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R. Reinisch, M. Nevière, H. Akhouayri, J. Danckaert, G. Vitrant. Modal theory of diffraction : a powerful tool for the study of grating couplers in nonlinear optics. *Journal de Physique III*, 1994, 4 (12), pp.2451-2458. 10.1051/jp3:1994290 . jpa-00249276

HAL Id: jpa-00249276

<https://hal.science/jpa-00249276>

Submitted on 4 Feb 2008

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Classification

Physics Abstracts

42.10H — 42.65K — 42.65P

Modal theory of diffraction : a powerful tool for the study of grating couplers in nonlinear optics

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(Received 22 December 1993, revised 29 March 1994, accepted 2 May 1994)

Abstract. — It is shown that the simultaneous use of the coupled-mode formalism and of the rigorous theory of diffraction in linear optics leads to a simple modal theory of grating couplers on nonlinear waveguides. Two examples are considered : optical Kerr effect and second harmonic generation. In Kerr-type grating couplers, despite its simplicity, this modal analysis is able to predict « exotic » behaviors such as, for example, chaos.

1. Introduction.

Grating couplers for nonlinear waveguides have been the object of many studies. This is mainly due to the fact that they allow the resonant excitation of normal modes of the structure : guided waves or surface plasmons. The associated electromagnetic (EM) resonance may give rise to different effects. For example, in second harmonic generation (SHG), grating couplers allow an enhancement of several orders of magnitude of the second harmonic intensity as compared to the off-resonance situation, whereas optical bistability may exist when dealing with the optical Kerr effect.

These nonlinear interactions at grating couplers may be studied, *a priori*, in two ways.

1) In the first one (referred to as method 1), no hypothesis is made regarding the transverse field map [1-3] (transverse : along the *y*-axis perpendicular to the mean plane of the grating). The differential equations, deduced from Maxwell equations, which describe the EM field(s) at the existing frequency(ies) are integrated along *y*. Thus method 1 applies whether or not the resonant conditions are fulfilled. As a result, this method is general but has the two following disadvantages : i) large amount of computer time is required and ii) is restricted to only plane waves. Point ii) is a serious drawback when dealing with Kerr type grating couplers. Indeed it is known that the finite width of the incident beam strongly modifies the response of this type of couplers ; to a lesser extent, this is also true for SHG in the presence of sharp resonances.

2) The second method (referred to as method 2) takes full advantage of the fact that these grating couplers in guided wave nonlinear optics are used in the *immediate vicinity* of an *isolated resonance* : of the order of a few times the width at half maximum of the resonance curve. Therefore the coupled mode formalism [4, 5] is particularly well suited for this kind of study since it shows that the transverse field map corresponds to that of the *linear* regime. Hence, in the *nonlinear* case, the two following important consequences :

a) the transverse field map *is known* and it is no longer necessary to integrate along y as for method 1 ;

b) The in-coupling of the pump field together with the phenomenon of diffraction is accounted for by using the rigorous theory of diffraction in *linear optics* [6] in which the groove depth of the grating is *not* considered as a perturbative parameter.

The *only* nonlinear quantity which enters the calculation is the overlap integral which involves the known transverse field maps and the nonlinear susceptibility.

The aim of this paper is to present method 2 and to apply it to optical Kerr effect and to SHG. The organization of the paper is as follows : section 2 is devoted to general considerations concerning the coupled-mode formalism. The grating coupler on a Kerr-type waveguide is considered in section 3 and SHG at a grating coupler is treated in section 4.

2. General considerations.

Use of the coupled mode formalism [4, 5] leads to the following procedure :

- a) define the unperturbed structure ;
- b) find its normal modes ;
- c) specify the source terms ;
- d) write down the equation(s) of evolution of the normal mode(s) amplitude and ;
- e) solve the evolution equations..

