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# Surface Flows of Granular Mixtures: I. General Principles and Minimal Model

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**Abstract.** — We discuss theoretically the filling of a (two-dimensional) silo when a mixture of particles is poured at the center. Our basic tool is a coupled set of equations (for the local density of rolling species, and for the profile) proposed by Bouchaud *et al.* We extend this to a pair of species, including various processes: amplification, capture, and exchange of rolling grains. Using the simplest form of the resulting equations, we obtain predictions for the steady state profile and for the concentration distribution. At the bottom side of the slope, we generally expect to have *complete purification*. Near this point, the concentrations should show unusual power laws.

**Résumé.** — Nous discutons ici le remplissage d'un "silo" (bidimensionnel) quand un mélange de grains est versé au centre, à débit constant. Notre point de départ est la description à deux variables couplées proposée par Bouchaud *et al.*, où figurent la pente locale et la densité locale d'espèces roulantes. Nous étendons d'abord cette description à un mélange de deux populations, en incorporant divers mécanismes: amplification, capture, et échange. Ensuite, après avoir réduit la description à une forme minimale, nous pouvons prédire des profils de pente et de concentration. Au bas de la pente, on attend en général une purification complète: une seule espèce doit être présente. Près de ce point, on attend des profils exhibant des exposants non triviaux, dépendants des constantes de couplage.

## 1. General Aims

1.1. A REMINDER ON SURFACE FLOWS. — Avalanches are fascinating [1, 2]. An understanding of the simplest case (avalanche of a sand pile) has progressed significantly when the necessity of using two distinct degrees of freedom became clear: namely a local slope  $\theta(x, t)$ , and a local density of rolling species  $R(x, t)$ . This was first appreciated (in a slightly different language) by Mehta *et al.* [3–6]. A lucid discussion was given by Bouchaud *et al.* [7, 8]. Some of their conclusions (in particular concerning the "spinodal limit" for the slope) are still open for discussion [9]. But the basic equations, in their simplest form, are clearly valid. For the one dimensional problem depicted in Figure 1, the rolling species is ruled by:

$$\frac{\partial R}{\partial t} = \gamma(\theta - \theta_r)R + v \frac{\partial R}{\partial x} \quad (1)$$

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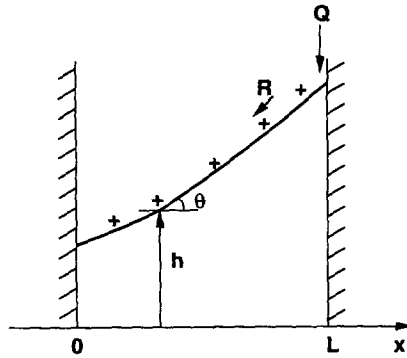


Fig. 1. — Steady filling of a two-dimensional silo by a source. Only the left half of the silo ( $0 \leq x \leq L$ ) is represented. The right half has a symmetric profile.

where  $\theta_r$  is the angle of repose: the first term describes capture if  $\theta < \theta_r$ , and amplification if  $\theta > \theta_r$ . The constant  $\gamma$  has the dimensions of a frequency, while  $R$  has the dimensions of a length (the “equivalent solid thickness” of the rolling species). The second term in equation (1) describes convection at a certain rolling velocity  $v$ . Dimensionally (for  $\theta_r \sim 1$ ) we expect  $v^2 \sim gd$  and  $\gamma \sim v/d$ , where  $d$  is the diameter of one grain.

The profile  $h(x, t)$  satisfies the conservation law:

$$\frac{\partial h}{\partial t} = -\gamma(\theta - \theta_r)R \tag{2}$$

and is related to the angle  $\theta$  by:

$$\frac{\partial h}{\partial x} = \tan \theta \sim \theta \tag{3}$$

(In all our discussion, we ignore the difference between  $\theta$  and  $\tan \theta$ ).

