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Monopoles in the Gauge Theory of the t-J Model

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Abstract. — We consider a RG approach for the plasma of magnetic monopoles of the Ioffe-Larkin approach to the t-J model. We first derive the interaction parameters of the 2+1 plasma of magnetic monopoles. The total charge along the time axis is constrained to be zero for each lattice plaquette. Under the one-plaquette approximation, the problem is equivalent to a one dimensional neutral plasma interacting *via* a potential $V(t) \sim t^\alpha$, with $\alpha = 1/3$. The plasma is in a dipolar phase if $\alpha \geq 1$ and a possibility of transition towards a Debye screening phase arises if $\alpha < 1$, so that there exists a critical Fermi wave vector k_f^* such as the plasma is Debye screening if $k_f < k_f^*$ and confined if $k_f > k_f^*$. The 2+1 dimensional problem is treated numerically. We show that k_f^* decreases and goes to zero as the number of colors increases. This suggests that the assumption of spin-charge decoupling within the slave-boson scheme is self-consistent at large enough values of N and small enough doping. Elsewhere, a confining force between spinons and antiholons appears, suggesting a transition to a Fermi liquid state.

1. Introduction

The physics of strongly interacting electron systems has received considerable attention over the recent years, and it still bears many challenges, especially on the theoretical side. Among the various methods and ideas which have been explored in this context, gauge theories seemed to offer a rather attractive approach [1–7]. Their essence is to focus on the presence of a non double occupancy constraint, which leads to a local $U(1)$ symmetry if a slave boson representation is used [1–3]. Although it is possible to derive these gauge theories from an expansion around a large N mean field theory of the t-J model [4,5], they could also be regarded as promising candidates for an effective low-energy theory in order to describe for instance the anomalous normal state properties of high- T_c superconductors. They seem to predict a phase diagram for the single band t-J model which qualitatively resembles the experimental ones for copper-oxide superconductors [6]. Furthermore, they reconcile the existence of a large Fermi surface corresponding to Luttinger’s theorem, as shown by photoemission experiments, and the anomalous transport properties, which are mostly governed by holes [6,7]. Thermodynamic properties have also been investigated, and a good agreement with high temperature expansions for the t-J model has been reached [8]. However, this work has also pointed out that fluctuations of the gauge field are large, in the sense that the variance of the local statistical flux around

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a given plaquette is not small in units of 2π , even down to low temperatures. This feature suggests that the presence of the lattice may not be inessential, since it induces a periodic action as a function of the time and space dependent flux per plaquette. As demonstrated by Polyakov, this periodic nature of the gauge field has dramatic consequences on 2+1 dimensional electrodynamics since it allows for non trivial space time configurations of the gauge field (monopoles), which induce charge confinement [9]. It should be emphasized that in the context of the t-J model, the gauge field Lagrangian density is not the usual $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ term, but it is generated upon integrating out fermionic and bosonic fluctuations [3]. A perturbative estimate of a single monopole action has also been derived in [3], and was found to diverge. However, it is clear that some globally neutral configurations (*i.e.* with the same number of instantons and anti-instantons) have a finite action, and the main question is whether the corresponding two-component plasma exhibits Debye-screening or not. This viewpoint has been developed by Nagaosa [10] where he assumed a dissipative-type action for the gauge field, which may be relevant for the t-J model at small temperatures, since it requires a finite dc conductivity for the fermions. His main conclusion is that no major instanton effect is present in the t-J model range of parameter since the dissipative nature of the gauge field dynamics strongly inhibits quantum tunneling. In the present paper, we address this question from a slightly different perspective, with emphasis on the possible zero temperature transitions. By contrast to the results of reference [10], we find that assuming a Ioffe-Larkin form for the monopole plasma action leads to a phase transition between a Debye screening phase (which corresponds to a confining force between spinons and antiholons), and a dipole phase (leading to unconfined spinons and antiholons). The control parameters are the band filling of the underlying t-J model, and the number N of fermion colors (the physical case being $N = 2$). In rather good agreement with physical intuition, the dipole phase of the plasma is found at large N and small doping. In this regime, spin-charge separation may then be a self-consistent hypothesis. We note however that this leads either to a renormalized Fermi liquid or an anomalous liquid depending on whether Bose condensation of holons occurs or not. In the other phase, the gauge field cannot be treated perturbatively, and the corresponding mesons (bound states of spinons and antiholons) are physical electrons. By contrast, a transition to the dipole phase is obtained in reference [10] in the presence of an infinitesimal dissipation. The main difficulty of the problem is the determination of the phase diagram of the monopole plasma which exhibits long range interactions (in space and imaginary time). Furthermore, unlike the case of the standard compact 2+1 electrodynamics, a strong anisotropy exists between space and time directions, reflecting the lack of Lorentz invariance in the model. This reduced symmetry increases the difficulty of real space RG analysis since the functional form of the monopole interaction potential is not stable under a RG transformation. Our approach has attempted to take advantage of the fact that the interaction is much stronger along the time direction, with a $\tau^{1/3}$ dependence. The corresponding one-dimensional problem exhibits a phase transition between a Debye-screening phase and a dipole phase. We argue and give some numerical indication that the unbinding of the monopole-antimonopole pairs along the time direction triggers a 2+1 dimensional unbinding, leading to a globally Debye-screening phase. The paper is organized as follows: Section 2 defines the statistical problem of the monopole plasma. The next section focuses on the 0+1 dimensional problem along a time direction, giving strong arguments in favor of a phase transition. The extension to the 2+1 dimensional situation is then discussed, leading to a phase diagram as a function of N and fermion filling, which is the main result of the paper. The conclusion is dedicated to a comparison with previous work and stresses open questions.

