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Short Communication

Distribution of conductance and its universal fluctuations in 1-D disordered, mesoscopic systems

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Abstract. — The evolution with length of the probability distribution for the two-probe conductance (g_2) of a disordered one-dimensional system is obtained numerically. It evolves from a strongly peaked distribution close to $g_2 = 1$ in the small length limit towards another strongly peaked distribution near $g_2 = 0$ in the large length limit. In the middle it goes through a quasi-diffusive regime where the distribution is nearly uniform. This is consistent with a universal conductance fluctuation of magnitude $\simeq 0.3\ e^2/h$ (as observed in a recent paper) occurs in such a domain.

The study of mesoscopic systems (the sizes of which are much larger than the atomic size but less than the localization length for electronic wave functions in disordered systems as well as the inelastic mean free path due to phonons) has gained a lot of prominence in recent years [1] because of their technological importance and some fundamental questions regarding the scaling theory of Anderson localization [2]. A knowledge of the full probability distribution [3, 4], and not just the individual moments [2], of the conductance (either four-probe or two-probe) of such samples at all length scales and the associated scaling (divergent) behaviour of the independent parameters involved in that description are of crucial importance for this purpose.

Historically, the second cumulant of the two-probe conductance in the metallic (for dimensionality $d > 2$) or the quasi-metallic (for $d \leq 2$) regime enjoys a special status. Disordered metals (in the diffusive or Ohmic regime) display a novel phenomenon of universal conductance fluctuations (UCF). This universality implies that in the diffusive regime, the standard deviation (i.e., the square root of second cumulant) of the two-probe conductance distribution is independent of any change in length (i.e., average conductance), Fermi energy or of any microscopic details of the system and consequently of the underlying Hamiltonian involved ; but depends only on the dimensionality of the system. The values of the UCF in 3D, 2D, and quasi-1D, for electrons of one spin variety, are 0.544, 0.431 and 0.365, respectively

(in units of e^2/h) as shown in [5]. Until recently, it was mostly believed ([1]) that this interesting feature can not be seen in strictly one-dimensional systems since in 1D, the localization length, ξ, is essentially equal to the elastic mean free path, ℓ (in fact, the localization length is totally dependent on the elastic mean free path because there is only one independent length or energy scale in the problem) and a diffusive regime (Ohmic behaviour) can never show up. But it has recently been shown by Gangopadthyay and Sen [6], referred to as GS from now on, that in an effective diffusive regime, Ohm's law is valid approximately in 1D and the UCF ($\simeq 0.3\ e^2/h$) does exist. GS show their results mostly for a tight binding Hamiltonian along with a site diagonal disorder, i.e., where the site energies are independent random variables chosen from a uniform distribution of width W. To show that their results are independent of the distribution or the Hamiltonian they did also check that the UCF exists when the Hamiltonian is a Schrodinger Hamiltonian with δ-function impurities or when the disorder distribution is Gaussian. The hopping term V for the tight binding Hamiltonian and the lattice constant were taken to be one to set the energy and the length scales respectively. For various disorders (W) and Fermi energies (E) of the electron, GS found that $\xi \simeq 5.5\ \ell$ and that if one allows a certain pre-specified tolerance, say a 2 % error, to Ohm's Law in this case, i.e., a $\pm$ 2 % deviation from constancy of the usually defined conductivity, then there is a length domain of about $2\ \ell - 4\ \ell$ within which Ohmic behaviour persists and a domain $0.45\ \xi - 1.1\ \xi$ within which the UCF of $0.28 - 0.31\ e^2/h$ exists. For example, we have shown in figure 1 the average conductivity and the standard deviation of the two-probe conductance

