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Low-frequency responses of a plasma to a large amplitude high-frequency wave polarized on the extraordinary mode

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Abstract. — An exhaustive classification of the density perturbations and magnetic field generation induced by a large amplitude HF wave is made in the steady state case in a moving frame associated to the HF wave packet in the case of one-dimensional spatial variation of the envelope of the HF electric field and the condition of the quasi-neutrality is also specified in this case. The classification depends on four parameters, the skin depth L_0, β the ratio of the kinetic to magnetic pressure, $\alpha = 1 - V_g^2/C_A^2$(V_g the velocity of the moving frame, C_A the Alfven velocity) and $\upsilon = 1 - C_A^2/c^2$. Explicit expressions of the low frequency density n^S and induced magnetic field $\mathbf{B}^S$ are given according to the value of L_0 compared to ∂_x^{-1}, the slow space scale of the perturbations, to the value of β compared to α and to the value of υ. A few consequences of this model are discussed.

1. Introduction.

Non-linear phenomena related to the propagation of a large amplitude high-frequency wave in a plasma have been studied extensively in the past years. Of particular importance is the knowledge of the low frequency density n^S and magnetic field $\mathbf{B}^S$ perturbations induced in the plasma by the so-called ponderomotive effects due to inhomogeneous high-frequency fields. The earlier works dealt with low frequency perturbations n^S only in unmagnetized homogeneous or inhomogeneous plasmas [1]. In anisotropic plasmas ($\mathbf{B}_0 \neq \mathbf{0}$), the situation is much more complex. Density perturbations n^S have been classified according to the nature of the low frequency modes in the electrostatic approximation [2] ; the classification was based upon an ordering scheme for characteristic lengths and times (Ω_i^{-1}, ∂_x, ∂_z). Porkolab *et al.* [3] have determined the influence of various parameters (C_A/V_g, β, r_L, ∂_x) on the expressions of n^S and $\mathbf{B}^S$ in the case of a homogeneous plasma and an incident electrostatic wave. Relativistic mass effects on the plasma motion have also been included in the analysis [4].

In this paper, we consider the ponderomotive interaction of a magnetised plasma with an extraordinary wave which is a Bernstein mode in our case. For the one-dimensional case, we show that explicit approximate expressions of the non-linear density and magnetic field

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perturbations can be derived in terms of the incident field amplitude. Under this assumption, only a Bernstein mode can propagate perpendicularly to the external magnetic field. The upper hybrid mode can not be studied because its group velocity is along the magnetic field. As one can see in the article of Porkolab *et al.* [3] no attention about this point is taken into account. Also we assume that the low frequency responses of the plasma are in equilibrium with the HF wave packet ; that is to say the non-linear perturbations follow the displacement of the HF wave packet. Here we are interesting particularly to write the expressions of the LF non-linear perturbations moving at the velocity V_g compared with the plasma. These LF (low frequency) responses may be classified in terms of four parameters describing the system ; situations studied in other works appear as particular cases of this classification.

An outline of the paper is as follows. In section 2 we derive, using the two-fluid plasma model, non-linear coupled equations for the low frequency responses n_e^S, n_i^S and $\mathbf{B}^S$. In order to close the system, we make the usual quasi-neutrality assumption, $n_e^S = n_i^S$ (i.e. $\nabla \cdot \mathbf{E}^S = 0$), and write the evolution equations of the non-linear perturbations n^S and $\mathbf{B}^S$. In section 3, we deal with a one-dimensional quasi-stationary case in a moving frame, with the Berstein wave (electrostatic wave) propagating perpendicularly to the applied magnetic field. At first the validity conditions leading to the usual quasi-neutrality assumption are established ; quasi-neutrality, $n_e^S \approx n_i^S$, is assumed throughout the rest of the paper. Then we classify the LF responses n^S and $\mathbf{B}^S$ in terms of the four parameters $L_0 = c/\omega_{pe}$ (the skin depth for current excitation by electromagnetic waves) ; β, the ratio of the kinetic to magnetic energy density ; $\alpha = 1 - C_A^2/V_g^2$ related to the group velocity of the LF motions and the Alfvèn velocity and $\upsilon = 1 - C_A^2/c^2$ depending on the ratio of the Alfven velocity to light velocity. We determine the different regimes that follow from the values of these parameters and give for each of them the expression for the non-linear driving terms and the relative magnitude of n^S and $\mathbf{B}^S$. In section 4, we discuss some conclusions that can be drawn from the classification, for example on the evolution of solitons and the physical meaning of the skin depth and the spatial scale of the LF perturbations in the expression of n^S and $\mathbf{B}^S$ in terms of $|E|^2$.

