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Conditional velocity pdf in 3-D turbulence.

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Abstract. — It is shown that the probability density functions of velocity increments at small scale in turbulent flows turn to an universal (Gaussian) shape when conditioned to a precisely defined energy transfer rate ε_l . The standard deviation $\sigma(\varepsilon_l)$ of this distribution depends on ε_l following a Kolmogorov like relation $\sigma^3 = C\varepsilon_l l$ with a Reynolds number dependent coefficient C .

Introduction.

One of the main properties of fully developed turbulence is the small scale intermittency. We usually consider that its qualitative signature is the non-Gaussian shape of the probability density function (hereafter noted p.d.f) of the velocity increments defined by $\delta\mathbf{u}(l) = \mathbf{u}(\mathbf{x} + l) - \mathbf{u}(\mathbf{x})$ for l lying within the inertial or dissipative range (for clarity, we shall consider the 1 parallel component of $\delta\mathbf{u}$: $\delta u(l)$).

This stretched exponential-like tail shape of the p.d.f of $\delta u(l)$ changes when the distance l decreases from the largest inertial scales to the smallest dissipation ones. In other words, the shape of the p.d.f. of $\delta u(l)$ normalized by its standard deviation $\sigma_{\delta u(l)}$ is not self-similar with l [1]. When analyzing *via* the scaling exponents ζ_p defined by $\langle \delta u(l) \rangle^p \sim l^{\zeta_p}$, the consequence of this change is the non-linear behaviour of ζ_p with p in contrast, for instance, with the predictions of the β -model [2].

Another problem concerns the use of the relation between the statistics of the velocity increments $\delta u(l)$ and the local energy transfer rate averaged over a ball of size l , ε_l :

$$\frac{u_l^3}{l} \sim \varepsilon_l \quad (1)$$

However, for some authors, (1) can be taken as a relation between two random variables if u_l is the typical velocity of structures of size l [3]. For others (Frisch [4]), u_l is $\delta u(l)$ but the equality

only means that the two quantities “have the same scaling properties, i.e. that moments of the same order have the same scaling exponents”. Is there a possibility to experimentally clarify this point?

The objectives of this paper are, first, to study the statistics of the velocity increment $\delta u(l)$ conditioned to the statistics of the energy transfer rate ε_l in the whole range of inertial and dissipation scales and, second, to test statistically the previous relation.

In section 1, we give experimental results on conditional p.d.f. of $\delta u(l)$ measured in two different Reynolds number turbulent flows. Section 2 is devoted to the dependence of the averaged value of the conditional variable $\delta u(l)_{/\varepsilon_l}$ on its standard deviation $\sigma_{\delta u(l)_{/\varepsilon_l}}$ and finally, in section 3 we give experimental data in order to test the Kolmogorov relation (1).

1. — As is said in the introduction, we have studied the statistical distribution of the velocity increments $\delta u(l)$ at a given scale l , conditioned to the value of ε_l .

Using the homogeneity and isotropy assumptions at small scale, the energy dissipation rate at a fixed point \mathbf{x} is defined by

$$\varepsilon^*(\mathbf{x}, t) = 15\nu \left(\frac{\partial u}{\partial x} \right)^2$$

Following Kolmogorov and Obukhov [5], the local average of the energy dissipation rate $\varepsilon_l^*(\mathbf{x}, t)$ taken over a ball of size l centered at \mathbf{x} is

$$\varepsilon_l^*(\mathbf{x}, t) \sim \frac{1}{l^3} \int_{|\mathbf{x}' - \mathbf{x}| < l} \varepsilon^*(\mathbf{x}', t) d^3 \mathbf{x}'$$

By also using the Taylor hypothesis to get spatial gradient from time derivative signal, experimentalists follow the 1D version of this definition to measure the local energy dissipation rate ε_l^* averaged over a linear interval of size l , for l lying in the inertial or dissipation range.

