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Pressure fluctuations in swirling turbulent flows

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Résumé. — Nous présentons des mesures de fluctuations de pression dues à des écoulements tourbillonnaires turbulents engendrés entre deux disques co-axiaux en rotation. Nous montrons que les spectres de pression dépendent fortement de la nature des différents écoulements observés en fonction du sens de rotation relatif des disques. Dans le cas contra-rotatif, nous montrons que la densité de probabilité des fluctuations de pression est fortement non-gaussienne, et présente de larges déviations vers les basses pressions que l'on peut attribuer à des concentrations intermittentes de vorticité sous forme de filaments. Le comportement des fluctuations vers les hautes pressions nous renseigne sur la structure des régions dissipatives.

Abstract. — We report measurements of pressure fluctuations in swirling turbulent flows generated in the gap between two rotating coaxial disks. We relate the pressure spectra to the observed flow regimes in the case of co-rotating and counter-rotating disks. In the latter case, we show that the pressure probability density function is strongly non-Gaussian and displays an exponential tail toward low pressures which might be ascribed to intermittent vorticity concentrated on tube-like structures. Similarly, the behaviour of the high pressure fluctuations results from the geometry of dissipative structures.

1. Introduction.

Pressure fluctuations in turbulent flows have been studied experimentally [1, 2] as well as theoretically for a long time [3, 4]. In the limit of an incompressible fluid of density ρ, it is well known that the pressure obeys a Poisson equation obtained by taking the divergence of the Navier-Stokes equation [2] which can be put in the form [5],

$$\frac{2}{\rho}\Delta p = -2\frac{\partial^2(v_i v_j)}{\partial x_i \partial x_j} = \omega^2 - \sigma^2, \tag{1}$$

thus showing that the source term is the difference between the squared vorticity, ω^2, and the squared rate of strain, σ^2. Since both of these quantities are highly intermittent, it was recently suggested, and shown by direct numerical simulations that the pressure also displays an intermittent behavior [6]. In this letter we present measurements of pressure fluctuations in swirling flows generated in the gap between two rotating co-axial disks. In the case of counter-rotating disks, we show that the pressure probability density function (p.d.f.) is strongly non-Gaussian in the turbulent regime, and displays a roughly exponential tail toward low pressures. The intermittent occurrence of sharp pressure drops, also clearly visible on direct time-recordings, might be ascribed to the vorticity concentrations ("filaments") observed in direct numerical simulations [7-10] and recently visualized experimentally in water seeded with air bubbles [11].

2. Experimental set-up and measurements.

The experimental set-up is shown in figure 1. It consists of a cylindrical container, 20 cm in diameter and 19 cm in height, filled with water. Its temperature is maintained constant with a regulated water circulation. Two rotating co-axial disks of diameter D, at a distance H apart from each other, are rotated independently by two DC motors at angular velocities $\Omega_{1,2}$ in the range $[0, 1800]$ RPM, i.e. $[0, 30]$ Hz for $\Omega_{1,2}/2\pi$. For the results presented here, $D = 17.5$ cm and $H = 8$ cm, but we have observed a similar qualitative behavior for the pressure fluctuations with smaller disks ($D = 10$ cm) or with closer disks ($H = 6$ cm and $H = 3.2$ cm). The importance of protruding rims around the disks has been emphasized in reference [11]. We have thus performed experiments with and without rims but we have observed no qualitative modification of the pressure fluctuations.

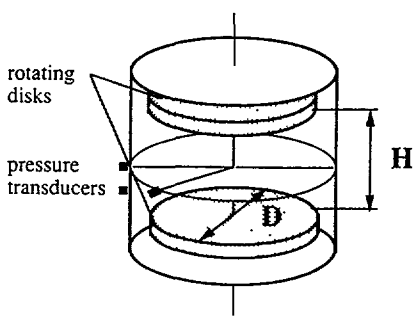

Fig. 1. — Sketch of the experimental set-up.