2. Basic equations.

The external magnetic field $\mathbf{B}_0$ is constant and parallel to the z axis. A large amplitude high frequency wave (E^F, B^F) propagates perpendicularly to the applied magnetic field. The HF wave is supposed to be extraordinary and corresponds effectively to the Bernstein mode. The plasma is described by the two-fluid model for ions and electrons :

$$\begin{cases} \dfrac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0 \\ m_\alpha n_\alpha \dfrac{\partial \mathbf{v}_\alpha}{\partial t} + m_\alpha n_\alpha \mathbf{v}_\alpha \quad \nabla \mathbf{v}_\alpha = n_\alpha q_\alpha (\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B}) - \nabla p_\alpha \end{cases} \tag{1}$$

where $\alpha =$ e, i indicates the particle species and the other symbols have the usual meaning and the system (1) is closed with the Maxwell equations

$$\begin{cases} \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = \dfrac{\partial \mathbf{E}}{\partial t} + \mu_0 \sum q_\alpha n_\alpha \mathbf{v}_\alpha \\ \nabla \cdot \mathbf{E} = \dfrac{1}{\varepsilon_0} \sum q_\alpha n_\alpha \\ \nabla \cdot \mathbf{B} = 0 \end{cases} \tag{2}$$

with $n = N_0 + n^{\mathrm{S}} + n^{\mathrm{F}}$, $\mathbf{v} = \mathbf{v}^{\mathrm{S}} + \mathbf{v}^{\mathrm{F}}$, $\mathbf{E} = \mathbf{E}^{\mathrm{S}} + \mathbf{E}^{\mathrm{F}}$ and $\mathrm{B} = \mathbf{B}_0 + \mathbf{B}^{\mathrm{S}} + \mathbf{B}^{\mathrm{F}}$ where F for the high frequency part (1st order) and S stands for the slowly varying par (2nd order). Since we are dealing with low frequency perturbations, we shall use the usual two-time scale expansion method. After averaging over the fast time scale, the fluid and Maxwell equations then write

$$\left\{\begin{array}{l} \partial_t n_\alpha^{\mathrm{S}} + \nabla \cdot N_0\, \mathbf{u}_\alpha^{\mathrm{S}} = 0 \\ N_0\, m_\alpha\, \partial_t\, \mathbf{u}_\alpha^{\mathrm{S}} = q_\alpha N_0 (\mathbf{E}^{\mathrm{S}} + \mathbf{u}_\alpha^{\mathrm{S}} \times \mathbf{B}_0) - \nabla P_\alpha^{\mathrm{S}} + \mathbf{F}_{\mathrm{p}\alpha} \\ \nabla \times \mathbf{E}^{\mathrm{S}} = - \dfrac{\partial \mathbf{B}^{\mathrm{S}}}{\partial t} \\ \nabla \times \mathbf{B}^{\mathrm{S}} = \dfrac{\partial \mathbf{E}^{\mathrm{S}}}{\partial t} + \mu_0 \sum q_\alpha N_0\, \mathbf{u}_\alpha^{\mathrm{S}} \\ \nabla \cdot \mathbf{E}^{\mathrm{S}} = \dfrac{1}{\varepsilon_0} \sum q_\alpha\, n_\alpha^{\mathrm{S}} \\ \nabla \cdot \mathbf{B}^{\mathrm{S}} = 0\,. \end{array}\right. \tag{3}$$

$P_\alpha = \gamma n_\alpha^{\mathrm{S}}\, T_\alpha$ is the thermal pressure ($\gamma = 1$ for simplicity) and $\mathbf{u}_\alpha^{\mathrm{S}}$ is the renormalized fluid velocity [5]

$$\mathbf{u}_\alpha^{\mathrm{S}} = \mathbf{v}_\alpha^{\mathrm{S}} + \frac{1}{N_0} \left\langle n_\alpha^{\mathrm{F}}\, \mathbf{v}_\alpha^{\mathrm{F}} \right\rangle .$$

The ponderomotive force $\mathbf{F}_{\mathrm{p}}$ is derived from the usual expression [5]

$$\mathbf{F}_{\mathrm{p}\alpha} = - N_0\, m_\alpha \left\langle \mathbf{v}_\alpha^{\mathrm{F}} \quad \nabla \mathbf{v}_\alpha^{\mathrm{F}} \right\rangle + q_\alpha \left\langle N_0\, \mathbf{v}_\alpha^{\mathrm{F}} \times \mathbf{B}^{\mathrm{F}} \right\rangle - q_\alpha \left\langle n_\alpha^{\mathrm{F}}\, \mathbf{v}_\alpha^{\mathrm{F}} \right\rangle \times \mathbf{B}_0 + m_\alpha\, \partial_t \left\langle n_\alpha^{\mathrm{F}}\, \mathbf{v}_\alpha^{\mathrm{F}} \right\rangle$$

