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## Power Fluctuations in Turbulent Swirling Flows

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**Abstract.** — We report an experimental study of the temporal fluctuations of the total power injected in a swirling flow generated in the gap between two coaxial disks, counter-rotating at a fixed velocity. We use either a free geometry in air or a shrouded one, *i.e.* with the disks enclosed in a cylindrical vessel. We show that the fluctuations of the power injection are large in both cases (rms amplitude up to 10% of the mean value), but that their statistical properties are qualitatively different. When the flow is confined in a cylindrical vessel, we find that the PDF of the power fluctuations is strongly non-Gaussian and we study the correlations between the power fluctuations and the pressure field at the boundary.

### 1. The Problem of Temporal Fluctuations of Spatially Averaged Quantities

The drag on a solid body moving in a turbulent flow, the torque on a disk or on a fan rotating at high speed in a fluid, or more generally the total power required by a turbulent flow, are quantities of great practical importance in fluid mechanics, and there exist a considerable amount of data on the behaviour of their mean values as functions of the flow Reynolds number [1]. However, the temporal fluctuations of these global, *i.e.* spatially averaged quantities, seem to have been overlooked. We consider this problem by performing measurements of power fluctuations in turbulent swirling flows generated in the gap between two counter-rotating co-axial disks [2].

We will first use the rotating disk flow example to set up the problem we are addressing. Consider a disk of radius  $R$  rotating about its axis with a uniform angular velocity  $\Omega$  in a fluid of density  $\rho$  and kinematic viscosity  $\nu$ . From dimensional analysis, we get

$$\bar{\Gamma} = \rho R^5 \Omega^2 f(\text{Re}), \quad (1)$$

for the average torque applied on the disk by the surrounding fluid, or equivalently,

$$\bar{P} = \bar{\Gamma} \Omega = \rho R^5 \Omega^3 f(\text{Re}), \quad (2)$$

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for the average power required to maintain the rotation rate  $\Omega$  constant. At this stage,  $f(\text{Re})$  is an arbitrary function of the Reynolds number,  $\text{Re} = R^2\Omega/\nu$ . Its exact expression is known only in the laminar regime where  $f(\text{Re}) \propto 1/\sqrt{\text{Re}}$ . It is strongly believed, and this is an important assumption of the phenomenology of fully developed turbulence, that the transport properties of the flow behave in a well defined way in the limit  $\text{Re} \rightarrow \infty$ ; the simplest choice  $f(\text{Re}) \rightarrow \text{constant}$  is supported by empirical evidence, in the case of a rotating fan or a disk fitted with blades for instance, and leads to a torque proportional to the square of the rotation rate, a behaviour known as the "centrifugal torque" in the engineering literature, and similar to the related law for the drag being proportional to the square of the velocity. The situation is much more complex for a flat disk for which  $f(\text{Re})$  decreases when  $\text{Re}$  is increased in the turbulent regime until the surface roughness becomes important [3].

Note that equation (2) also reads

$$\overline{\langle \epsilon_I \rangle} = R^2 \Omega^3 f(\text{Re}) = \frac{V^3}{R} f(\text{Re}), \quad (3)$$

where  $\overline{\langle \epsilon_I \rangle}$  is the power injected per unit mass of the fluid and  $V = R\Omega$  is the characteristic large scale velocity ( $\langle \cdot \rangle$  stands for the average in space and an overbar for the temporal average). In fully developed turbulent flows,  $V$  is usually replaced by  $u_{\text{rms}}$ , the rms value of turbulent velocity fluctuations, in the definitions of  $\overline{\langle \epsilon_I \rangle}$  and  $\text{Re}$ . This is an important distinction for some flow geometries, flows along a smooth plate or disk for instance, for which, as mentioned above,  $f(\text{Re})$  decreases until the surface roughness becomes important [3]; in these regimes, we do not expect  $u_{\text{rms}}$  to be proportional to  $V$ . Anyway, it is believed that at least for homogeneous isotropic turbulence [4, 5],  $\overline{\langle \epsilon_I \rangle} \propto u_{\text{rms}}^3/R$  when  $\text{Re} \rightarrow \infty$ . This quantity plays a central rôle in the phenomenology of turbulence, which usually considers the properties of small scale quantities such as the local viscous dissipation,  $\epsilon_d(\mathbf{r}, t)$ ,

$$\epsilon_d(\mathbf{r}, t) = \frac{\nu}{2} \sigma_{ij} \sigma_{ij} = \frac{\nu}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2, \quad (4)$$

or the velocity increments,  $\delta u_i \equiv u_i(\mathbf{r}) - u_i(\mathbf{r} + \mathbf{d})$ , and studies their statistical characteristics for a fixed value of  $\overline{\langle \epsilon_I \rangle}$ .

