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Instabilities of an elastic chain in a random potential

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Résumé. — Nous étudions un système élastique monodimensionnel dans un potentiel aléatoire. La réponse de ce milieu élastique à un ensemble de centres d'ancrage denses et aléatoirement répartis apparaît comme contrôlée par des instabilités élastiques. Nous étudions le seuil d'apparition de ces instabilités ainsi que leurs propriétés statistiques. La distribution des discontinuités en énergie et le spectre de puissance de la force d'ancrage suivent tous deux une loi de puissance. Nous discutons brièvement l'application éventuelle de ce modèle à la description de certaines expériences concernant la friction solide.

Abstract. — We investigate a one dimensional elastic system in a random potential. The response of the elastic medium to a set of sharp and dense pinning centers is shown to be controlled by non linear instabilities. We study the onset of these instabilities and their statistics. Both the distribution of energy discontinuities and the power spectrum of the pinning force are found to exhibit a power law behavior. The possible relevance of this model to some experiments on solid friction is briefly discussed.

Introduction.

Many physical systems consist of interacting particles imbedded in a quenched disordered potential. For instance flux lines in type II superconductors, charge density waves, Bloch walls in ferromagnets. The understanding of these systems involves at least two main aspects. The first problem is to understand the equilibrium configurations resulting from the competition between the interparticle interaction and the interaction with the random potential. Generically, these systems exhibit a very large number of possible nearly degenerate metastable states. A second aspect is the complex dynamics of various relaxation processes connected with this large number of available states.

It has been suggested that the common important feature of the physics of these systems is the presence of "avalanches" by which the system relaxes from one metastable state to another.

These avalanches involve many degrees of freedom. A necessary condition for the existence of avalanches is the presence of a finite threshold. As a consequence of this threshold, the collective dynamics exhibits hysteretic behavior. The emphasis on avalanches and threshold was presented in the paper by Bak, Tang, and Wiesenfeld. [1] These authors have proposed the name of *self organized criticality* for this class of phenomena. This viewpoint has triggered an avalanche of recent work on various systems such as sandpiles [2], flux lines [3, 4], charge density waves [5], earthquakes [6], and many others.

In this paper, we study a disordered version of the Frenkel-Kontorova model in one dimension. More specifically, we consider a homogeneous elastic chain moving in a random potential. As the chain is pulled from one end, stick-slip processes involving avalanches of particle displacements occur. Energy jumps connected to these avalanches are found to exhibit a power law distribution. Moreover, these jumps induce a finite pinning force. We also study the fluctuations of the force which is required to pull the system. Its power spectrum has the $1/f^\alpha$ form. The procedure of pulling the chain selects a small subset of "physical states" among the exponentially large number of possible metastable states. Pushing the chain back and forth through the random potential doesn't generate the same sequence of states. It is worth stressing that the present system differs in a significant way from previously studied cellular automata used as models for self organized criticality. These cellular automata are discrete in space and time, and a threshold enters explicitly in the definition of the dynamics. By contrast, the present model is continuous in both space and time, and threshold is the result of many particle effects.

This paper is organized as follows. We first define the model, and the iterative procedure used to generate all the possible metastable states. Instabilities are studied in a following section, with emphasis on an estimate of the threshold for their generation. We then define a dynamical process in which a sequence of such metastable states is selected, and study the statistics of energy and center of mass position discontinuities associated to the jumps from one metastable state to another. The power spectrum of the resulting pinning force is then investigated. We conclude by discussing the possible relevance of this model to actual experiments on stick slip phenomena.

Model.

We consider a harmonic chain consisting of N particles in a random potential [4]. The potential energy of the system is defined by

$$E = \sum_{i=1}^{N-1} \left[\frac{k}{2} (x_{i+1} - x_i - a)^2 + V(x_i) \right] \quad (1)$$

Here k is the spring constant, which we set equal to unity, x_i denotes the position of the particles, a is the lattice parameter, and $V(x_i)$ is the random potential of the following form

$$V(x) = -A_p \sum_p \exp(-[(x - y_p)/R_p]^2) \quad (2)$$

The positions y_p of the pinning centers are randomly distributed with a uniform density n_p . The strength and the range of the individual pinning centers are respectively A_p and R_p .

