HAL
open science

A cellular automaton model for freeway traffic

Kai Nagel, Michael Schreckenberg

To cite this version:

Kai Nagel, Michael Schreckenberg. A cellular automaton model for freeway traffic. Journal de Physique I, 1992, 2 (12), pp.2221-2229. 10.1051/jp1:1992277 . jpa-00246697

HAL Id: jpa-00246697

https://hal.science/jpa-00246697

Submitted on 4 Feb 2008

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

J. Phys. I France **2** (1992) 2221–2229

Classification
Physics Abstracts
02.50 — 02.70 — 03.40G

A cellular automaton model for freeway traffic

Kai Nagel([1]) and Michael Schreckenberg([2])

([1]) Mathematisches Institut, Universität zu Köln, Weyertal 86-90, W–5000 Köln 41, Germany
([2]) Institut für Theoretische Physik, Universität zu Köln, Zülpicher Str. 77, W–5000 Köln 41, Germany

(Received 3 September 1992, accepted 10 September 1992)

Abstract. — We introduce a stochastic discrete automaton model to simulate freeway traffic. Monte-Carlo simulations of the model show a transition from laminar traffic flow to start-stop-waves with increasing vehicle density, as is observed in real freeway traffic. For special cases analytical results can be obtained.

1. Introduction.

Fluid-dynamical approaches to traffic flow have been developed since the 1950's [1]. In recent times, the methods of nonlinear dynamics were succesfully applied to these models, stressing the notion of a phase transition from laminar flow to start-stop-waves with increasing car density [2]. Automatic detection of stronger fluctuations near this critical point has already been used to install better traffic control systems in Germany [3].

On the other hand, boolean simulation models for freeway traffic have been developed [4, 5]. For lattice gas automata, it is well known that boolean models can simulate fluids [6]. We show that indeed our boolean model for traffic flow has a transition from laminar to turbulent behavior, and our simulation results indicate that the system reaches a possibly critical state by itself in a bottleneck situation (reminiscent of self organizing criticality [7]). This point together with an extension to multi-lane traffic will be the subject of further investigations [12].

The outline of this paper is as follows: At the beginning, we describe our model (Sect. 2) and discuss its phenomenological behavior, especially the transition (Sect. 3). In section 4 results for the bottleneck situation are presented. Section 5 contains a discussion of the results and compares them to values in reality. Sections 6 and 7 give a short conclusion/outlook. A preliminary account of the model is given in [8].

2. The model.

Our computational model is defined on a one-dimensional array of L sites and with open or periodic boundary conditions. Each site may either be occupied by one vehicle, or it may be empty. Each vehicle has an integer velocity with values between zero and $v_{\max}$. For an arbitrary configuration, one update of the system consists of the following four consecutive steps, which are performed in parallel for all vehicles:

1) **Acceleration:** if the velocity v of a vehicle is lower than $v_{\max}$ and if the distance to the next car ahead is larger than $v+1$, the speed is advanced by one $[v \to v+1]$.

2) **Slowing down (due to other cars):** if a vehicle at site i sees the next vehicle at site $i+j$ (with $j \leq v$), it reduces its speed to $j-1$ $[v \to j-1]$.

3) **Randomization:** with probability p, the velocity of each vehicle (if greater than zero) is decreased by one $[v \to v-1]$.

4) **Car motion:** each vehicle is advanced v sites.

Through the steps one to four very general properties of single lane traffic are modelled on the basis of integer valued probabilistic cellular automaton rules [9, 10]. Already this simple model shows nontrivial and realistic behavior. Step 3 is essential in simulating realistic traffic flow since otherwise the dynamics is completely deterministic. It takes into account natural velocity fluctuations due to human behavior or due to varying external conditions. Without this randomness, every initial configuration of vehicles and corresponding velocities reaches very quickly a stationary pattern which is shifted backwards (i.e. opposite the vehicle motion) one site per time step.

The Monte Carlo simulations have mainly been carried out with the choice of $v_{\max} = 5$ for reasons stated below. The model has been implemented in FORTRAN, using a logical array for the positions of the cars and an integer array for the velocities. For the 'realistic case' of $v_{\max} = 5$ we made the following observations: in the interesting regime of a relatively low density of occupied sites (usually only about one fifth), an implementation using IF-statements has been about five times faster than an implementation using only boolean variables and operations (necessary for multispin coding for 32 cars at once). As the expected gain of multispin coding would therefore give only a factor of about six, we postponed the work on a bitwise implementation.

