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# Change in Neutron Polarization Induced by a Superconductor Rotated in a Magnetic Field 

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#### Abstract

Neutron depolarization (i.e. the change in the polarization vector projection onto the applied magnetic field) depends on neutron Larmor frequency $\omega_{L}$ in the superconductor and on a frequency $\omega, \omega$ being related to the directional variation of the magnetic induction inside the superconductor. Depolarization is small if $\eta=\omega_{L} / \omega \gg 1$. In the opposite limit $\eta \ll 1$, the depolarization is due to neutron spin rotation about the magnetic flux, which, in turn, rigidly rotates with a superconductor. A total spin reversal is possible at $\eta=1$. Depolarization at a stationary rotation angle in a superconducting plate carrying a transport critical current in a magnetic field is considered as well. Depolarisation is determined by either longitudinal or transversal critical currents depending on the angle between the applied field and the critical current direction.


## 1. Introduction

The rotation of a thin superconducting disk in a magnetic field parallel to its planar surfaces has been investigated both experimentally and theoretically. Boyer studied a rotation of Nb disk about the axis perpendicular to its planes and suggested the phenomenological model for rotation process [1]. Clem and Perez-Gonzalez [2,3] developed the critical state model for rotation of a superconductor in a magnetic field. Liu [4,5] has recently studied a rotation of both $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-x}$ and Nb disks and found very similar behavior of these superconductors during their rotation. This fact is difficult to understand in the frame of the critical state model $[2,3]$ as the magnetic flux penetration into low temperature and high temperature superconductors is quite different. Liu suggested another phenomenological model [6].

In our preliminary report [7] we used the neutron depolarization technique to study the magnetic flux distribution in a $\mathrm{YBa}_{2} \mathrm{Cu}_{3} \mathrm{O}_{7-x}$ disk rotated in a magnetic field. We observed the oscillations of neutron polarization with increasing the rotation angle after the superconductor was cooled down to 4.2 K in an applied magnetic field of 0.628 kOe . We suggested that they were due to the rigid rotation of the trapped magnetic flux with a superconductor. This rigidly rotated magnetic flux is present in only a small part of the superconductor volume and decreases with increasing the rotation angle.

[^0]

Fig. 1. - a) The neutron velocity $\mathbf{V}$ is parallel to the $x$ axis of rotation. $\mathbf{P}_{0}$ and $\mathbf{P}_{f}$ are the polarizations before and after the disk correspondingly. The polarization doesn't change its size: $\left|\mathbf{P}_{0}\right|=\left|\mathbf{P}_{\mathbf{f}}\right|$. Only $P=|\mathbf{P}|$ is measured by our neutron one-dimensional depolarization technique. b) In the peripheric regions 1 and 3 the rotating frame is used. Neutron spin rotates there about $\mathbf{B}_{\text {eff }}$ and acquires a phase $\varphi$ in each region. In the central region the polarization rotates about $\mathbf{B}$, which is of one direction, and gets a phase $\psi$ in the neutron frame.

Depolarization at large rotation angles where the oscillations were absent, was likely to be related to the complicated magnetic flux structure inside the rotated superconductor rather than to the rigidly rotated magnetic flux.

In this paper, we consider neutron precession in the rotated superconductor in the frame of the critical state model $[2,3]$. Besides, precession in a superconducting plate carrying the transport critical current is considered.

## 2. Neutron Precession in a Superconductor Rotated in a Magnetic Field

We consider a high irreversible type II superconducting thin disk in an applied magnetic field, which is much more than the first critical one. Therefore we can neglect the contribution of magnetization to the induction to good approximation.

According to the critical state model $[2,3]$ the profile of the magnetic induction inside a superconductor is the same along all the paths perpendicular to the planar disk surfaces. Therefore neutrons don't experience any intrinsic depolarization as they cross the superconducting disk along these trajectories. The neutron polarization $\mathbf{P}_{0}$ parallel to an applied magnetic field $\mathbf{B}_{0}$ before the superconducting disk changes its direction in a complicated way inside the superconductor, the size of the polarization vector being constant. As a matter of fact, we only measure the component $P$ of final polarization which is parallel to the applied magnetic field as shown in Figure 1a. The polarization vector component parallel to the applied magnetic field can be only measured in the field region mentioned above, typically more than 1 kOe . The finite spectral width of the monochromatic neutron beam which is used in one- or threedimensional depolarization techniques results in the strong instrumental depolarization of the perpendicular component in these high applied magnetic fields.