The minimization of the potential energy is performed iteratively by solving the equilibrium condition $\partial E/\partial x_i = 0$ for all i . More specifically, we have

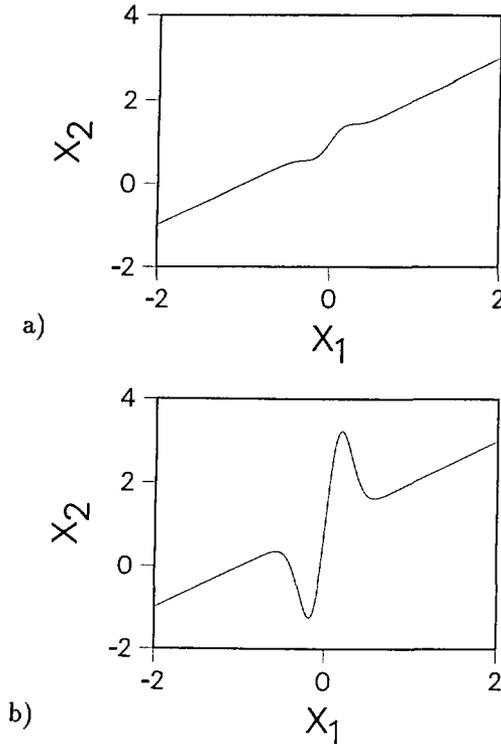


Fig.1. — Case of the two particle chain ($k = 1$) with a single pinning center ($R_p = 0.25$): graph of the position x_2 of the second particle *versus* the position x_1 of the first particle, for different values of the pinning strength A_p a) $A_p = 0.06 < A_{thr}$: under the threshold for elastic instabilities. b) $A_p = 0.6 > A_{thr}$: above the threshold for elastic instabilities. x_1 is a multivalued function of x_2

$$\begin{aligned}
 -k(x_2 - x_1 - a) + V'(x_1) &= 0 \\
 k(x_2 - x_1 - a) - k(x_3 - x_2 - a) + V'(x_2) &= 0 \\
 &= 0 \\
 k(x_{N-1} - x_{N-2} - a) - k(x_N - x_{N-1} - a) + V'(x_{N-1}) &= 0
 \end{aligned} \tag{3}$$

No equation is written for the N^{th} particle since we assumed that an additional force applied on the N^{th} particle is pulling the system in such a way that this N^{th} particle always experiences a zero net force. We prescribe the position x_1 of the first particle and express all physical quantities as a function of x_1 . The physical situation in which particle number N is forced to move with constant velocity (as in the experiments described in Ref. [7]) can be obtained by inverting the relation between x_1 and x_N . Even for weak potential $V(x_i)$ this relation is found to be extremely non-monotonous provided the chain is sufficiently long.