We address here the problem of the temporal fluctuations of  $\langle \epsilon_I \rangle$ . The evolution equation for the kinetic energy  $E$  per unit mass is easily obtained by multiplying the Navier-Stokes equation by the velocity  $u_i$ , and taking the average over the flow volume. One obtains

$$\dot{E} = \langle \epsilon_I \rangle - \langle \epsilon_d \rangle, \quad (5)$$

where

$$\mathcal{U} \langle \epsilon_I \rangle(t) = - \int_S \pi u_i n_i dS + \nu \int_S u_i \sigma_{ij} n_j dS, \quad (6)$$

is the rate of working of the forces exerted on the surface  $S$  enclosing the fluid volume  $\mathcal{U}$ , with

$$\pi = \frac{p}{\rho} + \frac{u^2}{2}, \quad \sigma_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$$

Both the pressure  $p$  and  $\sigma_{ij}$  are strongly fluctuating quantities at the solid boundary  $S$ .

In any experiment, the three quantities in equation (5) fluctuate in time. This is usually overlooked since the equivalence between spatial and temporal averages is assumed ("statistical stationarity"). Although it is obvious that the mean values in time are equal,  $\overline{\langle \epsilon_I \rangle} = \overline{\langle \epsilon_d \rangle}$ , large temporal fluctuations for  $\langle \epsilon_I \rangle$  should be expected in any realistic flow.

We thus expect a large scale intermittency related to temporal fluctuations of characteristic size  $\delta\epsilon_I$  of the power injected per unit mass in the flow, and we address the following problems:

- (i) what is the probability density function (PDF) of  $\langle\epsilon_I\rangle(t)$ ?
- (ii) What is the behaviour of  $\delta\epsilon_I/\langle\epsilon_I\rangle$  as  $Re \rightarrow \infty$ ?
- (iii) Do the fluctuations of  $\langle\epsilon_I\rangle(t)$  affect the dissipative range, *i.e.*  $\epsilon_d(r,t)$  or the measured small scale intermittency?

The power measurements in a von Karman flow generated in the gap between two counter-rotating disks presented below, are a first step toward the study of the above questions.

## 2. Experimental Set-Up

We use two aluminium horizontal coaxial disks of radius  $R = 10$  cm, a variable distance  $H$  apart. The disks are fitted with a set of 8 vertical blades, perpendicular to the disks surfaces, with height  $h_b = 2$  cm and thickness 0.5 cm, in order to increase entrainment. The disks are driven by independent 450 Watt d.c. motors, the rotation frequencies of which are adjustable from 0 to 45 Hz and controled by a feed-back loop. For some measurements (specified in the text), light wooden disks with 8 blades have been fitted directly on the motors' shafts in order to reduce inertia effect and thus increase the high frequency cut-off of the system. Air is the working fluid, in a free geometry [6] or in a shrouded one, *i.e.* the disks being enclosed in a cylinder 23.2 cm in diameter. The largest achieved values of the Reynolds number are thus about  $10^5$ .

Velocity measurements are performed using TSI subminiature hot-film probes with a sensing element  $10\ \mu\text{m}$  thick and 1 mm long, as already discussed in earlier studies [7,8].

Pressure fluctuations are measured by a piezoelectric transducer PCB103A02, mounted flush with the lateral wall. Its active diameter is 2.1 mm, its low frequency cut-off at  $-5\%$  is 0.05 Hz and its rise-time is  $25\ \mu\text{s}$  [9,10].

For the measurements of the fluctuations of the power consumption by the turbulent flow, it is important to discuss first the various possible parameters which one could maintain at a constant value:

- (i) a first possibility is to keep the rotation rate of the disks, *i.e.* the integral Reynolds number, constant.
- (ii) Another way of forcing the flow is to keep constant the torque applied on the disks. The rotation rate, *i.e.* the integral Reynolds number, then becomes a fluctuating quantity.
- (iii) A third, more complex possibility is to keep constant the injected power itself. Then, one has to look at fluctuations of other global quantities such as the torque or the rotation rate. The last two possibilities will be investigated in a forthcoming work, and the effects of different large scale forcings of the flow will be studied.

