A state variables method for the optimization of electrical machines. Application to the acceleration of an inertia

M. Poloujadoff, Z. Amine Es-Sbai

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A state variables method for the optimization of electrical machines. Application to the acceleration of an inertia

M. Poloujadoff and Z. Amine Es-Sbai

Université Paris VI, Laboratoire d'Electrotechnique, El-Tour 22-12, 4 place Jussieu, 75251 Paris Cedex 05, France

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Abstract. — The object of the paper is to define and illustrate a method to « synthetize » a motor before optimizing it. The example is the starting of an inertia from rest to full speed by switching on a squirrel cage machine ; supply voltage and frequency are kept constant during starting. It is shown that there are only seven independent variables, and that there is no attainable minimum of the starting time.

1. Introduction.

The duty of the design engineer is to design machines according to specifications. It may happen that a problem has no solution, but this case is not common. Generally, there is an infinity of solutions. In this latter case, it is natural to look for the best possible solution, at the condition that a quality factor has been clearly defined.

However, electrical machines are very complex, the number of parameters is large, and their relationship to performance characteristics are often implicit. This is why the most common practice to find a new design is to start from a previous one and to modify it by application of so-called « similitude rules ». However, this approach is progressively abandonned for various reasons. On the one hand, technology is permanenty changing, and this makes it difficult to apply similitude rules. On the other hand, there are many cases when there is no previous experience to be used as a starting point.

For those reasons, many authors have tried to use the methods called : « non linear programming ». This means that a machines is defined by a number n of main parameters $x_1, x_2, \ldots x_n$, arranged into a vector $\mathbf{X} = (x_1, x_2, \ldots x_n)^T$, and of « secondary variables » $k_1, \ldots, k_m$, defining a vector $\mathbf{K} = (k_1, \ldots, k_m)^T$. The secondary variables are quantities which do not influence the machines performances, or whose values cannot be freely chosen by the designer. The components of $\mathbf{K}$ will not be changed during the optimization process.

The design constraints can be expressed by

$$G_{im} \leqslant g_i(\mathbf{K}, \mathbf{X}) \leqslant G_{iM} \quad i = 1, 2, 3, \ldots, m .$$

The quality factor of a machine can be expressed as a function $C(\mathbf{K}, \mathbf{X})$.
Therefore, any design may be stated as :
« find $\mathbf{X}$ such as $C(\mathbf{K}, \mathbf{X})$ be minimum, subject to $G_{im} \leq g_i(\mathbf{K}, \mathbf{X}) \leq G_{iM}$ ».
Any machine meeting the specifications and satisfyng the constraints is called « feasible ».

2. Definition of the method.

The object of the present paper is to illustrate a particular kind of no linear programming, which we call « state variable method » and which has been for the first time in the simple problem of transformers [1], and later in the more difficult case of linear induction machines [2].

The first step is to replace the inequalities of the type

$$G_{im} \leq g_i(\mathbf{K}, \mathbf{X}) \leq G_{iM}$$

by equalities of the type :

$$g_i(\mathbf{K}, \mathbf{X}) = G_{im} + \beta_i (G_{iM} - G_{im})$$

$$0 \leq \beta_i \leq 1 .$$

Thus introducing β_i as a new parameter of the machine.

Then, we choose for $\mathbf{X} = (x_1, \ldots, x_n)$ a set of n parameters in such a way that each entirely arbitrary vector $\mathbf{X}$ defines one feasible machine and only one. Some of the quantities β_i may belong to the set $(x_1, x_2, \ldots, x_n)$. The choice of the components of $\mathbf{X}$, and the development of the algorithm which allows to determine a feasible machine from the numerical values of the x_i s is called the « synthesis » of the design. It is by far the most difficult step.

After the synthesis has been defined, the quality factor $C(\mathbf{K}, \mathbf{X})$ (also called « objective function ») becomes a function of the n independent variables $x_1, x_2, \ldots, x_n$. The search for the extremum of C is then very simple, because it may use any method to be found in classical textbooks under the title » unconstrained search methods ».

3. The problem.

In this paper, we try to determine the best induction motor to start an inertia from rest. Starting is performed by sudden switching on at fixed frequency. A motor is said to be « better » than another one if its starting time is shorter.

This example is particularly interesting because there are obviously no data in the open literature to be used as starting point. In addition, it will be seen that the method of state variables allows to discover that there is no optimum in this particular case, and helps to make a reasonable choice.

