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Short Communication

Hysteresis and Recovery Length in a Dry Solid Friction Experiment

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Abstract. — We report an experiment on dry friction between two elastic surfaces made of millimetric steel spheres embedded in an elastic medium. We study the hysteresis cycle of the tangential force induced by a quasi-static cyclic displacement. At low velocities, hysteresis effects are fully characterized by a *recovery length* comparable to the size of the steel spheres. This length is independent of the velocity and of the normal load. We propose an interpretation for this recovery length with a simple model of discrete contacts.

Dry solid friction occurs in a wide range of systems and is of enormous practical importance. The interest of the physicists in this subject has been renewed recently, due to the development of earth-quake models [1], as well as the analogies with pinning problems such as magnetic vortices, charge density waves, fracture fronts, wetting hysteresis, *etc.* From a macroscopic point of view solid friction is a phenomenon with a well defined threshold. It is very well known that two solids put in contact with each other remain at rest until the tangential force applied to them reaches a threshold value $F_a = \mu_s F_N$, where μ_s is the static friction coefficient and F_N the normal force applied to the bodies. Above this threshold the bodies enter into a relative motion. If the two bodies are moved periodically back and forth on a distance Δx, the existence of the threshold clearly produces a mechanical hysteresis because one needs to change the driving force from F_a to $-F_a$ in order to reverse the motion. Thus the energy E_d, dissipated to do back and forth the relative displacement Δx, is just $E_d = 2F_a\Delta x$, independently of the amplitude of Δx. This value of E_d is a consequence of the fact that, in the standard model, there is no relative displacement of the two solids if the tangential force is below threshold. However it has been known for a long time that surfaces undergoing dry solid friction exhibit displacements at the microscopic scale, called micro-slip, before the macroscopic sliding occurs [2, 3]. Micro-slip is of interest, not only because of its engineering importance in computer modelling of solid friction, but also in relation with the theories of mechanical friction based on elastic instabilities in a pinning potential [4–6]. In these theories,

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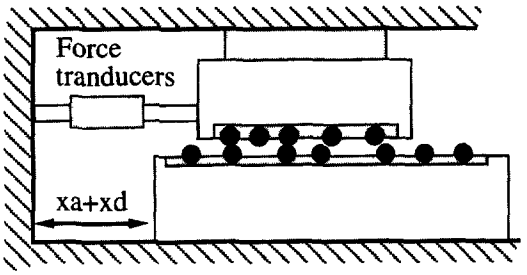

Fig. 1. — Schematic drawing of the experimental apparatus. Surfaces are made of millimetric steel spheres embedded in an elastic medium, mounted on aluminum plates. The lower plate is displaced with the sum of a harmonic and a continuous displacement. The tangential force and the normal force are measured by transducers fixed on the upper plate.

the competition between the internal elasticity of the bodies and the pinning forces induced by surface defects result in many metastable states. and friction is generated by the jump of portions of the system between those metastable positions. It is thus of interest to characterize the friction force for displacements of the surfaces of the order of those jumps. For this purpose, we construct artificial surfaces with controlled millimetric asperities [7, 8]. Those surfaces are submitted to cyclic displacements of various amplitude and frequency, and the hysteresis cycle of the tangential force is studied. Such a method is "currently" used in nanotribology [9, 10], and gives access to the energy dissipated during the cycle as well as the tangential stiffness of the contact. Our main result is that a significant relative displacement of the surfaces occurs below the threshold value of the friction force; the force-displacement characteristic is fully characterized by a length, independent of the frequency and of the normal load, and comparable to the size of the asperities.

Figure 1 shows a schematic drawing of the experimental set-up, which is described in detail elsewhere [7, 8]. The two sliding surfaces are mounted on aluminum plates of thickness one centimeter. These surfaces consist of randomly distributed steel spheres of radius 1.0 mm tightly embedded in a matrix of thickness 3 mm. The size of the lower plate is 10×30 cm, and the matrix is a rigid sty-cast glue. The upper plate is 10×10 cm, its mass is 190 g, and the elastic matrix is a plastic of Young Modulus $E = 17$ MPa. The sphere density on each surfaces is 3 spheres/cm^2, and the random distribution is achieved by hand. The spheres emerge from the elastic matrix of roughly half a millimeter. The surfaces are put in contact in such a way that only sphere to sphere contacts occur and the normal force is varied by monitoring the fixed gap between the two sliding surfaces. The upper plate is fixed on two force transducers of high internal stiffness ($k = 6.6$ N/μm), which measure the tangential force F_{T} and the normal force F_{N} exerted on it. The lower plate is submitted to a tangential displacement which is the sum of an harmonic displacement $x_{\mathrm{a}}(t) = A\cos\omega t$ and a continous drift $x_{\mathrm{d}}(t)$ of small velocity. The harmonic displacement x_{a} is recorded with an optical sensitive sensor. Due to the asperities on the solid surfaces, the tangential force exhibits fluctuations during a single displacement cycle. The continuous drift is used to realize a spatial average of the cycles over many configurations of the asperities. The drift velocity is always kept much smaller than the cycle velocity ($\dot{x}_{\mathrm{d}} < 5.10^{-2}$ Af), and we check that the results do not depend on its value in the range 0.5 to 20 μm/s. One hundred to one thousand cycles of the tangential force are averaged. The fluctuations of the force in a single cycle are at most 10% of its averaged value; the total error on the average value is estimated at less than 5%. Two types of measurements

