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MODULATED INDUCTANCE MAGNETOMETERS ⁽¹⁾

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Abstract. — The principles of operation of a modulated inductance magnetometer and the problems that have been encountered in an attempt to realize the expected sensitivity are discussed. A magnetometer for which inductance modulation is effected by a superconducting plane vibrating near a long inductance element is described. The suitability of this magnetometer to the measurement of small magnetic fields is considered in conjunction with the most recent experimental results.

For superconductors in a state where a complete Meissner effect is exhibited the fluxoid [1] :

$$\Phi + \mu_0 \oint \lambda^2 \mathbf{J} \cdot d\mathbf{l}$$

remains essentially constant in time; in this expression the line integral is taken around a path in the superconducting loop; \mathbf{J} is the current density, λ , the London penetration depth and Φ , the magnetic flux threading the loop. When the path of integration can be chosen to lie several penetration depths inside the superconductor, the fluxoid is nearly equal to the magnetic flux. In a modulated inductance magnetometer it is this conservation that is used to measure the magnetic flux externally applied to a superconducting loop. In this paper the principles of operation of such a modulated inductance magnetometer are described, and theoretical and practical limits to the sensitivity are discussed. A method to realize inductance modulation is then considered. Following this is a brief description of the results of the preliminary experiments and of the useful properties expected of such a magnetometer.

Figure 1 shows some of the important features of the magnetometer.

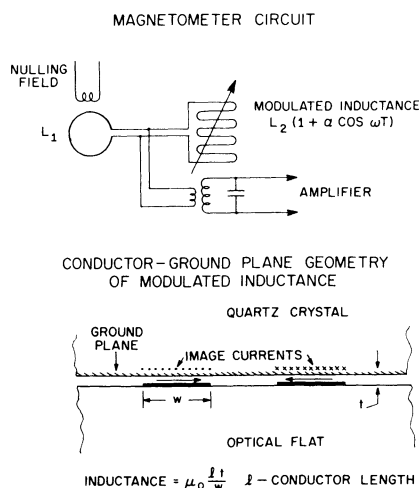


FIG. 1.

The magnetometer circuit is shown in figure 1. To understand the principle of operation consider the elementary magnetometer circuit first proposed by Deaver and Fairbank [2]. This consists only of the pickup loop with inductance L_1 and the modulator with inductance L_2 that together make up a single persistent superconducting loop. In general, a current i flows around the loop and the condition for conservation of flux is given by :

$$\Phi_{\text{ext}} - (L_1 + L_2) i = \Phi_i \quad (1)$$

where Φ_{ext} is the externally applied flux. The constant Φ_i is determined by the conditions that exist when the current loop becomes superconducting and is equal to the flux applied externally at that time if no currents are generated as the transition to the superconducting state occurs. One immediately has the following expression for the current :

$$i = (\Phi_{\text{ext}} - \Phi_i) / (L_1 + L_2). \quad (2)$$

It is seen that a time variation of the inductance L_2 is accompanied by a time variation of the current. This is observable as the voltage across L_1 equal in value to $L_1 di/dt$ and directly proportional to :

$$\Phi_{\text{ext}} - \Phi_i = \Delta\Phi.$$

If L_2 varies periodically in time as $L_2^0(1 + \alpha \cos \omega t)$, then the current i may be written, to first order in the parameter α , as :

$$i = \frac{\Delta\Phi}{L_1} \frac{\alpha x}{(1+x)^2} \cos \omega t \quad (3)$$

where x is defined as the ratio L_2^0/L_1 . The corresponding voltage e across L_1 is then given by :

$$e = \omega \alpha x (1+x)^{-2} \Delta\Phi \sin \omega t. \quad (4)$$

Now if a linear circuit element is connected across the terminals of L_1 , the resulting voltages and currents can be calculated approximately (to first order in α) by replacing the elementary magnetometer circuit with its Thevenin equivalent circuit. The voltage generator for this circuit has an output given by equation (4) and the output impedance is the parallel combination of L_1 and L_2 whose value L_p is :

$$L_p = L_1 x / (1+x). \quad (5)$$

Now because the output impedance is reactive, the output voltage can be increased by tuning the circuit