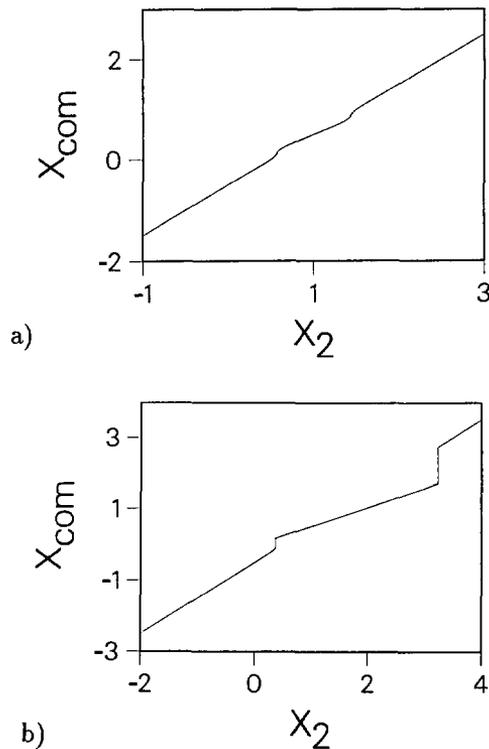


Fig.2. — Case of the two particle chain ($k = 1$) with a single pinning center ($R_p = 0.25$): graph of the center of mass position x_{com} versus the position x_2 of the second particle, for different values of the pinning strength A_p a) $A_p = 0.06 < A_{thr}$: under the threshold for elastic instabilities. b) $A_p = 0.6 > A_{thr}$: above the threshold for elastic instabilities. The dotted line corresponds to all the possible metastable states. The full line shows the succession of states actually reached upon increasing x_2 . This illustrates the existence of jumps in the center of mass position.

Generation of instabilities.

In order to gain some understanding of the complex behavior of the large N chain, it is helpful to start with the simplest $N = 2$ case. In figure 1, we show x_2 as a function of x_1 , for two different values of A_p . There is a critical value A_{thr} of A_p below which x_2 is a strictly increasing function of x_1 . When A_p is larger than A_{thr} , some parts of the curve correspond to unstable regions. This leads to jumps in x_1 when the control variable x_2 is smoothly increased. These jumps are the one dimensional equivalent of the previously studied elastic instabilities [8]. The evolution of the center of mass position, and the total energy versus x_2 are plotted in figures 2 and 3 respectively, for values of A_p smaller or larger than the threshold value A_{thr} . The presence of non monotonous regions in the x_2 versus x_1 leads to multiple functions of the control variable x_2 . In figures 2b and 3b, dotted lines represent metastable states, and full lines the ones which are actually reached as x_2 is increased. Upon decreasing x_2 , a different path in configuration space is followed. The pattern of figure 3b is quite generic, and will be pictorially referred to as the “cat” instability.

In the case of a longer chain, the multivalued nature of the energy and center of mass position

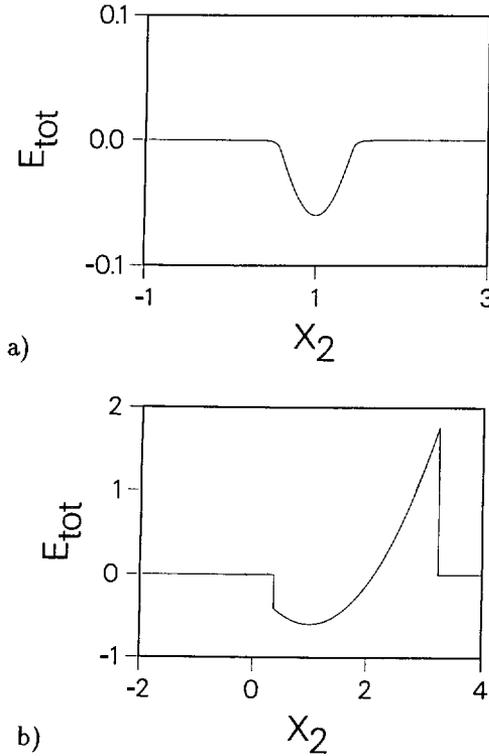


Fig.3. — Case of the two particle chain ($k = 1$) with a single pinning center ($R_p = 0.25$): graph of the total energy E_{tot} versus the position x_2 of the second particle, for different values of the pinning strength A_p a) $A_p = 0.06 < A_{thr}$: under the threshold for elastic instabilities. b) $A_p = 0.6 > A_{thr}$: above the threshold for elastic instabilities. The dotted line corresponds to all the possible metastable states. The full line shows the succession of states actually reached upon increasing x_2 . This has been labelled as the “cat instability” because of the shape of the $E_{tot}(x_2)$ curve.

