ν_p .

We have then $\{\sigma\} = [H_E]\{\varepsilon\}$ where $\{\sigma\}$ is the stress, $\{\varepsilon\}$ is the strain and $[H_E]$ is the constitutive matrix, involving E and ν_p .

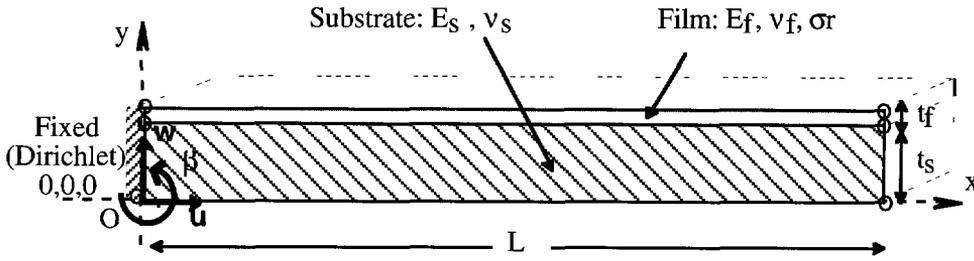


Fig. 7 — Thin film geometry

3.3.1. *Residual Stress and Thermal Strain.* — Total stress may come from different effects, such as:

- elastic stresses due to strains,
- residual stresses due to the solid “history”,
- stresses due to volume or surface force,
- thermal stresses due to a temperature field.

For an isotropic material, thermal deformations $\{\varepsilon_{th}\}$ depends on the thermal coefficient α and on the relative temperature T . The general Hooke’s law may thus be written:

$$\{\sigma\} = [H_E]\{\varepsilon\} + \{\sigma_0\} \tag{11}$$

where $\{\sigma_0\}$ represents the residual stress.

3.3.2 *Nonlinear Modelling.* — It is important to note that in the case of magnetostrictive materials, the range of calculation remains in the elastic domain, although these materials are nonlinear as there is no plastic deformation. Therefore, one may proceed in the same way as for the magnetic nonlinear case. This formulation is presented in the following.

3.4. NONLINEAR MECHANICAL MODELLING OF MAGNETOSTRICTIVE THIN FILMS. — Dimensions ratio of thin film devices are such that modelling must be achieved through the assumptions of thin beam bending in a plane (for a 2D approach) or thin plates. In a first step the problem can be simplified thanks to Reissner and Mindlin theories [2, 30, 35]:

- plane stress ($\sigma_{zz} = 0$),
- the plane sections are assumed to remain plane,
- no transverse shearing strains,
- the transverse displacement w does not vary through the film’s thickness,
- homogeneous and isotropic materials.

3.4.1. *The Nodal Variables.* — Nodal variables of this model are:

- u , displacement along x -axis,
- w , displacement along z -axis,
- β , rotation around y -axis.

The no-shear assumption involves that:

$$\beta = -w, x. \tag{12}$$

3.4.2. *Stress-Strain Relation.* — In this model, only ε_{xx} is non zero:

$$\sigma_{xx} = \frac{E}{1 - \nu_p^2} \varepsilon_{xx} + \sigma_r \quad (13)$$

where E is the Young modulus, ν_p the Poisson's ratio, σ_{xx} the stress along Ox , σ_r a residual stress and ε_{xx} the strain along Ox .

For magnetostrictive materials, these characteristics depend on the magnetic state.

The anisotropic character of magnetostrictive strain ($\varepsilon_{xx} = -\varepsilon_{zz}$) is taken into account by considering an effective Young modulus: $E_{\text{eff}} = E/(1 + \nu_p)$ instead of: $E/(1 - \nu_p^2)$.

3.4.3. *Nonlinear Modelling.* — The principle is the same as for the magnetic characteristics: the magnetic computation provides the magnetic field H at every integration point, then:

$$\sigma_0 = \sigma_0(\varepsilon_0, H_0) \text{ on the working point } (\varepsilon_0, H_0)$$

and

$$E_d = \frac{\partial \sigma_0}{\partial \varepsilon_0}(\varepsilon_0, H_0) \quad (14)$$

is the differential Young modulus at constant field.