Pressure fluctuations are measured with PCB 116B acceleration-compensated piezoelectric transducers, mounted flush with the lateral wall. Two of them are located in the mid-plane between the disks and the third one is 2.5 cm below. The diameter of the transducers being rather large (1 cm), we note that the resolution of high wavenumber turbulent structures is poor because of their averaging over the surface of the transducer [12]. However, the low pressure events, due to intermittent vorticity concentration in space, are well detected. We have checked the effect of transducer size using PONS 2Hz1000 hydrophones, the active diameter of which is less than 3 mm. We have found no qualitative difference. Since the low-frequency cut-off of the hydrophone is larger than for the PCB, it high-pass filters the signal and we have not used it for quantitative measurements. The low-frequency cut-off of the PCB is 0.05 Hz and its rise time is 5 μs.

3. Results.

The flows generated between two co-axial rotating disks, known as Von Karman swirling flows, are among the most widely studied ones in fluid mechanics [13]. We define the Reynolds number using the gap H as integral scale, thus $\mathrm{Re} = \Omega_1 H^2/\nu$ where ν is the kinematic viscosity. With our present experimental set-up, H is in the range $[D/6, D/2]$ and the Reynolds number based on the disk half-radius is of the same order of magnitude as the one defined above. For the results presented here, Re is in the range $[10^5, 10^6]$.

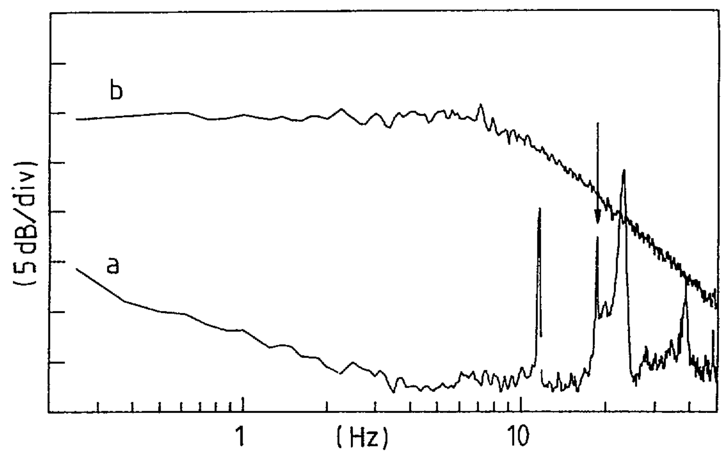

Fig. 2. — Pressure spectra in the co-rotating (a) and counter-rotating (b) cases. For both cases, the absolute value of the rotation rates is 1100 RPM. The peak shown by an arrow corresponds to the rotation frequency. The other peaks correspond to bursting frequencies of the axial vortex generated in the co-rotating case.

When the two disks are co-rotating (at the same or different rates), the flow mostly consists of a strong vortex located between the two disks in the vicinity of the rotation axis. Depending on the rotation rates $\Omega_{1,2}$, this vortex is straight or displays wavy instabilities which often lead to quasiperiodic burstings [14]. The pressure spectrum of figure 2a displays peaks at the bursting frequencies. Using a position sensitive detector, we have measured the deflection of a laser beam by the fluctuating axial vortex, and we have observed that the resulting signal is correlated with the pressure signal at the bursting frequencies [15]. Our direct pressure measurement thus correctly detects the vorticity dynamics.

The flow structure is completely different when the disks are counter-rotating. The axial vortex is suppressed when the disks are rotated in opposite directions, even if one has a much smaller velocity than the other. When the two disks are counter-rotating at the same rate, the flow in the median region of the cell looks rather homogeneous when a turbulent regime is reached, as described in reference [11]. The corresponding pressure spectrum is displayed in figure 2b for comparison with the co-rotating case. In both cases, the absolute value of the rotation rates is 1100 RPM. The comparison of spectra (a) and (b) shows that the much larger pressure fluctuations in (b) can be ascribed to the strong turbulent flow generated in the counter-rotating case. The spectrum is roughly flat up to 10 Hz. As said above, we should be cautious about any quantitative measurement of the high frequency part since high frequency perturbations are poorly resolved if their spatial wavenumber is large.

