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## Breakdown of the Fermi Liquid picture in one dimensional fermion systems: connection with the energy level statistics

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**Abstract.** — Using the adiabatic switching of interactions, we establish a condition for the existence of electronic quasiparticles in a Luttinger liquid. It involves a characteristic interaction strength proportional to the inverse square root of the system length. An investigation of the exact energy level separation probability distribution shows that this interaction scale also corresponds to a cross-over from the non interacting behaviour to a rather typical case for integrable systems, namely an exponential distribution. The level spacing statistics of a spin  $1/2$ , one branch Luttinger model are also analyzed, as well as the level statistics of a two coupled chain model.

The field of strongly correlated electron systems has recently stimulated interesting discussions which are sometimes challenging some more traditional ideas on the many body problem. For instance, Anderson has proposed that the low energy properties of a two dimensional Hubbard model are not properly described by a Fermi liquid theory [1]. In a recent paper [2], he emphasizes that this question requires a non-perturbative treatment, and a careful consideration of boundary conditions. As a consequence of the difficulty of the problem, much effort has been recently dedicated to numerical investigations either with Monte Carlo methods or exact diagonalizations [3]. However, the available sizes remain quite small, and the interpretation of these results is often delicate. A rather different approach has been proposed [4] recently with the hope to develop new tools for extracting more information from finite systems. These studies have shown that for a large class of low dimensionnal strongly correlated systems, the energy levels exhibit statistical properties rather well described by random matrix theory. For instance, a regime of energy level repulsion is clearly seen in most investigated cases, with the exception of integrable models such as the nearest neighbor or the  $1/r^2$  interaction Heisenberg spin chain. Such a behavior has been extensively discussed in the context of quantum chaos. More precisely, it has been verified that many time reversal symmetrical classically chaotic systems generate a spectrum in good agreement with the Gaussian Orthogonal ensemble predictions [5]. By contrast, simple integrable systems yield in general uncorrelated energy levels and the usual exponential distribution for energy level spacings [6].

In this paper, we are investigating some possible connections between simple physical properties of an interacting Fermi system, such as the existence of long lived electronic quasiparticles and the energy level distribution. Intuitively, if the energy levels of the interacting system keep a simple one-to-one correspondence with those of the non-interacting system, we expect on the one hand Fermi Liquid Theory to be valid and on the other hand the statistical properties of the spectrum to remain qualitatively similar as for the free electron case. The interpretation of random matrix behaviour is not straightforward. It may simply indicate that a model is non integrable. For a normal Fermi Liquid, electronic quasiparticles are expected only at low energies compared to the Fermi energy. Furthermore, already for the particle-hole phase spaces, interactions induce new collective modes, such as the zero sound, and the idea of a one-to-one correspondence with the non interacting gas does not hold for the whole spectrum. Clearly, it would be very interesting to see if the spectrum of a normal Fermi Liquid exhibits some features which would distinguish it from a random matrix Hamiltonian. However, this would likely require an intensive numerical effort (since best candidates would be at least two-dimensional systems). For the sake of simplicity, and the motivation of doing analytical calculations, in this paper we have concentrated on a one-dimensional model, namely the Luttinger model [7], which is integrable at any coupling strength. Interestingly, this feature holds for any system length [8]. Furthermore, it provides a good example of a non-Fermi liquid, which can be viewed as a non-translation invariant fixed point for many interacting systems in one dimension.

This paper is organized as follows. A first part investigates the condition for the existence of electronic quasiparticles, using the adiabatic generation of eigenstates. An existence condition is established, from the combined requirement of having a negligible generation of non adiabatic components and absence of decay. This criterion is satisfied if the interaction strength is less than a constant divided by the square root of the system length. As expected, no quasiparticles are found for an infinite system at any finite value of the interaction parameter. This result is also rederived from a simple analysis of the single particle Green function for a finite system. The second part is devoted to the study of the level spacing distribution as the interaction is gradually increased. We show that the typical interaction scale locating the departure from the highly degenerate non interacting system towards a more generic integrable model with a Poisson distribution is the same as the previous one. So, for this simple situation, noticeable change in the energy level distribution is reflected by the disappearance of electronic quasiparticles. Then, the last two sections of this paper are dedicated to variants of this model, namely in the spin 1/2 case and forward scattering only, for both one and two coupled one-dimensional systems. A brief conclusion summarizes our results.

## 1. Adiabatic switching on of interactions.

A formal way to generate quasiparticles in an interacting Fermi liquid is to apply the Landau switching on of interaction procedure, namely to start from a free particle added above the Fermi sea, and to switch on interactions adiabatically. The corresponding time dependent Hamiltonian is

$$H = H_0 + V_0 e^{\epsilon t}, \quad (1)$$

where the interaction term  $V_0$  is switched with a rate  $\epsilon$ .

Provided it is successful, this procedure establishes a one to one correspondence between the free gas excitations, and the dressed excitations of the Fermi liquid, namely, the quasiparticles. For a Fermi liquid, the validity condition of this procedure is [9]

$$\Gamma(\epsilon_k) \ll \epsilon \ll \epsilon_k, \quad (2)$$

where  $\epsilon_k$  is the energy of the quasiparticle, with respect to the Fermi surface, and  $\Gamma(\epsilon_k)$  is the decay rate of the quasiparticle. For a normal Fermi liquid, one can show [9] that  $\Gamma(\epsilon_k) \simeq \epsilon_k^2$ . At small energies,  $\Gamma(\epsilon_k) \ll \epsilon_k$ , so that it is possible to choose a rate  $\epsilon$  to perform the switching on procedure.

The aim of this section is to investigate under which conditions the switching on procedure is valid in a one-dimensional Luttinger liquid. We shall henceforth exhibit an inequality similar to equation (2) for the rate  $\epsilon$  in the case of a Luttinger liquid.

**1.1 INTRODUCTION.** — We first wish to sum up some results concerning the formalism developed in [8]. This will also permit us to fix the notation which shall be used in the rest of the paper.

The fermions are on a ring of perimeter  $L$ , with periodic boundary conditions, so that the wave vectors are quantized ( $k = \frac{2\pi}{L}n$ , with  $n$  an integer).

As we treat only low energy properties of a spinless, one dimensional Fermi gas, the curvature of the dispersion relation may be neglected. The two linear branches in the dispersion relation emerging from each extremity of the Fermi surface are extended to arbitrary energies. This linearized model is the Luttinger gas model, whose Hamiltonian is

$$H^0 = v_F \sum_{kp} (pk - k_F) : c_{kp}^+ c_{kp} \cdot, \quad (3)$$

where  $v_F$  is the Fermi velocity and  $p = +1$  or  $-1$  labels the branch (right or left). We shall also use the real space field  $\psi_p^+(x)$  associated with the right (left) free fermions. Furthermore,  $c_{kp}^+$  is the Fourier transform of  $\psi_p^+(x)$

$$c_{kp}^+ = L^{-1/2} \int_{-L/2}^{L/2} \psi_p^+(x) e^{ikx} dx. \quad (4)$$

Notice that the sign of the phase factor is not arbitrary, but is chosen such as right moving fermions with a positive wave vector propagate to the right.

Because of the presence of an infinite number of fermions in the ground state, the density operators

$$\rho_{qp} = \sum_k : c_{k+q,p}^+ c_{k,p} \cdot \quad (5)$$

have anomalous commutation relations (Schwinger terms)

$$[\rho_{qp}, \rho_{-q'p'}] = -\frac{Lpq}{2\pi} \delta_{pp'} \delta_{qq'}. \quad (6)$$

They may consequently be used to build a set of boson creators  $a_q^+$  ( $q \neq 0$ ). To handle the real space bosonic field, one needs to define

$$\Phi_p(x) = p \frac{\pi x}{L} N_p - i \sum_{q \neq 0} \theta(pq) \left( \frac{2\pi}{L|q|} \right)^{1/2} e^{iqx} a_q. \quad (7)$$

The  $q = 0$  modes correspond to charge and current excitations. Their algebra involves the unitary ladder operators  $U_p$  constructed in [8]. They act only in the  $q = 0$  sector, and increase by one the charge on the  $p$  branch. The complete form of the bosonic fields, including the  $q = 0$  modes, is

$$\theta_p(x) = \bar{\theta}_p + \Phi_p(x) + \Phi_p^+(x), \quad (8)$$

where  $\bar{\theta}_p$  is the phase conjugate to  $N_p$ .