Such a precession of the polarization vector has been calculated by Newton [8] as neutrons cross one domain wall by contrast to the intrinsic depolarization in magnetic domain materials [9]. We neglect the intrinsic depolarization in a superconductor due to the vortex line misalignments relative to the applied magnetic field direction. The experimental studies show that the intrinsic depolarization is very small in a thin superconducting disk [10].

The rotation of neutron polarization, which we call depolarization due to the limitation of our measurements, reflects the magnetic flux distribution inside the rotate superconductor and depends on its initial magnetic state before rotation. We consider the diamagnetic, paramagnetic and nonmagnetic initial states of a superconductor.
2.1. Diamagnetic Initial State. - The diamagnetic initial state is produced by cooling a superconductor through the critical temperature in zero field and then applying a magnetic field. According to the critical state theory $[2,3]$ the size $B(x)$ of the magnetic induction $\mathbf{B}$ and its direction $\theta(x)$ inside a thin disk of superconductor depend on the two parameters $\mu=k x_{\mathrm{m}}$ and $\chi=k x_{0}$. Here $2 x_{\mathrm{m}}$ is the thickness of a disk, $k=\mathrm{d} \theta / \mathrm{d} x$ and $x_{0}$ is the distance from the surface at which the induction $B$ is reduced from $B_{0}$ to zero before rotation.

The angle $\theta(x)$ is that between induction $\mathbf{B}$ and applied magnetic field $\mathbf{B}_{0} . B(x)$ and $\theta(x)$ follow relations [2,3].

$$
\begin{array}{rll}
B(x) & =B_{0}\left(1-x / x_{0}\right), & \\
B \leq x \leq x_{0} \\
B=0 & & x_{0} \leq x \leq x_{\mathrm{m}}  \tag{2}\\
& \theta(x)=k x, & x \leq x_{\mathrm{c}} \\
& \theta(x)=\theta, & x>x_{\mathrm{c}}
\end{array}
$$

Here $B_{0}$ is the applied magnetic field, $\theta$ is rotation angle of the superconducting disk, $x_{\mathrm{c}}=\theta / k$. It follows from relations $(1,2)$ that the magnetic flux in the central spatial region of thickness $2\left(x_{\mathrm{m}}-x_{\mathrm{c}}\right)$ rigidly rotates with the disk. A quasi steady distribution of $B$ is achieved when $\theta=\chi$ for $\chi \leq \mu$ and $\theta=\mu$ for $\chi>\mu[2,3]$.

We note that calculations of neutron polarization are only possible numerically when the induction $B$ is given by relations (1, 2). However a relation for neutron depolarization can be derived in the practically important case of the thin disk and sufficiently high applied magnetic fields. Relation (1) gives that the variation of induction is $\Delta B(x) / B_{0}=\mu / \chi \ll 1$ for this approach. Relation (2) shows that the induction turns in a spatial region $0 \leq x \leq x_{\mathrm{c}}$ with a constant frequency $\omega=k v$ in the neutron frame, where $v$ is the neutron velocity. It is now convenient to use the rotating frame, which rotates with a frequency $\omega$ about the $x$ axis, within two peripheric regions shown in Figure 1b. As it is well known [11], only the effective magnetic field $\mathbf{B}_{\text {eff }}=\mathbf{B} \pm \boldsymbol{\omega} / \gamma$ will affect the neutron spin in this frame. The magnetic induction in the central region is of one direction, that is $\omega=0$. Therefore the neutron frame should be used in this region. As the magnetic induction is pointing in one direction in each of the frames shown in Figure 1b, the following relations [9] can be used to predict the final polarization vectors $\mathbf{P}_{\mathrm{f}_{2}}(i=1,2,3)$ in the 1,2 and 3 spatial regions

$$
\begin{align*}
\mathbf{P}_{\mathrm{f}_{\imath}} & =\mathbf{P}_{0_{\imath}} \cos \beta_{\imath}+\left(\mathbf{m}_{\imath} \cdot \mathbf{P}_{0 \imath}\right) \mathbf{m}_{\imath}\left(1-\cos \beta_{\imath}\right)+\left[\mathbf{m}_{\imath} \mathbf{P}_{0_{\imath}}\right] \sin \beta_{\imath}  \tag{3}\\
\beta_{\imath} & =(\gamma / v) \int\left(B_{\mathrm{eff}}\right)_{\imath}(x) \mathrm{d} x \tag{4}
\end{align*}
$$

