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Short Communication

Crossover scaling in surface growth with truncated power-law noise

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Abstract. — We demonstrate the existence of a simple scaling form which describes the crossover from anomalous to Gaussian exponents as a function of cutoff in the Zhang model of surface growth for $\mu > 2$, where μ is the exponent characterising the power-law noise distribution. For $\mu = 2$ we find a novel scaling form with logarithmic corrections. Our results for cutoff scaling at $\mu = 2$ are supported by the scaling of the saturation growth velocity.

Zhang [1] has recently proposed a model of surface growth in which the noise η has a power-law distribution of the form,

$$P(\eta) \sim 1/\eta^{1+\mu} \quad (1)$$

for large η . From simulations of this model as well as of the related directed polymer model [1, 2] in $d = 2$, he obtained anomalously large values for the surface scaling exponents, which were found to depend on μ . In addition, he suggested that the presence of power-law noise may explain the results of recent experiments on porous media [3-6] and bacterial growth [7] in $d = 2$, for which anomalously high values of the growth exponents have also been reported. Recently, we have carried out extensive simulations [8] of several different surface growth models with power-law noise which strongly support the existence of anomalous exponents. However, there still exists some controversy over whether the observed anomalous exponents represent a new universality class for the Kardar-Parisi-Zhang (KPZ) equation [9]. In order to resolve some of these questions we have conducted a study of the Zhang model [1] with truncated noise.

In our simulations we used the power-law noise distribution given by equation (1), with a cutoff η_c such that $P(\eta) = 0$ for $\eta > \eta_c$. The noise η was determined at each odd (even) site at each odd (even) time step t , by generating an independent random number r at each site such that $\eta_c^{-\mu} < r < 1$ and calculating the quantity $\eta = r^{-1/\mu}$. Simulations were performed for a range of values of the cutoff, with $\eta_c^{-\mu}$ ranging from 2^{-31} to 10^{-3} in powers of 10 for each value of μ . The saturation value of the surface width, as measured by the r.m.s. fluctuation of the interface height, was determined for each value of the cutoff η_c , for system sizes $L = 16-1024$ in powers of 2. Averages were taken over times of the order of several million time steps.

The surface width $w(L, t)$ on length scale L at time t is expected to satisfy the scaling form [10], $w(L, t) = L^\alpha f(t/L^z)$, where $z = \alpha/\beta$ is the dynamic exponent, $f(x) \sim x^\beta$ for $x \ll 1$ and $f(x) \rightarrow \text{const}$ for $x \gg 1$, and α has the value of $1/2$ [9] for Gaussian noise. In order to scale our results for the saturation width $w(L, t = \infty, \eta_c)$ as a function of cutoff η_c , we propose the following crossover form for $\mu > 2$,

$$w(L, \infty, \eta_c) \sim L^\alpha f(L^{-x_c} \eta_c) \quad (2)$$

where $f(u) \sim u^{(\alpha-1/2)/x_c}$ for $u \ll 1$ and $f(u) \rightarrow \text{const}$ for $u \gg 1$ and α and x_c depend on μ . This is equivalent to the scaling form,

$$w(L, \infty, \eta_c) \sim L^{1/2} \eta_c^{(\alpha-1/2)/x_c} g(L^{x_c} \eta_c^{-1}) \quad (3)$$

with $g(u) \sim u^{(\alpha-1/2)/x_c}$ for $u \ll 1$ and $g(u) \rightarrow \text{const}$ for $u \gg 1$. Figure 1 shows scaling plots of the form of (2) for $\mu = 3-5$. For α we have used the values ($\alpha = 0.75, 0.62, 0.56$) obtained from a previous study [8], while the crossover exponents x_c were obtained from best fits ($x_c \simeq 0.62, 0.375$ and 0.2 respectively). We find good scaling for all three values of μ , except at small L for which the asymptotic region has not yet been reached. We note that with increasing μ the crossover scaling exponent x_c decreases towards zero. However, the quantity $(\alpha - 1/2)/x_c \simeq 0.33-0.4$ appears to remain approximately constant for $\mu > 2$.

