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## Application of the point-matched time domain finite element method to the analysis of transmission line transients

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**Abstract.** — A point-matched time domain finite element method (TDFE) applied to the analysis of electrical transients in transmission line networks is presented. In this method, the telegraph equation is solved numerically at discrete time steps. Absorbing boundary conditions are used to simulate an infinite line. Examples of voltage waveforms on lossless single-phase line with simple line termination and interfaces are shown.

### 1. Introduction.

During the past two decades electrical transients in arbitrary single- or multi-phase networks were solved by a nodal admittance matrix approach [1-3]. Before Dommel's method [4] became widely used, there have been attempts to employ the alternative lattice method [5] for travelling wave phenomena which required the knowledge of reflection coefficients. In recent years, an s-domain approach [6] has also been introduced in transient studies.

The response of transmission line networks can be found by accounting for all interactions between forward and backward travelling waves as a result of discontinuities or disturbances. In an extensive network the number of forward and backward waves caused by a single incident wave increases quickly as the wave goes through multiple reflections and refractions. With methods such as the lattice diagram a complicated bookkeeping scheme is required in order to obtain a solution [7]. Dommel's method transforms the network branches into their general impedance equivalents [1]. The evolution of the waveforms on the lines can be computed by solving a system of linear nodal equations in discrete time steps. However, some power transmission line problems cannot be adequately modelled by the equivalent circuit approach. For instance, transmission line corona problems [8], the effect of lightning strikes on high voltage lines [9] and the effects of non-uniform lines [10] require a distributed transmission line model.

This paper describes the point-matched time domain finite element method [11-13]. Given the success of this method in transient scattering applications, the present work is a logical extension to low frequency or transient power transmission line problems. In this approach, the

propagation of disturbance on transmission lines is simulated numerically by solving a one-dimensional boundary value problem at each time step.

## 2. Governing equations.

In a single-phase network, the current and voltage waves on a lossless transmission line are governed by

$$-\frac{\partial v}{\partial x} = L' \frac{\partial i}{\partial t} \quad (1)$$

$$-\frac{\partial i}{\partial x} = C' \frac{\partial v}{\partial t} \quad (2)$$

where  $L'$  and  $C'$  are the inductance and capacitance per unit length, respectively [14]. The general solution, by d'Alembert, is the combination of arbitrary waves which travel in both the forward and backward directions :

$$i(x, t) = f_1(x - \gamma t) + f_2(x + \gamma t) \quad (3)$$

$$v(x, t) = Zi(x, t) \quad (4)$$

where  $Z = \sqrt{\frac{L'}{C'}}$  is the characteristic impedance and  $\gamma = \sqrt{\frac{1}{L'C'}}$  is the speed of propagation.

## 3. Method of analysis.

**3.1 POINT-MATCHED FINITE ELEMENT METHOD.** — The point-matched finite element method requires the line to be discretized into a finite number of regions called elements. Each element has several points called interpolation nodes. This allows  $v$  and  $i$  to be written in the form

$$v(x, t) = \sum_{i=1}^M \Phi_i(x) V_i(t) \quad (5)$$

$$i(x, t) = \sum_{j=1}^N \psi_j(x) I_j(t) \quad (6)$$

where  $M$  and  $N$  are the number of nodes of the  $i$  and  $v$  finite element segments, respectively, and  $\Phi_i$  and  $\psi_j$  are basis functions which interpolate the voltage and current within each element using the values at the interpolation nodes. Substitution of (5) and (6) into (1) and (2) yields

$$-\frac{\partial}{\partial x} \sum_{i=1}^M \Phi_i(x) V_i(t) = L' \frac{\partial}{\partial t} \sum_{j=1}^N \psi_j(x) I_j(t) \quad (7)$$

$$-\frac{\partial}{\partial x} \sum_{j=1}^N \psi_j(x) I_j(t) = C' \frac{\partial}{\partial t} \sum_{i=1}^M \Phi_i(x) V_i(t). \quad (8)$$

The only unknowns in the above equations are the nodal values of the voltage  $V_i(t)$  and the current  $I_j(t)$  because  $\Phi_i(x)$  and  $\psi_j(x)$  are known functions of position, i.e. the unknowns are

$$V_i(t), \quad i = 1, 2, \dots, M \quad (9)$$

$$I_j(t), \quad j = 1, 2, \dots, N. \quad (10)$$

This method is called the point-matched finite element method because  $\Phi_i(x)$  and  $\psi_j(x)$  are defined to be

$$\begin{aligned}\Phi_i(x) &= 1 \quad \text{at } x = x_i \\ &= 0 \quad \text{at the other nodes, and}\end{aligned}\quad (11)$$

$$\begin{aligned}\psi_j(x) &= 1 \quad \text{at } x = x_i \\ &= 0 \quad \text{at the other nodes}\end{aligned}\quad (12)$$

such that (7) and (8) are enforced at each nodal point. As a result, (7) and (8) can be reduced to

$$\frac{\partial I_j}{\partial t} = -\frac{1}{L'} \sum_{i=1}^M \frac{\partial \Phi_i}{\partial x} V_i(t), \quad j = 1, 2, \dots, N \quad (13)$$

$$\frac{\partial V_i}{\partial t} = -\frac{1}{C'} \sum_{j=1}^N \frac{\partial \psi_j}{\partial x} I_j(t), \quad i = 1, 2, \dots, M \quad (14)$$

where  $\Phi_i = \Phi_i(x_j)$  and  $\psi_j = \psi_j(x_i)$ .