4. Machine spécification and models.

The inertia is to be accelerated from 0 to 3 000 rpm ; its value is 20 $kg.m^2$. In addition, it is specified that the supply is 380 V, 50 Hz. Each starting is performed with a cold motor (at room temperature, assumed 20 °C).

An obvious technical constraint is that the final temperatures of the stator and rotor conductors must be compatible with the nature of insulation ; we shall assume that limit values are 100 °C and 300 °C for the stator and rotor conductors respectively.

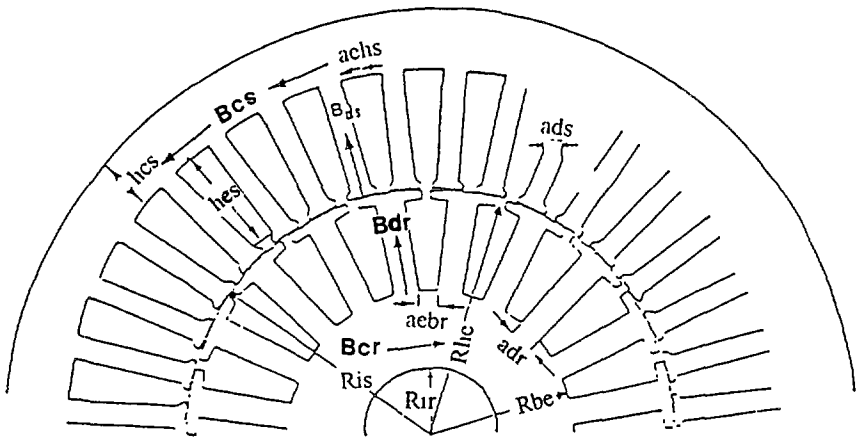

Fig. 1. — Definition related to punching and flux densities.

Then, design calculations must be based on a model of the machine. For the electromagnetic behaviour, we adopt the hypothesis in [3], with a modified presentation which has already been used in [4], and its developed again below formules for skin effect have been taken from [5]. The thermal behaviour is modeled by an adiabatic temperature rise ; such a choice is not simple ; it is made arbitrarily at the beginning of the study, because it is simple and seems logical ; however, it can be eventually retained because the final choices lead to a very small starting time (less than 2 s). The next step is to choose a geometry of the stator and rotor. We have chosen constant width teeth for both the rotor and stator ; naturally other choices may be tried.

A few quantities have little effect on the objective function ; they have been called « secondary variables » by Appelbaum [6] ; they are :

- number of rotor bars $Q = 32$;
- air gap length $e = 3$ mm ;
- number of stator slots per pole per phase $N_{epp} = 3$;
- heigth of insulating wedges stator $h_{ces} = 4$ mm ;
- stator pole pitch (full) ;
- filling factors of stator slot ($k_{rs} = 60\ \%$) ;
- height and width of rotor slot opening (4 and 2 mm) ;
- rotor skew ($a = 2$ stator slots) ;
- height and width of stator opening ($h_{is} = 3$ mm and $a_{is} = 3$ mm) ;
- flux density in the stator core ; 1.8 T ;
- the resistivity of aluminium and copper are 4.2×10^{-8} (320 °C) and 2.2×10^{-8} Ω.m (120 °C) respectively.

Other notations are given in figure 1.

5. Machine synthesis.

The general method presented in the introduction is very easy if the state variables are well chosen ; but this choice is generally very difficult, and demands a very deep knowledge of the physical relationships.

i) To design the rotor, we successively choose the following variables :

- a geometrical quantity, the shaft radius R_{ir} ,

• the ratio of the width a_{dr} of the tooth to the width a_{ebr} of the slot, both being measured at the bottom of the slot ; let $R_{edbr} = (a_{ebr}/a_{dr}) > 0$; if $R_{edbr} = 0$, the slots are triangular ;

• a magnetic quantity which is the peak value of the flux density in the rotor tooth at synchronism B_{dr}. There is a relationship between this magnetic flux density, the magnetic flux density in the rotor core B_{cr}, and the distance R_{be} of the bottom of the slot to the axis R_{be} .

$$R_{be} = \frac{B_{cr} \cdot R_{ir}}{B_{ce} - \dfrac{\pi \cdot B_{dr}}{Q(1 + R_{edbr}) \sin\left(\dfrac{p \cdot \pi}{Q}\right)}}$$

Since R_{be} must be positive, B_{cr} must be larger than

$$B_{crmin} = B_{cr} - \frac{\pi \cdot B_{dr}}{Q(1 + R_{edbr}) \sin\left(\dfrac{p \cdot \pi}{Q}\right)}$$

• as a consequence of the above, we are led to choose a dimensionless variable K_1 ($K_1 > 1$) such that :

$$B_{cr} = K_1 \cdot B_{crmin} .$$

The reason for this definition is that K_1 and B_{cr} can be defined independently, while B_{cr} and B_{dr} cannot.