We choose here to keep the rotation rates of the disks constant through a feedback loop. We measure the power required to keep the disks rotating at given angular speeds. The current in the stator is fixed while the voltage and current in the rotor are varied and measured for each rotation frequency. We have also repeated the measurements in the absence of disks and blades to obtain the power losses in the bearings and we have measured the voltage drop in the rotor to obtain the losses in the windings. These values are subtracted from the raw power measurements to obtain the power necessary to maintain the flow alone. Figure 1a displays the direct time recording of the power injection into the flow when the wooden disks are used; the rms fluctuations are roughly 10% of the mean, eventhough the Reynolds number is constant. The corresponding power spectrum is shown in Figure 1b; it is flat at low frequencies and displays a  $f^{-4}$  cut-off, related to the high frequency filtering effect due to the inertia of the disks. This clearly leads to an underestimation of the high frequency power fluctuations. Note

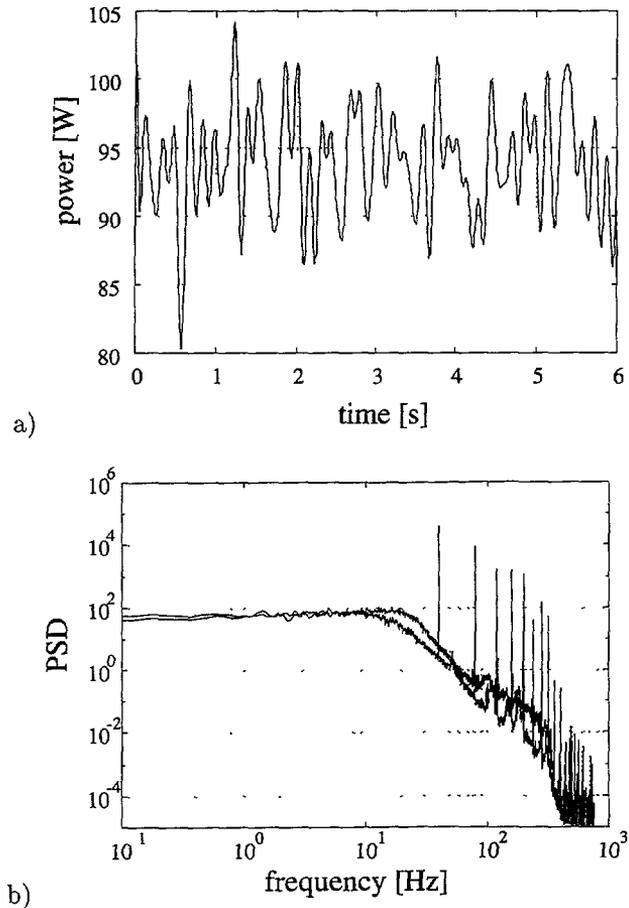


Fig. 1. — a) Time recording of the power consumption of the motors driving wooden disks in the shrouded geometry:  $f_1 = f_2 = 40$  Hz,  $H = 27.5$  cm. b) Frequency spectrum of the power fluctuations. The spectrum is flat up to 20 Hz above which the inertia of the disks generates a  $f^{-4}$  fall-off. Note that the rotation frequency and its harmonics are clearly visible.

that in the case of wooden disks, we have no filtering nearly up to the rotation frequency, so that fluctuations up to the integral timescale are recorded.

### 3. Power Fluctuations in the Free Geometry

3.1. DISKS WITHOUT RIMS. — We first present our results in the case of free disks with blades but without rims. The PDFs of the power fluctuations  $P_{1,2}$  of both disks are displayed in Figure 2a (solid and dashed lines) together with the PDF of the total power consumption,  $P = P_1 + P_2$  (dash-dotted line). The PDFs related to each disk are strongly asymmetric with a tail toward low powers. However, the PDF of the total power consumption,  $P$ , is almost Gaussian. This remarkable effect is due to a strong correlation between  $P_1$  and  $P_2$  as shown by the joint PDF displayed in Figure 2b; one clearly notices that the large values of  $P_1$  are related to the small ones of  $P_2$ , and *vice-versa*. An inspection of the flow shows that the large intensity shearing zone, roughly in the mid-plane between the two disks, undergoes coherent