1.2. FILLING OF A ONE COMPONENT SILO. — An interesting application of this picture is described in Figure 1, where a (two-dimensional) “silo” is filled from a source, providing a quantity  $2Q$  of matter per unit time at some central point ( $x = L$ ), or equivalently a quantity  $Q$  in the left half section ( $0 < x < L$ ). A steady state builds up, and follows the following equations (for the left half):

$$\frac{\partial h}{\partial t} = w = \frac{Q}{L} \tag{4}$$

$$0 = \frac{\partial R}{\partial t} = v \frac{\partial R}{\partial x} + \gamma(\theta - \theta_r)R = v \frac{\partial R}{\partial x} - w \tag{5}$$

The boundary conditions are:

$$R(x = 0) = 0 \tag{6}$$

$$R(x = L) = Q/v \tag{7}$$

Equation (6) expresses the requirement that all rolling species be stopped at the bottom end. Equation (7) states that all the source is sent in the form of rolling particles.

The solution of equations (4, 5) is simply:

$$R = \frac{wx}{v} \quad (8)$$

$$\theta - \theta_r = -\frac{v}{\gamma x} \quad (\sim -\frac{d}{x}) \quad (9)$$

Thus the slope is nearly equal to  $\theta_r$  in most of the sand pile. But there is an inner region, near the bottom end, where the profile flattens out. This inner region has a size of maybe 10 times the grain diameter  $d$ , and is hardly observable.

1.3. MIXTURES. — The above results (8, 9) are not very surprising. But a question immediately arises. If a certain polydispersity (in size, or in shape, or in surface properties) exists in the powder, will this alter seriously the profiles? Segregation in granular mixtures is a major phenomenon [10–14] (and is often a great practical nuisance). It has led to a number of theoretical essays [15–18].

A special attention has been focussed on 3-dimensional systems, such as a rotating cylinder, where granular mixtures display remarkable striations along the axis [19–23].

Our aim here is to understand first the (seemingly) simpler problem of *two* dimensional flows. Our attention was drawn to this case by a remarkable experiment performed in a “granular Hele Shaw cell”: two transparent plastic plates, separated by  $\sim 3$  mm, form the two-dimensional silo, and a mixture of “red sugar” and “white sand” (with slightly different grain shapes) is poured at one point [24]. A steady state profile builds up, but it shows unexpected oscillations! Layers of white and red powder appear successively. The authors of reference [24] have modeled their system through a cellular automaton, and provide a plausible explanation for the oscillations. However, it is useful to complement the cellular automaton approach by a direct analysis of collisions and transport.

Here again, our attitude is to return to the simplest problem: filling of a 2d cell by a granular mixture as described by equations (1-3): and looking first for steady state solutions, without any oscillation. This turns out to be, in itself, a rather rich subject: downhill segregation is predicted to be important, and may lead to remarkable effects.

In section 2, we construct the phenomenological equations for coupled flows of two granular species. This introduces *a priori* a rather large number of unknown functions, describing the various processes (amplification, capture, exchange) as a function of slope and concentration. Fortunately, they can be reduced to a relatively small number of phenomenological parameters. In section 3, we attack the filling of a silo (in steady state). When the collision features are simplified to the utmost (the “minimal model”), we arrive at very compact equations for the local slope and the local concentrations. Finally, we solve these equations and find the profiles. Near the bottom end, they show some interesting singular power laws.

One important feature of our approach is that it is *not restricted to mixtures of spheres*: the case of two populations of spherical grains, differing only by their size has been often discussed, but is somewhat special, because the angle of repose of each pure component is the same. We are particularly interested here in the opposite case, where the angles are different.

## 2. Coupled Flows

2.1. GENERAL PRINCIPLES. — Our grains belong to two species:

- the “up” species ( $\uparrow$ ) which is sticky and has a large angle of repose  $\theta_{\uparrow 0}$ ,
- the “down” species ( $\downarrow$ ) which is less sticky and has a smaller angle of repose  $\theta_{\downarrow 0}$ .

It will be convenient to assume that the difference between the two species is not too large; in particular, we want the angular difference  $\psi = \theta_{\uparrow 0} - \theta_{\downarrow 0}$  to be small. This will imply that the profiles always occur in a narrow angular region; then, certain phenomenological coefficients can be taken as independent of angle in this region.

In the "solid" phase (just below the rolling species) there will be volume fractions  $\Phi_{\uparrow}(x, t)$  and  $\Phi_{\downarrow}(x, t)$  of the two species ( $\Phi_{\uparrow} + \Phi_{\downarrow} \equiv 1$ ) while in the rolling phase, the local densities will be called  $R_{\uparrow}, R_{\downarrow}$ .