The nodes are placed in such a way that each voltage element contains an interpolation node for the current and each current element contains an interpolation node for the voltage. Moreover, only the interpolation functions associated with the surrounding nodes will contribute to the summation in (13) and (14). Hence, for first-order finite elements with just two interpolation nodes, (13) and (14) become

$$\frac{\partial I_j}{\partial t} = -\frac{1}{L'} \sum_{a=1}^2 \frac{\partial \Phi_a}{\partial x} V_a(t), \quad j = 1, 2, \dots, N \quad (15)$$

$$\frac{\partial V_i}{\partial t} = -\frac{1}{C'} \sum_{a=1}^2 \frac{\partial \psi_a}{\partial x} I_a(t), \quad i = 1, 2, \dots, M \quad (16)$$

where  $V_a$  represents the values of the voltage nodes surrounding the current node  $I_j$  and  $I_a$  represents the values at the current nodes surrounding the voltage node  $V_i$ .

**3.2 LEAP-FROG SCHEME IN TIME DOMAIN.** — In the leap-frog scheme, the time derivatives in (15) and (16) are represented by the forward Euler differencing

$$\frac{\partial I_j^n}{\partial t} \approx \frac{I_j^{n+\frac{1}{2}} - I_j^{n-\frac{1}{2}}}{\Delta t} \quad (17)$$

$$\frac{\partial V_i^{n+\frac{1}{2}}}{\partial t} \approx \frac{V_i^{n+1} - V_i^n}{\Delta t} \quad (18)$$

where  $I_j^n$  is the current at  $x_j$  at time  $n \Delta t$  and  $V_i^{n+\frac{1}{2}}$  is the voltage at  $x_i$  at time  $(n + 1/2) \Delta t$ .

Notice from (17) and (18) that the current time derivative is computed at one-half time step before the voltage time derivative. By substituting (17) and (18) into (15) and (16), respectively, the following explicit formulas for  $V_i$  and  $I_j$  are obtained :

$$I_j^{n+\frac{1}{2}} = I_j^{n-\frac{1}{2}} - \frac{\Delta t}{L'} \sum_{a=1}^2 \frac{\partial \Phi_a}{\partial x} V_a(t), \quad j = 1, 2, \dots, N \quad (19)$$

$$V_i^{n+1} = V_i^n - \frac{\Delta t}{C'} \sum_{a=1}^2 \frac{\partial \psi_a}{\partial x} I_a(t), \quad i = 1, 2, \dots, M. \quad (20)$$

Given the initial conditions at  $t = 0$ , (19) and (20) can be used alternately to update  $I_j$  and  $V_i$  at  $n = 1, 2, \dots$

**3.3 INTERPOLATION FUNCTIONS.** —  $\Phi_i(x)$  and  $\psi_j(x)$  are both chosen to be first-order interpolation polynomials. They have the form given by [15] :

$$U_b = U_1 \frac{x_2 - x_b}{x_2 - x_1} + U_2 \frac{x_b - x_1}{x_2 - x_1} \quad (21)$$

where

$x_b$  is the coordinate of a point between nodes 1 and 2

$X_1$  is the coordinate of node 1

$X_2$  is the coordinate of node 2

$U_b$  is the interpolated value at a position between nodes 1 and 2

$U_1$  corresponds to the value  $I_j^{n+\frac{1}{2}}$  shown in (19) and  $V_i^{n+1}$  shown in (20)

$U_2$  corresponds to the value  $I_{j+1}^{n+\frac{1}{2}}$  shown in (19) and  $V_{i+1}^{n+1}$  shown in (20).

By differentiating (21) with respect to  $x_b$  one obtains :

$$\frac{dU_b}{dx_b} = - \frac{U_2 - U_1}{x_2 - x_1} \quad (22)$$

where  $x_2 - x_1$  is the element length,  $\Delta x$ . Substituting (22) into (19) and (20), the following equations are obtained

$$I_j^{n+\frac{1}{2}} = I_j^{n-\frac{1}{2}} - \frac{\Delta t}{L' \Delta x} (V_{i+1}^n - V_i^n) \quad (23)$$

$$V_i^{n+1} = V_i^n - \frac{\Delta t}{C' \Delta x} \left( I_{j+1}^{n+\frac{1}{2}} - I_j^{n+\frac{1}{2}} \right). \quad (24)$$

#### 4. Numerical consideration.

The solution to the leap-frog scheme approximation is stable if [11]

$$u \Delta t/h \leq 1 \quad (25)$$

where  $u$  is the wave propagation speed,  $h$  is the length of each element and  $\Delta t$  is the duration of each time step. This implies that the wave must not propagate more than one subdivision in space during one time step.

#### 5. Numerical results.

The technique was tested for the cases of different simple line terminations. The excitation was chosen to be  $1 - e^{-bt}$ . For simplicity,  $L'$  and  $C'$  are 1 H/m and 1 F/m, respectively.

**5.1 SHORT CIRCUIT TERMINATION.** — For a grounded line, the voltage at the end of the line is zero. In travelling wave analysis, this implies that a voltage wave of equal amplitude but opposite sign is generated due to the grounding of the cable as shown in figure 1. The excitation in this case is  $1 - e^{-t}$ . To implement a grounded line, the voltage node at the end of the line is set to zero.