The speed on an IBM-RS/6000 320H workstation was about 0.2 site-updates per microsecond, which is about a factor of 7 slower than the speed of a non-vectorized, multispin coding Ising model [11]. A significant part of the Monte-Carlo simulation runs have been carried out on an iPSC-860 Hypercube using up to 16 nodes, with only slight modifications of the program. The model's speed on one processor of the Hypercube was about the same as on the workstation.

3. Traffic on a circle (closed system).

In this section we present results from systems with periodic boundary conditions (thus simulating traffic in a closed loop "as in car races" but only on a single lane). As the total number N of cars in the circle cannot change during the dynamics, it is possible to define a constant system density

$$\rho = \frac{N}{L} = \frac{\text{number of cars in the circle}}{\text{number of sites of the circle}} \quad , \tag{1}$$

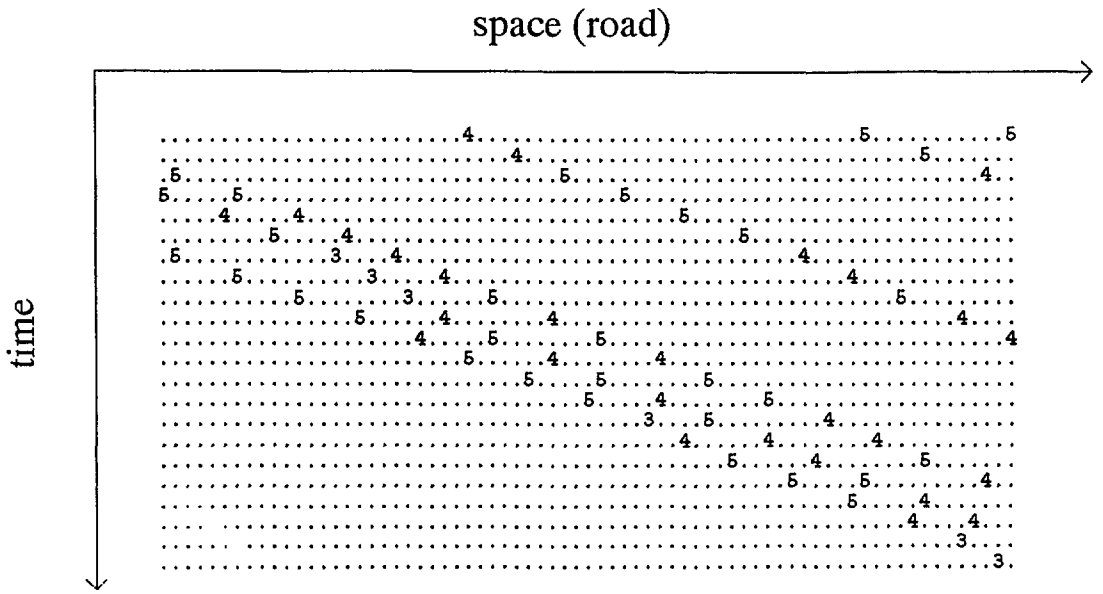

Fig.1. — Simulated traffic at a (low) density of 0.03 cars per site. Each new line shows the traffic lane after one further complete velocity-update and just before the car motion. Empty sites are represented by a dot, sites which are occupied by a car are represented by the integer number of its velocity. At low densities, we see undisturbed motion.

which is usually not possible in reality. Thus, in order to mimick real conditions, we measured densities (= occupancies) $\overline{\rho}^T$ on a fixed site i averaged over a time period T:

$$\overline{\rho}^T = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} n_i(t) \tag{2}$$

where $n_i(t) = 0(1)$ if site i is empty (occupied) at time step t. For large T we have

$$\lim_{T\to\infty} \overline{\rho}^T = \rho \ . \tag{3}$$

The time-averaged flow $\overline{q}$ between i and $i+1$ is defined by

$$\overline{q}^T = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} n_{i,i+1}(t) \tag{4}$$

where $n_{i,i+1}(t) = 1$ if a car motion is detected between sites i and $i+1$.

With these definitions, it was easy to perform many simulations with different densities ρ, thus — after relaxation to equilibrium — getting data for the commonly used fundamental diagrams which plot vehicle flow $\overline{q}$ *vs.* density $\overline{\rho}$.