If l belongs to the inertial range, ε_l^* roughly corresponds to the energy transfer rate ε_l averaged over the same interval l . When l is in the dissipation range, the local transfer ε_l can be estimated as

$$\varepsilon_l(x, t) \simeq 15\nu \left[\frac{1}{l} \int_x^{x+l} \left(\frac{\partial u}{\partial x} \right)^2 dx - \left(\frac{\delta u(l)}{l} \right)^2 \right]$$

in which $\left(\frac{\delta u(l)}{l} \right)^2$ takes into account the dissipation which occurs at the scale l itself. Clearly, this term increases when l goes down to the Kolmogorov scale η and it appears that its value is no longer negligible at such a scale.

We have calculated this ersatz of the local transfer rate ε_l on three different velocity signals obtained on the axis of a laboratory jet flow ($R_\lambda = 428$) and in the wind tunnel of O.N.E.R.A. in Modane, both on the axis ($R_\lambda = 2720$) and near the wall ($R_\lambda = 1689$) [6]. The ratio of the sampling interval Δx over the Kolmogorov scale η was about 0.15 in the jet and respectively 12 and 19 in the wind tunnel which means that the spatial resolution is good in the jet and very poor in the two flows at high Reynolds number.

Practically, ε_l has been calculated from the velocity data samples in the following way:

$$\varepsilon_l = \frac{1}{m} \left[\sum_{j=1}^{j=m} \left(\frac{u_{i+j} - u_{i+j-1}}{\Delta x} \right)^2 \right] - \left(\frac{u_{i+m} - u_i}{m\Delta x} \right)^2$$

in which $l = m\Delta x$.

Figure 1 gives a comparison between the shapes of the global p.d.f of $\delta u(l)$ and the conditional one at a "given value" of ε_l which means practically an interval of values centered at ε_l with a constant width that we have chosen equal to $\langle \varepsilon_l \rangle / 6$. In figure 1, the scale l belongs to the inertial range, respectively $l/\eta = 40$ in the jet (Fig. 1a), 120 on the axis of the wind tunnel (Fig. 1b) and 190 near the wall (Fig. 1c). The given value of ε_l is equal to $\langle \varepsilon_l \rangle$.