For nonlinear effects at grating couplers (Fig. 1), this leads to :

POINT a. — The unperturbed system consists of the grating coupler without any incident beam. Thus the corresponding structure is linear and leaky with modes having a longitudinal wavevector component γ_m which is complex (even in the absence of dielectric losses).

POINT b. — The normal modes are found using the rigorous theory of diffraction in linear optics [6].

POINT c. — The source terms depend on the geometry and also on the features of the nonlinear interaction : SHG, optical Kerr effect with the existence of diffusion, Kerr-type nonlinearity

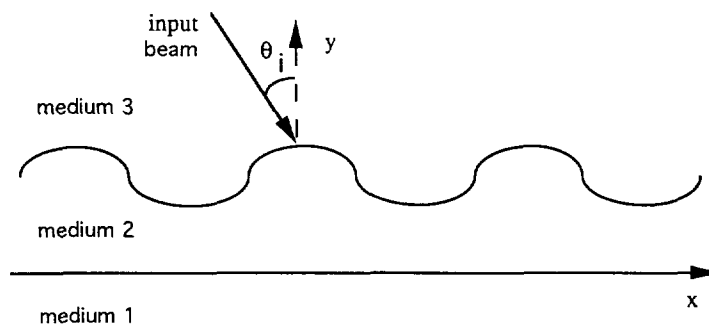


Fig. 1. — A grating coupler : region 2 represents the nonlinear medium.

which is instantaneous or not. One of these source terms describes the in-coupling of the incident beam(s) and, as already stated, is derived from the rigorous theory of diffraction in linear optics [6].

POINT d. — The existence of source terms leads to x -dependent mode amplitude $c_m(x)$ which obeys the following equation [4, 7] :

$$N_m^\pm \frac{dc_m^\pm}{dx} = -j\omega \langle \mathbf{E}_m^{\pm t}(y) \cdot \mathbf{P}^{\text{NL}}(x, y) \rangle e^{\pm j\gamma_m^t x} - j\omega \langle \mathbf{E}_m^{\pm t}(y) \cdot \mathbf{P}^{\text{L}}(x, y) \rangle e^{\pm j\gamma_m^t x} \quad (1a)$$

In equation (1a) :

i) \pm refers to forward (+) and backward (−) propagating modes of the unperturbed structure ($a \exp(-j\omega t)$ time dependence is assumed).

ii) $\langle .. \rangle$ stands for an integral in the cross-section plane :

$$\langle .. \rangle = \int . dy .$$

$$\text{iii) } N_m^\pm = \langle \mathbf{u} \cdot (\mathbf{E}_m^\pm \times \mathbf{H}_m^{\pm t} - \mathbf{E}_m^{\pm t} \times \mathbf{H}_m^\pm) \rangle \quad (1b)$$

\mathbf{u} : unit vector along the x -axis.

$\mathbf{E}_m^\pm, \mathbf{H}_m^\pm$: respectively y -dependence of the electric, magnetic field. The integral in the cross-section plane is calculated from $y = -\infty$ to $y = +\infty$. The superscript t denotes the adjoint structure deduced by transposing of the dielectric permittivity and the magnetic permeability matrices [4, 7]. Thus the quantity γ_m^t fulfills [4, 7] :

$$\gamma_m^t + \gamma_m = 0 . \quad (1c)$$

iv) The first bracket accounts for the existence of the nonlinear effect of the guiding layer which gives rise to the nonlinear polarization $\mathbf{P}^{\text{NL}}(x, y)$. The second one describes the in-coupling of the incident beam, the resulting source polarization being $\mathbf{P}^{\text{L}}(x, y)$.

POINT e. — This point is considered in the two next paragraphs.