2. Statistical Mechanics of the Monopole Plasma

As already stressed in the introduction, we shall assume a Lagrangian of the form

$$L = \sum_i \sum_{\sigma=1}^N \left(\bar{c}_{i\sigma}(\tau) \frac{\partial}{\partial \tau} c_{i\sigma}(\tau) + \bar{b}_{i\sigma}(\tau) \frac{\partial}{\partial \tau} b_{i\sigma}(\tau) \right) - t_f \sum_{\langle i,j \rangle} (e^{-ia_{ij}} c_{i\sigma}^+ c_{j\sigma} + \text{h.c.}) \quad (1)$$

$$- t_b \sum_{\langle i,j \rangle} (e^{-ia_{ij}} b_i^+ b_j + \text{h.c.}) + i \sum_j \lambda_j \left(c_{j\sigma}^+ c_{j\sigma} + b_j^+ b_j - \frac{N}{2} \right).$$

The fields $c_{i\sigma}(\tau)$ and $b_i(\tau)$ are respectively fermionic and bosonic, and they are defined on a two-dimensional square lattice with continuous imaginary time. The hopping constants t_f and t_b can be derived from a large N saddle point approximation of the one band t-J model [5,8]. We shall from now on focus on the effective dynamics of the $U(1)$ gauge field (a_{ij}, λ_i) , assuming that fermions and bosons have been traced out. As shown in the references [3,6], the gauge field action to Gaussian order is dominated by the fermion contribution at low doping, and with the assumptions that the holons have not condensed. Keeping only the transverse part which is responsible of the non-Fermi liquid behavior gives [3]

$$S_{\text{eff}}(a, \lambda) = T \sum_{\omega_n = 2\pi nT} \int_{BZ} \frac{1}{2} (\epsilon_1(k, \omega) \omega^2 + \mu(k, \omega) k^2) \left(\delta_{i,j} - \frac{k_i k_j}{k^2} a_i(k, \omega) a_j(-k, -\omega) \right). \quad (2)$$

In this equation,

$$\epsilon_1 = \frac{k_f}{2\pi k |\omega|} \quad (3)$$

for $|\omega| \ll 2t_f k_f k$, $k \ll k_f$ and $\mu(k, \omega) = t_f/12\pi$. The gauge field variables $a_{r,r+n}$, where n is a lattice vector and r a lattice site are denoted in the continuum limit by $a_n(r + n/2)$, in order to define the two component field $a_i(\rho)$. The time component of a is identified with λ . Most of the time, we shall use the axial gauge $a_0 = 0$. Latin indices such as i and j denote spatial components, whereas Greek indices correspond to arbitrary components. The quantities ϵ_1 and μ are derived with the approximation of a circular Fermi surface, and by taking the long wavelength, small frequency limit of the fermion current-current correlation function. The main drawback of this action is that the fundamental periodicity of the original action (1), namely its invariance under $a_{ij} \rightarrow a_{ij} + 2\pi$ is lost. This periodicity allows for non-trivial space-time configurations of the field corresponding to tunneling events where the flux threading a given plaquette may change by integer multiples of 2π . Ioffe and Larkin suggest to express (2) in terms of gauge invariant field strength $F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, with $\mu = 0, 1, 2$, and to replace the flux par plaquette F_{12} by its value *modulo* 2π . This algorithm sounds quite natural on physical grounds. However, (2) has been derived perturbatively for ‘flat’ configurations which satisfy Faraday’s law: $\partial_\mu b_\mu = 0$, where

$$b_\mu = \frac{1}{2} \epsilon_{\mu\nu\rho} F_{\nu\rho}. \quad (4)$$

After the field strength b_μ is taken *modulo* 2π , it satisfies

$$\partial_\mu b_\mu = \sum_i 2\pi n_i \delta(r - r_i) \delta(\tau - \tau_i), \quad (5)$$

where n_i are the integer charges located at r_i, τ_i . An ambiguity arises in extending the result (2) to non trivial configurations. We may add to equation (2) any quadratic form

$$T \sum_{\omega_n} \int \frac{d^2 k}{(2\pi)^2} C(k, \omega) (\omega^2 b(k, \omega) b(-k, -\omega) - k^2 e_{\perp}(k, \omega) e_{\perp}(-k, -\omega)) \tag{6}$$

without changing the result on “flat” configurations, but the action for non trivial configurations will depend on the kernel $C(k, \omega)$. In (6), e_{\perp} and b denote the transverse part of the electric field, and the magnetic field respectively. We also note that the perturbative evaluation of the fermion loop generates only the function $\epsilon_1(k, \omega)\omega^2 + \mu(k, \omega)k^2$. Physical intuition suggests that $\mu(k, \omega)$ is identical to the static diamagnetic susceptibility in the $\omega \rightarrow 0$ limit. This determines the two functions $\epsilon_1(k, \omega)$ and $\mu(k, \omega)$ as given above, and with such a determination, (2) becomes

$$S_{\text{eff}}(a, \lambda) = T \sum_{\omega_n} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{2} \epsilon_1(k, \omega) e_{\perp}(k, \omega) e_{\perp}(-k, -\omega) + \frac{1}{2} \mu(k, \omega) b(k, \omega) b(-k, -\omega). \tag{7}$$

Equation (7) is then extended to non-trivial configurations thus lifting the ambiguity in the choice of the kernel $C(k, \omega)$. But if the procedure seems perfectly sound at low frequencies, the separation between the electric and magnetic parts is less obvious to access at higher frequencies. In the bulk of this paper, we assume that this procedure is valid. The action for a many monopole configuration with a topological charge $q(r, \tau) = \pm 2\pi$ is then given by

$$S_{\text{plasma}} = \frac{T}{2} \sum_{\omega_n} \int \frac{d^2 k}{(2\pi)^2} q(k, \omega) \frac{\epsilon_1(k, \omega)\mu(k, \omega)}{\epsilon_1(k, \omega)\omega^2 + \mu(k, \omega)k^2} q(-k, -\omega). \tag{8}$$

where $q(k, \omega)$ is the Fourier transform of the charge density, namely

$$q(k, \omega) = \int_0^{\beta} d\tau \sum_r e^{-i(k \cdot r + \omega \tau)} q(r, \tau), \tag{9}$$

where r is a lattice site. More specifically, this gives

$$S_{\text{plasma}} = \frac{N t_f}{24\pi} T \sum_{\omega_n} \int \frac{d^2 k}{(2\pi)^2} \frac{q(k, \omega) q(-k, -\omega)}{|\omega| \left(|\omega| + \frac{t_f}{6k_f} k^3 \right)}. \tag{10}$$