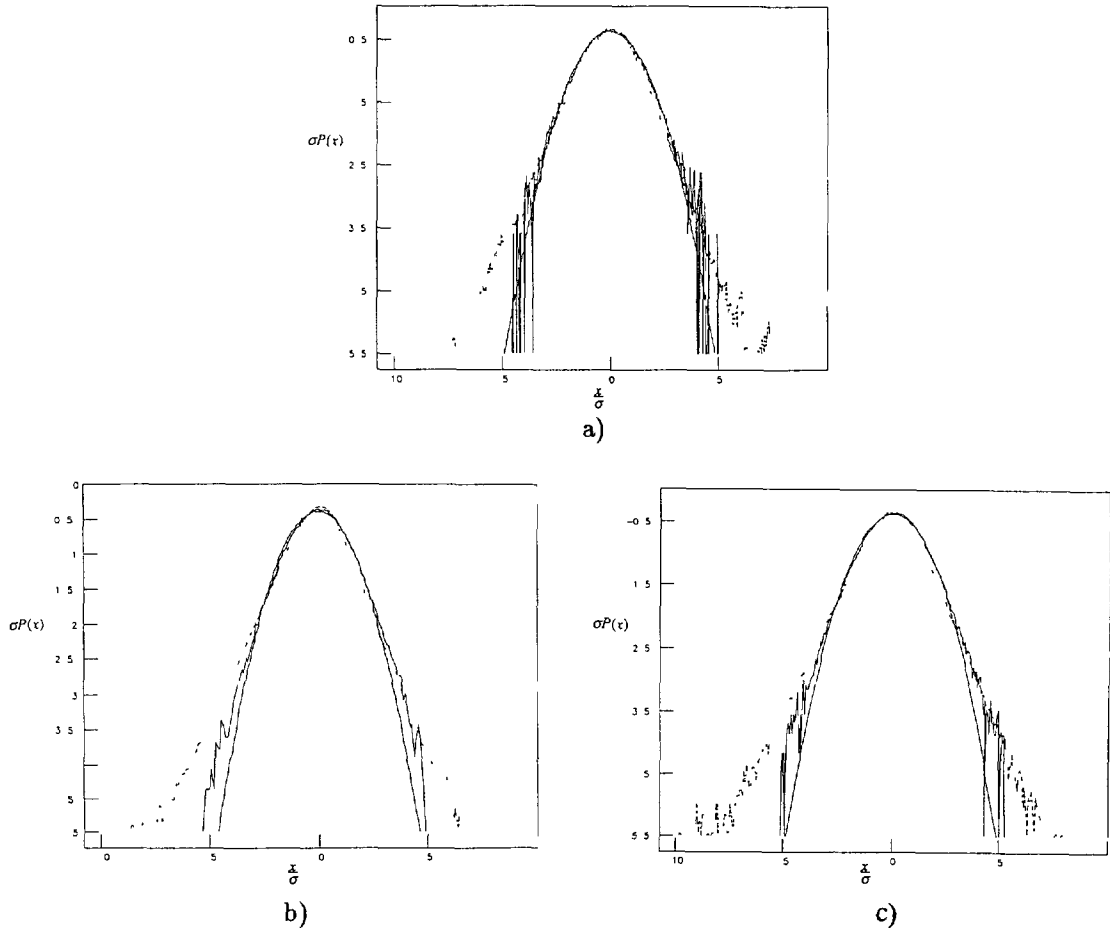


Fig. 1. — Global p.d.f of δu_l (dashed line) and p.d.f of δu_l conditioned to $\langle \varepsilon_l \rangle$ compared to a Gaussian shape (parabolic in lin-log coordinates), for l lying in the inertial range. a): jet, $R_\lambda = 428$, $l = 40\eta$. b): axis of the wind tunnel, $R_\lambda = 2720$, $l = 120\eta$. c): "boundary layer" of the wind tunnel, $R_\lambda = 1689$, $l = 190\eta$.

Figure 2 gives the same comparison for a scale l chosen in the dissipation range: $l/\eta = 10$ only in the jet (in the wind tunnel, the dissipation range has not been reached). As is well-known, the global p.d.f of $\delta u(l)$ changes from an exponential-like tail shape to a sharper and more stretched one when l goes down from the inertial to the dissipation range. On the contrary, the conditional p.d.f of $\delta u(l)$ has a quasi Gaussian shape which remains constant

when l is decreasing; "quasi" means that these p.d.f are not symmetric but always have a weak negative skewness like all the classical longitudinal velocity increments. Probably, in the case of transversal velocity increments, these conditional p.d.f would be purely Gaussian. In figures 1b and 1c, the tails of the experimental conditional p.d.f. move away from a parabolic shape, which suggests a remaining intermittency. That feature clearly comes from the poor spatial resolution ($\Delta x \simeq 12$ and 19) in the estimation of ε_l .

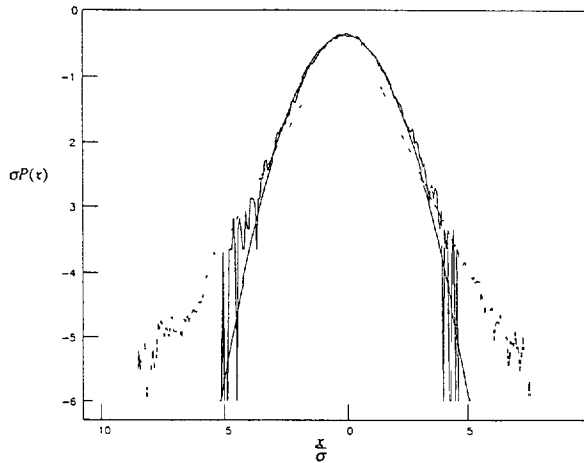


Fig. 2. — Same as figure 1, with l lying in the dissipation range, for the jet. $R_\lambda = 428$, $l = 10\eta$.

