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F. Ladieu, D. Mailly, M. Sanquer. Conductance statistics in small insulating GaAs:Si wires at low temperature. II: experimental study. *Journal de Physique I*, 1993, 3 (11), pp.2321-2341. 10.1051/jp1:1993248 . jpa-00246871

HAL Id: jpa-00246871

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Submitted on 4 Feb 2008

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Classification

Physics Abstracts

72.10B — 72.15R — 72.20M

Conductance statistics in small insulating GaAs:Si wires at low temperature. II: experimental study

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Résumé. — Nous avons observé des fluctuations de conductance reproductibles dans un petit fil de GaAs:Si auquel nous avons fait passer la transition d'Anderson par application d'une tension de grille. Nous analysons quantitativement la statistique log-normale de conductance en termes de fluctuations quantiques tronquées. Les fluctuations quantiques dues à de petites variations de l'énergie des électrons (contrôlée par la tension de grille) ne peuvent pas se développer complètement à cause des fluctuations géométriques du réseau de résistances associé à la conduction par sauts dans l'échantillon. L'évolution des fluctuations, suivant l'énergie des électrons ou le champ magnétique, montre que les fluctuations sont non ergodiques, sauf dans le domaine d'isolant critique de la transition d'Anderson où la longueur de localisation est grande devant la distance entre impuretés de Si. La magnétoconductance moyenne est en bon accord avec des simulations fondées sur l'analyse de "chemins dirigés", c'est-à-dire qu'elle sature à $\ln\left(\frac{\sigma(H > 1)}{\sigma(0)}\right) \simeq 1$ pour $\sigma(0)$ variant sur plusieurs ordres de grandeur dans le régime fortement localisé.

Abstract. — We have observed reproducible conductance fluctuations at low temperature in a small GaAs:Si wire driven across the Anderson transition by the application of a gate voltage. We analyse quantitatively the log-normal conductance statistics in terms of truncated quantum fluctuations. Quantum fluctuations due to small changes of the electron energy (controlled by the gate voltage) cannot develop fully due to identified geometrical fluctuations of the resistor network describing the hopping through the sample. The evolution of the fluctuations versus electron energy and magnetic field shows that the fluctuations are non-ergodic, except in the critical insulating region of the Anderson transition, where the localization length is larger than the distance between Si impurities. The mean magnetoconductance is in good accordance with simulations based on the Forward-Directed-Path analysis, i.e. it saturates to $\ln(\sigma(H > 1)/\sigma(0)) \simeq 1$, as $\sigma(0)$ decreases over orders of magnitude in the strongly localized regime.

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Introduction.

Quantum interference effects are not well understood in disordered insulators. This contrasts with the diffusive regime where their role in weak localization and of Universal Conductance Fluctuations phenomena has been largely clarified both theoretically and experimentally [1].

However, huge, reproducible conductance fluctuations have been observed for instance in the hopping regime of small Si:MOSFET [2] and in lightly doped GaAs:Si samples [3]; the conductance statistics are found to be very broad, giving rise to very high conductance and resistance peaks (as compared to the averaged value) when the Fermi level or the transverse applied magnetic field are varied.

The mechanism of electronic conduction at finite low temperatures in lightly doped semiconductors has been explained by Mott [4]. Let us note $k_B T_0$ the level spacing on the scale of the localization length ξ . At low temperatures the hopping electrons optimize the cost due to thermal activation between energy levels of the initial and final impurity states and the tunnelling term. This results in a Mott hopping length given r_M , on average, by:

$$r_0 = \langle r_M \rangle = \frac{\xi}{2} \left(\frac{T_0}{T} \right)^{\frac{1}{d+1}} \quad (1)$$

(d the dimensionality).

The mean energy difference between the final and initial impurity levels separated by r_0 is:

$$E_0 = \frac{1}{2} k_B T_0^{\frac{1}{d+1}} T^{\frac{d}{d+1}} \quad (2)$$

At very low temperatures r_0 diverges and becomes much larger than l , the distance between impurities. r_M is thought to be the phase coherence length in the insulating regime. The averaged conductance in a large macroscopic sample is given by:

$$g \sim \exp - \left(\frac{T_0}{T} \right)^{\frac{1}{d+1}} \simeq \exp \left(- \frac{2r_0}{\xi} \right) \quad (3)$$

One has to distinguish between two explanations to describe the conductance fluctuations versus electron energy in the hopping regime of small samples: fluctuations of geometrical origin due to a change of the impurity sites visited by the electrons travelling through the sample [5] (incoherent mesoscopic phenomena [6]), or quantum fluctuations due to interferences phenomena for a fixed geometry of hopping paths.

Firstly, changes in electronic energy could be sufficient to induce a change of the impurity sites i and j between which the electrons hop. In other words r_M fluctuates around r_0 when one shifts the electron energy. As we will see, the typical energy range associated with such a change is the Mott energy E_0 (Eq. (2)). The quantum tunnelling resistance depends exponentially on the distance and on the energy separation of these sites [7]:

$$R_{i,j} \sim \exp \left(\frac{|E_i| + |E_j| + |E_i - E_j|}{2k_B T} + \frac{|r_i - r_j|}{\xi} \right) \quad (4)$$

($E = 0$ corresponds to the Fermi level). Because few impurity levels are involved during hopping through a mesoscopic sample at very low temperatures, the logarithm of the conductance itself exhibits large fluctuations. The explanation of large fluctuations versus the applied magnetic field results, in this geometrical approach, only from Zeeman shifts of energy of the impurity states [2].

Secondly, there exist conductance fluctuations emerging from quantum interference effects for a *fixed* quantum coherent hop (fixed locations and energies for the initial and final impurity states) of typical size $r_M \gg l$. Because of quantum coherence, one has to consider all the Feynman paths connecting the initial and final states, consisting of multi-diffusion paths on intermediate impurity states. At $T = 0$ K, i.e. when the quantum coherence length is the length of the sample, only these quantum interferences persist. They can be regarded as fluctuations of ξ itself. These fluctuations are influenced by phase shifts induced by an applied magnetic flux.

Two models have been proposed to take into account the interference effects in the hopping regime.

The first approach, referred to as Forward Directed Path analysis (FDP), neglects explicitly the quantum interferences between returning loops due to backward scattering [8]. This approach is a perturbative treatment of the deeply localized electronic states by the intermediate scattering during hopping. A crucial assumption is that the localization length is smaller than the distance between impurities (which is itself much smaller than the hopping distance). In this situation, referred to in the rest of this paper as the regime of strong localization, one has to consider interferences between Feynman paths of steps $\sim l$, l being smaller than r_M . As suggested first by Nguyen, Spivak and Schklovskii (NSS) [8], only the shortest paths - the Forward Directed Paths - are important, because the amplitude of transmission along a Nl long path is affected by a prefactor $\exp(\frac{-Nl}{\xi}) \ll 1$ ($l/\xi > 1$), exponentially decreasing with N . So the Forward Directed Paths approaches are well adapted at least to the strongly localized regime. The hypothesis $\xi < l$ excludes the critical insulating regime described by the scaling theory of the Anderson transition.

The second approach is based on a Random Matrix Theory (RMT) applied to the transfer matrix of either conductors or insulators [9]. In this global approach resonances as well as quantum interferences between all sorts of Feynman paths are *a priori* included. To some extent, this theory indicates that returning loops within the localization domain are essential, and thus is well adapted to the critical regime of the Anderson transition, where $\xi \gg l$, i.e. when electrons are localized over many impurity sites.

FDP and RMT predictions differ drastically for strong spin-orbit scattering or for the effect of a magnetic field.

The FDP approach predicts the existence of a large positive mean magnetoconductance, which is not the consequence of interferences between Time Reversal conjugated returning loops (they are neglected). The mean magnetoconductance $\langle \ln \left(\frac{g(H)}{g(0)} \right) \rangle$ depends only on r_0 [10], and is always positive whatever the spin-orbit scattering strength. The FDP approaches also predict large log-normal conductance fluctuations which are smaller *versus* the magnetic field than *versus* the disorder configuration (non-ergodicity) [8]. Quantitatively, the amplitude of the fluctuations $\text{var}(\ln(g))$ *versus* disorder is given by [11] : $\text{var}(\ln(g)) \sim r_0^{2\omega}$ with $\omega = 1/3$ (resp. $1/5$) for $d = 2$ (resp. 3).

By contrast with the FDP approach, the basic symmetries, such as the Time Reversal and Spin Rotation symmetry, are just the essential ingredients in the Random Matrix Theory. This approach gives exact results only in quasi-1d geometry, and its implications have to be weakened in higher dimensions. Nevertheless numerical simulations in 2d and 3d samples, as well as previous experiments, yield conclusions which are similar to some extent to exact RMT results [12]. Moreover similar conclusions are obtained in $d = 1, 2, 3$ on a completely different model in [13]. The main predictions of the RMT approach are that the breaking of the time reversal symmetry induces changes in the localization length ξ , and consequently an exponential magnetoconductance [12]. The sign of this magnetoconductance depends critically

on the spin-orbit scattering strength, going from positive to negative when the spin-orbit scattering increases. This theory also predicts log-normal fluctuations but with a variance of the logarithm of the conductance which is related to the mean of the logarithm of conductance (this is a *one* -parameter theory): $\text{var}(\ln(g)) = - \langle \ln(g) \rangle \sim L/\xi$ [9]. Note that contrary to the FDP result, the fluctuation amplitude - as well as the mean magnetoconductance - depends on L/ξ , and not only on L ($L \sim r_0$ at finite temperature). The fluctuation is ergodic versus the magnetic field and the disorder [14].

