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# Global Mode Behavior of the Streamwise Velocity in Wakes

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**Abstract.** — We characterize experimentally and numerically the two-dimensional structure of the global mode of the longitudinal component of the velocity in flow behind obstacles of different geometrical shapes. Measurements in a low velocity water tunnel and numerical simulations of the longitudinal component  $V_x$  of the oscillating velocity are carried out in the near wake region of the flow where vortex shedding is regular. We find that in the range of Reynolds numbers studied, the position  $(X_{\max}, Y_{\max})$  of the maximum of the global mode of  $V_x$  shifts very little away from the central line in the  $y$  direction. The amplitude  $A_{\max}$  of the maximum of the global mode scales as  $(R - R_c)^{1/2}$  in a range of  $R - R_c$  larger than in previous studies. We also confirm that  $X_{\max}$  of the global mode scales as  $(R - R_c)^{-1/2}$  in the same range of Reynolds numbers. An analysis of the initial slope of the global mode is presented.

## 1. Introduction

A recent experimental observation of global modes of the Bénard-von Karman instability has triggered a renewed interest in the classical subject of vortex shedding from a body. Systematic experimental and numerical study have shown (see Refs. [1] and [2]) that the amplitude of the oscillations of the propagating wave instability grows from the origin (the shedding body), reaches a maximum at a downstream distance of the order of few diameters  $d$  of the shedder, and subsequently decays. The spatial envelope of this coherent oscillation gives the amplitude of the global mode. It has been shown that the maximum amplitude of this mode ( $A_{\max}$ ) and the position of this maximum ( $X_{\max}$ ) are functions of the Reynolds number. However, with a suitable renormalization scheme all spatial envelopes collapse to a universal curve independent of the Reynolds number.

The shear induced by the flow behind the body, which is at the origin of the instability, relaxes far away reducing its strength. It is therefore not surprising that the peak to peak amplitude of the temporal oscillations is spatially inhomogeneous and diminishes with the

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distance from the shedding body, as the instability gets weaker. This spatial inhomogeneity of the basic flow implies that the results of the local linear stability analysis change from one downstream station to the next. The stability analysis shows that the flow regions near the body are locally absolutely unstable, while further downstream local convective instability occurs [3].

It could be therefore expected that the local values of the linear growth rate and frequency will differ from one downstream station to the next. However, in reality vortex shedding selects unique frequency and acts as a synchronized nonlinear oscillator distributed in space. Because of its distribution in space the envelope of the temporal oscillations is called the global mode of the instability [4].

From the physical point of view it is legitimate to ask how this mode comes into existence. It is clear that some kind of feedback mechanism plays a role in this process, but more work is necessary in order to clarify this key point. In other hydrodynamic systems with self-sustained oscillations (jet-edge [5], confined jets [6], coupled jets [7] *etc.*) feedback mechanisms of the flow between downstream and upstream regions in the form of pressure waves or hydrodynamic recirculations are necessary to induce intrinsic dynamics. In wakes, the basic flow creates two recirculation lobes behind the body and they can play a role in inducing hydrodynamic feedback. However, the nature of this feedback remains to be elucidated in order to explain surprising effects of the presence of small objects or suction on the suppression of vortex shedding [8].

In previous studies of the Bénard-von Karman instability, experiments were performed by local measurement of the non linear evolution of the temporal oscillation as a function of the Reynolds number  $R$ . The obtained results were analyzed in terms of the Landau model for Hopf instabilities [9]. In reference [1] we showed, that in order to find the pertinent order parameter of the instability one needs to take into account the existence of the global mode. As a consequence we claimed that the maximum amplitude of this global mode as well as the position of this maximum are essential for a correct non linear study of the wake flow.

Our experimental studies motivated numerical simulations by Zielinska *et al.* [2] and Dušek [10]. These simulations confirm our experimental results concerning the form and behavior of the global mode. In addition they provide information about the spatial distribution of the mode out of the central line of the flow and on its evolution from the initial linear regime to the full non linear behavior.

Previous theoretical studies [11] of the evolution of global modes in space and their behavior as a function of the Reynolds number were restricted to linear global modes without taking into account their deformation, as observed by us in references [1, 2] and in the present work, due to nonlinear interactions and evolving Reynolds number. From the theoretical point of view a well founded proof of the applicability of the Landau model to spatially evolving open flows is a subject of intense research. A recent theoretical study [12] has shown that in the context of weakly nonlinear theory the Landau constant is a function of the degree of non parallelism of the flow.

It is our motivation here to give a full spatial description of the global mode of the velocity in the wake flow. For this purpose we have undertaken a two-dimensional characterization of this mode in the plane perpendicular to the shedding body. We have performed measurements in a low velocity water tunnel and carried out numerical simulations of the longitudinal (streamwise) component  $V_x$  of the oscillating velocity in the full region of the flow where vortex shedding exists.

Previously [1] we have presented measurements of the transversal component  $V_y$  on the central line of the flow. In these measurements the scaling law found for the  $A_{\max}$  had a critical exponent larger than 0.5 presented here, for  $V_x$  and in the numerical simulations [2].

This was perhaps due to lack of precision in the experimental determination of the  $R_c$  value in these preliminary experiments. One of the goals of this paper is to clarify this point.

The fundamental mode of the transversal component  $V_y$  is symmetric with respect to the central line and therefore the measurements along this line were sufficient to find the maximum of the global mode. However, the longitudinal component  $V_x$  of the oscillating velocity is antisymmetric with respect to the central line. Consequently its maximum lies off this line and its position is function of the Reynolds number. In order to study the scaling law based on the maximum amplitude of  $V_x$  a two-dimensional characterization in the streamwise as well as in the transverse direction was necessary.

In reference [1] we have established two scaling laws as a function of the reduced critical Reynolds number ( $R - R_c$ ) of onset of global oscillations: for the position of the maximum ( $X_{\max}$ ) of the global mode and for the evolution of the maximum of the peak to peak amplitude of the oscillations measured at  $X_{\max}$ . The existence of these two scaling laws allowed us to obtain for the first time a renormalized universal curve for the envelope of the global mode. In the present work we show the extension of these previous results for the streamwise component of the velocity  $V_x$ . It can be expected that the critical behavior of the  $V_x$  will be the same as of the  $V_y$ , because of the essentially two dimensional character of the flow. However, the  $V_x$  component on the symmetry axis ( $y = 0$ ) contains only higher pair harmonics [13] and is expected to follow a different scaling law than the fundamental mode. Therefore it was necessary to perform an additional full two-dimensional measurements of the  $V_x$  component in order to prove, that its scaling law is indeed the same as for the  $V_y$  in the extended range of Reynolds numbers. We also extend the study of the scaling behavior with an analysis of the initial slope of the global mode what emphasize the existence of scaling laws for  $A_{\max}$  and  $X_{\max}$ .

