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Method of moments as applied to arbitrarily shaped bounded nonlinear scatterers

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Résumé. — Dans cet article nous explorons la possibilité d'appliquer la méthode des moments pour déterminer la distribution du champ électromagnétique dans des objets tridimensionnels diélectriques, non-linéaires, limités et de formes arbitraires. La méthode des moments a été communément employée pour les problèmes de diffusion linéaire. Nous commençons par une formulation basée sur l'équation intégrale et nous dérivons un système non-linéaire d'équations algébriques qui nous permet d'obtenir une solution approximative pour les composantes harmoniques du vecteur du champ électrique. Les résultats préliminaires de quelques simulations numériques sont présentés.

Abstract. — In this paper, we explore the possibility of applying the moment method to determine the electromagnetic field distributions inside three-dimensional bounded nonlinear dielectric objects of arbitrary shapes. The moment method has usually been employed to solve linear scattering problems. We start with an integral equation formulation, and derive a nonlinear system of algebraic equations that allows us to obtain an approximate solution for the harmonic vector components of the electric field. Preliminary results of some numerical simulations are reported.

1. Introduction.

This paper deals with the issue of nonlinear electromagnetics. In the past, many aspects of the problem were addressed, from both the theoretical and practical points of view. Among the various topics that may be included in the term *nonlinear electromagnetics* (for which the reader can refer to classical books), the propagation of electromagnetic waves through nonlinear materials has been extensively investigated [1-6]. However, the associated phenomena have almost always been considered with reference to media of unbounded extent. In this paper, we address the problem arising from assuming bounded dielectric objects, i.e. a nonlinear direct scattering problem. In particular, we try to utilize a numerical technique that is widely used (and discussed) to determine scattering solutions in the case of linear dielectric bodies, i.e., the method of moments [7]. When applied to the scattering by linear dielectrics, this method allows one to reduce the integral equation(s) related to the formulation considered (for example, the electric field integral equation (EFIE)) to a linear system of algebraic equations that gives an approximate solution in the form of a series expansion for the unknown quantities in terms of suitable basis functions. In the past, for linear objects, this procedure was deeply discussed, mainly regarding the choice of the basis and weighting functions [8-15]. In the present paper, we consider dielectric scatterers whose dielectric permittivities depend on the internal electric fields. To simplify the addressed problem, the frequency dependence of the dielectric permittivity is neglected. Starting from an integral equation formulation (whose theoretical basis was first introduced in [16], with particular emphasis on slabs in rectangular waveguides), we derive a formal solution to the direct scattering problem, expressed as a series solution in which the nonlinear effect is taken into account by means of equivalent sources. As a result, we obtain a set of coupled integral equations, written in terms of the dyadic Green function for free space. The application of the moment method to these equations yields a nonlinear system of algebraic equations to be solved for the complex harmonic amplitudes of the electric field vector. Preliminary numerical examples are discussed ; they show the capability of the method for predicting, although in the case of simple scatterers, the harmonic production. The static field vector generated by the mixing of harmonic terms, due to the nonlinearity, is also considered. The computational load inherent in the complex three-dimensional full-vector nonlinear problem and in the moment method application is addressed in the section dealing with numerical simulations.

2. Mathematical formulation.

Let us assume a time-periodic incident electric field vector, $\mathbf{E}_i(\mathbf{r}, t)$, illuminating a bounded nonlinear dielectric object. The object occupies a three-dimensional space region Ω, and Σ denotes the closed surface containing Ω. The object is assumed to be nonmagnetic ($\mu(\mathbf{r}) = \mu_0$, $\mathbf{r} \in \Omega$, where μ_0, stands for the magnetic permeability of vacuum and $\mathbf{r}$ indicates the position vector), lossless, isotropic and inhomogeneous, the inhomogeneity being due to the nonlinear nature of the dielectric permittivity, $\varepsilon(\mathbf{r})$, $\mathbf{r} \in \Omega$. In particular, we assume $\varepsilon(\mathbf{r}, t) = \varepsilon_0[\varepsilon_r' + \varepsilon_r''(\mathbf{E}(\mathbf{r}, t))]$, $\mathbf{r} \in \Omega$, where ε_0 is the dielectric constant of vacuum, and ε_r' and $\varepsilon_r''(\mathbf{E}(\mathbf{r}, t))$ are the linear and nonlinear parts of the relative dielectric permittivity, respectively. $\varepsilon_r''(\mathbf{E}(\mathbf{r}, t))$ is assumed to depend on the total internal electric field, and, as an isotropic medium is assumed, it depends only on the field amplitude. Moreover, the propagation medium is assumed to be homogeneous and characterized by ε_0 and μ_0 (free space). Under these assumptions, at each point $\mathbf{r} \in \Omega$, the following Maxwell equations hold :

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu_0 D\mathbf{H}(\mathbf{r}, t) \tag{1}$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \varepsilon_0 D\left\{[\varepsilon_r' + \varepsilon_r''(\mathbf{E}(\mathbf{r}, t))]\, \mathbf{E}(\mathbf{r}, t)\right\} \tag{2}$$

where D indicates a time derivative ($D = \delta/\delta t$). For $\mathbf{r} \notin \Omega$, we have :

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu_0 D\mathbf{H}(\mathbf{r}, t) \tag{3}$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \varepsilon_0 D\mathbf{E}(\mathbf{r}, t) . \tag{4}$$