It is the aim of this work to test experimentally the validity domain of both approaches, by addressing the mean magnetoconductance effect, the distribution of the conductance fluctuations and the ergodicity. A submicronic disordered GaAs:Si wire is driven across the metal-insulator transition by application of a gate voltage. The conductance of the wire is measured over many orders of magnitude from the diffusive regime to the strongly localized regime at very low temperatures. To some extent our observations are similar to previously reported results [2, 3], but sample, analysis and interpretations differ noticeably. In short but wide GaAs MESFET used in reference [3], the conductance is dominated by a few most conductive paths, whereas in our 1D structure (see also Ref. [2]) the resistance is dominated by one most resistive hop. In Si-MOSFET used in reference [2], the effect of a magnetic field is interpreted in terms of Zeeman energy shifts in contrast to our observation of a quantum coherent contribution.

This paper is organized as follows: in the first part we describe our sample and the vicinity of the metal-insulator transition when the gate voltage V_G is varied. This part includes weak localization fits in the diffusive regime, which permit the determination of $L_\varphi = \sqrt{D\tau_\varphi}$, the phase coherence length and the effective width of the wire (D is the diffusion constant, τ_φ the phase-breaking time). The rest of the paper is devoted to the insulating regime.

First, we study the temperature dependence of the conductance. We show that, because of the one-dimensional geometry of our sample, its behavior with temperature is never given by the usual standard Mott law. Indeed, we explain that fluctuations of the hopping length around r_0 cannot be neglected. The conductance of our sample in the strongly localized regime is dominated by an exponentially small conductance corresponding to a hopping distance much larger than the mean Mott hopping length $r_0 = \langle r_M \rangle$. These considerations are important to explain some striking experimental observations.

We then turn to the study of the lognormal conductance fluctuations themselves. Those induced by varying the chemical potential are shown to result from a subtle interplay between geometrical and quantum fluctuations ("Truncated Quantum Fluctuations", [15]). Since quantum fluctuations cannot develop fully as the Fermi energy shifts, we turn to the study of fluctuations induced by the application of a magnetic field H and show that they are of quantum origin. Ergodicity and mean magnetoconductance behaviors change with the proximity of the metal-insulator transition, and this permits us to clarify the validity domains of FDP and RMT approaches.

1. The metal-insulator transition in our mesoscopic wire.

1.1 SAMPLE AND EXPERIMENT. — The sample is a standard Hall bar, with a distance between successive arms of $3 \mu\text{m}$, obtained by etching of a Si-doped GaAs layer. The layer is 400 nm thick grown by Molecular Beam Epitaxy with a Si concentration of 10^{23} m^{-3} on a GaAs semi-insulator substrate. Electron Beam Lithography has been used to pattern the sample. The subsequent mask was used to etch the active layer using 250 V argon ions. The width of the sample is approximately 400 nm . A 100 nm thick aluminium gate has been evaporated on the Hall bar.

The sample is placed in the plastic mixing chamber of a compact home-made dilution refrigerator. For electrical measurements, coaxial cables are used between 300 K and 4 K, and strip lines between 4 K and the mixing chamber. All the lines are properly filtered. The resistance is obtained by measuring the DC current passing through the sample with a Keithley 617 electrometer. The controlled excitation voltage supplied by the electrometer is divided, and the $I - V$ nonlinearities have been precisely studied (see later). The electrometer is controlled by computer, and each measurement cycle consists of 10 voltage inversions followed by a 3 s waiting time and 6 measurements (conversion time 0.3 s.). So the resistance results from an average of 60 measurements. The offset voltage is approximately $100 \mu\text{V}$ for very different measured resistances. We have not detected any offset current.

At very low temperatures in mesoscopic samples, one has to be very careful about excitation and offset voltages applied across the sample [1]. A common problem is to measure large resistances with excitation voltages small enough to be in the linear $I - V$ regime. Figure 1 shows a typical $I - V$ curve obtained at $T = 91 \text{ mK}$ in our sample. The characteristic is well fitted by:

$$I = A \operatorname{sh}\left(\frac{V_{ds} + V_{\text{offset}}}{B}\right) \quad \text{with} \quad A = 4 \times 10^{-12} \text{ A}, \quad B = 5 \times 10^{-4} \text{ V} \quad (5)$$

and $V_{\text{offset}} = -2 \times 10^{-4} \text{ V}$. The conductance is given by:

$$g = \frac{\partial I}{\partial V}_{V_{ds} + V_{\text{offset}} = 0} = \frac{A}{B} = 8 \times 10^{-9} \text{ S}. \quad (6)$$

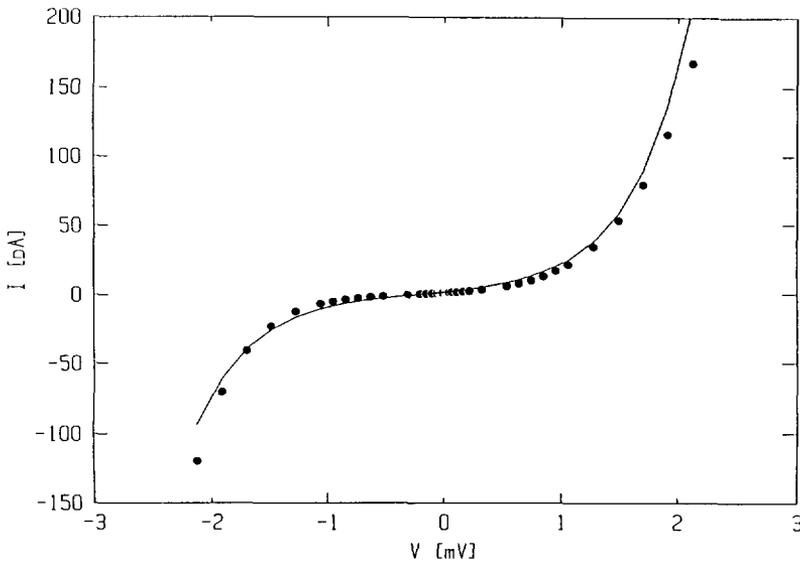


Fig. 1. — The $I - V$ characteristic at $T = 91 \text{ mK}$ for a typical low conductance. The solid line is a fit by an sh function (see Eqs. (5) and (6)).

The sh function is the simplest way to introduce the voltage nonlinearities; we do not see any rectifying behavior in our experiment. All the presented results are obtained in the $I - V$ linear regime.

The low-temperature conductance of the sample depends on the history of the cooling down from room temperature. In other words the conductance for $V_g = 0$ V depends for instance on whether the sample has been cooled under $V_g = +1$ V or under $V_g = -1$ V. The conductance is systematically larger in the latter case. There persist long time relaxations at $T = 4$ K after a large variation of V_g . A systematic study permits us to conclude that this relaxation is not due to a dynamic of disorder seen by the electrons, but to a slow variation of the Fermi level. In fact, after a large cycling in V_G , the observed conductance fluctuation patterns are translated in V_g but not at all decorrelated. This is consistent with a retarded response of the number of electrons to large changes of V_g , with the disorder configuration unchanged. One can qualitatively take the observed facts into account by supposing that the charge configuration of electronic traps inside the depletion barrier under the gate is not the equilibrium configuration corresponding to the nominal V_g at low temperatures. The difference results from the slow kinetics of trapping and release processes for the electrons at low temperature. The charge configuration in the depletion layer influences the number of electrons and the Fermi energy in the center of the wire.

These relaxations can be avoided by restricting the range of gate voltage changes in a given experiment at low temperature, or if not possible, by varying the gate voltage back and forth a few times in the corresponding range before the experiment. With the help of these experimental procedures the conductance pattern is fully reproducible as long as the sample is kept below $T = 4$ K.

1.2 THE DIFFUSIVE REGIME. — Figure 2 shows the magnetoconductance observed at low temperatures for a large gate voltage V_G , such as the conductance of the wire which is relatively large. For this value of V_g , the temperature dependence of the conductance is weak below $T = 4.2$ K. It is impossible to fit this dependence with a variable range hopping activation law (as we will do in the insulating regime), because it gives too small T_0 parameters (for instance $T_0 \simeq 50$ mK $< T$ for $V_G = 1.8$ V). We fit the mean behavior of the large positive magnetoconductance with standard 1D weak localization formula [16] and we find $L_\varphi = 130$ nm and an effective cross section $W^2 = (65 \text{ nm})^2$ (the sample has been rotated in the magnetic field and the magnetoconductance is found to be the same, which indicates that the cross section is isotropic). The effective length of the sample is evaluated to be $5 \mu\text{m}$, because in our two-probe measurement a part of two thin arms under the gate contributes to the conductance.

