



**HAL**  
open science

## Self-diffusion of particles in an alternatively sheared macroscopic suspension

Luc Petit, Philippe Gondret

► **To cite this version:**

Luc Petit, Philippe Gondret. Self-diffusion of particles in an alternatively sheared macroscopic suspension. Journal de Physique II, 1993, 3 (3), pp.301-307. 10.1051/jp2:1993134 . jpa-00247834

**HAL Id: jpa-00247834**

**<https://hal.science/jpa-00247834>**

Submitted on 4 Feb 2008

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Classification

Physics Abstracts

47.55K—51.20

Short Communication

## Self-diffusion of particles in an alternatively sheared macroscopic suspension

Luc Petit and Philippe Gondret

Laboratoire de Physique (\*), Ecole Normale Supérieure de Lyon, 46 Allée d'Italie, 69007 Lyon, France

(Received 16 November 1992, accepted 22 December 1992)

**Résumé.** — Nous présentons dans cette communication les premiers résultats que nous avons obtenus dans l'étude du mouvement de migration de particules sphériques macroscopiques (diamètre  $45\ \mu\text{m}$ ) à l'intérieur d'une suspension soumise à un écoulement de cisaillement plan alternatif. Le mouvement de cisaillement est appliqué dans deux directions orthogonales avec une différence de phase de  $\pi/2$  ("polarisation" circulaire de la vitesse). Un suivi des particules au cours du temps et une analyse de leurs positions successives nous donnent accès au déplacement quadratique moyen des particules  $\sigma^2 = \langle (\mathbf{R}(t) - \langle \mathbf{R}(t) \rangle)^2 \rangle$ , qui montre une dépendance linéaire avec le temps. Nous interprétons ce comportement de type diffusif comme résultant des interactions hydrodynamiques subies par chaque particule de la part des autres particules de la suspension. Ces interactions proviennent de corrections inertielles de l'écoulement appliqué, non nulles en moyenne sur des temps longs devant la période de l'écoulement alternatif. Nous proposons un modèle simple du type diffusion brownienne macroscopique pour obtenir une estimation du coefficient d'autodiffusion aux temps longs.

**Abstract.** — In this communication, we present the preliminary results of a study of the migration motions of macroscopic spherical particles (diameter  $45\ \mu\text{m}$ ) in a suspension submitted to an alternating plane shear flow. The shear flow is applied in two directions which are both perpendicular and in phase quadrature (circularly polarized shear). By following the particles in time and analysing their trajectory, we can compute the mean-square displacement  $\sigma^2 = \langle (\mathbf{R}(t) - \langle \mathbf{R}(t) \rangle)^2 \rangle$  which shows a linear behaviour with time. We interpret this diffusive behaviour as a result of the hydrodynamic interactions experienced by each particle from the other particles of the suspension. These interactions come from inertial corrections to the applied flow, of non-zero mean, over long times in comparison with the period of the alternating shear. We propose a simple model, based on macroscopic Brownian-like diffusion to give an estimate of the long-time self-diffusion coefficient.

---

(\*) Unité de Recherche Associée au C.N.R.S. n° 1325

## 1. Introduction.

Hydrodynamic diffusion of solid particles in flowing suspensions has been widely studied in the last two decades, by experimental [1-3], theoretical [4-7] and numerical techniques [8]. "Hydrodynamic diffusion" means that the diffusional behaviour is a consequence of the shear flow imposed to the suspension. The shear flow modifies the diffusion, which in the absence of flow is purely Brownian. The hydrodynamic interactions modify the microstructure which changes the particle mobility and hence the diffusion [4-6, 8]. This is the case when the Peclet number  $Pe = \dot{\gamma}a^2/D_B$  is small ( $\dot{\gamma}$  is the shear rate,  $a$  the radius of the particles, and  $D_B$  the Brownian diffusion coefficient of a single particle). In the opposite case of large Peclet number, the diffusion is a direct consequence of the flow, due to velocity fluctuations of a particle, as a consequence of the multiple interactions with the other particles [1-3, 7]. It is worth noting that in most of the studies corresponding to this last case, the particle interactions are purely hydrodynamic, and the inertia effects are negligible.

In this communication, we present our first results for the study of the migration of particles in a non-Brownian suspension, submitted to an alternating plane shear flow. The peculiarity of our study lies in the time-dependent nature of the shear flow. As a consequence, all linear effects are erased over long time compared with the period  $T$  of the shear, and only the non linear effects (arising from fluid inertia in our studies) are revealed. These effects are corrections to the primary flow (the particle Reynolds number -about  $10^{-3}$ - is small in our experiments).