Here $\mathbf{P}_{0_{2}}$ are the initial polarization vectors, $\gamma$ and $v$ are the gyromagnetic ratio and the velocity of the neutron respectively, $\mathbf{m}_{2}=\left(\mathbf{B}_{\text {eff }}\right)_{2} /\left(B_{\text {eff }}\right)_{2}$. Besides one should take into account that the polarizations at the boundary of the two adjacent regions are equal to each other, that is $\mathbf{P}_{0}=\mathbf{P}_{01}, \mathbf{P}_{\mathbf{f} \imath}=\mathbf{P}_{0 \imath+1}, \mathbf{P}_{\mathrm{f}}=\mathbf{P}_{\mathrm{f} 3}$. We find for the polarization $P=\left(\mathbf{P}_{\mathrm{f} 3} ; \mathbf{m}_{3}\right)$

$$
\begin{align*}
P / P_{0} & =d \cos \delta\left(P_{3} \cos \delta-P_{1} \sin \delta\right)+P_{3} \cos \varphi-P_{2} \sin \varphi \sin \delta  \tag{5}\\
P_{1} & =d \cos \delta \sin \delta \cos \psi+\sin \varphi \sin \delta \sin \psi  \tag{6}\\
P_{2} & =d \cos \delta \sin \delta \sin \psi-\sin \varphi \sin \delta \cos \psi  \tag{7}\\
P_{3} & =\cos \varphi \cos \psi+d \cos \psi \cos ^{2} \delta+c\left[\cos \varphi+d \cos ^{2} \delta\right]  \tag{8}\\
d & =1-\cos \varphi, \quad c=1-\cos \psi \tag{9}
\end{align*}
$$



Fig. 2. - The polarization $P / P_{0}$ as a function of rotation angle $\theta$ for a diamagnetic initial state. (1) and (2) are obtained from a numerical calculation and relation (13) respectively at $\chi=\mu=100$, $\eta_{0}=0.1$. (3) is for $\mu=\pi / \sqrt{2}, \chi / \mu=10, \eta_{0}=1$.

Introducing the parameter of adiabaticity $\eta_{0}=\gamma B_{0} / \omega$, the phases $\varphi, \delta$ can be written as follows

$$
\begin{align*}
\varphi & =\theta\left(1+\eta_{0}^{2}\right)^{1 / 2}, \quad \psi=2 \eta_{0}(\mu-\theta), \quad \theta \leq \mu  \tag{10}\\
\varphi & =\mu\left(1+\eta_{0}^{2}\right)^{1 / 2}, \quad \psi=0, \quad \theta>\mu  \tag{11}\\
\delta & =\operatorname{arccot}\left(\eta_{0}\right) \tag{12}
\end{align*}
$$

Relations (5-12) show that the polarization $P / P_{0} \Rightarrow 1$ if $\eta_{0} \Rightarrow \infty$, so that a neutron spin adiabatically follows the variations of $B(x)$ inside the superconductor. Another limit is $\eta_{0} \Rightarrow 0$. The polarization is given by

$$
\begin{equation*}
P / P_{0}=\cos ^{2} \theta+\sin ^{2} \theta \cos \psi \tag{13}
\end{equation*}
$$

Relation (13) means that the depolarization is due to the neutron spin precession about the direction of induction within the central part of superconductor, where the magnetic induction is rigidly rotated with the superconductor. It is evident that relation (13) holds for $\chi$ and $\mu$ other than $\chi \gg \mu$, if the condition $\eta(x)=\eta_{0}\left(1-x / x_{0}\right) \ll 1$ or $\eta_{0} \ll 1$ is satisfied throughout the superconductor. The formulas for the phase $\psi$ are easily deduced from relation (4). One obtains for $\chi \leq \mu$

$$
\begin{equation*}
\psi=\left(\eta_{0} / \chi\right)(\chi-\theta)^{2}, \quad \theta \leq \chi \quad \text { and } \quad \psi=0, \quad \theta>\chi \tag{14}
\end{equation*}
$$

and for $\ell>\mu$

$$
\begin{equation*}
\psi=\left(\eta_{0} / \chi\right)(\mu-\theta)(2 \chi-\theta-\mu), \theta \leq \mu \quad \text { and } \quad \psi=0, \quad \theta>\mu \tag{15}
\end{equation*}
$$