We shall also use the important relation to pass from a real space boson description to a real space fermion description

$$\begin{aligned}\Psi_p^+(x) &= L^{-1/2} e^{-ipk_F x} \cdot e^{-i\theta_p(x)} \\ &= L^{-1/2} e^{-ipk_F x} e^{-i\Phi_p^+(x)} U_p e^{-i\Phi_p(x)}\end{aligned}\quad (9)$$

Expressed on this new basis, the free hamiltonian becomes

$$H^0 = v_F \sum_{q \neq 0} |q| a_q^\dagger a_q + v_F \frac{\pi}{L} (N_R^2 + N_L^2), \quad (10)$$

where  $N_R$  ( $N_L$ ) denote the number of right (left) moving fermions added above the vacuum state. In terms of charge  $N = N_R + N_L$  and current  $J = N_R - N_L$  variables, the energy of the charge and current excitations is  $v_F \frac{\pi}{2L} (N^2 + J^2)$ .

Note that the action of the boson creation operators and of the ladder operators on the ground state generates a basis of the Hilbert space. The completeness may be shown [8] by comparing the generating functions of the degeneracies (i.e., the finite temperature partition functions) for both the free electron basis and the boson basis. The notation for the kets of the second basis is

$$|\{N_p\}, \{n_q\}\rangle = \prod_p (U_p)^{N_p} \prod_{q \neq 0} \frac{(a_q^+)^{n_q}}{(n_q!)^{1/2}} |0\rangle \quad (11)$$

We now briefly describe the formalism to deal with interactions. The term for two-particle interactions is written as

$$H^1 = \frac{\pi}{L} \sum_{pq} V_q \rho_{qp} \rho_{-q-p}. \quad (12)$$

For simplicity, our treatment does not include interactions between fermions lying on the same side of the Fermi surface. Only  $g_2$  interactions are relevant in the physics we shall develop. One important feature of the interactions  $V_q$  is that they are cut off for impulsions greater than the inverse of a length scale  $R$ . We shall use the following expression of  $V_q$  (for  $q < 1/R$ )

$$V_q = V(1 - (qR)^\alpha). \quad (13)$$

The intensity of the interactions is parametrized by  $V$ , and the shape of  $V_q$  is parametrized by  $\alpha$ . The bosonized form of the interaction Hamiltonian  $H^1$  is

$$H^1 = \frac{\pi}{2L} (v_N - v_F) N^2 + \frac{\pi}{2L} (v_J - v_F) J^2 + \sum_{q>0} q V_q (a_q^\dagger a_{-q}^\dagger + a_q a_{-q}) \quad (14)$$

The total Hamiltonian is diagonalized by the following Bogoliubov transformation,

$$b_q^+ = \cosh \varphi_q a_q^+ - \sinh \varphi_q a_{-q}, \quad (15)$$

where the angle  $\varphi_q$  is defined as

$$\tanh 2\varphi_q = -\frac{V_q}{v_F} \quad (16)$$

The total Hamiltonian reads, after the Bogoliubov transformation,

$$H = E_0 + \sum_{q \neq 0} \omega_q b_q^+ b_q + \frac{\pi}{2L} (v_N N^2 + v_J J^2). \quad (17)$$

The effect of the interactions is to give a non-zero ground state energy:

$$E_0 = \frac{1}{2} \sum_q (\omega_q - v_F q), \quad (18)$$

where

$$\omega_q = (v_F^2 - V_q^2)^{1/2} |q|. \quad (19)$$

Interactions also shift the energies of the oscillators from  $v_F |q|$  to  $\omega_q$ . Finally, charge and current excitations acquire different velocities  $v_N = v_S e^{-2\varphi}$  and  $v_J = v_S e^{2\varphi}$ . In these relations,  $\varphi$  is the infrared limit of  $\varphi_q$  and the sound velocity  $v_S$  is related to the infrared limit of the dispersion relation (19)

$$v_S = \lim_{q \rightarrow 0} (v_F^2 - V_q^2)^{1/2}. \quad (20)$$

In the presence of interactions, one needs to normal order the field  $\psi_p^+(x)$  in terms of  $b_q^+$  bosons, which leads to the following expression of  $\Phi_p(x)$

$$\Phi_p(x) = p \frac{\pi x}{L} N_p - i \sum_{q \neq 0} (\theta(pq) \cosh \varphi_q - \theta(-pq) \sinh \varphi_q) e^{iqx} b_q. \quad (21)$$

The fermion field reads, in terms of bosons

$$\psi_p^+(x) = L^{-1/2} \exp \left\{ - \sum_{q>0} \left( \frac{2\pi}{Lq} \right) (\sinh \varphi_q)^2 \right\} e^{-ipk_F x} e^{-i\Phi_p^+(x)} U_p e^{-i\Phi_p(x)}. \quad (22)$$

**1.2 INTERACTION PICTURE FOR  $c_{kR}^+ |\{N_p\}\rangle$ .** — As  $t \rightarrow -\infty$ , the system is made up of a right moving fermion, with an impulsion  $k$  added above a Dirac sea  $|\{N_p\}\rangle$ , and interactions are vanishing. This section deals with the propagation of this state,  $c_{kR}^+ |\{N_p\}\rangle$ , as interactions are switched on.

The first step is to decompose the state  $c_{kR}^+ |\{N_p\}\rangle$  into bosonic modes. The action of  $c_{kR}^+$  on the vacuum  $|\{N_p\}\rangle$  in the  $q = 0$  sector is simply to increase by one the number of right moving fermions, by the action of the ladder operator  $U_R$ .

To obtain the action of  $c_{kR}^+$  in the  $q \neq 0$  sectors, we first Fourier transform  $c_{kR}^+$  into the real space field  $\psi_R^+(x)$  for right-moving fermions.

We replace the expression of  $\psi_p^+(x)$  in (9) by its expression (7) in terms of bosonic modes  $a_q^+$ . The development of the exponential  $e^{-i\Phi_R(x)}$  leads then to an expression of  $c_{kR}^+ |\{N_p\}\rangle$  as a linear combination of bosonic states, with occupation numbers  $\{n_q\}$

$$c_{kR}^+ |\{N_p\}\rangle = \sum_{\{n_q\}} \delta \left( \sum_{q>0} q n_q - (k - (k_F + \frac{\pi}{L}(2N_R + 1))) \right) \quad (23)$$

$$\prod_{q>0} \frac{1}{\sqrt{n_q!}} \left( \frac{2\pi}{Lq} \right)^{\frac{n_q}{2}} |\{N_R + 1, N_L\}, \{n_q\}\rangle. \quad (24)$$

The delta function insures that only bosonic states with a total impulsion equal to  $k - k_F - \frac{\pi}{L}(2N_R + 1)$  survive in the decomposition. As no interaction couples the two branches, creating a right-moving fermion does not generate left moving bosons.

The second step is to propagate the bosonic wave packet (23). Instead of dealing with the rather complicated superposition (23) of bosonic states, we focus on the propagation of a single

term  $|\{N_p\}, \{n_q\}\rangle$ . We shall use the bosonized form of the two-body interaction Hamiltonian, and look for a solution of the time dependent Schrödinger equation

$$i \frac{d|\{N_p\}, \{n_q\}\rangle_{\text{int}}(t)}{dt} = H_{\text{int}}^1 |\{N_p\}, \{n_q\}\rangle_{\text{int}}(t). \quad (25)$$

The "int" label stands for an interaction picture. The initial conditions are

$$\lim_{t \rightarrow -\infty} |\{N_p\}, \{n_q\}\rangle_{\text{int}}(t) = |\{N_p\}, \{n_q\}\rangle. \quad (26)$$

The bosonic states are propagated under the form of a coherent state

$$|\{N_p\}, \{n_q\}\rangle_{\text{int}}(t) = N(z_q(t)) e^{-z\phi(\{n_q\}, t)} \prod_{q>0} e^{-z z_q(t) a_q^+ a_q^-} |\{N_p\}, \{n_q\}\rangle. \quad (27)$$

The prefactor  $N(z)$  normalizes  $|\{N_p\}, \{n_q\}\rangle_{\text{int}}(t)$

$$N(\{z_q\}) = \prod_{q>0} (1 - |z_q|^2)^{\frac{n_q+1}{2}} \quad (28)$$

To determine the time dependent  $\phi(\{n_q\}, t)$  and  $\{z_q(t)\}$  functions, we first change  $z_q(t)$  into  $u_q(t)$ , with  $z_q(t) = u_q(t) e^{2iv_F q t}$ , and then identify both sides of the Schrödinger equation. We obtain first order non linear differential equations for  $\{u_q(t)\}$  and  $\phi(\{n_q\}, t)$ :

$$\frac{du_q(t)}{dt} + 2iv_F q u_q(t) = q V_q(t) (1 - u_q^2(t)) \quad (29)$$

$$\frac{d\phi(\{n_q\}, t)}{dt} = \sum_{q>0} (n_q + 1) q V_q(t) \text{Im}(u_q(t)). \quad (30)$$