It is thus possible to linearize the relation:

$$\sigma_{xx} = E_d/(1 + \nu_p) \varepsilon_{xx} + \sigma_c \quad (15)$$

where

$$\sigma_c = \sigma_0 - E_d/(1 + \nu_p) \varepsilon_0 \quad (16)$$

is the “coercivity” stress induced by the applied magnetic field

3.5. **MAGNETOMECHANICAL STRONG COUPLING.** — The provided data are the residual stress in the film, the current density in the windings (which creates the magnetic field) and the characteristics of materials (linear for the substrate, magnetostrictive for the film). The coupling is done by a “strong” method, in which the magnetic and mechanical computations are simultaneously achieved, as presented in Figure 8. The term “strong coupling” has been chosen for this finite element method, because it simultaneously solves both phenomena that, in the case of magnetostrictive materials, are physically strongly coupled. This results in a global transfer matrix gathering magnetic and mechanical data (while they are successively solved in an iterative coupling method). Nonlinearity is taken into account by a Newton Raphson process. On each step of the process, the characteristics of the material are obtained thanks to the previous results.

The global functional is obtained by summing the magnetic and the mechanical functionals. The magnetomechanical coupling is achieved through the nonlinear characteristics of the materials by a linearization process. Indeed, a change in magnetic field is reported on the stress-strain characteristics and induces a mechanical residual coercitive stress, this stress acts to bend the cantilever. The linearization lets appear either a “coercivity” stress or a “coercivity” magnetic field, as presented in Figure 9. Thus, there is no direct coupling terms in the functional. The linearized characteristic of the material can be written:

$$\begin{bmatrix} \sigma \\ H \end{bmatrix} = \begin{bmatrix} E_{\text{eff}} & 0 \\ 0 & v \end{bmatrix} \cdot \begin{bmatrix} \varepsilon \\ B \end{bmatrix} + \begin{bmatrix} \sigma_c \\ H_c \end{bmatrix} \quad (17)$$

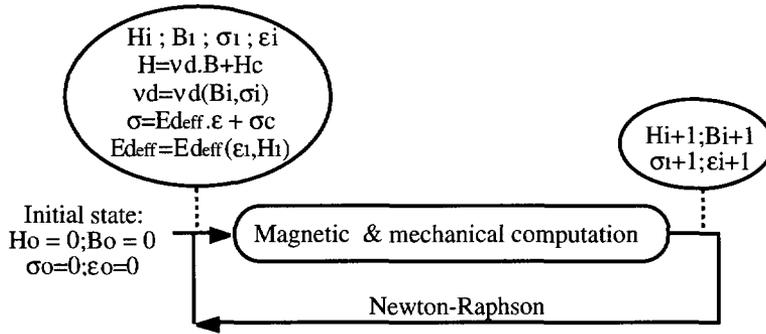


Fig. 8. — Newton Raphson process adapted for magnetomechanical strong coupling.

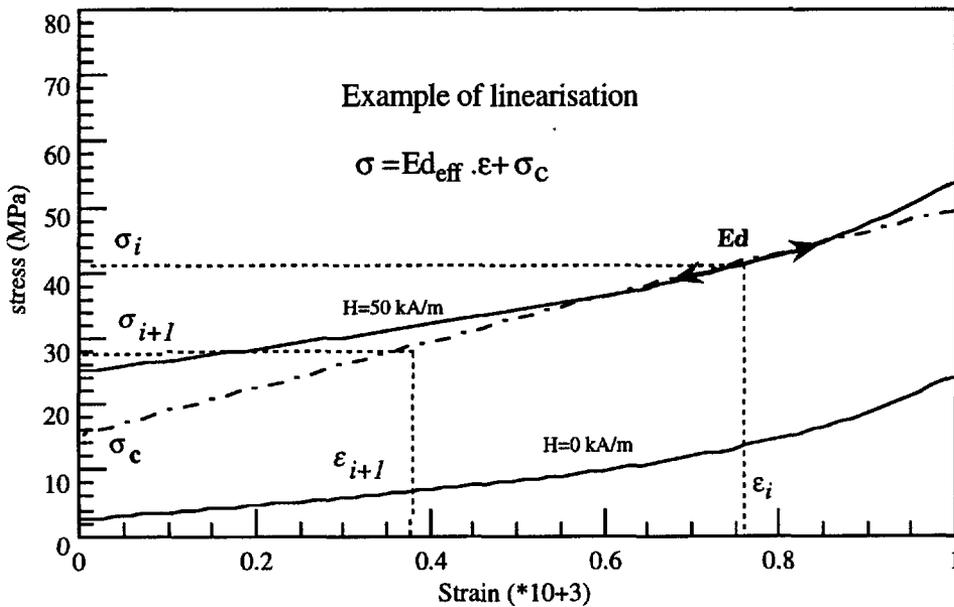


Fig. 9. — Newton Raphson linearization process adapted for magnetomechanical strong coupling.