Note that

$$a_{dr} = \frac{2 \cdot \pi \cdot R_{be}}{Q(1 + R_{edbr})}$$

and $a_{ebr} = R_{edbr} \cdot a_{dr}$,

• the outside diameter R_{he} cannot be defined independently of R_{be}, since R_{he} must be larger than R_{be}. However, the ration

$$R_{hbe} = \frac{R_{he}}{R_{be}} > 1$$

can be chosen independently of any other quantity ;

• the length of the rotor will be called L_t ;

• the cross section area S_{anc} of the short circuit rings is defined through its ration C_s to the cross section area S_{cr}, of the bars

$$C_s = \frac{S_{anc}}{S_{cr}} > 0 .$$

ii) To design the stator, we choose the following quantities :

• a dimensionsless factor F representing the relative saturation of stator and rotor teeth. If the peak flux density in the stator teeth is B_{ds},

$$F = \frac{B_{ds}}{B_{dr}} > 0$$

generally, $F < 1$;

- the height h_{es} of stator slots ;
- the inside radius of the stator will be the outside radius of the rotor, plus the air gap length.

It is easily seem that if all the above quantities have been chosen, the cross section area of one stator slot, the number of stator conductors determined from the voltage and frequency at the mains.

iii) Choice of the state variables.

Up to now, we have chosen to describe a motor by the following variable :

- seven secondary variables which cannot be changed, or which do not influence very much the result : air gap length, number of stator slots, width and height of the rotor and stator slot openings, skew, number of rotor slots ;
- nine variables which are defined above : R_{ir}, R_{edbr}, B_{dr}, K_1, R_{hbe}, L_f, C_s, F, h_{es}. We may call these : « primary variables ».

To any particular choice of the primary variables corresponds a motor, but this is not generally « feasible » because its temperature rises are not equal to the given limits. In the case of the present problem, it is easy to see that if we pick at random a value of the following seven variables :

$$R_{ir},\ R_{edbr},\ B_{dr},\ K_1,\ L_f,\ C_s,\ F\ .$$

There is one and only one value for R_{hbe} and one for h_{es} which allow to adjust exactly the temperature rises. This is clearly shown by figures 2a and 2b. At this point, any arbitrary choice of the values of the seven above quantities defines a feasible machine and only one. The starting time T_d of this machine can be evaluated by integrating the acceleration.

$$T_d = \int_0^{\Omega_s} J_t \frac{d\Omega}{C_{em} - C_r}$$

where C_{em} is the electromagnetic torque. C_r and J_t are the resistive torque and inertia constant of the load, respectively.

Therefore, T_d is a function of seven variables.

$$T_d = T_d(R_{ir},\ R_{edbr},\ B_{dr},\ K_1,\ L_f,\ C_s,\ F)\ .$$

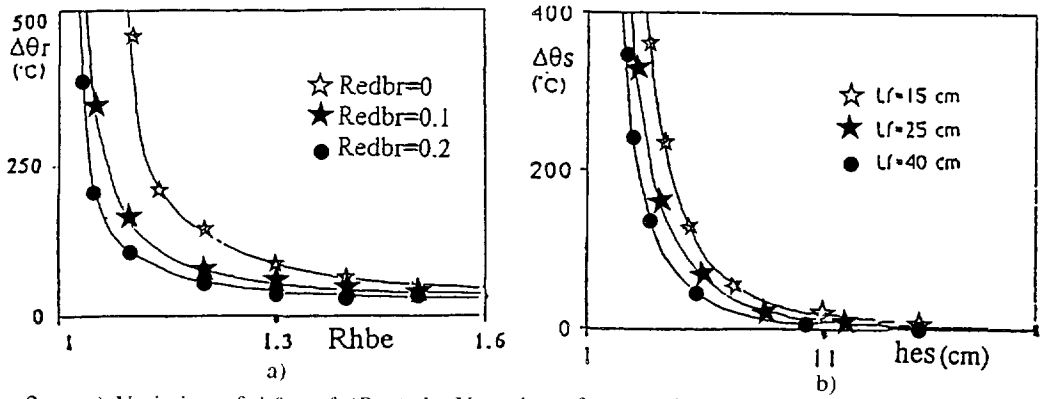

Fig. 2. — a) Variation of $\Delta\theta_r = f_1(R_{hbe})$. b) Variation of $\Delta\theta_s = f_2(h_{es})$.

