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## 4 — $d$ expansion for Anderson localization in a strong spin-orbit coupling case

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**Résumé.** — Le problème de la localisation d'Anderson en présence de fort couplage spin-orbite est étudié dans le développement en  $\varepsilon = 4 - d$  d'un modèle matriciel invariant de jauge. Un point fixe stable dans l'infrarouge est présent et ce modèle appartient à la même classe d'universalité que le modèle  $\sigma$  non linéaire. Une expression pour l'exposant de la conductivité est obtenue au premier ordre en  $\varepsilon$ .

**Abstract.** — The problem of Anderson localization in a strong spin-orbit coupling case is studied by using a gauge invariant matrix model in  $\varepsilon = 4 - d$  expansion. An infrared stable fixed point is present and this model belongs to the same universal class as a nonlinear  $\sigma$  model. An expression for the critical exponent of conductivity is obtained in the first order of the  $\varepsilon$ -expansion.

Recently Anderson localization has been investigated in  $2 + \varepsilon$  dimensions for three cases; the orthogonal, unitary, and symplectic cases [1-4]. The orthogonal case corresponds to the case with no spin-effect. Strong spin-spin interaction or strong magnetic field case belongs to the unitary case. The symplectic case is obtained by a strong spin-orbit interaction [5]. By an  $\varepsilon = d - 2$  expansion, the critical point for the metal-insulator transition is obtained for the orthogonal case and for the unitary case. However, for the symplectic case, the critical behaviour has not been discussed since, up to four-loop order, the renormalization group  $\beta$ -function is positive [3].

The Effective Hamiltonian for these three cases are non-compact nonlinear  $\sigma$ -models. As for usual critical phenomena, it is useful to consider also the  $\varepsilon = 4 - d$  expansion for the strong spin-orbit case based on a gauge invariant matrix model. The field-theoretical model which we consider in this letter near four dimensions has already been discussed as a model which belongs to the same universal class as the Grassmannian nonlinear  $\sigma$  model and indeed within the  $1/N$  expansion, the equivalence of the two models has been verified [6, 7]. The expression of the exponent  $\nu$  of the nonlinear  $\sigma$  model agrees with the expression of the gauge invariant matrix model in an

overlapping large  $N$  and small  $\varepsilon = 4 - d$  region. The Hamiltonian of the gauge invariant matrix model is given by

$$\begin{aligned}
 H = & \frac{1}{2} [\partial_\mu \Phi_i^\alpha + g A_{\mu ij} \Phi_j^\alpha]^2 + \frac{\lambda}{8} \sum_{i,j=1}^p \sum_{\alpha=1}^N \Phi_i^\alpha \Phi_i^\alpha \Phi_j^\alpha \Phi_j^\alpha \\
 & + \frac{v}{4} \sum_{i,j=1}^p \sum_{\alpha \neq \beta} \Phi_i^\alpha \Phi_j^\alpha \Phi_j^\beta \Phi_i^\beta + \frac{\rho}{8} \sum_{i,j=1}^p \sum_{\alpha \neq \beta} \Phi_i^\alpha \Phi_i^\alpha \Phi_j^\beta \Phi_j^\beta \\
 & + \frac{1}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g C_{abc} A_\mu^b A_\nu^c)^2
 \end{aligned} \quad (1)$$

where  $\Phi_i^\alpha$  ( $i = 1, \dots, p$ ;  $\alpha = 1, \dots, N$ ) is  $p \times N$  real matrix field, and  $A_\mu$  is a non-Abelian gauge field belonging to  $O(p)$ . We have considered a model in which  $\lambda = \rho + 2v$  where a global  $O(N)$  symmetry is present. If this model has an infrared stable fixed point, where bare coupling constants become infinite, the model is reduced to the  $O(N)/O(p) \times O(N - p)$  Grassmannian nonlinear  $\sigma$  model which has the following Hamiltonian [8]