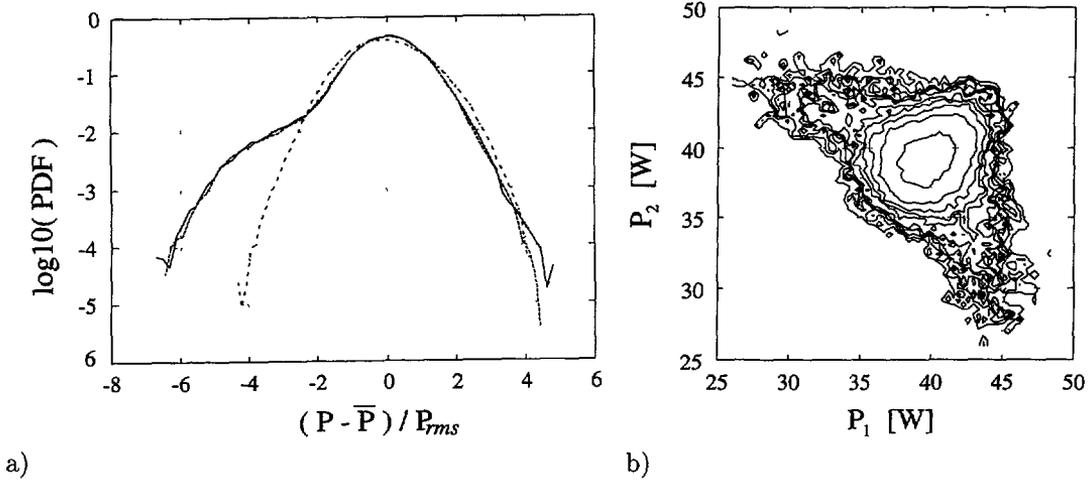


Fig. 2. — a) PDF of the power fluctuations  $P_{1,2}$  of each motors (solid and dashed lines) and PDF of the total power fluctuations  $P$  (dash-dotted line).  $f_1 = f_2 = 34$  Hz,  $H = 40$  cm, disks with 8 blades without rims. b): Joint PDF between  $P_1$  and  $P_2$ . The iso-probability lines are plotted for the following values: [0.0001, 0.0003, 0.0005, 0.0007, 0.0010, 0.0020, 0.0030, 0.0050, 0.0100, 0.0300].

vertical oscillations at about 1 Hz and is thus alternatively closer to one of the two disks. The coherence between  $P_1$  and  $P_2$  is 0.6 at 1 Hz but falls to less than 0.05 on each side of the oscillation frequency.  $P_1$  and  $P_2$  oscillate out of phase, thus their corresponding low frequency peak cancel each other in the frequency spectrum of the total power  $P$ . The above measurements show that the dynamics of this large scale structure generate a non-Gaussian behaviour of the PDFs of  $P_1$  or  $P_2$  considered alone, but the total power fluctuations are almost Gaussian.

3.2. POWER-VORTICITY CORRELATION. — Noting that equation (5) could also be written in the form

$$\mathcal{U} \dot{E} = - \int_S \pi u_i n_i dS + \nu \int_S (\mathbf{u} \times \boldsymbol{\omega}) \cdot \mathbf{n} dS - \nu \int_U \omega^2 d^3x, \quad (7)$$

where  $\boldsymbol{\omega}$  is the vorticity, it is tempting to compute the cross-correlation between the large scale vorticity and the fluctuations of power. The dynamics of the horizontal vorticity of the mid-plane shearing zone can be probed using ultrasound scattering [11,12]. The flow is probed with an ultrasonic beam at frequency 20 kHz generated by a Sell-type acoustic transducer [13] of size  $16 \times 16$  cm. The scattered wave at angle  $60^\circ$  is detected by an identical transducer. The scattering volume is about 25% of the total volume within the two disks. The scattering plane being vertical, it is an horizontal component of the vorticity, perpendicular to the scattering plane, which is probed. More precisely, in the Born approximation, the Fourier transform of the scattered pressure field is proportional to the Fourier transform of this vorticity component at the scattering wavenumber  $q = 3.7 \text{ cm}^{-1}$  [14]. The cross-correlation between the scattered pressure and the power fluctuations is displayed in Figure 3. As expected, we observe a correlation which shows that the dynamics of the large scale vorticity sheet in the vicinity of the mid-plane affects the power consumption. The time delay for which the cross-correlation is maximum does not provide any quantitative information, mostly because of the frequency filtering due to the inertia of the disks.

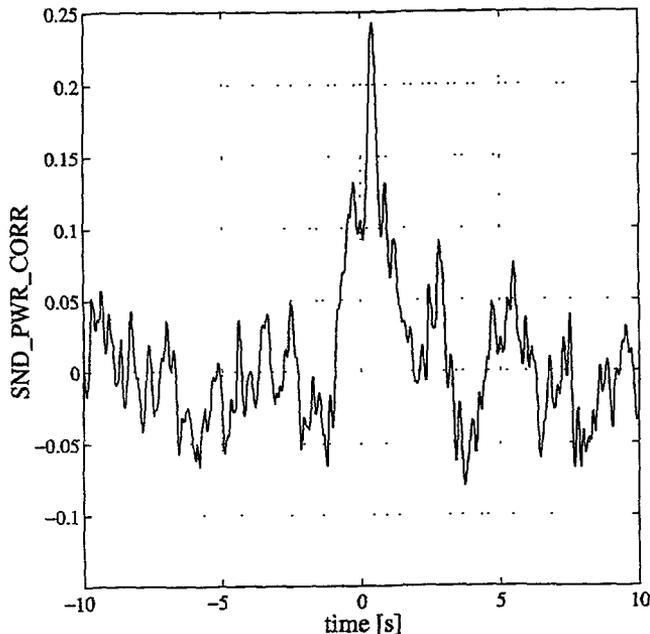


Fig. 3. — Cross correlation between the fluctuations of power and horizontal large-scale vorticity measured *via* ultrasound scattering.