The electromagnetic field fulfils Sommerfeld's radiation conditions [17], which impose that the field should represent an outward propagating wave. By applying the equivalence principle and denoting by $\mathbf{E}_s(\mathbf{r}, t)$ and $\mathbf{H}_s(\mathbf{r}, t)$ the contributions to the electromagnetic field vectors due to the presence of the scatterer (i.e., $\mathbf{E}_s(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) - \mathbf{E}_i(\mathbf{r}, t)$ and $\mathbf{H}_s(\mathbf{r}, t) = \mathbf{H}(\mathbf{r}, t) - \mathbf{H}_i(\mathbf{r}, t)$, where $\mathbf{E}_i(\mathbf{r}, t)$ and $\mathbf{H}_i(\mathbf{r}, t)$ indicate the e.m. field that exists everywhere without the scatterer), we obtain the equations :

$$\nabla \times \mathbf{E}_s(\mathbf{r}, t) = -\mu_0 D\mathbf{H}_s(\mathbf{r}, t) \tag{5}$$

$$\nabla \times \mathbf{H}_s(\mathbf{r}, t) = \varepsilon_0 D\mathbf{E}_s(\mathbf{r}, t) + \mathbf{K}(\mathbf{r}, t) \tag{6}$$

where $\mathbf{K}(\mathbf{r}, t)$ stands for an equivalent current density distribution given by $\mathbf{K}(\mathbf{r}, t) = \varepsilon_0 D\left\{(\varepsilon_r' - 1)\mathbf{E}(\mathbf{r}, t) + \varepsilon_r''(\mathbf{E}(\mathbf{r}, t))\mathbf{E}(\mathbf{r}, t)\right\}$, which corresponds to the well-known volume equivalent current density for direct scattering by linear dielectrics if $\varepsilon_r''(\mathbf{r}) = 0$. From the above equations, we can derive the wave equation :

$$\nabla \times \nabla \times \mathbf{E}_s(\mathbf{r}, t) + \varepsilon_0 \mu_0 D^2 \mathbf{E}_s(\mathbf{r}, t) = -\mu_0 D\mathbf{K}(\mathbf{r}, t). \tag{7}$$

We expand $\mathbf{E}_s(\mathbf{r}, t)$ in Fourier series with a fundamental frequency ω_1 and a generic n-th term indicated by $\mathbf{f}_n(\mathbf{r})\exp(jn\omega_1 t)$. Substitution into (7) yields, for each harmonic component [16] :

$$\nabla \times \nabla \times \mathbf{f}_n(\mathbf{r}) - \beta_n^2 \mathbf{f}_n(\mathbf{r}) = \beta_n^2(\varepsilon_r' - 1)\,[\mathbf{f}_n(\mathbf{r}) + \mathbf{e}_n(\mathbf{r})] + \beta_n^2 \mathbf{w}_n(\mathbf{r}) \tag{8}$$

where $\mathbf{e}_n(\mathbf{r})$ is the n-th harmonic term of the known periodic incident electric field vector, $\beta_n^2 = n^2 \omega_1^2 \varepsilon_0 \mu_0$, and $\mathbf{w}_n(\mathbf{r})$ is the n-th term of the series expansion of the product $\varepsilon_r''(\mathbf{E}(\mathbf{r}, t))\mathbf{E}(\mathbf{r}, t)$. The mathematical form of the dependence of $\mathbf{w}_n(\mathbf{r})$ on $\mathbf{f}_n(\mathbf{r})$ and $\mathbf{e}_n(\mathbf{r})$ can be detailed once the function $\varepsilon_r''(\mathbf{E}(\mathbf{r}, t))$ is made explicit. The operator $\varepsilon_r''(\mathbf{x})$ is assumed to be such as to provide a periodic function fulfilling the condition for the Fourier expansion. Under the following radiation conditions (for each n-th term, $n \neq 0$) [17] :

$$\left|\,|\mathbf{r}|\,[\mathbf{f}_n(\mathbf{r}) + \mathbf{e}_n(\mathbf{r})]\right| < \phi_n \quad (\phi_n \text{ real constant}) \tag{9}$$

$$\lim_{|\mathbf{r}| \to \infty} \left|\,|\mathbf{r}|\,[n\omega_1 \mu_0\, \mathbf{r} \times \mathbf{h}_n(\mathbf{r}) + \beta_n[\mathbf{f}_n(\mathbf{r}) + \mathbf{e}_n(\mathbf{r})]]\right| = \mathbf{0} \tag{10}$$

where $\mathbf{h}_n(\mathbf{r})$ stands for the magnetic field vector corresponding to $[\mathbf{f}_n(\mathbf{r}) + \mathbf{e}_n(\mathbf{r})]$, the following integral relation can be written [18] :

$$\mathbf{f}_n(\mathbf{r}) = -\int_\Omega \beta_n^2(\varepsilon_r' - 1)[\mathbf{f}_n(\mathbf{s}) + \mathbf{e}_n(\mathbf{s})] \cdot \Gamma_n(\mathbf{r}/\mathbf{s})\,\mathrm{d}\mathbf{s} - \int_\Omega \beta_n^2 \mathbf{w}_n(\mathbf{s}) \cdot \Gamma_n(\mathbf{r}/\mathbf{s})\,\mathrm{d}\mathbf{s} \tag{11}$$

where the terms that multiply $\Gamma_n(\mathbf{r}/\mathbf{s})$ may be viewed as equivalent current densities. $\Gamma_n(\mathbf{r}/\mathbf{s})$ denotes the dyadic Green function for free space [19] :