The experimental setup allows us to apply the alternating shear along two perpendicular directions  $X$  and  $Y$  with a phase-difference of  $\pi/2$  (Fig. 1a). The velocity at each point of the fluid is a vector rotating around a vertical axis at an angular frequency  $\omega = 2\pi/T$  (Fig. 1b). Under these conditions, one observes a slow migration of the particles of the suspension in addition to the motion at frequency  $\omega$ ; *slow* means that the characteristic time  $\tau_s$  ( $s$  stands for steady) of this motion (time required for a particle to wander over a distance of its diameter) is much greater than  $T$ . The results we present concern the study of the migration motion of the particles and their analysis. Figure 2 shows a top view photograph of the suspension submitted to the shear. It appears disordered. The typical value of the migration velocity  $v_s$  of the particles is about  $v_s = 10 \mu\text{m/s}$ , compared with the velocity  $v_0$  of the applied shear flow  $v_0 \approx 1 \text{ cm/s}$ . In section 3, we suggest that the difference between these two values is due to secondary flows created around the particles by the inertia of the fluid. The secondary flow is of non-zero mean, over long times compared with the period of the shear, because of the quadratic dependence of the velocity of the secondary flow on the velocity  $v_0$  of the primary flow. This is the same interpretation as for the formation of structures previously reported [9] in the case where the shear is applied only in one direction.

## 2. Analysis of the particle motion.

In order to study the nature and the origin of the migration motion of the particles, we have performed a digitizing of their successive positions <sup>(1)</sup>. A video movie of the suspension is displayed on a video monitor. One particle is followed during the migration by means of a digitizing ruler and its positions are recorded into files with a computer. The procedure is repeated for a few other particles in order to get a good statistics of the motion of the particles (in general about twenty particles were treated). Figure 3 shows the trajectories of the particles obtained from the files for a given shear rate.

---

<sup>(1)</sup> The set-up will be described in more detail in a subsequent paper.

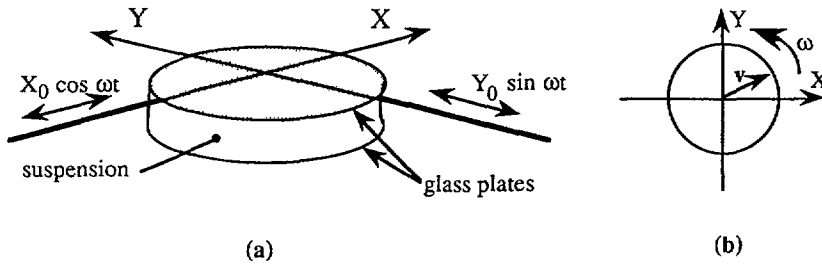


Fig. 1. — a) Schematic of the shear cell: the alternating shear is applied along two perpendicular directions  $X$  and  $Y$ , with a phase-difference of  $\pi/2$ . b) The velocity of the fluid is a vector rotating around the vertical axis at an angular frequency  $\omega$ .

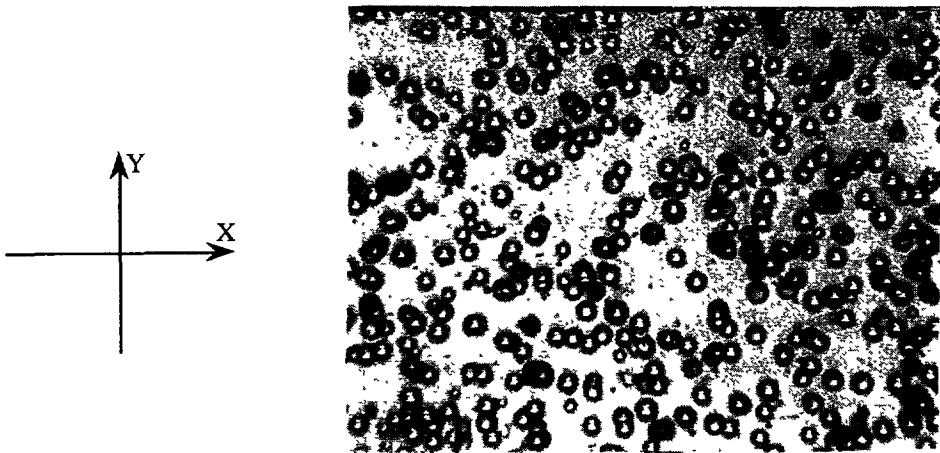


Fig. 2. — Instantaneous top view of the suspension under alternating shear (volume fraction is 5 %).