3.3. EFFECT OF RIMS FITTED ON THE DISKS. — We observe that the above coherent oscillation is prevented when the disks are fitted with rims 3 cm in height. Indeed, the power spectra of  $P_{1,2}$  do not show any characteristic frequency as above. Strikingly, the PDFs of  $P_1$  or  $P_2$  are then Gaussian (see Fig. 4a);  $P_1$  and  $P_2$  do not display any anticorrelation (see Fig. 4b) such as in Section 3.1.

#### 4. Power Fluctuations in the Confined Geometry

In the free geometry, we observe that the lateral extension of the flow is larger than the diameter of the disks and strongly fluctuates [8]. We thus consider the effect of confining the flow and enclose the disks in a cylinder 23.2 cm in diameter. The PDFs of  $P_{1,2}$  and  $P$  are superposable as in the free geometry with rims, but they are strongly asymmetric and skewed toward low values of the power, although the flow does not involve any large scale oscillation. The shape of the PDFs of power fluctuations looks like the one of the pressure PDF when measured far from the blades (see below). Thus, confining the von Karman flow generates a non Gaussian behaviour of power fluctuations. More precisely, the values of the skewness and the flatness are respectively,  $-0.72$  and  $4.10$  in the confined configuration, compared to  $0.04$  and  $2.94$  in the free geometry with disks fitted with rims. When the flow is confined, the relative rms fluctuations with respect to the mean power increases from 3% to almost 10%.

4.1. POWER FLUCTUATIONS VERSUS THE REYNOLDS NUMBER. — When the Reynolds number is varied, the PDFs of  $P/\Omega^3$  do not collapse on a single curve, but if the reduced variable,  $\Pi = (P - \bar{P})/P_{\text{rms}}$  is used, all PDFs obtained for different Reynolds numbers in the rotation frequency range [25, 45] Hz do collapse (see Fig. 5). This shows that, contrary to  $\bar{P}$  [15],

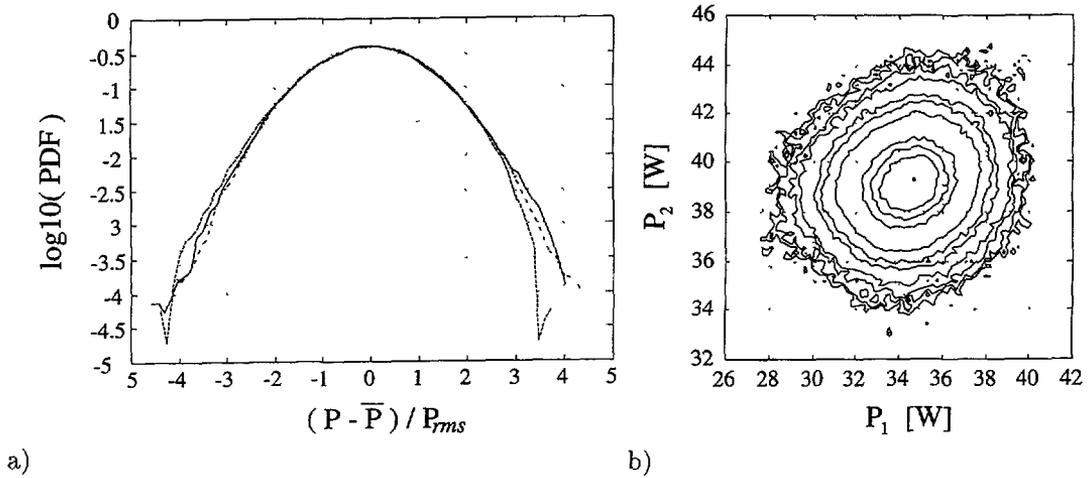


Fig. 4. — a): PDF of the power fluctuations  $P_{1,2}$  of each motors (solid and dashed lines) and PDF of the total power fluctuations  $P$  (dash-dotted line).  $f_1 = f_2 = 40$  Hz,  $H = 40$  cm, disks with 8 blades with rims. b): Joint PDF between  $P_1$  and  $P_2$ . The iso-probability lines are plotted for the following values: [0.0001, 0.0002, 0.0005, 0.0010, 0.0030, 0.0050, 0.0700, 0.0100, 0.0300, 0.0400, 0.0500].