This quasi Gaussian shape of the p.d.f conditioned to ε_l has already been obtained by Stolovitsky *et al.* [7] only for inertial scales. In contrast to us, these authors got bimodal conditioned p.d.f of $\delta u(l)$ for dissipative scales. It is due to the fact that, in their paper, the velocity $\delta u(l)$ has been conditioned to the quantity ε_l^* and not to ε_l as we did. We have checked their results using ε_l^* instead of ε_l and have found the same bimodal conditional distribution in the dissipative range.

The results of figure 1 suggest that the right quantity to condition the statistics of the velocity field is the energy transfer rate and not the dissipation one. If a non Gaussian shape is a statistical signature of intermittency, as many people think, then those results show that the small scale intermittency is fully contained in the distribution of the energy transfer rate and disappears at fixed ε_l . Moreover, this result is in very good agreement with the main physical assumption of the empirical model of velocity p.d.f proposed by Castaing *et al.* [1] which is theoretically supported by the variational approach (Castaing [8]).

2. — One of the consequences of the model quoted above is that the calculated distributions of $\delta u(l)$ have a non zero average value. Obviously, this point is in contradiction with experimental data for which the mean value $\langle \delta u(l) \rangle$ is always zero. In order to understand how this average of $\delta u(l)$ is restored to zero, we have studied the dependence of the mean value on the standard deviation of the conditional distribution of $\delta u(l)/\varepsilon_l$, namely $\langle \delta u(l)/\varepsilon_l \rangle$ vs. $\sigma_{\delta u(l)/\varepsilon_l}$. Figure 3 shows such a behaviour at a given $l/\eta = 60$ in the case of the wind tunnel. The experimental points correspond to different chosen values of ε_l equally distributed from zero to $2 \times \langle \varepsilon_l \rangle$ with a step of $\langle \varepsilon_l \rangle / 6$. This figure clearly displays a linear behaviour

of $\langle \delta u(l)/\varepsilon_l \rangle$ on $\sigma_{\delta u(l)/\varepsilon_l}$, decreasing from positive to negative values as $\sigma_{\delta u(l)/\varepsilon_l}$ increases. This remarkable feature occurs for any scale and any flow and leads to a linear coefficient K apparently independent of the scale l . A study of its dependence with the Reynolds number asks for more than our three examples but note anyway that K is roughly the same for the three of them.

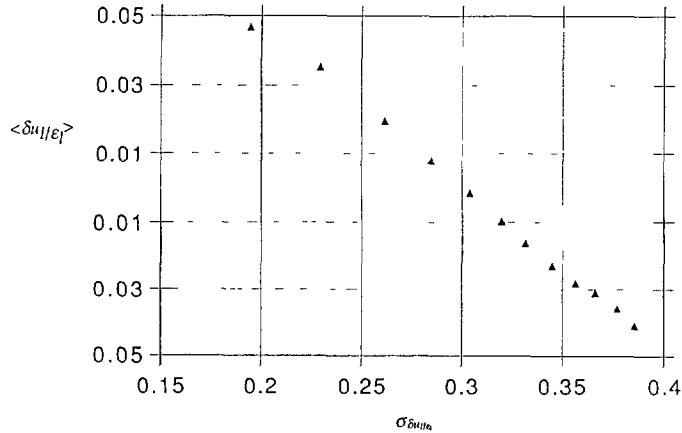


Fig. 3. — $\langle \delta u_l / \varepsilon_l \rangle$ versus $\sigma_{\delta u_l / \varepsilon_l}$ on the axis of the wind tunnel. $R_\lambda = 2720$, $l = 60\eta$.