The conservation equation gives:

$$\dot{R}|_{\text{coll}} = (\dot{R}_{\uparrow} + \dot{R}_{\downarrow})|_{\text{coll}} = -\dot{h} \quad (10)$$

where the dot denotes a time derivative, and  $\dot{R}|_{\text{coll}}$  denotes the contribution to  $\dot{R}$  due to collisions with the solid surface.

We can also write separate conservation equations for each species:

$$\dot{R}_{\alpha}|_{\text{coll}} = -\Phi_{\alpha}\dot{h} \quad (11)$$

where  $\alpha$  denotes  $\uparrow$  or  $\downarrow$ .

The rates will be described by a collision matrix  $\hat{M}$ :

$$\dot{R}_{\alpha}|_{\text{coll}} = M_{\alpha\beta}R_{\beta} \quad (12)$$

(where summation is performed over repeated indices). The elements  $M_{\alpha\beta}$  depend on the local slope  $\theta$ , and on the concentrations  $\Phi_{\alpha}$ . It is often convenient to write down the total rate  $\dot{R}|_{\text{coll}}$  in the form:

$$\dot{R}|_{\text{coll}} = M_{\uparrow}R_{\uparrow} + M_{\downarrow}R_{\downarrow} \quad (13)$$

where:

$$\left. \begin{aligned} M_{\uparrow} &= M_{\uparrow\uparrow} + M_{\downarrow\uparrow} \\ M_{\downarrow} &= M_{\uparrow\downarrow} + M_{\downarrow\downarrow} \end{aligned} \right\} \quad (14)$$

Finally, the transport equations for the rolling species will then have the form:

$$\dot{R}_{\alpha} = v_{\alpha} \frac{\partial R_{\alpha}}{\partial x} + \dot{R}_{\alpha}|_{\text{coll}} \quad (15)$$

where  $v_{\uparrow}, v_{\downarrow}$  are the rolling velocities for the two species. They can be taken as independent of the slope  $\theta$  since  $\psi$  is small. On the other hand, they may depend on  $\Phi_{\uparrow}, \Phi_{\downarrow}$ .

2.2. STRUCTURE OF THE COLLISION MATRIX. — Let us restrict our attention to *binary* collisions between one rolling grain and the immobile grains on the solid surface. Possible events are listed in Figure 2: they correspond to amplification, recoil, exchange, and capture.

a) *amplification*: the process where the two outgoing particles roll down, and both of them are  $\uparrow$  ( $\alpha = \uparrow$  in Fig. 2a) will contribute a term of the form  $a_{\uparrow}(\theta)\Phi_{\uparrow}R_{\uparrow}$  to  $\dot{R}_{\uparrow}|_{\text{coll}}$ . The analog process where the collision is between a  $\uparrow$  and a  $\downarrow$  particle gives a contribution  $x_{\uparrow}(\theta)\Phi_{\downarrow}R_{\uparrow}$  to  $\dot{R}_{\downarrow}|_{\text{coll}}$ ,

b) *recoil* does not contribute to  $\dot{R}_{\alpha}|_{\text{coll}}$ ,

c) *exchange* contributes a term  $y_{\uparrow}(\theta)\Phi_{\downarrow}R_{\uparrow}$  to  $\dot{R}_{\downarrow}|_{\text{coll}}$ , and a term  $-y_{\uparrow}(\theta)\Phi_{\downarrow}R_{\uparrow}$  to  $\dot{R}_{\uparrow}|_{\text{coll}}$ ,

d) *capture* gives a term  $-b_{\uparrow}(\theta)R_{\uparrow}$  in  $\dot{R}_{\uparrow}|_{\text{coll}}$ .