## 2. Experimental Set Up

Our experiments were carried out in a low velocity water tunnel shown schematically in Figure 1 and the velocity was measured using a Laser Doppler Anemometer. The details of the experimental set up can be found in reference [1].

The bluff body has a trapezoidal cross section with sharp edges to ensure constant initial span wise phase to the vortex shedding. Larger of the parallel sides of height  $d = 0.4$  cm faces the flow. The angle of the lateral faces is  $4^\circ$  and the ratio of the height  $l$  of the downstream to the upstream face is  $l/d = 0.875$ .

The flow in the channel is induced by gravitation from a constant level reservoir and exit gate modifies the flow rate. Two-dimensional nature of the flow behind the shedding body in the near wake region is checked using visualization. Fluoresceine dye is injected through the lateral faces of the bluff body and vortex shedding is observed using a sheet of argon laser light.

In order to characterize the global modes and their evolution as function of the Reynolds number we have measured the amplitude of the longitudinal velocity perturbations of the flow behind the obstacle scanning the  $x - y$  plane in the middle of the bluff body, at  $z = 0$  (see Fig. 1).

The aspect ratio of the body in the  $z$ -direction is 12.5; the ends of the obstacle are fixed to the side walls of the tunnel. It was shown [14–16], that in addition to the lowest mode of the velocity envelope, also higher modes with nodes distributed along the  $z$  axis can be excited. We performed velocity measurements along the  $z$ -direction at the central part of the body ( $\Delta z \approx L_z/2$ ) and did not find any evidence of the existence of the nodes. So we conclude

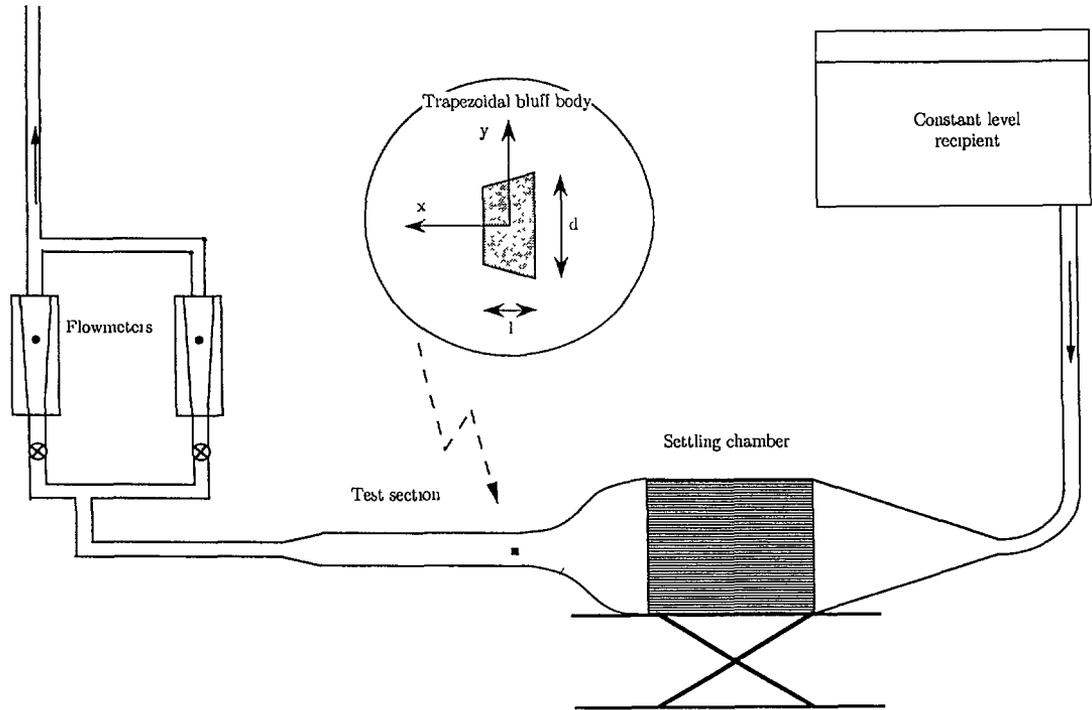


Fig. 1. — Experimental set up.

that in our case only the first mode of the velocity envelope is present, with zeros only in the extremity of the bluff body.

The longitudinal velocities ( $V_x$ ) from the trailing edge of the bluff body (from  $x = 0.7 d$  to  $x = 25 d$ , and from the center line  $y = 0$  to  $y = 2.8 d$ ) were scanned using a Laser Doppler Anemometer. The instantaneous value of the velocity is obtained by processing the Doppler bursts with a frequency counter and analyzed by a spectrum analyzer (FFT), in order to obtain the frequency and the amplitude of the oscillatory part of the velocity.

The measurements were carried out for various Reynolds numbers ranging from  $1.1R_c$  to  $1.6R_c$ , where  $R_c = 60.8$  is the onset of the global bifurcation.

### 3. Computations

Numerical simulations of two-dimensional incompressible Navier-Stokes equations were performed using a spectral finite element code NEKTON [17]. The computational domain is presented in Figure 2. In this figure the finite elements used in the simulations are shown. Within every element a solution based on 9th order polynomial is calculated.

The simulations were carried out with an equilateral triangular bluff body with blockage  $d/L_y = 1/15$  ( $d$  is the base of the triangle and  $L_y$  is the transverse extension of the domain of numerical simulations) and slip boundary conditions at the horizontal boundaries of the domain. The base of the triangle is perpendicular to the flow and the apex points upstream. The aspect ratio of the computational domain  $L_x/L_y = 3$ , where  $L_x$  is the longitudinal extension of the domain. A constant velocity  $U$  was introduced at the inflow and the Reynolds number