In equation (30) we have discarded a term depending only on  $N$  and  $J$ , which leads only to a global phase factor. Translated in terms of  $\phi$  and  $z_q$  variables, the initial conditions (26) simply mean that  $\phi(t)$  and  $z_q(t)$  are vanishing as  $t \rightarrow -\infty$ . These differential equations describe the propagation of a single component of the wave packet (30). The propagation of the summation is obtained as a superposition of the different components after propagation

$$(c_{kR}^+ |\{N_p\}\rangle)_{\text{int}}(t) = \sum_{\{n_q\}} \delta \left( \sum_{q>0} q n_q - (k - (k_F + \frac{\pi}{L}(2N_R + 1))) \right) \quad (31)$$

$$\prod_{q>0} \frac{1}{\sqrt{n_q!}} \left( \frac{2\pi}{Lq} \right)^{\frac{n_q}{2}} |\{N_R + 1, N_L\}, \{n_q\}\rangle_{\text{int}}(t). \quad (32)$$

**1.3 ADIABATICITY CONDITION.** — We are looking for a solution of equation (30) which depends only on the variable  $s = \epsilon t$ , in the small  $\epsilon$  limit. It is possible since the external time dependence in equation (30) involves only  $\epsilon t$ . We then assume  $u_q(s) = u_q^0(s) + \epsilon u_q^1(s) + O(\epsilon^2)$ . Neglecting the  $O(\epsilon^2)$  terms in equation (30) leads to

$$2iv_F q u_q^0(s) = q V_q(s) (1 - u_q^0(s)^2) \quad (33)$$

$$\frac{du_q^0(s)}{ds} + 2iv_F q u_q^1(s) = -2q V_q(s) u_q^0(s) u_q^1(s), \quad (34)$$

where

$$V_q(s) = V_q^0 e^s. \quad (35)$$

The purely adiabatic solution  $u_q^{(0)}(s)$  is given by

$$u_q^0(s) = \frac{i}{V_q(s)} (-v_F + \sqrt{v_F^2 - V_q(s)^2}) = i \tanh \varphi_q^0(s). \quad (36)$$

Using this solution in equation (34) gives the first finite  $\epsilon$  correction

$$u_q^{(1)}(s) = i \frac{v_F}{2q(v_F^2 - V_q(s)^2)} u_q^{(0)}(s). \quad (37)$$

The adiabatic preparation of eigenstates is achieved if  $|u_q^{(1)}(s)|\epsilon \ll |u_q^{(0)}(s)|$  for  $s = 0$ , which leads to

$$\frac{v_F \epsilon}{2q(v_F^2 - V_q^2)} \ll 1. \quad (38)$$

This condition depends explicitly on  $q$ , and is satisfied for any value of  $q$  if

$$\epsilon \ll 4\pi v_F / L. \quad (39)$$

Here, we assume a weak coupling, namely  $|V_q| \ll v_F$ . It should be noticed that this upper bound on  $\epsilon$  is a much more restrictive condition than the corresponding upper bound in equation (2) for a Fermi liquid. We interpret this as a consequence of the fact that the quasiparticles of the Landau theory are not exact eigenstates of the interacting system. They are obtained in a situation where the thermodynamic limit is taken first, whereas the generation of exact eigenstates would require  $\epsilon$  to go to zero as the typical spacing between energy levels. Our criterion (39) corresponds to this second situation. This choice has been motivated by the possibility to construct the exact eigenstates of a Luttinger liquid.

**1.4 ADIABATIC PROPAGATION IN A BOGOLIUBOV SUBSPACE.** — The aim of this section is to propagate a fermion during the switching on procedure. We suppose that the condition (39) is satisfied, and we now look for a minoration of  $\epsilon$ . We first search an approximation for the evolution operator in the limit  $\epsilon \ll \frac{2\pi}{L} v_F$ . At the order  $\epsilon^0$ , the evolution operator  $U_\epsilon(0, -\infty)$  realizes the Bogoliubov transformations of angles  $\{\varphi_q^0\}$ , corresponding to the rotation of the basis of eigenstates as interactions were switched on from zero at time  $t = -\infty$  to  $\{\varphi_q^0\}$  at time  $t = 0$ . We shall note  $U^0(0, -\infty)$  the corresponding part of the evolution operator.  $U^0$  must have the property that

$$U^0 a_q^+ (U^0)^{-1} = \cosh \varphi_q^0 a_q^+ - \sinh \varphi_q^0 a_{-q}. \quad (40)$$

This equality is verified if  $U^0$  has the following form

$$U^0 = \exp \left\{ \sum_{q>0} \varphi_q^0 (a_q^+ a_{-q}^+ - a_q a_{-q}) \right\}. \quad (41)$$

To see it, we differentiate each operator  $U^0 a_q^+ (U^0)^{-1}$  and  $U^0 a_{-q} (U^0)^{-1}$  with respect to  $\varphi_q^0$  and solve the differential system.

However, at higher orders in  $\epsilon$ , the evolution operator must take into account the phase factor  $\phi(\{n_q\}, t)$ , the evolution of which is given by equation (30). Assuming that the propagation is adiabatic, we approximate  $\text{Im } u_q(t)$  in (30) by  $\text{Im } u_q^0(t)$

$$\text{Im } u_q(t) \simeq \tanh \varphi_q^0(s = \epsilon t). \quad (42)$$

We use expression (35) for  $V_q(t)$ , and integrate the differential equation (30) for the phase factor  $\phi(\{n_q\}, t)$ . A constant (infinite) phase factor associated with the propagation of the ground state is factored out. Thus, we obtain the form of the evolution operator in the adiabatic limit (at order  $\epsilon^0$  for the operator  $U^0$ , and at order  $1/\epsilon$  for the phases)

$$U_\epsilon(0, -\infty) = U^0 \exp i \left\{ \sum_{q>0} \frac{qn_q v_F}{\epsilon} (\varphi_q^0)^2 \right\} \quad (43)$$

$$= \exp \left\{ \sum_{q>0} \varphi_q^0 (a_q^+ a_{-q}^+ - a_q a_{-q}) \right\} \exp \left\{ i \sum_{q>0} \frac{qn_q v_F}{\epsilon} (\varphi_q^0)^2 \right\}. \quad (44)$$

In the integrations, we have assumed that the interactions are weak, and the phase factors are given at the lowest order in  $\varphi_q^0$ .

The rest of this section is devoted to the calculation and the interpretation of the overlap

$$F(x - x') \sim F(x, x') = \langle \{N_p\} | \Psi_R(x') U_\epsilon^{-1}(0, -\infty) \Psi_R^+(x) U_\epsilon(0, -\infty) | \{N_p\} \rangle, \quad (45)$$

between the dressed fermions  $\Psi_R^+(x) U_\epsilon(0, -\infty) | \{N_p\} \rangle$ , and the bare ones:  $\Psi_R^+(x') U_\epsilon^{-1}(0, -\infty) | \{N_p\} \rangle$ . To perform it, we use expression (22) of the field for right moving fermions, and approximation (44) for the evolution operator. The computation is straightforward, and  $F(x, x')$  is the product of three terms:

1) a phase term

$$N = \exp \left\{ -i \left( k_F + \frac{\pi}{L} (2N_R + 1) \right) (x - x') \right\}, \quad (46)$$

corresponding to the propagation in the  $q = 0$  sector.