$$\Gamma_n(\mathbf{r}/\mathbf{s}) = (4\pi)^{-1}\,(I + \beta_n^{-2}\nabla\nabla)\,[|\mathbf{r} - \mathbf{s}|^{-1}\exp(-j\beta_n|\mathbf{r} - \mathbf{s}|)] \tag{12}$$

and I is the unit dyadic. For $n = 0$ (static component), the following relations involving the polarization vector, $\mathbf{P}_0(\mathbf{r})$, can be written [20] :

$$\mathbf{P}_0(\mathbf{r}) = \varepsilon_0(\varepsilon_r' - 1)[\mathbf{f}_0(\mathbf{r}) + \mathbf{e}_0(\mathbf{r})] + \varepsilon_0 \mathbf{w}_0(\mathbf{r})$$

$$\mathbf{f}_0(\mathbf{r}) = (4\pi\varepsilon_0)^{-1}\int_\Sigma \gamma(\mathbf{r}/\mathbf{s})[\varepsilon_0(\varepsilon_r' - 1)[\mathbf{f}_0(\mathbf{s}) + \mathbf{e}_0(\mathbf{s})] + \varepsilon_0 \mathbf{w}_0(\mathbf{s})] \cdot \mathbf{n}\,\mathrm{d}\mathbf{s} + \tag{13}$$

$$-(4\pi\varepsilon_0)^{-1}\int_\Omega \gamma(\mathbf{r}/\mathbf{s})\,\nabla \cdot [\varepsilon_0(\varepsilon_r' - 1)[\mathbf{f}_0(\mathbf{s}) + \mathbf{e}_0(\mathbf{s})] + \varepsilon_0 \mathbf{w}_0(\mathbf{s})]\,\mathrm{d}\mathbf{s} \tag{14}$$

where the terms $\nabla \cdot [\varepsilon_0(\varepsilon_r' - 1)[\mathbf{f}_0(\mathbf{s}) + \mathbf{e}_0(\mathbf{s})] + \varepsilon_0 \mathbf{w}_0(\mathbf{s})]$ and $[\varepsilon_0(\varepsilon_r' - 1)[\mathbf{f}_0(\mathbf{s}) + \mathbf{e}_0(\mathbf{s})] + \varepsilon_0 \mathbf{w}_0(\mathbf{s})] \cdot \mathbf{n}$ can be viewed as charge densities. In relation (14), $\gamma(\mathbf{r}/\mathbf{s})$ is given by

$\gamma(\mathbf{r}/\mathbf{s}) = |\mathbf{r}-\mathbf{s}|^{-3}(\mathbf{r}-\mathbf{s})$. Equations (11) and (14) constitute a set (infinite) of integral equations for the harmonic vector components. The nonlinear effect is given by the vectors $\mathbf{w}_n(\mathbf{r})$, which depend on the mixing of the unknown vector components $\mathbf{f}_0(\mathbf{r})$. If we assume the infinite series to be truncated at the N term, the functional relation $\mathbf{w}_n(\mathbf{r}) = \mathfrak{L}\{\mathbf{f}_m(\mathbf{r}), m = 1, \ldots, N\}$ can be made explicit.

The system of integral equations given by relation (11), written for $n = 1, \ldots, N$, and by relation (14), for $n = 0$, can be discretized by the moment method [7]. This is accomplished by expanding into basis functions all the Cartesian components of the unknown vectors $\mathbf{f}_n(\mathbf{r})$ and $\mathbf{w}_n(\mathbf{r})$. As a result, we obtain a set of unknown coefficients f_{nh}^p and w_{nh}^p, $p = x, y, z$, and h indicates the corresponding h-th basis function ($h = 1, \ldots, H$). In this paper, we assume, for simplicity, subsectional piecewise constant basis functions. Such functions were the subject of extensive investigations in the case of linear scattering [8, 11, 15]. In particular, the effectiveness of using them was judged questionable [9], especially for the block-model numerical prediction of the specific absorption rate in human bodies. Others believed that a suitable choice of the discretization procedure, according to appropriate criteria, might enable to obtain accurate solutions by using reduced computer resources [10]. Anyway, the approach developed in the present paper allows the use of other more sophisticated basis functions. An analogous discussion could hold true for the testing functions, which, in this paper, are Dirac delta functions.

Under the above assumptions, equations (14) and (11) result in the following system of algebraic equations :

$$f_{0h}^p = (4\pi)^{-1} \sum_{k=1}^{H} \sum_{q=x,y,z} \left\{ (\varepsilon_r' - 1)[f_{0k}^q + \mathbf{e}_{0k}\cdot\mathbf{q}] + w_{0k}^q \right\} \times$$
$$\times \left\{ (R_{hk}^p - \lambda_k^q \delta_{pq}) |\mathbf{R}_{hk} - \lambda_k^q \mathbf{q}|^{-3} - (R_{hk}^p + \lambda_k^q \delta_{pq}) |\mathbf{R}_{hk} + \lambda_k^q \mathbf{q}|^{-3} \right\} \Delta\Sigma_k^q \quad (15)$$

$$\sum_{k=1}^{H} \sum_{q=x,y,z} \gamma_{nhk}^{pq} [f_{nk}^q + \mathbf{e}_{nk}\cdot\mathbf{q}] + \sum_{k=1}^{H} \sum_{q=x,y,z} \phi_{nhk}^{pq} w_{nk}^q = -\mathbf{e}_{nk}\cdot\mathbf{p} \quad (16)$$