It follows from relations (13-15) that a large depolarization is expected for $\chi>1 / \eta_{0} \gg 1$. Depolarization due to the rigid magnetic flux rotation is absent at the large rotation angles, when the rotated superconductor is in a quasi steady state. Figure 2 represents the neutron polarization $P / P_{0}$ versus the rotation angle $\theta$ in the limit $\eta_{0} \ll 1$. Disagreement between numerical calculations and relation (13) is related to the contributions of the peripheric regions


Fig. 3. - Precession of neutron polarization in neutron rotating frame at $\eta_{0}=1$ and $\mu=\pi / \sqrt{2}$ for $\chi / \mu \gg 1$ after superconductor has reached a quasi steady state in which the central region shown in Figure 1 is absent.
shown in Figure 1b, which increase with the rotation angle. Note that all our numerical calculation are referred to the neutron velocity of $3.5 \times 10^{5} \mathrm{~cm} / \mathrm{s}$ and the superconductors thickness of $2 x_{\mathrm{m}}=0.02 \mathrm{~cm}$.

There is a very interesting situation at $\eta_{0}=1$ and $\chi \gg \mu$. The rotated superconductor enters a quasi steady state at $\theta=\mu[2,3]$. We find from relations (9-12) that $\ddot{\psi}=c=0$, $\delta=\pi / 4$ and $\varphi=\mu / \sqrt{2}$ at $\theta=\mu$. Substituting these values into relations (5-8) one obtains for the neutron polarization

$$
\begin{equation*}
P / P_{0}=\cos (\mu \sqrt{2})+0.5 \sin ^{2}(\mu \sqrt{2}) \tag{16}
\end{equation*}
$$

Relation (16) gives that there is a total spin reversal at $\mu=\pi / \sqrt{2}$. Figure 3 displays the precession of neutron polarization in the neutron rotating frame which results in a total spin flip. Approaching the quasi steady state is presented in Figure 2.

Apart from the total neutron spin reversal the numerical calculations show that remarkable depolarization is expected if there is a point $x_{1}$ at which $\eta\left(x_{1}\right)=1$ somewhere inside superconductor. The position of $x_{1}$ is given by

$$
\begin{equation*}
x_{1}=x_{0}\left(1-1 / \eta_{0}\right) \tag{17}
\end{equation*}
$$

Relation (17) is valid if $\eta_{0} \geq 1$ for $\chi \leq \mu$ and $1<\eta_{0} \leq(1-\mu / \chi)^{-1}$ for $\chi>\mu$. Depolarization increases when the point $x_{1}$ is near a peripheric region or inside it, that is $x_{\mathrm{c}}>x_{1}$ or $\theta \gtrsim$ $k x_{1}=\theta_{1}$. Figure 4 shows that depolarization increases for $\theta \gtrsim \theta_{1}=52^{\circ}$ and superconductor reaches the quasi steady state at $\theta_{\text {st }}=57^{\circ}$.
2.2. Paramagnetic Initial State. - The paramagnetic initial state is produced when the initial applied magnetic field $B_{0}^{\prime} \lesssim B_{\mathrm{c} 2}$ in diamagnetic state decreases to $B_{0} \ll B_{\mathrm{c} 2}$. According to the critical state model [13] induction increases with the distance from the planar disk surfaces by contrast to the diamagnetic initial state. While the disk is rotated the induction follows relation (1) but in contrast with the diamagnetic initial state only for $x<x_{\mathrm{v}}<x_{0}$ and it is given by

$$
\begin{equation*}
B(x)=B_{0}\left(1+x / x_{0}-2 x_{\mathrm{v}} / x_{0}\right) \quad \text { for } x>x_{\mathrm{v}} \tag{18}
\end{equation*}
$$

Here $x_{\mathrm{v}}$ is the position of the minimum for $B(x)$ and it can be obtained by solving the system of two differential equations [3]. The angle $\theta(x)$ is given by relations (2). The change in $B(x)$ is $\Delta B / B_{0}=\mu / \chi-k x_{v} / \chi \ll 1$ if $\chi \gg \mu$ as $\theta_{v}=k x_{v} \leq \mu$ [2]. Therefore relations (5-12) are valid for $\chi \gg \mu$.