To be specific, we start with a random initial configuration of cars with density ρ and velocity 0 and begin the collection of data after the first t_0 time steps, where we took $t_0 = 10 \times L$. In figures 1 and 2 we show typical situations at low and high densities. Whereas we find laminar traffic at low densities, there are congestion clusters (small jams) at higher densities, which are formed randomly due to velocity-fluctuations of the cars. If one follows (in Fig. 2) one

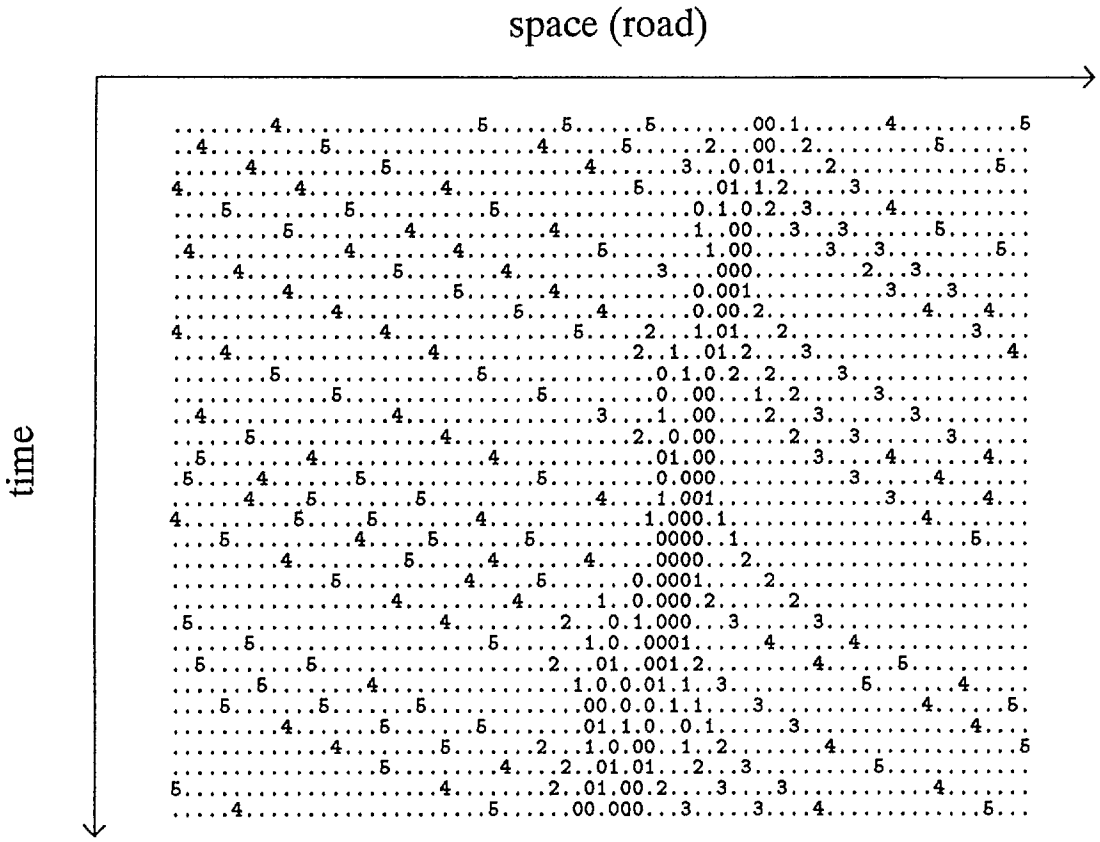

Fig.2. — Same picture as figure 1, but at a higher density of 0.1 cars per site. Note the backward motion of the traffic jam.

individual car coming from the left, one sees that the car comes in with a speed varying between four and five and then has to stop due to the congestion cluster. There it stays stuck in the queue for a certain time with some slow advances, and can accelerate to full speed after having left the cluster at its end. So the cluster represents a typical start-stop-wave as found in freeway traffic (cf. Fig. 3).

We present the fundamental diagram of our model in figure 4. Whereas the line indicates the results of averaging over 10^6 time steps, the dots represent averages over only 10^2 time steps and may be compared with results from real data (Fig. 5). It can clearly be seen that a change-over takes place near $\rho = 0.08$. Further simulations show that the position and the form of the maximum of $q(\rho)$ depend on the system size. (Simulations without randomization do not show a dependence on the system size.) However, it is not clear from these pictures where to locate an exact transition point.