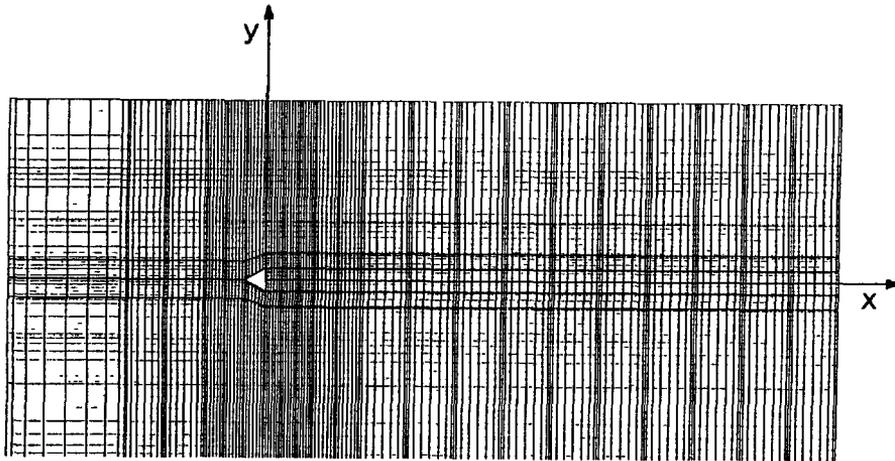


Fig. 2. — Computational domain used in numerical simulations.

was calculated using  $U$  and the base of the triangle  $d$ . The details of the numerical method can be found in reference [2]. The critical Reynolds number for the onset of the oscillations was found to be  $R_c = 36.2$  and is employed throughout the present study. We obtain the whole velocity field and in particular the  $V_x$ , component which is compared with the experiments.

The envelope of the global mode is determined as peak to peak amplitude of the oscillations.

#### 4. Results

As mentioned in the Introduction, the fundamental mode of the longitudinal component  $V_x$  of the oscillating velocity is antisymmetric with respect to the central line and consequently its maximum lies off this line. Therefore in order to establish a scaling law based on the maximum amplitude of  $V_x$ , the maximum of the oscillations has to be located in the  $x - y$  plane. For this purpose experimental measurements of  $V_x$  were performed in the region of  $0.7 d < x < 25 d$  and  $0 < y < 2.75 d$  for  $Re = 1.6 R_c$  and the contour lines of constant amplitude of the oscillations are shown in Figure 3a. No modal decomposition has been attempted here. For this Reynolds number the maximum of the global mode is situated at  $X_{max} = 7 d$  and  $Y_{max} = 1.25 d$ . The shape of the contour lines is elongated and reflects the non parallel aspect of the flow downstream of the shedding body. In the area of the flow field studied, the position of the maximum of the global mode  $X_{max}$  for consecutive cuts of constant  $y$  is a linear function of  $y$  as shown in Figure 3b.

As has been demonstrated previously (see Refs. [1] and [2]) the amplitude of the global modes  $A_{max}$  grows with increasing Reynolds number, while its position  $X_{max}$  moves closer to the shedding body. The same behavior can be observed for the envelope of  $V_x$ , as shown in Figure 4a (experimental results) and b (numerical simulations). In this figure the global modes for constant  $y$  and various Reynolds numbers are presented.

From the elongated shape of the contour lines in Figure 3a and from the monotone dependence of  $X_{max}$  as function of  $y$  presented in Figure 3b, it could be expected that the maximum of the global mode of  $V_x$  will move further away in  $y$  direction from the center line while moving further downstream in  $x$  direction with Reynolds number approaching  $R_c$ . Surprisingly this is not the case. In both numerical simulations and experiments we have observed that the