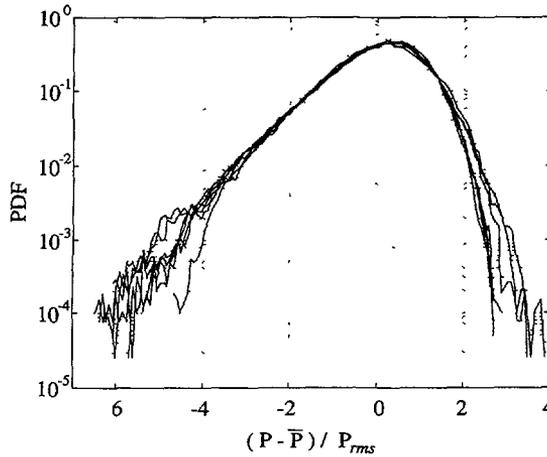


Fig. 5. — PDF of the total power fluctuations in the shrouded geometry.  $f_1 = f_2 = 25, 30, 35, 40, 45$  Hz,  $H = 27.5$  cm. When the reduced variable  $\Pi = (P - \bar{P}) / P_{rms}$  is considered, the PDFs obtained for different rotation frequencies collapse on a a single curve.

$P_{rms}$  does not scale like  $\Omega^3$ . However, this Reynolds number is too small to detect an asymptotic behaviour of  $P_{rms} / \bar{P}$ .

Thus, this range is too small to be able to answer the question (ii) we addressed above about the behaviour of  $\delta\epsilon_1 / \langle \epsilon_1 \rangle$  as  $\text{Re} \rightarrow \infty$ . However, even with this limited range, the following behaviour can be ruled out: a possible assumption is that  $\delta\epsilon_1 / \langle \epsilon_1 \rangle$  decreases with increasing Reynolds number because of the increase in weight of small scale motions which smooth out more and more efficiently the fluctuations  $\delta\epsilon_1$ . The most naive prediction would be to assume that, like in statistical mechanics,  $\delta\epsilon_1 / \langle \epsilon_1 \rangle$  is inversely proportional to the square root of the

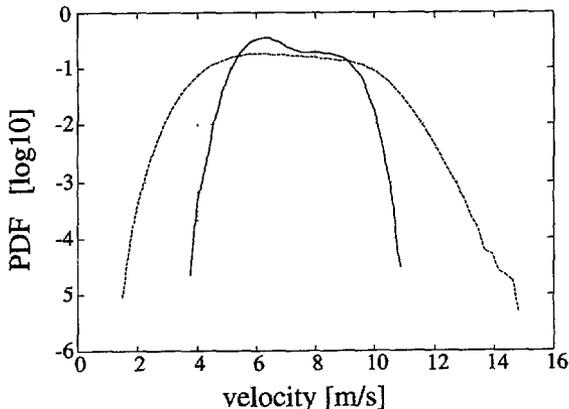


Fig. 6. — PDF of the coarse-grained velocity  $U_T$  for different values of the integration time  $T$ :  $T = 2\pi/\Omega$  (dashed line),  $T = 20\pi/\Omega$  (full line). The velocity  $u = \sqrt{u_r^2 + u_\theta^2}$  is measured in the mid-plane. The flat shape of the PDF is due to the large scale dynamics of the flow.

number of degrees of freedom, *i.e.* decreases like  $\text{Re}^{-9/8}$ . It is clear that, if relevant, such a mechanism would have produced a behaviour easily observed in our experiments.

Another way of thinking, pointed out by one of the referees, is to consider that the large scale flow is in a chaotic regime controlled by some eddy viscosity. The fluctuation of the input power is determined by the number  $N$  of degrees of freedom of this large scale chaotic flow. The scaling of  $N$  with the Reynolds number is difficult to evaluate. We have indeed tried to measure the dimension of the attractor corresponding to the power fluctuation signal as a function of the Reynolds number [16]. This dimension, larger than 7, is not measurable with conventional algorithms. Thus, the large scale flow cannot be described as a low dimensional chaotic system.

We believe that this smoothing of large scale quantities does not occur in most turbulent flows because they do not involve many independent integral lengths in the whole flow volume. This is clearly shown in the time domain by the following procedure: one defines a coarse-grained velocity as in reference [7],

$$U_T(t) \equiv \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} u(t') dt', \quad (8)$$

and compute the PDF of  $U_T$  for various values of the integration time  $T$ . The PDF of  $U_T$  for  $T_0 = 2\pi/\Omega$ , the integral timescale, and for  $T = 10T_0$  are displayed in Figure 6. We notice that the width of the PDF is changed by a factor less than  $\sqrt{10}$  when the velocity is coarse-grained on a 10 times longer duration. The variance of  $U_T$  decreases like  $1/\sqrt{T}$  only for  $T \gg 10T_0$ . This statistical behaviour is achieved for large  $T$  and this is compatible with the flat frequency spectrum of power fluctuations observed at low frequency (see Fig. 1). The fact that the PDF of velocity fluctuations is not significantly changed when the velocity is coarse-grained on the integral timescale is clearly connected with the large size of the fluctuations of global quantities observed in turbulence.

