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Ludovic Benguigui

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Short Communication

## The fractal dimension of some railway networks

L. Benguigui

Solid State Institute and Physics Department, Technion-Israel Institute of Technology, 32 000 Haifa, Israel

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**Abstract.** — We determine the fractal dimension  $D$  of four different railway networks. We found for Paris, Moscow and the Rhine towns in Germany,  $D$  between 1.7 and 1.8 but for the network connecting Paris to the suburbs,  $D$  is equal to 1.5. We discuss some implications of these results.

In this short note, we want to give a new example of fractal objects: railway networks. It is now well known that the concept of fractals is very useful in different fields in physics, chemistry [1] and biology [2]. However there are only few examples of fractals in man-made situations [3]. Here, we shall analyze four different railway networks and determine their fractal dimension. We conclude that railway networks we studied, seen as objects embedded in the plane (a two dimension space) have fractal dimension between 1.5 and 1.8. Preliminary results have been published [4] to attract the attention of geographers and in this paper we present more to the community of the physicists.

The four railway networks are the following (Fig. 1):

a) The interconnecting railway system of the Rhine towns in Germany. Along the Rhine river, large industrial and urban complex developed, including the towns of Essen, Dortmund, Dusseldorf, Bonn, Koln, Aachen etc. The network is displayed on a region that is approximately a square of  $130 \times 130 \text{ km}^2$ .

b) The railway network of the town of Moscow. The town must be distinguished from the whole urban ensemble (as in the case of the parisian urban ensemble, see below). It can be included in a circle with a radius of 15 km. We took together two distinct (but interconnected) networks: the underground and a regular railway network. Both have the same structure and they have the same function to ensure public transportation in Moscow itself.

c) The underground system of the town of Paris. It covers the center of the urban ensemble, and as in the precedent case its function is the transportation only inside the town. By simple visual inspection, it is possible to see that it is relatively dense, in a region with a constant density population. It extends approximately inside a circle of 6 km. It is known as the

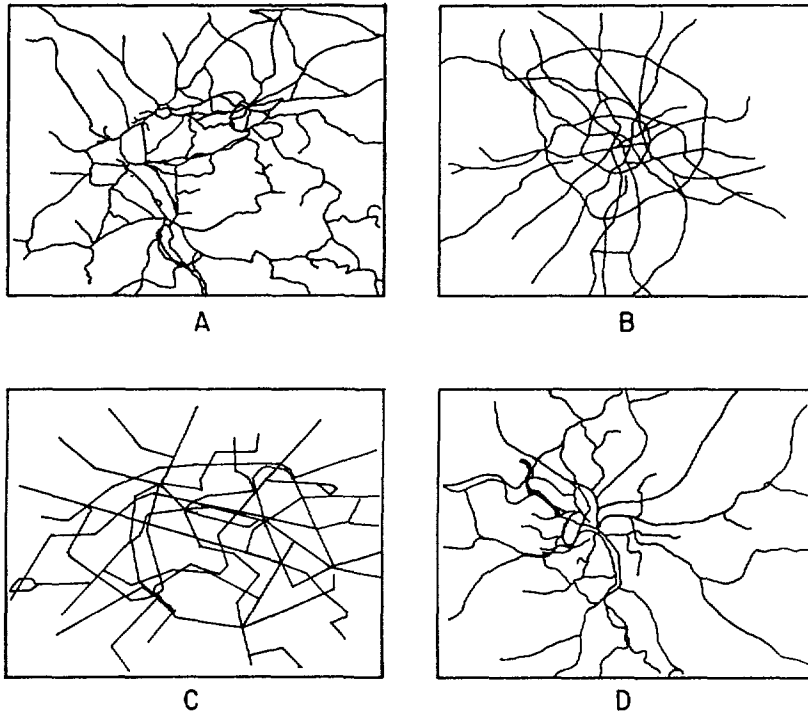


Fig. 1. — Railway networks studied in the paper: a) Rhine towns, b) Moscow, c) Paris, d) Suburbs around Paris.