In this formula, the global factor N has been added. It is simply the number of fermion colors in the large N approaches. Rescaling energies and frequencies by setting $t_f = \pm 1$, the model depends only on two dimensionless parameters, N and k_f . Assuming a circular Fermi surface, the maximal value of k_f corresponds to 1/2 electron per site for a given color, which gives $k_f \leq (2\pi)^{1/2}$

Before going further, we should mention that equation (8) is not the only candidate for the monopole plasma action. Developing an analogy with Josephson junction arrays, and emphasizing the dissipative nature of the gauge field, Nagaosa has also considered the following action [10]:

$$S_{\text{diss}} = \frac{\gamma}{4\pi} \int_{-\infty}^{+\infty} d\tau \int_0^{\beta} d\tau' \sum_{r, n} \frac{1}{(\tau - \tau')^2} (1 - \cos(a_{\perp}(r, n, \tau) - a_{\perp}(r, n, \tau'))). \tag{11}$$

for the dissipative part of the gauge field dynamics. As shown in reference [11], it is possible to map it into a statistical model in some regimes, but mostly a bidimensional model is obtained. The key variables are the winding number $m(r, n)$ along the time direction:

$$a_{\perp}(r, n, \beta) - a_{\perp}(r, n, 0) = 2\pi m(r, n) + \nu(r, n), \tag{12}$$

with $\nu(r, n) \in] - \pi, \pi]$ and $m(r, n)$ and integer. It seems that both approaches respect the 2π periodicity and the quadratic expansion of the gauge field around $a = 0$. In the absence of a fully first principle derivation, we shall adopt equation (8) as a working hypothesis, and hope to clarify this issue in a future work.

Going back to equation (10), it is important to stress that for any k value, the ω integral diverges if $\lim_{\omega \rightarrow 0} q(k, \omega)$ is non vanishing. Therefore, we shall impose a constraint on the allowed topological charge configurations, namely that $q(k, \omega = 0) = 0$ for any k . In real space, it means that

$$\int_0^{\beta} d\tau q(r, \tau) = 0 \tag{13}$$

for any plaquette located at r . The partition function of the plasma is then

$$Z = \sum_{n=0}^{+\infty} \frac{1}{(n!)^2} \prod_{i=1}^{2n} \left(\int_0^{\beta} \frac{d\tau_i}{\tau_0} \sum_{r_i} \right) \chi(r_1, \dots, r_n) \exp \left(-\frac{1}{2} \sum_{i,j} q_i q_j V(r_i - r_j, \tau_i - \tau_j) \right). \tag{14}$$

In this equation, r_i, τ_i denote the space-time coordinates of the monopoles with topological charge q_i . We set $q_i = 2\pi$ if $1 \leq i \leq n$ and $q_i = -2\pi$ if $n + 1 \leq i \leq 2n$. $\chi_{2n}(r_1, \dots, r_{2n})$ expresses the constraint and $\chi = 1$ if for any r we have

$$\sum_{i=1}^{2n} q_i \delta_{r, r_i} = 0. \tag{15}$$

The interaction potential $v(r, \tau)$ is obtained by Fourier transform of equation (10)

$$v(r, \tau) = \frac{N}{12\pi} T \sum_{\omega_n} \int \frac{d^2 k}{(2\pi)^2} \frac{e^{i(k \cdot r + \omega \tau)}}{|\omega| (|\omega| + \gamma |k|^3)}, \tag{16}$$

with $\gamma = 1/6k_f$ (we set $t_f = 1$). An important ingredient in (14) is the imaginary time scale τ_0 which is obtained by calculating the ratio of the two Gaussian determinants in the presence and in the absence of an instanton. We have carried out this calculation for the broken parabola model of Ioffe and Larkin, which leads to

$$\frac{1}{\tau_0} = \sqrt{2\pi} \mu \left(T \sum_{\omega_n} \int \frac{d^2 k}{(2\pi)^2} \frac{k^2}{\epsilon_1(k, \omega) \omega_n^2 + \mu k^2} \right)^{1/2} \tag{17}$$

The interested reader will find a derivation of this result in the appendix. We note that the integral is divergent at large frequencies. This may be another signal that we have not yet found a satisfactory derivation of this monopole plasma action. Since this τ_0 depends on the full non linear action of the gauge field, which is still unknown, we shall assume it equal to unity in the following discussion (since we have used $t_f = 1$ as energy unit). The following sections are now dedicated to an analysis of the classical statistical system given by equation (14).

3. Monopoles in Dimension 0+1

We deduce from the interaction (16) that the interaction between two monopoles is

$$-V(r, \tau) = \frac{Nt}{12\pi} \int \frac{d\omega}{2\pi} \int \frac{d^2k}{(2\pi)^2} \frac{\cos(k \cdot r) - \cos(k \cdot r + \omega\tau)}{|\omega|(|\omega| + \gamma|k|^3)} \quad (18)$$

We have shifted the interaction by an infinite constant so that the pair interaction between two monopoles is finite. The energy of a configuration of monopoles satisfying the neutrality condition $\int q(r, \tau) d\tau = 0$ is finite and does not depend on the regularization of the pair potential.

From now on, we are interested in the quantum problem at zero temperature, so the system is infinite along the imaginary time direction. We shall now use $\beta = Nt/12\pi$ to denote the inverse fictitious temperature of the monopole plasma, which should not introduce confusion. The two parameters associated to (18) are $\gamma = 1/6k_F$ and the prefactor β . The interaction (18) decreases as the distance $|r|$ between two plaquettes increases. In order to have an idea of the interaction ranges, we calculate the interaction $V(r, \tau)$ as a function of τ for different values of the interplaquette distance. We first rearrange the expression (18) using the change of variables $u = \omega\tau$ and $q = (\gamma\tau)^{1/3}k$. We obtain

$$-\frac{1}{\beta}V(r, \tau) = \frac{\tau^{1/3}}{\gamma^{2/3}} F\left(\frac{|r|}{(\gamma\tau)^{1/3}}\right), \quad (19)$$

with

$$F(x) = \int_0^{+\infty} \frac{qdq}{2\pi} J_0(qx) \int_{-\infty}^{+\infty} \frac{du}{2\pi} \frac{(1 - \cos u)}{|u|(|u| + q^3)}. \quad (20)$$

We plotted in Figure 1 the interaction for different values of the interplaquette distance $|r|$. In a first approximation, we take into account only the one-plaquette interaction along the time direction. In Section 4, we renormalize the bidimensional problem with a cut-off for the distance between two plaquettes.