For an analytical treatment of the circular traffic, one chooses as starting point the case $v_{\max} = 1$, where the situation simplifies considerably. Here step one of the four update steps is trivial since every car can accelerate in only one time step to its maximum speed $v_{\max} = 1$ and therefore, before step two, every car has speed 1. The question in the update procedure then is simply if the next site is free (step two) and if the speed is (randomly) set to zero (step three).

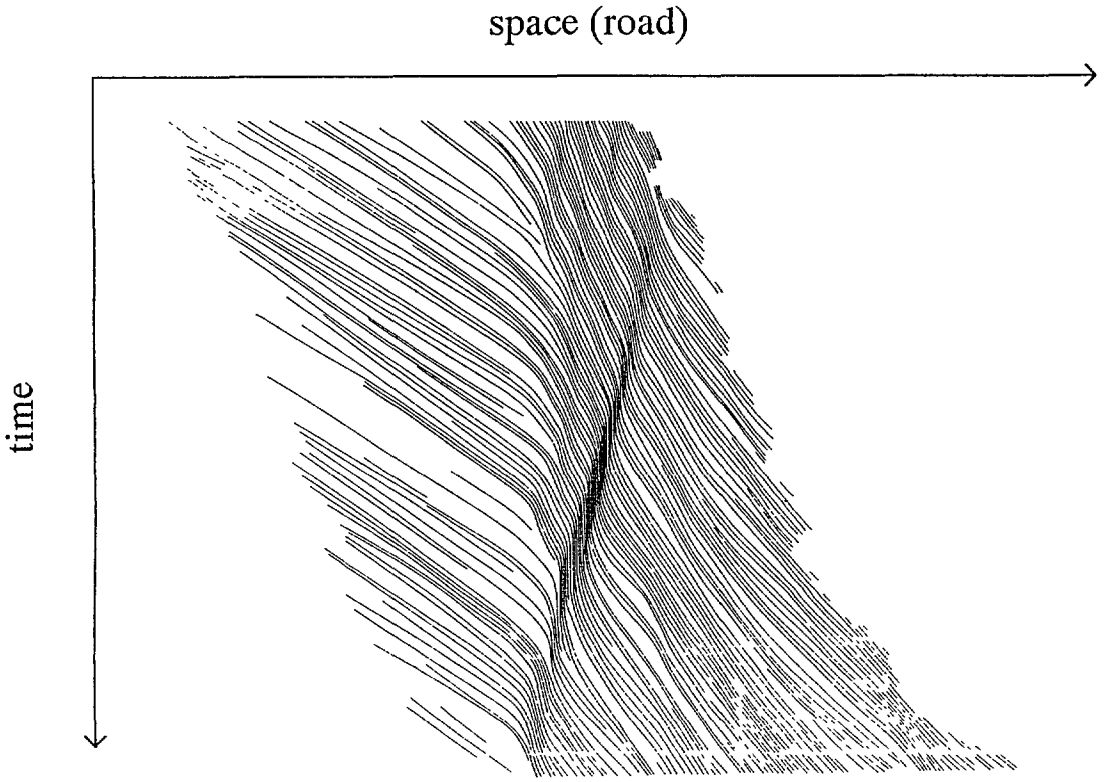

Fig.3. — Space-time-lines (trajectories) for cars from Aerial Photography (after [16]). Each line represents the movement of one vehicle in the space-time-domain.

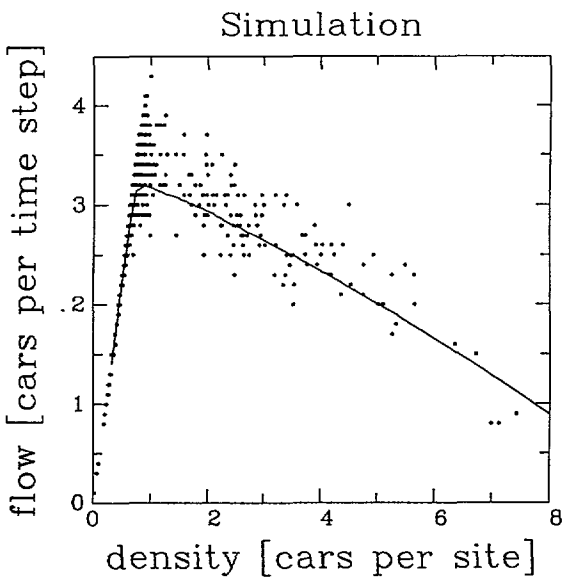

Fig.4. — Traffic flow q (in cars per time step) *vs.* density (in cars per site) from simulation results ($L = 10^4$). Dots are averages over 100 time steps, the line represents averages over 10^6 time steps.