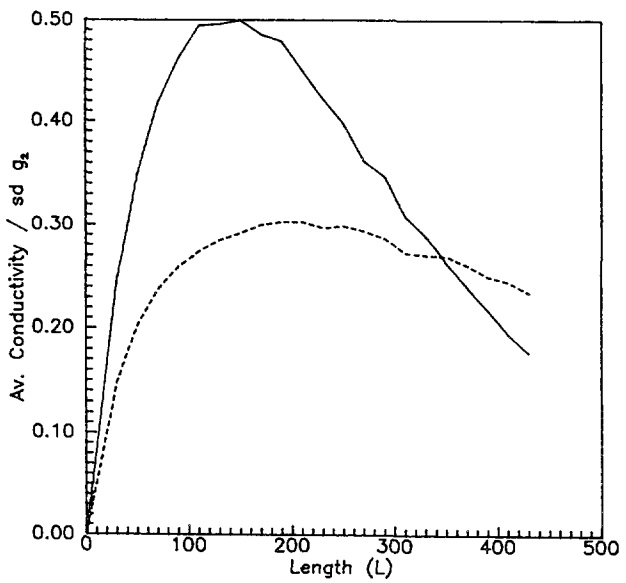

Fig. 1. — Average two-probe conductivity (scaled down by a factor of 100) shown in full line and the standard deviation of the two-probe conductance (sd g_2) shown in dashed line, as a function of length for $W = 0.6$, $E = 0.1$ (hopping term $V = 1$).

([1]) It will be noted that in Lee and Stone (1985), the work was done upto quasi-1D, and not in 1D. An « exact » analytical work was also recently done on quasi-1D, and it was not extended to exact 1D ; see Mello P. A., *Phys. Rev. Lett.* **60** (1988) 1089. Indeed, a very recent work intends to show that UCF in exact 1D can be sustained only in the subtle presence of an incoherent scattering such that the localization effects are not completely washed out and there is only one inelastic (phonon) scattering in the whole sample ; see D'Amato J. L., Pastawaski H. M., *Phys. Rev. B* **41** (1990) 7411.

g_2 against length using 5 000 sample configurations for each length. It may be noted that in this case $W = 0.6$, $E = 0.1$ and it turns out that $\xi \simeq 270$ and $\ell = 50$. The UCF of about $0.28 - 0.30\ e^2/h$ occur between the lengths 120 and 300 respectively.

For the purpose of this short communication, we have numerically calculated the full probability distribution function, $P(g_2)$, in an effort to understand the probabilistic origin of the constancy in the second cumulant irrespective of the value of the first one (i.e., the average) in the diffusive regime. We have shown in figure 2 the evolution of $P(g_2)$ as the length L of the system increases from a very small to a very large value. The parameters chosen for this figure are as in figure 1 and the lengths of the system chosen are $L = 10$, 135 and 400 respectively. It will be noted here that for a very small system size ($L \ll \ell$, as defined in GS), i.e., in the *nearly ballistic regime,* the peak of the very narrow distribution is centered around a g_2 very close to 1 (the upper cut-off being 1). In the other extreme ($L \gg \xi$), i.e., in the *strongly localized regime,* the peak is centred around a g_2 very close to 0 (the lower cut-off being 0) for obvious reasons. One very important distinguishing feature between these two regimes is that while in the nearly ballistic regime, the tail of the narrow distribution is almost non-existent, in the localized regime, there is a clearly discernible tail all the way upto the upper-cut off in g_2. This tail distribution makes a very significant contribution to the four-probe conductance such that in the large length limit, all the moments of the four-probe conductance, including the average, diverge [7] and the central limit theorem fails. This tail of the stationary distribution ($L \to \infty$ limit) is very important in the sense that the universality and the scaling behaviour is governed by it [8, 9]. The prevalent belief [7, 10] is that $\ln(1 + r_4)$ in the $L \to \infty$ limit, where r_4 is the four-probe resistance, is normally distributed. We have checked that our results for r_4 in the case of $L = 400$ of figure 2, is consistent with that. But for reasons described above we focus on the universal features of $P(g_2)$ in the mesoscopic regime.