The interactions of the one-plaquette problem are simply $V(\tau) = -\tau^{1/3}F(0)/\gamma^{2/3}$. We look for the phase diagram of the potential $V(\tau) \sim -\tau^\alpha$ in one dimension as a function of the exponent α . Fortunately, some exact results concerning the phase diagram of one-dimensional systems with long-range interactions are available [12]. The main result shows rigorously the existence of a finite temperature phase transition for the 1D ferromagnetic Ising model if the coupling $J(n - n') \propto 1/|n - n'|^\gamma$, with $1 < \gamma < 2$ [13]. To some extent, these results can be transposed to generalized Coulomb gas models, by considering a representation of the Ising model in terms of kink and antikink configurations. The potential energy for a single kink-antikink pair is then proportional to $|n - n'|^{2-\gamma}$, n and n' being the locations of the kink and antikink. The Ising and corresponding Coulomb gas problems are however not equivalent since the Ising model generates only rather special configurations where kinks and antikinks alternate. We expect intuitively the unrestricted Coulomb gas to be less ordered than the corresponding Ising model. By ordered state, we mean the dipolar phase. As a result, an unrestricted generalized Coulomb gas with $V(r) \propto r^\alpha$ is expected to have a high temperature Debye-screening phase if $\alpha < 1$. The fact that $\alpha = 1$ (the 1D genuine Coulomb potential) is the borderline is confirmed by several exact investigations [14, 15] showing that this system is always in the dipolar phase at any temperature. Our problem is a special case, with $\alpha = 1/3$. We shall now attempt to estimate the transition temperature to the Debye phase. It is then tempting to use a real space RG analysis along the lines of references [16–18]. For instance, the Coulomb potential in any dimension d ($\alpha = 2 - d$) has been analyzed in reference [18].

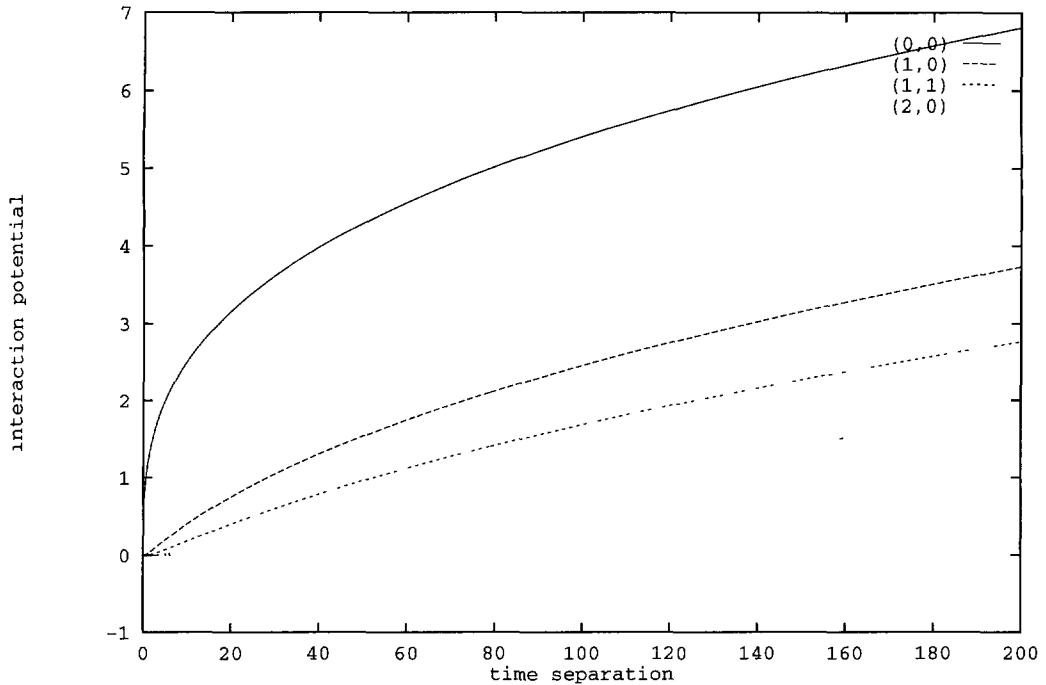


Fig. 1. — Interaction potential $V(|\mathbf{r}|, \tau)$ of equation (18) for $k_f = 4$ and for different values of the interplaquette spacing as a function of the time coordinate. The notation $(0,0)$ corresponds to $|\mathbf{r}| = 0$, $(1,0)$ stands for $|\mathbf{r}| = 1$, $(1,1)$ stands for $|\mathbf{r}| = \sqrt{2}$ and $(2,0)$ stands for $|\mathbf{r}| = 2$.

For $d > 2$, the system is always in a Debye screening phase, whereas for $d < 2$, there exists a finite temperature transition. We note that this simple RG analysis still predicts a non trivial fixed point for $d = 1$ and $\alpha = 1$, in discrepancy with the exact results of references [14] and [15]. But as d is decreased from 2 to 1, the unstable fixed point is found for higher values of the plasma fugacity, so that the dilute approximation leading to the RG equations is no longer valid. Hopefully, $\alpha = 1/3$ is not too large, so the usual RG procedure is consistent.

In order to analyse the one plaquette problem, we wish to treat the more general problem of the generalized Coulomb potential V_α in one dimension. We show that if $\alpha < 1$, the plasma has a Debye phase. We call Z_τ the partition function of the plasma with a minimal separation τ between the charges, which position is allowed to vary from $x = 0$ to $x = L$. For a neutral system of $2n$ particles, this defines an integration domain denoted $D_{2n}(L, d\tau)$. We wish to perform one renormalization step, that is to express Z_τ as a function of $Z_{\tau+\delta\tau}$. To do so, we write Z_τ under the form

$$\begin{aligned}
 Z_\tau = & \sum_{m=0}^{+\infty} \frac{K^{2m}}{(m!)^2} \sum_{p=0}^m \binom{m}{p}^2 p! (d\tau)^p \int_{D_{2(m-p)}(L, \tau+d\tau)} dr_1 \dots dr_{2(m-p)} W_B(r_1, \dots, r_{2(m-p)}) \\
 & \times \prod_{i=1}^p \int_0^L d\rho_i 2 \cosh(\beta q \tau E(\rho_i)). \tag{21}
 \end{aligned}$$