We process the data files to extract the coordinates  $X_i(t_j)$ ,  $Y_i(t_j)$  of the  $i^{\text{th}}$  particle at time  $t_j$  and the mean-square displacement  $\sigma^2(t_n) = \langle (\mathbf{R}(t_n) - \langle \mathbf{R}(t_n) \rangle)^2 \rangle$ , where  $t_n = t_{j+n} - t_j$  and the brackets denote an average over the particles and all the events of time  $t_n$  (in the following, we will omit the index  $n$  of the time  $t_n$ ). The time step  $\Delta t = t_{j+1} - t_j$  between two successive records of the position of each particle is chosen small in comparison with the characteristic time of the migration of the particles. Figure 4 shows the results for a suspension of 5 % in volume fraction and a shear rate amplitude of  $165 \text{ s}^{-1}$  for the two components  $\dot{\gamma}_X$  and  $\dot{\gamma}_Y$ . For this case, the choice of the time step  $\Delta t$  was 6 seconds. For times larger than a few  $\Delta t$ , one sees that the mean square displacement of the particles varies linearly with time, indicating that the motion of the particles is of a diffusive type. The non-linear behaviour observed for  $\sigma^2$  at short times ( $t \leq 5\Delta t$  for the present case) is ascribed to the small number of interactions experienced by each particle during this time. We get the long-time self-diffusivity  $D$  from the

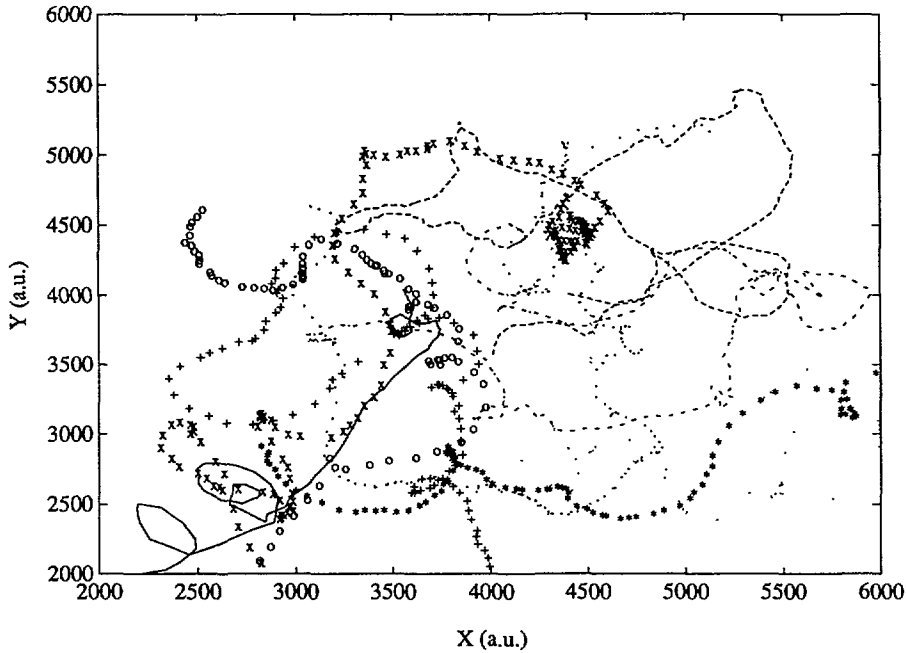


Fig. 3. — Particle trajectories in a suspension under alternating shear, with two components. Amplitude of each component is  $165 \text{ s}^{-1}$ . The various symbols correspond to the different particles (one can see at the bottom of the figure that some particles have gone out of the field of the camera before the end of digitizing).

rate of change of the mean-square displacement of the particles, in the limit  $t \gg \Delta t$  :

$$D = \lim_{t/\Delta t \rightarrow \infty} \frac{1}{2} \frac{d(\sigma^2)}{dt} \quad (1)$$

From the slope of the linear part of the curve of figure 4, one obtains:

$$D = 1.94 \frac{a^2}{\Delta t} = 3.2 \times 10^{-6} \text{ cm}^2/\text{s} \quad (2)$$

### 3. Interpretation of the results.

Here, we give a tentative interpretation of the diffusive-type behaviour of the migration motion of the particles. As mentioned in paragraph 1, there are two main experimental facts:

- the creation of a steady motion (migration of the particles) from alternating motion (applied shear flow);
- the smallness of the migration velocity  $v_s$  of the particles compared with the velocity  $v_0$  of the shear flow.