$$H = \frac{1}{2t} \{ [\partial_\mu \Phi_i^\alpha][\partial_\mu \Phi_i^\alpha] + [\Phi_i^\alpha \partial_\mu \Phi_j^\alpha][\Phi_j^\beta \partial_\mu \Phi_i^\beta] \} \quad (2)$$

with

$$\sum_{\alpha=1}^N \Phi_i^\alpha \Phi_j^\alpha = \delta_{ij}.$$

The renormalization group  $\beta$ -functions of (1) are ( $\varepsilon = 4 - d$ )

$$\begin{aligned}
 \beta_\lambda = & -\varepsilon \lambda + \frac{1}{16 \pi^2} \times \\
 & \times \left\{ (pN + 8) \lambda^2 - 4(N - 1)(p - 1) \lambda v + 4(p - 1)(N - 1) v^2 - 3(p - 1) g^2 \lambda + \frac{3}{4} (p - 1) g^4 \right\} \\
 \beta_v = & -\varepsilon v + \frac{1}{16 \pi^2} \left\{ (2N + 2p - 16) v^2 + 12 \lambda v - 3(p - 1) g^2 v + \frac{3}{8} (p - 2) g^4 \right\} \\
 \beta_g = & -\frac{\varepsilon}{2} g - \frac{1}{16 \pi^2} \left\{ \frac{11}{3} (p - 2) - \frac{N}{12} \right\} g^3.
 \end{aligned} \quad (3)$$

These  $\beta$ -functions have infrared stable fixed points for certain domains of the number of components  $p$  and  $N$ . In the large  $N$  limit for a fixed  $p$ , we have

$$\begin{aligned}
 g_c^2 = & 96 \pi^2 \varepsilon / N \\
 \lambda_c = & \frac{16 \pi^2 \varepsilon}{N} \left( 1 - \frac{10p}{N} + \frac{2}{N} \right) + \mathcal{O}\left(\frac{\varepsilon}{N^3}\right) \\
 v_c = & \frac{8 \pi^2 \varepsilon}{N} \left( 1 - \frac{10p}{N} + \frac{32}{N} \right) + \mathcal{O}\left(\frac{\varepsilon}{N^3}\right).
 \end{aligned} \quad (4)$$

This fixed point is stable for all directions and the critical exponent  $\nu$  agrees with the value of the nonlinear  $\sigma$  model in the large  $N$  and small  $\varepsilon$  limit. This fact shows that Hamiltonian (1) belongs to the same universal class as Hamiltonian (2).

The Anderson localizations are described by a non-compact nonlinear  $\sigma$  model. For the orthogonal case, the nonlinear  $\sigma$  model has  $O(p, N - p)/O(p) \times O(N - p)$  symmetry with  $p = N = 0$ . For the symplectic case, the symmetry becomes  $S_p(p, N - p)/S_p(p) \times S_p(N - p)$  with  $p = N = 0$ . There is an isomorphic relation between the orthogonal group and the symplectic group in the following

$$O(-2N) \simeq S_p(N) \tag{5}$$

and the non-compact case  $\beta$ -function is obtained from the compact case by changing a sign of the coupling [3, 9, 10] constant. Therefore, the strong spin-orbit case is described by the compact  $O(N)/O(p) \times O(N - p)$  nonlinear  $\sigma$  model with  $p = N = 0$ . If the Hamiltonian of (1) has an infrared stable fixed point for  $p = N = 0$ , then the critical behaviour represents the localization transition with a strong spin-orbit case. For  $p = N = 0$ , we have four different points. Two of them are reported here for the first time. They are  $(\tilde{\lambda}, \tilde{\nu}) = (0.10814, 0.01037), (0.114, 0.028), (-0.005, -0.040), (-0.0042, -0.0045)$  where  $\tilde{\lambda} = \lambda/16\pi^2 \varepsilon$  and  $\tilde{\nu} = \nu/16\pi^2 \varepsilon$ . The fixed point of the gauge coupling becomes  $g_c^2 = 48 \varepsilon\pi^2/44$ . By calculating the derivatives of  $\beta$ -functions, we find that the first fixed point is stable for all directions and the critical exponent  $\nu$  is obtained for general  $p$  and  $N$ .