The flow chart to compute T_d is shown in figure 3.
This is the end of the first part of the method : the synthesis of a feasible machine.

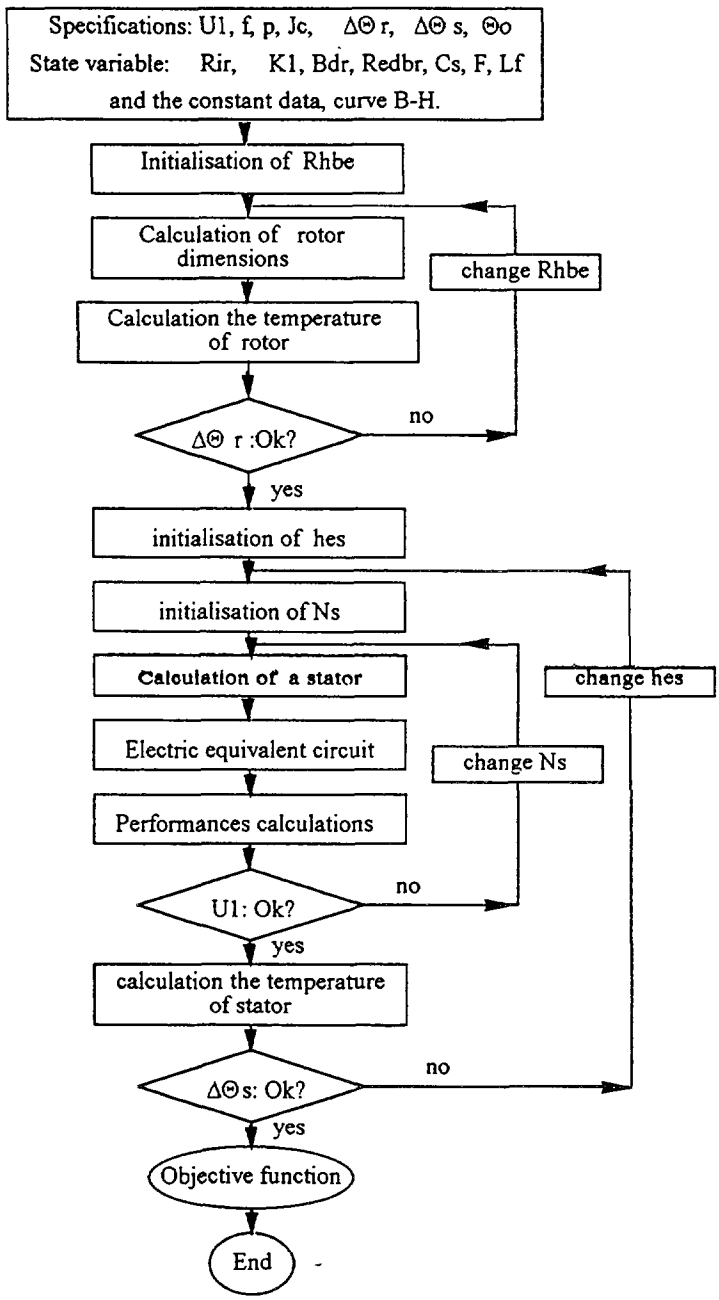

Fig. 3. — Flow chart of the main subroutine.

6. Optimization.

Now, it is extremely simple to find the machine which leads to the shortest possible starting time. Indeed, we have to find the vectors $\mathbf{X} = R_{ir}, K_1, R_{edbr}, B_{dr}, L_f, C_s, F)$ such that $T_d(\mathbf{X})$ is minimum, with :

$$R_{ir} > 0 ,$$
$$K_1 > 1$$
$$0 \leqslant R_{edbr} \leqslant 1$$
$$0 \leqslant B_{dr} \leqslant 2 \text{ T}$$
$$L_f > 0$$
$$0 < C_s \leqslant 1$$
$$0 < F < 1 .$$

This is a non constrained optimization problem.