where $\mathbf{R}_{hk} = \mathbf{r}_h - \mathbf{r}_k$ ($\mathbf{r}_s$ being the position vector of the s-th subdomain of Ω), $\Delta\Sigma_k^j$ denotes the area of the portion of the external surface of the s-th subdomain that is orthogonal to the j axis (whose unit vector is denoted by $\mathbf{j}$), λ_s^j is the edge of the s-th subdomain parallel to $\mathbf{j}$, and $\delta_{pq} = 1$, if $p = q$, else $\delta_{pq} = 0$. The coefficients γ_{nhk}^{pq} and ϕ_{nhk}^{pq} in expression (16) are given by :

$$\gamma_{nhk}^{pq} = jn\omega_0\, \varepsilon_0(\varepsilon_r' - 1)\, PV \int_{\Omega_k} \Gamma_{npq}(\mathbf{r}_h/\mathbf{s})\,\mathrm{ds} - (1 + 1/3(\varepsilon_r' - 1))\, \delta_{pq}\, \delta_{hk} \quad (17)$$

$$\phi_{nhk}^{pq} = jn\omega_0\, \varepsilon_0\, PV \int_{\Omega_k} \Gamma_{npq}(\mathbf{r}_h/\mathbf{s})\,\mathrm{ds} - 1/3\, \delta_{pq}\, \delta_{hk} \quad (18)$$

where Ω_s denotes the s-th subdomain. The system of equations (15) and (16) turns out to be nonlinear if the relation $\mathbf{w}_n(\mathbf{r}) = \mathfrak{L}\{\mathbf{f}_m(\mathbf{r}), m = 1, \ldots, N\}$, $n = 1, \ldots, N$, is taken into account. The scattering coefficients γ_{nhk}^{pq} and ϕ_{nhk}^{pq} are calculated by applying the Van Bladel theory for the principal value (« PV ») related to the singularity of the Green function [21]. By following the procedure considered in [22], they can be approximated as :

$$\gamma_{nhk}^{pq} \cong -j(4\pi)^{-1} |\mathbf{R}_{hk}|^{-2} k_n(\varepsilon_r' - 1)\, \Delta\Omega_k \left\{ [k_n^2 |\mathbf{R}_{hk}|^2 - jk_n |\mathbf{R}_{hk}| - 1]\, \delta_{pq} - \right.$$
$$\left. - |\mathbf{R}_{hk}|^2 (\mathbf{R}_{hk}\cdot\mathbf{p})(\mathbf{R}_{hk}\cdot\mathbf{q})[k_n^2 |\mathbf{R}_{hk}|^2 - j\,3\,k_n |\mathbf{R}_{hk}| - 3] \right\} \exp(-jk_n|\mathbf{R}_{hk}|) \quad h \neq k \quad (19)$$

$$\gamma_{nhk}^{pq} \cong \left\{(2/3)(\varepsilon_r' - 1)[(jk_n \rho_s + 1)\exp(-jk_n \rho_s) - 1] - [1 + (1/3)(\varepsilon_r' - 1)]\right\} \times \delta_{pq} \exp(-jk_n |\mathbf{R}_{hk}|) \quad h = k \tag{20}$$

$$\phi_{nhk}^{pq} \cong -j(4\pi)^{-1} |\mathbf{R}_{hk}|^{-2} k_n \Delta\Omega_k \left\{[k_n^2 |\mathbf{R}_{hk}|^2 - jk_n |\mathbf{R}_{hk}| - 1]\delta_{pq} - - |\mathbf{R}_{hk}|^2 (\mathbf{R}_{hk} \cdot \mathbf{p})(\mathbf{R}_{hk} \cdot \mathbf{q})[k_n^2 |\mathbf{R}_{hk}|^2 - j\,3\,k_n |\mathbf{R}_{hk}| - 3]\right\} \exp(-jk_n |\mathbf{R}_{hk}|) \quad h \neq k \tag{21}$$

$$\phi_{nhk}^{pq} \cong \left\{(2/3)(jk_n \rho_s + 1)\exp(-jk_n \rho_s) - 2\right\} \delta_{pq} \exp(-jk_n |\mathbf{R}_{hk}|) \quad h = k \tag{22}$$

where $\Delta\Omega_s$ is the volume of the s-th subdomain and $\rho_s = [(3/4)\,\pi^{-1}\,\Delta\Omega_s]^{1/3}$.
The solution of the above nonlinear system is obtained by Wolfe's method [23], which is a generalization of the secant method for a single function of one variable. This allowed us to obtain the preliminary results that are reported in the following section, and that seem to demonstrate the validity of extending the moment method to the computation of the electric field inside an arbitrarily shaped nonlinear dielectric object. Of course, due to the problem complexity, we have in general to solve a nonlinear system with a large number of unknowns, for which more suitable solution subroutines should be devised. For the authors, this represent the goal of future work.

3. Numerical examples.

In order to implement the mathematical model presented in the previous section, the nonlinear relationship between $\varepsilon_r''(\mathbf{x})$ and $\mathbf{E}(\mathbf{r}, t)$ must first be specified. In this paper, we assume a Kerr-like nonlinearity, and limit the power series to the second order. As a consequence, $\varepsilon_r''(\mathbf{E}(\mathbf{r}, t)) = \eta E^2(\mathbf{r}, t)$, where $E(\mathbf{r}, t)$ stands for the amplitude of the vector $\mathbf{E}(\mathbf{r}, t)$. In this case, we can write :

$$\varepsilon_r''(\mathbf{E}(\mathbf{r}, t)) = \sum_n q_n(\mathbf{r}) \exp(jn\omega_1 t) \tag{23}$$

where $q_n(\mathbf{r})$ is given by :