$$\begin{aligned} \frac{1}{\nu} &= 2 + \eta_{\Phi^2} - \eta_{\Phi} \\ \eta_{\Phi} &= \beta_g \frac{\partial}{\partial g} \ln Z_{\Phi} = \frac{g(p-1)}{44(p-2) - N} \varepsilon \\ \eta_{\Phi^2} &= -\frac{pN+2}{16\pi^2} \lambda_c + \frac{(p-1)(N-1)}{8\pi^2} v_c \end{aligned} \tag{6}$$

For  $p = N = 0$ , we have

$$\frac{1}{\nu} = 2 - 0.297 \varepsilon + \mathcal{O}(\varepsilon^2) \tag{7}$$

or

$$\nu = \frac{1}{2} + 0.0742 \varepsilon + \mathcal{O}(\varepsilon^2).$$

Stable fixed points are limited to some domains in the  $(p, N)$  plane. For the  $p = 0$  case, we have two regions of stability (i)  $8 < N < \infty$  and (ii)  $-88 < N < 1.2$ . For the  $N = 0$  case, we have a region of stability as  $10 - \infty < p < 0.5$ .

We now comment about the symmetry of  $p \leftrightarrow N - p$ . For the case of  $N = 2p$  (including  $p = N = 0$  case), we have a manifest  $p \leftrightarrow N - p$  symmetry which the Grassmannian nonlinear  $\sigma$  model (2) should preserve. However, in the case of  $N \neq 2p$ , Hamiltonian (1) has no manifest  $p \leftrightarrow N - p$  symmetry and we have not recovered this symmetry even at the fixed point due to the small size of the stability domain in the  $(p, N)$  plane. It is possible to write a Hamiltonian of manifest  $p \leftrightarrow N - p$  symmetry with two independent  $O(p)$  and  $O(N - p)$  gauge fields. However, the renormalization group calculation does not give us a stable fixed point for all coupling constants except for the  $N = 2p$  case.

If there exists a second order phase transition, the conductivity  $\sigma$  has the following behaviour near a mobility edge

$$\sigma = C |E - E_c|^s \tag{8}$$

where  $C$  is a constant and the exponent of the conductivity  $s$  is related to the previous exponent  $\nu$  as

$$s = (d - 2) \nu. \tag{9}$$

The symplectic case gives a positive  $\beta$ -function in two dimensions. Analogous behaviour for the  $\beta$ -function is obtained in the  $N$ -vector model ( $p = 1$ ) when we put  $N = 1$  which corresponds to the Ising model. It is known that Ising model has a lower critical dimensionality at  $d = 1$ . It is possible that the symplectic case has a lower critical dimensionality less than two and possibly one. Therefore, the critical exponent  $s$  in two dimensions is considered to be zero from (9) since the exponent  $\nu$  is finite. The assumption of finiteness for the exponent  $\nu$  in two dimensions is also consistent with the small correction of order  $\varepsilon$  as (7). The conductivity vanishes at the mobility edge discontinuously in two dimensions since the exponent  $s$  is vanishing. In a magneto-resistance experiment, the spin-orbit effect has been studied [11]. It is also interesting to compare the result of (7) with the experimental observations in doped semiconductors [12]. By putting  $\varepsilon = 1$  in (7) we have a conductivity exponent  $s = 0.57$ . Like ordinary critical phenomena, the mean field behaviour for the conductivity may be present. In our case, we take a mean field value of  $\nu$  as  $1/2$  and the conductivity exponent  $s$  is 1 since we put  $d = 4$  and  $\nu = 1/2$  in (9). Therefore, for the strong spin-orbit case, there appears a crossover phenomenon from the mean field behaviour with exponent  $s = 1$  to the critical behaviour with  $s = 0.57$  in three dimensions. Many materials are observed with exponent  $s = 1$  [13] and there may be exists such a behaviour among these materials.

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