It might be possible to use elaborated techniques such as the steepest descent method. We choose the univariant search method, associated with a dichotomy algorithm, for reasons which will be given below.

Table I. — *Optimization process resulting from figure* 4.

Curvess	Rir(cm)	Bdr(T)	K1	Redbr	Cs	F	Lf(m)	Td (sec)
Fig. 4-A	2<Rir >7	0.2	1.2	0.8	0.7	0.2	0.25	699
Fig. 4-B	4.3	0<Bdr<2	1.2	0.8	0.7	0.2	0.25	33.63
Fig. 4-C	4 3	1.2	K1>1	0.8	0.7	0.2	0.25	19.44
Fig. 4-D	4.3	1.2	1.1	0<Redbr<1	0.7	0.2	0.25	8.06
Fig. 4-E	4.3	1.2	1 1	0.05	0<Cs<1	0.2	0.25	6.99
Fig. 4-F	4.3	1.2	1.1	0.05	0.4	0<F<1	0.25	6.32
Fig. 4-G	4.3	1.2	1.1	0.05	0.4	0.95	Lf>0	5.95
Fig. 4-H	1<Rir<7	1.2	1.1	0.05	0.4	0.95	0.45	4.55
Fig. 4-I	3	0<Bdr<2	1.1	0.05	0.4	0.95	0.45	1.63
Fig. 4-J	3	2	K1>1	0.05	0.4	0.95	0.45	1.63
Fig. 4-K	3	2	1.1	0<Redbr<1	0.4	0.95	0.45	1.55
Fig. 4-L	3	2	1.1	0	0<Cs<1	0.95	0.45	1.38
Fig. 4-M	3	2	1.1	0	0.1	0<F<1	0.45	1.38
Fig. 4-N	3	2	1.1	0	0.1	0.95	Lf>0	1.3
final	**3**	**2**	**1.1**	**0**	**0.1**	**0.95**	**∞**	**1.3**

Starting with the following arbitrary values : $R_{ir} = 7$ cm, $B_{dr} = 0.2$ T, $K_1 = 1.2$, $R_{edbr} = 0.8$, $C_s = 0.7$, $F = 0.2$, $L_f = 25$ cm, we find $T_d = 3\,500$ s.

Let us vary R_{ir} for 7 cm to 2 cm, leaving all the other variables constant, a much better solution is found for $R_{ir} = 4.3$ cm (T_d is then equal to 699 s). This search is summarized by the curve in figure 4A and the first line of table I.

