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## Comment on “Stress Propagation and Arching in Static Sandpiles” by J.P. Wittmer *et al.*

### About the Scaling Hypothesis of the Stress Field in a Conic Sandpile

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**Abstract.** — This paper is a short comment about the paper “stress propagation and arching in static sandpiles” by Wittmer, Cates and Claudin [1]. It concerns the validity of the radial stress field (RSF) scaling assumption used to calculate the stress distribution in a conic granular material and its agreement with different experimental data.

**Résumé.** — On discute la validité de l’hypothèse d’un champ de contrainte radial utilisée par Wittmer, Cates et Claudin [1] pour prédire le champ de contrainte dans un milieu granulaire cône.

A step [1] has been performed recently in the understanding of the minimum of vertical stress which exists just at the center of a conic sandpile [2]. This is a nice result whose hypotheses deserve to be examined in detail. In particular, it seems worth identifying points which may not be completely satisfactory. This is the purpose of this paper, which does not want to be exhaustive however. It focuses only on discussing the symmetry of the phenomenon: in reference [1] this symmetry is assumed to be such as the stress distribution obeys the so-called radial stress field (RSF) scaling, which turns to be necessary to calculate the stress field; reference [1] mentions in particular that this assumption is supported by other theoretical works [3, 4], which used the same assumption, and claims endly that this hypothesis is supported by the experimental results of Smid and Novosad [2]. Here, I would just want to limit the validity of this symmetry assumption. I start discussing the real meaning of experimental data [2], I continue the discussion using other experimental results and ends with some theoretical argument.

As a matter of fact, in reference [2] Smid and Novosad measured the stress distribution at the bottom of a series of conic piles, built in the same manner on the same “rigid” basis, but

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which had different pile heights  $h$ . These authors found that the bottom stress distributions obey a scaling law whose scaling coefficient was  $\rho gh$  (where  $g$  is the gravity field,  $\rho$  the pile density). But no stress measurement was done deep in the pile itself in this paper; so, if there is no doubt that these results are compatible with some dilation symmetry and with RSF scaling, they cannot be considered as a proof of them.

We are then faced to discuss both assumptions separately. We start with the dilation symmetry and we note  $\rho_h(r, z)$  the stress field  $\sigma$  at a point of coordinates  $r, z$  in a pile of height  $h$ ; dilation symmetry assumes that  $\rho_h(r, z) = \sigma_{h_0}(rh_0/h, zh_0/h)h/h_0$ . According to the building process which does not change from pile to pile, the dilation symmetry is more or less imposed by the building process probably. However I will show in a while that avalanche experiments contradict this hypothesis; nevertheless, let us consider it as partly valid.

This does not mean yet that the stress distribution obeys RSF scaling, since this one requires that  $\sigma_h(\alpha r, \alpha z) = \alpha \sigma_h(r, z) = \alpha \sigma_{h_0}(r, z)$  as can be deduced from equation (11) of reference [1]. So, it requires that the pile does not exhibit a typical length scale (this is well stated in [1]). But the pile has a typical length scale, which is its height. It is then worth discussing this second assumption. In fact RSF scaling denies any specific effect induced by the bottom boundary condition, and it assumes that one shall get the same stress distribution starting from a large pile and cutting it by mind at a given height or by building it directly from the top at this height. So, i) it does not take into account the fact that the grains can move down at any point of the free inclined surface except at the bottom boundary, ii) this assumption attributes too little influence to the sand-plate friction probably [5]. This can be verified experimentally by changing the material of the bottom plate. This experiment has not been done by Smid and Novosad [4] unfortunately; but one can find in [6] few non exhaustive references of experimental works which did it and which have demonstrated that the stress dip does depend on the basis deflection, on the grain shape and density, on the gauges, . . . , which queries the RSF scaling.

Furthermore, what can be said from slope stability experiments: first of all, slope stability experiments show that avalanches start from the upper part of the pile [7]; this queries also the validity of RSF scaling. Secondly, centrifuge experiments in variable gravity have demonstrated that one can generate an avalanche increasing the gravity field [7, 8]; this last experiment is not only in contradiction with the RSF scaling, but also with dilation symmetry.

Let us now come to the theoretical discussion: In [1], the yield criterion of Coulomb is assumed to be satisfied in the outer volume of the pile, which is contained in between two cones; and the direction of yielding is parallel to the free surface and never parallel to the rigid basis. This condition of flow is possible everywhere in the bulk, but it cannot be satisfied at the boundary condition, since the flow shall be parallel to the plate there. So, the bottom boundary condition has to perturb the stress field in the pile bottom zone and the RSF scaling shall fail in this region.

This comment shows that more investigation has to be performed for a complete understanding of the problem. It is also aimed at emphasizing the difficulty of getting an exact rigorous solution to a "simple" experiment, and at recalling that the reconstruction of a stress field from few data points is always a difficult operation which requires important assumptions; this is due to the fact that the inverse problem has not a unique solution in this peculiar case so that we are just in a situation similar to that one explained by Platon in his myth of the cavern.

Anyway, the RSF scaling is probably not valid, and the Wittmer's approach too; this is not bothering really since the explanation of the dip does not rely on it: first we notice that the Wittmer *et al.* approach demonstrates the existence of a series of possible solutions exhibiting a dip in the center and obeying a high degree of symmetry. Second we note i) that reducing the symmetry of the problem increases the number of possible solutions and ii) that the solutions with higher symmetries fall as peculiar cases of this new series. So, we conclude

that few adequate stress field solutions shall also exist exhibiting a lower symmetry and a dip of vertical stress in the center. On the contrary, let us assume for a while that the RSF scaling is really valid in Smid and Novosad experiments [2]; the right question would be in this case to understand why.

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