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The deformation of a viscoelastic sphere in a time dependent shear flow

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Résumé. — Un modèle de Kelvin-Voigt tournant est introduit afin de décrire le comportement dans un écoulement instationnaire d'une sphère visco-élastique peu déformée. Les résultats de ce modèle sont en complet accord avec les calculs effectués par Cox [9] pour le cas d'une goutte liquide. Le mouvement de la particule dans un écoulement de cisaillement variant de façon sinusoïdale est ensuite analysé en détail.

Abstract. — A rotating Kelvin-Voigt model is introduced to describe the behaviour of a slightly deformed viscoelastic spherical particle in a time dependent shear flow. Complete agreement is found with the results obtained by Cox [9] for the case of a liquid drop. The particle motion in a sinusoidal shear flow is then extensively analysed.

Introduction.

Many problems in the field of dispersion rheology are related to the deformation of viscoelastic particles in a shear flow. A case often considered is that of particles spherical at rest, such as polymers, liquid drops, and microcapsules. These kinds of suspended particles have generally been analysed separately, because of the different concepts involved in each case : pure fluid mechanics for liquid drops [1-3], viscoelastic constitutive equations for solid spheres [4-6] and both descriptions for capsules [7, 8]. However, the study of the time dependent deformation of the different types of particles [9, 10] emphasizes the particular role of a common parameter : the relaxation time τ .

Our purpose in this paper is to introduce a simple but quite general analysis of the above mentioned class of phenomena. The possibility for the existence of internal degrees of freedom in continuum media was already considered in 1909 by E. and F. Cosserat [11], and later by many

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authors. Then, it was shown that the deformation or orientation of the fluid microstructure can be described by a vector [12]. The deformation of any particle is thus hereafter described by a rotating vector, behaving as a Kelvin-Voigt element, the detailed material properties of a particular particle entering only as parameters in the resulting equation. In section 1, we develop the differential equation, and we give its solution for any time-dependent shear flow. In section 2, we compare our results to those obtained by Cox for a liquid drop [9], and we show that, provided the same expression for the relaxation time is used, they are identical. The meaning of such identity is then discussed. Then (Sect. 3), we derive the deformation of the viscoelastic sphere in a sinusoidal flow, with some important applications in mind.

Finally, we suggest the possible extensions of the present analysis to the case of a particle characterized by several relaxation times.

1. The rotating Kelvin-Voigt model.

We choose the following flow pattern : $u = \dot{\gamma}y$, $v = 0$, $w = 0$, $\dot{\gamma}$ depending on time. Let O be the centre of the initially spherical particle. Then consider \mathbf{r} , a vector of the plane P which contains O and is parallel to the velocity vectors. The origin of \mathbf{r} is O , and its end is bound to the particle surface ; it rotates with an angular velocity ω : $\omega = \frac{1}{2} \dot{\gamma}$.

The two main assumptions of the model are the following :

a) \mathbf{r} behaves as a single Kelvin-Voigt element. (The complete material properties being not taken into account).

b) An element of external surface of the particle, bound to the end of the vector \mathbf{r} , is submitted to a normal stress depending on its angular position : $P_n = A\dot{\gamma} \cos 2\phi$ [1, 13] where A is a constant which has the dimension of a viscosity, and ϕ is defined in figure 1.

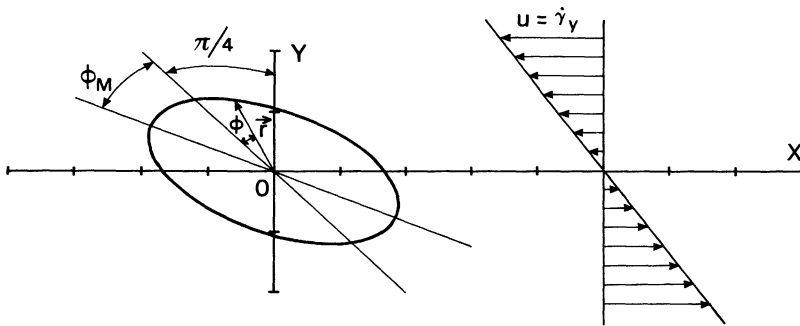


Fig. 1. — Schematic drawing of a spherical particle deformed in a shear flow.

The latter assumption is of course only valid for a very slightly deformed sphere.