3.6. CONCLUSION. — A computing method to solve the magnetostrictive thin film problems has been presented. The magnetostrictive characteristics of the material are interpolated by some simple nonlinear functions based on experimental characteristics of Terfenol-D rod. Magnetomechanical coupling is achieved through the Finite Element Method by a simultaneous “strong coupling” process. In the next part, this theory is applied to an example and results about magnetostriction modelling will be presented.

4. Application: A Magnetostrictive Micromembrane

The application device presented is a magnetostrictive micromembrane. It is controlled by a magnetic field created by a coil. Modelling is achieved due to the previously presented F.E.M. formulation.

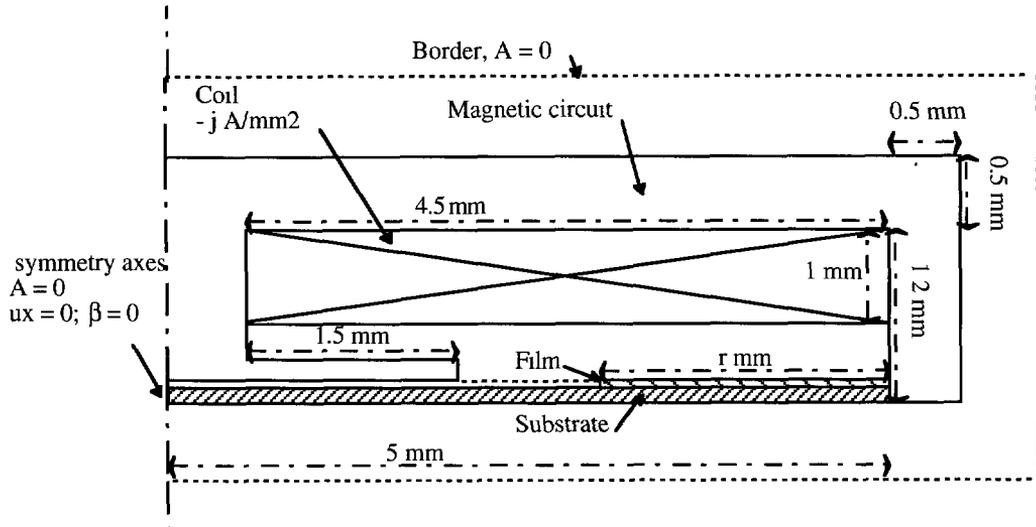


Fig. 10. — Geometry of the magnetostrictive membrane.

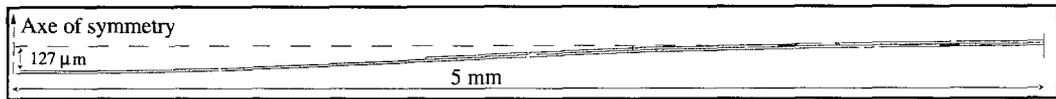


Fig. 11. — Example of the computed deflection of the magnetostrictive membrane.

The magnetostrictive material is analytically described as explained in Section 3.1. Results may be considered as a test. Since the characteristics of materials in thin films are not well known and may vary a lot from one compound to another, comparison with experimental results can only focus on the global behaviour of the system. Nevertheless, this study is a first step to nonlinear magneto-mechanical coupling modelling, and opens the way to future microdevices design.

The modelled device is presented in Figure 10. The membrane is composed of a substrate of Cu ($E_s = 130$ GPa, $\nu_p = 0.343$, $\nu = 1.1$) covered by a magnetostrictive film. The device is axisymmetrical so only half of a plane section is modelled. In order to increase the deflection by supporting the different sign of curvature of the cantilever, a part of the substrate has been left uncoated. A coil creates the magnetic field, which is guided through a magnetic circuit in order to cross the active film, and bend the membrane. The infinite is modelled with a dirichlet boundary condition on the vector potential: $A = 0$. The mesh is composed of second degree elements (triangular for air and coils, rectangular for the cantilever). The current density in the coil is perpendicular to the plane. The substrate's radius is 5 mm, thickness is $15 \mu\text{m}$.

The micromembrane modelling is achieved through an axisymmetrical formulation. This one is developed similarly to the 2D Cartesian method previously presented, and assumes axisymmetrical assumptions

An example of a computed deflection of the membrane is shown in Figure 11. Membrane and deflection are at the same scale. The radius of substrate uncoated is 2 mm, and for a current density in the coil equals to 50 A mm^{-2} the deflection in the middle of the membrane is $127 \mu\text{m}$.