Fig. 4. - Polarization $P / P_{0}$ versus rotation angle $\theta$ at $\eta_{0}=10$ : (1) is for diamagnetic initial state at $\ell=\mu=1 ;(2)$ is for nonmagnetic initial state at $\chi=1$ and $\mu=\eta_{0}=10$.

As in Section 2.1 the depolarization is given by relation (13) if the neutron crosses nonadiabatically the peripheric regions. ..e. on condition that $\eta(x) \ll 1$. This inequality is valid if $\eta_{0} \ll(1+\mu / \chi)^{-1}$

If $\chi<\mu$, there is a rotation angle $\theta_{\text {st }}$ at which a superconductor enters the quasi steady state [2]. Depending on $\theta_{\text {st }}$ and $\chi$ two cases for the phase $\psi$ are possible. The first one takes place if $\theta_{\text {st }} \leq \mu$. The phase $\psi$ then follows the relations

$$
\begin{array}{ll}
\psi=\left(\eta_{0} / \chi\right)(\mu-\theta)\left(2 \chi+\mu+\theta-4 \theta_{\mathrm{v}}\right) \equiv \psi_{1}(\theta) & \theta \leq \mu  \tag{19}\\
\psi=\psi_{1}\left(\theta_{\mathrm{st}}\right) & \theta \geq \mu
\end{array}
$$

The second one occurs if $\theta_{\text {st }} \geq \mu$. Then $\psi=\psi_{1}(\theta)$ for $\theta \leq \mu$ and $\psi=0$ for $\theta>\mu$. This latter dependence $\psi$ on $\theta$ is also valid for $\chi \geq \mu$ as there is no $\theta_{\text {st }}$ [3]. As in Section 2.1 a noticeable depolarization is expected for $\chi>1 / \eta_{0} \gg 1$.

A strong depolarization should be observed for $\eta(x)>1$, when a point $x_{\mathrm{v}}(\theta)$ approaches $x_{1}$ given by relation (17) during rotation. In turn, relation (17) is valid if $\eta_{0} \geq 1$ for $\chi<\mu$ and $1<\eta_{0} \leq(1-\mu / \chi)^{-1}$ for $\chi \geq \mu$. An increase of depolarization is expected as $\theta$ approaches $\theta_{1}$, where $\theta_{1}$ is a solution of equation $x_{\mathrm{v}}(\theta)=x_{1}$. Figure 5 shows that polarization changes strongly near $\theta \lesssim \theta_{1}=192^{\circ}$ and superconductor reaches the quasi steady state at $\theta_{\mathrm{st}}=293^{\circ}$.
2.3. Nonmagnetic Initial State. - The nonmagnetic state is produced by cooling a superconductor through the critical temperature in applied magnetic field.

We confine our consideration to the case of $\chi \leq 1$. Induction follows relation (1) for $x \leq x_{\mathrm{v}}$ $[2,3]$ and it is given by

$$
\begin{equation*}
B(x)=B_{0} \cos k\left(x-x_{c}\right) \tag{20}
\end{equation*}
$$

for $x_{\mathrm{v}}<x \leq x_{\mathrm{c}}$ and $B(x)=B_{0}$ for $x_{\mathrm{c}}<x \leq x_{\mathrm{m}}$. Here $x_{\mathrm{v}}$ is a position of $B(x)$ minimum, which is a solution of the functional equation derived in [3]. The angle $\theta(x)$ follows relation (2). A variation of $B(x)$ is $\left[B_{0}-B\left(x_{\mathrm{v}}\right)\right] / B_{0}=k x_{\mathrm{v}} / \chi \leq \mu / \chi \ll 1$ if $\chi \gg \mu$. Therefore relations (5-12) are valid for $\chi \gg \mu$.
If $\eta_{0} \ll 1$, the phase $\psi$ is

$$
\begin{equation*}
\psi=2 \eta_{0} \mu(\mu-\theta) \tag{21}
\end{equation*}
$$



Fig. 5. - Polarization $P / P_{0}$ as a function of rotation angle $\theta$ : (1) for paramagnetic initial state at $\chi=0.5, \mu=10$ and $\eta_{0}=5$. (2) and (3) are relation (13) and numerical calculation respectively for nonmagnetic initial state at $\chi=1, \mu=100$ and $\eta_{0}=0.1$.