2) A term corresponding to the left moving bosons normal ordering in (45):

$$G_1 = \exp \left\{ - \sum_{q>0} \frac{2\pi}{Lq} (\sinh \varphi_q^0)^2 \right\} \quad (47)$$

and 3) A term coming from the right moving bosons normal ordering

$$G_2(x, x') = \exp \left\{ \sum_{q>0} \frac{2\pi}{Lq} e^{-iq(x-x' - \frac{v_F(\varphi_q^0)^2}{\epsilon})} \right\} \quad (48)$$

The result for the overlap is

$$F(x, x') = \frac{1}{L} N G_1 G_2(x, x'). \quad (49)$$

The  $G_1$  term contains the usual physics of the orthogonality catastrophe [10]. If  $\varphi_q^0$  is assumed to be constant between  $q = 2\pi/L$  and  $q = 1/R$ , and zero afterwards, and if  $L \gg R$ ,  $G_1$  can be calculated as

$$G_1 = \left( \frac{L}{R} \right)^{-\sinh^2 \varphi^0} \quad (50)$$

In the weak coupling limit, one can deduce the characteristic interaction scale associated with the orthogonality catastrophe

$$V_{o.c.} = v_F \left( \ln \frac{L}{2\pi R} \right)^{-1/2} \quad (51)$$

To obtain the energy scale associated with the  $G_2$  term, we use relation (13) and approximate the phase as

$$- \frac{qv_F(\varphi_q^0)^2}{\epsilon} = - \frac{q(V^0)^2}{4v_F\epsilon} + \frac{q(V^0)^2}{2v_F\epsilon} (qR)^\alpha \quad (52)$$

The first term is linear in  $q$  up to the impulsion scale  $1/R$ . The second term is associated with smaller impulsion scales. The former are relevant for a quasiparticle. If  $k$  is the impulsion of the quasiparticle with respect with the Fermi level, the energy scale  $V_{\text{deph}}$  associated with the dephasing is given by

$$\frac{kV_{\text{deph}}^2}{2v_F\epsilon}(kR)^\alpha = 2\pi, \quad (53)$$

that is

$$V_{\text{deph}}(k) = \left(\frac{4\pi v_F\epsilon}{k(kR)^\alpha}\right)^{1/2} \quad (54)$$

The switching-on procedure shall henceforth be successful provided the intensity of interactions  $V$  is much smaller than  $V_{\text{deph}}(k)$ , that is

$$\frac{kV^2(kR)^\alpha}{4\pi v_F} \ll \epsilon. \quad (55)$$

**1.5 CONCLUSIONS.** — For the switching-on procedure to create a quasiparticle, conditions (39) and (55) have to be simultaneously met, that is

$$\frac{kV^2(kR)^\alpha}{4\pi v_F} \ll \epsilon \ll \frac{4\pi}{L}v_F. \quad (56)$$

This inequality is satisfied if the following consistency condition is fulfilled

$$V \ll \frac{4\pi v_F}{(kL(kR)^\alpha)^{1/2}}. \quad (57)$$

As we shall see, this condition has a simple interpretation on the spectrum of the Luttinger model. At this stage, we should again emphasize that the upper bound on  $\epsilon$  is more restrictive than in Landau theory. If we use the more usual condition that the spread in energy is smaller than the average density of the wave packet, equation (56) is replaced by

$$\frac{kV^2(kR)^\alpha}{4\pi v_F} \ll \epsilon \ll kv_F, \quad (58)$$

and the consistency condition is

$$V \ll \left(\frac{4\pi}{(kR)^\alpha}\right)^{1/2}v_F. \quad (59)$$

The absence of Landau quasiparticles in the thermodynamic limit is then attributed to orthogonality catastrophe, as indicated by equation (51).

**1.6 COMPARISON WITH THE GREEN FUNCTION.** — In this section, we calculate the Green function for the finite size Luttinger model

$$G_{\mathbf{R}}(x, t; x', t') = -i\{\{\{N_p\}\}e^{iH(t'-t)}\psi_{\mathbf{R}}(x')e^{-iH(t'-t)}\psi_{\mathbf{R}}^+(x)\{\{N_p\}\}\theta(t'-t) - (x \leftrightarrow x'; t \leftrightarrow t')\} \quad (60)$$

and reestablish the consistency condition (57). Note that in equation (60),  $|\{N_p\}\rangle$  denotes an eigenstate of the interacting system.

To calculate the Green function, we use the expression (9) of the field  $\psi_R^+(x)$  in terms of the Bose field, and normal order the expression (60) of the Green function with respect to the bosonic modes  $b_q^+$ . The computation is straightforward, and the result is

$$\begin{aligned}
 G_R(x, t; x', t') &= \frac{-i}{L} e^{i(k_F + \pi/L)(x' - x)} e^{i\frac{\pi}{L}(v_N(2N+1) + v_J(2J+1))(t' - t)} \\
 &\exp\left(-2 \sum_{q>0} \frac{2\pi}{Lq} (\sinh \varphi_q)^2\right) \\
 &\{[\exp\left(\sum_{q>0} \left(\frac{2\pi}{Lq}\right) (\cosh \varphi_q)^2 e^{iq(x' - x)} e^{-i\omega_q(t' - t)}\right) \right. \\
 &\left. \exp\left(\sum_{q>0} \left(\frac{2\pi}{Lq}\right) (\sinh \varphi_q)^2 e^{-iq(x' - x)} e^{-i\omega_q(t' - t)}\right)\right] \\
 &\left. - [x \leftrightarrow x'; t \leftrightarrow t'] \theta(t - t')\right\}
 \end{aligned} \tag{61}$$

The dispersion in the frequencies leads to decoherence after a time  $t_k$ . ( $k$  is the impulsion of the quasiparticle, with respect to the Fermi level).  $t_k$  may be estimated in the same way as we did for  $U_{\text{deph}}$ , and one finds

$$t_k = \frac{2\pi v_F}{V^2 k (kR)^\alpha} \tag{62}$$

For a system of size  $L$ , the wave packet is stable, provided it can cross the ring without decoherence

$$v_F t_k > L, \tag{63}$$

that is

$$V < \left(\frac{2\pi}{kL(kR)^\alpha}\right)^{1/2} v_F. \tag{64}$$

Up to some numerical dimensionless constants, this criterion is the same as the consistency condition (57) for the switching on of interactions.

## 2. Level statistics of the interacting Luttinger model.

**2.1 INTRODUCTION.** — We first need to find out a proper sector of the Hilbert space, in which we shall compute the level statistics. We note  $H_{JN}$  the subspace with given current  $J$  and charge  $N$ .

In the free case, the boson basis of  $H_{JN}$  can be organized as follows: consider all the sets of occupation numbers  $\{n_q^0\}$  such as, for all  $q$ ,  $n_q^0 = 0$  or  $n_{-q}^0 = 0$ . The corresponding states  $|\{n_q^0\}\rangle$  are annihilated by any pair destruction operator:  $a_q a_{-q} |\{n_q^0\}\rangle = 0$ . Starting from  $|\{n_q^0\}\rangle$ , and creating pairs generates a subspace  $H_{\text{pairs}}(\{n_q^0\})$ . A basis of  $H_{\text{pairs}}(\{n_q^0\})$  is made up of all the states  $|\{n_q^0 + p_{|q}\}\rangle$  with arbitrary occupation numbers for the pairs  $\{p_q\}_{q>0}$ .  $H_{NJ}$  is the direct sum of all the  $H_{\text{pairs}}(\{n_q^0\})$  subspaces.

The subspaces  $H_{\text{pairs}}(\{n_q^0\})$  remain stable under the action of the interaction Hamiltonian  $H^1$ , so that they are appropriate to the study of the level evolution.

We choose  $N = J = 0$  and drop the energy term associated with  $\{n_q^0\}$ , since we always handle differences between consecutive levels. The energy levels are given by

$$E(\{n_q\}) = \sum_{q>0} 2v_F q n_q \left(1 - \left(\frac{V_q}{v_F}\right)^2\right)^{1/2} \tag{65}$$

where we use the expression (13) for  $V_q$ .

**2.2 DESCRIPTION OF THE ALGORITHMS.** — In this section and the next paragraph, we use reduced units for the energies and impulsions:  $\omega$  is an energy divided by  $\frac{2\pi}{L}v_F$  and  $q$  is an impulsion divided by  $2\pi/L$ .

The degeneracies of the Luttinger model are given by

$$g(\omega) = \sum_{\{n_q\}} \delta(\omega - \sum_{q>0} qn_q). \quad (66)$$

Replacing the  $\delta$  function by its integral representation leads to

$$g(\omega) = \int_{-L/2}^{L/2} \frac{dx}{L} \prod_{q>0} \frac{1}{1 - e^{iqx}} e^{-i\omega x} \quad (67)$$

Let  $g^{(k)}(\omega)$  be the number of different sets of occupation numbers, having the property that

$$\omega = \sum_{l=k}^{\omega} ln_l. \quad (68)$$

Of course,  $g^{(1)}(\omega) = g(\omega)$ . The integral representation for  $g^{(k)}(\omega)$  reads

$$g^{(k)}(\omega) = \int_{-L/2}^{L/2} \frac{dx}{L} \prod_{q \geq k} \frac{1}{1 - e^{iqx}} e^{-i\omega x}. \quad (69)$$

Using the integral representations for  $g^{(k)}(\omega)$ , we obtain the following recurrences

$$g^{(k)}(\omega) = \sum_{\nu=k}^{\omega} g^{(\nu)}(\omega - \nu), \quad (70)$$

which allows us to compute  $g(\omega)$  numerically.