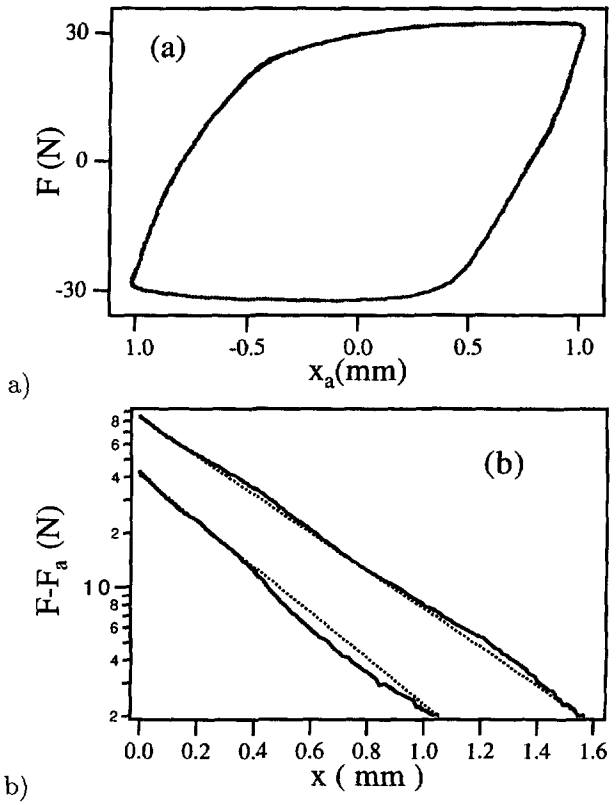

Fig. 2. — a) Averaged tangential force during an oscillating displacement of the surfaces. Starting from the minimum displacement, the force increases over a millimetric displacement of the surfaces, and saturates at a constant level. A symmetric behavior occurs when the surfaces recede. b) log-lin plot of the relaxation of the force toward its constant value F_a, *versus* the displacement x of the solids, for two different loads. The relaxation is exponential over one and a half decades, with a memory length $\xi = 340$ μm for the lowest load and $\xi = 380$ μm for the highest load.

are performed: i) the direct measurement of the (averaged) tangential force cycle and ii) the harmonic components of the force response, *i.e.* the real and imaginary part of the component of the force at the exciting frequency. This last measurement is taken by a spectrum analyzer HP 3562A.

At frequencies lower than 50 mHz, the experimental results are independent of the displacement frequency. This absence of temporal dependance [1,11,12] is due to the elastic properties of the material used which does not exhibit visco-plastic effects in the condition of the experiment [7,8]. In the following we restrict to this low frequency limit and focus on the quasistatic behaviour of the tangential force versus the displacement amplitude only.

Figure 2a shows the averaged tangential force as a function of the alternative displacement for an excitation of amplitude 1 mm at a frequency 50 mHz. The tangential force is clearly hysteretic during a cycle. Starting from the minimal abscissa, the force increases on a distance of the order of half a millimeter, and saturates at an advancing force $F_a \approx 32$ N. When the displacement is reversed a symmetric behavior is observed, with $F_r = -F_a$. Notice that the surfaces undergo a relative displacement much before the tangential force reaches its maxi-