$$q_n(\mathbf{r}) = \sum_i \sum_j \alpha_{ij}^n [\mathbf{f}_i(\mathbf{r}) + \mathbf{e}_i(\mathbf{r})] \cdot [\mathbf{f}_j(\mathbf{r}) + \mathbf{e}_j(\mathbf{r})] \tag{24}$$

where $\alpha_{ij}^n = 1$, if $i + j = n$; $\alpha_{ij}^n = 0$, otherwise. At this point, the nth nonlinear vector component $\mathbf{w}_n(\mathbf{r})$ is given by :

$$\mathbf{w}_n(\mathbf{r}) = \sum_i \sum_j \alpha_{ij}^n\, q_i(\mathbf{r})[\mathbf{f}_j(\mathbf{r}) + \mathbf{e}_j(\mathbf{r})]\,. \tag{25}$$

Simple scattering objects were used for initial tests. A parallelepiped, whose dimensions are specified in figure 1, was considered for the first example ($\lambda_1 = 2\,\pi k_1^{-1}$ stands for the wavelength of the fundamental frequency ν_1). We assumed $\varepsilon_r' = 2.0$. The linear part of the relative dielectric permittivity was assumed to be homogeneous, even though, after minor modifications, the method could also be used for inhomogeneous distributions. Figure 1 gives the values of the amplitudes of the various harmonic vector components generated by the nonlinearity, for different values of the index η. The static field ($n = 0$) was also considered. The scatterer was illuminated by an incident electric field represented by two uniform unit plane waves with propagation constants k_1 and k_2, with $k_2 = 2\,k_1$ (the approach can be applied only if the ratio $k_2/2\,k_1$ is an integer). The incident waves propagate in the **z** direction with the electric field polarized in the **y** direction. The values presented in figure 1 were calculated at

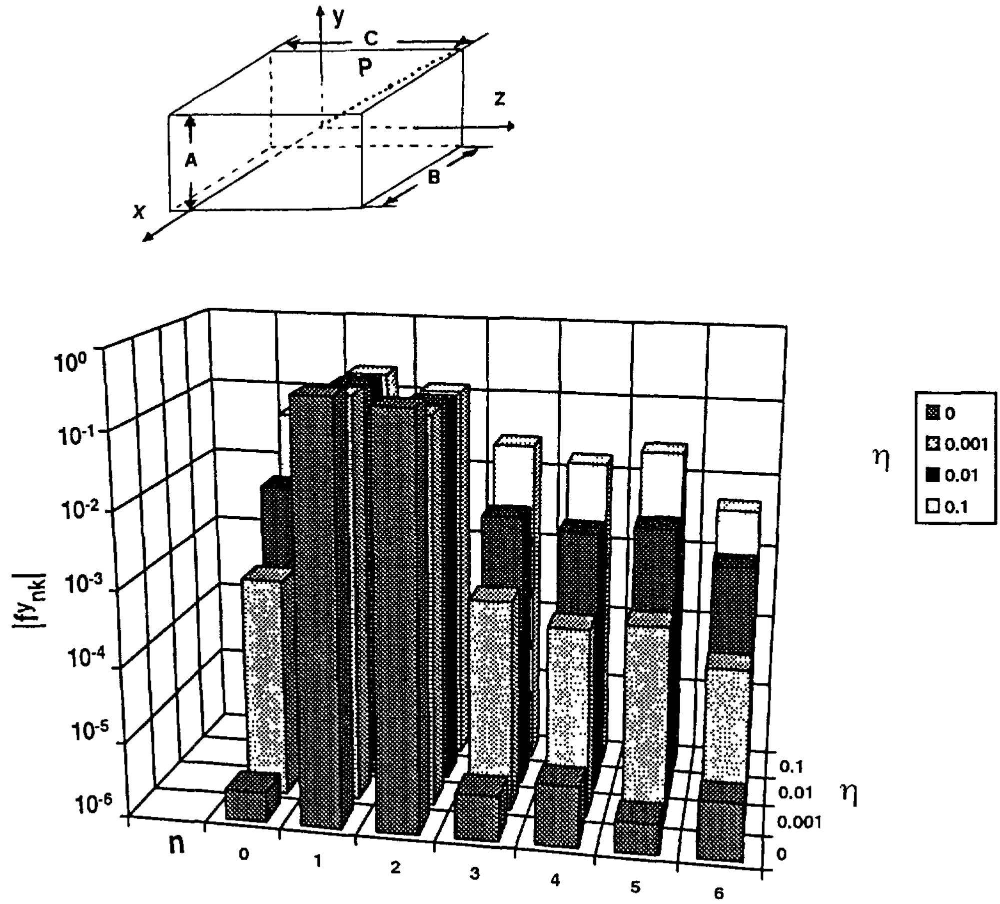

Fig. 1. — Nonlinear dielectric parallelepiped ($A = 1/2\,B = 1/2\,C = \lambda_1/30$) and coefficients $|f^y_{nk}|$ *versus* the nonlinear parameter η, for $\varepsilon'_r = 2.0$, $N = 6$; $X = 40$.