With a similar recursion, we may generate all the states of the free Luttinger model: the states with an energy  $\omega$  are obtained by adding a boson with an impulsion  $\nu$  on the states with an energy  $\omega - \nu$ .

As far as the interacting Luttinger model is concerned, we need to generate all the energy levels with an energy inferior to a given cut-off  $\omega_0$ . Since there are an infinite number of levels in the sector under consideration, we need to introduce such a cut-off to compute the statistics. We shall then compute the statistical properties of this set of levels. If a sufficient number of levels with an energy inferior to  $\omega_0$  has been generated, the statistical properties are independent of  $\omega_0$ . To generate the levels, we remark that the frequencies of the oscillators increase with their impulsion. So that we successively fill up the individual oscillator levels, starting with the smallest frequencies.

### 2.3 LEVEL STATISTICS.

**2.3.1 Degeneracies of the free Luttinger model.** — Using the recursion relation (70), we computed the degeneracies of the first 800 levels of the free Luttinger model. The asymptotic form

of the density of states may be derived in terms of initial fermions. The partial degeneracies for  $n$ -particles  $n$ -holes excitations in a one branch model are

$$g^{(n)}(\omega) = \sum_{\{k_i\}_{i=1}^n} \sum_{\{k'_i\}_{i=1}^n} \delta(\omega - \sum_{i=1}^n \omega(k_i) - \sum_{i=1}^n \omega(k'_i)). \quad (71)$$

The sets  $\{k_i\}$  ( $\{k'_i\}$ ) are the impulsions of the holes (particules), and are constrained by the Pauli principle  $k_i \neq k_j$  ( $k'_i \neq k'_j$ ) for all indices  $i \neq j$ . This sum is approximated by assuming a constant density of states, neglecting the Pauli exclusion principle, replacing the discrete sum by an integral

$$g^{(n)}(\omega) = \frac{1}{(n!)^2} \int_0^\omega d\omega_1 \int_0^{\omega-\omega_1} d\omega_2 \dots \int_0^{\omega-(\omega_1+\dots+\omega_{2n-1})} d\omega_{2n} \delta(\omega - (\omega_1 + \dots + \omega_{2n})) \quad (72)$$

The multiple integral is readily evaluated and leads to

$$g(\omega) = \sum_{n=1}^{+\infty} g^{(n)}(\omega) = \sum_{n=1}^{+\infty} \frac{\omega^{2n-1}}{(n!)^2 (2n-1)!}. \quad (73)$$

For sufficiently large energies, the sum may be approximated by its saddle point value, approximately reached for the following value of  $n$

$$n^* = \sqrt{\frac{\omega}{2}}. \quad (74)$$

The degeneracy evaluated at  $n = n^*$  is

$$g^{n^*}(\omega) \sim \frac{2^{3/4}}{(2\pi)^{3/2}} \frac{1}{\omega^{5/4}} \exp \sqrt{8\omega}. \quad (75)$$

We computed the summation (73) in order to test the accuracy of the saddle point approximation, which is plotted in figure 1. The exact degeneracies of the Luttinger model reveal to be inferior to the saddle point asymptotic form, which is imputed to the exclusion principle (Fig. 1).

**2.3.2 Qualitative structure of the spectrum.** — The evolution of some energy levels as a function of the interactions is plotted in figure 2. In this spectrum, we distinguish two regions:

1) No level crossings are present at sufficiently small energies and interactions. The free Luttinger model ( $V = 0$ ) belongs to this part of the spectrum. In this region, the statistics are ill defined for they strongly depend on the energy cut-off.

2) If  $E$  and  $V$  are large enough, level crossings occur, and level statistics are Poisson statistics. The convergence of the statistics as a function of the energy cut off  $e_0$  is shown in figure 3. Here, we emphasize that these level crossings occur because the Luttinger model remains integrable for any value of the coupling constant.

To characterize the separation between these two regions of the spectrum, the location of the crossings is estimated in the following way: as the intensity of interactions  $V$  is equal to zero, the spectrum is made up of equidistant degenerate levels, separated by an amount of energy  $\Delta E = \frac{2\pi}{L} v_F$ . As  $V$  is turned on, the degeneracies are lifted. We focus on a single fan of levels. All the levels are degenerate if  $V = 0$ , and their energy is  $E^0 = 2v_F k$ , where  $k$  is the

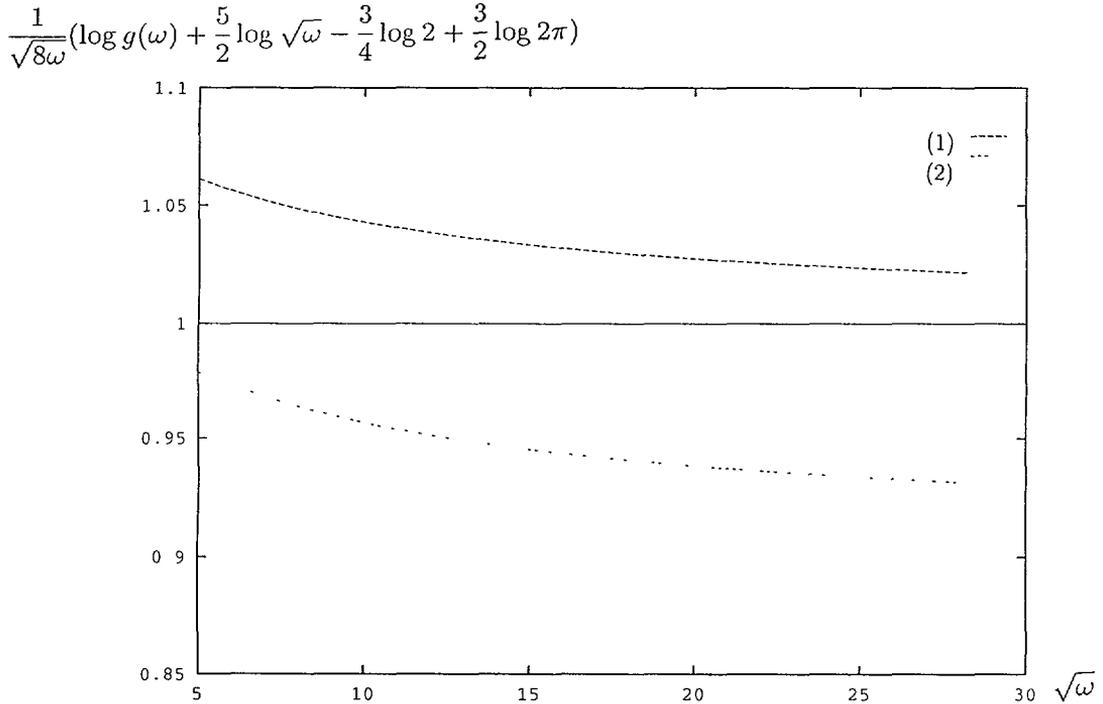


Fig. 1. — Degeneracies of the free Luttinger model, compared to the saddle point approximation.  $\{\log g(\omega) - \frac{3}{4} \log 2 + \frac{3}{2} \log 2\pi + \frac{5}{2} \log \sqrt{\omega}\} / \sqrt{8\omega}$  is plotted as a function of  $\sqrt{\omega}$ . This function equals 1 for the saddle point approximation. In plot (1),  $g(\omega)$  is the exact degeneracy. As expected, the saddle point approximation overevaluates the degeneracies since it takes into account particle-hole excitations forbidden by the exclusion principle. In plot (2), all the terms of the summation (73) are taken into account. The saddle point approximation in (73) underevaluates the degeneracies, and becomes exact at high energies.

total impulsion of the states. For a given value of  $V$ , all the levels lie between  $E_{\min}$  and  $E_{\max}$ .  $E_{\min}$  is obtained as all the quanta are in the smallest energy state (namely  $q = \frac{2\pi}{L}v_F$ ), so that

$$E_{\min} = 2v_F k \left(1 - \frac{V^2}{v_F^2} \frac{q = \frac{2\pi}{L}v_F}{k}\right)^{1/2} \tag{76}$$