The magnetic field H_c which gives a flux quantum through $L_\varphi W$ is $H_c = \frac{h}{e} \frac{1}{L_\varphi W} = 0.42$ T.

This gives the good order of magnitude for the correlation field of the magnetoconductance fluctuations. The amplitude of the fluctuations, if they are supposed to be the Universal Conductance Fluctuation, is given by [17] : $\delta g(H) \simeq \frac{e^2}{h} \sqrt{\frac{4}{15} \left(\frac{L_\varphi}{L}\right)^3} \simeq 2.2 \times 10^{-3} \left(\frac{e^2}{h}\right)$ in good accordance with the experiment.

Figure 3 shows the variation of the conductance (in units of e^2/h) as a function of the applied gate voltage for $T = 100$ mK. The conductance exhibits reproducible Gaussian fluctuations as a function of V_G , of amplitude similar to the conductance fluctuations induced by the transverse applied magnetic field, and so in accordance with the estimate of the Universal Conductance Fluctuation.

In accordance with the scaling theory of the Anderson transition, we expect that the transition occurs for a conductance at the phase coherence length of order e^2/h . For our sample consisting approximately of $L/L_\varphi \simeq 40$ quantum boxes in series, this criterion corresponds to a conductance of order $2.5 \times 10^{-2} e^2/h$. This corresponds to a gate voltage of approximately

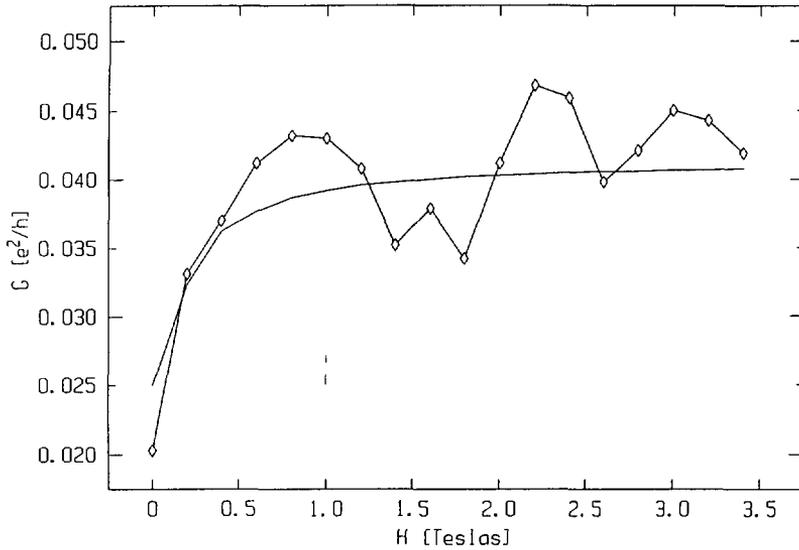


Fig. 2. — Magnetoconductance (in units of e^2/h) at $T = 100$ mK for large positive $V_G = +1.8$ V (diffusive regime). The solid line is the 1D Weak localization fit. The vertical bar is the UCF estimate.

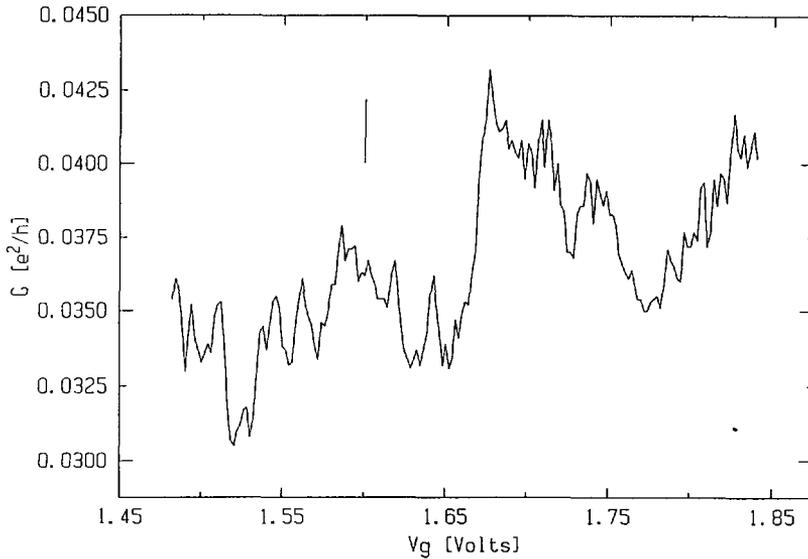


Fig. 3. — The conductance (in quantum units) as function of V_G in the diffusive regime. The vertical bar is the UCF estimate.

$V_g = +0.5$ V above which the temperature dependence of g is weak between $T = 4.2$ K and $T = 100$ mK and roughly independent of V_g . Below $V_g = +0.5$ V, g becomes activated: for instance, from figure 5, we obtain typically that the resistance ratio between $T = 4.2$ K and $T = 100$ mK is 2.4, 2.5, 2.75, 3.7, 6.5 and 10 respectively for $V_g = +0.7, +0.6, +0.5, +0.4,$

+0.3 and +0.2 V. From the experiment it will be pointless to argue any further about the exact position of the transition.

Note nevertheless that in this range of conductances, the conductance fluctuation departs from its value in the diffusive regime, growing and becoming asymmetric with tails to low conductances.

With the estimated effective cross section, and supposing that the concentration of electrons is close to the critical concentration in GaAs for the Metal-Insulator Transition $n_c = 1.6 \times 10^{22} \text{ m}^{-3}$, we find a mobility of $\mu \simeq 3600 \text{ cm}^2/\text{Vs}$. Close to the transition, we obtain that $\lambda_F \simeq 65 \text{ nm}$, which is comparable to the width of the sample, $E_F \simeq 45 \text{ K}$, the elastic mean free path $l \simeq 24 \text{ nm}$ comparable to the distance between Si atoms, and $k_F l \simeq 2$ (Ioffe-Regel criterion for the Metal-Insulator transition).

1.3 THE ANDERSON TRANSITION. — As the gate voltage is reduced, the number of electrons in the wire decreases as their Fermi energy:

$$eN = \int C_{\text{gate}}(V_g) dV_g \quad (7)$$

Typically, we estimate that $C_{\text{gate}} \simeq 1.5 \times 10^{-16} \text{ F}$ and we neglect its gate voltage dependence. Near the critical Mott concentration $n_c \simeq 1.6 \times 10^{22} \text{ m}^{-3}$ and taking a 3D density of states, we estimate that a variation $\Delta V_g \simeq 10 \text{ mV}$ corresponds to $\Delta E_F \simeq 1 \text{ K}$ (Note that, with this crude estimate, the gate voltage range needed to deplete the wire completely from the n_c value is $\simeq 0.5 \text{ V}$).

The Anderson transition takes place below a certain critical gate voltage, and the temperature dependence of the conductance becomes activated. This is apparent in figure 4a, where $\ln(G)$ is plotted *versus* $T^{-1/2}$ for various gate voltages. (The choice of the exponent $-1/2$ or -1 is rather arbitrary as explained and discussed in the following). An interesting point is that the activated behavior saturates below a temperature which increases when the sample becomes more insulating. For instance in figure 4a, there is a complete saturation of g below $T \sim 450 \text{ mK}$ for $T_0 \sim 93 \text{ K}$, as for $T_0 = 3.8 \text{ K}$, g does not saturate at low temperatures and typically, $g(T = 450 \text{ mK}) \sim 1.5g(T = 70 \text{ mK})$. We will discuss this saturation in section 1.5.

In the restricted range of temperatures where the Mott hopping regime is seen (Eq. (2)), it is difficult to evaluate precisely the actual value of the exponent $\frac{1}{d+1}$. One first point is that the exponent must give a reasonable estimate for the parameter T_0 , i.e. it cannot exceed 60 K, the energy of a single Si impurity state in GaAs. For this reason, one cannot choose an exponent of $1/4$ ($d=3$) since this would give a T_0 of order of a thousand K. Moreover, since the effective cross section of our sample at the M.I.T. is only 65 nm^2 and since it decreases when V_g is diminished, it is not surprising that, below M.I.T., our sample should be a 1D wire ($r_0 > W$, $d = 1$).

1.4 THE ONE-DIMENSIONAL HOPPING REGIME. — It has been first pointed out by Kurkijarvi [18], that one has a simple T^{-1} activation law for the conductance for a given 1D wire in Mott's regime ($r_0 > W$). This results from the fact that a single hop dominates the measured resistance. *A priori*, the slope of this single activation law only gives the energy activation of the dominant link and not directly T_0 , the mean energy spacing on the scale of the localization domain. We will see later that when averaging over disorder is made, one recovers an exponent $1/2$ whose slope is a function of both the length of wire and of T_0 . Let us explain why.