as a function of x_N becomes more dramatic with a very large number of folds. However, this intricate pattern can be decomposed in a series of nested elementary cat instabilities. A qualitative understanding of this behavior comes from the fact that if say x_1 lies within the range of a pinning center, it generates displacements in x_p which grow linearly with p . As a result, the probability for particle number p to sweep through a pinning center increases with p . In order to compensate the pinning force acting on particle number p , x_N oscillates with an amplitude proportional to $N - p$. Furthermore, the corresponding displacement in x_1 required for particle number p to sweep through the given pinning center goes as $1/p$. After inverting the x_N versus x_1 curve, this leads to small and numerous cat instabilities induced by particles close to particle number N , superimposed on larger and less frequent instabilities due to particles closer to particle number 1. This phenomenon is well illustrated on figure 4, where x_N and total energy are plotted as a function of x_1 . The oscillations in figures 4a and 4b lead to the multivalued energy function $E(x_N)$ shown in figure 4c.

As shown in figure 4c, because of these jumps the energy, E_{tot} , as function of x_N has always an upward curvature locally. Hence, a finite average force is required in order to induce a global displacement of the chain. This is the microscopic origin of a finite friction force between the

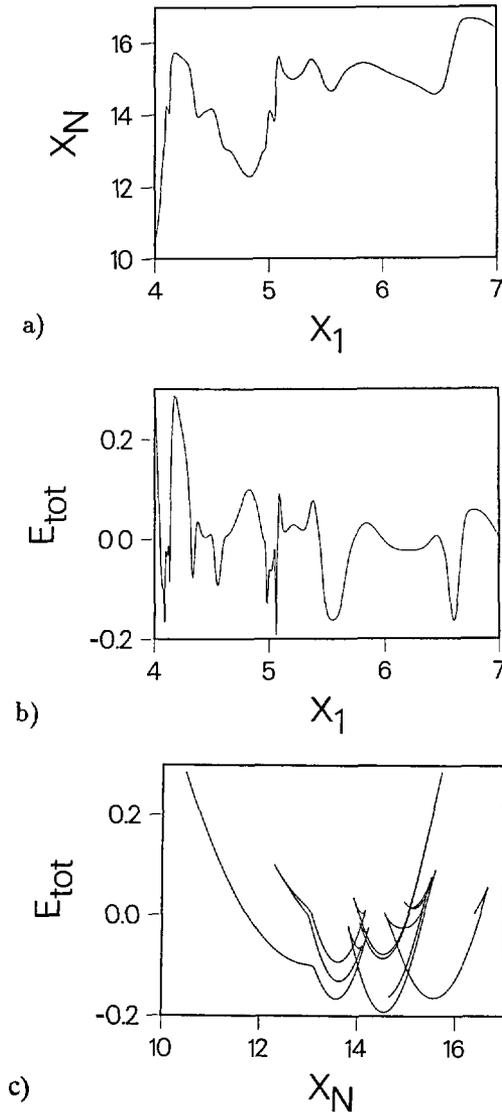


Fig.4. — General case: chain of length $N = 10$ with a density $n_p = 0.5$ of pinning centers with $R_p = 0.25$ and $A_p = 0.05$ above the instability threshold A_{thr} . a) Position of the N^{th} particle x_N versus position of the first particle x_1 . b) Total energy E_{tot} versus x_1 . c) E_{tot} versus x_N from combining the two previous graphs. The rather complicated shape of this curve can be analyzed as a superposition of “elementary cats”.

chain and the disordered substrate. The threshold for onset of these instabilities decreases as $N^{-\sigma}$, where $\sigma = 3/2$ in agreement with the general argument as follows. Instabilities occur when dx_N/dx_1 becomes negative. We have

$$\frac{dx_N}{dx_1} = 1 + k^{-1} \sum_{p=1}^{N-1} pV''(x_{N-p}) \frac{dx_{N-p}}{dx_1} \quad (4)$$