$E_{\max}$  corresponds to a state with one quantum in the highest  $q = k$  state

$$E_{\max} = 2v_F k \left(1 - \frac{V^2}{v_F^2} \frac{q=k}{k}\right)^{1/2} \tag{77}$$

As the interaction parameter  $V$  increases, the levels evolve and the first crossings occur as the width of the fan  $E_{\max} - E_{\min}$  is of order  $\Delta E$ . This condition defines the interaction energy beyond which crossings exist

$$V^* = \left(\frac{\pi}{kL(kR)^\alpha}\right)^{1/2} v_F. \tag{78}$$

Via bosonization, the free Luttinger liquid is described as a set of harmonic oscillators with commensurable frequencies. As interactions are switched on, the oscillator frequencies vary and

reduced energy

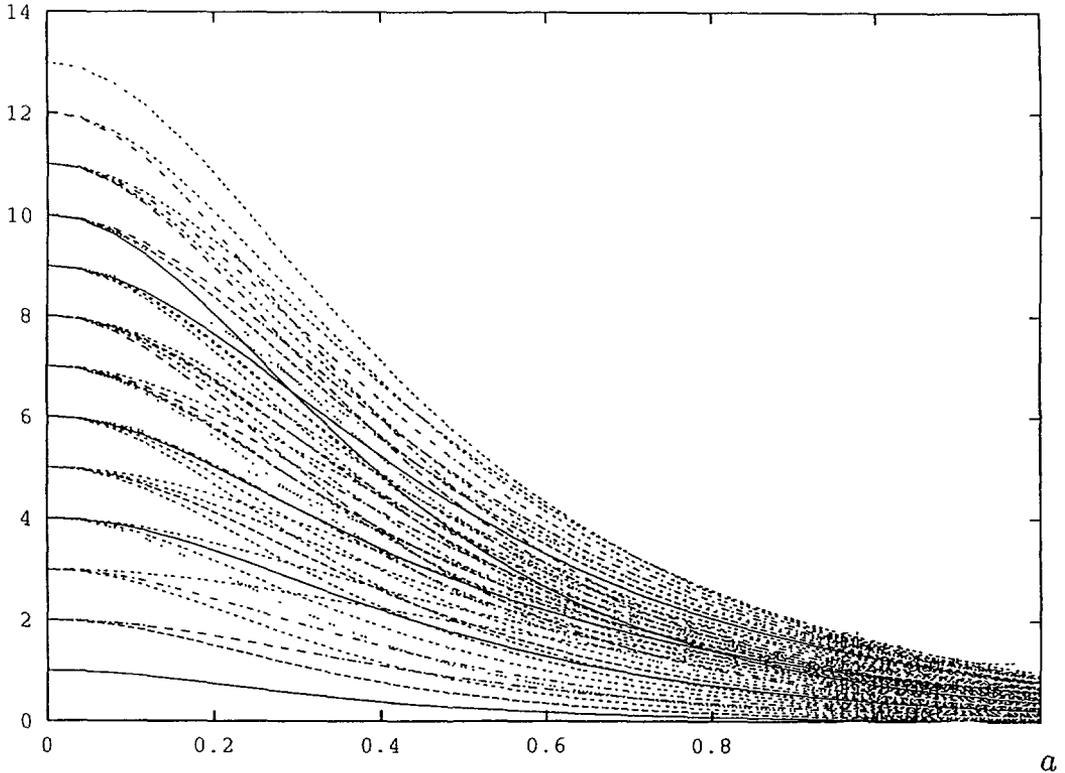


Fig. 2. — Evolution of some levels as a function of interactions.  $\varphi_q$  is a linear decreasing function of  $q$ , such as  $\varphi_{q=0} = 2.5a$ , and  $\varphi_{q \geq 26L/2\pi} = 0$ .  $a$  parametrizes the interaction strength, and the energy is in units of  $v_F 2\pi/L$ . For the plot to be readable, all the levels are not shown.

become incommensurable. In [6], Berry and Tabor show that a system with a finite number of generic harmonic oscillators does not exhibit level clustering. It appears that increasing the number of oscillators with incommensurable frequencies generates clustering.

**2.3.3 Quasi particle destruction and level spacing statistics.** — The condition (78) separates two regions of the spectrum. The same energy scale controls the existence or the absence of a quasiparticle in a Luttinger liquid, in the sense of adiabatic continuation of exact eigenstates. We have thus shown that the structure of the spectrum of the finite size Luttinger liquid is related to the success or the failure of adiabatic generation of eigenstates from the non interacting fermion system.

**2.3.4 Limit  $R = 0$ .** — Consider the case of the two-branch Luttinger liquid with  $1/R = +\infty$ , and a constant interaction, namely, for all  $q$ ,  $V_q = V$ . In this case, all the bosonic modes keep their coherence whatever the value of  $\epsilon$ . The decoherence time  $t_k$ , given in (62), is infinite. The condition (55) associated with the dephasings is always verified whatever the value of  $\epsilon$ .

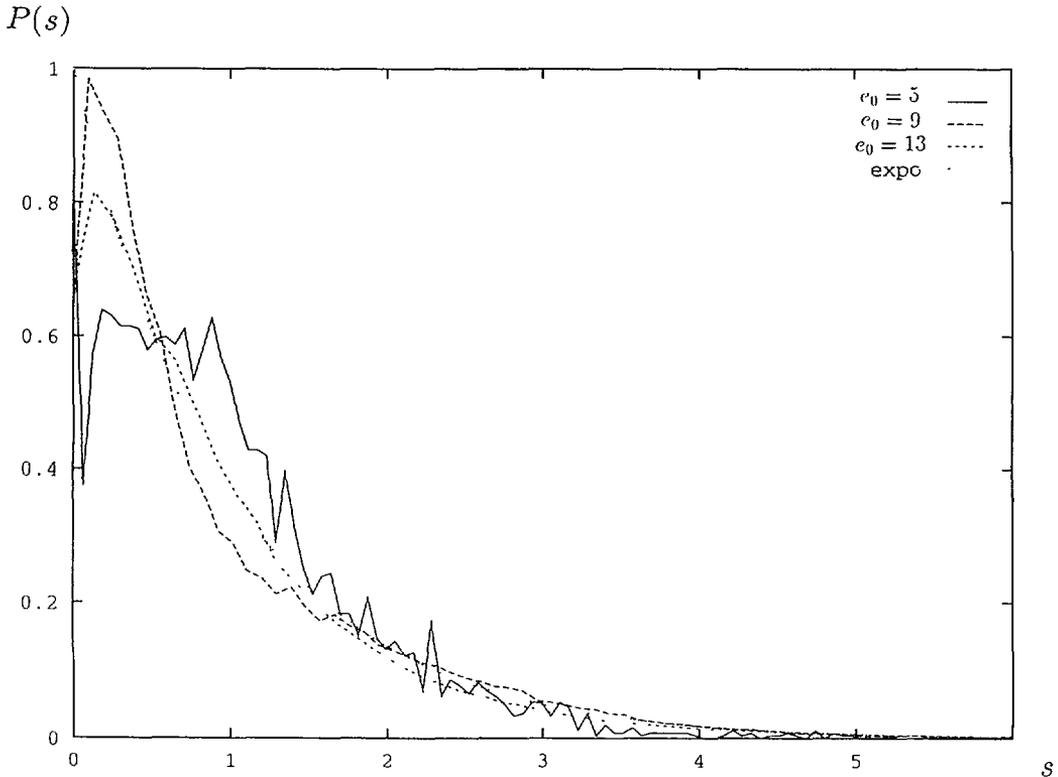


Fig. 3. — Evolution of the level spacing statistic as a function of the cut-off  $e_0$ .  $\varphi_q$  is a decreasing linear function, such as  $\varphi_{q=0} = 0.25$  and  $\varphi_{262\pi/L} = 0$ . The statistics converge slowly to a Poissonian distribution ("expo"). The statistics are plotted for  $e_0$  equal to 5, 9, 13. The number of levels taken into account in the statistics is 4196, 97438, 1048214, respectively.

The only remaining restriction for the switching-on procedure to be successful is thus

$$\epsilon \ll \frac{2v_F}{L}. \quad (79)$$

The level statistics are singular in this limit. The degeneracies of the fan of levels are never lifted, whatever the intensity of interactions  $V$ . The degenerate levels depend on  $V$  in the following way

$$E(\{n_q\}) = \sum_{q>0} 2v_F q n_q \left(1 - \left(\frac{V}{v_F}\right)^2\right)^{1/2} \quad (80)$$

However, we note that the overlap between the eigenstate thus constructed and the state obtained from the action of the bare electron operator on the interacting ground state is vanishing according to equation (50) since  $R = 0$ .