A direct time-recording of pressure fluctuations in the counter-rotating case is shown in figure 3. It is clearly visible that pressure fluctuations are asymmetric, with random occurrence of strong pressure drops. Using equation (1), we relate the low pressure events in the direct time-recording of figure 3 to intermittent vorticity concentrations in the vicinity of the pressure transducer. Although we have not yet performed detailed spatial measurements, we have checked that the pressure signals recorded by two transducers 2.5 cm far apart display no coherence. The intermittent pressure drops therefore correspond to localized events and not to a large scale coherent vorticity burst.

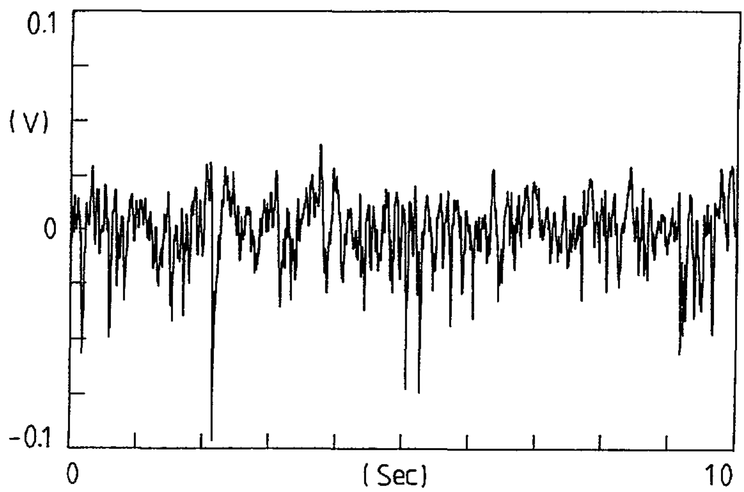

Fig. 3. — Time-recording of pressure fluctuations in the counter-rotating case, $\Omega_1 = -\Omega_2 = 810$ RPM. (0.1 V corresponds to 2600 pa).

It has also been reported in numerical simulations that the correlation of low pressures with intense vorticity regions generates a non-Gaussian pressure p.d.f. strongly skewed toward low pressures [6, 16]. The p.d.f. of p/Ω^2 obtained with counter-rotating disks is displayed in figure 4 for two different rotation rates. It is strongly non-Gaussian with a roughly exponential tail toward low pressures, due to the intermittent occurrence of pressure drops. It compares very well with the p.d.f. obtained in numerical simulations (see Fig. 9 of Ref. [6] and Fig. 5f of Ref. [16]). The p.d.f.s of p/Ω^2 for $\Omega = 810$ RPM and $\Omega = 1250$ RPM roughly collapse on a single curve thus showing that large scale velocity fluctuations give the largest contribution to the pressure fluctuations.