In between the two domains discussed above, i.e., inside the *weakly localized regime,* an interesting situation develops. It was shown in GS that the *quasi-Ohmic regime* occurs around the stationary value of the conductivity around $L_{max} = \xi/2$. For the case of figure 2, L_{max} is about 135. It will be noted that the $P(g_2)$ for this length is almost uniform. Indeed the

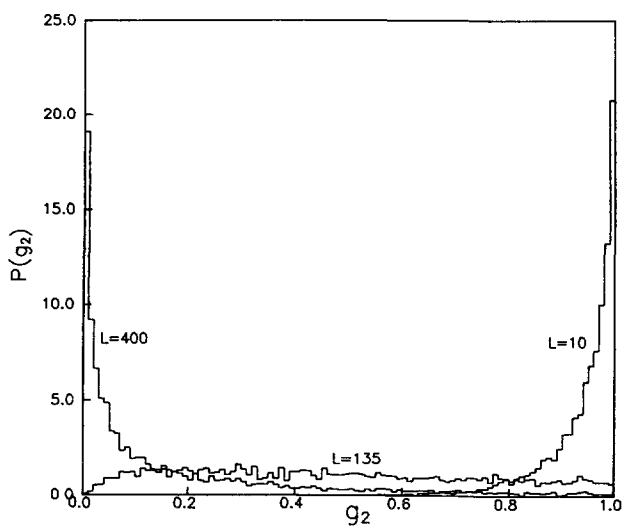

Fig. 2. — The probability distribution, $P(g_2)$ for the same parameters as in figure 1 and for three different lengths to show its evolution through three different regimes.

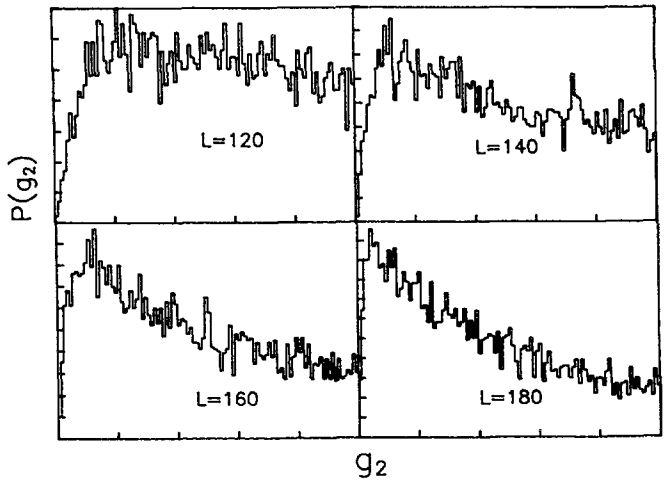

Fig. 3. — $P(g_2)$ for the same parameters as in figure 1 but for four different lengths in the UCF regime to show its broad nature. For each of the graphs, the range of g_2 is [0, 1] and $P(g_2)$ axis is scaled in such a way that the area below the histogram is normalized to unity.

distribution is very broad in the quasi-Ohmic regime as we have shown in figures 3a-3d, and this near-uniformity of the distribution remains intact as we change the length scale within our quasi Ohmic range. The existence of UCF in 1D can now be justified by noting that the standard deviation of a uniform probability distribution of width s is $s/\sqrt{12}$. Here we note that the distribution of g_2 shows approximately uniform behaviour for a definite range of system sizes and we have $s = 1$. Had the distribution been exactly uniform in the whole quasi-Ohmic range we would have obtained a standard deviation of $1/\sqrt{12} \simeq 0.29$, and this value happens to be close to that obtained in GS.

In this connection, we would like to comment on the insufficiency of random phase model (RPM) in obtaining the distribution analytically [9] even in the UCF (or the weakly localized) regime. Actually the distribution in reference [9] is for the four-probe resistance $r_4 = (1 - g_2)/g_2$. When one makes a transformation of random variables, one obtains for this distribution *in the RPM :*

$$P(g_2) = \frac{1}{\lambda g_2^2} \exp\left[- \left(\frac{1}{g_2} - 1 \right) / \lambda \right], \tag{1}$$

where λ is the average four-probe resistance of the finite sample considered. To see how it compares with our distribution, we have chosen the parameters $W = 0.6$, $E = 0.9$ when the localization length $\xi \simeq 210$. In this case we have chosen $L = 100$, which is clearly within our UCF regime and we find that for this length $\lambda \simeq 1$. In figure 4, one notes a very significant difference between our « exact » distribution and the one obtained in RPM. To see how the standard deviation (sd) of g_2 compares between these two cases, we have calculated the same for the distribution (1) numerically for this length. It turns out that the value is about 0.22 e^2/h, which is also significantly different from the value in GS. It may be noted here that Abrikosov [10] has found an « exact » expression (in the form of an integral) for the distribution of four-probe resistance in 1D. When evaluated numerically we find that this distribution fits through our histogram extremely well (not shown here to avoir clumsiness). It is thus very important to find the actual probability distribution for the conductances carefully because it plays a very important role in determining the scaling behaviour and any universal property (e.g. UCF) thereof.