These facts led us to look for an explanation of the migration from the effect of secondary flows of non-zero mean created around the particles (steady streaming at low Reynolds number). Let us consider the trajectory of the center of a particle under the effect of the alternating

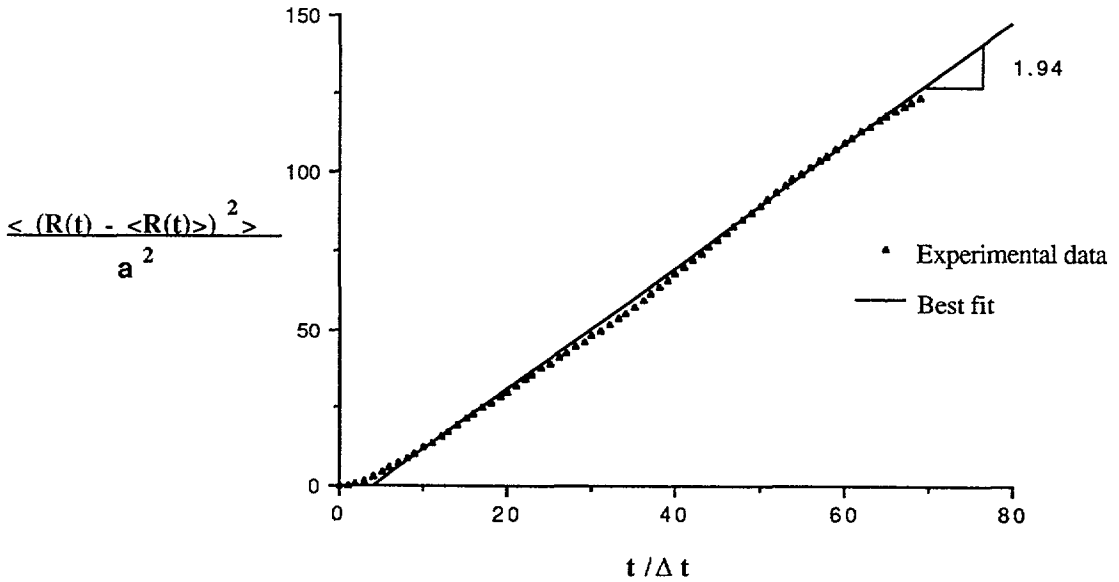


Fig. 4. — Mean-square displacement  $\sigma^2(t) = \langle (\mathbf{R}(t) - \langle \mathbf{R}(t) \rangle)^2 \rangle$  of the particles as a function of time in the experimental case of figure 3.  $\sigma^2$  is rendered dimensionless by the square of particle diameter  $a^2$ , and the time  $t$  by the time step  $\Delta t$  used for the acquisition of the successive positions of the particles (here  $\Delta t = 6$  s).

shear flow: it is a circle described in a time  $T$ , the period of the shear (Fig. 5). Along the trajectory, the particle has a rotatory motion around a radial axis. Owing to this motion and the presence of the other particles, a secondary flow is created around the particle whose origin is fluid inertia: some fluid is expelled from the particle in the plane perpendicular to the axis of rotation, and sucked into the particle along this axis. If two particles are not too far from each other, and if the line joining their centers is in the direction of their axis of rotation at one time (case of the spheres in solid line in Fig. 5), they will experience a mutual attraction. A fraction of the period  $T$  later (dashed positions of the spheres in Fig. 5), the interaction will be repulsive; in the same time, the particles could interact with other particles. As time progresses, each particle experiences a series of such interactions with its neighbours, in turn repulsive and attractive, depending on their positions on their circular orbits. Contrarily to the uniaxial oscillating shear, the reversal of the interaction forces between neighbours during one period prevents the formation of a long-range order and acts as a source of random motion. The cumulative effect of these interactions, over a time much larger than  $T$ , results in a Brownian-like behaviour of the particles.