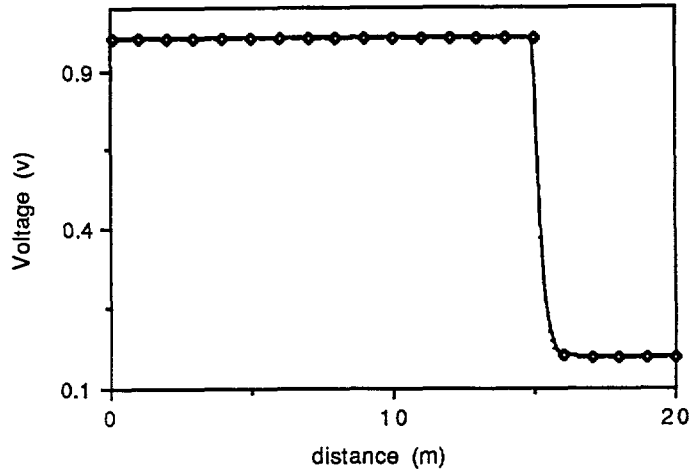


Fig. 1. — Voltage distribution along a grounded line at  $t = 25$  s. (—) Analytical. ( $\diamond$ ) TDFE.

**5.2 OPEN CIRCUIT TERMINATION.** — The current at the end of an open circuit is always zero. The negative-current reflected wave accompanies a positive-voltage reflected wave which will double the amplitude of the voltage waveform as shown in figure 2. The excitation in this case is  $1 - e^{-5t}$ . Moreover, the current node at the end of the line is set to zero for all  $t > 0$ .

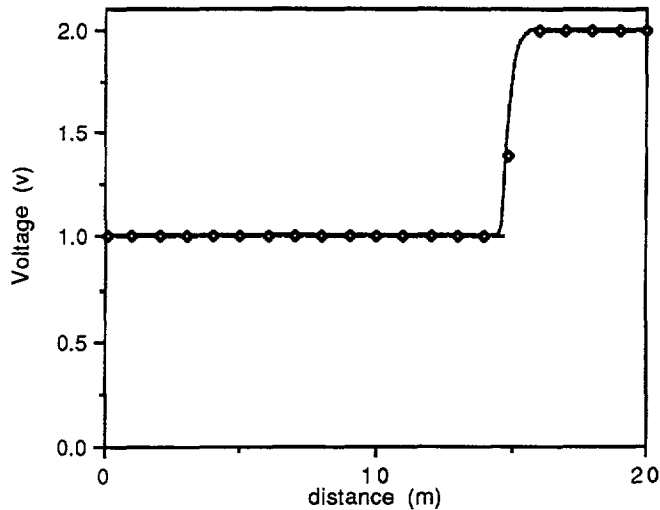


Fig. 2. — Voltage distribution on an open circuit line at  $t = 25$  s. (—) Analytical. ( $\diamond$ ) TDFE.

**5.3 LINE TERMINATED BY A CAPACITOR.** — Before the capacitor is charged up, the termination is similar to a short circuit. When the capacitor is fully charged, it will act as an open circuit. Hence the voltage transient will experience a gradual change from zero volts to a voltage which

is double the source input, as shown in figure 3. The excitation in this case is  $1 - e^{-5t}$ . The voltage/current relationship between the terminals of a capacitor is given by :

$$i = C \frac{\partial V}{\partial t} \quad (26)$$

Using forward Euler differencing, (26) is discretized and becomes

$$\frac{\Delta t}{C} i \left[ \left( n + \frac{1}{2} \right) t \right] + V(n \Delta t) = V[(n+1) \Delta t]. \quad (27)$$

The current at the voltage node is approximated by the nodal current at one-half  $\Delta x$  in front of the voltage node.

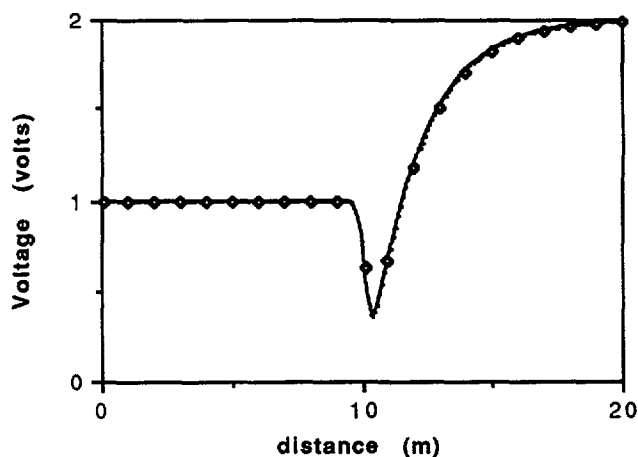


Fig. 3. — Voltage along a line terminated by a capacitor at  $t = 30$  s. (—) Analytical. ( $\diamond$ ) TDFE.

**5.4 LINE TERMINATED BY AN INDUCTOR.** — An inductor acts like an open line when a current wave front is first impressed on it. Then gradually the current will reach a steady state and the inductor responds as if it were a short termination. The voltage transient in this case will reach a maximum and then decay to zero as shown in figure 4. The input source in this case is  $1 - e^{-5t}$ . The voltage/current relationship between the terminals of a capacitor is given by :

$$V = L \frac{di}{dt} \quad (28)$$

Using forward Euler differencing, (28) is discretized and becomes

$$\frac{\Delta t}{L} V(n \Delta t) + i \left[ \left( n - \frac{1}{2} \right) \Delta t \right] = i \left[ \left( n + \frac{1}{2} \right) \Delta t \right]. \quad (29)$$

The voltage at the current node is approximated by the nodal voltage at one-half  $\Delta x$  in front of the current node.