⁽¹⁾ This research was supported in part by the U.S. Office of Naval Research.

to resonance with a capacitance C across the terminals. The transformer shown in figure 1, sometimes required to transform impedances, is not necessary if the capacitance C is large enough for tuning. In terms of the combined Q of the circuit and capacitor, the power output into a matched load $R_L = \omega L_p Q$ is given by :

$$P = \frac{e_{\max}^2 Q^2}{8R_L} = \frac{1}{8} \frac{\omega \alpha^2 x Q}{L_1(1+x)^3} (\Delta\Phi)^2. \quad (6)$$

The power output is maximum when $x = 1/2$; its value P_{\max} is :

$$P_{\max} = \frac{\omega \alpha^2 Q}{54L_1} (\Delta Q)^2. \quad (7)$$

It is useful here to calculate the level of flux change that corresponds to the situation where signal power is equal to thermal noise power. This level can be taken as the theoretical limit to the sensitivity obtainable with this kind of magnetometer. Equating P_{\max} with the thermal noise power $k_B T_N \Delta\nu$ gives the result for $(\Delta\Phi)_{\min}$:

$$(\Delta\Phi)_{\min} = \left(\frac{54L_1 k_B T_N \Delta\nu}{\omega \alpha^2 Q} \right)^{1/2} \quad (8)$$

where T_N is the noise temperature, $\Delta\nu$ the noise bandwidth, and k_B Boltzmann's constant. For $L_1 = 1.2 \mu\text{H}$; $Q = 2.5 \times 10^4$; $\alpha = 0.1$; $\Delta\nu = 1 \text{ Hz}$; $\omega = 2 \times 10^5 \text{ rad/s}$; and $T_N = 4 \text{ }^\circ\text{K}$:

$$(\Delta\Phi)_{\min} = 6 \times 10^{-10} \text{ G.cm}^2. \quad (9)$$

The field change corresponding to this $(\Delta\Phi)_{\min}$ is $4 \times 10^{-11} \text{ Oe}$ for a pickup coil with three turns and an area of 5 cm^2 . The parameters used to calculate this sensitivity are realizable, for the most part. The value of $Q = 2.5 \times 10^4$ is determined by the 6 k Ω optimum source impedance of a Brookdeal 431 amplifier which has an equivalent noise source resistance of 40 ohms. How a value of $\alpha = 0.1$ is achieved in modulating, a 0.6 μH inductance at 100 kHz will be described.

Here it must be observed that no modulated inductance magnetometer has been operated with sensitivity approaching the theoretical limit just calculated. Two important factors not yet considered have limited the sensitivity up to this time. The first is non-periodicity in the value of the modulated inductance. This has been a problem particularly associated with magnetometers in which inductance is modulated by causing the core of an inductor alternately to become superconducting and normal by periodic heating and cooling. The essence of the problem, as discussed by Pierce [3], is that the transition time of the core does not have a definite period even though the heating current is periodic. Also, the state of the core is not always the same in each period of the heating current. The second factor limiting the magnetometer sensitivity has been direct pickup of signals at frequency ω . In all of the proposed designs the inductance modulation is accomplished by driving some device such as a heater or piezoelectric crystal at frequency ω or a subharmonic. Excessive leakage of the drive signal into the region of the pickup loop makes very sensitive measurements impossible if the leakage is at all unstable. To eliminate the problem of non-periodicity, a method of inductance modulation that does not involve a transition between the superconducting and

normal state can be used. It uses a vibrating superconducting plane to modulate the inductance of a nearby conductor, and it is the method with which this paper is primarily concerned. The technique used to minimize the direct pickup will become clearer in the course of the discussion.