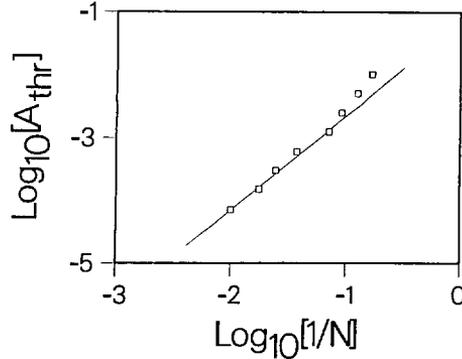


Fig.5. — Dependence of the threshold for elastic instabilities A_{thr} on chain length N . Parameters are: $n_p = 0.5$, $R_p = 0.25$, $k = 1$. This log log plot shows the scaling $A_{\text{thr}} \sim N^{-3/2}$

dx_N/dx_1 is able to change sign when the variance of the second term in the right hand side of equation number 4 is equal to 1. In weak disorder, we may assume the derivatives dx_{N-p}/dx_1 to be of the order of 1. Furthermore, we assume the different $V''(x_p)$ to be independent random variables for different values of p . This leads to a variance proportional to $N^3 n_p R_p^{-3} A_p^2 k^{-2}$. Hence,

$$A_{\text{thr}} \sim n_p^{-1/2} R_p^{3/2} k N^{-3/2} \quad (5)$$

This scaling for A_{thr} can also be derived from a criterium on the magnitude of the square displacements of the individual particles (see Ref. [10]). Figure 5 shows an example of this scaling of A_{thr} with the chain length N .

Dynamics.

In this section, we study the motion of the chain as x_N is gradually increased by applying an external force on particle number N so that the system is always in equilibrium. The previous discussion has shown that the evolution of the system cannot be always smooth. Jumps have to occur, corresponding to cusps in the energy as a function of x_N . However, because of the large number of metastable states, an additional prescription is required in order to determine the final state after a jump. It certainly depends on the specific choice of dynamics. For the sake of simplicity, we wanted to construct a dynamics which can be directly related to the knowledge of the various equilibrium states of the system. Our criterion of selection has been to pick the states which minimize the jump of the center of mass position. This simple criterion corresponds to choosing a situation where the center of mass dynamics is over damped. This *a priori* is not equivalent to assuming that each particle motion is itself over damped. It would be interesting to study the influence of the chosen dynamics on the behavior of the chain. However, this requires a full simulation and we would lose the simplicity of the present approach. The implementation of our criterion is illustrated in figure 6. Figure 6a shows the center of mass position x_{com} as a function of x_N . The corresponding energy branch is displayed as the full drawn line in figure 6b, where the dotted line refers to the full set of metastable states.

We observed that the states which are selected by this procedure differ qualitatively from the typical metastable states. For instance, we have checked that in these selected states x_p is an increasing function of p . However most typical states do not satisfy this property. The