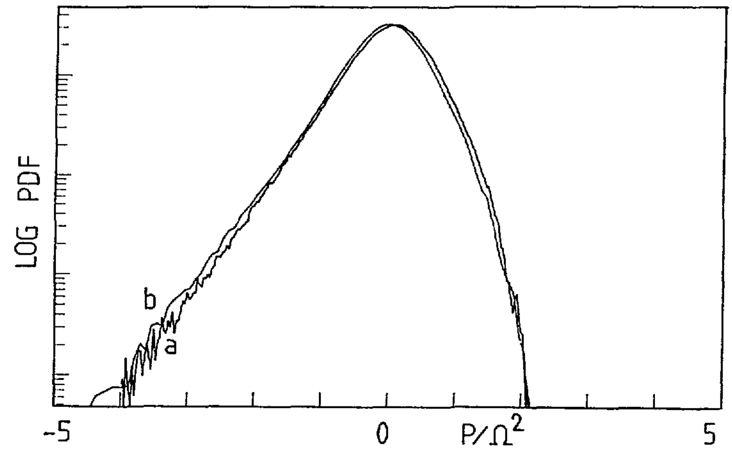

Fig. 4. — Probability density functions of the pressure in the counter-rotating case: (a) $\Omega_1 = -\Omega_2 =$ 810 RPM, (b) $\Omega_1 = -\Omega_2 = 1250$ RPM. (One unit corresponds to 0.1 pa.s^2).

4. Discussion.

The low pressure events occur for high enough rotation rates, when direct flow-visualization with air bubbles shows the intermittent formation of filaments of bubbles, which result from vorticity concentration [11]. Tube-like high vorticity regions were observed first in direct numerical simulations of three-dimensional turbulence [7], and studied in great detail recently [8-10]. As for the numerical simulation of the Taylor-Green flow of reference [6], we relate the intermittent pressure drops to the formation of tube-like vortical structures.

More specifically, we will show that we can study the geometry of structures where strong velocity-gradients develop from the shape of the pressure p.d.f. From equation (1) which looks like the Poisson equation for the electrostatic potential, low pressures are generated in the vicinity of structures with $\omega^2 - \sigma^2$ maximum, i.e. in high vorticity regions, whereas high pressures correspond to minima of $\omega^2 - \sigma^2$, i.e. to dissipative structures with large σ^2 [5, 6].

Let us call δ the capacity dimension of these structures, i.e.

$$\int_B (\omega^2 - \sigma^2) \mathrm{d}^3 r \propto \pm r^\delta, \tag{2}$$

on a ball B of size r, where the $\pm$ signs are related to vortical (respectively dissipative) structures. Then Gauss's theorem and equation (1) give, $|\nabla p| \propto r^{\delta-2}$ Thus,

$$p = p_- \operatorname{Log} \frac{r}{L}, \quad \text{if } \delta = 1, \tag{3}$$

and

$$p = p_+ \left[1 - \left(\frac{r}{L}\right)^{\delta-1}\right], \quad \text{if } \delta > 1, \tag{4}$$

where r is the distance to the corresponding structure and L some scale, larger than the characteristic one of the high velocity-gradient structure. The probability $\pi(p)\mathrm{d}p$ of observing a low (respectively large) pressure in the range $[p,\ p+\mathrm{d}p]$ is proportional to the probability of being at a distance r within $\mathrm{d}r$ from vortical (respectively dissipative) structures, which scales as $\mathrm{d}\left(r^{3-\delta}\right)$. The p.d.f. for large pressure deviations is therefore

$$\pi(p) \propto \frac{1}{p_-} \exp \frac{2p}{p_-}, \quad \text{if } \delta = 1, \tag{5}$$

and

$$\pi(p) \propto (p_+ - p)^{\alpha}, \quad \text{if } \delta > 1, \tag{6}$$

with

$$\alpha = \frac{4-2\delta}{\delta-1}, \quad \text{or} \quad \delta = \frac{\alpha+4}{\alpha+2}$$

As said above, the low pressure part of the p.d.f. displays an exponential tail. Therefore $\delta = 1$, and in agreement with numerical simulations [6-10], we find that vorticity concentrates on tube-like structures [17].

Let us now consider the high pressure part of the p.d.f. Figure 5 shows that $[\pi(p)]^{1/4}$ displays a roughly linear behaviour *versus* p. Thus :

$$\alpha \approx 4 \quad \text{and} \quad \delta \approx \frac{4}{3} \approx 1.33.$$

This is again compatible with the results of numerical simulations which show that dissipative structures occur on nearly two-dimensional sheets [6].

