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### Short Communication

## Crossover scaling in surface growth with truncated power-law noise

Jacques G. Amar and Fereydoon Family

Department of Physics, Emory University, Atlanta, GA 30322, U.S.A.

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**Abstract.** — We demonstrate the existence of a simple scaling form which describes the crossover from anomalous to Gaussian exponents as a function of cutoff in the Zhang model of surface growth for  $\mu > 2$ , where  $\mu$  is the exponent characterising the power-law noise distribution. For  $\mu = 2$  we find a novel scaling form with logarithmic corrections. Our results for cutoff scaling at  $\mu = 2$  are supported by the scaling of the saturation growth velocity.

Zhang [1] has recently proposed a model of surface growth in which the noise  $\eta$  has a power-law distribution of the form,

$$P(\eta) \sim 1/\eta^{1+\mu} \quad (1)$$

for large  $\eta$ . From simulations of this model as well as of the related directed polymer model [1, 2] in  $d = 2$ , he obtained anomalously large values for the surface scaling exponents, which were found to depend on  $\mu$ . In addition, he suggested that the presence of power-law noise may explain the results of recent experiments on porous media [3-6] and bacterial growth [7] in  $d = 2$ , for which anomalously high values of the growth exponents have also been reported. Recently, we have carried out extensive simulations [8] of several different surface growth models with power-law noise which strongly support the existence of anomalous exponents. However, there still exists some controversy over whether the observed anomalous exponents represent a new universality class for the Kardar-Parisi-Zhang (KPZ) equation [9]. In order to resolve some of these questions we have conducted a study of the Zhang model [1] with truncated noise.

In our simulations we used the power-law noise distribution given by equation (1), with a cutoff  $\eta_c$  such that  $P(\eta) = 0$  for  $\eta > \eta_c$ . The noise  $\eta$  was determined at each odd (even) site at each odd (even) time step  $t$ , by generating an independent random number  $r$  at each site such that  $\eta_c^{-\mu} < r < 1$  and calculating the quantity  $\eta = r^{-1/\mu}$ . Simulations were performed for a range of values of the cutoff, with  $\eta_c^{-\mu}$  ranging from  $2^{-31}$  to  $10^{-3}$  in powers of 10 for each value of  $\mu$ . The saturation value of the surface width, as measured by the r.m.s. fluctuation of the interface height, was determined for each value of the cutoff  $\eta_c$ , for system sizes  $L = 16-1024$  in powers of 2. Averages were taken over times of the order of several million time steps.

The surface width  $w(L, t)$  on length scale  $L$  at time  $t$  is expected to satisfy the scaling form [10],  $w(L, t) = L^\alpha f(t/L^z)$ , where  $z = \alpha/\beta$  is the dynamic exponent,  $f(x) \sim x^\beta$  for  $x \ll 1$  and  $f(x) \rightarrow \text{const}$  for  $x \gg 1$ , and  $\alpha$  has the value of  $1/2$  [9] for Gaussian noise. In order to scale our results for the saturation width  $w(L, t = \infty, \eta_c)$  as a function of cutoff  $\eta_c$ , we propose the following crossover form for  $\mu > 2$ ,

$$w(L, \infty, \eta_c) \sim L^\alpha f(L^{-x_c} \eta_c) \quad (2)$$

where  $f(u) \sim u^{(\alpha-1/2)/x_c}$  for  $u \ll 1$  and  $f(u) \rightarrow \text{const}$  for  $u \gg 1$  and  $\alpha$  and  $x_c$  depend on  $\mu$ . This is equivalent to the scaling form,

$$w(L, \infty, \eta_c) \sim L^{1/2} \eta_c^{(\alpha-1/2)/x_c} g(L^{x_c} \eta_c^{-1}) \quad (3)$$

with  $g(u) \sim u^{(\alpha-1/2)/x_c}$  for  $u \ll 1$  and  $g(u) \rightarrow \text{const}$  for  $u \gg 1$ . Figure 1 shows scaling plots of the form of (2) for  $\mu = 3-5$ . For  $\alpha$  we have used the values ( $\alpha = 0.75, 0.62, 0.56$ ) obtained from a previous study [8], while the crossover exponents  $x_c$  were obtained from best fits ( $x_c \simeq 0.62, 0.375$  and  $0.2$  respectively). We find good scaling for all three values of  $\mu$ , except at small  $L$  for which the asymptotic region has not yet been reached. We note that with increasing  $\mu$  the crossover scaling exponent  $x_c$  decreases towards zero. However, the quantity  $(\alpha - 1/2)/x_c \simeq 0.33-0.4$  appears to remain approximately constant for  $\mu > 2$ .