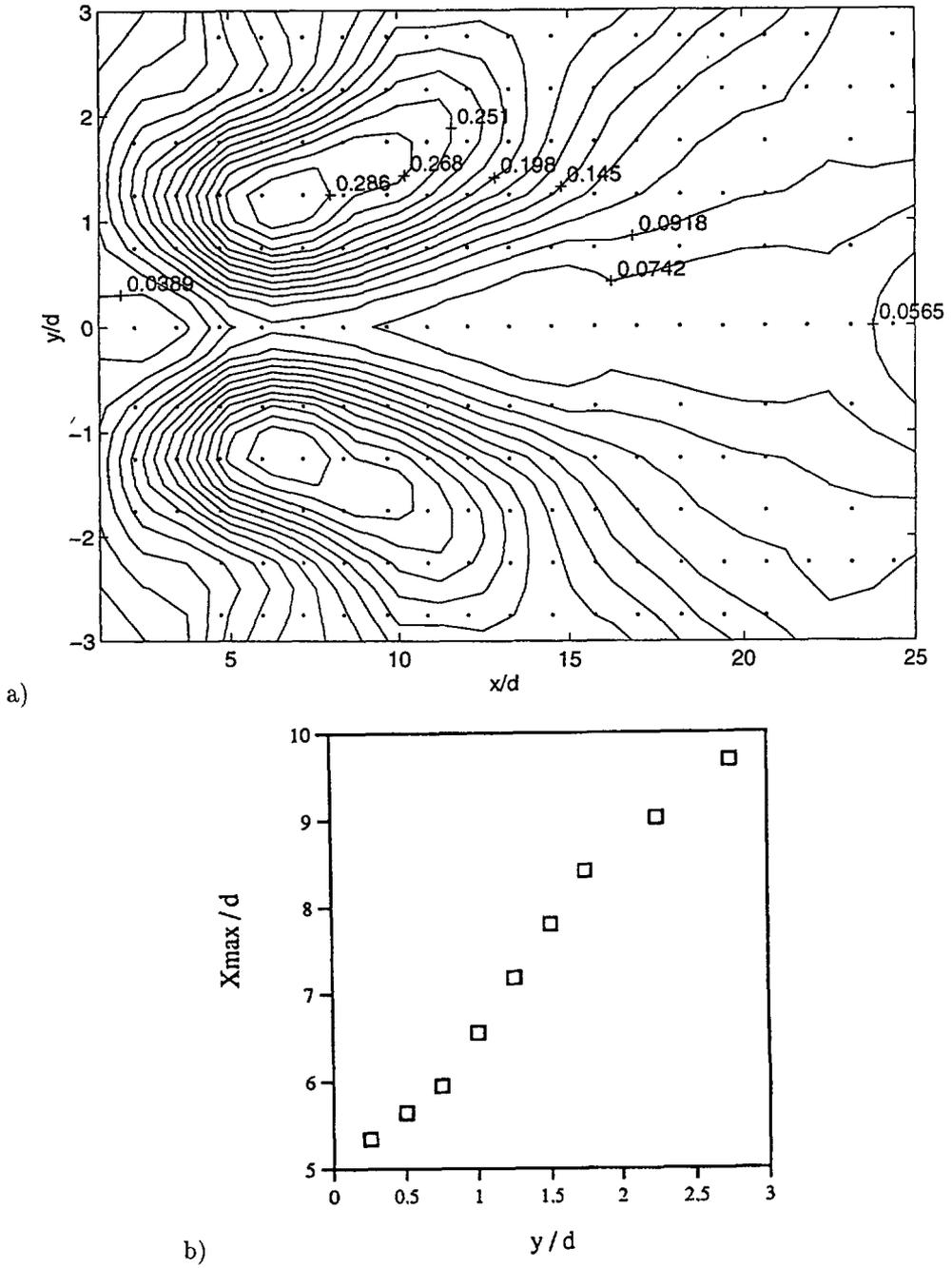
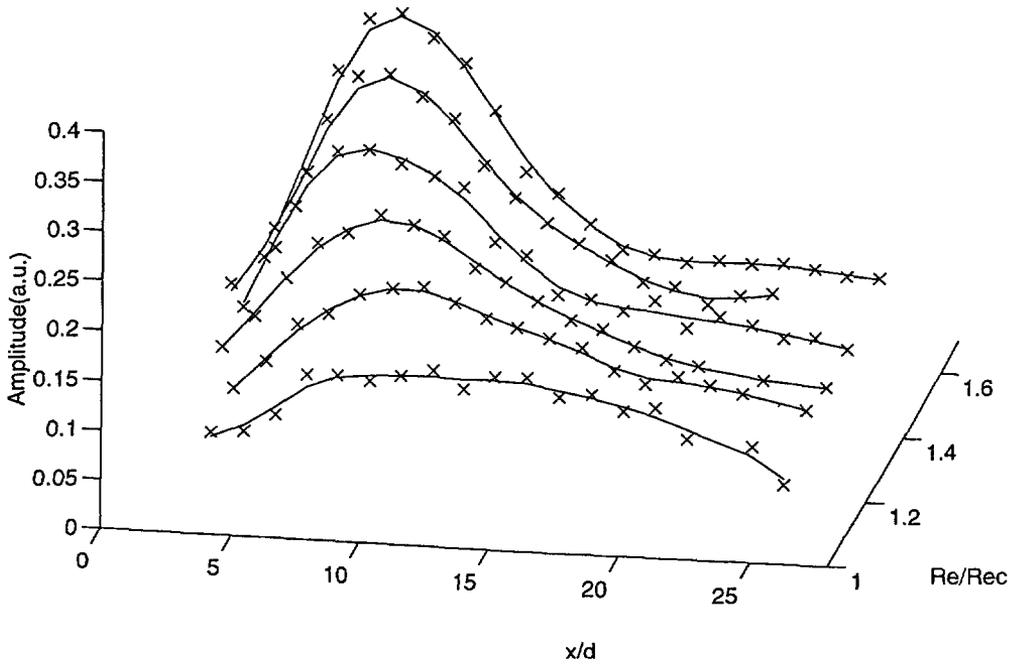
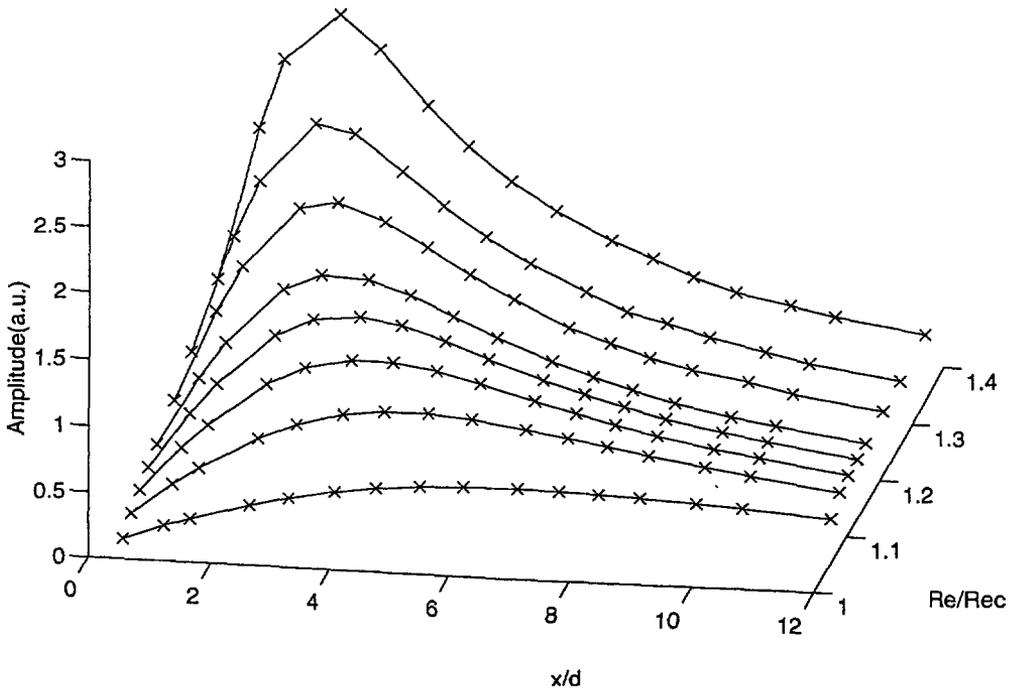


Fig. 3. — a) Contour lines of constant  $V_x$  amplitude of oscillations (experimental results). b) Position of the maximum of the global mode  $X_{max}$  as a function of  $y$  for  $R = 1.6 R_c$ .



a)



b)

Fig. 4. — Global modes of  $V_x$  a) for  $y = Y_{max}$  (experimental results), b) for  $y = 0.5 d$  (numerical simulations).