3. Grating coupler on a Kerr-type waveguide.

For the sake of generality, we consider a non-instantaneous, non-local Kerr-type nonlinearity. In that case, equation (1a), generalized in the time domain, has to be associated to a material equation which describes the spatio-temporal evolution of the nonlinearity. Use of reduced units leads to the following set of spatio-temporal equations [8, 9] :

$$i \frac{\partial F(x, t)}{\partial t} + \frac{\partial^2 F(x, t)}{\partial x^2} + 2i\alpha_0 \frac{\partial F(x, t)}{\partial x} - \eta [\Delta - U(x, t)] F + iF = iS(x, t) \quad (2a)$$

$$L^2 \frac{\partial^2 U(x, t)}{\partial x^2} - \tau \frac{\partial U(x, t)}{\partial t} = U(x, t) - |F(x, t)|^2 \quad (2b)$$

where [8, 9] :

F : total electric field of the resonantly excited guided mode, S : input beam, α_0 . angle of incidence, Δ : detuning, $\eta = \pm 1$ whether the nonlinear medium is self-focusing ($\eta = +1$) or self-defocusing ($\eta = -1$), U : nonlinear term, L : diffusion length, τ : nonlinear response time.

The space x and time t variables are scaled respectively to the width of the resonance curve and to the resonator build-up time.

It can be immediately checked that in the limit of $L = 0$ and $\tau = 0$, equations (2) yield the usual spatio-temporal modal equation.

It is worth noticing that as explained in the introduction, the validity of equations (2) rests on the possibility of isolating a single resonance and of remaining « close » to it. Thus, as long as reduced quantities are used, equations (2) apply whatever the geometry of the Kerr-type resonator may be (prism coupler with guided mode resonances, Fabry-Pérot with Airy resonances, etc.). In other words, the solution F of equations (2) gives the response of *all* the optical resonators corresponding the *same* set of *reduced* parameters. Of course, the relation between the quantity F and the EM field radiated outside the resonator does depend on the geometry of the device [7, 10]. Thus it is no longer necessary to specify the geometry of the Kerr-type resonator provided it is the quantity F which is of interest.

The set of equations (2) allows us to study the optical response of Kerr-type resonators (for example a grating coupler) illuminated by a Gaussian beam. The results [9] are plotted in figure 2a for a defocusing nonlinearity and in figure 2b for a positive one. As can be seen on these figures, instabilities are present in both cases leading, for positive Kerr media, even to a chaotic regime.

4. EM resonance enhanced SHG at grating couplers.

As in the previous section, the starting point in equation (1a) written for the angular frequencies ω and 2ω . It is assumed that the EM resonance occurs both at the pump frequency ω for a mode p with longitudinal wavevector component γ_{1p} , and at the SH frequency 2ω for a mode m with longitudinal wavevector component γ_{2m} i.e. that phase-matching takes place.

We make the usual undepleted pump approximation but take into account the finite-width of the pump beam. Thus its envelope \mathcal{A}_1 depends on x : $\mathcal{A}_i = \mathcal{A}_i(x)$. Then, according to equation (1a), the amplitudes $c_{1p}(x)$ and $c_{2m}(x)$ of the resonantly excited guided modes at frequencies ω and 2ω obey the following set of equations :

$$\frac{dc_{1p}}{dx} = -j \mathcal{A}_1(x) t_{p, q_0} e^{j(\beta_0 + q_0 \sigma - \gamma_{1p})x} \quad (3a)$$

$$\frac{dc_{2m}}{dx} = -j 2 \omega \xi_m(x) c_{1p}^2 e^{j(2\gamma_{1p} - \gamma_{2m})x} \quad (3b)$$

with : $\beta_0 = k_0 \sin \theta_1$, $k_0 = \omega/c$, θ_1 : angle of incidence of the pump beam, $\sigma = \frac{2\pi}{d}$ (d : periodicity of the grating), q_0 : integer which labels the resonantly excited evanescent diffracted order at the pump frequency, t_{p, q_0} : in-coupling coefficient for this diffracted order q_0 at the pump frequency ω calculated using one of the methods developed in reference [6], $\xi_m(x)$: nonlinear coefficient directly related to the overlap integral [4, 5]. The existence of a modulated region in the grating coupler explains the x -dependence of ξ_m .

