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Short Communication

A theory of magnetoconductance in Anderson insulators

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Abstract. — We present a simple theory to understand the effect of magnetic field on the localisation length ξ in Anderson insulators. For thin wires, we find that ξ is doubled, a result recently derived through random matrix theories. For films or bulk samples, new results are obtained. In this case, the localisation length is multiplied by a non universal factor. We discuss *quantitatively* the full dependence of ξ on the magnetic field.

Interest in quantum transport phenomena has soared during the past few years (see e.g. [1-3]). Among other interesting progress, one may cite the discovery of persistent currents in mesoscopic rings [4], the development of "Diffusing Wave Spectroscopy" [2] (which takes advantage of weak localisation effects to probe in a new manner e.g. concentrated suspensions), or the unveiling of the peculiar nature of conductance fluctuations in disordered metals (universal amplitude $\partial g \approx e^2/h$, sensitivity to single impurity rearrangements [1,5], relevance to $1/f$ noise [6]). The theoretical understanding of these fluctuations was greatly improved after Imry, and Alt'shuler and Shklovskii [7,8] suggested that the random matrix theory of Dyson and Mehta was the natural language to describe these effects: the universal character of the conductance fluctuations is intimately related to the spectral rigidity of random matrices. It was then realized that breaking of a basic symmetry (e.g. imposing a magnetic field to break time reversal symmetry) simply changed the relevant ensemble of random matrices, which ultimately leads to the following simple and spectacular prediction [5,9,10,18]: when a magnetic field is imposed, the conductance fluctuations of a *metallic* sample are reduced by a factor 2 (in absence of spin-orbit coupling) compared to the zero field case. This prediction was confirmed experimentally [10].

Very recently, Pichard and collaborators [11] (see also [12]) pointed out that a similar phenomenon also occurs in the highly disordered, insulating case. Again using random matrix theory, they showed that - at least for quasi 1 D wires - the localisation length ξ should simply be multiplied by 2 when a sufficiently strong magnetic field B is applied (or divided by 2 if strong spin orbit scattering is present). This should have dramatic consequences on the conductivity of these insulating samples [given, in the Mott régime, by $\sigma \propto \exp-(T_0/T)^{1/(d+1)}$ with $T_0 \propto \xi^{-d}$], or on the static

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dielectric constant $\epsilon \propto \xi^2$. Numerical simulations on 1d strips and Mott conductivity experiments with 3d samples [11] are indeed in qualitative agreement with the prediction $\xi(B) = 2\xi(0)$.

In this Short Communication we develop an approximate theory which describes quantitatively the effect on magnetic field on the localisation length in the insulating régime. Our discussion is based on Sornette and Souillard’s heuristic picture of localisation [13]- which is essentially a simple physical way to recover the results of the self consistent diagrammatic theory of Vollhardt and Wolfe [14]. A similar approach for weak localisation effects was proposed in [15]. Our main results are the following: while we indeed recover the factor 2 for quasi 1d samples, we predict that, at least for weak disorder, $\xi(B) \approx \xi^2(0)/l_e$ in films (l_e is the mean free path), and that $\xi(B)/\xi(0)$ may be anything between 1 and ∞ in three dimensions, depending upon the distance from the localisation transition. We furthermore provide a prediction for the full dependence of $\xi(B)$: the crossover field B^* is, as expected, $B^*\xi^2(0) \approx \hbar/e$. We obtain $\xi(B) - \xi(0) = A_d (B/B^*)^2 \xi(0)$ for $B \ll B^*$ (where A_d is a numerical factor depending upon dimensionality). For 1d wires, we find $2\xi(0) - \xi(B) \propto B^{-1}$ for large fields.