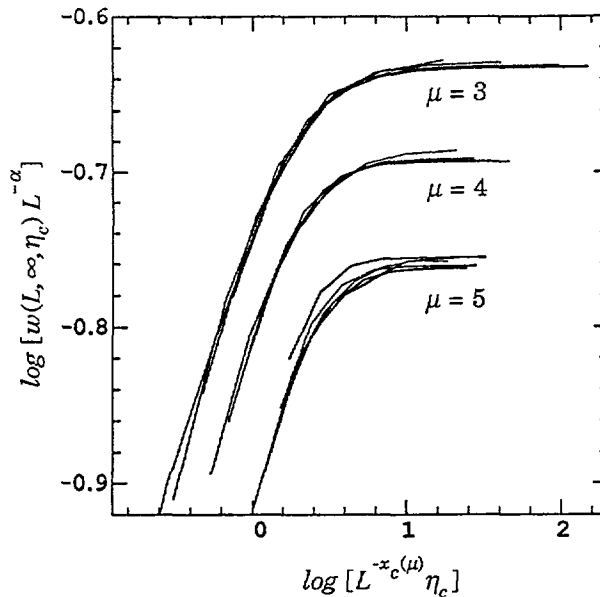


Fig. 1. — Scaling plots for $\mu = 3$ ($L = 32-1024$), 4 and 5 ($L = 64-1024$). Highest curve for $\mu = 5$ is for $L = 64$.

We now consider cutoff scaling for $\mu = 2$. This is the borderline for the existence of self-invariant noise, because for $\mu = 2$ the second moment of the noise is logarithmically divergent. Because of this, the surface is fractal and α has been assumed to be exactly 1 for $\mu = 2$ [1, 11]. The inset of figure 2 shows the best scaling of the form (2) obtained with $\alpha = 1$, which occurred for $x_c = 1$. The scaling is relatively poor even though for $\mu = 2$ the data for the saturation width

appears to show asymptotic behavior for very small L [1, 8]. This led us to consider an alternate form with logarithmic corrections at $\mu = 2$,

$$w(L, \infty, \eta_c) \sim L(\ln L)^p F\left([L(\ln L)^{2p}]^{-x_c} \eta_c\right) \tag{4}$$

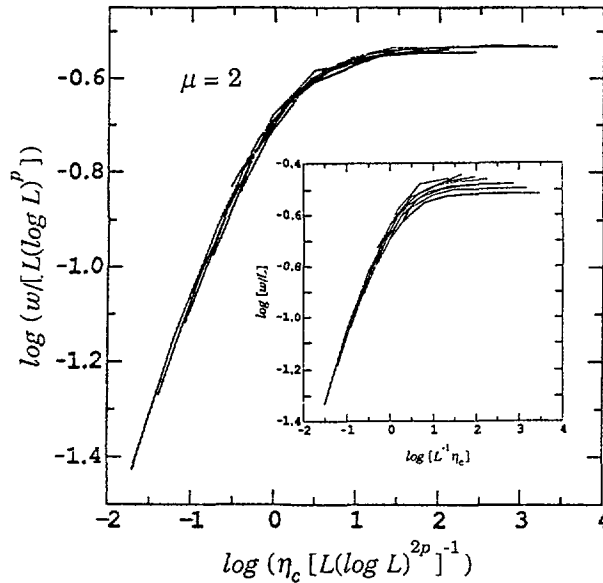


Fig. 2. — Scaling plot of form of equation (4) for $\mu = 2$, with $p = 0.20$ and $x_c = 1$. L ranges from 16 to 1024 and η_c varies from 31.6 to 46340. Inset shows scaling plot of form of equation (3).

where $F(u) \sim u^{1/2x_c}$ for $u \ll 1$ and $F(u) \rightarrow \text{const}$ for $u \gg 1$. Using this scaling form with $x_c = 1$ and $p \simeq 0.20-0.25$ we find very good scaling for the entire range of system sizes L . Thus, there appears to exist a logarithmic correction at $\mu = 2$. The existence of such a correction may explain why values of α slightly higher than 1 ($\alpha \simeq 1.04$) were obtained for several different growth models with power-law noise at $\mu = 2$ in reference [8].