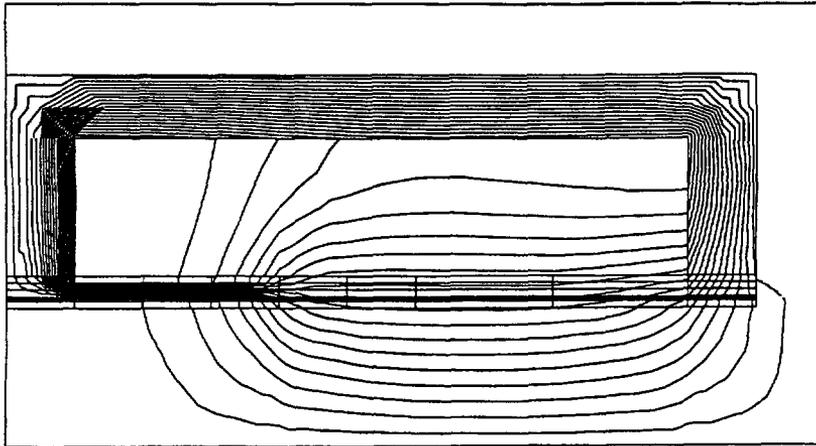


Fig 12 — Magnetic flux lines.

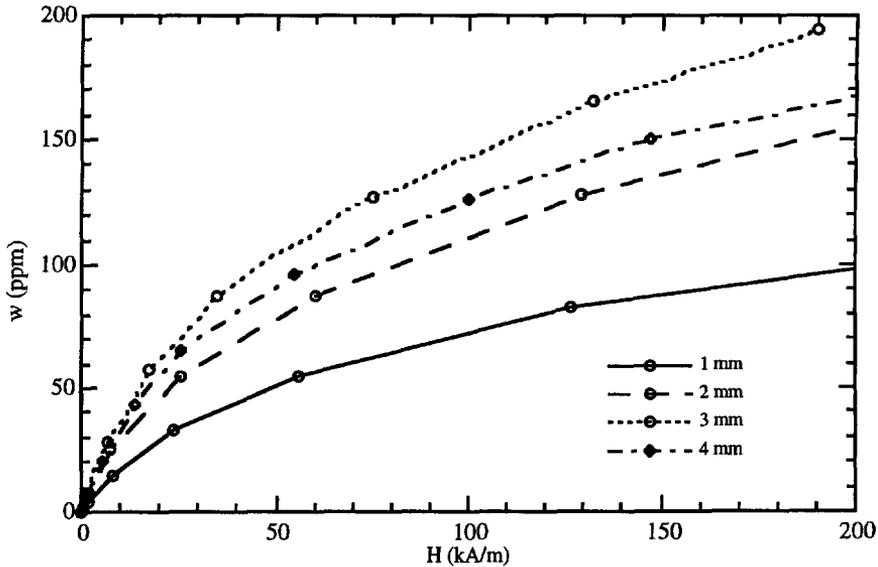


Fig. 13 — Deflection of the membrane *versus* magnetic field, for different values of radius of the film.

The magnetic flux lines and the meshing are shown in Figure 12 (due to its thickness, the membrane can not be seen at this scale), important leaks appear. The magnetostrictive film as well as the substrate have a weak permeability, hence an important part of the flux is lost, though it is guided by the magnetic circuit. Such computation confirms the importance of the geometrical design of the device.

4.1. STUDY OF THE RATIO OF SUBSTRATE COATED BY THE FILM. — First, the performances of the device are studied depending on the ratio of substrate covered by the film. The thickness of the film is $10 \mu\text{m}$, and $15 \mu\text{m}$ for the substrate. The radius of the substrate is 5 mm, it is partially coated on a r mm radius by the magnetostrictive thin film. The deflection *versus* the