### 3. Level spacing statistics for a spin 1/2, one branch Luttinger model.

The rest of the article is devoted to the study of some models derived from the two branch, spinless Luttinger liquid model. We begin with the one-branch Luttinger model, with spin 1/2,

and a  $g_4$  interaction. The kinetic energy term is

$$H^0 = \sum_{k\sigma} v_F(k - k_F) c_{k\sigma}^+ c_{k\sigma}, \tag{81}$$

where the label  $\sigma$  denotes the spin component along the  $z$  axis. The interaction is given by

$$H^4 = \frac{g_4}{2L} \sum_{q\sigma} \rho_{q\sigma} \rho_{q-\sigma}^+. \tag{82}$$

The usual spin and charge combinations

$$C_q^+ = \left(\frac{\pi}{Lq}\right)^{1/2} (\rho_{q\uparrow} + \rho_{q\downarrow}) \tag{83}$$

$$S_q^+ = \left(\frac{\pi}{Lq}\right)^{1/2} (\rho_{q\uparrow} - \rho_{q\downarrow}), \tag{84}$$

have bosonic commutation relations, and the total Hamiltonian  $H = H^0 + H^4$  is diagonal in terms of spin and charge variables:

$$H = v_C \sum_{q>0} q C_q^+ C_q + v_S \sum_{q>0} q S_q^+ S_q + v_F \frac{\pi}{2L} (N_{\uparrow}^2 + N_{\downarrow}^2). \tag{85}$$

The charge and spin velocities are:  $v_C = v_F + g_4/2\pi$  and  $v_S = v_F - g_4/2\pi$ .

The  $g_4$  interaction is switched on adiabatically:

$$g_4(t) = g_4^0 e^{\epsilon t} \tag{86}$$

The evolution operator is

$$U_{\epsilon}(0, -\infty) = \exp \left\{ -i \sum_{q>0} \frac{g_4^0}{2\pi\epsilon} q (n_{Cq} - n_{Sq}) \right\}, \tag{87}$$

where  $n_{Cq} = C_q^+ C_q$  and  $n_{Sq} = S_q^+ S_q$ . The overlap

$$F(x, x') = \langle \{N_p\} | \Psi_{\uparrow}(x') U_{\epsilon}^{-1}(0, -\infty) \Psi_{\uparrow}^+(x) U_{\epsilon}(0, -\infty) | \{N_p\} \rangle \tag{88}$$

is found to be equal to

$$F(x, x') = \frac{1}{L} e^{i(\frac{\pi}{L}(2N_{\uparrow}+1)+k_F)(x-x')} \frac{1}{(1 - e^{i\frac{2\pi}{L}(x-x' - \frac{g_4^0}{2\pi\epsilon})})^{1/2}} \frac{1}{(1 - e^{i\frac{2\pi}{L}(x-x' + \frac{g_4^0}{2\pi\epsilon})})^{1/2}}. \tag{89}$$

Spin-charge separation is effective if the real space separation is of order  $\frac{g_4^0}{4\pi\epsilon}$ , which leads to the energy scale for spin charge decoupling

$$g_4^* = \frac{4\pi^2\epsilon}{k}, \tag{90}$$

where  $k$  is the impulsion of the quasiparticle with respect to the Fermi surface. The switching-on procedure is successful provided  $g_4^0 \ll g_4^*$ . Since the transformation (83) is independent of the interactions, there is no upper limit for the rate of switching on  $\epsilon$ .

In the same way as for the Luttinger liquid, the sector of the Hilbert space has to remain stable under the action of the evolution operator (87). Since  $U_{\epsilon}(0, -\infty)$  is diagonal in terms of

charge and spin variables, the relevant sector has a given impulsion  $k$ . This sector corresponds to a single fan of levels, with no crossings, except for  $g_4 = 0$ , leading to singular statistics. One may compute the statistics in the whole Hilbert space, namely to superpose the uncorrelated blocks with different impulsions. The statistics still remain singular. The degeneracies of some levels are not lifted for any value of the interaction  $g_4$ . These singularities correspond to remaining degeneracies as the impulsions of the charge and spin part are specified independently, and are reminiscent of the degeneracies of the free Luttinger model. The spectrum exhibits further singularities at non-zero level spacings, due to the linear dependence of the levels in  $g_4$ : the statistics do not become Poissonian even though uncorrelated sectors are superposed. Note that many degeneracies, and the singularities at non-zero level spacings are expected to disappear if  $g_4$  is not a constant as a function of  $q$ . In this more generic case, the Poisson statistics is expected.

**4. Level spacing statistics for a model of 2 coupled chains.**

We now discuss the level statistics for a model of two coupled Luttinger liquids. This model is solved in [11] and we first remind some results.

The two-chain kinetic energy is given by

$$H^0 = v_F \sum_{k\alpha\sigma} (k - k_F) \cdot c_{k\alpha\sigma}^+ c_{k\alpha\sigma} \quad (91)$$

where  $\alpha$  labels the chain and  $\sigma$  the spin. The interactions consist of a  $g_4$  term

$$H^4 = \frac{g_4}{2L} \sum_{k\sigma\alpha} \rho_{k\alpha\sigma} \rho_{k\alpha-\sigma}^+ \quad (92)$$

and of a hopping term between the two chains

$$H^\perp = -t_\perp \sum_{k\sigma\alpha} c_{k\alpha\sigma}^+ c_{k-\alpha\sigma} \quad (93)$$

Only the case of two coupled one-branch models is treated. This is sufficient since no interaction couples right and left fermions. Fabrizio and Parola [11] were able to diagonalize the Hamiltonian  $H = H^0 + H^4 + H^\perp$ . The excitation spectrum of the model exhibits four branches:

$$\epsilon_\rho(q) = u_\rho q \quad (94)$$

$$\epsilon_\sigma(q) = u_\sigma q \quad (95)$$

$$\epsilon_+(q) = \frac{1}{2}(u_\rho + u_\sigma)q + \sqrt{\left(\frac{1}{2}(u_\rho - u_\sigma)q\right)^2 + 4t_\perp^2} \quad (96)$$

$$\epsilon_-(q) = \frac{1}{2}(u_\rho + u_\sigma)q - \sqrt{\left(\frac{1}{2}(u_\rho - u_\sigma)q\right)^2 + 4t_\perp^2} \quad (97)$$

The ground state is such that all the states with a negative energy are occupied, and all the states with a positive energy are empty. We computed the level statistics for a toy model with only the  $\epsilon_-(q)$  branch, in a sector of given total impulsion  $q$ . We study the evolution of the statistics as the dimensionless hopping constant  $\tilde{t}_\perp = \frac{Lt_\perp}{\pi v_F}$  is fixed, and  $\tilde{g}_4 = \frac{g_4}{2\pi v_F}$  varies. The statistics exhibit a cross-over between two regimes as  $\tilde{g}_4$  decreases. This cross-over is controlled