which can be written under the well-known following form [6]

$$\mathbf{F}_{\mathrm{p}\alpha} = - N_0\, \nabla \Phi_\alpha + \mathbf{B}_0 \times (\nabla \times \mathbf{M}_\alpha)$$

where the ponderomotive potential and the induced magnetisation are, respectively

$$\Phi_\alpha = \frac{m_\alpha}{4} \left\{ \mathbf{w}_\alpha \quad \mathbf{w}_\alpha^* + \frac{i q_\alpha}{\omega m_\alpha}\, \mathbf{B}_0 \cdot (\mathbf{w}_\alpha \times \mathbf{w}_\alpha^*) \right\}$$

$$\mathbf{M}_\alpha = i\, \frac{N_0\, q_\alpha}{4\, \omega}\, (\mathbf{w}_\alpha \times \mathbf{w}_\alpha^*)$$

where $\mathbf{w}_\alpha = \boldsymbol{\mu}_\alpha \quad \mathbf{E}^{\mathrm{F}}$, $\mathbf{w}_\alpha^*$ is its conjugate and $\boldsymbol{\mu}_\alpha$ the mobility tensor. The brackets mean averaging over a period of the high frequency motion. To obtain (3) we have neglected terms assumed of a higher order, such that $n^{\mathrm{S}}\, v^{\mathrm{S}}$.

Using Heuraux's formalism [7], we obtain after some algebra the equations coupling the low frequency responses $n_{\mathrm{e}}^{\mathrm{S}}$, $n_{\mathrm{i}}^{\mathrm{S}}$ and $\mathbf{B}^{\mathrm{S}}$

$$\begin{aligned} &\frac{1}{c^2}\, (\partial_t^2 + \omega_{\mathrm{pe}}^2 + \omega_{\mathrm{pi}}^2) \left\{ \nabla \times \tau_{\mathrm{e}}\, \tau_{\mathrm{i}}\, \nabla \times \mathbf{B}^{\mathrm{S}} + \frac{1}{c^2}\, \partial_t^2\, \tau_{\mathrm{e}}\, \tau_{\mathrm{i}}\, \mathbf{B}^{\mathrm{S}} + \left(\frac{\omega_{\mathrm{pe}}^2}{c^2} + \frac{\omega_{\mathrm{pi}}^2}{c^2} \right) \partial_t^2 \mathbf{B}^{\mathrm{S}} + \right. \\ &\qquad + \mu_{0\mathrm{e}} ([\nabla \times\,,\, \tau_{\mathrm{e}}]\, v_{\mathrm{ti}}^2\, \nabla n_{\mathrm{i}}^{\mathrm{S}} - [\nabla \times\,,\, \tau_{\mathrm{i}}]\, v_{\mathrm{te}}^2\, \nabla n_{\mathrm{e}}^{\mathrm{S}} \\ &\qquad \left. - \omega_{\mathrm{ce}}\, \partial_t^2 (n_{\mathrm{i}}^{\mathrm{S}} - n_{\mathrm{e}}^{\mathrm{S}})\, \hat{\mathbf{z}} - \nabla \times \tau_{\mathrm{e}} \frac{\mathbf{F}_{\mathrm{pe}}}{m_{\mathrm{i}}} + \nabla \times \tau_{\mathrm{i}} \frac{\mathbf{F}_{\mathrm{pe}}}{m_{\mathrm{e}}} \right) \right\} \times \\ &\qquad \times \left\{ \mu_{0\mathrm{e}} (v_{\mathrm{ti}}^2\, \nabla n_{\mathrm{i}}^{\mathrm{S}} - v_{\mathrm{te}}^2\, \nabla n_{\mathrm{e}}^{\mathrm{S}}) + \partial_t \nabla \times \mathbf{B}^{\mathrm{S}} \right\} - \frac{1}{c^2}\, \omega_{\mathrm{ci}}\, \omega_{\mathrm{ce}}\, \partial_t^2 (\mathbf{B}^{\mathrm{S}} \quad \hat{\mathbf{z}})\, \hat{\mathbf{z}} = \mathbf{0} \end{aligned} \tag{4}$$

where v_{te} and v_{ti} are the thermal velocities of electron and ion, c is the light velocity, $\tau_e \equiv \partial_t - \omega_{ce}\,\hat{\mathbf{z}}\times$, $\tau_i \equiv \partial_t - \omega_{ci}\,\hat{\mathbf{z}}\times$ and $\mathfrak{D}_z$, $[\nabla\times, \tau_\alpha]$ are operators which can be written in the following form