We consider a wave (of velocity c , wavelength λ) propagating in a random medium. The elastic mean free path is l_e and we suppose for simplicity isotropic scattering at each collision. Adapting from [13], we examine the stationary situation schematized in figure 1. An energy current J_O is injected at O along Ox and eventually (since stationarity is assumed) leaves the sample far from O. The local “equilibrium” current J_{loc} is built by all “Feynman paths” reinjecting energy at O along the initial direction Ox . Following [13], this initial direction is in fact, due to the wave character of the problem we consider, not precisely defined: it lies within a cone of solid angle $\delta\Omega = (\delta\theta)^{d-1}$ with $(l_e/\lambda) \delta\theta \approx 1$. Only the fraction $\delta\Omega/\Omega$ of the reinjected energy will thus be *coherently* backscattered. J_{loc} is then obtained as:

$$J_{loc} = J_O + R(\delta\Omega/\Omega)J_{loc} = J_O/[1 - R(\delta\Omega/\Omega)] \tag{1}$$

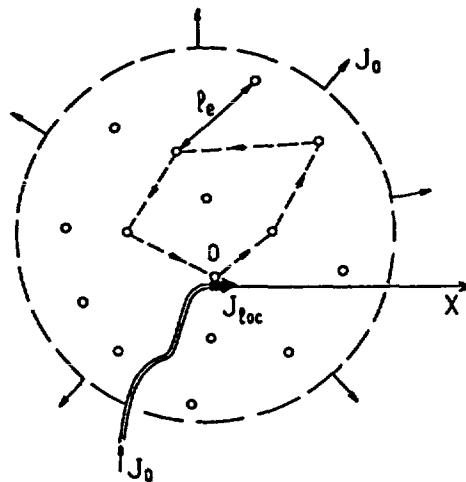


Fig. 1. — We consider the stationary situation where an energy flux J_O is injected at O, building through closed paths the equilibrium local current J_{loc} . Localisation occurs when $J_{loc} \neq 0$ even when $J_O = 0$.

(see Fig.1). R is the total energy reinjection coefficient, obtained as:

$$R = \left| \sum_t \sum_{C^t} \sqrt{\mathcal{P}(C^t)} e^{i\varphi(C^t)} \right|^2 \tag{2}$$

where $\mathcal{P}(C^t)$ is the weight of the t long path C^t - which is in fact precisely what we are looking for, since the large scale transport of energy will be determined by those weights - and $\varphi(C_t)$ the phase acquired by the wave along this path. We then i) group together pairs of time reversed paths C_t and C_{-t} , and ii) suppose that the phases of two otherwise different paths are completely decorrelated (this of course would be completely false if the medium were periodic). Hence:

$$R \approx 1/2 \sum_t \sum_{C^t} \mathcal{P}(C^t) \left| e^{i\varphi(C_t)} + e^{i\varphi(C_{-t})} \right|^2 \tag{3}$$

In the presence of a magnetic field, one has:

$$\varphi(C_{\pm t}) = \varphi_0(C_{|t|}) \pm e/\hbar BS(C_t)$$

where $S(C_t)$ is the algebraic area enclosed by the projection of C_t in a plane orthogonal to B . Thus (3) may be written

$$R \approx \sum_t P(O, t) \langle 1 + \cos [2e/\hbar BS(C_t)] \rangle \tag{4}$$

where $P(O, t)$ is the probability of finding t steps closed paths and $\langle \dots \rangle$ denotes the average over closed paths of length t . Let us suppose for a *moment* that energy diffuses classically, that is, the weights $\mathcal{P}(C^t)$ are those pertaining to Brownian paths. Then R can be explicitly calculated, since:

a) One has, in d dimensions:

$$P(O, t) = l_e^d (4\pi D_O t)^{-d/2} \tag{5}$$

where $D_O = l_e^2/2d\tau_e$ is the diffusion constant, and $\tau_e = l_e/c$ is the mean free time. For one dimensional strips or bars, one has rather:

$$P(O, t) = (l_e/W)^{d-1} (l_e/4\pi D_O t)^{1/2} \tag{6}$$

where W is the width of the strip or bar.