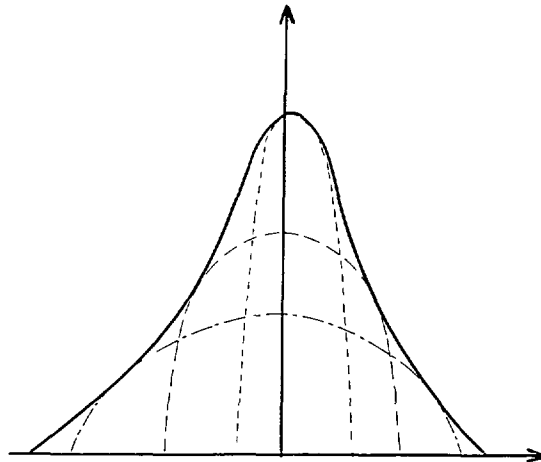


Fig. 4. — p.d.f. of $\delta u(l)$ obtained by the superposition of quasi-Gaussian p.d.f. with several modal and standard deviation values.

As we discussed previously, Castaing *et al.* [1] have shown that the p.d.f of $\delta u(l)$ can be fitted by a superposition of Gaussian p.d.f with different standard deviations $\sigma_{\delta u(l)}$ increasing from very small values corresponding to the top of the global p.d.f to large ones taking into account the tails as is shown in figure 4.

Figure 3 indicates that the narrowest Gaussian which contributes to the most probable value of the global $\delta u(l)$ are positively shifted, whereas the largest Gaussian are negatively shifted leading to the negative skewness of the longitudinal p.d.f. This result clearly explains what was first seen by van Atta and Park [9], that is, for a given scale l , the most probable value of the global distribution of $\delta u(l)$ is always positive and the negative skewness is only due to the largest and rare negative fluctuations of $\delta u(l)$.