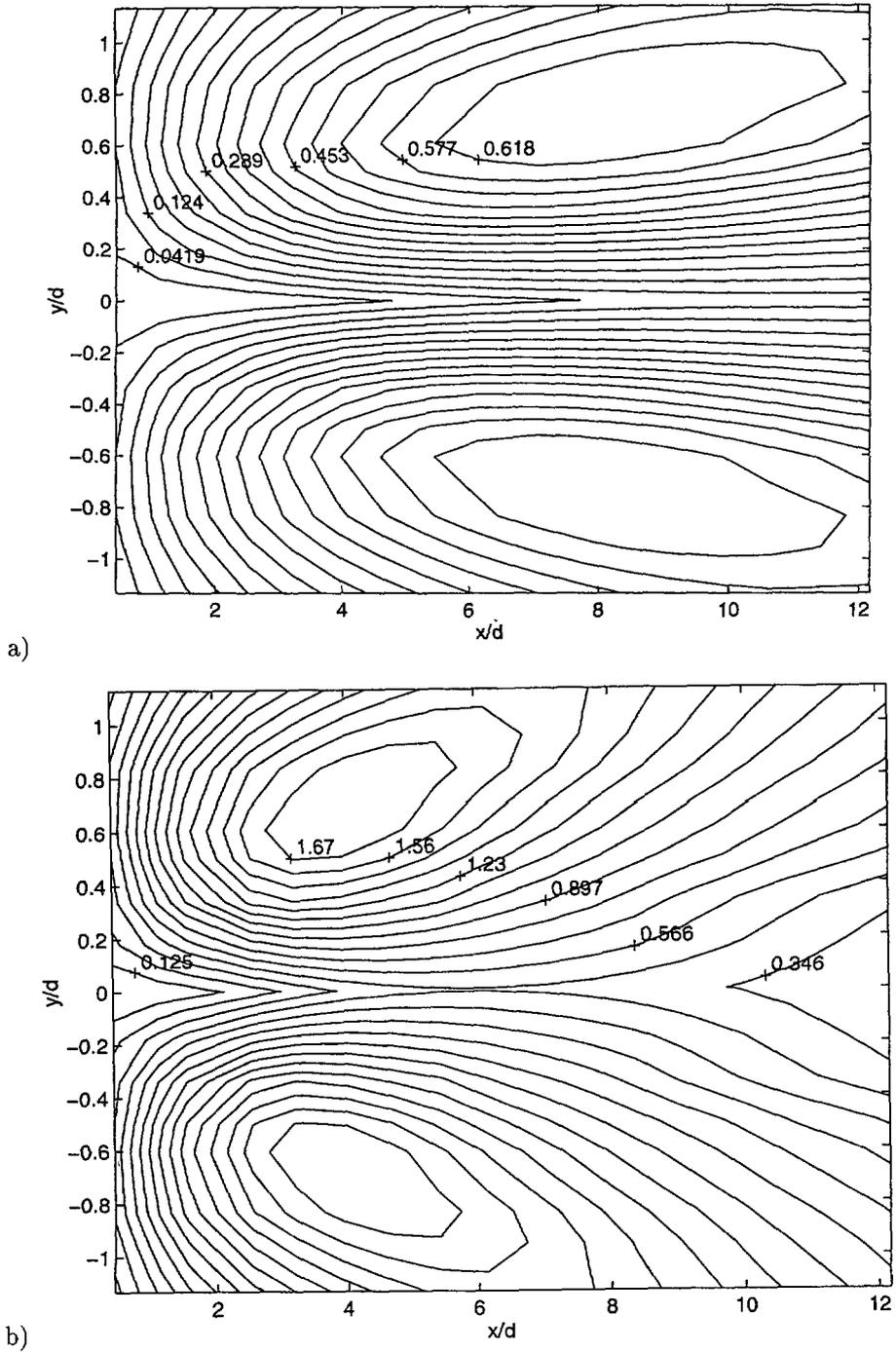


Fig. 5. — Contour lines of the global mode obtained by numerical simulation of the  $V_x$  for: a)  $Re = 1.017 Re_c$  and b)  $Re = 1.213 Re_c$

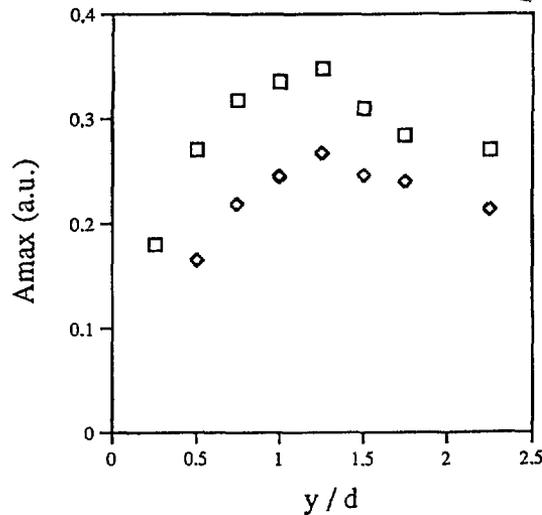


Fig. 6. — Amplitude of the global mode of  $V_x$  at  $x = X_{\max}$  as the function of  $y$  for  $Re = 1.41 R_c$  and  $1.6 R_c$ .

value of  $Y_{\max}$  stays relatively constant when the Reynolds number is changed. The contour lines of the global mode of the  $V_x$  obtained from numerical simulations for  $Re = 1.02 R_c$  and  $Re = 1.21 R_c$  are shown in Figures 5a and b respectively. It can be seen that while the  $X_{\max}$  changes from  $8.5 d$  for  $Re = 1.017 R_c$  to  $4 d$  for  $Re = 1.213 R_c$ ,  $Y_{\max}$  remains quite constant and equal to  $0.7 d$ . The negligible variation of  $Y_{\max}$  with the Reynolds number is confirmed by experiments. In Figure 6 the amplitude of the global mode at  $x = X_{\max}$  is shown as the function of  $y$  for two different Reynolds numbers:  $Re = 1.41 R_c$  and  $1.6 R_c$ . The value of  $Y_{\max}$  does not change appreciably in this range of Reynolds numbers and stays equal to  $1.25 d$ . The difference in the value of  $Y_{\max}$  between the trapeze used in experiments and triangular obstacle employed in numerical simulations is not surprising, the streamlines of the basic flow behind the bluff body being quite different in both cases. The flat face of the trapeze facing the flow forces the impinging streamlines to diverge from the parallel flow more than in the case of the flow around a triangle with the apex facing upstream. For comparison  $Y_{\max} = 0.5 d$  for a cylindrical obstacle was found in numerical simulations [10] for a Reynolds number of  $Re = 1.04 R_c$ . On the other hand  $Y_{\max} = 2 d$  for the cylindrical obstacle and various Reynolds numbers was observed in recent experiments [18]. The difference in the value of  $Y_{\max}$  between the experimental and numerical results rests to be elucidated.

Previous results presented in references [1] and [2] show that global modes of the transversal component  $V_y$  measured on the center line renormalized with  $A_{\max}$  and  $X_{\max}$  collapse to an universal curve. This means that the shape of the global modes remains the same when the Reynolds number is changed. Similar behavior can be found for global modes of the  $V_x$ , as shown in Figure 7. In Figure 7a measurements taken at  $y = Y_{\max}$  are presented, while in Figure 7b numerical data are compiled for  $y = 0.5 d < Y_{\max} = 0.7 d$ . It is interesting to note that the data for a constant value of  $y$  different from  $Y_{\max}$  also renormalize to one curve. This fact together with the negligible dependence of  $Y_{\max}$  on the Reynolds number allows construction of scaling laws with  $A_{\max}$  and  $X_{\max}$  obtained for constant values of  $y$ , for  $y$  not much different from  $Y_{\max}$ .