In order to obtain further support for our results, we also studied the scaling of the saturation velocity $V(L)$ which is expected to have a finite-size correction of the form [12],

$$V(L) - V(\infty) \sim \langle \lambda |\nabla h|^2 \rangle \sim A(\lambda) L^{2\alpha-2} \tag{5}$$

where $A(\lambda)$ is proportional to the nonlinearity parameter λ in the KPZ equation. Very good fits of the form of equation (5) were obtained for $\mu = 3-5$, using the values of α previously obtained from the scaling of the interface width. For $\mu = 2$, however, the saturation velocity does not approach a constant value as expected for $\alpha = 1$ and in fact appears to be diverging with L . We conjecture a logarithmic correction of the form $V(L) \sim (\ln L)^{2p}$. Figure 3 shows a plot of $\ln(V(L))$ versus $\ln(\ln(L))$. The slope of the straight-line fit is $2p \simeq 0.58$. This supports the existence of a logarithmic correction at $\mu = 2$, which is most likely due to the fact that $\mu = 2$ is the critical value for the transition from non-invariant to self-invariant noise.

Our results for cutoff scaling have implications for experiments as well as for simulations of the Zhang model with large system sizes. The reason is that typically 32-bit or 46-bit integer

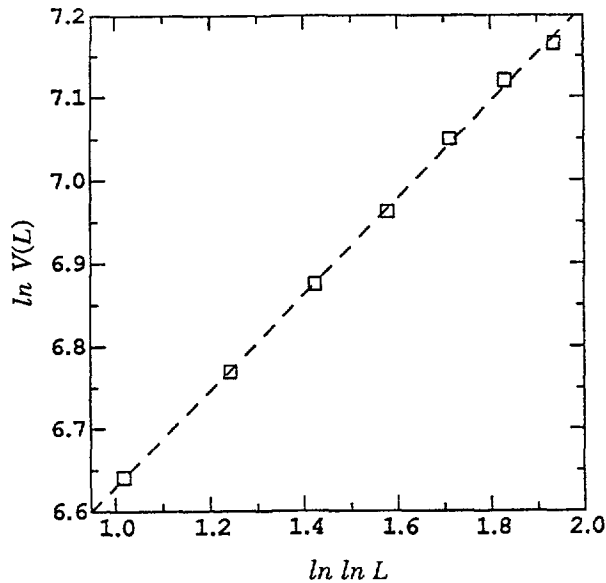


Fig. 3. — Plot of $\ln V(L)$ versus $\ln \ln L$ showing logarithmic divergence of $V(L)$ at $\mu = 2$. Dashed line fit has slope $2p = 0.58$.

random number generators, for which the cutoff r_c is 2^{-31} (4.65×10^{-10}) and 2^{-45} (2.8×10^{-14}) respectively, are used. Equation (2) implies that the scaling region for which the surface width is not affected by the cutoff occurs for $u = \eta_c/L^{x_c} = r_c^{-1/\mu}/L^{x_c} > u_c$ where $u_c \gg 1$. For a 32-bit random number generator, using the value $u_c \simeq 40$ ($\log(u_c) = 1.6$, see Fig. 1), we find for $\mu = 3, 4, 5$ that $L_c = 257, 88, 21$ respectively. For a 46-bit random number generator for which $r_c = 2.8 \times 10^{-14}$, we obtain substantially larger values for L_c (of the order of 10^4). However, on the order of 10^{14} random numbers must be generated to sample the full range of the random number generator, while a reasonable estimate of the number of random numbers accessible in current computer simulations is only of the order of 10^9 - 10^{11} . Thus, the effects of a cutoff should begin to show up for systems of order 10^3 even in large-scale simulations. Crossover to Gaussian exponents, however, ($u < 1$) should only occur for much larger system sizes (10^5 or larger).

Summary.

We have demonstrated the existence of a simple scaling form which describes the crossover to Gaussian exponents as a function of cutoff in the Zhang model of surface growth for $\mu > 2$. This crossover form strongly supports the existence of anomalous exponents in this model. In addition, for $\mu = 3, 4$ and 5 we have obtained approximate values for the crossover exponents $x_c(\mu)$. With increasing μ , x_c decreases towards zero while the quantity $(\alpha - 1/2)/x_c$ remains approximately constant.

For $\mu = 2$ we find a novel scaling form with logarithmic corrections. Our results for cutoff scaling at $\mu = 2$ are further supported by the scaling of the saturation growth velocity $V(L)$ which scales as $(\ln L)^{2p}$ with $p \simeq 0.29$. For $\mu > 2$, our results for the finite-size correction to the growth velocity agree with previously obtained estimates for the saturation exponent α as a function of μ , thus verifying the validity of the nonlinear KPZ equation as a description of the Zhang model.

Due to the natural existence of cutoffs, our results may be useful in the analysis of experiments with power-law noise.

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