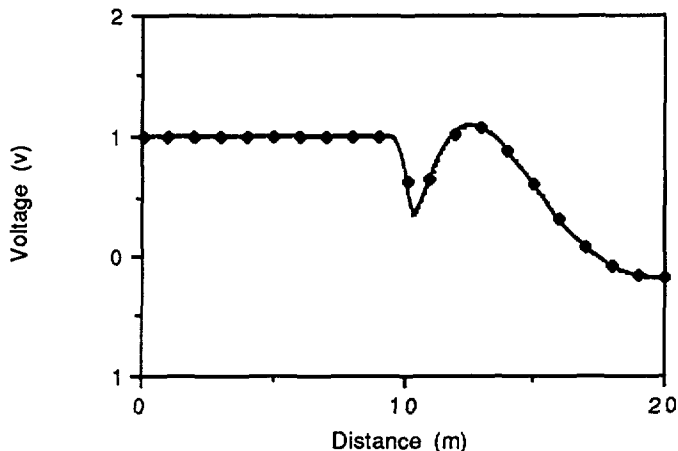


Fig. 4. — Voltage along a line terminated by an inductor at  $t = 25$  s. (—) Analytical. (◆) TDFE.

**5.5 LINE TERMINATED BY A RESISTOR AND A CAPACITOR PARALLEL.** — Using Kirchoff’s voltage and current laws, the current/voltage relationship of the parallel configuration is expressed as :

$$i_c = C \frac{dV}{dt} \tag{30a}$$

$$V = i_r R \tag{30b}$$

$$i_c + i_r = i \tag{30c}$$

where

$i_c$  is the current through the capacitor,

$i_r$  is the current through the resistor,

$V$  is the voltage across the resistor and capacitor, and

$i$  is the current from the line.

Using the classical integration formulas and combining (30a), (30b) and (30c), the discretized form of system (30) is given by :

$$V [(n + 1) \Delta t] = \frac{\Delta t}{C} \sum_{j=1}^n \left( i_j - \frac{V_j}{R} \right) \tag{31}$$

where  $n$  is the number of time steps,  $i_j$  and  $V_j$  are the current and voltage at time step  $j$ , respectively. The analytical and TDFE results are plotted in figure 5. The source in this case is  $1 - e^{-5t}$   $R$  is  $2 \Omega$  and  $C$  is  $2$  F.

**5.6 LINE TERMINATED BY AN INDUCTOR AND A CAPACITOR IN PARALLEL.** — The current/voltage relationship of the parallel configuration is expressed as :

$$i_c = C \frac{dV}{dt} \tag{32a}$$

$$V = L \frac{di_l}{dt} \tag{32b}$$

$$i_c + i_l = i \tag{32c}$$



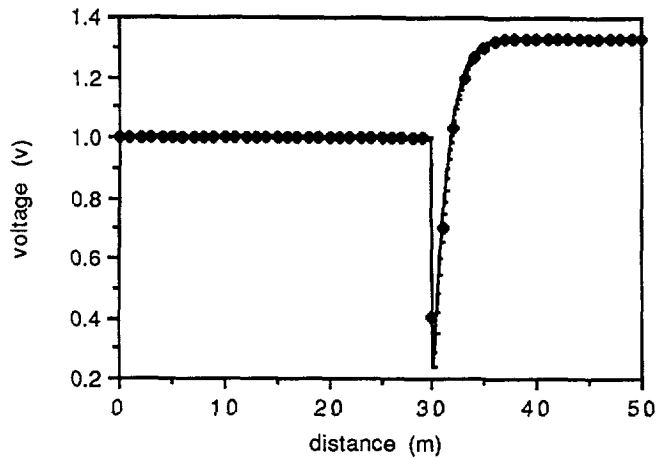


Fig. 5. — Voltage along a line terminated by a capacitor and resistor in parallel at  $t = 70$  s. (—) Analytical. (◆) Numerical.

where

$i_c$  is the current through the capacitor

$i_l$  is the current through the inductor

$L$  is the inductance

$C$  is the capacitance

$i$  is the current from the line, and

$V$  is the voltage across the termination.

(32a) is discretized by forward Euler differencing and (32b) is discretized by the classical formulas for integration. Combining with (32c), we obtain

$$V[(n+1)\Delta t] = V(n\Delta t) + \frac{\Delta t}{C} i \left[ \left( n + \frac{1}{2} \right) \Delta t \right] - \frac{(\Delta t)^2}{LC} \sum_{j=1}^n V_j \quad (33)$$

where  $V_j$  is the voltage at time step  $j$  and  $n$  is the number at time steps. The analytical and TDFE results are plotted in figure 6. The source in this case is  $1 - e^{-5t}$   $L$  is 2H and  $C$  is 2F.