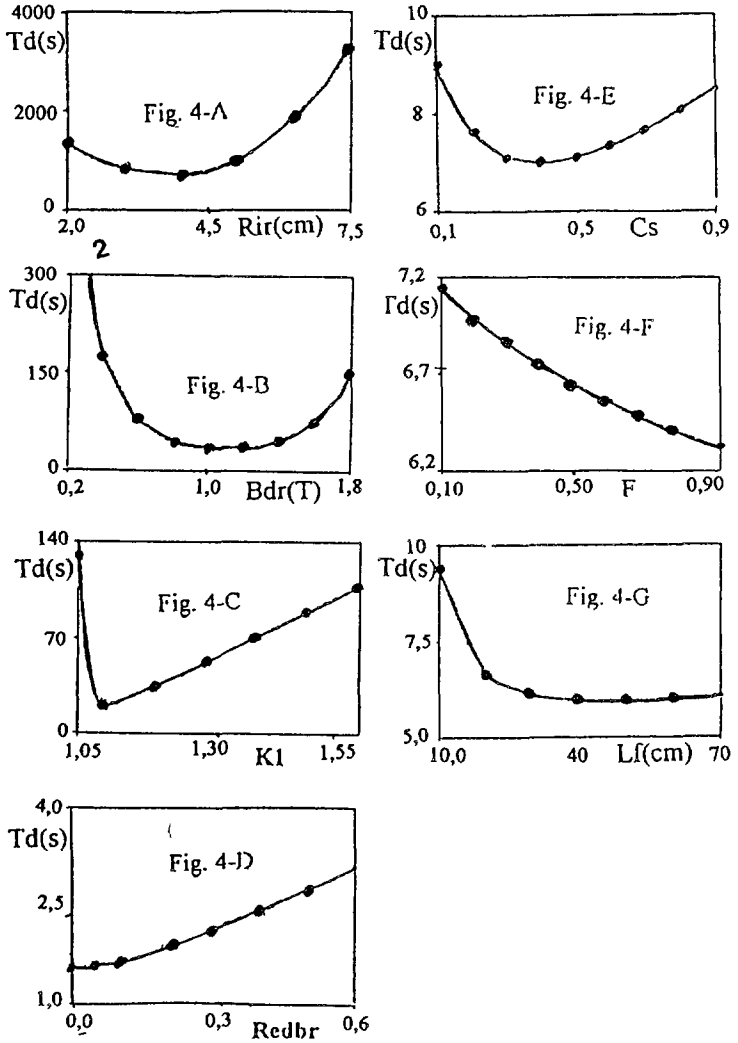

Fig. 4. — Variation of the objective function against the state variables.

Let us keep now R_{ir} constant equal to 4.3 cm, let us vary B_{dr} between 0 and 2 T, and keep the other variables equal to the above values, a much better solution is found for B_{dr} = 1.2 T (T_d = 33.63 s). This step is illustrated by the curve in figure 4B and the second line of table I. When all variables have been changed in turn, a starting time of 6 s has been obtained.

Then R_{ir} can be varied again between 1 and 7 cm. After all the variables have been changed two times, a value $T_d < 1.38$ can be obtained by accepting a very large value of L_f : see figure 4N. This indicates that the length of the machine should be very large, in fact, the length should be infinite.

It appears that the best choice is probably :

(R_{ir}, B_{dr}, K_1, R_{edbr}, C_s, F, L_f) = (3 ; 2 ; 1.1 ; 0, 0.1, 0.95, 0.25) which leads to T_d = 1.4 s.

Data for the final motor are shown in table II.