Qualitatively, let us note that in samples at $d = 2$ or 3, Mott's law is observed without averaging over many samples. This is because when $d > 1$ self-averaging occurs within each

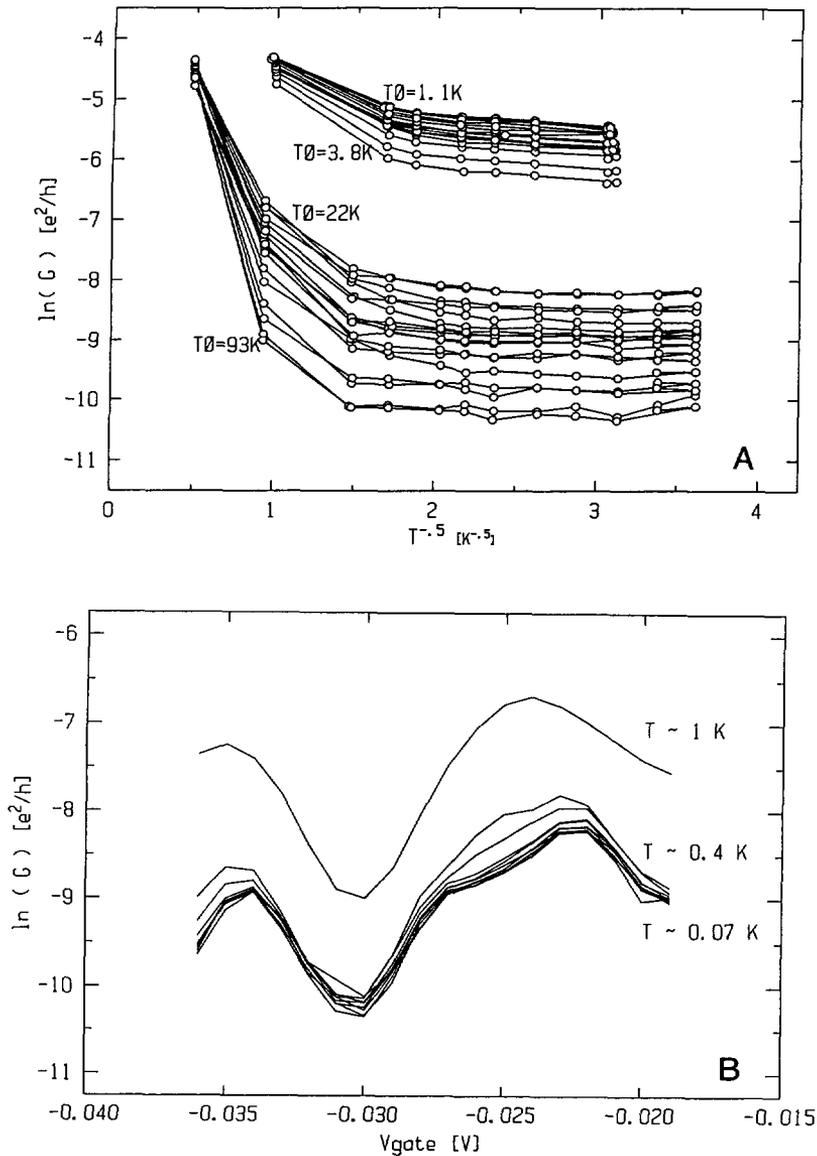


Fig. 4. — A) $\ln(g)$ (in quantum units) versus $T^{-1/2}$ for various V_G . The T_0 parameter values for the extremal curves are indicated. B) $\ln(g)$ versus V_G at various temperatures between $T \simeq 1$ K and $T \simeq 70$ mK. The range of V_G corresponds to the curves at the bottom of figure 4a.

sample, allowing us to consider only a typical resistor (r_0, E_0) given by Mott's law (see Eqs. (1-3)) in order to calculate the resistance of the whole sample. But in 1D wires, such an averaging does not take place: since elementary resistors are always added in series, one has to consider the strongest one (and not the mean one) in order to evaluate the resistance of the wire.

Such an idea can be quantitatively developed. We now summarize what comes out of a

detailed analysis of the Mott VRH in 1D wires [5, 6, 15]. Let us consider a long wire without fluctuations of quantum origin which allows us to use equation (4) for each elementary resistance $R_{i,j}$ and to get their values as soon as the distribution (x_i, E_i) of localized states is known. One statistically neglects resonant or direct tunnelling since we assume $L \gg r_0$. Using an assumption of local optimisation (at each step the electron chooses the less resistive hop), one can self consistently solve the problem of 1D hopping [15]. Due to a possible local lack of levels near the chemical potential μ , lengths of elementary hops fluctuate around r_0 , giving for $R_{i,j}$ a distribution whose width $w_{i,j}$ is so large that the addition of $N = L/r_0$ resistances $R_{i,j}$ in series does not self-average (as long as N is not extremely large). Note that such a method is consistent only if $w_{i,j} \gg w_q$, where w_q is the width of the distribution of resistances due to quantum interferences (w_q can be regarded as the fluctuation of $1/\xi$ in (1)).

One can show that, if $N < N^* = \frac{1}{a} e^{2(2\frac{T_0}{\xi})^2}$ ($a \simeq 2$), the resistance of a wire is entirely dominated by only one elementary most resistive hop: $R_{\max} = \text{Max}_N(R_{i,j})$ whose average value is size dependent. Estimation of R_{\max} gives:

$$\ln R \simeq \ln R_{\max} = \frac{\tau_{\max}}{\xi} \quad (8)$$

with

$$\langle \tau_{\max} \rangle = 2r_0 \sqrt{2 \ln(aN)} = \xi \left(\frac{T_0}{T} \right)^{1/2} \sqrt{2 \ln(aN)} \quad (8\text{bis})$$

Note that *in average over disorder*, one still has a $T^{-1/2}$ law. The measured $\ln R$ does not directly give T_0 but features of the dominant hop. Nevertheless T_0 — the important averaged microscopic energy — can be estimated for our experimental parameter $\ln R$ and for reasonable ξ : in the companion paper [15] a simulation of our wire for $\ln R \simeq +9$ at $T = 0.45$ K is presented with: $\xi = 2l \simeq 50$ nm and $T_0 = 6$ K (see the comments in [15] on the slight discrepancy between calculated and measured $\ln R$). $\xi \simeq l$, so we call this regime “strongly localized”, by contrast with the “barely insulating regime” that one encounters near M.I.T. where T_0 is not large enough compared to T to allow a description in terms of variable range hopping. In this regime ξ must be given in order of magnitude by $L_\varphi \simeq 130$ nm (at very low temperatures), i.e. $\xi \gg l$.

Moreover, we found numerically that the whole experimental range of conductances corresponds to variations of T_0 between 2 K and 10 K. Let us emphasize that these values are significantly lower than those naively extracted from data in $T^{-1/2}$ scale (see Fig. 4a) which, as we explained, is definitely not relevant for a *given* wire in Mott’s regime. Nevertheless, as already pointed, no precise determination of the temperature exponent can be extracted from the experiment, and the choice of the abscissa in figure 4a is rather arbitrary.

1.5 SATURATION OF THE CONDUCTANCE AT LOW TEMPERATURE. — As noted before, the temperature dependence of the conductance exhibits a saturation below a temperature which increases when the gate voltage decreases. Because all the measured conductance properties become temperature independent, it is likely to incriminate electron heating by radiofrequency voltage sources (let us recall that the conductance is recorded in the $I - V$ linear regime). Voltage radiofrequency noise is *a priori* more efficient to heat electrons when conductance is high. However the conductance saturation is clear only when conductance is low (small V_G). Moreover the saturation temperature is the same for the peaks and the valleys of the conductance pattern even for peak-to-valley ratio as large as 10^2 , in the strongly localized regime. This is hardly compatible with simple heating.

Even if it is difficult to rule out heating by radiofrequency pickup, the observed saturation up to $T = 400$ mK seen in the strongly localized regime could be due to intrinsic physical effects: either resonant tunnelling processes or the existence of plateaus in the temperature dependence of a mesoscopic 1D wire in the hopping regime [15].

A crossover from hopping at high temperatures to T -independent tunnelling at low temperatures should happen if the diverging Mott hopping length r_0 (more precisely r_{\max}) becomes of the order of the sample length at low T [2, 19]. But the estimate of $r_{\max} \simeq 600$ nm obtained from the reported estimate of T_0 is about 10 times smaller than our sample length when the saturation of g occurs. The resonant tunnelling through the sample is negligible under this condition. Another observation against the resonant tunnelling picture is that the measured conductance is always decreasing when the temperature decreases, even for sharp conductance peaks. However it is well known that inelastic processes always decrease the resonant conductance in the tunnelling processes, whereas phonons always increase the hopping conductance. For these reasons we do not believe that resonant or direct tunnelling processes are of importance in our geometry.