Equations (3a) and (3b) are important since they constitute the basic set of equations for SHG at a nonlinear grating coupler illuminated by a finite width pump beam and account for the angular dependence and spatial evolution of the amplitude of the resonantly excited evanescent diffracted orders at ω and 2ω respectively. Thus these two equations describe the diffraction process in nonlinear optics at the pump and SH frequencies.

In general the nonlinear medium includes a modulated region and a homogeneous one. Therefore $\xi_m(x)$ is the sum of two terms, one for each region :

$$\xi_m(x) = \xi_m^{(1)}(x) + \xi_m^{(2)} \quad (4)$$

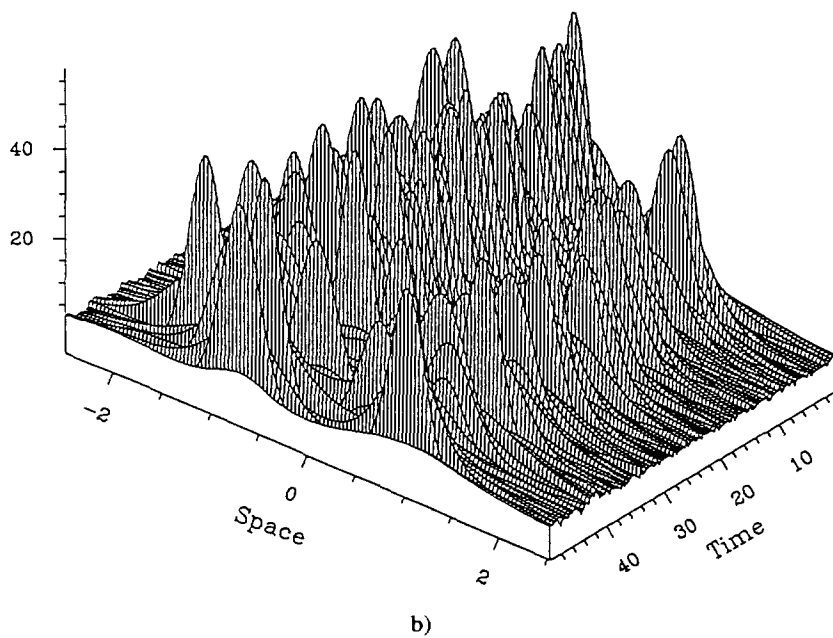
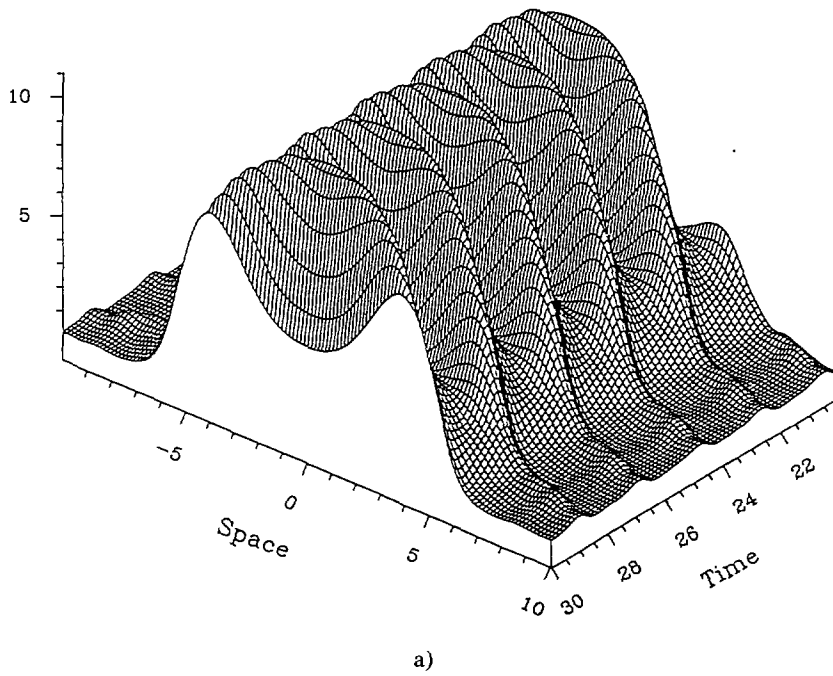