“metropolitain” or in short the “metro”.

d) The railway network of the suburbs in the parisian urban ensemble. The purpose of this network is to connect the town of Paris to all the other much smaller towns which compose (with Paris) the parisian urban ensemble. It covers an approximately circular region with a radius of 50 km.

The maps were digitized to get pictures which can be displayed on a computer screen. From the digitized pictures, determination of the fractal density have been made. We counted the total length  $L$  of the railway tracks inside a circle of radius  $R$  centered on one point of the network. The relationship between  $L$  and  $R$  is obtained by varying  $R$ .

In the cases a (Rhine towns) and c (town of Paris), the network seems to extend on all the plane without special point. We took several points as the center of the circles and for each  $R$  a mean value is adopted for  $L$ . For the cases b (Moscow) and d (suburbs of the parisian ensemble), the network seems constructed from a central point (it is true especially for the case d) and we choose this point as the center of the circles.

The minimum and maximum values of  $R$  are as follows: a) from 2 km to 60 km; b) from 0.7 km to 12 km; c) from 0.25 km to 6 km; d) from 2 km to 50 km.

In figure 2, we show the log-log plots of  $L$  versus  $R$  and in all the cases we get a straight line i.e.  $L \sim R^D$ . For the three first cases (a, b, c) we found values of  $D$  equal to  $1.70 \pm 0.05$ ,  $1.70 \pm 0.05$  and  $1.80 \pm 0.05$  respectively. The last case is distinct with  $D$  equal to  $1.50 \pm 0.05$ . May be we have not to take too seriously the exact figures since the samples are relatively small:  $R$  varies by slightly more than one order of magnitude. We can say that in the three

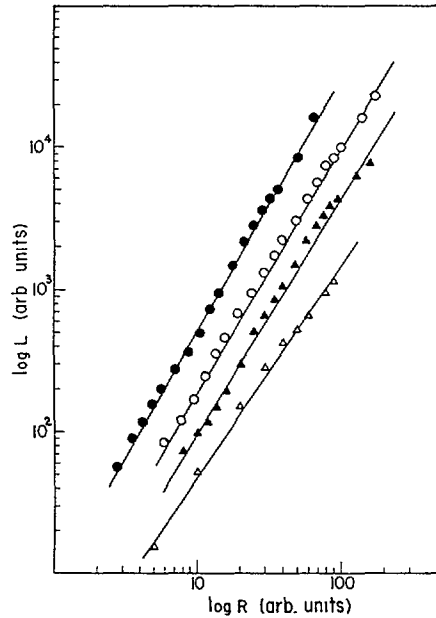


Fig. 2. —  $L$  versus  $R$  in a log-log plot showing the relationship  $L \sim R^D$ . (●) Paris, (▲) Moscow, (○) Rhine towns, (△) suburbs of Paris. The different straight lines are moved upward for better clarity.

first cases  $D$  is near to 1.7 – 1.8. The near values of  $D$  can be easily understood for Paris and the Rhine towns since the network covers a region without a marked center. The case of Moscow is less clear.

The difference in the last network with  $D = 1.5$  is easily understood when one thinks that its function is to transport people from or to the center of urban ensemble, the town of Paris, and it is therefore less dense.

The results are a little surprising since this means that in these networks there is no particular length scale (in the range of  $R$  which were investigated). Secondly, each network has the property to invade the space in all places with the same pattern. One possible conclusion is that, whatever the real process (economic, financial, political, etc.) which work in building the network, a unique pattern emerges in its formation. So the important point is to understand why these networks are fractal or in other words to find the hidden pattern characteristic of their structures. This may give a new insight in the study of these networks not only as complicated ensemble of lines but as transportation systems. Clearly, a similar study of other big towns like New-York, London or Tokyo can help to decide the universality of the phenomenon.

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