$$\mathfrak{D}_Z = \begin{cases} -\,\omega_{ce}\,\omega_{ci}\,\partial_y\hat{\mathbf{z}}\cdot + \omega_{ce}\,\partial_t\,\partial_x\hat{\mathbf{z}}\cdot \\ \\ \omega_{ce}\,\omega_{ci}\,\partial_x\hat{\mathbf{z}}\cdot + \omega_{ce}\,\partial_t\,\partial_y\hat{\mathbf{z}}\cdot \\ \\ -\,\omega_{ce}\,\partial_t\,\partial_z\,\hat{\mathbf{z}}\cdot \end{cases} \text{and } [\nabla\times, \tau_\alpha] = -\,\varepsilon\omega_{c\alpha}\begin{pmatrix} \partial_x\hat{\mathbf{z}}\cdot \\ \partial_y\hat{\mathbf{z}}\cdot \\ \nabla\cdot - \partial_z\hat{\mathbf{z}}\cdot \end{pmatrix}.$$

The variables have the following meaning : ω_{pe} and ω_{pi} are the electron and ion plasma frequencies, ω_{ce} and ω_{ci} are the (unperturbed) electron and ion cyclotron frequencies, and $\hat{\mathbf{z}}$ the unit vector along $\mathbf{B}_0$. It is easy to show that the operators τ_e and τ_i commute, i.e. the commutator $[\tau_i, \tau_e] = \tau_i\,\tau_e - \tau_e\,\tau_i = 0$ and we have used simplications as $\sum_{e\,i} [\nabla\times, \tau_\alpha]\dfrac{e}{m_\alpha}\mathbf{E}^{\text{s}} = \mathbf{0}$ and $\dfrac{F_{\text{Pez}}}{m_i} - \dfrac{F_{\text{Piz}}}{m_e} = 0$.

The second evolution equation is obtained more easily from the scalar equations and reads

$$\frac{1}{c^2}\,\partial_t^2\mathbf{B}_0\cdot\mathbf{B}^{\mathrm{S}} - \mathbf{B}_0\cdot\nabla\times\nabla\times\mathbf{B}^{\mathrm{S}} + \mu_0(m_i\,\partial_t^2\,n_i^{\mathrm{S}} + m_e\,\partial_t^2\,n_e^{\mathrm{S}} - T_i\,\nabla^2 n_i^{\mathrm{S}} - T_e\,\nabla^2 n_e^{\mathrm{S}}) =$$

$$= -\,\mu_0(\nabla\cdot\mathbf{F}_{pe} + \nabla\cdot\mathbf{F}_{pi})\,. \quad (5)$$

In order to close the set (4, 5), we make the usual quasi-neutrality assumption, $n_e^{\mathrm{S}} = n_i^{\mathrm{S}}$ (i.e. $\nabla\cdot\mathbf{E}^{\mathrm{S}} = 0$). We shall give in section 3 the validity condition of this assumption in the one-dimensional case. Changing to the normalized variables $n_{e,i}^{\mathrm{S}} = n_{e,i}^{\mathrm{S}}/N_0$, $\mathbf{b}^{\mathrm{S}} = \mathbf{B}^{\mathrm{S}}/B_0$, we obtain after some tedious algebra two equations coupling the low frequency responses n^{S} and $\mathbf{B}^{\mathrm{S}}$ given in equation (10) of Heuraux *et al.* [7].

Strictly speaking, in deriving equation (10) in Heuraux *et al.* [7], it is made two more assumptions, namely $m_e \ll m_i$ and $F_{pe} \gg F_{pi}$, but these are not necessary and used only to simplify the expressions. The second inequality may be wrong if $\omega_0 \approx \omega_{ci}$, but the validity of the two-time scale analysis ($\omega_0 \gg \partial_t$) and of the two-fluid model ($\omega_{pe} \gtrsim \omega_{ce}$, i.e. $\omega_{pi} \gg \omega_{ci}$) excludes this case.