point P. In this case, we assumed $N = 6$, $M = 4$, for a number of complex unknowns equal to 78 for 78 equations. The Wolfe subroutine was started by generating random solutions for the unknown coefficients of the harmonic vector components. Such solutions were given by independent sequences (for the real and imaginary parts) of stochastic variables uniformly distributed between -1 and 1. This choice was made in accordance with the values of the amplitude of the incident electric field vector. Of course, in the case of weak nonlinearities, if one uses as starting solutions those obtained by applying the moment method to solve the problem related related to the linear part of the dielectric permittivity, one can expect a faster convergence. We assumed that the solution would be reached when the values obtained at a given step, and substituted into the resulting non-linear system, give rise to a residual that, in norm, was less than a fixed threshold (in all the simulations reported, this threshold was assumed to be equal to 10^{-4}). If, after a fixed number, X, of iterations, the desired accuracy had not been achieved, for example, because a local minimum had been reached, the algorithm was made to restart, considering other initial solutions. It should be noted that this procedure has been used only for the simple cases considered in this work ; however, when one faces a multiple solution problem, only physical constraints may lead to the right solution. For more complex cases, the authors are currently studying the application of algorithms for global optimization (e.g., simulated annealing), which, if correctly implemented, seem able to reach global minima.

Figure 2 shows the plots of the numbers of trials and iterations *vs.* different values of η. In this case, we assumed $\varepsilon_r' = 2.0$ and $X = 40$. The same figure also gives the numbers of trials and iterations *vs.* different values of the linear part of the dielectric permittivity, for $\eta = 0.1$ and $X = 40$. One can see that, as the values of ε_r' increase, the solution of the nonlinear system becomes easier. This is perhaps to be ascribed to the fact that nonlinear phenomena are blinded when the linear part of the dielectric permittivity becomes large. Figure 3 gives the values of the coefficients $|f_{nk}^y|$, $n = 0, \ldots, N$, $N = 6$, computed at point P

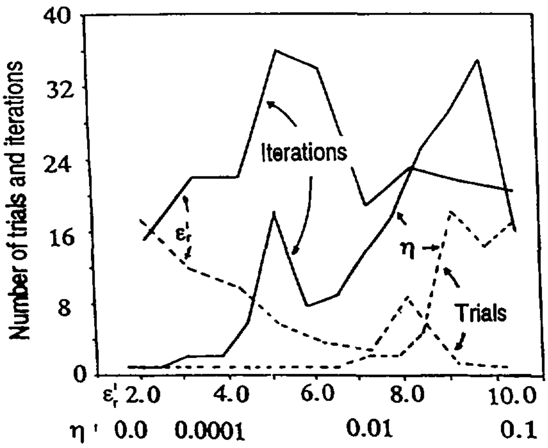

Fig. 2. — Numbers of trials and iterations *versus* the η values, for $\varepsilon_r' = 2.0$, $N = 6$, $X = 40$, and *versus* the ε_r' values, for $\eta = 0.1$, $N = 6$, $X = 40$.

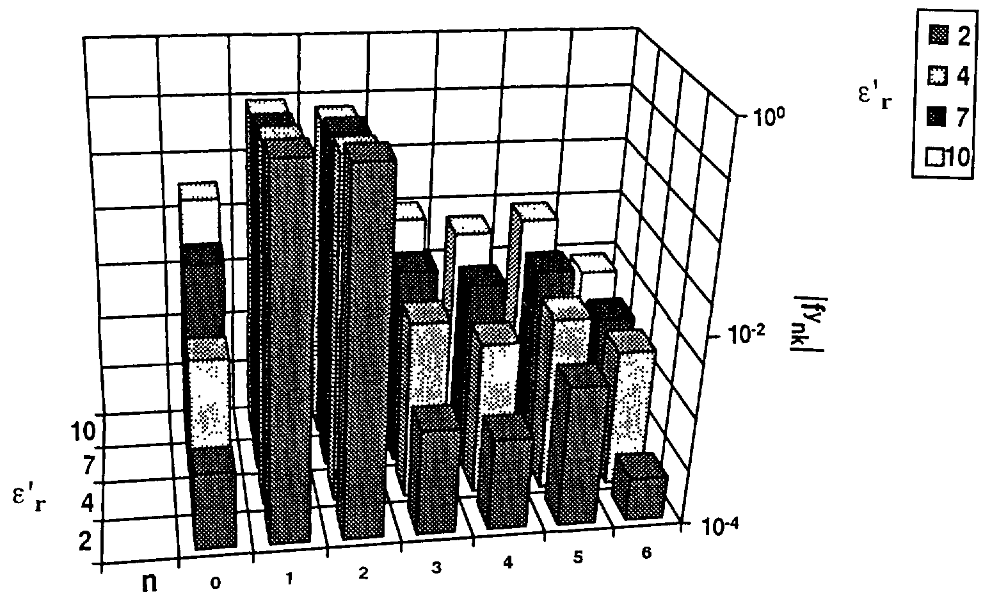

Fig. 3. — Coefficients $|f_{nk}^y|$ *versus* the linear part of the dielectric permittivity ε_b, for $\eta = 0.1$, $N = 6$, $X = 40$.

under the same conditions as previously specified, for different values of the linear part of the dielectric permittivity and for $\eta = 0.1$. The effect of the truncation of the series for the harmonic vector components is displayed in figure 4, with reference to the scatterer shown in the upper right portion of the figure. The figure gives the numbers of trials and iterations, for $X = 25$. Finally, as an example, figure 5 gives the norm of the residual error (obtained at each iteration when substituting the obtained solution into the resulting nonlinear system) *versus* the number of iterations (for $X = 40$).