Let us now give an estimate of the self-diffusion coefficient  $D$  of this Brownian-like walk. We build the coefficient  $D$  as the product of the energy associated with the interactions between particles,  $\rho (v_0)^2 a^3$  ( $\rho$  is the density of the fluid and  $v_0$  the velocity of the applied flow), and the mobility of the particles,  $(6\pi\eta a g(\phi))^{-1}$  ( $\eta$  is the viscosity of the fluid and  $g(\phi)$  an unknown function of the particle volume fraction  $\phi$ , which contains the decrease of the particle mobility as  $\phi$  increase). Hence, we get:

$$D \approx \frac{\rho (v_0)^2 a^3}{6\pi\eta a g(\phi)} \approx \frac{\rho a^4 \dot{\gamma}_X \dot{\gamma}_Y}{6\pi\eta g(\phi)} \tag{3}$$

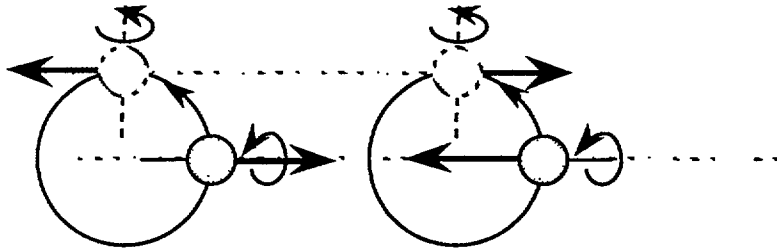


Fig. 5. — Trajectories of two particles under the effect of the applied shear flow. Along their circular trajectory, the particles rotate around a horizontal axis. The secondary steady flow created by this rotation induces an attraction between two particles if their center-line is parallel to the axis of rotation (case for the particles in solid line in the figure), and a repulsion if the center-line is perpendicular to the axis of rotation: this is the case for the same spheres (drawn in dashed lines) at a time  $T/4$  later.

where we have replaced  $v_0^2$  by the product of the amplitudes of the velocities  $v_{0X}$  and  $v_{0Y}$  in the two directions  $X$  and  $Y$ , because migration disappears as soon as one component of shear is suppressed. A numerical estimate of  $D$  with the values corresponding to the experimental results presented above and with  $g(\phi) = 1$  gives an order of magnitude for  $D \approx 10^{-7} \text{ cm}^2/\text{s}$ , compared with the experimental value  $D = 3.2 \times 10^{-6} \text{ cm}^2/\text{s}$  (expression (2)). The difference between these two values may come from the approximate evaluation of  $D$  (Eq. (3)) and the existence of unknown prefactors. It will be interesting to study the dependence of  $D$  with the various parameters of the problem; particularly the amplitude of the shear rates, the particle diameter and the volume fraction  $\phi$ . Further analysis of the migration motion is in progress to study the influence of these parameters and to test the validity of this model.

#### 4. Conclusion.

We have presented a particle migration phenomenon observed in a macroscopic suspension, submitted to an alternating plane shear flow. The analysis of the particle motion shows a diffusive-type behaviour that we interpret as due to hydrodynamic inertial interactions between the particles of non-zero mean over the time-scale of observation. A simple model of macroscopic Brownian-like diffusion is proposed to account for these results.

#### Acknowledgements.

We gratefully acknowledge B. Noetinger for his help during the experiments and J. Prost for stimulating discussions on this problem.

#### References

- [1] ECKSTEIN E.C., BAILEY D.G., SHAPIRO A.H., Self-diffusion of particles in shear flow of a suspension, *J. Fluid Mech.* **79** (1977) 191-208.

- [2] LEIGHTON D., ACRIVOS A., Measurement of the shear-induced self-diffusion in concentrated suspensions of spheres, *J. Fluid Mech.* **177** (1987) 109-131.
- [3] LEIGHTON D., ACRIVOS A., The shear-induced migration of particles in concentrated suspensions, *J. Fluid Mech.* **181** (1987) 415-439.
- [4] HINCH E.J., Application of the Langevin equation to fluid suspensions, *J. Fluid Mech.* **72** (1975) 499-511.
- [5] BATCHELOR G.K., Brownian diffusion of particles with hydrodynamic interaction, *J. Fluid Mech.* **74** (1976) 1-29.
- [6] RALLISON J.M., Brownian diffusion in concentrated suspensions of interacting particles *J. Fluid Mech.* **186** (1988) 471-500.
- [7] KOCH D.L., On hydrodynamic diffusion and drift in sheared suspensions, *Phys. Fluids A* **1** (1989) 1742-1745.
- [8] BOSSIS G., BRADY J.F., Self diffusion of Brownian particles in concentrated suspensions under shear, *J. Chem. Phys.* **87** (1987) 5437-5448.
- [9] PETIT L., NOETINGER B., Shear-induced structures in macroscopic dispersions, *Rheol. Acta* **27** (1988) 437-441.