Apart from resonant tunnelling or heating, special features of temperature dependence in 1D V.R.H. could give rise to temperature saturation. As reported in figure 2 of [15], one has to distinguish two main cases for the temperature dependence. First, when the temperature is such that values of $N = L/r_0$ are large enough (precisely when $N > N^*$ defined above), $\ln R$ should vary as $1/T$. Since r_0 diminishes as T increases, such a regime only arises at quite high temperatures, let us say: $T > T^*$. T^* is given by: $T^* = \frac{2T_0}{\ln\left(2\frac{aL}{\xi}\sqrt{\frac{T^*}{T_0}}\right)}$ [15] roughly

proportional to T_0 , such that T^* increases when the sample becomes more insulating. As already noted, one can indeed see in figure 4a - and this is a general trend - that activated behavior is valid above a temperature which grows as the sample is driven to a more insulating regime. We estimate, using the definition of N^* with the experimental parameters, that: $T^* \simeq 1 - 2$ K in the strongly insulating regime.

What happens if $N < N^*$? As discussed in [15] we think that in this case the activation energy of the dominant link can be very weak, leading to an apparent saturation of R with decreasing T . If this happens, such a non-activated link will remain dominant as long as the second-dominant activated link becomes more resistive because of decreasing T . Thus T -dependence of R will be a succession of "activated segment - apparent plateau" and reference [15] shows that in a logarithmic scale of T plateaus and segments are of same size. When averaging over many samples, one should however recover Mott's 1D law due to random location of segments and plateaus for different samples.

However, the observed saturation of R is larger than the size of plateaus predicted in [15] and moreover we never see an activated segment at temperatures lower than the temperature at which saturation begins. Therefore, we think that heating by rf pick-up could be partly responsible for the observed saturation.

Up to now, the study of the temperature dependence in the localised regime has been carried out without taking into account any quantum fluctuations. We now focus on conductance fluctuations versus Fermi energy and on the effect of magnetic field, which will give us much more insight into the relevance of zero temperature theories for our experiment.

2. Conductance fluctuations in the localized regime.

Figure 5 shows the conductance as a function of the gate voltage (over a large range of V_G) for

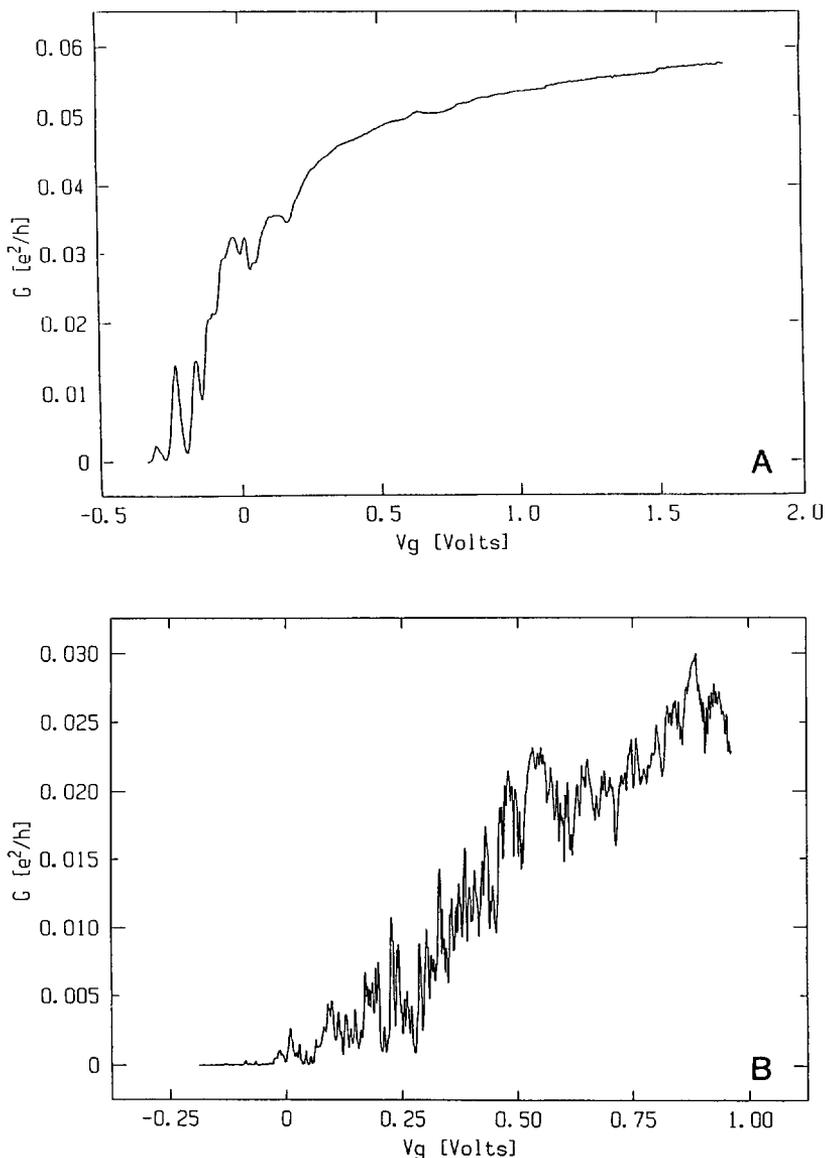


Fig. 5. — A) Conductance in quantum units versus the gate voltage at $T = 4.2$ K. B) The same at $T = 100$ mK.

two temperatures: $T = 4.2$ K and $T = 100$ mK (a thermal cycling up to room temperature has been applied between the two records). The relative fluctuation becomes enormous for small values of the conductance (sometimes exceeding two orders of magnitude), so that a semilog representation is more adapted (Fig. 6).

2.1 QUANTITATIVE ANALYSIS OF THE LOG-NORMAL CONDUCTANCE FLUCTUATIONS. — In this section we develop a quantitative analysis of the log-normal conductance fluctuations,

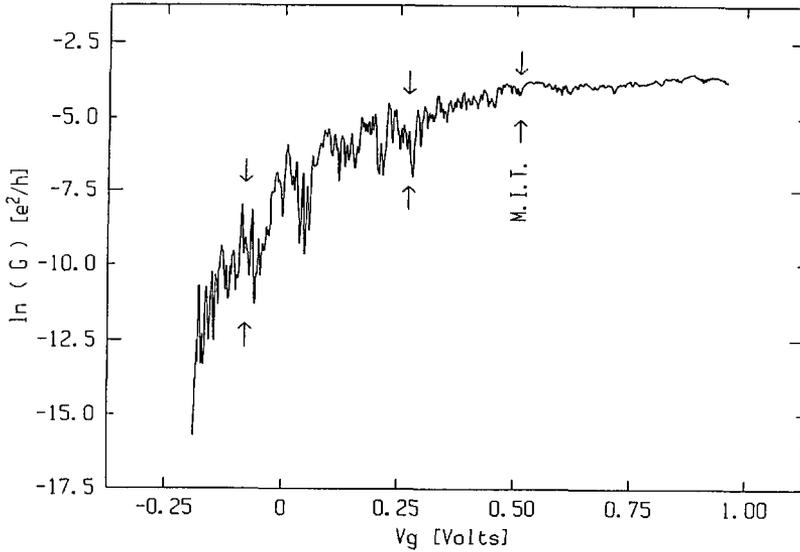


Fig. 6. — Figure 5B in a semi-logarithmic plot. The arrows indicate the estimated Anderson transition and the barely and strongly insulating regimes where the magnetic field dependence has been precisely studied (see Fig. 8).

based on the considerations developed successively by Lee [5], Raikh and Ruzin [6], and Ladiou and Bouchaud [15].

Figure 7 shows $\delta \ln(R)$ versus $\langle \ln(R) \rangle$ for $T = 100$ mK and $H = 0$ T. $\langle \ln(R) \rangle$ is obtained by numerical smoothing of $\ln(R)$ to remove the short V_G -range fluctuations. Two experiments differing only by a thermal cycling to room temperature are presented in order to improve the statistics.

As we reported in the preceding section, the measured $\ln R$ is dominated by the most resistive link R_{\max} whose value is size dependent. Thus amplitude of fluctuations is given by the width w_N of R_{\max} distribution. Estimation of w_N leads to $w_N \ll w_{ij}$, and gives:

$$\frac{\Delta \ln R}{\ln R} = \frac{1}{2 \ln(aN)} \tag{9}$$

Fortunately, this prediction depends weakly on the single adjustable parameter N , for realistic large values of N . We numerically found (see Fig. 4 of [15]) that $N \simeq 53$, but even taking $N = 25 - 100$ ($r_0 = 50 - 200$ nm), we get a small dispersion:

$$\frac{\Delta \ln R}{\ln R} = 0.11 \pm .015 \tag{10}$$

This prediction is reported in figure 7, in very good accordance with the experimental data.

Therefore, at this point, one does not need to invoke the quantum coherence to explain the observed amplitude of $\delta \ln(R)$. We now detail the arguments which justify the introduction of quantum fluctuations within the most resistive hop.