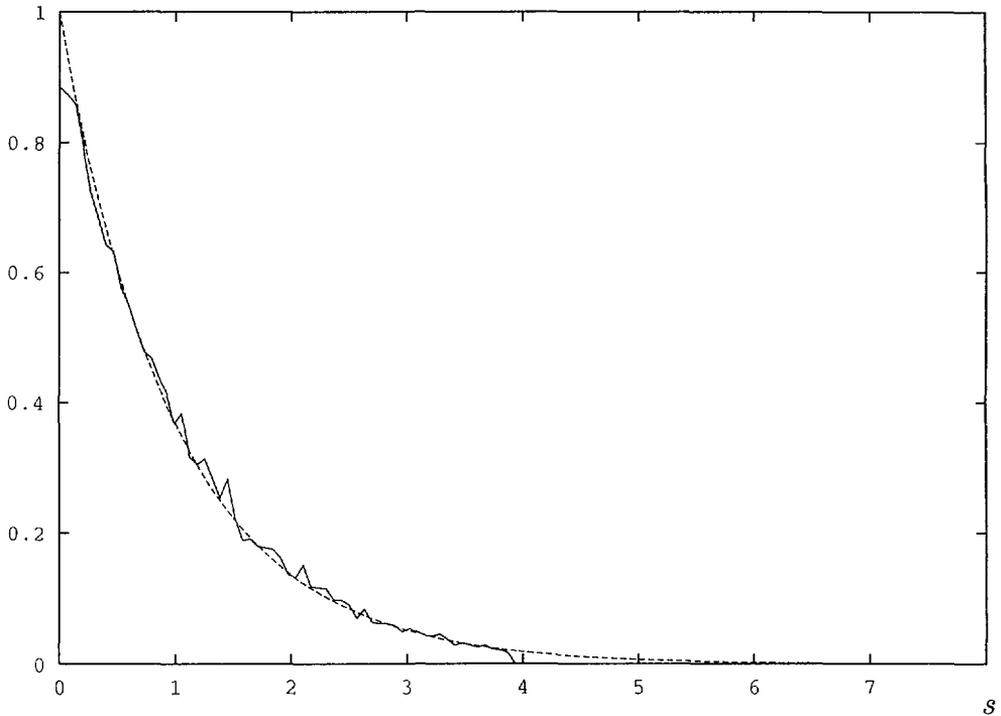
$P(s)$ 

Fig. 4. — Level spacing statistics for the model of two coupled chains in the regime  $q\xi \gg 1$ . Only excitations of the lowest energy branch  $\epsilon_-(q)$  are taken into account, and the analysis is restricted to the  $1p - 1h$  and  $2p - 2h$  excitations only, for parameters equal to:  $p = 200$ ,  $t_{\text{per}} = 20$ ,  $g = 0.5$ , with the following notation:  $p$  is the total impulsion divided by  $2\pi/L$ ,  $t_{\text{per}} = \frac{Lt_{\perp}}{\pi v_F}$  and  $g = \frac{g_4}{2\pi v_F}$ . 24000 states were generated. The value of the parameter  $q\xi$  is 100.

by the same length scale  $\xi = \frac{u_{\rho} - u_{\sigma}}{4t_{\perp}}$  as in [11]. If  $q\xi \ll 1$ , the statistics are singular, with a sharp peak at  $s = 0$ . In this regime, the dispersion relation  $\epsilon_-(q)$  may be approximated as

$$\epsilon_-(q) = \frac{1}{2}(u_{\rho} + u_{\sigma})q - 2t_{\perp}. \quad (98)$$

The linear  $q$  dependence induces high degeneracies in the excitation spectrum, leading to a sharp peak for zero separation.

In the opposite regime ( $q\xi \gg 1$ ), the statistics are Poissonian. The corresponding spectrum is plotted in figure 4. In this case, the curvature of the dispersion relation  $\epsilon_-(q)$  is no longer negligible, and individual fermion levels can no longer be considered as equidistant.

Notice that the cross-over observed here is similar to the case of the one-dimensional, one-branch Luttinger liquid with  $q$ -dependent interactions. In both cases, the dispersion relation is linear as the interaction parameter is set to zero (corresponding to a highly degenerate spectrum), and becomes non-linear as interactions are switched on (leading to a random spectrum). This transition is independent of the bosonic or fermionic nature of the particles. In the one-dimensional Luttinger liquid, we dealt with bosons, and the particles under consideration in

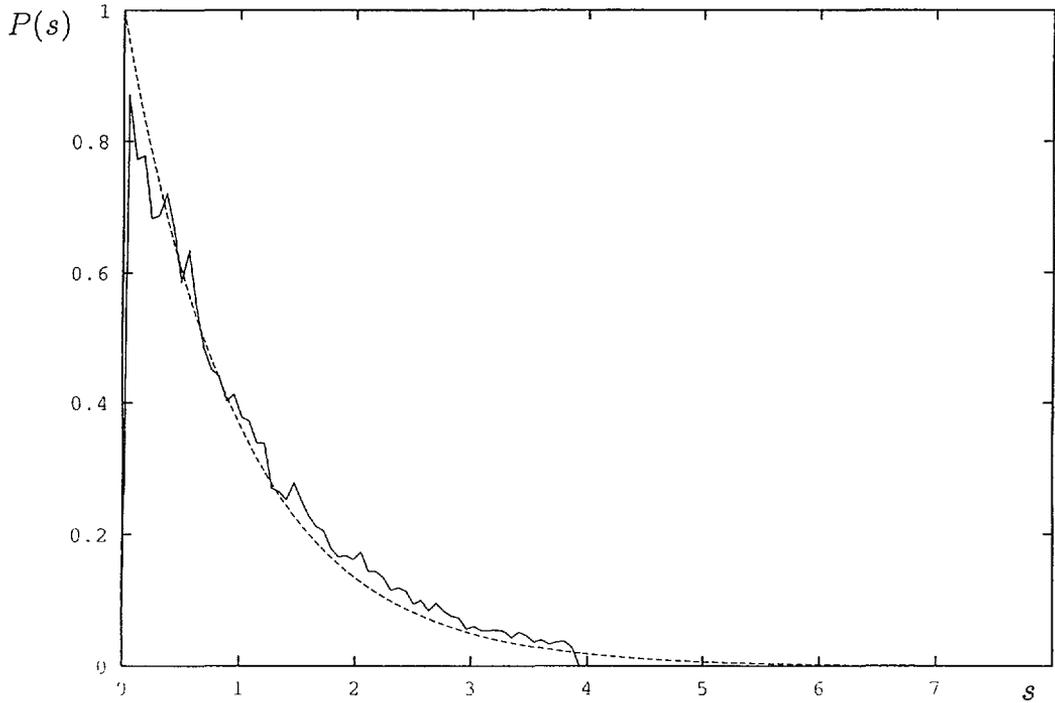


Fig. 5. — Level spacing statistics for the model of two coupled chains in the regime  $q\xi \gg 1$ , with the four excitation branches. Only  $1p - 1h$  and  $2p - 2h$  excitations were taken into account. The parameters are set to:  $p = 100$ ,  $t_{\text{per}} = 100$ ,  $g = 0.5$  and  $q\xi = 50$ . The number of computed levels is 42692. Among them, 8293 separations are equal to zero. For visibility, the level statistics is cut off for separations inferior to 0.05, which suppresses the large peak at zero separations.

the case of the two coupled chains are fermionic.

What happens if we now take the four branches into account? In the regime  $q\xi \gg 1$ , we observe a peak for  $s = 0$ , coexisting with a Poissonian distribution for non zero separations (see Fig. 5, where the peak is suppressed for clarity). The peak for  $s = 0$  is due to the degeneracies in the excitation spectrum, induced by the presence of the two linear branches. An example of such degenerate configurations, with 2 particle hole excitations is as follows: the two holes have impulsions  $h_1$  and  $h_2$ , and belong to the  $\epsilon_-$  branch. The particles with impulsions  $p_1$  and  $p_2$  are on the linear  $\epsilon_\rho$  branch. Consider another excitation, deduced from the previous one as follows: the holes have the same impulsions ( $h'_1 = h_1$  and  $h'_2 = h_2$ ). The impulsions of the particles are such that  $p_1 + p_2 = p'_1 + p'_2$ . Since all the particules belong to the same linear branch, these configurations are degenerate.

Thus, the existence of the two regimes  $q\xi \gg 1$  and  $q\xi \ll 1$  in the coupled chains is reflected in the statistical properties of the spectrum. To summarize, we have studied a special class of models, since they are integrable for any value of the coupling constant. In general, a non-interacting fermionic quasiparticle can be described as a linear combination of degenerate eigenstates, which undergo an energy splitting as interactions are switched on. This is responsible for the decay of such a quasiparticle state, and provides a lower bound for the switching rate  $\epsilon$ , in the process of adiabatic construction of quasiparticles. The same degeneracy lifting has been found to modify the energy level spacing distribution, from a singular behaviour for

a degenerate, non-interacting system, to a more generic Poisson distribution already observed in many integrable systems. We should stress that both aspects are non-universal features of the models. More precisely, they depend on the complete  $q$ -dependence of the interaction functions  $g_2$  and  $g_4$ . By contrast, universal properties such as correlation function exponents depend only on the  $q = 0$  limit of the couplings. We have seen that the vanishing of the quasi-particle residue, due to orthogonality catastrophe is also such a universal property, independent of the fine structure of the spectrum and its statistics.

In this paper, we could not address the question of strongly correlated fermion systems leading to Gaussian orthogonal ensemble (G.O.E.) statistics. However, the present study indicates that one of the most interesting questions is whether the difference between G.O.E. or Poisson distribution is a universal feature of a low-energy fixed point or not. Our paper has been dedicated to fine tuning phenomena within an integrable class of models, and the lack of universality found here is not surprising. Intuitively, the difference between Poisson and G.O.E. statistics is much more robust and might still be a way to distinguish between several physically non equivalent fixed points.

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