The consequence of assumptions *a*) and *b*) is that the time-dependence of $r = |\mathbf{r}|$ is described by the following differential equation :

$$\frac{dr}{dt} = \frac{A}{\eta_m} a \dot{\gamma} \cos 2\phi - \frac{r - a}{\tau} \quad (1)$$

where τ is the characteristic relaxation time of the Kelvin-Voigt element. a is the radius of the particle at rest. η_m is an equivalent viscosity accounting for the overall dissipative properties of the particle.

If we follow the rotation of a given vector \mathbf{r} , the angle ϕ depends on time :

$$\frac{d\phi}{dt} = \frac{1}{2} \dot{\gamma}(t) \Rightarrow \phi = \frac{1}{2} P(\dot{\gamma}) + \psi \tag{2}$$

where $P(\dot{\gamma})$ is the primitive of the function $\dot{\gamma}(t)$ such that $P(\dot{\gamma}) = 0$ for $t = 0$, and ψ a constant.

Equations (1) and (2), together with initial conditions, completely specify the motion of the viscoelastic particle under consideration. It is convenient to use non-dimensional quantities throughout. Consequently, we put :

$$R = \frac{r - a}{a}; \quad \theta = \frac{t}{\tau}; \quad \Lambda^{-1} = A/\eta_m; \quad \Gamma = \dot{\gamma}\tau.$$

Then, (1) and (2) become :

$$\frac{dR}{d\theta} + R = \Lambda^{-1} \Gamma \cos 2\phi; \quad \phi = \frac{1}{2} P(\Gamma) + \psi. \tag{3}$$

The general solution of (3) if $\Gamma = 0$ for $\theta < 0$, and $\Gamma = \Gamma(\theta)$ for $\theta \geq 0$ is :

$$R(\theta) = \int_0^\theta \Lambda^{-1} \Gamma(x) e^{-(\theta-x)} \cos (P[\Gamma(x)] + 2\psi) dx. \tag{4}$$

We shall focus our attention on the maximal deformation R_M , and the angle of maximal deformation ϕ_M . For that, it is easier first to find the values of the angle ψ_M for which $dR/d\psi = 0$. Indeed, deriving equation (4) and equating this derivative to zero, we find :

$$2\psi_M = \tan^{-1} \left(-\frac{I_1}{I_2} \right) \quad \text{and} \quad R_M = \Lambda^{-1} e^{-\theta} \sqrt{I_1^2 + I_2^2} \tag{5}$$

where

$$I_1 = \int_0^\theta \Gamma(x) e^x \sin (P[\Gamma(x)]) dx,$$

and

$$I_2 = \int_0^\theta \Gamma(x) e^x \cos (P[\Gamma(x)]) dx.$$

ϕ_M can then be obtained from ψ_M and the second equation (3). For the particular case of a constant shear rate : $\Gamma(\theta) = \Gamma_0 \forall \theta \geq 0$, we can find the steady solution ($\theta \rightarrow \infty$) :

$$\phi_M = \frac{1}{2} \tan^{-1} \Gamma_0; \quad R = \frac{\Gamma_0}{(1 + \Gamma_0^2)^{1/2}} \cos 2(\phi - \phi_M) \tag{6}$$

and thus $R_M = \frac{\Lambda^{-1} \Gamma_0}{(1 + \Gamma_0^2)^{1/2}}$.

2. The liquid drop.

The behaviour of a liquid drop depends on the internal viscosity η_i , the suspending fluid viscosity η_0 , and the interfacial tension σ . These numbers are generally included within two dimensionless

parameters : $\lambda = \eta_i/\eta_0$ and $k = \sigma/\eta_0 \dot{\gamma}a$. The characteristic time of relaxation of a drop, for instance, is of the order of $\frac{\lambda}{k\dot{\gamma}}$.

It is now interesting to investigate the relevance of our general rheological model when applied to the extensively studied case of such liquid drops. The first point to notice is that the solutions (5) and (6) are exactly (after some minor algebraic modifications) identical to those obtained by Cox [9] if we put :

$$\tau = \frac{19}{20} \frac{\lambda}{\dot{\gamma}k} \quad \text{and} \quad A^{-1} = \frac{5(19\lambda + 16)}{76\lambda(\lambda + 1)}.$$

It is then useful to ask for the domain of validity of Cox's calculations since it can also indicate something about the limitations of our own model. Cox presents its results as valid for low particle Reynolds number, for any value of λ and k , but only for small deformation. The latter condition is very restrictive in our model, since we have not even considered, as Cox did, the first order perturbation terms of the normal stresses which are due to the small departure of the particle from the spherical shape. However, though these perturbation terms were not included in our analysis, we find essentially the same solution as Cox. Otherwise, the author's claim, according to which his calculations are valid for any value of λ and k , was recently challenged by Rallison [14] who showed that Cox's results are valid only for λ and $k \gg 1$, and are not complete even in this case. These results, however, give a qualitatively reasonable account of the motion of a drop. An analysis of our equation (1) based on a Kelvin-Voigt model would show the same limitations, together with the necessity of small temporal variations [15].