All our functions  $a_{\alpha}(\theta), b_{\alpha}(\theta), x_{\alpha}(\theta), y_{\alpha}(\theta)$  are positive:  $a_{\alpha}(\theta)$  and  $x_{\alpha}(\theta)$  are expected to increase rapidly with  $\theta$ , while  $b_{\alpha}(\theta)$  is a decreasing function. Ultimately, we shall write:

$$M_{\downarrow\uparrow} = [x_{\uparrow}(\theta) + y_{\uparrow}(\theta)]\Phi_{\downarrow} = m_{\uparrow}\Phi_{\downarrow} \quad (16)$$

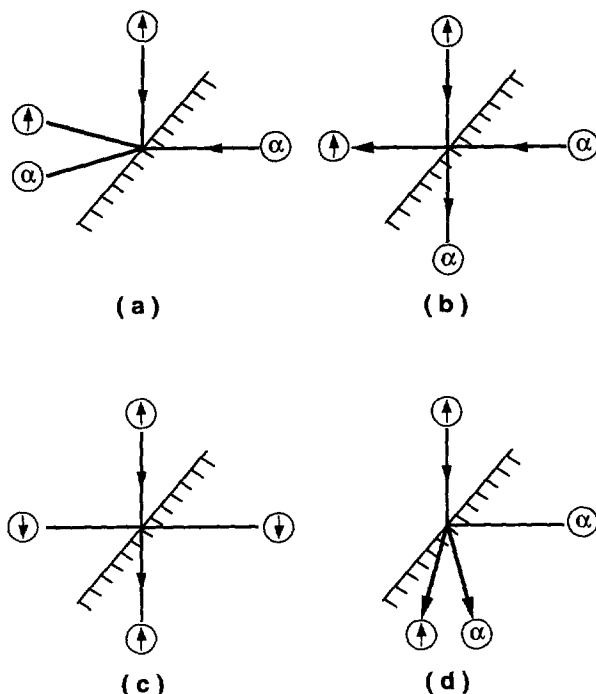


Fig. 2. — Various “diagrams” contributing to the rate processes involving two rolling species (↑) and (↓). The discussion is restricted to binary collisions between a rolling grain (here, a ↑ grain) and one immobile grain in the heap (which may be ↑ or ↓): a) amplification: two (↑) grains roll out, b) simple recoil (no effect on the rates), c) exchange, d) capture.

and we shall omit the angular dependence in  $m_\alpha$ , since it is always positive in the narrow angular interval of interest.

Let us now turn to the parameter  $M_\uparrow = M_{\uparrow\uparrow} + M_{\downarrow\uparrow}$ . This is more delicate, because there are positive and negative contributions. We shall write:

$$\begin{aligned}
 M_\uparrow &\equiv a_\uparrow \Phi_\uparrow + m_\uparrow \Phi_\downarrow - b_\uparrow - y_\uparrow \Phi_\downarrow \\
 &= \gamma_\uparrow (\theta - \theta_\uparrow)
 \end{aligned}
 \tag{17}$$

Here,  $\gamma_\uparrow$  and  $\theta_\uparrow$  are still function of the concentrations  $\Phi_\alpha$ , but they are expected to be positive and smooth. In particular  $\theta_\uparrow(\Phi_\downarrow = 0) = \theta_{\uparrow 0}$ , the equilibrium angle for the pure (↑) species. In one limit, the significance of  $\theta_\uparrow$  becomes particularly simple: if we may neglect the exchange term (c) in comparison to the amplification terms ( $a + b$ ); then it is plausible to put:

$$a_\uparrow(\theta) = x_\uparrow(\theta) = m_\uparrow(\theta)
 \tag{18}$$

Then, the condition defining  $\theta_\uparrow$ , namely  $M_\uparrow = 0$ , or:

$$\theta_\uparrow a_\uparrow(\theta_\uparrow) + \theta_\downarrow m_\uparrow(\theta_\uparrow) = b(\theta_\uparrow)$$

reduces to:

$$m_\uparrow(\theta_\uparrow) = b(\theta_\uparrow)
 \tag{19}$$

and is independent of  $\Phi$ : in this case,  $\theta_{\uparrow} = \theta_{\uparrow 0}$  all the time.

Equations (16) and (17) ultimately lead us to a collision matrix of the form:

$$\hat{M} = \begin{vmatrix} \gamma_{\uparrow}(\theta - \theta_{\uparrow}) - m_{\uparrow}\Phi_{\downarrow} & m_{\downarrow}\Phi_{\uparrow} \\ m_{\uparrow}\Phi_{\downarrow} & \gamma_{\downarrow}(\theta - \theta_{\downarrow}) - m_{\downarrow}\Phi_{\uparrow} \end{vmatrix} \quad (20)$$

We call equation (20) the *canonical* form of the collision matrix.