We have introduced a fugacity denoted by $z = K\tau$. In this equation, we have taken into account p dipoles with their center of gravity located at ρ_i ($i = 1, \dots, p$) and with a size between τ and

$\tau + d\tau$. W_B is the Boltzmann weight of the $2(m-p)$ remaining isolated charges located at $r_1, \dots, r_{2(m-p)}$. $E(\rho)$ is the electric field at ρ , created by the $2(m-p)$ isolated charges. More specifically,

$$W_B(r_1, \dots, r_{2n}) = \exp \left(-\beta^{-1} \sum_{i < j} q_i q_j V_{\alpha, \tau}(r_i - r_j) \right), \quad (22)$$

with

$$V_{\alpha, \tau}(r) = -\frac{1}{\alpha} \left(\left| \frac{r}{\tau} \right|^\alpha - 1 \right), \quad (23)$$

and

$$E(\rho) = -\sum_i q_i \nabla_\rho V_{\alpha, \tau}(\rho - r_i) = \sum_i \frac{q_i}{\tau} \left| \frac{\rho - r_i}{\tau} \right|^{\alpha-1} \quad (24)$$

The p dipoles are assumed to be independent. The integration over the dipole coordinates ρ_i must take into account the position of the other charges located at $r_1, \dots, r_{2(m-p)}$. We expand the cosh in (21) up to second order in the electric field, and we write

$$\int_0^L 2 \cosh(\beta q \tau E(\rho)) d\rho = 2L + \varphi(r_1, \dots, r_{2n}), \quad (25)$$

where we have used the notation $n = m - p$ and where the integration domain takes into account the presence of a hard core condition. We first need to determine the function φ . To do so, we write

$$\begin{aligned} \int_0^L 2 \cosh(\beta q \tau E(\rho)) d\rho &= \sum_{i=1}^{2n} \theta(x_{i+1} - x_i - 3\tau) \\ &\times \int_{x_i+2\tau/2}^{x_{i+1}-3\tau/2} d\rho \left(2 + \beta^2 \tau^2 q^2 \sum_{j=1}^{2n} \sum_{k=1}^{2n} q_j q_k V'_\tau(\rho - x_j) V'_\tau(\rho - x_k) \right). \end{aligned} \quad (26)$$

If we take only the two-body interactions, and the thermodynamic limit, the expression of φ takes the form

$$\begin{aligned} \varphi(r_1, \dots, r_{2n}) &= -3\tau(2n) + \lim_{L \rightarrow +\infty} \beta^2 q^2 \left[\sum_{i \neq j} q_i q_j \int_0^L d\rho \left| \frac{\rho - r_i}{\tau} \right|^{\alpha-1} \left| \frac{\rho - r_j}{\tau} \right|^{\alpha-1} \right. \\ &\times \text{sign}((\rho - r_i)(\rho - r_j)) \theta \left(\left| \frac{\rho - r_i}{\tau} \right| - \frac{3}{2} \right) \theta \left(\left| \frac{\rho - r_j}{\tau} \right| - \frac{3}{2} \right) \\ &\left. + \sum_i q^2 \int_0^L d\rho \left| \frac{\rho - r_i}{\tau} \right|^{2(\alpha-1)} \theta \left(\left| \frac{\rho - r_i}{\tau} \right| - \frac{3}{2} \right) \right]. \end{aligned} \quad (27)$$

The $L \rightarrow +\infty$ limit exists provided the system is neutral and $\alpha < 3/2$. Indeed, the $2n$ charges create a dipolar field at large distances which decays as $\rho^{\alpha-2}$ or faster. Taking the square gives the upper bound on α for long distance convergency. This property also enables us to shift variables and recast the previous expression as a sum of pair contributions which all converge

separately. We thus obtain

$$\begin{aligned} \varphi(r_1, \dots, r_{2n}) = & -3\tau(2n) + \beta^2 q^2 \sum_{i \neq j} q_i q_j \int_{-\infty}^{+\infty} d\rho \left[\left| \frac{\rho - r_i}{\tau} \right|^{\alpha-1} \left| \frac{\rho - r_j}{\tau} \right|^{\alpha-1} \right. \\ & \times \text{sign}((\rho - r_i)(\rho - r_j)) \theta \left(\left| \frac{\rho - r_i}{\tau} \right| - \frac{3}{2} \right) \theta \left(\left| \frac{\rho - r_j}{\tau} \right| - \frac{3}{2} \right) \\ & \left. - \left| \frac{\rho - r_i + r_j}{2\tau} \right|^{2(\alpha-1)} \theta \left(\left| \rho - \frac{r_i + r_j}{2} \right| - \frac{3}{2}\tau \right) \right]. \end{aligned} \quad (28)$$

After some rather simple calculations, and extracting the dominant behavior, we have

$$\varphi(r_1, \dots, r_{2n}) = \beta^2 q^2 \tau \sum_{i \neq j} q_i q_j c(\alpha) \left[\left| \frac{r_i - r_j}{\tau} \right|^{2\alpha-1} - 1 \right] - 2n\tau (3 + (c(\alpha) + d(\alpha))\beta^2 q^2). \quad (29)$$

The coefficients $c(\alpha)$ and $d(\alpha)$ are given in terms of the Euler B function:

$$c(\alpha) = \frac{2(\alpha-1)}{2\alpha-1} B(\alpha, 2(1-\alpha)) - B(\alpha, \alpha) \quad (30)$$

$$d(\alpha) = \frac{2}{2\alpha-1} \left(\frac{3}{2} \right)^{2\alpha-1} \quad (31)$$