3. A classification of the LF responses in the one-dimensional case.

Now we assume a weak inhomogeneity, $\partial_x \gg L^{-1}$ (L, the inhomogeneity length), and $\partial_x \gg \partial_z$. These assumptions are justified if the plasma is lighted up with an extended HF source compared to the plasma. That is to say we look at the density and magnetic field perturbations associated to a soliton which moves toward a magnetized plasma. This situation is associated to the quasi-steady state assumptions in a moving frame because, under the quasi-adiabatic assumption, the LF perturbations follow the soliton with the same velocity. The advantage of this one-dimensional restriction can permit to reduce the set of Zakharov equations to NLSE whichever gives the velocity of the soliton [8] and permits to describ all the cases which can be found under the assumption of the weak inhomogeneity. Under these conditions, it is also possible to have a picture about the evolution of the soliton during its motion through the density gradient. However this study is closed to few particular cases applied for Bernstein wave. In fact in more realistic cases, the HF wave has a finite spatial extension which imposes to treat this problem in 2D space and time to take account the relaxation phenomena which can appear. Our study deals with only nonlinear slow perturbations associated to a soliton propagating perpendicularly to the external magnetic field. Under these considerations, one can exhibit a mecanism which can explain the destruction of the soliton due to the change of the nature of slow perturbations (modulational instability criterion) induced by a increase or decrease of the soliton velocity through the density gradient, this in the limits of the validity of the model.

Under the assumption $\partial_x \gg \partial_z, \partial_y$ one can see after some algebra from equations (4), (5) that the components B_x^S, B_y^S are smaller than B_z^S in the steady-state case. This results can also be derived from an article on the 3D non-linear magnetic field generation [9].

From equations (4, 5), assuming B_x^S, $B_y^S \ll B_z^S$ the evolution equations become

$$\left[(\partial_t^2 + \omega_{ce}\,\omega_{ci})\left(\frac{1}{c^2}\partial_x^2 - \partial_t^2\right) + \frac{\omega_{pe}^2}{c^2}\partial_t^2\right] b_z^S = \frac{\omega_{pe}^2}{c^2}\left(\frac{T_e}{m_i}\partial_x^2 n_e^S + \frac{T_i}{m_i}\partial_x^2 n_i^S + \partial_t^2(n_i^S - n_e^S)\right) + S_{NL}$$

$$C_A^2\left(\frac{1}{c^2}\partial_x^2 - \partial_t^2\right) b_z^S - \frac{T_e}{m_i}\partial_x^2 n_e^S - v_{ti}^2\,\partial_x^2 n_i^S + \partial_t^2 n_i^S = -\frac{1}{m_i N_0}\partial_x F_{pex} \tag{6}$$

where S_{NL} corresponds to the terms containing $\mathbf{F}_{pe}$ in equation (4), C_A is the Alfven velocity and C_S is the ion acoustic velocity.

After eliminating b_z^S in these evolution equations (it is faster to start from the fluid and Maxwell equations to do it) and if we take into account the fact that the low frequency fluctuations have characteristic times much longer than the inverse electron frequencies, we may write a simplified expression relating the ion and electron densities

$$n_e^S = n_i^S + \frac{\partial_t^2 n_i^S}{\omega_{pi}^2 + \omega_{ci}^2}. \tag{7}$$

This condition encompasses the cases analysed by Porkolab and Berezhiani [3, 4]. From equation (5) it immediately follows that LF responses exhibit significant departures from the neutrality only in two cases : *a*) strongly magnetized, tenuous plasma, $\omega_{pe}^2 < \omega_{ce}\,\omega_{ci}$ and *b*) in other cases ($\omega_{pi} \gg \omega_{ci}$) when LF time scale much less than ω_{pi}^{-1}. In most cases, however, the LF responses are essentially quasi-neutral, $n_e^S \approx n_i^S \approx n^S$. We shall use this hypothesis in the following paragraphs.

We will focus now on the situation where the incident HF wave is electrostatic in the one-dimensional case. The system of equations (6) rewrites in two coupled equations for the low frequency density and magnetic field perturbations, n^S and $\mathbf{B}^S$ which can be decoupled. The decoupled equations for n^S and $\mathbf{B}^S$ write

$$\begin{aligned} \mathcal{L} n^L &= \beta\left\{\partial_x^2(Z_P - S_P) - \frac{S_P}{L_0^2\,\Gamma}\,\upsilon\right\}, \\ \mathcal{L} B^L &= \beta\left\{\frac{\alpha + \beta - 1}{\Gamma}\,\partial_x^2 Z_P - \frac{S_P}{L_0^2\,\Gamma}\right\}. \end{aligned} \tag{8}$$

Here $n^L = n^S/N_0$ and $B^L = B_z^S/B_0$ are the relative density and magnetic field perturbations and $\mathcal{L}$ is a linear operator that writes

$$\mathcal{L} = \left\{\frac{1}{L_0^2} + \frac{\alpha + \beta - 1}{L_0^2\,\Gamma}\,\upsilon - (\alpha + \beta - 1)\,\partial_x^2\right\}.$$

The expressions of S_P and Z_P are given in the appendix for an extraordinary wave. For simplicity, we shall restrict ourselves here to the case of an electrostatic wave $E^F(E_x, 0, 0)$. Then the expressions of S_P and Z_P write

$$S_P = \frac{\omega_{pe}^2}{(\omega^2 - \omega_{ce}^2)^2} \left((\omega^2 + \omega_{ce}^2) + \omega^2 \frac{V_g}{V_\varphi} \right) \frac{\varepsilon_0 |E_x|^2}{N_0(T_e + T_i)} \quad \text{and} \quad Z_P = \frac{\omega_{ce}^2 \, \omega_{pe}^2}{(\omega^2 - \omega_{ce}^2)^2} \frac{\varepsilon_0 |E_x|^2}{N_0(T_e + T_i)}$$

where V_φ is the phase velocity of the HF wave.