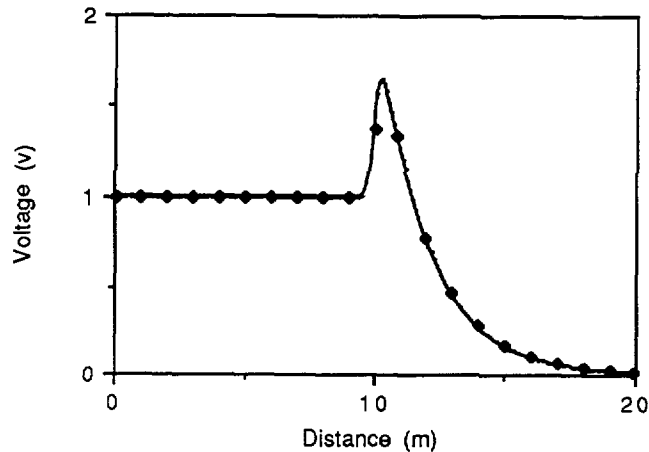


Fig. 6. — Voltage along a line terminated by a capacitor and an inductor in parallel at  $t = 30$  s. (—) Analytical. (◆) TDFE.

**5.7 MISMATCHED IMPEDANCES.** — When two lines with different surge impedances are joined together, reflection and refraction occurs. The model of this kind of transition point in TDFE requires the intersection or boundary located at one of the voltage or current nodes. It is important to note that  $C'$  and  $L'$  cannot change at the same location, or the value used in the computation will be ambiguous.

In figure 7, a 30 m line is connected to a 40 m line with different  $C'$ . The  $L'$  for both lines is set to unity.  $C'$  for the first line is 1 F/m and for the second line is 2 F/m. The reflection and refraction coefficients for this case are 0.172 and 0.828, respectively. The speed of propagation for the first line is 1 m/s and for the second line it is 0.707 m/s. The source in this case is  $1 - e^{-5t}$

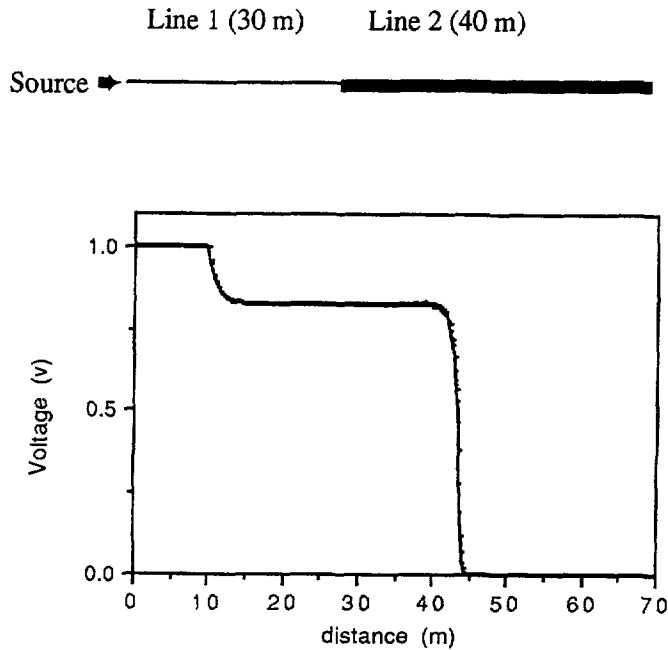


Fig. 7. — Voltage along a line with mismatched impedance at  $t = 50$  s.

From figure 7 the transmitted wave has an amplitude of 0.83. The distance of the transmitted wave travelled in 20 s is 14 m. Therefore, the speed of propagation is 0.7 m/s. The percentage difference between the TDFE solution and the analytical solution is 0.2 percent for the amplitude and 1 percent for the speed of propagation.

The reason for choosing a small  $b$  for the source in this case is due to the unstable numerical solution created by a steep wavefront. This effect is demonstrated in the following figures. Three sources with different rise times, as shown in figure 8, are used. The TDFE solution for  $b = 10$  is shown in figures 9a, 9b, and 9c. The solution for  $b = 5$  is shown in figures 10a and 10b. The solution for  $b = 1$  is shown in figures 11a and 11b.

**5.8 BRANCHING.** — Branching is a common feature in a transmission system network. In TDFE computation, branching is modelled by a system of equations. To illustrate the procedure, a junction joining three lines is considered. Its configuration is shown in figure 12.

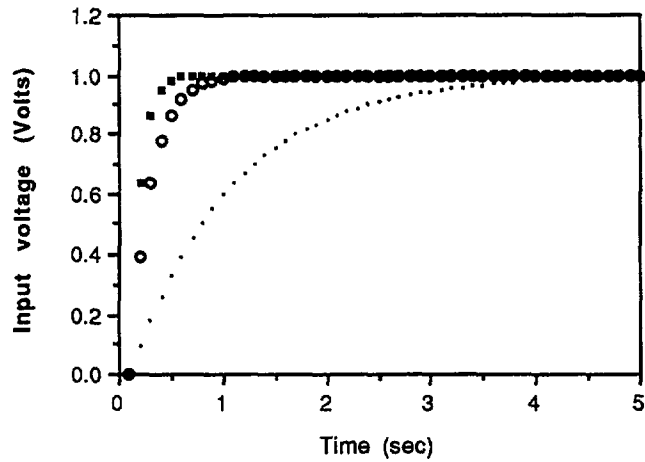


Fig. 8. — Sources with different rise times. (·)  $b = 1$ . (○)  $b = 5$ . (■)  $b = 10$ .