We note that φ has the dimension of a length, so that Z_τ is dimensionless. The scaling equations are now obtained and read

$$\frac{d \ln \beta}{d \ln \tau} = \alpha + 2(2\alpha-1)c(\alpha)K^2\tau^2\beta q^2 \quad (32)$$

$$\frac{d \ln K}{d \ln \tau} = -\frac{\beta q^2}{2} - (3 + (c(\alpha) + d(\alpha))\beta^2 q^4) K^2\tau^2, \quad (33)$$

and the interaction function is modified by

$$\frac{dV}{d \ln \tau} = 2(2\alpha-1)c(\alpha)K^2\tau^2\beta q^2 \left[\frac{1}{2\alpha-1} \left(\left| \frac{\Delta}{\tau} \right|^{2\alpha-1} - 1 \right) - \frac{1}{\alpha} \left(\left| \frac{\Delta}{\tau} \right|^\alpha - 1 \right) \right], \quad (34)$$

where Δ is the particle separation. These equations are obtained by imposing the normalization constraints $V(\tau) = 0$ and

$$\frac{dV}{d\Delta}(\Delta = \tau) = -1. \quad (35)$$

Since the fundamental form of the interactions is preserved only for the Coulomb potential ($\alpha = 1$), these prescriptions are meaningful mostly near $\alpha = 1$. In the case $\alpha = 1$, we recover Kosterlitz's RG equations as derived in [18]. We note that the second term in the r.h.s. of equation (33) is not given in [18], but it does not change the critical behavior. Its meaning is a natural reduction of fugacity because of excluded volume effects. The structure of these equations, and in particular, the fact that $\alpha > 0$ and $2(2\alpha-1)c(\alpha)K^2\tau^2\beta q^2 < 0$ shows that the model keeps a finite temperature transition, and that the size of the Debye screening phase increases as α decreases. Coming back to our problem, α is fixed to $1/3$ for the single plaquette problem. The interaction strength is larger if N increases and if γ decreases, so if k_f increases. The dipolar phase is then expected at large N and large electron filling, in agreement with the physical intuition that spin-charge separation is more likely to occur in the vicinity of the Mott insulator and in the large N limit. This defines a critical Fermi wave vector $k_f^*(N)$ such that spin-charge separation occurs for $k_f > k_f^*(N)$. The aim of Section 4 is to obtain quantitative results for the variation of $k_f^*(N)$ with the number of colors N .

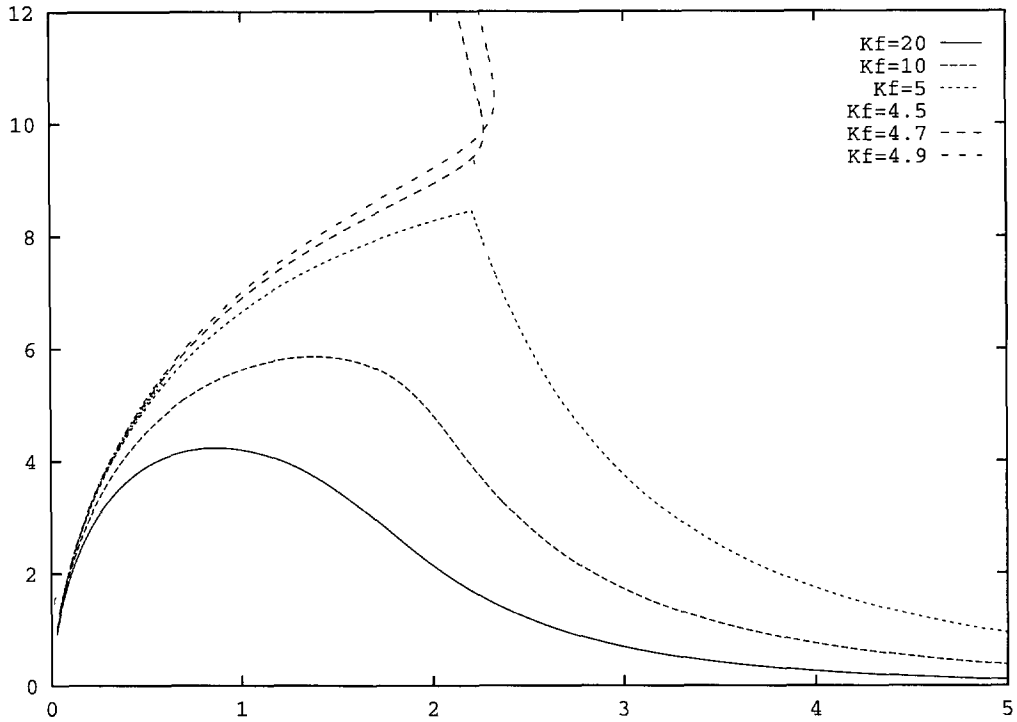


Fig. 2. — RG trajectories for $N = 1$ and different values of the Fermi wave vector. The time coordinate is compactified on a circle of length 400. We took $\tau = 1$. Some trajectories cross each other. This is due to the fact that the potential (19, 20) depends explicitly on γ and thus on the initial conditions. We have plotted the square of the fugacity $z^2 = K^2 \tau^2$ as a function of the effective inverse fictitious temperature of the monopole plasma. The dipolar phase corresponds to the fixed point $(\beta, z) = (\infty, 0)$ and the Debye screening phase corresponds to the fixed point $(\beta, z) = (0, \infty)$.

4. Monopoles in Dimension 2+1

We now consider the 2+1 dimensional problem. The derivation of the RG equations is a straightforward generalization of what has already been done in the 0+1 dimensional case. The potential $V(r, \tau)$ is given by equations (19) and (20). We use periodic boundary conditions in time, with a period L . This regularization will also be used in the numerical calculations. Small dipoles (with a length between τ and $\tau + d\tau$) can only be parallel to the temporal direction since we require the local neutrality condition $\int q(r, \tau) d\tau = 0$, so that the cut-off τ is only introduced in the temporal direction. The function φ is given by

$$\begin{aligned} \varphi(r_1 t_1, \dots, r_{2n} t_{2n}) &= 2\beta^2 q^2 \tau^2 \sum_r \sum_{j \neq k} q_j q_k \int_{3\tau/2}^{L/2} d\tau \partial_0 V(0, \tau) \partial_0 V(r_k - r_j - r, \Delta_{j,k} - \tau) \\ &\quad + 2\beta^2 q^2 \tau^2 \sum_r \sum_j q_j^2 \int_{3\tau/2}^{L/2} (\partial_0 V(r, \tau))^2 d\tau. \end{aligned} \quad (36)$$

In this expression, the summation over r is a summation over the plaquettes which contain the small dipoles (with a separation in the temporal direction between τ and $\tau + d\tau$). $\Delta_{j,k}$ is