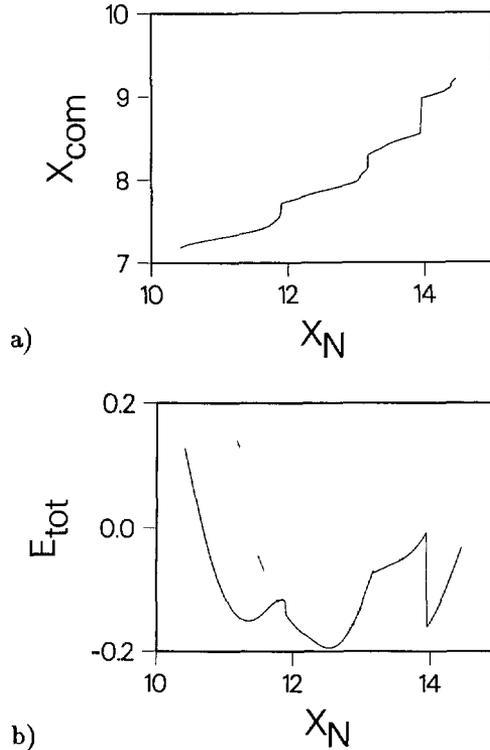


Fig.6. — Selection of states in the over damped center of mass dynamics. The parameters are the same as in figure 4. a) Position of the center of mass x_{com} versus x_N . When the system jumps, it is assumed to select the closest value for x_{com} associated to the same x_N . b) Total energy E_{tot} versus x_N . Note that the selected states after a jump are neither the closest nor the lowest ones in energy.

fact that the states selected in different dynamics differ has already been observed in other complex systems such as charge density waves [9]. However, in the work by Tang et al, the preparation method is quite different than ours. It would be interesting to investigate if our models exhibits this phenomenon of pulse memory.

As already stated for the two particle case, the motion of the chain is strongly hysteretic. If the control variable x_N is cycled in a fixed interval around a given initial value, the system evolves in the following way. After the first cycle, it does not return in general to its initial state. However, it afterwards evolves along a closed loop in configuration space. We show an example of this behavior in figure 7. We should also note that these periodic cycles depend on the minimal and maximal values of x_N .

Statistics of discontinuities.

In the present section, we discuss the statistical properties of the motion of the chain. In particular, we are concerned with the discontinuities in the total energy and center of mass position which occur as the system jumps from one metastable state to another. The distribution of energy discontinuities is shown in figure 8 for different chain lengths.

Power law behavior is observed except in the high energy region. The exponent of the scaling

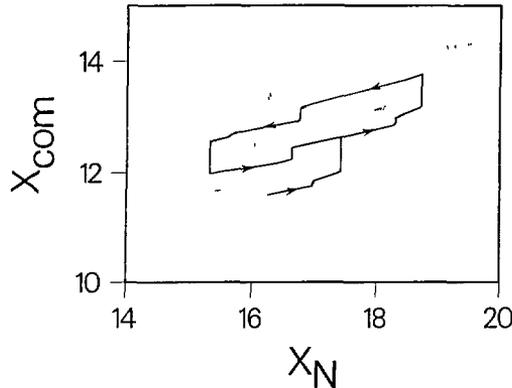


Fig.7. — Hysteretic behavior. Center of mass position x_{com} as a function of x_N , with the same parameters as in figure 4. Note that after a first cycle in x_N , the final state is in general different from the initial state. However, a second identical cycle in x_N , starting from this new state leads to a closed hysteresis loop.

region, $b \approx 0.8$, is within the numerical accuracy independent of A_p , n_p , R_p and N (see Fig. 8). The cross over from scaling for higher energies seems to be related to depinning events involving a single or a small number of pinning centers. To check this idea, we have calculated the distribution of displacements of individual particles during jumps. This shows that high energy events are connected with a broad distribution of displacements centered around zero and with a width of order several times a . The low energy discontinuities are connected with a narrow displacement distribution centered around zero. Intuitively, displacements induced by few pinning centers can propagate far away and grow linearly with the distance to the relevant pinning centers. This corresponds to the high energy events, which then resemble to the elementary cats discussed above. By contrast, low energy events involve many pinning centers, so that induced displacements cannot propagate very far. A more systematic study of this cross over from scaling to high energy regimes is in progress. One trend, among others, is the increase in the cross over energy upon increasing A_p , as can be seen in figure 8.

The corresponding distribution of discontinuities in the center of mass, $D(X_{com})$, is nearly uniform (over a range from about $10^{-4}a$ up to about $a/2$) for the events in the scaling region of $D(\Delta E)$. An example of this behavior is given in figure 9. The local peak in $D(\Delta E)$ is connected with a peak in $D(X_{com})$ at about a , thus corresponding to large energy events.

Power spectrum of pinning force.