3. Viscoelastic particle in a sinusoidal flow.

Many important rheological studies deal with suspensions in pulsatile flow. If the length scale of time-dependent boundary layer is much larger than the radius of the particle (i.e. if $\sqrt{\eta_0/\rho\omega} \gg a$, where ω is the greatest pulsation of the flow, and ρ is the mass density), the pulsatile flow can be reduced to a local shear flow around the particle. We thus choose to treat the case of a drop in a time varying shear flow. The time variation is chosen, for the sake of simplicity to be : $\Gamma(\theta) = \Gamma_0 \cos(\Omega\theta + \beta)$ where $\Omega = \omega\tau$ and β is an arbitrary phase factor. In this case, the integrals in (5) were computed numerically, with $P(\Gamma) = \frac{\Gamma_0}{\Omega} \sin(\Omega\theta + \beta)$.

The results for $\Gamma_0 = 5$, $\Omega = 0.2$, $\beta = 0$, $A^{-1} = 0.1$ are plotted in figures 2 and 3. Such values of the parameters have been selected because they characterize a physical situation in which an equilibrium have enough time to be reached between two inversions of the shear sign. Some important features can be noticed (Fig. 2). First, there appears a transient behaviour of the deformation R_M and of the angle ϕ_M at each inversion of the shear rate sign (but slightly-out-of phase with it). The initial large transient is due to the fact that the shear flow is started impulsively from rest. Secondly, R_M and ϕ_M reach a quite stable value as soon as the shear rate exceeds a certain threshold. Their variation do not follow the sinusoidal law of the shear rate, but is rather a succession of plateaux demonstrating the existence of stable values of the deformation and the angular position of the viscoelastic particles. However, there is only one stable value of R_M , while there are two stable values of ϕ_M symmetric about $\phi = \frac{\pi}{4}$ i.e. symmetrical about the velocity direction. In figure 3, the same data are plotted on a polar diagram. It is seen that two symmetrical equilibrium position (R_M, ϕ_M) can be reached and that between these two positions, some nice transient curves occur.