The source terms are obtained from the definition of the ponderomotive force written in terms of ponderomotive potential and magnetization current and it is easy to deduce that Z_P depends only the induced magnetization.

We have introduced the parameters $L_0 = c/\omega_{pe}(x)$ (the plasma skin depth with respect to current generation by electromagnetic waves), $\alpha = 1 - V_g^2/C_A^2$ (C_A is the Alfven velocity), $\beta = \mu_0 N_0(x)(T_e + T_i)/B_0^2$, $\upsilon = 1 - C_A^2/c^2$ and $\Gamma = 1 - V_g^2/c^2$. In the following, we shall study the different types of LF responses in terms of L_0, α, β, υ, Γ and the slow space scale of the perturbations ∂_x^{-1}

Note that α can be either positive (sub-Alfvenic regime) or negative (super-Alfvenic regime). Most situations correspond to the first case. Although the present analysis is not relativistic, it must be noted that Berezhiani *et al.* [4] give a criterion for the existence of relativistic effects on the soliton formation in the incident field. The criterion takes into account two types of nonlinearities in $|E|^2$, one is associated to the ponderomotive force and the other corresponds to the relativistic effects [4] ; with our notations, it writes

$$\alpha < 1 - \frac{2}{3} \frac{\omega_{pe}^2}{\omega_{ce}^2} . \tag{9}$$

Hence, for non relativistic plasmas submitted to intense HF fields, relativistic effects can modify terms in the non-linear evolution equation for the HF mode, but leave unchanged the LF equations.

Therefore, we will ignore condition (9) in the subsequent classification and put $\Gamma = 1$.

3.1 CASE $\upsilon \approx 1$, $C_A^2 \ll c^2$. — In the limit of validity of fluid model $\partial_x^2 \ll 1/(L_0^2 \beta)$, the spatial scale length is much larger than the Larmor radius. The nonlinear source terms are important only for $\omega \approx \omega_{ce}$, thus we have $S_P \approx 2 Z_P$. Then equations (8) reduce to

$$\begin{aligned} (\alpha + \beta)\, n^L &= \beta \left\{ L_0^2 \, \partial_x^2 (Z_P - S_P) - S_P \right\} , \\ (\alpha + \beta)\, B^L &= \beta \left\{ (\alpha + \beta - 1)\, L_0^2 \, \partial_x^2 Z_P - S_P \right\} . \end{aligned} \tag{10}$$

i) The most frequent situation corresponds to a low-β plasma with a sub-Alfvenic mode ($\alpha \approx 1$).

Here we have two subcases :

a) If $L_0^2 \, \partial_x^2 \gg 1$ (short wavelength perturbations), then we have

$$n^L = \beta L_0^2 \, \partial_x^2 (Z_P - S_P) \quad \text{and} \quad B^L = -\beta S_P .$$

In this case, $B^L \ll n^L$

b) else $L_0^2 \, \partial_x^2 \ll 1$ (long wavelength perturbations), we have

$$n^L = -\beta S_P \quad \text{and} \quad B^L = -\beta S_P .$$

Here $B^L \approx n^L$.

These cases can be found in previous works [3] but with no clear classification parameters.

ii) For a high-β plasma with a sub-Alfvenic or trans-Alfvenic mode if $\beta \gg |\alpha|$, since in this case the validity of the fluid model excludes the inequality $L_0^2\,\partial_x^2 \gg 1$, one finds for the case $L_0^2\,\partial_x^2 \ll 1$

$$\tilde{n}^{\mathrm{L}} = -S_{\mathrm{P}} \quad \text{and} \quad B^{\mathrm{L}} = -S_{\mathrm{P}}\,.$$

Again we have $B^{\mathrm{L}} \approx n^{\mathrm{L}}$. This situation includes the case when the LF mode is the Alfven mode.

iii) Case $\beta \ll |\alpha - 1|$, $\upsilon \approx 1$. This situation occurs in low-β plasmas when the LF mode is the fast mode (super Alfvenic perturbations).