For many qualitative investigations, it will be useful to go even further, and to construct a *minimal* form of the equations. We obtain this by setting  $y_{\alpha}(\theta) = 0$ , and postulating equation (18). Then  $\theta_{\uparrow}$  and  $\theta_{\downarrow}$  are independent of concentration. Furthermore, we set:

$$\left. \begin{aligned} m_{\uparrow} &= m_{\downarrow} = m \\ \gamma_{\uparrow} &= \gamma_{\downarrow} = \gamma \\ v_{\uparrow} &= v_{\downarrow} = v \end{aligned} \right\} \quad (21)$$

and we assume all these parameters to be concentration independent. Most of our discussion in the present paper is based on this minimal form. But for detailed applications, the complete canonical form will be required: this shall be discussed in a second paper.

### 3. Steady State Filling of a Silo

3.1. STARTING POINT. — Here, we start from equations (10-12) using the minimal form for the collision matrix. Just as in equation (4) we look for a steady state regime  $\dot{h} = w = Q/L$ . At the point of injection ( $x = L$ ), we impose a certain ingoing flux  $Q_{\uparrow}$ ,  $Q_{\downarrow}$  for each species. This, in turn, fixes the amount  $R_{\alpha}$  at this point:

$$vR_{\alpha}(x = L) = Q_{\alpha} \quad (22)$$

Equation (5) is replaced by:

$$v \frac{\partial}{\partial x} R_{\alpha} = w \Phi_{\alpha} = -M_{\alpha\beta} R_{\beta} \quad (23)$$

We cannot directly integrate  $R_{\alpha}$  from the first equality, because the concentrations  $\Phi_{\alpha}$  will depend on the position  $x$ . But we can integrate the sum, and recover a result similar to equation (8):

$$R \equiv R_{\uparrow} + R_{\downarrow} = \frac{w}{v} x \quad (24)$$

3.2. EQUATIONS FOR THE SLOPE. — Equation (13) can be rewritten as:

$$(\theta - \theta_{\uparrow})R_{\uparrow} + (\theta - \theta_{\downarrow})R_{\downarrow} = -\frac{w}{\gamma} \quad (25)$$

giving:

$$\theta = \frac{\theta_{\uparrow}R_{\uparrow} + \theta_{\downarrow}R_{\downarrow}}{R} - \frac{w}{\gamma R} \quad (26)$$

In our minimal structure,  $\theta_{\uparrow}$  and  $\theta_{\downarrow}$  are independent of  $\Phi$ . It is then easy to differentiate (26), using equation (23) to eliminate  $\partial R_{\alpha}/\partial x$ . We then get:

$$x \frac{\partial \theta}{\partial x} = \bar{\theta}_{\Phi}(x) - \theta \quad (27)$$

where  $\bar{\theta}_{\Phi}(x)$  is the (local) weighted average:

$$\bar{\theta}_{\Phi} = \theta_{\uparrow}\Phi_{\uparrow}(x) + \theta_{\downarrow}\Phi_{\downarrow}(x) \quad (28)$$

3.3. THE CONCENTRATION PROFILE  $\Phi_{\uparrow}(x)$ . — Let us assume that we know the local slope  $\theta(x)$ . This implies that we know the running populations  $R_{\uparrow}(x)$  and  $R_{\downarrow}(x)$  via equation (25). We can in fact simplify equation (25) because the right hand side ( $\sim w/\gamma$ ) is much smaller in order of magnitude than the left hand side ( $\sim \psi R \sim \psi wx/v$ ). The ratio of the two is  $\sim d/\psi x$ . Thus whenever:

$$x \gg d/\psi \tag{29}$$

we can replace equation (25) by:

$$(\theta - \theta_{\uparrow})R_{\uparrow} + (\theta - \theta_{\downarrow})R_{\downarrow} = 0 \tag{30}$$