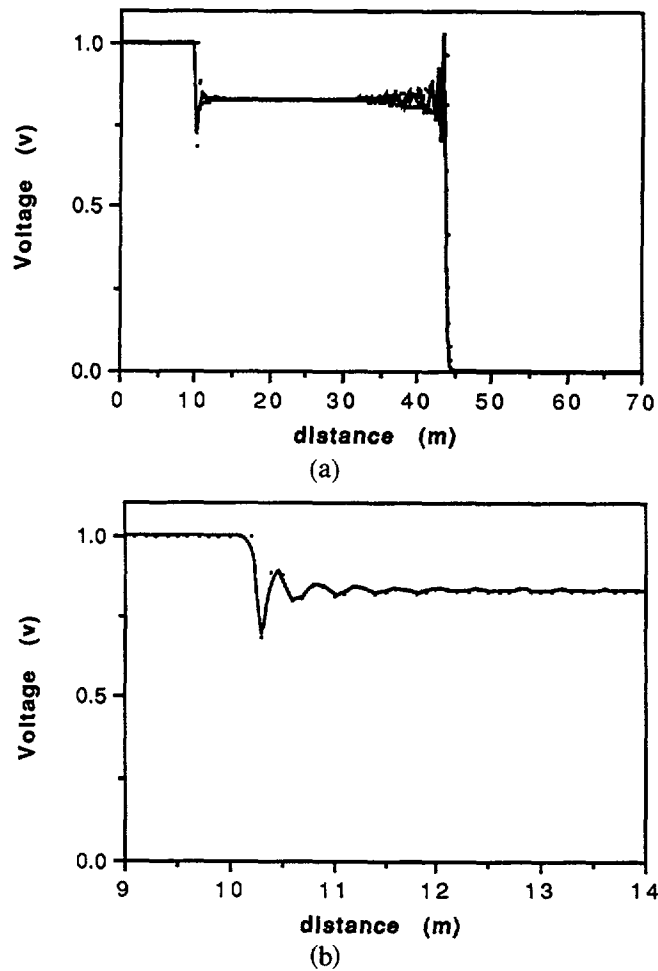
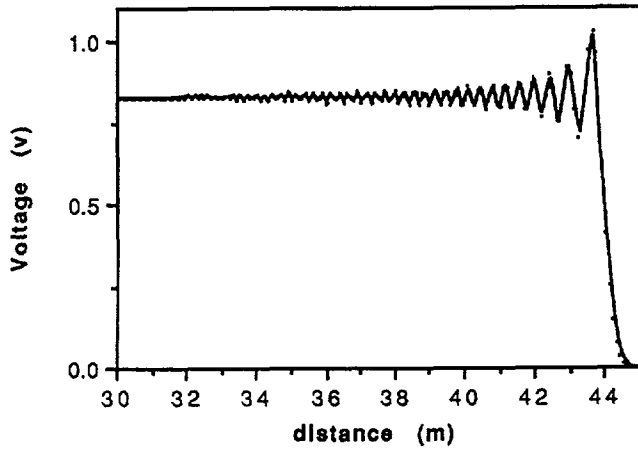
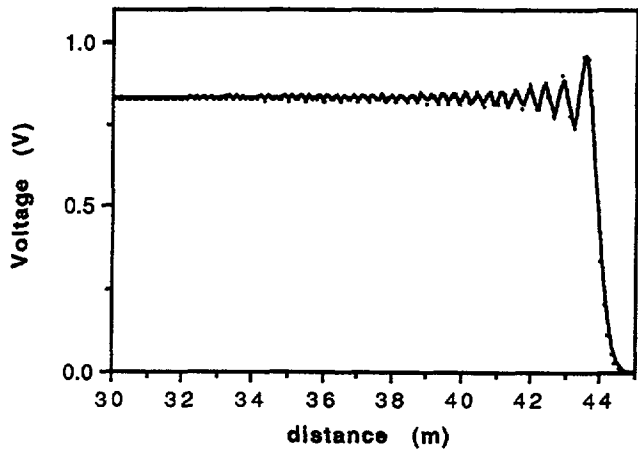


Fig. 9. — a) TDFE solution for  $b = 10$  at  $t = 50$  s. b) Reflected voltage waveform for  $b = 10$ . c) Transmitted voltage waveform for  $b = 10$ .

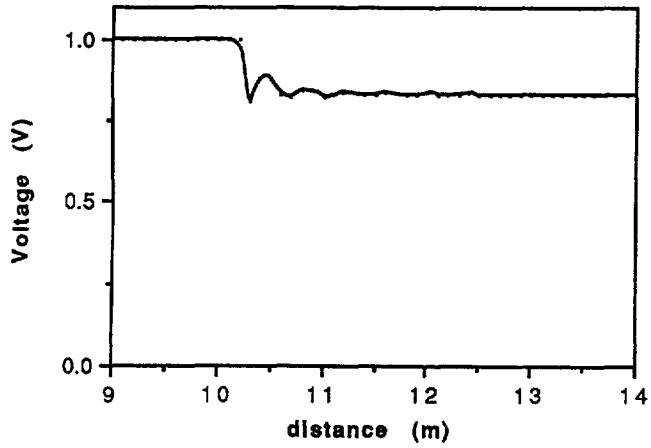


(c)

Fig. 9 (continued).