The predicted energy width for the geometrical fluctuations is given by [6-15]:

$$\Delta E_{\text{geo}} = 4E_0 \frac{1}{\sqrt{2 \ln(2N)}} \simeq 1 \text{ K} \tag{11}$$

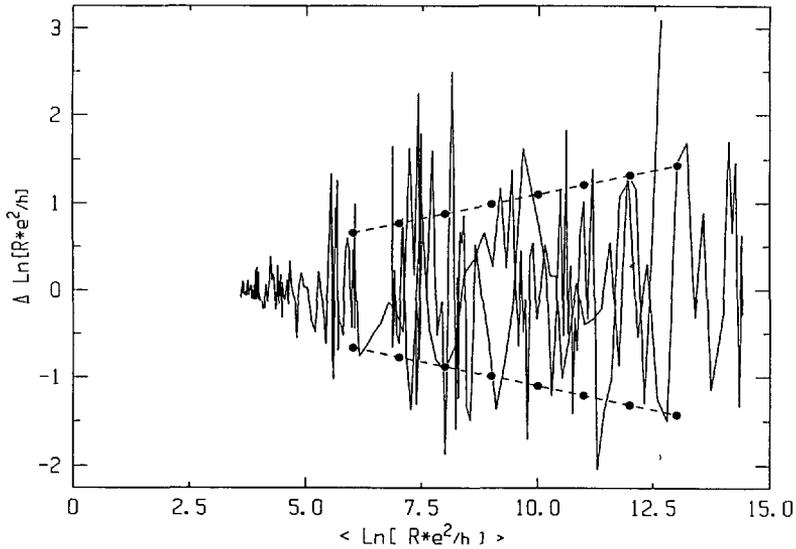


Fig. 7. — $\delta \ln(R)$ versus $\langle \ln R \rangle$ in quantum units at $T = 100$ mK. Two experiments are represented to improve the statistics. $\langle \ln g \rangle$ is obtained after smoothing of the experimental curves $g(V_G)$. Dotted lines are the prediction of reference [15]. Note nevertheless the tendency of $\delta \ln R$ to saturate at $\simeq 1$ for high resistances.

typically in our strongly localized regime (for $T \simeq 0.5$ K). This energy scale is in fact twice the mean energy spacing of levels lying within r_{\max} . However, numerical simulations of quantum fluctuations versus Fermi energy at $T = 0$ K have been carried out very recently ([20] companion paper). They have suggested that the typical width in energy ΔE_{qu} of these fluctuations is of the order of the mean energy level spacing within the finite quantum coherent system. Let us assume, as usual, that quantum coherence is preserved on the scale of each hop at finite temperature. Then, we get that quantum interferences in the dominant link change completely within a scale in energy given by the mean level spacing within r_{\max} at finite temperature. Therefore, we get $\Delta E_{\text{qu}} \simeq \Delta E_{\text{geo}}$ at finite temperature. Crudely speaking, this means that within r_{\max} quantum interferences are dominated by diffusion on levels whose energy is the closest to initial and final energies of hop. This energy is simply $\simeq \Delta E_{\text{geo}}$.

Of course, the latter statement is concerned with only mean energy scales. Therefore, we think that observed fluctuations are partly of quantum origin, depending on each particular fluctuation: if for a given hop, quantum interferences change with energy faster than geometrical fluctuations, then the fluctuation will be of quantum origin and therefore T independent. If the inverse situation takes place we will get a strongly T dependent fluctuation just given by geometrical considerations. Indeed, even for a given hop, providing that the value of the resistance is given exclusively by equation (4), the fluctuation induced by varying Fermi energy is very sensitive to any shift of temperature.

Figure 4b gives an example of a fluctuation of quantum origin. Indeed, one can see that this conductance fluctuation $\delta \ln g$ exhibits no or a very weak temperature dependence, even in a temperature range where the mean conductance keeps on decreasing with decreasing T (here, e.g. between $T = 1$ K and $T = 400$ mK). This behavior suggests that finite temperature models totally removing quantum interferences are incomplete.

Let us now consider the amplitude of a quantum fluctuation on the dominant resistor. It is

worth noting that the zero temperature RMT or FDP approaches predict: $\Delta \ln R \simeq \left(\frac{T_0}{T}\right)^\alpha > 1$ (see [15], $\alpha = 1/4$ or $1/10$ for R.M.T. and F.D.P. respectively), whereas the geometrical one is always $\lesssim 1$ in our experiment. This quantitative analysis shows that the fluctuation that we observe cannot be the full quantum one, but is truncated by the geometrical fluctuation. This means that when a quantum fluctuation inside the largest (dominating) resistor yields a large increase of the resistance, the electrons hop to a different final impurity site. On the contrary, when a quantum fluctuation yields a large decrease of the resistance, the second largest resistor starts playing a leading role, therefore limiting again the fluctuation of measured $\ln R$.

Near the Anderson transition, $w_{i,j}$ is no longer much larger than the estimated quantum fluctuations [15], which means that the above considerations breakdown since the method used is no longer valid. Physically, this means that the effect of interferences within ξ itself can no longer be ignored (quantum fluctuations can be regarded as fluctuations of ξ). Moreover because $w_{i,j}$ decreases, the whole conductance is less and less controlled by the weakest link. In this regime, the quantum fluctuation should develop fully, but this range is too narrow to allow a quantitative test. Moreover, the temperature dependence of fluctuations in this regime is much more marked than in the regime of figure 4b. This emphasizes that the description of the vicinity of the transition requires a model where quantum fluctuations are fully taken into account, and not only considered on the dominant link.

The study of fluctuations versus the Fermi energy shows the subtle interplay between quantum and geometrical fluctuations. The application of a magnetic field can induce Zeeman shifts of energy levels E_i in (4), and consequently induce geometrical fluctuations. On the other hand magnetic flux can change the quantum interferences and induce quantum fluctuations. We will see in the next section that magnetoconductance fluctuations are purely due to quantum interference effect in our sample.

2.2 THE FLUCTUATIONS IN APPLIED MAGNETIC FIELD VERSUS THE FLUCTUATIONS IN V_G .

— Figure 8 presents a detail of the conductance fluctuation versus gate voltage and applied magnetic field for both very low and moderately low conductances at $T = 100$ mK (see Fig. 6).

2.2.1 Strongly localized regime: non ergodicity. — For the very low conductances in a linear scale representation (Fig. 8A), conductance peaks seem to appear just by application of the magnetic field, as in reference [2]. In a logarithmic representation (Fig. 8B), however, such conductance peaks correspond to maxima of the conductance in zero field. Moreover, the applied magnetic field is unable to decorrelate the pattern of the conductance fluctuations versus the gate voltage. This situation is precisely referred to as non-ergodic [3]. With the data of figure 8B we find indeed:

$$\text{var}(\ln R)_H \simeq 0.22 < \text{var}(\ln R)_{V_G} \simeq 1.10$$

This is not, strictly speaking, a proof that there is non-ergodicity in this strongly localized situation because one first has to know if the field scale appearing in the problem is not too large or, equivalently, if the statistics over the magnetic field is complete. Our experimental field range is limited below 4 or 5 T because of the large negative mean magnetoconductance associated with the shrinking of atomic orbitals for higher field [21]. In fact when the condition:

$H \gg \frac{\hbar}{e a l} \simeq 3$ T (for $a = a_{\text{Bohr}}$ and $l = 20$ nm, the distance between Silicon impurities) is satisfied, the magnetic field modifies the shape of each wave function, and not only the phase along the Feynman paths. So we restrict our interpretation to the low field range.

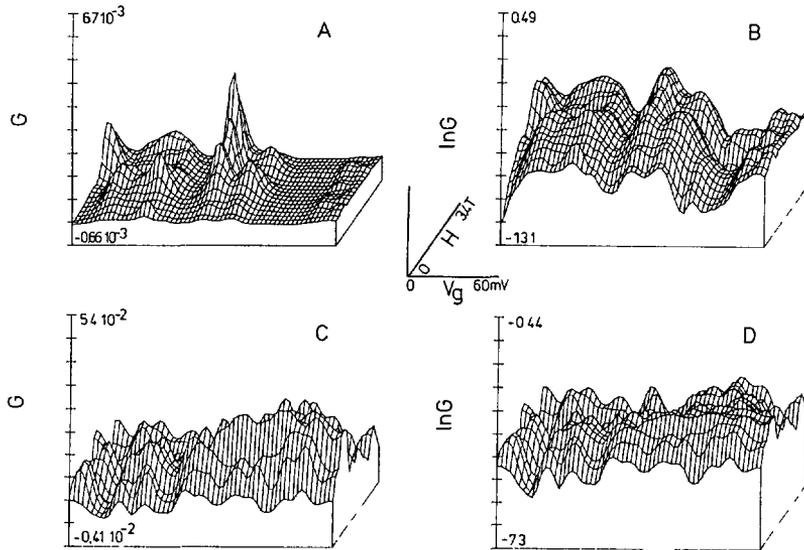


Fig. 8. — Conductance as a function of V_G and H in a 3D-Plot. The V_G ranges are indicated in figure 6. A) Low Conductances in a linear scale. B) Low Conductances in a logarithmic scale. C) Moderate conductances in a linear scale. D) Moderate Conductances in a logarithmic scale.