4.2. POWER-PRESSURE CORRELATIONS. — From equation (6) we expect correlations between power fluctuations and pressure. The pressure dependent term in the expression for  $\langle \epsilon_1 \rangle$  is non zero only on the blades because the velocity  $u_i$  is perpendicular to the normal  $n_i$  on the lateral boundaries or on smooth disks without blades. In the case of disks with blades, this term is

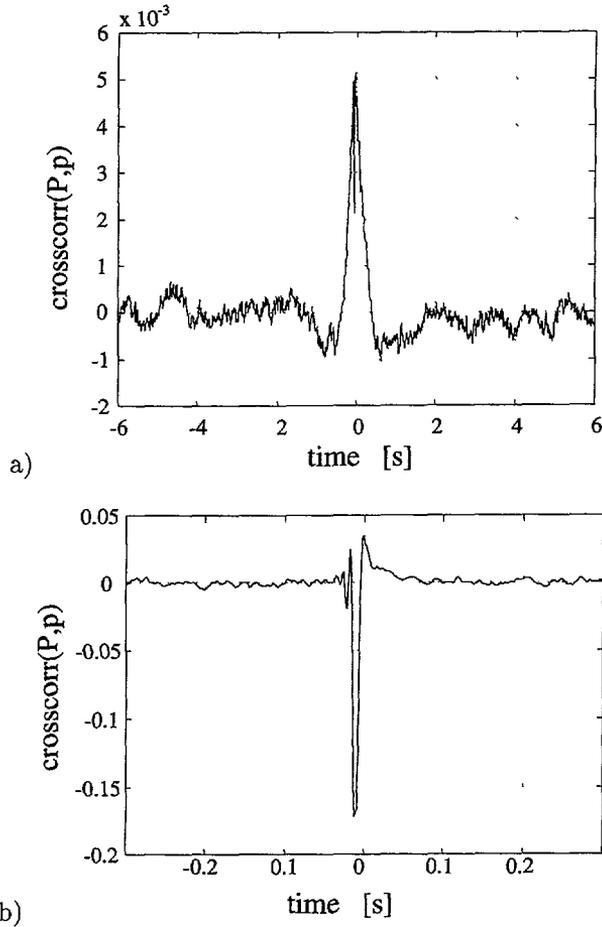


Fig. 7. — Cross-correlation between the fluctuations of the pressure,  $p$ , and the power,  $P$ , in the shrouded geometry. The pressure is measured with a transducer flush to the cylindrical wall enclosing the disks: a) transducer 3 cm above the mid-plane; b) transducer 4.5 cm above the lower disk. Note that the correlation is larger close to the disk as expected, but also that its sign is reversed.

the dominant contribution to  $\langle \epsilon_1 \rangle$ . We have measured the pressure 4.5 cm above the lower disk and 3 cm above the mid-plane. As expected, the cross-correlation between pressure and power fluctuations is non zero for some finite time lag, and the corresponding peak decreases when the distance from the disk increases (see Fig. 7). Although the correlation is rather small at large distance, we have checked that the correlation peak remains constant whereas the background noise decreases when the averaging time on which the cross-correlation is computed is increased. Again, the exact value of the time lag does not provide any information because of the response time of the mechanical device due to the inertia of the disks. Note that the sign of the correlation is reversed when the pressure is measured close to the blades. The shape of the pressure PDF also depends on the location of the transducer (see Fig. 8); when recorded in the vicinity of the mid-plane, it is strongly asymmetric with a stretched exponential tail toward low pressure due to vorticity filaments, as already observed in previous studies [9, 10, 17]. On the contrary, the pressure PDF measured in the vicinity of the blades (4.5 cm above the lower