The actual pinning force $F_p(x_N)$ is obtained as

$$F_p(x_N) = \sum_{i=1}^{N-1} V'(x_i) = -k(x_N - x_{N-1} - a). \tag{6}$$

We have found that $F_p(x_N)$ exhibits a similar saw-toothed behavior (see Fig. 10) as was observed in the experiments on solid friction [7]. It is then illuminating to calculate the power spectrum as shown in figure 11. This is done with the assumption that the chain is pulled at a constant velocity, so that x_N is proportional to time. We find a $1/f^{1.5}$ behavior for frequencies smaller than the characteristic frequency, f_2 , connected with the duration of the sawtooth, and

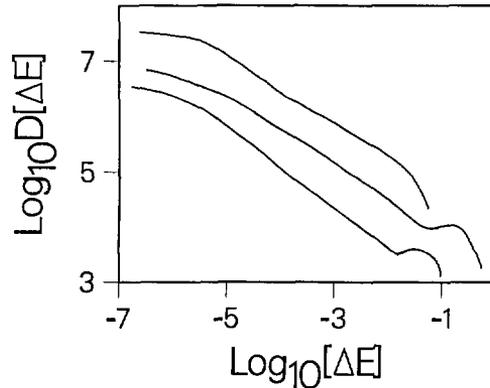


Fig.8. — Distribution of energy discontinuities. Parameters are: $n_p = 0.5$, $R_p = 0.25$, $k = 1$. From top to bottom: $N = 10$, $A_p = 0.01$; $N = 10$, $A_p = 0.05$; and $N = 20$, $A_p = 0.01$. The three curves have been arbitrarily shifted vertically for the sake of clarity. This log log plot shows the power law behavior in the low energy region.

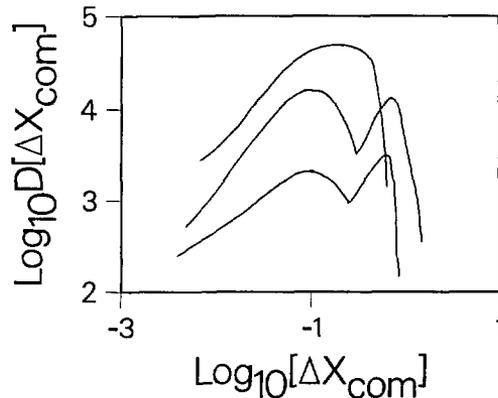


Fig.9. — Distribution of discontinuities in the center of mass position x_{com} . The parameters are the same as in figure 8. Again, the three curves have been arbitrarily shifted vertically for the sake of clarity.

larger than a frequency f_1 which decreases as N increases. The exponent 1.5 in this scaling regime appears to be independent of the system size N and the pinning strength A_p . For f larger than f_2 , the power spectrum follows a power law decay, with an exponent between 2 and 2.5. This exponent seems to decrease when N is increased, and to saturate at the value 2 in the large N limit. For f smaller than f_1 , we find a white noise, i.e. a power spectrum independent of f .

It is tempting to compare our results for this simple model to some experiments on solid friction. It is first useful to note that the $1/f^2$ behavior observed at high frequencies in our model is a simple consequence of the fact that the pinning force is a discontinuous function of time. Indeed, the power spectrum generated by a single kink is $1/f^2$, and at high frequencies, we can neglect interferences coming from different kinks in the calculation of the power spectrum. In the experiment of reference [7], such a $1/f^2$ is observed at high frequencies. This is likely to stem from the fact that in the actual experiment, the jumps occur on distances much larger than the substrate roughness. We think that observation of a $1/f^2$ behavior at high frequencies

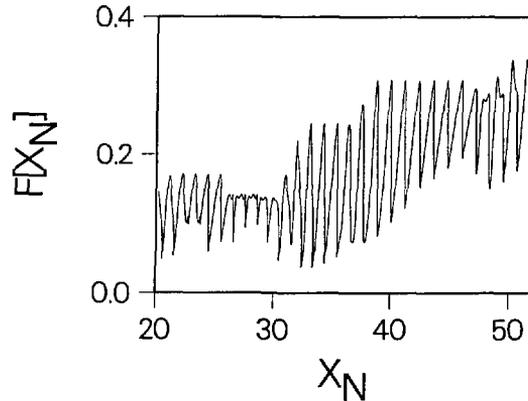


Fig.10. — Sawtooth shape of the pinning force as x_N is increased gradually.