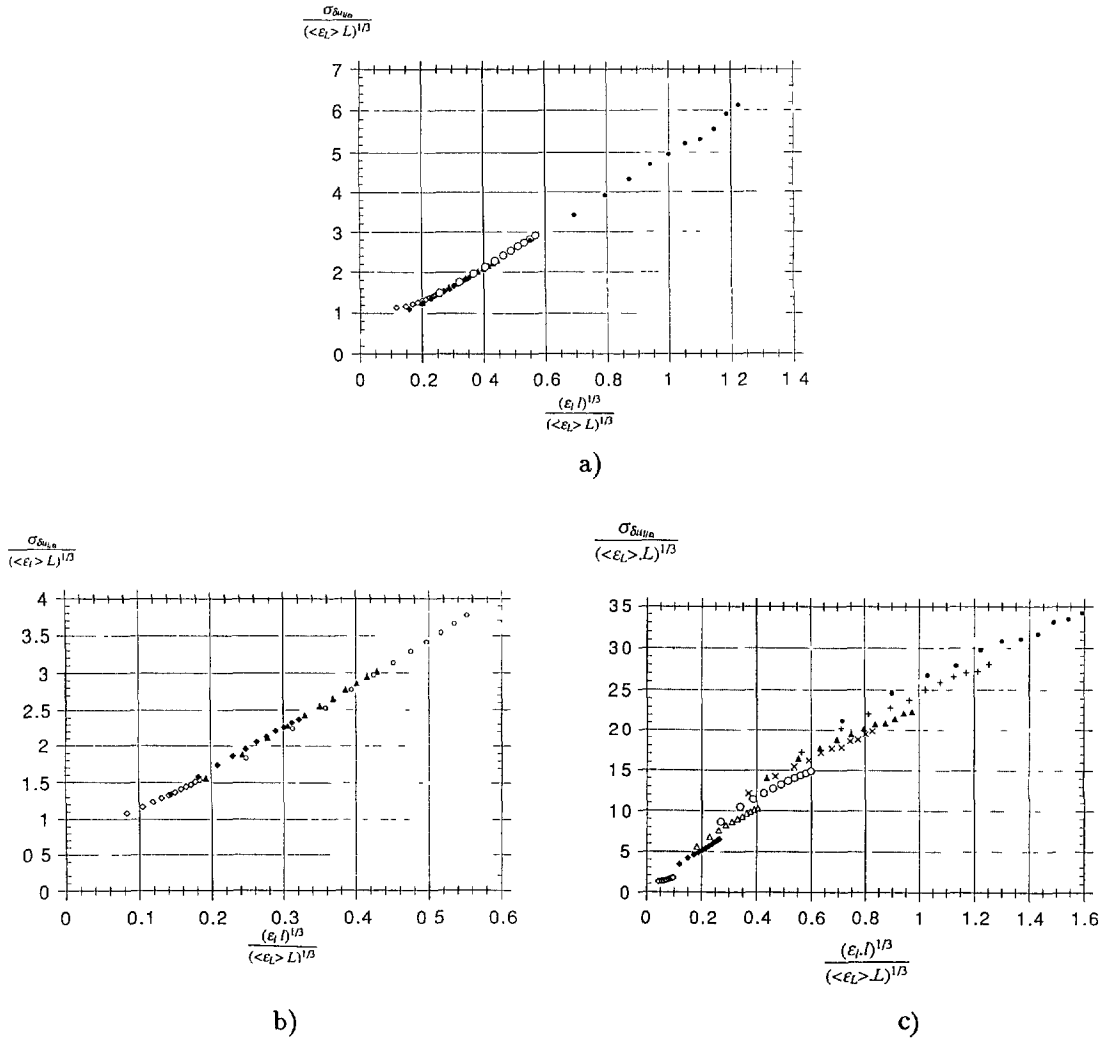


Fig. 5. — $\sigma_{\delta u_l / \epsilon_l}$ versus $(\epsilon_l l)^{1/3}$ normalized by $(\langle \epsilon_L \rangle \cdot L)^{1/3}$, where L is the integral scale of each flow. a): axis of the wind tunnel. $R_\lambda = 2720$. (\diamond): $l = 24\eta$; (\blacklozenge): $l = 60\eta$; (\blacktriangle): $l = 120\eta$; (\circ): $l = 250\eta$; (\bullet): $l = 2500\eta$. b): "boundary layer" of the wind tunnel. $R_\lambda = 1689$. (\diamond): $l = 38\eta$; (\blacklozenge): $l = 95\eta$; (\blacktriangle): $l = 190\eta$; (\circ): $l = 380\eta$. c): jet. $R_\lambda = 428$. (\diamond): $l = 1.5\eta$; (\blacklozenge): $l = 5.28\eta$; (\triangle): $l = 9.8\eta$; (\circ): $l = 20\eta$; (\times): $l = 40\eta$; (\blacktriangle): $l = 60\eta$; (+): $l = 120\eta$; (\bullet): $l = 240\eta$.

3. — Knowing the value of $\sigma_{\delta u(l)}$ corresponding to each ε_l , we have studied their mutual behaviour in order to test relation (1). In figure 5, we have plotted $\sigma_{\delta u(l)/\varepsilon_l}$ versus $(l.\varepsilon_l)^{1/3}$; variables have been normalized with the integral scale L and the corresponding energy transfer rate $\langle \varepsilon_L \rangle$. At very large Reynolds numbers (Fig. 5a, $R_\lambda = 2720$ and Fig. 5b, $R_\lambda = 1689$), for different scales ranging in the inertial range, relation (1) is well verified by experimental data; we have namely

$$\sigma_{\delta u(l)/\varepsilon_l} = C(l.\varepsilon_l)^{1/3}$$

where the coefficient C does not depend on the scale l . However, the $\sigma_{\delta u(l)/\varepsilon_l}$ decreases to a non zero value when the scale l goes to zero. Again, it is due to the poor estimate of ε_l at small scale l which overestimates the value of $\sigma_{\delta u(l)/\varepsilon_l}$. The experimental values of the factor C are roughly equal to: $C(2720) \simeq 4.8$ and $C(1689) \simeq 6.2$. Even though these $C(R_\lambda)$ coefficients are underestimated, it seems that they decrease with the Reynolds number since the factor $C(1689)$ is *a priori* more reduced by experimental error than the $C(2720)$ one.