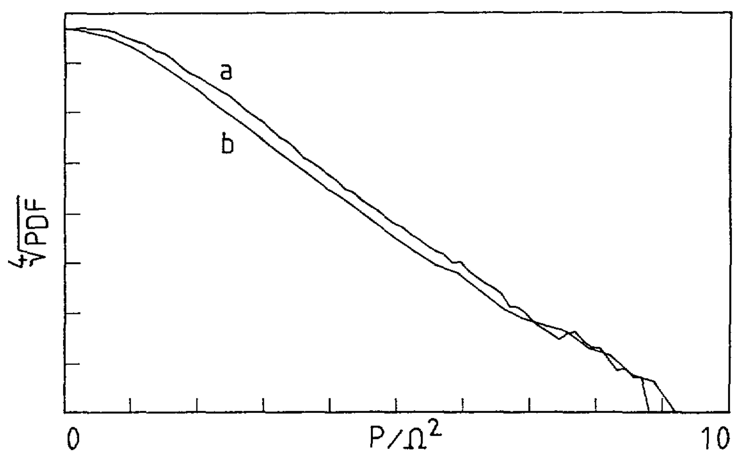

Fig. 5. — Probability density functions of the positive pressure fluctuations to the power 1/4 in the counter-rotating case: (a) $\Omega_1 = -\Omega_2 = 810$ RPM, (b) $\Omega_1 = -\Omega_2 = 1250$ RPM. (One unit corresponds to 0.025 pa.s^2).

In summary, starting with the ansatz (2), we have shown that one can obtain insights on the geometry of structures where strong velocity-gradients develop. More specifically, we have related the asymmetry of the pressure p.d.f. to the different dimensions of structures where vorticity (respectively dissipation) concentrates.

Finally, if no parallelism occurs between vortical and dissipative structures, the dimension of the space orthogonal to structures where strong velocity-gradients develop is given by $3 - (1 + 1.33) \approx 0.66$. In a recent paper [18], the intermittency of two-point velocity differences was analyzed, introducing an exponent β. It was suggested that its physical meaning would be the dimension of a space orthogonal to structures where strong velocity-gradients develop. For Taylor-scale based Reynolds number $R_\lambda < 400$, β was found to be nearly constant : $\beta \approx 0.6$. The present experiment thus confirms the physical interpretation of β.

In conclusion, we have measured the pressure p.d.f. and showed that it provides a simple way to study the geometry of the vortical and dissipative structures in turbulent flows. Our model is simple and its starting assumption (2) can be checked on existing numerical data. It shows that the important quantity to look at is the distribution of $\omega^2 - \sigma^2$ in the vicinity of the vortical or dissipative structures. From the experimental point of view, the most questionable point is that one needs to assume that the one-point pressure distribution is identical to the one of the whole cell. This is usually done for many turbulence measurements, but here one might expect modification of the flow in the vicinity of the boundary where the pressure transducers are located. We can argue that the pressure is non-local, i.e. it is generated by velocity-gradients in a whole volume in the vicinity of the transducer. Anyway, the striking similarity of our results with the numerically computed p.d.f.s tends to show that this last assumption is also reasonable. We hope that this discussion will trigger new experimental and numerical works to check our model.

Acknowledgements.

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[17] It is not realistic to use a simple model of a tube-like vortical structure (stationary axisymmetric vortex solutions of the Euler equation or exact time-dependent axisymmetric vortex solutions of the Navier-Stokes equation), to estimate the pressure field distributions. First, vortical stuctures are generated from fully 3D-flows on a very short time-scale. Second, these simple models have velocity-increments within r linear in r, whereas velocity-increments of a 3D turbulent flow are in r^{α}, with $\alpha = 1/3$ in Kolmogorov picture and α distributed around $1/3$ in experimental measurements. One has thus to invoke either time-dependence or a fancy spatial distribution which will completely modify the pressure scaling. The velocity-gradients being badly approximated, the above simple models must be quite off the mark for predictions about the pressure field.

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