b) the probability distribution of $S(C_t)$ is explicitly known in two dimensions [16,17]:

$$P(S, t) = \pi / (4D_O t) \cosh^{-2} [\pi S/2D_O t] \tag{7}$$

We emphasize that this result will be useful in three dimensions since only the projection of the walks in a two dimensional plane orthogonal to B will matter. For quasi-1d geometries, the total area is the sum of t/t_W terms of order W^2 , distributed according to $P(S, t_W)$, with $t_W = W^2/2dD_O$. Hence (7) will be replaced by

$$P_{1D}(S, t) = (2\pi\mu W^2 D_O t)^{-1/2} \exp - [S^2/2\mu W^2 D_O t] \tag{8}$$

The validity of equation (8) and the exact value of μ have in fact been very recently obtained by A. Comtet and J. Desbois, and is, for $d = 2$: $\mu = 1/12$.

Let us first consider the case $B = 0$. Using (5), one sees immediately that R diverges in dimensions $d = 1$ and 2 (but is finite for $d > 2$). If $\delta\Omega/\Omega$ is not zero (i.e. for non zero wavelengths), this is clearly inconsistent with equation (1): in dimensions $d \leq 2$, energy cannot diffuse classically but is rather expected to be localised in some finite volume. The assumption that $\mathcal{P}(C^t)$ is given by the random walk theory is thus wrong. A simple way [13] to account for the appearance of an exponential bound state of size ξ is to modify the diffusion (propagator) equation as $\partial P/\partial t = D_0 [\Delta P - \xi^{-2} P]$: the weight of each path of length t is reduced by a factor $\exp - (t/\tau)$, with $\tau = \tau_e (\xi/l_e)^2$ ξ must then be self consistently estimated in such a way that $R(\xi)(\delta\Omega/\Omega) = 1$ which is the condition for which $J_{loc} \neq 0$ even if $J_0 = 0$ (see equation (1)). The results given below would not be deeply affected if instead of the exponential cutoff $\exp - (t/\tau)$ was replaced by another rapidly decaying function of (t/τ) . Let us first focus on the one-dimensional case. ξ must then satisfy the following equation (in the following, we shall use units such that $\tau_e = 1$) :

$$2 \int_1^\infty dt t^{-1/2} \exp - (t/\tau) = A_1 \tag{9}$$

where A_1 is a numerical constant and the factor two comes from equation (4) for $B = 0$. The lower bound of course corresponds to the mean free path. Hence in one dimensional strips or bars of width W , one finds (using (6) and $\delta\Omega \approx (\lambda/l_e)^{d-1}$) :

$$\xi(0) = N A_1 / (2\sqrt{\pi}) l_e \tag{10}$$

where N is the number of channels ($N = (W/\lambda)^{d-1}$). This is, up to numerical factors, *precisely the exact result* found e.g. in [18,11]. Note that $\xi(0)$ diverges in the classical limit ($\lambda = 0$), as it must. Now, in the presence of magnetic field, and in the limit $\xi \gg W$, (9) will read (using (8)):

$$\int_1^\infty dt t^{-1/2} \exp - (t/\tau) + \int_1^\infty dt t^{-1/2} \exp - (t/\theta) = A_1 \tag{11}$$

with $\theta = \tau / (1 + \tau b^2)$, b is the reduced field $b = (\mu/d)^{1/2} e / \hbar B W l_e$.

From equation (11), one obtains the crossover field $B^* = (d/\mu)^{1/2} (\hbar/e) / W \xi(0)$ by writing $\tau b^2 \approx 1$, that is that one quantum flux is contained in a localisation area $W\xi$. For $B \ll B^*$, (9) leads to:

$$\xi(B) - \xi(0) = (1/4) (B/B^*)^2 \xi(0) \tag{12}$$

For $B \gg B^*$, one finds:

$$\xi(B)/2\xi(0) = 1 - b^{-1} \tag{13}$$

The full curve $\xi(B)/\xi(0)$ versus B/B^* (corresponding to equation (11)) is plotted in figure 2 and compared to the numerical results of [11]. The agreement is not very good, but this is probably because $\xi(0)$ is comparable to W in the numerical simulation. A much better fit is achieved if one argues that equation (8) only holds for $t \gg t_W$ and thus rather uses the two dimensional area distribution (Eq. (7)) for $P(S, t)$: see Fig. 2, insert.