Then:

$$\theta = \bar{\theta}_R \equiv \frac{\theta_{\uparrow}R_{\uparrow} + \theta_{\downarrow}R_{\downarrow}}{R}$$

or equivalently:

$$\left. \begin{aligned} \frac{R_{\uparrow}}{R} &= \frac{\theta - \theta_{\downarrow}}{\psi} \\ \frac{R_{\downarrow}}{R} &= \frac{\theta_{\uparrow} - \theta}{\psi} \end{aligned} \right\} \tag{31}$$

We call the region defined by the inequality (29) the *outer region*. In the following text, we focus our attention on this sector. Then we can obtain the concentrations  $\Phi_{\alpha}(x)$  by returning to equation (23) in the form:

$$-w\Phi_{\alpha} = M_{\alpha\beta}R_{\beta} \tag{32}$$

Combining (32) and (31) we arrive at very simple forms:

$$\left. \begin{aligned} \Phi_{\uparrow} &= \left( 1 + \frac{\gamma\psi}{m} \frac{R_{\downarrow}}{R} \right) \frac{R_{\uparrow}}{R} \\ \Phi_{\downarrow} &= \left( 1 - \frac{\gamma\psi}{m} \frac{R_{\uparrow}}{R} \right) \frac{R_{\downarrow}}{R} \end{aligned} \right\} \tag{33}$$

where  $R_{\alpha}/R$  can, if desired, be expressed in terms of  $\theta$  via equation (31).

3.4. THE ROLLING SPECIES  $R_{\alpha}(x)$ . — The profile is completely described by 3 variables:  $R_{\alpha}$ ,  $\theta$  and  $\Phi_{\alpha}$ . Equations (31) and (33) show that when we know only one of these variables, we can immediately derive the two others.  $\theta(x)$  could be found by solving (27). We shall calculate  $R_{\alpha}(x)$ , in the outer region defined by the inequality (29).

Equation (23) for  $\alpha = \uparrow$  implies:

$$v \frac{\partial R_{\uparrow}}{\partial x} = w\Phi_{\uparrow} \tag{34}$$

where  $\Phi_{\uparrow}$  is given by (33):

$$\Phi_{\uparrow} = \left( 1 + r \frac{R_{\downarrow}}{R} \right) \frac{R_{\uparrow}}{R} \tag{35}$$

with:

$$r = \frac{\gamma\psi}{m} \tag{36}$$

This leads to a differential equation for the proportion of ( $\uparrow$ ) in the rolling phase  $R_{\uparrow}/R$ :

$$x \frac{d}{dx} \left( \frac{R_{\uparrow}}{R} \right) = r \frac{R_{\uparrow}}{R} \left( 1 - \frac{R_{\uparrow}}{R} \right) \tag{37}$$



The solution is:

$$\frac{R_{\uparrow}}{R} = \frac{1}{1 + \left(\frac{x_0}{x}\right)^r} \quad (38)$$

where  $x_0$  is determined by the condition from equation (22):

$$\frac{R_{\uparrow}}{R}(x=L) = \frac{Q_{\uparrow}}{Q_{\uparrow} + Q_{\downarrow}}$$

Finally we get:

$$\left. \begin{aligned} R_{\uparrow}(x) &= \frac{R(x)}{1 + \frac{Q_{\downarrow}}{Q_{\uparrow}} \left(\frac{L}{x}\right)^r} \\ R_{\downarrow}(x) &= \frac{R(x)}{1 + \frac{Q_{\uparrow}}{Q_{\downarrow}} \left(\frac{x}{L}\right)^r} \end{aligned} \right\} \quad (39)$$

with  $R(x) = \frac{w}{v}x$  (Eq. (24)).

**3.5. DOWNHILL SEGREGATION.** — The proportion of the sticky particles ( $\uparrow$ ) decreases in the rolling phase as the particles go down. How is the mixture at the lower end of the slope, when  $d/\psi \leq x \ll L$ ?