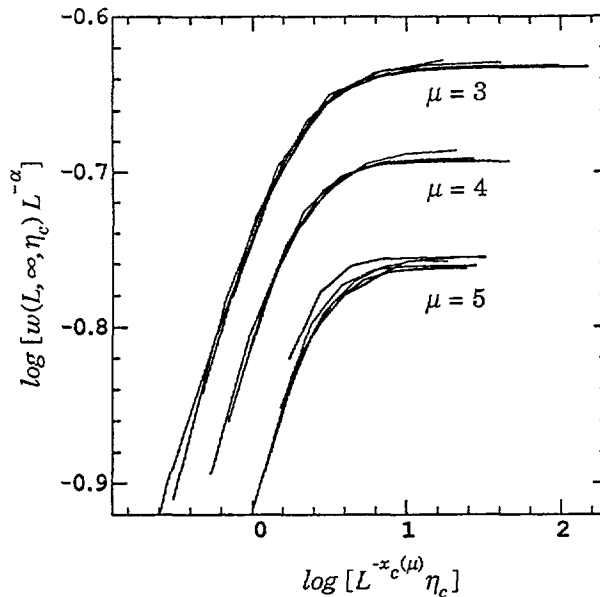


Fig. 1. — Scaling plots for  $\mu = 3$  ( $L = 32-1024$ ), 4 and 5 ( $L = 64-1024$ ). Highest curve for  $\mu = 5$  is for  $L = 64$ .

We now consider cutoff scaling for  $\mu = 2$ . This is the borderline for the existence of self-invariant noise, because for  $\mu = 2$  the second moment of the noise is logarithmically divergent. Because of this, the surface is fractal and  $\alpha$  has been assumed to be exactly 1 for  $\mu = 2$  [1, 11]. The inset of figure 2 shows the best scaling of the form (2) obtained with  $\alpha = 1$ , which occurred for  $x_c = 1$ . The scaling is relatively poor even though for  $\mu = 2$  the data for the saturation width

appears to show asymptotic behavior for very small  $L$  [1, 8]. This led us to consider an alternate form with logarithmic corrections at  $\mu = 2$ ,

$$w(L, \infty, \eta_c) \sim L(\ln L)^p F\left([L(\ln L)^{2p}]^{-x_c} \eta_c\right) \tag{4}$$

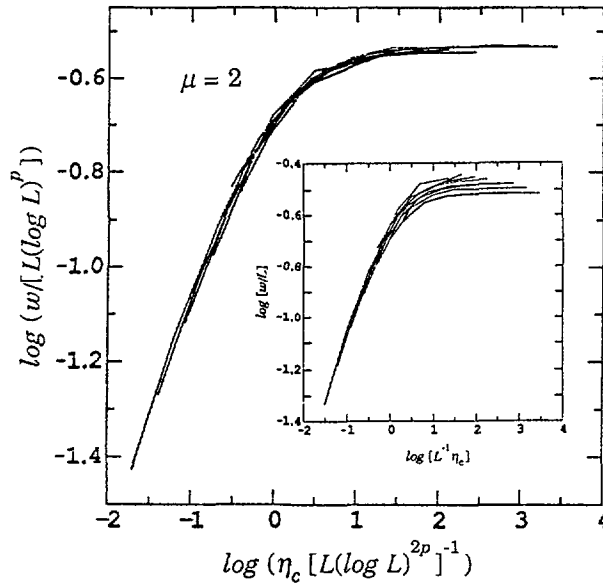


Fig. 2. — Scaling plot of form of equation (4) for  $\mu = 2$ , with  $p = 0.20$  and  $x_c = 1$ .  $L$  ranges from 16 to 1024 and  $\eta_c$  varies from 31.6 to 46340. Inset shows scaling plot of form of equation (3).

where  $F(u) \sim u^{1/2x_c}$  for  $u \ll 1$  and  $F(u) \rightarrow \text{const}$  for  $u \gg 1$ . Using this scaling form with  $x_c = 1$  and  $p \simeq 0.20-0.25$  we find very good scaling for the entire range of system sizes  $L$ . Thus, there appears to exist a logarithmic correction at  $\mu = 2$ . The existence of such a correction may explain why values of  $\alpha$  slightly higher than 1 ( $\alpha \simeq 1.04$ ) were obtained for several different growth models with power-law noise at  $\mu = 2$  in reference [8].