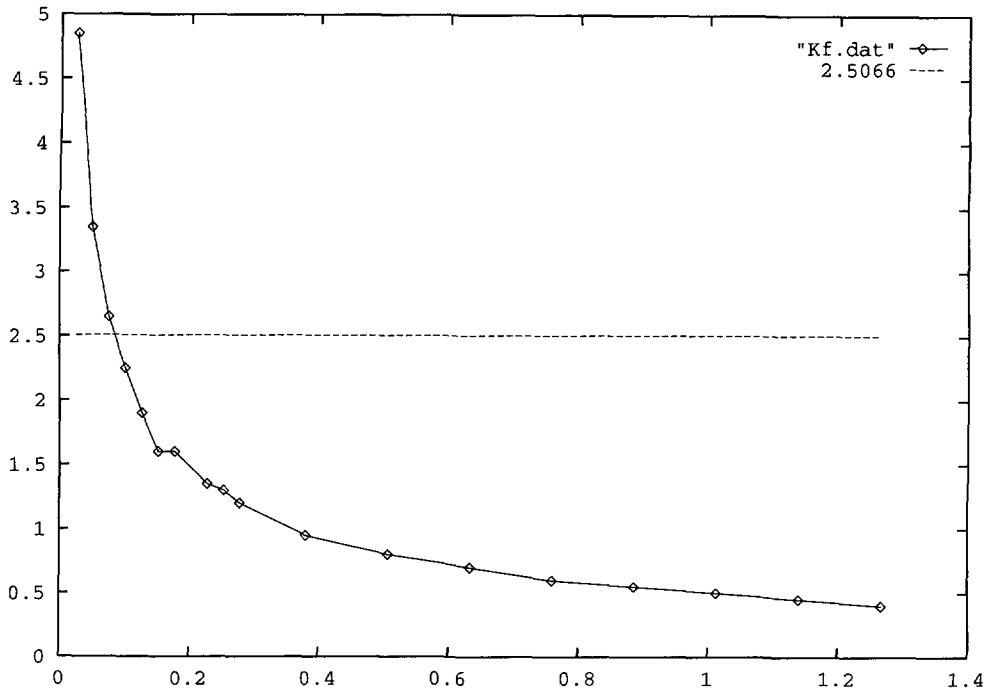


Fig. 3. — Critical Fermi wave vector as a function of the number of colors. The errorbars indicate the precision in the location of the fixed point. The dashed line indicates the maximal value of the Fermi wave vector ($\sqrt{2\pi}$).

the difference between the time coordinates of the monopole j and the monopole k . As in the expression (27), the integration over the time coordinate of the small dipole contains a hard core condition. We evaluated numerically the integrals in (36), and took into account only the potentials $V(r, \tau)$ such as $|r| < \Lambda$, with Λ a lattice cut-off. The RG trajectories are plotted in Figure 2 for different values of the Fermi wave vector. Notice that some trajectories are free to cross each other since the potential depends on the initial conditions *via* γ . We can check the validity of the predictions of the 0+1 dimensional approach: if $k_f < k_f^*$, the plasma is deconfined whereas it is confined if $k_f > k_f^*$. For a one-color model, we find $k_f^* = 0.5 \pm 0.05$. However, the Fermi wave vector is bounded above by $\sqrt{2\pi}$ since there is less than 1/2 electron of a given color per plaquette. We conclude that there exists a transition even for the one-color model. We now address the question of the N colors monopole model. The action is simply multiplied by N , inducing a change in the initial conditions of the renormalization procedure. We plotted in Figure 3 the critical Fermi wave vector k_f^* as a function of the number of colors. We see that for $N = 2$, which is the case of physical interest, that a possibility of a non Fermi liquid arises as the doping increases.

5. Conclusion

To conclude, the main result of this investigation is the possibility of tuning the microscopic parameters of the model (here the number of colors and the filling factor) in such a way that confinement between spinons and antiholons arises or not, depending on these parameters.

Using a different approach, it had been previously claimed that the Ioffe-Larkin type of plasma relevant to the t-J model is always in the dipolar phase, so that spinons and holons have a chance not to form a Fermi liquid [10]. It is true that our plasma action (Eq. (10)) is obtained from the simplest fermion loop, without dressing the fermion Green's function, and this may be the source of the difference between the results. We were guided here by the very strong anisotropy of the intermonopole potential, making it much stronger along the time direction, and requiring the neutrality constraint for each plaquette along the time direction. Our intuition is that screening may only be more effective if the two spatial dimensions are added to the one plaquette problem, thus weakening the strength of the large distance inter-monopole interaction. We hope that these ideas may lead to a more rigorous approach, and possibly Monte-Carlo studies of this plasma. Moreover, we have also presented arguments showing that a satisfactory microscopic derivation of the plasma action is still missing. One difficulty is connected to the ambiguity present while implementing the necessary periodicity requirement from a perturbative calculation which by essence assumes "flat" field configurations. A second one, and maybe related to the previous remark, is the diverging bare fugacity which results from the Villain-type treatment developed in the Appendix.

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Appendix

Derivation of the Monopole Fugacity in the Ioffe-Larkin Approach

We start from the quadratic action in the transverse subspace

$$S = \frac{1}{2} \frac{T}{N_s} \sum_{\omega} \sum_k \epsilon_1(k, \omega) e_{\perp}(k, \omega) e_{\perp}(-k, -\omega) + \mu b(k, \omega) b(-k, -\omega). \quad (37)$$

In this expression, b is a scalar field and e_{\perp} is related to b by the Faraday equation

$$\omega b(k, \omega) = k \times e_{\perp}(k, \omega) \cdot \hat{z}. \quad (38)$$

Suppose we consider an instanton located on a given plaquette r_0 at time τ_0 . The idea is to replace $b(r, \tau)$ by $b(r, \tau) - 2\pi\theta(b(r, \tau) - \pi)\delta_{r, r_0}$. Minimizing over transverse configurations of b leads to the instanton profile

$$e_{\perp}(k, \omega) = -i \frac{\mu \hat{z} \times k}{\epsilon_1(k, \omega) \omega^2 + \mu k^2} q(k, \omega) \quad (39)$$

$$b(k, \omega) = \left(-\frac{i}{\omega} + i \frac{\epsilon_1(k, \omega) \omega}{\epsilon_1(k, \omega) \omega^2 + \mu k^2} \right) q(k, \omega). \quad (40)$$