Note that for $B l_e^2 \approx \hbar/e$, transport is already affected by the magnetic field at scales $< l_e$ ("Hofstadter régime"), and our theory ceases to apply (it assumes that all relevant length scales are much larger than l_e).

Let us briefly describe the situation in higher dimensions. For $d = 2$, one may write an equation analogous to (11):

$$\int_1^\infty dt t^{-1} \exp - (t/\tau) + b \int_1^\infty dt \exp - (t/\tau) / \sinh(b\tau) = A_2 \tag{14}$$

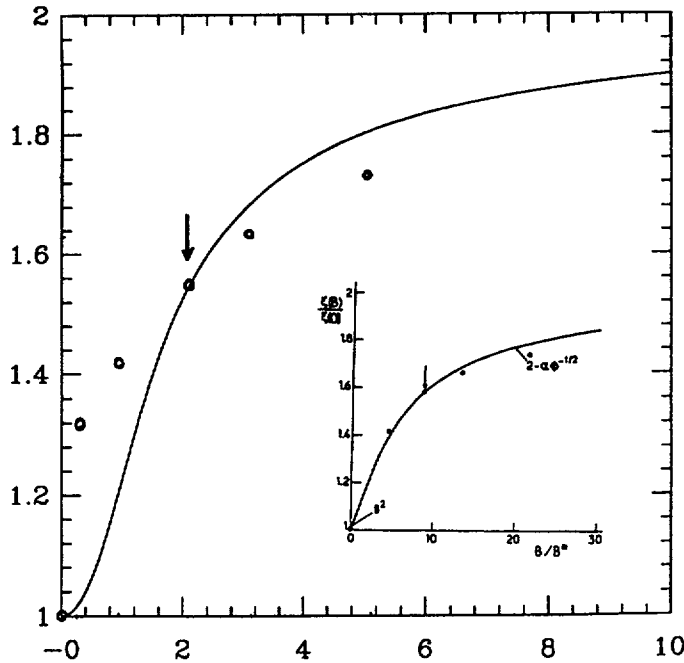


Fig. 2. — Plot of $\xi(B)/\xi(0)$ versus B/B^* in quasi one dimensional geometries (strips). Note that the limit $\xi(B^{**})\xi(0) = 2$ is slowly reached (as $1/B$), while the behaviour at low fields is B^2 . To compare with the numerical simulations of [11] (circles), we have arbitrarily imposed that the point indicated by an arrow should lie on the theoretical curve. The agreement is poor, which is probably related to the fact that ξ is not very large compared to W in reference [11]. The insert shows $\xi(B)/\xi(0)$ versus B/B^* using the two dimensional area distribution, equation (7). The fit of the numerical results is much better. This allows us to obtain from the data a value for the crossover field: $B^* \approx 4.6 \times 10^{-3}$ (in units were $B = 1$ corresponds to one quantum flux per unit cell), which qualitatively agrees with the theoretical prediction $B^* = 2(l_e/\xi)^2 \approx 4.8 \times 10^{-3}$ (using the numerically determined localisation length $\xi \approx 20.5$ lattice spacings [11], and assuming - since disorder is strong - that l_e is equal to the lattice spacing).