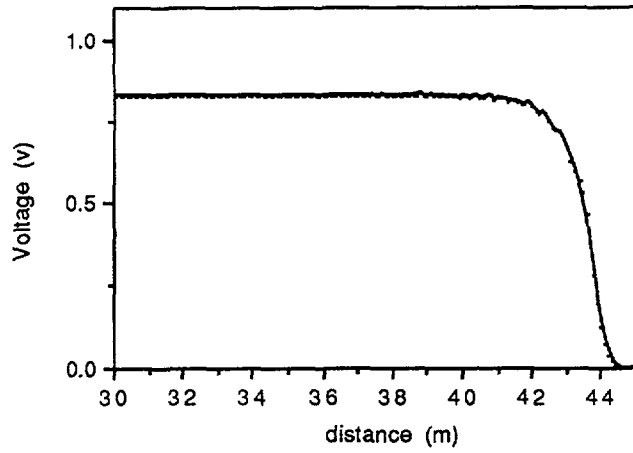


(a)

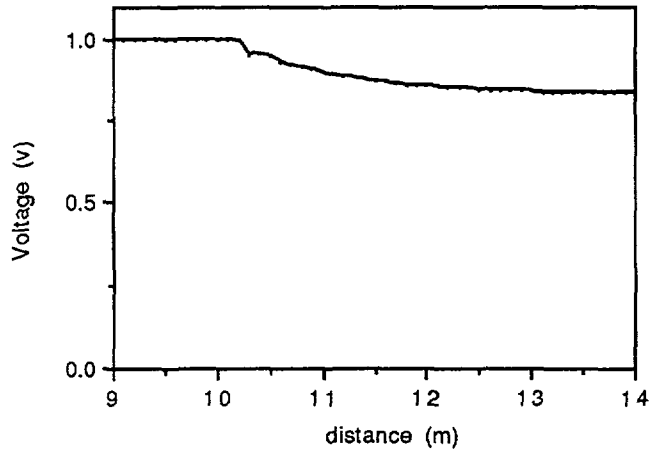


(b)

Fig. 10. — a) Transmitted voltage wave for  $b = 5$ . b) Reflected voltage waveform for  $b = 5$ .



(a)



(b)

Fig. 11. — a) Transmitted waveform for  $b = 1$ . b) Reflected wave for  $b = 1$ .

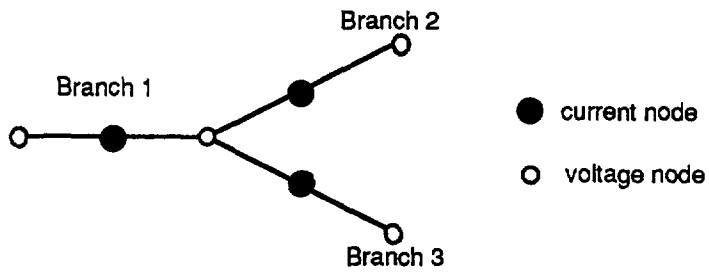


Fig. 12. — Single-phase transmission line network.

Let the junction be a voltage node. A system of differential equations is obtained :

$$I_{j1}^{n+\frac{1}{2}} = I_{j1}^{n-\frac{1}{2}} - \frac{\Delta t}{L'} \left[ \frac{1}{\Delta x} (V_{i1+1}^n - V_{i1}^n) \right] \tag{34a}$$

$$I_{j2}^{n+\frac{1}{2}} = I_{j2}^{n-\frac{1}{2}} - \frac{\Delta t}{L'} \left[ \frac{1}{\Delta x} (V_{i2+1}^n - V_{i2}^n) \right] \tag{34b}$$

$$I_{j3}^{n+\frac{1}{2}} = I_{j3}^{n-\frac{1}{2}} - \frac{\Delta t}{L'} \left[ \frac{1}{\Delta x} (V_{i3+1}^n - V_{i3}^n) \right] \tag{34c}$$

where  $I_{j1}, I_{j2}, I_{j3}$  are the currents and  $V_{i1}, V_{i2}, V_{i3}$  are the voltages on lines one, two and three, respectively. For simplicity,  $L'$  is chosen to be the same for all three lines. Using the continuity equations for both current and voltages, it follows that

$$I_{j1}^{n-\frac{1}{2}} + I_{j2}^{n-\frac{1}{2}} + I_{j3}^{n-\frac{1}{2}} = 0 \tag{35a}$$

$$V_{i3}^n = V_{i2}^n = V_{i1+1}^n = V. \tag{35b}$$

Substituting (35a), (35b) into (34a), (34b), (34c), the following system is obtained

$$J_1 = -\Gamma_1 + \theta_1 - \frac{\Delta t}{\Delta x L'} V \tag{36a}$$

$$J_2 = -\Gamma_2 + \theta_2 + \frac{\Delta t}{\Delta x L'} V \tag{36b}$$

$$J_3 = -\Gamma_3 - \theta_3 + \frac{\Delta t}{\Delta x L'} V \tag{36c}$$

$$J_1 + J_2 + J_3 = 0 \tag{36d}$$

where

$$J_1 = I_{j1}^{n+\frac{1}{2}}, \quad J_2 = I_{j2}^{n+\frac{1}{2}}, \quad J_3 = I_{j3}^{n+\frac{1}{2}},$$