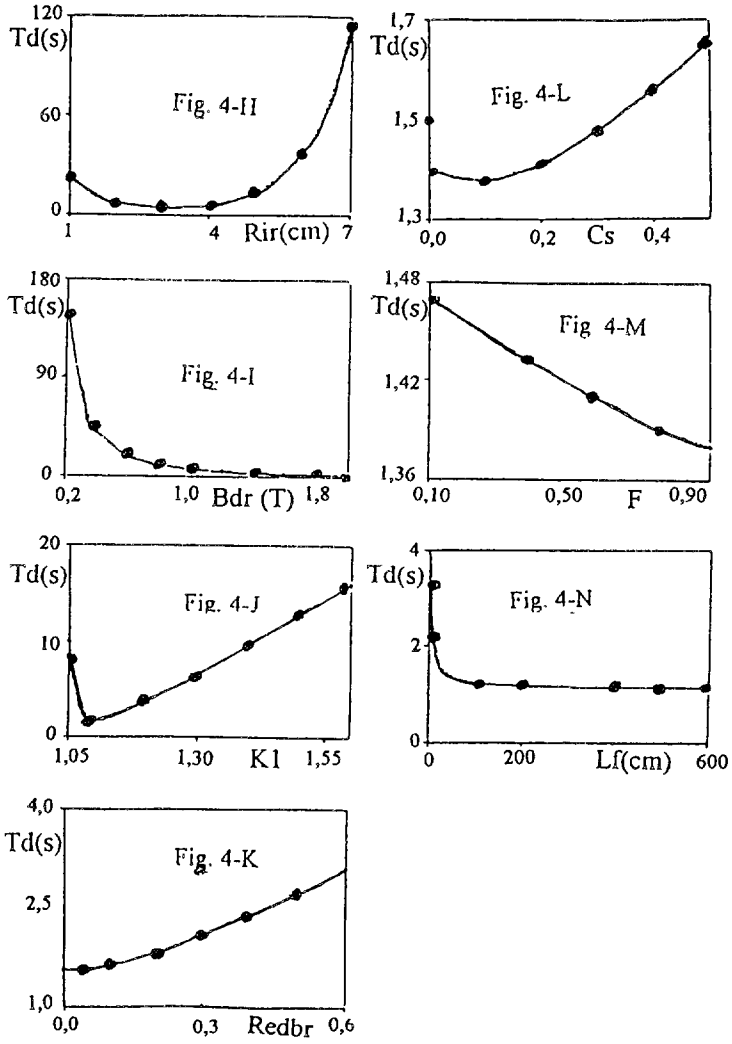

Fig. 4 *(continued).*

Table II. — *Dimensions for the final motor*

Rir (mm)	30	Rıs (mm)	379.3
Rbe (mm)	330	aebs (mm)	7.96
Rhe (mm)	374	achs (mm)	17
her (mm)	44.2	hes (mm)	52 2
aebr (mm)	0	ads (mm)	59.4
aehr (mm)	8.67	Scs (mm²)	196
Scr (mm²)	191 8	Res (mm)	62.7
Sanc (mm²)	1918)	hcs (mm)	188 8
adr (mm)	64,8	e (mm)	3
Lf (mm)	250	Ns	3

7. Discussion.

To arrive at a minimum, the above dichotomy method demands the analysis of 112 different motors. The use of the steepest descent demands the study of seven motors at each step. Therefore, it is not obvious that the steepest descent is significally more performant in terms of computing time.

It would be also interesting to make comparisons with other deterministic or stochastic methods ; but they are too recent [7-10].

In fact, the advantage of using a dichotomy method resides in the clarity of the process. Most optimization methods just assume the existence of a minimum of the objective function, but are not able to give a proof. In contrast, figure 4 and table I clearly show that there is no minimum. If the length of the machine had been limited by a constraint such as $0 < L_f < L_f M$, then, there would have been an optimum which would have probably correspond to $L_f = L_f M$. This is obviously an advantage.

Another advantage is to show that the decrease in starting time leads to a conflict of interest. Changing the length of the machine from 0.25 m to 6 m decreases the starting time from 1.4 to 1.12 s, at the cost of multiplying the price by around 25 (and, may be introducing critical speed problems). Drawing the intermediate curves as in figure 4 greatly clarifies the nature of the choices.

8. Conclusion.

In the present paper, we have defined a « state variable » approach to the problem of « synthesis » of « feasible » electrical machine. As an example, we have discussed the problem of starting an inertia from rest ; the motor is a squirrel cage machine, switched on at zero speed, and operating at constant voltage and frequency. The method allows to show that there is an infinite number of solutions. If the « best » machine is defined by the maximum or minimum of an « objective function », it is then found by any classical method of non linear programming.

The method allows to prove the existence of a minimum, if any. In the present example, the best machine is the one which ensures the shortest possible and it clearly appears that there exists no optimum. Eventually, the method allows to make a reasonable choice, in spite of the fact that there is no minimum of the objective function.

References

[1] Poloujadoff M., Findlay M. R. D., A procedure for illustrating the effect of variation of parameters on optimal transformer design, *IEEE Trans. PWRS* **1n** (1986) 202-206.

[2] Poloujadoff M., Amine Es-Sbai Z., Optimum design of a current fed linear induction motor, IMACS-TC1'90 Nancy (1990) pp. 117-122.

[3] Poloujadoff M., The theory of three phase induction squirrel cage motors , *Elec. Machines Power Systems* **13** (1987) 245-264.

[4] Poloujadoff M., Nurdin M., Faure A., Synthesis of squirrel cage motors : A key to optimization. *IEEE Trans Energy Conversion* **E.C-6** (1991) 327-335.

[5] Liwsckitz-Garik M., Skin-effect bars of squirrel cage rotors, **73**, pt. III-A (1954) 255-258.

[6] Erlicki M. S., Appelbaum J., Optimized parameter analysis of an induction machine, *IEEE Trans. Power Apparatus Systems* **PAS-84** (1965) 1017-1024.

[7] Fuchs E. F., Vandenput A. J., Holl J., White J. C., Design analysis of capacitor-start, capacitor single-phase induction motors, *IEEE Trans Energy Conversion* **5**, n 2 (June 1990).

[8] Xian Liu, Slemon G. R., An improved method of optimization for electrical machines, *IEEE Trans. Energy Conversion* **6**, n 3 (September, 1991).

[9] Moses J. A., Kirtely J. L., Lang Jr. H, Tabors R. D., de Cuadra F., A computer-based design assistant for induction motors, conference record, IEEE IAS (October 1991).

[10] Saldanha R. R., Pelissier S., Kadded K., Yonnet Y. P., Coulomb J. L., Nonlinear optimization methods applied to magnetic actuators design, *IEEE Trans Mag* **28**, n 2 (March, 1992).