Between 0 and 3.4 T, typically we only see 2 or 3 oscillations of $\ln(g(H))$ in the strongly localized regime. The correlation field (difficult to be estimated) is of order 1 T (a quantum of flux h/e is put through $(64 \text{ nm})^2$ for 1 T). Nevertheless, the comparison with the barely localized situation shows that the experiment distinguishes, in practice, the ergodic and non-ergodic cases - even for one or two oscillations of magnetoconductance.

The observed non-ergodicity implies that the magnetic field is unable to induce geometrical fluctuations. On the contrary, a strong Zeeman shift would change all the impurity energies and thus the geometry of hopping paths [2-3] inducing geometrical fluctuations. We do not see the magnetic field translating the maxima of $\ln g$ [2], and so Zeeman effects are negligible in our sample for our field range.

The experiment shows that the quantum fluctuation versus magnetic field ($\Delta \ln R_H < 1$), is smaller than the geometrical fluctuation ($\Delta \ln R_{\text{geo}} \simeq 1$). This is in the spirit of the Nguyen, Spivak and Shklovskii model [8], where the quantum fluctuation versus magnetic flux is smaller than any other kind of fluctuation. This has been already noticed by Orlov *et al.* in reference [3]. Furthermore we have suggested in section 2.1 that the geometrical fluctuation is smaller than the quantum fluctuation versus energy ($\Delta \ln R_{\text{qu}} > 1$) ("truncated quantum fluctuation"). This allows us to conclude that the quantum fluctuation is larger versus energy than versus magnetic field. To our knowledge, there is no attempt to model the fluctuation versus energy at $T = 0 \text{ K}$ apart from that of Avishai and Pichard [20]. In the strongly localized regime, their numerical results show a similar non-ergodic behavior, precisely when standard RMT results start to fail.

2.2.2 Barely localized regime: ergodicity. — Figures 8C and 8D show the conductance as a function of V_G and applied magnetic field at $T = 70 \text{ mK}$ for a range of conductance just on the insulating side of the Anderson transition: typically $\langle \ln(g(H=0)) \rangle \sim -5$ ($g \sim 7 \times 10^{-3}$), whereas the transition takes place for $\langle \ln(g(H=0)) \rangle \sim -3.7$ ($g \sim 2.5 \times 10^{-2}$). For these

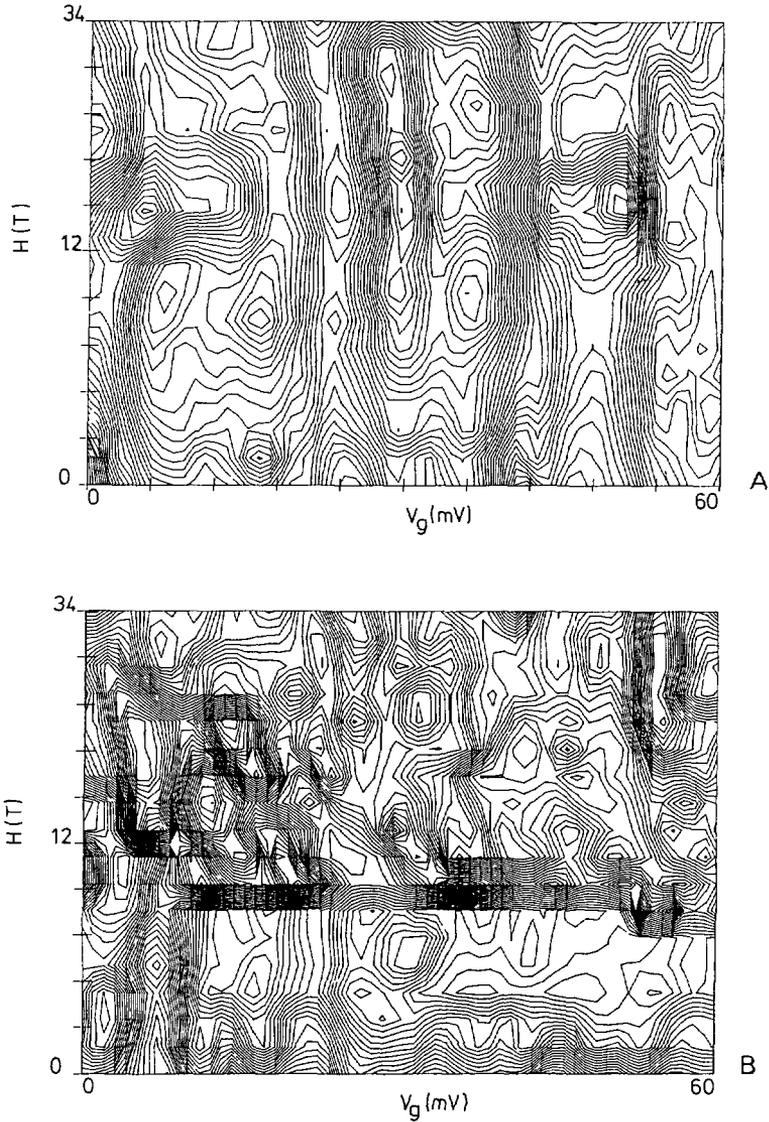


Fig. 9. — Contour plots of figures 8b and 8d. The magnetic field does not decorrelate the conductance pattern versus gate voltage in the strongly localized regime (9A). On the contrary the situation is ergodic in the barely localized regime (9B).

conductances, $T_0 \simeq 2$ K, so that we are in the limiting case of the VRH regime. In this range of conductance, the shape of the fluctuations is reminiscent of what is observed more deeply in the insulating regime.

By contrast to the strongly insulating regime, near the Anderson transition, the experiment indicates the validity of the ergodic hypothesis - formulated first in the diffusive regime for small disorder parameter $(k_F l)^{-1}$.

$$\text{var}(\ln R)_H \simeq 0.19 \simeq \text{var}(\ln R)_{V_G} \simeq 0.27$$

Our experiment shows that it is still valid at least very close to the transition, and by continuity

in the critical insulating regime. We have seen that near the Anderson transition it is no longer relevant to separate geometrical and quantum fluctuations: the analysis performed in the strongly localized regime fails as already mentioned in section 2.1.

Because the estimated ξ becomes quite large with respect to the distance between impurities, the electrons are no longer fixed to a given impurity but localized in shallow regions, which are changed by the application of a magnetic field. Because of this redistribution, the ergodic hypothesis is realistic. It is indeed numerically obtained by Avishai and Pichard near the Anderson transition [20].

But as we mentioned in 2.1, the extension of this critical regime ($\xi \gg l$) in our MBE grown GaAs:Si sample appears to be quite narrow. We believe that it is much more developed in less pure samples like amorphous alloys. Because of this narrowness, it is difficult to be more quantitative.

In both NSS model and RMT model, there exists a close connection between the quantum fluctuations and the averaged magnetoconductance effect; let us now turn to the analysis of the mean magnetoconductance effect.

2.3 THE MEAN MAGNETOCONDUCTANCE EFFECT. — Positive magnetoconductance at low temperatures in insulating GaAs:Si was reported long ago [3, 13, 22]. Amongst the models which have been proposed, Spivak and Shklovskii [8, 21] predict at the macroscopic limit, that:

$$\ln\left(\frac{\sigma(H \gg H_c)}{\sigma(0)}\right) \simeq 1 \quad (12)$$

(H_c is given by $\frac{\pi c \hbar}{r_0^{3/2} \xi^{1/2} e}$), which compares very well with numerical simulations [8].

Zhao *et al.* [10] argue that simulations performed within the same framework of FDP analysis but on larger samples, show no saturation of the magnetoconductance in the limit of very large quantum coherent sample (i.e. very low temperatures). Moreover they give a universal estimate:

$$\ln\left(\frac{\sigma(H)}{\sigma(0)}\right) \simeq 0.1 \frac{r_0}{L_H} \quad \text{where} \quad L_H = \left(\frac{\hbar c}{e} H\right)^{-1/2} \quad (13)$$

Their simulations corroborate the results obtained by Medina *et al.* [11].

As noted in the introduction, RMT predictions differ from the FDP model because the positive magnetoconductance (in case of negligible spin-orbit scattering) depends on r_0/ξ and not only on r_0 (the predictions differ completely in the case of strong spin-orbit scattering). For instance at $T = 0$ K (to avoid the introduction of the phase coherent hop and its magnetic field dependence):

$$\ln\left(\frac{g(H > H_c^*)}{g(0)}\right) \sim -\frac{L}{\xi(H > H_c)} + \frac{L}{\xi(0)} = \frac{L}{2\xi(0)} = -\frac{1}{2}\ln(g(0)) \quad (14)$$

if $\xi(H > H_c^*) = 2\xi(0)$ [12] (quasi 1D RMT result; H_c^* is given by $H_c^* \xi^2 \simeq \frac{\hbar c}{e}$). At finite temperature, the expression is less simple because r_0 depends weakly on H via $\xi(H)$ [12]. Nevertheless, the mean magnetoconductance is very sensitive to the mean conductance in zero field in this RMT approach.