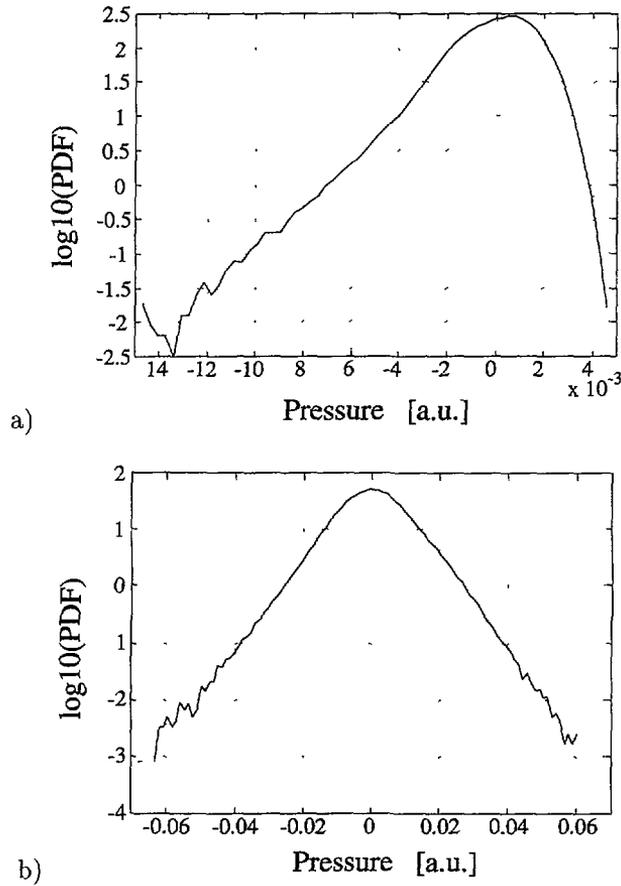


Fig. 8. — PDF of the pressure measured with a transducer flush to the cylindrical wall enclosing the disks: a) transducer 3 cm above the mid-plane; b) transducer 4.5 cm above the lower disk.

disk) is roughly symmetric with exponential tails on both sides. We believe that the high pressure part is due to stagnation points generated by the flow ejected by the disks toward the lateral boundary where the pressure transducer is fitted.

## 5. Concluding Remarks

We want to emphasize that the total power injection  $\langle \epsilon_I \rangle$  or dissipation  $\langle \epsilon_d \rangle$  in a turbulent flow are not constant in time, as often assumed in the phenomenology of turbulence, but display temporal fluctuations. Correspondingly, the mean kinetic energy per unit mass  $E$  also fluctuates according to equation (5)

$$\dot{E} = \langle \epsilon_I \rangle - \langle \epsilon_d \rangle. \quad (5)$$

It would be interesting to compare the PDFs of  $\langle \epsilon_I \rangle$  and  $\langle \epsilon_d \rangle$ . Although their mean values in time are obviously equal,  $\epsilon_d$  is a positive quantity whereas one could expect rare events with  $\langle \epsilon_I \rangle < 0$ , *i.e.* energy given back by the fluid to the driving device through the work of the forces exerted on the surface enclosing the fluid. Thus, one expects a wider distribution for  $\langle \epsilon_I \rangle$  and

we emphasize that viscous dissipation is not the only mechanism to remove instantaneously kinetic energy from the flow [8].

Although power fluctuations are clearly related to the geometry and the dynamics of the large scale flow, the effect of fluctuations of  $\langle \epsilon_1 \rangle$  on the small scale dynamics of turbulence is not clear, and the statistical properties of dissipation and velocity increments conditioned to  $\langle \epsilon_1 \rangle$  will be made to answer this question.

We only considered in this paper the problem of the fluctuations of  $\langle \epsilon_1 \rangle$  and showed the importance of the confinement of the flow. The way the flow is forced may also have some effect. In the case of a rotating flat disk, either term on the right hand side of equation (6) could be dominant depending on the magnitude of the roughness compared to the thickness of the boundary layer. When the roughness is important, and in particular in the presence of blades,  $\overline{\langle \epsilon_1 \rangle}$  is proportional to  $V^3/R$ . On the contrary, in the case of a smooth disk, only the second term on the right hand side of equation (6) contributes to the injection of energy, and  $f(\text{Re})$  in equation (3) is a decreasing function [3]. The influence of the roughness of the disks on the fluctuations of  $\langle \epsilon_1 \rangle$  will be investigated.

In the case considered in this paper, the flow is forced through moving boundaries whereas energy is dissipated in volume. In the case of a flow generated by a force per unit mass  $\mathbf{f}(\mathbf{r}, t)$ ,

$$\langle \epsilon_1 \rangle(t) = \langle \mathbf{f} \cdot \mathbf{u} \rangle, \quad (9)$$

so that power fluctuations depend on the fluctuations of the velocity itself (both in amplitude and direction) instead of the fluctuations of the velocity gradients or the pressure at the boundaries as in equation (6). It might be interesting to study the influence of a volumic forcing on the energy injection dynamics. Moreover, in a numerical simulation it could be possible to tune  $\mathbf{f}(t)$  in order to keep  $\langle \epsilon_1 \rangle$  constant and to study the resulting dynamics of the flow.

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