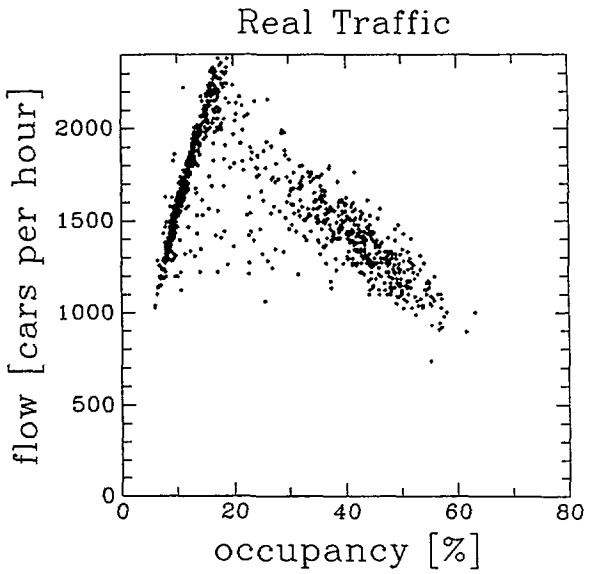

Fig.5. — Traffic flow q (in cars per hour) *vs.* occupancy (in cars per hour) from measurements in reality. Occupancy is the percentage of the road which is covered by vehicles (after [17]).

For speeds larger than 1 an additional parameter for the current speed is needed which gives serious difficulties for analytical treatments. However, even in this case a kind of mean-field approximation is possible and the results will be reported elsewhere [12].

For direct calculations the easiest way to formulate the dynamics is on the basis of a master equation with continuous time and random sequential update. This makes of course a difference to the parallel updating (which can simply be seen by simulating the two different updates) but the results should be qualitatively similar and the randomization parameter p plays a particular simple role for random sequential update.

With the use of the more familiar spin variables $\sigma_i = \pm 1$ with $+1$ for occupied and -1 for empty sites, the transition probability $W(-\sigma_i, -\sigma_{i+1}|\sigma_i, \sigma_{i+1})$ from (σ_i, σ_{i+1}) to $(-\sigma_i, -\sigma_{i+1})$ (i.e. a car moves from i to $i+1$) simply reads

$$W(-\sigma_i, -\sigma_{i+1}|\sigma_i, \sigma_{i+1}) = (1-p)\,\frac{1+\sigma_i}{2} \tag{5}$$

With periodic boundary conditions this transition probability ensures the conservation of the total magnetization $\sum_i \sigma_i$ in the system. The master equation for the probability $P(\{\sigma_j\}, t)$ to find configuration $\{\sigma_j\}$ at time t is given by

$$\begin{aligned}\frac{\mathrm{d}P(\{\sigma_j\})}{\mathrm{d}t} &= -\sum_i W(-\sigma_i, -\sigma_{i+1}|\sigma_i, \sigma_{i+1})\, P(\{\sigma_i, \sigma_{i+1}, t\} \\ &\quad + \sum_i W(\sigma_i, \sigma_{i+1}| -\sigma_i, -\sigma_{i+1})\, P(\{-\sigma_i, -\sigma_{i+1}, t\}).\end{aligned} \tag{6}$$

From this formula it can easily be seen that the factor $(1-p)$ only gives a simple scaling factor of the time axis. Therefore, in continuous time, the systems with different $p < 1$ are equivalent in the sense that they have the same equilibrium distribution. Analyzing this equilibrium

distribution one finds that, given a fixed total magnetization, every configuration of the spin is equally probable. Therefore, in this simple situation, one has

$$q = \rho(1-\rho) \tag{7}$$

which is just the probablity that a car has a free site in front of it. The property that every configuration of the cars is equally probable is certainly not true for $v_{\max} > 0$. The parallel update gives a different function $q(\rho)$ but it is also symmetric with respect to $\rho = 1/2$ [12].