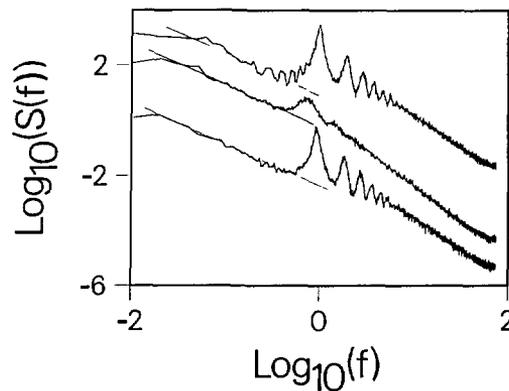


Fig.11. — Power spectrum for the pinning force. The parameters are the same as in figure 8. The chain is assumed to be pulled with a constant velocity, so that the frequency f can be interpreted as an inverse length scale for x_N . This log-log plot shows a $1/f^{1.5}$ in an intermediate frequency range.

for both our model and the experiment is not relevant in order to probe the collective nature of solid friction. In the intermediate scaling regime $f_1 < f < f_2$, the power law reflects some correlations between different avalanches, due to the fact that the same impurity configuration is at the origin of the many metastable states sampled during the system evolution. This may be observed in a solid friction experiment similar to the one described in reference [7], provided the force measuring apparatus is sensitive enough to measure small jumps so that the memory of the substrate configuration is not lost after each jump.

Conclusions.

In conclusion, we have investigated one dimensional elastic systems in random potentials. We found that the response of an elastic medium to a set of sharp and dense pinning centers is controlled by non-linear instabilities. We studied the statistics of the instabilities in the case where the system is moved through the random potential. We found that the energy discontinuities obey a power law distribution. The power spectrum is also characterized by the presence of a non trivial scaling regime. We should stress that these results do not seem a priori obvious. The existence of instabilities can be rather simply established analytically

because of the one dimensional nature of the system. However, it is much more difficult to deal analytically with the hierarchy of different metastable states which are generated as the system size or the strength of the pinning potential are increased. Our results indicate the existence of avalanches on all length scales in the thermodynamic limit. It would be interesting to try to extend the present approach to higher dimensions. A first possibility would be to couple several chains together. However, the dimension of the parameter space becomes much larger, and some new features are expected. For instance, we no longer expect the system to be attracted by hysteresis loops after one cycle as is the case for our model.

It may be worth noticing that the present model is strongly reminiscent of the picture already proposed by Coulomb to explain solid friction [11]. The idea was that two random surfaces had to glide on each other, the threshold being the force needed to unlock the asperities. However, it is known that for most of metal-metal friction coefficients, this picture is incorrect. The solid friction then comes from the necessity to shear the plastic junctions between asperities [12]. But for strong metals with a non plastic oxide layer as chromium, or with ceramics [13], or in the case of hard metals on emery paper [14], the picture proposed by Coulomb may still be relevant. Our model could be meaningfully compared with experiments consisting in studying the friction force and its noise on such friction couples, with no lubricant, and a low load. It is encouraging to notice for instance that in the case of friction on emery paper [14], the experimental results are in qualitative agreement with what our model would predict: the higher the elastic modulus of the metal, and the smaller the grain size of the emery paper, the lower is the resulting friction force. This suggests that careful noise measurements would be interesting to perform.

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