The principle employed in modulating an inductance with a moving superconducting plane is indicated in figure 1. Two sections of a thin film conductor of width w carrying current in opposite directions are represented. At a distance t from the conductors is a superconducting plane that confines most of the magnetic field to the space between the conductors and the plane. It is assumed that the field in the superconductor vanishes; the field below the plane is approximately that arising from the actual currents and the image currents shown. The resulting approximate value of the inductance L for a length of conductor l can be written as :

$$L/l = \mu_0/tw. \quad (10)$$

More details concerning this calculation are given by Newhouse [4]. A sinusoidal variation of the spacing gives the desired sinusoidal time variation of the inductance L . For practical conductor cross-sections and spacings, two additional contributions to the inductance are observable. A non-vanishing penetration depth in the superconductor results in a somewhat larger inductance because the field penetrates into the superconducting plane. Related to this is a finite contribution to the inductance arising from the kinetic energy of the superconducting electrons. For typical dimensions $t = 1 \mu$, $w = 10 \mu$, and a conductor thickness of 0.15μ , these effects contribute about 10 % of the total inductance. It remains true that a sinusoidal variation of the spacing t gives a nearly sinusoidal variation of the inductance L .

Two things are required to implement this idea of modulation : first, a transducer to provide motion of the superconducting plane, and second, a thin film inductance element with sufficient inductance to allow tuning at the frequency of operation. The transducer is shown in figures 2 and 3, which also show other parts of an inductance modulator. The transducer is a piezoelectric quartz crystal resonant at 100 kHz. It is shown in figure 3 as it appears before a superconducting film is deposited onto it. In the assembly drawing of the modulator in figure 2 the crystal is represented in cross-section. The quartz resonator is essentially a 3/8 inch diameter y-cut rod connected to a large clamping ring by a membrane broadly resonant at the same frequency as the center rod. The composite structure, cut from a single quartz crystal, has a well defined resonant frequency very nearly equal to that of the center rod alone. The bottom surface of the crystal is optically flat and includes an area of maximum motion in the center. Motion at the outer edge is negligible. The crystal, driven with electrodes shown in figure 3, exhibits a mechanical Q between 10^3 and 10^4 when the outer ring is clamped as indicated in figure 2. The crystal can provide up to 1μ of motion of the active area relative to the fixed clamp. For a more typical motion amplitude of 0.1μ , the power dissipation is $3 \text{ W}/Q$ or 1 mW if the mechanical Q is 3×10^3 . Two features of the transducer are important. First the design facilitates locating an inductance element

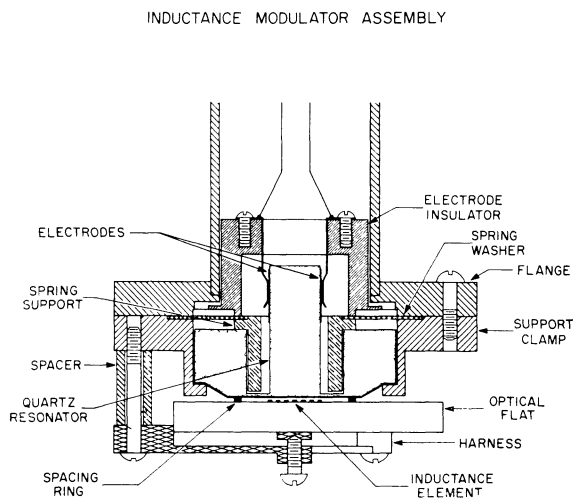


FIG. 2.