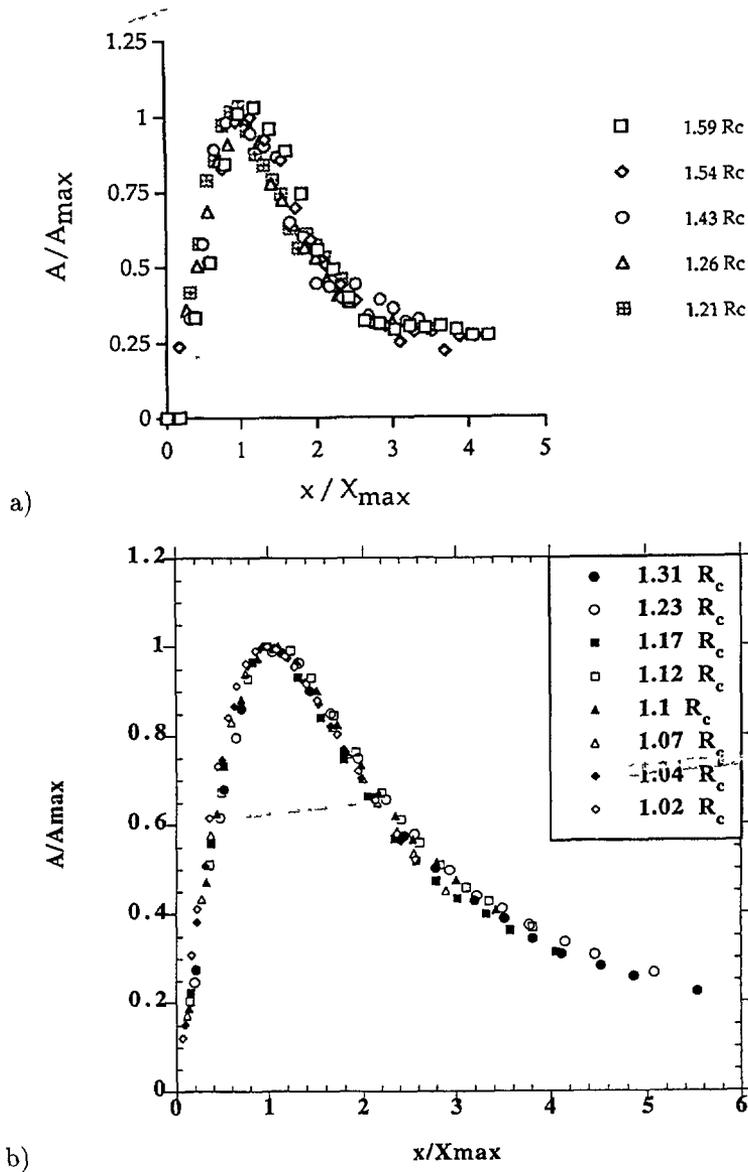


Fig. 7. — Global modes of  $V_z$  renormalized with  $A_{max}$  and  $X_{max}$ . a) for  $y = Y_{max}$  (experimental results), b) for  $y = 0.5 d < Y_{max} = 0.7 d$  ( numerical simulations).

In Figures 8a, 8b and 8c, 8d the log-log plots of  $A_{max}$  and  $X_{max}$  as function of the reduced Reynolds number are presented using experimental data (for  $y = Y_{max}$ ) and numerical data (for  $y = 0.5 d$ ) respectively. We observe that the laws  $A_{max} \sim (R - R_c)^{1/2}$  and  $X_{max} \sim (R - R_c)^{-1/2}$  can be fitted in both cases. We note however, that in the case of numerical simulations the value of  $X_{max}$  is not large enough for the lowest value of the Reynolds number. This can be attributed to finite size effects due to the limited size of the computational domain. Both scaling laws are consistent with Landau-Ginzburg model, if  $X_{max}$  is interpreted as a correlation length [19]. However, the authors are aware of the fact that this equation was not applied to flows with spatial dependence in the streamwise direction.