a) $L_0^2\,\partial_x^2 \gg 1$, we obtain

$$n^{\mathrm{L}} = \frac{\beta}{|\alpha - 1|}(Z_{\mathrm{P}} - S_{\mathrm{P}}) \quad \text{and} \quad B^{\mathrm{L}} = \beta Z_{\mathrm{P}}\,.$$

Then $B^{\mathrm{L}} \approx (\alpha - 1)\,n^{\mathrm{L}}$ can be much greater than n^{L}.

b) $L_0^2\,\partial_x^2 \ll 1$. One finds

$$n^{\mathrm{L}} = \frac{\beta}{|\alpha|} S_{\mathrm{P}}$$

and we have two subcases

- $(\alpha - 1)\,L_0^2\,\partial_x^2 \ll 1$

$$B^{\mathrm{L}} = \frac{\beta}{|\alpha|} S_{\mathrm{P}}$$

for which $B^{\mathrm{L}} \approx n^{\mathrm{L}}$. One recovers the case studied by Kaufman *et al.* [10].

- $|(\alpha - 1)\,L_0^2\,\partial_x^2| \approx 1$

$$B^{\mathrm{L}} = \beta\,\frac{(\alpha - 1)}{\alpha}\,L_0^2\,\partial_x^2\,Z_{\mathrm{P}} + \frac{\beta}{|\alpha|} S_{\mathrm{P}}\,.$$

Hence, $B^{\mathrm{L}} > n^{\mathrm{L}}$. In this case, corrections due to the V_g/V_φ term in S_{P} can become important. The density and magnetic field perturbations are comparable, but weak since $\beta/\alpha < 1$.

Since α may become negative, the change in the sign of α will change the sign of the dispersion term. It results that rarefaction waves (cavitons) correspond to sub-Alfvenic perturbations and compression waves to super-Alfvenic perturbations, as already noted by Kaufman *et al.* [10]. Similarly, we point out that in the super-Alfvenic regime, the generated magnetic field adds to the external magnetic field, in contrast with the most common case of sub-Alfvenic perturbations.

3.2 CASE $\upsilon \approx 0$, $C_{\mathrm{A}}^2 \approx c^2$. — This case has never been studied so far. Since $\alpha \approx 1$, equations (8) reduce to

$$\begin{aligned} n^{\mathrm{L}} &= r_{\mathrm{L}}^2\,\partial_x^2(Z_{\mathrm{P}} - S_{\mathrm{P}}) = \beta L_0^2\,\partial_x^2(Z_{\mathrm{P}} - S_{\mathrm{P}})\,, \\ B^{\mathrm{L}} &= -\beta S_{\mathrm{P}}\,. \end{aligned} \tag{11}$$

Hence, when $L_0^2\,\partial_x^2 \gg 1$, one has $B^{\mathrm{L}} \ll n^{\mathrm{L}}$, and when $L_0^2\,\partial_x^2 \ll 1$, one finds $B^{\mathrm{L}} \gg n^{\mathrm{L}}$. This case exhibits very well the effect of the skin depth on the determination of the nature of the nonlinear responses.

If we compare these results to the case of an unmagnetized plasma, it is easy to show that the perturbations are smaller (for a given pump) except for the case of high β plasma.

4. Discussion.

The equations (8) may be used, together with the evolution equation of the HF wave, to write a self-consistent set of evolution equations (Zakharov-like equations) in the slab model under the WKB hypothesis. It would be interesting to solve numerically these equations ; however, from the above simplifications of equations (8), we can make a qualitative analysis of the field evolution. With the help of these simplified solutions the HF evolution equation may be reduced to a generalized NLSE equation, whose solutions are envelope solitons (Langmuir-like or cusped) [11, 12]. In the inhomogeneous plasma all types of solitons are accelerated in the density gradient [8]. If we assume that the nonlinear perturbations follow the HF wave packet (quasi-adiabatic hypothesis) and that the velocity V_g of the moving frame is slowly varying in the density gradient, it is clear that the linear operator on the lhs of (8) changes *via* α because the velocity of the soliton V_g depends on the position in the density gradient [8]. The nonlinear source terms are little modified by the variations of the soliton velocity. The soliton can not indefinitely accelerate. Indeed, when the soliton velocity increases, the nature of the responses changes according to the above classification. In particular, when α becomes negative, it changes from a rarefaction wave to a compressional wave. Then the criterion for modulational instability (dispersive and nonlinear terms should have the same sign) is no longer satisfied and the soliton is destroyed by dispersive effects. A temperature or magnetic field inhomogeneity can produce the same effect *via* the parameters α, β, L_0 and υ.