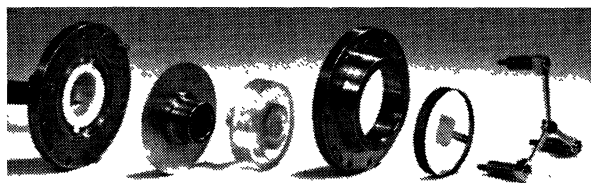


FIG. 3.

a fixed distance from the average position of the vibrating superconducting ground plane that is deposited on the bottom surface. Because the outer ring does not move, an inductance element on an optical flat can be clamped against it. A spacing ring between the optical flat and the clamping ring provides the desired average separation as indicated in figure 2. The second important feature is the possibility of shielding against direct pickup of the driving signal. Shielding is provided in the following manner: The superconducting film deposited onto the quartz crystal is extended to the stationary clamping ring where it makes contact with the support clamp. The support clamp is in turn connected to a shield surrounding the conductors carrying current to the driving electrodes. In this manner the driving oscillator, the electrodes and the intermediate conductors are entirely enclosed in a shield which can be superconducting if necessary. This geometry makes possible shielding that is limited only by the number of penetration depths in the shield itself.

The inductance element shown in figure 3 is similar in appearance to the actual inductance that is used. The element consists of a 5 meter long niobium thin film conductor on an optical flat two inches in diameter. It is made up of 500 lines, whose width and spacing are each 10μ , contained in a one centimeter square area at the center of the optical flat. Terminal pads extend to the outer edge of the optical flat; they are coated with a thin lead film to allow soldering of lead wires to the niobium. A 1μ thick copper spacing ring, not shown in figure 3, partially surrounds the inductance element. With this 1μ spacing the inductance calculated from eq. (10) is $0.6 \mu\text{H}$, with a corresponding value of $\alpha = 0.1$, for 0.1μ motion of the superconducting plane. The capacitance needed to resonate with this inductance in parallel with a

$1.2 \mu\text{H}$ pickup coil is $6 \mu\text{F}$ — a capacitance that can be made up with superconducting lead foil and mylar insulation.

Fabrication of the inductance element is a difficult and somewhat involved process that can be described only briefly here. The micro-circuit technique involves sputtering two layers of niobium onto an optically flat fused quartz substrate and subsequent *rf* sputter-etching of the portion of the film not protected by a photo-resist pattern. The photo-resist pattern is made in the usual way by exposing a resist coating to ultraviolet light through a photographic mask and then developing the pattern with suitable solvents. The deposition of two layers of niobium is required to obtain a film that is free of unwanted pinholes. Sputter-etching of the first layer prior to deposition of the second prevents formation of an oxide barrier between the two layers. Sputter-etching also provides a suitably controlled etch rate for removal of the niobium film. In the fabrication process a good deal of attention must be paid to the quality of the polished surface on the quartz substrate in order to avoid breaks in the inductance element. Also, it is difficult to obtain a defect-free photographic mask to use in the photo-etching process. However, the difficulties encountered have been circumvented and inductance elements have been fabricated with no bridges across parallel lines and with a critical current in excess of 10 mA.

At the time of the writing of this paper a magnetometer, incorporating the inductance modulator just described, has been assembled. In the first experimental tests the connection between the pickup loop and the modulator was not persistent, and as a consequence the unit could not be tested as a magnetometer. (Other tests have shown that such a connection can be made to carry persistent currents as large as one ampere.) It was possible, however, to observe the level of direct pickup which was measured to be 200 nV at the input terminals of the amplifier with a normal driving voltage applied to the quartz crystal. In this apparatus the superconducting part of the shielding did not go beyond the ground plane itself. For purposes of comparison, 200 nV is the expected signal level from the magnetometer for $\alpha = 0.1$ and $Q = 10^4$ when ΔQ is one quantum of flux. It remains possible to extend the superconducting shielding and to buck out the stable portion of the residual pickup. If this residual pickup is then stable to ± 1 nV for a period of one second, it imposes no greater limit on the sensitivity than does thermal noise as calculated for eq. (9).