- If  $Q_{\downarrow}/Q_{\uparrow}(\psi L/d)^r \leq 1$ , the lower end of the slope is richer in ( $\downarrow$ ) particles than what we pour at  $x=L$ , but remains a mixture.
- If  $Q_{\downarrow}/Q_{\uparrow}(\psi L/d)^r \gg 1$ , at the lower end, we have:

$$\begin{aligned} \frac{R_{\downarrow}}{R} &\simeq 1, \quad \Phi_{\downarrow} \simeq 1 \\ \frac{R_{\uparrow}}{R} &\simeq \frac{Q_{\uparrow}}{Q_{\downarrow}} \left(\frac{x}{L}\right)^r \ll 1 \end{aligned} \quad (40)$$

$$\theta \simeq \theta_{\downarrow} + \psi \frac{Q_{\uparrow}}{Q_{\downarrow}} \left(\frac{x}{L}\right)^r \quad (41)$$

We then expect complete purification: the bottom part is being nearly a pure species. The concentrations and the slope have a *singular power law*. The exponent  $r$  depends on the structure of the collision matrix! In practice, we expect  $r \sim \psi$ , thus  $r$  would be rather small.

**3.6. THE INNER REGION.** — What happens when  $x < d/\psi$ ? It turns out that the calculations can still be without the approximation (29); and we get simply:

$$R_{\uparrow}(x) = \frac{R(x)}{1 + \frac{Q_{\downarrow}}{Q_{\uparrow}} \left(\frac{L}{x + r\frac{d}{\psi}}\right)^r}$$

*i.e.* a shift in the origin. Thus, in the inner region, for  $x < r(d/\psi)$ , the concentrations and the slope do not change any more, keeping the value they had at  $x \simeq r(d/\psi)$ . In practice,  $rd/\psi$  is expected to be not much larger than the grain size  $d$ ; this effect should not be measurable.

#### 4. Concluding Remarks

1) Pouring sand mixtures should automatically lead to a certain form of filtration: the more sticky species appearing near the top, and the less sticky species appearing as asymptotically pure near the bottom.

2) At first sight, one might have expected exponential relaxations towards the bottom state. Rewriting the transport equation:

$$v \frac{\partial R_\alpha}{\partial x} = -M_{\alpha\beta} R_\beta \quad (42)$$

and  $M \sim \gamma\psi + m$ , we would indeed expect forms such as  $\exp -(\kappa x)$  with a short characteristic length:

$$\kappa^{-1} \sim \frac{v}{M} \sim \frac{d}{\psi}$$

But this is not correct, because (in the outer region) the populations  $R_\alpha$ , which are realised locally, manage to satisfy the condition (30), or more generally:

$$M_\uparrow R_\uparrow + M_\downarrow R_\downarrow \cong 0 \quad (43)$$

We are using an eigenmode of the  $M$  matrix which gives very slow rates. Physically, equation (43) means that at each point, the system is nearly in conditions of 0 growth: the overall input  $Q$  does not influence the profile. Only the compositions  $Q_\alpha/Q$  matter.

3) When the grains ( $\uparrow$ ) and ( $\downarrow$ ) are both spherical in shape, and differ only in their diameter, the equilibrium angles  $\theta_\uparrow$ ,  $\theta_\downarrow$  for the pure species are equal. The main driving force for segregation is then probably the differences in rolling velocities: large spheres roll more easily over small spheres than the reverse. This effect has been considered by Meakin [15] and by Jullien and Meakin [17]. To incorporate it properly in our presentation, we need to return to the “canonical form” of equation (18), allowing  $v_\uparrow \neq v_\downarrow$  (and also allowing these velocities to depend on concentration); finally incorporating the exchange terms. This will be done in a forthcoming article (part II of this series).

4) Is the steady state profile stable? The experiments of reference [24] suggest that it is not: as mentioned in the introduction in certain conditions, pouring a mixture of “red” and “white” grains leads to a succession of red and white *strata* in the 2d silo (actually in a vertical Hele Shaw cell with thickness  $\sim 10d$ ). In any case, it will be interesting to look for the linearised modes of fluctuations around our steady state solution. Here again, we shall need the complete canonical form.

5) Our model has been applied here only to the “2d silo”. It is of course possible to transpose it to rotating drums and in particular to the thin rotating drums of granular mixtures studied in Paris [25] and in Rennes [26]. We have already constructed the steady state profiles for  $R(x)$  in a rotating drum with continuous avalanche flows [9].

6) Even nominally pure granular materials may segregate in avalanche flows, whenever the grains are in fact slightly different in size or shape. This may cause some uncertainty in the definition of an angle of repose.

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