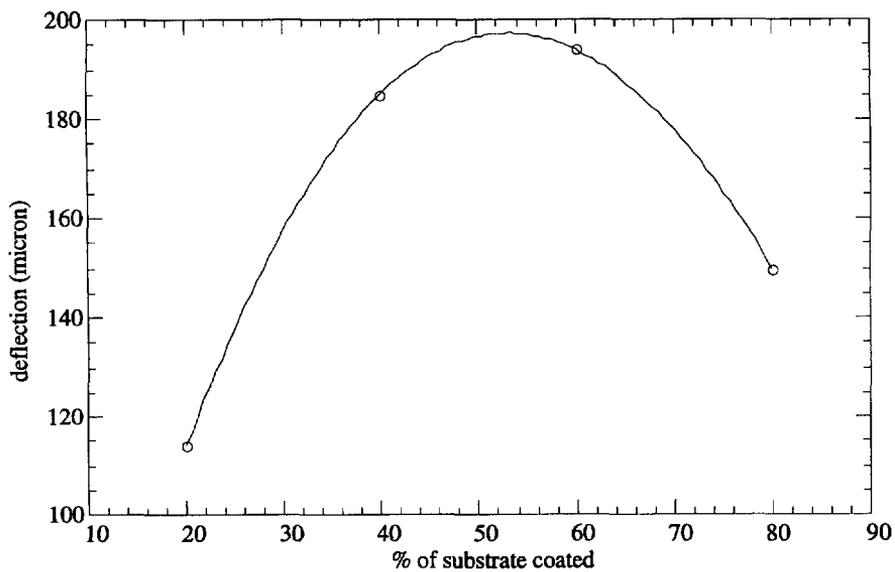


Fig. 14. — Maximum deflection *versus* % of the radius of coated substrate.

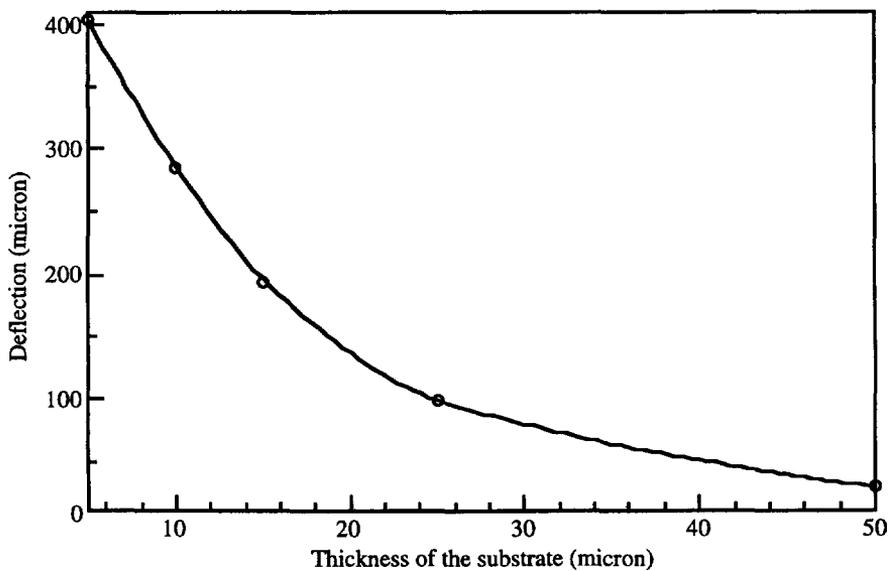


Fig. 15. — Maximum deflection of the membrane *versus* thickness of the substrate, for a magnetic field of 180 kA m^{-1} in the film

ratio of coated substrate is plotted in Figure 14. These results show that there is an optimal % value of coated substrate, which is around 50-55%. Similar results have been obtained by [34] on double clamped cantilevers. F.E.M. calculations enable to optimize the geometry of devices in order to obtain maximum performances.

4.2. STUDY OF THE THICKNESS OF THE SUBSTRATE. — A parameter to be optimized is the thickness' ratio of both layers. In the following curves, the thickness of the film remains $10\ \mu\text{m}$, and the substrate is coated on $r = 3\ \text{mm}$. The thickness of the substrate range from $5\ \mu\text{m}$ to $50\ \mu\text{m}$. The maximum deflection obtained *versus* the thickness of the substrate is plotted in Figure 15.

Similar results have been obtained by [19]. Values for very small thicknesses (inferior to the thickness of the film) may not be considered as physically pertinent. Nevertheless this study clearly shows that film and substrate should have an equivalent thickness in order to get a maximum deflection. Unfortunately magnetostrictive film deposition on thin substrate cannot easily be achieved, as many physical problems occur. Moreover, it is not sure that a thin film on a thin substrate can provide enough power to be an interesting device.

5. Conclusion

We showed that modelling results prove in good agreement with the global behaviour of magnetostrictive thin films. Due to the simulation, many magnetic and mechanical characteristics are available. They refer to the behaviour of thin films, such as deflection dependence with the thickness of the film, magnetic field or Young modulus variation in the film, strain variation with residual stress. As the characteristics of the material used here are only a rough interpolation of bulk Terfenol-D one, no precise comparison with experiments has been made. Precise data regarding magnetostrictive thin films are to be expected. Once these data are available, they can easily be implemented in the model. Nevertheless, a model is studied in order to be able to directly take into account the experimental magnetostrictive curves available for thin films [48–50]. This work is an important step on the way to magnetostrictive microdevices modelling. New materials need adapted design tools, the “strong coupling” F.E.M. formulation proposed is an efficient method to solve nonlinear magnetomechanical coupled problems.

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