In figure 5c (jet, $R_\lambda = 428$), experimental data tends to the origin of the axis which denotes a good experimental estimate of ε_l at small scale l . But the merging of experimental data obtained at different values of the scale l is not so good, in particular we get a systematic shift for each scale l . We have no satisfactory explanation for this systematic shift. In particular, it cannot be fully explained by the finite size of the ε_l interval corresponding to a single point. It could be artificially corrected by a different dependence of σ versus l (l^α with $\alpha > 1/3$). Note however that the dispersion of the points is of the order of the shift which yields to think to a sampling artifact. The slopes of the different pieces of this curve are roughly the same, at least for inertial scales and their common value is about $C(428) = 15$.

Thus, relation (1) is experimentally well verified only for inertial scales as assumed by Kolmogorov. This suggests that the energy transfer rate ε_l averaged over an inertial separation l is indeed an inertial quantity (and not an inertial and dissipative one as proposed by Kraichnan [10]).

Concluding remarks.

We have shown that the velocity field $\delta u(l)$ at small scale is intrinsically Gaussian (within the skewness of the longitudinal case) and the intermittent non Gaussian shape of its p.d.f is only due to the intermittency of the energy transfer rate ε_l . This result confirms that quantitative intermittency can be defined only from energetical quantities as assumed in the variational approach proposed by Castaing [8].

As these conditional p.d.f are Gaussian, they are completely characterized by their standard deviation, even for the longitudinal case, where we have shown that the mean values $\langle \delta u(l)/\varepsilon_l \rangle$ are not zero but depend on a unique way on the standard deviation. The scaling law of the standard deviations $\sigma_{\delta u(l)/\varepsilon_l}$ on the value of ε_l is experimentally, for inertial scales, in good agreement with the well-known relation of Kolmogorov. Our results clearly indicate that this scaling depends on the Reynolds number value. In the inertial range, our experimental data lead to similar results than those reported by Praskovskiy [11], Thoroddsen and van Atta [12] and Chen *et al.* [13]. Since in these papers the velocity increment $\delta u(l)$ has been conditionally averaged over a fixed value of ε_l^* (and not of ε_l as done by us), no comparison can be made for scales l lying in the dissipation range.

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References

- [1] Castaing B., Gagne Y. and Hopfinger E.J., *Physica D* **46** (North-Holland, 1990) 177-200.
- [2] Frisch U., Sulem P.L. and Nelkin M., *J. Fluid Mech.* **87** (1978) 719-736.
- [3] Meneveau C. and Sreenivasan K.R., *Nuclear Phys. (Suppl.)* **B2** (North-Holland, 1987) 49-67.
- [4] Frisch U., *Turbulence and Stochastic Processes: Kolmogorov's ideas 50 years on* (The Royal Society, London, 1992) p. 89.
- [5] Kolmogorov A.N., *J. Fluid Mech.* **13** (1962) 82-85;
Obukhov A.M., *J. Fluid Mech.* **13** (1962) 77-81.
- [6] Castaing B., Gagne Y. and Marchand M., *Physica D* (1993) to be published.
- [7] Stolovitsky G., Kailasnath P. and Sreenivasan K.R., *Phys. Rev. Lett.* **69** (1992) 1178-1181.
- [8] Castaing B., *J. Phys. France* **50** (1989) 147.
- [9] Van Atta C.W. and Park J., *Statistical Models and Turbulence, Lecture Notes in Physics*, **12** (Springer-Verlag, 1972) pp. 402-426.
- [10] Kraichnan R.H., *J. Fluid Mech.* **62** (1974) 305-330.
- [11] Praskovsky A. A., *Phys. Fluids A* **4** (1992) 2589-2591.
- [12] Thoroddsen S. J. and Van Atta C. W., *Phys. Fluids A* **4** (1992) 2592-2594.
- [13] Chen S., Doolen G., Kraichnan R. H. and She Z. S., *Phys. Fluids A* **5** (1993) 458-463.