4. Traffic in a bottleneck situation (Open system).

For this section, we apply different boundary conditions, leaving the rest of the model unchanged:

- When the leftmost site (site 1) is empty, we occupy it with a car of velocity 0. As our traffic is going from left to right, one may imagine a bottleneck situation where a saturated two-lane-street feeds a street of only one lane (which we simulate).
- At the right side (i.e. the end of the street), we simply delete cars on the rightmost six sites, thus producing an open boundary. (This simulates the beginning of an expanded (four-lane) freeway.)

Our simulations included grid length up to 10000 sites with durations up to 5×10^6 time steps. After relaxation, the model shows traffic at a density of $<\rho_c> = 0.069 \pm 0.002$ and a flow of $<q_c> = 0.304 \pm 0.001$.

Again the situation here simplifies considerably for $v_{\max} = 1$. The case of random sequential update is exactly equivalent to an asymmetric exclusion model investigated in detail in references [13, 14] where the equilibrium distribution in a system with open boundaries is calculated. The case $\alpha = \beta = 1$ (in the notation of Refs. [13, 14]) corresponds then to our bottleneck situation. It can be seen that in this simple case the system drives itself to the state of maximum flow at $\rho = 1/2$. However this seems not to be a general property also valid for larger $v_{\max} > 1$ (cf. Fig. 6).

5. Quantitative comparison with realistic traffic.

In this section, we make some rough arguments concerning the length scale and time scale of the simulation model. The easiest approach to scale the model is the claim that — in a complete jam — each car occupies about 7.5 m of place, which is thus the length of one site. As the average velocity in free traffic of 4.5 sites per time step should correspond to a velocity of about 120 km/h (in Germany), we arrive naturally at a time for one iteration of

$$7.5\,\frac{\mathrm{m}}{\mathrm{site}} \times 4.5\,\frac{\mathrm{sites}}{\text{time-step}}/(120/3.6)\,\frac{\mathrm{s}}{\mathrm{m}} \tag{8}$$

$\approx$ (1 second per time-step) which agrees with [5].

A second possibility is to scale the model by the position of the maximum in the fundamental diagram. From traffic measurements, this maximum is found at about $\rho \sim$ (30 vehicles per lane and kilometer) = (0.225 vehicles/7.5 m), which is by a factor of about 2 higher than the position of the maximum in the scatterplot for our model.

Similarly, freeways have a maximum capacity of about (2000 vehicles per hour and lane) = (0.56 vehicles per second). As our maximum of the flow is only 0.32 vehicles per time step,

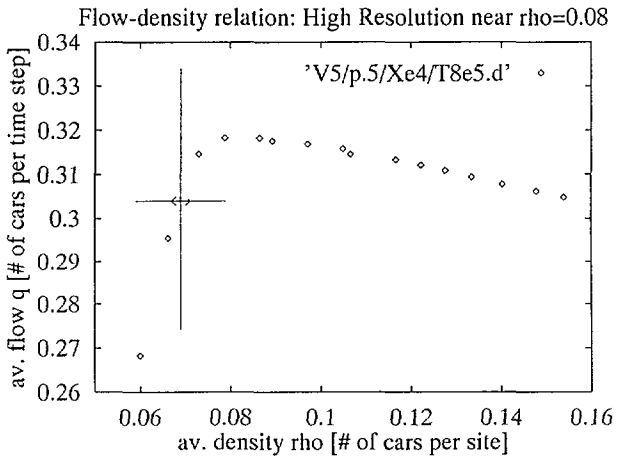

Fig.6. — Interesting part (near the change-over) of figure 4 (only long time averages). The cross indicates the point from the bottleneck situation (i.e. open system with maximal input of cars from the left and an absorbing boundary at the right, see text); tips of the arrow indicate the error of ρ. The error of q is too small to be visible on this scale. The simulation clearly indicates that the self-organizing bottleneck state does not correspond to the maximum of the flow, as it is the case for $v_{\max} = 1$.

our model time step should correspond to $0.32/0.56 \approx 0.5$ seconds, thus being by a factor of two lower from the value presented above.

A fourth possibility uses the value of the velocity of the back-travelling start-stop-waves, where a value of about 15 km/h $\approx$ 4.2 m/s has been measured on freeways. As the maximum capacity of our simulated system is about 0.3 cars per time step, the maximum speed of the backpropagating wave is 0.3 sites ($\approx$ 2.25 m) per time step (i.e., about every third time step a new car arrives at the back of the traffic jam). This would fix one model time step at $2.25/4.2 \approx 0.7$ seconds, thus yielding a value between those of the first and of the third method. Thus all these estimates agree in order of magnitude: time steps correspond to seconds.