In order to obtain further support for our results, we also studied the scaling of the saturation velocity  $V(L)$  which is expected to have a finite-size correction of the form [12],

$$V(L) - V(\infty) \sim \langle \lambda |\nabla h|^2 \rangle \sim A(\lambda) L^{2\alpha-2} \tag{5}$$

where  $A(\lambda)$  is proportional to the nonlinearity parameter  $\lambda$  in the KPZ equation. Very good fits of the form of equation (5) were obtained for  $\mu = 3-5$ , using the values of  $\alpha$  previously obtained from the scaling of the interface width. For  $\mu = 2$ , however, the saturation velocity does not approach a constant value as expected for  $\alpha = 1$  and in fact appears to be diverging with  $L$ . We conjecture a logarithmic correction of the form  $V(L) \sim (\ln L)^{2p}$  Figure 3 shows a plot of  $\ln(V(L))$  versus  $\ln(\ln(L))$ . The slope of the straight-line fit is  $2p \simeq 0.58$ . This supports the existence of a logarithmic correction at  $\mu = 2$ , which is most likely due to the fact that  $\mu = 2$  is the critical value for the transition from non-invariant to self-invariant noise.

Our results for cutoff scaling have implications for experiments as well as for simulations of the Zhang model with large system sizes. The reason is that typically 32-bit or 46-bit integer

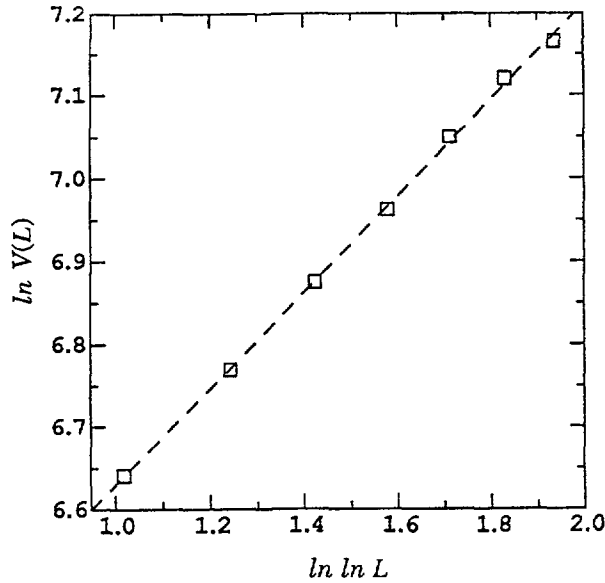


Fig. 3. — Plot of  $\ln V(L)$  versus  $\ln \ln L$  showing logarithmic divergence of  $V(L)$  at  $\mu = 2$ . Dashed line fit has slope  $2p = 0.58$ .

random number generators, for which the cutoff  $r_c$  is  $2^{-31}$  ( $4.65 \times 10^{-10}$ ) and  $2^{-45}$  ( $2.8 \times 10^{-14}$ ) respectively, are used. Equation (2) implies that the scaling region for which the surface width is not affected by the cutoff occurs for  $u = \eta_c/L^{x_c} = r_c^{-1/\mu}/L^{x_c} > u_c$  where  $u_c \gg 1$ . For a 32-bit random number generator, using the value  $u_c \simeq 40$  ( $\log(u_c) = 1.6$ , see Fig. 1), we find for  $\mu = 3, 4, 5$  that  $L_c = 257, 88, 21$  respectively. For a 46-bit random number generator for which  $r_c = 2.8 \times 10^{-14}$ , we obtain substantially larger values for  $L_c$  (of the order of  $10^4$ ). However, on the order of  $10^{14}$  random numbers must be generated to sample the full range of the random number generator, while a reasonable estimate of the number of random numbers accessible in current computer simulations is only of the order of  $10^9$ - $10^{11}$ . Thus, the effects of a cutoff should begin to show up for systems of order  $10^3$  even in large-scale simulations. Crossover to Gaussian exponents, however, ( $u < 1$ ) should only occur for much larger system sizes ( $10^5$  or larger).

### Summary.

We have demonstrated the existence of a simple scaling form which describes the crossover to Gaussian exponents as a function of cutoff in the Zhang model of surface growth for  $\mu > 2$ . This crossover form strongly supports the existence of anomalous exponents in this model. In addition, for  $\mu = 3, 4$  and  $5$  we have obtained approximate values for the crossover exponents  $x_c(\mu)$ . With increasing  $\mu$ ,  $x_c$  decreases towards zero while the quantity  $(\alpha - 1/2)/x_c$  remains approximately constant.

For  $\mu = 2$  we find a novel scaling form with logarithmic corrections. Our results for cutoff scaling at  $\mu = 2$  are further supported by the scaling of the saturation growth velocity  $V(L)$  which scales as  $(\ln L)^{2p}$  with  $p \simeq 0.29$ . For  $\mu > 2$ , our results for the finite-size correction to the growth velocity agree with previously obtained estimates for the saturation exponent  $\alpha$  as a function of  $\mu$ , thus verifying the validity of the nonlinear KPZ equation as a description of the Zhang model.

Due to the natural existence of cutoffs, our results may be useful in the analysis of experiments with power-law noise.

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