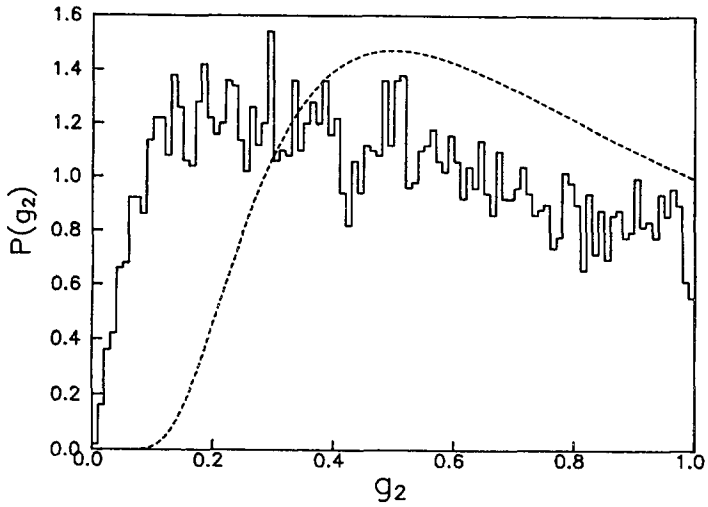

Fig. 4. — Comparison of $P(g_2)$ for a length in the UCF regime as obtained by us (histogram) and as obtained analytically [9] in the random phase model (RPM). The parameters chosen are $W = 0.6$, $E = 0.9$ (where localisation length $\simeq 210$) and $L = 100$.

References

[1] Anderson Transition and Mesoscopic Fluctuations, B. Kramer, G. Schon Eds. (North Holland, Amsterdam, 1990) ;
Mesoscopic Phenomena in Solids, B. L. Al'tshuler, P. A. Lee, R. A. Webb Eds. (North Holland, Amsterdam, 1991).

[2] Al'tshuler B. L., Kravtsov V. E., Lerner I. V., *Zh. Eksp. Teor. Fiz.* **91** (1986) 2276 (*Sov. Phys. JETP* **64** (1986) 1352).

[3] Shapiro B., *Phys. Rev. Lett.* **65** (1990) 1510.

[4] Jayannavar A. M., Pramana, *J. Phys. France* **36** (1991) 611.

[5] Lee P. A., Stone A. D., *Phys. Rev. Lett.* **55** (1985) 1622 ;
See also Al'tshuler B. L., *Pis ma Zh. Eksp. Teor. Fiz.* **41** (1985) 530 (*JETP Lett.* **41** (1985) 648).

[6] Gangopadhyay S., Sen A. K., *Phys. Rev. B* **46** (1992) 4020 (also referred to as GS in the paper).

[7] Anderson P. W., Thouless D. J., Abrahams E., Fisher D. S., *Phys. Rev. B* **22** (1980) 3519 ;
Abrahams E., Stephen M. J., *J. Phys. C* **13** (1980) L377 ;
Mel'nikov V. I., *Fiz. Tverd. Tela* (Leningrad) **23** (1981) 782 (*Sov. Phys. Solid State* **23** (1981) 444).

[8] Kumar N., Jayannavar A. M., *J. Phys. C Solid State Phys.* **19** (1986) L85.

[9] Shapiro B., *Phys. Rev. B* **34** (1986) 4394 ;
Shapiro B., *Philos. Mag B* **56** (1987) 1031.

[10] Abrikosov A. A., *Phys. Scr.* **T27** (1989) 148.