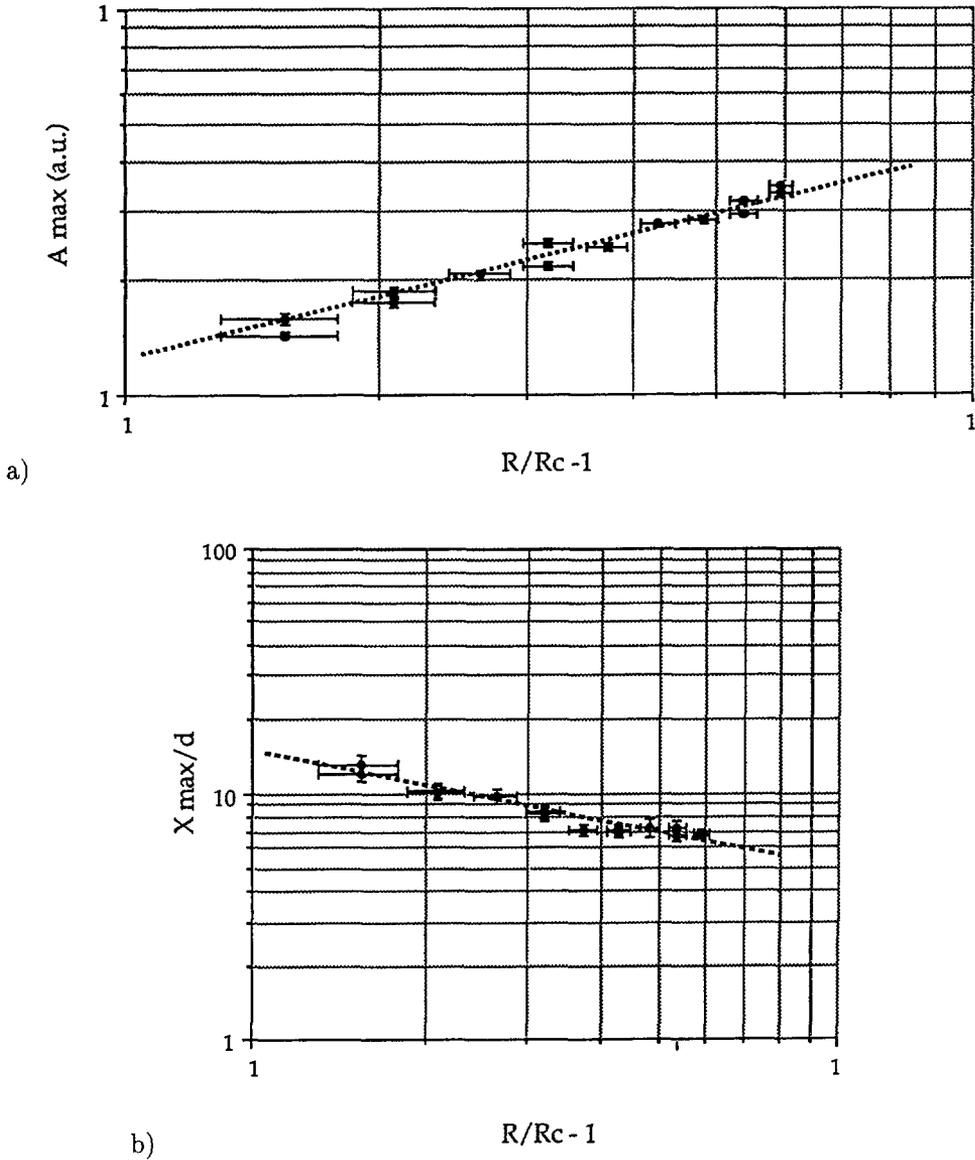


Fig. 8. — log-log plots of  $A_{max}$  and  $X_{max}$  as function of the reduced Reynolds number: (a,b) experimental results, (c,d) numerical simulations.

An interesting feature is observed in Figures 9a and b where it is shown that the ratio of  $A_{max}$  to  $X_{max}$  is a linear function of the Reynolds number for the experimental and numerical data respectively. On the other hand the renormalized global modes presented in Figure 7 grow linearly with  $X_{max}$  for  $x < X_{max}$ . This means that the angle of the initial spatial growth of the global modes shown in Figure 4 depends linearly on the Reynolds number.

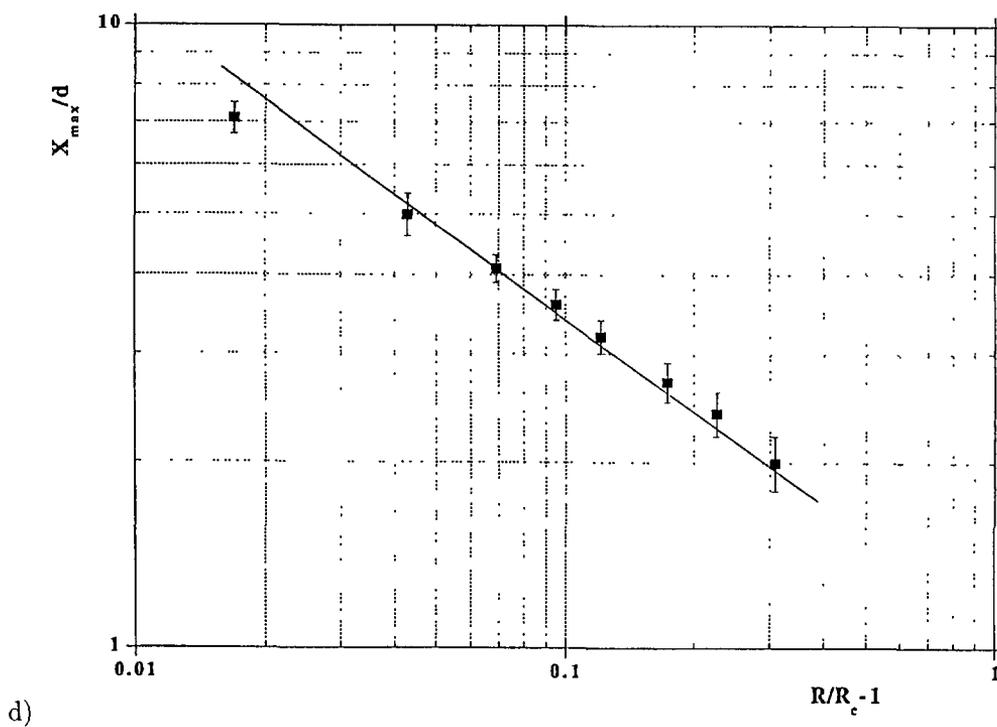
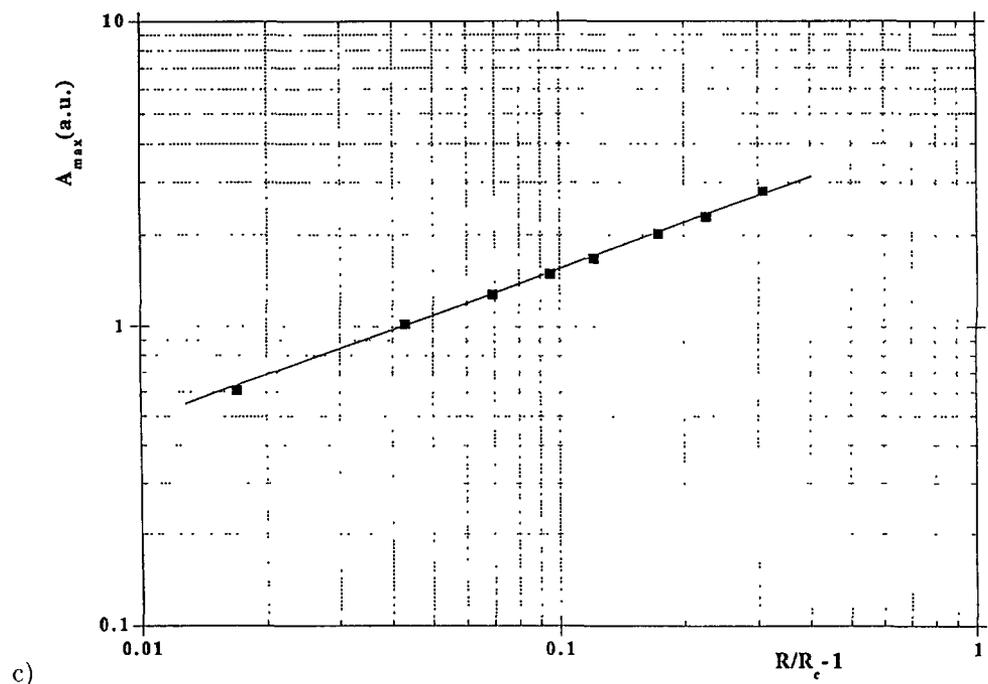


Fig. 8. — (Continued.)