Another interesting result of the last argument is that start-stop-waves might intrinsically be a queueing phenomenon where only the formation takes place dynamically: once a congestion cluster of length L has formed, there is a probability p^{arr} per time step that a new car arrives at the end, and a probability p^{dep} per time step (determined in a nontrivial way by the dynamics of the model) that a car departs at the front of the jam (cluster). As p^{dep} at one cluster dictates p^{arr} for the next cluster, all clusters act like queues at the critical threshold where $p^{\mathrm{arr}} = p^{\mathrm{dep}}$. But the formation of the clusters as well as the self-organization of the critical value of p^{arr} take place without external control.

One should note that, contrarily to intuition, the parameters for a discrete traffic flow model are relatively fixed once one accepts "one car per lattice site in a jam". The parameters used for the simulations seem relatively reasonable.

6. Conclusion.

It has been shown that a discrete model approach for traffic flow is not only computationally advantageous [5, 4], but that it contains some of the important aspects of the fluid-dynamical

approach to traffic flow such as the transition from laminar to start-stop-traffic in a natural way (see particularly the end of the last section). Thereby, it retains more elements of individual (though statistical) behavior of the driver, which might lead to better usefulness for traffic simulations where individual behavior is concerned (e.g. dynamic routing).

An additional interesting observation is that sand falling down in a long and narrow tube shows very similar behavior, i.e. "start-stop-waves" originally caused by fluctuations in the velocity of the freely moving particles. In the case of sand these fluctuations are due to dissipation at the boundary [15].

Acknowledgements.

We want to thank A. Bachem for initiation to the theme and, together with R. Kühne, P. Konhäuser and M. Rödiger, for helpful discussions, D. Stauffer for numerical and methodological help, and H.J. Herrmann for interesting hints. ZAM at the Jülich Research Center provided computing time on the newly installed Intel iPSC/860, and R. Knecht helped in using it. This work was partly performed within the research program of the Sonderforschungsbereich 341 Köln-Aachen-Jülich supported by the Deutsche Forschungsgemeinschaft.

References

[1] LIGHTHILL M.J., WHITHAM G.B., *Proc. R. Soc. Lond.* **A229** (1955) 317.

[2] KÜHNE R., in: Highway Capacity and Level of Service, U. Brannolte Ed., Proc. Int. Symp. Highway Capacity, Karlsruhe (Rotterdam: Balkema, 1991).

[3] ZACKOR H., KÜHNE R., BALZ W., *Forschung Straßenbau und Straßenverkehrstechnik* **524** (1988), (Bundesanstalt für Straßenwesen, Bonn–Bad Godesberg).

[4] CREMER M., LUDWIG J., *Mathematics and Computers in Simulation* **28** (1986) 297.

[5] SCHÜTT H., *Schriftenreihe der AG Automatisierungstechnik TU Hamburg-Harburg* **6** (1991).

[6] FRISCH U., HASSLACHER B., POMEAU Y., *Phys. Rev. Lett.* **56** (1986) 1505.

[7] BAK P., TANG C., WIESENFELD K., *Phys. Rev.* A **38** (1988) 368.

[8] NAGEL K., Proc. Phys. Comp. '92 Conf., Praha (Singapore: World Scientific, 1992).

[9] WOLFRAM S., Theory and Applications of Cellular Automata (Singapore: World Scientific, 1986).

[10] STAUFFER D., *J. Phys. A: Math Gen.* **24** (1991) 909.

[11] KOHRING G.A., STAUFFER D., *Int. J. Mod. Phys. C* (October 1992).

[12] NAGEL K., SCHRECKENBERG, M., in preparation.

[13] DERRIDA B., DOMANY E., MUKAMEL D., *J. Stat. Phys.* **69** (November 1992).

[14] DERRIDA B., EVANS M.R., preprint 1992.

[15] HERRMANN H.J., personal communication.

[16] TREITERER J., TR **PB 246 094** (Columbus, Ohio State University, 1975).

[17] HALL F.L., BRIAN L.A., GUNTER M.A., *Transpn. Res. A* **20A** (1986) 3, 197.