Depolarization is expected to be large if $\mu>1 / \eta_{0} \gg 1$. Figure 5 represents the behavior of polarization for nonmagnetic initial state in the limit $\eta_{0} \ll 1$. Dependences of $P / P_{0}$ are very similar to those in Figure 2 for the diamagnetic initial state.

In our study [7] mentioned above we found that polarization at low applied fields follows relation (13). We should point out that our consideration is not valid for the ceramic superconductors. Our experimental studies of rotation process in nonceramic low temperature superconductors are in progress.

As in Section 2.2 depolarization increases for $\eta(x)>1$ or $\eta_{0} \geq 1$ when a point $x_{\mathrm{v}}$ approaches $x_{1}$ that is for $\theta \lesssim \theta_{1}$. Here $\theta_{1}$ as earlier is a solution of the functional equation $x_{\mathrm{v}}(\theta)=$ $x_{1}$. Figure 4 shows that polarization strongly decreases as $\theta$ is close to $\theta_{1}=135^{\circ}$. The superconductor enters the quasi steady state at $\theta_{\text {st }}=\pi / 2+\chi=147^{\circ}[3]$.

## 3. Neutron Depolarization in a Superconductor Carrying Transport Critical Current in a Magnetic Field

We now consider neutron depolarization in a thin superconductor plate carrying transport critical current in applied magnetic field parallel to its planes. The distributions of $\theta(x)$ and $B(x)$ have been derived in [14] for the case of $|B(x)-\bar{B}| / \bar{B} \ll 1$ and $|\theta(x)-\bar{\theta}| \ll 1$. Taking into account these inequalities we neglect the variations of $B(x)$ inside superconductor in the first approximation. According to [14] the angle $\theta(x)$ between induction $B(x)$ and the transport critical current direction depends on the angle $\theta_{0}$ of applied magnetic field and on the critical angle $\theta_{\mathrm{c}}=\arctan \left(I_{\mathrm{c} \perp} / I_{\mathrm{c} \|}\right)$. Here $I_{\mathrm{c}| |}$ and $I_{\mathrm{c} \perp}$ are the critical currents parallel and perpendicular to induction $\mathbf{B}$ relatively.

The angle $\theta(x)$ in a plate of thickness $2 x_{\mathrm{m}}$ is given by [14]

$$
\begin{equation*}
\theta(x)=\theta_{0} \pm \mu\left(x / x_{\mathrm{m}}-1\right) \tag{22}
\end{equation*}
$$

where the signs $\pm$ are referred to the regions $0 \leq \theta_{0} \leq \theta_{c}$ and $\pi-\theta_{c} \leq \theta_{0} \leq \pi$ correspondingly.


Fig. 6. - Polarization $P / P_{0}$ versus angle $\theta$ between applied magnetic field and transport critical current direction in superconductor plate with $I_{\mathbf{c} \|}=I_{\mathbf{c} \perp}$ and $\mu=0.1$ at $\eta_{0}=1$.

In the region $\theta_{\mathrm{c}} \leq \theta_{0} \leq \pi-\theta_{\mathrm{c}}, \theta(x)$ follows relations

$$
\begin{array}{ll}
\theta(x)=\theta_{0} \pm k\left(x-0.5 x_{\mathrm{d}}\right) & 0 \leq x \leq x_{\mathrm{d}}  \tag{23}\\
\theta(x)=\theta_{0} \pm x_{\mathrm{d}} / 2 x_{\mathrm{m}} & 0 \leq x \leq 2 x_{\mathrm{m}}
\end{array}
$$

where the signs $\pm$ are referred to the region $\theta_{c} \leq \theta_{0} \leq \pi / 2$ and $\pi / 2 \leq \theta_{0} \leq \pi-\theta_{c}$ relatively and $x_{\mathrm{d}}=2 x_{\mathrm{m}}\left|\cot \theta_{0}\right| / \cot \theta_{\mathrm{c}}$.