Although the present magnetometer has not yet functioned properly, applicability of the principles involved has been demonstrated with the aid of another apparatus. This apparatus is represented schematically in figures 4 and 5. In figure 4 a magnetometer circuit with an inductance element and a pickup loop is represented. Conductor line widths and spacings are again 10μ . Capacitor plates are included to provide a means of measuring the separation of the circuit from the superconducting ground plane. Figure 5 represents the apparatus designed to position a ground plane above the circuit. Here the niobium plane is deposited onto the end of the quartz crystal; it is continued to make contact with the crystal holder to provide sufficient shielding to allow measurement of voltages generated across the pickup loop. Three differential screws adjust the spacing and parallelism

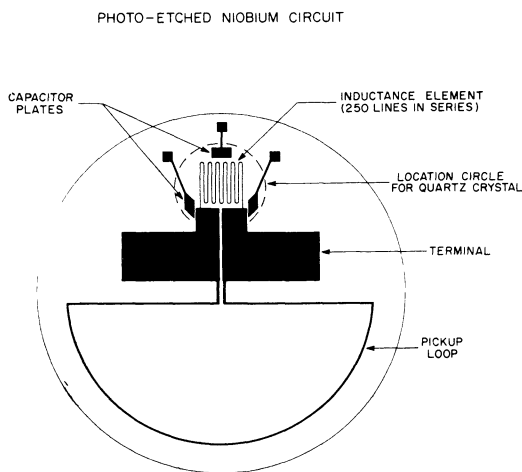


FIG. 4.

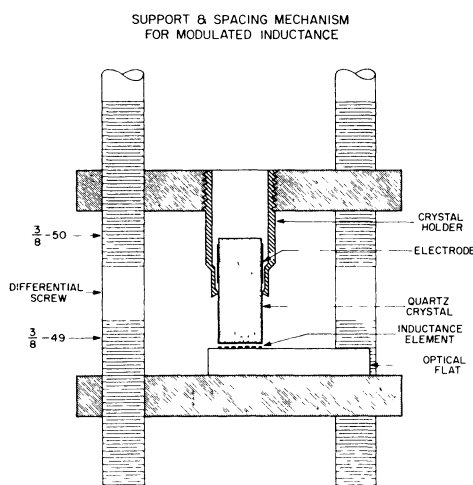


FIG. 5.

of the ground plane relative to the inductance element. With this apparatus a spacing of 2μ between the crystal and the optical flat was obtained. It was possible to drive the crystal to attain an inductance modulation corresponding to $\alpha = 0.1$. The expected voltage (eq. (4)) was observed when a relatively large flux ($0.1 \text{ G}\cdot\text{cm}^2$) was applied to the loop; this indicated that the moving ground plane did indeed shuttle flux

back and forth between the pickup loop and inductance element. Persistent currents as large as 100 mA were observed. In short, all of the principles of operation were shown to be valid at relatively high levels of magnetic flux.

The magnetometer described in the preceding paragraphs should have several useful properties. If the theoretical limit of sensitivity is attained, then the magnetometer will combine good sensitivity with a large dynamic range of operation. When operated at maximum sensitivity, the device requires a fast transient flux of $0.2 \text{ G}\cdot\text{cm}^2$ to cause a critical current of 100 mA to be exceeded. A transient persisting for many cycles of the driving frequency has the same effect at an amplitude $\alpha Q \sim 10^3$ times lower. Here αQ is the ratio of ac loop current to dc current in steady state operation. Even if the magnetometer is used as a null-indicating device, the large dynamic range aids in the design of feedback circuitry. Another feature is the potential stability that may be realized if direct pickup is reduced to a small stable value. All of the superconducting parts of the device operate in a pure superconducting state where any trapped flux remains constant in time and it may be expected that signals arising from the trapped flux remain constant in time also. Finally it should be possible to make such a magnetometer operate at the highest temperatures at which films can be made to superconduct, although the sensitivity will be reduced somewhat.

The preceding has been a description of a modulated inductance magnetometer that is designed to eliminate the problems encountered in previous models of this type of magnetometer. Preliminary measurements of direct pickup indicate that flux changes as small as one flux quantum in a $1.2 \mu\text{H}$ pickup loop should be measurable, and substantial improvements are still possible. A complete evaluation of relative advantages and disadvantages of this magnetometer require further testing.

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