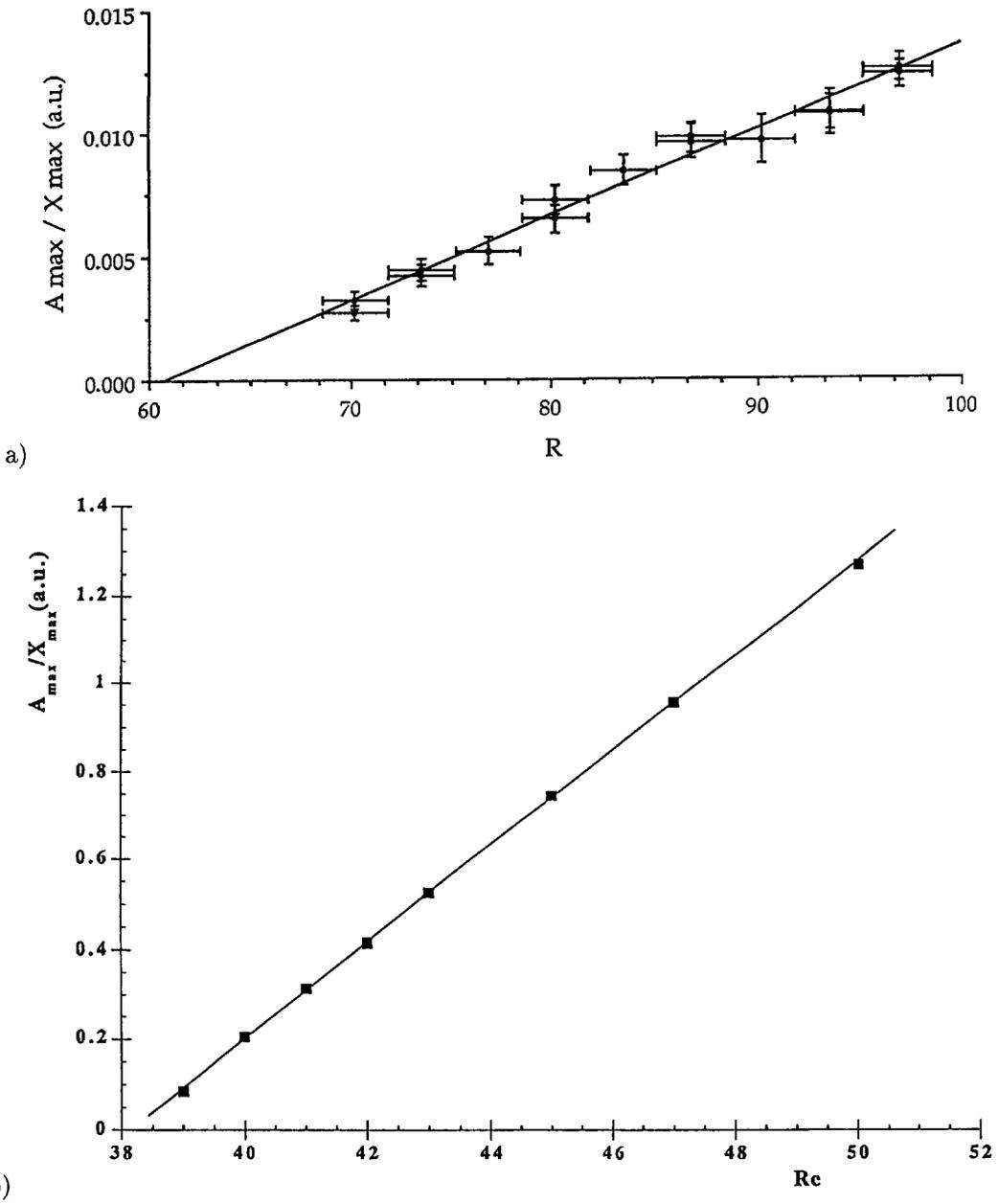


Fig. 9. —  $A_{max}/X_{max}$  as function of the Reynolds number: a) experimental results, b) numerical simulations.

### 5. Conclusions

We have characterized experimentally and numerically the two-dimensional structure of the global mode of the longitudinal component of the velocity in the wake flow. We have performed measurements in a low velocity water tunnel and carried out numerical simulations of the

longitudinal component  $V_x$  of the oscillating velocity in the full region of the flow where regular vortex shedding exists.

We did not attempt modal decomposition of the obtained oscillation amplitudes. This means that all the temporal harmonics are present in the envelope of the global mode of  $V_x$ . However, the scaling laws derived from our results apply effectively only to the fundamental mode. As shown in references [2] and [10], the position of  $A_{\max}$  of the second harmonic of  $V_x$  (oscillating with double frequency  $2f_o$ ) is located on the central line and further away in the wake than the position of the maximum of the fundamental mode (oscillating with  $f_o$ ). Moreover, the second harmonic, being already weaker than the fundamental mode by its nature, has a local minimum around the maximum of the fundamental. This means that its contribution to the global mode at  $X_{\max}$  and  $Y_{\max}$  is negligible. Since the third and higher harmonics are very small and can be neglected for practical purposes [13], the by far dominant contribution to the global mode around  $(X_{\max}, Y_{\max})$  comes from the fundamental mode.

It has been found in this study that the position  $(X_{\max}, Y_{\max})$  of the maximum of the global mode of  $V_x$  moves very little in the transverse  $y$  direction away from the central line. In fact we were able to assume that  $Y_{\max}$  stays constant in the range of Reynolds numbers studied. However, the value of  $Y_{\max}$  might change for larger Reynolds numbers. Dušek [10] performed modal decomposition of the wake flow past a cylinder and found that between  $R = 1.04 R_c$  and  $R = 2.17 R_c$ , the value  $Y_{\max}$  of the fundamental mode changes from  $0.5 d$  to  $0.75 d$ . It is of practical importance, however, that the position of  $Y_{\max}$  stays relatively constant in the range of the applicability of the scaling laws. This reduces considerably the number of measurements to be taken in order to establish the scaling law. It is enough to measure the amplitude of the global mode along a line of constant  $y$  approximately equal to  $Y_{\max}$  and not in the whole  $x - y$  plane.

The scaling laws for the amplitude of the global mode  $A_{\max}$  of  $V_x$  established in this study are consistent with a Landau model in a range of Reynolds numbers larger than in previous studies if the pertinent order parameter of the transition is the amplitude  $A_{\max}$  of the global mode (the position of this maximum is Reynolds number dependent) and not a local measurement of the amplitude  $A(x)$  (for comparison with previous studies, see Ref. [2]).

From the measurements of the  $V_x$  component of the velocity presented here (which is a pertinent order parameter) we can now definitely confirm the value  $1/2$  for the critical exponent of the  $A_{\max}$  as a function of the relative Reynolds number.

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