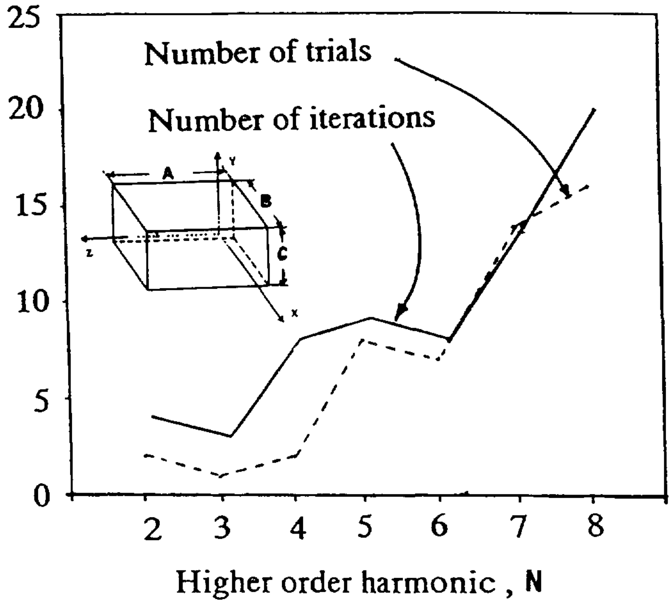

Fig. 4. — Numbers of trials and iterations for different values of N (highest-order harmonic component). In this case : $\eta = 0.01$, $\varepsilon_r' = 2.0$, $N = 5$, $X = 25$ $(A = 2B = 2C = \lambda_l/15)$.

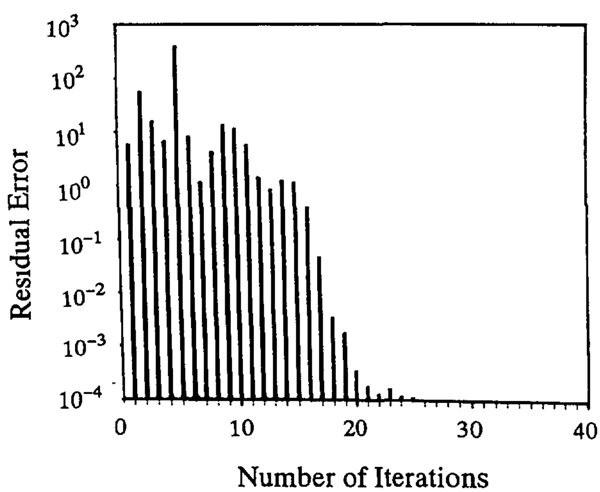

Fig. 5. — Residual errors (in norm) *versus* number of iterations.

From a computational point of view, it should be noted that the classic moment method for dielectrics requires a computer time proportional to $(3\ H)^3$ and a storage memory proportional to $(3\ H)^2$. The extension of this method to a nonlinear case, as proposed in this paper, requires that, for each trial, a matrix inversion (which is by far the heaviest part of the computation) be performed. So, for each trial, the computer time is proportional to $(3\ NH + 3\ H + 1)^3$ (the final unit increment is due to the Wolfe procedure). The computer time required for each iteration is very short, like the time required by the matrix formation (random numbers). Moreover, as the number of unknowns increases, the time required to compute the equation coefficients becomes significant (independently of the numbers of trials and iterations). This time is similar to that taken by the computation of the Green matrix elements for the moment method solution of linear direct scattering.

Other simulations were performed. Figure 6 provides the values of the coefficients $|f^{y}_{nk}|$, $n = 0, \ldots, N$, $N = 4$, for the scatterer schematically represented in the upper left portion of the figure. The figure gives the values of such coefficients at points P, Q, and R, for $\eta = 0.01$, $\varepsilon_r' = 2.0$, and $X = 40$. The incident electric field is the same as in the example presented in figure 1, but, in this case, the amplitude of the plane wave $|\mathbf{e}_2|$, with the propagation constant k_2, was assumed to range between 1.0 (V/m) and 0.4 (V/m). Finally, we would like to point out that the proposed method is able to compute the approximate distribution of the scattered electric field vector even outside a nonlinear dielectric object. In particular, for $\mathbf{r} \notin \Omega$, the harmonic vector components of the scattered electric field vector can