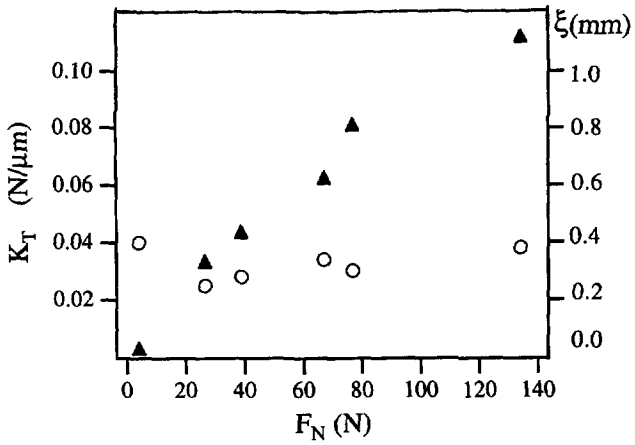

Fig. 3. — Tangential stiffness of the interface (▲) and recovery length (○) as a function of the normal load.

mum value, which is not described by the classical laws of dry friction. We want to stress that this displacement really occurs at the level of the surfaces themselves, since the stiffness of the force transducer and the shear stiffness of the supporting elastic medium do not allow a tangential stretch larger than 6 μm at the maximum tangential load F_a. Hereafter, we describe the shape of the hysteresis cycle by two independent parameters: the value F_a of the force saturation, determined by the extremum value of F_T in the cycle, and a *recovery length* ξ. This recovery length is determined from the log-lin plot of $F_a - F_T$ *versus* x_a as shown in Figure 2b: the relaxation of the tangential force toward its extremum value is approximately exponential, over a range of displacement extending to three or four times the decay length ξ. As mentionned above, this recovery length is independant of the displacement velocity and does not vary when the frequency is changed by one order of magnitude. Therefore ξ appears as the typical length along which the surfaces have to be displaced, in order to loose memory of their previous displacement (advancing or receding). The memory of the previous displacement is here characterized by a length and not by a time, as mentionned in studies of rate effect on friction [11].

Another characteristic of the recovery length is to depend very weakly on the normal load. Experiments have been performed with normal loads varying by more than one order of magnitude, and the recovery length does not change significantly (Fig. 3). The other important parameter to consider is the tangential stiffness of the interface K_T which is the derivative of the tangential force *versus* the displacement just after the motion has been reversed. The tangential stiffness of the interface increases almost linearly with the normal load (Fig. 3). Therefore, the recovery length appears as the relevant parameter to describe the force-displacement characteristic under the threshold, rather than the tangential stiffness.

Before discussing these results, we describe an alternative method to investigate hysteresis effects, based on the harmonic analysis of the tangential force. The spectrum analyser measures the component in phase, a, and out of phase, b, of the tangential force with respect to the imposed displacement $x(t) = x_d(t) + A\cos\omega t$:

$$a = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} F(t)\cos\omega t dt \qquad b = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} F(t)\sin\omega t dt.$$

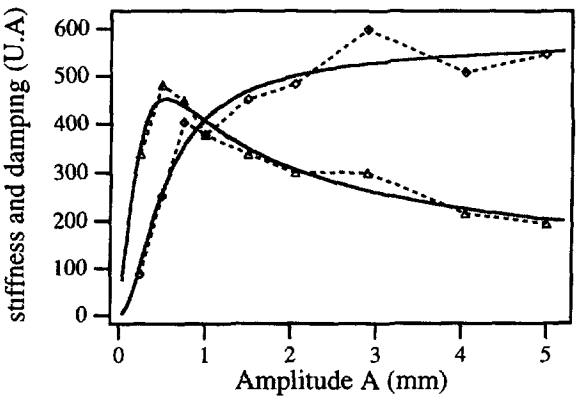

Fig. 4. — Real (△) and imaginary (◇) components of the tangential force, as a function of the amplitude of the oscillating displacement (dotted lines are just a guide). Plain lines show the best fit of the experimental data with equation (2), the two parameters $F_a - F_r$ and ξ being the same for both components. In this case $\xi = 300$ μm.

The in-phase component measures the reversible part of the tangential force; in the limit of small amplitude excitations it is related to the contact stiffness by $a/A = K_T$. The out-of-phase component measures the energy dissipated during the cycle: $E_d = 2\pi Ab$.