with $b = B/B^{**}$ and $B^{**} = d(\hbar/e)/l_e^2$. For $b = 0$, one finds:

$$\ln[\xi(0)/l_e] = A_2/2(l_e/\lambda) \tag{15}$$

and one recovers the fact that all states are localised in two dimensions with an exponentially large localisation length for small wavelengths, which again is the exact result. Equation (14) shows that the localisation length changes when $b\tau \approx 1$, i.e. at the crossover field $B^* = d(\hbar/e)/\xi^2 (\approx 1 \text{ T for } \xi = 30 \text{ nm})$, we find, for $B \ll B^*$ equation (12) with $1/6$ replacing $1/4$, while for $B^* \ll B \ll B^{**}$, one obtains

$$\xi(B)/l_e \approx (\tanh b/2)^{1/2} [\xi(0)/l_e]^2 \tag{16}$$

Thus we find that the effect of a magnetic field is quite dramatic in weakly disordered films; we predict in particular that the localisation length increases as the square root of the field in quite a large range of fields. This dependence is stronger than that observed experimentally on disordered films of $\text{In}_2\text{O}_{3-x}$ [19]. However our assumption that Brownian statistics holds for $l < \xi$ is probably incorrect if disorder is not weak: as shown in e.g. [20], the wave function becomes “fractal” in this regime, with a fractal dimension decreasing from 2 as the disorder increases [20]. It can be argued

that this fractal dimension plays the role of an effective dimensionality for the problem, and thus that the effect of a magnetic field becomes weaker, and similar to the one dimensional case for sufficiently strong disorder.

Finally, in three dimensions, one finds [13,14] a critical value of the wavelength (or the disorder) above which states are localised, given by the Ioffe-Regel condition: $\lambda^* \approx l_e$. For $\lambda > \lambda^*$, the localisation length is given by $\xi(0)/l_e = A_3 \lambda^2 / (\lambda^2 - \lambda^{*2})$, thus predicting a critical exponent $\nu=1$ in 3d (and more generally $\nu = (d-2)^{-1}$). This exponent is however not expected to be exact [21], since this argument completely discards the possibility of a non trivial (fractal) support of the wave function just at threshold.

Applying a strong magnetic field $B \approx B^{**}$ is found to change the localisation length according to:

$$\xi(B)/\xi(0) = (\lambda^2 - \lambda^{*2}) / (\lambda^2 - 2\lambda^{*2}) \quad (17)$$

which means that

- i) the first localised wavelength is multiplied by $\sqrt{2}$ in the presence of a magnetic field and
- ii) the resulting ratio of localisation lengths can be either very large if $\lambda \approx \sqrt{2}\lambda^*$ (mobility edge) or very close to 1 if $\lambda \gg \sqrt{2}\lambda^*$ (strong disorder). Note that for all $\lambda \geq \sqrt{2}\lambda^*$, one has $B^* \approx B^{**}$, since in this case $\xi(0) \approx l_e$.

For $B \ll B^*$, one obtains $\xi(B) - \xi(0) = (1/24) (B/B^*)^2 \xi(0)$. Experimentally [11], the ratio $\xi(B^*)/\xi(0)$ has been found equal to ≈ 2.5 for samples rather close to the mobility edge and to ≈ 1.7 for more strongly disordered samples, which qualitatively agrees with equation (16). It would be extremely interesting to observe the field-driven delocalisation transition predicted by equation (16).

As a conclusion, we have presented a very simple theory to understand the effect of a magnetic field on an Anderson insulator. The rôle of *closed loops* is crucial in our approach, at variance with theories focussing on "directed paths" only [22,25]. While our results corroborate those obtained through the random matrix theory in 1d, they quite strongly differ from it in higher dimension—at least when $\xi \gg l_e$ (It would be very interesting to find the precise relation between these two theories). We are also able to predict the full dependence of the localisation length on magnetic field—which could perhaps be obtained within the random matrix approach (see e.g. [23]). We find in particular that the localisation length should vary as B^2 for weak fields. We suggest that more experiments should be done, in particular in films where quite strong effects are expected (see Eq. (13)).

Finally, the effect of spin-orbit scattering (see [11]) or of an external electric field may be treated along similar lines. For the former problem, we recover [24] the results of [11] in the one dimensional case, that is $\xi(B)/\xi(0) = 1/2$ for strong magnetic fields and in the limit of strong spin orbit coupling, which contrasts with the recent results of [25].

In the latter case, one finds strong distortions of the localisation length when $E\xi \approx (\text{Bandwidth})$, where E is the electric field.

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