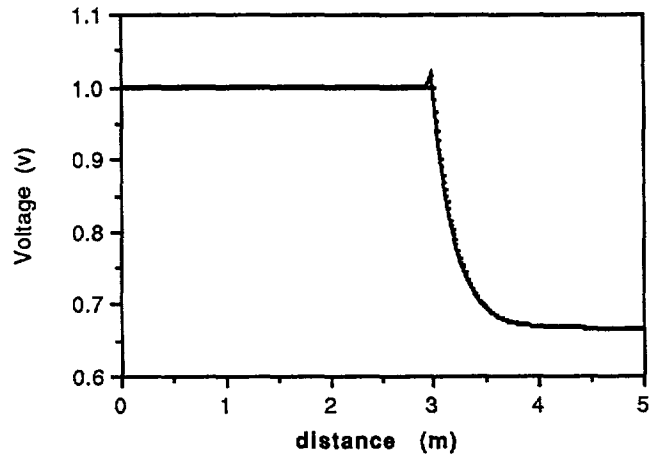
$$\theta_1 = \frac{\Delta t}{\Delta x L'} V_{i1}^n, \quad \theta_2 = \frac{\Delta t}{\Delta x L'} V_{i2+1}^n, \quad \theta_3 = \frac{\Delta t}{\Delta x L'} V_{i3+1}^n,$$

$$\Gamma_1 = I_{j1}^{n-\frac{1}{2}}, \quad \Gamma_2 = I_{j2}^{n-\frac{1}{2}}, \quad \Gamma_3 = I_{j3}^{n-\frac{1}{2}}$$

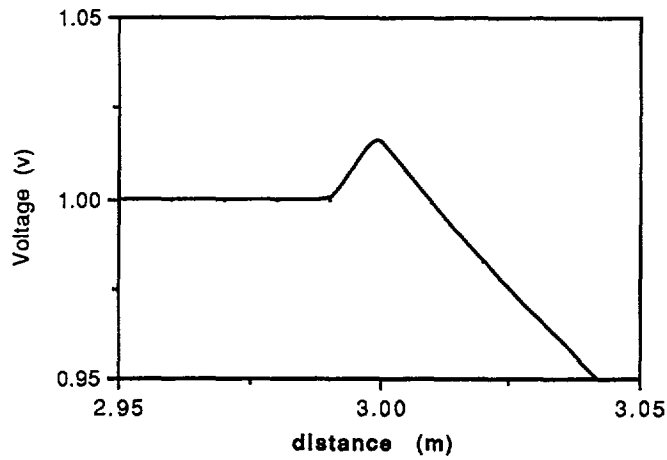
Since  $\Gamma_1, \Gamma_2, \Gamma_3, \theta_1, \theta_2,$  and  $\theta_3$  are known quantities, the system in equations (36a) through (36d) has only four unknowns, namely  $J_1, J_2, J_3$  and  $V$ . This formulation can easily be extended to a junction joining multiple branches.

Although the numbering of the nodes is arbitrary, it is important to keep track of the reference branch ending node at the junction. For example, in the above illustration, branch one is chosen to be the reference branch and hence  $V_{i1+1}$  instead of  $V_{i1}$ , is the common voltage at the junction.

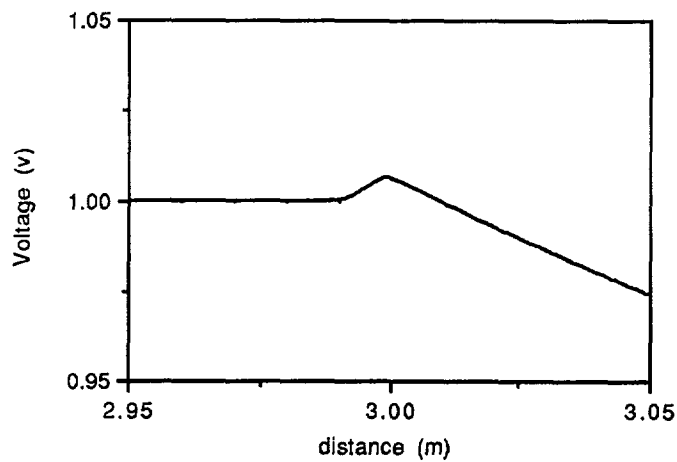
A source of  $1 - \exp(-bt)$ ,  $b$  is 5.0, is applied to branch one. The voltage along branch one at  $t = 7$  s is shown in figure 13a. The overshoot, shown in figure 13b, is the numerical error due to a fast rise time of the source. Figure 13c shows the reduction of the numerical error when  $b$  is 1.0. Figure 14 shows the voltage along the other two branches. Analytically the transmitted and reflected wave should have an amplitude of 0.66 V.



(a)



(b)



(c)

Fig. 13. — a) Voltage along branch one at  $t = 7$  s. b) Numerical error due to high source rise time. c) Reduction of numerical error due to the reduction of the slope of the input source.

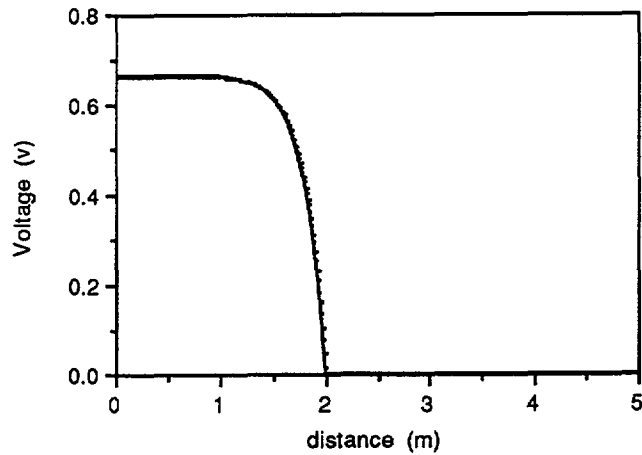


Fig. 14. — Voltage distribution along branch two and three at  $t = 7$  s.

## 6. Conclusion.

Preliminary results indicate that the TDFE method works well for single-phase transmission line transient analysis. Work is in progress to extend the method to problems which include losses, coupled and multiphase lines, nonlinear line terminations and frequency dependent parameters.

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