Figure 10 shows the mean magnetoconductance effect between $H = 0$ and $H = 2.5$ T in the strongly localized regime. The zero field mean conductance – experimentally the smoothed

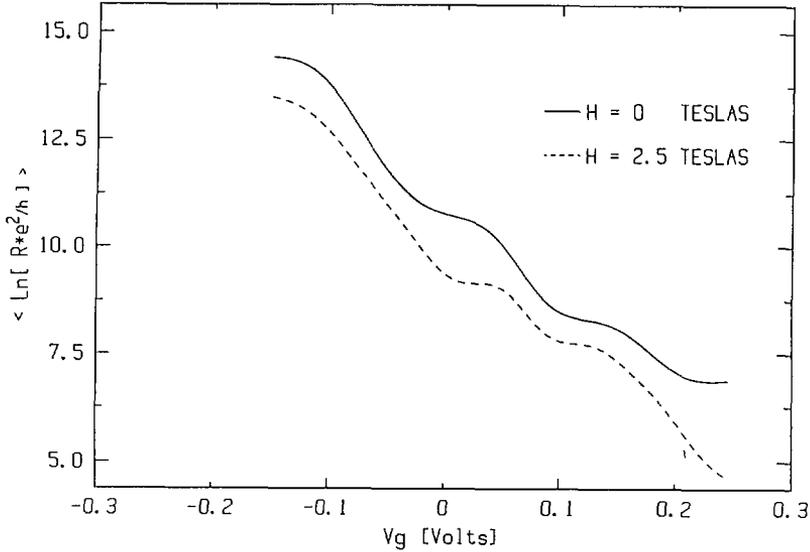


Fig. 10. — The smoothed conductance for $H = 0$ and $H = 2.5$ T in the strongly localized regime. The observed mean magnetoconductance effect is roughly insensitive to the conductance value (at $H = 0$) and fluctuates around: $\ln\left(\frac{g(H = 2.5 \text{ T})}{g(H = 0)}\right) \simeq 1$, the mean magnetoconductance value after averaging over the whole range of V_G .

conductance after numerical averaging of the fluctuations in V_G — varies over 3 orders of magnitude. Nevertheless $\langle \ln\left(\frac{g(H = 2.5 \text{ T})}{g(0)}\right) \rangle$ is roughly unchanged and approximately equal to 1. The averaging over the whole range V_G gives $\ln\left(\frac{g(2.5 \text{ T})}{g(0)}\right) \simeq 1$. This is just the prediction of NSS [8-21] (the averaging needed for this prediction is obtained by smoothing in V_G which extends over several fluctuations). We note that it is also in good accordance with the result of Zhao *et al.* [11] if we suppose that $r_0 \simeq 160$ nm, a realistic value in our experiment, is roughly insensitive to $\langle g(0) \rangle$. However we are not able to test their analytical universal result. In any case, the insensitivity of the mean magnetoconductance to the mean conductance value stresses the fact that FDP approaches are more adapted than RMT approaches in this regime.

As one approaches the Anderson transition, the mean magnetoconductance tends smoothly to the weak antilocalization contribution in the diffusive regime. Contrarily to the strongly insulating regime where the mean magnetoconductance and the conductance fluctuations are of the same order of magnitude, near the transition the mean magnetoconductance becomes much larger than the fluctuations. The analysis in the barely localized regime in terms of changes of the localization length is restricted because of the small range of conductance where this regime occurs. Nevertheless the observed magnetoconductance is compatible with a small increase of ξ (for instance $\xi(2.5 \text{ T}) = 1.3\xi(0)$ for $T_0 \simeq 2$ K), as predicted by RMT approach [9]:

$$\xi = (\beta N + 2 - \beta)l \quad (15)$$

where N is the number of transverse channels and β is one or two in the absence or in the

presence of applied magnetic field respectively. In our experiment N is close to one, so that the crossover from $\beta = 1$ to $\beta = 2$ does not imply a doubling of ξ , as predicted in the macroscopic limit ($N \mapsto \infty$).

Conclusion.

The initial aim of this work was to gain more insight into quantum interference phenomena in a mesoscopic, disordered insulator. We have studied a small wire where enormous reproducible conductance fluctuations versus the Fermi energy of electrons or versus applied magnetic field are observed at very low temperatures.

The fluctuation versus Fermi energy results from an interplay between geometrical incoherent and quantum mechanically coherent mesoscopic effects. The fluctuation versus magnetic field, on the contrary, is purely due to interference effects.

We can distinguish two insulating regimes. When ξ is comparable to the distance between impurities, the observed non-ergodicity and the analysis of the mean magnetoconductance ($\langle \ln \left(\frac{g(H > 1)}{g(0)} \right) \rangle \simeq 1$) indicate that interferences between Time Reversal conjugated loops are not essential to describe the properties of the conductance distribution. On the other hand, close to the Anderson transition, the localization radius includes many impurity sites. Unfortunately, this critical regime is narrow in our sample, so that a precise comparison with the predictions of the RMT approach is not available, except for the important fact that the fluctuation is ergodic. We believe that, in the experiment, we do not mistake this critical insulating regime for the critical diffusive regime near the Anderson transition. In any case, as far as a finite temperature experiment can determine the critical transition point, the ergodicity holds beyond the diffusive regime.

In the variable range hopping regime of our 1D sample deep enough in the insulating regime, a theory where only one elementary long hop dominates the resistance gives a good quantitative prediction for the fluctuation versus energy.

Finally, the study of mesoscopic insulators with a larger disorder, like amorphous alloys, will give us more insight into the critical Anderson insulating phase.

Acknowledgements.

We would particularly like to thank J-L. Pichard and J-P Bouchaud for their constant collaboration. We are very grateful to B. Etienne for providing the GaAs:Si layer and to R. Tourbot for his technical support. We also acknowledge fruitful discussions with R. Jalabert, K. Slevin and B.I. Shklovskii, and the members of the E.E.C. Science program contract No.SCC-CT90-0020.

References

- [1] Washburn S., *Quantum Coherence in Mesoscopic Systems*, Kramer Ed. (NATO ASI Serie Plenum Press, 1991) and reference therein.
- [2] Fowler A., Wainer J. and Webb R. *Hopping Transport in Solids*, Pollack and Shklovskii Eds. (Elsevier Science Publisher, 1991);
Popovic D., Fowler A.B. and Washburn S., *Phys. Rev. Lett.* **67** (1991) 2870.
- [3] Orlov A., Savchenko A. and Koslov A., *Solid State Comm.* **72** (1989) 743;

- Laiko E.I., Orlov A.O., Savchenko A.K., Ilyichev E.A. and Poltoratsky E.A., *Zh Eksp Teor Fiz.* **93** (1987) 2204 (*Sov. Phys. JETP* **66** (1987) 1264).
- [4] Mott N.F. *J. Non-Crystal. Solids* **1** (1968) 1.
- [5] Lee P.A. *Phys. Rev. Lett.* **53** (1984) 2042.
- [6] Raikh M.E. and Ruzin I.M., *Mesoscopic Phenomena in Solids*, B.L. Altshuler, P.A. Lee and R.A. Webb Eds. (North Holland, 1991).
- [7] Ambegaokar V., Halperin B. and Langer J.S., *Phys. Rev.* **B4** (1971) 2612.
- [8] Nguen V.L., Spivak B.Z. and Shklovskii B.I., *Piz'ma Zh Eksp Teor Fiz.* **43** (1986) 35 (*JETP Lett.* **43** (1986) 44).
- [9] Pichard J.L., *Quantum Coherence in Mesoscopic Systems*, Kramer Ed. (NATO ASI Serie Plenum Press 1991) and reference therein.
- [10] Zhao H.L., Spivak B.Z., Gelfand M.P. and Feng S., *Phys. Rev.* **B44** (1991) 10760.
- [11] Medina E. and Kardar M., *Phys. Rev. Lett.* **66** (1991) 3187;
Medina E., Kardar M., Shapir Y. and Wang X.R., *Phys. Rev. Lett.* **64** (1990) 1816.
- [12] Pichard J.L., Sanquer M., Slevin K. and Debray P., *Phys. Rev. Lett.* **65** (1990) 1812.
- [13] Bouchaud J.P., *J. Phys. I France* **1** (1991) 985.
- [14] Feng S. and Pichard J.L., *Phys. Rev. Lett.* **67** (1991) 753.
- [15] Ladiou F. and Bouchaud J.P., Companion Paper, submitted to *J. Phys. I France* **33** (1993).
- [16] Altshuler B.L. and Aronov A.G., *JETP Lett.* **33** (1981) 499.
- [17] Mailly D. and Sanquer M., *J. Phys. I France* **2** (1992) 357.
- [18] Kurkijarvi J., *Phys. Rev.* **B8** (1973) 922.
- [19] Stone A.D. and Lee P.A., *Phys. Rev. Lett.* **54** (1985) 1196.
- [20] Avishai Y. and Pichard J.L. and Muttalib K.A., *J. Phys. I France* **3** (1993).
- [21] Shklovskii B.I. and Efros A.L., *Electronic Properties of Doped Semiconductors.* (Springer, 1984).
- [22] Tremblay F., Pepper M., Ritchie D., Peacock D.C., Frost J.E.F. and Jones G.A.C., *Phys. Rev.* **B39** (1989) 8059.