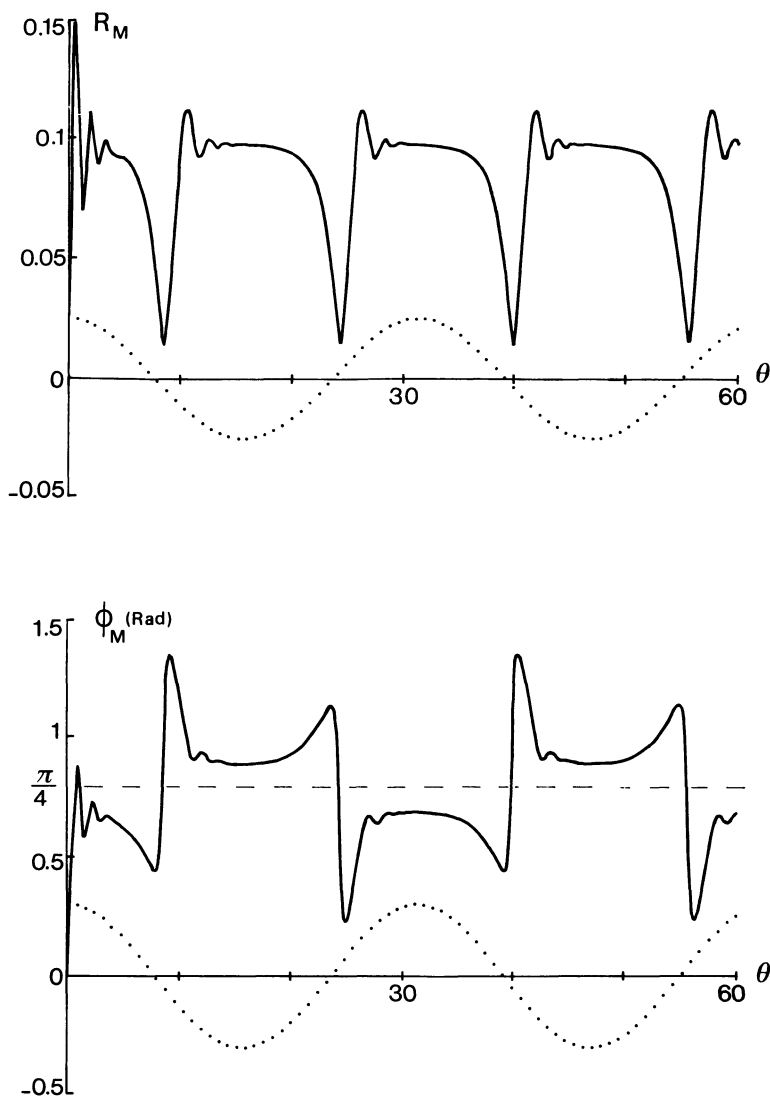


Fig. 2. — Variation of R_M and ϕ_M as a function of time, for $\Gamma_0 = 5$, $\Omega = 0.2$, $\beta = 0$, $\Lambda^{-1} = 0.1$. The time varying shear-rate is plotted in dotted curves.

Conclusion.

In this paper, we have presented the concept of a rotating Kelvin-Voigt model in order to describe the behaviour of an initially spherical, slightly deformed viscoelastic particle in a time dependent shear flow. This Kelvin-Voigt model, characterized by a single relaxation time τ has been then shown to give a description of the liquid drop behaviour essentially similar to Cox's one, provided the same expressions for the time τ and the viscosity ratio are chosen. Other simple viscoelastic spherical body submitted to an unsteady flow could be described by such rotating Kelvin-Voigt element. However, what has been more generally suggested in this work is the possibility to use

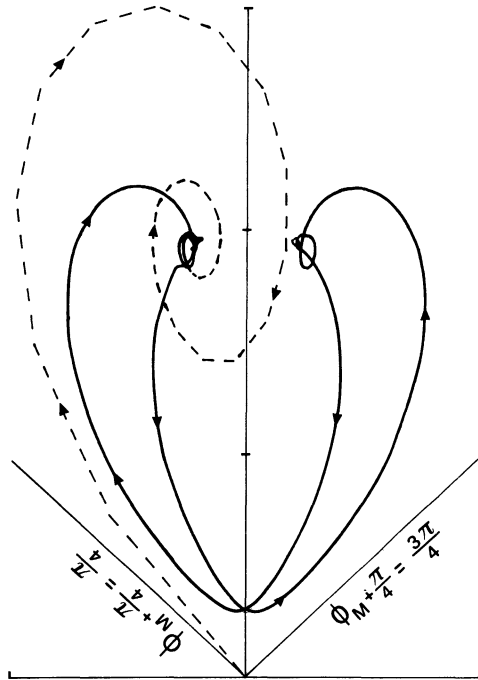


Fig. 3. — Polar plot of R_M versus ϕ_M for the same values of the parameters as in Fig. 2. Dotted line represents the transient.

any appropriate rotating rheological model including several relaxation time and even plastic properties to represent the first-order deformation of complex bodies.

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References

- [1] TAYLOR, G. I., *Proc. R. Soc. London, Ser. A* **146** (1934) 501.
- [2] FRANKEL, N. A., ACRIVOS, A., *J. Fluid Mech.* **44** (1970) 65.
- [3] RALLISON, J. M., ACRIVOS, A., *J. Fluid Mech.* **89** (1978) 191.
- [4] CERF, R., *J. Chim. Phys.* **48** (1951) 59.
- [5] ROSCOE, R., *J. Fluid Mech.* **28** (1967) 273.
- [6] LIGHTHILL, M. J., *J. Fluid Mech.* **34** (1968) 113.
- [7] BRUNN, P., *Biorheology* **17** (1980) 419.
- [8] BARTHES-BIESEL, D., *J. Fluid Mech.* **100** (1980) 831.
- [9] COX, R. G., *J. Fluid Mech.* **37** (1969) 601.
- [10] BARTHES-BIESEL, D., RALLISON, J. M., *J. Fluid Mech.* **113** (1981) 251.
- [11] COSSERAT, E., COSSERAT, F., *Théorie des corps déformables*, (Hermann, Paris) 1909.
- [12] OLBRIGHT, W. L., RALLISON, J. M., LEAL, L. G., *J. Non-Newtonian Mech.* **10** (1982) 291.
- [13] GOLDSMITH, H. L., MASON, S. G., *Rheology*, Ed. F. R. Eirich (Academic Press) **4** (1967) 85.
- [14] RALLISON, J. M., *J. Fluid Mech.* **98** (1980) 625.
- [15] BARTHES-BIESEL, D., 1984, private communication.