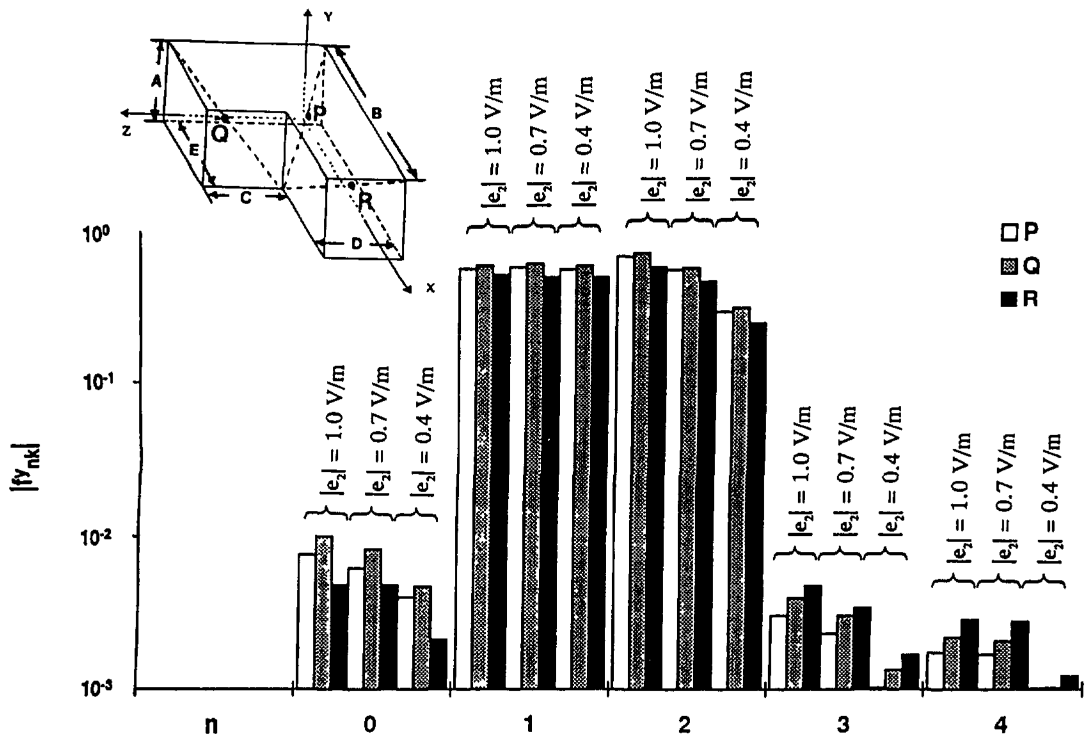

Fig. 6. — Nonlinear scatterer ($A = 1/2\ B = C = D = E = \lambda_{\,l}/30$) and coefficients $|f^{y}_{nk}|$ for different values of $|\mathbf{e}_2|$. In this case : $\eta = 0.01$, $\varepsilon_r' = 2.0$, $N = 4$, $X = 25$.

be computed by numerically calculating the following integrals :

$$- \int_{\Omega} \beta_n^2(\varepsilon_r' - 1)[\mathbf{f}_n(\mathbf{s}) + \mathbf{e}_n(\mathbf{s})] \cdot \Gamma_n(\mathbf{r}/\mathbf{s})\, ds - \int_{\Omega} \beta_n^2 \mathbf{w}_n(\mathbf{s}) \cdot \Gamma_{n'}(\mathbf{r}/\mathbf{s})\, ds = \mathbf{f}_n(\mathbf{r}) \quad n = 1, \dots, N \tag{26}$$

$$(4\pi\varepsilon_0)^{-1} \int_{\Sigma} \gamma(\mathbf{r}/\mathbf{s})[\varepsilon_0(\varepsilon_r' - 1)[\mathbf{f}_0(\mathbf{s}) + \mathbf{e}_0(\mathbf{s})] + \varepsilon_0 \mathbf{w}_0(\mathbf{s})] \cdot \mathbf{n}\, ds - \\ - (4\pi\varepsilon_0)^{-1} \int_{\Omega} \gamma(\mathbf{r}/\mathbf{s}) \nabla \cdot [\varepsilon_0(\varepsilon_r' - 1)[\mathbf{f}_0(\mathbf{s}) + \mathbf{e}_0(\mathbf{s})] + \varepsilon_0 \mathbf{w}_0(\mathbf{s})]\, ds = \mathbf{f}_0(\mathbf{r}) \quad n = 0 \tag{27}$$

for which integrand functions are approximately known. As an example of this computation, figure 7 gives the values of the time-dependent total electric field (y component) calculated at 1 000 points along the propagation axis (forward scattering), for $0 < z < \lambda_1$. In this case, we considered the scatterer in figure 1, for $\eta = 0.1$, $\varepsilon_r' = 2.0$, $X = 40$, $N = 6$, $H = 4$, $|\mathbf{e}_1| = |\mathbf{e}_2| = 1.0$ (V/m), $\alpha_i = (i - 1)/5\ \nu_1$.

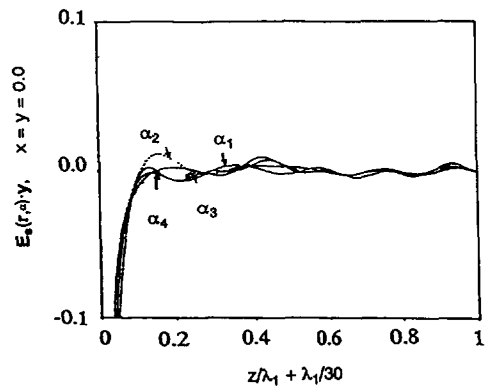

Fig. 7. — External scattered electric field (y-component) along the propagation axis (z-axis), for $\eta = 0.1$, $\varepsilon_r' = 2.0$, $N = 6$, $X = 40$, $\alpha_i = (i - 1)/5\ \nu_1$.

4. Conclusions.

The classic moment method, extensively used to solve the direct scattering problem for linear dielectric objects, has been applied in this paper to determine the harmonic components of the electric field vectors inside three-dimensional bounded scatterers whose dielectric permittivities depended on the internal electric fields. This has been accomplished by starting from a formal solution of the nonlinear scattering problem, using an integral-equation formalism in which the nonlinear effect is taken into account on the basis of the distributions of equivalent current densities. The application of the moment method has reduced the problem to the solution of a nonlinear system of algebraic equations. In the case of simple scatterers, the proposed method has proved able to predict the generation of the harmonic terms. Future work will be aimed at considering more realistic scattering objects. To this end, more efficient subroutines for finding adequate solutions of nonlinear systems with a large number of unknowns will have to be devised. Although results are still preliminary, they are interesting and seem to indicate the possibility of successfully applying the moment method to nonlinear objects of arbitrary shapes, for which nonnumerical solutions cannot be adopted. Therefore, it will be important to establish the best operating conditions in terms of numbers and kinds of testing and weighting functions. This in order to study more efficient versions of the moment method from a computational point of view.

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