Fig. 2. — Kerr type optical resonator (for example : Kerr type grating coupler). Gaussian input beam with a reduced waist equal to $5\sqrt{2}$ and a peak value equal to $\sqrt{35}$, $\alpha_0 = 0$, $\Delta = 5$. a) $L = 0.6$, $\tau = 1$, $\eta = -1$. b) $L = 0.1$, $\tau = 0.1$, $\eta = 1$.

where the first term of the right hand side of equation (4) refers to the modulated region. According to the periodicity of the grating, $\xi_m^{(1)}(x)$ can be expanded in Fourier series :

$$\xi_m^{(1)}(x) = \sum_{\ell} \xi_{m,\ell}^{(1)} e^{i\ell\sigma x}$$

In order to gain some physical insight into SHG at grating couplers, let us consider a plane wave pump field : $\mathcal{A}_1(x)$ no longer depends on x . Thus :

$$c_{1p}(x) = \mathcal{A}_1 C_{q_0} e^{i(\beta_0 + q_0 \sigma - \gamma_{1p}) x}$$

$$C_{q_0} = - \frac{t_{p,q_0}}{\beta_0 + q_0 \sigma - \gamma_{1p}}$$

and

$$c_{2m} = c_{2m}^{(1)} + c_{2m}^{(2)}$$

with :

$$c_{2m}^{(1)} = -2 \omega \mathcal{A}_1^2 \sum_{\ell} \xi_{m,\ell}^{(1)} \frac{t_{p,q_0}^2}{(\beta_0 + q_0 \sigma - \gamma_{1p})^2 [2\beta_0 + (2q_0 + \ell)\sigma - \gamma_{2m}]} \times e^{i[2\beta_0 + (2q_0 + \ell)\sigma - \gamma_{2m}] x} \quad (5a)$$

$$c_{2m}^{(2)} = -2 \omega \mathcal{A}_1^2 \xi_m^{(2)} \frac{t_{p,q_0}^2}{(\beta_0 + q_0 \sigma - \gamma_{1p})^2 [2(\beta_0 + q_0 \sigma) - \gamma_{2m}]} e^{i[2(\beta_0 + q_0 \sigma) - \gamma_{2m}] x} \quad (5b)$$

Phase matching takes place when EM resonance occurs both at ω and at 2ω . For usual dispersive media, this is only possible for $c_{2m}^{(1)}$ which arises from the modulation of the nonlinear susceptibility. Let ℓ_0 be the integer which labels the Fourier-component of the nonlinear susceptibility which leads to phase matching. In this case, the phase matched contribution to the SH amplitude comes from $c_{2m}^{(1)}$ and corresponds to $\ell = \ell_0$. Then, according to equation (5a), the periodicity of the grating fulfills :

$$d = \ell_0 \frac{\lambda/2}{n_{\text{eff},m}(2\omega) - n_{\text{eff},p}(\omega)} \quad (6)$$

$n_{\text{eff},m}(2\omega)$, $n_{\text{eff},p}(\omega)$ being the effective indices for the modes m and p at 2ω and ω respectively.

The calculation of $\xi_{m,\ell_0}^{(1)}$ is achieved by recalling that :

- $\xi_{m,\ell_0}^{(1)}$ only involves the normal modes [4, 5] of the grating couplers [6] at ω and 2ω ,
- there is no analytical expression of the transverse (along y) field map in the modulated region of the grating couplers [6].