Here, $q(k, \omega) = 2\pi \exp(-i(k \cdot r_0 + \omega \tau_0))$ is the corresponding topological charge density. The fugacity is obtained from considering quadratic fluctuations around the single-instanton solution and integrating them out. We have to single out the zero mode which corresponds to a global translation along the time direction of this solution. This is a standard procedure, and

we just quote the result [19]

$$K = \frac{A}{\sqrt{2\pi}} \left(\frac{\prod \epsilon_i^{(0)}}{\prod_{i \neq 0} \epsilon_i} \right)^{1/2} \tag{41}$$

In this formula, A is the norm of the zero mode function

$$(r, \tau) \mapsto \frac{\partial b(r, \tau)}{\partial \tau} \tag{42}$$

From equation (40), we obtain

$$A = 2\pi\mu \left(\frac{T}{N_s} \sum_{k,\omega} \frac{k^4}{(\epsilon_1(k, \omega)\omega^2 + \mu k^2)^2} \right)^{1/2} \tag{43}$$

The second term is related to the ratio of the product of eigenvalues of the Hessian matrix in the vacuum and in the presence of the instanton. In the denominator, it is necessary to exclude the zero eigenvalue coming from translation invariance. The Hessian matrix is found by expanding the action (37) with the shift $b(r, \tau) \rightarrow b(r, \tau) - 2\pi\theta(b(r, \tau) - \pi)$, up to quadratic order in field deviations $\delta b(r, \tau)$ around the instanton solution. Written in Fourier space, the quadratic part of the action is

$$\begin{aligned} \delta^2 S &= \frac{1}{2} \frac{T}{N_s} \sum_{k,\omega} \Delta(k, \omega) \delta b(k, \omega) \delta b(-k, -\omega) \\ &\quad - \frac{\pi\mu}{|b'_0|} \frac{T^2}{N_s^2} \sum_{k,\omega} \sum_{k',\omega'} e^{i((k-k')r_0 + (\omega-\omega')\tau_0)} \delta b(k, \omega) \delta b(-k', -\omega'). \end{aligned} \tag{44}$$

We have used the notations $\Delta(k, \omega) = \mu + \epsilon_1(k, \omega)\omega^2/k^2$, and

$$b'_0 = \frac{\partial b}{\partial \tau}(r_0, \tau_0), \tag{45}$$

where b is the instanton profile. The eigenmodes corresponding to equation (44) are obtained from a rational secular equation since the scattering potential is separable. This equation reads

$$- \frac{2\pi\mu}{|b'_0|} \frac{T}{N_s} \sum_{k,\omega} \frac{1}{\epsilon - \Delta(k, \omega)} = 1. \tag{46}$$

From the structure of this equation, it is possible to derive the determinant ratio as

$$\frac{\prod_{i \neq 0} \epsilon_i}{\prod \epsilon_i^{(0)}} = \frac{2\pi\mu}{|b'_0|} \frac{T}{N_s} \sum_{k,\omega} \frac{k^4}{(\epsilon_1(k, \omega)\omega^2 + \mu k^2)^2}. \tag{47}$$

Using equation (41), and the expression for A leads to $K = (\mu|b'_0|)^{1/2}$. From equation (40), we finally get

$$K = \sqrt{2\pi\mu} \left(\frac{T}{N_s} \sum_{k,\omega} \frac{k^2}{\epsilon_1(k, \omega)\omega^2 + \mu k^2} \right)^{1/2} \tag{48}$$

References

- [1] Baskaran G. and Anderson P.W., *Phys. Rev. B* **37** (1988) 580.
- [2] Zou Z. and Anderson P.W., *Phys. Rev. B* **37** (1988) 627.
- [3] Ioffe L.B. and Larkin A.I., *Phys. Rev. B* **39** (1989) 8988.
- [4] Grilli M. and Kotliar G., *Phys. Rev. Lett.* **64** (1990) 1170.
- [5] Rodriguez J.P. and Douçot B., *Europhys. Lett.* **11** (1990) 451.
- [6] Nagaosa N. and Lee P.A., *Phys. Rev. Lett.* **64** (1990) 2450.
- [7] Ioffe L.B. and Wiegmann P.B., *Phys. Rev. Lett.* **65** (1990) 653; Ioffe L.B. and Kotliar G., *Phys. Rev. B* **42** (1990) 10348.
- [8] Hlubina R., Putikka W.O., Rice T.M. and Khveshchenko D.V., *Phys. Rev. B* **46** (1992) 11224.
- [9] See for instance Polyakov A.M., *Gauge fields and strings* (Hardwood Academic Publishers, Chur, 1987) chapters 4 and 5.
- [10] Nagaosa N., *Phys. Rev. Lett.* **71** (1993) 4210.
- [11] Fazio R. and Schön G., *Phys. Rev. B* **43** (1991) 5307.
- [12] For a brief review, see for instance Lieb E.H., in "Mathematical problems in theoretical physics", Lecture notes in Physics, volume 80, G. Dell'Antonio, S. Doplicher and G. Jona-Lasinio Eds. (Springer-Verlag, 1978); and Toda M., Kubo R. and Saito N. in "Statistical physics I", Springer Series in Solid State Science 30, (Springer-Verlag, 1983) pp. 135-137.
- [13] Dyson F.J., *Commun. Math. Phys.* **12** (1969) 212; Fröhlich J., Israel R., Lieb E. and Simon B., *Commun. Math. Phys.* **62** (1978) 1.
- [14] Lenard A., *J. Math. Phys.* **2** (1961) 682.
- [15] Aizenman M. and Fröhlich J., *J. Stat. Phys.* **26** (1981) 347.
- [16] Anderson P.W., Yuval G. and Hamman D.R., *Phys. Rev. B* **1** (1970) 4464.
- [17] Kosterlitz J.M., *J. Phys. C* **7** (1974) 1046.
- [18] Kosterlitz J.M., *J. Phys. C* **10** (1977) 3753.
- [19] See for instance Langer J.S., *Ann. Phys.* **41** (1967) 108.