In general, when the LF perturbations evolve over a characteristic length ∂_x^{-1} much greater than the skin depth L_0, the excited magnetic field is negligible compared to the density fluctuation. This is due to the fact that the nonlinear currents can not reach large values over distances that are greater than the skin depth. Hence, there can be efficient magnetic field generation only for cases where the skin depth is of the order of the scale length of the LF fluctuations $L_0\,\partial_x \gtrsim 1$. The evaluation of the relative variations of perturbations are essentially determined by the value of υ, α compared to β, and L_0 compared to ∂_x^{-1} because the validity of fluid model imposes $r_L\,\partial_x \ll 1$. The introduction of the υ parameter leads to new cases with nonlinear responses varying as $|E_x|^2$ or $\partial_x^2|E_x|^2$.

In conclusion, for one-dimensional propagation perpendicular to the external magnetic field, we have derived *explicit* solutions for the nonlinear low frequency responses of a plasma to a high frequency wave which can only be a Bernstein wave because we have taken into account the group velocity of the HF wave which has not been made in the previous studies [3, 9]. These density and magnetic field perturbations are proportional either to $|E_x|^2$ or $\partial_x^2|E_x|^2$. Hence, over a large range of parameters (determined by the values of α, β, υ and L_0), the HF field envelope perturbation can have the particular form of cusped or Langmuir solitons like.

Appendix.

We express in the following the non-linear source terms in two different cases : the electromagnetic wave (extraordinary mode) or its electrostatic limit.

ELECTROSTATIC WAVE $E^F(E_x, 0, 0)$. — From the expression of the ponderomotive force F_{pe}, one can find

$$S_P = \frac{\omega_{pe}^2}{(\omega^2 - \omega_{ce}^2)^2}\left((\omega^2 + \omega_{ce}^2) + \omega^2\frac{V_g}{V_\varphi}\right)K_P$$

and

$$Z_P = \frac{\omega_{ce}^2\,\omega_{pe}^2}{(\omega^2 - \omega_{ce}^2)^2} K_P \quad \text{where} \quad K_P = \frac{\varepsilon_0 |E_x|^2}{N_0(T_e + T_i)}$$

where V_φ is the phase velocity of the HF wave.

In the vicinity of the upper hybrid resonance $S_P \approx 2\, Z_P$ when $\omega_{pe} \approx \omega_{ce}$.

ELECTROMAGNETIC WAVE $E^F(E_x, E_y, 0)$. — After some tedious algebra, the ponderomotive sources can be written as

$$S_P = \frac{\omega_{pe}}{(\omega^2 - \omega_{ce}^2)^2} \left((\omega^2 + \omega_{ce}^2) K_P + 2\, \omega\omega_{ce} E_P - \frac{V_g}{V_\varphi} (P_P - \omega\omega_{ce} E_P) \right)$$

$$Z_P = \frac{\omega_{ce}^2\,\omega_{pe}^2}{\omega(\omega^2 - \omega_{ce}^2)^2} \left(\omega\omega_{ce} K_P - \omega^2 E_P - (\omega^2 - \omega_{ce}^2) I_P - \frac{V_g}{V_\varphi} \times \right.$$
$$\left. \times \left((\omega^2 - \omega_{ce}^2) I_P + 2 \frac{\omega^2}{\omega_{ce}^2} H_P + 2\, \omega^2 L_P \right) \right)$$

where $K_P = \dfrac{\varepsilon_0 |E_x|^2 + |E_x|^2}{4\, N_0(T_e + T_i)}$; $L_P = \dfrac{\varepsilon_0 |E_y|^2}{4\, N_0(T_e + T_i)}$; $P_P = \dfrac{\varepsilon_0(\omega^2 |E_x|^2 + \omega_{ce}^2 |E_y|^2)}{4\, N_0(T_e + T_i)}$;

$$E_P = \frac{\varepsilon_0 \operatorname{Im}(E_x E_y^* - cc)}{4\, N_0(T_e + T_i)}; \quad I_P = \frac{\varepsilon_0}{4\, N_0(T_e + T_i)} \int dx \operatorname{Im} \langle E_x\, \partial_x E_y^* - cc \rangle$$

and
$$H_P = \frac{\varepsilon_0}{4\, N_0(T_e + T_i)} \int dx \operatorname{Im} \langle E_y\, \partial_x E_x^* - cc \rangle .$$

In this derivation, the polarisation of the HF extraordinary mode is taken account, that is to say the terms proportional to Im $(E_x E_y^* + cc)$ vanish.

The polarisation of the HF wave modifies the source terms of equations (8), and implies that the classification can be changed. For example when $\omega \approx \omega_{ce}$, an electrostatic mode gives $S_P \approx 2\, Z_P$ and $2\, S_P \approx Z_P$ for the extraordinary mode in an underdense plasma (purely electromagnetic mode).

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