Thus the calculation of $\xi_{m,\ell_0}^{(1)}$ is performed using the rigorous theory of diffraction in *linear* optics [6] at ω and 2ω .

The simplification introduced by the modal analysis can be better appreciated by comparing with the rigorous theory of diffraction in nonlinear optics developed in reference [3] : instead of the complicated flow chart figure 5 of reference [3], the SH amplitude c_{2m} is given by equations (5). In these expressions, the only computations correspond to a *linear* diffraction

study at ω and 2ω in order to get the numerical values of t_{p,q_0} and $\xi_{m,\ell_0}^{(1)}$. The formalism presented here has the additional advantage of allowing an easy physical insight in the SHG process at grating couplers.

Some numerical calculations have been performed considering a $\omega \rightarrow 2\omega$ interaction between TE_0 modes at these two frequencies. The refractive indices of the different layers of the grating coupler for SHG (Fig. 1) have been chosen in such a way that phase matching occurs inside the homogeneous guiding layer. Under these conditions, it is the Fourier coefficient $\ell_0 = 0$ which has to be retained in the expression of $c_{2m}^{(1)}$ since it is this Fourier coefficient which leads to phase matching in the modulated zone. The results are plotted in figure 3: the solid curve and the crosses correspond respectively to the modal approach developed in this paragraph and to the full numerical method 1. The agreement between both methods can be considered as excellent.

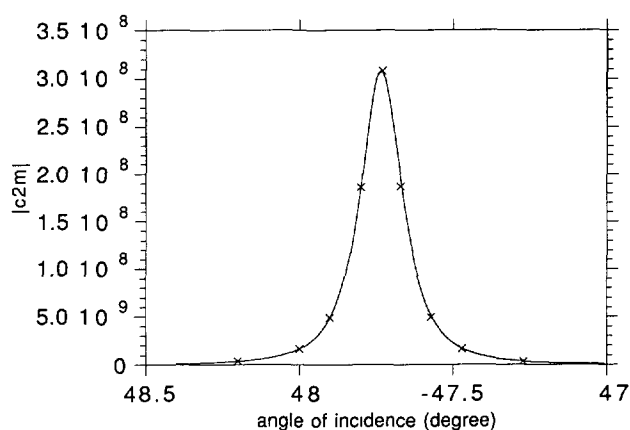


Fig. 3. — SHG at a grating coupler. Pump wavelength : $1.06 \mu\text{m}$. Grating : profile : sinusoidal, periodicity : $0.4 \mu\text{m}$, groove depth : $0.12 \mu\text{m}$. Waveguide : thickness : $0.58 \mu\text{m}$. Indices of the different media : at the pump frequency : medium 1 : 1.7 ; medium 2 : $2.0 + j0.0005$; medium 3 : 1. At the SH frequency : medium 1 : 1.7 ; medium 2 : $1.94175 + j0.0005$; medium 3 : 1. Solid curve : modal approach, crosses : method 1.

5. Conclusion.

It has been shown that the simultaneous use of the coupled mode formalism and of the rigorous theory of diffraction in linear optics provides a very convenient means to study nonlinear effects at grating couplers on waveguides. For stationary plane wave studies, analytical expressions are obtained where method 1 requires large computer calculations. The obvious advantage is that a physical insight is gained. The finite width of the input beam as well as diffusion and transient effects of the nonlinearity are known to be very important when dealing with optical Kerr effect. Indeed, the plane wave solution is never recovered even for an infinitely wide incident beam. In addition self-pulsing or chaos may be present.

The geometry of the resonator is not important but it is the ability to isolate a single resonance that is important. Thus the analysis presented in this paper applies not only to nonlinear grating couplers but also to other nonlinear optical resonators such as : nonlinear prism couplers, nonlinear Fabry-Pérot, nonlinear interference filters, nonlinear distributed feedback devices. Moreover, this method has been extended to the time domain leading, in

that way, to a powerful spatio-temporal theory of phenomena occurring in nonlinear optical resonators.

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