It follows from relations $(22,23)$ that the behavior of neutron polarization can be easily considered in the neutron rotating frame. The effective magnetic field in this frame is $\mathbf{B}_{\text {eff }}=$ $\mathbf{B}+\omega / \gamma$, where $\omega=v \mathrm{~d} \theta / \mathrm{d} x=k v$. As $P_{0}=\left(\mathbf{P}_{0} \mathbf{m}\right)$ and $P=\left(\mathbf{P}_{\mathrm{f}} \mathbf{m}\right)$ one obtains from relation (3) that

$$
\begin{align*}
P / P_{0} & =\left(1 / \eta_{0}^{2}\right)^{-1}\left(\eta_{0}^{2}+\cos \varphi\right)  \tag{24}\\
\varphi & =2 \mu \xi\left(1+\eta_{0}^{2}\right)^{1 / 2} \tag{25}
\end{align*}
$$

where $\xi=\left|\cot \theta_{0}\right| / \cot \theta_{\mathrm{c}}$ in the region $\theta_{\mathrm{c}} \leq \theta_{0} \leq \pi-\theta_{\mathrm{c}}$ and $\xi=1$ outside this region, $\eta_{0}=\gamma B_{0} / \omega$.

Relations $(24,25)$ give that $P / P_{0} \Rightarrow 1$ for $\eta_{0} \Rightarrow \infty$ like the behavior of $P / P_{0}$ in the rotated superconductors, whereas $P / P_{0}=2 \mu \xi$ for $\eta_{0} \Rightarrow 0$. As $2 \mu \xi$ is simply a rotation angle of $\mathbf{B}$ inside superconductor, we conclude that polarization $\mathbf{P}$ does not change in this limit and there is only projecting $\mathbf{P}_{0}$ on $\mathbf{B}$ when neutrons leave the superconductor.

As $|\theta(x)-\bar{\theta}| \leq \mu \ll 1$ one finds from relations $(24,25)$ that

$$
\begin{equation*}
P / P_{0}=1-2(\xi \mu)^{2} \tag{26}
\end{equation*}
$$

Finally taking into account $\mu=\left.\mu_{0} I_{\mathrm{c} \mid}\right|_{\mathrm{m}} / B_{0}$ [10] and $\xi$ one gets

$$
\begin{array}{rc}
P / P_{0}=1-A\left(I_{\mathrm{c} \mid}\right)^{2} & 0 \leq \theta_{0} \leq \theta_{c}, \pi-\theta_{\mathrm{c}} \leq \theta_{0} \leq \pi \\
P / P_{0}=1-A\left(I_{\mathrm{c} \perp} \cot \theta_{0}\right)^{2}, & \theta_{\mathrm{c}} \leq \theta_{0} \leq \pi-\theta_{\mathrm{c}} \tag{27}
\end{array}
$$

where $A=2\left(\mu_{0} x_{\mathrm{m}} / B_{0}\right)^{2}$.
Relation (27) shows that neutron depolarization measurements can be used to find both longitudinal and transversal critical currents in a superconductor. Figure 6 represents neutron
polarization for the superconductor with $I_{\mathrm{c}| |}=I_{\mathrm{c} \perp}$ and $\mu=0.1$ that corresponds to the critical current of $10^{5} \mathrm{~A} / \mathrm{cm}^{2}$ at $B_{0}=10 \mathrm{kOe}$ in the plate of thickness $2 x_{\mathrm{m}}=2 \times 10^{-2} \mathrm{~cm}$.

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## References

[1] Boyer R. and Le Blanc M.A.R., Solid. State Commun. 24 (1977) 261.
[2] Clem J.R. and Perez-Gonzales A., Phys. Rev. B 30 (1984) 5041.
[3] Perez-Gonzales A. and Clem J.R., Phys. Rev. B 31 (1985) 7048.
[4] Liu L. and Kouvel J.S., J. Appl. Phys. 67 (1990) 4527.
[5] Liu L., Kouvel J.S. and Brun T.O., Phys. Rev. B 38 (1988) 11799.
[6] Liu L. and Kouvel J.S., Phys. Rev. B 43 (1991) 7859.
[7] Zhuchenko N.K. and Jagood R.Z., Abstracts of International Conference on Neutron Scattering (Japan, 1994).
[8] Newton R.R. and Kittel Ch., Phys. Rev. 74 (1948) 1604.
[9] Maleev S.V., J. Phys. France 43 (1982) 7.
[10] Weber H.W., J. Low Temp. Phys. 17 (1974) 49.
[11] Farrar T.C. and Becker E.D., Pulse and Fourier Transform NMR (Acad. press, New York and London, 1971).
[12] Sbitnev V.I., Z. Phys. B 74 (1989) 321.
[13] Bean C.P., Rev. Mod. Phys. 36 (1964) 31.
[14] Clem J.R. and Perez-Gonzales A., Phys. Rev. B 33 (1986) 1601.


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