In the classical model of dry friction, and if we ignore velocity effects, the friction force acting on a solid depends only on the sign of velocity. Thus, the energy dissipated during one cycle varies linearly with the displacement amplitude. It follows that the dissipative component b is constant, and the reversible component a is zero. However, as long as the tangential force keeps memory of the displacement history, b is expected to vary with the amplitude of the cycle. Figure 4 shows the components of the force as a function of the amplitude A of the displacement (the frequency is $f = 19.4$ mHz). As expected, at large amplitude, the dissipative component reaches a constant value, and the reversible component decreases, revealing that the force is essentially in phase with the relative velocity of the surfaces. The deviation from this asymptotic behaviour at small amplitudes contains information about the recovery length. For a quantitative analysis, a phenomenological law must be prescribed for the variation of the tangential force. Since the data do not depend on frequency, we only take into account variation of the tangential force with the surface displacement x. Starting from an initial force F_i for an initial position x_i, it is assumed that the variation of the force follows an exponential law:

$$F_T(x) = F_a + (F_i - F_a)\exp\left(-(x - x_i)/\xi\right) \tag{1}$$

when $\dot{x} > 0$, and a symmetric evolution law when the surface recedes. Such an evolution law is a single state variable law as described by Ruina [13], with velocity effects omitted. With this law, the reversible and dissipative components of the tangential force are:

$$a = (F_a - F_r)\frac{I_1(A/\xi) - I_1(-A/\xi)}{4\cosh(A/\xi)} \qquad b = (F_a - F_r)\frac{1}{\pi}\left(1 - \frac{\xi}{A}\tanh(A/\xi)\right) \tag{2}$$

where I_1 is the first order Hankel function.

The comparison of this phenomenogical law with the experimental data is shown in Figure 4. The set of two independent parameters F_a and ξ is adjusted by a least-square fit on both a and b. The agreement with the theoretical equation (2) is excellent. Furthermore, the value obtained for the recovery length ξ and the hysteresis amplitude F_a are in very good agreement with the values found by the direct analysis of the hysteresis cycle.

In conclusion, these measurements of the tangential force induced by a small displacement of the solids show the existence of a recovery length, independent of the velocity and of the normal load, which characterizes the memory that the surfaces keep of their previous displacement. It must be stressed that the memory effect observed here is purely quasi-static, and independent of any velocity or temporal dependence of the friction force, as observed in other experiments [1, 11, 12]. We also want to mention that similar properties are observed in some nanotribology experiments [9, 14], although the scales involved are very different. In nanotribology experiments, the recovery length is usually defined as the ratio of the threshold tangential force to the tangential stiffness of the interface: $\xi = 2F_a/K_T$. However, this tangential stiffness increases as the normal load, whereas the recovery length depends little on it. It is striking that our experiment reproduces the same phenomenological behaviour at a much larger scale.

We propose here a connection between the observed recovery length and the theories of mechanical friction based on elastic instabilities in random pinning potential. In these theories, the competition between the internal elasticity of the medium and the existence of pinning force (here induced by sphere-sphere contact at the interface) results in many metastable states for the system. In a displacement, elastic instabilities occur, during which small portions of the surfaces undergo a jump of finite size. For a single pinning force, the size of the jump is the anchoring length Λ defined by de Gennes [5]; it is equal in first approximation, to the maximum amplitude of the pinning force divided by the stiffness λ of the restoring force exerted by the elastic medium. We assume that the number N of sphere-sphere contacts increases linearly with the normal load, and that these contacts act as independent pinning defects. Instead the pinning potential does not depend on the normal load. In a quasi-static continuous displacement, the number of contacts involved in a jump for a relative displacement Δx of the surfaces is $N\Delta x/\xi$, since the recovery length is the typical distance needed to renew the contact population. If the defects are stiff, which is the case in our experiment [15], the energy dissipated in a jump is $e \propto \lambda\Lambda^2/2$. Therefore the total energy dissipated, in a cycle of amplitude $\Delta x \gg \xi$, is $E_d = 2F_a\Delta x = eN\Delta x/\xi$. The threshold friction force is then $F_a = N\lambda\Lambda^2/4\xi$. We need to know an estimate of K_T, and this can be done by considering that when the motion is reversed, no instability occurs at an early stage. The tangential force then undergoes an elastic discharge in which all the N contacts participate and $K_T \propto N\lambda$. From the definition of $\xi = 2F_a/K_T$, one obtains $\xi \propto \Lambda$. In this description, the recovery length is related to the anchoring length of the pinning defects and appears as a parameter intrinsic to the solid surfaces, whereas the tangential stiffness of the interface depends on the number of contacts.

Such a crude description is of course limited. It does not explain the detailed shape of the force-displacement dependency, and it does not take into account possible correlations among contacts [16]. Experiments are in progress, for a better understanding of the relation between the recovery length and the sliding surfaces characteristics. Also, similar experiments are performed in another case of mechanical hysteresis, the contact angle hysteresis of a fluid interface on a solid surface, for which recovery effects are also encountered [17].

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